Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.2/43-1.1.3.2-a

Nasser M. Abbasi

May 18, 2024 Compiled on May 18, 2024 at 6:24pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**456**]. This is test number [43].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (456)	0.00 (0)
Mathematica	100.00 (456)	0.00 (0)
Sympy	96.93 (442)	3.07 (14)
Maple	91.67 (418)	8.33 (38)
Fricas	87.94 (401)	12.06 (55)
Mupad	74.56 (340)	25.44 (116)
Maxima	63.60 (290)	36.40 (166)
Giac	60.31 (275)	39.69 (181)
Reduce	58.55 (267)	41.45 (189)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

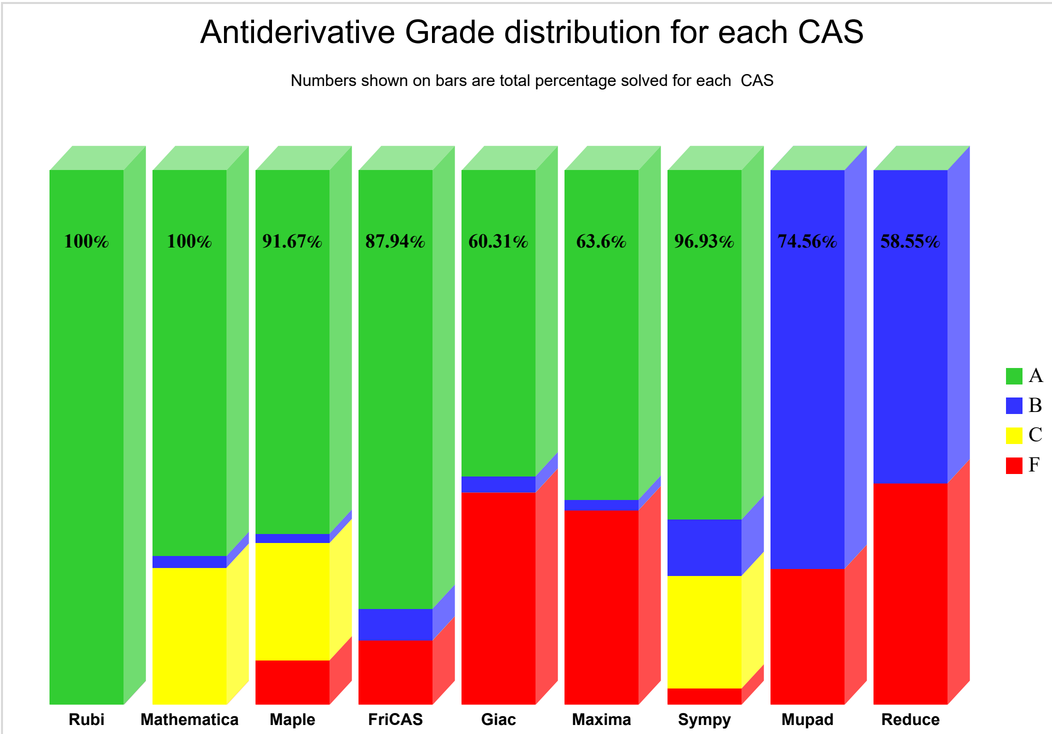
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

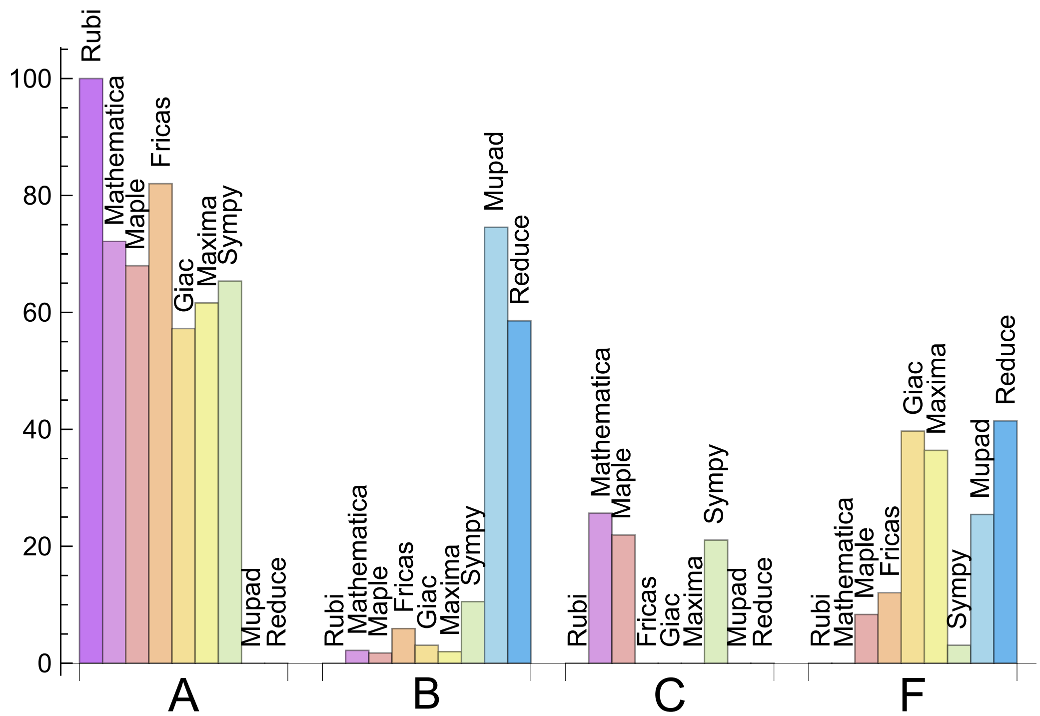
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Fricas	82.018	5.921	0.000	12.061
Mathematica	72.149	2.193	25.658	0.000
Maple	67.982	1.754	21.930	8.333
Sympy	65.351	10.526	21.053	3.070
Maxima	61.623	1.974	0.000	36.404
Giac	57.237	3.070	0.000	39.693
Mupad	0.000	74.561	0.000	25.439
Reduce	0.000	58.553	0.000	41.447

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	14	0.00	100.00	0.00
Maple	38	100.00	0.00	0.00
Fricas	55	98.18	1.82	0.00
Mupad	116	0.00	100.00	0.00
Maxima	166	100.00	0.00	0.00
Giac	181	99.45	0.00	0.55
Reduce	189	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Giac	0.13
Mupad	0.19
Reduce	0.23
Fricas	0.26
Rubi	0.38
Maple	0.68
Mathematica	2.57
Sympy	3.45

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	59.34	0.85	51.00	0.93
Maxima	59.47	0.98	53.00	0.88
Giac	65.33	1.04	53.00	0.91
Reduce	70.54	1.20	57.00	0.92
Fricas	78.00	1.01	48.00	0.85
Mupad	84.75	1.23	57.00	0.89
Sympy	94.69	1.40	42.00	0.86
Rubi	132.72	1.04	71.50	1.02
Maple	149.18	0.94	43.50	0.86

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

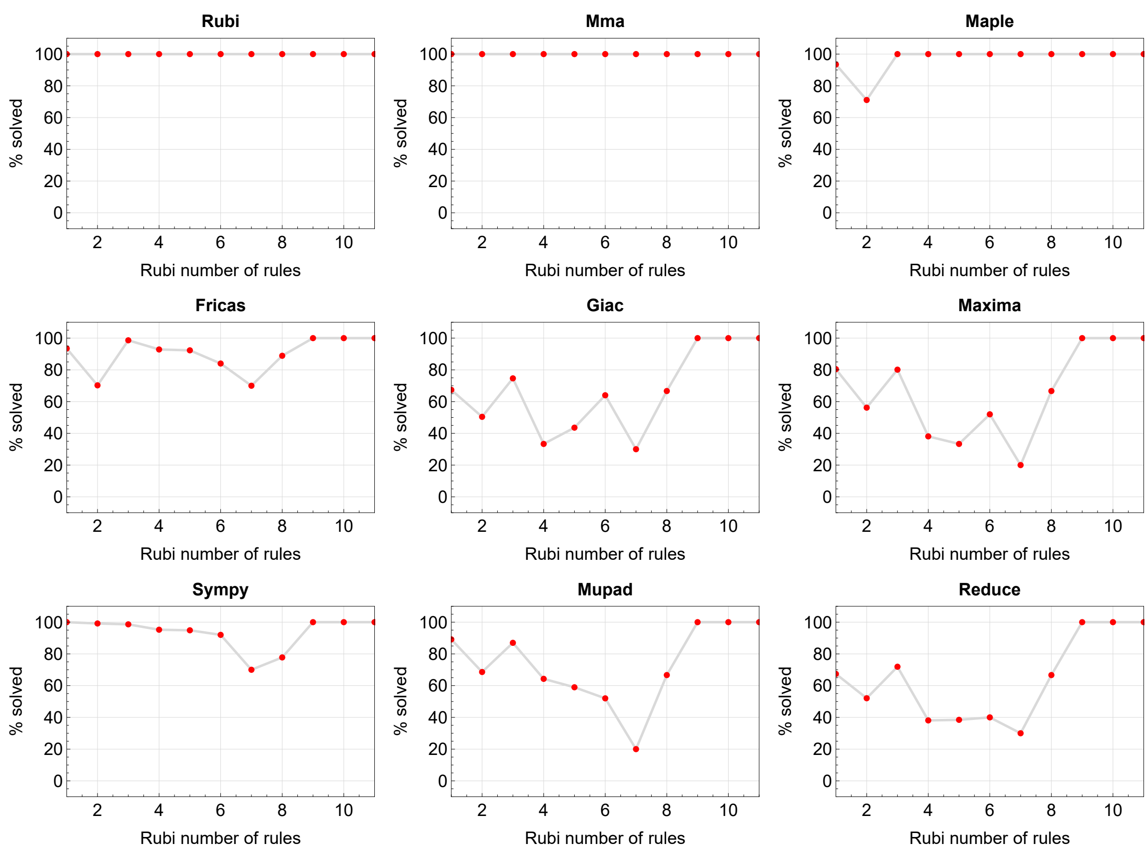


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

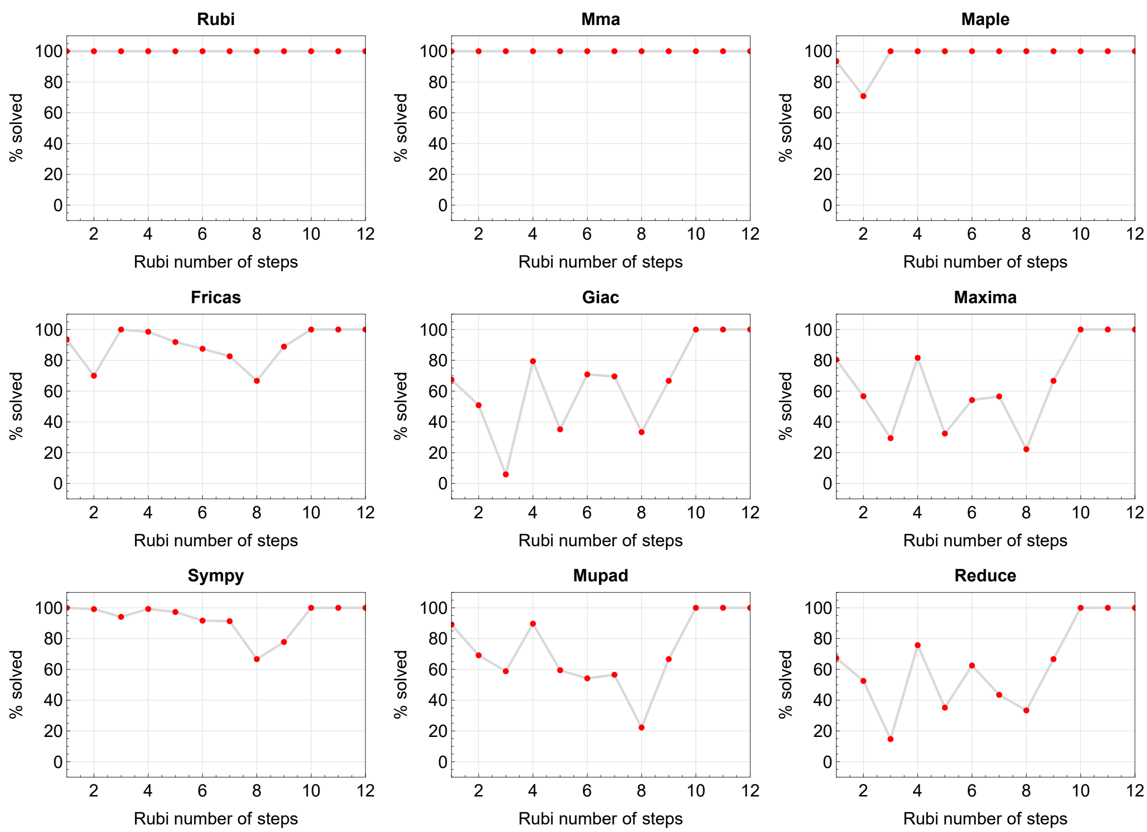


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

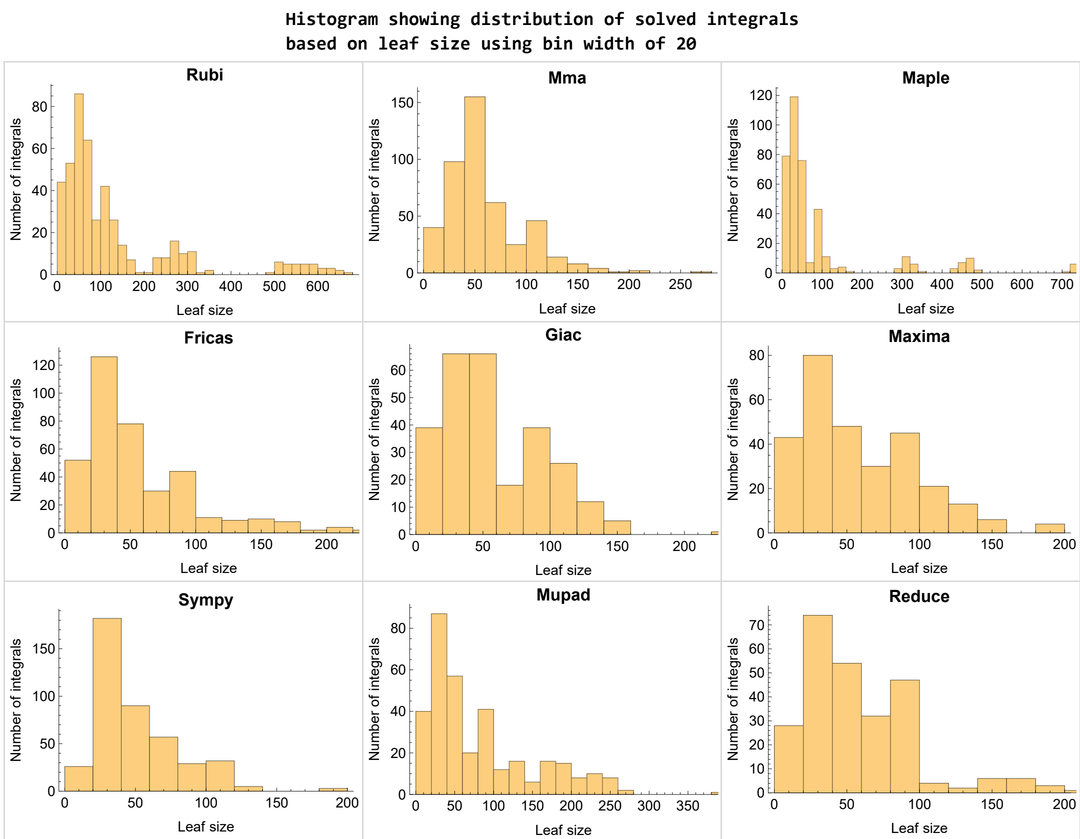


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

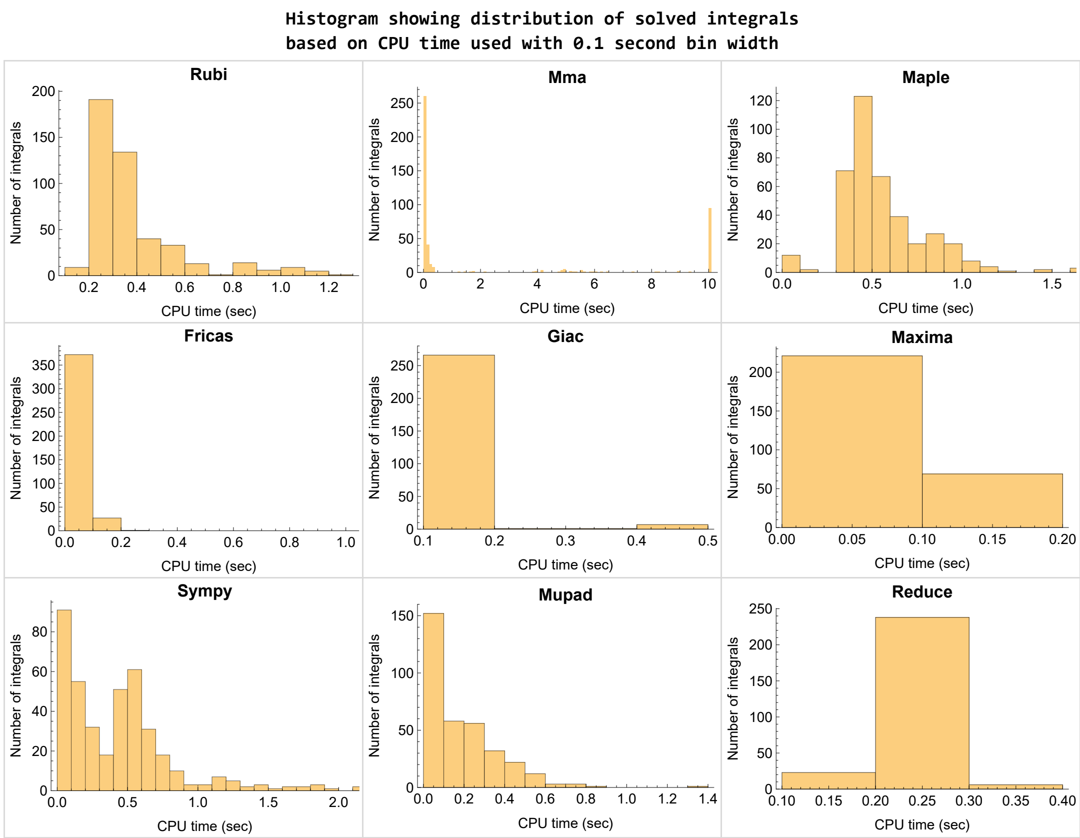


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

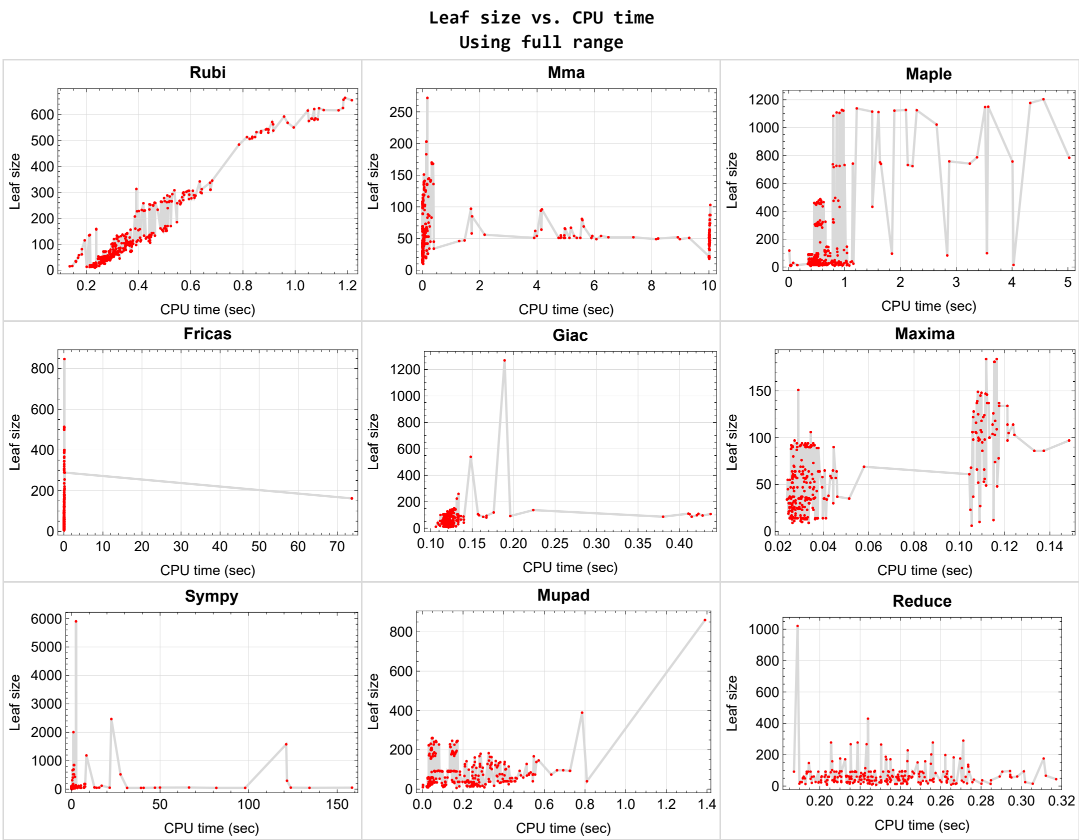


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {176, 177, 178, 179, 180, 193, 194, 195, 196, 197, 210, 211, 212, 213, 214, 227, 228, 229, 230, 231, 245, 246, 247, 264, 265, 266, 267, 281, 282, 283, 284, 285, 299, 300, 301, 302, 303, 304, 305, 306, 320, 321, 322, 336, 337, 338}

Mathematica {369, 379, 390, 427}

Maple $\{7, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

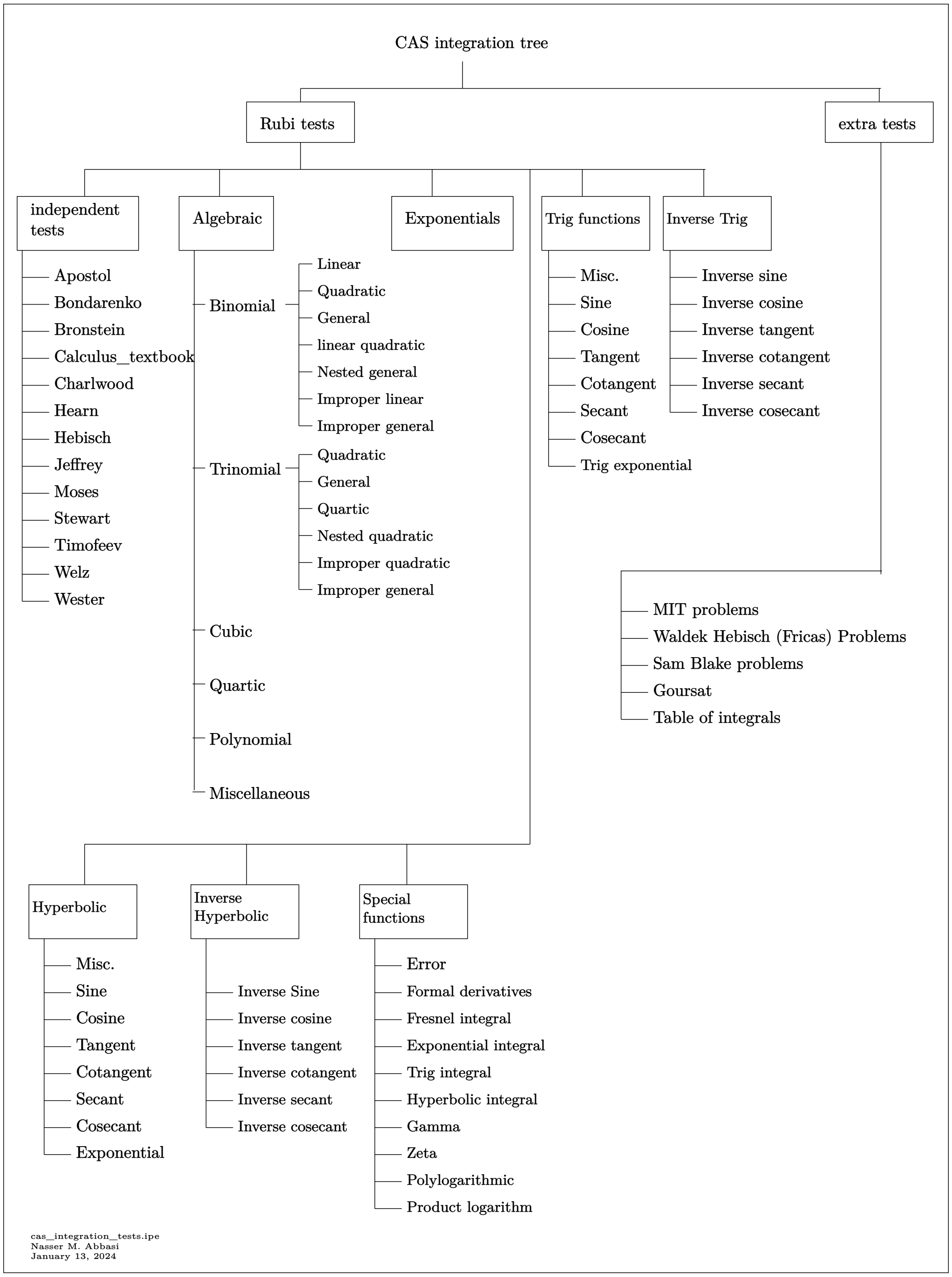
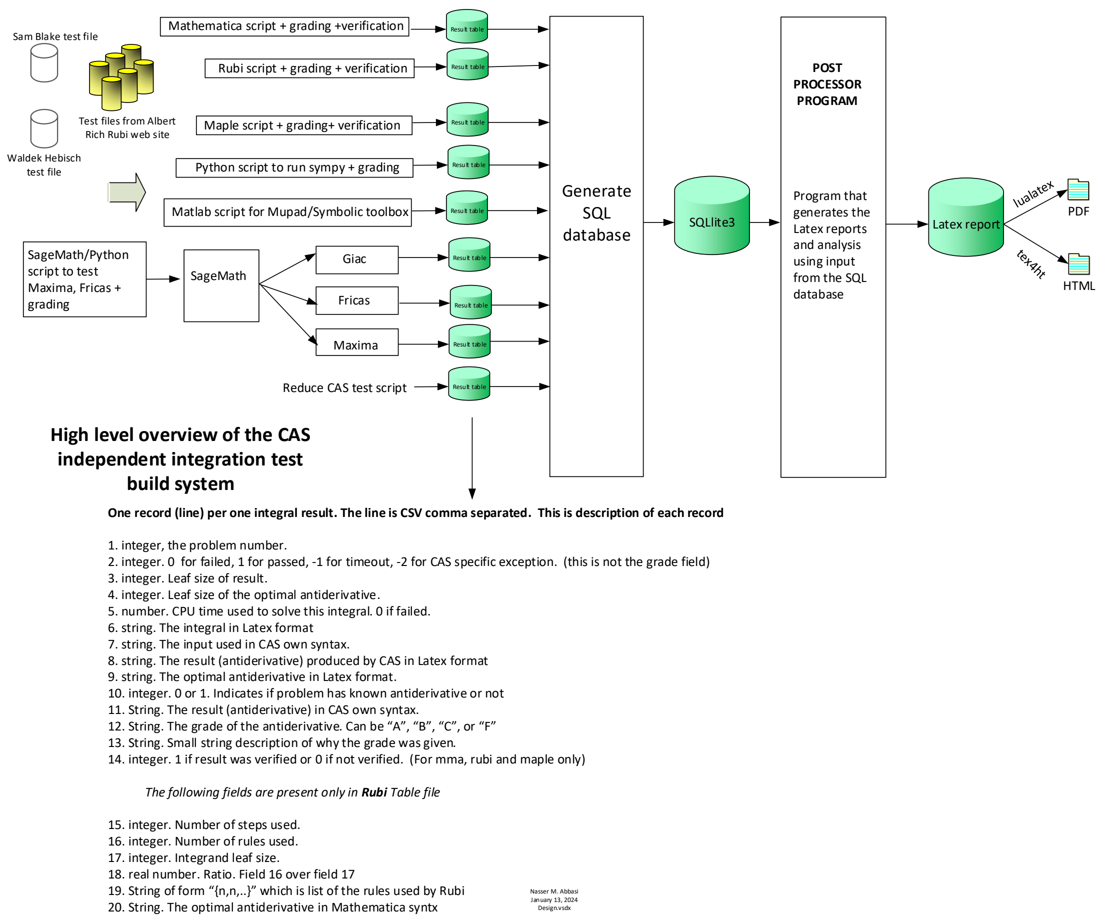


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 181, 182, 183, 184, 185, 186, 187, 198, 199, 200, 201, 202, 203, 204, 215, 216, 217, 218, 219, 220, 221, 232, 233, 234, 235, 236, 237, 238, 239, 250, 251, 252, 253, 254, 255, 256, 257, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 304, 305, 306, 307, 308, 309, 320, 321, 322, 323, 324, 325, 336, 337, 338, 339, 340, 341, 342, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456 }

B grade { 35, 40, 57, 58, 65, 84, 85, 86, 96, 97 }

C grade { 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 369, 379, 390, 427 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 125, 126, 127, 128, 129, 137, 138, 139, 140, 141, 142, 150, 151, 152, 153, 154, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 250, 251, 252, 253, 254, 255, 256, 257, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 304, 305, 306, 307, 308, 309, 320, 321, 322, 323, 324, 325, 336, 337, 338, 339, 340, 341, 342, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 436, 440, 441, 452, 453, 454, 455 }

B grade { 40, 65, 85, 96, 97, 437, 438, 439 }

C grade { 119, 120, 121, 122, 123, 124, 130, 131, 132, 133, 134, 135, 136, 143, 144, 145, 146, 147, 148, 149, 155, 156, 157, 158, 159, 160, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 393, 435 }

F normal fail { 368, 369, 370, 371, 385, 386, 387, 388, 389, 390, 391, 392, 407, 408, 409, 410, 411, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 313, 314, 318, 319, 320, 321, 322, 323, 324, 325, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 393, 394, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 412, 413, 414, 415, 416, 418, 421, 422, 423, 424, 435, 436, 440, 441, 452, 453, 454, 455 }

B grade { 35, 40, 58, 65, 85, 86, 96, 97, 139, 143, 144, 146, 147, 148, 184, 236, 254, 362, 363, 379, 402, 417, 419, 420, 437, 438, 439 }

C grade { }

F normal fail { 310, 311, 312, 315, 316, 317, 326, 327, 331, 332, 333, 343, 344, 349, 350, 351, 368, 369, 370, 371, 385, 386, 387, 388, 389, 390, 391, 392, 407, 408, 409, 410, 411, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F(-1) timedout fail { 380 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 181, 182, 183, 184, 185, 186, 187, 198, 199, 200, 201, 202, 203, 204, 215, 216, 217, 218, 219, 220, 221, 232, 233, 234, 235, 237, 238, 239, 250, 251, 252, 253, 255, 256, 257, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 307, 309, 323, 325, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 379, 380, 381, 382, 383, 384, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 435, 436, 437, 438, 439, 440, 441, 452, 453, 454, 455 }

B grade { 40, 65, 85, 96, 97, 139, 236, 254, 378 }

C grade { }

F normal fail { 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 368, 369, 370, 371, 385, 386, 387, 388, 389, 390, 391, 392, 407, 408, 409, 410, 411, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 181, 182, 183, 184, 185, 186, 187, 198, 199, 200, 201, 202, 203, 204, 215, 216, 217, 218, 219, 220, 221, 232, 233, 234, 235, 237, 238, 239, 250, 251, 252, 253, 255, 256, 257, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 304, 305, 306, 307, 308, 309, 320, 321, 322, 323, 324, 325, 336, 337, 338, 339, 355, 356, 357, 358, 359, 360, 372, 373, 374, 375, 376, 377, 393, 394, 395, 396, 397, 398, 399, 412, 413, 414, 415, 416, 417, 436, 441, 452, 453, 454 }

B grade { 40, 65, 85, 96, 97, 236, 254, 340, 342, 437, 438, 439, 440, 455 }

C grade { }

F normal fail { 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F(-1) timedout fail { }

F(-2) exception fail { 341 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 172, 179, 181, 182, 183, 184, 185, 186, 187, 190, 196, 198, 199, 200, 201, 202, 203, 204, 207, 213, 215, 216, 217, 218, 219, 220, 221, 224, 230, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 307, 308, 309, 323, 324, 325, 339, 340, 341, 342, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 369, 372, 373, 374, 375, 376, 377, 379, 381, 382, 383, 384, 387, 390, 393, 394, 395, 396, 397, 398, 399, 402, 403, 404, 405, 406, 410, 412, 413, 414, 415, 416, 417, 421, 422, 423, 424, 427, 433, 436, 437, 438, 439, 440, 441, 452, 453, 454, 455 }

C grade { }

F normal fail { }

F(-1) timedout fail { 170, 171, 173, 174, 175, 176, 177, 178, 180, 188, 189, 191, 192, 193, 194, 195, 197, 205, 206, 208, 209, 210, 211, 212, 214, 222, 223, 225, 226, 227, 228, 229, 231, 304, 305, 306, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 361, 362, 368, 370, 371, 378, 380, 385, 386, 388, 389, 391, 392, 400, 401, 407, 408, 409, 411, 418, 419, 420, 425, 426, 428, 429, 430, 431, 432, 434, 435, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 321, 322, 323, 324, 338, 339, 340, 355, 356, 372, 373, 394, 395, 396, 397, 403, 404, 412, 413, 414, 415, 422, 436 }

B grade { 15, 35, 40, 57, 58, 65, 84, 85, 86, 96, 97, 139, 161, 165, 166, 182, 183, 184, 219, 307, 341, 357, 358, 364, 365, 366, 367, 374, 375, 381, 382, 383, 384, 393, 405, 406, 421, 423, 424, 437, 438, 439, 440, 441, 452, 453, 454, 455 }

C grade { 254, 255, 256, 257, 272, 273, 274, 275, 290, 291, 292, 293, 310, 311, 312, 313, 314, 315, 316, 317, 318, 326, 327, 328, 329, 332, 333, 334, 335, 344, 345, 346, 347, 351, 352, 353, 354, 359, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 380, 385, 386, 387, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 407, 408, 409, 410, 411, 416, 417, 418, 419, 420, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F normal fail { }

F(-1) timedout fail { 308, 309, 319, 320, 325, 330, 331, 336, 337, 342, 343, 348, 349, 350 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 181, 182, 183, 184, 185, 186, 187, 198, 199, 200, 201, 202, 203, 204, 215, 216, 217, 218, 219, 220, 221, 232, 233, 234, 235, 236, 237, 238, 239, 250, 251, 252, 253, 254, 255, 256, 257, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 304, 305, 306, 307, 308, 309, 320, 321, 322, 323, 324, 325, 336, 337, 338, 339, 340, 341, 342, 355, 356, 357, 358, 364, 365, 366, 367, 372, 373, 374, 375, 381, 382, 383, 384, 415, 437, 438, 439, 440, 441, 452, 453, 454, 455 }

C grade { }

F normal fail { 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 360, 361, 362, 363, 368, 369, 370, 371, 376, 377, 378, 379, 380, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 456 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.220	0.002	0.155	0.036	0.068	0.019	0.127	0.225	0.023

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	17	14	14	13	12	13	14	13
N.S.	1	1.06	1.06	0.88	0.88	0.81	0.75	0.81	0.88	0.81
time (sec)	N/A	0.223	0.002	0.151	0.029	0.064	0.017	0.124	0.226	0.022

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.228	0.001	0.037	0.030	0.062	0.021	0.120	0.215	0.023

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.213	0.000	0.026	0.031	0.063	0.023	0.119	0.234	0.002

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	12	11	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	0.92	0.85	0.85
time (sec)	N/A	0.227	0.003	0.040	0.041	0.069	0.035	0.122	0.250	0.024

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	14	8	13	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.93	0.53	0.87	0.93	0.87
time (sec)	N/A	0.233	0.001	0.041	0.032	0.068	0.044	0.127	0.285	0.029

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	8	10	15	10
N.S.	1	1.00	1.00	0.92	0.83	1.25	0.67	0.83	1.25	0.83
time (sec)	N/A	0.228	0.001	0.039	0.030	0.064	0.046	0.119	0.306	0.024

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	18	17	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.38	1.31	0.85
time (sec)	N/A	0.237	0.003	0.046	0.032	0.063	0.062	0.120	0.273	0.103

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	1.00	0.87
time (sec)	N/A	0.234	0.003	0.036	0.033	0.066	0.069	0.115	0.203	0.026

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.235	0.003	0.035	0.033	0.078	0.063	0.124	0.190	0.031

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.88	0.76
time (sec)	N/A	0.229	0.003	0.035	0.026	0.068	0.074	0.121	0.280	0.029

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.234	0.002	0.035	0.027	0.063	0.077	0.123	0.266	0.029

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.255	0.001	0.402	0.029	0.065	0.020	0.124	0.209	0.038

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.265	0.001	0.400	0.030	0.062	0.020	0.122	0.263	0.034

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	15	14	24	24	14	25	24
N.S.	1	1.00	1.88	0.94	0.88	1.50	1.50	0.88	1.56	1.50
time (sec)	N/A	0.217	0.001	0.382	0.026	0.058	0.024	0.122	0.276	0.035

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.249	0.002	0.401	0.027	0.063	0.026	0.121	0.233	0.034

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	20	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.80	0.84	0.96	0.84
time (sec)	N/A	0.251	0.001	0.342	0.026	0.076	0.018	0.120	0.249	0.003

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	30	26	23	25	22	24	23	22	22
N.S.	1	1.15	1.00	0.88	0.96	0.85	0.92	0.88	0.85	0.85
time (sec)	N/A	0.257	0.001	0.408	0.026	0.066	0.049	0.116	0.230	0.031

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	23	25	19	23	25	24
N.S.	1	1.00	1.00	0.96	0.92	1.00	0.76	0.92	1.00	0.96
time (sec)	N/A	0.255	0.001	0.366	0.028	0.062	0.051	0.119	0.271	0.037

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	25	22	22	25	22
N.S.	1	1.00	1.00	0.88	0.85	0.96	0.85	0.85	0.96	0.85
time (sec)	N/A	0.250	0.001	0.371	0.025	0.071	0.058	0.119	0.191	0.037

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	25	27	24	32	27	23
N.S.	1	1.04	1.00	0.89	0.93	1.00	0.89	1.19	1.00	0.85
time (sec)	N/A	0.258	0.001	0.371	0.035	0.068	0.068	0.120	0.219	0.035

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	25	26	26	25	26	27
N.S.	1	1.00	1.00	0.89	0.89	0.93	0.93	0.89	0.93	0.96
time (sec)	N/A	0.251	0.001	0.364	0.027	0.059	0.069	0.118	0.199	0.029

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	26	23
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.13	1.00
time (sec)	N/A	0.252	0.001	0.365	0.026	0.062	0.071	0.118	0.211	0.030

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	30	26	23	26	28	24	32	28	25
N.S.	1	1.15	1.00	0.88	1.00	1.08	0.92	1.23	1.08	0.96
time (sec)	N/A	0.265	0.001	0.364	0.025	0.061	0.101	0.118	0.203	0.048

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.93	0.89
time (sec)	N/A	0.255	0.001	0.358	0.027	0.066	0.105	0.117	0.235	0.038

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.260	0.001	0.349	0.031	0.068	0.123	0.120	0.209	0.040

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	1.37
time (sec)	N/A	0.219	0.001	0.355	0.029	0.060	0.116	0.122	0.190	0.038

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.256	0.001	0.362	0.029	0.063	0.123	0.121	0.198	0.038

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.256	0.001	0.352	0.032	0.060	0.117	0.119	0.231	0.042

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.269	0.001	0.358	0.026	0.065	0.133	0.117	0.260	0.040

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	36	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.84	0.81	0.86	0.81
time (sec)	N/A	0.290	0.002	0.406	0.040	0.063	0.023	0.124	0.234	0.045

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	36	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.84	0.81	0.86	0.81
time (sec)	N/A	0.286	0.002	0.405	0.036	0.070	0.022	0.119	0.235	0.043

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	36	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.84	0.81	0.86	0.81
time (sec)	N/A	0.288	0.002	0.404	0.036	0.081	0.021	0.124	0.239	0.043

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	43	36	35	35	36	35	37	35
N.S.	1	1.12	1.26	1.06	1.03	1.03	1.06	1.03	1.09	1.03
time (sec)	N/A	0.266	0.002	0.396	0.034	0.084	0.026	0.120	0.285	0.044

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	15	14	35	36	14	36	35
N.S.	1	1.00	2.69	0.94	0.88	2.19	2.25	0.88	2.25	2.19
time (sec)	N/A	0.218	0.002	0.398	0.031	0.065	0.022	0.118	0.267	0.044

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	43	36	33	35	32	32	33	32	32
N.S.	1	1.19	1.00	0.92	0.97	0.89	0.89	0.92	0.89	0.89
time (sec)	N/A	0.274	0.004	0.373	0.034	0.069	0.044	0.118	0.232	0.038

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	42	37	34	34	38	34	44	38	33
N.S.	1	1.14	1.00	0.92	0.92	1.03	0.92	1.19	1.03	0.89
time (sec)	N/A	0.278	0.004	0.423	0.041	0.069	0.070	0.123	0.204	0.037

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	41	38	35	36	39	37	45	39	36
N.S.	1	1.08	1.00	0.92	0.95	1.03	0.97	1.18	1.03	0.95
time (sec)	N/A	0.282	0.004	0.369	0.025	0.066	0.108	0.121	0.224	0.037

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	37	34	39	39	37	45	39	35
N.S.	1	1.16	1.00	0.92	1.05	1.05	1.00	1.22	1.05	0.95
time (sec)	N/A	0.284	0.004	0.364	0.035	0.064	0.161	0.114	0.220	0.114

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37	37
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95	1.95
time (sec)	N/A	0.220	0.004	0.362	0.026	0.068	0.168	0.123	0.228	0.031

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	43	36	37	37	39	37	37	37
N.S.	1	1.10	1.08	0.90	0.92	0.92	0.98	0.92	0.92	0.92
time (sec)	N/A	0.245	0.004	0.387	0.030	0.060	0.219	0.120	0.201	0.035

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	37	37	39	37	37	37
N.S.	1	1.09	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.279	0.004	0.373	0.026	0.063	0.239	0.119	0.240	0.038

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	37	37	39	37	37	37
N.S.	1	1.09	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.284	0.004	0.381	0.031	0.064	0.219	0.122	0.223	0.039

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.267	0.003	0.423	0.030	0.063	0.020	0.116	0.217	0.045

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.273	0.002	0.423	0.030	0.068	0.019	0.128	0.281	0.047

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.269	0.002	0.421	0.025	0.059	0.020	0.125	0.206	0.045

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	32	32	36	32	35	32
N.S.	1	1.00	1.00	0.87	0.84	0.84	0.95	0.84	0.92	0.84
time (sec)	N/A	0.265	0.001	0.533	0.027	0.061	0.023	0.126	0.225	0.046

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	35	37	36	35	37	35
N.S.	1	1.00	1.00	0.88	0.85	0.90	0.88	0.85	0.90	0.85
time (sec)	N/A	0.269	0.004	0.381	0.032	0.073	0.043	0.113	0.217	0.048

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	33	37	36	33	37	33
N.S.	1	1.00	1.00	0.87	0.85	0.95	0.92	0.85	0.95	0.85
time (sec)	N/A	0.263	0.007	0.386	0.032	0.063	0.056	0.122	0.256	0.046

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	36	37	39	36	37	38
N.S.	1	1.00	1.00	0.88	0.88	0.90	0.95	0.88	0.90	0.93
time (sec)	N/A	0.271	0.004	0.508	0.025	0.072	0.076	0.121	0.204	0.112

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	36	37	37	36	37	36
N.S.	1	1.00	1.00	0.87	0.92	0.95	0.95	0.92	0.95	0.92
time (sec)	N/A	0.268	0.005	0.405	0.033	0.060	0.093	0.122	0.230	0.043

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	38	37	39	38	37	38
N.S.	1	1.00	1.00	0.88	0.93	0.90	0.95	0.93	0.90	0.93
time (sec)	N/A	0.261	0.004	0.407	0.042	0.063	0.131	0.119	0.260	0.035

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	69	58	57	57	65	57	59	57
N.S.	1	1.06	1.00	0.84	0.83	0.83	0.94	0.83	0.86	0.83
time (sec)	N/A	0.321	0.003	0.437	0.045	0.079	0.028	0.124	0.246	0.028

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	69	58	57	57	66	57	59	57
N.S.	1	1.06	1.00	0.84	0.83	0.83	0.96	0.83	0.86	0.83
time (sec)	N/A	0.330	0.003	0.463	0.044	0.060	0.043	0.123	0.226	0.026

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	69	58	57	57	65	57	59	57
N.S.	1	1.06	0.96	0.81	0.79	0.79	0.90	0.79	0.82	0.79
time (sec)	N/A	0.328	0.003	0.407	0.028	0.081	0.028	0.117	0.204	0.026

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	69	58	57	57	66	57	59	57
N.S.	1	1.08	1.30	1.09	1.08	1.08	1.25	1.08	1.11	1.08
time (sec)	N/A	0.304	0.002	0.412	0.028	0.067	0.027	0.116	0.294	0.027

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	69	58	57	57	66	57	59	57
N.S.	1	1.12	2.03	1.71	1.68	1.68	1.94	1.68	1.74	1.68
time (sec)	N/A	0.263	0.003	0.401	0.031	0.059	0.032	0.120	0.196	0.027

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	69	15	14	57	65	14	58	57
N.S.	1	1.00	4.31	0.94	0.88	3.56	4.06	0.88	3.62	3.56
time (sec)	N/A	0.221	0.002	0.394	0.040	0.060	0.042	0.118	0.220	0.027

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	65	56	58	55	65	56	55	55
N.S.	1	1.03	1.00	0.86	0.89	0.85	1.00	0.86	0.85	0.85
time (sec)	N/A	0.309	0.004	0.383	0.040	0.068	0.054	0.118	0.235	0.031

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	58	61	65	67	61	56
N.S.	1	1.00	1.00	0.86	0.88	0.92	0.98	1.02	0.92	0.85
time (sec)	N/A	0.307	0.004	0.389	0.028	0.065	0.077	0.123	0.226	0.034

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	66	57	59	61	65	69	61	59
N.S.	1	1.03	1.00	0.86	0.89	0.92	0.98	1.05	0.92	0.89
time (sec)	N/A	0.316	0.004	0.362	0.043	0.073	0.176	0.120	0.216	0.032

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	66	57	61	61	65	71	61	59
N.S.	1	1.03	1.00	0.86	0.92	0.92	0.98	1.08	0.92	0.89
time (sec)	N/A	0.313	0.006	0.464	0.031	0.063	0.162	0.123	0.197	0.043

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	66	57	61	61	63	69	61	59
N.S.	1	0.98	1.00	0.86	0.92	0.92	0.95	1.05	0.92	0.89
time (sec)	N/A	0.312	0.004	0.376	0.027	0.114	0.221	0.120	0.241	0.046

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	65	56	61	61	61	67	61	58
N.S.	1	1.03	1.00	0.86	0.94	0.94	0.94	1.03	0.94	0.89
time (sec)	N/A	0.315	0.004	0.362	0.030	0.073	0.282	0.121	0.214	0.052

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	57	57	61	57	59	59
N.S.	1	1.00	3.63	3.05	3.00	3.00	3.21	3.00	3.11	3.11
time (sec)	N/A	0.226	0.004	0.367	0.026	0.068	0.294	0.126	0.236	0.044

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	69	58	59	59	63	59	59	59
N.S.	1	1.10	1.72	1.45	1.48	1.48	1.58	1.48	1.48	1.48
time (sec)	N/A	0.250	0.004	0.378	0.027	0.060	0.332	0.122	0.208	0.111

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	69	58	59	59	63	59	59	59
N.S.	1	1.16	1.11	0.94	0.95	0.95	1.02	0.95	0.95	0.95
time (sec)	N/A	0.265	0.006	0.362	0.027	0.064	0.346	0.114	0.265	0.047

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	69	58	59	59	63	59	59	59
N.S.	1	1.03	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.324	0.004	0.375	0.026	0.068	0.364	0.131	0.213	0.124

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	69	58	59	59	63	59	59	59
N.S.	1	1.06	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.321	0.006	0.381	0.026	0.065	0.401	0.119	0.202	0.050

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	59	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.86	0.83
time (sec)	N/A	0.314	0.003	0.409	0.032	0.066	0.025	0.117	0.224	0.028

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	59	56
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.89	0.85
time (sec)	N/A	0.312	0.002	0.414	0.026	0.064	0.022	0.114	0.272	0.026

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	59	56
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.89	0.85
time (sec)	N/A	0.313	0.002	0.419	0.032	0.066	0.021	0.126	0.211	0.027

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	54	53	53	60	53	57	53
N.S.	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.93	0.87
time (sec)	N/A	0.309	0.001	0.358	0.032	0.065	0.024	0.121	0.263	0.026

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	57	59	61	57	59	57
N.S.	1	1.00	1.00	0.89	0.88	0.91	0.94	0.88	0.91	0.88
time (sec)	N/A	0.304	0.004	0.375	0.032	0.068	0.053	0.115	0.239	0.030

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	55	59	61	55	59	55
N.S.	1	1.00	1.00	0.86	0.85	0.91	0.94	0.85	0.91	0.85
time (sec)	N/A	0.308	0.004	0.371	0.024	0.058	0.065	0.119	0.220	0.030

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	58	58	59	63	58	59	60
N.S.	1	1.00	1.00	0.92	0.92	0.94	1.00	0.92	0.94	0.95
time (sec)	N/A	0.307	0.005	0.371	0.033	0.063	0.104	0.121	0.205	0.030

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	56	58	59	65	58	59	58
N.S.	1	1.00	1.00	0.86	0.89	0.91	1.00	0.89	0.91	0.89
time (sec)	N/A	0.309	0.004	0.374	0.026	0.058	0.118	0.119	0.230	0.028

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	57	59	59	61	59	59	59
N.S.	1	1.00	1.00	0.92	0.95	0.95	0.98	0.95	0.95	0.95
time (sec)	N/A	0.313	0.004	0.372	0.033	0.059	0.127	0.127	0.262	0.049

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	56	59	61	56	59	57
N.S.	1	1.00	1.00	0.92	0.92	0.97	1.00	0.92	0.97	0.93
time (sec)	N/A	0.299	0.004	0.362	0.035	0.061	0.119	0.122	0.238	0.052

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	133	108	91	90	90	105	90	92	90
N.S.	1	1.03	0.84	0.71	0.70	0.70	0.81	0.70	0.71	0.70
time (sec)	N/A	0.421	0.003	0.414	0.034	0.063	0.038	0.121	0.194	0.125

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	108	91	90	90	107	90	92	90
N.S.	1	1.04	0.98	0.83	0.82	0.82	0.97	0.82	0.84	0.82
time (sec)	N/A	0.378	0.003	0.404	0.032	0.068	0.034	0.119	0.195	0.097

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	108	91	90	90	107	90	92	90
N.S.	1	1.04	1.19	1.00	0.99	0.99	1.18	0.99	1.01	0.99
time (sec)	N/A	0.365	0.003	0.416	0.045	0.059	0.031	0.121	0.234	0.163

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	108	91	90	90	107	90	92	90
N.S.	1	1.06	1.50	1.26	1.25	1.25	1.49	1.25	1.28	1.25
time (sec)	N/A	0.344	0.003	0.418	0.026	0.059	0.032	0.126	0.207	0.097

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	108	91	90	90	107	90	92	90
N.S.	1	1.08	2.04	1.72	1.70	1.70	2.02	1.70	1.74	1.70
time (sec)	N/A	0.307	0.003	0.407	0.028	0.073	0.031	0.123	0.231	0.169

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	108	91	90	90	107	90	92	90
N.S.	1	1.12	3.18	2.68	2.65	2.65	3.15	2.65	2.71	2.65
time (sec)	N/A	0.274	0.004	0.398	0.034	0.065	0.053	0.117	0.250	0.099

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	108	15	14	90	105	14	91	14
N.S.	1	1.00	6.75	0.94	0.88	5.62	6.56	0.88	5.69	0.88
time (sec)	N/A	0.224	0.003	0.404	0.025	0.062	0.037	0.125	0.234	0.182

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	106	104	89	91	88	105	89	88	88
N.S.	1	1.02	1.00	0.86	0.88	0.85	1.01	0.86	0.85	0.85
time (sec)	N/A	0.364	0.005	0.525	0.035	0.073	0.071	0.119	0.205	0.136

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	105	90	91	94	105	100	94	89
N.S.	1	0.98	1.00	0.86	0.87	0.90	1.00	0.95	0.90	0.85
time (sec)	N/A	0.371	0.009	0.392	0.032	0.067	0.095	0.121	0.218	0.062

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	105	90	92	94	105	102	94	92
N.S.	1	0.98	1.00	0.86	0.88	0.90	1.00	0.97	0.90	0.88
time (sec)	N/A	0.372	0.004	0.362	0.032	0.063	0.144	0.120	0.301	0.061

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	105	90	92	94	104	102	94	92
N.S.	1	0.96	1.00	0.86	0.88	0.90	0.99	0.97	0.90	0.88
time (sec)	N/A	0.370	0.009	0.355	0.036	0.070	0.211	0.116	0.263	0.058

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	105	90	94	94	104	104	94	92
N.S.	1	0.98	1.00	0.86	0.90	0.90	0.99	0.99	0.90	0.88
time (sec)	N/A	0.366	0.005	0.360	0.031	0.069	0.246	0.123	0.234	0.136

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	105	90	94	94	102	104	94	92
N.S.	1	0.96	1.00	0.86	0.90	0.90	0.97	0.99	0.90	0.88
time (sec)	N/A	0.368	0.006	0.365	0.033	0.070	0.300	0.133	0.242	0.135

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	105	90	94	94	100	104	94	92
N.S.	1	0.98	1.00	0.86	0.90	0.90	0.95	0.99	0.90	0.88
time (sec)	N/A	0.368	0.004	0.411	0.035	0.070	0.398	0.115	0.243	0.073

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	102	105	90	94	94	99	102	94	94
N.S.	1	0.97	1.00	0.86	0.90	0.90	0.94	0.97	0.90	0.90
time (sec)	N/A	0.365	0.009	0.358	0.032	0.065	0.465	0.118	0.211	0.079

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	106	104	89	94	94	97	100	94	91
N.S.	1	1.02	1.00	0.86	0.90	0.90	0.93	0.96	0.90	0.88
time (sec)	N/A	0.368	0.005	0.350	0.028	0.064	0.568	0.112	0.210	0.142

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	108	91	90	90	97	90	92	92
N.S.	1	1.00	5.68	4.79	4.74	4.74	5.11	4.74	4.84	4.84
time (sec)	N/A	0.213	0.008	0.355	0.034	0.062	0.567	0.123	0.225	0.086

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	108	91	92	92	99	92	92	92
N.S.	1	1.10	2.70	2.28	2.30	2.30	2.48	2.30	2.30	2.30
time (sec)	N/A	0.245	0.004	0.362	0.035	0.063	0.681	0.121	0.227	0.153

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	108	91	92	92	99	92	92	92
N.S.	1	1.16	1.74	1.47	1.48	1.48	1.60	1.48	1.48	1.48
time (sec)	N/A	0.270	0.009	0.353	0.026	0.066	0.661	0.116	0.203	0.087

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	100	108	91	92	92	99	92	92	92
N.S.	1	1.19	1.29	1.08	1.10	1.10	1.18	1.10	1.10	1.10
time (sec)	N/A	0.296	0.004	0.358	0.030	0.065	0.673	0.118	0.193	0.156

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	128	108	91	92	92	99	92	92	92
N.S.	1	1.21	1.02	0.86	0.87	0.87	0.93	0.87	0.87	0.87
time (sec)	N/A	0.323	0.006	0.365	0.033	0.063	0.776	0.121	0.291	0.086

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	108	91	92	92	99	92	92	92
N.S.	1	1.02	1.00	0.84	0.85	0.85	0.92	0.85	0.85	0.85
time (sec)	N/A	0.376	0.007	0.360	0.030	0.059	0.759	0.122	0.227	0.159

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	108	91	92	92	99	92	92	92
N.S.	1	1.02	1.00	0.84	0.85	0.85	0.92	0.85	0.85	0.85
time (sec)	N/A	0.368	0.013	0.365	0.036	0.062	0.819	0.136	0.251	0.095

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	89	89	102	89	92	89
N.S.	1	1.00	1.00	0.87	0.86	0.86	0.99	0.86	0.89	0.86
time (sec)	N/A	0.367	0.003	0.386	0.038	0.063	0.031	0.128	0.216	0.185

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	92	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.85	0.83
time (sec)	N/A	0.364	0.003	0.400	0.026	0.063	0.029	0.120	0.224	0.097

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	92	90
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.87	0.85
time (sec)	N/A	0.361	0.003	0.398	0.034	0.081	0.035	0.125	0.203	0.136

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	87	87	100	87	90	87
N.S.	1	1.00	1.00	0.89	0.88	0.88	1.01	0.88	0.91	0.88
time (sec)	N/A	0.363	0.002	0.359	0.028	0.063	0.029	0.124	0.215	0.046

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	91	90	92	99	90	92	90
N.S.	1	1.00	1.00	0.91	0.90	0.92	0.99	0.90	0.92	0.90
time (sec)	N/A	0.358	0.009	0.497	0.033	0.097	0.076	0.117	0.246	0.062

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	88	92	99	88	92	88
N.S.	1	1.00	1.00	0.91	0.90	0.94	1.01	0.90	0.94	0.90
time (sec)	N/A	0.361	0.005	0.501	0.026	0.063	0.084	0.122	0.261	0.127

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	91	91	92	104	91	92	93
N.S.	1	1.00	1.00	0.89	0.89	0.90	1.02	0.89	0.90	0.91
time (sec)	N/A	0.359	0.004	0.361	0.030	0.100	0.110	0.121	0.187	0.141

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	89	92	99	89	92	91
N.S.	1	1.00	1.00	0.91	0.91	0.94	1.01	0.91	0.94	0.93
time (sec)	N/A	0.365	0.009	0.364	0.037	0.064	0.104	0.123	0.208	0.126

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	91	91	92	100	91	92	93
N.S.	1	1.00	1.00	0.93	0.93	0.94	1.02	0.93	0.94	0.95
time (sec)	N/A	0.354	0.006	0.362	0.026	0.087	0.186	0.123	0.240	0.128

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	102	91	92	91
N.S.	1	1.00	1.00	0.89	0.91	0.92	1.02	0.91	0.92	0.91
time (sec)	N/A	0.350	0.004	0.361	0.034	0.094	0.138	0.117	0.241	0.124

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	15	16	17	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.68	0.73	0.77	0.77
time (sec)	N/A	0.238	0.001	0.394	0.026	0.064	0.020	0.120	0.241	0.036

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	34	34	33	32	35	61	33
N.S.	1	0.98	1.00	0.85	0.85	0.82	0.80	0.88	1.52	0.82
time (sec)	N/A	0.279	0.006	0.473	0.033	0.070	0.110	0.127	0.249	0.058

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	48	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	1.78	0.81
time (sec)	N/A	0.266	0.005	0.429	0.031	0.091	0.092	0.116	0.239	0.116

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	37	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	2.47	0.87
time (sec)	N/A	0.217	0.003	0.450	0.032	0.065	0.092	0.130	0.207	0.107

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	15	22	45	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.68	1.00	2.05	0.82
time (sec)	N/A	0.232	0.005	0.417	0.026	0.068	0.137	0.123	0.239	0.098

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	35	32	33	33	31	42	61	31
N.S.	1	1.03	1.00	0.91	0.94	0.94	0.89	1.20	1.74	0.89
time (sec)	N/A	0.278	0.008	0.421	0.028	0.073	0.180	0.128	0.298	0.096

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	133	111	37	109	123	32	114	81	120
N.S.	1	1.07	0.90	0.30	0.88	0.99	0.26	0.92	0.65	0.97
time (sec)	N/A	0.463	0.027	0.443	0.117	0.082	0.092	0.128	0.247	0.335

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	123	108	34	106	106	22	111	78	114
N.S.	1	1.03	0.91	0.29	0.89	0.89	0.18	0.93	0.66	0.96
time (sec)	N/A	0.441	0.014	0.437	0.106	0.075	0.091	0.124	0.263	0.283

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	116	89	27	98	304	24	112	65	111
N.S.	1	1.01	0.77	0.23	0.85	2.64	0.21	0.97	0.57	0.97
time (sec)	N/A	0.409	0.009	0.428	0.111	0.084	0.069	0.121	0.236	0.367

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	111	89	27	98	299	20	112	67	99
N.S.	1	0.97	0.77	0.23	0.85	2.60	0.17	0.97	0.58	0.86
time (sec)	N/A	0.407	0.009	0.411	0.109	0.083	0.079	0.121	0.222	0.228

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	131	114	53	106	103	29	121	84	102
N.S.	1	1.07	0.93	0.43	0.87	0.84	0.24	0.99	0.69	0.84
time (sec)	N/A	0.442	0.014	0.441	0.110	0.109	0.102	0.123	0.221	0.341

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	128	119	54	106	143	32	115	96	128
N.S.	1	1.03	0.96	0.44	0.85	1.15	0.26	0.93	0.77	1.03
time (sec)	N/A	0.447	0.016	0.445	0.107	0.109	0.126	0.125	0.241	0.320

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	38	41	43	56	42	49	115	45
N.S.	1	0.93	0.83	0.89	0.93	1.22	0.91	1.07	2.50	0.98
time (sec)	N/A	0.285	0.014	0.430	0.037	0.066	0.159	0.127	0.271	0.056

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	48	97	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	1.45	2.94	0.88
time (sec)	N/A	0.266	0.008	0.437	0.027	0.079	0.155	0.125	0.227	0.047

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.216	0.004	0.414	0.031	0.064	0.109	0.117	0.224	0.030

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	37	47	34	45	114	34
N.S.	1	1.03	0.87	0.92	0.97	1.24	0.89	1.18	3.00	0.89
time (sec)	N/A	0.287	0.012	0.421	0.025	0.072	0.191	0.118	0.223	0.071

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	54	53	73	54	51	142	53
N.S.	1	1.00	0.79	1.04	1.02	1.40	1.04	0.98	2.73	1.02
time (sec)	N/A	0.305	0.035	0.447	0.036	0.078	0.255	0.125	0.255	0.085

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	127	50	125	146	48	127	176	132
N.S.	1	1.07	0.91	0.36	0.89	1.04	0.34	0.91	1.26	0.94
time (sec)	N/A	0.494	0.056	0.445	0.108	0.078	0.179	0.123	0.270	0.376

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	143	119	45	116	400	44	132	158	138
N.S.	1	1.05	0.88	0.33	0.85	2.94	0.32	0.97	1.16	1.01
time (sec)	N/A	0.456	0.058	0.443	0.107	0.151	0.140	0.126	0.252	0.258

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	136	118	43	114	391	39	130	171	108
N.S.	1	1.01	0.88	0.32	0.85	2.92	0.29	0.97	1.28	0.81
time (sec)	N/A	0.442	0.048	0.461	0.124	0.120	0.118	0.124	0.235	0.334

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	143	119	48	124	386	44	129	158	138
N.S.	1	1.05	0.88	0.35	0.91	2.84	0.32	0.95	1.16	1.01
time (sec)	N/A	0.451	0.043	0.444	0.111	0.094	0.116	0.123	0.206	0.405

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	136	118	46	122	389	39	127	171	128
N.S.	1	1.01	0.88	0.34	0.91	2.90	0.29	0.95	1.28	0.96
time (sec)	N/A	0.449	0.042	0.412	0.106	0.091	0.138	0.120	0.213	0.283

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	158	131	73	126	146	56	139	183	120
N.S.	1	1.09	0.90	0.50	0.87	1.01	0.39	0.96	1.26	0.83
time (sec)	N/A	0.506	0.061	0.457	0.115	0.080	0.169	0.123	0.266	0.372

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	155	129	74	128	180	58	131	202	146
N.S.	1	1.07	0.89	0.51	0.88	1.24	0.40	0.90	1.39	1.01
time (sec)	N/A	0.491	0.058	0.470	0.106	0.082	0.193	0.127	0.255	0.572

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	62	48	54	66	91	65	62	187	67
N.S.	1	1.02	0.79	0.89	1.08	1.49	1.07	1.02	3.07	1.10
time (sec)	N/A	0.316	0.053	0.428	0.032	0.073	0.244	0.120	0.233	0.207

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	39	43	55	69	53	42	170	53
N.S.	1	0.98	0.75	0.83	1.06	1.33	1.02	0.81	3.27	1.02
time (sec)	N/A	0.300	0.016	0.427	0.025	0.065	0.230	0.127	0.233	0.067

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.222	0.010	0.425	0.032	0.079	0.183	0.126	0.243	0.113

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	25	28
N.S.	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.56	1.75
time (sec)	N/A	0.221	0.004	0.435	0.038	0.061	0.176	0.125	0.215	0.035

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	43	46	60	90	56	57	198	56
N.S.	1	1.02	0.80	0.85	1.11	1.67	1.04	1.06	3.67	1.04
time (sec)	N/A	0.298	0.025	0.424	0.027	0.067	0.319	0.133	0.262	0.167

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	69	59	64	76	119	76	80	228	73
N.S.	1	1.05	0.89	0.97	1.15	1.80	1.15	1.21	3.45	1.11
time (sec)	N/A	0.334	0.040	0.447	0.030	0.071	0.321	0.130	0.244	0.185

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	170	140	56	139	510	70	142	267	161
N.S.	1	1.10	0.90	0.36	0.90	3.29	0.45	0.92	1.72	1.04
time (sec)	N/A	0.523	0.057	0.474	0.108	0.142	0.232	0.126	0.223	0.258

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	163	136	54	137	503	68	140	278	127
N.S.	1	1.07	0.89	0.35	0.90	3.29	0.44	0.92	1.82	0.83
time (sec)	N/A	0.503	0.049	0.443	0.108	0.087	0.220	0.129	0.206	0.403

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	170	141	58	149	512	70	145	265	168
N.S.	1	1.08	0.89	0.37	0.94	3.24	0.44	0.92	1.68	1.06
time (sec)	N/A	0.511	0.077	0.442	0.108	0.090	0.209	0.129	0.230	0.281

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	138	56	146	503	65	142	278	145
N.S.	1	1.05	0.90	0.36	0.95	3.27	0.42	0.92	1.81	0.94
time (sec)	N/A	0.484	0.050	0.448	0.112	0.082	0.220	0.130	0.256	0.328

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	170	139	59	147	514	70	139	267	163
N.S.	1	1.10	0.90	0.38	0.95	3.32	0.45	0.90	1.72	1.05
time (sec)	N/A	0.502	0.047	0.446	0.112	0.086	0.190	0.126	0.215	0.340

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	161	135	57	145	499	63	137	278	142
N.S.	1	1.07	0.89	0.38	0.96	3.30	0.42	0.91	1.84	0.94
time (sec)	N/A	0.498	0.042	0.425	0.109	0.092	0.219	0.125	0.219	0.217

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	185	151	84	148	201	80	151	290	143
N.S.	1	1.12	0.92	0.51	0.90	1.22	0.48	0.92	1.76	0.87
time (sec)	N/A	0.547	0.050	0.470	0.110	0.079	0.257	0.128	0.271	0.361

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	36	36	35	34	37	62	35
N.S.	1	1.00	1.00	0.88	0.88	0.85	0.83	0.90	1.51	0.85
time (sec)	N/A	0.282	0.007	0.488	0.036	0.068	0.099	0.128	0.201	0.064

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	25	23	22	26	49	23
N.S.	1	1.00	1.00	0.86	0.89	0.82	0.79	0.93	1.75	0.82
time (sec)	N/A	0.261	0.005	0.435	0.026	0.064	0.085	0.127	0.207	0.043

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	12	16	41	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.75	1.00	2.56	0.94
time (sec)	N/A	0.218	0.003	0.434	0.030	0.065	0.090	0.124	0.221	0.111

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	23	22	25	20	15	24	45	21
N.S.	1	1.17	1.00	0.96	1.09	0.87	0.65	1.04	1.96	0.91
time (sec)	N/A	0.228	0.006	0.427	0.026	0.072	0.130	0.119	0.241	0.108

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	35	32	35	33	31	41	63	31
N.S.	1	1.06	1.00	0.91	1.00	0.94	0.89	1.17	1.80	0.89
time (sec)	N/A	0.270	0.007	0.428	0.029	0.072	0.209	0.118	0.244	0.158

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	131	111	39	108	130	34	106	80	134
N.S.	1	1.05	0.89	0.31	0.86	1.04	0.27	0.85	0.64	1.07
time (sec)	N/A	0.441	0.023	0.447	0.109	0.090	0.086	0.117	0.194	0.341

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	108	37	106	103	24	104	78	116
N.S.	1	1.02	0.90	0.31	0.88	0.86	0.20	0.87	0.65	0.97
time (sec)	N/A	0.428	0.011	0.448	0.106	0.081	0.088	0.119	0.249	0.210

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	29	97	307	26	104	65	109
N.S.	1	1.00	0.77	0.25	0.84	2.67	0.23	0.90	0.57	0.95
time (sec)	N/A	0.398	0.009	0.453	0.121	0.083	0.086	0.132	0.220	0.297

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	110	89	29	97	320	22	104	65	115
N.S.	1	0.96	0.78	0.25	0.85	2.81	0.19	0.91	0.57	1.01
time (sec)	N/A	0.393	0.008	0.441	0.108	0.086	0.093	0.115	0.213	0.425

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	129	114	50	105	111	31	113	83	127
N.S.	1	1.05	0.93	0.41	0.85	0.90	0.25	0.92	0.67	1.03
time (sec)	N/A	0.437	0.019	0.462	0.122	0.077	0.105	0.120	0.251	0.340

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	126	119	54	105	141	34	107	96	127
N.S.	1	1.02	0.96	0.44	0.85	1.14	0.27	0.86	0.77	1.02
time (sec)	N/A	0.430	0.017	0.444	0.108	0.082	0.112	0.125	0.294	0.209

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	42	6	28	6
N.S.	1	1.00	1.00	0.70	0.60	0.60	4.20	0.60	2.80	0.60
time (sec)	N/A	0.233	0.025	0.394	0.105	0.072	0.274	0.121	0.225	0.168

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	61	47	64	57	114	57	56	90
N.S.	1	1.05	0.76	0.59	0.80	0.71	1.42	0.71	0.70	1.12
time (sec)	N/A	0.324	0.032	0.460	0.029	0.110	0.432	0.121	0.209	0.292

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	50	36	47	46	90	43	45	70
N.S.	1	1.07	0.85	0.61	0.80	0.78	1.53	0.73	0.76	1.19
time (sec)	N/A	0.290	0.031	0.464	0.036	0.085	0.301	0.123	0.256	0.262

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	25	30	34	66	29	33	29
N.S.	1	1.11	1.00	0.66	0.79	0.89	1.74	0.76	0.87	0.76
time (sec)	N/A	0.268	0.026	0.449	0.034	0.074	0.228	0.121	0.251	0.255

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	20	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	1.11	0.78
time (sec)	N/A	0.218	0.017	0.456	0.026	0.087	0.092	0.120	0.216	0.240

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	32	49	81	76	36	45	52
N.S.	1	1.00	1.00	0.74	1.14	1.88	1.77	0.84	1.05	1.21
time (sec)	N/A	0.263	0.032	0.461	0.112	0.086	0.855	0.114	0.215	0.395

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	53	105	49	43	61	56
N.S.	1	1.00	1.00	0.77	1.13	2.23	1.04	0.91	1.30	1.19
time (sec)	N/A	0.255	0.062	0.499	0.111	0.080	1.011	0.126	0.262	0.441

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	62	50	100	130	100	72	82	76
N.S.	1	1.03	0.87	0.70	1.41	1.83	1.41	1.01	1.15	1.07
time (sec)	N/A	0.286	0.094	0.501	0.108	0.084	2.186	0.119	0.210	0.545

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	103	73	62	137	156	129	80	102	96
N.S.	1	1.08	0.77	0.65	1.44	1.64	1.36	0.84	1.07	1.01
time (sec)	N/A	0.309	0.114	0.494	0.113	0.092	5.279	0.117	0.206	0.693

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	284	94	323	0	58	39	0	76	0
N.S.	1	1.03	0.34	1.17	0.00	0.21	0.14	0.00	0.28	0.00
time (sec)	N/A	0.519	4.132	0.608	0.000	0.081	0.520	0.000	0.300	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	254	62	312	0	48	39	0	57	0
N.S.	1	1.01	0.25	1.24	0.00	0.19	0.16	0.00	0.23	0.00
time (sec)	N/A	0.442	3.935	0.500	0.000	0.074	0.520	0.000	0.233	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	46	297	0	33	37	0	35	37
N.S.	1	1.00	0.20	1.31	0.00	0.15	0.16	0.00	0.15	0.16
time (sec)	N/A	0.394	0.004	0.478	0.000	0.069	0.504	0.000	0.230	0.176

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	51	295	0	34	42	0	45	0
N.S.	1	1.00	0.22	1.29	0.00	0.15	0.18	0.00	0.20	0.00
time (sec)	N/A	0.401	9.311	0.510	0.000	0.074	0.531	0.000	0.245	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	258	51	312	0	46	46	0	46	0
N.S.	1	1.02	0.20	1.23	0.00	0.18	0.18	0.00	0.18	0.00
time (sec)	N/A	0.419	10.011	0.540	0.000	0.074	0.646	0.000	0.261	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	288	51	325	0	57	46	0	46	0
N.S.	1	1.04	0.18	1.17	0.00	0.21	0.17	0.00	0.17	0.00
time (sec)	N/A	0.510	10.011	0.657	0.000	0.077	0.868	0.000	0.282	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	568	96	477	0	69	39	0	79	0
N.S.	1	1.06	0.18	0.89	0.00	0.13	0.07	0.00	0.15	0.00
time (sec)	N/A	0.971	4.169	0.580	0.000	0.078	0.588	0.000	0.238	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	538	64	466	0	57	39	0	60	0
N.S.	1	1.05	0.13	0.91	0.00	0.11	0.08	0.00	0.12	0.00
time (sec)	N/A	0.916	4.152	0.506	0.000	0.077	0.531	0.000	0.289	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	508	51	451	0	43	39	0	38	0
N.S.	1	1.04	0.10	0.93	0.00	0.09	0.08	0.00	0.08	0.00
time (sec)	N/A	0.835	3.899	0.483	0.000	0.071	0.519	0.000	0.254	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	506	49	447	0	38	41	0	43	40
N.S.	1	1.06	0.10	0.93	0.00	0.08	0.09	0.00	0.09	0.08
time (sec)	N/A	0.826	6.071	0.491	0.000	0.104	0.508	0.000	0.236	0.457

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	536	51	464	0	54	46	0	46	0
N.S.	1	1.05	0.10	0.91	0.00	0.11	0.09	0.00	0.09	0.00
time (sec)	N/A	0.869	10.011	0.518	0.000	0.078	0.584	0.000	0.315	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	68	136	57	67	108
N.S.	1	1.05	0.62	0.59	0.80	0.85	1.70	0.71	0.84	1.35
time (sec)	N/A	0.341	0.056	0.462	0.027	0.076	0.703	0.131	0.234	0.266

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	57	112	43	56	88
N.S.	1	1.07	0.66	0.61	0.80	0.97	1.90	0.73	0.95	1.49
time (sec)	N/A	0.308	0.049	0.490	0.026	0.071	0.430	0.117	0.224	0.247

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	45	88	29	44	68
N.S.	1	1.11	0.74	0.66	0.79	1.18	2.32	0.76	1.16	1.79
time (sec)	N/A	0.277	0.038	0.520	0.025	0.071	0.328	0.124	0.233	0.276

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	65	14	31	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.61	0.78	1.72	0.78
time (sec)	N/A	0.230	0.017	0.480	0.037	0.076	0.152	0.118	0.242	0.284

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	52	41	61	100	83	50	62	68
N.S.	1	1.03	0.88	0.69	1.03	1.69	1.41	0.85	1.05	1.15
time (sec)	N/A	0.292	0.044	0.495	0.104	0.082	1.253	0.120	0.244	0.366

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	55	49	66	118	100	57	73	69
N.S.	1	1.05	0.89	0.79	1.06	1.90	1.61	0.92	1.18	1.11
time (sec)	N/A	0.289	0.074	0.517	0.108	0.082	1.514	0.121	0.239	0.514

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	59	48	98	135	78	70	83	74
N.S.	1	1.03	0.86	0.70	1.42	1.96	1.13	1.01	1.20	1.07
time (sec)	N/A	0.291	0.111	0.530	0.106	0.104	1.712	0.133	0.257	0.634

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	308	79	334	0	69	39	0	95	0
N.S.	1	1.04	0.27	1.13	0.00	0.23	0.13	0.00	0.32	0.00
time (sec)	N/A	0.537	5.584	0.559	0.000	0.071	0.640	0.000	0.238	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	278	67	323	0	59	39	0	76	0
N.S.	1	1.02	0.25	1.19	0.00	0.22	0.14	0.00	0.28	0.00
time (sec)	N/A	0.498	5.182	0.523	0.000	0.113	0.600	0.000	0.227	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	249	47	309	0	47	37	0	52	37
N.S.	1	1.01	0.19	1.26	0.00	0.19	0.15	0.00	0.21	0.15
time (sec)	N/A	0.456	0.003	0.482	0.000	0.090	0.554	0.000	0.309	0.198

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	251	52	306	0	44	42	0	63	0
N.S.	1	1.02	0.21	1.24	0.00	0.18	0.17	0.00	0.26	0.00
time (sec)	N/A	0.456	10.011	0.525	0.000	0.085	0.622	0.000	0.255	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	252	52	306	0	44	46	0	63	0
N.S.	1	1.02	0.21	1.23	0.00	0.18	0.19	0.00	0.25	0.00
time (sec)	N/A	0.446	10.010	0.556	0.000	0.074	0.656	0.000	0.250	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	592	81	488	0	80	39	0	98	0
N.S.	1	1.06	0.15	0.88	0.00	0.14	0.07	0.00	0.18	0.00
time (sec)	N/A	0.958	5.566	0.564	0.000	0.076	0.804	0.000	0.291	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	562	69	477	0	68	39	0	79	0
N.S.	1	1.06	0.13	0.90	0.00	0.13	0.07	0.00	0.15	0.00
time (sec)	N/A	0.916	5.629	0.536	0.000	0.080	0.614	0.000	0.262	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	532	52	463	0	58	39	0	55	0
N.S.	1	1.05	0.10	0.91	0.00	0.11	0.08	0.00	0.11	0.00
time (sec)	N/A	0.852	4.835	0.518	0.000	0.072	0.519	0.000	0.260	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	530	50	458	0	51	41	0	61	40
N.S.	1	1.05	0.10	0.91	0.00	0.10	0.08	0.00	0.12	0.08
time (sec)	N/A	0.897	8.226	0.528	0.000	0.088	0.596	0.000	0.304	0.810

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	530	52	458	0	51	46	0	63	0
N.S.	1	1.05	0.10	0.91	0.00	0.10	0.09	0.00	0.12	0.00
time (sec)	N/A	0.869	10.010	0.555	0.000	0.094	0.592	0.000	0.266	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	50	47	64	46	94	61	45	73
N.S.	1	1.00	0.62	0.59	0.80	0.58	1.18	0.76	0.56	0.91
time (sec)	N/A	0.350	0.033	0.536	0.031	0.100	0.381	0.127	0.222	0.299

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	39	36	47	35	70	47	34	35
N.S.	1	1.03	0.66	0.61	0.80	0.59	1.19	0.80	0.58	0.59
time (sec)	N/A	0.317	0.029	0.487	0.027	0.073	0.318	0.122	0.208	0.279

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	27	25	30	23	46	30	22	24
N.S.	1	1.05	0.71	0.66	0.79	0.61	1.21	0.79	0.58	0.63
time (sec)	N/A	0.282	0.025	0.496	0.044	0.071	0.219	0.121	0.263	0.264

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	13	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.72	0.78
time (sec)	N/A	0.232	0.013	0.495	0.032	0.100	0.092	0.119	0.193	0.251

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	37	60	26	23	36	40
N.S.	1	1.00	1.00	0.74	1.37	2.22	0.96	0.85	1.33	1.48
time (sec)	N/A	0.254	0.021	0.483	0.106	0.075	0.610	0.121	0.207	0.413

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	50	39	68	104	49	48	61	59
N.S.	1	0.98	1.00	0.78	1.36	2.08	0.98	0.96	1.22	1.18
time (sec)	N/A	0.272	0.064	0.506	0.105	0.079	1.228	0.122	0.289	0.496

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	62	51	104	134	104	75	83	79
N.S.	1	1.07	0.84	0.69	1.41	1.81	1.41	1.01	1.12	1.07
time (sec)	N/A	0.293	0.096	0.506	0.115	0.080	2.630	0.115	0.222	0.525

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	260	79	312	0	47	37	0	57	0
N.S.	1	1.02	0.31	1.23	0.00	0.19	0.15	0.00	0.22	0.00
time (sec)	N/A	0.461	10.026	0.595	0.000	0.075	0.538	0.000	0.221	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	62	300	0	33	37	0	39	0
N.S.	1	1.00	0.27	1.30	0.00	0.14	0.16	0.00	0.17	0.00
time (sec)	N/A	0.424	10.015	0.577	0.000	0.068	0.461	0.000	0.219	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	46	283	0	14	36	0	20	37
N.S.	1	1.00	0.22	1.37	0.00	0.07	0.17	0.00	0.10	0.18
time (sec)	N/A	0.386	0.004	0.599	0.000	0.085	0.475	0.000	0.216	0.182

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	51	301	0	34	41	0	24	0
N.S.	1	1.00	0.22	1.29	0.00	0.15	0.18	0.00	0.10	0.00
time (sec)	N/A	0.442	10.010	0.526	0.000	0.087	0.564	0.000	0.224	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	264	51	312	0	46	44	0	24	0
N.S.	1	1.03	0.20	1.22	0.00	0.18	0.17	0.00	0.09	0.00
time (sec)	N/A	0.486	10.010	0.553	0.000	0.076	0.591	0.000	0.199	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	544	80	466	0	58	37	0	60	0
N.S.	1	1.06	0.16	0.91	0.00	0.11	0.07	0.00	0.12	0.00
time (sec)	N/A	0.892	10.030	0.624	0.000	0.076	0.552	0.000	0.295	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	514	64	454	0	43	37	0	42	0
N.S.	1	1.05	0.13	0.93	0.00	0.09	0.08	0.00	0.09	0.00
time (sec)	N/A	0.836	10.025	0.553	0.000	0.070	0.491	0.000	0.225	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	484	51	435	0	22	37	0	21	0
N.S.	1	1.05	0.11	0.94	0.00	0.05	0.08	0.00	0.05	0.00
time (sec)	N/A	0.785	10.010	0.608	0.000	0.079	0.430	0.000	0.203	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	512	49	453	0	40	39	0	24	40
N.S.	1	1.06	0.10	0.94	0.00	0.08	0.08	0.00	0.05	0.08
time (sec)	N/A	0.836	10.016	0.576	0.000	0.074	0.507	0.000	0.212	0.401

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	542	51	464	0	54	44	0	24	0
N.S.	1	1.05	0.10	0.90	0.00	0.11	0.09	0.00	0.05	0.00
time (sec)	N/A	0.888	10.010	0.582	0.000	0.078	0.717	0.000	0.244	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	49	46	64	57	94	69	53	70
N.S.	1	1.00	0.63	0.59	0.82	0.73	1.21	0.88	0.68	0.90
time (sec)	N/A	0.351	0.036	0.519	0.028	0.069	0.472	0.123	0.215	0.404

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	35	47	46	70	57	42	41
N.S.	1	1.00	0.64	0.59	0.80	0.78	1.19	0.97	0.71	0.69
time (sec)	N/A	0.324	0.044	0.496	0.031	0.069	0.346	0.127	0.220	0.371

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	30	35	46	33	31	24
N.S.	1	1.00	0.71	0.63	0.79	0.92	1.21	0.87	0.82	0.63
time (sec)	N/A	0.291	0.031	0.488	0.026	0.075	0.224	0.118	0.253	0.311

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	22	14
N.S.	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	1.22	0.78
time (sec)	N/A	0.237	0.020	0.572	0.027	0.072	0.149	0.122	0.199	0.271

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	52	126	184	41	102	55
N.S.	1	1.00	1.00	0.76	1.13	2.74	4.00	0.89	2.22	1.20
time (sec)	N/A	0.282	0.054	0.487	0.108	0.083	0.909	0.116	0.236	0.431

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	74	57	55	86	170	75	72	136	74
N.S.	1	1.16	0.89	0.86	1.34	2.66	1.17	1.12	2.12	1.16
time (sec)	N/A	0.304	0.077	0.562	0.133	0.088	1.870	0.127	0.274	0.555

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	104	73	74	122	200	112	88	159	96
N.S.	1	1.09	0.77	0.78	1.28	2.11	1.18	0.93	1.67	1.01
time (sec)	N/A	0.335	0.101	0.520	0.111	0.103	4.006	0.126	0.243	0.662

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	259	65	320	0	67	37	0	116	0
N.S.	1	1.03	0.26	1.27	0.00	0.27	0.15	0.00	0.46	0.00
time (sec)	N/A	0.464	4.982	0.806	0.000	0.108	0.520	0.000	0.238	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	55	303	0	51	37	0	98	0
N.S.	1	1.00	0.24	1.32	0.00	0.22	0.16	0.00	0.43	0.00
time (sec)	N/A	0.420	4.952	0.551	0.000	0.077	0.487	0.000	0.284	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	55	306	0	51	36	0	31	37
N.S.	1	1.00	0.24	1.32	0.00	0.22	0.16	0.00	0.13	0.16
time (sec)	N/A	0.412	0.010	0.467	0.000	0.074	0.506	0.000	0.247	0.236

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	263	54	321	0	66	41	0	35	0
N.S.	1	1.03	0.21	1.26	0.00	0.26	0.16	0.00	0.14	0.00
time (sec)	N/A	0.489	10.012	0.795	0.000	0.073	0.602	0.000	0.223	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	293	54	342	0	80	44	0	35	0
N.S.	1	1.06	0.19	1.23	0.00	0.29	0.16	0.00	0.13	0.00
time (sec)	N/A	0.529	10.010	0.795	0.000	0.072	0.680	0.000	0.229	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	511	543	66	476	0	76	37	0	120	0
N.S.	1	1.06	0.13	0.93	0.00	0.15	0.07	0.00	0.23	0.00
time (sec)	N/A	0.897	4.969	0.887	0.000	0.077	0.595	0.000	0.283	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	513	55	457	0	61	37	0	103	0
N.S.	1	1.05	0.11	0.94	0.00	0.13	0.08	0.00	0.21	0.00
time (sec)	N/A	0.815	4.830	0.467	0.000	0.085	0.514	0.000	0.246	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	513	54	460	0	61	37	0	32	0
N.S.	1	1.05	0.11	0.94	0.00	0.12	0.08	0.00	0.07	0.00
time (sec)	N/A	0.846	3.992	0.455	0.000	0.092	0.478	0.000	0.232	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	541	52	475	0	70	39	0	35	40
N.S.	1	1.05	0.10	0.93	0.00	0.14	0.08	0.00	0.07	0.08
time (sec)	N/A	0.898	7.363	0.802	0.000	0.075	0.594	0.000	0.293	0.543

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	571	54	496	0	88	44	0	35	0
N.S.	1	1.07	0.10	0.93	0.00	0.16	0.08	0.00	0.07	0.00
time (sec)	N/A	0.912	10.012	0.831	0.000	0.073	0.635	0.000	0.239	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	26	37	26	56	37	25	46
N.S.	1	1.00	0.57	0.49	0.70	0.49	1.06	0.70	0.47	0.87
time (sec)	N/A	0.267	0.018	0.879	0.046	0.072	0.214	0.121	0.213	0.146

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	25	21	28	21	41	28	20	21
N.S.	1	1.05	0.62	0.52	0.70	0.52	1.02	0.70	0.50	0.52
time (sec)	N/A	0.253	0.017	0.779	0.031	0.066	0.144	0.123	0.250	0.035

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	18	15	19	14	26	19	13	14
N.S.	1	1.07	0.67	0.56	0.70	0.52	0.96	0.70	0.48	0.52
time (sec)	N/A	0.251	0.015	0.793	0.025	0.084	0.108	0.111	0.243	0.028

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62	0.69
time (sec)	N/A	0.201	0.012	0.822	0.034	0.078	0.072	0.122	0.204	0.028

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	12	26	23	164
N.S.	1	1.00	1.00	0.79	1.79	1.79	0.86	1.86	1.64	11.71
time (sec)	N/A	0.228	0.015	0.961	0.025	0.075	0.496	0.124	0.201	0.332

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	24	37	44	26	38	41	176
N.S.	1	0.94	1.00	0.77	1.19	1.42	0.84	1.23	1.32	5.68
time (sec)	N/A	0.236	0.038	1.050	0.025	0.077	1.320	0.125	0.276	0.158

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	50	38	34	64	52	65	50	53	189
N.S.	1	1.06	0.81	0.72	1.36	1.11	1.38	1.06	1.13	4.02
time (sec)	N/A	0.251	0.046	1.023	0.026	0.070	2.290	0.126	0.228	0.166

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	71	43	41	80	57	85	59	64	201
N.S.	1	1.13	0.68	0.65	1.27	0.90	1.35	0.94	1.02	3.19
time (sec)	N/A	0.271	0.047	1.035	0.027	0.073	7.146	0.117	0.210	0.047

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	141	40	17	0	25	29	0	39	179
N.S.	1	1.04	0.29	0.12	0.00	0.18	0.21	0.00	0.29	1.32
time (sec)	N/A	0.357	10.020	0.973	0.000	0.074	0.493	0.000	0.249	0.151

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	32	17	0	17	29	0	28	167
N.S.	1	1.00	0.27	0.14	0.00	0.14	0.24	0.00	0.23	1.39
time (sec)	N/A	0.318	10.018	0.911	0.000	0.072	0.460	0.000	0.241	0.052

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	17	14	0	6	27	0	16	155
N.S.	1	1.00	0.17	0.14	0.00	0.06	0.26	0.00	0.16	1.50
time (sec)	N/A	0.300	0.001	0.382	0.000	0.078	0.453	0.000	0.212	0.041

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	22	17	0	21	32	0	18	169
N.S.	1	1.00	0.18	0.14	0.00	0.17	0.26	0.00	0.15	1.39
time (sec)	N/A	0.315	10.004	0.840	0.000	0.083	0.457	0.000	0.230	0.157

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	143	22	17	0	30	36	0	18	181
N.S.	1	1.04	0.16	0.12	0.00	0.22	0.26	0.00	0.13	1.31
time (sec)	N/A	0.356	10.005	0.939	0.000	0.075	0.501	0.000	0.276	0.162

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	281	42	17	0	30	29	0	42	239
N.S.	1	1.07	0.16	0.06	0.00	0.11	0.11	0.00	0.16	0.91
time (sec)	N/A	0.567	10.019	0.973	0.000	0.094	0.484	0.000	0.234	0.060

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	260	34	17	0	22	29	0	31	227
N.S.	1	1.06	0.14	0.07	0.00	0.09	0.12	0.00	0.13	0.92
time (sec)	N/A	0.562	10.009	0.929	0.000	0.077	0.416	0.000	0.209	0.056

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	239	22	17	0	9	29	0	17	214
N.S.	1	1.07	0.10	0.08	0.00	0.04	0.13	0.00	0.08	0.96
time (sec)	N/A	0.512	10.014	0.839	0.000	0.081	0.344	0.000	0.204	0.139

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	258	20	17	0	22	31	0	18	227
N.S.	1	1.08	0.08	0.07	0.00	0.09	0.13	0.00	0.08	0.95
time (sec)	N/A	0.548	10.017	0.952	0.000	0.076	0.417	0.000	0.252	0.082

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	279	22	17	0	33	36	0	18	239
N.S.	1	1.06	0.08	0.06	0.00	0.13	0.14	0.00	0.07	0.91
time (sec)	N/A	0.614	10.005	0.901	0.000	0.076	0.489	0.000	0.220	0.168

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	32	28	45	28	58	59	27	54
N.S.	1	1.00	0.52	0.46	0.74	0.46	0.95	0.97	0.44	0.89
time (sec)	N/A	0.183	0.018	0.780	0.024	0.073	0.226	0.123	0.217	0.040

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	27	23	34	23	42	41	22	23
N.S.	1	1.04	0.59	0.50	0.74	0.50	0.91	0.89	0.48	0.50
time (sec)	N/A	0.168	0.018	0.623	0.034	0.076	0.141	0.108	0.250	0.033

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	20	17	23	16	27	23	17	16
N.S.	1	1.06	0.65	0.55	0.74	0.52	0.87	0.74	0.55	0.52
time (sec)	N/A	0.160	0.014	0.786	0.036	0.072	0.105	0.119	0.243	0.032

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	10	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	0.67	0.73
time (sec)	N/A	0.136	0.010	0.603	0.027	0.089	0.068	0.106	0.226	0.025

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	29	29	31	30	27	180
N.S.	1	1.00	1.00	0.81	1.81	1.81	1.94	1.88	1.69	11.25
time (sec)	N/A	0.146	0.014	0.891	0.026	0.071	0.548	0.120	0.214	0.286

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	43	50	82	44	47	195
N.S.	1	1.00	1.00	0.80	1.23	1.43	2.34	1.26	1.34	5.57
time (sec)	N/A	0.159	0.031	0.777	0.032	0.071	1.117	0.121	0.256	0.169

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	58	42	40	70	58	138	58	61	209
N.S.	1	1.09	0.79	0.75	1.32	1.09	2.60	1.09	1.15	3.94
time (sec)	N/A	0.172	0.031	0.898	0.027	0.076	2.342	0.126	0.222	0.046

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	81	47	45	90	63	182	76	74	223
N.S.	1	1.14	0.66	0.63	1.27	0.89	2.56	1.07	1.04	3.14
time (sec)	N/A	0.183	0.047	0.774	0.032	0.075	7.249	0.117	0.222	0.032

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	157	40	15	0	27	31	0	45	199
N.S.	1	1.03	0.26	0.10	0.00	0.18	0.20	0.00	0.30	1.31
time (sec)	N/A	0.238	10.019	0.677	0.000	0.076	0.459	0.000	0.197	0.045

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	32	15	0	19	31	0	32	185
N.S.	1	1.00	0.24	0.11	0.00	0.14	0.23	0.00	0.24	1.38
time (sec)	N/A	0.211	10.024	0.613	0.000	0.110	0.412	0.000	0.244	0.146

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	15	12	0	6	29	0	20	172
N.S.	1	1.00	0.13	0.10	0.00	0.05	0.25	0.00	0.17	1.50
time (sec)	N/A	0.194	0.001	0.384	0.000	0.076	0.560	0.000	0.207	0.035

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	20	15	0	26	34	0	24	187
N.S.	1	1.00	0.15	0.11	0.00	0.19	0.25	0.00	0.18	1.38
time (sec)	N/A	0.213	10.004	0.612	0.000	0.078	0.498	0.000	0.215	0.058

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	159	20	15	0	33	37	0	24	201
N.S.	1	1.03	0.13	0.10	0.00	0.21	0.24	0.00	0.16	1.31
time (sec)	N/A	0.238	10.005	0.631	0.000	0.075	0.578	0.000	0.268	0.045

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	313	42	15	0	32	31	0	48	260
N.S.	1	1.06	0.14	0.05	0.00	0.11	0.11	0.00	0.16	0.88
time (sec)	N/A	0.392	10.019	0.668	0.000	0.073	0.510	0.000	0.244	0.050

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	290	34	15	0	24	31	0	35	246
N.S.	1	1.05	0.12	0.05	0.00	0.09	0.11	0.00	0.13	0.89
time (sec)	N/A	0.574	10.008	0.627	0.000	0.073	0.451	0.000	0.207	0.146

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	267	20	15	0	9	31	0	21	231
N.S.	1	1.06	0.08	0.06	0.00	0.04	0.12	0.00	0.08	0.92
time (sec)	N/A	0.557	10.014	0.569	0.000	0.086	0.401	0.000	0.199	0.041

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	287	18	15	0	26	32	0	24	245
N.S.	1	1.06	0.07	0.06	0.00	0.10	0.12	0.00	0.09	0.91
time (sec)	N/A	0.618	10.010	0.628	0.000	0.070	0.449	0.000	0.260	0.173

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	310	20	15	0	36	37	0	24	260
N.S.	1	1.05	0.07	0.05	0.00	0.12	0.13	0.00	0.08	0.88
time (sec)	N/A	0.675	10.005	0.630	0.000	0.086	0.569	0.000	0.219	0.048

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	30	26	37	26	56	37	25	46
N.S.	1	1.00	0.57	0.49	0.70	0.49	1.06	0.70	0.47	0.87
time (sec)	N/A	0.281	0.017	0.843	0.027	0.070	0.217	0.124	0.222	0.038

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	25	21	28	21	41	28	20	21
N.S.	1	1.05	0.62	0.52	0.70	0.52	1.02	0.70	0.50	0.52
time (sec)	N/A	0.277	0.016	0.837	0.026	0.067	0.152	0.124	0.196	0.030

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	18	15	19	14	26	19	13	14
N.S.	1	1.07	0.67	0.56	0.70	0.52	0.96	0.70	0.48	0.52
time (sec)	N/A	0.263	0.014	0.686	0.028	0.074	0.108	0.127	0.238	0.023

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	8	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62	0.69
time (sec)	N/A	0.216	0.010	0.822	0.033	0.071	0.070	0.121	0.228	0.025

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	31	10	32	164
N.S.	1	1.00	1.00	0.79	0.71	0.71	2.21	0.71	2.29	11.71
time (sec)	N/A	0.239	0.015	1.084	0.109	0.100	0.582	0.125	0.211	0.042

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	28	28	24	23	25	80	23	48	177
N.S.	1	0.90	0.90	0.77	0.74	0.81	2.58	0.74	1.55	5.71
time (sec)	N/A	0.251	0.024	1.158	0.105	0.068	1.140	0.125	0.244	0.142

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	38	36	48	34	138	35	60	189
N.S.	1	1.04	0.81	0.77	1.02	0.72	2.94	0.74	1.28	4.02
time (sec)	N/A	0.261	0.037	1.108	0.117	0.078	2.456	0.124	0.218	0.147

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	43	40	66	39	182	44	71	201
N.S.	1	1.11	0.68	0.63	1.05	0.62	2.89	0.70	1.13	3.19
time (sec)	N/A	0.304	0.041	1.132	0.111	0.077	7.702	0.126	0.203	0.041

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	153	158	52	33	0	25	27	0	39	179
N.S.	1	1.03	0.34	0.22	0.00	0.16	0.18	0.00	0.25	1.17
time (sec)	N/A	0.405	10.017	0.934	0.000	0.066	0.499	0.000	0.200	0.052

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	44	33	0	17	27	0	28	167
N.S.	1	1.00	0.32	0.24	0.00	0.12	0.20	0.00	0.20	1.22
time (sec)	N/A	0.338	10.030	0.823	0.000	0.074	0.416	0.000	0.238	0.050

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	35	30	0	6	26	0	16	156
N.S.	1	1.00	0.29	0.25	0.00	0.05	0.22	0.00	0.13	1.30
time (sec)	N/A	0.306	0.002	0.387	0.000	0.066	0.415	0.000	0.252	0.043

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	40	33	0	21	31	0	20	169
N.S.	1	1.00	0.29	0.24	0.00	0.15	0.22	0.00	0.14	1.22
time (sec)	N/A	0.352	10.007	0.916	0.000	0.073	0.524	0.000	0.245	0.063

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	155	160	40	33	0	30	34	0	20	181
N.S.	1	1.03	0.26	0.21	0.00	0.19	0.22	0.00	0.13	1.17
time (sec)	N/A	0.388	10.007	0.897	0.000	0.079	0.577	0.000	0.218	0.065

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	294	312	54	33	0	30	27	0	42	240
N.S.	1	1.06	0.18	0.11	0.00	0.10	0.09	0.00	0.14	0.82
time (sec)	N/A	0.642	10.016	0.926	0.000	0.071	0.490	0.000	0.233	0.058

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	278	291	46	33	0	22	27	0	31	228
N.S.	1	1.05	0.17	0.12	0.00	0.08	0.10	0.00	0.11	0.82
time (sec)	N/A	0.594	10.009	0.758	0.000	0.090	0.471	0.000	0.230	0.136

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	255	270	40	33	0	9	27	0	17	215
N.S.	1	1.06	0.16	0.13	0.00	0.04	0.11	0.00	0.07	0.84
time (sec)	N/A	0.571	10.014	0.767	0.000	0.071	0.403	0.000	0.223	0.137

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	269	289	38	33	0	21	29	0	20	226
N.S.	1	1.07	0.14	0.12	0.00	0.08	0.11	0.00	0.07	0.84
time (sec)	N/A	0.612	10.017	0.886	0.000	0.069	0.432	0.000	0.261	0.158

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	294	310	40	33	0	33	34	0	20	240
N.S.	1	1.05	0.14	0.11	0.00	0.11	0.12	0.00	0.07	0.82
time (sec)	N/A	0.676	10.010	0.839	0.000	0.070	0.615	0.000	0.254	0.069

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	32	28	45	28	63	59	26	54
N.S.	1	1.00	0.52	0.46	0.74	0.46	1.03	0.97	0.43	0.89
time (sec)	N/A	0.308	0.020	0.710	0.035	0.071	0.219	0.119	0.211	0.036

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	27	23	34	23	46	41	21	23
N.S.	1	1.04	0.59	0.50	0.74	0.50	1.00	0.89	0.46	0.50
time (sec)	N/A	0.286	0.019	0.551	0.034	0.081	0.153	0.119	0.214	0.031

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	20	17	23	16	29	23	16	16
N.S.	1	1.06	0.65	0.55	0.74	0.52	0.94	0.74	0.52	0.52
time (sec)	N/A	0.266	0.016	0.566	0.033	0.073	0.098	0.119	0.240	0.029

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	14	11	9	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.93	0.73	0.60	0.73
time (sec)	N/A	0.225	0.011	0.558	0.030	0.072	0.082	0.117	0.265	0.024

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	12	12	24	180
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	1.50	11.25
time (sec)	N/A	0.249	0.017	1.040	0.115	0.071	0.544	0.116	0.230	0.035

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	35	28	27	31	29	27	42	194
N.S.	1	0.97	1.00	0.80	0.77	0.89	0.83	0.77	1.20	5.54
time (sec)	N/A	0.265	0.026	0.994	0.109	0.068	1.141	0.126	0.244	0.148

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	42	38	56	39	66	41	54	209
N.S.	1	1.08	0.79	0.72	1.06	0.74	1.25	0.77	1.02	3.94
time (sec)	N/A	0.282	0.038	0.814	0.111	0.074	2.474	0.122	0.245	0.044

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	47	44	74	44	90	59	65	223
N.S.	1	1.13	0.66	0.62	1.04	0.62	1.27	0.83	0.92	3.14
time (sec)	N/A	0.303	0.038	1.027	0.116	0.090	7.676	0.116	0.214	0.031

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	154	54	18	0	27	32	0	42	199
N.S.	1	1.03	0.36	0.12	0.00	0.18	0.21	0.00	0.28	1.34
time (sec)	N/A	0.399	10.018	0.685	0.000	0.076	0.464	0.000	0.222	0.042

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	47	18	0	19	32	0	29	185
N.S.	1	1.00	0.36	0.14	0.00	0.15	0.24	0.00	0.22	1.41
time (sec)	N/A	0.332	10.033	0.599	0.000	0.083	0.422	0.000	0.277	0.058

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	37	16	0	6	31	0	19	172
N.S.	1	1.00	0.33	0.14	0.00	0.05	0.28	0.00	0.17	1.54
time (sec)	N/A	0.323	0.002	0.376	0.000	0.065	0.472	0.000	0.268	0.032

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	42	18	0	24	36	0	21	187
N.S.	1	1.00	0.32	0.14	0.00	0.18	0.27	0.00	0.16	1.41
time (sec)	N/A	0.317	10.008	0.592	0.000	0.081	0.482	0.000	0.220	0.153

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	156	42	18	0	33	39	0	21	201
N.S.	1	1.03	0.28	0.12	0.00	0.22	0.26	0.00	0.14	1.33
time (sec)	N/A	0.369	10.007	0.634	0.000	0.075	0.548	0.000	0.213	0.042

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	300	56	18	0	32	32	0	45	259
N.S.	1	1.06	0.20	0.06	0.00	0.11	0.11	0.00	0.16	0.92
time (sec)	N/A	0.638	10.017	0.652	0.000	0.073	0.500	0.000	0.255	0.047

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	277	49	18	0	24	32	0	34	245
N.S.	1	1.05	0.19	0.07	0.00	0.09	0.12	0.00	0.13	0.93
time (sec)	N/A	0.602	10.012	0.842	0.000	0.076	0.451	0.000	0.248	0.067

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	254	42	18	0	9	32	0	20	230
N.S.	1	1.06	0.18	0.08	0.00	0.04	0.13	0.00	0.08	0.96
time (sec)	N/A	0.560	10.014	0.583	0.000	0.081	0.414	0.000	0.244	0.042

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	274	40	18	0	24	34	0	21	244
N.S.	1	1.07	0.16	0.07	0.00	0.09	0.13	0.00	0.08	0.95
time (sec)	N/A	0.593	10.010	0.698	0.000	0.082	0.437	0.000	0.256	0.170

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	297	42	18	0	36	39	0	21	259
N.S.	1	1.05	0.15	0.06	0.00	0.13	0.14	0.00	0.07	0.92
time (sec)	N/A	0.642	10.012	0.630	0.000	0.076	0.568	0.000	0.248	0.048

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	106	85	99	0	207	121	98	90	0
N.S.	1	1.04	0.83	0.97	0.00	2.03	1.19	0.96	0.88	0.00
time (sec)	N/A	0.444	1.732	3.550	0.000	0.172	17.016	0.158	0.218	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	72	65	79	0	159	58	78	64	0
N.S.	1	1.01	0.92	1.11	0.00	2.24	0.82	1.10	0.90	0.00
time (sec)	N/A	0.368	0.160	0.741	0.000	0.168	1.166	0.133	0.241	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	71	62	83	0	167	92	94	71	0
N.S.	1	1.01	0.89	1.19	0.00	2.39	1.31	1.34	1.01	0.00
time (sec)	N/A	0.371	0.173	0.875	0.000	0.164	1.881	0.134	0.294	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	56	41	32	25
N.S.	1	1.00	0.93	0.75	1.25	0.89	2.00	1.46	1.14	0.89
time (sec)	N/A	0.252	0.187	0.453	0.051	0.073	49.956	0.128	0.242	0.310

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	46	31	0	46	0	65	46	51
N.S.	1	0.98	0.79	0.53	0.00	0.79	0.00	1.12	0.79	0.88
time (sec)	N/A	0.301	1.281	0.478	0.000	0.084	0.000	0.136	0.235	0.326

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	58	42	65	57	0	85	57	65
N.S.	1	1.04	0.65	0.47	0.73	0.64	0.00	0.96	0.64	0.73
time (sec)	N/A	0.349	1.717	0.534	0.044	0.080	0.000	0.133	0.252	0.369

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	337	103	751	0	0	46	0	90	0
N.S.	1	1.14	0.35	2.54	0.00	0.00	0.16	0.00	0.30	0.00
time (sec)	N/A	0.674	10.048	1.634	0.000	0.000	134.384	0.000	0.321	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	297	87	737	0	0	46	0	69	0
N.S.	1	1.12	0.33	2.78	0.00	0.00	0.17	0.00	0.26	0.00
time (sec)	N/A	0.570	10.036	0.789	0.000	0.000	5.378	0.000	0.324	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	262	54	717	0	0	46	0	47	0
N.S.	1	1.12	0.23	3.08	0.00	0.00	0.20	0.00	0.20	0.00
time (sec)	N/A	0.508	10.011	0.804	0.000	0.000	0.590	0.000	0.354	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	263	56	726	0	50	49	0	59	0
N.S.	1	1.12	0.24	3.10	0.00	0.21	0.21	0.00	0.25	0.00
time (sec)	N/A	0.517	10.017	0.853	0.000	0.102	5.038	0.000	0.431	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	300	56	742	0	65	53	0	59	0
N.S.	1	1.13	0.21	2.80	0.00	0.25	0.20	0.00	0.22	0.00
time (sec)	N/A	0.572	10.013	1.148	0.000	0.088	158.388	0.000	0.556	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	616	87	1127	0	0	46	0	71	0
N.S.	1	1.12	0.16	2.05	0.00	0.00	0.08	0.00	0.13	0.00
time (sec)	N/A	1.166	10.036	0.952	0.000	0.000	40.707	0.000	0.324	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	579	56	1109	0	0	46	0	47	0
N.S.	1	1.13	0.11	2.16	0.00	0.00	0.09	0.00	0.09	0.00
time (sec)	N/A	1.074	10.011	0.857	0.000	0.000	1.332	0.000	0.350	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	575	54	1106	0	0	49	0	53	0
N.S.	1	1.12	0.11	2.16	0.00	0.00	0.10	0.00	0.10	0.00
time (sec)	N/A	1.053	10.015	0.908	0.000	0.000	0.706	0.000	0.346	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	615	56	1125	0	59	53	0	59	0
N.S.	1	1.12	0.10	2.05	0.00	0.11	0.10	0.00	0.11	0.00
time (sec)	N/A	1.049	10.014	0.951	0.000	0.077	15.607	0.000	0.486	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	655	56	1138	0	72	0	0	59	0
N.S.	1	1.13	0.10	1.96	0.00	0.12	0.00	0.00	0.10	0.00
time (sec)	N/A	1.217	10.014	1.218	0.000	0.077	0.000	0.000	0.585	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	115	86	101	0	211	0	106	91	0
N.S.	1	1.10	0.82	0.96	0.00	2.01	0.00	1.01	0.87	0.00
time (sec)	N/A	0.444	0.239	1.043	0.000	0.169	0.000	0.157	0.274	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	75	86	0	179	60	86	67	0
N.S.	1	1.01	1.01	1.16	0.00	2.42	0.81	1.16	0.91	0.00
time (sec)	N/A	0.371	0.212	0.646	0.000	0.180	13.007	0.163	0.242	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	45	45	57	0	121	29	47	45	0
N.S.	1	1.02	1.02	1.30	0.00	2.75	0.66	1.07	1.02	0.00
time (sec)	N/A	0.327	0.278	0.500	0.000	0.162	0.622	0.133	0.235	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	27	42	25	25
N.S.	1	1.00	0.93	0.75	1.25	0.89	0.96	1.50	0.89	0.89
time (sec)	N/A	0.251	0.188	0.623	0.040	0.076	1.889	0.140	0.302	0.425

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	34	29	0	35	58	65	35	40
N.S.	1	0.98	0.59	0.50	0.00	0.60	1.00	1.12	0.60	0.69
time (sec)	N/A	0.297	0.239	0.609	0.000	0.082	66.370	0.140	0.227	0.433

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	47	42	65	46	0	88	46	54
N.S.	1	1.02	0.53	0.47	0.73	0.52	0.00	0.99	0.52	0.61
time (sec)	N/A	0.350	1.470	0.638	0.041	0.087	0.000	0.140	0.264	0.465

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	306	87	740	0	0	44	0	69	0
N.S.	1	1.14	0.32	2.76	0.00	0.00	0.16	0.00	0.26	0.00
time (sec)	N/A	0.608	10.026	1.651	0.000	0.000	81.769	0.000	0.362	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	266	71	725	0	0	44	0	50	0
N.S.	1	1.11	0.30	3.03	0.00	0.00	0.18	0.00	0.21	0.00
time (sec)	N/A	0.527	10.026	0.911	0.000	0.000	3.202	0.000	0.343	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	233	54	432	0	24	44	0	30	0
N.S.	1	1.14	0.26	2.12	0.00	0.12	0.22	0.00	0.15	0.00
time (sec)	N/A	0.473	10.011	1.497	0.000	0.077	0.625	0.000	0.278	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	266	56	732	0	51	48	0	32	0
N.S.	1	1.12	0.24	3.09	0.00	0.22	0.20	0.00	0.14	0.00
time (sec)	N/A	0.519	10.012	2.132	0.000	0.079	7.084	0.000	0.374	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	306	56	742	0	65	0	0	32	0
N.S.	1	1.14	0.21	2.77	0.00	0.24	0.00	0.00	0.12	0.00
time (sec)	N/A	0.593	10.018	3.243	0.000	0.095	0.000	0.000	0.403	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	625	87	1127	0	0	0	0	71	0
N.S.	1	1.13	0.16	2.04	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.183	10.028	2.098	0.000	0.000	0.000	0.000	0.456	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	585	71	1115	0	0	44	0	52	0
N.S.	1	1.12	0.14	2.14	0.00	0.00	0.08	0.00	0.10	0.00
time (sec)	N/A	1.078	10.022	1.494	0.000	0.000	31.386	0.000	0.394	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	550	56	1085	0	0	44	0	27	0
N.S.	1	1.14	0.12	2.25	0.00	0.00	0.09	0.00	0.06	0.00
time (sec)	N/A	0.995	10.024	0.798	0.000	0.000	0.964	0.000	0.398	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	581	54	1112	0	32	48	0	32	0
N.S.	1	1.13	0.10	2.16	0.00	0.06	0.09	0.00	0.06	0.00
time (sec)	N/A	1.086	10.011	1.607	0.000	0.074	0.874	0.000	0.312	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	621	56	1125	0	58	51	0	32	0
N.S.	1	1.12	0.10	2.03	0.00	0.10	0.09	0.00	0.06	0.00
time (sec)	N/A	1.073	10.012	2.293	0.000	0.100	21.544	0.000	0.373	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	152	97	109	0	289	0	119	176	0
N.S.	1	1.14	0.73	0.82	0.00	2.17	0.00	0.89	1.32	0.00
time (sec)	N/A	0.508	1.692	0.870	0.000	0.214	0.000	0.176	0.311	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	112	83	91	0	255	0	94	151	0
N.S.	1	1.15	0.86	0.94	0.00	2.63	0.00	0.97	1.56	0.00
time (sec)	N/A	0.439	0.316	0.658	0.000	0.184	0.000	0.167	0.249	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	69	77	0	220	61	81	124	0
N.S.	1	1.01	0.95	1.05	0.00	3.01	0.84	1.11	1.70	0.00
time (sec)	N/A	0.374	0.252	0.498	0.000	0.165	12.938	0.167	0.274	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	29	29	30	27	20
N.S.	1	1.00	0.93	0.75	0.00	1.04	1.04	1.07	0.96	0.71
time (sec)	N/A	0.248	0.179	0.548	0.000	0.090	0.590	0.133	0.256	0.276

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	34	29	0	48	60	92	44	57
N.S.	1	1.00	0.60	0.51	0.00	0.84	1.05	1.61	0.77	1.00
time (sec)	N/A	0.302	0.396	0.665	0.000	0.085	4.523	0.196	0.317	0.459

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	91	45	40	0	61	296	0	55	70
N.S.	1	1.06	0.52	0.47	0.00	0.71	3.44	0.00	0.64	0.81
time (sec)	N/A	0.374	0.395	0.649	0.000	0.084	121.673	0.000	0.295	0.484

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	125	56	51	0	70	0	137	66	85
N.S.	1	1.07	0.48	0.44	0.00	0.60	0.00	1.17	0.56	0.73
time (sec)	N/A	0.406	2.169	0.663	0.000	0.093	0.000	0.224	0.312	0.533

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	345	87	787	0	0	0	0	154	0
N.S.	1	1.17	0.29	2.66	0.00	0.00	0.00	0.00	0.52	0.00
time (sec)	N/A	0.682	10.038	3.372	0.000	0.000	0.000	0.000	0.584	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	305	74	758	0	0	44	0	133	0
N.S.	1	1.14	0.28	2.84	0.00	0.00	0.16	0.00	0.50	0.00
time (sec)	N/A	0.589	10.025	2.875	0.000	0.000	98.060	0.000	0.539	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	266	64	732	0	69	44	0	112	0
N.S.	1	1.11	0.27	3.06	0.00	0.29	0.18	0.00	0.47	0.00
time (sec)	N/A	0.500	10.020	0.999	0.000	0.093	3.357	0.000	0.530	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	265	63	724	0	62	44	0	41	0
N.S.	1	1.12	0.27	3.07	0.00	0.26	0.19	0.00	0.17	0.00
time (sec)	N/A	0.527	10.020	2.216	0.000	0.079	0.926	0.000	0.439	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	302	59	757	0	88	48	0	43	0
N.S.	1	1.14	0.22	2.86	0.00	0.33	0.18	0.00	0.16	0.00
time (sec)	N/A	0.583	10.014	4.007	0.000	0.083	14.180	0.000	0.499	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	342	59	784	0	101	0	0	43	0
N.S.	1	1.16	0.20	2.65	0.00	0.34	0.00	0.00	0.15	0.00
time (sec)	N/A	0.634	10.013	5.024	0.000	0.082	0.000	0.000	0.740	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	664	87	1177	0	0	0	0	155	0
N.S.	1	1.14	0.15	2.03	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	1.191	10.052	4.325	0.000	0.000	0.000	0.000	0.659	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	624	74	1148	0	0	0	0	134	0
N.S.	1	1.13	0.13	2.09	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	1.092	10.038	3.517	0.000	0.000	0.000	0.000	0.559	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	584	62	1122	0	0	44	0	115	0
N.S.	1	1.13	0.12	2.16	0.00	0.00	0.08	0.00	0.22	0.00
time (sec)	N/A	1.065	10.027	1.892	0.000	0.000	39.231	0.000	0.576	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	584	59	1122	0	73	44	0	38	0
N.S.	1	1.13	0.11	2.17	0.00	0.14	0.09	0.00	0.07	0.00
time (sec)	N/A	1.062	10.012	0.978	0.000	0.087	1.161	0.000	0.417	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	617	57	1150	0	76	48	0	43	0
N.S.	1	1.12	0.10	2.09	0.00	0.14	0.09	0.00	0.08	0.00
time (sec)	N/A	1.109	10.012	3.573	0.000	0.105	1.765	0.000	0.423	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	657	59	1204	0	95	51	0	43	0
N.S.	1	1.13	0.10	2.07	0.00	0.16	0.09	0.00	0.07	0.00
time (sec)	N/A	1.185	10.014	4.562	0.000	0.087	47.092	0.000	0.587	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	57	110	57	57	55
N.S.	1	1.05	0.62	0.59	0.80	0.71	1.38	0.71	0.71	0.69
time (sec)	N/A	0.348	0.039	0.469	0.038	0.088	0.461	0.121	0.201	0.259

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	46	87	43	46	44
N.S.	1	1.07	0.66	0.61	0.80	0.78	1.47	0.73	0.78	0.75
time (sec)	N/A	0.319	0.030	0.460	0.034	0.069	0.314	0.119	0.195	0.225

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	25	30	34	63	29	34	33
N.S.	1	1.11	1.00	0.66	0.79	0.89	1.66	0.76	0.89	0.87
time (sec)	N/A	0.292	0.028	0.451	0.030	0.094	0.251	0.109	0.263	0.211

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78	0.78
time (sec)	N/A	0.239	0.014	0.447	0.028	0.074	0.122	0.120	0.220	0.221

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	102	124	96	95	100	42	96	35	112
N.S.	1	1.07	1.31	1.01	1.00	1.05	0.44	1.01	0.37	1.18
time (sec)	N/A	0.379	0.109	1.848	0.110	0.074	0.673	0.428	0.222	0.246

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	108	135	111	103	151	41	106	45	122
N.S.	1	1.01	1.26	1.04	0.96	1.41	0.38	0.99	0.42	1.14
time (sec)	N/A	0.360	0.167	0.647	0.124	0.081	0.743	0.412	0.253	0.359

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	126	168	146	181	177	39	0	53	0
N.S.	1	1.05	1.40	1.22	1.51	1.48	0.32	0.00	0.44	0.00
time (sec)	N/A	0.366	0.345	0.683	0.115	0.115	1.223	0.000	0.228	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	96	141	121	134	190	39	0	31	0
N.S.	1	1.02	1.50	1.29	1.43	2.02	0.41	0.00	0.33	0.00
time (sec)	N/A	0.314	0.279	0.641	0.121	0.082	0.719	0.000	0.218	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	91	144	116	114	162	41	0	15	40
N.S.	1	1.03	1.64	1.32	1.30	1.84	0.47	0.00	0.17	0.45
time (sec)	N/A	0.312	0.201	0.656	0.121	73.726	0.698	0.000	0.217	0.407

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81	0.81
time (sec)	N/A	0.229	0.101	0.480	0.030	0.071	0.505	0.000	0.268	0.296

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	28	35	38	109	0	38	32
N.S.	1	1.00	0.95	0.64	0.80	0.86	2.48	0.00	0.86	0.73
time (sec)	N/A	0.273	0.108	0.490	0.030	0.078	0.655	0.000	0.255	0.381

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	53	39	52	49	520	0	49	73
N.S.	1	1.09	0.78	0.57	0.76	0.72	7.65	0.00	0.72	1.07
time (sec)	N/A	0.309	0.125	0.510	0.026	0.083	0.877	0.000	0.192	0.524

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	60	847	0	60	93
N.S.	1	1.13	0.58	0.54	0.75	0.65	9.21	0.00	0.65	1.01
time (sec)	N/A	0.351	0.136	0.540	0.058	0.076	1.281	0.000	0.215	0.727

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	39	0	49	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	0.96	0.00
time (sec)	N/A	0.271	4.939	0.000	0.000	0.000	0.529	0.000	0.232	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	203	0	0	0	37	0	27	37
N.S.	1	1.00	4.41	0.00	0.00	0.00	0.80	0.00	0.59	0.80
time (sec)	N/A	0.257	0.138	0.000	0.000	0.000	0.664	0.000	0.224	0.156

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	42	0	47	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.82	0.00	0.92	0.00
time (sec)	N/A	0.273	10.013	0.000	0.000	0.000	0.541	0.000	0.233	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	42	0	48	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.82	0.00	0.94	0.00
time (sec)	N/A	0.279	10.009	0.000	0.000	0.000	0.630	0.000	0.250	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	57	110	57	57	55
N.S.	1	1.05	0.62	0.59	0.80	0.71	1.38	0.71	0.71	0.69
time (sec)	N/A	0.330	0.038	0.463	0.046	0.075	0.557	0.114	0.215	0.250

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	46	87	43	46	44
N.S.	1	1.07	0.66	0.61	0.80	0.78	1.47	0.73	0.78	0.75
time (sec)	N/A	0.309	0.031	0.464	0.034	0.071	0.384	0.127	0.216	0.221

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	39	25	30	35	63	29	35	33
N.S.	1	1.11	1.03	0.66	0.79	0.92	1.66	0.76	0.92	0.87
time (sec)	N/A	0.282	0.028	0.461	0.025	0.068	0.281	0.121	0.270	0.229

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	0.78	0.78
time (sec)	N/A	0.228	0.014	0.462	0.036	0.070	0.124	0.121	0.267	0.235

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	126	98	97	122	44	98	37	122
N.S.	1	1.05	1.29	1.00	0.99	1.24	0.45	1.00	0.38	1.24
time (sec)	N/A	0.350	0.113	0.523	0.148	0.078	0.816	0.421	0.241	0.253

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	136	112	103	290	39	107	46	137
N.S.	1	1.00	1.27	1.05	0.96	2.71	0.36	1.00	0.43	1.28
time (sec)	N/A	0.368	0.155	0.643	0.116	0.085	0.825	0.438	0.224	0.383

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	122	168	144	181	345	39	0	50	0
N.S.	1	1.04	1.44	1.23	1.55	2.95	0.33	0.00	0.43	0.00
time (sec)	N/A	0.349	0.369	0.673	0.116	0.089	1.228	0.000	0.257	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	203	118	134	362	37	0	27	37
N.S.	1	1.01	2.23	1.30	1.47	3.98	0.41	0.00	0.30	0.41
time (sec)	N/A	0.304	0.135	0.010	0.118	0.089	0.734	0.000	0.255	0.149

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	91	145	116	114	0	42	0	15	0
N.S.	1	1.03	1.65	1.32	1.30	0.00	0.48	0.00	0.17	0.00
time (sec)	N/A	0.301	0.228	0.713	0.115	0.000	0.795	0.000	0.213	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81	0.81
time (sec)	N/A	0.233	0.105	0.519	0.026	0.086	0.528	0.000	0.254	0.353

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	28	35	38	110	0	38	38
N.S.	1	1.00	0.95	0.64	0.80	0.86	2.50	0.00	0.86	0.86
time (sec)	N/A	0.269	0.124	0.528	0.034	0.075	0.795	0.000	0.242	0.398

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	53	39	52	49	520	0	49	73
N.S.	1	1.09	0.78	0.57	0.76	0.72	7.65	0.00	0.72	1.07
time (sec)	N/A	0.310	0.126	0.539	0.033	0.077	1.424	0.000	0.193	0.543

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	60	847	0	60	93
N.S.	1	1.13	0.58	0.54	0.75	0.65	9.21	0.00	0.65	1.01
time (sec)	N/A	0.358	0.141	0.589	0.027	0.077	1.494	0.000	0.192	0.726

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	39	0	53	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	1.04	0.00
time (sec)	N/A	0.282	5.897	0.000	0.000	0.000	0.677	0.000	0.222	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	39	0	31	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	0.61	0.00
time (sec)	N/A	0.270	5.387	0.000	0.000	0.000	0.587	0.000	0.221	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	41	0	43	39
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.84	0.00	0.88	0.80
time (sec)	N/A	0.267	8.985	0.000	0.000	0.000	0.659	0.000	0.218	0.439

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	46	0	48	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.90	0.00	0.94	0.00
time (sec)	N/A	0.289	10.010	0.000	0.000	0.000	0.692	0.000	0.233	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	39	0	70	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	1.35	0.00
time (sec)	N/A	0.271	6.489	0.000	0.000	0.000	0.619	0.000	0.229	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	272	0	0	0	37	0	45	37
N.S.	1	1.00	5.79	0.00	0.00	0.00	0.79	0.00	0.96	0.79
time (sec)	N/A	0.253	0.172	0.000	0.000	0.000	0.659	0.000	0.213	0.175

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	42	0	65	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.81	0.00	1.25	0.00
time (sec)	N/A	0.270	10.016	0.000	0.000	0.000	0.623	0.000	0.239	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	46	0	65	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.88	0.00	1.25	0.00
time (sec)	N/A	0.269	10.013	0.000	0.000	0.000	0.681	0.000	0.250	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	15	34	33	71	55	33	33
N.S.	1	1.09	0.59	0.33	0.74	0.72	1.54	1.20	0.72	0.72
time (sec)	N/A	0.265	0.026	0.477	0.024	0.069	0.724	0.111	0.221	0.248

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	46	92	61	15	48
N.S.	1	1.05	0.62	0.59	0.80	0.58	1.15	0.76	0.19	0.60
time (sec)	N/A	0.329	0.035	0.496	0.033	0.079	0.359	0.118	0.239	0.276

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	35	68	47	15	36
N.S.	1	1.07	0.66	0.61	0.80	0.59	1.15	0.80	0.25	0.61
time (sec)	N/A	0.308	0.027	0.494	0.029	0.071	0.318	0.124	0.230	0.253

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	24	44	30	15	24
N.S.	1	1.11	0.74	0.66	0.79	0.63	1.16	0.79	0.39	0.63
time (sec)	N/A	0.280	0.025	0.563	0.034	0.073	0.211	0.120	0.231	0.256

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	15	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.83	0.78
time (sec)	N/A	0.227	0.012	0.480	0.030	0.070	0.085	0.120	0.214	0.248

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	85	103	83	86	236	37	87	15	100
N.S.	1	1.02	1.24	1.00	1.04	2.84	0.45	1.05	0.18	1.20
time (sec)	N/A	0.326	0.086	0.507	0.110	0.086	0.548	0.415	0.208	0.408

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	113	136	111	118	344	39	110	15	138
N.S.	1	1.03	1.24	1.01	1.07	3.13	0.35	1.00	0.14	1.25
time (sec)	N/A	0.352	0.161	0.672	0.116	0.088	0.785	0.411	0.218	0.561

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	128	168	144	184	399	37	0	15	0
N.S.	1	1.07	1.40	1.20	1.53	3.32	0.31	0.00	0.12	0.00
time (sec)	N/A	0.351	0.327	0.709	0.112	0.093	1.604	0.000	0.207	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	98	140	118	137	309	37	0	15	0
N.S.	1	1.04	1.49	1.26	1.46	3.29	0.39	0.00	0.16	0.00
time (sec)	N/A	0.304	0.278	0.702	0.112	0.093	0.845	0.000	0.220	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	110	99	100	330	36	0	11	37
N.S.	1	1.00	1.57	1.41	1.43	4.71	0.51	0.00	0.16	0.53
time (sec)	N/A	0.267	0.030	0.500	0.113	0.090	0.551	0.000	0.198	0.218

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	31	0	15	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	1.48	0.00	0.71	0.81
time (sec)	N/A	0.232	0.124	0.492	0.028	0.075	0.468	0.000	0.231	0.218

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	27	70	0	15	27
N.S.	1	1.00	0.70	0.64	0.80	0.61	1.59	0.00	0.34	0.61
time (sec)	N/A	0.267	0.140	0.485	0.030	0.082	0.531	0.000	0.211	0.267

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	38	406	0	15	38
N.S.	1	1.09	0.62	0.57	0.76	0.56	5.97	0.00	0.22	0.56
time (sec)	N/A	0.311	0.162	0.500	0.026	0.088	0.773	0.000	0.203	0.327

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	49	692	0	15	76
N.S.	1	1.13	0.58	0.54	0.75	0.53	7.52	0.00	0.16	0.83
time (sec)	N/A	0.355	0.158	0.538	0.030	0.094	1.153	0.000	0.208	0.393

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	0.29	0.00
time (sec)	N/A	0.278	4.822	0.000	0.000	0.000	0.524	0.000	0.213	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	0.29	0.00
time (sec)	N/A	0.262	4.972	0.000	0.000	0.000	0.484	0.000	0.247	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	13	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	0.25	0.00
time (sec)	N/A	0.257	4.752	0.000	0.000	0.000	0.429	0.000	0.212	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	39	0	15	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	0.31	0.82
time (sec)	N/A	0.265	8.157	0.000	0.000	0.000	0.487	0.000	0.206	0.420

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	44	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.86	0.00	0.29	0.00
time (sec)	N/A	0.266	10.011	0.000	0.000	0.000	0.614	0.000	0.222	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	50	47	64	46	92	61	15	48
N.S.	1	1.05	0.64	0.60	0.82	0.59	1.18	0.78	0.19	0.62
time (sec)	N/A	0.313	0.032	0.514	0.038	0.077	0.440	0.122	0.200	0.274

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	39	36	46	35	68	46	15	36
N.S.	1	1.09	0.70	0.64	0.82	0.62	1.21	0.82	0.27	0.64
time (sec)	N/A	0.298	0.028	0.562	0.027	0.074	0.328	0.123	0.197	0.242

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	27	24	30	23	44	30	15	24
N.S.	1	1.11	0.75	0.67	0.83	0.64	1.22	0.83	0.42	0.67
time (sec)	N/A	0.270	0.025	0.518	0.028	0.072	0.216	0.116	0.213	0.243

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87	0.87
time (sec)	N/A	0.223	0.014	0.509	0.026	0.073	0.086	0.120	0.237	0.234

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	86	101	83	86	122	39	87	15	102
N.S.	1	1.02	1.20	0.99	1.02	1.45	0.46	1.04	0.18	1.21
time (sec)	N/A	0.331	0.080	2.839	0.137	0.083	0.580	0.380	0.202	0.411

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	135	111	118	179	39	109	15	128
N.S.	1	1.04	1.23	1.01	1.07	1.63	0.35	0.99	0.14	1.16
time (sec)	N/A	0.355	0.150	0.664	0.110	0.081	0.777	0.423	0.194	0.513

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	132	170	146	184	179	37	0	15	0
N.S.	1	1.07	1.38	1.19	1.50	1.46	0.30	0.00	0.12	0.00
time (sec)	N/A	0.345	0.322	1.041	0.117	0.092	1.626	0.000	0.187	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	102	142	122	137	161	37	0	15	0
N.S.	1	1.05	1.46	1.26	1.41	1.66	0.38	0.00	0.15	0.00
time (sec)	N/A	0.307	0.290	0.989	0.118	0.085	0.776	0.000	0.198	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	116	100	100	140	37	0	13	0
N.S.	1	1.00	1.61	1.39	1.39	1.94	0.51	0.00	0.18	0.00
time (sec)	N/A	0.274	0.203	0.741	0.113	0.092	0.550	0.000	0.205	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	31	0	15	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.63	0.00	0.79	0.89
time (sec)	N/A	0.231	0.125	0.671	0.032	0.082	0.399	0.000	0.234	0.222

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	26	35	27	68	0	15	25
N.S.	1	1.00	0.70	0.59	0.80	0.61	1.55	0.00	0.34	0.57
time (sec)	N/A	0.261	0.153	0.727	0.033	0.081	0.561	0.000	0.240	0.255

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	38	406	0	15	38
N.S.	1	1.09	0.62	0.57	0.76	0.56	5.97	0.00	0.22	0.56
time (sec)	N/A	0.310	0.178	0.805	0.034	0.086	0.789	0.000	0.200	0.334

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	49	692	0	15	76
N.S.	1	1.13	0.58	0.54	0.75	0.53	7.52	0.00	0.16	0.83
time (sec)	N/A	0.344	0.212	0.822	0.031	0.078	1.146	0.000	0.200	0.380

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	0.29	0.00
time (sec)	N/A	0.278	5.511	0.000	0.000	0.000	0.531	0.000	0.198	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	0.29	0.00
time (sec)	N/A	0.263	5.130	0.000	0.000	0.000	0.506	0.000	0.193	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	183	0	0	0	36	0	11	37
N.S.	1	1.00	3.98	0.00	0.00	0.00	0.78	0.00	0.24	0.80
time (sec)	N/A	0.247	0.133	0.000	0.000	0.000	0.447	0.000	0.227	0.211

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	41	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	0.29	0.00
time (sec)	N/A	0.266	10.013	0.000	0.000	0.000	0.554	0.000	0.211	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	44	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.86	0.00	0.29	0.00
time (sec)	N/A	0.273	10.013	0.000	0.000	0.000	0.667	0.000	0.201	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	37	0	34	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.69	0.00	0.63	0.00
time (sec)	N/A	0.273	6.233	0.000	0.000	0.000	0.538	0.000	0.228	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	37	0	34	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.69	0.00	0.63	0.00
time (sec)	N/A	0.292	5.928	0.000	0.000	0.000	0.479	0.000	0.218	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	37	0	32	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.69	0.00	0.59	0.00
time (sec)	N/A	0.272	5.293	0.000	0.000	0.000	0.627	0.000	0.186	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	39	0	33	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.75	0.00	0.63	0.77
time (sec)	N/A	0.269	8.914	0.000	0.000	0.000	0.587	0.000	0.209	0.453

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	44	0	33	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.81	0.00	0.61	0.00
time (sec)	N/A	0.270	10.012	0.000	0.000	0.000	0.652	0.000	0.201	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	83	15	78	82	31	0	13	0
N.S.	1	1.00	1.57	0.28	1.47	1.55	0.58	0.00	0.25	0.00
time (sec)	N/A	0.246	0.173	4.026	0.117	0.081	0.484	0.000	0.200	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	13	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	1.00	0.69
time (sec)	N/A	0.219	0.011	0.537	0.026	0.087	0.077	0.124	0.188	0.305

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	136	1022	151	847	5902	1269	1021	860
N.S.	1	1.00	0.90	6.77	1.00	5.61	39.09	8.40	6.76	5.70
time (sec)	N/A	0.505	0.384	2.648	0.029	0.141	2.525	0.189	0.189	1.390

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	431	97	367	2006	540	430	389
N.S.	1	1.00	0.91	4.44	1.00	3.78	20.68	5.57	4.43	4.01
time (sec)	N/A	0.391	0.190	0.911	0.027	0.104	1.074	0.148	0.224	0.786

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	177	61	157	666	224	176	167
N.S.	1	1.00	0.92	2.90	1.00	2.57	10.92	3.67	2.89	2.74
time (sec)	N/A	0.320	0.062	0.562	0.035	0.083	0.571	0.131	0.210	0.548

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	51	43	85	313	117	91	93
N.S.	1	1.00	0.93	1.19	1.00	1.98	7.28	2.72	2.12	2.16
time (sec)	N/A	0.292	0.044	0.486	0.027	0.098	0.406	0.126	0.217	0.478

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	25	33	94	43	33	34
N.S.	1	1.00	1.00	1.20	1.00	1.32	3.76	1.72	1.32	1.36
time (sec)	N/A	0.271	0.028	0.079	0.028	0.086	0.282	0.122	0.226	0.386

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0	15	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	2.26	0.00	0.38	0.00
time (sec)	N/A	0.268	0.026	0.000	0.000	0.000	2.112	0.000	0.213	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	520	0	26	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	13.33	0.00	0.67	0.00
time (sec)	N/A	0.261	0.029	0.000	0.000	0.000	27.654	0.000	0.195	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	1578	0	37	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	40.46	0.00	0.95	0.00
time (sec)	N/A	0.257	0.033	0.000	0.000	0.000	121.230	0.000	0.191	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	66	0	0	0	54	0	255	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.84	0.00	3.98	0.00
time (sec)	N/A	0.310	0.134	0.000	0.000	0.000	1.439	0.000	0.255	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	54	0	104	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.86	0.00	1.65	0.00
time (sec)	N/A	0.300	0.094	0.000	0.000	0.000	0.719	0.000	0.222	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	63	65	0	0	0	53	0	23	0
N.S.	1	1.21	1.25	0.00	0.00	0.00	1.02	0.00	0.44	0.00
time (sec)	N/A	0.302	0.107	0.000	0.000	0.000	0.591	0.000	0.200	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	53	0	34	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.80	0.00	0.52	0.00
time (sec)	N/A	0.306	0.141	0.000	0.000	0.000	0.790	0.000	0.204	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	58	0	255	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.84	0.00	3.70	0.00
time (sec)	N/A	0.315	0.141	0.000	0.000	0.000	3.109	0.000	0.238	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	66	0	0	0	58	0	103	0
N.S.	1	1.11	1.08	0.00	0.00	0.00	0.95	0.00	1.69	0.00
time (sec)	N/A	0.304	0.125	0.000	0.000	0.000	1.056	0.000	0.221	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	68	66	0	0	0	58	0	102	0
N.S.	1	1.11	1.08	0.00	0.00	0.00	0.95	0.00	1.67	0.00
time (sec)	N/A	0.288	0.120	0.000	0.000	0.000	0.824	0.000	0.219	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	25	104	21	26	21
N.S.	1	1.00	1.00	0.96	0.91	1.09	4.52	0.91	1.13	0.91
time (sec)	N/A	0.233	0.033	0.613	0.025	0.095	0.584	0.121	0.207	0.258

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	47	40	42	47	58	432	51	52	68
N.S.	1	0.98	0.83	0.88	0.98	1.21	9.00	1.06	1.08	1.42
time (sec)	N/A	0.296	0.054	0.634	0.039	0.082	1.940	0.119	0.192	0.246

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	70	64	80	73	98	1187	132	92	118
N.S.	1	0.95	0.86	1.08	0.99	1.32	16.04	1.78	1.24	1.59
time (sec)	N/A	0.337	0.065	0.702	0.036	0.080	8.407	0.124	0.199	0.284

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	132	106	148	2467	260	147	183
N.S.	1	1.00	1.00	1.39	1.12	1.56	25.97	2.74	1.55	1.93
time (sec)	N/A	0.374	0.073	0.866	0.034	0.086	22.486	0.133	0.195	0.326

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	174	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	2.64	0.00
time (sec)	N/A	0.310	0.029	0.000	0.000	0.000	123.816	0.000	0.203	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [148] had the largest ratio of [1.1111100000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.06	11	0.182
3	A	2	2	1.00	9	0.222
4	A	1	1	1.00	7	0.143
5	A	2	2	1.00	11	0.182
6	A	2	2	1.00	11	0.182
7	A	2	2	1.00	11	0.182
8	A	2	2	1.00	11	0.182
9	A	2	2	1.00	11	0.182
10	A	2	2	1.00	11	0.182
11	A	2	2	1.00	11	0.182
12	A	2	2	1.00	11	0.182
13	A	2	2	1.00	13	0.154
14	A	2	2	1.00	13	0.154
15	A	1	1	1.00	13	0.077
16	A	2	2	1.00	11	0.182
17	A	2	2	1.00	9	0.222
18	A	4	3	1.15	13	0.231
19	A	2	2	1.00	13	0.154
20	A	2	2	1.00	13	0.154
21	A	4	3	1.04	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	2	2	1.00	13	0.154
23	A	2	2	1.00	13	0.154
24	A	4	3	1.15	13	0.231
25	A	2	2	1.00	13	0.154
26	A	2	2	1.00	13	0.154
27	A	1	1	1.00	13	0.077
28	A	2	2	1.00	13	0.154
29	A	2	2	1.00	13	0.154
30	A	4	3	1.13	13	0.231
31	A	4	3	1.09	13	0.231
32	A	4	3	1.09	13	0.231
33	A	4	3	1.09	13	0.231
34	A	4	3	1.12	13	0.231
35	A	1	1	1.00	13	0.077
36	A	4	3	1.19	13	0.231
37	A	4	3	1.14	13	0.231
38	A	4	3	1.08	13	0.231
39	A	4	3	1.16	13	0.231
40	A	1	1	1.00	13	0.077
41	A	4	3	1.10	13	0.231
42	A	4	3	1.09	13	0.231
43	A	4	3	1.09	13	0.231
44	A	2	2	1.00	13	0.154
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	11	0.182
47	A	2	2	1.00	9	0.222
48	A	2	2	1.00	13	0.154
49	A	2	2	1.00	13	0.154
50	A	2	2	1.00	13	0.154
51	A	2	2	1.00	13	0.154
52	A	2	2	1.00	13	0.154
53	A	4	3	1.06	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	4	3	1.06	13	0.231
55	A	4	3	1.06	13	0.231
56	A	4	3	1.08	13	0.231
57	A	4	3	1.12	13	0.231
58	A	1	1	1.00	13	0.077
59	A	4	3	1.03	13	0.231
60	A	4	3	1.00	13	0.231
61	A	4	3	1.03	13	0.231
62	A	4	3	1.03	13	0.231
63	A	4	3	0.98	13	0.231
64	A	4	3	1.03	13	0.231
65	A	1	1	1.00	13	0.077
66	A	4	3	1.10	13	0.231
67	A	5	4	1.16	13	0.308
68	A	4	3	1.03	13	0.231
69	A	4	3	1.06	13	0.231
70	A	2	2	1.00	13	0.154
71	A	2	2	1.00	13	0.154
72	A	2	2	1.00	11	0.182
73	A	2	2	1.00	9	0.222
74	A	2	2	1.00	13	0.154
75	A	2	2	1.00	13	0.154
76	A	2	2	1.00	13	0.154
77	A	2	2	1.00	13	0.154
78	A	2	2	1.00	13	0.154
79	A	2	2	1.00	13	0.154
80	A	4	3	1.03	13	0.231
81	A	4	3	1.04	13	0.231
82	A	4	3	1.04	13	0.231
83	A	4	3	1.06	13	0.231
84	A	4	3	1.08	13	0.231
85	A	4	3	1.12	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.00	13	0.077
87	A	4	3	1.02	13	0.231
88	A	4	3	0.98	13	0.231
89	A	4	3	0.98	13	0.231
90	A	4	3	0.96	13	0.231
91	A	4	3	0.98	13	0.231
92	A	4	3	0.96	13	0.231
93	A	4	3	0.98	13	0.231
94	A	4	3	0.97	13	0.231
95	A	4	3	1.02	13	0.231
96	A	1	1	1.00	13	0.077
97	A	4	3	1.10	13	0.231
98	A	5	4	1.16	13	0.308
99	A	6	5	1.19	13	0.385
100	A	7	6	1.21	13	0.462
101	A	4	3	1.02	13	0.231
102	A	4	3	1.02	13	0.231
103	A	2	2	1.00	13	0.154
104	A	2	2	1.00	13	0.154
105	A	2	2	1.00	11	0.182
106	A	2	2	1.00	9	0.222
107	A	2	2	1.00	13	0.154
108	A	2	2	1.00	13	0.154
109	A	2	2	1.00	13	0.154
110	A	2	2	1.00	13	0.154
111	A	2	2	1.00	13	0.154
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	11	0.182
114	A	4	3	0.98	13	0.231
115	A	4	3	0.96	13	0.231
116	A	1	1	1.00	13	0.077
117	A	5	4	1.18	13	0.308

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	4	3	1.03	13	0.231
119	A	10	9	1.07	13	0.692
120	A	10	9	1.03	13	0.692
121	A	9	8	1.01	11	0.727
122	A	9	8	0.97	9	0.889
123	A	10	9	1.07	13	0.692
124	A	10	9	1.03	13	0.692
125	A	4	3	0.93	13	0.231
126	A	4	3	0.94	13	0.231
127	A	1	1	1.00	13	0.077
128	A	4	3	1.03	13	0.231
129	A	4	3	1.00	13	0.231
130	A	11	10	1.07	13	0.769
131	A	10	9	1.05	13	0.692
132	A	10	9	1.01	13	0.692
133	A	10	9	1.05	11	0.818
134	A	10	9	1.01	9	1.000
135	A	11	10	1.09	13	0.769
136	A	11	10	1.07	13	0.769
137	A	4	3	1.02	13	0.231
138	A	4	3	0.98	13	0.231
139	A	1	1	1.00	13	0.077
140	A	1	1	1.00	13	0.077
141	A	4	3	1.02	13	0.231
142	A	4	3	1.05	13	0.231
143	A	11	10	1.10	13	0.769
144	A	11	10	1.07	13	0.769
145	A	11	10	1.08	13	0.769
146	A	11	10	1.05	13	0.769
147	A	11	10	1.10	11	0.909
148	A	11	10	1.07	9	1.111
149	A	12	11	1.12	13	0.846

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	3	1.00	14	0.214
151	A	4	3	1.00	14	0.214
152	A	1	1	1.00	14	0.071
153	A	5	4	1.17	14	0.286
154	A	4	3	1.06	14	0.214
155	A	9	8	1.05	14	0.571
156	A	9	8	1.02	14	0.571
157	A	8	7	1.00	12	0.583
158	A	8	7	0.96	10	0.700
159	A	9	8	1.05	14	0.571
160	A	9	8	1.02	14	0.571
161	A	4	3	1.00	13	0.231
162	A	4	3	1.05	15	0.200
163	A	4	3	1.07	15	0.200
164	A	4	3	1.11	15	0.200
165	A	1	1	1.00	15	0.067
166	A	5	4	1.00	15	0.267
167	A	5	4	1.00	15	0.267
168	A	6	5	1.03	15	0.333
169	A	7	6	1.08	15	0.400
170	A	4	4	1.03	15	0.267
171	A	3	3	1.01	15	0.200
172	A	2	2	1.00	11	0.182
173	A	2	2	1.00	15	0.133
174	A	3	3	1.02	15	0.200
175	A	4	4	1.04	15	0.267
176	A	6	6	1.06	15	0.400
177	A	5	5	1.05	15	0.333
178	A	4	4	1.04	13	0.308
179	A	4	4	1.06	15	0.267
180	A	5	5	1.05	15	0.333
181	A	4	3	1.05	15	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	4	3	1.07	15	0.200
183	A	4	3	1.11	15	0.200
184	A	1	1	1.00	15	0.067
185	A	6	5	1.03	15	0.333
186	A	6	5	1.05	15	0.333
187	A	6	5	1.03	15	0.333
188	A	5	5	1.04	15	0.333
189	A	4	4	1.02	15	0.267
190	A	3	3	1.01	11	0.273
191	A	3	3	1.02	15	0.200
192	A	3	3	1.02	15	0.200
193	A	7	7	1.06	15	0.467
194	A	6	6	1.06	15	0.400
195	A	5	5	1.05	13	0.385
196	A	5	5	1.05	15	0.333
197	A	5	5	1.05	15	0.333
198	A	4	3	1.00	15	0.200
199	A	4	3	1.03	15	0.200
200	A	4	3	1.05	15	0.200
201	A	1	1	1.00	15	0.067
202	A	4	3	1.00	15	0.200
203	A	5	4	0.98	15	0.267
204	A	6	5	1.07	15	0.333
205	A	3	3	1.02	15	0.200
206	A	2	2	1.00	15	0.133
207	A	1	1	1.00	11	0.091
208	A	2	2	1.00	15	0.133
209	A	3	3	1.03	15	0.200
210	A	5	5	1.06	15	0.333
211	A	4	4	1.05	15	0.267
212	A	3	3	1.05	13	0.231
213	A	4	4	1.06	15	0.267

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	5	5	1.05	15	0.333
215	A	4	3	1.00	15	0.200
216	A	4	3	1.00	15	0.200
217	A	4	3	1.00	15	0.200
218	A	1	1	1.00	15	0.067
219	A	5	4	1.00	15	0.267
220	A	6	5	1.16	15	0.333
221	A	7	6	1.09	15	0.400
222	A	3	3	1.03	15	0.200
223	A	2	2	1.00	15	0.133
224	A	2	2	1.00	11	0.182
225	A	3	3	1.03	15	0.200
226	A	4	4	1.06	15	0.267
227	A	5	5	1.06	15	0.333
228	A	4	4	1.05	15	0.267
229	A	4	4	1.05	13	0.308
230	A	5	5	1.05	15	0.333
231	A	6	6	1.07	15	0.400
232	A	4	3	1.00	13	0.231
233	A	4	3	1.05	13	0.231
234	A	4	3	1.07	13	0.231
235	A	1	1	1.00	13	0.077
236	A	4	3	1.00	13	0.231
237	A	5	4	0.94	13	0.308
238	A	6	5	1.06	13	0.385
239	A	7	6	1.13	13	0.462
240	A	3	3	1.04	13	0.231
241	A	2	2	1.00	13	0.154
242	A	1	1	1.00	9	0.111
243	A	2	2	1.00	13	0.154
244	A	3	3	1.04	13	0.231
245	A	5	5	1.07	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	4	4	1.06	13	0.308
247	A	3	3	1.07	11	0.273
248	A	4	4	1.08	13	0.308
249	A	5	5	1.06	13	0.385
250	A	4	3	1.00	15	0.200
251	A	4	3	1.04	15	0.200
252	A	4	3	1.06	15	0.200
253	A	1	1	1.00	15	0.067
254	A	4	3	1.00	15	0.200
255	A	5	4	1.00	15	0.267
256	A	6	5	1.09	15	0.333
257	A	7	6	1.14	15	0.400
258	A	3	3	1.03	15	0.200
259	A	2	2	1.00	15	0.133
260	A	1	1	1.00	11	0.091
261	A	2	2	1.00	15	0.133
262	A	3	3	1.03	15	0.200
263	A	5	5	1.06	15	0.333
264	A	4	4	1.05	15	0.267
265	A	3	3	1.06	13	0.231
266	A	4	4	1.06	15	0.267
267	A	5	5	1.05	15	0.333
268	A	4	3	1.00	13	0.231
269	A	4	3	1.05	13	0.231
270	A	4	3	1.07	13	0.231
271	A	1	1	1.00	13	0.077
272	A	4	3	1.00	13	0.231
273	A	5	4	0.90	13	0.308
274	A	6	5	1.04	13	0.385
275	A	7	6	1.11	13	0.462
276	A	3	3	1.03	13	0.231
277	A	2	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	1	1	1.00	9	0.111
279	A	2	2	1.00	13	0.154
280	A	3	3	1.03	13	0.231
281	A	5	5	1.06	13	0.385
282	A	4	4	1.05	13	0.308
283	A	3	3	1.06	11	0.273
284	A	4	4	1.07	13	0.308
285	A	5	5	1.05	13	0.385
286	A	4	3	1.00	15	0.200
287	A	4	3	1.04	15	0.200
288	A	4	3	1.06	15	0.200
289	A	1	1	1.00	15	0.067
290	A	4	3	1.00	15	0.200
291	A	5	4	0.97	15	0.267
292	A	6	5	1.08	15	0.333
293	A	7	6	1.13	15	0.400
294	A	3	3	1.03	15	0.200
295	A	2	2	1.00	15	0.133
296	A	1	1	1.00	11	0.091
297	A	2	2	1.00	15	0.133
298	A	3	3	1.03	15	0.200
299	A	5	5	1.06	15	0.333
300	A	4	4	1.05	15	0.267
301	A	3	3	1.06	13	0.231
302	A	4	4	1.07	15	0.267
303	A	5	5	1.05	15	0.333
304	A	7	6	1.04	19	0.316
305	A	6	5	1.01	19	0.263
306	A	6	5	1.01	19	0.263
307	A	1	1	1.00	19	0.053
308	A	2	2	0.98	19	0.105
309	A	3	3	1.04	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	6	5	1.14	19	0.263
311	A	5	4	1.12	19	0.211
312	A	4	3	1.12	19	0.158
313	A	4	3	1.12	19	0.158
314	A	5	4	1.13	19	0.211
315	A	8	7	1.12	19	0.368
316	A	7	6	1.13	19	0.316
317	A	7	6	1.12	19	0.316
318	A	8	7	1.12	19	0.368
319	A	9	8	1.13	19	0.421
320	A	7	6	1.10	19	0.316
321	A	6	5	1.01	19	0.263
322	A	5	4	1.02	19	0.211
323	A	1	1	1.00	19	0.053
324	A	2	2	0.98	19	0.105
325	A	3	3	1.02	19	0.158
326	A	5	4	1.14	19	0.211
327	A	4	3	1.11	19	0.158
328	A	3	2	1.14	19	0.105
329	A	4	3	1.12	19	0.158
330	A	5	4	1.14	19	0.211
331	A	8	7	1.13	19	0.368
332	A	7	6	1.12	19	0.316
333	A	6	5	1.14	19	0.263
334	A	7	6	1.13	19	0.316
335	A	8	7	1.12	19	0.368
336	A	8	7	1.14	19	0.368
337	A	7	6	1.15	19	0.316
338	A	6	5	1.01	19	0.263
339	A	1	1	1.00	19	0.053
340	A	2	2	1.00	19	0.105
341	A	3	3	1.06	19	0.158

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	4	4	1.07	19	0.211
343	A	6	5	1.17	19	0.263
344	A	5	4	1.14	19	0.211
345	A	4	3	1.11	19	0.158
346	A	4	3	1.12	19	0.158
347	A	5	4	1.14	19	0.211
348	A	6	5	1.16	19	0.263
349	A	9	8	1.14	19	0.421
350	A	8	7	1.13	19	0.368
351	A	7	6	1.13	19	0.316
352	A	7	6	1.13	19	0.316
353	A	8	7	1.12	19	0.368
354	A	9	8	1.13	19	0.421
355	A	4	3	1.05	15	0.200
356	A	4	3	1.07	15	0.200
357	A	4	3	1.11	15	0.200
358	A	1	1	1.00	15	0.067
359	A	7	6	1.07	15	0.400
360	A	7	6	1.01	15	0.400
361	A	3	3	1.05	15	0.200
362	A	2	2	1.02	13	0.154
363	A	2	2	1.03	15	0.133
364	A	1	1	1.00	15	0.067
365	A	2	2	1.00	15	0.133
366	A	3	3	1.09	15	0.200
367	A	4	4	1.13	15	0.267
368	A	2	2	1.00	15	0.133
369	A	2	2	1.00	11	0.182
370	A	2	2	1.00	15	0.133
371	A	2	2	1.00	15	0.133
372	A	4	3	1.05	15	0.200
373	A	4	3	1.07	15	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	4	3	1.11	15	0.200
375	A	1	1	1.00	15	0.067
376	A	7	6	1.05	15	0.400
377	A	7	6	1.00	15	0.400
378	A	3	3	1.04	15	0.200
379	A	2	2	1.01	11	0.182
380	A	2	2	1.03	15	0.133
381	A	1	1	1.00	15	0.067
382	A	2	2	1.00	15	0.133
383	A	3	3	1.09	15	0.200
384	A	4	4	1.13	15	0.267
385	A	2	2	1.00	15	0.133
386	A	2	2	1.00	13	0.154
387	A	2	2	1.00	15	0.133
388	A	2	2	1.00	15	0.133
389	A	2	2	1.00	15	0.133
390	A	2	2	1.00	11	0.182
391	A	2	2	1.00	15	0.133
392	A	2	2	1.00	15	0.133
393	A	4	3	1.09	15	0.200
394	A	4	3	1.05	15	0.200
395	A	4	3	1.07	15	0.200
396	A	4	3	1.11	15	0.200
397	A	1	1	1.00	15	0.067
398	A	6	5	1.02	15	0.333
399	A	7	6	1.03	15	0.400
400	A	3	3	1.07	15	0.200
401	A	2	2	1.04	15	0.133
402	A	1	1	1.00	11	0.091
403	A	1	1	1.00	15	0.067
404	A	2	2	1.00	15	0.133
405	A	3	3	1.09	15	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
406	A	4	4	1.13	15	0.267
407	A	2	2	1.00	15	0.133
408	A	2	2	1.00	15	0.133
409	A	2	2	1.00	13	0.154
410	A	2	2	1.00	15	0.133
411	A	2	2	1.00	15	0.133
412	A	4	3	1.05	15	0.200
413	A	4	3	1.09	15	0.200
414	A	4	3	1.11	15	0.200
415	A	1	1	1.00	15	0.067
416	A	6	5	1.02	15	0.333
417	A	7	6	1.04	15	0.400
418	A	3	3	1.07	15	0.200
419	A	2	2	1.05	15	0.133
420	A	1	1	1.00	13	0.077
421	A	1	1	1.00	15	0.067
422	A	2	2	1.00	15	0.133
423	A	3	3	1.09	15	0.200
424	A	4	4	1.13	15	0.267
425	A	2	2	1.00	15	0.133
426	A	2	2	1.00	15	0.133
427	A	2	2	1.00	11	0.182
428	A	2	2	1.00	15	0.133
429	A	2	2	1.00	15	0.133
430	A	2	2	1.00	15	0.133
431	A	2	2	1.00	15	0.133
432	A	2	2	1.00	13	0.154
433	A	2	2	1.00	15	0.133
434	A	2	2	1.00	15	0.133
435	A	1	1	1.00	13	0.077
436	A	1	1	1.00	13	0.077
437	A	2	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	2	2	1.00	13	0.154
439	A	2	2	1.00	13	0.154
440	A	2	2	1.00	13	0.154
441	A	2	2	1.00	11	0.182
442	A	1	1	1.00	13	0.077
443	A	1	1	1.00	13	0.077
444	A	1	1	1.00	13	0.077
445	A	2	2	1.00	15	0.133
446	A	2	2	1.00	15	0.133
447	A	2	2	1.21	15	0.133
448	A	2	2	1.00	15	0.133
449	A	2	2	1.00	17	0.118
450	A	2	2	1.11	17	0.118
451	A	2	2	1.11	17	0.118
452	A	1	1	1.00	13	0.077
453	A	4	3	0.98	13	0.231
454	A	4	3	0.95	13	0.231
455	A	4	3	1.00	13	0.231
456	A	2	2	1.00	15	0.133

CHAPTER 3

Listing of integrals

3.1 $\int x^3(a+bx^3)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^3(a+bx^3)\,dx=\frac{ax^4}{4}+\frac{bx^7}{7}$$

output
```
1/4*a*x^4+1/7*b*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^3)\,dx=\frac{ax^4}{4}+\frac{bx^7}{7}$$

input
```
Integrate[x^3*(a + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^7)/7
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^3) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int (ax^3+bx^6) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4}+\frac{bx^7}{7}$$

input
```
Int[x^3*(a + b*x^3),x]
```

output
```
(a*x^4)/4 + (b*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{4}a\,x^{4}+\frac{1}{7}b\,x^{7}$	14
default	$\frac{1}{4}a\,x^{4}+\frac{1}{7}b\,x^{7}$	14
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{7}b\,x^{7}$	14
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{7}b\,x^{7}$	14
parallelrisch	$\frac{1}{4}a\,x^{4}+\frac{1}{7}b\,x^{7}$	14
orering	$\frac{x^{4}\left(4b\,x^{3}+7a\right)}{28}$	16

input
```
int(x^3*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/7*b*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3\left(a+bx^3\right)\,dx=\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x^3+a),x, algorithm="fricas")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^3(a+bx^3)\ dx=\frac{ax^4}{4}+\frac{bx^7}{7}$$

input
```
integrate(x**3*(b*x**3+a),x)
```

output
```
a*x**4/4 + b*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx^3)\ dx=\frac{1}{7}bx^7+\frac{1}{4}ax^4$$

input
```
integrate(x^3*(b*x^3+a),x, algorithm="maxima")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx^3)\ dx=\frac{1}{7}bx^7+\frac{1}{4}ax^4$$

input
```
integrate(x^3*(b*x^3+a),x, algorithm="giac")
```

output
```
1/7*b*x^7 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3 \left(a + bx^3\right)\, dx = \frac{b\,x^7}{7} + \frac{a\,x^4}{4}$$

input
```
int(x^3*(a + b*x^3),x)
```

output
```
(a*x^4)/4 + (b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^3 \left(a + bx^3\right)\, dx = \frac{x^4(4b\,x^3 + 7a)}{28}$$

input
```
int(x^3*(b*x^3+a),x)
```

output
```
(x**4*(7*a + 4*b*x**3))/28
```

3.2 $\int x^2(a+bx^3)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x^2\left(a+bx^3\right)\,dx=\frac{\left(a+bx^3\right)^2}{6b}$$

output
```
1/6*(b*x^3+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^2\left(a+bx^3\right)\,dx=\frac{ax^3}{3}+\frac{bx^6}{6}$$

input
```
Integrate[x^2*(a + b*x^3),x]
```

output
```
(a*x^3)/3 + (b*x^6)/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^3)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int (ax^2+bx^5)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^6}{6}$$

input
```
Int[x^2*(a + b*x^3),x]
```

output
```
(a*x^3)/3 + (b*x^6)/6
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{1}{6}b\,x^{6}+\frac{1}{3}a\,x^{3}$	14
norman	$\frac{1}{6}b\,x^{6}+\frac{1}{3}a\,x^{3}$	14
parallelrisch	$\frac{1}{6}b\,x^{6}+\frac{1}{3}a\,x^{3}$	14
default	$\frac{(b\,x^{3}+a)^{2}}{6b}$	15
orering	$\frac{x^{3}(b\,x^{3}+2a)}{6}$	15
risch	$\frac{b\,x^{6}}{6}+\frac{a\,x^{3}}{3}+\frac{a^{2}}{6b}$	22

input
```
int(x^2*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*x^6+1/3*a*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^2(a+bx^3)\ dx=\frac{1}{6}\,bx^6+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^2\left(a+bx^3\right)\,dx=\frac{ax^3}{3}+\frac{bx^6}{6}$$

input
```
integrate(x**2*(b*x**3+a),x)
```

output
```
a*x**3/3 + b*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2\left(a+bx^3\right)\,dx=\frac{\left(bx^3+a\right)^2}{6\,b}$$

input
```
integrate(x^2*(b*x^3+a),x, algorithm="maxima")
```

output
```
1/6*(b*x^3 + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^2\left(a+bx^3\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x^3+a),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^2(a+bx^3)\,dx=\frac{b\,x^6}{6}+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a + b*x^3),x)
```

output
```
(a*x^3)/3 + (b*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)\,dx=\frac{x^3(b\,x^3+2a)}{6}$$

input
```
int(x^2*(b*x^3+a),x)
```

output
```
(x**3*(2*a + b*x**3))/6
```

3.3 $\int x(a+bx^3)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int x(a+bx^3)\,dx=\frac{ax^2}{2}+\frac{bx^5}{5}$$

output
```
1/2*a*x^2+1/5*b*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)\,dx=\frac{ax^2}{2}+\frac{bx^5}{5}$$

input
```
Integrate[x*(a + b*x^3),x]
```

output
```
(a*x^2)/2 + (b*x^5)/5
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)\,dx$$
$$\Big\downarrow \text{802}$$
$$\int (ax+bx^4)\,dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^2}{2}+\frac{bx^5}{5}$$

input
```
Int[x*(a + b*x^3),x]
```

output
```
(a*x^2)/2 + (b*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{2}a\,x^2+\frac{1}{5}b\,x^5$	14
default	$\frac{1}{2}a\,x^2+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{2}a\,x^2+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{2}a\,x^2+\frac{1}{5}b\,x^5$	14
parallelrisch	$\frac{1}{2}a\,x^2+\frac{1}{5}b\,x^5$	14
orering	$\frac{x^2(2b\,x^3+5a)}{10}$	16

input
```
int(x*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/5*b*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^3)\,dx=\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^3+a),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(a+bx^3)\,dx = \frac{ax^2}{2} + \frac{bx^5}{5}$$

input
```
integrate(x*(b*x**3+a),x)
```

output
```
a*x**2/2 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^3)\,dx = \frac{1}{5}\,bx^5 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^3+a),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^3)\,dx = \frac{1}{5}\,bx^5 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^3+a),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^3)\,dx=\frac{b\,x^5}{5}+\frac{a\,x^2}{2}$$

input
```
int(x*(a + b*x^3),x)
```

output
```
(a*x^2)/2 + (b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x(a+bx^3)\,dx=\frac{x^2(2b\,x^3+5a)}{10}$$

input
```
int(x*(b*x^3+a),x)
```

output
```
(x**2*(5*a + 2*b*x**3))/10
```

3.4 $\int (a+bx^3)\,dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (a+bx^3)\,dx = ax+\frac{bx^4}{4}$$

output
```
a*x+1/4*b*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+bx^3)\,dx = ax+\frac{bx^4}{4}$$

input
```
Integrate[a + b*x^3,x]
```

output
```
a*x + (b*x^4)/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^3) \, dx$$

$$\Big\downarrow \text{2009}$$

$$ax + \frac{bx^4}{4}$$

input
```
Int[a + b*x^3,x]
```

output
```
a*x + (b*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$ax+\frac{1}{4}b\,x^{4}$	11
default	$ax+\frac{1}{4}b\,x^{4}$	11
norman	$ax+\frac{1}{4}b\,x^{4}$	11
risch	$ax+\frac{1}{4}b\,x^{4}$	11
parallelrisch	$ax+\frac{1}{4}b\,x^{4}$	11
parts	$ax+\frac{1}{4}b\,x^{4}$	11
orering	$\frac{x\left(b\,x^{3}+4a\right)}{4}$	13

input
```
int(b*x^3+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/4*b*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a+bx^3\right)\,dx=\frac{1}{4}\,bx^4+ax$$

input
```
integrate(b*x^3+a,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (a + bx^3) \, dx = ax + \frac{bx^4}{4}$$

input
```
integrate(b*x**3+a,x)
```

output
```
a*x + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + bx^3) \, dx = \frac{1}{4} bx^4 + ax$$

input
```
integrate(b*x^3+a,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + bx^3) \, dx = \frac{1}{4} bx^4 + ax$$

input
```
integrate(b*x^3+a,x, algorithm="giac")
```

output
```
1/4*b*x^4 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + bx^3) \, dx = \frac{b\,x^4}{4} + a\,x$$

input
```
int(a + b*x^3,x)
```

output
```
a*x + (b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + bx^3) \, dx = \frac{x(b\,x^3 + 4a)}{4}$$

input
```
int(b*x^3+a,x)
```

output
```
(x*(4*a + b*x**3))/4
```

3.5 $\int \frac{a+bx^3}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^3}{x} \, dx = \frac{bx^3}{3} + a\log(x)$$

output
```
1/3*b*x^3+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x} \, dx = \frac{bx^3}{3} + a\log(x)$$

input
```
Integrate[(a + b*x^3)/x,x]
```

output
```
(b*x^3)/3 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x}+bx^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+\frac{bx^3}{3}$$

input
```
Int[(a + b*x^3)/x,x]
```

output
```
(b*x^3)/3 + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b\,x^{3}}{3}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^{3}}{3}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^{3}}{3}+a\ln\left(x\right)$	12
parallelrisch	$\frac{b\,x^{3}}{3}+a\ln\left(x\right)$	12

input
```
int((b*x^3+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*b*x^3+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{x}\,dx = \frac{1}{3}\,bx^3 + a\log\left(x\right)$$

input
```
integrate((b*x^3+a)/x,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^3}{x}\,dx = a\log\left(x\right) + \frac{bx^3}{3}$$

input
```
integrate((b*x**3+a)/x,x)
```

output
```
a*log(x) + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^3}{x}\,dx = \frac{1}{3}\,bx^3 + \frac{1}{3}\,a\log\left(x^3\right)$$

input
```
integrate((b*x^3+a)/x,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + 1/3*a*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{a+bx^3}{x}\,dx = \frac{1}{3}\,bx^3 + a\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)/x,x, algorithm="giac")
```

output
```
1/3*b*x^3 + a*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{x}\,dx = \frac{b\,x^3}{3} + a\,\ln\left(x\right)$$

input
```
int((a + b*x^3)/x,x)
```

output
```
(b*x^3)/3 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{x}\,dx = \log(x)\,a + \frac{b\,x^3}{3}$$

input
```
int((b*x^3+a)/x,x)
```

output
```
(3*log(x)*a + b*x**3)/3
```

3.6 $\int \frac{a+bx^3}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{a+bx^3}{x^2} dx = -\frac{a}{x} + \frac{bx^2}{2}$$

output
```
-a/x+1/2*b*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^2} dx = -\frac{a}{x} + \frac{bx^2}{2}$$

input
```
Integrate[(a + b*x^3)/x^2,x]
```

output
```
-(a/x) + (b*x^2)/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^2}+bx\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^2}{2}-\frac{a}{x}$$

input
```
Int[(a + b*x^3)/x^2,x]
```

output
```
-(a/x) + (b*x^2)/2
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a}{x}+\frac{b\,x^{2}}{2}$	14
risch	$-\frac{a}{x}+\frac{b\,x^{2}}{2}$	14
norman	$\frac{\frac{b\,x^{3}}{2}-a}{x}$	15
parallelrisch	$\frac{b\,x^{3}-2a}{2x}$	15
gosper	$-\frac{-b\,x^{3}+2a}{2x}$	16
orering	$-\frac{-b\,x^{3}+2a}{2x}$	16

input
```
int((b*x^3+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+1/2*b*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3}{x^2}\,dx=\frac{bx^3-2\,a}{2\,x}$$

input
```
integrate((b*x^3+a)/x^2,x, algorithm="fricas")
```

output
```
1/2*(b*x^3 - 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{a+bx^3}{x^2}\,dx = -\frac{a}{x}+\frac{bx^2}{2}$$

input
```
integrate((b*x**3+a)/x**2,x)
```

output
```
-a/x + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2-\frac{a}{x}$$

input
```
integrate((b*x^3+a)/x^2,x, algorithm="maxima")
```

output
```
1/2*b*x^2 - a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^2}\,dx = \frac{1}{2}\,bx^2-\frac{a}{x}$$

input
```
integrate((b*x^3+a)/x^2,x, algorithm="giac")
```

output
```
1/2*b*x^2 - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^2}\,dx=\frac{b\,x^2}{2}-\frac{a}{x}$$

input
```
int((a + b*x^3)/x^2,x)
```

output
```
(b*x^2)/2 - a/x
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3}{x^2}\,dx=\frac{b\,x^3-2a}{2x}$$

input
```
int((b*x^3+a)/x^2,x)
```

output
```
( - 2*a + b*x**3)/(2*x)
```

3.7 $\int \frac{a+bx^3}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{a+bx^3}{x^3} dx = -\frac{a}{2x^2} + bx$$

output
```
-1/2*a/x^2+b*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^3} dx = -\frac{a}{2x^2} + bx$$

input
```
Integrate[(a + b*x^3)/x^3,x]
```

output
```
-1/2*a/x^2 + b*x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^3} \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a}{x^3}+b\right)dx$$

$$\Big\downarrow \; 2009$$

$$bx-\frac{a}{2x^2}$$

input
```
Int[(a + b*x^3)/x^3,x]
```

output
```
-1/2*a/x^2 + b*x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a}{2x^2}+bx$	11
risch	$-\frac{a}{2x^2}+bx$	11
gosper	$-\frac{-2b\,x^3+a}{2x^2}$	14
norman	$\frac{b\,x^3-\frac{a}{2}}{x^2}$	14
orering	$-\frac{-2b\,x^3+a}{2x^2}$	14
parallelrisch	$\frac{2b\,x^3-a}{2x^2}$	16

input
```
int((b*x^3+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{a+bx^3}{x^3}\,dx=\frac{2\,bx^3-a}{2\,x^2}$$

input
```
integrate((b*x^3+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*x^3 - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^3}{x^3}\,dx = -\frac{a}{2x^2}+bx$$

input
```
integrate((b*x**3+a)/x**3,x)
```

output
```
-a/(2*x**2) + b*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{x^3}\,dx = bx-\frac{a}{2\,x^2}$$

input
```
integrate((b*x^3+a)/x^3,x, algorithm="maxima")
```

output
```
b*x - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{x^3}\,dx = bx-\frac{a}{2\,x^2}$$

input
```
integrate((b*x^3+a)/x^3,x, algorithm="giac")
```

output
```
b*x - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^3}{x^3}\,dx = b\,x - \frac{a}{2\,x^2}$$

input
```
int((a + b*x^3)/x^3,x)
```

output
```
b*x - a/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{a+bx^3}{x^3}\,dx = \frac{2b\,x^3-a}{2x^2}$$

input
```
int((b*x^3+a)/x^3,x)
```

output
```
( - a + 2*b*x**3)/(2*x**2)
```

3.8 $\int \frac{a+bx^3}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^3}{x^4} \, dx = -\frac{a}{3x^3} + b \log(x)$$

output
```
-1/3*a/x^3+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^4} \, dx = -\frac{a}{3x^3} + b \log(x)$$

input
```
Integrate[(a + b*x^3)/x^4,x]
```

output
```
-1/3*a/x^3 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^4} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$b\log(x)-\frac{a}{3x^3}$$

input
```
Int[(a + b*x^3)/x^4,x]
```

output
```
-1/3*a/x^3 + b*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a}{3x^3}+b\ln(x)$	12
norman	$-\frac{a}{3x^3}+b\ln(x)$	12
risch	$-\frac{a}{3x^3}+b\ln(x)$	12
parallelrisch	$\frac{3b\ln(x)x^3-a}{3x^3}$	18

input
```
int((b*x^3+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^3}{x^4}\,dx=\frac{3\,bx^3\log(x)-a}{3\,x^3}$$

input
```
integrate((b*x^3+a)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*x^3*log(x) - a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^3}{x^4}\,dx=-\frac{a}{3x^3}+b\log(x)$$

input
```
integrate((b*x**3+a)/x**4,x)
```

output
```
-a/(3*x**3) + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^3}{x^4}\,dx = \frac{1}{3}\,b\log\left(x^3\right) - \frac{a}{3\,x^3}$$

input
```
integrate((b*x^3+a)/x^4,x, algorithm="maxima")
```

output
```
1/3*b*log(x^3) - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{a+bx^3}{x^4}\,dx = b\log\left(|x|\right) - \frac{bx^3+a}{3\,x^3}$$

input
```
integrate((b*x^3+a)/x^4,x, algorithm="giac")
```

output
```
b*log(abs(x)) - 1/3*(b*x^3 + a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^3}{x^4}\,dx = b\,\ln\left(x\right) - \frac{a}{3\,x^3}$$

input
```
int((a + b*x^3)/x^4,x)
```

output
```
b*log(x) - a/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^3}{x^4} dx = \frac{3\log(x)\,b\,x^3 - a}{3x^3}$$

input
```
int((b*x^3+a)/x^4,x)
```

output
```
(3*log(x)*b*x**3 - a)/(3*x**3)
```

3.9 $\int \frac{a+bx^3}{x^5} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{a+bx^3}{x^5} \, dx = -\frac{a}{4x^4} - \frac{b}{x}$$

output
```
-1/4*a/x^4-b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^5} \, dx = -\frac{a}{4x^4} - \frac{b}{x}$$

input
```
Integrate[(a + b*x^3)/x^5,x]
```

output
```
-1/4*a/x^4 - b/x
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^5} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{4x^4}-\frac{b}{x}$$

input
```
Int[(a + b*x^3)/x^5,x]
```

output
```
-1/4*a/x^4 - b/x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{4b\,x^{3}+a}{4x^{4}}$	14
default	$-\frac{a}{4x^{4}}-\frac{b}{x}$	14
orering	$-\frac{4b\,x^{3}+a}{4x^{4}}$	14
norman	$\frac{-b\,x^{3}-\frac{a}{4}}{x^{4}}$	15
risch	$\frac{-b\,x^{3}-\frac{a}{4}}{x^{4}}$	15
parallelrisch	$\frac{-4b\,x^{3}-a}{4x^{4}}$	16

input
```
int((b*x^3+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*b*x^3+a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^5}\,dx=-\frac{4\,bx^3+a}{4\,x^4}$$

input
```
integrate((b*x^3+a)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^3}{x^5}\,dx = \frac{-a-4bx^3}{4x^4}$$

input
```
integrate((b*x**3+a)/x**5,x)
```

output
```
(-a - 4*b*x**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^5}\,dx = -\frac{4\,bx^3+a}{4\,x^4}$$

input
```
integrate((b*x^3+a)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^5}\,dx = -\frac{4\,bx^3+a}{4\,x^4}$$

input
```
integrate((b*x^3+a)/x^5,x, algorithm="giac")
```

output
```
-1/4*(4*b*x^3 + a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^3}{x^5}\,dx = -\frac{4\,b\,x^3+a}{4\,x^4}$$

input
```
int((a + b*x^3)/x^5,x)
```

output
```
-(a + 4*b*x^3)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^5}\,dx = \frac{-4b\,x^3-a}{4x^4}$$

input
```
int((b*x^3+a)/x^5,x)
```

output
```
( - a - 4*b*x**3)/(4*x**4)
```

3.10 $\int \frac{a+bx^3}{x^6} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^3}{x^6} \, dx = -\frac{a}{5x^5} - \frac{b}{2x^2}$$

output
```
-1/5*a/x^5-1/2*b/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^6} \, dx = -\frac{a}{5x^5} - \frac{b}{2x^2}$$

input
```
Integrate[(a + b*x^3)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(2*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^6} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^6}+\frac{b}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{b}{2x^2}$$

input
```
Int[(a + b*x^3)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(2*x^2)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{5x^5}-\frac{b}{2x^2}$	14
norman	$\frac{-\frac{b\,x^3}{2}-\frac{a}{5}}{x^5}$	15
risch	$\frac{-\frac{b\,x^3}{2}-\frac{a}{5}}{x^5}$	15
gosper	$-\frac{5b\,x^3+2a}{10x^5}$	16
parallelrisch	$\frac{-5b\,x^3-2a}{10x^5}$	16
orering	$-\frac{5b\,x^3+2a}{10x^5}$	16

input
```
int((b*x^3+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/2*b/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^6}\,dx=-\frac{5\,bx^3+2\,a}{10\,x^5}$$

input
```
integrate((b*x^3+a)/x^6,x, algorithm="fricas")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^6}\,dx = \frac{-2a-5bx^3}{10x^5}$$

input
```
integrate((b*x**3+a)/x**6,x)
```

output
```
(-2*a - 5*b*x**3)/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^6}\,dx = -\frac{5\,bx^3+2\,a}{10\,x^5}$$

input
```
integrate((b*x^3+a)/x^6,x, algorithm="maxima")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^6}\,dx = -\frac{5\,bx^3+2\,a}{10\,x^5}$$

input
```
integrate((b*x^3+a)/x^6,x, algorithm="giac")
```

output
```
-1/10*(5*b*x^3 + 2*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^6}\,dx = -\frac{5\,b\,x^3+2\,a}{10\,x^5}$$

input
```
int((a + b*x^3)/x^6,x)
```

output
```
-(2*a + 5*b*x^3)/(10*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^6}\,dx = \frac{-5b\,x^3-2a}{10x^5}$$

input
```
int((b*x^3+a)/x^6,x)
```

output
```
( - 2*a - 5*b*x**3)/(10*x**5)
```

3.11 $\int \frac{a+bx^3}{x^7} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^3}{x^7} \, dx = -\frac{a}{6x^6} - \frac{b}{3x^3}$$

output
```
-1/6*a/x^6-1/3*b/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^7} \, dx = -\frac{a}{6x^6} - \frac{b}{3x^3}$$

input
```
Integrate[(a + b*x^3)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(3*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^7} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x^7}+\frac{b}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{6x^6}-\frac{b}{3x^3}$$

input
```
Int[(a + b*x^3)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(3*x^3)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{2b\,x^{3}+a}{6x^{6}}$	14
default	$-\frac{a}{6x^{6}}-\frac{b}{3x^{3}}$	14
orering	$-\frac{2b\,x^{3}+a}{6x^{6}}$	14
norman	$\frac{-\frac{b\,x^{3}}{3}-\frac{a}{6}}{x^{6}}$	15
risch	$\frac{-\frac{b\,x^{3}}{3}-\frac{a}{6}}{x^{6}}$	15
parallelrisch	$\frac{-2b\,x^{3}-a}{6x^{6}}$	16

input
```
int((b*x^3+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*b*x^3+a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^3}{x^7}\,dx=-\frac{2\,bx^3+a}{6\,x^6}$$

input
```
integrate((b*x^3+a)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^3}{x^7}\,dx = \frac{-a-2bx^3}{6x^6}$$

input
```
integrate((b*x**3+a)/x**7,x)
```

output
```
(-a - 2*b*x**3)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^3}{x^7}\,dx = -\frac{2\,bx^3+a}{6\,x^6}$$

input
```
integrate((b*x^3+a)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(2*b*x^3 + a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^3}{x^7}\,dx = -\frac{2\,bx^3+a}{6\,x^6}$$

input
```
integrate((b*x^3+a)/x^7,x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3 + a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^3}{x^7}\,dx=-\frac{2\,b\,x^3+a}{6\,x^6}$$

input
```
int((a + b*x^3)/x^7,x)
```

output
```
-(a + 2*b*x^3)/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^7}\,dx=\frac{-2b\,x^3-a}{6x^6}$$

input
```
int((b*x^3+a)/x^7,x)
```

output
```
( - a - 2*b*x**3)/(6*x**6)
```

3.12 $\int \frac{a+bx^3}{x^8} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^3}{x^8} dx = -\frac{a}{7x^7} - \frac{b}{4x^4}$$

output
```
-1/7*a/x^7-1/4*b/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^3}{x^8} dx = -\frac{a}{7x^7} - \frac{b}{4x^4}$$

input
```
Integrate[(a + b*x^3)/x^8,x]
```

output
```
-1/7*a/x^7 - b/(4*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^3}{x^8} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^8}+\frac{b}{x^5}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{7x^7}-\frac{b}{4x^4}$$

input
```
Int[(a + b*x^3)/x^8,x]
```

output
```
-1/7*a/x^7 - b/(4*x^4)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{7x^{7}}-\frac{b}{4x^{4}}$	14
norman	$\frac{-\frac{b\,x^{3}}{4}-\frac{a}{7}}{x^{7}}$	15
risch	$\frac{-\frac{b\,x^{3}}{4}-\frac{a}{7}}{x^{7}}$	15
gosper	$-\frac{7b\,x^{3}+4a}{28x^{7}}$	16
parallelrisch	$\frac{-7b\,x^{3}-4a}{28x^{7}}$	16
orering	$-\frac{7b\,x^{3}+4a}{28x^{7}}$	16

input
```
int((b*x^3+a)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a/x^7-1/4*b/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^8}\,dx=-\frac{7\,bx^3+4\,a}{28\,x^7}$$

input
```
integrate((b*x^3+a)/x^8,x, algorithm="fricas")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^8}\,dx = \frac{-4a-7bx^3}{28x^7}$$

input
```
integrate((b*x**3+a)/x**8,x)
```

output
```
(-4*a - 7*b*x**3)/(28*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^8}\,dx = -\frac{7\,bx^3+4\,a}{28\,x^7}$$

input
```
integrate((b*x^3+a)/x^8,x, algorithm="maxima")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^8}\,dx = -\frac{7\,bx^3+4\,a}{28\,x^7}$$

input
```
integrate((b*x^3+a)/x^8,x, algorithm="giac")
```

output
```
-1/28*(7*b*x^3 + 4*a)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^8} \, dx = -\frac{7\,b\,x^3+4\,a}{28\,x^7}$$

input
```
int((a + b*x^3)/x^8,x)
```

output
```
-(4*a + 7*b*x^3)/(28*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^3}{x^8} \, dx = \frac{-7b\,x^3-4a}{28x^7}$$

input
```
int((b*x^3+a)/x^8,x)
```

output
```
( - 4*a - 7*b*x**3)/(28*x**7)
```

3.13 $\int x^4(a+bx^3)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^4(a+bx^3)^2 \, dx = \frac{a^2x^5}{5} + \frac{1}{4}abx^8 + \frac{b^2x^{11}}{11}$$

output
```
1/5*a^2*x^5+1/4*a*b*x^8+1/11*b^2*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^3)^2 \, dx = \frac{a^2x^5}{5} + \frac{1}{4}abx^8 + \frac{b^2x^{11}}{11}$$

input
```
Integrate[x^4*(a + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^8)/4 + (b^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^4+2abx^7+b^2x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^5}{5}+\frac{1}{4}abx^8+\frac{b^2x^{11}}{11}$$

input
```
Int[x^4*(a + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^8)/4 + (b^2*x^11)/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{5}a^2x^5 + \frac{1}{4}ab\,x^8 + \frac{1}{11}b^2x^{11}$	25
default	$\frac{1}{5}a^2x^5 + \frac{1}{4}ab\,x^8 + \frac{1}{11}b^2x^{11}$	25
norman	$\frac{1}{5}a^2x^5 + \frac{1}{4}ab\,x^8 + \frac{1}{11}b^2x^{11}$	25
risch	$\frac{1}{5}a^2x^5 + \frac{1}{4}ab\,x^8 + \frac{1}{11}b^2x^{11}$	25
parallelrisch	$\frac{1}{5}a^2x^5 + \frac{1}{4}ab\,x^8 + \frac{1}{11}b^2x^{11}$	25
orering	$\frac{x^5(20b^2x^6+55ab\,x^3+44a^2)}{220}$	27

input
```
int(x^4*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+1/4*a*b*x^8+1/11*b^2*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4(a+bx^3)^2\,dx = \frac{1}{11}\,b^2x^{11} + \frac{1}{4}\,abx^8 + \frac{1}{5}\,a^2x^5$$

input
```
integrate(x^4*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/11*b^2*x^11 + 1/4*a*b*x^8 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^3)^2 \, dx = \frac{a^2x^5}{5} + \frac{abx^8}{4} + \frac{b^2x^{11}}{11}$$

input
```
integrate(x**4*(b*x**3+a)**2,x)
```

output
```
a**2*x**5/5 + a*b*x**8/4 + b**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^3)^2 \, dx = \frac{1}{11} b^2x^{11} + \frac{1}{4} abx^8 + \frac{1}{5} a^2x^5$$

input
```
integrate(x^4*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/11*b^2*x^11 + 1/4*a*b*x^8 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^3)^2 \, dx = \frac{1}{11} b^2x^{11} + \frac{1}{4} abx^8 + \frac{1}{5} a^2x^5$$

input
```
integrate(x^4*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/11*b^2*x^11 + 1/4*a*b*x^8 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4\left(a+bx^3\right)^2\,dx=\frac{a^2\,x^5}{5}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^{11}}{11}$$

input
```
int(x^4*(a + b*x^3)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^11)/11 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4\left(a+bx^3\right)^2\,dx=\frac{x^5(20b^2x^6+55ab\,x^3+44a^2)}{220}$$

input
```
int(x^4*(b*x^3+a)^2,x)
```

output
```
(x**5*(44*a**2 + 55*a*b*x**3 + 20*b**2*x**6))/220
```

3.14 $\int x^3(a+bx^3)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^3\left(a+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{2}{7}abx^7+\frac{b^2x^{10}}{10}$$

output
```
1/4*a^2*x^4+2/7*a*b*x^7+1/10*b^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^3\right)^2\,dx=\frac{a^2x^4}{4}+\frac{2}{7}abx^7+\frac{b^2x^{10}}{10}$$

input
```
Integrate[x^3*(a + b*x^3)^2,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^7)/7 + (b^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx^3)^2 \, dx$$

$$\Big\downarrow \; 802$$

$$\int (a^2x^3 + 2abx^6 + b^2x^9) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^4}{4} + \frac{2}{7}abx^7 + \frac{b^2x^{10}}{10}$$

input
```
Int[x^3*(a + b*x^3)^2,x]
```

output
```
(a^2*x^4)/4 + (2*a*b*x^7)/7 + (b^2*x^10)/10
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{4}a^2x^4+\frac{2}{7}ab\,x^7+\frac{1}{10}b^2x^{10}$	25
default	$\frac{1}{4}a^2x^4+\frac{2}{7}ab\,x^7+\frac{1}{10}b^2x^{10}$	25
norman	$\frac{1}{4}a^2x^4+\frac{2}{7}ab\,x^7+\frac{1}{10}b^2x^{10}$	25
risch	$\frac{1}{4}a^2x^4+\frac{2}{7}ab\,x^7+\frac{1}{10}b^2x^{10}$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{2}{7}ab\,x^7+\frac{1}{10}b^2x^{10}$	25
orering	$\frac{x^4(14b^2x^6+40ab\,x^3+35a^2)}{140}$	27

input
```
int(x^3*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+2/7*a*b*x^7+1/10*b^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx^3)^2\,dx=\frac{1}{10}\,b^2x^{10}+\frac{2}{7}\,abx^7+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*x^10 + 2/7*a*b*x^7 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^3 (a+bx^3)^2\,dx = \frac{a^2x^4}{4} + \frac{2abx^7}{7} + \frac{b^2x^{10}}{10}$$

input
```
integrate(x**3*(b*x**3+a)**2,x)
```

output
```
a**2*x**4/4 + 2*a*b*x**7/7 + b**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a+bx^3)^2\,dx = \frac{1}{10}\,b^2x^{10} + \frac{2}{7}\,abx^7 + \frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*x^10 + 2/7*a*b*x^7 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a+bx^3)^2\,dx = \frac{1}{10}\,b^2x^{10} + \frac{2}{7}\,abx^7 + \frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/10*b^2*x^10 + 2/7*a*b*x^7 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a + bx^3)^2 \, dx = \frac{a^2 x^4}{4} + \frac{2 a b x^7}{7} + \frac{b^2 x^{10}}{10}$$

input
```
int(x^3*(a + b*x^3)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^10)/10 + (2*a*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^3 (a + bx^3)^2 \, dx = \frac{x^4(14b^2x^6 + 40ab\,x^3 + 35a^2)}{140}$$

input
```
int(x^3*(b*x^3+a)^2,x)
```

output
```
(x**4*(35*a**2 + 40*a*b*x**3 + 14*b**2*x**6))/140
```

3.15 $\int x^2(a+bx^3)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^2\left(a+bx^3\right)^2\,dx=\frac{\left(a+bx^3\right)^3}{9b}$$

output
```
1/9*(b*x^3+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int x^2\left(a+bx^3\right)^2\,dx=\frac{a^2x^3}{3}+\frac{1}{3}abx^6+\frac{b^2x^9}{9}$$

input
```
Integrate[x^2*(a + b*x^3)^2,x]
```

output
```
(a^2*x^3)/3 + (a*b*x^6)/3 + (b^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^3)^2 \, dx$$

$$\Big\downarrow \, 793$$

$$\frac{(a + bx^3)^3}{9b}$$

input
```
Int[x^2*(a + b*x^3)^2,x]
```

output
```
(a + b*x^3)^3/(9*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^3+a)^3}{9b}$	15
gosper	$\frac{1}{9}b^2x^9+\frac{1}{3}ab\,x^6+\frac{1}{3}a^2x^3$	25
norman	$\frac{1}{9}b^2x^9+\frac{1}{3}ab\,x^6+\frac{1}{3}a^2x^3$	25
parallelrisch	$\frac{1}{9}b^2x^9+\frac{1}{3}ab\,x^6+\frac{1}{3}a^2x^3$	25
orering	$\frac{x^3(b^2x^6+3ab\,x^3+3a^2)}{9}$	26
risch	$\frac{b^2x^9}{9}+\frac{ab\,x^6}{3}+\frac{a^2x^3}{3}+\frac{a^3}{9b}$	33

input
```
int(x^2*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*(b*x^3+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^2(a+bx^3)^2\,dx=\frac{1}{9}\,b^2x^9+\frac{1}{3}\,abx^6+\frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 1/3*a*b*x^6 + 1/3*a^2*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^2 (a + bx^3)^2 \, dx = \frac{a^2x^3}{3} + \frac{abx^6}{3} + \frac{b^2x^9}{9}$$

input
```
integrate(x**2*(b*x**3+a)**2,x)
```

output
```
a**2*x**3/3 + a*b*x**6/3 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 (a + bx^3)^2 \, dx = \frac{(bx^3 + a)^3}{9\,b}$$

input
```
integrate(x^2*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/9*(b*x^3 + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 (a + bx^3)^2 \, dx = \frac{(bx^3 + a)^3}{9\,b}$$

input
```
integrate(x^2*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/9*(b*x^3 + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^2(a+bx^3)^2\,dx=\frac{a^2\,x^3}{3}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^9}{9}$$

input
```
int(x^2*(a + b*x^3)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^9)/9 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int x^2(a+bx^3)^2\,dx=\frac{x^3(b^2x^6+3ab\,x^3+3a^2)}{9}$$

input
```
int(x^2*(b*x^3+a)^2,x)
```

output
```
(x**3*(3*a**2 + 3*a*b*x**3 + b**2*x**6))/9
```

3.16 $\int x(a+bx^3)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int x(a+bx^3)^2\,dx=\frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}$$

output
```
1/2*a^2*x^2+2/5*a*b*x^5+1/8*b^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)^2\,dx=\frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}$$

input
```
Integrate[x*(a + b*x^3)^2,x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^5)/5 + (b^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2 dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x+2abx^4+b^2x^7\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+\frac{2}{5}abx^5+\frac{b^2x^8}{8}$$

input
```
Int[x*(a + b*x^3)^2,x]
```

output
```
(a^2*x^2)/2 + (2*a*b*x^5)/5 + (b^2*x^8)/8
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{2}a^2x^2 + \frac{2}{5}ab\,x^5 + \frac{1}{8}b^2x^8$	25
default	$\frac{1}{2}a^2x^2 + \frac{2}{5}ab\,x^5 + \frac{1}{8}b^2x^8$	25
norman	$\frac{1}{2}a^2x^2 + \frac{2}{5}ab\,x^5 + \frac{1}{8}b^2x^8$	25
risch	$\frac{1}{2}a^2x^2 + \frac{2}{5}ab\,x^5 + \frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{2}a^2x^2 + \frac{2}{5}ab\,x^5 + \frac{1}{8}b^2x^8$	25
orering	$\frac{x^2(5b^2x^6+16ab\,x^3+20a^2)}{40}$	27

input
```
int(x*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+2/5*a*b*x^5+1/8*b^2*x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^3)^2\,dx = \frac{1}{8}\,b^2x^8 + \frac{2}{5}\,abx^5 + \frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 2/5*a*b*x^5 + 1/2*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(a+bx^3)^2 \, dx = \frac{a^2x^2}{2} + \frac{2abx^5}{5} + \frac{b^2x^8}{8}$$

input
```
integrate(x*(b*x**3+a)**2,x)
```

output
```
a**2*x**2/2 + 2*a*b*x**5/5 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^3)^2 \, dx = \frac{1}{8}\,b^2x^8 + \frac{2}{5}\,abx^5 + \frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 2/5*a*b*x^5 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^3)^2 \, dx = \frac{1}{8}\,b^2x^8 + \frac{2}{5}\,abx^5 + \frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 2/5*a*b*x^5 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^3)^2\,dx=\frac{a^2\,x^2}{2}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^8}{8}$$

input
```
int(x*(a + b*x^3)^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^8)/8 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(a+bx^3)^2\,dx=\frac{x^2(5b^2x^6+16ab\,x^3+20a^2)}{40}$$

input
```
int(x*(b*x^3+a)^2,x)
```

output
```
(x**2*(20*a**2 + 16*a*b*x**3 + 5*b**2*x**6))/40
```

3.17 $\int (a+bx^3)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+bx^3)^2 \, dx = a^2x + \frac{1}{2}abx^4 + \frac{b^2x^7}{7}$$

output
```
a^2*x+1/2*a*b*x^4+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^2 \, dx = a^2x + \frac{1}{2}abx^4 + \frac{b^2x^7}{7}$$

input
```
Integrate[(a + b*x^3)^2,x]
```

output
```
a^2*x + (a*b*x^4)/2 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^3)^2 \, dx$$

$$\Big\downarrow \text{747}$$

$$\int (a^2 + 2abx^3 + b^2x^6) \, dx$$

$$\Big\downarrow \text{2009}$$

$$a^2x + \frac{1}{2}abx^4 + \frac{b^2x^7}{7}$$

input
```
Int[(a + b*x^3)^2,x]
```

output
```
a^2*x + (a*b*x^4)/2 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x+\frac{1}{2}ab\,x^4+\frac{1}{7}b^2x^7$	22
default	$a^2x+\frac{1}{2}ab\,x^4+\frac{1}{7}b^2x^7$	22
norman	$a^2x+\frac{1}{2}ab\,x^4+\frac{1}{7}b^2x^7$	22
risch	$a^2x+\frac{1}{2}ab\,x^4+\frac{1}{7}b^2x^7$	22
parallelrisch	$a^2x+\frac{1}{2}ab\,x^4+\frac{1}{7}b^2x^7$	22
orering	$\frac{x(2b^2x^6+7ab\,x^3+14a^2)}{14}$	25

input
```
int((b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+1/2*a*b*x^4+1/7*b^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{2}\,abx^4+a^2x$$

input
```
integrate((b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 1/2*a*b*x^4 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \left(a+bx^3\right)^2\,dx=a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}$$

input
```
integrate((b*x**3+a)**2,x)
```

output
```
a**2*x + a*b*x**4/2 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{2}\,abx^4+a^2x$$

input
```
integrate((b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 1/2*a*b*x^4 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{2}\,abx^4+a^2x$$

input
```
integrate((b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 1/2*a*b*x^4 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^2\,dx=a^2\,x+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^7}{7}$$

input
```
int((a + b*x^3)^2,x)
```

output
```
a^2*x + (b^2*x^7)/7 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a+bx^3\right)^2\,dx=\frac{x(2b^2x^6+7ab\,x^3+14a^2)}{14}$$

input
```
int((b*x^3+a)^2,x)
```

output
```
(x*(14*a**2 + 7*a*b*x**3 + 2*b**2*x**6))/14
```

3.18 $\int \frac{(a+bx^3)^2}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(a+bx^3)^2}{x} \, dx = \frac{2}{3}abx^3 + \frac{b^2x^6}{6} + a^2\log(x)$$

output
```
2/3*a*b*x^3+1/6*b^2*x^6+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x} \, dx = \frac{2}{3}abx^3 + \frac{b^2x^6}{6} + a^2\log(x)$$

input
```
Integrate[(a + b*x^3)^2/x,x]
```

output
```
(2*a*b*x^3)/3 + (b^2*x^6)/6 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2}{x^3}dx^3$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}\int \left(b^2x^3+2ab+\frac{a^2}{x^3}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(a^2\log\left(x^3\right)+2abx^3+\frac{b^2x^6}{2}\right)$$

input
```
Int[(a + b*x^3)^2/x,x]
```

output
```
(2*a*b*x^3 + (b^2*x^6)/2 + a^2*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2ab\,x^3}{3}+\frac{b^2x^6}{6}+a^2\ln\left(x\right)$	23
norman	$\frac{2ab\,x^3}{3}+\frac{b^2x^6}{6}+a^2\ln\left(x\right)$	23
parallelrisch	$\frac{2ab\,x^3}{3}+\frac{b^2x^6}{6}+a^2\ln\left(x\right)$	23
risch	$\frac{b^2x^6}{6}+\frac{2ab\,x^3}{3}+\frac{2a^2}{3}+a^2\ln\left(x\right)$	28

input
```
int((b*x^3+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*a*b*x^3+1/6*b^2*x^6+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\frac{(a+bx^3)^2}{x}\,dx=\frac{1}{6}\,b^2x^6+\frac{2}{3}\,abx^3+a^2\log\left(x\right)$$

input
```
integrate((b*x^3+a)^2/x,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 2/3*a*b*x^3 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2}{x}\,dx = a^2\log{(x)} + \frac{2abx^3}{3} + \frac{b^2x^6}{6}$$

input
```
integrate((b*x**3+a)**2/x,x)
```

output
```
a**2*log(x) + 2*a*b*x**3/3 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{3}\,abx^3 + \frac{1}{3}\,a^2\log\left(x^3\right)$$

input
```
integrate((b*x^3+a)^2/x,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + 2/3*a*b*x^3 + 1/3*a^2*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^2}{x}\,dx = \frac{1}{6}\,b^2x^6 + \frac{2}{3}\,abx^3 + a^2\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)^2/x,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + 2/3*a*b*x^3 + a^2*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x}\,dx = a^2\,\ln(x) + \frac{b^2\,x^6}{6} + \frac{2\,a\,b\,x^3}{3}$$

input
```
int((a + b*x^3)^2/x,x)
```

output
```
a^2*log(x) + (b^2*x^6)/6 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x}\,dx = \log(x)\,a^2 + \frac{2ab\,x^3}{3} + \frac{b^2x^6}{6}$$

input
```
int((b*x^3+a)^2/x,x)
```

output
```
(6*log(x)*a**2 + 4*a*b*x**3 + b**2*x**6)/6
```

3.19 $\int \frac{(a+bx^3)^2}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = -\frac{a^2}{x} + abx^2 + \frac{b^2x^5}{5}$$

output
```
-a^2/x+a*b*x^2+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = -\frac{a^2}{x} + abx^2 + \frac{b^2x^5}{5}$$

input
```
Integrate[(a + b*x^3)^2/x^2,x]
```

output
```
-(a^2/x) + a*b*x^2 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{x^2}+2abx+b^2x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+abx^2+\frac{b^2x^5}{5}$$

input
```
Int[(a + b*x^3)^2/x^2,x]
```

output
```
-(a^2/x) + a*b*x^2 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+ab\,x^2+\frac{b^2x^5}{5}$	24
risch	$-\frac{a^2}{x}+ab\,x^2+\frac{b^2x^5}{5}$	24
norman	$\frac{\frac{1}{5}b^2x^6+ab\,x^3-a^2}{x}$	25
parallelrisch	$\frac{b^2x^6+5ab\,x^3-5a^2}{5x}$	26
gosper	$-\frac{-b^2x^6-5ab\,x^3+5a^2}{5x}$	27
orering	$-\frac{-b^2x^6-5ab\,x^3+5a^2}{5x}$	27

input
```
int((b*x^3+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+a*b*x^2+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^2}\,dx=\frac{b^2x^6+5\,abx^3-5\,a^2}{5\,x}$$

input
```
integrate((b*x^3+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/5*(b^2*x^6 + 5*a*b*x^3 - 5*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = -\frac{a^2}{x} + abx^2 + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**3+a)**2/x**2,x)
```

output
```
-a**2/x + a*b*x**2 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + abx^2 - \frac{a^2}{x}$$

input
```
integrate((b*x^3+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + a*b*x^2 - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = \frac{1}{5}\,b^2x^5 + abx^2 - \frac{a^2}{x}$$

input
```
integrate((b*x^3+a)^2/x^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + a*b*x^2 - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = \frac{-a^2+a\,b\,x^3+\frac{b^2\,x^6}{5}}{x}$$

input
```
int((a + b*x^3)^2/x^2,x)
```

output
```
((b^2*x^6)/5 - a^2 + a*b*x^3)/x
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^2}\,dx = \frac{b^2x^6+5ab\,x^3-5a^2}{5x}$$

input
```
int((b*x^3+a)^2/x^2,x)
```

output
```
( - 5*a**2 + 5*a*b*x**3 + b**2*x**6)/(5*x)
```

3.20 $\int \frac{(a+bx^3)^2}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(a+bx^3)^2}{x^3}\,dx = -\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$$

output
```
-1/2*a^2/x^2+2*a*b*x+1/4*b^2*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^3}\,dx = -\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$$

input
```
Integrate[(a + b*x^3)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + 2*a*b*x + (b^2*x^4)/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^2}{x^3}+2ab+b^2x^3\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$$

input
```
Int[(a + b*x^3)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + 2*a*b*x + (b^2*x^4)/4
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$	23
risch	$-\frac{a^2}{2x^2}+2abx+\frac{b^2x^4}{4}$	23
norman	$\frac{\frac{1}{4}b^2x^6+2ab\,x^3-\frac{1}{2}a^2}{x^2}$	26
parallelrisch	$\frac{b^2x^6+8ab\,x^3-2a^2}{4x^2}$	26
gosper	$-\frac{-b^2x^6-8ab\,x^3+2a^2}{4x^2}$	27
orering	$-\frac{-b^2x^6-8ab\,x^3+2a^2}{4x^2}$	27

input
```
int((b*x^3+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+2*a*b*x+1/4*b^2*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^3}\,dx = \frac{b^2x^6+8\,abx^3-2\,a^2}{4\,x^2}$$

input
```
integrate((b*x^3+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/4*(b^2*x^6 + 8*a*b*x^3 - 2*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x^3}\,dx = -\frac{a^2}{2x^2} + 2abx + \frac{b^2x^4}{4}$$

input
```
integrate((b*x**3+a)**2/x**3,x)
```

output
```
-a**2/(2*x**2) + 2*a*b*x + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x^3}\,dx = \frac{1}{4}\,b^2x^4 + 2\,abx - \frac{a^2}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + 2*a*b*x - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x^3}\,dx = \frac{1}{4}\,b^2x^4 + 2\,abx - \frac{a^2}{2\,x^2}$$

input
```
integrate((b*x^3+a)^2/x^3,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + 2*a*b*x - 1/2*a^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x^3}\,dx=\frac{b^2\,x^4}{4}-\frac{a^2}{2\,x^2}+2\,a\,b\,x$$

input
```
int((a + b*x^3)^2/x^3,x)
```

output
```
(b^2*x^4)/4 - a^2/(2*x^2) + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^3}\,dx=\frac{b^2x^6+8ab\,x^3-2a^2}{4x^2}$$

input
```
int((b*x^3+a)^2/x^3,x)
```

output
```
( - 2*a**2 + 8*a*b*x**3 + b**2*x**6)/(4*x**2)
```

3.21 $\int \frac{(a+bx^3)^2}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{(a+bx^3)^2}{x^4} dx = -\frac{a^2}{3x^3} + \frac{b^2x^3}{3} + 2ab\log(x)$$

output
```
-1/3*a^2/x^3+1/3*b^2*x^3+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^4} dx = -\frac{a^2}{3x^3} + \frac{b^2x^3}{3} + 2ab\log(x)$$

input
```
Integrate[(a + b*x^3)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 + (b^2*x^3)/3 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2}{x^6}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(\frac{a^2}{x^6}+\frac{2ba}{x^3}+b^2\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^2}{x^3}+2ab\log\left(x^3\right)+b^2x^3\right)$$

input
```
Int[(a + b*x^3)^2/x^4,x]
```

output
```
(-(a^2/x^3) + b^2*x^3 + 2*a*b*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{3x^3}+\frac{b^2x^3}{3}+2ab\ln\left(x\right)$	24
risch	$-\frac{a^2}{3x^3}+\frac{b^2x^3}{3}+2ab\ln\left(x\right)$	24
norman	$\frac{-\frac{a^2}{3}+\frac{b^2x^6}{3}}{x^3}+2ab\ln\left(x\right)$	26
parallelrisch	$\frac{b^2x^6+6ab\ln(x)x^3-a^2}{3x^3}$	28

input
```
int((b*x^3+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+1/3*b^2*x^3+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int\frac{\left(a+bx^3\right)^2}{x^4}\,dx=\frac{b^2x^6+6\,abx^3\log\left(x\right)-a^2}{3\,x^3}$$

input
```
integrate((b*x^3+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^6 + 6*a*b*x^3*log(x) - a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2}{x^4}\,dx = -\frac{a^2}{3x^3} + 2ab\log(x) + \frac{b^2x^3}{3}$$

input
```
integrate((b*x**3+a)**2/x**4,x)
```

output
```
-a**2/(3*x**3) + 2*a*b*log(x) + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^4}\,dx = \frac{1}{3}\,b^2x^3 + \frac{2}{3}\,ab\log\left(x^3\right) - \frac{a^2}{3\,x^3}$$

input
```
integrate((b*x^3+a)^2/x^4,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 2/3*a*b*log(x^3) - 1/3*a^2/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^2}{x^4}\,dx = \frac{1}{3}\,b^2x^3 + 2\,ab\log(|x|) - \frac{2\,abx^3 + a^2}{3\,x^3}$$

input
```
integrate((b*x^3+a)^2/x^4,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 2*a*b*log(abs(x)) - 1/3*(2*a*b*x^3 + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^2}{x^4}\,dx=\frac{b^2\,x^3}{3}-\frac{a^2}{3\,x^3}+2\,a\,b\,\ln\left(x\right)$$

input
```
int((a + b*x^3)^2/x^4,x)
```

output
```
(b^2*x^3)/3 - a^2/(3*x^3) + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^4}\,dx=\frac{6\log(x)\,ab\,x^3-a^2+b^2x^6}{3x^3}$$

input
```
int((b*x^3+a)^2/x^4,x)
```

output
```
(6*log(x)*a*b*x**3 - a**2 + b**2*x**6)/(3*x**3)
```

3.22 $\int \frac{(a+bx^3)^2}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(a+bx^3)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{2ab}{x} + \frac{b^2x^2}{2}$$

output
```
-1/4*a^2/x^4-2*a*b/x+1/2*b^2*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{2ab}{x} + \frac{b^2x^2}{2}$$

input
```
Integrate[(a + b*x^3)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/x + (b^2*x^2)/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^5} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^2}+b^2x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{4x^4}-\frac{2ab}{x}+\frac{b^2x^2}{2}$$

input
```
Int[(a + b*x^3)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 - (2*a*b)/x + (b^2*x^2)/2
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{-2b^2x^6+8ab\,x^3+a^2}{4x^4}$	25
default	$-\frac{a^2}{4x^4}-\frac{2ab}{x}+\frac{b^2x^2}{2}$	25
orering	$-\frac{-2b^2x^6+8ab\,x^3+a^2}{4x^4}$	25
norman	$\frac{\frac{1}{2}b^2x^6-2ab\,x^3-\frac{1}{4}a^2}{x^4}$	26
risch	$\frac{b^2x^2}{2}+\frac{-2ab\,x^3-\frac{1}{4}a^2}{x^4}$	27
parallelrisch	$\frac{2b^2x^6-8ab\,x^3-a^2}{4x^4}$	27

input
```
int((b*x^3+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b^2*x^6+8*a*b*x^3+a^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^5}\,dx = \frac{2\,b^2x^6-8\,abx^3-a^2}{4\,x^4}$$

input
```
integrate((b*x^3+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^6 - 8*a*b*x^3 - a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^5}\,dx = \frac{b^2x^2}{2} + \frac{-a^2-8abx^3}{4x^4}$$

input
```
integrate((b*x**3+a)**2/x**5,x)
```

output
```
b**2*x**2/2 + (-a**2 - 8*a*b*x**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2}{x^5}\,dx = \frac{1}{2}\,b^2x^2 - \frac{8\,abx^3+a^2}{4\,x^4}$$

input
```
integrate((b*x^3+a)^2/x^5,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 - 1/4*(8*a*b*x^3 + a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2}{x^5}\,dx = \frac{1}{2}\,b^2x^2 - \frac{8\,abx^3+a^2}{4\,x^4}$$

input
```
integrate((b*x^3+a)^2/x^5,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 - 1/4*(8*a*b*x^3 + a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^5}\,dx = \frac{b^2\,x^2}{2} - \frac{\frac{a^2}{4} + 2\,b\,a\,x^3}{x^4}$$

input
```
int((a + b*x^3)^2/x^5,x)
```

output
```
(b^2*x^2)/2 - (a^2/4 + 2*a*b*x^3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^5}\,dx = \frac{2b^2x^6 - 8ab\,x^3 - a^2}{4x^4}$$

input
```
int((b*x^3+a)^2/x^5,x)
```

output
```
( - a**2 - 8*a*b*x**3 + 2*b**2*x**6)/(4*x**4)
```

3.23 $\int \frac{(a+bx^3)^2}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{(a+bx^3)^2}{x^6} dx = -\frac{a^2}{5x^5} - \frac{ab}{x^2} + b^2x$$

output
```
-1/5*a^2/x^5-a*b/x^2+b^2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^6} dx = -\frac{a^2}{5x^5} - \frac{ab}{x^2} + b^2x$$

input
```
Integrate[(a + b*x^3)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/x^2 + b^2*x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^6} \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^3}+b^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2}{5x^5}-\frac{ab}{x^2}+b^2x$$

input
```
Int[(a + b*x^3)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (a*b)/x^2 + b^2*x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{5x^5}-\frac{ab}{x^2}+b^2x$	22
risch	$b^2x+\frac{-ab\,x^3-\frac{1}{5}a^2}{x^5}$	24
gosper	$-\frac{-5b^2x^6+5ab\,x^3+a^2}{5x^5}$	25
norman	$\frac{b^2x^6-ab\,x^3-\frac{1}{5}a^2}{x^5}$	25
orering	$-\frac{-5b^2x^6+5ab\,x^3+a^2}{5x^5}$	25
parallelrisch	$\frac{5b^2x^6-5ab\,x^3-a^2}{5x^5}$	27

input
```
int((b*x^3+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2/x^5-a*b/x^2+b^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^2}{x^6}\,dx=\frac{5\,b^2x^6-5\,abx^3-a^2}{5\,x^5}$$

input
```
integrate((b*x^3+a)^2/x^6,x, algorithm="fricas")
```

output
```
1/5*(5*b^2*x^6 - 5*a*b*x^3 - a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^6}\,dx = b^2x + \frac{-a^2-5abx^3}{5x^5}$$

input
```
integrate((b*x**3+a)**2/x**6,x)
```

output
```
b**2*x + (-a**2 - 5*a*b*x**3)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^6}\,dx = b^2x - \frac{5\,abx^3+a^2}{5\,x^5}$$

input
```
integrate((b*x^3+a)^2/x^6,x, algorithm="maxima")
```

output
```
b^2*x - 1/5*(5*a*b*x^3 + a^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^6}\,dx = b^2x - \frac{5\,abx^3+a^2}{5\,x^5}$$

input
```
integrate((b*x^3+a)^2/x^6,x, algorithm="giac")
```

output
```
b^2*x - 1/5*(5*a*b*x^3 + a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^6}\,dx = b^2\,x - \frac{\frac{a^2}{5} + b\,a\,x^3}{x^5}$$

input
```
int((a + b*x^3)^2/x^6,x)
```

output
```
b^2*x - (a^2/5 + a*b*x^3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^2}{x^6}\,dx = \frac{5b^2x^6 - 5ab\,x^3 - a^2}{5x^5}$$

input
```
int((b*x^3+a)^2/x^6,x)
```

output
```
( - a**2 - 5*a*b*x**3 + 5*b**2*x**6)/(5*x**5)
```

3.24 $\int \frac{(a+bx^3)^2}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = -\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)$$

output
```
-1/6*a^2/x^6-2/3*a*b/x^3+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = -\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\log(x)$$

input
```
Integrate[(a + b*x^3)^2/x^7,x]
```

output
```
-1/6*a^2/x^6 - (2*a*b)/(3*x^3) + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^7}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2}{x^9}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^2}{x^9}+\frac{2ba}{x^6}+\frac{b^2}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2}{2x^6}-\frac{2ab}{x^3}+b^2\log\left(x^3\right)\right)$$

input
```
Int[(a + b*x^3)^2/x^7,x]
```

output
```
(-1/2*a^2/x^6 - (2*a*b)/x^3 + b^2*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^2}{6x^6}-\frac{2ab}{3x^3}+b^2\ln(x)$	23
norman	$\frac{-\frac{1}{6}a^2-\frac{2}{3}ab\,x^3}{x^6}+b^2\ln(x)$	25
risch	$\frac{-\frac{1}{6}a^2-\frac{2}{3}ab\,x^3}{x^6}+b^2\ln(x)$	25
parallelrisch	$\frac{6b^2\ln(x)x^6-4ab\,x^3-a^2}{6x^6}$	29

input
```
int((b*x^3+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^2/x^6-2/3*a*b/x^3+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int\frac{(a+bx^3)^2}{x^7}\,dx=\frac{6\,b^2x^6\log(x)-4\,abx^3-a^2}{6\,x^6}$$

input
```
integrate((b*x^3+a)^2/x^7,x, algorithm="fricas")
```

output
```
1/6*(6*b^2*x^6*log(x) - 4*a*b*x^3 - a^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = b^2\log(x) + \frac{-a^2-4abx^3}{6x^6}$$

input
```
integrate((b*x**3+a)**2/x**7,x)
```

output
```
b**2*log(x) + (-a**2 - 4*a*b*x**3)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = \frac{1}{3}\,b^2\log\left(x^3\right) - \frac{4\,abx^3+a^2}{6\,x^6}$$

input
```
integrate((b*x^3+a)^2/x^7,x, algorithm="maxima")
```

output
```
1/3*b^2*log(x^3) - 1/6*(4*a*b*x^3 + a^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = b^2\log(|x|) - \frac{3\,b^2x^6+4\,abx^3+a^2}{6\,x^6}$$

input
```
integrate((b*x^3+a)^2/x^7,x, algorithm="giac")
```

output
```
b^2*log(abs(x)) - 1/6*(3*b^2*x^6 + 4*a*b*x^3 + a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = b^2\,\ln(x) - \frac{\frac{a^2}{6} + \frac{2\,b\,a\,x^3}{3}}{x^6}$$

input
```
int((a + b*x^3)^2/x^7,x)
```

output
```
b^2*log(x) - (a^2/6 + (2*a*b*x^3)/3)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^2}{x^7}\,dx = \frac{6\log(x)\,b^2x^6 - a^2 - 4ab\,x^3}{6x^6}$$

input
```
int((b*x^3+a)^2/x^7,x)
```

output
```
(6*log(x)*b**2*x**6 - a**2 - 4*a*b*x**3)/(6*x**6)
```

3.25 $\int \frac{(a+bx^3)^2}{x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = -\frac{a^2}{7x^7} - \frac{ab}{2x^4} - \frac{b^2}{x}$$

output
```
-1/7*a^2/x^7-1/2*a*b/x^4-b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = -\frac{a^2}{7x^7} - \frac{ab}{2x^4} - \frac{b^2}{x}$$

input
```
Integrate[(a + b*x^3)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (a*b)/(2*x^4) - b^2/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^8}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^5}+\frac{b^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{7x^7}-\frac{ab}{2x^4}-\frac{b^2}{x}$$

input
```
Int[(a + b*x^3)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (a*b)/(2*x^4) - b^2/x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{7x^7}-\frac{ab}{2x^4}-\frac{b^2}{x}$	25
norman	$\frac{-b^2x^6-\frac{1}{2}ab\,x^3-\frac{1}{7}a^2}{x^7}$	26
risch	$\frac{-b^2x^6-\frac{1}{2}ab\,x^3-\frac{1}{7}a^2}{x^7}$	26
gosper	$-\frac{14b^2x^6+7ab\,x^3+2a^2}{14x^7}$	27
parallelrisch	$\frac{-14b^2x^6-7ab\,x^3-2a^2}{14x^7}$	27
orering	$-\frac{14b^2x^6+7ab\,x^3+2a^2}{14x^7}$	27

input
```
int((b*x^3+a)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2/x^7-1/2*a*b/x^4-b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = -\frac{14\,b^2x^6+7\,abx^3+2\,a^2}{14\,x^7}$$

input
```
integrate((b*x^3+a)^2/x^8,x, algorithm="fricas")
```

output
```
-1/14*(14*b^2*x^6 + 7*a*b*x^3 + 2*a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = \frac{-2a^2-7abx^3-14b^2x^6}{14x^7}$$

input
```
integrate((b*x**3+a)**2/x**8,x)
```

output
```
(-2*a**2 - 7*a*b*x**3 - 14*b**2*x**6)/(14*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = -\frac{14\,b^2x^6+7\,abx^3+2\,a^2}{14\,x^7}$$

input
```
integrate((b*x^3+a)^2/x^8,x, algorithm="maxima")
```

output
```
-1/14*(14*b^2*x^6 + 7*a*b*x^3 + 2*a^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = -\frac{14\,b^2x^6+7\,abx^3+2\,a^2}{14\,x^7}$$

input
```
integrate((b*x^3+a)^2/x^8,x, algorithm="giac")
```

output
```
-1/14*(14*b^2*x^6 + 7*a*b*x^3 + 2*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = -\frac{\frac{a^2}{7}+\frac{a\,b\,x^3}{2}+b^2\,x^6}{x^7}$$

input
```
int((a + b*x^3)^2/x^8,x)
```

output
```
-(a^2/7 + b^2*x^6 + (a*b*x^3)/2)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^2}{x^8}\,dx = \frac{-14b^2x^6-7ab\,x^3-2a^2}{14x^7}$$

input
```
int((b*x^3+a)^2/x^8,x)
```

output
```
( - 2*a**2 - 7*a*b*x**3 - 14*b**2*x**6)/(14*x**7)
```

3.26 $\int \frac{(a+bx^3)^2}{x^9} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^3)^2}{x^9} dx = -\frac{a^2}{8x^8} - \frac{2ab}{5x^5} - \frac{b^2}{2x^2}$$

output
```
-1/8*a^2/x^8-2/5*a*b/x^5-1/2*b^2/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^9} dx = -\frac{a^2}{8x^8} - \frac{2ab}{5x^5} - \frac{b^2}{2x^2}$$

input
```
Integrate[(a + b*x^3)^2/x^9,x]
```

output
```
-1/8*a^2/x^8 - (2*a*b)/(5*x^5) - b^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^9} \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a^2}{x^9}+\frac{2ab}{x^6}+\frac{b^2}{x^3}\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2}{8x^8}-\frac{2ab}{5x^5}-\frac{b^2}{2x^2}$$

input
```
Int[(a + b*x^3)^2/x^9,x]
```

output
```
-1/8*a^2/x^8 - (2*a*b)/(5*x^5) - b^2/(2*x^2)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{8x^8}-\frac{2ab}{5x^5}-\frac{b^2}{2x^2}$	25
norman	$\frac{-\frac{1}{2}b^2x^6-\frac{2}{5}ab\,x^3-\frac{1}{8}a^2}{x^8}$	26
risch	$\frac{-\frac{1}{2}b^2x^6-\frac{2}{5}ab\,x^3-\frac{1}{8}a^2}{x^8}$	26
gosper	$-\frac{20b^2x^6+16ab\,x^3+5a^2}{40x^8}$	27
parallelrisch	$\frac{-20b^2x^6-16ab\,x^3-5a^2}{40x^8}$	27
orering	$-\frac{20b^2x^6+16ab\,x^3+5a^2}{40x^8}$	27

input
```
int((b*x^3+a)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2/x^8-2/5*a*b/x^5-1/2*b^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^9}\,dx = -\frac{20\,b^2x^6+16\,abx^3+5\,a^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2/x^9,x, algorithm="fricas")
```

output
```
-1/40*(20*b^2*x^6 + 16*a*b*x^3 + 5*a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{x^9}\,dx = \frac{-5a^2-16abx^3-20b^2x^6}{40x^8}$$

input
```
integrate((b*x**3+a)**2/x**9,x)
```

output
```
(-5*a**2 - 16*a*b*x**3 - 20*b**2*x**6)/(40*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^9}\,dx = -\frac{20\,b^2x^6+16\,abx^3+5\,a^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2/x^9,x, algorithm="maxima")
```

output
```
-1/40*(20*b^2*x^6 + 16*a*b*x^3 + 5*a^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^9}\,dx = -\frac{20\,b^2x^6+16\,abx^3+5\,a^2}{40\,x^8}$$

input
```
integrate((b*x^3+a)^2/x^9,x, algorithm="giac")
```

output
```
-1/40*(20*b^2*x^6 + 16*a*b*x^3 + 5*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^9}\,dx = -\frac{\frac{a^2}{8}+\frac{2\,a\,b\,x^3}{5}+\frac{b^2\,x^6}{2}}{x^8}$$

input
```
int((a + b*x^3)^2/x^9,x)
```

output
```
-(a^2/8 + (b^2*x^6)/2 + (2*a*b*x^3)/5)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^9}\,dx = \frac{-20b^2x^6-16ab\,x^3-5a^2}{40x^8}$$

input
```
int((b*x^3+a)^2/x^9,x)
```

output
```
( - 5*a**2 - 16*a*b*x**3 - 20*b**2*x**6)/(40*x**8)
```

3.27 $\int \frac{(a+bx^3)^2}{x^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^3)^2}{x^{10}} \, dx = -\frac{(a+bx^3)^3}{9ax^9}$$

output
```
-1/9*(b*x^3+a)^3/a/x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^3)^2}{x^{10}} \, dx = -\frac{a^2}{9x^9} - \frac{ab}{3x^6} - \frac{b^2}{3x^3}$$

input
```
Integrate[(a + b*x^3)^2/x^10,x]
```

output
```
-1/9*a^2/x^9 - (a*b)/(3*x^6) - b^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^{10}} \, dx$$
$$\Big\downarrow \; 796$$
$$-\frac{(a+bx^3)^3}{9ax^9}$$

input
```
Int[(a + b*x^3)^2/x^10,x]
```

output
```
-1/9*(a + b*x^3)^3/(a*x^9)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
gosper	$-\frac{3b^2x^6+3ab\,x^3+a^2}{9x^9}$	25
default	$-\frac{b^2}{3x^3}-\frac{ab}{3x^6}-\frac{a^2}{9x^9}$	25
orering	$-\frac{3b^2x^6+3ab\,x^3+a^2}{9x^9}$	25
norman	$\frac{-\frac{1}{3}b^2x^6-\frac{1}{3}ab\,x^3-\frac{1}{9}a^2}{x^9}$	26
risch	$\frac{-\frac{1}{3}b^2x^6-\frac{1}{3}ab\,x^3-\frac{1}{9}a^2}{x^9}$	26
parallelrisch	$\frac{-3b^2x^6-3ab\,x^3-a^2}{9x^9}$	27

input
```
int((b*x^3+a)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(3*b^2*x^6+3*a*b*x^3+a^2)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^3)^2}{x^{10}}\,dx = -\frac{3\,b^2x^6+3\,abx^3+a^2}{9\,x^9}$$

input
```
integrate((b*x^3+a)^2/x^10,x, algorithm="fricas")
```

output
```
-1/9*(3*b^2*x^6 + 3*a*b*x^3 + a^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^2}{x^{10}}\,dx = \frac{-a^2-3abx^3-3b^2x^6}{9x^9}$$

input
```
integrate((b*x**3+a)**2/x**10,x)
```

output
```
(-a**2 - 3*a*b*x**3 - 3*b**2*x**6)/(9*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^3)^2}{x^{10}}\,dx = -\frac{3\,b^2x^6+3\,abx^3+a^2}{9\,x^9}$$

input
```
integrate((b*x^3+a)^2/x^10,x, algorithm="maxima")
```

output
```
-1/9*(3*b^2*x^6 + 3*a*b*x^3 + a^2)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^3)^2}{x^{10}}\,dx = -\frac{3\,b^2x^6+3\,abx^3+a^2}{9\,x^9}$$

input
```
integrate((b*x^3+a)^2/x^10,x, algorithm="giac")
```

output
```
-1/9*(3*b^2*x^6 + 3*a*b*x^3 + a^2)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^2}{x^{10}}\,dx = -\frac{\frac{a^2}{9}+\frac{a\,b\,x^3}{3}+\frac{b^2\,x^6}{3}}{x^9}$$

input
```
int((a + b*x^3)^2/x^10,x)
```

output
```
-(a^2/9 + (b^2*x^6)/3 + (a*b*x^3)/3)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^3)^2}{x^{10}}\,dx = \frac{-3b^2x^6 - 3ab\,x^3 - a^2}{9x^9}$$

input
```
int((b*x^3+a)^2/x^10,x)
```

output
```
( - a**2 - 3*a*b*x**3 - 3*b**2*x**6)/(9*x**9)
```

3.28 $\int \frac{(a+bx^3)^2}{x^{11}} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^3)^2}{x^{11}} dx = -\frac{a^2}{10x^{10}} - \frac{2ab}{7x^7} - \frac{b^2}{4x^4}$$

output
```
-1/10*a^2/x^10-2/7*a*b/x^7-1/4*b^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^{11}} dx = -\frac{a^2}{10x^{10}} - \frac{2ab}{7x^7} - \frac{b^2}{4x^4}$$

input
```
Integrate[(a + b*x^3)^2/x^11,x]
```

output
```
-1/10*a^2/x^10 - (2*a*b)/(7*x^7) - b^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx$$

$$\Big\downarrow \ 802$$

$$\int \left(\frac{a^2}{x^{11}}+\frac{2ab}{x^8}+\frac{b^2}{x^5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^2}{10x^{10}}-\frac{2ab}{7x^7}-\frac{b^2}{4x^4}$$

input
```
Int[(a + b*x^3)^2/x^11,x]
```

output
```
-1/10*a^2/x^10 - (2*a*b)/(7*x^7) - b^2/(4*x^4)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{10x^{10}}-\frac{2ab}{7x^7}-\frac{b^2}{4x^4}$	25
norman	$\frac{-\frac{1}{4}b^2x^6-\frac{2}{7}ab\,x^3-\frac{1}{10}a^2}{x^{10}}$	26
risch	$\frac{-\frac{1}{4}b^2x^6-\frac{2}{7}ab\,x^3-\frac{1}{10}a^2}{x^{10}}$	26
gosper	$-\frac{35b^2x^6+40ab\,x^3+14a^2}{140x^{10}}$	27
parallelrisch	$\frac{-35b^2x^6-40ab\,x^3-14a^2}{140x^{10}}$	27
orering	$-\frac{35b^2x^6+40ab\,x^3+14a^2}{140x^{10}}$	27

input
```
int((b*x^3+a)^2/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^2/x^10-2/7*a*b/x^7-1/4*b^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx = -\frac{35\,b^2x^6+40\,abx^3+14\,a^2}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^2/x^11,x, algorithm="fricas")
```

output
```
-1/140*(35*b^2*x^6 + 40*a*b*x^3 + 14*a^2)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx = \frac{-14a^2-40abx^3-35b^2x^6}{140x^{10}}$$

input
```
integrate((b*x**3+a)**2/x**11,x)
```

output
```
(-14*a**2 - 40*a*b*x**3 - 35*b**2*x**6)/(140*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx = -\frac{35\,b^2x^6+40\,abx^3+14\,a^2}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^2/x^11,x, algorithm="maxima")
```

output
```
-1/140*(35*b^2*x^6 + 40*a*b*x^3 + 14*a^2)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx = -\frac{35\,b^2x^6+40\,abx^3+14\,a^2}{140\,x^{10}}$$

input
```
integrate((b*x^3+a)^2/x^11,x, algorithm="giac")
```

output
```
-1/140*(35*b^2*x^6 + 40*a*b*x^3 + 14*a^2)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx = -\frac{\frac{a^2}{10}+\frac{2\,a\,b\,x^3}{7}+\frac{b^2\,x^6}{4}}{x^{10}}$$

input
```
int((a + b*x^3)^2/x^11,x)
```

output
```
-(a^2/10 + (b^2*x^6)/4 + (2*a*b*x^3)/7)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{11}}\,dx = \frac{-35b^2x^6-40ab\,x^3-14a^2}{140x^{10}}$$

input
```
int((b*x^3+a)^2/x^11,x)
```

output
```
( - 14*a**2 - 40*a*b*x**3 - 35*b**2*x**6)/(140*x**10)
```

3.29 $\int \frac{(a+bx^3)^2}{x^{12}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = -\frac{a^2}{11x^{11}} - \frac{ab}{4x^8} - \frac{b^2}{5x^5}$$

output
```
-1/11*a^2/x^11-1/4*a*b/x^8-1/5*b^2/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = -\frac{a^2}{11x^{11}} - \frac{ab}{4x^8} - \frac{b^2}{5x^5}$$

input
```
Integrate[(a + b*x^3)^2/x^12,x]
```

output
```
-1/11*a^2/x^11 - (a*b)/(4*x^8) - b^2/(5*x^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^2}{x^{12}}+\frac{2ab}{x^9}+\frac{b^2}{x^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{11x^{11}}-\frac{ab}{4x^8}-\frac{b^2}{5x^5}$$

input
```
Int[(a + b*x^3)^2/x^12,x]
```

output
```
-1/11*a^2/x^11 - (a*b)/(4*x^8) - b^2/(5*x^5)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{11 x^{11}}-\frac{a b}{4 x^{8}}-\frac{b^{2}}{5 x^{5}}$	25
norman	$\frac{-\frac{1}{5} b^{2} x^{6}-\frac{1}{4} a b\, x^{3}-\frac{1}{11} a^{2}}{x^{11}}$	26
risch	$\frac{-\frac{1}{5} b^{2} x^{6}-\frac{1}{4} a b\, x^{3}-\frac{1}{11} a^{2}}{x^{11}}$	26
gosper	$-\frac{44 b^{2} x^{6}+55 a b\, x^{3}+20 a^{2}}{220 x^{11}}$	27
parallelrisch	$\frac{-44 b^{2} x^{6}-55 a b\, x^{3}-20 a^{2}}{220 x^{11}}$	27
orering	$-\frac{44 b^{2} x^{6}+55 a b\, x^{3}+20 a^{2}}{220 x^{11}}$	27

input
```
int((b*x^3+a)^2/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^2/x^11-1/4*a*b/x^8-1/5*b^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = -\frac{44\,b^2x^6+55\,abx^3+20\,a^2}{220\,x^{11}}$$

input
```
integrate((b*x^3+a)^2/x^12,x, algorithm="fricas")
```

output
```
-1/220*(44*b^2*x^6 + 55*a*b*x^3 + 20*a^2)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = \frac{-20a^2-55abx^3-44b^2x^6}{220x^{11}}$$

input
```
integrate((b*x**3+a)**2/x**12,x)
```

output
```
(-20*a**2 - 55*a*b*x**3 - 44*b**2*x**6)/(220*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = -\frac{44\,b^2x^6+55\,abx^3+20\,a^2}{220\,x^{11}}$$

input
```
integrate((b*x^3+a)^2/x^12,x, algorithm="maxima")
```

output
```
-1/220*(44*b^2*x^6 + 55*a*b*x^3 + 20*a^2)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = -\frac{44\,b^2x^6+55\,abx^3+20\,a^2}{220\,x^{11}}$$

input
```
integrate((b*x^3+a)^2/x^12,x, algorithm="giac")
```

output
```
-1/220*(44*b^2*x^6 + 55*a*b*x^3 + 20*a^2)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = -\frac{\frac{a^2}{11}+\frac{a\,b\,x^3}{4}+\frac{b^2\,x^6}{5}}{x^{11}}$$

input
```
int((a + b*x^3)^2/x^12,x)
```

output
```
-(a^2/11 + (b^2*x^6)/5 + (a*b*x^3)/4)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{12}}\,dx = \frac{-44b^2x^6-55ab\,x^3-20a^2}{220x^{11}}$$

input
```
int((b*x^3+a)^2/x^12,x)
```

output
```
( - 20*a**2 - 55*a*b*x**3 - 44*b**2*x**6)/(220*x**11)
```

3.30 $\int \frac{(a+bx^3)^2}{x^{13}} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^3)^2}{x^{13}} dx = -\frac{a^2}{12x^{12}} - \frac{2ab}{9x^9} - \frac{b^2}{6x^6}$$

output
```
-1/12*a^2/x^12-2/9*a*b/x^9-1/6*b^2/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2}{x^{13}} dx = -\frac{a^2}{12x^{12}} - \frac{2ab}{9x^9} - \frac{b^2}{6x^6}$$

input
```
Integrate[(a + b*x^3)^2/x^13,x]
```

output
```
-1/12*a^2/x^12 - (2*a*b)/(9*x^9) - b^2/(6*x^6)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2}{x^{13}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^2}{x^{15}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{a^2}{x^{15}}+\frac{2ba}{x^{12}}+\frac{b^2}{x^9}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2}{4x^{12}}-\frac{2ab}{3x^9}-\frac{b^2}{2x^6}\right)$$

input
```
Int[(a + b*x^3)^2/x^13,x]
```

output
```
(-1/4*a^2/x^12 - (2*a*b)/(3*x^9) - b^2/(2*x^6))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{12x^{12}}-\frac{2ab}{9x^9}-\frac{b^2}{6x^6}$	25
norman	$\frac{-\frac{1}{6}b^2x^6-\frac{2}{9}ab\,x^3-\frac{1}{12}a^2}{x^{12}}$	26
risch	$\frac{-\frac{1}{6}b^2x^6-\frac{2}{9}ab\,x^3-\frac{1}{12}a^2}{x^{12}}$	26
gosper	$-\frac{6b^2x^6+8ab\,x^3+3a^2}{36x^{12}}$	27
parallelrisch	$\frac{-6b^2x^6-8ab\,x^3-3a^2}{36x^{12}}$	27
orering	$-\frac{6b^2x^6+8ab\,x^3+3a^2}{36x^{12}}$	27

input
```
int((b*x^3+a)^2/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^2/x^12-2/9*a*b/x^9-1/6*b^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{13}}\,dx=-\frac{6\,b^2x^6+8\,abx^3+3\,a^2}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^2/x^13,x, algorithm="fricas")
```

output
```
-1/36*(6*b^2*x^6 + 8*a*b*x^3 + 3*a^2)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2}{x^{13}}\,dx = \frac{-3a^2-8abx^3-6b^2x^6}{36x^{12}}$$

input
```
integrate((b*x**3+a)**2/x**13,x)
```

output
```
(-3*a**2 - 8*a*b*x**3 - 6*b**2*x**6)/(36*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{13}}\,dx = -\frac{6\,b^2x^6+8\,abx^3+3\,a^2}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^2/x^13,x, algorithm="maxima")
```

output
```
-1/36*(6*b^2*x^6 + 8*a*b*x^3 + 3*a^2)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{13}}\,dx = -\frac{6\,b^2x^6+8\,abx^3+3\,a^2}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^2/x^13,x, algorithm="giac")
```

output
```
-1/36*(6*b^2*x^6 + 8*a*b*x^3 + 3*a^2)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{13}}\,dx = -\frac{\frac{a^2}{12}+\frac{2\,a\,b\,x^3}{9}+\frac{b^2\,x^6}{6}}{x^{12}}$$

input
```
int((a + b*x^3)^2/x^13,x)
```

output
```
-(a^2/12 + (b^2*x^6)/6 + (2*a*b*x^3)/9)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^2}{x^{13}}\,dx = \frac{-6b^2x^6-8ab\,x^3-3a^2}{36x^{12}}$$

input
```
int((b*x^3+a)^2/x^13,x)
```

output
```
( - 3*a**2 - 8*a*b*x**3 - 6*b**2*x**6)/(36*x**12)
```

3.31 $\int x^{14}(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^{14}(a+bx^3)^3\,dx=\frac{a^3x^{15}}{15}+\frac{1}{6}a^2bx^{18}+\frac{1}{7}ab^2x^{21}+\frac{b^3x^{24}}{24}$$

output
```
1/15*a^3*x^15+1/6*a^2*b*x^18+1/7*a*b^2*x^21+1/24*b^3*x^24
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^{14}(a+bx^3)^3\,dx=\frac{a^3x^{15}}{15}+\frac{1}{6}a^2bx^{18}+\frac{1}{7}ab^2x^{21}+\frac{b^3x^{24}}{24}$$

input
```
Integrate[x^14*(a + b*x^3)^3,x]
```

output
```
(a^3*x^15)/15 + (a^2*b*x^18)/6 + (a*b^2*x^21)/7 + (b^3*x^24)/24
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14} \left(a+bx^3\right)^3 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^{12} \left(bx^3+a\right)^3 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(b^3x^{21}+3ab^2x^{18}+3a^2bx^{15}+a^3x^{12}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^3x^{15}}{5}+\frac{1}{2}a^2bx^{18}+\frac{3}{7}ab^2x^{21}+\frac{b^3x^{24}}{8}\right)$$

input
```
Int[x^14*(a + b*x^3)^3,x]
```

output
```
((a^3*x^15)/5 + (a^2*b*x^18)/2 + (3*a*b^2*x^21)/7 + (b^3*x^24)/8)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{15}a^3x^{15}+\frac{1}{6}a^2b\,x^{18}+\frac{1}{7}a\,b^2x^{21}+\frac{1}{24}b^3x^{24}$	36
default	$\frac{1}{15}a^3x^{15}+\frac{1}{6}a^2b\,x^{18}+\frac{1}{7}a\,b^2x^{21}+\frac{1}{24}b^3x^{24}$	36
norman	$\frac{1}{15}a^3x^{15}+\frac{1}{6}a^2b\,x^{18}+\frac{1}{7}a\,b^2x^{21}+\frac{1}{24}b^3x^{24}$	36
risch	$\frac{1}{15}a^3x^{15}+\frac{1}{6}a^2b\,x^{18}+\frac{1}{7}a\,b^2x^{21}+\frac{1}{24}b^3x^{24}$	36
parallelrisch	$\frac{1}{15}a^3x^{15}+\frac{1}{6}a^2b\,x^{18}+\frac{1}{7}a\,b^2x^{21}+\frac{1}{24}b^3x^{24}$	36
orering	$\frac{x^{15}(35b^3x^9+120a\,b^2x^6+140a^2b\,x^3+56a^3)}{840}$	38

input
```
int(x^14*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*x^15+1/6*a^2*b*x^18+1/7*a*b^2*x^21+1/24*b^3*x^24
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{14}(a+bx^3)^3\,dx=\frac{1}{24}\,b^3x^{24}+\frac{1}{7}\,ab^2x^{21}+\frac{1}{6}\,a^2bx^{18}+\frac{1}{15}\,a^3x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/24*b^3*x^24 + 1/7*a*b^2*x^21 + 1/6*a^2*b*x^18 + 1/15*a^3*x^15
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int x^{14}\left(a+bx^3\right)^3\,dx=\frac{a^3x^{15}}{15}+\frac{a^2bx^{18}}{6}+\frac{ab^2x^{21}}{7}+\frac{b^3x^{24}}{24}$$

input
```
integrate(x**14*(b*x**3+a)**3,x)
```

output
```
a**3*x**15/15 + a**2*b*x**18/6 + a*b**2*x**21/7 + b**3*x**24/24
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{14}\left(a+bx^3\right)^3\,dx=\frac{1}{24}\,b^3x^{24}+\frac{1}{7}\,ab^2x^{21}+\frac{1}{6}\,a^2bx^{18}+\frac{1}{15}\,a^3x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/24*b^3*x^24 + 1/7*a*b^2*x^21 + 1/6*a^2*b*x^18 + 1/15*a^3*x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{14}\left(a+bx^3\right)^3\,dx=\frac{1}{24}\,b^3x^{24}+\frac{1}{7}\,ab^2x^{21}+\frac{1}{6}\,a^2bx^{18}+\frac{1}{15}\,a^3x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/24*b^3*x^24 + 1/7*a*b^2*x^21 + 1/6*a^2*b*x^18 + 1/15*a^3*x^15
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{14}(a+bx^3)^3\,dx=\frac{a^3\,x^{15}}{15}+\frac{a^2\,b\,x^{18}}{6}+\frac{a\,b^2\,x^{21}}{7}+\frac{b^3\,x^{24}}{24}$$

input
```
int(x^14*(a + b*x^3)^3,x)
```

output
```
(a^3*x^15)/15 + (b^3*x^24)/24 + (a^2*b*x^18)/6 + (a*b^2*x^21)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^{14}(a+bx^3)^3\,dx=\frac{x^{15}(35b^3x^9+120a\,b^2x^6+140a^2b\,x^3+56a^3)}{840}$$

input
```
int(x^14*(b*x^3+a)^3,x)
```

output
```
(x**15*(56*a**3 + 140*a**2*b*x**3 + 120*a*b**2*x**6 + 35*b**3*x**9))/840
```

3.32 $\int x^{11}(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^{11}\left(a+bx^3\right)^3\,dx=\frac{a^3x^{12}}{12}+\frac{1}{5}a^2bx^{15}+\frac{1}{6}ab^2x^{18}+\frac{b^3x^{21}}{21}$$

output
```
1/12*a^3*x^12+1/5*a^2*b*x^15+1/6*a*b^2*x^18+1/21*b^3*x^21
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^{11}\left(a+bx^3\right)^3\,dx=\frac{a^3x^{12}}{12}+\frac{1}{5}a^2bx^{15}+\frac{1}{6}ab^2x^{18}+\frac{b^3x^{21}}{21}$$

input
```
Integrate[x^11*(a + b*x^3)^3,x]
```

output
```
(a^3*x^12)/12 + (a^2*b*x^15)/5 + (a*b^2*x^18)/6 + (b^3*x^21)/21
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \left(a+b x^3\right)^3 \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3} \int x^9 \left(b x^3+a\right)^3 dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} \int \left(b^3 x^{18}+3 a b^2 x^{15}+3 a^2 b x^{12}+a^3 x^9\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{a^3 x^{12}}{4}+\frac{3}{5} a^2 b x^{15}+\frac{1}{2} a b^2 x^{18}+\frac{b^3 x^{21}}{7}\right)$$

input
```
Int[x^11*(a + b*x^3)^3,x]
```

output
```
((a^3*x^12)/4 + (3*a^2*b*x^15)/5 + (a*b^2*x^18)/2 + (b^3*x^21)/7)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{12}a^3x^{12}+\frac{1}{5}a^2b\,x^{15}+\frac{1}{6}a\,b^2x^{18}+\frac{1}{21}b^3x^{21}$	36
default	$\frac{1}{12}a^3x^{12}+\frac{1}{5}a^2b\,x^{15}+\frac{1}{6}a\,b^2x^{18}+\frac{1}{21}b^3x^{21}$	36
norman	$\frac{1}{12}a^3x^{12}+\frac{1}{5}a^2b\,x^{15}+\frac{1}{6}a\,b^2x^{18}+\frac{1}{21}b^3x^{21}$	36
risch	$\frac{1}{12}a^3x^{12}+\frac{1}{5}a^2b\,x^{15}+\frac{1}{6}a\,b^2x^{18}+\frac{1}{21}b^3x^{21}$	36
parallelrisch	$\frac{1}{12}a^3x^{12}+\frac{1}{5}a^2b\,x^{15}+\frac{1}{6}a\,b^2x^{18}+\frac{1}{21}b^3x^{21}$	36
orering	$\frac{x^{12}(20b^3x^9+70a\,b^2x^6+84a^2b\,x^3+35a^3)}{420}$	38

input
```
int(x^11*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^3*x^12+1/5*a^2*b*x^15+1/6*a*b^2*x^18+1/21*b^3*x^21
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{11}\left(a+bx^3\right)^3\,dx=\frac{1}{21}\,b^3x^{21}+\frac{1}{6}\,ab^2x^{18}+\frac{1}{5}\,a^2bx^{15}+\frac{1}{12}\,a^3x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/21*b^3*x^21 + 1/6*a*b^2*x^18 + 1/5*a^2*b*x^15 + 1/12*a^3*x^12
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int x^{11}(a+bx^3)^3\,dx = \frac{a^3x^{12}}{12} + \frac{a^2bx^{15}}{5} + \frac{ab^2x^{18}}{6} + \frac{b^3x^{21}}{21}$$

input
```
integrate(x**11*(b*x**3+a)**3,x)
```

output
```
a**3*x**12/12 + a**2*b*x**15/5 + a*b**2*x**18/6 + b**3*x**21/21
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{11}(a+bx^3)^3\,dx = \frac{1}{21}\,b^3x^{21} + \frac{1}{6}\,ab^2x^{18} + \frac{1}{5}\,a^2bx^{15} + \frac{1}{12}\,a^3x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/21*b^3*x^21 + 1/6*a*b^2*x^18 + 1/5*a^2*b*x^15 + 1/12*a^3*x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{11}(a+bx^3)^3\,dx = \frac{1}{21}\,b^3x^{21} + \frac{1}{6}\,ab^2x^{18} + \frac{1}{5}\,a^2bx^{15} + \frac{1}{12}\,a^3x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/21*b^3*x^21 + 1/6*a*b^2*x^18 + 1/5*a^2*b*x^15 + 1/12*a^3*x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^{11}\left(a+bx^3\right)^3\,dx=\frac{a^3\,x^{12}}{12}+\frac{a^2\,b\,x^{15}}{5}+\frac{a\,b^2\,x^{18}}{6}+\frac{b^3\,x^{21}}{21}$$

input
```
int(x^11*(a + b*x^3)^3,x)
```

output
```
(a^3*x^12)/12 + (b^3*x^21)/21 + (a^2*b*x^15)/5 + (a*b^2*x^18)/6
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^{11}\left(a+bx^3\right)^3\,dx=\frac{x^{12}(20b^3x^9+70a\,b^2x^6+84a^2b\,x^3+35a^3)}{420}$$

input
```
int(x^11*(b*x^3+a)^3,x)
```

output
```
(x**12*(35*a**3 + 84*a**2*b*x**3 + 70*a*b**2*x**6 + 20*b**3*x**9))/420
```

3.33 $\int x^8(a+bx^3)^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^8\left(a+bx^3\right)^3 \, dx = \frac{a^3x^9}{9}+\frac{1}{4}a^2bx^{12}+\frac{1}{5}ab^2x^{15}+\frac{b^3x^{18}}{18}$$

output
```
1/9*a^3*x^9+1/4*a^2*b*x^12+1/5*a*b^2*x^15+1/18*b^3*x^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^8\left(a+bx^3\right)^3 \, dx = \frac{a^3x^9}{9}+\frac{1}{4}a^2bx^{12}+\frac{1}{5}ab^2x^{15}+\frac{b^3x^{18}}{18}$$

input
```
Integrate[x^8*(a + b*x^3)^3,x]
```

output
```
(a^3*x^9)/9 + (a^2*b*x^12)/4 + (a*b^2*x^15)/5 + (b^3*x^18)/18
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^3 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^3 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(b^3x^{15}+3ab^2x^{12}+3a^2bx^9+a^3x^6\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^3x^9}{3}+\frac{3}{4}a^2bx^{12}+\frac{3}{5}ab^2x^{15}+\frac{b^3x^{18}}{6}\right)$$

input
```
Int[x^8*(a + b*x^3)^3,x]
```

output
```
((a^3*x^9)/3 + (3*a^2*b*x^12)/4 + (3*a*b^2*x^15)/5 + (b^3*x^18)/6)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{9}a^3x^9+\frac{1}{4}a^2b\,x^{12}+\frac{1}{5}a\,b^2x^{15}+\frac{1}{18}b^3x^{18}$	36
default	$\frac{1}{9}a^3x^9+\frac{1}{4}a^2b\,x^{12}+\frac{1}{5}a\,b^2x^{15}+\frac{1}{18}b^3x^{18}$	36
norman	$\frac{1}{9}a^3x^9+\frac{1}{4}a^2b\,x^{12}+\frac{1}{5}a\,b^2x^{15}+\frac{1}{18}b^3x^{18}$	36
risch	$\frac{1}{9}a^3x^9+\frac{1}{4}a^2b\,x^{12}+\frac{1}{5}a\,b^2x^{15}+\frac{1}{18}b^3x^{18}$	36
parallelrisch	$\frac{1}{9}a^3x^9+\frac{1}{4}a^2b\,x^{12}+\frac{1}{5}a\,b^2x^{15}+\frac{1}{18}b^3x^{18}$	36
orering	$\frac{x^9\left(10b^3x^9+36a\,b^2x^6+45a^2b\,x^3+20a^3\right)}{180}$	38

input
```
int(x^8*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^3*x^9+1/4*a^2*b*x^12+1/5*a*b^2*x^15+1/18*b^3*x^18
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^8\left(a+bx^3\right)^3\,dx=\frac{1}{18}\,b^3x^{18}+\frac{1}{5}\,ab^2x^{15}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{9}\,a^3x^9$$

input
```
integrate(x^8*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int x^8(a+bx^3)^3\,dx = \frac{a^3x^9}{9} + \frac{a^2bx^{12}}{4} + \frac{ab^2x^{15}}{5} + \frac{b^3x^{18}}{18}$$

input
```
integrate(x**8*(b*x**3+a)**3,x)
```

output
```
a**3*x**9/9 + a**2*b*x**12/4 + a*b**2*x**15/5 + b**3*x**18/18
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^8(a+bx^3)^3\,dx = \frac{1}{18}\,b^3x^{18} + \frac{1}{5}\,ab^2x^{15} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{9}\,a^3x^9$$

input
```
integrate(x^8*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^8(a+bx^3)^3\,dx = \frac{1}{18}\,b^3x^{18} + \frac{1}{5}\,ab^2x^{15} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{9}\,a^3x^9$$

input
```
integrate(x^8*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^8 \left(a+bx^3\right)^3 \, dx = \frac{a^3 x^9}{9} + \frac{a^2 b x^{12}}{4} + \frac{a b^2 x^{15}}{5} + \frac{b^3 x^{18}}{18}$$

input
```
int(x^8*(a + b*x^3)^3,x)
```

output
```
(a^3*x^9)/9 + (b^3*x^18)/18 + (a^2*b*x^12)/4 + (a*b^2*x^15)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^8 \left(a+bx^3\right)^3 \, dx = \frac{x^9 (10b^3 x^9 + 36a b^2 x^6 + 45a^2 b x^3 + 20a^3)}{180}$$

input
```
int(x^8*(b*x^3+a)^3,x)
```

output
```
(x**9*(20*a**3 + 45*a**2*b*x**3 + 36*a*b**2*x**6 + 10*b**3*x**9))/180
```

3.34 $\int x^5(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^5\left(a+bx^3\right)^3\,dx=-\frac{a(a+bx^3)^4}{12b^2}+\frac{\left(a+bx^3\right)^5}{15b^2}$$

output
```
-1/12*a*(b*x^3+a)^4/b^2+1/15*(b*x^3+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int x^5\left(a+bx^3\right)^3\,dx=\frac{a^3x^6}{6}+\frac{1}{3}a^2bx^9+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{15}}{15}$$

input
```
Integrate[x^5*(a + b*x^3)^3,x]
```

output
```
(a^3*x^6)/6 + (a^2*b*x^9)/3 + (a*b^2*x^12)/4 + (b^3*x^15)/15
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^3 dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^3 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^4}{b}-\frac{a\left(bx^3+a\right)^3}{b}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3\right)^5}{5b^2}-\frac{a\left(a+bx^3\right)^4}{4b^2}\right)$$

input
```
Int[x^5*(a + b*x^3)^3,x]
```

output
```
(-1/4*(a*(a + b*x^3)^4)/b^2 + (a + b*x^3)^5/(5*b^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{1}{6}a^3x^6+\frac{1}{3}a^2b\,x^9+\frac{1}{4}a\,b^2x^{12}+\frac{1}{15}b^3x^{15}$	36
default	$\frac{1}{6}a^3x^6+\frac{1}{3}a^2b\,x^9+\frac{1}{4}a\,b^2x^{12}+\frac{1}{15}b^3x^{15}$	36
norman	$\frac{1}{6}a^3x^6+\frac{1}{3}a^2b\,x^9+\frac{1}{4}a\,b^2x^{12}+\frac{1}{15}b^3x^{15}$	36
risch	$\frac{1}{6}a^3x^6+\frac{1}{3}a^2b\,x^9+\frac{1}{4}a\,b^2x^{12}+\frac{1}{15}b^3x^{15}$	36
parallelrisch	$\frac{1}{6}a^3x^6+\frac{1}{3}a^2b\,x^9+\frac{1}{4}a\,b^2x^{12}+\frac{1}{15}b^3x^{15}$	36
orering	$\frac{x^6\left(4b^3x^9+15a\,b^2x^6+20a^2b\,x^3+10a^3\right)}{60}$	38

input
```
int(x^5*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^3*x^6+1/3*a^2*b*x^9+1/4*a*b^2*x^12+1/15*b^3*x^15
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^5\left(a+bx^3\right)^3\,dx=\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x^5(a+bx^3)^3\,dx=\frac{a^3x^6}{6}+\frac{a^2bx^9}{3}+\frac{ab^2x^{12}}{4}+\frac{b^3x^{15}}{15}$$

input
```
integrate(x**5*(b*x**3+a)**3,x)
```

output
```
a**3*x**6/6 + a**2*b*x**9/3 + a*b**2*x**12/4 + b**3*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^5(a+bx^3)^3\,dx=\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^5(a+bx^3)^3\,dx=\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^5\left(a+bx^3\right)^3\,dx=\frac{a^3\,x^6}{6}+\frac{a^2\,b\,x^9}{3}+\frac{a\,b^2\,x^{12}}{4}+\frac{b^3\,x^{15}}{15}$$

input
```
int(x^5*(a + b*x^3)^3,x)
```

output
```
(a^3*x^6)/6 + (b^3*x^15)/15 + (a^2*b*x^9)/3 + (a*b^2*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int x^5\left(a+bx^3\right)^3\,dx=\frac{x^6(4b^3x^9+15a\,b^2x^6+20a^2b\,x^3+10a^3)}{60}$$

input
```
int(x^5*(b*x^3+a)^3,x)
```

output
```
(x**6*(10*a**3 + 20*a**2*b*x**3 + 15*a*b**2*x**6 + 4*b**3*x**9))/60
```

3.35 $\int x^2(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^2\left(a+bx^3\right)^3\,dx=\frac{\left(a+bx^3\right)^4}{12b}$$

output
```
1/12*(b*x^3+a)^4/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int x^2\left(a+bx^3\right)^3\,dx=\frac{a^3x^3}{3}+\frac{1}{2}a^2bx^6+\frac{1}{3}ab^2x^9+\frac{b^3x^{12}}{12}$$

input
```
Integrate[x^2*(a + b*x^3)^3,x]
```

output
```
(a^3*x^3)/3 + (a^2*b*x^6)/2 + (a*b^2*x^9)/3 + (b^3*x^12)/12
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^3)^3 \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{(a + bx^3)^4}{12b}$$

input
```
Int[x^2*(a + b*x^3)^3,x]
```

output
```
(a + b*x^3)^4/(12*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^3+a)^4}{12b}$	15
gosper	$\frac{1}{12}b^3x^{12}+\frac{1}{3}a\,b^2x^9+\frac{1}{2}a^2b\,x^6+\frac{1}{3}a^3x^3$	36
norman	$\frac{1}{12}b^3x^{12}+\frac{1}{3}a\,b^2x^9+\frac{1}{2}a^2b\,x^6+\frac{1}{3}a^3x^3$	36
parallelrisch	$\frac{1}{12}b^3x^{12}+\frac{1}{3}a\,b^2x^9+\frac{1}{2}a^2b\,x^6+\frac{1}{3}a^3x^3$	36
orering	$\frac{x^3(b^3x^9+4a\,b^2x^6+6a^2b\,x^3+4a^3)}{12}$	37
risch	$\frac{b^3x^{12}}{12}+\frac{a\,b^2x^9}{3}+\frac{a^2b\,x^6}{2}+\frac{a^3x^3}{3}+\frac{a^4}{12b}$	44

input
```
int(x^2*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(b*x^3+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int x^2(a+bx^3)^3\,dx=\frac{1}{12}\,b^3x^{12}+\frac{1}{3}\,ab^2x^9+\frac{1}{2}\,a^2bx^6+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int x^2 (a + bx^3)^3 \, dx = \frac{a^3x^3}{3} + \frac{a^2bx^6}{2} + \frac{ab^2x^9}{3} + \frac{b^3x^{12}}{12}$$

input
```
integrate(x**2*(b*x**3+a)**3,x)
```

output
```
a**3*x**3/3 + a**2*b*x**6/2 + a*b**2*x**9/3 + b**3*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 (a + bx^3)^3 \, dx = \frac{(bx^3 + a)^4}{12\,b}$$

input
```
integrate(x^2*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/12*(b*x^3 + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 (a + bx^3)^3 \, dx = \frac{(bx^3 + a)^4}{12\,b}$$

input
```
integrate(x^2*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/12*(b*x^3 + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int x^2 (a + bx^3)^3 \, dx = \frac{a^3 x^3}{3} + \frac{a^2 b x^6}{2} + \frac{a b^2 x^9}{3} + \frac{b^3 x^{12}}{12}$$

input
```
int(x^2*(a + b*x^3)^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^12)/12 + (a^2*b*x^6)/2 + (a*b^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int x^2 (a + bx^3)^3 \, dx = \frac{x^3(b^3x^9 + 4a b^2x^6 + 6a^2b x^3 + 4a^3)}{12}$$

input
```
int(x^2*(b*x^3+a)^3,x)
```

output
```
(x**3*(4*a**3 + 6*a**2*b*x**3 + 4*a*b**2*x**6 + b**3*x**9))/12
```

3.36 $\int \frac{(a+bx^3)^3}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{(a+bx^3)^3}{x} \, dx = a^2bx^3 + \frac{1}{2}ab^2x^6 + \frac{b^3x^9}{9} + a^3\log(x)$$

output
```
a^2*b*x^3+1/2*a*b^2*x^6+1/9*b^3*x^9+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x} \, dx = a^2bx^3 + \frac{1}{2}ab^2x^6 + \frac{b^3x^9}{9} + a^3\log(x)$$

input
```
Integrate[(a + b*x^3)^3/x,x]
```

output
```
a^2*b*x^3 + (a*b^2*x^6)/2 + (b^3*x^9)/9 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^3}{x^3}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^3x^6+3ab^2x^3+3a^2b+\frac{a^3}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(a^3\log\left(x^3\right)+3a^2bx^3+\frac{3}{2}ab^2x^6+\frac{b^3x^9}{3}\right)$$

input
```
Int[(a + b*x^3)^3/x,x]
```

output
```
(3*a^2*b*x^3 + (3*a*b^2*x^6)/2 + (b^3*x^9)/3 + a^3*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
default	$a^{2}b\,x^{3}+\frac{a\,b^{2}x^{6}}{2}+\frac{b^{3}x^{9}}{9}+a^{3}\ln\left(x\right)$	33
norman	$a^{2}b\,x^{3}+\frac{a\,b^{2}x^{6}}{2}+\frac{b^{3}x^{9}}{9}+a^{3}\ln\left(x\right)$	33
risch	$a^{2}b\,x^{3}+\frac{a\,b^{2}x^{6}}{2}+\frac{b^{3}x^{9}}{9}+a^{3}\ln\left(x\right)$	33
parallelrisch	$a^{2}b\,x^{3}+\frac{a\,b^{2}x^{6}}{2}+\frac{b^{3}x^{9}}{9}+a^{3}\ln\left(x\right)$	33

input
```
int((b*x^3+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*b*x^3+1/2*a*b^2*x^6+1/9*b^3*x^9+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^3}{x}\,dx=\frac{1}{9}\,b^3x^9+\frac{1}{2}\,ab^2x^6+a^2bx^3+a^3\log\left(x\right)$$

input
```
integrate((b*x^3+a)^3/x,x, algorithm="fricas")
```

output
```
1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^3}{x}\,dx = a^3\log(x) + a^2bx^3 + \frac{ab^2x^6}{2} + \frac{b^3x^9}{9}$$

input
```
integrate((b*x**3+a)**3/x,x)
```

output
```
a**3*log(x) + a**2*b*x**3 + a*b**2*x**6/2 + b**3*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^3}{x}\,dx = \frac{1}{9}\,b^3x^9 + \frac{1}{2}\,ab^2x^6 + a^2bx^3 + \frac{1}{3}\,a^3\log(x^3)$$

input
```
integrate((b*x^3+a)^3/x,x, algorithm="maxima")
```

output
```
1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + 1/3*a^3*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x}\,dx = \frac{1}{9}\,b^3x^9 + \frac{1}{2}\,ab^2x^6 + a^2bx^3 + a^3\log(|x|)$$

input
```
integrate((b*x^3+a)^3/x,x, algorithm="giac")
```

output
```
1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^3}{x}\,dx = a^3\,\ln(x) + \frac{b^3\,x^9}{9} + a^2\,b\,x^3 + \frac{a\,b^2\,x^6}{2}$$

input
```
int((a + b*x^3)^3/x,x)
```

output
```
a^3*log(x) + (b^3*x^9)/9 + a^2*b*x^3 + (a*b^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^3}{x}\,dx = \log(x)\,a^3 + a^2b\,x^3 + \frac{a\,b^2x^6}{2} + \frac{b^3x^9}{9}$$

input
```
int((b*x^3+a)^3/x,x)
```

output
```
(18*log(x)*a**3 + 18*a**2*b*x**3 + 9*a*b**2*x**6 + 2*b**3*x**9)/18
```

3.37 $\int \frac{(a+bx^3)^3}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{(a+bx^3)^3}{x^4} dx = -\frac{a^3}{3x^3} + ab^2x^3 + \frac{b^3x^6}{6} + 3a^2b\log(x)$$

output
```
-1/3*a^3/x^3+a*b^2*x^3+1/6*b^3*x^6+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^4} dx = -\frac{a^3}{3x^3} + ab^2x^3 + \frac{b^3x^6}{6} + 3a^2b\log(x)$$

input
```
Integrate[(a + b*x^3)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 + a*b^2*x^3 + (b^3*x^6)/6 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^4}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^3}{x^6}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^3}+3b^2a+b^3x^3\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^3}{x^3}+3a^2b\log\left(x^3\right)+3ab^2x^3+\frac{b^3x^6}{2}\right)$$

input
```
Int[(a + b*x^3)^3/x^4,x]
```

output
```
(-(a^3/x^3) + 3*a*b^2*x^3 + (b^3*x^6)/2 + 3*a^2*b*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^3}{3x^3}+a\,b^2x^3+\frac{b^3x^6}{6}+3a^2b\ln\left(x\right)$	34
norman	$\frac{a\,b^2x^6-\frac{1}{3}a^3+\frac{1}{6}b^3x^9}{x^3}+3a^2b\ln\left(x\right)$	36
parallelrisch	$\frac{b^3x^9+6a\,b^2x^6+18a^2b\ln(x)x^3-2a^3}{6x^3}$	39
risch	$\frac{b^3x^6}{6}+a\,b^2x^3+\frac{3a^2b}{2}-\frac{a^3}{3x^3}+3a^2b\ln\left(x\right)$	40

input
```
int((b*x^3+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3/x^3+a*b^2*x^3+1/6*b^3*x^6+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int\frac{\left(a+bx^3\right)^3}{x^4}\,dx=\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log\left(x\right)-2\,a^3}{6\,x^3}$$

input
```
integrate((b*x^3+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^4}\,dx = -\frac{a^3}{3x^3} + 3a^2b\log(x) + ab^2x^3 + \frac{b^3x^6}{6}$$

input
```
integrate((b*x**3+a)**3/x**4,x)
```

output
```
-a**3/(3*x**3) + 3*a**2*b*log(x) + a*b**2*x**3 + b**3*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^4}\,dx = \frac{1}{6}\,b^3x^6 + ab^2x^3 + a^2b\log\left(x^3\right) - \frac{a^3}{3\,x^3}$$

input
```
integrate((b*x^3+a)^3/x^4,x, algorithm="maxima")
```

output
```
1/6*b^3*x^6 + a*b^2*x^3 + a^2*b*log(x^3) - 1/3*a^3/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^3)^3}{x^4}\,dx = \frac{1}{6}\,b^3x^6 + ab^2x^3 + 3\,a^2b\log(|x|) - \frac{3\,a^2bx^3 + a^3}{3\,x^3}$$

input
```
integrate((b*x^3+a)^3/x^4,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 + a*b^2*x^3 + 3*a^2*b*log(abs(x)) - 1/3*(3*a^2*b*x^3 + a^3)/x^
3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^3}{x^4}\,dx = \frac{b^3\,x^6}{6} - \frac{a^3}{3\,x^3} + a\,b^2\,x^3 + 3\,a^2\,b\,\ln\left(x\right)$$

input
```
int((a + b*x^3)^3/x^4,x)
```

output
```
(b^3*x^6)/6 - a^3/(3*x^3) + a*b^2*x^3 + 3*a^2*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^3}{x^4}\,dx = \frac{18\log(x)\,a^2b\,x^3 - 2a^3 + 6a\,b^2x^6 + b^3x^9}{6x^3}$$

input
```
int((b*x^3+a)^3/x^4,x)
```

output
```
(18*log(x)*a**2*b*x**3 - 2*a**3 + 6*a*b**2*x**6 + b**3*x**9)/(6*x**3)
```

3.38 $\int \frac{(a+bx^3)^3}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = -\frac{a^3}{6x^6} - \frac{a^2b}{x^3} + \frac{b^3x^3}{3} + 3ab^2\log(x)$$

output
```
-1/6*a^3/x^6-a^2*b/x^3+1/3*b^3*x^3+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = -\frac{a^3}{6x^6} - \frac{a^2b}{x^3} + \frac{b^3x^3}{3} + 3ab^2\log(x)$$

input
```
Integrate[(a + b*x^3)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (a^2*b)/x^3 + (b^3*x^3)/3 + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^3}{x^7} \, dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^3}{x^9}dx^3$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}\int \left(\frac{a^3}{x^9}+\frac{3ba^2}{x^6}+\frac{3b^2a}{x^3}+b^3\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^3}{2x^6}-\frac{3a^2b}{x^3}+3ab^2\log\left(x^3\right)+b^3x^3\right)$$

input
```
Int[(a + b*x^3)^3/x^7,x]
```

output
```
(-1/2*a^3/x^6 - (3*a^2*b)/x^3 + b^3*x^3 + 3*a*b^2*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{3}}{6 x^{6}}-\frac{a^{2} b}{x^{3}}+\frac{b^{3} x^{3}}{3}+3 a\,b^{2} \ln \left(x \right)$	35
norman	$\frac{-\frac{1}{6} a^{3}+\frac{1}{3} b^{3} x^{9}-a^{2} b\,x^{3}}{x^{6}}+3 a\,b^{2} \ln \left(x \right)$	37
risch	$\frac{b^{3} x^{3}}{3}+\frac{-a^{2} b\,x^{3}-\frac{1}{6} a^{3}}{x^{6}}+3 a\,b^{2} \ln \left(x \right)$	37
parallelrisch	$\frac{2 b^{3} x^{9}+18 a\,b^{2} \ln \left(x \right) x^{6}-6 a^{2} b\,x^{3}-a^{3}}{6 x^{6}}$	40

input
```
int((b*x^3+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^3/x^6-a^2*b/x^3+1/3*b^3*x^3+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^3}{x^7}\,dx=\frac{2\,b^3x^9+18\,ab^2x^6\log{(x)}-6\,a^2bx^3-a^3}{6\,x^6}$$

input
```
integrate((b*x^3+a)^3/x^7,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = 3ab^2\log{(x)} + \frac{b^3x^3}{3} + \frac{-a^3-6a^2bx^3}{6x^6}$$

input
```
integrate((b*x**3+a)**3/x**7,x)
```

output
```
3*a*b**2*log(x) + b**3*x**3/3 + (-a**3 - 6*a**2*b*x**3)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = \frac{1}{3}\,b^3x^3 + ab^2\log\left(x^3\right) - \frac{6\,a^2bx^3+a^3}{6\,x^6}$$

input
```
integrate((b*x^3+a)^3/x^7,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 + a*b^2*log(x^3) - 1/6*(6*a^2*b*x^3 + a^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = \frac{1}{3}\,b^3x^3 + 3\,ab^2\log\left(|x|\right) - \frac{9\,ab^2x^6+6\,a^2bx^3+a^3}{6\,x^6}$$

input
```
integrate((b*x^3+a)^3/x^7,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 + 3*a*b^2*log(abs(x)) - 1/6*(9*a*b^2*x^6 + 6*a^2*b*x^3 + a^3)/
x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = \frac{b^3\,x^3}{3} - \frac{\frac{a^3}{6} + b\,a^2\,x^3}{x^6} + 3\,a\,b^2\,\ln(x)$$

input
```
int((a + b*x^3)^3/x^7,x)
```

output
```
(b^3*x^3)/3 - (a^3/6 + a^2*b*x^3)/x^6 + 3*a*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^3}{x^7}\,dx = \frac{18\log(x)\,a\,b^2x^6 - a^3 - 6a^2b\,x^3 + 2b^3x^9}{6x^6}$$

input
```
int((b*x^3+a)^3/x^7,x)
```

output
```
(18*log(x)*a*b**2*x**6 - a**3 - 6*a**2*b*x**3 + 2*b**3*x**9)/(6*x**6)
```

3.39 $\int \frac{(a+bx^3)^3}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = -\frac{a^3}{9x^9}-\frac{a^2b}{2x^6}-\frac{ab^2}{x^3}+b^3\log(x)$$

output
```
-1/9*a^3/x^9-1/2*a^2*b/x^6-a*b^2/x^3+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = -\frac{a^3}{9x^9}-\frac{a^2b}{2x^6}-\frac{ab^2}{x^3}+b^3\log(x)$$

input
```
Integrate[(a + b*x^3)^3/x^10,x]
```

output
```
-1/9*a^3/x^9 - (a^2*b)/(2*x^6) - (a*b^2)/x^3 + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^3}{x^{12}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^3}{x^{12}}+\frac{3ba^2}{x^9}+\frac{3b^2a}{x^6}+\frac{b^3}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^3}{3x^9}-\frac{3a^2b}{2x^6}-\frac{3ab^2}{x^3}+b^3\log\left(x^3\right)\right)$$

input
```
Int[(a + b*x^3)^3/x^10,x]
```

output
```
(-1/3*a^3/x^9 - (3*a^2*b)/(2*x^6) - (3*a*b^2)/x^3 + b^3*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{3}}{9 x^{9}}-\frac{a^{2} b}{2 x^{6}}-\frac{a\,b^{2}}{x^{3}}+b^{3} \ln \left(x \right)$	34
norman	$\frac{-\frac{1}{9} a^{3}-a\,b^{2} x^{6}-\frac{1}{2} a^{2} b\,x^{3}}{x^{9}}+b^{3} \ln \left(x \right)$	36
risch	$\frac{-\frac{1}{9} a^{3}-a\,b^{2} x^{6}-\frac{1}{2} a^{2} b\,x^{3}}{x^{9}}+b^{3} \ln \left(x \right)$	36
parallelrisch	$\frac{18 b^{3} \ln \left(x \right) x^{9}-18 a\,b^{2} x^{6}-9 a^{2} b\,x^{3}-2 a^{3}}{18 x^{9}}$	40

input
```
int((b*x^3+a)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^3/x^9-1/2*a^2*b/x^6-a*b^2/x^3+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx=\frac{18\,b^3x^9\log(x)-18\,ab^2x^6-9\,a^2bx^3-2\,a^3}{18\,x^9}$$

input
```
integrate((b*x^3+a)^3/x^10,x, algorithm="fricas")
```

output
```
1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = b^3\log(x) + \frac{-2a^3 - 9a^2bx^3 - 18ab^2x^6}{18x^9}$$

input
```
integrate((b*x**3+a)**3/x**10,x)
```

output
```
b**3*log(x) + (-2*a**3 - 9*a**2*b*x**3 - 18*a*b**2*x**6)/(18*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = \frac{1}{3}b^3\log\left(x^3\right) - \frac{18\,ab^2x^6 + 9\,a^2bx^3 + 2\,a^3}{18\,x^9}$$

input
```
integrate((b*x^3+a)^3/x^10,x, algorithm="maxima")
```

output
```
1/3*b^3*log(x^3) - 1/18*(18*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3)/x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = b^3\log(|x|) - \frac{11\,b^3x^9 + 18\,ab^2x^6 + 9\,a^2bx^3 + 2\,a^3}{18\,x^9}$$

input
```
integrate((b*x^3+a)^3/x^10,x, algorithm="giac")
```

output
```
b^3*log(abs(x)) - 1/18*(11*b^3*x^9 + 18*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3)/x
^9
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = b^3\,\ln(x) - \frac{\frac{a^3}{9} + \frac{a^2\,b\,x^3}{2} + a\,b^2\,x^6}{x^9}$$

input
```
int((a + b*x^3)^3/x^10,x)
```

output
```
b^3*log(x) - (a^3/9 + (a^2*b*x^3)/2 + a*b^2*x^6)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^3}{x^{10}}\,dx = \frac{18\log(x)\,b^3x^9 - 2a^3 - 9a^2b\,x^3 - 18a\,b^2x^6}{18x^9}$$

input
```
int((b*x^3+a)^3/x^10,x)
```

output
```
(18*log(x)*b**3*x**9 - 2*a**3 - 9*a**2*b*x**3 - 18*a*b**2*x**6)/(18*x**9)
```

3.40 $\int \frac{(a+bx^3)^3}{x^{13}} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^3)^3}{x^{13}} dx = -\frac{(a+bx^3)^4}{12ax^{12}}$$

output
```
-1/12*(b*x^3+a)^4/a/x^12
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{(a+bx^3)^3}{x^{13}} dx = -\frac{a^3}{12x^{12}} - \frac{a^2b}{3x^9} - \frac{ab^2}{2x^6} - \frac{b^3}{3x^3}$$

input
```
Integrate[(a + b*x^3)^3/x^13,x]
```

output
```
-1/12*a^3/x^12 - (a^2*b)/(3*x^9) - (a*b^2)/(2*x^6) - b^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^{13}} \, dx$$

$$\Big\downarrow \ 796$$

$$-\frac{(a+bx^3)^4}{12ax^{12}}$$

input
```
Int[(a + b*x^3)^3/x^13,x]
```

output
```
-1/12*(a + b*x^3)^4/(a*x^12)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result	size
gosper	$-\frac{4b^3x^9+6a\,b^2x^6+4a^2b\,x^3+a^3}{12x^{12}}$	36
default	$-\frac{b^3}{3x^3}-\frac{a\,b^2}{2x^6}-\frac{a^2b}{3x^9}-\frac{a^3}{12x^{12}}$	36
orering	$-\frac{4b^3x^9+6a\,b^2x^6+4a^2b\,x^3+a^3}{12x^{12}}$	36
norman	$\frac{-\frac{1}{3}b^3x^9-\frac{1}{2}a\,b^2x^6-\frac{1}{3}a^2b\,x^3-\frac{1}{12}a^3}{x^{12}}$	37
risch	$\frac{-\frac{1}{3}b^3x^9-\frac{1}{2}a\,b^2x^6-\frac{1}{3}a^2b\,x^3-\frac{1}{12}a^3}{x^{12}}$	37
parallelrisch	$\frac{-4b^3x^9-6a\,b^2x^6-4a^2b\,x^3-a^3}{12x^{12}}$	38

input
```
int((b*x^3+a)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^3)^3}{x^{13}}\,dx = -\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

input
```
integrate((b*x^3+a)^3/x^13,x, algorithm="fricas")
```

output
```
-1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^3)^3}{x^{13}}\,dx = \frac{-a^3-4a^2bx^3-6ab^2x^6-4b^3x^9}{12x^{12}}$$

input
```
integrate((b*x**3+a)**3/x**13,x)
```

output
```
(-a**3 - 4*a**2*b*x**3 - 6*a*b**2*x**6 - 4*b**3*x**9)/(12*x**12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^3)^3}{x^{13}}\,dx = -\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

input
```
integrate((b*x^3+a)^3/x^13,x, algorithm="maxima")
```

output
```
-1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^3)^3}{x^{13}}\,dx = -\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

input
```
integrate((b*x^3+a)^3/x^13,x, algorithm="giac")
```

output
```
-1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^3)^3}{x^{13}}\,dx = -\frac{\frac{a^3}{12}+\frac{a^2\,b\,x^3}{3}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^9}{3}}{x^{12}}$$

input
```
int((a + b*x^3)^3/x^13,x)
```

output
```
-(a^3/12 + (b^3*x^9)/3 + (a^2*b*x^3)/3 + (a*b^2*x^6)/2)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^3)^3}{x^{13}}\,dx = \frac{-4b^3x^9-6a\,b^2x^6-4a^2b\,x^3-a^3}{12x^{12}}$$

input
```
int((b*x^3+a)^3/x^13,x)
```

output
```
( - a**3 - 4*a**2*b*x**3 - 6*a*b**2*x**6 - 4*b**3*x**9)/(12*x**12)
```

3.41 $\int \frac{(a+bx^3)^3}{x^{16}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = -\frac{(a+bx^3)^4}{15ax^{15}} + \frac{b(a+bx^3)^4}{60a^2x^{12}}$$

output
```
-1/15*(b*x^3+a)^4/a/x^15+1/60*b*(b*x^3+a)^4/a^2/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = -\frac{a^3}{15x^{15}} - \frac{a^2b}{4x^{12}} - \frac{ab^2}{3x^9} - \frac{b^3}{6x^6}$$

input
```
Integrate[(a + b*x^3)^3/x^16,x]
```

output
```
-1/15*a^3/x^15 - (a^2*b)/(4*x^12) - (a*b^2)/(3*x^9) - b^3/(6*x^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^3}{x^{18}}dx^3$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{(bx^3+a)^3}{x^{15}}dx^3}{5a}-\frac{(a+bx^3)^4}{5ax^{15}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{3}\left(\frac{b(a+bx^3)^4}{20a^2x^{12}}-\frac{(a+bx^3)^4}{5ax^{15}}\right)$$

input
```
Int[(a + b*x^3)^3/x^16,x]
```

output
```
(-1/5*(a + b*x^3)^4/(a*x^15) + (b*(a + b*x^3)^4)/(20*a^2*x^12))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a^{3}}{15 x^{15}}-\frac{b^{3}}{6 x^{6}}-\frac{a \,b^{2}}{3 x^{9}}-\frac{a^{2} b}{4 x^{12}}$	36
norman	$\frac{-\frac{1}{15} a^{3}-\frac{1}{4} a^{2} b \,x^{3}-\frac{1}{3} a \,b^{2} x^{6}-\frac{1}{6} b^{3} x^{9}}{x^{15}}$	37
risch	$\frac{-\frac{1}{15} a^{3}-\frac{1}{4} a^{2} b \,x^{3}-\frac{1}{3} a \,b^{2} x^{6}-\frac{1}{6} b^{3} x^{9}}{x^{15}}$	37
gosper	$-\frac{10 b^{3} x^{9}+20 a \,b^{2} x^{6}+15 a^{2} b \,x^{3}+4 a^{3}}{60 x^{15}}$	38
parallelrisch	$\frac{-10 b^{3} x^{9}-20 a \,b^{2} x^{6}-15 a^{2} b \,x^{3}-4 a^{3}}{60 x^{15}}$	38
orering	$-\frac{10 b^{3} x^{9}+20 a \,b^{2} x^{6}+15 a^{2} b \,x^{3}+4 a^{3}}{60 x^{15}}$	38

input
```
int((b*x^3+a)^3/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^3/x^15-1/6*b^3/x^6-1/3*a*b^2/x^9-1/4*a^2*b/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = -\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

input
```
integrate((b*x^3+a)^3/x^16,x, algorithm="fricas")
```

output
```
-1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = \frac{-4a^3-15a^2bx^3-20ab^2x^6-10b^3x^9}{60x^{15}}$$

input
```
integrate((b*x**3+a)**3/x**16,x)
```

output
```
(-4*a**3 - 15*a**2*b*x**3 - 20*a*b**2*x**6 - 10*b**3*x**9)/(60*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = -\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

input
```
integrate((b*x^3+a)^3/x^16,x, algorithm="maxima")
```

output
```
-1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = -\frac{10\,b^3x^9+20\,ab^2x^6+15\,a^2bx^3+4\,a^3}{60\,x^{15}}$$

input
```
integrate((b*x^3+a)^3/x^16,x, algorithm="giac")
```

output
```
-1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = -\frac{\frac{a^3}{15}+\frac{a^2\,b\,x^3}{4}+\frac{a\,b^2\,x^6}{3}+\frac{b^3\,x^9}{6}}{x^{15}}$$

input
```
int((a + b*x^3)^3/x^16,x)
```

output
```
-(a^3/15 + (b^3*x^9)/6 + (a^2*b*x^3)/4 + (a*b^2*x^6)/3)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^{16}}\,dx = \frac{-10b^3x^9-20a\,b^2x^6-15a^2b\,x^3-4a^3}{60x^{15}}$$

input
```
int((b*x^3+a)^3/x^16,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x**3 - 20*a*b**2*x**6 - 10*b**3*x**9)/(60*x**15)
```

3.42 $\int \frac{(a+bx^3)^3}{x^{19}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = -\frac{a^3}{18x^{18}} - \frac{a^2b}{5x^{15}} - \frac{ab^2}{4x^{12}} - \frac{b^3}{9x^9}$$

output
```
-1/18*a^3/x^18-1/5*a^2*b/x^15-1/4*a*b^2/x^12-1/9*b^3/x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = -\frac{a^3}{18x^{18}} - \frac{a^2b}{5x^{15}} - \frac{ab^2}{4x^{12}} - \frac{b^3}{9x^9}$$

input
```
Integrate[(a + b*x^3)^3/x^19,x]
```

output
```
-1/18*a^3/x^18 - (a^2*b)/(5*x^15) - (a*b^2)/(4*x^12) - b^3/(9*x^9)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^3}{x^{21}}dx^3$$

$$\Big\downarrow \ 53$$

$$\frac{1}{3}\int \left(\frac{a^3}{x^{21}}+\frac{3ba^2}{x^{18}}+\frac{3b^2a}{x^{15}}+\frac{b^3}{x^{12}}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^3}{6x^{18}}-\frac{3a^2b}{5x^{15}}-\frac{3ab^2}{4x^{12}}-\frac{b^3}{3x^9}\right)$$

input
```
Int[(a + b*x^3)^3/x^19,x]
```

output
```
(-1/6*a^3/x^18 - (3*a^2*b)/(5*x^15) - (3*a*b^2)/(4*x^12) - b^3/(3*x^9))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^3}{18x^{18}}-\frac{a^2b}{5x^{15}}-\frac{a\,b^2}{4x^{12}}-\frac{b^3}{9x^9}$	36
norman	$\frac{-\frac{1}{18}a^3-\frac{1}{5}a^2b\,x^3-\frac{1}{4}a\,b^2x^6-\frac{1}{9}b^3x^9}{x^{18}}$	37
risch	$\frac{-\frac{1}{18}a^3-\frac{1}{5}a^2b\,x^3-\frac{1}{4}a\,b^2x^6-\frac{1}{9}b^3x^9}{x^{18}}$	37
gosper	$-\frac{20b^3x^9+45a\,b^2x^6+36a^2b\,x^3+10a^3}{180x^{18}}$	38
parallelrisch	$\frac{-20b^3x^9-45a\,b^2x^6-36a^2b\,x^3-10a^3}{180x^{18}}$	38
orering	$-\frac{20b^3x^9+45a\,b^2x^6+36a^2b\,x^3+10a^3}{180x^{18}}$	38

input
```
int((b*x^3+a)^3/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*a^3/x^18-1/5*a^2*b/x^15-1/4*a*b^2/x^12-1/9*b^3/x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = -\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^3/x^19,x, algorithm="fricas")
```

output
```
-1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = \frac{-10a^3-36a^2bx^3-45ab^2x^6-20b^3x^9}{180x^{18}}$$

input
```
integrate((b*x**3+a)**3/x**19,x)
```

output
```
(-10*a**3 - 36*a**2*b*x**3 - 45*a*b**2*x**6 - 20*b**3*x**9)/(180*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = -\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^3/x^19,x, algorithm="maxima")
```

output
```
-1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = -\frac{20\,b^3x^9+45\,ab^2x^6+36\,a^2bx^3+10\,a^3}{180\,x^{18}}$$

input
```
integrate((b*x^3+a)^3/x^19,x, algorithm="giac")
```

output
```
-1/180*(20*b^3*x^9 + 45*a*b^2*x^6 + 36*a^2*b*x^3 + 10*a^3)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = -\frac{\frac{a^3}{18}+\frac{a^2\,b\,x^3}{5}+\frac{a\,b^2\,x^6}{4}+\frac{b^3\,x^9}{9}}{x^{18}}$$

input
```
int((a + b*x^3)^3/x^19,x)
```

output
```
-(a^3/18 + (b^3*x^9)/9 + (a^2*b*x^3)/5 + (a*b^2*x^6)/4)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{19}}\,dx = \frac{-20b^3x^9-45a\,b^2x^6-36a^2b\,x^3-10a^3}{180x^{18}}$$

input
```
int((b*x^3+a)^3/x^19,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x**3 - 45*a*b**2*x**6 - 20*b**3*x**9)/(180*x**18)
```

3.43 $\int \frac{(a+bx^3)^3}{x^{22}} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^3)^3}{x^{22}} dx = -\frac{a^3}{21x^{21}} - \frac{a^2b}{6x^{18}} - \frac{ab^2}{5x^{15}} - \frac{b^3}{12x^{12}}$$

output
```
-1/21*a^3/x^21-1/6*a^2*b/x^18-1/5*a*b^2/x^15-1/12*b^3/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^{22}} dx = -\frac{a^3}{21x^{21}} - \frac{a^2b}{6x^{18}} - \frac{ab^2}{5x^{15}} - \frac{b^3}{12x^{12}}$$

input
```
Integrate[(a + b*x^3)^3/x^22,x]
```

output
```
-1/21*a^3/x^21 - (a^2*b)/(6*x^18) - (a*b^2)/(5*x^15) - b^3/(12*x^12)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^{22}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{(bx^3+a)^3}{x^{24}} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{a^3}{x^{24}} + \frac{3ba^2}{x^{21}} + \frac{3b^2a}{x^{18}} + \frac{b^3}{x^{15}}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^3}{7x^{21}} - \frac{a^2b}{2x^{18}} - \frac{3ab^2}{5x^{15}} - \frac{b^3}{4x^{12}}\right)$$

input
```
Int[(a + b*x^3)^3/x^22,x]
```

output
```
(-1/7*a^3/x^21 - (a^2*b)/(2*x^18) - (3*a*b^2)/(5*x^15) - b^3/(4*x^12))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{3}}{21 x^{21}}-\frac{a^{2} b}{6 x^{18}}-\frac{a\,b^{2}}{5 x^{15}}-\frac{b^{3}}{12 x^{12}}$	36
norman	$\frac{-\frac{1}{21} a^{3}-\frac{1}{6} a^{2} b\,x^{3}-\frac{1}{5} a\,b^{2} x^{6}-\frac{1}{12} b^{3} x^{9}}{x^{21}}$	37
risch	$\frac{-\frac{1}{21} a^{3}-\frac{1}{6} a^{2} b\,x^{3}-\frac{1}{5} a\,b^{2} x^{6}-\frac{1}{12} b^{3} x^{9}}{x^{21}}$	37
gosper	$-\frac{35 b^{3} x^{9}+84 a\,b^{2} x^{6}+70 a^{2} b\,x^{3}+20 a^{3}}{420 x^{21}}$	38
parallelrisch	$\frac{-35 b^{3} x^{9}-84 a\,b^{2} x^{6}-70 a^{2} b\,x^{3}-20 a^{3}}{420 x^{21}}$	38
orering	$-\frac{35 b^{3} x^{9}+84 a\,b^{2} x^{6}+70 a^{2} b\,x^{3}+20 a^{3}}{420 x^{21}}$	38

input
```
int((b*x^3+a)^3/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/21*a^3/x^21-1/6*a^2*b/x^18-1/5*a*b^2/x^15-1/12*b^3/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{22}}\,dx=-\frac{35\,b^3x^9+84\,ab^2x^6+70\,a^2bx^3+20\,a^3}{420\,x^{21}}$$

input
```
integrate((b*x^3+a)^3/x^22,x, algorithm="fricas")
```

output
```
-1/420*(35*b^3*x^9 + 84*a*b^2*x^6 + 70*a^2*b*x^3 + 20*a^3)/x^21
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^3}{x^{22}}\,dx = \frac{-20a^3-70a^2bx^3-84ab^2x^6-35b^3x^9}{420x^{21}}$$

input
```
integrate((b*x**3+a)**3/x**22,x)
```

output
```
(-20*a**3 - 70*a**2*b*x**3 - 84*a*b**2*x**6 - 35*b**3*x**9)/(420*x**21)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{22}}\,dx = -\frac{35\,b^3x^9+84\,ab^2x^6+70\,a^2bx^3+20\,a^3}{420\,x^{21}}$$

input
```
integrate((b*x^3+a)^3/x^22,x, algorithm="maxima")
```

output
```
-1/420*(35*b^3*x^9 + 84*a*b^2*x^6 + 70*a^2*b*x^3 + 20*a^3)/x^21
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{22}}\,dx = -\frac{35\,b^3x^9+84\,ab^2x^6+70\,a^2bx^3+20\,a^3}{420\,x^{21}}$$

input
```
integrate((b*x^3+a)^3/x^22,x, algorithm="giac")
```

output
```
-1/420*(35*b^3*x^9 + 84*a*b^2*x^6 + 70*a^2*b*x^3 + 20*a^3)/x^21
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{22}}\,dx = -\frac{\frac{a^3}{21}+\frac{a^2\,b\,x^3}{6}+\frac{a\,b^2\,x^6}{5}+\frac{b^3\,x^9}{12}}{x^{21}}$$

input
```
int((a + b*x^3)^3/x^22,x)
```

output
```
-(a^3/21 + (b^3*x^9)/12 + (a^2*b*x^3)/6 + (a*b^2*x^6)/5)/x^21
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^3}{x^{22}}\,dx = \frac{-35b^3x^9-84a\,b^2x^6-70a^2b\,x^3-20a^3}{420x^{21}}$$

input
```
int((b*x^3+a)^3/x^22,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x**3 - 84*a*b**2*x**6 - 35*b**3*x**9)/(420*x**21)
```

3.44 $\int x^4(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^4\left(a+bx^3\right)^3\,dx=\frac{a^3x^5}{5}+\frac{3}{8}a^2bx^8+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{14}}{14}$$

output
```
1/5*a^3*x^5+3/8*a^2*b*x^8+3/11*a*b^2*x^11+1/14*b^3*x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^3\right)^3\,dx=\frac{a^3x^5}{5}+\frac{3}{8}a^2bx^8+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{14}}{14}$$

input
```
Integrate[x^4*(a + b*x^3)^3,x]
```

output
```
(a^3*x^5)/5 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^11)/11 + (b^3*x^14)/14
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^3 \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(a^3x^4+3a^2bx^7+3ab^2x^{10}+b^3x^{13}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3x^5}{5}+\frac{3}{8}a^2bx^8+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{14}}{14}$$

input
```
Int[x^4*(a + b*x^3)^3,x]
```

output
```
(a^3*x^5)/5 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^11)/11 + (b^3*x^14)/14
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5+\frac{3}{8}a^2b\,x^8+\frac{3}{11}a\,b^2x^{11}+\frac{1}{14}b^3x^{14}$	36
default	$\frac{1}{5}a^3x^5+\frac{3}{8}a^2b\,x^8+\frac{3}{11}a\,b^2x^{11}+\frac{1}{14}b^3x^{14}$	36
norman	$\frac{1}{5}a^3x^5+\frac{3}{8}a^2b\,x^8+\frac{3}{11}a\,b^2x^{11}+\frac{1}{14}b^3x^{14}$	36
risch	$\frac{1}{5}a^3x^5+\frac{3}{8}a^2b\,x^8+\frac{3}{11}a\,b^2x^{11}+\frac{1}{14}b^3x^{14}$	36
parallelrisch	$\frac{1}{5}a^3x^5+\frac{3}{8}a^2b\,x^8+\frac{3}{11}a\,b^2x^{11}+\frac{1}{14}b^3x^{14}$	36
orering	$\frac{x^5\left(220b^3x^9+840a\,b^2x^6+1155a^2b\,x^3+616a^3\right)}{3080}$	38

input
```
int(x^4*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+3/8*a^2*b*x^8+3/11*a*b^2*x^11+1/14*b^3*x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4\left(a+bx^3\right)^3\,dx=\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^4 (a + bx^3)^3 \, dx = \frac{a^3x^5}{5} + \frac{3a^2bx^8}{8} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{14}}{14}$$

input
```
integrate(x**4*(b*x**3+a)**3,x)
```

output
```
a**3*x**5/5 + 3*a**2*b*x**8/8 + 3*a*b**2*x**11/11 + b**3*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 (a + bx^3)^3 \, dx = \frac{1}{14} b^3x^{14} + \frac{3}{11} ab^2x^{11} + \frac{3}{8} a^2bx^8 + \frac{1}{5} a^3x^5$$

input
```
integrate(x^4*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 (a + bx^3)^3 \, dx = \frac{1}{14} b^3x^{14} + \frac{3}{11} ab^2x^{11} + \frac{3}{8} a^2bx^8 + \frac{1}{5} a^3x^5$$

input
```
integrate(x^4*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 (a+bx^3)^3 \, dx = \frac{a^3 x^5}{5} + \frac{3 a^2 b x^8}{8} + \frac{3 a b^2 x^{11}}{11} + \frac{b^3 x^{14}}{14}$$

input
```
int(x^4*(a + b*x^3)^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^14)/14 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^4 (a+bx^3)^3 \, dx = \frac{x^5(220b^3x^9 + 840a b^2x^6 + 1155a^2b x^3 + 616a^3)}{3080}$$

input
```
int(x^4*(b*x^3+a)^3,x)
```

output
```
(x**5*(616*a**3 + 1155*a**2*b*x**3 + 840*a*b**2*x**6 + 220*b**3*x**9))/308
0
```

3.45 $\int x^3(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^3\left(a+bx^3\right)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{7}a^2bx^7+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{13}}{13}$$

output
```
1/4*a^3*x^4+3/7*a^2*b*x^7+3/10*a*b^2*x^10+1/13*b^3*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^3\right)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{7}a^2bx^7+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{13}}{13}$$

input
```
Integrate[x^3*(a + b*x^3)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^10)/10 + (b^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^3 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^3+3a^2bx^6+3ab^2x^9+b^3x^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^4}{4}+\frac{3}{7}a^2bx^7+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{13}}{13}$$

input
```
Int[x^3*(a + b*x^3)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^10)/10 + (b^3*x^13)/13
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{4}a^3x^4+\frac{3}{7}a^2b\,x^7+\frac{3}{10}a\,b^2x^{10}+\frac{1}{13}b^3x^{13}$	36
default	$\frac{1}{4}a^3x^4+\frac{3}{7}a^2b\,x^7+\frac{3}{10}a\,b^2x^{10}+\frac{1}{13}b^3x^{13}$	36
norman	$\frac{1}{4}a^3x^4+\frac{3}{7}a^2b\,x^7+\frac{3}{10}a\,b^2x^{10}+\frac{1}{13}b^3x^{13}$	36
risch	$\frac{1}{4}a^3x^4+\frac{3}{7}a^2b\,x^7+\frac{3}{10}a\,b^2x^{10}+\frac{1}{13}b^3x^{13}$	36
parallelrisch	$\frac{1}{4}a^3x^4+\frac{3}{7}a^2b\,x^7+\frac{3}{10}a\,b^2x^{10}+\frac{1}{13}b^3x^{13}$	36
orering	$\frac{x^4\left(140b^3x^9+546a\,b^2x^6+780a^2b\,x^3+455a^3\right)}{1820}$	38

input
```
int(x^3*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^3*x^4+3/7*a^2*b*x^7+3/10*a*b^2*x^10+1/13*b^3*x^13
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3\left(a+bx^3\right)^3\,dx=\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^3 (a + bx^3)^3 \, dx = \frac{a^3x^4}{4} + \frac{3a^2bx^7}{7} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{13}}{13}$$

input
```
integrate(x**3*(b*x**3+a)**3,x)
```

output
```
a**3*x**4/4 + 3*a**2*b*x**7/7 + 3*a*b**2*x**10/10 + b**3*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3 (a + bx^3)^3 \, dx = \frac{1}{13} b^3x^{13} + \frac{3}{10} ab^2x^{10} + \frac{3}{7} a^2bx^7 + \frac{1}{4} a^3x^4$$

input
```
integrate(x^3*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3 (a + bx^3)^3 \, dx = \frac{1}{13} b^3x^{13} + \frac{3}{10} ab^2x^{10} + \frac{3}{7} a^2bx^7 + \frac{1}{4} a^3x^4$$

input
```
integrate(x^3*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^3 (a + bx^3)^3 \, dx = \frac{a^3 x^4}{4} + \frac{3 a^2 b x^7}{7} + \frac{3 a b^2 x^{10}}{10} + \frac{b^3 x^{13}}{13}$$

input
```
int(x^3*(a + b*x^3)^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^13)/13 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^3 (a + bx^3)^3 \, dx = \frac{x^4(140b^3x^9 + 546a b^2x^6 + 780a^2b x^3 + 455a^3)}{1820}$$

input
```
int(x^3*(b*x^3+a)^3,x)
```

output
```
(x**4*(455*a**3 + 780*a**2*b*x**3 + 546*a*b**2*x**6 + 140*b**3*x**9))/1820
```

3.46 $\int x(a+bx^3)^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int x(a+bx^3)^3\,dx=\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}$$

output
```
1/2*a^3*x^2+3/5*a^2*b*x^5+3/8*a*b^2*x^8+1/11*b^3*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)^3\,dx=\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}$$

input
```
Integrate[x*(a + b*x^3)^3,x]
```

output
```
(a^3*x^2)/2 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^8)/8 + (b^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int (a^3x+3a^2bx^4+3ab^2x^7+b^3x^{10})\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2}+\frac{3}{5}a^2bx^5+\frac{3}{8}ab^2x^8+\frac{b^3x^{11}}{11}$$

input
```
Int[x*(a + b*x^3)^3,x]
```

output
```
(a^3*x^2)/2 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^8)/8 + (b^3*x^11)/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{2}a^3x^2+\frac{3}{5}a^2b\,x^5+\frac{3}{8}a\,b^2x^8+\frac{1}{11}b^3x^{11}$	36
default	$\frac{1}{2}a^3x^2+\frac{3}{5}a^2b\,x^5+\frac{3}{8}a\,b^2x^8+\frac{1}{11}b^3x^{11}$	36
norman	$\frac{1}{2}a^3x^2+\frac{3}{5}a^2b\,x^5+\frac{3}{8}a\,b^2x^8+\frac{1}{11}b^3x^{11}$	36
risch	$\frac{1}{2}a^3x^2+\frac{3}{5}a^2b\,x^5+\frac{3}{8}a\,b^2x^8+\frac{1}{11}b^3x^{11}$	36
parallelrisch	$\frac{1}{2}a^3x^2+\frac{3}{5}a^2b\,x^5+\frac{3}{8}a\,b^2x^8+\frac{1}{11}b^3x^{11}$	36
orering	$\frac{x^2(40b^3x^9+165a\,b^2x^6+264a^2b\,x^3+220a^3)}{440}$	38

input
```
int(x*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+3/5*a^2*b*x^5+3/8*a*b^2*x^8+1/11*b^3*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)^3\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x(a+bx^3)^3\,dx=\frac{a^3x^2}{2}+\frac{3a^2bx^5}{5}+\frac{3ab^2x^8}{8}+\frac{b^3x^{11}}{11}$$

input
```
integrate(x*(b*x**3+a)**3,x)
```

output
```
a**3*x**2/2 + 3*a**2*b*x**5/5 + 3*a*b**2*x**8/8 + b**3*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)^3\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)^3\,dx=\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^3)^3\,dx=\frac{a^3\,x^2}{2}+\frac{3\,a^2\,b\,x^5}{5}+\frac{3\,a\,b^2\,x^8}{8}+\frac{b^3\,x^{11}}{11}$$

input
```
int(x*(a + b*x^3)^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^11)/11 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x(a+bx^3)^3\,dx=\frac{x^2(40b^3x^9+165a\,b^2x^6+264a^2b\,x^3+220a^3)}{440}$$

input
```
int(x*(b*x^3+a)^3,x)
```

output
```
(x**2*(220*a**3 + 264*a**2*b*x**3 + 165*a*b**2*x**6 + 40*b**3*x**9))/440
```

3.47 $\int (a+bx^3)^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int (a+bx^3)^3 \, dx = a^3x + \frac{3}{4}a^2bx^4 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{10}}{10}$$

output
```
a^3*x+3/4*a^2*b*x^4+3/7*a*b^2*x^7+1/10*b^3*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^3 \, dx = a^3x + \frac{3}{4}a^2bx^4 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{10}}{10}$$

input
```
Integrate[(a + b*x^3)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^4)/4 + (3*a*b^2*x^7)/7 + (b^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^3 dx$$

$$\Big\downarrow \text{747}$$

$$\int \left(a^3+3a^2bx^3+3ab^2x^6+b^3x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+\frac{3}{4}a^2bx^4+\frac{3}{7}ab^2x^7+\frac{b^3x^{10}}{10}$$

input
```
Int[(a + b*x^3)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^4)/4 + (3*a*b^2*x^7)/7 + (b^3*x^10)/10
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
gosper	$a^3x+\frac{3}{4}a^2b\,x^4+\frac{3}{7}a\,b^2x^7+\frac{1}{10}b^3x^{10}$	33
default	$a^3x+\frac{3}{4}a^2b\,x^4+\frac{3}{7}a\,b^2x^7+\frac{1}{10}b^3x^{10}$	33
norman	$a^3x+\frac{3}{4}a^2b\,x^4+\frac{3}{7}a\,b^2x^7+\frac{1}{10}b^3x^{10}$	33
risch	$a^3x+\frac{3}{4}a^2b\,x^4+\frac{3}{7}a\,b^2x^7+\frac{1}{10}b^3x^{10}$	33
parallelrisch	$a^3x+\frac{3}{4}a^2b\,x^4+\frac{3}{7}a\,b^2x^7+\frac{1}{10}b^3x^{10}$	33
orering	$\frac{x\left(14b^3x^9+60a\,b^2x^6+105a^2b\,x^3+140a^3\right)}{140}$	36

input
```
int((b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+3/4*a^2*b*x^4+3/7*a*b^2*x^7+1/10*b^3*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^3\,dx=\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

input
```
integrate((b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \left(a+bx^3\right)^3 \, dx = a^3x + \frac{3a^2bx^4}{4} + \frac{3ab^2x^7}{7} + \frac{b^3x^{10}}{10}$$

input
```
integrate((b*x**3+a)**3,x)
```

output
```
a**3*x + 3*a**2*b*x**4/4 + 3*a*b**2*x**7/7 + b**3*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^3 \, dx = \frac{1}{10}\,b^3x^{10} + \frac{3}{7}\,ab^2x^7 + \frac{3}{4}\,a^2bx^4 + a^3x$$

input
```
integrate((b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^3 \, dx = \frac{1}{10}\,b^3x^{10} + \frac{3}{7}\,ab^2x^7 + \frac{3}{4}\,a^2bx^4 + a^3x$$

input
```
integrate((b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^3\right)^3 \, dx = a^3\,x+\frac{3\,a^2\,b\,x^4}{4}+\frac{3\,a\,b^2\,x^7}{7}+\frac{b^3\,x^{10}}{10}$$

input
```
int((a + b*x^3)^3,x)
```

output
```
a^3*x + (b^3*x^10)/10 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \left(a+bx^3\right)^3 \, dx = \frac{x(14b^3x^9+60a\,b^2x^6+105a^2b\,x^3+140a^3)}{140}$$

input
```
int((b*x^3+a)^3,x)
```

output
```
(x*(140*a**3 + 105*a**2*b*x**3 + 60*a*b**2*x**6 + 14*b**3*x**9))/140
```

3.48 $\int \frac{(a+bx^3)^3}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{(a+bx^3)^3}{x^2} dx = -\frac{a^3}{x} + \frac{3}{2}a^2bx^2 + \frac{3}{5}ab^2x^5 + \frac{b^3x^8}{8}$$

output
```
-a^3/x+3/2*a^2*b*x^2+3/5*a*b^2*x^5+1/8*b^3*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^2} dx = -\frac{a^3}{x} + \frac{3}{2}a^2bx^2 + \frac{3}{5}ab^2x^5 + \frac{b^3x^8}{8}$$

input
```
Integrate[(a + b*x^3)^3/x^2,x]
```

output
```
-(a^3/x) + (3*a^2*b*x^2)/2 + (3*a*b^2*x^5)/5 + (b^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^2} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^3}{x^2}+3a^2bx+3ab^2x^4+b^3x^7\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{x}+\frac{3}{2}a^2bx^2+\frac{3}{5}ab^2x^5+\frac{b^3x^8}{8}$$

input
```
Int[(a + b*x^3)^3/x^2,x]
```

output
```
-(a^3/x) + (3*a^2*b*x^2)/2 + (3*a*b^2*x^5)/5 + (b^3*x^8)/8
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{x}+\frac{3a^{2}b\,x^{2}}{2}+\frac{3a\,b^{2}x^{5}}{5}+\frac{b^{3}x^{8}}{8}$	36
risch	$-\frac{a^{3}}{x}+\frac{3a^{2}b\,x^{2}}{2}+\frac{3a\,b^{2}x^{5}}{5}+\frac{b^{3}x^{8}}{8}$	36
norman	$\frac{\frac{1}{8}b^{3}x^{9}+\frac{3}{5}a\,b^{2}x^{6}+\frac{3}{2}a^{2}b\,x^{3}-a^{3}}{x}$	37
gosper	$-\frac{-5b^{3}x^{9}-24a\,b^{2}x^{6}-60a^{2}b\,x^{3}+40a^{3}}{40x}$	38
parallelrisch	$\frac{5b^{3}x^{9}+24a\,b^{2}x^{6}+60a^{2}b\,x^{3}-40a^{3}}{40x}$	38
orering	$-\frac{-5b^{3}x^{9}-24a\,b^{2}x^{6}-60a^{2}b\,x^{3}+40a^{3}}{40x}$	38

input
```
int((b*x^3+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+3/2*a^2*b*x^2+3/5*a*b^2*x^5+1/8*b^3*x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3}{x^2}\,dx = \frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

input
```
integrate((b*x^3+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3}{x^2}\,dx = -\frac{a^3}{x} + \frac{3a^2bx^2}{2} + \frac{3ab^2x^5}{5} + \frac{b^3x^8}{8}$$

input
```
integrate((b*x**3+a)**3/x**2,x)
```

output
```
-a**3/x + 3*a**2*b*x**2/2 + 3*a*b**2*x**5/5 + b**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^3}{x^2}\,dx = \frac{1}{8}\,b^3x^8 + \frac{3}{5}\,ab^2x^5 + \frac{3}{2}\,a^2bx^2 - \frac{a^3}{x}$$

input
```
integrate((b*x^3+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/8*b^3*x^8 + 3/5*a*b^2*x^5 + 3/2*a^2*b*x^2 - a^3/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^3}{x^2}\,dx = \frac{1}{8}\,b^3x^8 + \frac{3}{5}\,ab^2x^5 + \frac{3}{2}\,a^2bx^2 - \frac{a^3}{x}$$

input
```
integrate((b*x^3+a)^3/x^2,x, algorithm="giac")
```

output
```
1/8*b^3*x^8 + 3/5*a*b^2*x^5 + 3/2*a^2*b*x^2 - a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^3}{x^2}\,dx = \frac{b^3\,x^8}{8} - \frac{a^3}{x} + \frac{3\,a^2\,b\,x^2}{2} + \frac{3\,a\,b^2\,x^5}{5}$$

input
```
int((a + b*x^3)^3/x^2,x)
```

output
```
(b^3*x^8)/8 - a^3/x + (3*a^2*b*x^2)/2 + (3*a*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3}{x^2}\,dx = \frac{5b^3x^9 + 24a\,b^2x^6 + 60a^2b\,x^3 - 40a^3}{40x}$$

input
```
int((b*x^3+a)^3/x^2,x)
```

output
```
( - 40*a**3 + 60*a**2*b*x**3 + 24*a*b**2*x**6 + 5*b**3*x**9)/(40*x)
```

3.49 $\int \frac{(a+bx^3)^3}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^3)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + 3a^2bx + \frac{3}{4}ab^2x^4 + \frac{b^3x^7}{7}$$

output
```
-1/2*a^3/x^2+3*a^2*b*x+3/4*a*b^2*x^4+1/7*b^3*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + 3a^2bx + \frac{3}{4}ab^2x^4 + \frac{b^3x^7}{7}$$

input
```
Integrate[(a + b*x^3)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 + 3*a^2*b*x + (3*a*b^2*x^4)/4 + (b^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^3}+3a^2b+3ab^2x^3+b^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{2x^2}+3a^2bx+\frac{3}{4}ab^2x^4+\frac{b^3x^7}{7}$$

input
```
Int[(a + b*x^3)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 + 3*a^2*b*x + (3*a*b^2*x^4)/4 + (b^3*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{3}}{2x^{2}}+3a^{2}bx+\frac{3a\,b^{2}x^{4}}{4}+\frac{b^{3}x^{7}}{7}$	34
risch	$-\frac{a^{3}}{2x^{2}}+3a^{2}bx+\frac{3a\,b^{2}x^{4}}{4}+\frac{b^{3}x^{7}}{7}$	34
norman	$\frac{\frac{1}{7}b^{3}x^{9}+\frac{3}{4}a\,b^{2}x^{6}+3a^{2}b\,x^{3}-\frac{1}{2}a^{3}}{x^{2}}$	37
gosper	$-\frac{-4b^{3}x^{9}-21a\,b^{2}x^{6}-84a^{2}b\,x^{3}+14a^{3}}{28x^{2}}$	38
parallelrisch	$\frac{4b^{3}x^{9}+21a\,b^{2}x^{6}+84a^{2}b\,x^{3}-14a^{3}}{28x^{2}}$	38
orering	$-\frac{-4b^{3}x^{9}-21a\,b^{2}x^{6}-84a^{2}b\,x^{3}+14a^{3}}{28x^{2}}$	38

input
```
int((b*x^3+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2+3*a^2*b*x+3/4*a*b^2*x^4+1/7*b^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^3}\,dx=\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

input
```
integrate((b*x^3+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + 3a^2bx + \frac{3ab^2x^4}{4} + \frac{b^3x^7}{7}$$

input
```
integrate((b*x**3+a)**3/x**3,x)
```

output
```
-a**3/(2*x**2) + 3*a**2*b*x + 3*a*b**2*x**4/4 + b**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^3}{x^3}\,dx = \frac{1}{7}\,b^3x^7 + \frac{3}{4}\,ab^2x^4 + 3\,a^2bx - \frac{a^3}{2\,x^2}$$

input
```
integrate((b*x^3+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/7*b^3*x^7 + 3/4*a*b^2*x^4 + 3*a^2*b*x - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^3}{x^3}\,dx = \frac{1}{7}\,b^3x^7 + \frac{3}{4}\,ab^2x^4 + 3\,a^2bx - \frac{a^3}{2\,x^2}$$

input
```
integrate((b*x^3+a)^3/x^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 + 3/4*a*b^2*x^4 + 3*a^2*b*x - 1/2*a^3/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^3}{x^3}\,dx=\frac{b^3\,x^7}{7}-\frac{a^3}{2\,x^2}+\frac{3\,a\,b^2\,x^4}{4}+3\,a^2\,b\,x$$

input
```
int((a + b*x^3)^3/x^3,x)
```

output
```
(b^3*x^7)/7 - a^3/(2*x^2) + (3*a*b^2*x^4)/4 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^3}\,dx=\frac{4b^3x^9+21a\,b^2x^6+84a^2b\,x^3-14a^3}{28x^2}$$

input
```
int((b*x^3+a)^3/x^3,x)
```

output
```
( - 14*a**3 + 84*a**2*b*x**3 + 21*a*b**2*x**6 + 4*b**3*x**9)/(28*x**2)
```

3.50 $\int \frac{(a+bx^3)^3}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{(a+bx^3)^3}{x^5} dx = -\frac{a^3}{4x^4} - \frac{3a^2b}{x} + \frac{3}{2}ab^2x^2 + \frac{b^3x^5}{5}$$

output
```
-1/4*a^3/x^4-3*a^2*b/x+3/2*a*b^2*x^2+1/5*b^3*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^5} dx = -\frac{a^3}{4x^4} - \frac{3a^2b}{x} + \frac{3}{2}ab^2x^2 + \frac{b^3x^5}{5}$$

input
```
Integrate[(a + b*x^3)^3/x^5,x]
```

output
```
-1/4*a^3/x^4 - (3*a^2*b)/x + (3*a*b^2*x^2)/2 + (b^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^5}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^5}+\frac{3a^2b}{x^2}+3ab^2x+b^3x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{4x^4}-\frac{3a^2b}{x}+\frac{3}{2}ab^2x^2+\frac{b^3x^5}{5}$$

input
```
Int[(a + b*x^3)^3/x^5,x]
```

output
```
-1/4*a^3/x^4 - (3*a^2*b)/x + (3*a*b^2*x^2)/2 + (b^3*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{4x^{4}}-\frac{3a^{2}b}{x}+\frac{3a\,b^{2}x^{2}}{2}+\frac{b^{3}x^{5}}{5}$	36
norman	$\frac{\frac{1}{5}b^{3}x^{9}+\frac{3}{2}a\,b^{2}x^{6}-3a^{2}b\,x^{3}-\frac{1}{4}a^{3}}{x^{4}}$	37
gosper	$-\frac{-4b^{3}x^{9}-30a\,b^{2}x^{6}+60a^{2}b\,x^{3}+5a^{3}}{20x^{4}}$	38
risch	$\frac{b^{3}x^{5}}{5}+\frac{3a\,b^{2}x^{2}}{2}+\frac{-3a^{2}b\,x^{3}-\frac{1}{4}a^{3}}{x^{4}}$	38
parallelrisch	$\frac{4b^{3}x^{9}+30a\,b^{2}x^{6}-60a^{2}b\,x^{3}-5a^{3}}{20x^{4}}$	38
orering	$-\frac{-4b^{3}x^{9}-30a\,b^{2}x^{6}+60a^{2}b\,x^{3}+5a^{3}}{20x^{4}}$	38

input
```
int((b*x^3+a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-3*a^2*b/x+3/2*a*b^2*x^2+1/5*b^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3}{x^5}\,dx=\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

input
```
integrate((b*x^3+a)^3/x^5,x, algorithm="fricas")
```

output
```
1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^5}\,dx = \frac{3ab^2x^2}{2} + \frac{b^3x^5}{5} + \frac{-a^3-12a^2bx^3}{4x^4}$$

input
```
integrate((b*x**3+a)**3/x**5,x)
```

output
```
3*a*b**2*x**2/2 + b**3*x**5/5 + (-a**3 - 12*a**2*b*x**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3}{x^5}\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{2}\,ab^2x^2 - \frac{12\,a^2bx^3+a^3}{4\,x^4}$$

input
```
integrate((b*x^3+a)^3/x^5,x, algorithm="maxima")
```

output
```
1/5*b^3*x^5 + 3/2*a*b^2*x^2 - 1/4*(12*a^2*b*x^3 + a^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^3}{x^5}\,dx = \frac{1}{5}\,b^3x^5 + \frac{3}{2}\,ab^2x^2 - \frac{12\,a^2bx^3+a^3}{4\,x^4}$$

input
```
integrate((b*x^3+a)^3/x^5,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 + 3/2*a*b^2*x^2 - 1/4*(12*a^2*b*x^3 + a^3)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3}{x^5}\,dx = \frac{b^3\,x^5}{5} - \frac{\frac{a^3}{4} + 3\,b\,a^2\,x^3}{x^4} + \frac{3\,a\,b^2\,x^2}{2}$$

input
```
int((a + b*x^3)^3/x^5,x)
```

output
```
(b^3*x^5)/5 - (a^3/4 + 3*a^2*b*x^3)/x^4 + (3*a*b^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3}{x^5}\,dx = \frac{4b^3x^9 + 30a\,b^2x^6 - 60a^2b\,x^3 - 5a^3}{20x^4}$$

input
```
int((b*x^3+a)^3/x^5,x)
```

output
```
( - 5*a**3 - 60*a**2*b*x**3 + 30*a*b**2*x**6 + 4*b**3*x**9)/(20*x**4)
```

3.51 $\int \frac{(a+bx^3)^3}{x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^3)^3}{x^6}\,dx=-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}$$

output
```
-1/5*a^3/x^5-3/2*a^2*b/x^2+3*a*b^2*x+1/4*b^3*x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^6}\,dx=-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}$$

input
```
Integrate[(a + b*x^3)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/(2*x^2) + 3*a*b^2*x + (b^3*x^4)/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^6}\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a^3}{x^6}+\frac{3a^2b}{x^3}+3ab^2+b^3x^3\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^3}{5x^5}-\frac{3a^2b}{2x^2}+3ab^2x+\frac{b^3x^4}{4}$$

input
```
Int[(a + b*x^3)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/(2*x^2) + 3*a*b^2*x + (b^3*x^4)/4
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{3}}{5x^{5}}-\frac{3a^{2}b}{2x^{2}}+3a\,b^{2}x+\frac{b^{3}x^{4}}{4}$	34
risch	$\frac{b^{3}x^{4}}{4}+3a\,b^{2}x+\frac{-\frac{3}{2}a^{2}b\,x^{3}-\frac{1}{5}a^{3}}{x^{5}}$	36
norman	$\frac{\frac{1}{4}b^{3}x^{9}+3a\,b^{2}x^{6}-\frac{3}{2}a^{2}b\,x^{3}-\frac{1}{5}a^{3}}{x^{5}}$	37
gosper	$-\frac{-5b^{3}x^{9}-60a\,b^{2}x^{6}+30a^{2}b\,x^{3}+4a^{3}}{20x^{5}}$	38
parallelrisch	$\frac{5b^{3}x^{9}+60a\,b^{2}x^{6}-30a^{2}b\,x^{3}-4a^{3}}{20x^{5}}$	38
orering	$-\frac{-5b^{3}x^{9}-60a\,b^{2}x^{6}+30a^{2}b\,x^{3}+4a^{3}}{20x^{5}}$	38

input
```
int((b*x^3+a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^3/x^5-3/2*a^2*b/x^2+3*a*b^2*x+1/4*b^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^6}\,dx=\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

input
```
integrate((b*x^3+a)^3/x^6,x, algorithm="fricas")
```

output
```
1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^6}\,dx = 3ab^2x + \frac{b^3x^4}{4} + \frac{-2a^3 - 15a^2bx^3}{10x^5}$$

input
```
integrate((b*x**3+a)**3/x**6,x)
```

output
```
3*a*b**2*x + b**3*x**4/4 + (-2*a**3 - 15*a**2*b*x**3)/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^6}\,dx = \frac{1}{4}\,b^3x^4 + 3\,ab^2x - \frac{15\,a^2bx^3 + 2\,a^3}{10\,x^5}$$

input
```
integrate((b*x^3+a)^3/x^6,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 3*a*b^2*x - 1/10*(15*a^2*b*x^3 + 2*a^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^6}\,dx = \frac{1}{4}\,b^3x^4 + 3\,ab^2x - \frac{15\,a^2bx^3 + 2\,a^3}{10\,x^5}$$

input
```
integrate((b*x^3+a)^3/x^6,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + 3*a*b^2*x - 1/10*(15*a^2*b*x^3 + 2*a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^3}{x^6}\,dx = \frac{b^3\,x^4}{4} - \frac{\frac{a^3}{5}+\frac{3\,b\,a^2\,x^3}{2}}{x^5} + 3\,a\,b^2\,x$$

input
```
int((a + b*x^3)^3/x^6,x)
```

output
```
(b^3*x^4)/4 - (a^3/5 + (3*a^2*b*x^3)/2)/x^5 + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^6}\,dx = \frac{5b^3x^9+60a\,b^2x^6-30a^2b\,x^3-4a^3}{20x^5}$$

input
```
int((b*x^3+a)^3/x^6,x)
```

output
```
( - 4*a**3 - 30*a**2*b*x**3 + 60*a*b**2*x**6 + 5*b**3*x**9)/(20*x**5)
```

3.52 $\int \frac{(a+bx^3)^3}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{(a+bx^3)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{x} + \frac{b^3x^2}{2}$$

output
```
-1/7*a^3/x^7-3/4*a^2*b/x^4-3*a*b^2/x+1/2*b^3*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{x} + \frac{b^3x^2}{2}$$

input
```
Integrate[(a + b*x^3)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/x + (b^3*x^2)/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^3}{x^8} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^5}+\frac{3ab^2}{x^2}+b^3x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{4x^4}-\frac{3ab^2}{x}+\frac{b^3x^2}{2}$$

input
```
Int[(a + b*x^3)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/x + (b^3*x^2)/2
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{7x^{7}}-\frac{3a^{2}b}{4x^{4}}-\frac{3a\,b^{2}}{x}+\frac{b^{3}x^{2}}{2}$	36
norman	$\frac{\frac{1}{2}b^{3}x^{9}-3a\,b^{2}x^{6}-\frac{3}{4}a^{2}b\,x^{3}-\frac{1}{7}a^{3}}{x^{7}}$	37
gosper	$-\frac{-14b^{3}x^{9}+84a\,b^{2}x^{6}+21a^{2}b\,x^{3}+4a^{3}}{28x^{7}}$	38
risch	$\frac{b^{3}x^{2}}{2}+\frac{-3a\,b^{2}x^{6}-\frac{3}{4}a^{2}b\,x^{3}-\frac{1}{7}a^{3}}{x^{7}}$	38
parallelrisch	$\frac{14b^{3}x^{9}-84a\,b^{2}x^{6}-21a^{2}b\,x^{3}-4a^{3}}{28x^{7}}$	38
orering	$-\frac{-14b^{3}x^{9}+84a\,b^{2}x^{6}+21a^{2}b\,x^{3}+4a^{3}}{28x^{7}}$	38

input
```
int((b*x^3+a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^3/x^7-3/4*a^2*b/x^4-3*a*b^2/x+1/2*b^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3}{x^8}\,dx = \frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

input
```
integrate((b*x^3+a)^3/x^8,x, algorithm="fricas")
```

output
```
1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^3}{x^8}\,dx = \frac{b^3x^2}{2} + \frac{-4a^3-21a^2bx^3-84ab^2x^6}{28x^7}$$

input
```
integrate((b*x**3+a)**3/x**8,x)
```

output
```
b**3*x**2/2 + (-4*a**3 - 21*a**2*b*x**3 - 84*a*b**2*x**6)/(28*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3}{x^8}\,dx = \frac{1}{2}\,b^3x^2 - \frac{84\,ab^2x^6+21\,a^2bx^3+4\,a^3}{28\,x^7}$$

input
```
integrate((b*x^3+a)^3/x^8,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2 - 1/28*(84*a*b^2*x^6 + 21*a^2*b*x^3 + 4*a^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3}{x^8}\,dx = \frac{1}{2}\,b^3x^2 - \frac{84\,ab^2x^6+21\,a^2bx^3+4\,a^3}{28\,x^7}$$

input
```
integrate((b*x^3+a)^3/x^8,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 - 1/28*(84*a*b^2*x^6 + 21*a^2*b*x^3 + 4*a^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^3}{x^8}\,dx = \frac{b^3\,x^2}{2} - \frac{\frac{a^3}{7} + \frac{3\,a^2\,b\,x^3}{4} + 3\,a\,b^2\,x^6}{x^7}$$

input
```
int((a + b*x^3)^3/x^8,x)
```

output
```
(b^3*x^2)/2 - (a^3/7 + (3*a^2*b*x^3)/4 + 3*a*b^2*x^6)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^3}{x^8}\,dx = \frac{14b^3x^9 - 84a\,b^2x^6 - 21a^2b\,x^3 - 4a^3}{28x^7}$$

input
```
int((b*x^3+a)^3/x^8,x)
```

output
```
( - 4*a**3 - 21*a**2*b*x**3 - 84*a*b**2*x**6 + 14*b**3*x**9)/(28*x**7)
```

3.53 $\int x^{17}(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{18}}{18}+\frac{5}{21}a^4bx^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}ab^4x^{30}+\frac{b^5x^{33}}{33}$$

output
```
1/18*a^5*x^18+5/21*a^4*b*x^21+5/12*a^3*b^2*x^24+10/27*a^2*b^3*x^27+1/6*a*b
^4*x^30+1/33*b^5*x^33
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{18}}{18}+\frac{5}{21}a^4bx^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}ab^4x^{30}+\frac{b^5x^{33}}{33}$$

input
```
Integrate[x^17*(a + b*x^3)^5,x]
```

output
```
(a^5*x^18)/18 + (5*a^4*b*x^21)/21 + (5*a^3*b^2*x^24)/12 + (10*a^2*b^3*x^27
)/27 + (a*b^4*x^30)/6 + (b^5*x^33)/33
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{17} \left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^{15} \left(bx^3+a\right)^5 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(b^5x^{30}+5ab^4x^{27}+10a^2b^3x^{24}+10a^3b^2x^{21}+5a^4bx^{18}+a^5x^{15}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^5x^{18}}{6}+\frac{5}{7}a^4bx^{21}+\frac{5}{4}a^3b^2x^{24}+\frac{10}{9}a^2b^3x^{27}+\frac{1}{2}ab^4x^{30}+\frac{b^5x^{33}}{11}\right)$$

input
```
Int[x^17*(a + b*x^3)^5,x]
```

output
```
((a^5*x^18)/6 + (5*a^4*b*x^21)/7 + (5*a^3*b^2*x^24)/4 + (10*a^2*b^3*x^27)/
9 + (a*b^4*x^30)/2 + (b^5*x^33)/11)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{18}a^5x^{18}+\frac{5}{21}a^4b\,x^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}a\,b^4x^{30}+\frac{1}{33}b^5x^{33}$	58
default	$\frac{1}{18}a^5x^{18}+\frac{5}{21}a^4b\,x^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}a\,b^4x^{30}+\frac{1}{33}b^5x^{33}$	58
norman	$\frac{1}{18}a^5x^{18}+\frac{5}{21}a^4b\,x^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}a\,b^4x^{30}+\frac{1}{33}b^5x^{33}$	58
risch	$\frac{1}{18}a^5x^{18}+\frac{5}{21}a^4b\,x^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}a\,b^4x^{30}+\frac{1}{33}b^5x^{33}$	58
parallelrisch	$\frac{1}{18}a^5x^{18}+\frac{5}{21}a^4b\,x^{21}+\frac{5}{12}a^3b^2x^{24}+\frac{10}{27}a^2b^3x^{27}+\frac{1}{6}a\,b^4x^{30}+\frac{1}{33}b^5x^{33}$	58
orering	$\frac{x^{18}\left(252b^5x^{15}+1386a\,b^4x^{12}+3080a^2b^3x^9+3465a^3b^2x^6+1980a^4b\,x^3+462a^5\right)}{8316}$	60

input
```
int(x^17*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/18*a^5*x^18+5/21*a^4*b*x^21+5/12*a^3*b^2*x^24+10/27*a^2*b^3*x^27+1/6*a*b
^4*x^30+1/33*b^5*x^33
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{1}{33}\,b^5x^{33}+\frac{1}{6}\,ab^4x^{30}+\frac{10}{27}\,a^2b^3x^{27}+\frac{5}{12}\,a^3b^2x^{24}+\frac{5}{21}\,a^4bx^{21}+\frac{1}{18}\,a^5x^{18}$$

input
```
integrate(x^17*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/33*b^5*x^33 + 1/6*a*b^4*x^30 + 10/27*a^2*b^3*x^27 + 5/12*a^3*b^2*x^24 +
5/21*a^4*b*x^21 + 1/18*a^5*x^18
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{18}}{18}+\frac{5a^4bx^{21}}{21}+\frac{5a^3b^2x^{24}}{12}+\frac{10a^2b^3x^{27}}{27}+\frac{ab^4x^{30}}{6}+\frac{b^5x^{33}}{33}$$

input
```
integrate(x**17*(b*x**3+a)**5,x)
```

output
```
a**5*x**18/18 + 5*a**4*b*x**21/21 + 5*a**3*b**2*x**24/12 + 10*a**2*b**3*x*
*27/27 + a*b**4*x**30/6 + b**5*x**33/33
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{1}{33}\,b^5x^{33}+\frac{1}{6}\,ab^4x^{30}+\frac{10}{27}\,a^2b^3x^{27}+\frac{5}{12}\,a^3b^2x^{24}+\frac{5}{21}\,a^4bx^{21}+\frac{1}{18}\,a^5x^{18}$$

input
```
integrate(x^17*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/33*b^5*x^33 + 1/6*a*b^4*x^30 + 10/27*a^2*b^3*x^27 + 5/12*a^3*b^2*x^24 +
5/21*a^4*b*x^21 + 1/18*a^5*x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{1}{33}\,b^5x^{33}+\frac{1}{6}\,ab^4x^{30}+\frac{10}{27}\,a^2b^3x^{27}+\frac{5}{12}\,a^3b^2x^{24}+\frac{5}{21}\,a^4bx^{21}+\frac{1}{18}\,a^5x^{18}$$

input
```
integrate(x^17*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/33*b^5*x^33 + 1/6*a*b^4*x^30 + 10/27*a^2*b^3*x^27 + 5/12*a^3*b^2*x^24 +
5/21*a^4*b*x^21 + 1/18*a^5*x^18
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{a^5\,x^{18}}{18}+\frac{5\,a^4\,b\,x^{21}}{21}+\frac{5\,a^3\,b^2\,x^{24}}{12}+\frac{10\,a^2\,b^3\,x^{27}}{27}+\frac{a\,b^4\,x^{30}}{6}+\frac{b^5\,x^{33}}{33}$$

input
```
int(x^17*(a + b*x^3)^5,x)
```

output
```
(a^5*x^18)/18 + (b^5*x^33)/33 + (5*a^4*b*x^21)/21 + (a*b^4*x^30)/6 + (5*a^
3*b^2*x^24)/12 + (10*a^2*b^3*x^27)/27
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^{17}\left(a+bx^3\right)^5\,dx=\frac{x^{18}(252b^5x^{15}+1386a\,b^4x^{12}+3080a^2b^3x^9+3465a^3b^2x^6+1980a^4b\,x^3+462a^5)}{8316}$$

input
```
int(x^17*(b*x^3+a)^5,x)
```

output
```
(x**18*(462*a**5 + 1980*a**4*b*x**3 + 3465*a**3*b**2*x**6 + 3080*a**2*b**3
*x**9 + 1386*a*b**4*x**12 + 252*b**5*x**15))/8316
```

3.54 $\int x^{14}(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^{14}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{15}}{15}+\frac{5}{18}a^4bx^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}ab^4x^{27}+\frac{b^5x^{30}}{30}$$

output
```
1/15*a^5*x^15+5/18*a^4*b*x^18+10/21*a^3*b^2*x^21+5/12*a^2*b^3*x^24+5/27*a*
b^4*x^27+1/30*b^5*x^30
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^{14}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{15}}{15}+\frac{5}{18}a^4bx^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}ab^4x^{27}+\frac{b^5x^{30}}{30}$$

input
```
Integrate[x^14*(a + b*x^3)^5,x]
```

output
```
(a^5*x^15)/15 + (5*a^4*b*x^18)/18 + (10*a^3*b^2*x^21)/21 + (5*a^2*b^3*x^24
)/12 + (5*a*b^4*x^27)/27 + (b^5*x^30)/30
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}\left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^{12}\left(bx^3+a\right)^5dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^5x^{27}+5ab^4x^{24}+10a^2b^3x^{21}+10a^3b^2x^{18}+5a^4bx^{15}+a^5x^{12}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^5x^{15}}{5}+\frac{5}{6}a^4bx^{18}+\frac{10}{7}a^3b^2x^{21}+\frac{5}{4}a^2b^3x^{24}+\frac{5}{9}ab^4x^{27}+\frac{b^5x^{30}}{10}\right)$$

input
```
Int[x^14*(a + b*x^3)^5,x]
```

output
```
((a^5*x^15)/5 + (5*a^4*b*x^18)/6 + (10*a^3*b^2*x^21)/7 + (5*a^2*b^3*x^24)/
4 + (5*a*b^4*x^27)/9 + (b^5*x^30)/10)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{15}a^5x^{15}+\frac{5}{18}a^4b\,x^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}a\,b^4x^{27}+\frac{1}{30}b^5x^{30}$	58
default	$\frac{1}{15}a^5x^{15}+\frac{5}{18}a^4b\,x^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}a\,b^4x^{27}+\frac{1}{30}b^5x^{30}$	58
norman	$\frac{1}{15}a^5x^{15}+\frac{5}{18}a^4b\,x^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}a\,b^4x^{27}+\frac{1}{30}b^5x^{30}$	58
risch	$\frac{1}{15}a^5x^{15}+\frac{5}{18}a^4b\,x^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}a\,b^4x^{27}+\frac{1}{30}b^5x^{30}$	58
parallelrisch	$\frac{1}{15}a^5x^{15}+\frac{5}{18}a^4b\,x^{18}+\frac{10}{21}a^3b^2x^{21}+\frac{5}{12}a^2b^3x^{24}+\frac{5}{27}a\,b^4x^{27}+\frac{1}{30}b^5x^{30}$	58
orering	$\frac{x^{15}\left(126b^5x^{15}+700a\,b^4x^{12}+1575a^2b^3x^9+1800a^3b^2x^6+1050a^4b\,x^3+252a^5\right)}{3780}$	60

input
```
int(x^14*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/15*a^5*x^15+5/18*a^4*b*x^18+10/21*a^3*b^2*x^21+5/12*a^2*b^3*x^24+5/27*a*
b^4*x^27+1/30*b^5*x^30
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{14}\left(a+bx^3\right)^5\,dx=\frac{1}{30}\,b^5x^{30}+\frac{5}{27}\,ab^4x^{27}+\frac{5}{12}\,a^2b^3x^{24}+\frac{10}{21}\,a^3b^2x^{21}+\frac{5}{18}\,a^4bx^{18}+\frac{1}{15}\,a^5x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/30*b^5*x^30 + 5/27*a*b^4*x^27 + 5/12*a^2*b^3*x^24 + 10/21*a^3*b^2*x^21 +
 5/18*a^4*b*x^18 + 1/15*a^5*x^15
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^{14}(a+bx^3)^5\,dx=\frac{a^5x^{15}}{15}+\frac{5a^4bx^{18}}{18}+\frac{10a^3b^2x^{21}}{21}+\frac{5a^2b^3x^{24}}{12}+\frac{5ab^4x^{27}}{27}+\frac{b^5x^{30}}{30}$$

input
```
integrate(x**14*(b*x**3+a)**5,x)
```

output
```
a**5*x**15/15 + 5*a**4*b*x**18/18 + 10*a**3*b**2*x**21/21 + 5*a**2*b**3*x*
*24/12 + 5*a*b**4*x**27/27 + b**5*x**30/30
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{14}(a+bx^3)^5\,dx=\frac{1}{30}\,b^5x^{30}+\frac{5}{27}\,ab^4x^{27}+\frac{5}{12}\,a^2b^3x^{24}+\frac{10}{21}\,a^3b^2x^{21}+\frac{5}{18}\,a^4bx^{18}+\frac{1}{15}\,a^5x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/30*b^5*x^30 + 5/27*a*b^4*x^27 + 5/12*a^2*b^3*x^24 + 10/21*a^3*b^2*x^21 +
 5/18*a^4*b*x^18 + 1/15*a^5*x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{14}\left(a+bx^3\right)^5\,dx=\frac{1}{30}\,b^5x^{30}+\frac{5}{27}\,ab^4x^{27}+\frac{5}{12}\,a^2b^3x^{24}+\frac{10}{21}\,a^3b^2x^{21}+\frac{5}{18}\,a^4bx^{18}+\frac{1}{15}\,a^5x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/30*b^5*x^30 + 5/27*a*b^4*x^27 + 5/12*a^2*b^3*x^24 + 10/21*a^3*b^2*x^21 +
 5/18*a^4*b*x^18 + 1/15*a^5*x^15
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{14}\left(a+bx^3\right)^5\,dx=\frac{a^5\,x^{15}}{15}+\frac{5\,a^4\,b\,x^{18}}{18}+\frac{10\,a^3\,b^2\,x^{21}}{21}+\frac{5\,a^2\,b^3\,x^{24}}{12}+\frac{5\,a\,b^4\,x^{27}}{27}+\frac{b^5\,x^{30}}{30}$$

input
```
int(x^14*(a + b*x^3)^5,x)
```

output
```
(a^5*x^15)/15 + (b^5*x^30)/30 + (5*a^4*b*x^18)/18 + (5*a*b^4*x^27)/27 + (1
0*a^3*b^2*x^21)/21 + (5*a^2*b^3*x^24)/12
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^{14}\left(a+bx^3\right)^5\,dx=\frac{x^{15}\left(126b^5x^{15}+700a\,b^4x^{12}+1575a^2b^3x^9+1800a^3b^2x^6+1050a^4b\,x^3+252a^5\right)}{3780}$$

input
```
int(x^14*(b*x^3+a)^5,x)
```

output

```
(x**15*(252*a**5 + 1050*a**4*b*x**3 + 1800*a**3*b**2*x**6 + 1575*a**2*b**3
*x**9 + 700*a*b**4*x**12 + 126*b**5*x**15))/3780
```

3.55 $\int x^{11}(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^{11}\left(a+bx^3\right)^5\,dx=-\frac{a^3(a+bx^3)^6}{18b^4}+\frac{a^2(a+bx^3)^7}{7b^4}-\frac{a(a+bx^3)^8}{8b^4}+\frac{(a+bx^3)^9}{27b^4}$$

output
```
-1/18*a^3*(b*x^3+a)^6/b^4+1/7*a^2*(b*x^3+a)^7/b^4-1/8*a*(b*x^3+a)^8/b^4+1/
27*(b*x^3+a)^9/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int x^{11}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{12}}{12}+\frac{1}{3}a^4bx^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{10}{21}a^2b^3x^{21}+\frac{5}{24}ab^4x^{24}+\frac{b^5x^{27}}{27}$$

input
```
Integrate[x^11*(a + b*x^3)^5,x]
```

output
```
(a^5*x^12)/12 + (a^4*b*x^15)/3 + (5*a^3*b^2*x^18)/9 + (10*a^2*b^3*x^21)/21
 + (5*a*b^4*x^24)/24 + (b^5*x^27)/27
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \left(a+b x^3\right)^5 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^9 \left(b x^3+a\right)^5 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(\frac{\left(b x^3+a\right)^8}{b^3}-\frac{3 a \left(b x^3+a\right)^7}{b^3}+\frac{3 a^2 \left(b x^3+a\right)^6}{b^3}-\frac{a^3 \left(b x^3+a\right)^5}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{a^3 \left(a+b x^3\right)^6}{6 b^4}+\frac{3 a^2 \left(a+b x^3\right)^7}{7 b^4}+\frac{\left(a+b x^3\right)^9}{9 b^4}-\frac{3 a \left(a+b x^3\right)^8}{8 b^4}\right)$$

input
```
Int[x^11*(a + b*x^3)^5,x]
```

output
```
(-1/6*(a^3*(a + b*x^3)^6)/b^4 + (3*a^2*(a + b*x^3)^7)/(7*b^4) - (3*a*(a +
b*x^3)^8)/(8*b^4) + (a + b*x^3)^9/(9*b^4))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{1}{3}a^4b\,x^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{24}a\,b^4x^{24}+\frac{1}{27}b^5x^{27}+\frac{10}{21}a^2b^3x^{21}+\frac{1}{12}a^5x^{12}$	58
default	$\frac{1}{3}a^4b\,x^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{24}a\,b^4x^{24}+\frac{1}{27}b^5x^{27}+\frac{10}{21}a^2b^3x^{21}+\frac{1}{12}a^5x^{12}$	58
norman	$\frac{1}{3}a^4b\,x^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{24}a\,b^4x^{24}+\frac{1}{27}b^5x^{27}+\frac{10}{21}a^2b^3x^{21}+\frac{1}{12}a^5x^{12}$	58
risch	$\frac{1}{3}a^4b\,x^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{24}a\,b^4x^{24}+\frac{1}{27}b^5x^{27}+\frac{10}{21}a^2b^3x^{21}+\frac{1}{12}a^5x^{12}$	58
parallelrisch	$\frac{1}{3}a^4b\,x^{15}+\frac{5}{9}a^3b^2x^{18}+\frac{5}{24}a\,b^4x^{24}+\frac{1}{27}b^5x^{27}+\frac{10}{21}a^2b^3x^{21}+\frac{1}{12}a^5x^{12}$	58
orering	$\frac{x^{12}\left(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5\right)}{1512}$	60

input
```
int(x^11*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*b*x^15+5/9*a^3*b^2*x^18+5/24*a*b^4*x^24+1/27*b^5*x^27+10/21*a^2*b^
3*x^21+1/12*a^5*x^12
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^{11}\left(a+bx^3\right)^5\,dx=\frac{1}{27}\,b^5x^{27}+\frac{5}{24}\,ab^4x^{24}+\frac{10}{21}\,a^2b^3x^{21}+\frac{5}{9}\,a^3b^2x^{18}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 +
1/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int x^{11}\left(a+bx^3\right)^5\,dx=\frac{a^5x^{12}}{12}+\frac{a^4bx^{15}}{3}+\frac{5a^3b^2x^{18}}{9}+\frac{10a^2b^3x^{21}}{21}+\frac{5ab^4x^{24}}{24}+\frac{b^5x^{27}}{27}$$

input
```
integrate(x**11*(b*x**3+a)**5,x)
```

output
```
a**5*x**12/12 + a**4*b*x**15/3 + 5*a**3*b**2*x**18/9 + 10*a**2*b**3*x**21/
21 + 5*a*b**4*x**24/24 + b**5*x**27/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^{11}\left(a+bx^3\right)^5\,dx=\frac{1}{27}\,b^5x^{27}+\frac{5}{24}\,ab^4x^{24}+\frac{10}{21}\,a^2b^3x^{21}+\frac{5}{9}\,a^3b^2x^{18}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 +
1/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^{11}(a+bx^3)^5\,dx=\frac{1}{27}b^5x^{27}+\frac{5}{24}ab^4x^{24}+\frac{10}{21}a^2b^3x^{21}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{3}a^4bx^{15}+\frac{1}{12}a^5x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 +
1/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^{11}(a+bx^3)^5\,dx=\frac{a^5\,x^{12}}{12}+\frac{a^4\,b\,x^{15}}{3}+\frac{5\,a^3\,b^2\,x^{18}}{9}+\frac{10\,a^2\,b^3\,x^{21}}{21}+\frac{5\,a\,b^4\,x^{24}}{24}+\frac{b^5\,x^{27}}{27}$$

input
```
int(x^11*(a + b*x^3)^5,x)
```

output
```
(a^5*x^12)/12 + (b^5*x^27)/27 + (a^4*b*x^15)/3 + (5*a*b^4*x^24)/24 + (5*a^
3*b^2*x^18)/9 + (10*a^2*b^3*x^21)/21
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int x^{11}(a+bx^3)^5\,dx=\frac{x^{12}(56b^5x^{15}+315a\,b^4x^{12}+720a^2b^3x^9+840a^3b^2x^6+504a^4b\,x^3+126a^5)}{1512}$$

input
```
int(x^11*(b*x^3+a)^5,x)
```

output
```
(x**12*(126*a**5 + 504*a**4*b*x**3 + 840*a**3*b**2*x**6 + 720*a**2*b**3*x*
*9 + 315*a*b**4*x**12 + 56*b**5*x**15))/1512
```

3.56 $\int x^8(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^8\left(a+bx^3\right)^5\,dx=\frac{a^2(a+bx^3)^6}{18b^3}-\frac{2a(a+bx^3)^7}{21b^3}+\frac{(a+bx^3)^8}{24b^3}$$

output
```
1/18*a^2*(b*x^3+a)^6/b^3-2/21*a*(b*x^3+a)^7/b^3+1/24*(b*x^3+a)^8/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int x^8\left(a+bx^3\right)^5\,dx=\frac{a^5x^9}{9}+\frac{5}{12}a^4bx^{12}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{5}{21}ab^4x^{21}+\frac{b^5x^{24}}{24}$$

input
```
Integrate[x^8*(a + b*x^3)^5,x]
```

output
```
(a^5*x^9)/9 + (5*a^4*b*x^12)/12 + (2*a^3*b^2*x^15)/3 + (5*a^2*b^3*x^18)/9
+ (5*a*b^4*x^21)/21 + (b^5*x^24)/24
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^6 \left(bx^3+a\right)^5 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^7}{b^2}-\frac{2a\left(bx^3+a\right)^6}{b^2}+\frac{a^2\left(bx^3+a\right)^5}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^2\left(a+bx^3\right)^6}{6b^3}+\frac{\left(a+bx^3\right)^8}{8b^3}-\frac{2a\left(a+bx^3\right)^7}{7b^3}\right)$$

input
```
Int[x^8*(a + b*x^3)^5,x]
```

output
```
((a^2*(a + b*x^3)^6)/(6*b^3) - (2*a*(a + b*x^3)^7)/(7*b^3) + (a + b*x^3)^8
/(8*b^3))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{5}{12}a^4b\,x^{12}+\frac{1}{24}b^5x^{24}+\frac{5}{21}a\,b^4x^{21}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{9}a^5x^9$	58
default	$\frac{5}{12}a^4b\,x^{12}+\frac{1}{24}b^5x^{24}+\frac{5}{21}a\,b^4x^{21}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{9}a^5x^9$	58
norman	$\frac{5}{12}a^4b\,x^{12}+\frac{1}{24}b^5x^{24}+\frac{5}{21}a\,b^4x^{21}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{9}a^5x^9$	58
risch	$\frac{5}{12}a^4b\,x^{12}+\frac{1}{24}b^5x^{24}+\frac{5}{21}a\,b^4x^{21}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{9}a^5x^9$	58
parallelrisch	$\frac{5}{12}a^4b\,x^{12}+\frac{1}{24}b^5x^{24}+\frac{5}{21}a\,b^4x^{21}+\frac{2}{3}a^3b^2x^{15}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{9}a^5x^9$	58
orering	$\frac{x^9\left(21b^5x^{15}+120a\,b^4x^{12}+280a^2b^3x^9+336a^3b^2x^6+210a^4b\,x^3+56a^5\right)}{504}$	60

input
```
int(x^8*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
5/12*a^4*b*x^12+1/24*b^5*x^24+5/21*a*b^4*x^21+2/3*a^3*b^2*x^15+5/9*a^2*b^3
*x^18+1/9*a^5*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x^8\left(a+bx^3\right)^5\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{21}\,ab^4x^{21}+\frac{5}{9}\,a^2b^3x^{18}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{9}\,a^5x^9$$

input
```
integrate(x^8*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/
12*a^4*b*x^12 + 1/9*a^5*x^9
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int x^8 (a + bx^3)^5 \, dx = \frac{a^5 x^9}{9} + \frac{5a^4 b x^{12}}{12} + \frac{2a^3 b^2 x^{15}}{3} + \frac{5a^2 b^3 x^{18}}{9} + \frac{5ab^4 x^{21}}{21} + \frac{b^5 x^{24}}{24}$$

input
```
integrate(x**8*(b*x**3+a)**5,x)
```

output
```
a**5*x**9/9 + 5*a**4*b*x**12/12 + 2*a**3*b**2*x**15/3 + 5*a**2*b**3*x**18/
9 + 5*a*b**4*x**21/21 + b**5*x**24/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x^8 (a + bx^3)^5 \, dx = \frac{1}{24} b^5 x^{24} + \frac{5}{21} ab^4 x^{21} + \frac{5}{9} a^2 b^3 x^{18} + \frac{2}{3} a^3 b^2 x^{15} + \frac{5}{12} a^4 b x^{12} + \frac{1}{9} a^5 x^9$$

input
```
integrate(x^8*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/
12*a^4*b*x^12 + 1/9*a^5*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x^8 \left(a+bx^3\right)^5 \, dx = \frac{1}{24} b^5 x^{24} + \frac{5}{21} a b^4 x^{21} + \frac{5}{9} a^2 b^3 x^{18} + \frac{2}{3} a^3 b^2 x^{15} + \frac{5}{12} a^4 b x^{12} + \frac{1}{9} a^5 x^9$$

input
```
integrate(x^8*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/
12*a^4*b*x^12 + 1/9*a^5*x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x^8 \left(a+bx^3\right)^5 \, dx = \frac{a^5\, x^9}{9} + \frac{5\, a^4\, b\, x^{12}}{12} + \frac{2\, a^3\, b^2\, x^{15}}{3} + \frac{5\, a^2\, b^3\, x^{18}}{9} + \frac{5\, a\, b^4\, x^{21}}{21} + \frac{b^5\, x^{24}}{24}$$

input
```
int(x^8*(a + b*x^3)^5,x)
```

output
```
(a^5*x^9)/9 + (b^5*x^24)/24 + (5*a^4*b*x^12)/12 + (5*a*b^4*x^21)/21 + (2*a
^3*b^2*x^15)/3 + (5*a^2*b^3*x^18)/9
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int x^8 \left(a+bx^3\right)^5 \, dx = \frac{x^9 (21 b^5 x^{15} + 120 a\, b^4 x^{12} + 280 a^2 b^3 x^9 + 336 a^3 b^2 x^6 + 210 a^4 b\, x^3 + 56 a^5)}{504}$$

input
```
int(x^8*(b*x^3+a)^5,x)
```

output
```
(x**9*(56*a**5 + 210*a**4*b*x**3 + 336*a**3*b**2*x**6 + 280*a**2*b**3*x**9
 + 120*a*b**4*x**12 + 21*b**5*x**15))/504
```

3.57 $\int x^5(a+bx^3)^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^5\left(a+bx^3\right)^5 \, dx = -\frac{a(a+bx^3)^6}{18b^2}+\frac{(a+bx^3)^7}{21b^2}$$

output
```
-1/18*a*(b*x^3+a)^6/b^2+1/21*(b*x^3+a)^7/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(34) = 68.

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int x^5\left(a+bx^3\right)^5 \, dx = \frac{a^5x^6}{6}+\frac{5}{9}a^4bx^9+\frac{5}{6}a^3b^2x^{12}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{21}}{21}$$

input
```
Integrate[x^5*(a + b*x^3)^5,x]
```

output
```
(a^5*x^6)/6 + (5*a^4*b*x^9)/9 + (5*a^3*b^2*x^12)/6 + (2*a^2*b^3*x^15)/3 +
(5*a*b^4*x^18)/18 + (b^5*x^21)/21
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^3 \left(bx^3+a\right)^5 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^6}{b}-\frac{a\left(bx^3+a\right)^5}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3\right)^7}{7b^2}-\frac{a\left(a+bx^3\right)^6}{6b^2}\right)$$

input
```
Int[x^5*(a + b*x^3)^5,x]
```

output
```
(-1/6*(a*(a + b*x^3)^6)/b^2 + (a + b*x^3)^7/(7*b^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
gosper	$\frac{1}{6}a^5x^6+\frac{5}{9}a^4b\,x^9+\frac{5}{6}a^3b^2x^{12}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{21}b^5x^{21}$	58
default	$\frac{1}{6}a^5x^6+\frac{5}{9}a^4b\,x^9+\frac{5}{6}a^3b^2x^{12}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{21}b^5x^{21}$	58
norman	$\frac{1}{6}a^5x^6+\frac{5}{9}a^4b\,x^9+\frac{5}{6}a^3b^2x^{12}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{21}b^5x^{21}$	58
risch	$\frac{1}{6}a^5x^6+\frac{5}{9}a^4b\,x^9+\frac{5}{6}a^3b^2x^{12}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{21}b^5x^{21}$	58
parallelrisch	$\frac{1}{6}a^5x^6+\frac{5}{9}a^4b\,x^9+\frac{5}{6}a^3b^2x^{12}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{21}b^5x^{21}$	58
orering	$\frac{x^6(6b^5x^{15}+35a\,b^4x^{12}+84a^2b^3x^9+105a^3b^2x^6+70a^4b\,x^3+21a^5)}{126}$	60

input
```
int(x^5*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^5*x^6+5/9*a^4*b*x^9+5/6*a^3*b^2*x^12+2/3*a^2*b^3*x^15+5/18*a*b^4*x^1
8+1/21*b^5*x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int x^5(a+bx^3)^5\,dx=\frac{1}{21}\,b^5x^{21}+\frac{5}{18}\,ab^4x^{18}+\frac{2}{3}\,a^2b^3x^{15}+\frac{5}{6}\,a^3b^2x^{12}+\frac{5}{9}\,a^4bx^9+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/
9*a^4*b*x^9 + 1/6*a^5*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int x^5 (a + bx^3)^5 \, dx = \frac{a^5x^6}{6} + \frac{5a^4bx^9}{9} + \frac{5a^3b^2x^{12}}{6} + \frac{2a^2b^3x^{15}}{3} + \frac{5ab^4x^{18}}{18} + \frac{b^5x^{21}}{21}$$

input
```
integrate(x**5*(b*x**3+a)**5,x)
```

output
```
a**5*x**6/6 + 5*a**4*b*x**9/9 + 5*a**3*b**2*x**12/6 + 2*a**2*b**3*x**15/3
+ 5*a*b**4*x**18/18 + b**5*x**21/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int x^5 (a + bx^3)^5 \, dx = \frac{1}{21} b^5x^{21} + \frac{5}{18} ab^4x^{18} + \frac{2}{3} a^2b^3x^{15} + \frac{5}{6} a^3b^2x^{12} + \frac{5}{9} a^4bx^9 + \frac{1}{6} a^5x^6$$

input
```
integrate(x^5*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/
9*a^4*b*x^9 + 1/6*a^5*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int x^5 (a + bx^3)^5 \, dx = \frac{1}{21} b^5 x^{21} + \frac{5}{18} ab^4 x^{18} + \frac{2}{3} a^2 b^3 x^{15} + \frac{5}{6} a^3 b^2 x^{12} + \frac{5}{9} a^4 b x^9 + \frac{1}{6} a^5 x^6$$

input
```
integrate(x^5*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/
9*a^4*b*x^9 + 1/6*a^5*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int x^5 (a + bx^3)^5 \, dx = \frac{a^5 x^6}{6} + \frac{5 a^4 b x^9}{9} + \frac{5 a^3 b^2 x^{12}}{6} + \frac{2 a^2 b^3 x^{15}}{3} + \frac{5 a b^4 x^{18}}{18} + \frac{b^5 x^{21}}{21}$$

input
```
int(x^5*(a + b*x^3)^5,x)
```

output
```
(a^5*x^6)/6 + (b^5*x^21)/21 + (5*a^4*b*x^9)/9 + (5*a*b^4*x^18)/18 + (5*a^3
*b^2*x^12)/6 + (2*a^2*b^3*x^15)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int x^5 (a + bx^3)^5 \, dx = \frac{x^6 (6b^5 x^{15} + 35a b^4 x^{12} + 84a^2 b^3 x^9 + 105a^3 b^2 x^6 + 70a^4 b x^3 + 21a^5)}{126}$$

input
```
int(x^5*(b*x^3+a)^5,x)
```

output
```
(x**6*(21*a**5 + 70*a**4*b*x**3 + 105*a**3*b**2*x**6 + 84*a**2*b**3*x**9 +
 35*a*b**4*x**12 + 6*b**5*x**15))/126
```

3.58 $\int x^2(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^2\left(a+bx^3\right)^5\,dx=\frac{\left(a+bx^3\right)^6}{18b}$$

output
```
1/18*(b*x^3+a)^6/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.31

$$\int x^2\left(a+bx^3\right)^5\,dx=\frac{a^5x^3}{3}+\frac{5}{6}a^4bx^6+\frac{10}{9}a^3b^2x^9+\frac{5}{6}a^2b^3x^{12}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{18}}{18}$$

input
```
Integrate[x^2*(a + b*x^3)^5,x]
```

output
```
(a^5*x^3)/3 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^9)/9 + (5*a^2*b^3*x^12)/6 +
(a*b^4*x^15)/3 + (b^5*x^18)/18
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^3)^5\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{(a+bx^3)^6}{18b}$$

input
```
Int[x^2*(a + b*x^3)^5,x]
```

output
```
(a + b*x^3)^6/(18*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(b\,x^{3}+a\right)^{6}}{18b}$	15
gosper	$\frac{1}{3}a^{5}x^{3}+\frac{5}{6}a^{4}b\,x^{6}+\frac{10}{9}a^{3}b^{2}x^{9}+\frac{5}{6}a^{2}b^{3}x^{12}+\frac{1}{3}a\,b^{4}x^{15}+\frac{1}{18}b^{5}x^{18}$	58
norman	$\frac{1}{3}a^{5}x^{3}+\frac{5}{6}a^{4}b\,x^{6}+\frac{10}{9}a^{3}b^{2}x^{9}+\frac{5}{6}a^{2}b^{3}x^{12}+\frac{1}{3}a\,b^{4}x^{15}+\frac{1}{18}b^{5}x^{18}$	58
parallelrisch	$\frac{1}{3}a^{5}x^{3}+\frac{5}{6}a^{4}b\,x^{6}+\frac{10}{9}a^{3}b^{2}x^{9}+\frac{5}{6}a^{2}b^{3}x^{12}+\frac{1}{3}a\,b^{4}x^{15}+\frac{1}{18}b^{5}x^{18}$	58
orering	$\frac{x^{3}\left(b^{5}x^{15}+6a\,b^{4}x^{12}+15a^{2}b^{3}x^{9}+20a^{3}b^{2}x^{6}+15a^{4}b\,x^{3}+6a^{5}\right)}{18}$	59
risch	$\frac{b^{5}x^{18}}{18}+\frac{a\,b^{4}x^{15}}{3}+\frac{5a^{2}b^{3}x^{12}}{6}+\frac{10a^{3}b^{2}x^{9}}{9}+\frac{5a^{4}b\,x^{6}}{6}+\frac{a^{5}x^{3}}{3}+\frac{a^{6}}{18b}$	66

input
```
int(x^2*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/18*(b*x^3+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int x^2\left(a+bx^3\right)^5\,dx=\frac{1}{18}\,b^5x^{18}+\frac{1}{3}\,ab^4x^{15}+\frac{5}{6}\,a^2b^3x^{12}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{6}\,a^4bx^6+\frac{1}{3}\,a^5x^3$$

input
```
integrate(x^2*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6
*a^4*b*x^6 + 1/3*a^5*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(10) = 20$.

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int x^2 (a + bx^3)^5 \, dx = \frac{a^5x^3}{3} + \frac{5a^4bx^6}{6} + \frac{10a^3b^2x^9}{9} + \frac{5a^2b^3x^{12}}{6} + \frac{ab^4x^{15}}{3} + \frac{b^5x^{18}}{18}$$

input
```
integrate(x**2*(b*x**3+a)**5,x)
```

output
```
a**5*x**3/3 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**9/9 + 5*a**2*b**3*x**12/6
+ a*b**4*x**15/3 + b**5*x**18/18
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 (a + bx^3)^5 \, dx = \frac{(bx^3 + a)^6}{18 \, b}$$

input
```
integrate(x^2*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/18*(b*x^3 + a)^6/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2 (a + bx^3)^5 \, dx = \frac{(bx^3 + a)^6}{18 \, b}$$

input
```
integrate(x^2*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/18*(b*x^3 + a)^6/b
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int x^2 (a + bx^3)^5 \, dx = \frac{a^5 x^3}{3} + \frac{5 a^4 b x^6}{6} + \frac{10 a^3 b^2 x^9}{9} + \frac{5 a^2 b^3 x^{12}}{6} + \frac{a b^4 x^{15}}{3} + \frac{b^5 x^{18}}{18}$$

input
```
int(x^2*(a + b*x^3)^5,x)
```

output
```
(a^5*x^3)/3 + (b^5*x^18)/18 + (5*a^4*b*x^6)/6 + (a*b^4*x^15)/3 + (10*a^3*b
^2*x^9)/9 + (5*a^2*b^3*x^12)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int x^2 (a + bx^3)^5 \, dx = \frac{x^3 (b^5 x^{15} + 6a b^4 x^{12} + 15a^2 b^3 x^9 + 20a^3 b^2 x^6 + 15a^4 b x^3 + 6a^5)}{18}$$

input
```
int(x^2*(b*x^3+a)^5,x)
```

output
```
(x**3*(6*a**5 + 15*a**4*b*x**3 + 20*a**3*b**2*x**6 + 15*a**2*b**3*x**9 + 6
*a*b**4*x**12 + b**5*x**15))/18
```

3.59 $\int \frac{(a+bx^3)^5}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^3)^5}{x} dx = \frac{5}{3}a^4bx^3 + \frac{5}{3}a^3b^2x^6 + \frac{10}{9}a^2b^3x^9 + \frac{5}{12}ab^4x^{12} + \frac{b^5x^{15}}{15} + a^5\log(x)$$

output
```
5/3*a^4*b*x^3+5/3*a^3*b^2*x^6+10/9*a^2*b^3*x^9+5/12*a*b^4*x^12+1/15*b^5*x^
15+a^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x} dx = \frac{5}{3}a^4bx^3 + \frac{5}{3}a^3b^2x^6 + \frac{10}{9}a^2b^3x^9 + \frac{5}{12}ab^4x^{12} + \frac{b^5x^{15}}{15} + a^5\log(x)$$

input
```
Integrate[(a + b*x^3)^5/x,x]
```

output
```
(5*a^4*b*x^3)/3 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^9)/9 + (5*a*b^4*x^12)/
12 + (b^5*x^15)/15 + a^5*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^3}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^5x^{12}+5ab^4x^9+10a^2b^3x^6+10a^3b^2x^3+5a^4b+\frac{a^5}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(a^5\log\left(x^3\right)+5a^4bx^3+5a^3b^2x^6+\frac{10}{3}a^2b^3x^9+\frac{5}{4}ab^4x^{12}+\frac{b^5x^{15}}{5}\right)$$

input
```
Int[(a + b*x^3)^5/x,x]
```

output
```
(5*a^4*b*x^3 + 5*a^3*b^2*x^6 + (10*a^2*b^3*x^9)/3 + (5*a*b^4*x^12)/4 + (b^
5*x^15)/5 + a^5*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{5a^4b\,x^3}{3}+\frac{5a^3b^2x^6}{3}+\frac{10a^2b^3x^9}{9}+\frac{5a\,b^4x^{12}}{12}+\frac{b^5x^{15}}{15}+a^5\ln\left(x\right)$	56
norman	$\frac{5a^4b\,x^3}{3}+\frac{5a^3b^2x^6}{3}+\frac{10a^2b^3x^9}{9}+\frac{5a\,b^4x^{12}}{12}+\frac{b^5x^{15}}{15}+a^5\ln\left(x\right)$	56
risch	$\frac{5a^4b\,x^3}{3}+\frac{5a^3b^2x^6}{3}+\frac{10a^2b^3x^9}{9}+\frac{5a\,b^4x^{12}}{12}+\frac{b^5x^{15}}{15}+a^5\ln\left(x\right)$	56
parallelrisch	$\frac{5a^4b\,x^3}{3}+\frac{5a^3b^2x^6}{3}+\frac{10a^2b^3x^9}{9}+\frac{5a\,b^4x^{12}}{12}+\frac{b^5x^{15}}{15}+a^5\ln\left(x\right)$	56

input
```
int((b*x^3+a)^5/x,x,method=_RETURNVERBOSE)
```

output
```
5/3*a^4*b*x^3+5/3*a^3*b^2*x^6+10/9*a^2*b^3*x^9+5/12*a*b^4*x^12+1/15*b^5*x^
15+a^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int\frac{\left(a+bx^3\right)^5}{x}\,dx=\frac{1}{15}\,b^5x^{15}+\frac{5}{12}\,ab^4x^{12}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{3}\,a^3b^2x^6+\frac{5}{3}\,a^4bx^3+a^5\log\left(x\right)$$

input
```
integrate((b*x^3+a)^5/x,x, algorithm="fricas")
```

output
```
1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3
*a^4*b*x^3 + a^5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x}\,dx = a^5\log(x) + \frac{5a^4bx^3}{3} + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^9}{9} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{15}}{15}$$

input
```
integrate((b*x**3+a)**5/x,x)
```

output
```
a**5*log(x) + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**9/9 +
 5*a*b**4*x**12/12 + b**5*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x}\,dx = \frac{1}{15}\,b^5x^{15} + \frac{5}{12}\,ab^4x^{12} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{3}\,a^3b^2x^6 + \frac{5}{3}\,a^4bx^3 + \frac{1}{3}\,a^5\log(x^3)$$

input
```
integrate((b*x^3+a)^5/x,x, algorithm="maxima")
```

output
```
1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3
*a^4*b*x^3 + 1/3*a^5*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x}\,dx = \frac{1}{15}\,b^5x^{15} + \frac{5}{12}\,ab^4x^{12} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{3}\,a^3b^2x^6 + \frac{5}{3}\,a^4bx^3 + a^5\log(|x|)$$

input
```
integrate((b*x^3+a)^5/x,x, algorithm="giac")
```

output
```
1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3
*a^4*b*x^3 + a^5*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^5}{x}\,dx = a^5\,\ln(x) + \frac{b^5\,x^{15}}{15} + \frac{5\,a^4\,b\,x^3}{3} + \frac{5\,a\,b^4\,x^{12}}{12} + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{10\,a^2\,b^3\,x^9}{9}$$

input
```
int((a + b*x^3)^5/x,x)
```

output
```
a^5*log(x) + (b^5*x^15)/15 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^12)/12 + (5*a^3*
b^2*x^6)/3 + (10*a^2*b^3*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^5}{x}\,dx = \log(x)\,a^5 + \frac{5a^4b\,x^3}{3} + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^9}{9} + \frac{5a\,b^4x^{12}}{12} + \frac{b^5x^{15}}{15}$$

input
```
int((b*x^3+a)^5/x,x)
```

output
```
(180*log(x)*a**5 + 300*a**4*b*x**3 + 300*a**3*b**2*x**6 + 200*a**2*b**3*x*
*9 + 75*a*b**4*x**12 + 12*b**5*x**15)/180
```

3.60 $\int \frac{(a+bx^3)^5}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{(a+bx^3)^5}{x^4}\,dx = -\frac{a^5}{3x^3} + \frac{10}{3}a^3b^2x^3 + \frac{5}{3}a^2b^3x^6 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{12}}{12} + 5a^4b\log(x)$$

output
```
-1/3*a^5/x^3+10/3*a^3*b^2*x^3+5/3*a^2*b^3*x^6+5/9*a*b^4*x^9+1/12*b^5*x^12+
5*a^4*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^4}\,dx = -\frac{a^5}{3x^3} + \frac{10}{3}a^3b^2x^3 + \frac{5}{3}a^2b^3x^6 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{12}}{12} + 5a^4b\log(x)$$

input
```
Integrate[(a + b*x^3)^5/x^4,x]
```

output
```
-1/3*a^5/x^3 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^9)/9 +
(b^5*x^12)/12 + 5*a^4*b*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^6}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^5x^9+5ab^4x^6+10a^2b^3x^3+10a^3b^2+\frac{5a^4b}{x^3}+\frac{a^5}{x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5}{x^3}+5a^4b\log\left(x^3\right)+10a^3b^2x^3+5a^2b^3x^6+\frac{5}{3}ab^4x^9+\frac{b^5x^{12}}{4}\right)$$

input
```
Int[(a + b*x^3)^5/x^4,x]
```

output
```
(-(a^5/x^3) + 10*a^3*b^2*x^3 + 5*a^2*b^3*x^6 + (5*a*b^4*x^9)/3 + (b^5*x^12
)/4 + 5*a^4*b*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{5}}{3x^{3}}+\frac{10a^{3}b^{2}x^{3}}{3}+\frac{5a^{2}b^{3}x^{6}}{3}+\frac{5a\,b^{4}x^{9}}{9}+\frac{b^{5}x^{12}}{12}+5a^{4}b\ln\left(x\right)$	57
risch	$-\frac{a^{5}}{3x^{3}}+\frac{10a^{3}b^{2}x^{3}}{3}+\frac{5a^{2}b^{3}x^{6}}{3}+\frac{5a\,b^{4}x^{9}}{9}+\frac{b^{5}x^{12}}{12}+5a^{4}b\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{3}a^{5}+\frac{1}{12}b^{5}x^{15}+\frac{5}{9}a\,b^{4}x^{12}+\frac{5}{3}a^{2}b^{3}x^{9}+\frac{10}{3}a^{3}b^{2}x^{6}}{x^{3}}+5a^{4}b\ln\left(x\right)$	59
parallelrisch	$\frac{3b^{5}x^{15}+20a\,b^{4}x^{12}+60a^{2}b^{3}x^{9}+120a^{3}b^{2}x^{6}+180a^{4}b\ln\left(x\right)x^{3}-12a^{5}}{36x^{3}}$	62

input
```
int((b*x^3+a)^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^5/x^3+10/3*a^3*b^2*x^3+5/3*a^2*b^3*x^6+5/9*a*b^4*x^9+1/12*b^5*x^12+
5*a^4*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int\frac{(a+bx^3)^5}{x^4}\,dx=\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log\left(x\right)-12\,a^5}{36\,x^3}$$

input
```
integrate((b*x^3+a)^5/x^4,x, algorithm="fricas")
```

output
```
1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*
a^4*b*x^3*log(x) - 12*a^5)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^5}{x^4} dx = -\frac{a^5}{3x^3} + 5a^4b\log(x) + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^9}{9} + \frac{b^5x^{12}}{12}$$

input
```
integrate((b*x**3+a)**5/x**4,x)
```

output
```
-a**5/(3*x**3) + 5*a**4*b*log(x) + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**6/
3 + 5*a*b**4*x**9/9 + b**5*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^5}{x^4} dx = \frac{1}{12}b^5x^{12} + \frac{5}{9}ab^4x^9 + \frac{5}{3}a^2b^3x^6 + \frac{10}{3}a^3b^2x^3 + \frac{5}{3}a^4b\log(x^3) - \frac{a^5}{3x^3}$$

input
```
integrate((b*x^3+a)^5/x^4,x, algorithm="maxima")
```

output
```
1/12*b^5*x^12 + 5/9*a*b^4*x^9 + 5/3*a^2*b^3*x^6 + 10/3*a^3*b^2*x^3 + 5/3*a
^4*b*log(x^3) - 1/3*a^5/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5}{x^4}\,dx = \frac{1}{12}\,b^5x^{12} + \frac{5}{9}\,ab^4x^9 + \frac{5}{3}\,a^2b^3x^6 + \frac{10}{3}\,a^3b^2x^3 + 5\,a^4b\log\left(|x|\right) - \frac{5\,a^4bx^3 + a^5}{3\,x^3}$$

input
```
integrate((b*x^3+a)^5/x^4,x, algorithm="giac")
```

output
```
1/12*b^5*x^12 + 5/9*a*b^4*x^9 + 5/3*a^2*b^3*x^6 + 10/3*a^3*b^2*x^3 + 5*a^4
*b*log(abs(x)) - 1/3*(5*a^4*b*x^3 + a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^5}{x^4}\,dx = \frac{b^5\,x^{12}}{12} - \frac{a^5}{3\,x^3} + \frac{5\,a\,b^4\,x^9}{9} + 5\,a^4\,b\,\ln\left(x\right) + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^6}{3}$$

input
```
int((a + b*x^3)^5/x^4,x)
```

output
```
(b^5*x^12)/12 - a^5/(3*x^3) + (5*a*b^4*x^9)/9 + 5*a^4*b*log(x) + (10*a^3*b
^2*x^3)/3 + (5*a^2*b^3*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^4}\,dx = \frac{180\log(x)\,a^4b\,x^3 - 12a^5 + 120a^3b^2x^6 + 60a^2b^3x^9 + 20a\,b^4x^{12} + 3b^5x^{15}}{36x^3}$$

input
```
int((b*x^3+a)^5/x^4,x)
```

output
```
(180*log(x)*a**4*b*x**3 - 12*a**5 + 120*a**3*b**2*x**6 + 60*a**2*b**3*x**9
 + 20*a*b**4*x**12 + 3*b**5*x**15)/(36*x**3)
```

3.61 $\int \frac{(a+bx^3)^5}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{(a+bx^3)^5}{x^7} dx = -\frac{a^5}{6x^6} - \frac{5a^4b}{3x^3} + \frac{10}{3}a^2b^3x^3 + \frac{5}{6}ab^4x^6 + \frac{b^5x^9}{9} + 10a^3b^2\log(x)$$

output
```
-1/6*a^5/x^6-5/3*a^4*b/x^3+10/3*a^2*b^3*x^3+5/6*a*b^4*x^6+1/9*b^5*x^9+10*a
^3*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^7} dx = -\frac{a^5}{6x^6} - \frac{5a^4b}{3x^3} + \frac{10}{3}a^2b^3x^3 + \frac{5}{6}ab^4x^6 + \frac{b^5x^9}{9} + 10a^3b^2\log(x)$$

input
```
Integrate[(a + b*x^3)^5/x^7,x]
```

output
```
-1/6*a^5/x^6 - (5*a^4*b)/(3*x^3) + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^6)/6 +
(b^5*x^9)/9 + 10*a^3*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^7}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^9}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^5x^6+5ab^4x^3+10a^2b^3+\frac{10a^3b^2}{x^3}+\frac{5a^4b}{x^6}+\frac{a^5}{x^9}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5}{2x^6}-\frac{5a^4b}{x^3}+10a^3b^2\log\left(x^3\right)+10a^2b^3x^3+\frac{5}{2}ab^4x^6+\frac{b^5x^9}{3}\right)$$

input
```
Int[(a + b*x^3)^5/x^7,x]
```

output
```
(-1/2*a^5/x^6 - (5*a^4*b)/x^3 + 10*a^2*b^3*x^3 + (5*a*b^4*x^6)/2 + (b^5*x^
9)/3 + 10*a^3*b^2*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{5}}{6x^{6}}-\frac{5a^{4}b}{3x^{3}}+\frac{10a^{2}b^{3}x^{3}}{3}+\frac{5a\,b^{4}x^{6}}{6}+\frac{b^{5}x^{9}}{9}+10a^{3}b^{2}\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{6}a^{5}+\frac{1}{9}b^{5}x^{15}+\frac{5}{6}a\,b^{4}x^{12}+\frac{10}{3}a^{2}b^{3}x^{9}-\frac{5}{3}a^{4}b\,x^{3}}{x^{6}}+10a^{3}b^{2}\ln\left(x\right)$	59
risch	$\frac{b^{5}x^{9}}{9}+\frac{5a\,b^{4}x^{6}}{6}+\frac{10a^{2}b^{3}x^{3}}{3}+\frac{-\frac{5}{3}a^{4}b\,x^{3}-\frac{1}{6}a^{5}}{x^{6}}+10a^{3}b^{2}\ln\left(x\right)$	59
parallelrisch	$\frac{2b^{5}x^{15}+15a\,b^{4}x^{12}+60a^{2}b^{3}x^{9}+180a^{3}b^{2}\ln\left(x\right)x^{6}-30a^{4}b\,x^{3}-3a^{5}}{18x^{6}}$	62

input
```
int((b*x^3+a)^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^5/x^6-5/3*a^4*b/x^3+10/3*a^2*b^3*x^3+5/6*a*b^4*x^6+1/9*b^5*x^9+10*a
^3*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+bx^{3}\right)^{5}}{x^{7}}\,dx=\frac{2\,b^{5}x^{15}+15\,ab^{4}x^{12}+60\,a^{2}b^{3}x^{9}+180\,a^{3}b^{2}x^{6}\log\left(x\right)-30\,a^{4}bx^{3}-3\,a^{5}}{18\,x^{6}}$$

input
```
integrate((b*x^3+a)^5/x^7,x, algorithm="fricas")
```

output
```
1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x)
 - 30*a^4*b*x^3 - 3*a^5)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^5}{x^7}\,dx = 10a^3b^2\log(x) + \frac{10a^2b^3x^3}{3} + \frac{5ab^4x^6}{6} + \frac{b^5x^9}{9} + \frac{-a^5-10a^4bx^3}{6x^6}$$

input
```
integrate((b*x**3+a)**5/x**7,x)
```

output
```
10*a**3*b**2*log(x) + 10*a**2*b**3*x**3/3 + 5*a*b**4*x**6/6 + b**5*x**9/9
+ (-a**5 - 10*a**4*b*x**3)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^7}\,dx = \frac{1}{9}\,b^5x^9 + \frac{5}{6}\,ab^4x^6 + \frac{10}{3}\,a^2b^3x^3 + \frac{10}{3}\,a^3b^2\log\left(x^3\right) - \frac{10\,a^4bx^3+a^5}{6\,x^6}$$

input
```
integrate((b*x^3+a)^5/x^7,x, algorithm="maxima")
```

output
```
1/9*b^5*x^9 + 5/6*a*b^4*x^6 + 10/3*a^2*b^3*x^3 + 10/3*a^3*b^2*log(x^3) - 1
/6*(10*a^4*b*x^3 + a^5)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5}{x^7}\,dx = \frac{1}{9}\,b^5x^9 + \frac{5}{6}\,ab^4x^6 + \frac{10}{3}\,a^2b^3x^3 + 10\,a^3b^2\log(|x|) - \frac{30\,a^3b^2x^6+10\,a^4bx^3+a^5}{6\,x^6}$$

input
```
integrate((b*x^3+a)^5/x^7,x, algorithm="giac")
```

output
```
1/9*b^5*x^9 + 5/6*a*b^4*x^6 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*log(abs(x)) -
1/6*(30*a^3*b^2*x^6 + 10*a^4*b*x^3 + a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^7}\,dx = \frac{b^5\,x^9}{9} - \frac{\frac{a^5}{6}+\frac{5\,b\,a^4\,x^3}{3}}{x^6} + \frac{5\,a\,b^4\,x^6}{6} + \frac{10\,a^2\,b^3\,x^3}{3} + 10\,a^3\,b^2\,\ln(x)$$

input
```
int((a + b*x^3)^5/x^7,x)
```

output
```
(b^5*x^9)/9 - (a^5/6 + (5*a^4*b*x^3)/3)/x^6 + (5*a*b^4*x^6)/6 + (10*a^2*b^
3*x^3)/3 + 10*a^3*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^7}\,dx = \frac{180\log(x)\,a^3b^2x^6 - 3a^5 - 30a^4b\,x^3 + 60a^2b^3x^9 + 15a\,b^4x^{12} + 2b^5x^{15}}{18x^6}$$

input
```
int((b*x^3+a)^5/x^7,x)
```

output
```
(180*log(x)*a**3*b**2*x**6 - 3*a**5 - 30*a**4*b*x**3 + 60*a**2*b**3*x**9 +
 15*a*b**4*x**12 + 2*b**5*x**15)/(18*x**6)
```

3.62 $\int \frac{(a+bx^3)^5}{x^{10}} dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{(a+bx^3)^5}{x^{10}} dx = -\frac{a^5}{9x^9} - \frac{5a^4b}{6x^6} - \frac{10a^3b^2}{3x^3} + \frac{5}{3}ab^4x^3 + \frac{b^5x^6}{6} + 10a^2b^3\log(x)$$

output
```
-1/9*a^5/x^9-5/6*a^4*b/x^6-10/3*a^3*b^2/x^3+5/3*a*b^4*x^3+1/6*b^5*x^6+10*a
^2*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^{10}} dx = -\frac{a^5}{9x^9} - \frac{5a^4b}{6x^6} - \frac{10a^3b^2}{3x^3} + \frac{5}{3}ab^4x^3 + \frac{b^5x^6}{6} + 10a^2b^3\log(x)$$

input
```
Integrate[(a + b*x^3)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(6*x^6) - (10*a^3*b^2)/(3*x^3) + (5*a*b^4*x^3)/3
+ (b^5*x^6)/6 + 10*a^2*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{10}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{12}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^9}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^3}+5b^4a+b^5x^3\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5}{3x^9}-\frac{5a^4b}{2x^6}-\frac{10a^3b^2}{x^3}+10a^2b^3\log\left(x^3\right)+5ab^4x^3+\frac{b^5x^6}{2}\right)$$

input
```
Int[(a + b*x^3)^5/x^10,x]
```

output
```
(-1/3*a^5/x^9 - (5*a^4*b)/(2*x^6) - (10*a^3*b^2)/x^3 + 5*a*b^4*x^3 + (b^5*
x^6)/2 + 10*a^2*b^3*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{6x^6}-\frac{10a^3b^2}{3x^3}+\frac{5a\,b^4x^3}{3}+\frac{b^5x^6}{6}+10a^2b^3\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{9}a^5+\frac{1}{6}b^5x^{15}+\frac{5}{3}a\,b^4x^{12}-\frac{10}{3}a^3b^2x^6-\frac{5}{6}a^4b\,x^3}{x^9}+10a^2b^3\ln\left(x\right)$	59
parallelrisch	$\frac{3b^5x^{15}+30a\,b^4x^{12}+180a^2b^3\ln(x)x^9-60a^3b^2x^6-15a^4b\,x^3-2a^5}{18x^9}$	62
risch	$\frac{b^5x^6}{6}+\frac{5a\,b^4x^3}{3}+\frac{25a^2b^3}{6}+\frac{-\frac{10}{3}a^3b^2x^6-\frac{5}{6}a^4b\,x^3-\frac{1}{9}a^5}{x^9}+10a^2b^3\ln\left(x\right)$	67

input
```
int((b*x^3+a)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^5/x^9-5/6*a^4*b/x^6-10/3*a^3*b^2/x^3+5/3*a*b^4*x^3+1/6*b^5*x^6+10*a
^2*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int\frac{\left(a+bx^3\right)^5}{x^{10}}\,dx=\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log\left(x\right)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5/x^10,x, algorithm="fricas")
```

output
```
1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6
 - 15*a^4*b*x^3 - 2*a^5)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^5}{x^{10}}\,dx = 10a^2b^3\log(x) + \frac{5ab^4x^3}{3} + \frac{b^5x^6}{6} + \frac{-2a^5 - 15a^4bx^3 - 60a^3b^2x^6}{18x^9}$$

input
```
integrate((b*x**3+a)**5/x**10,x)
```

output
```
10*a**2*b**3*log(x) + 5*a*b**4*x**3/3 + b**5*x**6/6 + (-2*a**5 - 15*a**4*b
*x**3 - 60*a**3*b**2*x**6)/(18*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^{10}}\,dx = \frac{1}{6}\,b^5x^6 + \frac{5}{3}\,ab^4x^3 + \frac{10}{3}\,a^2b^3\log\left(x^3\right) - \frac{60\,a^3b^2x^6 + 15\,a^4bx^3 + 2\,a^5}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5/x^10,x, algorithm="maxima")
```

output
```
1/6*b^5*x^6 + 5/3*a*b^4*x^3 + 10/3*a^2*b^3*log(x^3) - 1/18*(60*a^3*b^2*x^6
 + 15*a^4*b*x^3 + 2*a^5)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^3)^5}{x^{10}}\,dx = \frac{1}{6}\,b^5x^6 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3\log(|x|) - \frac{110\,a^2b^3x^9 + 60\,a^3b^2x^6 + 15\,a^4bx^3 + 2\,a^5}{18\,x^9}$$

input
```
integrate((b*x^3+a)^5/x^10,x, algorithm="giac")
```

output
```
1/6*b^5*x^6 + 5/3*a*b^4*x^3 + 10*a^2*b^3*log(abs(x)) - 1/18*(110*a^2*b^3*x
^9 + 60*a^3*b^2*x^6 + 15*a^4*b*x^3 + 2*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^{10}}\,dx = \frac{b^5\,x^6}{6} - \frac{\frac{a^5}{9}+\frac{5\,a^4\,b\,x^3}{6}+\frac{10\,a^3\,b^2\,x^6}{3}}{x^9} + \frac{5\,a\,b^4\,x^3}{3} + 10\,a^2\,b^3\,\ln(x)$$

input
```
int((a + b*x^3)^5/x^10,x)
```

output
```
(b^5*x^6)/6 - (a^5/9 + (5*a^4*b*x^3)/6 + (10*a^3*b^2*x^6)/3)/x^9 + (5*a*b^
4*x^3)/3 + 10*a^2*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^{10}}\,dx = \frac{180\log(x)\,a^2b^3x^9 - 2a^5 - 15a^4b\,x^3 - 60a^3b^2x^6 + 30a\,b^4x^{12} + 3b^5x^{15}}{18x^9}$$

input
```
int((b*x^3+a)^5/x^10,x)
```

output
```
(180*log(x)*a**2*b**3*x**9 - 2*a**5 - 15*a**4*b*x**3 - 60*a**3*b**2*x**6 +
 30*a*b**4*x**12 + 3*b**5*x**15)/(18*x**9)
```

3.63 $\int \frac{(a+bx^3)^5}{x^{13}} dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{(a+bx^3)^5}{x^{13}} dx = -\frac{a^5}{12x^{12}} - \frac{5a^4b}{9x^9} - \frac{5a^3b^2}{3x^6} - \frac{10a^2b^3}{3x^3} + \frac{b^5x^3}{3} + 5ab^4\log(x)$$

output
```
-1/12*a^5/x^12-5/9*a^4*b/x^9-5/3*a^3*b^2/x^6-10/3*a^2*b^3/x^3+1/3*b^5*x^3+
5*a*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^{13}} dx = -\frac{a^5}{12x^{12}} - \frac{5a^4b}{9x^9} - \frac{5a^3b^2}{3x^6} - \frac{10a^2b^3}{3x^3} + \frac{b^5x^3}{3} + 5ab^4\log(x)$$

input
```
Integrate[(a + b*x^3)^5/x^13,x]
```

output
```
-1/12*a^5/x^12 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(3*x^6) - (10*a^2*b^3)/(3
*x^3) + (b^5*x^3)/3 + 5*a*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{15}}dx^3$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}\int \left(\frac{a^5}{x^{15}}+\frac{5ba^4}{x^{12}}+\frac{10b^2a^3}{x^9}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^3}+b^5\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^5}{4x^{12}}-\frac{5a^4b}{3x^9}-\frac{5a^3b^2}{x^6}-\frac{10a^2b^3}{x^3}+5ab^4\log\left(x^3\right)+b^5x^3\right)$$

input
```
Int[(a + b*x^3)^5/x^13,x]
```

output
```
(-1/4*a^5/x^12 - (5*a^4*b)/(3*x^9) - (5*a^3*b^2)/x^6 - (10*a^2*b^3)/x^3 +
b^5*x^3 + 5*a*b^4*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{5}}{12 x^{12}}-\frac{5 a^{4} b}{9 x^{9}}-\frac{5 a^{3} b^{2}}{3 x^{6}}-\frac{10 a^{2} b^{3}}{3 x^{3}}+\frac{x^{3} b^{5}}{3}+5 a\, b^{4} \ln \left(x \right)$	57
norman	$\frac{-\frac{1}{12} a^{5}+\frac{1}{3} b^{5} x^{15}-\frac{10}{3} a^{2} b^{3} x^{9}-\frac{5}{3} a^{3} b^{2} x^{6}-\frac{5}{9} a^{4} b\, x^{3}}{x^{12}}+5 a\, b^{4} \ln \left(x \right)$	59
risch	$\frac{x^{3} b^{5}}{3}+\frac{-\frac{10}{3} a^{2} b^{3} x^{9}-\frac{5}{3} a^{3} b^{2} x^{6}-\frac{5}{9} a^{4} b\, x^{3}-\frac{1}{12} a^{5}}{x^{12}}+5 a\, b^{4} \ln \left(x \right)$	59
parallelrisch	$\frac{12 b^{5} x^{15}+180 a\, b^{4} \ln \left(x \right) x^{12}-120 a^{2} b^{3} x^{9}-60 a^{3} b^{2} x^{6}-20 a^{4} b\, x^{3}-3 a^{5}}{36 x^{12}}$	62

input
```
int((b*x^3+a)^5/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^5/x^12-5/9*a^4*b/x^9-5/3*a^3*b^2/x^6-10/3*a^2*b^3/x^3+1/3*x^3*b^5+
5*a*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx = \frac{12\,b^5x^{15}+180\,ab^4x^{12}\log(x)-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5/x^13,x, algorithm="fricas")
```

output
```
1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x
^6 - 20*a^4*b*x^3 - 3*a^5)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx = 5ab^4\log(x) + \frac{b^5x^3}{3} + \frac{-3a^5 - 20a^4bx^3 - 60a^3b^2x^6 - 120a^2b^3x^9}{36x^{12}}$$

input
```
integrate((b*x**3+a)**5/x**13,x)
```

output
```
5*a*b**4*log(x) + b**5*x**3/3 + (-3*a**5 - 20*a**4*b*x**3 - 60*a**3*b**2*x
**6 - 120*a**2*b**3*x**9)/(36*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx = \frac{1}{3}\,b^5x^3 + \frac{5}{3}\,ab^4\log(x^3) - \frac{120\,a^2b^3x^9 + 60\,a^3b^2x^6 + 20\,a^4bx^3 + 3\,a^5}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5/x^13,x, algorithm="maxima")
```

output
```
1/3*b^5*x^3 + 5/3*a*b^4*log(x^3) - 1/36*(120*a^2*b^3*x^9 + 60*a^3*b^2*x^6
+ 20*a^4*b*x^3 + 3*a^5)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx = \frac{1}{3}\,b^5x^3 + 5\,ab^4\log\left(|x|\right) - \frac{125\,ab^4x^{12} + 120\,a^2b^3x^9 + 60\,a^3b^2x^6 + 20\,a^4bx^3 + 3\,a^5}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^5/x^13,x, algorithm="giac")
```

output
```
1/3*b^5*x^3 + 5*a*b^4*log(abs(x)) - 1/36*(125*a*b^4*x^12 + 120*a^2*b^3*x^9
 + 60*a^3*b^2*x^6 + 20*a^4*b*x^3 + 3*a^5)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx = \frac{b^5\,x^3}{3} - \frac{\frac{a^5}{12} + \frac{5\,a^4\,b\,x^3}{9} + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{10\,a^2\,b^3\,x^9}{3}}{x^{12}} + 5\,a\,b^4\,\ln\left(x\right)$$

input
```
int((a + b*x^3)^5/x^13,x)
```

output
```
(b^5*x^3)/3 - (a^5/12 + (5*a^4*b*x^3)/9 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*
x^9)/3)/x^12 + 5*a*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^{13}}\,dx = \frac{180\log(x)\,a\,b^4x^{12} - 3a^5 - 20a^4b\,x^3 - 60a^3b^2x^6 - 120a^2b^3x^9 + 12b^5x^{15}}{36x^{12}}$$

input
```
int((b*x^3+a)^5/x^13,x)
```

output
```
(180*log(x)*a*b**4*x**12 - 3*a**5 - 20*a**4*b*x**3 - 60*a**3*b**2*x**6 - 1
20*a**2*b**3*x**9 + 12*b**5*x**15)/(36*x**12)
```

3.64 $\int \frac{(a+bx^3)^5}{x^{16}} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^3)^5}{x^{16}} dx = -\frac{a^5}{15x^{15}} - \frac{5a^4b}{12x^{12}} - \frac{10a^3b^2}{9x^9} - \frac{5a^2b^3}{3x^6} - \frac{5ab^4}{3x^3} + b^5\log(x)$$

output
```
-1/15*a^5/x^15-5/12*a^4*b/x^12-10/9*a^3*b^2/x^9-5/3*a^2*b^3/x^6-5/3*a*b^4/
x^3+b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^{16}} dx = -\frac{a^5}{15x^{15}} - \frac{5a^4b}{12x^{12}} - \frac{10a^3b^2}{9x^9} - \frac{5a^2b^3}{3x^6} - \frac{5ab^4}{3x^3} + b^5\log(x)$$

input
```
Integrate[(a + b*x^3)^5/x^16,x]
```

output
```
-1/15*a^5/x^15 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/
(3*x^6) - (5*a*b^4)/(3*x^3) + b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{16}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{18}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^5}{x^{18}}+\frac{5ba^4}{x^{15}}+\frac{10b^2a^3}{x^{12}}+\frac{10b^3a^2}{x^9}+\frac{5b^4a}{x^6}+\frac{b^5}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5}{5x^{15}}-\frac{5a^4b}{4x^{12}}-\frac{10a^3b^2}{3x^9}-\frac{5a^2b^3}{x^6}-\frac{5ab^4}{x^3}+b^5\log\left(x^3\right)\right)$$

input
```
Int[(a + b*x^3)^5/x^16,x]
```

output
```
(-1/5*a^5/x^15 - (5*a^4*b)/(4*x^12) - (10*a^3*b^2)/(3*x^9) - (5*a^2*b^3)/x
^6 - (5*a*b^4)/x^3 + b^5*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{3x^6}-\frac{5a\,b^4}{3x^3}+b^5\ln\left(x\right)$	56
norman	$\frac{-\frac{1}{15}a^5-\frac{5}{3}a\,b^4x^{12}-\frac{5}{3}a^2b^3x^9-\frac{10}{9}a^3b^2x^6-\frac{5}{12}a^4b\,x^3}{x^{15}}+b^5\ln\left(x\right)$	58
risch	$\frac{-\frac{1}{15}a^5-\frac{5}{3}a\,b^4x^{12}-\frac{5}{3}a^2b^3x^9-\frac{10}{9}a^3b^2x^6-\frac{5}{12}a^4b\,x^3}{x^{15}}+b^5\ln\left(x\right)$	58
parallelrisch	$\frac{180b^5\ln(x)x^{15}-300a\,b^4x^{12}-300a^2b^3x^9-200a^3b^2x^6-75a^4b\,x^3-12a^5}{180x^{15}}$	62

input
```
int((b*x^3+a)^5/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^5/x^15-5/12*a^4*b/x^12-10/9*a^3*b^2/x^9-5/3*a^2*b^3/x^6-5/3*a*b^4/
x^3+b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int\frac{(a+bx^3)^5}{x^{16}}\,dx=\frac{180\,b^5x^{15}\log\left(x\right)-300\,ab^4x^{12}-300\,a^2b^3x^9-200\,a^3b^2x^6-75\,a^4bx^3-12\,a^5}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5/x^16,x, algorithm="fricas")
```

output
```
1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^
2*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^{16}}\,dx = b^5\log(x) + \frac{-12a^5 - 75a^4bx^3 - 200a^3b^2x^6 - 300a^2b^3x^9 - 300ab^4x^{12}}{180x^{15}}$$

input
```
integrate((b*x**3+a)**5/x**16,x)
```

output
```
b**5*log(x) + (-12*a**5 - 75*a**4*b*x**3 - 200*a**3*b**2*x**6 - 300*a**2*b
**3*x**9 - 300*a*b**4*x**12)/(180*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^{16}}\,dx = \frac{1}{3}\,b^5\log\left(x^3\right) - \frac{300\,ab^4x^{12} + 300\,a^2b^3x^9 + 200\,a^3b^2x^6 + 75\,a^4bx^3 + 12\,a^5}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5/x^16,x, algorithm="maxima")
```

output
```
1/3*b^5*log(x^3) - 1/180*(300*a*b^4*x^12 + 300*a^2*b^3*x^9 + 200*a^3*b^2*x
^6 + 75*a^4*b*x^3 + 12*a^5)/x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^5}{x^{16}}\,dx = b^5\log(|x|) - \frac{137\,b^5x^{15} + 300\,ab^4x^{12} + 300\,a^2b^3x^9 + 200\,a^3b^2x^6 + 75\,a^4bx^3 + 12\,a^5}{180\,x^{15}}$$

input
```
integrate((b*x^3+a)^5/x^16,x, algorithm="giac")
```

output
```
b^5*log(abs(x)) - 1/180*(137*b^5*x^15 + 300*a*b^4*x^12 + 300*a^2*b^3*x^9 +
 200*a^3*b^2*x^6 + 75*a^4*b*x^3 + 12*a^5)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^{16}}\,dx = b^5\,\ln(x) - \frac{\frac{a^5}{15} + \frac{5\,a^4\,b\,x^3}{12} + \frac{10\,a^3\,b^2\,x^6}{9} + \frac{5\,a^2\,b^3\,x^9}{3} + \frac{5\,a\,b^4\,x^{12}}{3}}{x^{15}}$$

input
```
int((a + b*x^3)^5/x^16,x)
```

output
```
b^5*log(x) - (a^5/15 + (5*a^4*b*x^3)/12 + (5*a*b^4*x^12)/3 + (10*a^3*b^2*x
^6)/9 + (5*a^2*b^3*x^9)/3)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^{16}}\,dx = \frac{180\log(x)\,b^5x^{15} - 12a^5 - 75a^4b\,x^3 - 200a^3b^2x^6 - 300a^2b^3x^9 - 300a\,b^4x^{12}}{180x^{15}}$$

input
```
int((b*x^3+a)^5/x^16,x)
```

output
```
(180*log(x)*b**5*x**15 - 12*a**5 - 75*a**4*b*x**3 - 200*a**3*b**2*x**6 - 3
00*a**2*b**3*x**9 - 300*a*b**4*x**12)/(180*x**15)
```

3.65 $\int \frac{(a+bx^3)^5}{x^{19}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = -\frac{(a+bx^3)^6}{18ax^{18}}$$

output
```
-1/18*(b*x^3+a)^6/a/x^18
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = -\frac{a^5}{18x^{18}} - \frac{a^4b}{3x^{15}} - \frac{5a^3b^2}{6x^{12}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{6x^6} - \frac{b^5}{3x^3}$$

input
```
Integrate[(a + b*x^3)^5/x^19,x]
```

output
```
-1/18*a^5/x^18 - (a^4*b)/(3*x^15) - (5*a^3*b^2)/(6*x^12) - (10*a^2*b^3)/(9
*x^9) - (5*a*b^4)/(6*x^6) - b^5/(3*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{19}} \, dx$$

$$\Big\downarrow \; 796$$

$$-\frac{(a+bx^3)^6}{18ax^{18}}$$

input
```
Int[(a + b*x^3)^5/x^19,x]
```

output
```
-1/18*(a + b*x^3)^6/(a*x^18)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

method	result	size
gosper	$-\frac{6b^5x^{15}+15a\,b^4x^{12}+20a^2b^3x^9+15a^3b^2x^6+6a^4b\,x^3+a^5}{18x^{18}}$	58
default	$-\frac{b^5}{3x^3}-\frac{a^4b}{3x^{15}}-\frac{5a\,b^4}{6x^6}-\frac{a^5}{18x^{18}}-\frac{10a^2b^3}{9x^9}-\frac{5a^3b^2}{6x^{12}}$	58
orering	$-\frac{6b^5x^{15}+15a\,b^4x^{12}+20a^2b^3x^9+15a^3b^2x^6+6a^4b\,x^3+a^5}{18x^{18}}$	58
norman	$\frac{-\frac{1}{18}a^5-\frac{1}{3}a^4b\,x^3-\frac{5}{6}a^3b^2x^6-\frac{10}{9}a^2b^3x^9-\frac{5}{6}a\,b^4x^{12}-\frac{1}{3}b^5x^{15}}{x^{18}}$	59
risch	$\frac{-\frac{1}{18}a^5-\frac{1}{3}a^4b\,x^3-\frac{5}{6}a^3b^2x^6-\frac{10}{9}a^2b^3x^9-\frac{5}{6}a\,b^4x^{12}-\frac{1}{3}b^5x^{15}}{x^{18}}$	59
parallelrisch	$\frac{-6b^5x^{15}-15a\,b^4x^{12}-20a^2b^3x^9-15a^3b^2x^6-6a^4b\,x^3-a^5}{18x^{18}}$	60

input
```
int((b*x^3+a)^5/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a^4*b*x^3+
a^5)/x^18
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = -\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

input
```
integrate((b*x^3+a)^5/x^19,x, algorithm="fricas")
```

output
```
-1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^
4*b*x^3 + a^5)/x^18
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = \frac{-a^5-6a^4bx^3-15a^3b^2x^6-20a^2b^3x^9-15ab^4x^{12}-6b^5x^{15}}{18x^{18}}$$

input
```
integrate((b*x**3+a)**5/x**19,x)
```

output
```
(-a**5 - 6*a**4*b*x**3 - 15*a**3*b**2*x**6 - 20*a**2*b**3*x**9 - 15*a*b**4
*x**12 - 6*b**5*x**15)/(18*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = -\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

input
```
integrate((b*x^3+a)^5/x^19,x, algorithm="maxima")
```

output
```
-1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^
4*b*x^3 + a^5)/x^18
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = -\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

input
```
integrate((b*x^3+a)^5/x^19,x, algorithm="giac")
```

output
```
-1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^
4*b*x^3 + a^5)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = -\frac{\frac{a^5}{18}+\frac{a^4\,b\,x^3}{3}+\frac{5\,a^3\,b^2\,x^6}{6}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{5\,a\,b^4\,x^{12}}{6}+\frac{b^5\,x^{15}}{3}}{x^{18}}$$

input
```
int((a + b*x^3)^5/x^19,x)
```

output
```
-(a^5/18 + (b^5*x^15)/3 + (a^4*b*x^3)/3 + (5*a*b^4*x^12)/6 + (5*a^3*b^2*x^
6)/6 + (10*a^2*b^3*x^9)/9)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx^3)^5}{x^{19}}\,dx = \frac{-6b^5x^{15}-15a\,b^4x^{12}-20a^2b^3x^9-15a^3b^2x^6-6a^4b\,x^3-a^5}{18x^{18}}$$

input
```
int((b*x^3+a)^5/x^19,x)
```

output
```
( - a**5 - 6*a**4*b*x**3 - 15*a**3*b**2*x**6 - 20*a**2*b**3*x**9 - 15*a*b*
*4*x**12 - 6*b**5*x**15)/(18*x**18)
```

3.66 $\int \frac{(a+bx^3)^5}{x^{22}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = -\frac{(a+bx^3)^6}{21ax^{21}} + \frac{b(a+bx^3)^6}{126a^2x^{18}}$$

output
```
-1/21*(b*x^3+a)^6/a/x^21+1/126*b*(b*x^3+a)^6/a^2/x^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = -\frac{a^5}{21x^{21}} - \frac{5a^4b}{18x^{18}} - \frac{2a^3b^2}{3x^{15}} - \frac{5a^2b^3}{6x^{12}} - \frac{5ab^4}{9x^9} - \frac{b^5}{6x^6}$$

input
```
Integrate[(a + b*x^3)^5/x^22,x]
```

output
```
-1/21*a^5/x^21 - (5*a^4*b)/(18*x^18) - (2*a^3*b^2)/(3*x^15) - (5*a^2*b^3)/
(6*x^12) - (5*a*b^4)/(9*x^9) - b^5/(6*x^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{22}}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{24}}dx^3$$

$$\Big\downarrow \ 55$$

$$\frac{1}{3}\left(-\frac{b\int \frac{(bx^3+a)^5}{x^{21}}dx^3}{7a}-\frac{(a+bx^3)^6}{7ax^{21}}\right)$$

$$\Big\downarrow \ 48$$

$$\frac{1}{3}\left(\frac{b(a+bx^3)^6}{42a^2x^{18}}-\frac{(a+bx^3)^6}{7ax^{21}}\right)$$

input
```
Int[(a + b*x^3)^5/x^22,x]
```

output
```
(-1/7*(a + b*x^3)^6/(a*x^21) + (b*(a + b*x^3)^6)/(42*a^2*x^18))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{a^{5}}{21x^{21}}-\frac{2a^{3}b^{2}}{3x^{15}}-\frac{b^{5}}{6x^{6}}-\frac{5a^{4}b}{18x^{18}}-\frac{5a\,b^{4}}{9x^{9}}-\frac{5a^{2}b^{3}}{6x^{12}}$	58
norman	$\frac{-\frac{1}{21}a^{5}-\frac{5}{18}a^{4}b\,x^{3}-\frac{2}{3}a^{3}b^{2}x^{6}-\frac{5}{6}a^{2}b^{3}x^{9}-\frac{5}{9}a\,b^{4}x^{12}-\frac{1}{6}b^{5}x^{15}}{x^{21}}$	59
risch	$\frac{-\frac{1}{21}a^{5}-\frac{5}{18}a^{4}b\,x^{3}-\frac{2}{3}a^{3}b^{2}x^{6}-\frac{5}{6}a^{2}b^{3}x^{9}-\frac{5}{9}a\,b^{4}x^{12}-\frac{1}{6}b^{5}x^{15}}{x^{21}}$	59
gosper	$-\frac{21b^{5}x^{15}+70a\,b^{4}x^{12}+105a^{2}b^{3}x^{9}+84a^{3}b^{2}x^{6}+35a^{4}b\,x^{3}+6a^{5}}{126x^{21}}$	60
parallelrisch	$\frac{-21b^{5}x^{15}-70a\,b^{4}x^{12}-105a^{2}b^{3}x^{9}-84a^{3}b^{2}x^{6}-35a^{4}b\,x^{3}-6a^{5}}{126x^{21}}$	60
orering	$-\frac{21b^{5}x^{15}+70a\,b^{4}x^{12}+105a^{2}b^{3}x^{9}+84a^{3}b^{2}x^{6}+35a^{4}b\,x^{3}+6a^{5}}{126x^{21}}$	60

input
```
int((b*x^3+a)^5/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/21*a^5/x^21-2/3*a^3*b^2/x^15-1/6*b^5/x^6-5/18*a^4*b/x^18-5/9*a*b^4/x^9-
5/6*a^2*b^3/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = -\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5/x^22,x, algorithm="fricas")
```

output
```
-1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 3
5*a^4*b*x^3 + 6*a^5)/x^21
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = \frac{-6a^5-35a^4bx^3-84a^3b^2x^6-105a^2b^3x^9-70ab^4x^{12}-21b^5x^{15}}{126x^{21}}$$

input
```
integrate((b*x**3+a)**5/x**22,x)
```

output
```
(-6*a**5 - 35*a**4*b*x**3 - 84*a**3*b**2*x**6 - 105*a**2*b**3*x**9 - 70*a*
b**4*x**12 - 21*b**5*x**15)/(126*x**21)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = -\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5/x^22,x, algorithm="maxima")
```

output
```
-1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 3
5*a^4*b*x^3 + 6*a^5)/x^21
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = -\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

input
```
integrate((b*x^3+a)^5/x^22,x, algorithm="giac")
```

output
```
-1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 3
5*a^4*b*x^3 + 6*a^5)/x^21
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = -\frac{\frac{a^5}{21}+\frac{5\,a^4\,b\,x^3}{18}+\frac{2\,a^3\,b^2\,x^6}{3}+\frac{5\,a^2\,b^3\,x^9}{6}+\frac{5\,a\,b^4\,x^{12}}{9}+\frac{b^5\,x^{15}}{6}}{x^{21}}$$

input
```
int((a + b*x^3)^5/x^22,x)
```

output
```
-(a^5/21 + (b^5*x^15)/6 + (5*a^4*b*x^3)/18 + (5*a*b^4*x^12)/9 + (2*a^3*b^2
*x^6)/3 + (5*a^2*b^3*x^9)/6)/x^21
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^5}{x^{22}}\,dx = \frac{-21b^5x^{15}-70a\,b^4x^{12}-105a^2b^3x^9-84a^3b^2x^6-35a^4b\,x^3-6a^5}{126x^{21}}$$

input
```
int((b*x^3+a)^5/x^22,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x**3 - 84*a**3*b**2*x**6 - 105*a**2*b**3*x**9 - 70*
a*b**4*x**12 - 21*b**5*x**15)/(126*x**21)
```

3.67 $\int \frac{(a+bx^3)^5}{x^{25}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = -\frac{(a+bx^3)^6}{24ax^{24}}+\frac{b(a+bx^3)^6}{84a^2x^{21}}-\frac{b^2(a+bx^3)^6}{504a^3x^{18}}$$

output
```
-1/24*(b*x^3+a)^6/a/x^24+1/84*b*(b*x^3+a)^6/a^2/x^21-1/504*b^2*(b*x^3+a)^6
/a^3/x^18
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = -\frac{a^5}{24x^{24}}-\frac{5a^4b}{21x^{21}}-\frac{5a^3b^2}{9x^{18}}-\frac{2a^2b^3}{3x^{15}}-\frac{5ab^4}{12x^{12}}-\frac{b^5}{9x^9}$$

input
```
Integrate[(a + b*x^3)^5/x^25,x]
```

output
```
-1/24*a^5/x^24 - (5*a^4*b)/(21*x^21) - (5*a^3*b^2)/(9*x^18) - (2*a^2*b^3)/
(3*x^15) - (5*a*b^4)/(12*x^12) - b^5/(9*x^9)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{25}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{27}}dx^3$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{(bx^3+a)^5}{x^{24}}dx^3}{4a}-\frac{(a+bx^3)^6}{8ax^{24}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{b\int \frac{(bx^3+a)^5}{x^{21}}dx^3}{7a}-\frac{(a+bx^3)^6}{7ax^{21}}\right)}{4a}-\frac{(a+bx^3)^6}{8ax^{24}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{b(a+bx^3)^6}{42a^2x^{18}}-\frac{(a+bx^3)^6}{7ax^{21}}\right)}{4a}-\frac{(a+bx^3)^6}{8ax^{24}}\right)$$

input
```
Int[(a + b*x^3)^5/x^25,x]
```

output
```
(-1/8*(a + b*x^3)^6/(a*x^24) - (b*(-1/7*(a + b*x^3)^6/(a*x^21) + (b*(a + b
*x^3)^6)/(42*a^2*x^18)))/(4*a))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{5a^4b}{21x^{21}}-\frac{2a^2b^3}{3x^{15}}-\frac{a^5}{24x^{24}}-\frac{5a^3b^2}{9x^{18}}-\frac{b^5}{9x^9}-\frac{5a\,b^4}{12x^{12}}$	58
norman	$\frac{-\frac{1}{24}a^5-\frac{5}{9}a^3b^2x^6-\frac{2}{3}a^2b^3x^9-\frac{5}{12}a\,b^4x^{12}-\frac{1}{9}b^5x^{15}-\frac{5}{21}a^4b\,x^3}{x^{24}}$	59
risch	$\frac{-\frac{1}{24}a^5-\frac{5}{9}a^3b^2x^6-\frac{2}{3}a^2b^3x^9-\frac{5}{12}a\,b^4x^{12}-\frac{1}{9}b^5x^{15}-\frac{5}{21}a^4b\,x^3}{x^{24}}$	59
gosper	$-\frac{56b^5x^{15}+210a\,b^4x^{12}+336a^2b^3x^9+280a^3b^2x^6+120a^4b\,x^3+21a^5}{504x^{24}}$	60
parallelrisch	$\frac{-56b^5x^{15}-210a\,b^4x^{12}-336a^2b^3x^9-280a^3b^2x^6-120a^4b\,x^3-21a^5}{504x^{24}}$	60
orering	$-\frac{56b^5x^{15}+210a\,b^4x^{12}+336a^2b^3x^9+280a^3b^2x^6+120a^4b\,x^3+21a^5}{504x^{24}}$	60

input
```
int((b*x^3+a)^5/x^25,x,method=_RETURNVERBOSE)
```

output
```
-5/21*a^4*b/x^21-2/3*a^2*b^3/x^15-1/24*a^5/x^24-5/9*a^3*b^2/x^18-1/9*b^5/x
^9-5/12*a*b^4/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = -\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

input
```
integrate((b*x^3+a)^5/x^25,x, algorithm="fricas")
```

output
```
-1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
 120*a^4*b*x^3 + 21*a^5)/x^24
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = \frac{-21a^5-120a^4bx^3-280a^3b^2x^6-336a^2b^3x^9-210ab^4x^{12}-56b^5x^{15}}{504x^{24}}$$

input
```
integrate((b*x**3+a)**5/x**25,x)
```

output
```
(-21*a**5 - 120*a**4*b*x**3 - 280*a**3*b**2*x**6 - 336*a**2*b**3*x**9 - 21
0*a*b**4*x**12 - 56*b**5*x**15)/(504*x**24)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = -\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

input
```
integrate((b*x^3+a)^5/x^25,x, algorithm="maxima")
```

output
```
-1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
 120*a^4*b*x^3 + 21*a^5)/x^24
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = -\frac{56\,b^5x^{15}+210\,ab^4x^{12}+336\,a^2b^3x^9+280\,a^3b^2x^6+120\,a^4bx^3+21\,a^5}{504\,x^{24}}$$

input
```
integrate((b*x^3+a)^5/x^25,x, algorithm="giac")
```

output
```
-1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
 120*a^4*b*x^3 + 21*a^5)/x^24
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = -\frac{\frac{a^5}{24}+\frac{5\,a^4\,b\,x^3}{21}+\frac{5\,a^3\,b^2\,x^6}{9}+\frac{2\,a^2\,b^3\,x^9}{3}+\frac{5\,a\,b^4\,x^{12}}{12}+\frac{b^5\,x^{15}}{9}}{x^{24}}$$

input
```
int((a + b*x^3)^5/x^25,x)
```

output
```
-(a^5/24 + (b^5*x^15)/9 + (5*a^4*b*x^3)/21 + (5*a*b^4*x^12)/12 + (5*a^3*b^
2*x^6)/9 + (2*a^2*b^3*x^9)/3)/x^24
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^{25}}\,dx = \frac{-56b^5x^{15}-210a\,b^4x^{12}-336a^2b^3x^9-280a^3b^2x^6-120a^4b\,x^3-21a^5}{504x^{24}}$$

input
```
int((b*x^3+a)^5/x^25,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x**3 - 280*a**3*b**2*x**6 - 336*a**2*b**3*x**9 -
210*a*b**4*x**12 - 56*b**5*x**15)/(504*x**24)
```

3.68 $\int \frac{(a+bx^3)^5}{x^{28}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = -\frac{a^5}{27x^{27}} - \frac{5a^4b}{24x^{24}} - \frac{10a^3b^2}{21x^{21}} - \frac{5a^2b^3}{9x^{18}} - \frac{ab^4}{3x^{15}} - \frac{b^5}{12x^{12}}$$

output
```
-1/27*a^5/x^27-5/24*a^4*b/x^24-10/21*a^3*b^2/x^21-5/9*a^2*b^3/x^18-1/3*a*b
^4/x^15-1/12*b^5/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = -\frac{a^5}{27x^{27}} - \frac{5a^4b}{24x^{24}} - \frac{10a^3b^2}{21x^{21}} - \frac{5a^2b^3}{9x^{18}} - \frac{ab^4}{3x^{15}} - \frac{b^5}{12x^{12}}$$

input
```
Integrate[(a + b*x^3)^5/x^28,x]
```

output
```
-1/27*a^5/x^27 - (5*a^4*b)/(24*x^24) - (10*a^3*b^2)/(21*x^21) - (5*a^2*b^3
)/(9*x^18) - (a*b^4)/(3*x^15) - b^5/(12*x^12)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{28}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{30}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{a^5}{x^{30}}+\frac{5ba^4}{x^{27}}+\frac{10b^2a^3}{x^{24}}+\frac{10b^3a^2}{x^{21}}+\frac{5b^4a}{x^{18}}+\frac{b^5}{x^{15}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5}{9x^{27}}-\frac{5a^4b}{8x^{24}}-\frac{10a^3b^2}{7x^{21}}-\frac{5a^2b^3}{3x^{18}}-\frac{ab^4}{x^{15}}-\frac{b^5}{4x^{12}}\right)$$

input
```
Int[(a + b*x^3)^5/x^28,x]
```

output
```
(-1/9*a^5/x^27 - (5*a^4*b)/(8*x^24) - (10*a^3*b^2)/(7*x^21) - (5*a^2*b^3)/
(3*x^18) - (a*b^4)/x^15 - b^5/(4*x^12))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^5}{27x^{27}}-\frac{5a^4b}{24x^{24}}-\frac{10a^3b^2}{21x^{21}}-\frac{5a^2b^3}{9x^{18}}-\frac{a\,b^4}{3x^{15}}-\frac{b^5}{12x^{12}}$	58
norman	$\frac{-\frac{1}{27}a^5-\frac{1}{12}b^5x^{15}-\frac{5}{24}a^4b\,x^3-\frac{10}{21}a^3b^2x^6-\frac{5}{9}a^2b^3x^9-\frac{1}{3}a\,b^4x^{12}}{x^{27}}$	59
risch	$\frac{-\frac{1}{27}a^5-\frac{1}{12}b^5x^{15}-\frac{5}{24}a^4b\,x^3-\frac{10}{21}a^3b^2x^6-\frac{5}{9}a^2b^3x^9-\frac{1}{3}a\,b^4x^{12}}{x^{27}}$	59
gosper	$-\frac{126b^5x^{15}+504a\,b^4x^{12}+840a^2b^3x^9+720a^3b^2x^6+315a^4b\,x^3+56a^5}{1512x^{27}}$	60
parallelrisch	$\frac{-126b^5x^{15}-504a\,b^4x^{12}-840a^2b^3x^9-720a^3b^2x^6-315a^4b\,x^3-56a^5}{1512x^{27}}$	60
orering	$-\frac{126b^5x^{15}+504a\,b^4x^{12}+840a^2b^3x^9+720a^3b^2x^6+315a^4b\,x^3+56a^5}{1512x^{27}}$	60

input
```
int((b*x^3+a)^5/x^28,x,method=_RETURNVERBOSE)
```

output
```
-1/27*a^5/x^27-5/24*a^4*b/x^24-10/21*a^3*b^2/x^21-5/9*a^2*b^3/x^18-1/3*a*b
^4/x^15-1/12*b^5/x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = -\frac{126\,b^5x^{15}+504\,ab^4x^{12}+840\,a^2b^3x^9+720\,a^3b^2x^6+315\,a^4bx^3+56\,a^5}{1512\,x^{27}}$$

input
```
integrate((b*x^3+a)^5/x^28,x, algorithm="fricas")
```

output
```
-1/1512*(126*b^5*x^15 + 504*a*b^4*x^12 + 840*a^2*b^3*x^9 + 720*a^3*b^2*x^6
 + 315*a^4*b*x^3 + 56*a^5)/x^27
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = \frac{-56a^5-315a^4bx^3-720a^3b^2x^6-840a^2b^3x^9-504ab^4x^{12}-126b^5x^{15}}{1512x^{27}}$$

input
```
integrate((b*x**3+a)**5/x**28,x)
```

output
```
(-56*a**5 - 315*a**4*b*x**3 - 720*a**3*b**2*x**6 - 840*a**2*b**3*x**9 - 50
4*a*b**4*x**12 - 126*b**5*x**15)/(1512*x**27)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = -\frac{126\,b^5x^{15}+504\,ab^4x^{12}+840\,a^2b^3x^9+720\,a^3b^2x^6+315\,a^4bx^3+56\,a^5}{1512\,x^{27}}$$

input
```
integrate((b*x^3+a)^5/x^28,x, algorithm="maxima")
```

output
```
-1/1512*(126*b^5*x^15 + 504*a*b^4*x^12 + 840*a^2*b^3*x^9 + 720*a^3*b^2*x^6
 + 315*a^4*b*x^3 + 56*a^5)/x^27
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = -\frac{126\,b^5x^{15}+504\,ab^4x^{12}+840\,a^2b^3x^9+720\,a^3b^2x^6+315\,a^4bx^3+56\,a^5}{1512\,x^{27}}$$

input
```
integrate((b*x^3+a)^5/x^28,x, algorithm="giac")
```

output
```
-1/1512*(126*b^5*x^15 + 504*a*b^4*x^12 + 840*a^2*b^3*x^9 + 720*a^3*b^2*x^6
 + 315*a^4*b*x^3 + 56*a^5)/x^27
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = -\frac{\frac{a^5}{27}+\frac{5\,a^4\,b\,x^3}{24}+\frac{10\,a^3\,b^2\,x^6}{21}+\frac{5\,a^2\,b^3\,x^9}{9}+\frac{a\,b^4\,x^{12}}{3}+\frac{b^5\,x^{15}}{12}}{x^{27}}$$

input
```
int((a + b*x^3)^5/x^28,x)
```

output
```
-(a^5/27 + (b^5*x^15)/12 + (5*a^4*b*x^3)/24 + (a*b^4*x^12)/3 + (10*a^3*b^2
*x^6)/21 + (5*a^2*b^3*x^9)/9)/x^27
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{28}}\,dx = \frac{-126b^5x^{15}-504a\,b^4x^{12}-840a^2b^3x^9-720a^3b^2x^6-315a^4b\,x^3-56a^5}{1512x^{27}}$$

input
```
int((b*x^3+a)^5/x^28,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x**3 - 720*a**3*b**2*x**6 - 840*a**2*b**3*x**9 -
504*a*b**4*x**12 - 126*b**5*x**15)/(1512*x**27)
```

3.69 $\int \frac{(a+bx^3)^5}{x^{31}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = -\frac{a^5}{30x^{30}} - \frac{5a^4b}{27x^{27}} - \frac{5a^3b^2}{12x^{24}} - \frac{10a^2b^3}{21x^{21}} - \frac{5ab^4}{18x^{18}} - \frac{b^5}{15x^{15}}$$

output
```
-1/30*a^5/x^30-5/27*a^4*b/x^27-5/12*a^3*b^2/x^24-10/21*a^2*b^3/x^21-5/18*a
*b^4/x^18-1/15*b^5/x^15
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = -\frac{a^5}{30x^{30}} - \frac{5a^4b}{27x^{27}} - \frac{5a^3b^2}{12x^{24}} - \frac{10a^2b^3}{21x^{21}} - \frac{5ab^4}{18x^{18}} - \frac{b^5}{15x^{15}}$$

input
```
Integrate[(a + b*x^3)^5/x^31,x]
```

output
```
-1/30*a^5/x^30 - (5*a^4*b)/(27*x^27) - (5*a^3*b^2)/(12*x^24) - (10*a^2*b^3
)/(21*x^21) - (5*a*b^4)/(18*x^18) - b^5/(15*x^15)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^5}{x^{33}}dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3}\int \left(\frac{a^5}{x^{33}}+\frac{5ba^4}{x^{30}}+\frac{10b^2a^3}{x^{27}}+\frac{10b^3a^2}{x^{24}}+\frac{5b^4a}{x^{21}}+\frac{b^5}{x^{18}}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{a^5}{10x^{30}}-\frac{5a^4b}{9x^{27}}-\frac{5a^3b^2}{4x^{24}}-\frac{10a^2b^3}{7x^{21}}-\frac{5ab^4}{6x^{18}}-\frac{b^5}{5x^{15}}\right)$$

input
```
Int[(a + b*x^3)^5/x^31,x]
```

output
```
(-1/10*a^5/x^30 - (5*a^4*b)/(9*x^27) - (5*a^3*b^2)/(4*x^24) - (10*a^2*b^3)
/(7*x^21) - (5*a*b^4)/(6*x^18) - b^5/(5*x^15))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^5}{30x^{30}}-\frac{5a^4b}{27x^{27}}-\frac{5a^3b^2}{12x^{24}}-\frac{10a^2b^3}{21x^{21}}-\frac{5a\,b^4}{18x^{18}}-\frac{b^5}{15x^{15}}$	58
norman	$\frac{-\frac{1}{30}a^5-\frac{1}{15}b^5x^{15}-\frac{5}{27}a^4b\,x^3-\frac{5}{12}a^3b^2x^6-\frac{10}{21}a^2b^3x^9-\frac{5}{18}a\,b^4x^{12}}{x^{30}}$	59
risch	$\frac{-\frac{1}{30}a^5-\frac{1}{15}b^5x^{15}-\frac{5}{27}a^4b\,x^3-\frac{5}{12}a^3b^2x^6-\frac{10}{21}a^2b^3x^9-\frac{5}{18}a\,b^4x^{12}}{x^{30}}$	59
gosper	$-\frac{252b^5x^{15}+1050a\,b^4x^{12}+1800a^2b^3x^9+1575a^3b^2x^6+700a^4b\,x^3+126a^5}{3780x^{30}}$	60
parallelrisch	$\frac{-252b^5x^{15}-1050a\,b^4x^{12}-1800a^2b^3x^9-1575a^3b^2x^6-700a^4b\,x^3-126a^5}{3780x^{30}}$	60
orering	$-\frac{252b^5x^{15}+1050a\,b^4x^{12}+1800a^2b^3x^9+1575a^3b^2x^6+700a^4b\,x^3+126a^5}{3780x^{30}}$	60

input
```
int((b*x^3+a)^5/x^31,x,method=_RETURNVERBOSE)
```

output
```
-1/30*a^5/x^30-5/27*a^4*b/x^27-5/12*a^3*b^2/x^24-10/21*a^2*b^3/x^21-5/18*a
*b^4/x^18-1/15*b^5/x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = -\frac{252\,b^5x^{15}+1050\,ab^4x^{12}+1800\,a^2b^3x^9+1575\,a^3b^2x^6+700\,a^4bx^3+126\,a^5}{3780\,x^{30}}$$

input
```
integrate((b*x^3+a)^5/x^31,x, algorithm="fricas")
```

output
```
-1/3780*(252*b^5*x^15 + 1050*a*b^4*x^12 + 1800*a^2*b^3*x^9 + 1575*a^3*b^2*
x^6 + 700*a^4*b*x^3 + 126*a^5)/x^30
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = \frac{-126a^5-700a^4bx^3-1575a^3b^2x^6-1800a^2b^3x^9-1050ab^4x^{12}-252b^5x^{15}}{3780x^{30}}$$

input
```
integrate((b*x**3+a)**5/x**31,x)
```

output
```
(-126*a**5 - 700*a**4*b*x**3 - 1575*a**3*b**2*x**6 - 1800*a**2*b**3*x**9 -
 1050*a*b**4*x**12 - 252*b**5*x**15)/(3780*x**30)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = -\frac{252\,b^5x^{15}+1050\,ab^4x^{12}+1800\,a^2b^3x^9+1575\,a^3b^2x^6+700\,a^4bx^3+126\,a^5}{3780\,x^{30}}$$

input
```
integrate((b*x^3+a)^5/x^31,x, algorithm="maxima")
```

output
```
-1/3780*(252*b^5*x^15 + 1050*a*b^4*x^12 + 1800*a^2*b^3*x^9 + 1575*a^3*b^2*
x^6 + 700*a^4*b*x^3 + 126*a^5)/x^30
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = -\frac{252\,b^5x^{15}+1050\,ab^4x^{12}+1800\,a^2b^3x^9+1575\,a^3b^2x^6+700\,a^4bx^3+126\,a^5}{3780\,x^{30}}$$

input
```
integrate((b*x^3+a)^5/x^31,x, algorithm="giac")
```

output
```
-1/3780*(252*b^5*x^15 + 1050*a*b^4*x^12 + 1800*a^2*b^3*x^9 + 1575*a^3*b^2*
x^6 + 700*a^4*b*x^3 + 126*a^5)/x^30
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = -\frac{\frac{a^5}{30}+\frac{5\,a^4\,b\,x^3}{27}+\frac{5\,a^3\,b^2\,x^6}{12}+\frac{10\,a^2\,b^3\,x^9}{21}+\frac{5\,a\,b^4\,x^{12}}{18}+\frac{b^5\,x^{15}}{15}}{x^{30}}$$

input
```
int((a + b*x^3)^5/x^31,x)
```

output
```
-(a^5/30 + (b^5*x^15)/15 + (5*a^4*b*x^3)/27 + (5*a*b^4*x^12)/18 + (5*a^3*b
^2*x^6)/12 + (10*a^2*b^3*x^9)/21)/x^30
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^5}{x^{31}}\,dx = \frac{-252b^5x^{15}-1050a\,b^4x^{12}-1800a^2b^3x^9-1575a^3b^2x^6-700a^4b\,x^3-126a^5}{3780x^{30}}$$

input
```
int((b*x^3+a)^5/x^31,x)
```

output
```
( - 126*a**5 - 700*a**4*b*x**3 - 1575*a**3*b**2*x**6 - 1800*a**2*b**3*x**9
 - 1050*a*b**4*x**12 - 252*b**5*x**15)/(3780*x**30)
```

3.70 $\int x^4(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^4\left(a+bx^3\right)^5\,dx=\frac{a^5x^5}{5}+\frac{5}{8}a^4bx^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{20}}{20}$$

output
```
1/5*a^5*x^5+5/8*a^4*b*x^8+10/11*a^3*b^2*x^11+5/7*a^2*b^3*x^14+5/17*a*b^4*x
^17+1/20*b^5*x^20
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^3\right)^5\,dx=\frac{a^5x^5}{5}+\frac{5}{8}a^4bx^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{20}}{20}$$

input
```
Integrate[x^4*(a + b*x^3)^5,x]
```

output
```
(a^5*x^5)/5 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^11)/11 + (5*a^2*b^3*x^14)/7
+ (5*a*b^4*x^17)/17 + (b^5*x^20)/20
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + bx^3)^5 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int (a^5x^4 + 5a^4bx^7 + 10a^3b^2x^{10} + 10a^2b^3x^{13} + 5ab^4x^{16} + b^5x^{19}) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^5}{5} + \frac{5}{8}a^4bx^8 + \frac{10}{11}a^3b^2x^{11} + \frac{5}{7}a^2b^3x^{14} + \frac{5}{17}ab^4x^{17} + \frac{b^5x^{20}}{20}$$

input
```
Int[x^4*(a + b*x^3)^5,x]
```

output
```
(a^5*x^5)/5 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^11)/11 + (5*a^2*b^3*x^14)/7
+ (5*a*b^4*x^17)/17 + (b^5*x^20)/20
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^5x^5+\frac{5}{8}a^4b\,x^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{20}b^5x^{20}$	58
default	$\frac{1}{5}a^5x^5+\frac{5}{8}a^4b\,x^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{20}b^5x^{20}$	58
norman	$\frac{1}{5}a^5x^5+\frac{5}{8}a^4b\,x^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{20}b^5x^{20}$	58
risch	$\frac{1}{5}a^5x^5+\frac{5}{8}a^4b\,x^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{20}b^5x^{20}$	58
parallelrisch	$\frac{1}{5}a^5x^5+\frac{5}{8}a^4b\,x^8+\frac{10}{11}a^3b^2x^{11}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{20}b^5x^{20}$	58
orering	$\frac{x^5\left(2618b^5x^{15}+15400a\,b^4x^{12}+37400a^2b^3x^9+47600a^3b^2x^6+32725a^4b\,x^3+10472a^5\right)}{52360}$	60

input
```
int(x^4*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*x^5+5/8*a^4*b*x^8+10/11*a^3*b^2*x^11+5/7*a^2*b^3*x^14+5/17*a*b^4*x
^17+1/20*b^5*x^20
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4\left(a+bx^3\right)^5\,dx=\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^4 (a + bx^3)^5 \, dx = \frac{a^5x^5}{5} + \frac{5a^4bx^8}{8} + \frac{10a^3b^2x^{11}}{11} + \frac{5a^2b^3x^{14}}{7} + \frac{5ab^4x^{17}}{17} + \frac{b^5x^{20}}{20}$$

input
```
integrate(x**4*(b*x**3+a)**5,x)
```

output
```
a**5*x**5/5 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**11/11 + 5*a**2*b**3*x**14/
7 + 5*a*b**4*x**17/17 + b**5*x**20/20
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4 (a + bx^3)^5 \, dx = \frac{1}{20} b^5x^{20} + \frac{5}{17} ab^4x^{17} + \frac{5}{7} a^2b^3x^{14} + \frac{10}{11} a^3b^2x^{11} + \frac{5}{8} a^4bx^8 + \frac{1}{5} a^5x^5$$

input
```
integrate(x^4*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4 (a + bx^3)^5 \, dx = \frac{1}{20} b^5x^{20} + \frac{5}{17} ab^4x^{17} + \frac{5}{7} a^2b^3x^{14} + \frac{10}{11} a^3b^2x^{11} + \frac{5}{8} a^4bx^8 + \frac{1}{5} a^5x^5$$

input
```
integrate(x^4*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 +
5/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4 (a + bx^3)^5 \, dx = \frac{a^5 x^5}{5} + \frac{5 a^4 b x^8}{8} + \frac{10 a^3 b^2 x^{11}}{11} + \frac{5 a^2 b^3 x^{14}}{7} + \frac{5 a b^4 x^{17}}{17} + \frac{b^5 x^{20}}{20}$$

input
```
int(x^4*(a + b*x^3)^5,x)
```

output
```
(a^5*x^5)/5 + (b^5*x^20)/20 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^17)/17 + (10*a^
3*b^2*x^11)/11 + (5*a^2*b^3*x^14)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^4 (a + bx^3)^5 \, dx$$
$$= \frac{x^5 (2618 b^5 x^{15} + 15400 a b^4 x^{12} + 37400 a^2 b^3 x^9 + 47600 a^3 b^2 x^6 + 32725 a^4 b x^3 + 10472 a^5)}{52360}$$

input
```
int(x^4*(b*x^3+a)^5,x)
```

output
```
(x**5*(10472*a**5 + 32725*a**4*b*x**3 + 47600*a**3*b**2*x**6 + 37400*a**2*
b**3*x**9 + 15400*a*b**4*x**12 + 2618*b**5*x**15))/52360
```

3.71 $\int x^3(a+bx^3)^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int x^3 \left(a+bx^3\right)^5 \, dx = \frac{a^5x^4}{4} + \frac{5}{7}a^4bx^7 + a^3b^2x^{10} + \frac{10}{13}a^2b^3x^{13} + \frac{5}{16}ab^4x^{16} + \frac{b^5x^{19}}{19}$$

output
```
1/4*a^5*x^4+5/7*a^4*b*x^7+a^3*b^2*x^10+10/13*a^2*b^3*x^13+5/16*a*b^4*x^16+
1/19*b^5*x^19
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^3 \left(a+bx^3\right)^5 \, dx = \frac{a^5x^4}{4} + \frac{5}{7}a^4bx^7 + a^3b^2x^{10} + \frac{10}{13}a^2b^3x^{13} + \frac{5}{16}ab^4x^{16} + \frac{b^5x^{19}}{19}$$

input
```
Integrate[x^3*(a + b*x^3)^5,x]
```

output
```
(a^5*x^4)/4 + (5*a^4*b*x^7)/7 + a^3*b^2*x^10 + (10*a^2*b^3*x^13)/13 + (5*a
*b^4*x^16)/16 + (b^5*x^19)/19
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^5x^3+5a^4bx^6+10a^3b^2x^9+10a^2b^3x^{12}+5ab^4x^{15}+b^5x^{18}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^4}{4}+\frac{5}{7}a^4bx^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{19}}{19}$$

input
```
Int[x^3*(a + b*x^3)^5,x]
```

output
```
(a^5*x^4)/4 + (5*a^4*b*x^7)/7 + a^3*b^2*x^10 + (10*a^2*b^3*x^13)/13 + (5*a
*b^4*x^16)/16 + (b^5*x^19)/19
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{1}{4}a^5x^4+\frac{5}{7}a^4b\,x^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{19}b^5x^{19}$	57
default	$\frac{1}{4}a^5x^4+\frac{5}{7}a^4b\,x^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{19}b^5x^{19}$	57
norman	$\frac{1}{4}a^5x^4+\frac{5}{7}a^4b\,x^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{19}b^5x^{19}$	57
risch	$\frac{1}{4}a^5x^4+\frac{5}{7}a^4b\,x^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{19}b^5x^{19}$	57
parallelrisch	$\frac{1}{4}a^5x^4+\frac{5}{7}a^4b\,x^7+a^3b^2x^{10}+\frac{10}{13}a^2b^3x^{13}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{19}b^5x^{19}$	57
orering	$\frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)}{27664}$	60

input
```
int(x^3*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*x^4+5/7*a^4*b*x^7+a^3*b^2*x^10+10/13*a^2*b^3*x^13+5/16*a*b^4*x^16+
1/19*b^5*x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^3(a+bx^3)^5\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^3 (a+bx^3)^5 \, dx = \frac{a^5x^4}{4} + \frac{5a^4bx^7}{7} + a^3b^2x^{10} + \frac{10a^2b^3x^{13}}{13} + \frac{5ab^4x^{16}}{16} + \frac{b^5x^{19}}{19}$$

input
```
integrate(x**3*(b*x**3+a)**5,x)
```

output
```
a**5*x**4/4 + 5*a**4*b*x**7/7 + a**3*b**2*x**10 + 10*a**2*b**3*x**13/13 +
5*a*b**4*x**16/16 + b**5*x**19/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^3 (a+bx^3)^5 \, dx = \frac{1}{19} b^5x^{19} + \frac{5}{16} ab^4x^{16} + \frac{10}{13} a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7} a^4bx^7 + \frac{1}{4} a^5x^4$$

input
```
integrate(x^3*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^3 (a+bx^3)^5 \, dx = \frac{1}{19} b^5x^{19} + \frac{5}{16} ab^4x^{16} + \frac{10}{13} a^2b^3x^{13} + a^3b^2x^{10} + \frac{5}{7} a^4bx^7 + \frac{1}{4} a^5x^4$$

input
```
integrate(x^3*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*
a^4*b*x^7 + 1/4*a^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^3\left(a+bx^3\right)^5\,dx=\frac{a^5\,x^4}{4}+\frac{5\,a^4\,b\,x^7}{7}+a^3\,b^2\,x^{10}+\frac{10\,a^2\,b^3\,x^{13}}{13}+\frac{5\,a\,b^4\,x^{16}}{16}+\frac{b^5\,x^{19}}{19}$$

input
```
int(x^3*(a + b*x^3)^5,x)
```

output
```
(a^5*x^4)/4 + (b^5*x^19)/19 + (5*a^4*b*x^7)/7 + (5*a*b^4*x^16)/16 + a^3*b^
2*x^10 + (10*a^2*b^3*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^3\left(a+bx^3\right)^5\,dx$$
$$=\frac{x^4(1456b^5x^{15}+8645a\,b^4x^{12}+21280a^2b^3x^9+27664a^3b^2x^6+19760a^4b\,x^3+6916a^5)}{27664}$$

input
```
int(x^3*(b*x^3+a)^5,x)
```

output
```
(x**4*(6916*a**5 + 19760*a**4*b*x**3 + 27664*a**3*b**2*x**6 + 21280*a**2*b
**3*x**9 + 8645*a*b**4*x**12 + 1456*b**5*x**15))/27664
```

3.72 $\int x(a+bx^3)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int x(a+bx^3)^5\,dx=\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}$$

output
```
1/2*a^5*x^2+a^4*b*x^5+5/4*a^3*b^2*x^8+10/11*a^2*b^3*x^11+5/14*a*b^4*x^14+1
/17*b^5*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)^5\,dx=\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}$$

input
```
Integrate[x*(a + b*x^3)^5,x]
```

output
```
(a^5*x^2)/2 + a^4*b*x^5 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^11)/11 + (5*a*
b^4*x^14)/14 + (b^5*x^17)/17
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^5x+5a^4bx^4+10a^3b^2x^7+10a^2b^3x^{10}+5ab^4x^{13}+b^5x^{16}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^2}{2}+a^4bx^5+\frac{5}{4}a^3b^2x^8+\frac{10}{11}a^2b^3x^{11}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{17}}{17}$$

input
```
Int[x*(a + b*x^3)^5,x]
```

output
```
(a^5*x^2)/2 + a^4*b*x^5 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^11)/11 + (5*a*
b^4*x^14)/14 + (b^5*x^17)/17
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{1}{2}a^5x^2 + a^4b\,x^5 + \frac{5}{4}a^3b^2x^8 + \frac{10}{11}a^2b^3x^{11} + \frac{5}{14}a\,b^4x^{14} + \frac{1}{17}b^5x^{17}$	57
default	$\frac{1}{2}a^5x^2 + a^4b\,x^5 + \frac{5}{4}a^3b^2x^8 + \frac{10}{11}a^2b^3x^{11} + \frac{5}{14}a\,b^4x^{14} + \frac{1}{17}b^5x^{17}$	57
norman	$\frac{1}{2}a^5x^2 + a^4b\,x^5 + \frac{5}{4}a^3b^2x^8 + \frac{10}{11}a^2b^3x^{11} + \frac{5}{14}a\,b^4x^{14} + \frac{1}{17}b^5x^{17}$	57
risch	$\frac{1}{2}a^5x^2 + a^4b\,x^5 + \frac{5}{4}a^3b^2x^8 + \frac{10}{11}a^2b^3x^{11} + \frac{5}{14}a\,b^4x^{14} + \frac{1}{17}b^5x^{17}$	57
parallelrisch	$\frac{1}{2}a^5x^2 + a^4b\,x^5 + \frac{5}{4}a^3b^2x^8 + \frac{10}{11}a^2b^3x^{11} + \frac{5}{14}a\,b^4x^{14} + \frac{1}{17}b^5x^{17}$	57
orering	$\frac{x^2(308b^5x^{15}+1870a\,b^4x^{12}+4760a^2b^3x^9+6545a^3b^2x^6+5236a^4b\,x^3+2618a^5)}{5236}$	60

input
```
int(x*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^5*x^2+a^4*b*x^5+5/4*a^3*b^2*x^8+10/11*a^2*b^3*x^11+5/14*a*b^4*x^14+1
/17*b^5*x^17
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x\left(a+bx^3\right)^5\,dx = \frac{1}{17}\,b^5x^{17} + \frac{5}{14}\,ab^4x^{14} + \frac{10}{11}\,a^2b^3x^{11} + \frac{5}{4}\,a^3b^2x^8 + a^4bx^5 + \frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x\left(a+bx^3\right)^5\,dx=\frac{a^5x^2}{2}+a^4bx^5+\frac{5a^3b^2x^8}{4}+\frac{10a^2b^3x^{11}}{11}+\frac{5ab^4x^{14}}{14}+\frac{b^5x^{17}}{17}$$

input
```
integrate(x*(b*x**3+a)**5,x)
```

output
```
a**5*x**2/2 + a**4*b*x**5 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**11/11 + 5
*a*b**4*x**14/14 + b**5*x**17/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x\left(a+bx^3\right)^5\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x\left(a+bx^3\right)^5\,dx=\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a
^4*b*x^5 + 1/2*a^5*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)^5\,dx=\frac{a^5\,x^2}{2}+a^4\,b\,x^5+\frac{5\,a^3\,b^2\,x^8}{4}+\frac{10\,a^2\,b^3\,x^{11}}{11}+\frac{5\,a\,b^4\,x^{14}}{14}+\frac{b^5\,x^{17}}{17}$$

input
```
int(x*(a + b*x^3)^5,x)
```

output
```
(a^5*x^2)/2 + (b^5*x^17)/17 + a^4*b*x^5 + (5*a*b^4*x^14)/14 + (5*a^3*b^2*x
^8)/4 + (10*a^2*b^3*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x(a+bx^3)^5\,dx$$
$$=\frac{x^2(308b^5x^{15}+1870a\,b^4x^{12}+4760a^2b^3x^9+6545a^3b^2x^6+5236a^4b\,x^3+2618a^5)}{5236}$$

input
```
int(x*(b*x^3+a)^5,x)
```

output
```
(x**2*(2618*a**5 + 5236*a**4*b*x**3 + 6545*a**3*b**2*x**6 + 4760*a**2*b**3
*x**9 + 1870*a*b**4*x**12 + 308*b**5*x**15))/5236
```

3.73 $\int (a+bx^3)^5 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int (a+bx^3)^5 \, dx = a^5x + \frac{5}{4}a^4bx^4 + \frac{10}{7}a^3b^2x^7 + a^2b^3x^{10} + \frac{5}{13}ab^4x^{13} + \frac{b^5x^{16}}{16}$$

output
```
a^5*x+5/4*a^4*b*x^4+10/7*a^3*b^2*x^7+a^2*b^3*x^10+5/13*a*b^4*x^13+1/16*b^5
*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^5 \, dx = a^5x + \frac{5}{4}a^4bx^4 + \frac{10}{7}a^3b^2x^7 + a^2b^3x^{10} + \frac{5}{13}ab^4x^{13} + \frac{b^5x^{16}}{16}$$

input
```
Integrate[(a + b*x^3)^5,x]
```

output
```
a^5*x + (5*a^4*b*x^4)/4 + (10*a^3*b^2*x^7)/7 + a^2*b^3*x^10 + (5*a*b^4*x^1
3)/13 + (b^5*x^16)/16
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{747}$$

$$\int \left(a^5+5a^4bx^3+10a^3b^2x^6+10a^2b^3x^9+5ab^4x^{12}+b^5x^{15}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^5x+\frac{5}{4}a^4bx^4+\frac{10}{7}a^3b^2x^7+a^2b^3x^{10}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{16}}{16}$$

input
```
Int[(a + b*x^3)^5,x]
```

output
```
a^5*x + (5*a^4*b*x^4)/4 + (10*a^3*b^2*x^7)/7 + a^2*b^3*x^10 + (5*a*b^4*x^1
3)/13 + (b^5*x^16)/16
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
gosper	$a^5x+\frac{5}{4}a^4b\,x^4+\frac{10}{7}a^3b^2x^7+a^2b^3x^{10}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{16}b^5x^{16}$	54
default	$a^5x+\frac{5}{4}a^4b\,x^4+\frac{10}{7}a^3b^2x^7+a^2b^3x^{10}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{16}b^5x^{16}$	54
norman	$a^5x+\frac{5}{4}a^4b\,x^4+\frac{10}{7}a^3b^2x^7+a^2b^3x^{10}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{16}b^5x^{16}$	54
risch	$a^5x+\frac{5}{4}a^4b\,x^4+\frac{10}{7}a^3b^2x^7+a^2b^3x^{10}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{16}b^5x^{16}$	54
parallelrisch	$a^5x+\frac{5}{4}a^4b\,x^4+\frac{10}{7}a^3b^2x^7+a^2b^3x^{10}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{16}b^5x^{16}$	54
orering	$\frac{x\left(91b^5x^{15}+560a\,b^4x^{12}+1456a^2b^3x^9+2080a^3b^2x^6+1820a^4b\,x^3+1456a^5\right)}{1456}$	58

input
```
int((b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*x+5/4*a^4*b*x^4+10/7*a^3*b^2*x^7+a^2*b^3*x^10+5/13*a*b^4*x^13+1/16*b^5
*x^16
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \left(a+bx^3\right)^5\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

input
```
integrate((b*x^3+a)^5,x, algorithm="fricas")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \left(a+bx^3\right)^5 \, dx = a^5x + \frac{5a^4bx^4}{4} + \frac{10a^3b^2x^7}{7} + a^2b^3x^{10} + \frac{5ab^4x^{13}}{13} + \frac{b^5x^{16}}{16}$$

input
```
integrate((b*x**3+a)**5,x)
```

output
```
a**5*x + 5*a**4*b*x**4/4 + 10*a**3*b**2*x**7/7 + a**2*b**3*x**10 + 5*a*b**
4*x**13/13 + b**5*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \left(a+bx^3\right)^5 \, dx = \frac{1}{16}\,b^5x^{16} + \frac{5}{13}\,ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7}\,a^3b^2x^7 + \frac{5}{4}\,a^4bx^4 + a^5x$$

input
```
integrate((b*x^3+a)^5,x, algorithm="maxima")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \left(a+bx^3\right)^5 \, dx = \frac{1}{16}\,b^5x^{16} + \frac{5}{13}\,ab^4x^{13} + a^2b^3x^{10} + \frac{10}{7}\,a^3b^2x^7 + \frac{5}{4}\,a^4bx^4 + a^5x$$

input
```
integrate((b*x^3+a)^5,x, algorithm="giac")
```

output
```
1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^
4*b*x^4 + a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int (a+bx^3)^5\,dx = a^5\,x + \frac{5\,a^4\,b\,x^4}{4} + \frac{10\,a^3\,b^2\,x^7}{7} + a^2\,b^3\,x^{10} + \frac{5\,a\,b^4\,x^{13}}{13} + \frac{b^5\,x^{16}}{16}$$

input
```
int((a + b*x^3)^5,x)
```

output
```
a^5*x + (b^5*x^16)/16 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^13)/13 + (10*a^3*b^2*
x^7)/7 + a^2*b^3*x^10
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int (a+bx^3)^5\,dx = \frac{x(91b^5x^{15} + 560a\,b^4x^{12} + 1456a^2b^3x^9 + 2080a^3b^2x^6 + 1820a^4b\,x^3 + 1456a^5)}{1456}$$

input
```
int((b*x^3+a)^5,x)
```

output
```
(x*(1456*a**5 + 1820*a**4*b*x**3 + 2080*a**3*b**2*x**6 + 1456*a**2*b**3*x*
*9 + 560*a*b**4*x**12 + 91*b**5*x**15))/1456
```

3.74 $\int \frac{(a+bx^3)^5}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^3)^5}{x^2}\,dx=-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}$$

output
```
-a^5/x+5/2*a^4*b*x^2+2*a^3*b^2*x^5+5/4*a^2*b^3*x^8+5/11*a*b^4*x^11+1/14*b^
5*x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^2}\,dx=-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}$$

input
```
Integrate[(a + b*x^3)^5/x^2,x]
```

output
```
-(a^5/x) + (5*a^4*b*x^2)/2 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^8)/4 + (5*a*b^4*
x^11)/11 + (b^5*x^14)/14
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^2} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^5}{x^2}+5a^4bx+10a^3b^2x^4+10a^2b^3x^7+5ab^4x^{10}+b^5x^{13}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5}{x}+\frac{5}{2}a^4bx^2+2a^3b^2x^5+\frac{5}{4}a^2b^3x^8+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{14}}{14}$$

input
```
Int[(a + b*x^3)^5/x^2,x]
```

output
```
-(a^5/x) + (5*a^4*b*x^2)/2 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^8)/4 + (5*a*b^4*
x^11)/11 + (b^5*x^14)/14
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^{5}}{x}+\frac{5a^{4}b\,x^{2}}{2}+2a^{3}b^{2}x^{5}+\frac{5a^{2}b^{3}x^{8}}{4}+\frac{5a\,b^{4}x^{11}}{11}+\frac{b^{5}x^{14}}{14}$	58
risch	$-\frac{a^{5}}{x}+\frac{5a^{4}b\,x^{2}}{2}+2a^{3}b^{2}x^{5}+\frac{5a^{2}b^{3}x^{8}}{4}+\frac{5a\,b^{4}x^{11}}{11}+\frac{b^{5}x^{14}}{14}$	58
norman	$\frac{-a^{5}+\frac{5}{2}a^{4}b\,x^{3}+2a^{3}b^{2}x^{6}+\frac{5}{4}a^{2}b^{3}x^{9}+\frac{5}{11}a\,b^{4}x^{12}+\frac{1}{14}b^{5}x^{15}}{x}$	59
gosper	$-\frac{-22b^{5}x^{15}-140a\,b^{4}x^{12}-385a^{2}b^{3}x^{9}-616a^{3}b^{2}x^{6}-770a^{4}b\,x^{3}+308a^{5}}{308x}$	60
parallelrisch	$\frac{22b^{5}x^{15}+140a\,b^{4}x^{12}+385a^{2}b^{3}x^{9}+616a^{3}b^{2}x^{6}+770a^{4}b\,x^{3}-308a^{5}}{308x}$	60
orering	$-\frac{-22b^{5}x^{15}-140a\,b^{4}x^{12}-385a^{2}b^{3}x^{9}-616a^{3}b^{2}x^{6}-770a^{4}b\,x^{3}+308a^{5}}{308x}$	60

input
```
int((b*x^3+a)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5/x+5/2*a^4*b*x^2+2*a^3*b^2*x^5+5/4*a^2*b^3*x^8+5/11*a*b^4*x^11+1/14*b^
5*x^14
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^2}\,dx = \frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

input
```
integrate((b*x^3+a)^5/x^2,x, algorithm="fricas")
```

output
```
1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 +
770*a^4*b*x^3 - 308*a^5)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^2}\,dx = -\frac{a^5}{x} + \frac{5a^4bx^2}{2} + 2a^3b^2x^5 + \frac{5a^2b^3x^8}{4} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{14}}{14}$$

input
```
integrate((b*x**3+a)**5/x**2,x)
```

output
```
-a**5/x + 5*a**4*b*x**2/2 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**8/4 + 5*a*b*
*4*x**11/11 + b**5*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^5}{x^2}\,dx = \frac{1}{14}\,b^5x^{14} + \frac{5}{11}\,ab^4x^{11} + \frac{5}{4}\,a^2b^3x^8 + 2\,a^3b^2x^5 + \frac{5}{2}\,a^4bx^2 - \frac{a^5}{x}$$

input
```
integrate((b*x^3+a)^5/x^2,x, algorithm="maxima")
```

output
```
1/14*b^5*x^14 + 5/11*a*b^4*x^11 + 5/4*a^2*b^3*x^8 + 2*a^3*b^2*x^5 + 5/2*a^
4*b*x^2 - a^5/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^5}{x^2}\,dx = \frac{1}{14}\,b^5x^{14} + \frac{5}{11}\,ab^4x^{11} + \frac{5}{4}\,a^2b^3x^8 + 2\,a^3b^2x^5 + \frac{5}{2}\,a^4bx^2 - \frac{a^5}{x}$$

input
```
integrate((b*x^3+a)^5/x^2,x, algorithm="giac")
```

output
```
1/14*b^5*x^14 + 5/11*a*b^4*x^11 + 5/4*a^2*b^3*x^8 + 2*a^3*b^2*x^5 + 5/2*a^
4*b*x^2 - a^5/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^5}{x^2}\,dx=\frac{b^5\,x^{14}}{14}-\frac{a^5}{x}+\frac{5\,a^4\,b\,x^2}{2}+\frac{5\,a\,b^4\,x^{11}}{11}+2\,a^3\,b^2\,x^5+\frac{5\,a^2\,b^3\,x^8}{4}$$

input
```
int((a + b*x^3)^5/x^2,x)
```

output
```
(b^5*x^14)/14 - a^5/x + (5*a^4*b*x^2)/2 + (5*a*b^4*x^11)/11 + 2*a^3*b^2*x^
5 + (5*a^2*b^3*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^2}\,dx=\frac{22b^5x^{15}+140a\,b^4x^{12}+385a^2b^3x^9+616a^3b^2x^6+770a^4b\,x^3-308a^5}{308x}$$

input
```
int((b*x^3+a)^5/x^2,x)
```

output
```
( - 308*a**5 + 770*a**4*b*x**3 + 616*a**3*b**2*x**6 + 385*a**2*b**3*x**9 +
 140*a*b**4*x**12 + 22*b**5*x**15)/(308*x)
```

3.75 $\int \frac{(a+bx^3)^5}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^3)^5}{x^3} dx = -\frac{a^5}{2x^2} + 5a^4bx + \frac{5}{2}a^3b^2x^4 + \frac{10}{7}a^2b^3x^7 + \frac{1}{2}ab^4x^{10} + \frac{b^5x^{13}}{13}$$

output
```
-1/2*a^5/x^2+5*a^4*b*x+5/2*a^3*b^2*x^4+10/7*a^2*b^3*x^7+1/2*a*b^4*x^10+1/1
3*b^5*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^3} dx = -\frac{a^5}{2x^2} + 5a^4bx + \frac{5}{2}a^3b^2x^4 + \frac{10}{7}a^2b^3x^7 + \frac{1}{2}ab^4x^{10} + \frac{b^5x^{13}}{13}$$

input
```
Integrate[(a + b*x^3)^5/x^3,x]
```

output
```
-1/2*a^5/x^2 + 5*a^4*b*x + (5*a^3*b^2*x^4)/2 + (10*a^2*b^3*x^7)/7 + (a*b^4
*x^10)/2 + (b^5*x^13)/13
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^3}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^5}{x^3}+5a^4b+10a^3b^2x^3+10a^2b^3x^6+5ab^4x^9+b^5x^{12}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{2x^2}+5a^4bx+\frac{5}{2}a^3b^2x^4+\frac{10}{7}a^2b^3x^7+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{13}}{13}$$

input
```
Int[(a + b*x^3)^5/x^3,x]
```

output
```
-1/2*a^5/x^2 + 5*a^4*b*x + (5*a^3*b^2*x^4)/2 + (10*a^2*b^3*x^7)/7 + (a*b^4
*x^10)/2 + (b^5*x^13)/13
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{5}}{2x^{2}}+5a^{4}bx+\frac{5a^{3}b^{2}x^{4}}{2}+\frac{10a^{2}b^{3}x^{7}}{7}+\frac{a\,b^{4}x^{10}}{2}+\frac{b^{5}x^{13}}{13}$	56
risch	$-\frac{a^{5}}{2x^{2}}+5a^{4}bx+\frac{5a^{3}b^{2}x^{4}}{2}+\frac{10a^{2}b^{3}x^{7}}{7}+\frac{a\,b^{4}x^{10}}{2}+\frac{b^{5}x^{13}}{13}$	56
norman	$\frac{-\frac{1}{2}a^{5}+5a^{4}b\,x^{3}+\frac{5}{2}a^{3}b^{2}x^{6}+\frac{10}{7}a^{2}b^{3}x^{9}+\frac{1}{2}a\,b^{4}x^{12}+\frac{1}{13}b^{5}x^{15}}{x^{2}}$	59
gosper	$-\frac{-14b^{5}x^{15}-91a\,b^{4}x^{12}-260a^{2}b^{3}x^{9}-455a^{3}b^{2}x^{6}-910a^{4}b\,x^{3}+91a^{5}}{182x^{2}}$	60
parallelrisch	$\frac{14b^{5}x^{15}+91a\,b^{4}x^{12}+260a^{2}b^{3}x^{9}+455a^{3}b^{2}x^{6}+910a^{4}b\,x^{3}-91a^{5}}{182x^{2}}$	60
orering	$-\frac{-14b^{5}x^{15}-91a\,b^{4}x^{12}-260a^{2}b^{3}x^{9}-455a^{3}b^{2}x^{6}-910a^{4}b\,x^{3}+91a^{5}}{182x^{2}}$	60

input
```
int((b*x^3+a)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^5/x^2+5*a^4*b*x+5/2*a^3*b^2*x^4+10/7*a^2*b^3*x^7+1/2*a*b^4*x^10+1/1
3*b^5*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^3}\,dx = \frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

input
```
integrate((b*x^3+a)^5/x^3,x, algorithm="fricas")
```

output
```
1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 9
10*a^4*b*x^3 - 91*a^5)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^3}\,dx = -\frac{a^5}{2x^2} + 5a^4bx + \frac{5a^3b^2x^4}{2} + \frac{10a^2b^3x^7}{7} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{13}}{13}$$

input
```
integrate((b*x**3+a)**5/x**3,x)
```

output
```
-a**5/(2*x**2) + 5*a**4*b*x + 5*a**3*b**2*x**4/2 + 10*a**2*b**3*x**7/7 + a
*b**4*x**10/2 + b**5*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^5}{x^3}\,dx = \frac{1}{13}\,b^5x^{13} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{2}\,a^3b^2x^4 + 5\,a^4bx - \frac{a^5}{2\,x^2}$$

input
```
integrate((b*x^3+a)^5/x^3,x, algorithm="maxima")
```

output
```
1/13*b^5*x^13 + 1/2*a*b^4*x^10 + 10/7*a^2*b^3*x^7 + 5/2*a^3*b^2*x^4 + 5*a^
4*b*x - 1/2*a^5/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^5}{x^3}\,dx = \frac{1}{13}\,b^5x^{13} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{2}\,a^3b^2x^4 + 5\,a^4bx - \frac{a^5}{2\,x^2}$$

input
```
integrate((b*x^3+a)^5/x^3,x, algorithm="giac")
```

output
```
1/13*b^5*x^13 + 1/2*a*b^4*x^10 + 10/7*a^2*b^3*x^7 + 5/2*a^3*b^2*x^4 + 5*a^
4*b*x - 1/2*a^5/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^5}{x^3}\,dx = \frac{b^5\,x^{13}}{13} - \frac{a^5}{2\,x^2} + \frac{a\,b^4\,x^{10}}{2} + \frac{5\,a^3\,b^2\,x^4}{2} + \frac{10\,a^2\,b^3\,x^7}{7} + 5\,a^4\,b\,x$$

input
```
int((a + b*x^3)^5/x^3,x)
```

output
```
(b^5*x^13)/13 - a^5/(2*x^2) + (a*b^4*x^10)/2 + (5*a^3*b^2*x^4)/2 + (10*a^2
*b^3*x^7)/7 + 5*a^4*b*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^3}\,dx = \frac{14b^5x^{15} + 91a\,b^4x^{12} + 260a^2b^3x^9 + 455a^3b^2x^6 + 910a^4b\,x^3 - 91a^5}{182x^2}$$

input
```
int((b*x^3+a)^5/x^3,x)
```

output
```
( - 91*a**5 + 910*a**4*b*x**3 + 455*a**3*b**2*x**6 + 260*a**2*b**3*x**9 +
91*a*b**4*x**12 + 14*b**5*x**15)/(182*x**2)
```

3.76 $\int \frac{(a+bx^3)^5}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{(a+bx^3)^5}{x^5} dx = -\frac{a^5}{4x^4} - \frac{5a^4b}{x} + 5a^3b^2x^2 + 2a^2b^3x^5 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{11}}{11}$$

output
```
-1/4*a^5/x^4-5*a^4*b/x+5*a^3*b^2*x^2+2*a^2*b^3*x^5+5/8*a*b^4*x^8+1/11*b^5*
x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^5} dx = -\frac{a^5}{4x^4} - \frac{5a^4b}{x} + 5a^3b^2x^2 + 2a^2b^3x^5 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{11}}{11}$$

input
```
Integrate[(a + b*x^3)^5/x^5,x]
```

output
```
-1/4*a^5/x^4 - (5*a^4*b)/x + 5*a^3*b^2*x^2 + 2*a^2*b^3*x^5 + (5*a*b^4*x^8)
/8 + (b^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^5} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^5}{x^5}+\frac{5a^4b}{x^2}+10a^3b^2x+10a^2b^3x^4+5ab^4x^7+b^5x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5}{8}ab^4x^8+\frac{b^5x^{11}}{11}$$

input
```
Int[(a + b*x^3)^5/x^5,x]
```

output
```
-1/4*a^5/x^4 - (5*a^4*b)/x + 5*a^3*b^2*x^2 + 2*a^2*b^3*x^5 + (5*a*b^4*x^8)
/8 + (b^5*x^11)/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{4x^4}-\frac{5a^4b}{x}+5a^3b^2x^2+2a^2b^3x^5+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{11}}{11}$	58
norman	$\frac{-\frac{1}{4}a^5-5a^4b\,x^3+5a^3b^2x^6+2a^2b^3x^9+\frac{5}{8}a\,b^4x^{12}+\frac{1}{11}b^5x^{15}}{x^4}$	59
gosper	$-\frac{-8b^5x^{15}-55a\,b^4x^{12}-176a^2b^3x^9-440a^3b^2x^6+440a^4b\,x^3+22a^5}{88x^4}$	60
risch	$\frac{b^5x^{11}}{11}+\frac{5a\,b^4x^8}{8}+2a^2b^3x^5+5a^3b^2x^2+\frac{-5a^4b\,x^3-\frac{1}{4}a^5}{x^4}$	60
parallelrisch	$\frac{8b^5x^{15}+55a\,b^4x^{12}+176a^2b^3x^9+440a^3b^2x^6-440a^4b\,x^3-22a^5}{88x^4}$	60
orering	$-\frac{-8b^5x^{15}-55a\,b^4x^{12}-176a^2b^3x^9-440a^3b^2x^6+440a^4b\,x^3+22a^5}{88x^4}$	60

input
```
int((b*x^3+a)^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^5/x^4-5*a^4*b/x+5*a^3*b^2*x^2+2*a^2*b^3*x^5+5/8*a*b^4*x^8+1/11*b^5*
x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^5}\,dx = \frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

input
```
integrate((b*x^3+a)^5/x^5,x, algorithm="fricas")
```

output
```
1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440
*a^4*b*x^3 - 22*a^5)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^5}\,dx = 5a^3b^2x^2 + 2a^2b^3x^5 + \frac{5ab^4x^8}{8} + \frac{b^5x^{11}}{11} + \frac{-a^5 - 20a^4bx^3}{4x^4}$$

input
```
integrate((b*x**3+a)**5/x**5,x)
```

output
```
5*a**3*b**2*x**2 + 2*a**2*b**3*x**5 + 5*a*b**4*x**8/8 + b**5*x**11/11 + (-
a**5 - 20*a**4*b*x**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^5}\,dx = \frac{1}{11}\,b^5x^{11} + \frac{5}{8}\,ab^4x^8 + 2\,a^2b^3x^5 + 5\,a^3b^2x^2 - \frac{20\,a^4bx^3 + a^5}{4\,x^4}$$

input
```
integrate((b*x^3+a)^5/x^5,x, algorithm="maxima")
```

output
```
1/11*b^5*x^11 + 5/8*a*b^4*x^8 + 2*a^2*b^3*x^5 + 5*a^3*b^2*x^2 - 1/4*(20*a^
4*b*x^3 + a^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^5}\,dx = \frac{1}{11}\,b^5x^{11} + \frac{5}{8}\,ab^4x^8 + 2\,a^2b^3x^5 + 5\,a^3b^2x^2 - \frac{20\,a^4bx^3 + a^5}{4\,x^4}$$

input
```
integrate((b*x^3+a)^5/x^5,x, algorithm="giac")
```

output
```
1/11*b^5*x^11 + 5/8*a*b^4*x^8 + 2*a^2*b^3*x^5 + 5*a^3*b^2*x^2 - 1/4*(20*a^
4*b*x^3 + a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^5}\,dx = \frac{b^5\,x^{11}}{11} - \frac{\frac{a^5}{4}+5\,b\,a^4\,x^3}{x^4} + \frac{5\,a\,b^4\,x^8}{8} + 5\,a^3\,b^2\,x^2 + 2\,a^2\,b^3\,x^5$$

input
```
int((a + b*x^3)^5/x^5,x)
```

output
```
(b^5*x^11)/11 - (a^5/4 + 5*a^4*b*x^3)/x^4 + (5*a*b^4*x^8)/8 + 5*a^3*b^2*x^
2 + 2*a^2*b^3*x^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^5}{x^5}\,dx = \frac{8b^5x^{15}+55a\,b^4x^{12}+176a^2b^3x^9+440a^3b^2x^6-440a^4b\,x^3-22a^5}{88x^4}$$

input
```
int((b*x^3+a)^5/x^5,x)
```

output
```
( - 22*a**5 - 440*a**4*b*x**3 + 440*a**3*b**2*x**6 + 176*a**2*b**3*x**9 +
55*a*b**4*x**12 + 8*b**5*x**15)/(88*x**4)
```

3.77 $\int \frac{(a+bx^3)^5}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^3)^5}{x^6} dx = -\frac{a^5}{5x^5} - \frac{5a^4b}{2x^2} + 10a^3b^2x + \frac{5}{2}a^2b^3x^4 + \frac{5}{7}ab^4x^7 + \frac{b^5x^{10}}{10}$$

output
```
-1/5*a^5/x^5-5/2*a^4*b/x^2+10*a^3*b^2*x+5/2*a^2*b^3*x^4+5/7*a*b^4*x^7+1/10
*b^5*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^6} dx = -\frac{a^5}{5x^5} - \frac{5a^4b}{2x^2} + 10a^3b^2x + \frac{5}{2}a^2b^3x^4 + \frac{5}{7}ab^4x^7 + \frac{b^5x^{10}}{10}$$

input
```
Integrate[(a + b*x^3)^5/x^6,x]
```

output
```
-1/5*a^5/x^5 - (5*a^4*b)/(2*x^2) + 10*a^3*b^2*x + (5*a^2*b^3*x^4)/2 + (5*a
*b^4*x^7)/7 + (b^5*x^10)/10
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^6}\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^3}+10a^3b^2+10a^2b^3x^3+5ab^4x^6+b^5x^9\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5}{2}a^2b^3x^4+\frac{5}{7}ab^4x^7+\frac{b^5x^{10}}{10}$$

input
```
Int[(a + b*x^3)^5/x^6,x]
```

output
```
-1/5*a^5/x^5 - (5*a^4*b)/(2*x^2) + 10*a^3*b^2*x + (5*a^2*b^3*x^4)/2 + (5*a
*b^4*x^7)/7 + (b^5*x^10)/10
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^5}{5x^5}-\frac{5a^4b}{2x^2}+10a^3b^2x+\frac{5a^2b^3x^4}{2}+\frac{5a\,b^4x^7}{7}+\frac{b^5x^{10}}{10}$	56
risch	$\frac{b^5x^{10}}{10}+\frac{5a\,b^4x^7}{7}+\frac{5a^2b^3x^4}{2}+10a^3b^2x+\frac{-\frac{5}{2}a^4b\,x^3-\frac{1}{5}a^5}{x^5}$	58
norman	$\frac{-\frac{1}{5}a^5-\frac{5}{2}a^4b\,x^3+10a^3b^2x^6+\frac{5}{2}a^2b^3x^9+\frac{5}{7}a\,b^4x^{12}+\frac{1}{10}b^5x^{15}}{x^5}$	59
gosper	$-\frac{-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5}{70x^5}$	60
parallelrisch	$\frac{7b^5x^{15}+50a\,b^4x^{12}+175a^2b^3x^9+700a^3b^2x^6-175a^4b\,x^3-14a^5}{70x^5}$	60
orering	$-\frac{-7b^5x^{15}-50a\,b^4x^{12}-175a^2b^3x^9-700a^3b^2x^6+175a^4b\,x^3+14a^5}{70x^5}$	60

input
```
int((b*x^3+a)^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^5/x^5-5/2*a^4*b/x^2+10*a^3*b^2*x+5/2*a^2*b^3*x^4+5/7*a*b^4*x^7+1/10
*b^5*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^6}\,dx = \frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

input
```
integrate((b*x^3+a)^5/x^6,x, algorithm="fricas")
```

output
```
1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175
*a^4*b*x^3 - 14*a^5)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^6}\,dx = 10a^3b^2x + \frac{5a^2b^3x^4}{2} + \frac{5ab^4x^7}{7} + \frac{b^5x^{10}}{10} + \frac{-2a^5-25a^4bx^3}{10x^5}$$

input
```
integrate((b*x**3+a)**5/x**6,x)
```

output
```
10*a**3*b**2*x + 5*a**2*b**3*x**4/2 + 5*a*b**4*x**7/7 + b**5*x**10/10 + (-
2*a**5 - 25*a**4*b*x**3)/(10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^6}\,dx = \frac{1}{10}\,b^5x^{10} + \frac{5}{7}\,ab^4x^7 + \frac{5}{2}\,a^2b^3x^4 + 10\,a^3b^2x - \frac{25\,a^4bx^3 + 2\,a^5}{10\,x^5}$$

input
```
integrate((b*x^3+a)^5/x^6,x, algorithm="maxima")
```

output
```
1/10*b^5*x^10 + 5/7*a*b^4*x^7 + 5/2*a^2*b^3*x^4 + 10*a^3*b^2*x - 1/10*(25*
a^4*b*x^3 + 2*a^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^6}\,dx = \frac{1}{10}\,b^5x^{10} + \frac{5}{7}\,ab^4x^7 + \frac{5}{2}\,a^2b^3x^4 + 10\,a^3b^2x - \frac{25\,a^4bx^3 + 2\,a^5}{10\,x^5}$$

input
```
integrate((b*x^3+a)^5/x^6,x, algorithm="giac")
```

output
```
1/10*b^5*x^10 + 5/7*a*b^4*x^7 + 5/2*a^2*b^3*x^4 + 10*a^3*b^2*x - 1/10*(25*
a^4*b*x^3 + 2*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^5}{x^6}\,dx = \frac{b^5\,x^{10}}{10} - \frac{\frac{a^5}{5}+\frac{5\,b\,a^4\,x^3}{2}}{x^5} + 10\,a^3\,b^2\,x + \frac{5\,a\,b^4\,x^7}{7} + \frac{5\,a^2\,b^3\,x^4}{2}$$

input
```
int((a + b*x^3)^5/x^6,x)
```

output
```
(b^5*x^10)/10 - (a^5/5 + (5*a^4*b*x^3)/2)/x^5 + 10*a^3*b^2*x + (5*a*b^4*x^
7)/7 + (5*a^2*b^3*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^5}{x^6}\,dx = \frac{7b^5x^{15}+50a\,b^4x^{12}+175a^2b^3x^9+700a^3b^2x^6-175a^4b\,x^3-14a^5}{70x^5}$$

input
```
int((b*x^3+a)^5/x^6,x)
```

output
```
( - 14*a**5 - 175*a**4*b*x**3 + 700*a**3*b**2*x**6 + 175*a**2*b**3*x**9 +
50*a*b**4*x**12 + 7*b**5*x**15)/(70*x**5)
```

3.78 $\int \frac{(a+bx^3)^5}{x^8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{(a+bx^3)^5}{x^8} \, dx = -\frac{a^5}{7x^7} - \frac{5a^4b}{4x^4} - \frac{10a^3b^2}{x} + 5a^2b^3x^2 + ab^4x^5 + \frac{b^5x^8}{8}$$

output
```
-1/7*a^5/x^7-5/4*a^4*b/x^4-10*a^3*b^2/x+5*a^2*b^3*x^2+a*b^4*x^5+1/8*b^5*x^
8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^8} \, dx = -\frac{a^5}{7x^7} - \frac{5a^4b}{4x^4} - \frac{10a^3b^2}{x} + 5a^2b^3x^2 + ab^4x^5 + \frac{b^5x^8}{8}$$

input
```
Integrate[(a + b*x^3)^5/x^8,x]
```

output
```
-1/7*a^5/x^7 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/x + 5*a^2*b^3*x^2 + a*b^4*
x^5 + (b^5*x^8)/8
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^8} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^5}{x^8}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^2}+10a^2b^3x+5ab^4x^4+b^5x^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+ab^4x^5+\frac{b^5x^8}{8}$$

input
```
Int[(a + b*x^3)^5/x^8,x]
```

output
```
-1/7*a^5/x^7 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/x + 5*a^2*b^3*x^2 + a*b^4*
x^5 + (b^5*x^8)/8
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{7x^7}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{x}+5a^2b^3x^2+a\,b^4x^5+\frac{b^5x^8}{8}$	57
norman	$\frac{-\frac{1}{7}a^5-\frac{5}{4}a^4b\,x^3-10a^3b^2x^6+5a^2b^3x^9+a\,b^4x^{12}+\frac{1}{8}b^5x^{15}}{x^7}$	58
risch	$\frac{b^5x^8}{8}+a\,b^4x^5+5a^2b^3x^2+\frac{-10a^3b^2x^6-\frac{5}{4}a^4b\,x^3-\frac{1}{7}a^5}{x^7}$	59
gosper	$-\frac{-7b^5x^{15}-56a\,b^4x^{12}-280a^2b^3x^9+560a^3b^2x^6+70a^4b\,x^3+8a^5}{56x^7}$	60
parallelrisch	$\frac{7b^5x^{15}+56a\,b^4x^{12}+280a^2b^3x^9-560a^3b^2x^6-70a^4b\,x^3-8a^5}{56x^7}$	60
orering	$-\frac{-7b^5x^{15}-56a\,b^4x^{12}-280a^2b^3x^9+560a^3b^2x^6+70a^4b\,x^3+8a^5}{56x^7}$	60

input
```
int((b*x^3+a)^5/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^5/x^7-5/4*a^4*b/x^4-10*a^3*b^2/x+5*a^2*b^3*x^2+a*b^4*x^5+1/8*b^5*x^
8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^8}\,dx = \frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

input
```
integrate((b*x^3+a)^5/x^8,x, algorithm="fricas")
```

output
```
1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*
a^4*b*x^3 - 8*a^5)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^3)^5}{x^8}\,dx = 5a^2b^3x^2 + ab^4x^5 + \frac{b^5x^8}{8} + \frac{-4a^5 - 35a^4bx^3 - 280a^3b^2x^6}{28x^7}$$

input
```
integrate((b*x**3+a)**5/x**8,x)
```

output
```
5*a**2*b**3*x**2 + a*b**4*x**5 + b**5*x**8/8 + (-4*a**5 - 35*a**4*b*x**3 -
 280*a**3*b**2*x**6)/(28*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^8}\,dx = \frac{1}{8}\,b^5x^8 + ab^4x^5 + 5\,a^2b^3x^2 - \frac{280\,a^3b^2x^6 + 35\,a^4bx^3 + 4\,a^5}{28\,x^7}$$

input
```
integrate((b*x^3+a)^5/x^8,x, algorithm="maxima")
```

output
```
1/8*b^5*x^8 + a*b^4*x^5 + 5*a^2*b^3*x^2 - 1/28*(280*a^3*b^2*x^6 + 35*a^4*b
*x^3 + 4*a^5)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^8}\,dx = \frac{1}{8}\,b^5x^8 + ab^4x^5 + 5\,a^2b^3x^2 - \frac{280\,a^3b^2x^6 + 35\,a^4bx^3 + 4\,a^5}{28\,x^7}$$

input
```
integrate((b*x^3+a)^5/x^8,x, algorithm="giac")
```

output
```
1/8*b^5*x^8 + a*b^4*x^5 + 5*a^2*b^3*x^2 - 1/28*(280*a^3*b^2*x^6 + 35*a^4*b
*x^3 + 4*a^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^8}\,dx = \frac{b^5\,x^8}{8} - \frac{\frac{a^5}{7} + \frac{5\,a^4\,b\,x^3}{4} + 10\,a^3\,b^2\,x^6}{x^7} + a\,b^4\,x^5 + 5\,a^2\,b^3\,x^2$$

input
```
int((a + b*x^3)^5/x^8,x)
```

output
```
(b^5*x^8)/8 - (a^5/7 + (5*a^4*b*x^3)/4 + 10*a^3*b^2*x^6)/x^7 + a*b^4*x^5 +
 5*a^2*b^3*x^2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^5}{x^8}\,dx = \frac{7b^5x^{15} + 56a\,b^4x^{12} + 280a^2b^3x^9 - 560a^3b^2x^6 - 70a^4b\,x^3 - 8a^5}{56x^7}$$

input
```
int((b*x^3+a)^5/x^8,x)
```

output
```
( - 8*a**5 - 70*a**4*b*x**3 - 560*a**3*b**2*x**6 + 280*a**2*b**3*x**9 + 56
*a*b**4*x**12 + 7*b**5*x**15)/(56*x**7)
```

3.79 $\int \frac{(a+bx^3)^5}{x^9} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{(a+bx^3)^5}{x^9} \, dx = -\frac{a^5}{8x^8} - \frac{a^4b}{x^5} - \frac{5a^3b^2}{x^2} + 10a^2b^3x + \frac{5}{4}ab^4x^4 + \frac{b^5x^7}{7}$$

output
```
-1/8*a^5/x^8-a^4*b/x^5-5*a^3*b^2/x^2+10*a^2*b^3*x+5/4*a*b^4*x^4+1/7*b^5*x^
7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^9} \, dx = -\frac{a^5}{8x^8} - \frac{a^4b}{x^5} - \frac{5a^3b^2}{x^2} + 10a^2b^3x + \frac{5}{4}ab^4x^4 + \frac{b^5x^7}{7}$$

input
```
Integrate[(a + b*x^3)^5/x^9,x]
```

output
```
-1/8*a^5/x^8 - (a^4*b)/x^5 - (5*a^3*b^2)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^4
)/4 + (b^5*x^7)/7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^5}{x^9} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^5}{x^9}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^3}+10a^2b^3+5ab^4x^3+b^5x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{8x^8}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{x^2}+10a^2b^3x+\frac{5}{4}ab^4x^4+\frac{b^5x^7}{7}$$

input
```
Int[(a + b*x^3)^5/x^9,x]
```

output
```
-1/8*a^5/x^8 - (a^4*b)/x^5 - (5*a^3*b^2)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^4
)/4 + (b^5*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{8x^8}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{x^2}+10a^2b^3x+\frac{5a\,b^4x^4}{4}+\frac{b^5x^7}{7}$	56
risch	$\frac{b^5x^7}{7}+\frac{5a\,b^4x^4}{4}+10a^2b^3x+\frac{-5a^3b^2x^6-a^4b\,x^3-\frac{1}{8}a^5}{x^8}$	58
norman	$\frac{-\frac{1}{8}a^5-a^4b\,x^3-5a^3b^2x^6+10a^2b^3x^9+\frac{5}{4}a\,b^4x^{12}+\frac{1}{7}b^5x^{15}}{x^8}$	59
gosper	$-\frac{-8b^5x^{15}-70a\,b^4x^{12}-560a^2b^3x^9+280a^3b^2x^6+56a^4b\,x^3+7a^5}{56x^8}$	60
parallelrisch	$\frac{8b^5x^{15}+70a\,b^4x^{12}+560a^2b^3x^9-280a^3b^2x^6-56a^4b\,x^3-7a^5}{56x^8}$	60
orering	$-\frac{-8b^5x^{15}-70a\,b^4x^{12}-560a^2b^3x^9+280a^3b^2x^6+56a^4b\,x^3+7a^5}{56x^8}$	60

input
```
int((b*x^3+a)^5/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^5/x^8-a^4*b/x^5-5*a^3*b^2/x^2+10*a^2*b^3*x+5/4*a*b^4*x^4+1/7*b^5*x^
7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^5}{x^9}\,dx = \frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

input
```
integrate((b*x^3+a)^5/x^9,x, algorithm="fricas")
```

output
```
1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*
a^4*b*x^3 - 7*a^5)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^5}{x^9}\,dx = 10a^2b^3x + \frac{5ab^4x^4}{4} + \frac{b^5x^7}{7} + \frac{-a^5-8a^4bx^3-40a^3b^2x^6}{8x^8}$$

input
```
integrate((b*x**3+a)**5/x**9,x)
```

output
```
10*a**2*b**3*x + 5*a*b**4*x**4/4 + b**5*x**7/7 + (-a**5 - 8*a**4*b*x**3 -
40*a**3*b**2*x**6)/(8*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^9}\,dx = \frac{1}{7}\,b^5x^7 + \frac{5}{4}\,ab^4x^4 + 10\,a^2b^3x - \frac{40\,a^3b^2x^6+8\,a^4bx^3+a^5}{8\,x^8}$$

input
```
integrate((b*x^3+a)^5/x^9,x, algorithm="maxima")
```

output
```
1/7*b^5*x^7 + 5/4*a*b^4*x^4 + 10*a^2*b^3*x - 1/8*(40*a^3*b^2*x^6 + 8*a^4*b
*x^3 + a^5)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^5}{x^9}\,dx = \frac{1}{7}\,b^5x^7 + \frac{5}{4}\,ab^4x^4 + 10\,a^2b^3x - \frac{40\,a^3b^2x^6+8\,a^4bx^3+a^5}{8\,x^8}$$

input
```
integrate((b*x^3+a)^5/x^9,x, algorithm="giac")
```

output
```
1/7*b^5*x^7 + 5/4*a*b^4*x^4 + 10*a^2*b^3*x - 1/8*(40*a^3*b^2*x^6 + 8*a^4*b
*x^3 + a^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^5}{x^9}\,dx = \frac{b^5\,x^7}{7} - \frac{\frac{a^5}{8} + a^4\,b\,x^3 + 5\,a^3\,b^2\,x^6}{x^8} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^4}{4}$$

input
```
int((a + b*x^3)^5/x^9,x)
```

output
```
(b^5*x^7)/7 - (a^5/8 + a^4*b*x^3 + 5*a^3*b^2*x^6)/x^8 + 10*a^2*b^3*x + (5*
a*b^4*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^5}{x^9}\,dx = \frac{8b^5x^{15} + 70a\,b^4x^{12} + 560a^2b^3x^9 - 280a^3b^2x^6 - 56a^4b\,x^3 - 7a^5}{56x^8}$$

input
```
int((b*x^3+a)^5/x^9,x)
```

output
```
( - 7*a**5 - 56*a**4*b*x**3 - 280*a**3*b**2*x**6 + 560*a**2*b**3*x**9 + 70
*a*b**4*x**12 + 8*b**5*x**15)/(56*x**8)
```

3.80 $\int x^{20}(a+bx^3)^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 129

$$\int x^{20}\left(a+bx^3\right)^8 \, dx = \frac{a^6(a+bx^3)^9}{27b^7} - \frac{a^5(a+bx^3)^{10}}{5b^7} + \frac{5a^4(a+bx^3)^{11}}{11b^7} - \frac{5a^3(a+bx^3)^{12}}{9b^7} + \frac{5a^2(a+bx^3)^{13}}{13b^7} - \frac{a(a+bx^3)^{14}}{7b^7} + \frac{(a+bx^3)^{15}}{45b^7}$$

output
```
1/27*a^6*(b*x^3+a)^9/b^7-1/5*a^5*(b*x^3+a)^10/b^7+5/11*a^4*(b*x^3+a)^11/b^
7-5/9*a^3*(b*x^3+a)^12/b^7+5/13*a^2*(b*x^3+a)^13/b^7-1/7*a*(b*x^3+a)^14/b^
7+1/45*(b*x^3+a)^15/b^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int x^{20}\left(a+bx^3\right)^8 \, dx = \frac{a^8x^{21}}{21} + \frac{1}{3}a^7bx^{24} + \frac{28}{27}a^6b^2x^{27} + \frac{28}{15}a^5b^3x^{30} + \frac{70}{33}a^4b^4x^{33} + \frac{14}{9}a^3b^5x^{36} + \frac{28}{39}a^2b^6x^{39} + \frac{4}{21}ab^7x^{42} + \frac{b^8x^{45}}{45}$$

input
```
Integrate[x^20*(a + b*x^3)^8,x]
```

output
```
(a^8*x^21)/21 + (a^7*b*x^24)/3 + (28*a^6*b^2*x^27)/27 + (28*a^5*b^3*x^30)/
15 + (70*a^4*b^4*x^33)/33 + (14*a^3*b^5*x^36)/9 + (28*a^2*b^6*x^39)/39 + (
4*a*b^7*x^42)/21 + (b^8*x^45)/45
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{20} \left(a+bx^3\right)^8 dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^{18} \left(bx^3+a\right)^8 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{14}}{b^6} - \frac{6a\left(bx^3+a\right)^{13}}{b^6} + \frac{15a^2\left(bx^3+a\right)^{12}}{b^6} - \frac{20a^3\left(bx^3+a\right)^{11}}{b^6} + \frac{15a^4\left(bx^3+a\right)^{10}}{b^6} - \frac{6a^5\left(bx^3+a\right)^9}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^6\left(a+bx^3\right)^9}{9b^7} - \frac{3a^5\left(a+bx^3\right)^{10}}{5b^7} + \frac{15a^4\left(a+bx^3\right)^{11}}{11b^7} - \frac{5a^3\left(a+bx^3\right)^{12}}{3b^7} + \frac{15a^2\left(a+bx^3\right)^{13}}{13b^7} + \frac{\left(a+bx^3\right)^{15}}{15b^7} - \right.$$

input
```
Int[x^20*(a + b*x^3)^8,x]
```

output
```
((a^6*(a + b*x^3)^9)/(9*b^7) - (3*a^5*(a + b*x^3)^10)/(5*b^7) + (15*a^4*(a
 + b*x^3)^11)/(11*b^7) - (5*a^3*(a + b*x^3)^12)/(3*b^7) + (15*a^2*(a + b*x
^3)^13)/(13*b^7) - (3*a*(a + b*x^3)^14)/(7*b^7) + (a + b*x^3)^15/(15*b^7))
/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

method	result
gosper	$\frac{4}{21}a\,b^7x^{42}+\frac{14}{9}a^3b^5x^{36}+\frac{1}{21}a^8x^{21}+\frac{70}{33}a^4b^4x^{33}+\frac{28}{39}a^2b^6x^{39}+\frac{1}{3}a^7b\,x^{24}+\frac{28}{27}a^6b^2x^{27}+\frac{28}{15}a^5b^3x$
default	$\frac{4}{21}a\,b^7x^{42}+\frac{14}{9}a^3b^5x^{36}+\frac{1}{21}a^8x^{21}+\frac{70}{33}a^4b^4x^{33}+\frac{28}{39}a^2b^6x^{39}+\frac{1}{3}a^7b\,x^{24}+\frac{28}{27}a^6b^2x^{27}+\frac{28}{15}a^5b^3x$
risch	$\frac{4}{21}a\,b^7x^{42}+\frac{14}{9}a^3b^5x^{36}+\frac{1}{21}a^8x^{21}+\frac{70}{33}a^4b^4x^{33}+\frac{28}{39}a^2b^6x^{39}+\frac{1}{3}a^7b\,x^{24}+\frac{28}{27}a^6b^2x^{27}+\frac{28}{15}a^5b^3x$
parallelrisch	$\frac{4}{21}a\,b^7x^{42}+\frac{14}{9}a^3b^5x^{36}+\frac{1}{21}a^8x^{21}+\frac{70}{33}a^4b^4x^{33}+\frac{28}{39}a^2b^6x^{39}+\frac{1}{3}a^7b\,x^{24}+\frac{28}{27}a^6b^2x^{27}+\frac{28}{15}a^5b^3x$
orering	$\frac{x^{21}(3003b^8x^{24}+25740a\,b^7x^{21}+97020a^2b^6x^{18}+210210a^3b^5x^{15}+286650a^4b^4x^{12}+252252a^5b^3x^9+140140a^6b^2x^6+45045a^7b\,x}{135135}$

input
```
int(x^20*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
4/21*a*b^7*x^42+14/9*a^3*b^5*x^36+1/21*a^8*x^21+70/33*a^4*b^4*x^33+28/39*a
^2*b^6*x^39+1/3*a^7*b*x^24+28/27*a^6*b^2*x^27+28/15*a^5*b^3*x^30+1/45*b^8*
x^45
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{20}(a+bx^3)^8\,dx=\frac{1}{45}\,b^8x^{45}+\frac{4}{21}\,ab^7x^{42}+\frac{28}{39}\,a^2b^6x^{39}+\frac{14}{9}\,a^3b^5x^{36}+\frac{70}{33}\,a^4b^4x^{33}+\frac{28}{15}\,a^5b^3x^{30}+\frac{28}{27}\,a^6b^2x^{27}+\frac{1}{3}\,a^7bx^{24}+\frac{1}{21}\,a^8x^{21}$$

input
```
integrate(x^20*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/45*b^8*x^45 + 4/21*a*b^7*x^42 + 28/39*a^2*b^6*x^39 + 14/9*a^3*b^5*x^36 +
 70/33*a^4*b^4*x^33 + 28/15*a^5*b^3*x^30 + 28/27*a^6*b^2*x^27 + 1/3*a^7*b*
x^24 + 1/21*a^8*x^21
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int x^{20}(a+bx^3)^8\,dx=\frac{a^8x^{21}}{21}+\frac{a^7bx^{24}}{3}+\frac{28a^6b^2x^{27}}{27}+\frac{28a^5b^3x^{30}}{15}+\frac{70a^4b^4x^{33}}{33}+\frac{14a^3b^5x^{36}}{9}+\frac{28a^2b^6x^{39}}{39}+\frac{4ab^7x^{42}}{21}+\frac{b^8x^{45}}{45}$$

input
```
integrate(x**20*(b*x**3+a)**8,x)
```

output
```
a**8*x**21/21 + a**7*b*x**24/3 + 28*a**6*b**2*x**27/27 + 28*a**5*b**3*x**3
0/15 + 70*a**4*b**4*x**33/33 + 14*a**3*b**5*x**36/9 + 28*a**2*b**6*x**39/3
9 + 4*a*b**7*x**42/21 + b**8*x**45/45
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{20}\left(a+bx^3\right)^8\,dx=\frac{1}{45}\,b^8x^{45}+\frac{4}{21}\,ab^7x^{42}+\frac{28}{39}\,a^2b^6x^{39}+\frac{14}{9}\,a^3b^5x^{36}+\frac{70}{33}\,a^4b^4x^{33}+\frac{28}{15}\,a^5b^3x^{30}+\frac{28}{27}\,a^6b^2x^{27}+\frac{1}{3}\,a^7bx^{24}+\frac{1}{21}\,a^8x^{21}$$

input
```
integrate(x^20*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/45*b^8*x^45 + 4/21*a*b^7*x^42 + 28/39*a^2*b^6*x^39 + 14/9*a^3*b^5*x^36 +
 70/33*a^4*b^4*x^33 + 28/15*a^5*b^3*x^30 + 28/27*a^6*b^2*x^27 + 1/3*a^7*b*
x^24 + 1/21*a^8*x^21
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{20}\left(a+bx^3\right)^8\,dx=\frac{1}{45}\,b^8x^{45}+\frac{4}{21}\,ab^7x^{42}+\frac{28}{39}\,a^2b^6x^{39}+\frac{14}{9}\,a^3b^5x^{36}+\frac{70}{33}\,a^4b^4x^{33}+\frac{28}{15}\,a^5b^3x^{30}+\frac{28}{27}\,a^6b^2x^{27}+\frac{1}{3}\,a^7bx^{24}+\frac{1}{21}\,a^8x^{21}$$

input
```
integrate(x^20*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/45*b^8*x^45 + 4/21*a*b^7*x^42 + 28/39*a^2*b^6*x^39 + 14/9*a^3*b^5*x^36 +
 70/33*a^4*b^4*x^33 + 28/15*a^5*b^3*x^30 + 28/27*a^6*b^2*x^27 + 1/3*a^7*b*
x^24 + 1/21*a^8*x^21
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{20}(a+bx^3)^8\,dx=\frac{a^8\,x^{21}}{21}+\frac{a^7\,b\,x^{24}}{3}+\frac{28\,a^6\,b^2\,x^{27}}{27}+\frac{28\,a^5\,b^3\,x^{30}}{15}+\frac{70\,a^4\,b^4\,x^{33}}{33}+\frac{14\,a^3\,b^5\,x^{36}}{9}+\frac{28\,a^2\,b^6\,x^{39}}{39}+\frac{4\,a\,b^7\,x^{42}}{21}+\frac{b^8\,x^{45}}{45}$$

input
```
int(x^20*(a + b*x^3)^8,x)
```

output
```
(a^8*x^21)/21 + (b^8*x^45)/45 + (a^7*b*x^24)/3 + (4*a*b^7*x^42)/21 + (28*a
^6*b^2*x^27)/27 + (28*a^5*b^3*x^30)/15 + (70*a^4*b^4*x^33)/33 + (14*a^3*b^
5*x^36)/9 + (28*a^2*b^6*x^39)/39
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int x^{20}(a+bx^3)^8\,dx=\frac{x^{21}(3003b^8x^{24}+25740a\,b^7x^{21}+97020a^2b^6x^{18}+210210a^3b^5x^{15}+286650a^4b^4x^{12}+252252a^5b^3x^9+1401}{135135}$$

input
```
int(x^20*(b*x^3+a)^8,x)
```

output
```
(x**21*(6435*a**8 + 45045*a**7*b*x**3 + 140140*a**6*b**2*x**6 + 252252*a**
5*b**3*x**9 + 286650*a**4*b**4*x**12 + 210210*a**3*b**5*x**15 + 97020*a**2
*b**6*x**18 + 25740*a*b**7*x**21 + 3003*b**8*x**24))/135135
```

3.81 $\int x^{17}(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int x^{17}(a+bx^3)^8\,dx=-\frac{a^5(a+bx^3)^9}{27b^6}+\frac{a^4(a+bx^3)^{10}}{6b^6}-\frac{10a^3(a+bx^3)^{11}}{33b^6}+\frac{5a^2(a+bx^3)^{12}}{18b^6}-\frac{5a(a+bx^3)^{13}}{39b^6}+\frac{(a+bx^3)^{14}}{42b^6}$$

output
```
-1/27*a^5*(b*x^3+a)^9/b^6+1/6*a^4*(b*x^3+a)^10/b^6-10/33*a^3*(b*x^3+a)^11/
b^6+5/18*a^2*(b*x^3+a)^12/b^6-5/39*a*(b*x^3+a)^13/b^6+1/42*(b*x^3+a)^14/b^
6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int x^{17}(a+bx^3)^8\,dx=\frac{a^8x^{18}}{18}+\frac{8}{21}a^7bx^{21}+\frac{7}{6}a^6b^2x^{24}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{9}a^2b^6x^{36}+\frac{8}{39}ab^7x^{39}+\frac{b^8x^{42}}{42}$$

input
```
Integrate[x^17*(a + b*x^3)^8,x]
```

output
```
(a^8*x^18)/18 + (8*a^7*b*x^21)/21 + (7*a^6*b^2*x^24)/6 + (56*a^5*b^3*x^27)
/27 + (7*a^4*b^4*x^30)/3 + (56*a^3*b^5*x^33)/33 + (7*a^2*b^6*x^36)/9 + (8*
a*b^7*x^39)/39 + (b^8*x^42)/42
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{17}\left(a+bx^3\right)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^{15}\left(bx^3+a\right)^8dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{13}}{b^5}-\frac{5a\left(bx^3+a\right)^{12}}{b^5}+\frac{10a^2\left(bx^3+a\right)^{11}}{b^5}-\frac{10a^3\left(bx^3+a\right)^{10}}{b^5}+\frac{5a^4\left(bx^3+a\right)^9}{b^5}-\frac{a^5\left(bx^3+a\right)^8}{b^5}\right)d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^5\left(a+bx^3\right)^9}{9b^6}+\frac{a^4\left(a+bx^3\right)^{10}}{2b^6}-\frac{10a^3\left(a+bx^3\right)^{11}}{11b^6}+\frac{5a^2\left(a+bx^3\right)^{12}}{6b^6}+\frac{\left(a+bx^3\right)^{14}}{14b^6}-\frac{5a\left(a+bx^3\right)^{13}}{13b^6}\right)$$

input
```
Int[x^17*(a + b*x^3)^8,x]
```

output
```
(-1/9*(a^5*(a + b*x^3)^9)/b^6 + (a^4*(a + b*x^3)^10)/(2*b^6) - (10*a^3*(a
+ b*x^3)^11)/(11*b^6) + (5*a^2*(a + b*x^3)^12)/(6*b^6) - (5*a*(a + b*x^3)^
13)/(13*b^6) + (a + b*x^3)^14/(14*b^6))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{1}{42}b^8x^{42}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{6}a^6b^2x^{24}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{9}a^2b^6x^{36}+\frac{8}{39}a\,b^7x^{39}+\frac{8}{21}a^7b\,x^{21}$
default	$\frac{1}{42}b^8x^{42}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{6}a^6b^2x^{24}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{9}a^2b^6x^{36}+\frac{8}{39}a\,b^7x^{39}+\frac{8}{21}a^7b\,x^{21}$
risch	$\frac{1}{42}b^8x^{42}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{6}a^6b^2x^{24}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{9}a^2b^6x^{36}+\frac{8}{39}a\,b^7x^{39}+\frac{8}{21}a^7b\,x^{21}$
parallelrisch	$\frac{1}{42}b^8x^{42}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{6}a^6b^2x^{24}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{9}a^2b^6x^{36}+\frac{8}{39}a\,b^7x^{39}+\frac{8}{21}a^7b\,x^{21}$
orering	$\frac{x^{18}(1287b^8x^{24}+11088a\,b^7x^{21}+42042a^2b^6x^{18}+91728a^3b^5x^{15}+126126a^4b^4x^{12}+112112a^5b^3x^9+63063a^6b^2x^6+20592a^7b\,x^3}{54054}$

input
```
int(x^17*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/42*b^8*x^42+56/33*a^3*b^5*x^33+7/6*a^6*b^2*x^24+7/3*a^4*b^4*x^30+56/27*a
^5*b^3*x^27+7/9*a^2*b^6*x^36+8/39*a*b^7*x^39+8/21*a^7*b*x^21+1/18*a^8*x^18
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{17}(a+bx^3)^8\,dx = \frac{1}{42}\,b^8x^{42} + \frac{8}{39}\,ab^7x^{39} + \frac{7}{9}\,a^2b^6x^{36} + \frac{56}{33}\,a^3b^5x^{33} + \frac{7}{3}\,a^4b^4x^{30} + \frac{56}{27}\,a^5b^3x^{27} + \frac{7}{6}\,a^6b^2x^{24} + \frac{8}{21}\,a^7bx^{21} + \frac{1}{18}\,a^8x^{18}$$

input
```
integrate(x^17*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/42*b^8*x^42 + 8/39*a*b^7*x^39 + 7/9*a^2*b^6*x^36 + 56/33*a^3*b^5*x^33 +
7/3*a^4*b^4*x^30 + 56/27*a^5*b^3*x^27 + 7/6*a^6*b^2*x^24 + 8/21*a^7*b*x^21
 + 1/18*a^8*x^18
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int x^{17}(a+bx^3)^8\,dx = \frac{a^8x^{18}}{18} + \frac{8a^7bx^{21}}{21} + \frac{7a^6b^2x^{24}}{6} + \frac{56a^5b^3x^{27}}{27} + \frac{7a^4b^4x^{30}}{3} + \frac{56a^3b^5x^{33}}{33} + \frac{7a^2b^6x^{36}}{9} + \frac{8ab^7x^{39}}{39} + \frac{b^8x^{42}}{42}$$

input
```
integrate(x**17*(b*x**3+a)**8,x)
```

output
```
a**8*x**18/18 + 8*a**7*b*x**21/21 + 7*a**6*b**2*x**24/6 + 56*a**5*b**3*x**
27/27 + 7*a**4*b**4*x**30/3 + 56*a**3*b**5*x**33/33 + 7*a**2*b**6*x**36/9
+ 8*a*b**7*x**39/39 + b**8*x**42/42
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{17}(a+bx^3)^8\,dx=\frac{1}{42}b^8x^{42}+\frac{8}{39}ab^7x^{39}+\frac{7}{9}a^2b^6x^{36}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{6}a^6b^2x^{24}+\frac{8}{21}a^7bx^{21}+\frac{1}{18}a^8x^{18}$$

input
```
integrate(x^17*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/42*b^8*x^42 + 8/39*a*b^7*x^39 + 7/9*a^2*b^6*x^36 + 56/33*a^3*b^5*x^33 +
7/3*a^4*b^4*x^30 + 56/27*a^5*b^3*x^27 + 7/6*a^6*b^2*x^24 + 8/21*a^7*b*x^21
 + 1/18*a^8*x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{17}(a+bx^3)^8\,dx=\frac{1}{42}b^8x^{42}+\frac{8}{39}ab^7x^{39}+\frac{7}{9}a^2b^6x^{36}+\frac{56}{33}a^3b^5x^{33}+\frac{7}{3}a^4b^4x^{30}+\frac{56}{27}a^5b^3x^{27}+\frac{7}{6}a^6b^2x^{24}+\frac{8}{21}a^7bx^{21}+\frac{1}{18}a^8x^{18}$$

input
```
integrate(x^17*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/42*b^8*x^42 + 8/39*a*b^7*x^39 + 7/9*a^2*b^6*x^36 + 56/33*a^3*b^5*x^33 +
7/3*a^4*b^4*x^30 + 56/27*a^5*b^3*x^27 + 7/6*a^6*b^2*x^24 + 8/21*a^7*b*x^21
 + 1/18*a^8*x^18
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{17}\left(a+bx^3\right)^8\,dx=\frac{a^8\,x^{18}}{18}+\frac{8\,a^7\,b\,x^{21}}{21}+\frac{7\,a^6\,b^2\,x^{24}}{6}+\frac{56\,a^5\,b^3\,x^{27}}{27}+\frac{7\,a^4\,b^4\,x^{30}}{3}+\frac{56\,a^3\,b^5\,x^{33}}{33}+\frac{7\,a^2\,b^6\,x^{36}}{9}+\frac{8\,a\,b^7\,x^{39}}{39}+\frac{b^8\,x^{42}}{42}$$

input
```
int(x^17*(a + b*x^3)^8,x)
```

output
```
(a^8*x^18)/18 + (b^8*x^42)/42 + (8*a^7*b*x^21)/21 + (8*a*b^7*x^39)/39 + (7
*a^6*b^2*x^24)/6 + (56*a^5*b^3*x^27)/27 + (7*a^4*b^4*x^30)/3 + (56*a^3*b^5
*x^33)/33 + (7*a^2*b^6*x^36)/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int x^{17}\left(a+bx^3\right)^8\,dx=\frac{x^{18}(1287b^8x^{24}+11088a\,b^7x^{21}+42042a^2b^6x^{18}+91728a^3b^5x^{15}+126126a^4b^4x^{12}+112112a^5b^3x^9+63063}{54054}$$

input
```
int(x^17*(b*x^3+a)^8,x)
```

output
```
(x**18*(3003*a**8 + 20592*a**7*b*x**3 + 63063*a**6*b**2*x**6 + 112112*a**5
*b**3*x**9 + 126126*a**4*b**4*x**12 + 91728*a**3*b**5*x**15 + 42042*a**2*b
**6*x**18 + 11088*a*b**7*x**21 + 1287*b**8*x**24))/54054
```

3.82 $\int x^{14}(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int x^{14}\left(a+bx^3\right)^8\,dx=\frac{a^4(a+bx^3)^9}{27b^5}-\frac{2a^3(a+bx^3)^{10}}{15b^5}+\frac{2a^2(a+bx^3)^{11}}{11b^5}-\frac{a(a+bx^3)^{12}}{9b^5}+\frac{(a+bx^3)^{13}}{39b^5}$$

output
```
1/27*a^4*(b*x^3+a)^9/b^5-2/15*a^3*(b*x^3+a)^10/b^5+2/11*a^2*(b*x^3+a)^11/b
^5-1/9*a*(b*x^3+a)^12/b^5+1/39*(b*x^3+a)^13/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int x^{14}\left(a+bx^3\right)^8\,dx=\frac{a^8x^{15}}{15}+\frac{4}{9}a^7bx^{18}+\frac{4}{3}a^6b^2x^{21}+\frac{7}{3}a^5b^3x^{24}+\frac{70}{27}a^4b^4x^{27}+\frac{28}{15}a^3b^5x^{30}+\frac{28}{33}a^2b^6x^{33}+\frac{2}{9}ab^7x^{36}+\frac{b^8x^{39}}{39}$$

input
```
Integrate[x^14*(a + b*x^3)^8,x]
```

output
```
(a^8*x^15)/15 + (4*a^7*b*x^18)/9 + (4*a^6*b^2*x^21)/3 + (7*a^5*b^3*x^24)/3
 + (70*a^4*b^4*x^27)/27 + (28*a^3*b^5*x^30)/15 + (28*a^2*b^6*x^33)/33 + (2
*a*b^7*x^36)/9 + (b^8*x^39)/39
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14}(a+bx^3)^8\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^{12}(bx^3+a)^8\,dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{(bx^3+a)^{12}}{b^4}-\frac{4a(bx^3+a)^{11}}{b^4}+\frac{6a^2(bx^3+a)^{10}}{b^4}-\frac{4a^3(bx^3+a)^9}{b^4}+\frac{a^4(bx^3+a)^8}{b^4}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^4(a+bx^3)^9}{9b^5}-\frac{2a^3(a+bx^3)^{10}}{5b^5}+\frac{6a^2(a+bx^3)^{11}}{11b^5}+\frac{(a+bx^3)^{13}}{13b^5}-\frac{a(a+bx^3)^{12}}{3b^5}\right)$$

input
```
Int[x^14*(a + b*x^3)^8,x]
```

output
```
((a^4*(a + b*x^3)^9)/(9*b^5) - (2*a^3*(a + b*x^3)^10)/(5*b^5) + (6*a^2*(a
+ b*x^3)^11)/(11*b^5) - (a*(a + b*x^3)^12)/(3*b^5) + (a + b*x^3)^13/(13*b^
5))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{4}{9}a^7b\,x^{18}+\frac{70}{27}a^4b^4x^{27}+\frac{28}{15}a^3b^5x^{30}+\frac{1}{15}a^8x^{15}+\frac{1}{39}b^8x^{39}+\frac{2}{9}a\,b^7x^{36}+\frac{7}{3}a^5b^3x^{24}+\frac{4}{3}a^6b^2x^{21}+$
default	$\frac{4}{9}a^7b\,x^{18}+\frac{70}{27}a^4b^4x^{27}+\frac{28}{15}a^3b^5x^{30}+\frac{1}{15}a^8x^{15}+\frac{1}{39}b^8x^{39}+\frac{2}{9}a\,b^7x^{36}+\frac{7}{3}a^5b^3x^{24}+\frac{4}{3}a^6b^2x^{21}+$
risch	$\frac{4}{9}a^7b\,x^{18}+\frac{70}{27}a^4b^4x^{27}+\frac{28}{15}a^3b^5x^{30}+\frac{1}{15}a^8x^{15}+\frac{1}{39}b^8x^{39}+\frac{2}{9}a\,b^7x^{36}+\frac{7}{3}a^5b^3x^{24}+\frac{4}{3}a^6b^2x^{21}+$
parallelrisch	$\frac{4}{9}a^7b\,x^{18}+\frac{70}{27}a^4b^4x^{27}+\frac{28}{15}a^3b^5x^{30}+\frac{1}{15}a^8x^{15}+\frac{1}{39}b^8x^{39}+\frac{2}{9}a\,b^7x^{36}+\frac{7}{3}a^5b^3x^{24}+\frac{4}{3}a^6b^2x^{21}+$
orering	$\frac{x^{15}(495b^8x^{24}+4290a\,b^7x^{21}+16380a^2b^6x^{18}+36036a^3b^5x^{15}+50050a^4b^4x^{12}+45045a^5b^3x^9+25740a^6b^2x^6+8580a^7b\,x^3+1287}{19305}$

input
```
int(x^14*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
4/9*a^7*b*x^18+70/27*a^4*b^4*x^27+28/15*a^3*b^5*x^30+1/15*a^8*x^15+1/39*b^
8*x^39+2/9*a*b^7*x^36+7/3*a^5*b^3*x^24+4/3*a^6*b^2*x^21+28/33*a^2*b^6*x^33
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3)^8\,dx = \frac{1}{39}\,b^8x^{39} + \frac{2}{9}\,ab^7x^{36} + \frac{28}{33}\,a^2b^6x^{33} + \frac{28}{15}\,a^3b^5x^{30} + \frac{70}{27}\,a^4b^4x^{27} + \frac{7}{3}\,a^5b^3x^{24} + \frac{4}{3}\,a^6b^2x^{21} + \frac{4}{9}\,a^7bx^{18} + \frac{1}{15}\,a^8x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/39*b^8*x^39 + 2/9*a*b^7*x^36 + 28/33*a^2*b^6*x^33 + 28/15*a^3*b^5*x^30 +
 70/27*a^4*b^4*x^27 + 7/3*a^5*b^3*x^24 + 4/3*a^6*b^2*x^21 + 4/9*a^7*b*x^18
 + 1/15*a^8*x^15
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int x^{14}(a+bx^3)^8\,dx = \frac{a^8x^{15}}{15} + \frac{4a^7bx^{18}}{9} + \frac{4a^6b^2x^{21}}{3} + \frac{7a^5b^3x^{24}}{3} + \frac{70a^4b^4x^{27}}{27} + \frac{28a^3b^5x^{30}}{15} + \frac{28a^2b^6x^{33}}{33} + \frac{2ab^7x^{36}}{9} + \frac{b^8x^{39}}{39}$$

input
```
integrate(x**14*(b*x**3+a)**8,x)
```

output
```
a**8*x**15/15 + 4*a**7*b*x**18/9 + 4*a**6*b**2*x**21/3 + 7*a**5*b**3*x**24
/3 + 70*a**4*b**4*x**27/27 + 28*a**3*b**5*x**30/15 + 28*a**2*b**6*x**33/33
 + 2*a*b**7*x**36/9 + b**8*x**39/39
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3)^8\,dx = \frac{1}{39}b^8x^{39} + \frac{2}{9}ab^7x^{36} + \frac{28}{33}a^2b^6x^{33} + \frac{28}{15}a^3b^5x^{30} + \frac{70}{27}a^4b^4x^{27} + \frac{7}{3}a^5b^3x^{24} + \frac{4}{3}a^6b^2x^{21} + \frac{4}{9}a^7bx^{18} + \frac{1}{15}a^8x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/39*b^8*x^39 + 2/9*a*b^7*x^36 + 28/33*a^2*b^6*x^33 + 28/15*a^3*b^5*x^30 +
 70/27*a^4*b^4*x^27 + 7/3*a^5*b^3*x^24 + 4/3*a^6*b^2*x^21 + 4/9*a^7*b*x^18
 + 1/15*a^8*x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3)^8\,dx = \frac{1}{39}b^8x^{39} + \frac{2}{9}ab^7x^{36} + \frac{28}{33}a^2b^6x^{33} + \frac{28}{15}a^3b^5x^{30} + \frac{70}{27}a^4b^4x^{27} + \frac{7}{3}a^5b^3x^{24} + \frac{4}{3}a^6b^2x^{21} + \frac{4}{9}a^7bx^{18} + \frac{1}{15}a^8x^{15}$$

input
```
integrate(x^14*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/39*b^8*x^39 + 2/9*a*b^7*x^36 + 28/33*a^2*b^6*x^33 + 28/15*a^3*b^5*x^30 +
 70/27*a^4*b^4*x^27 + 7/3*a^5*b^3*x^24 + 4/3*a^6*b^2*x^21 + 4/9*a^7*b*x^18
 + 1/15*a^8*x^15
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^{14}(a+bx^3)^8\,dx=\frac{a^8\,x^{15}}{15}+\frac{4\,a^7\,b\,x^{18}}{9}+\frac{4\,a^6\,b^2\,x^{21}}{3}+\frac{7\,a^5\,b^3\,x^{24}}{3}+\frac{70\,a^4\,b^4\,x^{27}}{27}+\frac{28\,a^3\,b^5\,x^{30}}{15}+\frac{28\,a^2\,b^6\,x^{33}}{33}+\frac{2\,a\,b^7\,x^{36}}{9}+\frac{b^8\,x^{39}}{39}$$

input
```
int(x^14*(a + b*x^3)^8,x)
```

output
```
(a^8*x^15)/15 + (b^8*x^39)/39 + (4*a^7*b*x^18)/9 + (2*a*b^7*x^36)/9 + (4*a
^6*b^2*x^21)/3 + (7*a^5*b^3*x^24)/3 + (70*a^4*b^4*x^27)/27 + (28*a^3*b^5*x
^30)/15 + (28*a^2*b^6*x^33)/33
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int x^{14}(a+bx^3)^8\,dx$$
$$=\frac{x^{15}(495b^8x^{24}+4290a\,b^7x^{21}+16380a^2b^6x^{18}+36036a^3b^5x^{15}+50050a^4b^4x^{12}+45045a^5b^3x^9+25740a^6b^2}{19305}$$

input
```
int(x^14*(b*x^3+a)^8,x)
```

output
```
(x**15*(1287*a**8 + 8580*a**7*b*x**3 + 25740*a**6*b**2*x**6 + 45045*a**5*b
**3*x**9 + 50050*a**4*b**4*x**12 + 36036*a**3*b**5*x**15 + 16380*a**2*b**6
*x**18 + 4290*a*b**7*x**21 + 495*b**8*x**24))/19305
```

3.83 $\int x^{11}(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^{11}\left(a+bx^3\right)^8\,dx=-\frac{a^3(a+bx^3)^9}{27b^4}+\frac{a^2(a+bx^3)^{10}}{10b^4}-\frac{a(a+bx^3)^{11}}{11b^4}+\frac{(a+bx^3)^{12}}{36b^4}$$

output
```
-1/27*a^3*(b*x^3+a)^9/b^4+1/10*a^2*(b*x^3+a)^10/b^4-1/11*a*(b*x^3+a)^11/b^
4+1/36*(b*x^3+a)^12/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.50

$$\int x^{11}\left(a+bx^3\right)^8\,dx=\frac{a^8x^{12}}{12}+\frac{8}{15}a^7bx^{15}+\frac{14}{9}a^6b^2x^{18}+\frac{8}{3}a^5b^3x^{21}+\frac{35}{12}a^4b^4x^{24}+\frac{56}{27}a^3b^5x^{27}+\frac{14}{15}a^2b^6x^{30}+\frac{8}{33}ab^7x^{33}+\frac{b^8x^{36}}{36}$$

input
```
Integrate[x^11*(a + b*x^3)^8,x]
```

output
```
(a^8*x^12)/12 + (8*a^7*b*x^15)/15 + (14*a^6*b^2*x^18)/9 + (8*a^5*b^3*x^21)
/3 + (35*a^4*b^4*x^24)/12 + (56*a^3*b^5*x^27)/27 + (14*a^2*b^6*x^30)/15 +
(8*a*b^7*x^33)/33 + (b^8*x^36)/36
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^3\right)^8 dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^9\left(bx^3+a\right)^8 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{11}}{b^3}-\frac{3a\left(bx^3+a\right)^{10}}{b^3}+\frac{3a^2\left(bx^3+a\right)^9}{b^3}-\frac{a^3\left(bx^3+a\right)^8}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^3\left(a+bx^3\right)^9}{9b^4}+\frac{3a^2\left(a+bx^3\right)^{10}}{10b^4}+\frac{\left(a+bx^3\right)^{12}}{12b^4}-\frac{3a\left(a+bx^3\right)^{11}}{11b^4}\right)$$

input
```
Int[x^11*(a + b*x^3)^8,x]
```

output
```
(-1/9*(a^3*(a + b*x^3)^9)/b^4 + (3*a^2*(a + b*x^3)^10)/(10*b^4) - (3*a*(a
+ b*x^3)^11)/(11*b^4) + (a + b*x^3)^12/(12*b^4))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

method	result
gosper	$\frac{14}{15}a^2b^6x^{30}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{36}b^8x^{36}+\frac{1}{12}a^8x^{12}+\frac{56}{27}a^3b^5x^{27}+\frac{35}{12}a^4b^4x^{24}+\frac{8}{33}a\,b^7x^{33}+\frac{8}{15}a^7b\,x^1$
default	$\frac{14}{15}a^2b^6x^{30}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{36}b^8x^{36}+\frac{1}{12}a^8x^{12}+\frac{56}{27}a^3b^5x^{27}+\frac{35}{12}a^4b^4x^{24}+\frac{8}{33}a\,b^7x^{33}+\frac{8}{15}a^7b\,x^1$
norman	$\frac{14}{15}a^2b^6x^{30}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{36}b^8x^{36}+\frac{1}{12}a^8x^{12}+\frac{56}{27}a^3b^5x^{27}+\frac{35}{12}a^4b^4x^{24}+\frac{8}{33}a\,b^7x^{33}+\frac{8}{15}a^7b\,x^1$
risch	$\frac{14}{15}a^2b^6x^{30}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{36}b^8x^{36}+\frac{1}{12}a^8x^{12}+\frac{56}{27}a^3b^5x^{27}+\frac{35}{12}a^4b^4x^{24}+\frac{8}{33}a\,b^7x^{33}+\frac{8}{15}a^7b\,x^1$
parallelrisch	$\frac{14}{15}a^2b^6x^{30}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{36}b^8x^{36}+\frac{1}{12}a^8x^{12}+\frac{56}{27}a^3b^5x^{27}+\frac{35}{12}a^4b^4x^{24}+\frac{8}{33}a\,b^7x^{33}+\frac{8}{15}a^7b\,x^1$
orering	$\frac{x^{12}(165b^8x^{24}+1440a\,b^7x^{21}+5544a^2b^6x^{18}+12320a^3b^5x^{15}+17325a^4b^4x^{12}+15840a^5b^3x^9+9240a^6b^2x^6+3168a^7b\,x^3+495a^8)}{5940}$

input
```
int(x^11*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
14/15*a^2*b^6*x^30+14/9*a^6*b^2*x^18+1/36*b^8*x^36+1/12*a^8*x^12+56/27*a^3
*b^5*x^27+35/12*a^4*b^4*x^24+8/33*a*b^7*x^33+8/15*a^7*b*x^15+8/3*a^5*b^3*x
^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^{11}(a+bx^3)^8\,dx = \frac{1}{36}\,b^8x^{36} + \frac{8}{33}\,ab^7x^{33} + \frac{14}{15}\,a^2b^6x^{30} + \frac{56}{27}\,a^3b^5x^{27} + \frac{35}{12}\,a^4b^4x^{24} + \frac{8}{3}\,a^5b^3x^{21} + \frac{14}{9}\,a^6b^2x^{18} + \frac{8}{15}\,a^7bx^{15} + \frac{1}{12}\,a^8x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/36*b^8*x^36 + 8/33*a*b^7*x^33 + 14/15*a^2*b^6*x^30 + 56/27*a^3*b^5*x^27
+ 35/12*a^4*b^4*x^24 + 8/3*a^5*b^3*x^21 + 14/9*a^6*b^2*x^18 + 8/15*a^7*b*x
^15 + 1/12*a^8*x^12
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.49

$$\int x^{11}(a+bx^3)^8\,dx = \frac{a^8x^{12}}{12} + \frac{8a^7bx^{15}}{15} + \frac{14a^6b^2x^{18}}{9} + \frac{8a^5b^3x^{21}}{3} + \frac{35a^4b^4x^{24}}{12} + \frac{56a^3b^5x^{27}}{27} + \frac{14a^2b^6x^{30}}{15} + \frac{8ab^7x^{33}}{33} + \frac{b^8x^{36}}{36}$$

input
```
integrate(x**11*(b*x**3+a)**8,x)
```

output
```
a**8*x**12/12 + 8*a**7*b*x**15/15 + 14*a**6*b**2*x**18/9 + 8*a**5*b**3*x**
21/3 + 35*a**4*b**4*x**24/12 + 56*a**3*b**5*x**27/27 + 14*a**2*b**6*x**30/
15 + 8*a*b**7*x**33/33 + b**8*x**36/36
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^{11}\left(a+bx^3\right)^8\,dx=\frac{1}{36}\,b^8x^{36}+\frac{8}{33}\,ab^7x^{33}+\frac{14}{15}\,a^2b^6x^{30}+\frac{56}{27}\,a^3b^5x^{27}+\frac{35}{12}\,a^4b^4x^{24}+\frac{8}{3}\,a^5b^3x^{21}+\frac{14}{9}\,a^6b^2x^{18}+\frac{8}{15}\,a^7bx^{15}+\frac{1}{12}\,a^8x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/36*b^8*x^36 + 8/33*a*b^7*x^33 + 14/15*a^2*b^6*x^30 + 56/27*a^3*b^5*x^27
+ 35/12*a^4*b^4*x^24 + 8/3*a^5*b^3*x^21 + 14/9*a^6*b^2*x^18 + 8/15*a^7*b*x
^15 + 1/12*a^8*x^12
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^{11}\left(a+bx^3\right)^8\,dx=\frac{1}{36}\,b^8x^{36}+\frac{8}{33}\,ab^7x^{33}+\frac{14}{15}\,a^2b^6x^{30}+\frac{56}{27}\,a^3b^5x^{27}+\frac{35}{12}\,a^4b^4x^{24}+\frac{8}{3}\,a^5b^3x^{21}+\frac{14}{9}\,a^6b^2x^{18}+\frac{8}{15}\,a^7bx^{15}+\frac{1}{12}\,a^8x^{12}$$

input
```
integrate(x^11*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/36*b^8*x^36 + 8/33*a*b^7*x^33 + 14/15*a^2*b^6*x^30 + 56/27*a^3*b^5*x^27
+ 35/12*a^4*b^4*x^24 + 8/3*a^5*b^3*x^21 + 14/9*a^6*b^2*x^18 + 8/15*a^7*b*x
^15 + 1/12*a^8*x^12
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^{11}(a+bx^3)^8\,dx=\frac{a^8\,x^{12}}{12}+\frac{8\,a^7\,b\,x^{15}}{15}+\frac{14\,a^6\,b^2\,x^{18}}{9}+\frac{8\,a^5\,b^3\,x^{21}}{3}+\frac{35\,a^4\,b^4\,x^{24}}{12}+\frac{56\,a^3\,b^5\,x^{27}}{27}+\frac{14\,a^2\,b^6\,x^{30}}{15}+\frac{8\,a\,b^7\,x^{33}}{33}+\frac{b^8\,x^{36}}{36}$$

input
```
int(x^11*(a + b*x^3)^8,x)
```

output
```
(a^8*x^12)/12 + (b^8*x^36)/36 + (8*a^7*b*x^15)/15 + (8*a*b^7*x^33)/33 + (1
4*a^6*b^2*x^18)/9 + (8*a^5*b^3*x^21)/3 + (35*a^4*b^4*x^24)/12 + (56*a^3*b^
5*x^27)/27 + (14*a^2*b^6*x^30)/15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int x^{11}(a+bx^3)^8\,dx$$
$$=\frac{x^{12}(165b^8x^{24}+1440a\,b^7x^{21}+5544a^2b^6x^{18}+12320a^3b^5x^{15}+17325a^4b^4x^{12}+15840a^5b^3x^9+9240a^6b^2x^6}{5940}$$

input
```
int(x^11*(b*x^3+a)^8,x)
```

output
```
(x**12*(495*a**8 + 3168*a**7*b*x**3 + 9240*a**6*b**2*x**6 + 15840*a**5*b**
3*x**9 + 17325*a**4*b**4*x**12 + 12320*a**3*b**5*x**15 + 5544*a**2*b**6*x*
*18 + 1440*a*b**7*x**21 + 165*b**8*x**24))/5940
```

3.84 $\int x^8(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^8(a+bx^3)^8\,dx=\frac{a^2(a+bx^3)^9}{27b^3}-\frac{a(a+bx^3)^{10}}{15b^3}+\frac{(a+bx^3)^{11}}{33b^3}$$

output
```
1/27*a^2*(b*x^3+a)^9/b^3-1/15*a*(b*x^3+a)^10/b^3+1/33*(b*x^3+a)^11/b^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(53) = 106.

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.04

$$\int x^8(a+bx^3)^8\,dx=\frac{a^8x^9}{9}+\frac{2}{3}a^7bx^{12}+\frac{28}{15}a^6b^2x^{15}+\frac{28}{9}a^5b^3x^{18}+\frac{10}{3}a^4b^4x^{21}+\frac{7}{3}a^3b^5x^{24}+\frac{28}{27}a^2b^6x^{27}+\frac{4}{15}ab^7x^{30}+\frac{b^8x^{33}}{33}$$

input
```
Integrate[x^8*(a + b*x^3)^8,x]
```

output
```
(a^8*x^9)/9 + (2*a^7*b*x^12)/3 + (28*a^6*b^2*x^15)/15 + (28*a^5*b^3*x^18)/
9 + (10*a^4*b^4*x^21)/3 + (7*a^3*b^5*x^24)/3 + (28*a^2*b^6*x^27)/27 + (4*a
*b^7*x^30)/15 + (b^8*x^33)/33
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^8 dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^8 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{10}}{b^2} - \frac{2a\left(bx^3+a\right)^9}{b^2} + \frac{a^2\left(bx^3+a\right)^8}{b^2}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{a^2\left(a+bx^3\right)^9}{9b^3} + \frac{\left(a+bx^3\right)^{11}}{11b^3} - \frac{a\left(a+bx^3\right)^{10}}{5b^3}\right)$$

input
```
Int[x^8*(a + b*x^3)^8,x]
```

output
```
((a^2*(a + b*x^3)^9)/(9*b^3) - (a*(a + b*x^3)^10)/(5*b^3) + (a + b*x^3)^11
/(11*b^3))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

method	result
gosper	$\frac{1}{9}a^8x^9 + \frac{28}{9}a^5b^3x^{18} + \frac{28}{27}a^2b^6x^{27} + \frac{1}{33}b^8x^{33} + \frac{10}{3}a^4b^4x^{21} + \frac{7}{3}a^3b^5x^{24} + \frac{4}{15}a\,b^7x^{30} + \frac{28}{15}a^6b^2x^{15}$
default	$\frac{1}{9}a^8x^9 + \frac{28}{9}a^5b^3x^{18} + \frac{28}{27}a^2b^6x^{27} + \frac{1}{33}b^8x^{33} + \frac{10}{3}a^4b^4x^{21} + \frac{7}{3}a^3b^5x^{24} + \frac{4}{15}a\,b^7x^{30} + \frac{28}{15}a^6b^2x^{15}$
norman	$\frac{1}{9}a^8x^9 + \frac{28}{9}a^5b^3x^{18} + \frac{28}{27}a^2b^6x^{27} + \frac{1}{33}b^8x^{33} + \frac{10}{3}a^4b^4x^{21} + \frac{7}{3}a^3b^5x^{24} + \frac{4}{15}a\,b^7x^{30} + \frac{28}{15}a^6b^2x^{15}$
risch	$\frac{1}{9}a^8x^9 + \frac{28}{9}a^5b^3x^{18} + \frac{28}{27}a^2b^6x^{27} + \frac{1}{33}b^8x^{33} + \frac{10}{3}a^4b^4x^{21} + \frac{7}{3}a^3b^5x^{24} + \frac{4}{15}a\,b^7x^{30} + \frac{28}{15}a^6b^2x^{15}$
parallelrisch	$\frac{1}{9}a^8x^9 + \frac{28}{9}a^5b^3x^{18} + \frac{28}{27}a^2b^6x^{27} + \frac{1}{33}b^8x^{33} + \frac{10}{3}a^4b^4x^{21} + \frac{7}{3}a^3b^5x^{24} + \frac{4}{15}a\,b^7x^{30} + \frac{28}{15}a^6b^2x^{15}$
orering	$\frac{x^9(45b^8x^{24}+396a\,b^7x^{21}+1540a^2b^6x^{18}+3465a^3b^5x^{15}+4950a^4b^4x^{12}+4620a^5b^3x^9+2772a^6b^2x^6+990a^7b\,x^3+165a^8)}{1485}$

input
```
int(x^8*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^8*x^9+28/9*a^5*b^3*x^18+28/27*a^2*b^6*x^27+1/33*b^8*x^33+10/3*a^4*b^
4*x^21+7/3*a^3*b^5*x^24+4/15*a*b^7*x^30+28/15*a^6*b^2*x^15+2/3*a^7*b*x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int x^8 (a+bx^3)^8 \, dx = \frac{1}{33} b^8 x^{33} + \frac{4}{15} a b^7 x^{30} + \frac{28}{27} a^2 b^6 x^{27} + \frac{7}{3} a^3 b^5 x^{24} + \frac{10}{3} a^4 b^4 x^{21} + \frac{28}{9} a^5 b^3 x^{18} + \frac{28}{15} a^6 b^2 x^{15} + \frac{2}{3} a^7 b x^{12} + \frac{1}{9} a^8 x^9$$

input
```
integrate(x^8*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/33*b^8*x^33 + 4/15*a*b^7*x^30 + 28/27*a^2*b^6*x^27 + 7/3*a^3*b^5*x^24 +
10/3*a^4*b^4*x^21 + 28/9*a^5*b^3*x^18 + 28/15*a^6*b^2*x^15 + 2/3*a^7*b*x^1
2 + 1/9*a^8*x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(44) = 88.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int x^8 (a+bx^3)^8 \, dx = \frac{a^8 x^9}{9} + \frac{2a^7 b x^{12}}{3} + \frac{28a^6 b^2 x^{15}}{15} + \frac{28a^5 b^3 x^{18}}{9} + \frac{10a^4 b^4 x^{21}}{3} + \frac{7a^3 b^5 x^{24}}{3} + \frac{28a^2 b^6 x^{27}}{27} + \frac{4ab^7 x^{30}}{15} + \frac{b^8 x^{33}}{33}$$

input
```
integrate(x**8*(b*x**3+a)**8,x)
```

output
```
a**8*x**9/9 + 2*a**7*b*x**12/3 + 28*a**6*b**2*x**15/15 + 28*a**5*b**3*x**1
8/9 + 10*a**4*b**4*x**21/3 + 7*a**3*b**5*x**24/3 + 28*a**2*b**6*x**27/27 +
 4*a*b**7*x**30/15 + b**8*x**33/33
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int x^8 (a+bx^3)^8 \, dx = \frac{1}{33} b^8 x^{33} + \frac{4}{15} a b^7 x^{30} + \frac{28}{27} a^2 b^6 x^{27} + \frac{7}{3} a^3 b^5 x^{24} + \frac{10}{3} a^4 b^4 x^{21} + \frac{28}{9} a^5 b^3 x^{18} + \frac{28}{15} a^6 b^2 x^{15} + \frac{2}{3} a^7 b x^{12} + \frac{1}{9} a^8 x^9$$

input
```
integrate(x^8*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/33*b^8*x^33 + 4/15*a*b^7*x^30 + 28/27*a^2*b^6*x^27 + 7/3*a^3*b^5*x^24 +
10/3*a^4*b^4*x^21 + 28/9*a^5*b^3*x^18 + 28/15*a^6*b^2*x^15 + 2/3*a^7*b*x^1
2 + 1/9*a^8*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int x^8 (a+bx^3)^8 \, dx = \frac{1}{33} b^8 x^{33} + \frac{4}{15} a b^7 x^{30} + \frac{28}{27} a^2 b^6 x^{27} + \frac{7}{3} a^3 b^5 x^{24} + \frac{10}{3} a^4 b^4 x^{21} + \frac{28}{9} a^5 b^3 x^{18} + \frac{28}{15} a^6 b^2 x^{15} + \frac{2}{3} a^7 b x^{12} + \frac{1}{9} a^8 x^9$$

input
```
integrate(x^8*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/33*b^8*x^33 + 4/15*a*b^7*x^30 + 28/27*a^2*b^6*x^27 + 7/3*a^3*b^5*x^24 +
10/3*a^4*b^4*x^21 + 28/9*a^5*b^3*x^18 + 28/15*a^6*b^2*x^15 + 2/3*a^7*b*x^1
2 + 1/9*a^8*x^9
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

$$\int x^8 (a + bx^3)^8 \, dx = \frac{a^8 x^9}{9} + \frac{2 a^7 b x^{12}}{3} + \frac{28 a^6 b^2 x^{15}}{15} + \frac{28 a^5 b^3 x^{18}}{9} + \frac{10 a^4 b^4 x^{21}}{3} + \frac{7 a^3 b^5 x^{24}}{3} + \frac{28 a^2 b^6 x^{27}}{27} + \frac{4 a b^7 x^{30}}{15} + \frac{b^8 x^{33}}{33}$$

input
```
int(x^8*(a + b*x^3)^8,x)
```

output
```
(a^8*x^9)/9 + (b^8*x^33)/33 + (2*a^7*b*x^12)/3 + (4*a*b^7*x^30)/15 + (28*a
^6*b^2*x^15)/15 + (28*a^5*b^3*x^18)/9 + (10*a^4*b^4*x^21)/3 + (7*a^3*b^5*x
^24)/3 + (28*a^2*b^6*x^27)/27
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int x^8 (a + bx^3)^8 \, dx = \frac{x^9 (45 b^8 x^{24} + 396 a b^7 x^{21} + 1540 a^2 b^6 x^{18} + 3465 a^3 b^5 x^{15} + 4950 a^4 b^4 x^{12} + 4620 a^5 b^3 x^9 + 2772 a^6 b^2 x^6 + 990}{1485}$$

input
```
int(x^8*(b*x^3+a)^8,x)
```

output
```
(x**9*(165*a**8 + 990*a**7*b*x**3 + 2772*a**6*b**2*x**6 + 4620*a**5*b**3*x
**9 + 4950*a**4*b**4*x**12 + 3465*a**3*b**5*x**15 + 1540*a**2*b**6*x**18 +
 396*a*b**7*x**21 + 45*b**8*x**24))/1485
```

3.85 $\int x^5(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^5(a+bx^3)^8\,dx = -\frac{a(a+bx^3)^9}{27b^2}+\frac{(a+bx^3)^{10}}{30b^2}$$

output
```
-1/27*a*(b*x^3+a)^9/b^2+1/30*(b*x^3+a)^10/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(34) = 68.

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.18

$$\int x^5(a+bx^3)^8\,dx = \frac{a^8x^6}{6}+\frac{8}{9}a^7bx^9+\frac{7}{3}a^6b^2x^{12}+\frac{56}{15}a^5b^3x^{15}+\frac{35}{9}a^4b^4x^{18} + \frac{8}{3}a^3b^5x^{21}+\frac{7}{6}a^2b^6x^{24}+\frac{8}{27}ab^7x^{27}+\frac{b^8x^{30}}{30}$$

input
```
Integrate[x^5*(a + b*x^3)^8,x]
```

output
```
(a^8*x^6)/6 + (8*a^7*b*x^9)/9 + (7*a^6*b^2*x^12)/3 + (56*a^5*b^3*x^15)/15
+ (35*a^4*b^4*x^18)/9 + (8*a^3*b^5*x^21)/3 + (7*a^2*b^6*x^24)/6 + (8*a*b^7
*x^27)/27 + (b^8*x^30)/30
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^8 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^8 dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^9}{b} - \frac{a\left(bx^3+a\right)^8}{b}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{\left(a+bx^3\right)^{10}}{10b^2} - \frac{a\left(a+bx^3\right)^9}{9b^2}\right)$$

input
```
Int[x^5*(a + b*x^3)^8,x]
```

output
```
(-1/9*(a*(a + b*x^3)^9)/b^2 + (a + b*x^3)^10/(10*b^2))/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(30) = 60.

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.68

method	result
gosper	$\frac{8}{9}a^7b\,x^9+\frac{7}{3}a^6b^2x^{12}+\frac{8}{3}a^3b^5x^{21}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{6}a^2b^6x^{24}+\frac{1}{30}b^8x^{30}+\frac{1}{6}a^8x^6+\frac{56}{15}a^5b^3x^{15}+\frac{8}{27}$
default	$\frac{8}{9}a^7b\,x^9+\frac{7}{3}a^6b^2x^{12}+\frac{8}{3}a^3b^5x^{21}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{6}a^2b^6x^{24}+\frac{1}{30}b^8x^{30}+\frac{1}{6}a^8x^6+\frac{56}{15}a^5b^3x^{15}+\frac{8}{27}$
norman	$\frac{8}{9}a^7b\,x^9+\frac{7}{3}a^6b^2x^{12}+\frac{8}{3}a^3b^5x^{21}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{6}a^2b^6x^{24}+\frac{1}{30}b^8x^{30}+\frac{1}{6}a^8x^6+\frac{56}{15}a^5b^3x^{15}+\frac{8}{27}$
risch	$\frac{8}{9}a^7b\,x^9+\frac{7}{3}a^6b^2x^{12}+\frac{8}{3}a^3b^5x^{21}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{6}a^2b^6x^{24}+\frac{1}{30}b^8x^{30}+\frac{1}{6}a^8x^6+\frac{56}{15}a^5b^3x^{15}+\frac{8}{27}$
parallelrisch	$\frac{8}{9}a^7b\,x^9+\frac{7}{3}a^6b^2x^{12}+\frac{8}{3}a^3b^5x^{21}+\frac{35}{9}a^4b^4x^{18}+\frac{7}{6}a^2b^6x^{24}+\frac{1}{30}b^8x^{30}+\frac{1}{6}a^8x^6+\frac{56}{15}a^5b^3x^{15}+\frac{8}{27}$
orering	$\frac{x^6\left(9b^8x^{24}+80a\,b^7x^{21}+315a^2b^6x^{18}+720a^3b^5x^{15}+1050a^4b^4x^{12}+1008a^5b^3x^9+630a^6b^2x^6+240a^7b\,x^3+45a^8\right)}{270}$

input
```
int(x^5*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
8/9*a^7*b*x^9+7/3*a^6*b^2*x^12+8/3*a^3*b^5*x^21+35/9*a^4*b^4*x^18+7/6*a^2*
b^6*x^24+1/30*b^8*x^30+1/6*a^8*x^6+56/15*a^5*b^3*x^15+8/27*a*b^7*x^27
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^5 (a + bx^3)^8 \, dx = \frac{1}{30} b^8 x^{30} + \frac{8}{27} a b^7 x^{27} + \frac{7}{6} a^2 b^6 x^{24} + \frac{8}{3} a^3 b^5 x^{21} + \frac{35}{9} a^4 b^4 x^{18} + \frac{56}{15} a^5 b^3 x^{15} + \frac{7}{3} a^6 b^2 x^{12} + \frac{8}{9} a^7 b x^9 + \frac{1}{6} a^8 x^6$$

input
```
integrate(x^5*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/30*b^8*x^30 + 8/27*a*b^7*x^27 + 7/6*a^2*b^6*x^24 + 8/3*a^3*b^5*x^21 + 35
/9*a^4*b^4*x^18 + 56/15*a^5*b^3*x^15 + 7/3*a^6*b^2*x^12 + 8/9*a^7*b*x^9 +
1/6*a^8*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(27) = 54$.

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.15

$$\int x^5 (a + bx^3)^8 \, dx = \frac{a^8 x^6}{6} + \frac{8 a^7 b x^9}{9} + \frac{7 a^6 b^2 x^{12}}{3} + \frac{56 a^5 b^3 x^{15}}{15} + \frac{35 a^4 b^4 x^{18}}{9} + \frac{8 a^3 b^5 x^{21}}{3} + \frac{7 a^2 b^6 x^{24}}{6} + \frac{8 a b^7 x^{27}}{27} + \frac{b^8 x^{30}}{30}$$

input
```
integrate(x**5*(b*x**3+a)**8,x)
```

output
```
a**8*x**6/6 + 8*a**7*b*x**9/9 + 7*a**6*b**2*x**12/3 + 56*a**5*b**3*x**15/1
5 + 35*a**4*b**4*x**18/9 + 8*a**3*b**5*x**21/3 + 7*a**2*b**6*x**24/6 + 8*a
*b**7*x**27/27 + b**8*x**30/30
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^5 (a + bx^3)^8 \, dx = \frac{1}{30} b^8 x^{30} + \frac{8}{27} a b^7 x^{27} + \frac{7}{6} a^2 b^6 x^{24} + \frac{8}{3} a^3 b^5 x^{21} + \frac{35}{9} a^4 b^4 x^{18} + \frac{56}{15} a^5 b^3 x^{15} + \frac{7}{3} a^6 b^2 x^{12} + \frac{8}{9} a^7 b x^9 + \frac{1}{6} a^8 x^6$$

input
```
integrate(x^5*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/30*b^8*x^30 + 8/27*a*b^7*x^27 + 7/6*a^2*b^6*x^24 + 8/3*a^3*b^5*x^21 + 35
/9*a^4*b^4*x^18 + 56/15*a^5*b^3*x^15 + 7/3*a^6*b^2*x^12 + 8/9*a^7*b*x^9 +
1/6*a^8*x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^5 (a + bx^3)^8 \, dx = \frac{1}{30} b^8 x^{30} + \frac{8}{27} a b^7 x^{27} + \frac{7}{6} a^2 b^6 x^{24} + \frac{8}{3} a^3 b^5 x^{21} + \frac{35}{9} a^4 b^4 x^{18} + \frac{56}{15} a^5 b^3 x^{15} + \frac{7}{3} a^6 b^2 x^{12} + \frac{8}{9} a^7 b x^9 + \frac{1}{6} a^8 x^6$$

input
```
integrate(x^5*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/30*b^8*x^30 + 8/27*a*b^7*x^27 + 7/6*a^2*b^6*x^24 + 8/3*a^3*b^5*x^21 + 35
/9*a^4*b^4*x^18 + 56/15*a^5*b^3*x^15 + 7/3*a^6*b^2*x^12 + 8/9*a^7*b*x^9 +
1/6*a^8*x^6
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^5 (a + bx^3)^8 \, dx = \frac{a^8 x^6}{6} + \frac{8 a^7 b x^9}{9} + \frac{7 a^6 b^2 x^{12}}{3} + \frac{56 a^5 b^3 x^{15}}{15} + \frac{35 a^4 b^4 x^{18}}{9} + \frac{8 a^3 b^5 x^{21}}{3} + \frac{7 a^2 b^6 x^{24}}{6} + \frac{8 a b^7 x^{27}}{27} + \frac{b^8 x^{30}}{30}$$

input
```
int(x^5*(a + b*x^3)^8,x)
```

output
```
(a^8*x^6)/6 + (b^8*x^30)/30 + (8*a^7*b*x^9)/9 + (8*a*b^7*x^27)/27 + (7*a^6
*b^2*x^12)/3 + (56*a^5*b^3*x^15)/15 + (35*a^4*b^4*x^18)/9 + (8*a^3*b^5*x^2
1)/3 + (7*a^2*b^6*x^24)/6
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.71

$$\int x^5 (a + bx^3)^8 \, dx = \frac{x^6 (9b^8 x^{24} + 80a b^7 x^{21} + 315a^2 b^6 x^{18} + 720a^3 b^5 x^{15} + 1050a^4 b^4 x^{12} + 1008a^5 b^3 x^9 + 630a^6 b^2 x^6 + 240a^7 b x^3}{270}$$

input
```
int(x^5*(b*x^3+a)^8,x)
```

output
```
(x**6*(45*a**8 + 240*a**7*b*x**3 + 630*a**6*b**2*x**6 + 1008*a**5*b**3*x**
9 + 1050*a**4*b**4*x**12 + 720*a**3*b**5*x**15 + 315*a**2*b**6*x**18 + 80*
a*b**7*x**21 + 9*b**8*x**24))/270
```

3.86 $\int x^2(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^2\left(a+bx^3\right)^8\,dx=\frac{\left(a+bx^3\right)^9}{27b}$$

output
```
1/27*(b*x^3+a)^9/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 6.75

$$\int x^2\left(a+bx^3\right)^8\,dx=\frac{a^8x^3}{3}+\frac{4}{3}a^7bx^6+\frac{28}{9}a^6b^2x^9+\frac{14}{3}a^5b^3x^{12}+\frac{14}{3}a^4b^4x^{15}+\frac{28}{9}a^3b^5x^{18}+\frac{4}{3}a^2b^6x^{21}+\frac{1}{3}ab^7x^{24}+\frac{b^8x^{27}}{27}$$

input
```
Integrate[x^2*(a + b*x^3)^8,x]
```

output
```
(a^8*x^3)/3 + (4*a^7*b*x^6)/3 + (28*a^6*b^2*x^9)/9 + (14*a^5*b^3*x^12)/3 +
 (14*a^4*b^4*x^15)/3 + (28*a^3*b^5*x^18)/9 + (4*a^2*b^6*x^21)/3 + (a*b^7*x
^24)/3 + (b^8*x^27)/27
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^3)^8 \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{(a + bx^3)^9}{27b}$$

input
```
Int[x^2*(a + b*x^3)^8,x]
```

output
```
(a + b*x^3)^9/(27*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{(b\,x^3+a)^9}{27b}$
gosper	$\frac{4}{3}a^7b\,x^6+\frac{14}{3}a^5b^3x^{12}+\frac{4}{3}a^2b^6x^{21}+\frac{1}{3}a^8x^3+\frac{1}{27}b^8x^{27}+\frac{1}{3}a\,b^7x^{24}+\frac{28}{9}a^6b^2x^9+\frac{28}{9}a^3b^5x^{18}+\frac{14}{3}$
norman	$\frac{4}{3}a^7b\,x^6+\frac{14}{3}a^5b^3x^{12}+\frac{4}{3}a^2b^6x^{21}+\frac{1}{3}a^8x^3+\frac{1}{27}b^8x^{27}+\frac{1}{3}a\,b^7x^{24}+\frac{28}{9}a^6b^2x^9+\frac{28}{9}a^3b^5x^{18}+\frac{14}{3}$
parallelrisch	$\frac{4}{3}a^7b\,x^6+\frac{14}{3}a^5b^3x^{12}+\frac{4}{3}a^2b^6x^{21}+\frac{1}{3}a^8x^3+\frac{1}{27}b^8x^{27}+\frac{1}{3}a\,b^7x^{24}+\frac{28}{9}a^6b^2x^9+\frac{28}{9}a^3b^5x^{18}+\frac{14}{3}$
orering	$\frac{x^3(b^8x^{24}+9a\,b^7x^{21}+36a^2b^6x^{18}+84a^3b^5x^{15}+126a^4b^4x^{12}+126a^5b^3x^9+84a^6b^2x^6+36a^7b\,x^3+9a^8)}{27}$
risch	$\frac{b^8x^{27}}{27}+\frac{a\,b^7x^{24}}{3}+\frac{4a^2b^6x^{21}}{3}+\frac{28a^3b^5x^{18}}{9}+\frac{14a^4b^4x^{15}}{3}+\frac{14a^5b^3x^{12}}{3}+\frac{28a^6b^2x^9}{9}+\frac{4a^7b\,x^6}{3}+\frac{a^8x^3}{3}+\frac{a^9}{27b}$

input
```
int(x^2*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/27*(b*x^3+a)^9/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.62

$$\int x^2(a+bx^3)^8\,dx=\frac{1}{27}b^8x^{27}+\frac{1}{3}ab^7x^{24}+\frac{4}{3}a^2b^6x^{21}+\frac{28}{9}a^3b^5x^{18}+\frac{14}{3}a^4b^4x^{15}+\frac{14}{3}a^5b^3x^{12}+\frac{28}{9}a^6b^2x^9+\frac{4}{3}a^7bx^6+\frac{1}{3}a^8x^3$$

input
```
integrate(x^2*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/27*b^8*x^27 + 1/3*a*b^7*x^24 + 4/3*a^2*b^6*x^21 + 28/9*a^3*b^5*x^18 + 14
/3*a^4*b^4*x^15 + 14/3*a^5*b^3*x^12 + 28/9*a^6*b^2*x^9 + 4/3*a^7*b*x^6 + 1
/3*a^8*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(10) = 20.

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 6.56

$$\int x^2(a+bx^3)^8\,dx=\frac{a^8x^3}{3}+\frac{4a^7bx^6}{3}+\frac{28a^6b^2x^9}{9}+\frac{14a^5b^3x^{12}}{3}+\frac{14a^4b^4x^{15}}{3}+\frac{28a^3b^5x^{18}}{9}+\frac{4a^2b^6x^{21}}{3}+\frac{ab^7x^{24}}{3}+\frac{b^8x^{27}}{27}$$

input
```
integrate(x**2*(b*x**3+a)**8,x)
```

output
```
a**8*x**3/3 + 4*a**7*b*x**6/3 + 28*a**6*b**2*x**9/9 + 14*a**5*b**3*x**12/3
 + 14*a**4*b**4*x**15/3 + 28*a**3*b**5*x**18/9 + 4*a**2*b**6*x**21/3 + a*b
**7*x**24/3 + b**8*x**27/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)^8\,dx=\frac{(bx^3+a)^9}{27\,b}$$

input
```
integrate(x^2*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/27*(b*x^3 + a)^9/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)^8\,dx=\frac{(bx^3+a)^9}{27\,b}$$

input
```
integrate(x^2*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/27*(b*x^3 + a)^9/b
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^3)^8\,dx=\frac{(b\,x^3+a)^9}{27\,b}$$

input
```
int(x^2*(a + b*x^3)^8,x)
```

output
```
(a + b*x^3)^9/(27*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 5.69

$$\int x^2(a+bx^3)^8\,dx$$
$$=\frac{x^3(b^8x^{24}+9a\,b^7x^{21}+36a^2b^6x^{18}+84a^3b^5x^{15}+126a^4b^4x^{12}+126a^5b^3x^9+84a^6b^2x^6+36a^7b\,x^3+9a^8)}{27}$$

input
```
int(x^2*(b*x^3+a)^8,x)
```

output
```
(x**3*(9*a**8 + 36*a**7*b*x**3 + 84*a**6*b**2*x**6 + 126*a**5*b**3*x**9 +
126*a**4*b**4*x**12 + 84*a**3*b**5*x**15 + 36*a**2*b**6*x**18 + 9*a*b**7*x
**21 + b**8*x**24))/27
```

3.87 $\int \frac{(a+bx^3)^8}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{(a+bx^3)^8}{x}\,dx=\frac{8}{3}a^7bx^3+\frac{14}{3}a^6b^2x^6+\frac{56}{9}a^5b^3x^9+\frac{35}{6}a^4b^4x^{12}+\frac{56}{15}a^3b^5x^{15}+\frac{14}{9}a^2b^6x^{18}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{24}}{24}+a^8\log(x)$$

output
```
8/3*a^7*b*x^3+14/3*a^6*b^2*x^6+56/9*a^5*b^3*x^9+35/6*a^4*b^4*x^12+56/15*a^
3*b^5*x^15+14/9*a^2*b^6*x^18+8/21*a*b^7*x^21+1/24*b^8*x^24+a^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x}\,dx=\frac{8}{3}a^7bx^3+\frac{14}{3}a^6b^2x^6+\frac{56}{9}a^5b^3x^9+\frac{35}{6}a^4b^4x^{12}+\frac{56}{15}a^3b^5x^{15}+\frac{14}{9}a^2b^6x^{18}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{24}}{24}+a^8\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x,x]
```

output
```
(8*a^7*b*x^3)/3 + (14*a^6*b^2*x^6)/3 + (56*a^5*b^3*x^9)/9 + (35*a^4*b^4*x^
12)/6 + (56*a^3*b^5*x^15)/15 + (14*a^2*b^6*x^18)/9 + (8*a*b^7*x^21)/21 + (
b^8*x^24)/24 + a^8*Log[x]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^3}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^8x^{21}+8ab^7x^{18}+28a^2b^6x^{15}+56a^3b^5x^{12}+70a^4b^4x^9+56a^5b^3x^6+28a^6b^2x^3+8a^7b+\frac{a^8}{x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(a^8\log\left(x^3\right)+8a^7bx^3+14a^6b^2x^6+\frac{56}{3}a^5b^3x^9+\frac{35}{2}a^4b^4x^{12}+\frac{56}{5}a^3b^5x^{15}+\frac{14}{3}a^2b^6x^{18}+\frac{8}{7}ab^7x^{21}+\frac{b^8x^{24}}{8}\right)$$

input
```
Int[(a + b*x^3)^8/x,x]
```

output
```
(8*a^7*b*x^3 + 14*a^6*b^2*x^6 + (56*a^5*b^3*x^9)/3 + (35*a^4*b^4*x^12)/2 +
 (56*a^3*b^5*x^15)/5 + (14*a^2*b^6*x^18)/3 + (8*a*b^7*x^21)/7 + (b^8*x^24)
/8 + a^8*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result
default	$\frac{8a^7b\,x^3}{3}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^9}{9}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^3b^5x^{15}}{15}+\frac{14a^2b^6x^{18}}{9}+\frac{8a\,b^7x^{21}}{21}+\frac{b^8x^{24}}{24}+a^8\ln\left(x\right)$
norman	$\frac{8a^7b\,x^3}{3}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^9}{9}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^3b^5x^{15}}{15}+\frac{14a^2b^6x^{18}}{9}+\frac{8a\,b^7x^{21}}{21}+\frac{b^8x^{24}}{24}+a^8\ln\left(x\right)$
parallelrisch	$\frac{8a^7b\,x^3}{3}+\frac{14a^6b^2x^6}{3}+\frac{56a^5b^3x^9}{9}+\frac{35a^4b^4x^{12}}{6}+\frac{56a^3b^5x^{15}}{15}+\frac{14a^2b^6x^{18}}{9}+\frac{8a\,b^7x^{21}}{21}+\frac{b^8x^{24}}{24}+a^8\ln\left(x\right)$
risch	$\frac{352a^8}{315}+\frac{b^8x^{24}}{24}+a^8\ln\left(x\right)+\frac{14a^2b^6x^{18}}{9}+\frac{56a^3b^5x^{15}}{15}+\frac{35a^4b^4x^{12}}{6}+\frac{8a^7b\,x^3}{3}+\frac{56a^5b^3x^9}{9}+\frac{8a\,b^7x^{21}}{21}+$ 14

input
```
int((b*x^3+a)^8/x,x,method=_RETURNVERBOSE)
```

output
```
8/3*a^7*b*x^3+14/3*a^6*b^2*x^6+56/9*a^5*b^3*x^9+35/6*a^4*b^4*x^12+56/15*a^
3*b^5*x^15+14/9*a^2*b^6*x^18+8/21*a*b^7*x^21+1/24*b^8*x^24+a^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x}\,dx = \frac{1}{24}\,b^8x^{24} + \frac{8}{21}\,ab^7x^{21} + \frac{14}{9}\,a^2b^6x^{18} + \frac{56}{15}\,a^3b^5x^{15} + \frac{35}{6}\,a^4b^4x^{12} + \frac{56}{9}\,a^5b^3x^9 + \frac{14}{3}\,a^6b^2x^6 + \frac{8}{3}\,a^7bx^3 + a^8\log(x)$$

input
```
integrate((b*x^3+a)^8/x,x, algorithm="fricas")
```

output
```
1/24*b^8*x^24 + 8/21*a*b^7*x^21 + 14/9*a^2*b^6*x^18 + 56/15*a^3*b^5*x^15 +
 35/6*a^4*b^4*x^12 + 56/9*a^5*b^3*x^9 + 14/3*a^6*b^2*x^6 + 8/3*a^7*b*x^3 +
 a^8*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^8}{x}\,dx = a^8\log(x) + \frac{8a^7bx^3}{3} + \frac{14a^6b^2x^6}{3} + \frac{56a^5b^3x^9}{9} + \frac{35a^4b^4x^{12}}{6} + \frac{56a^3b^5x^{15}}{15} + \frac{14a^2b^6x^{18}}{9} + \frac{8ab^7x^{21}}{21} + \frac{b^8x^{24}}{24}$$

input
```
integrate((b*x**3+a)**8/x,x)
```

output
```
a**8*log(x) + 8*a**7*b*x**3/3 + 14*a**6*b**2*x**6/3 + 56*a**5*b**3*x**9/9
+ 35*a**4*b**4*x**12/6 + 56*a**3*b**5*x**15/15 + 14*a**2*b**6*x**18/9 + 8*
a*b**7*x**21/21 + b**8*x**24/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x}\,dx = \frac{1}{24}\,b^8x^{24} + \frac{8}{21}\,ab^7x^{21} + \frac{14}{9}\,a^2b^6x^{18} + \frac{56}{15}\,a^3b^5x^{15} + \frac{35}{6}\,a^4b^4x^{12} + \frac{56}{9}\,a^5b^3x^9 + \frac{14}{3}\,a^6b^2x^6 + \frac{8}{3}\,a^7bx^3 + \frac{1}{3}\,a^8\log\left(x^3\right)$$

input
```
integrate((b*x^3+a)^8/x,x, algorithm="maxima")
```

output
```
1/24*b^8*x^24 + 8/21*a*b^7*x^21 + 14/9*a^2*b^6*x^18 + 56/15*a^3*b^5*x^15 +
 35/6*a^4*b^4*x^12 + 56/9*a^5*b^3*x^9 + 14/3*a^6*b^2*x^6 + 8/3*a^7*b*x^3 +
 1/3*a^8*log(x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^8}{x}\,dx = \frac{1}{24}\,b^8x^{24} + \frac{8}{21}\,ab^7x^{21} + \frac{14}{9}\,a^2b^6x^{18} + \frac{56}{15}\,a^3b^5x^{15} + \frac{35}{6}\,a^4b^4x^{12} + \frac{56}{9}\,a^5b^3x^9 + \frac{14}{3}\,a^6b^2x^6 + \frac{8}{3}\,a^7bx^3 + a^8\log\left(|x|\right)$$

input
```
integrate((b*x^3+a)^8/x,x, algorithm="giac")
```

output
```
1/24*b^8*x^24 + 8/21*a*b^7*x^21 + 14/9*a^2*b^6*x^18 + 56/15*a^3*b^5*x^15 +
 35/6*a^4*b^4*x^12 + 56/9*a^5*b^3*x^9 + 14/3*a^6*b^2*x^6 + 8/3*a^7*b*x^3 +
 a^8*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x}\,dx = a^8\,\ln(x) + \frac{b^8\,x^{24}}{24} + \frac{8\,a^7\,b\,x^3}{3} + \frac{8\,a\,b^7\,x^{21}}{21} + \frac{14\,a^6\,b^2\,x^6}{3} + \frac{56\,a^5\,b^3\,x^9}{9} + \frac{35\,a^4\,b^4\,x^{12}}{6} + \frac{56\,a^3\,b^5\,x^{15}}{15} + \frac{14\,a^2\,b^6\,x^{18}}{9}$$

input
```
int((a + b*x^3)^8/x,x)
```

output
```
a^8*log(x) + (b^8*x^24)/24 + (8*a^7*b*x^3)/3 + (8*a*b^7*x^21)/21 + (14*a^6
*b^2*x^6)/3 + (56*a^5*b^3*x^9)/9 + (35*a^4*b^4*x^12)/6 + (56*a^3*b^5*x^15)
/15 + (14*a^2*b^6*x^18)/9
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x}\,dx = \log(x)\,a^8 + \frac{8a^7b\,x^3}{3} + \frac{14a^6b^2x^6}{3} + \frac{56a^5b^3x^9}{9} + \frac{35a^4b^4x^{12}}{6} + \frac{56a^3b^5x^{15}}{15} + \frac{14a^2b^6x^{18}}{9} + \frac{8a\,b^7x^{21}}{21} + \frac{b^8x^{24}}{24}$$

input
```
int((b*x^3+a)^8/x,x)
```

output
```
(2520*log(x)*a**8 + 6720*a**7*b*x**3 + 11760*a**6*b**2*x**6 + 15680*a**5*b
**3*x**9 + 14700*a**4*b**4*x**12 + 9408*a**3*b**5*x**15 + 3920*a**2*b**6*x
**18 + 960*a*b**7*x**21 + 105*b**8*x**24)/2520
```

3.88 $\int \frac{(a+bx^3)^8}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = -\frac{a^8}{3x^3}+\frac{28}{3}a^6b^2x^3+\frac{28}{3}a^5b^3x^6+\frac{70}{9}a^4b^4x^9+\frac{14}{3}a^3b^5x^{12}+\frac{28}{15}a^2b^6x^{15}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{21}}{21}+8a^7b\log(x)$$

output
```
-1/3*a^8/x^3+28/3*a^6*b^2*x^3+28/3*a^5*b^3*x^6+70/9*a^4*b^4*x^9+14/3*a^3*b
^5*x^12+28/15*a^2*b^6*x^15+4/9*a*b^7*x^18+1/21*b^8*x^21+8*a^7*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = -\frac{a^8}{3x^3}+\frac{28}{3}a^6b^2x^3+\frac{28}{3}a^5b^3x^6+\frac{70}{9}a^4b^4x^9+\frac{14}{3}a^3b^5x^{12}+\frac{28}{15}a^2b^6x^{15}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{21}}{21}+8a^7b\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^4,x]
```

output
```
-1/3*a^8/x^3 + (28*a^6*b^2*x^3)/3 + (28*a^5*b^3*x^6)/3 + (70*a^4*b^4*x^9)/
9 + (14*a^3*b^5*x^12)/3 + (28*a^2*b^6*x^15)/15 + (4*a*b^7*x^18)/9 + (b^8*x
^21)/21 + 8*a^7*b*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^4}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^6}dx^3$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}\int \left(b^8x^{18}+8ab^7x^{15}+28a^2b^6x^{12}+56a^3b^5x^9+70a^4b^4x^6+56a^5b^3x^3+28a^6b^2+\frac{8a^7b}{x^3}+\frac{a^8}{x^6}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^8}{x^3}+8a^7b\log\left(x^3\right)+28a^6b^2x^3+28a^5b^3x^6+\frac{70}{3}a^4b^4x^9+14a^3b^5x^{12}+\frac{28}{5}a^2b^6x^{15}+\frac{4}{3}ab^7x^{18}+\frac{b^8x^{21}}{7}\right)$$

input
```
Int[(a + b*x^3)^8/x^4,x]
```

output
```
(-(a^8/x^3) + 28*a^6*b^2*x^3 + 28*a^5*b^3*x^6 + (70*a^4*b^4*x^9)/3 + 14*a^
3*b^5*x^12 + (28*a^2*b^6*x^15)/5 + (4*a*b^7*x^18)/3 + (b^8*x^21)/7 + 8*a^7
*b*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result
default	$-\frac{a^{8}}{3x^{3}}+\frac{28a^{6}b^{2}x^{3}}{3}+\frac{28a^{5}b^{3}x^{6}}{3}+\frac{70a^{4}b^{4}x^{9}}{9}+\frac{14a^{3}b^{5}x^{12}}{3}+\frac{28a^{2}b^{6}x^{15}}{15}+\frac{4a\,b^{7}x^{18}}{9}+\frac{b^{8}x^{21}}{21}+8a^{7}b\ln\left(x\right)$
risch	$-\frac{a^{8}}{3x^{3}}+\frac{28a^{6}b^{2}x^{3}}{3}+\frac{28a^{5}b^{3}x^{6}}{3}+\frac{70a^{4}b^{4}x^{9}}{9}+\frac{14a^{3}b^{5}x^{12}}{3}+\frac{28a^{2}b^{6}x^{15}}{15}+\frac{4a\,b^{7}x^{18}}{9}+\frac{b^{8}x^{21}}{21}+8a^{7}b\ln\left(x\right)$
norman	$\frac{-\frac{1}{3}a^{8}+\frac{1}{21}b^{8}x^{24}+\frac{4}{9}a\,b^{7}x^{21}+\frac{28}{15}a^{2}b^{6}x^{18}+\frac{14}{3}a^{3}b^{5}x^{15}+\frac{70}{9}a^{4}b^{4}x^{12}+\frac{28}{3}a^{5}b^{3}x^{9}+\frac{28}{3}a^{6}b^{2}x^{6}}{x^{3}}+8a^{7}b\ln\left(x\right)$
parallelrisch	$\frac{15b^{8}x^{24}+140a\,b^{7}x^{21}+588a^{2}b^{6}x^{18}+1470a^{3}b^{5}x^{15}+2450a^{4}b^{4}x^{12}+2940a^{5}b^{3}x^{9}+2940a^{6}b^{2}x^{6}+2520a^{7}b\ln\left(x\right)x^{3}-105a^{8}}{315x^{3}}$

input
```
int((b*x^3+a)^8/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^8/x^3+28/3*a^6*b^2*x^3+28/3*a^5*b^3*x^6+70/9*a^4*b^4*x^9+14/3*a^3*b
^5*x^12+28/15*a^2*b^6*x^15+4/9*a*b^7*x^18+1/21*b^8*x^21+8*a^7*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = \frac{15\,b^8x^{24} + 140\,ab^7x^{21} + 588\,a^2b^6x^{18} + 1470\,a^3b^5x^{15} + 2450\,a^4b^4x^{12} + 2940\,a^5b^3x^9 + 2940\,a^6b^2x^6 + 2520\,a}{315\,x^3}$$

input
```
integrate((b*x^3+a)^8/x^4,x, algorithm="fricas")
```

output
```
1/315*(15*b^8*x^24 + 140*a*b^7*x^21 + 588*a^2*b^6*x^18 + 1470*a^3*b^5*x^15
 + 2450*a^4*b^4*x^12 + 2940*a^5*b^3*x^9 + 2940*a^6*b^2*x^6 + 2520*a^7*b*x^
3*log(x) - 105*a^8)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = -\frac{a^8}{3x^3} + 8a^7b\log(x) + \frac{28a^6b^2x^3}{3} + \frac{28a^5b^3x^6}{3} + \frac{70a^4b^4x^9}{9} + \frac{14a^3b^5x^{12}}{3} + \frac{28a^2b^6x^{15}}{15} + \frac{4ab^7x^{18}}{9} + \frac{b^8x^{21}}{21}$$

input
```
integrate((b*x**3+a)**8/x**4,x)
```

output
```
-a**8/(3*x**3) + 8*a**7*b*log(x) + 28*a**6*b**2*x**3/3 + 28*a**5*b**3*x**6
/3 + 70*a**4*b**4*x**9/9 + 14*a**3*b**5*x**12/3 + 28*a**2*b**6*x**15/15 +
4*a*b**7*x**18/9 + b**8*x**21/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = \frac{1}{21}\,b^8x^{21} + \frac{4}{9}\,ab^7x^{18} + \frac{28}{15}\,a^2b^6x^{15} + \frac{14}{3}\,a^3b^5x^{12} + \frac{70}{9}\,a^4b^4x^9 + \frac{28}{3}\,a^5b^3x^6 + \frac{28}{3}\,a^6b^2x^3 + \frac{8}{3}\,a^7b\log\left(x^3\right) - \frac{a^8}{3\,x^3}$$

input
```
integrate((b*x^3+a)^8/x^4,x, algorithm="maxima")
```

output
```
1/21*b^8*x^21 + 4/9*a*b^7*x^18 + 28/15*a^2*b^6*x^15 + 14/3*a^3*b^5*x^12 +
70/9*a^4*b^4*x^9 + 28/3*a^5*b^3*x^6 + 28/3*a^6*b^2*x^3 + 8/3*a^7*b*log(x^3
) - 1/3*a^8/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = \frac{1}{21}\,b^8x^{21} + \frac{4}{9}\,ab^7x^{18} + \frac{28}{15}\,a^2b^6x^{15} + \frac{14}{3}\,a^3b^5x^{12} + \frac{70}{9}\,a^4b^4x^9 + \frac{28}{3}\,a^5b^3x^6 + \frac{28}{3}\,a^6b^2x^3 + 8\,a^7b\log\left(|x|\right) - \frac{8\,a^7bx^3 + a^8}{3\,x^3}$$

input
```
integrate((b*x^3+a)^8/x^4,x, algorithm="giac")
```

output
```
1/21*b^8*x^21 + 4/9*a*b^7*x^18 + 28/15*a^2*b^6*x^15 + 14/3*a^3*b^5*x^12 +
70/9*a^4*b^4*x^9 + 28/3*a^5*b^3*x^6 + 28/3*a^6*b^2*x^3 + 8*a^7*b*log(abs(x
)) - 1/3*(8*a^7*b*x^3 + a^8)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = \frac{b^8\,x^{21}}{21} - \frac{a^8}{3\,x^3} + \frac{4\,a\,b^7\,x^{18}}{9} + 8\,a^7\,b\,\ln(x) + \frac{28\,a^6\,b^2\,x^3}{3} + \frac{28\,a^5\,b^3\,x^6}{3} + \frac{70\,a^4\,b^4\,x^9}{9} + \frac{14\,a^3\,b^5\,x^{12}}{3} + \frac{28\,a^2\,b^6\,x^{15}}{15}$$

input
```
int((a + b*x^3)^8/x^4,x)
```

output
```
(b^8*x^21)/21 - a^8/(3*x^3) + (4*a*b^7*x^18)/9 + 8*a^7*b*log(x) + (28*a^6*
b^2*x^3)/3 + (28*a^5*b^3*x^6)/3 + (70*a^4*b^4*x^9)/9 + (14*a^3*b^5*x^12)/3
 + (28*a^2*b^6*x^15)/15
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^4}\,dx = \frac{2520\log(x)\,a^7b\,x^3 - 105a^8 + 2940a^6b^2x^6 + 2940a^5b^3x^9 + 2450a^4b^4x^{12} + 1470a^3b^5x^{15} + 588a^2b^6x^{18} + 14}{315x^3}$$

input
```
int((b*x^3+a)^8/x^4,x)
```

output
```
(2520*log(x)*a**7*b*x**3 - 105*a**8 + 2940*a**6*b**2*x**6 + 2940*a**5*b**3
*x**9 + 2450*a**4*b**4*x**12 + 1470*a**3*b**5*x**15 + 588*a**2*b**6*x**18
+ 140*a*b**7*x**21 + 15*b**8*x**24)/(315*x**3)
```

3.89 $\int \frac{(a+bx^3)^8}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^7} dx = -\frac{a^8}{6x^6} - \frac{8a^7b}{3x^3} + \frac{56}{3}a^5b^3x^3 + \frac{35}{3}a^4b^4x^6 + \frac{56}{9}a^3b^5x^9 + \frac{7}{3}a^2b^6x^{12} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{18}}{18} + 28a^6b^2\log(x)$$

output
```
-1/6*a^8/x^6-8/3*a^7*b/x^3+56/3*a^5*b^3*x^3+35/3*a^4*b^4*x^6+56/9*a^3*b^5*
x^9+7/3*a^2*b^6*x^12+8/15*a*b^7*x^15+1/18*b^8*x^18+28*a^6*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^7} dx = -\frac{a^8}{6x^6} - \frac{8a^7b}{3x^3} + \frac{56}{3}a^5b^3x^3 + \frac{35}{3}a^4b^4x^6 + \frac{56}{9}a^3b^5x^9 + \frac{7}{3}a^2b^6x^{12} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{18}}{18} + 28a^6b^2\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^7,x]
```

output
```
-1/6*a^8/x^6 - (8*a^7*b)/(3*x^3) + (56*a^5*b^3*x^3)/3 + (35*a^4*b^4*x^6)/3
 + (56*a^3*b^5*x^9)/9 + (7*a^2*b^6*x^12)/3 + (8*a*b^7*x^15)/15 + (b^8*x^18
)/18 + 28*a^6*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^7}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^9}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^8x^{15}+8ab^7x^{12}+28a^2b^6x^9+56a^3b^5x^6+70a^4b^4x^3+56a^5b^3+\frac{28a^6b^2}{x^3}+\frac{8a^7b}{x^6}+\frac{a^8}{x^9}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^8}{2x^6}-\frac{8a^7b}{x^3}+28a^6b^2\log\left(x^3\right)+56a^5b^3x^3+35a^4b^4x^6+\frac{56}{3}a^3b^5x^9+7a^2b^6x^{12}+\frac{8}{5}ab^7x^{15}+\frac{b^8x^{18}}{6}\right)$$

input
```
Int[(a + b*x^3)^8/x^7,x]
```

output
```
(-1/2*a^8/x^6 - (8*a^7*b)/x^3 + 56*a^5*b^3*x^3 + 35*a^4*b^4*x^6 + (56*a^3*
b^5*x^9)/3 + 7*a^2*b^6*x^12 + (8*a*b^7*x^15)/5 + (b^8*x^18)/6 + 28*a^6*b^2
*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	siz
default	$-\frac{a^{8}}{6 x^{6}}-\frac{8 a^{7} b}{3 x^{3}}+\frac{56 a^{5} b^{3} x^{3}}{3}+\frac{35 a^{4} b^{4} x^{6}}{3}+\frac{56 a^{3} b^{5} x^{9}}{9}+\frac{7 a^{2} b^{6} x^{12}}{3}+\frac{8 a\, b^{7} x^{15}}{15}+\frac{b^{8} x^{18}}{18}+28 a^{6} b^{2} \ln \left(x \right)$	90
norman	$\frac{-\frac{1}{6} a^{8}+\frac{1}{18} b^{8} x^{24}+\frac{8}{15} a\, b^{7} x^{21}+\frac{7}{3} a^{2} b^{6} x^{18}+\frac{56}{9} a^{3} b^{5} x^{15}+\frac{35}{3} a^{4} b^{4} x^{12}+\frac{56}{3} a^{5} b^{3} x^{9}-\frac{8}{3} a^{7} b\, x^{3}}{x^{6}}+28 a^{6} b^{2} \ln \left(x \right)$	92
risch	$\frac{b^{8} x^{18}}{18}+\frac{8 a\, b^{7} x^{15}}{15}+\frac{7 a^{2} b^{6} x^{12}}{3}+\frac{56 a^{3} b^{5} x^{9}}{9}+\frac{35 a^{4} b^{4} x^{6}}{3}+\frac{56 a^{5} b^{3} x^{3}}{3}+\frac{-\frac{8}{3} a^{7} b\, x^{3}-\frac{1}{6} a^{8}}{x^{6}}+28 a^{6} b^{2} \ln \left(x \right)$	92
parallelrisch	$\frac{5 b^{8} x^{24}+48 a\, b^{7} x^{21}+210 a^{2} b^{6} x^{18}+560 a^{3} b^{5} x^{15}+1050 a^{4} b^{4} x^{12}+1680 a^{5} b^{3} x^{9}+2520 a^{6} b^{2} \ln \left(x \right) x^{6}-240 a^{7} b\, x^{3}-15 a^{8}}{90 x^{6}}$	95

input
```
int((b*x^3+a)^8/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^8/x^6-8/3*a^7*b/x^3+56/3*a^5*b^3*x^3+35/3*a^4*b^4*x^6+56/9*a^3*b^5*
x^9+7/3*a^2*b^6*x^12+8/15*a*b^7*x^15+1/18*b^8*x^18+28*a^6*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^7}\,dx = \frac{5\,b^8x^{24}+48\,ab^7x^{21}+210\,a^2b^6x^{18}+560\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+1680\,a^5b^3x^9+2520\,a^6b^2x^6\log(x)-24}{90\,x^6}$$

input
```
integrate((b*x^3+a)^8/x^7,x, algorithm="fricas")
```

output
```
1/90*(5*b^8*x^24 + 48*a*b^7*x^21 + 210*a^2*b^6*x^18 + 560*a^3*b^5*x^15 + 1
050*a^4*b^4*x^12 + 1680*a^5*b^3*x^9 + 2520*a^6*b^2*x^6*log(x) - 240*a^7*b*
x^3 - 15*a^8)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^7}\,dx = 28a^6b^2\log(x)+\frac{56a^5b^3x^3}{3}+\frac{35a^4b^4x^6}{3}+\frac{56a^3b^5x^9}{9}+\frac{7a^2b^6x^{12}}{3}+\frac{8ab^7x^{15}}{15}+\frac{b^8x^{18}}{18}+\frac{-a^8-16a^7bx^3}{6x^6}$$

input
```
integrate((b*x**3+a)**8/x**7,x)
```

output
```
28*a**6*b**2*log(x) + 56*a**5*b**3*x**3/3 + 35*a**4*b**4*x**6/3 + 56*a**3*
b**5*x**9/9 + 7*a**2*b**6*x**12/3 + 8*a*b**7*x**15/15 + b**8*x**18/18 + (-
a**8 - 16*a**7*b*x**3)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^7}\,dx = \frac{1}{18}\,b^8x^{18} + \frac{8}{15}\,ab^7x^{15} + \frac{7}{3}\,a^2b^6x^{12} + \frac{56}{9}\,a^3b^5x^9 + \frac{35}{3}\,a^4b^4x^6 + \frac{56}{3}\,a^5b^3x^3 + \frac{28}{3}\,a^6b^2\log\left(x^3\right) - \frac{16\,a^7bx^3 + a^8}{6\,x^6}$$

input
```
integrate((b*x^3+a)^8/x^7,x, algorithm="maxima")
```

output
```
1/18*b^8*x^18 + 8/15*a*b^7*x^15 + 7/3*a^2*b^6*x^12 + 56/9*a^3*b^5*x^9 + 35
/3*a^4*b^4*x^6 + 56/3*a^5*b^3*x^3 + 28/3*a^6*b^2*log(x^3) - 1/6*(16*a^7*b*
x^3 + a^8)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^8}{x^7}\,dx = \frac{1}{18}\,b^8x^{18} + \frac{8}{15}\,ab^7x^{15} + \frac{7}{3}\,a^2b^6x^{12} + \frac{56}{9}\,a^3b^5x^9 + \frac{35}{3}\,a^4b^4x^6 + \frac{56}{3}\,a^5b^3x^3 + 28\,a^6b^2\log\left(|x|\right) - \frac{84\,a^6b^2x^6 + 16\,a^7bx^3 + a^8}{6\,x^6}$$

input
```
integrate((b*x^3+a)^8/x^7,x, algorithm="giac")
```

output
```
1/18*b^8*x^18 + 8/15*a*b^7*x^15 + 7/3*a^2*b^6*x^12 + 56/9*a^3*b^5*x^9 + 35
/3*a^4*b^4*x^6 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*log(abs(x)) - 1/6*(84*a^6*b
^2*x^6 + 16*a^7*b*x^3 + a^8)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^7}\,dx = \frac{b^8\,x^{18}}{18} - \frac{\frac{a^8}{6} + \frac{8\,b\,a^7\,x^3}{3}}{x^6} + \frac{8\,a\,b^7\,x^{15}}{15} + \frac{56\,a^5\,b^3\,x^3}{3} + \frac{35\,a^4\,b^4\,x^6}{3} + \frac{56\,a^3\,b^5\,x^9}{9} + \frac{7\,a^2\,b^6\,x^{12}}{3} + 28\,a^6\,b^2\,\ln(x)$$

input
```
int((a + b*x^3)^8/x^7,x)
```

output
```
(b^8*x^18)/18 - (a^8/6 + (8*a^7*b*x^3)/3)/x^6 + (8*a*b^7*x^15)/15 + (56*a^
5*b^3*x^3)/3 + (35*a^4*b^4*x^6)/3 + (56*a^3*b^5*x^9)/9 + (7*a^2*b^6*x^12)/
3 + 28*a^6*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^7}\,dx = \frac{2520\log(x)\,a^6b^2x^6 - 15a^8 - 240a^7b\,x^3 + 1680a^5b^3x^9 + 1050a^4b^4x^{12} + 560a^3b^5x^{15} + 210a^2b^6x^{18} + 48a\,b^7}{90x^6}$$

input
```
int((b*x^3+a)^8/x^7,x)
```

output
```
(2520*log(x)*a**6*b**2*x**6 - 15*a**8 - 240*a**7*b*x**3 + 1680*a**5*b**3*x
**9 + 1050*a**4*b**4*x**12 + 560*a**3*b**5*x**15 + 210*a**2*b**6*x**18 + 4
8*a*b**7*x**21 + 5*b**8*x**24)/(90*x**6)
```

3.90 $\int \frac{(a+bx^3)^8}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = -\frac{a^8}{9x^9} - \frac{4a^7b}{3x^6} - \frac{28a^6b^2}{3x^3} + \frac{70}{3}a^4b^4x^3 + \frac{28}{3}a^3b^5x^6 + \frac{28}{9}a^2b^6x^9 + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{15}}{15} + 56a^5b^3\log(x)$$

output
```
-1/9*a^8/x^9-4/3*a^7*b/x^6-28/3*a^6*b^2/x^3+70/3*a^4*b^4*x^3+28/3*a^3*b^5*
x^6+28/9*a^2*b^6*x^9+2/3*a*b^7*x^12+1/15*b^8*x^15+56*a^5*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = -\frac{a^8}{9x^9} - \frac{4a^7b}{3x^6} - \frac{28a^6b^2}{3x^3} + \frac{70}{3}a^4b^4x^3 + \frac{28}{3}a^3b^5x^6 + \frac{28}{9}a^2b^6x^9 + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{15}}{15} + 56a^5b^3\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^10,x]
```

output
```
-1/9*a^8/x^9 - (4*a^7*b)/(3*x^6) - (28*a^6*b^2)/(3*x^3) + (70*a^4*b^4*x^3)
/3 + (28*a^3*b^5*x^6)/3 + (28*a^2*b^6*x^9)/9 + (2*a*b^7*x^12)/3 + (b^8*x^1
5)/15 + 56*a^5*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{12}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^8x^{12}+8ab^7x^9+28a^2b^6x^6+56a^3b^5x^3+70a^4b^4+\frac{56a^5b^3}{x^3}+\frac{28a^6b^2}{x^6}+\frac{8a^7b}{x^9}+\frac{a^8}{x^{12}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^8}{3x^9}-\frac{4a^7b}{x^6}-\frac{28a^6b^2}{x^3}+56a^5b^3\log\left(x^3\right)+70a^4b^4x^3+28a^3b^5x^6+\frac{28}{3}a^2b^6x^9+2ab^7x^{12}+\frac{b^8x^{15}}{5}\right)$$

input
```
Int[(a + b*x^3)^8/x^10,x]
```

output
```
(-1/3*a^8/x^9 - (4*a^7*b)/x^6 - (28*a^6*b^2)/x^3 + 70*a^4*b^4*x^3 + 28*a^3
*b^5*x^6 + (28*a^2*b^6*x^9)/3 + 2*a*b^7*x^12 + (b^8*x^15)/5 + 56*a^5*b^3*L
og[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^8}{9x^9}-\frac{4a^7b}{3x^6}-\frac{28a^6b^2}{3x^3}+\frac{70a^4b^4x^3}{3}+\frac{28a^3b^5x^6}{3}+\frac{28a^2b^6x^9}{9}+\frac{2a\,b^7x^{12}}{3}+\frac{b^8x^{15}}{15}+56a^5b^3\ln(x)$	90
norman	$\frac{-\frac{1}{9}a^8+\frac{1}{15}b^8x^{24}+\frac{2}{3}a\,b^7x^{21}+\frac{28}{9}a^2b^6x^{18}+\frac{28}{3}a^3b^5x^{15}+\frac{70}{3}a^4b^4x^{12}-\frac{28}{3}a^6b^2x^6-\frac{4}{3}a^7b\,x^3}{x^9}+56a^5b^3\ln(x)$	92
risch	$\frac{b^8x^{15}}{15}+\frac{2a\,b^7x^{12}}{3}+\frac{28a^2b^6x^9}{9}+\frac{28a^3b^5x^6}{3}+\frac{70a^4b^4x^3}{3}+\frac{-\frac{28}{3}a^6b^2x^6-\frac{4}{3}a^7b\,x^3-\frac{1}{9}a^8}{x^9}+56a^5b^3\ln(x)$	92
parallelrisch	$\frac{3b^8x^{24}+30a\,b^7x^{21}+140a^2b^6x^{18}+420a^3b^5x^{15}+1050a^4b^4x^{12}+2520a^5b^3\ln(x)x^9-420a^6b^2x^6-60a^7b\,x^3-5a^8}{45x^9}$	95

input
```
int((b*x^3+a)^8/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^8/x^9-4/3*a^7*b/x^6-28/3*a^6*b^2/x^3+70/3*a^4*b^4*x^3+28/3*a^3*b^5*
x^6+28/9*a^2*b^6*x^9+2/3*a*b^7*x^12+1/15*b^8*x^15+56*a^5*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = \frac{3\,b^8x^{24}+30\,ab^7x^{21}+140\,a^2b^6x^{18}+420\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+2520\,a^5b^3x^9\log(x)-420\,a^6b^2x^6-60\,a}{45\,x^9}$$

input
```
integrate((b*x^3+a)^8/x^10,x, algorithm="fricas")
```

output
```
1/45*(3*b^8*x^24 + 30*a*b^7*x^21 + 140*a^2*b^6*x^18 + 420*a^3*b^5*x^15 + 1
050*a^4*b^4*x^12 + 2520*a^5*b^3*x^9*log(x) - 420*a^6*b^2*x^6 - 60*a^7*b*x^
3 - 5*a^8)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = 56a^5b^3\log(x)+\frac{70a^4b^4x^3}{3}+\frac{28a^3b^5x^6}{3}+\frac{28a^2b^6x^9}{9}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{15}}{15}+\frac{-a^8-12a^7bx^3-84a^6b^2x^6}{9x^9}$$

input
```
integrate((b*x**3+a)**8/x**10,x)
```

output
```
56*a**5*b**3*log(x) + 70*a**4*b**4*x**3/3 + 28*a**3*b**5*x**6/3 + 28*a**2*
b**6*x**9/9 + 2*a*b**7*x**12/3 + b**8*x**15/15 + (-a**8 - 12*a**7*b*x**3 -
 84*a**6*b**2*x**6)/(9*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = \frac{1}{15}\,b^8x^{15}+\frac{2}{3}\,ab^7x^{12}+\frac{28}{9}\,a^2b^6x^9+\frac{28}{3}\,a^3b^5x^6+\frac{70}{3}\,a^4b^4x^3 + \frac{56}{3}\,a^5b^3\log\left(x^3\right)-\frac{84\,a^6b^2x^6+12\,a^7bx^3+a^8}{9\,x^9}$$

input
```
integrate((b*x^3+a)^8/x^10,x, algorithm="maxima")
```

output
```
1/15*b^8*x^15 + 2/3*a*b^7*x^12 + 28/9*a^2*b^6*x^9 + 28/3*a^3*b^5*x^6 + 70/
3*a^4*b^4*x^3 + 56/3*a^5*b^3*log(x^3) - 1/9*(84*a^6*b^2*x^6 + 12*a^7*b*x^3
 + a^8)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = \frac{1}{15}\,b^8x^{15}+\frac{2}{3}\,ab^7x^{12}+\frac{28}{9}\,a^2b^6x^9+\frac{28}{3}\,a^3b^5x^6+\frac{70}{3}\,a^4b^4x^3 + 56\,a^5b^3\log\left(|x|\right)-\frac{308\,a^5b^3x^9+84\,a^6b^2x^6+12\,a^7bx^3+a^8}{9\,x^9}$$

input
```
integrate((b*x^3+a)^8/x^10,x, algorithm="giac")
```

output
```
1/15*b^8*x^15 + 2/3*a*b^7*x^12 + 28/9*a^2*b^6*x^9 + 28/3*a^3*b^5*x^6 + 70/
3*a^4*b^4*x^3 + 56*a^5*b^3*log(abs(x)) - 1/9*(308*a^5*b^3*x^9 + 84*a^6*b^2
*x^6 + 12*a^7*b*x^3 + a^8)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = \frac{b^8\,x^{15}}{15} - \frac{\frac{a^8}{9}+\frac{4\,a^7\,b\,x^3}{3}+\frac{28\,a^6\,b^2\,x^6}{3}}{x^9} + \frac{2\,a\,b^7\,x^{12}}{3} + \frac{70\,a^4\,b^4\,x^3}{3} + \frac{28\,a^3\,b^5\,x^6}{3} + \frac{28\,a^2\,b^6\,x^9}{9} + 56\,a^5\,b^3\,\ln(x)$$

input
```
int((a + b*x^3)^8/x^10,x)
```

output
```
(b^8*x^15)/15 - (a^8/9 + (4*a^7*b*x^3)/3 + (28*a^6*b^2*x^6)/3)/x^9 + (2*a*
b^7*x^12)/3 + (70*a^4*b^4*x^3)/3 + (28*a^3*b^5*x^6)/3 + (28*a^2*b^6*x^9)/9
 + 56*a^5*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{10}}\,dx = \frac{2520\log(x)\,a^5b^3x^9 - 5a^8 - 60a^7b\,x^3 - 420a^6b^2x^6 + 1050a^4b^4x^{12} + 420a^3b^5x^{15} + 140a^2b^6x^{18} + 30a\,b^7x^{21}}{45x^9}$$

input
```
int((b*x^3+a)^8/x^10,x)
```

output
```
(2520*log(x)*a**5*b**3*x**9 - 5*a**8 - 60*a**7*b*x**3 - 420*a**6*b**2*x**6
 + 1050*a**4*b**4*x**12 + 420*a**3*b**5*x**15 + 140*a**2*b**6*x**18 + 30*a
*b**7*x**21 + 3*b**8*x**24)/(45*x**9)
```

3.91 $\int \frac{(a+bx^3)^8}{x^{13}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = -\frac{a^8}{12x^{12}} - \frac{8a^7b}{9x^9} - \frac{14a^6b^2}{3x^6} - \frac{56a^5b^3}{3x^3} + \frac{56}{3}a^3b^5x^3 + \frac{14}{3}a^2b^6x^6 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{12}}{12} + 70a^4b^4\log(x)$$

output
```
-1/12*a^8/x^12-8/9*a^7*b/x^9-14/3*a^6*b^2/x^6-56/3*a^5*b^3/x^3+56/3*a^3*b^
5*x^3+14/3*a^2*b^6*x^6+8/9*a*b^7*x^9+1/12*b^8*x^12+70*a^4*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = -\frac{a^8}{12x^{12}} - \frac{8a^7b}{9x^9} - \frac{14a^6b^2}{3x^6} - \frac{56a^5b^3}{3x^3} + \frac{56}{3}a^3b^5x^3 + \frac{14}{3}a^2b^6x^6 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{12}}{12} + 70a^4b^4\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^13,x]
```

output
```
-1/12*a^8/x^12 - (8*a^7*b)/(9*x^9) - (14*a^6*b^2)/(3*x^6) - (56*a^5*b^3)/(
3*x^3) + (56*a^3*b^5*x^3)/3 + (14*a^2*b^6*x^6)/3 + (8*a*b^7*x^9)/9 + (b^8*
x^12)/12 + 70*a^4*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{15}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(b^8x^9+8ab^7x^6+28a^2b^6x^3+56a^3b^5+\frac{70a^4b^4}{x^3}+\frac{56a^5b^3}{x^6}+\frac{28a^6b^2}{x^9}+\frac{8a^7b}{x^{12}}+\frac{a^8}{x^{15}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^8}{4x^{12}}-\frac{8a^7b}{3x^9}-\frac{14a^6b^2}{x^6}-\frac{56a^5b^3}{x^3}+70a^4b^4\log\left(x^3\right)+56a^3b^5x^3+14a^2b^6x^6+\frac{8}{3}ab^7x^9+\frac{b^8x^{12}}{4}\right)$$

input
```
Int[(a + b*x^3)^8/x^13,x]
```

output
```
(-1/4*a^8/x^12 - (8*a^7*b)/(3*x^9) - (14*a^6*b^2)/x^6 - (56*a^5*b^3)/x^3 +
 56*a^3*b^5*x^3 + 14*a^2*b^6*x^6 + (8*a*b^7*x^9)/3 + (b^8*x^12)/4 + 70*a^4
*b^4*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{8}}{12x^{12}}-\frac{8a^{7}b}{9x^{9}}-\frac{14a^{6}b^{2}}{3x^{6}}-\frac{56a^{5}b^{3}}{3x^{3}}+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{14a^{2}x^{6}b^{6}}{3}+\frac{8a\,b^{7}x^{9}}{9}+\frac{b^{8}x^{12}}{12}+70a^{4}b^{4}\ln\left(x\right)$	90
norman	$\frac{-\frac{1}{12}a^{8}+\frac{1}{12}b^{8}x^{24}+\frac{8}{9}a\,b^{7}x^{21}+\frac{14}{3}a^{2}b^{6}x^{18}+\frac{56}{3}a^{3}b^{5}x^{15}-\frac{56}{3}a^{5}b^{3}x^{9}-\frac{14}{3}a^{6}b^{2}x^{6}-\frac{8}{9}a^{7}b\,x^{3}}{x^{12}}+70a^{4}b^{4}\ln\left(x\right)$	92
risch	$\frac{b^{8}x^{12}}{12}+\frac{8a\,b^{7}x^{9}}{9}+\frac{14a^{2}x^{6}b^{6}}{3}+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{-\frac{56}{3}a^{5}b^{3}x^{9}-\frac{14}{3}a^{6}b^{2}x^{6}-\frac{8}{9}a^{7}b\,x^{3}-\frac{1}{12}a^{8}}{x^{12}}+70a^{4}b^{4}\ln\left(x\right)$	92
parallelrisch	$\frac{3b^{8}x^{24}+32a\,b^{7}x^{21}+168a^{2}b^{6}x^{18}+672a^{3}b^{5}x^{15}+2520a^{4}b^{4}\ln\left(x\right)x^{12}-672a^{5}b^{3}x^{9}-168a^{6}b^{2}x^{6}-32a^{7}b\,x^{3}-3a^{8}}{36x^{12}}$	95

input
```
int((b*x^3+a)^8/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^8/x^12-8/9*a^7*b/x^9-14/3*a^6*b^2/x^6-56/3*a^5*b^3/x^3+56/3*a^3*b^
5*x^3+14/3*a^2*x^6*b^6+8/9*a*b^7*x^9+1/12*b^8*x^12+70*a^4*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx$$
$$= \frac{3\,b^8x^{24}+32\,ab^7x^{21}+168\,a^2b^6x^{18}+672\,a^3b^5x^{15}+2520\,a^4b^4x^{12}\log(x)-672\,a^5b^3x^9-168\,a^6b^2x^6-32\,a^7}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^8/x^13,x, algorithm="fricas")
```

output
```
1/36*(3*b^8*x^24 + 32*a*b^7*x^21 + 168*a^2*b^6*x^18 + 672*a^3*b^5*x^15 + 2
520*a^4*b^4*x^12*log(x) - 672*a^5*b^3*x^9 - 168*a^6*b^2*x^6 - 32*a^7*b*x^3
 - 3*a^8)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = 70a^4b^4\log(x)+\frac{56a^3b^5x^3}{3}+\frac{14a^2b^6x^6}{3}+\frac{8ab^7x^9}{9}+\frac{b^8x^{12}}{12}+\frac{-3a^8-32a^7bx^3-168a^6b^2x^6-672a^5b^3x^9}{36x^{12}}$$

input
```
integrate((b*x**3+a)**8/x**13,x)
```

output
```
70*a**4*b**4*log(x) + 56*a**3*b**5*x**3/3 + 14*a**2*b**6*x**6/3 + 8*a*b**7
*x**9/9 + b**8*x**12/12 + (-3*a**8 - 32*a**7*b*x**3 - 168*a**6*b**2*x**6 -
 672*a**5*b**3*x**9)/(36*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = \frac{1}{12}\,b^8x^{12} + \frac{8}{9}\,ab^7x^9 + \frac{14}{3}\,a^2b^6x^6 + \frac{56}{3}\,a^3b^5x^3 + \frac{70}{3}\,a^4b^4\log\left(x^3\right) - \frac{672\,a^5b^3x^9 + 168\,a^6b^2x^6 + 32\,a^7bx^3 + 3\,a^8}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^8/x^13,x, algorithm="maxima")
```

output
```
1/12*b^8*x^12 + 8/9*a*b^7*x^9 + 14/3*a^2*b^6*x^6 + 56/3*a^3*b^5*x^3 + 70/3
*a^4*b^4*log(x^3) - 1/36*(672*a^5*b^3*x^9 + 168*a^6*b^2*x^6 + 32*a^7*b*x^3
 + 3*a^8)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = \frac{1}{12}\,b^8x^{12} + \frac{8}{9}\,ab^7x^9 + \frac{14}{3}\,a^2b^6x^6 + \frac{56}{3}\,a^3b^5x^3 + 70\,a^4b^4\log\left(|x|\right) - \frac{1750\,a^4b^4x^{12} + 672\,a^5b^3x^9 + 168\,a^6b^2x^6 + 32\,a^7bx^3 + 3\,a^8}{36\,x^{12}}$$

input
```
integrate((b*x^3+a)^8/x^13,x, algorithm="giac")
```

output
```
1/12*b^8*x^12 + 8/9*a*b^7*x^9 + 14/3*a^2*b^6*x^6 + 56/3*a^3*b^5*x^3 + 70*a
^4*b^4*log(abs(x)) - 1/36*(1750*a^4*b^4*x^12 + 672*a^5*b^3*x^9 + 168*a^6*b
^2*x^6 + 32*a^7*b*x^3 + 3*a^8)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = \frac{b^8\,x^{12}}{12} - \frac{\frac{a^8}{12} + \frac{8\,a^7\,b\,x^3}{9} + \frac{14\,a^6\,b^2\,x^6}{3} + \frac{56\,a^5\,b^3\,x^9}{3}}{x^{12}} + \frac{8\,a\,b^7\,x^9}{9} + \frac{56\,a^3\,b^5\,x^3}{3} + \frac{14\,a^2\,b^6\,x^6}{3} + 70\,a^4\,b^4\,\ln(x)$$

input
```
int((a + b*x^3)^8/x^13,x)
```

output
```
(b^8*x^12)/12 - (a^8/12 + (8*a^7*b*x^3)/9 + (14*a^6*b^2*x^6)/3 + (56*a^5*b
^3*x^9)/3)/x^12 + (8*a*b^7*x^9)/9 + (56*a^3*b^5*x^3)/3 + (14*a^2*b^6*x^6)/
3 + 70*a^4*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{13}}\,dx = \frac{2520\log(x)\,a^4b^4x^{12} - 3a^8 - 32a^7b\,x^3 - 168a^6b^2x^6 - 672a^5b^3x^9 + 672a^3b^5x^{15} + 168a^2b^6x^{18} + 32a\,b^7x^{21} + \cdots}{36x^{12}}$$

input
```
int((b*x^3+a)^8/x^13,x)
```

output
```
(2520*log(x)*a**4*b**4*x**12 - 3*a**8 - 32*a**7*b*x**3 - 168*a**6*b**2*x**
6 - 672*a**5*b**3*x**9 + 672*a**3*b**5*x**15 + 168*a**2*b**6*x**18 + 32*a*
b**7*x**21 + 3*b**8*x**24)/(36*x**12)
```

3.92 $\int \frac{(a+bx^3)^8}{x^{16}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = -\frac{a^8}{15x^{15}} - \frac{2a^7b}{3x^{12}} - \frac{28a^6b^2}{9x^9} - \frac{28a^5b^3}{3x^6} - \frac{70a^4b^4}{3x^3} + \frac{28}{3}a^2b^6x^3 + \frac{4}{3}ab^7x^6 + \frac{b^8x^9}{9} + 56a^3b^5\log(x)$$

output
```
-1/15*a^8/x^15-2/3*a^7*b/x^12-28/9*a^6*b^2/x^9-28/3*a^5*b^3/x^6-70/3*a^4*b
^4/x^3+28/3*a^2*b^6*x^3+4/3*a*b^7*x^6+1/9*b^8*x^9+56*a^3*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = -\frac{a^8}{15x^{15}} - \frac{2a^7b}{3x^{12}} - \frac{28a^6b^2}{9x^9} - \frac{28a^5b^3}{3x^6} - \frac{70a^4b^4}{3x^3} + \frac{28}{3}a^2b^6x^3 + \frac{4}{3}ab^7x^6 + \frac{b^8x^9}{9} + 56a^3b^5\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^16,x]
```

output
```
-1/15*a^8/x^15 - (2*a^7*b)/(3*x^12) - (28*a^6*b^2)/(9*x^9) - (28*a^5*b^3)/
(3*x^6) - (70*a^4*b^4)/(3*x^3) + (28*a^2*b^6*x^3)/3 + (4*a*b^7*x^6)/3 + (b
^8*x^9)/9 + 56*a^3*b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{18}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^8}{x^{18}}+\frac{8ba^7}{x^{15}}+\frac{28b^2a^6}{x^{12}}+\frac{56b^3a^5}{x^9}+\frac{70b^4a^4}{x^6}+\frac{56b^5a^3}{x^3}+28b^6a^2+8b^7x^3a+b^8x^6\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^8}{5x^{15}}-\frac{2a^7b}{x^{12}}-\frac{28a^6b^2}{3x^9}-\frac{28a^5b^3}{x^6}-\frac{70a^4b^4}{x^3}+56a^3b^5\log\left(x^3\right)+28a^2b^6x^3+4ab^7x^6+\frac{b^8x^9}{3}\right)$$

input
```
Int[(a + b*x^3)^8/x^16,x]
```

output
```
(-1/5*a^8/x^15 - (2*a^7*b)/x^12 - (28*a^6*b^2)/(3*x^9) - (28*a^5*b^3)/x^6
- (70*a^4*b^4)/x^3 + 28*a^2*b^6*x^3 + 4*a*b^7*x^6 + (b^8*x^9)/3 + 56*a^3*b
^5*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{8}}{15x^{15}}-\frac{2a^{7}b}{3x^{12}}-\frac{28a^{6}b^{2}}{9x^{9}}-\frac{28a^{5}b^{3}}{3x^{6}}-\frac{70a^{4}b^{4}}{3x^{3}}+\frac{28a^{2}b^{6}x^{3}}{3}+\frac{4a\,b^{7}x^{6}}{3}+\frac{b^{8}x^{9}}{9}+56a^{3}b^{5}\ln\left(x\right)$	90
norman	$\frac{-\frac{1}{15}a^{8}+\frac{1}{9}b^{8}x^{24}+\frac{4}{3}a\,b^{7}x^{21}+\frac{28}{3}a^{2}b^{6}x^{18}-\frac{70}{3}a^{4}b^{4}x^{12}-\frac{28}{3}a^{5}b^{3}x^{9}-\frac{28}{9}a^{6}b^{2}x^{6}-\frac{2}{3}a^{7}b\,x^{3}}{x^{15}}+56a^{3}b^{5}\ln\left(x\right)$	92
risch	$\frac{b^{8}x^{9}}{9}+\frac{4a\,b^{7}x^{6}}{3}+\frac{28a^{2}b^{6}x^{3}}{3}+\frac{-\frac{1}{15}a^{8}-\frac{2}{3}a^{7}b\,x^{3}-\frac{28}{9}a^{6}b^{2}x^{6}-\frac{28}{3}a^{5}b^{3}x^{9}-\frac{70}{3}a^{4}b^{4}x^{12}}{x^{15}}+56a^{3}b^{5}\ln\left(x\right)$	92
parallelrisch	$\frac{5b^{8}x^{24}+60a\,b^{7}x^{21}+420a^{2}b^{6}x^{18}+2520a^{3}b^{5}\ln\left(x\right)x^{15}-1050a^{4}b^{4}x^{12}-420a^{5}b^{3}x^{9}-140a^{6}b^{2}x^{6}-30a^{7}b\,x^{3}-3a^{8}}{45x^{15}}$	95

input
```
int((b*x^3+a)^8/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^8/x^15-2/3*a^7*b/x^12-28/9*a^6*b^2/x^9-28/3*a^5*b^3/x^6-70/3*a^4*b
^4/x^3+28/3*a^2*b^6*x^3+4/3*a*b^7*x^6+1/9*b^8*x^9+56*a^3*b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = \frac{5\,b^8x^{24}+60\,ab^7x^{21}+420\,a^2b^6x^{18}+2520\,a^3b^5x^{15}\log(x)-1050\,a^4b^4x^{12}-420\,a^5b^3x^9-140\,a^6b^2x^6-30\,a^7bx^3-3\,a^8}{45\,x^{15}}$$

input
```
integrate((b*x^3+a)^8/x^16,x, algorithm="fricas")
```

output
```
1/45*(5*b^8*x^24 + 60*a*b^7*x^21 + 420*a^2*b^6*x^18 + 2520*a^3*b^5*x^15*lo
g(x) - 1050*a^4*b^4*x^12 - 420*a^5*b^3*x^9 - 140*a^6*b^2*x^6 - 30*a^7*b*x^
3 - 3*a^8)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = 56a^3b^5\log(x) + \frac{28a^2b^6x^3}{3} + \frac{4ab^7x^6}{3} + \frac{b^8x^9}{9} + \frac{-3a^8-30a^7bx^3-140a^6b^2x^6-420a^5b^3x^9-1050a^4b^4x^{12}}{45x^{15}}$$

input
```
integrate((b*x**3+a)**8/x**16,x)
```

output
```
56*a**3*b**5*log(x) + 28*a**2*b**6*x**3/3 + 4*a*b**7*x**6/3 + b**8*x**9/9
+ (-3*a**8 - 30*a**7*b*x**3 - 140*a**6*b**2*x**6 - 420*a**5*b**3*x**9 - 10
50*a**4*b**4*x**12)/(45*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = \frac{1}{9}\,b^8x^9 + \frac{4}{3}\,ab^7x^6 + \frac{28}{3}\,a^2b^6x^3 + \frac{56}{3}\,a^3b^5\log\left(x^3\right) - \frac{1050\,a^4b^4x^{12} + 420\,a^5b^3x^9 + 140\,a^6b^2x^6 + 30\,a^7bx^3 + 3\,a^8}{45\,x^{15}}$$

input
```
integrate((b*x^3+a)^8/x^16,x, algorithm="maxima")
```

output
```
1/9*b^8*x^9 + 4/3*a*b^7*x^6 + 28/3*a^2*b^6*x^3 + 56/3*a^3*b^5*log(x^3) - 1
/45*(1050*a^4*b^4*x^12 + 420*a^5*b^3*x^9 + 140*a^6*b^2*x^6 + 30*a^7*b*x^3
+ 3*a^8)/x^15
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = \frac{1}{9}\,b^8x^9 + \frac{4}{3}\,ab^7x^6 + \frac{28}{3}\,a^2b^6x^3 + 56\,a^3b^5\log\left(|x|\right) - \frac{1918\,a^3b^5x^{15} + 1050\,a^4b^4x^{12} + 420\,a^5b^3x^9 + 140\,a^6b^2x^6 + 30\,a^7bx^3 + 3\,a^8}{45\,x^{15}}$$

input
```
integrate((b*x^3+a)^8/x^16,x, algorithm="giac")
```

output
```
1/9*b^8*x^9 + 4/3*a*b^7*x^6 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*log(abs(x)) -
1/45*(1918*a^3*b^5*x^15 + 1050*a^4*b^4*x^12 + 420*a^5*b^3*x^9 + 140*a^6*b^
2*x^6 + 30*a^7*b*x^3 + 3*a^8)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = \frac{b^8\,x^9}{9} - \frac{\frac{a^8}{15} + \frac{2\,a^7\,b\,x^3}{3} + \frac{28\,a^6\,b^2\,x^6}{9} + \frac{28\,a^5\,b^3\,x^9}{3} + \frac{70\,a^4\,b^4\,x^{12}}{3}}{x^{15}} + \frac{4\,a\,b^7\,x^6}{3} + \frac{28\,a^2\,b^6\,x^3}{3} + 56\,a^3\,b^5\,\ln(x)$$

input
```
int((a + b*x^3)^8/x^16,x)
```

output
```
(b^8*x^9)/9 - (a^8/15 + (2*a^7*b*x^3)/3 + (28*a^6*b^2*x^6)/9 + (28*a^5*b^3
*x^9)/3 + (70*a^4*b^4*x^12)/3)/x^15 + (4*a*b^7*x^6)/3 + (28*a^2*b^6*x^3)/3
 + 56*a^3*b^5*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{16}}\,dx = \frac{2520\log(x)\,a^3b^5x^{15} - 3a^8 - 30a^7b\,x^3 - 140a^6b^2x^6 - 420a^5b^3x^9 - 1050a^4b^4x^{12} + 420a^2b^6x^{18} + 60a\,b^7x^{21}}{45x^{15}}$$

input
```
int((b*x^3+a)^8/x^16,x)
```

output
```
(2520*log(x)*a**3*b**5*x**15 - 3*a**8 - 30*a**7*b*x**3 - 140*a**6*b**2*x**
6 - 420*a**5*b**3*x**9 - 1050*a**4*b**4*x**12 + 420*a**2*b**6*x**18 + 60*a
*b**7*x**21 + 5*b**8*x**24)/(45*x**15)
```

3.93 $\int \frac{(a+bx^3)^8}{x^{19}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = -\frac{a^8}{18x^{18}} - \frac{8a^7b}{15x^{15}} - \frac{7a^6b^2}{3x^{12}} - \frac{56a^5b^3}{9x^9} - \frac{35a^4b^4}{3x^6} - \frac{56a^3b^5}{3x^3} + \frac{8}{3}ab^7x^3 + \frac{b^8x^6}{6} + 28a^2b^6\log(x)$$

output
```
-1/18*a^8/x^18-8/15*a^7*b/x^15-7/3*a^6*b^2/x^12-56/9*a^5*b^3/x^9-35/3*a^4*
b^4/x^6-56/3*a^3*b^5/x^3+8/3*a*b^7*x^3+1/6*b^8*x^6+28*a^2*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = -\frac{a^8}{18x^{18}} - \frac{8a^7b}{15x^{15}} - \frac{7a^6b^2}{3x^{12}} - \frac{56a^5b^3}{9x^9} - \frac{35a^4b^4}{3x^6} - \frac{56a^3b^5}{3x^3} + \frac{8}{3}ab^7x^3 + \frac{b^8x^6}{6} + 28a^2b^6\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^19,x]
```

output
```
-1/18*a^8/x^18 - (8*a^7*b)/(15*x^15) - (7*a^6*b^2)/(3*x^12) - (56*a^5*b^3)
/(9*x^9) - (35*a^4*b^4)/(3*x^6) - (56*a^3*b^5)/(3*x^3) + (8*a*b^7*x^3)/3 +
 (b^8*x^6)/6 + 28*a^2*b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{21}}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(\frac{a^8}{x^{21}}+\frac{8ba^7}{x^{18}}+\frac{28b^2a^6}{x^{15}}+\frac{56b^3a^5}{x^{12}}+\frac{70b^4a^4}{x^9}+\frac{56b^5a^3}{x^6}+\frac{28b^6a^2}{x^3}+8b^7a+b^8x^3\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^8}{6x^{18}}-\frac{8a^7b}{5x^{15}}-\frac{7a^6b^2}{x^{12}}-\frac{56a^5b^3}{3x^9}-\frac{35a^4b^4}{x^6}-\frac{56a^3b^5}{x^3}+28a^2b^6\log\left(x^3\right)+8ab^7x^3+\frac{b^8x^6}{2}\right)$$

input
```
Int[(a + b*x^3)^8/x^19,x]
```

output
```
(-1/6*a^8/x^18 - (8*a^7*b)/(5*x^15) - (7*a^6*b^2)/x^12 - (56*a^5*b^3)/(3*x
^9) - (35*a^4*b^4)/x^6 - (56*a^3*b^5)/x^3 + 8*a*b^7*x^3 + (b^8*x^6)/2 + 28
*a^2*b^6*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	s
default	$-\frac{a^{8}}{18 x^{18}}-\frac{8 a^{7} b}{15 x^{15}}-\frac{7 a^{6} b^{2}}{3 x^{12}}-\frac{56 a^{5} b^{3}}{9 x^{9}}-\frac{35 a^{4} b^{4}}{3 x^{6}}-\frac{56 a^{3} b^{5}}{3 x^{3}}+\frac{8 a\,b^{7} x^{3}}{3}+\frac{b^{8} x^{6}}{6}+28 a^{2} b^{6} \ln \left(x \right)$	9
norman	$\frac{-\frac{1}{18} a^{8}+\frac{1}{6} b^{8} x^{24}+\frac{8}{3} a\,b^{7} x^{21}-\frac{56}{3} a^{3} b^{5} x^{15}-\frac{35}{3} a^{4} b^{4} x^{12}-\frac{56}{9} a^{5} b^{3} x^{9}-\frac{7}{3} a^{6} b^{2} x^{6}-\frac{8}{15} a^{7} b\,x^{3}}{x^{18}}+28 a^{2} b^{6} \ln \left(x \right)$	9
parallelrisch	$\frac{15 b^{8} x^{24}+240 a\,b^{7} x^{21}+2520 a^{2} b^{6} \ln \left(x \right) x^{18}-1680 a^{3} b^{5} x^{15}-1050 a^{4} b^{4} x^{12}-560 a^{5} b^{3} x^{9}-210 a^{6} b^{2} x^{6}-48 a^{7} b\,x^{3}-5 a^{8}}{90 x^{18}}$	9
risch	$\frac{b^{8} x^{6}}{6}+\frac{8 a\,b^{7} x^{3}}{3}+\frac{32 a^{2} b^{6}}{3}+\frac{-\frac{1}{18} a^{8}-\frac{8}{15} a^{7} b\,x^{3}-\frac{7}{3} a^{6} b^{2} x^{6}-\frac{56}{9} a^{5} b^{3} x^{9}-\frac{35}{3} a^{4} b^{4} x^{12}-\frac{56}{3} a^{3} b^{5} x^{15}}{x^{18}}+28 a^{2} b^{6} \ln \left(x \right)$	1

input
```
int((b*x^3+a)^8/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*a^8/x^18-8/15*a^7*b/x^15-7/3*a^6*b^2/x^12-56/9*a^5*b^3/x^9-35/3*a^4*
b^4/x^6-56/3*a^3*b^5/x^3+8/3*a*b^7*x^3+1/6*b^8*x^6+28*a^2*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = \frac{15\,b^8x^{24}+240\,ab^7x^{21}+2520\,a^2b^6x^{18}\log(x)-1680\,a^3b^5x^{15}-1050\,a^4b^4x^{12}-560\,a^5b^3x^9-210\,a^6b^2x^6-}{90\,x^{18}}$$

input
```
integrate((b*x^3+a)^8/x^19,x, algorithm="fricas")
```

output
```
1/90*(15*b^8*x^24 + 240*a*b^7*x^21 + 2520*a^2*b^6*x^18*log(x) - 1680*a^3*b
^5*x^15 - 1050*a^4*b^4*x^12 - 560*a^5*b^3*x^9 - 210*a^6*b^2*x^6 - 48*a^7*b
*x^3 - 5*a^8)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = 28a^2b^6\log(x)+\frac{8ab^7x^3}{3}+\frac{b^8x^6}{6} + \frac{-5a^8-48a^7bx^3-210a^6b^2x^6-560a^5b^3x^9-1050a^4b^4x^{12}-1680a^3b^5x^{15}}{90x^{18}}$$

input
```
integrate((b*x**3+a)**8/x**19,x)
```

output
```
28*a**2*b**6*log(x) + 8*a*b**7*x**3/3 + b**8*x**6/6 + (-5*a**8 - 48*a**7*b
*x**3 - 210*a**6*b**2*x**6 - 560*a**5*b**3*x**9 - 1050*a**4*b**4*x**12 - 1
680*a**3*b**5*x**15)/(90*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = \frac{1}{6}\,b^8x^6 + \frac{8}{3}\,ab^7x^3 + \frac{28}{3}\,a^2b^6\log\left(x^3\right) - \frac{1680\,a^3b^5x^{15} + 1050\,a^4b^4x^{12} + 560\,a^5b^3x^9 + 210\,a^6b^2x^6 + 48\,a^7bx^3 + 5\,a^8}{90\,x^{18}}$$

input
```
integrate((b*x^3+a)^8/x^19,x, algorithm="maxima")
```

output
```
1/6*b^8*x^6 + 8/3*a*b^7*x^3 + 28/3*a^2*b^6*log(x^3) - 1/90*(1680*a^3*b^5*x
^15 + 1050*a^4*b^4*x^12 + 560*a^5*b^3*x^9 + 210*a^6*b^2*x^6 + 48*a^7*b*x^3
 + 5*a^8)/x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = \frac{1}{6}\,b^8x^6 + \frac{8}{3}\,ab^7x^3 + 28\,a^2b^6\log\left(|x|\right) - \frac{2058\,a^2b^6x^{18} + 1680\,a^3b^5x^{15} + 1050\,a^4b^4x^{12} + 560\,a^5b^3x^9 + 210\,a^6b^2x^6 + 48\,a^7bx^3 + 5\,a^8}{90\,x^{18}}$$

input
```
integrate((b*x^3+a)^8/x^19,x, algorithm="giac")
```

output
```
1/6*b^8*x^6 + 8/3*a*b^7*x^3 + 28*a^2*b^6*log(abs(x)) - 1/90*(2058*a^2*b^6*
x^18 + 1680*a^3*b^5*x^15 + 1050*a^4*b^4*x^12 + 560*a^5*b^3*x^9 + 210*a^6*b
^2*x^6 + 48*a^7*b*x^3 + 5*a^8)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = \frac{b^8\,x^6}{6} - \frac{\frac{a^8}{18} + \frac{8\,a^7\,b\,x^3}{15} + \frac{7\,a^6\,b^2\,x^6}{3} + \frac{56\,a^5\,b^3\,x^9}{9} + \frac{35\,a^4\,b^4\,x^{12}}{3} + \frac{56\,a^3\,b^5\,x^{15}}{3}}{x^{18}} + \frac{8\,a\,b^7\,x^3}{3} + 28\,a^2\,b^6\,\ln(x)$$

input
```
int((a + b*x^3)^8/x^19,x)
```

output
```
(b^8*x^6)/6 - (a^8/18 + (8*a^7*b*x^3)/15 + (7*a^6*b^2*x^6)/3 + (56*a^5*b^3
*x^9)/9 + (35*a^4*b^4*x^12)/3 + (56*a^3*b^5*x^15)/3)/x^18 + (8*a*b^7*x^3)/
3 + 28*a^2*b^6*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{19}}\,dx = \frac{2520\log(x)\,a^2b^6x^{18} - 5a^8 - 48a^7b\,x^3 - 210a^6b^2x^6 - 560a^5b^3x^9 - 1050a^4b^4x^{12} - 1680a^3b^5x^{15} + 240a\,b^7x}{90x^{18}}$$

input
```
int((b*x^3+a)^8/x^19,x)
```

output
```
(2520*log(x)*a**2*b**6*x**18 - 5*a**8 - 48*a**7*b*x**3 - 210*a**6*b**2*x**
6 - 560*a**5*b**3*x**9 - 1050*a**4*b**4*x**12 - 1680*a**3*b**5*x**15 + 240
*a*b**7*x**21 + 15*b**8*x**24)/(90*x**18)
```

3.94 $\int \frac{(a+bx^3)^8}{x^{22}} dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int \frac{(a+bx^3)^8}{x^{22}} dx = -\frac{a^8}{21x^{21}} - \frac{4a^7b}{9x^{18}} - \frac{28a^6b^2}{15x^{15}} - \frac{14a^5b^3}{3x^{12}} - \frac{70a^4b^4}{9x^9} - \frac{28a^3b^5}{3x^6} - \frac{28a^2b^6}{3x^3} + \frac{b^8x^3}{3} + 8ab^7\log(x)$$

output
```
-1/21*a^8/x^21-4/9*a^7*b/x^18-28/15*a^6*b^2/x^15-14/3*a^5*b^3/x^12-70/9*a^
4*b^4/x^9-28/3*a^3*b^5/x^6-28/3*a^2*b^6/x^3+1/3*b^8*x^3+8*a*b^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{22}} dx = -\frac{a^8}{21x^{21}} - \frac{4a^7b}{9x^{18}} - \frac{28a^6b^2}{15x^{15}} - \frac{14a^5b^3}{3x^{12}} - \frac{70a^4b^4}{9x^9} - \frac{28a^3b^5}{3x^6} - \frac{28a^2b^6}{3x^3} + \frac{b^8x^3}{3} + 8ab^7\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^22,x]
```

output
```
-1/21*a^8/x^21 - (4*a^7*b)/(9*x^18) - (28*a^6*b^2)/(15*x^15) - (14*a^5*b^3
)/(3*x^12) - (70*a^4*b^4)/(9*x^9) - (28*a^3*b^5)/(3*x^6) - (28*a^2*b^6)/(3
*x^3) + (b^8*x^3)/3 + 8*a*b^7*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{24}}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^8}{x^{24}}+\frac{8ba^7}{x^{21}}+\frac{28b^2a^6}{x^{18}}+\frac{56b^3a^5}{x^{15}}+\frac{70b^4a^4}{x^{12}}+\frac{56b^5a^3}{x^9}+\frac{28b^6a^2}{x^6}+\frac{8b^7a}{x^3}+b^8\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^8}{7x^{21}}-\frac{4a^7b}{3x^{18}}-\frac{28a^6b^2}{5x^{15}}-\frac{14a^5b^3}{x^{12}}-\frac{70a^4b^4}{3x^9}-\frac{28a^3b^5}{x^6}-\frac{28a^2b^6}{x^3}+8ab^7\log\left(x^3\right)+b^8x^3\right)$$

input
```
Int[(a + b*x^3)^8/x^22,x]
```

output
```
(-1/7*a^8/x^21 - (4*a^7*b)/(3*x^18) - (28*a^6*b^2)/(5*x^15) - (14*a^5*b^3)
/x^12 - (70*a^4*b^4)/(3*x^9) - (28*a^3*b^5)/x^6 - (28*a^2*b^6)/x^3 + b^8*x
^3 + 8*a*b^7*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	siz
default	$-\frac{a^{8}}{21 x^{21}}-\frac{4 a^{7} b}{9 x^{18}}-\frac{28 a^{6} b^{2}}{15 x^{15}}-\frac{14 a^{5} b^{3}}{3 x^{12}}-\frac{70 a^{4} b^{4}}{9 x^{9}}-\frac{28 a^{3} b^{5}}{3 x^{6}}-\frac{28 a^{2} b^{6}}{3 x^{3}}+\frac{b^{8} x^{3}}{3}+8 a\, b^{7} \ln \left(x \right)$	90
norman	$\frac{-\frac{1}{21} a^{8}+\frac{1}{3} b^{8} x^{24}-\frac{28}{3} a^{2} b^{6} x^{18}-\frac{28}{3} a^{3} b^{5} x^{15}-\frac{70}{9} a^{4} b^{4} x^{12}-\frac{14}{3} a^{5} b^{3} x^{9}-\frac{28}{15} a^{6} b^{2} x^{6}-\frac{4}{9} a^{7} b\, x^{3}}{x^{21}}+8 a\, b^{7} \ln \left(x \right)$	92
risch	$\frac{b^{8} x^{3}}{3}+\frac{-\frac{1}{21} a^{8}-\frac{4}{9} a^{7} b\, x^{3}-\frac{28}{15} a^{6} b^{2} x^{6}-\frac{14}{3} a^{5} b^{3} x^{9}-\frac{70}{9} a^{4} b^{4} x^{12}-\frac{28}{3} a^{3} b^{5} x^{15}-\frac{28}{3} a^{2} b^{6} x^{18}}{x^{21}}+8 a\, b^{7} \ln \left(x \right)$	92
parallelrisch	$\frac{105 b^{8} x^{24}+2520 a\, b^{7} \ln \left(x \right) x^{21}-2940 a^{2} b^{6} x^{18}-2940 a^{3} b^{5} x^{15}-2450 a^{4} b^{4} x^{12}-1470 a^{5} b^{3} x^{9}-588 a^{6} b^{2} x^{6}-140 a^{7} b\, x^{3}-15 a^{8}}{315 x^{21}}$	95

input
```
int((b*x^3+a)^8/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/21*a^8/x^21-4/9*a^7*b/x^18-28/15*a^6*b^2/x^15-14/3*a^5*b^3/x^12-70/9*a^
4*b^4/x^9-28/3*a^3*b^5/x^6-28/3*a^2*b^6/x^3+1/3*b^8*x^3+8*a*b^7*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx = \frac{105\,b^8x^{24}+2520\,ab^7x^{21}\log(x)-2940\,a^2b^6x^{18}-2940\,a^3b^5x^{15}-2450\,a^4b^4x^{12}-1470\,a^5b^3x^9-588\,a^6b^2x}{315\,x^{21}}$$

input
```
integrate((b*x^3+a)^8/x^22,x, algorithm="fricas")
```

output
```
1/315*(105*b^8*x^24 + 2520*a*b^7*x^21*log(x) - 2940*a^2*b^6*x^18 - 2940*a^
3*b^5*x^15 - 2450*a^4*b^4*x^12 - 1470*a^5*b^3*x^9 - 588*a^6*b^2*x^6 - 140*
a^7*b*x^3 - 15*a^8)/x^21
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx = 8ab^7\log(x)+\frac{b^8x^3}{3} + \frac{-15a^8-140a^7bx^3-588a^6b^2x^6-1470a^5b^3x^9-2450a^4b^4x^{12}-2940a^3b^5x^{15}-2940a^2b^6x^{18}}{315x^{21}}$$

input
```
integrate((b*x**3+a)**8/x**22,x)
```

output
```
8*a*b**7*log(x) + b**8*x**3/3 + (-15*a**8 - 140*a**7*b*x**3 - 588*a**6*b**
2*x**6 - 1470*a**5*b**3*x**9 - 2450*a**4*b**4*x**12 - 2940*a**3*b**5*x**15
 - 2940*a**2*b**6*x**18)/(315*x**21)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx = \frac{1}{3}\,b^8x^3 + \frac{8}{3}\,ab^7\log\left(x^3\right) - \frac{2940\,a^2b^6x^{18} + 2940\,a^3b^5x^{15} + 2450\,a^4b^4x^{12} + 1470\,a^5b^3x^9 + 588\,a^6b^2x^6 + 140\,a^7bx^3 + 15\,a^8}{315\,x^{21}}$$

input
```
integrate((b*x^3+a)^8/x^22,x, algorithm="maxima")
```

output
```
1/3*b^8*x^3 + 8/3*a*b^7*log(x^3) - 1/315*(2940*a^2*b^6*x^18 + 2940*a^3*b^5
*x^15 + 2450*a^4*b^4*x^12 + 1470*a^5*b^3*x^9 + 588*a^6*b^2*x^6 + 140*a^7*b
*x^3 + 15*a^8)/x^21
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx = \frac{1}{3}\,b^8x^3 + 8\,ab^7\log\left(|x|\right) - \frac{2178\,ab^7x^{21} + 2940\,a^2b^6x^{18} + 2940\,a^3b^5x^{15} + 2450\,a^4b^4x^{12} + 1470\,a^5b^3x^9 + 588\,a^6b^2x^6 + 140\,a^7bx^3 + 1}{315\,x^{21}}$$

input
```
integrate((b*x^3+a)^8/x^22,x, algorithm="giac")
```

output
```
1/3*b^8*x^3 + 8*a*b^7*log(abs(x)) - 1/315*(2178*a*b^7*x^21 + 2940*a^2*b^6*
x^18 + 2940*a^3*b^5*x^15 + 2450*a^4*b^4*x^12 + 1470*a^5*b^3*x^9 + 588*a^6*
b^2*x^6 + 140*a^7*b*x^3 + 15*a^8)/x^21
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx = -\frac{15\,a^8-105\,b^8\,x^{24}+140\,a^7\,b\,x^3+588\,a^6\,b^2\,x^6+1470\,a^5\,b^3\,x^9+2450\,a^4\,b^4\,x^{12}+2940\,a^3\,b^5\,x^{15}+2940\,a}{315\,x^{21}}$$

input
```
int((a + b*x^3)^8/x^22,x)
```

output
```
-(15*a^8 - 105*b^8*x^24 + 140*a^7*b*x^3 + 588*a^6*b^2*x^6 + 1470*a^5*b^3*x
^9 + 2450*a^4*b^4*x^12 + 2940*a^3*b^5*x^15 + 2940*a^2*b^6*x^18 - 2520*a*b^
7*x^21*log(x))/(315*x^21)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{22}}\,dx = \frac{2520\log(x)\,a\,b^7x^{21}-15a^8-140a^7b\,x^3-588a^6b^2x^6-1470a^5b^3x^9-2450a^4b^4x^{12}-2940a^3b^5x^{15}-2940a}{315x^{21}}$$

input
```
int((b*x^3+a)^8/x^22,x)
```

output
```
(2520*log(x)*a*b**7*x**21 - 15*a**8 - 140*a**7*b*x**3 - 588*a**6*b**2*x**6
 - 1470*a**5*b**3*x**9 - 2450*a**4*b**4*x**12 - 2940*a**3*b**5*x**15 - 294
0*a**2*b**6*x**18 + 105*b**8*x**24)/(315*x**21)
```

3.95 $\int \frac{(a+bx^3)^8}{x^{25}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = -\frac{a^8}{24x^{24}} - \frac{8a^7b}{21x^{21}} - \frac{14a^6b^2}{9x^{18}} - \frac{56a^5b^3}{15x^{15}} - \frac{35a^4b^4}{6x^{12}} - \frac{56a^3b^5}{9x^9} - \frac{14a^2b^6}{3x^6} - \frac{8ab^7}{3x^3} + b^8\log(x)$$

output
```
-1/24*a^8/x^24-8/21*a^7*b/x^21-14/9*a^6*b^2/x^18-56/15*a^5*b^3/x^15-35/6*a
^4*b^4/x^12-56/9*a^3*b^5/x^9-14/3*a^2*b^6/x^6-8/3*a*b^7/x^3+b^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = -\frac{a^8}{24x^{24}} - \frac{8a^7b}{21x^{21}} - \frac{14a^6b^2}{9x^{18}} - \frac{56a^5b^3}{15x^{15}} - \frac{35a^4b^4}{6x^{12}} - \frac{56a^3b^5}{9x^9} - \frac{14a^2b^6}{3x^6} - \frac{8ab^7}{3x^3} + b^8\log(x)$$

input
```
Integrate[(a + b*x^3)^8/x^25,x]
```

output
```
-1/24*a^8/x^24 - (8*a^7*b)/(21*x^21) - (14*a^6*b^2)/(9*x^18) - (56*a^5*b^3
)/(15*x^15) - (35*a^4*b^4)/(6*x^12) - (56*a^3*b^5)/(9*x^9) - (14*a^2*b^6)/
(3*x^6) - (8*a*b^7)/(3*x^3) + b^8*Log[x]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{27}}dx^3$$

$$\Big\downarrow \ 49$$

$$\frac{1}{3}\int \left(\frac{a^8}{x^{27}}+\frac{8ba^7}{x^{24}}+\frac{28b^2a^6}{x^{21}}+\frac{56b^3a^5}{x^{18}}+\frac{70b^4a^4}{x^{15}}+\frac{56b^5a^3}{x^{12}}+\frac{28b^6a^2}{x^9}+\frac{8b^7a}{x^6}+\frac{b^8}{x^3}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^8}{8x^{24}}-\frac{8a^7b}{7x^{21}}-\frac{14a^6b^2}{3x^{18}}-\frac{56a^5b^3}{5x^{15}}-\frac{35a^4b^4}{2x^{12}}-\frac{56a^3b^5}{3x^9}-\frac{14a^2b^6}{x^6}-\frac{8ab^7}{x^3}+b^8\log\left(x^3\right)\right)$$

input
```
Int[(a + b*x^3)^8/x^25,x]
```

output
```
(-1/8*a^8/x^24 - (8*a^7*b)/(7*x^21) - (14*a^6*b^2)/(3*x^18) - (56*a^5*b^3)
/(5*x^15) - (35*a^4*b^4)/(2*x^12) - (56*a^3*b^5)/(3*x^9) - (14*a^2*b^6)/x^
6 - (8*a*b^7)/x^3 + b^8*Log[x^3])/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result
default	$-\frac{a^{8}}{24 x^{24}}-\frac{8 a^{7} b}{21 x^{21}}-\frac{14 a^{6} b^{2}}{9 x^{18}}-\frac{56 a^{5} b^{3}}{15 x^{15}}-\frac{35 a^{4} b^{4}}{6 x^{12}}-\frac{56 a^{3} b^{5}}{9 x^{9}}-\frac{14 a^{2} b^{6}}{3 x^{6}}-\frac{8 a \,b^{7}}{3 x^{3}}+b^{8} \ln \left(x \right)$
norman	$\frac{-\frac{1}{24} a^{8}-\frac{8}{3} a \,b^{7} x^{21}-\frac{14}{3} a^{2} b^{6} x^{18}-\frac{56}{9} a^{3} b^{5} x^{15}-\frac{35}{6} a^{4} b^{4} x^{12}-\frac{56}{15} a^{5} b^{3} x^{9}-\frac{14}{9} a^{6} b^{2} x^{6}-\frac{8}{21} a^{7} b \,x^{3}}{x^{24}}+b^{8} \ln \left(x \right)$
risch	$\frac{-\frac{1}{24} a^{8}-\frac{8}{3} a \,b^{7} x^{21}-\frac{14}{3} a^{2} b^{6} x^{18}-\frac{56}{9} a^{3} b^{5} x^{15}-\frac{35}{6} a^{4} b^{4} x^{12}-\frac{56}{15} a^{5} b^{3} x^{9}-\frac{14}{9} a^{6} b^{2} x^{6}-\frac{8}{21} a^{7} b \,x^{3}}{x^{24}}+b^{8} \ln \left(x \right)$
parallelrisch	$\frac{2520 b^{8} \ln \left(x \right) x^{24}-6720 a \,b^{7} x^{21}-11760 a^{2} b^{6} x^{18}-15680 a^{3} b^{5} x^{15}-14700 a^{4} b^{4} x^{12}-9408 a^{5} b^{3} x^{9}-3920 a^{6} b^{2} x^{6}-960 a^{7} b \,x^{3}-105 a^{8}}{2520 x^{24}}$

input
```
int((b*x^3+a)^8/x^25,x,method=_RETURNVERBOSE)
```

output
```
-1/24*a^8/x^24-8/21*a^7*b/x^21-14/9*a^6*b^2/x^18-56/15*a^5*b^3/x^15-35/6*a
^4*b^4/x^12-56/9*a^3*b^5/x^9-14/3*a^2*b^6/x^6-8/3*a*b^7/x^3+b^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = \frac{2520\,b^8x^{24}\log(x) - 6720\,ab^7x^{21} - 11760\,a^2b^6x^{18} - 15680\,a^3b^5x^{15} - 14700\,a^4b^4x^{12} - 9408\,a^5b^3x^9 - 3920}{2520\,x^{24}}$$

input
```
integrate((b*x^3+a)^8/x^25,x, algorithm="fricas")
```

output
```
1/2520*(2520*b^8*x^24*log(x) - 6720*a*b^7*x^21 - 11760*a^2*b^6*x^18 - 1568
0*a^3*b^5*x^15 - 14700*a^4*b^4*x^12 - 9408*a^5*b^3*x^9 - 3920*a^6*b^2*x^6
- 960*a^7*b*x^3 - 105*a^8)/x^24
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = b^8\log(x) + \frac{-105a^8 - 960a^7bx^3 - 3920a^6b^2x^6 - 9408a^5b^3x^9 - 14700a^4b^4x^{12} - 15680a^3b^5x^{15} - 11760a^2b^6x^{18} - 67}{2520x^{24}}$$

input
```
integrate((b*x**3+a)**8/x**25,x)
```

output
```
b**8*log(x) + (-105*a**8 - 960*a**7*b*x**3 - 3920*a**6*b**2*x**6 - 9408*a*
*5*b**3*x**9 - 14700*a**4*b**4*x**12 - 15680*a**3*b**5*x**15 - 11760*a**2*
b**6*x**18 - 6720*a*b**7*x**21)/(2520*x**24)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = \frac{1}{3}\,b^8\log\left(x^3\right) - \frac{6720\,ab^7x^{21} + 11760\,a^2b^6x^{18} + 15680\,a^3b^5x^{15} + 14700\,a^4b^4x^{12} + 9408\,a^5b^3x^9 + 3920\,a^6b^2x^6 + 960\,a^7bx}{2520\,x^{24}}$$

input
```
integrate((b*x^3+a)^8/x^25,x, algorithm="maxima")
```

output
```
1/3*b^8*log(x^3) - 1/2520*(6720*a*b^7*x^21 + 11760*a^2*b^6*x^18 + 15680*a^
3*b^5*x^15 + 14700*a^4*b^4*x^12 + 9408*a^5*b^3*x^9 + 3920*a^6*b^2*x^6 + 96
0*a^7*b*x^3 + 105*a^8)/x^24
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = b^8\log\left(|x|\right) - \frac{2283\,b^8x^{24} + 6720\,ab^7x^{21} + 11760\,a^2b^6x^{18} + 15680\,a^3b^5x^{15} + 14700\,a^4b^4x^{12} + 9408\,a^5b^3x^9 + 3920\,a^6b^2}{2520\,x^{24}}$$

input
```
integrate((b*x^3+a)^8/x^25,x, algorithm="giac")
```

output
```
b^8*log(abs(x)) - 1/2520*(2283*b^8*x^24 + 6720*a*b^7*x^21 + 11760*a^2*b^6*
x^18 + 15680*a^3*b^5*x^15 + 14700*a^4*b^4*x^12 + 9408*a^5*b^3*x^9 + 3920*a
^6*b^2*x^6 + 960*a^7*b*x^3 + 105*a^8)/x^24
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = b^8\ \ln(x) - \frac{\frac{a^8}{24}+\frac{8\,a^7\,b\,x^3}{21}+\frac{14\,a^6\,b^2\,x^6}{9}+\frac{56\,a^5\,b^3\,x^9}{15}+\frac{35\,a^4\,b^4\,x^{12}}{6}+\frac{56\,a^3\,b^5\,x^{15}}{9}+\frac{14\,a^2\,b^6\,x^{18}}{3}+\frac{8\,a\,b^7\,x^{21}}{3}}{x^{24}}$$

input
```
int((a + b*x^3)^8/x^25,x)
```

output
```
b^8*log(x) - (a^8/24 + (8*a^7*b*x^3)/21 + (8*a*b^7*x^21)/3 + (14*a^6*b^2*x
^6)/9 + (56*a^5*b^3*x^9)/15 + (35*a^4*b^4*x^12)/6 + (56*a^3*b^5*x^15)/9 +
(14*a^2*b^6*x^18)/3)/x^24
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^{25}}\,dx = \frac{2520\log(x)\,b^8x^{24}-105a^8-960a^7b\,x^3-3920a^6b^2x^6-9408a^5b^3x^9-14700a^4b^4x^{12}-15680a^3b^5x^{15}-11}{2520x^{24}}$$

input
```
int((b*x^3+a)^8/x^25,x)
```

output
```
(2520*log(x)*b**8*x**24 - 105*a**8 - 960*a**7*b*x**3 - 3920*a**6*b**2*x**6
 - 9408*a**5*b**3*x**9 - 14700*a**4*b**4*x**12 - 15680*a**3*b**5*x**15 - 1
1760*a**2*b**6*x**18 - 6720*a*b**7*x**21)/(2520*x**24)
```

3.96 $\int \frac{(a+bx^3)^8}{x^{28}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{(a+bx^3)^9}{27ax^{27}}$$

output
```
-1/27*(b*x^3+a)^9/a/x^27
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.68

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{a^8}{27x^{27}}-\frac{a^7b}{3x^{24}}-\frac{4a^6b^2}{3x^{21}}-\frac{28a^5b^3}{9x^{18}}-\frac{14a^4b^4}{3x^{15}}-\frac{14a^3b^5}{3x^{12}}-\frac{28a^2b^6}{9x^9}-\frac{4ab^7}{3x^6}-\frac{b^8}{3x^3}$$

input
```
Integrate[(a + b*x^3)^8/x^28,x]
```

output
```
-1/27*a^8/x^27 - (a^7*b)/(3*x^24) - (4*a^6*b^2)/(3*x^21) - (28*a^5*b^3)/(9
*x^18) - (14*a^4*b^4)/(3*x^15) - (14*a^3*b^5)/(3*x^12) - (28*a^2*b^6)/(9*x
^9) - (4*a*b^7)/(3*x^6) - b^8/(3*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+bx^3)^9}{27ax^{27}}$$

input
```
Int[(a + b*x^3)^8/x^28,x]
```

output
```
-1/27*(a + b*x^3)^9/(a*x^27)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.79

method	result	size
gosper	$-\frac{9b^8x^{24}+36a\,b^7x^{21}+84a^2b^6x^{18}+126a^3b^5x^{15}+126a^4b^4x^{12}+84a^5b^3x^9+36a^6b^2x^6+9a^7b\,x^3+a^8}{27x^{27}}$	91
default	$-\frac{b^8}{3x^3}-\frac{4a^6b^2}{3x^{21}}-\frac{14a^4b^4}{3x^{15}}-\frac{a^7b}{3x^{24}}-\frac{a^8}{27x^{27}}-\frac{4a\,b^7}{3x^6}-\frac{28a^5b^3}{9x^{18}}-\frac{28a^2b^6}{9x^9}-\frac{14a^3b^5}{3x^{12}}$	91
orering	$-\frac{9b^8x^{24}+36a\,b^7x^{21}+84a^2b^6x^{18}+126a^3b^5x^{15}+126a^4b^4x^{12}+84a^5b^3x^9+36a^6b^2x^6+9a^7b\,x^3+a^8}{27x^{27}}$	91
norman	$\frac{-\frac{1}{27}a^8-\frac{4}{3}a\,b^7x^{21}-\frac{4}{3}a^6b^2x^6-\frac{1}{3}a^7b\,x^3-\frac{1}{3}b^8x^{24}-\frac{28}{9}a^5b^3x^9-\frac{14}{3}a^4b^4x^{12}-\frac{28}{9}a^2b^6x^{18}-\frac{14}{3}a^3b^5x^{15}}{x^{27}}$	92
risch	$\frac{-\frac{1}{27}a^8-\frac{4}{3}a\,b^7x^{21}-\frac{4}{3}a^6b^2x^6-\frac{1}{3}a^7b\,x^3-\frac{1}{3}b^8x^{24}-\frac{28}{9}a^5b^3x^9-\frac{14}{3}a^4b^4x^{12}-\frac{28}{9}a^2b^6x^{18}-\frac{14}{3}a^3b^5x^{15}}{x^{27}}$	92
parallelrisch	$\frac{-9b^8x^{24}-36a\,b^7x^{21}-84a^2b^6x^{18}-126a^3b^5x^{15}-126a^4b^4x^{12}-84a^5b^3x^9-36a^6b^2x^6-9a^7b\,x^3-a^8}{27x^{27}}$	93

input
```
int((b*x^3+a)^8/x^28,x,method=_RETURNVERBOSE)
```

output
```
-1/27*(9*b^8*x^24+36*a*b^7*x^21+84*a^2*b^6*x^18+126*a^3*b^5*x^15+126*a^4*b
^4*x^12+84*a^5*b^3*x^9+36*a^6*b^2*x^6+9*a^7*b*x^3+a^8)/x^27
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

input
```
integrate((b*x^3+a)^8/x^28,x, algorithm="fricas")
```

output
```
-1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 1
26*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^2
7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(15) = 30$.

Time = 0.57 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.11

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = \frac{-a^8 - 9a^7bx^3 - 36a^6b^2x^6 - 84a^5b^3x^9 - 126a^4b^4x^{12} - 126a^3b^5x^{15} - 84a^2b^6x^{18} - 36ab^7x^{21} - 9b^8x^{24}}{27x^{27}}$$

input
```
integrate((b*x**3+a)**8/x**28,x)
```

output
```
(-a**8 - 9*a**7*b*x**3 - 36*a**6*b**2*x**6 - 84*a**5*b**3*x**9 - 126*a**4*
b**4*x**12 - 126*a**3*b**5*x**15 - 84*a**2*b**6*x**18 - 36*a*b**7*x**21 -
9*b**8*x**24)/(27*x**27)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{9\,b^8x^{24} + 36\,ab^7x^{21} + 84\,a^2b^6x^{18} + 126\,a^3b^5x^{15} + 126\,a^4b^4x^{12} + 84\,a^5b^3x^9 + 36\,a^6b^2x^6 + 9\,a^7bx^3 + a^8}{27\,x^{27}}$$

input
```
integrate((b*x^3+a)^8/x^28,x, algorithm="maxima")
```

output
```
-1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 1
26*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^2
7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{9\,b^8x^{24}+36\,ab^7x^{21}+84\,a^2b^6x^{18}+126\,a^3b^5x^{15}+126\,a^4b^4x^{12}+84\,a^5b^3x^9+36\,a^6b^2x^6+9\,a^7bx^3+a^8}{27\,x^{27}}$$

input
```
integrate((b*x^3+a)^8/x^28,x, algorithm="giac")
```

output
```
-1/27*(9*b^8*x^24 + 36*a*b^7*x^21 + 84*a^2*b^6*x^18 + 126*a^3*b^5*x^15 + 1
26*a^4*b^4*x^12 + 84*a^5*b^3*x^9 + 36*a^6*b^2*x^6 + 9*a^7*b*x^3 + a^8)/x^2
7
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = -\frac{\frac{a^8}{27}+\frac{a^7\,b\,x^3}{3}+\frac{4\,a^6\,b^2\,x^6}{3}+\frac{28\,a^5\,b^3\,x^9}{9}+\frac{14\,a^4\,b^4\,x^{12}}{3}+\frac{14\,a^3\,b^5\,x^{15}}{3}+\frac{28\,a^2\,b^6\,x^{18}}{9}+\frac{4\,a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{3}}{x^{27}}$$

input
```
int((a + b*x^3)^8/x^28,x)
```

output
```
-(a^8/27 + (b^8*x^24)/3 + (a^7*b*x^3)/3 + (4*a*b^7*x^21)/3 + (4*a^6*b^2*x^
6)/3 + (28*a^5*b^3*x^9)/9 + (14*a^4*b^4*x^12)/3 + (14*a^3*b^5*x^15)/3 + (2
8*a^2*b^6*x^18)/9)/x^27
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(a+bx^3)^8}{x^{28}}\,dx = \frac{-9b^8x^{24}-36a\,b^7x^{21}-84a^2b^6x^{18}-126a^3b^5x^{15}-126a^4b^4x^{12}-84a^5b^3x^9-36a^6b^2x^6-9a^7b\,x^3-a^8}{27x^{27}}$$

input
```
int((b*x^3+a)^8/x^28,x)
```

output
```
( - a**8 - 9*a**7*b*x**3 - 36*a**6*b**2*x**6 - 84*a**5*b**3*x**9 - 126*a**
4*b**4*x**12 - 126*a**3*b**5*x**15 - 84*a**2*b**6*x**18 - 36*a*b**7*x**21
- 9*b**8*x**24)/(27*x**27)
```

3.97 $\int \frac{(a+bx^3)^8}{x^{31}} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^3)^8}{x^{31}} dx = -\frac{(a+bx^3)^9}{30ax^{30}} + \frac{b(a+bx^3)^9}{270a^2x^{27}}$$

output
```
-1/30*(b*x^3+a)^9/a/x^30+1/270*b*(b*x^3+a)^9/a^2/x^27
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 108 vs. 2(40) = 80.

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.70

$$\int \frac{(a+bx^3)^8}{x^{31}} dx = -\frac{a^8}{30x^{30}} - \frac{8a^7b}{27x^{27}} - \frac{7a^6b^2}{6x^{24}} - \frac{8a^5b^3}{3x^{21}} - \frac{35a^4b^4}{9x^{18}} - \frac{56a^3b^5}{15x^{15}} - \frac{7a^2b^6}{3x^{12}} - \frac{8ab^7}{9x^9} - \frac{b^8}{6x^6}$$

input
```
Integrate[(a + b*x^3)^8/x^31,x]
```

output
```
-1/30*a^8/x^30 - (8*a^7*b)/(27*x^27) - (7*a^6*b^2)/(6*x^24) - (8*a^5*b^3)/
(3*x^21) - (35*a^4*b^4)/(9*x^18) - (56*a^3*b^5)/(15*x^15) - (7*a^2*b^6)/(3
*x^12) - (8*a*b^7)/(9*x^9) - b^8/(6*x^6)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{33}}dx^3$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{(bx^3+a)^8}{x^{30}}dx^3}{10a}-\frac{(a+bx^3)^9}{10ax^{30}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{3}\left(\frac{b(a+bx^3)^9}{90a^2x^{27}}-\frac{(a+bx^3)^9}{10ax^{30}}\right)$$

input
```
Int[(a + b*x^3)^8/x^31,x]
```

output
```
(-1/10*(a + b*x^3)^9/(a*x^30) + (b*(a + b*x^3)^9)/(90*a^2*x^27))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

method	result	size
default	$-\frac{8a^5b^3}{3x^{21}}-\frac{56a^3b^5}{15x^{15}}-\frac{7a^6b^2}{6x^{24}}-\frac{8a^7b}{27x^{27}}-\frac{b^8}{6x^6}-\frac{a^8}{30x^{30}}-\frac{35a^4b^4}{9x^{18}}-\frac{8a\,b^7}{9x^9}-\frac{7a^2b^6}{3x^{12}}$	91
norman	$\frac{-\frac{1}{30}a^8-\frac{56}{15}a^3b^5x^{15}-\frac{7}{3}a^2b^6x^{18}-\frac{8}{9}a\,b^7x^{21}-\frac{1}{6}b^8x^{24}-\frac{8}{27}a^7b\,x^3-\frac{7}{6}a^6b^2x^6-\frac{35}{9}a^4b^4x^{12}-\frac{8}{3}a^5b^3x^9}{x^{30}}$	92
risch	$\frac{-\frac{1}{30}a^8-\frac{56}{15}a^3b^5x^{15}-\frac{7}{3}a^2b^6x^{18}-\frac{8}{9}a\,b^7x^{21}-\frac{1}{6}b^8x^{24}-\frac{8}{27}a^7b\,x^3-\frac{7}{6}a^6b^2x^6-\frac{35}{9}a^4b^4x^{12}-\frac{8}{3}a^5b^3x^9}{x^{30}}$	92
gosper	$-\frac{45b^8x^{24}+240a\,b^7x^{21}+630a^2b^6x^{18}+1008a^3b^5x^{15}+1050a^4b^4x^{12}+720a^5b^3x^9+315a^6b^2x^6+80a^7b\,x^3+9a^8}{270x^{30}}$	93
parallelrisch	$\frac{-45b^8x^{24}-240a\,b^7x^{21}-630a^2b^6x^{18}-1008a^3b^5x^{15}-1050a^4b^4x^{12}-720a^5b^3x^9-315a^6b^2x^6-80a^7b\,x^3-9a^8}{270x^{30}}$	93
orering	$-\frac{45b^8x^{24}+240a\,b^7x^{21}+630a^2b^6x^{18}+1008a^3b^5x^{15}+1050a^4b^4x^{12}+720a^5b^3x^9+315a^6b^2x^6+80a^7b\,x^3+9a^8}{270x^{30}}$	93

input
```
int((b*x^3+a)^8/x^31,x,method=_RETURNVERBOSE)
```

output
```
-8/3*a^5*b^3/x^21-56/15*a^3*b^5/x^15-7/6*a^6*b^2/x^24-8/27*a^7*b/x^27-1/6*
b^8/x^6-1/30*a^8/x^30-35/9*a^4*b^4/x^18-8/9*a*b^7/x^9-7/3*a^2*b^6/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(36) = 72.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx = -\frac{45\,b^8x^{24}+240\,ab^7x^{21}+630\,a^2b^6x^{18}+1008\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+720\,a^5b^3x^9+315\,a^6b^2x^6+80\,a^7b}{270\,x^{30}}$$

input
```
integrate((b*x^3+a)^8/x^31,x, algorithm="fricas")
```

output
```
-1/270*(45*b^8*x^24 + 240*a*b^7*x^21 + 630*a^2*b^6*x^18 + 1008*a^3*b^5*x^1
5 + 1050*a^4*b^4*x^12 + 720*a^5*b^3*x^9 + 315*a^6*b^2*x^6 + 80*a^7*b*x^3 +
 9*a^8)/x^30
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(32) = 64.

Time = 0.68 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx = \frac{-9a^8-80a^7bx^3-315a^6b^2x^6-720a^5b^3x^9-1050a^4b^4x^{12}-1008a^3b^5x^{15}-630a^2b^6x^{18}-240ab^7x^{21}-45}{270x^{30}}$$

input
```
integrate((b*x**3+a)**8/x**31,x)
```

output
```
(-9*a**8 - 80*a**7*b*x**3 - 315*a**6*b**2*x**6 - 720*a**5*b**3*x**9 - 1050
*a**4*b**4*x**12 - 1008*a**3*b**5*x**15 - 630*a**2*b**6*x**18 - 240*a*b**7
*x**21 - 45*b**8*x**24)/(270*x**30)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx = -\frac{45\,b^8x^{24}+240\,ab^7x^{21}+630\,a^2b^6x^{18}+1008\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+720\,a^5b^3x^9+315\,a^6b^2x^6+80\,a^7b}{270\,x^{30}}$$

input
```
integrate((b*x^3+a)^8/x^31,x, algorithm="maxima")
```

output
```
-1/270*(45*b^8*x^24 + 240*a*b^7*x^21 + 630*a^2*b^6*x^18 + 1008*a^3*b^5*x^1
5 + 1050*a^4*b^4*x^12 + 720*a^5*b^3*x^9 + 315*a^6*b^2*x^6 + 80*a^7*b*x^3 +
 9*a^8)/x^30
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx = -\frac{45\,b^8x^{24}+240\,ab^7x^{21}+630\,a^2b^6x^{18}+1008\,a^3b^5x^{15}+1050\,a^4b^4x^{12}+720\,a^5b^3x^9+315\,a^6b^2x^6+80\,a^7b}{270\,x^{30}}$$

input
```
integrate((b*x^3+a)^8/x^31,x, algorithm="giac")
```

output
```
-1/270*(45*b^8*x^24 + 240*a*b^7*x^21 + 630*a^2*b^6*x^18 + 1008*a^3*b^5*x^1
5 + 1050*a^4*b^4*x^12 + 720*a^5*b^3*x^9 + 315*a^6*b^2*x^6 + 80*a^7*b*x^3 +
 9*a^8)/x^30
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx = -\frac{\frac{a^8}{30}+\frac{8\,a^7\,b\,x^3}{27}+\frac{7\,a^6\,b^2\,x^6}{6}+\frac{8\,a^5\,b^3\,x^9}{3}+\frac{35\,a^4\,b^4\,x^{12}}{9}+\frac{56\,a^3\,b^5\,x^{15}}{15}+\frac{7\,a^2\,b^6\,x^{18}}{3}+\frac{8\,a\,b^7\,x^{21}}{9}+\frac{b^8\,x^{24}}{6}}{x^{30}}$$

input
```
int((a + b*x^3)^8/x^31,x)
```

output
```
-(a^8/30 + (b^8*x^24)/6 + (8*a^7*b*x^3)/27 + (8*a*b^7*x^21)/9 + (7*a^6*b^2
*x^6)/6 + (8*a^5*b^3*x^9)/3 + (35*a^4*b^4*x^12)/9 + (56*a^3*b^5*x^15)/15 +
 (7*a^2*b^6*x^18)/3)/x^30
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^3)^8}{x^{31}}\,dx = \frac{-45b^8x^{24}-240a\,b^7x^{21}-630a^2b^6x^{18}-1008a^3b^5x^{15}-1050a^4b^4x^{12}-720a^5b^3x^9-315a^6b^2x^6-80a^7b\,x^3}{270x^{30}}$$

input
```
int((b*x^3+a)^8/x^31,x)
```

output
```
( - 9*a**8 - 80*a**7*b*x**3 - 315*a**6*b**2*x**6 - 720*a**5*b**3*x**9 - 10
50*a**4*b**4*x**12 - 1008*a**3*b**5*x**15 - 630*a**2*b**6*x**18 - 240*a*b*
*7*x**21 - 45*b**8*x**24)/(270*x**30)
```

3.98 $\int \frac{(a+bx^3)^8}{x^{34}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{(a+bx^3)^8}{x^{34}} \, dx = -\frac{(a+bx^3)^9}{33ax^{33}} + \frac{b(a+bx^3)^9}{165a^2x^{30}} - \frac{b^2(a+bx^3)^9}{1485a^3x^{27}}$$

output
```
-1/33*(b*x^3+a)^9/a/x^33+1/165*b*(b*x^3+a)^9/a^2/x^30-1/1485*b^2*(b*x^3+a)
^9/a^3/x^27
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^3)^8}{x^{34}} \, dx = -\frac{a^8}{33x^{33}} - \frac{4a^7b}{15x^{30}} - \frac{28a^6b^2}{27x^{27}} - \frac{7a^5b^3}{3x^{24}} - \frac{10a^4b^4}{3x^{21}} - \frac{28a^3b^5}{9x^{18}} - \frac{28a^2b^6}{15x^{15}} - \frac{2ab^7}{3x^{12}} - \frac{b^8}{9x^9}$$

input
```
Integrate[(a + b*x^3)^8/x^34,x]
```

output
```
-1/33*a^8/x^33 - (4*a^7*b)/(15*x^30) - (28*a^6*b^2)/(27*x^27) - (7*a^5*b^3
)/(3*x^24) - (10*a^4*b^4)/(3*x^21) - (28*a^3*b^5)/(9*x^18) - (28*a^2*b^6)/
(15*x^15) - (2*a*b^7)/(3*x^12) - b^8/(9*x^9)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx$$
$$\Big\downarrow \ 798$$
$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{36}}dx^3$$
$$\Big\downarrow \ 55$$
$$\frac{1}{3}\left(-\frac{2b\int \frac{(bx^3+a)^8}{x^{33}}dx^3}{11a}-\frac{(a+bx^3)^9}{11ax^{33}}\right)$$
$$\Big\downarrow \ 55$$
$$\frac{1}{3}\left(-\frac{2b\left(-\frac{b\int \frac{(bx^3+a)^8}{x^{30}}dx^3}{10a}-\frac{(a+bx^3)^9}{10ax^{30}}\right)}{11a}-\frac{(a+bx^3)^9}{11ax^{33}}\right)$$
$$\Big\downarrow \ 48$$
$$\frac{1}{3}\left(-\frac{2b\left(\frac{b(a+bx^3)^9}{90a^2x^{27}}-\frac{(a+bx^3)^9}{10ax^{30}}\right)}{11a}-\frac{(a+bx^3)^9}{11ax^{33}}\right)$$

input
```
Int[(a + b*x^3)^8/x^34,x]
```

output
```
(-1/11*(a + b*x^3)^9/(a*x^33) - (2*b*(-1/10*(a + b*x^3)^9/(a*x^30) + (b*(a
 + b*x^3)^9)/(90*a^2*x^27)))/(11*a))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{10a^4b^4}{3x^{21}}-\frac{28a^2b^6}{15x^{15}}-\frac{7a^5b^3}{3x^{24}}-\frac{a^8}{33x^{33}}-\frac{28a^6b^2}{27x^{27}}-\frac{4a^7b}{15x^{30}}-\frac{28a^3b^5}{9x^{18}}-\frac{b^8}{9x^9}-\frac{2a\,b^7}{3x^{12}}$	91
norman	$\frac{-\frac{1}{33}a^8-\frac{7}{3}a^5b^3x^9-\frac{1}{9}b^8x^{24}-\frac{10}{3}a^4b^4x^{12}-\frac{28}{9}a^3b^5x^{15}-\frac{28}{15}a^2b^6x^{18}-\frac{2}{3}a\,b^7x^{21}-\frac{28}{27}a^6b^2x^6-\frac{4}{15}a^7b\,x^3}{x^{33}}$	92
risch	$\frac{-\frac{1}{33}a^8-\frac{7}{3}a^5b^3x^9-\frac{1}{9}b^8x^{24}-\frac{10}{3}a^4b^4x^{12}-\frac{28}{9}a^3b^5x^{15}-\frac{28}{15}a^2b^6x^{18}-\frac{2}{3}a\,b^7x^{21}-\frac{28}{27}a^6b^2x^6-\frac{4}{15}a^7b\,x^3}{x^{33}}$	92
gosper	$-\frac{165b^8x^{24}+990a\,b^7x^{21}+2772a^2b^6x^{18}+4620a^3b^5x^{15}+4950a^4b^4x^{12}+3465a^5b^3x^9+1540a^6b^2x^6+396a^7b\,x^3+45a^8}{1485x^{33}}$	93
parallelrisch	$\frac{-165b^8x^{24}-990a\,b^7x^{21}-2772a^2b^6x^{18}-4620a^3b^5x^{15}-4950a^4b^4x^{12}-3465a^5b^3x^9-1540a^6b^2x^6-396a^7b\,x^3-45a^8}{1485x^{33}}$	93
orering	$-\frac{165b^8x^{24}+990a\,b^7x^{21}+2772a^2b^6x^{18}+4620a^3b^5x^{15}+4950a^4b^4x^{12}+3465a^5b^3x^9+1540a^6b^2x^6+396a^7b\,x^3+45a^8}{1485x^{33}}$	93

input
```
int((b*x^3+a)^8/x^34,x,method=_RETURNVERBOSE)
```

output
```
-10/3*a^4*b^4/x^21-28/15*a^2*b^6/x^15-7/3*a^5*b^3/x^24-1/33*a^8/x^33-28/27
*a^6*b^2/x^27-4/15*a^7*b/x^30-28/9*a^3*b^5/x^18-1/9*b^8/x^9-2/3*a*b^7/x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx = -\frac{165\,b^8x^{24}+990\,ab^7x^{21}+2772\,a^2b^6x^{18}+4620\,a^3b^5x^{15}+4950\,a^4b^4x^{12}+3465\,a^5b^3x^9+1540\,a^6b^2x^6+3}{1485\,x^{33}}$$

input
```
integrate((b*x^3+a)^8/x^34,x, algorithm="fricas")
```

output
```
-1/1485*(165*b^8*x^24 + 990*a*b^7*x^21 + 2772*a^2*b^6*x^18 + 4620*a^3*b^5*
x^15 + 4950*a^4*b^4*x^12 + 3465*a^5*b^3*x^9 + 1540*a^6*b^2*x^6 + 396*a^7*b
*x^3 + 45*a^8)/x^33
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx = \frac{-45a^8-396a^7bx^3-1540a^6b^2x^6-3465a^5b^3x^9-4950a^4b^4x^{12}-4620a^3b^5x^{15}-2772a^2b^6x^{18}-990ab^7x^2}{1485x^{33}}$$

input
```
integrate((b*x**3+a)**8/x**34,x)
```

output
```
(-45*a**8 - 396*a**7*b*x**3 - 1540*a**6*b**2*x**6 - 3465*a**5*b**3*x**9 -
4950*a**4*b**4*x**12 - 4620*a**3*b**5*x**15 - 2772*a**2*b**6*x**18 - 990*a
*b**7*x**21 - 165*b**8*x**24)/(1485*x**33)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx = -\frac{165\,b^8x^{24}+990\,ab^7x^{21}+2772\,a^2b^6x^{18}+4620\,a^3b^5x^{15}+4950\,a^4b^4x^{12}+3465\,a^5b^3x^9+1540\,a^6b^2x^6+3}{1485\,x^{33}}$$

input
```
integrate((b*x^3+a)^8/x^34,x, algorithm="maxima")
```

output
```
-1/1485*(165*b^8*x^24 + 990*a*b^7*x^21 + 2772*a^2*b^6*x^18 + 4620*a^3*b^5*
x^15 + 4950*a^4*b^4*x^12 + 3465*a^5*b^3*x^9 + 1540*a^6*b^2*x^6 + 396*a^7*b
*x^3 + 45*a^8)/x^33
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx = -\frac{165\,b^8x^{24}+990\,ab^7x^{21}+2772\,a^2b^6x^{18}+4620\,a^3b^5x^{15}+4950\,a^4b^4x^{12}+3465\,a^5b^3x^9+1540\,a^6b^2x^6+3}{1485\,x^{33}}$$

input
```
integrate((b*x^3+a)^8/x^34,x, algorithm="giac")
```

output
```
-1/1485*(165*b^8*x^24 + 990*a*b^7*x^21 + 2772*a^2*b^6*x^18 + 4620*a^3*b^5*
x^15 + 4950*a^4*b^4*x^12 + 3465*a^5*b^3*x^9 + 1540*a^6*b^2*x^6 + 396*a^7*b
*x^3 + 45*a^8)/x^33
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx = -\frac{\frac{a^8}{33}+\frac{4\,a^7\,b\,x^3}{15}+\frac{28\,a^6\,b^2\,x^6}{27}+\frac{7\,a^5\,b^3\,x^9}{3}+\frac{10\,a^4\,b^4\,x^{12}}{3}+\frac{28\,a^3\,b^5\,x^{15}}{9}+\frac{28\,a^2\,b^6\,x^{18}}{15}+\frac{2\,a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{9}}{x^{33}}$$

input
```
int((a + b*x^3)^8/x^34,x)
```

output
```
-(a^8/33 + (b^8*x^24)/9 + (4*a^7*b*x^3)/15 + (2*a*b^7*x^21)/3 + (28*a^6*b^
2*x^6)/27 + (7*a^5*b^3*x^9)/3 + (10*a^4*b^4*x^12)/3 + (28*a^3*b^5*x^15)/9
+ (28*a^2*b^6*x^18)/15)/x^33
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^3)^8}{x^{34}}\,dx$$
$$= \frac{-165b^8x^{24} - 990a\,b^7x^{21} - 2772a^2b^6x^{18} - 4620a^3b^5x^{15} - 4950a^4b^4x^{12} - 3465a^5b^3x^9 - 1540a^6b^2x^6 - 396}{1485x^{33}}$$

input
```
int((b*x^3+a)^8/x^34,x)
```

output
```
( - 45*a**8 - 396*a**7*b*x**3 - 1540*a**6*b**2*x**6 - 3465*a**5*b**3*x**9
- 4950*a**4*b**4*x**12 - 4620*a**3*b**5*x**15 - 2772*a**2*b**6*x**18 - 990
*a*b**7*x**21 - 165*b**8*x**24)/(1485*x**33)
```

3.99 $\int \frac{(a+bx^3)^8}{x^{37}} dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{(a+bx^3)^8}{x^{37}} dx = -\frac{(a+bx^3)^9}{36ax^{36}} + \frac{b(a+bx^3)^9}{132a^2x^{33}} - \frac{b^2(a+bx^3)^9}{660a^3x^{30}} + \frac{b^3(a+bx^3)^9}{5940a^4x^{27}}$$

output
```
-1/36*(b*x^3+a)^9/a/x^36+1/132*b*(b*x^3+a)^9/a^2/x^33-1/660*b^2*(b*x^3+a)^
9/a^3/x^30+1/5940*b^3*(b*x^3+a)^9/a^4/x^27
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^3)^8}{x^{37}} dx = -\frac{a^8}{36x^{36}} - \frac{8a^7b}{33x^{33}} - \frac{14a^6b^2}{15x^{30}} - \frac{56a^5b^3}{27x^{27}} - \frac{35a^4b^4}{12x^{24}} - \frac{8a^3b^5}{3x^{21}} - \frac{14a^2b^6}{9x^{18}} - \frac{8ab^7}{15x^{15}} - \frac{b^8}{12x^{12}}$$

input
```
Integrate[(a + b*x^3)^8/x^37,x]
```

output
```
-1/36*a^8/x^36 - (8*a^7*b)/(33*x^33) - (14*a^6*b^2)/(15*x^30) - (56*a^5*b^
3)/(27*x^27) - (35*a^4*b^4)/(12*x^24) - (8*a^3*b^5)/(3*x^21) - (14*a^2*b^6
)/(9*x^18) - (8*a*b^7)/(15*x^15) - b^8/(12*x^12)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {798, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{39}}dx^3$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{(bx^3+a)^8}{x^{36}}dx^3}{4a}-\frac{(a+bx^3)^9}{12ax^{36}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{2b\int \frac{(bx^3+a)^8}{x^{33}}dx^3}{11a}-\frac{(a+bx^3)^9}{11ax^{33}}\right)}{4a}-\frac{(a+bx^3)^9}{12ax^{36}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{2b\left(-\frac{b\int\frac{\left(bx^3+a\right)^8}{x^{30}}dx^3}{10a}-\frac{\left(a+bx^3\right)^9}{10ax^{30}}\right)}{11a}-\frac{\left(a+bx^3\right)^9}{11ax^{33}}\right)}{4a}-\frac{\left(a+bx^3\right)^9}{12ax^{36}}\right)$$

$$\Big\downarrow \; 48$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{2b\left(\frac{b\left(a+bx^3\right)^9}{90a^2x^{27}}-\frac{\left(a+bx^3\right)^9}{10ax^{30}}\right)}{11a}-\frac{\left(a+bx^3\right)^9}{11ax^{33}}\right)}{4a}-\frac{\left(a+bx^3\right)^9}{12ax^{36}}\right)$$

input
```
Int[(a + b*x^3)^8/x^37,x]
```

output
```
(-1/12*(a + b*x^3)^9/(a*x^36) - (b*(-1/11*(a + b*x^3)^9/(a*x^33) - (2*b*(-
1/10*(a + b*x^3)^9/(a*x^30) + (b*(a + b*x^3)^9)/(90*a^2*x^27)))/(11*a)))/(
4*a))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

method	result
default	$-\frac{8a^3b^5}{3x^{21}}-\frac{8a\,b^7}{15x^{15}}-\frac{35a^4b^4}{12x^{24}}-\frac{8a^7b}{33x^{33}}-\frac{56a^5b^3}{27x^{27}}-\frac{a^8}{36x^{36}}-\frac{14a^6b^2}{15x^{30}}-\frac{14a^2b^6}{9x^{18}}-\frac{b^8}{12x^{12}}$
norman	$\frac{-\frac{1}{36}a^8-\frac{8}{15}a\,b^7x^{21}-\frac{8}{3}a^3b^5x^{15}-\frac{35}{12}a^4b^4x^{12}-\frac{14}{9}a^2b^6x^{18}-\frac{1}{12}b^8x^{24}-\frac{8}{33}a^7b\,x^3-\frac{14}{15}a^6b^2x^6-\frac{56}{27}a^5b^3x^9}{x^{36}}$
risch	$\frac{-\frac{1}{36}a^8-\frac{8}{15}a\,b^7x^{21}-\frac{8}{3}a^3b^5x^{15}-\frac{35}{12}a^4b^4x^{12}-\frac{14}{9}a^2b^6x^{18}-\frac{1}{12}b^8x^{24}-\frac{8}{33}a^7b\,x^3-\frac{14}{15}a^6b^2x^6-\frac{56}{27}a^5b^3x^9}{x^{36}}$
gosper	$-\frac{495b^8x^{24}+3168a\,b^7x^{21}+9240a^2b^6x^{18}+15840a^3b^5x^{15}+17325a^4b^4x^{12}+12320a^5b^3x^9+5544a^6b^2x^6+1440a^7b\,x^3+165a^8}{5940x^{36}}$
parallelrisch	$\frac{-495b^8x^{24}-3168a\,b^7x^{21}-9240a^2b^6x^{18}-15840a^3b^5x^{15}-17325a^4b^4x^{12}-12320a^5b^3x^9-5544a^6b^2x^6-1440a^7b\,x^3-165a^8}{5940x^{36}}$
orering	$-\frac{495b^8x^{24}+3168a\,b^7x^{21}+9240a^2b^6x^{18}+15840a^3b^5x^{15}+17325a^4b^4x^{12}+12320a^5b^3x^9+5544a^6b^2x^6+1440a^7b\,x^3+165a^8}{5940x^{36}}$

input

```
int((b*x^3+a)^8/x^37,x,method=_RETURNVERBOSE)
```

output

```
-8/3*a^3*b^5/x^21-8/15*a*b^7/x^15-35/12*a^4*b^4/x^24-8/33*a^7*b/x^33-56/27
*a^5*b^3/x^27-1/36*a^8/x^36-14/15*a^6*b^2/x^30-14/9*a^2*b^6/x^18-1/12*b^8/
x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx = -\frac{495\,b^8x^{24}+3168\,ab^7x^{21}+9240\,a^2b^6x^{18}+15840\,a^3b^5x^{15}+17325\,a^4b^4x^{12}+12320\,a^5b^3x^9+5544\,a^6b^2x^{6}}{5940\,x^{36}}$$

input
```
integrate((b*x^3+a)^8/x^37,x, algorithm="fricas")
```

output
```
-1/5940*(495*b^8*x^24 + 3168*a*b^7*x^21 + 9240*a^2*b^6*x^18 + 15840*a^3*b^
5*x^15 + 17325*a^4*b^4*x^12 + 12320*a^5*b^3*x^9 + 5544*a^6*b^2*x^6 + 1440*
a^7*b*x^3 + 165*a^8)/x^36
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx = \frac{-165a^8-1440a^7bx^3-5544a^6b^2x^6-12320a^5b^3x^9-17325a^4b^4x^{12}-15840a^3b^5x^{15}-9240a^2b^6x^{18}-316}{5940x^{36}}$$

input
```
integrate((b*x**3+a)**8/x**37,x)
```

output
```
(-165*a**8 - 1440*a**7*b*x**3 - 5544*a**6*b**2*x**6 - 12320*a**5*b**3*x**9
 - 17325*a**4*b**4*x**12 - 15840*a**3*b**5*x**15 - 9240*a**2*b**6*x**18 -
3168*a*b**7*x**21 - 495*b**8*x**24)/(5940*x**36)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx = -\frac{495\,b^8x^{24}+3168\,ab^7x^{21}+9240\,a^2b^6x^{18}+15840\,a^3b^5x^{15}+17325\,a^4b^4x^{12}+12320\,a^5b^3x^9+5544\,a^6b^2x}{5940\,x^{36}}$$

input
```
integrate((b*x^3+a)^8/x^37,x, algorithm="maxima")
```

output
```
-1/5940*(495*b^8*x^24 + 3168*a*b^7*x^21 + 9240*a^2*b^6*x^18 + 15840*a^3*b^
5*x^15 + 17325*a^4*b^4*x^12 + 12320*a^5*b^3*x^9 + 5544*a^6*b^2*x^6 + 1440*
a^7*b*x^3 + 165*a^8)/x^36
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx = -\frac{495\,b^8x^{24}+3168\,ab^7x^{21}+9240\,a^2b^6x^{18}+15840\,a^3b^5x^{15}+17325\,a^4b^4x^{12}+12320\,a^5b^3x^9+5544\,a^6b^2x}{5940\,x^{36}}$$

input
```
integrate((b*x^3+a)^8/x^37,x, algorithm="giac")
```

output
```
-1/5940*(495*b^8*x^24 + 3168*a*b^7*x^21 + 9240*a^2*b^6*x^18 + 15840*a^3*b^
5*x^15 + 17325*a^4*b^4*x^12 + 12320*a^5*b^3*x^9 + 5544*a^6*b^2*x^6 + 1440*
a^7*b*x^3 + 165*a^8)/x^36
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx = -\frac{\frac{a^8}{36}+\frac{8\,a^7\,b\,x^3}{33}+\frac{14\,a^6\,b^2\,x^6}{15}+\frac{56\,a^5\,b^3\,x^9}{27}+\frac{35\,a^4\,b^4\,x^{12}}{12}+\frac{8\,a^3\,b^5\,x^{15}}{3}+\frac{14\,a^2\,b^6\,x^{18}}{9}+\frac{8\,a\,b^7\,x^{21}}{15}+\frac{b^8\,x^{24}}{12}}{x^{36}}$$

input
```
int((a + b*x^3)^8/x^37,x)
```

output
```
-(a^8/36 + (b^8*x^24)/12 + (8*a^7*b*x^3)/33 + (8*a*b^7*x^21)/15 + (14*a^6*
b^2*x^6)/15 + (56*a^5*b^3*x^9)/27 + (35*a^4*b^4*x^12)/12 + (8*a^3*b^5*x^15
)/3 + (14*a^2*b^6*x^18)/9)/x^36
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^3)^8}{x^{37}}\,dx = \frac{-495b^8x^{24}-3168a\,b^7x^{21}-9240a^2b^6x^{18}-15840a^3b^5x^{15}-17325a^4b^4x^{12}-12320a^5b^3x^9-5544a^6b^2x^6-}{5940x^{36}}$$

input
```
int((b*x^3+a)^8/x^37,x)
```

output
```
( - 165*a**8 - 1440*a**7*b*x**3 - 5544*a**6*b**2*x**6 - 12320*a**5*b**3*x*
*9 - 17325*a**4*b**4*x**12 - 15840*a**3*b**5*x**15 - 9240*a**2*b**6*x**18
- 3168*a*b**7*x**21 - 495*b**8*x**24)/(5940*x**36)
```

3.100 $\int \frac{(a+bx^3)^8}{x^{40}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = -\frac{(a+bx^3)^9}{39ax^{39}} + \frac{b(a+bx^3)^9}{117a^2x^{36}} - \frac{b^2(a+bx^3)^9}{429a^3x^{33}} + \frac{b^3(a+bx^3)^9}{2145a^4x^{30}} - \frac{b^4(a+bx^3)^9}{19305a^5x^{27}}$$

output
```
-1/39*(b*x^3+a)^9/a/x^39+1/117*b*(b*x^3+a)^9/a^2/x^36-1/429*b^2*(b*x^3+a)^
9/a^3/x^33+1/2145*b^3*(b*x^3+a)^9/a^4/x^30-1/19305*b^4*(b*x^3+a)^9/a^5/x^2
7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = -\frac{a^8}{39x^{39}} - \frac{2a^7b}{9x^{36}} - \frac{28a^6b^2}{33x^{33}} - \frac{28a^5b^3}{15x^{30}} - \frac{70a^4b^4}{27x^{27}} - \frac{7a^3b^5}{3x^{24}} - \frac{4a^2b^6}{3x^{21}} - \frac{4ab^7}{9x^{18}} - \frac{b^8}{15x^{15}}$$

input
```
Integrate[(a + b*x^3)^8/x^40,x]
```

output
```
-1/39*a^8/x^39 - (2*a^7*b)/(9*x^36) - (28*a^6*b^2)/(33*x^33) - (28*a^5*b^3
)/(15*x^30) - (70*a^4*b^4)/(27*x^27) - (7*a^3*b^5)/(3*x^24) - (4*a^2*b^6)/
(3*x^21) - (4*a*b^7)/(9*x^18) - b^8/(15*x^15)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {798, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{42}}dx^3$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{4b\int \frac{(bx^3+a)^8}{x^{39}}dx^3}{13a}-\frac{(a+bx^3)^9}{13ax^{39}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{4b\left(-\frac{b\int \frac{(bx^3+a)^8}{x^{36}}dx^3}{4a}-\frac{(a+bx^3)^9}{12ax^{36}}\right)}{13a}-\frac{(a+bx^3)^9}{13ax^{39}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{3}\left(-\frac{4b\left(-\frac{b\left(-\frac{2b\int\frac{\left(bx^3+a\right)^8}{x^{33}}dx^3}{11a}-\frac{\left(a+bx^3\right)^9}{11ax^{33}}\right)}{4a}-\frac{\left(a+bx^3\right)^9}{12ax^{36}}\right)}{13a}-\frac{\left(a+bx^3\right)^9}{13ax^{39}}\right)$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{3}\left(-\frac{4b\left(-\frac{b\left(-\frac{2b\left(-\frac{b\int\frac{\left(bx^3+a\right)^8}{x^{30}}dx^3}{10a}-\frac{\left(a+bx^3\right)^9}{10ax^{30}}\right)}{11a}-\frac{\left(a+bx^3\right)^9}{11ax^{33}}\right)}{4a}-\frac{\left(a+bx^3\right)^9}{12ax^{36}}\right)}{13a}-\frac{\left(a+bx^3\right)^9}{13ax^{39}}\right)$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{3}\left(-\frac{4b\left(-\frac{b\left(-\frac{2b\left(\frac{b(a+bx^3)^9}{90a^2x^{27}}-\frac{(a+bx^3)^9}{10ax^{30}}\right)}{11a}-\frac{(a+bx^3)^9}{11ax^{33}}\right)}{4a}-\frac{(a+bx^3)^9}{12ax^{36}}\right)}{13a}-\frac{(a+bx^3)^9}{13ax^{39}}\right)$$

input
```
Int[(a + b*x^3)^8/x^40,x]
```

output
```
(-1/13*(a + b*x^3)^9/(a*x^39) - (4*b*(-1/12*(a + b*x^3)^9/(a*x^36) - (b*(-
1/11*(a + b*x^3)^9/(a*x^33) - (2*b*(-1/10*(a + b*x^3)^9/(a*x^30) + (b*(a +
 b*x^3)^9)/(90*a^2*x^27)))/(11*a)))/(4*a)))/(13*a))/3
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
default	$-\frac{4a^2b^6}{3x^{21}}-\frac{b^8}{15x^{15}}-\frac{7a^3b^5}{3x^{24}}-\frac{28a^6b^2}{33x^{33}}-\frac{70a^4b^4}{27x^{27}}-\frac{2a^7b}{9x^{36}}-\frac{a^8}{39x^{39}}-\frac{28a^5b^3}{15x^{30}}-\frac{4a\,b^7}{9x^{18}}$
risch	$\frac{-\frac{1}{39}a^8-\frac{28}{33}a^6b^2x^6-\frac{28}{15}a^5b^3x^9-\frac{7}{3}a^3b^5x^{15}-\frac{4}{9}a\,b^7x^{21}-\frac{4}{3}a^2b^6x^{18}-\frac{70}{27}a^4b^4x^{12}-\frac{2}{9}a^7b\,x^3-\frac{1}{15}b^8x^{24}}{x^{39}}$
gosper	$-\frac{1287b^8x^{24}+8580a\,b^7x^{21}+25740a^2b^6x^{18}+45045a^3b^5x^{15}+50050a^4b^4x^{12}+36036a^5b^3x^9+16380a^6b^2x^6+4290a^7b\,x^3+495a^8}{19305x^{39}}$
parallelrisch	$\frac{-1287b^8x^{24}-8580a\,b^7x^{21}-25740a^2b^6x^{18}-45045a^3b^5x^{15}-50050a^4b^4x^{12}-36036a^5b^3x^9-16380a^6b^2x^6-4290a^7b\,x^3-495a^8}{19305x^{39}}$
orering	$-\frac{1287b^8x^{24}+8580a\,b^7x^{21}+25740a^2b^6x^{18}+45045a^3b^5x^{15}+50050a^4b^4x^{12}+36036a^5b^3x^9+16380a^6b^2x^6+4290a^7b\,x^3+495a^8}{19305x^{39}}$

input
```
int((b*x^3+a)^8/x^40,x,method=_RETURNVERBOSE)
```

output
```
-4/3*a^2*b^6/x^21-1/15*b^8/x^15-7/3*a^3*b^5/x^24-28/33*a^6*b^2/x^33-70/27*
a^4*b^4/x^27-2/9*a^7*b/x^36-1/39*a^8/x^39-28/15*a^5*b^3/x^30-4/9*a*b^7/x^1
8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = -\frac{1287\,b^8x^{24}+8580\,ab^7x^{21}+25740\,a^2b^6x^{18}+45045\,a^3b^5x^{15}+50050\,a^4b^4x^{12}+36036\,a^5b^3x^9+16380\,a^6}{19305\,x^{39}}$$

input
```
integrate((b*x^3+a)^8/x^40,x, algorithm="fricas")
```

output
```
-1/19305*(1287*b^8*x^24 + 8580*a*b^7*x^21 + 25740*a^2*b^6*x^18 + 45045*a^3
*b^5*x^15 + 50050*a^4*b^4*x^12 + 36036*a^5*b^3*x^9 + 16380*a^6*b^2*x^6 + 4
290*a^7*b*x^3 + 495*a^8)/x^39
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = \frac{-495a^8 - 4290a^7bx^3 - 16380a^6b^2x^6 - 36036a^5b^3x^9 - 50050a^4b^4x^{12} - 45045a^3b^5x^{15} - 25740a^2b^6x^{18} - 8}{19305x^{39}}$$

input
```
integrate((b*x**3+a)**8/x**40,x)
```

output
```
(-495*a**8 - 4290*a**7*b*x**3 - 16380*a**6*b**2*x**6 - 36036*a**5*b**3*x**
9 - 50050*a**4*b**4*x**12 - 45045*a**3*b**5*x**15 - 25740*a**2*b**6*x**18
- 8580*a*b**7*x**21 - 1287*b**8*x**24)/(19305*x**39)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = -\frac{1287\,b^8x^{24} + 8580\,ab^7x^{21} + 25740\,a^2b^6x^{18} + 45045\,a^3b^5x^{15} + 50050\,a^4b^4x^{12} + 36036\,a^5b^3x^9 + 16380\,a^6}{19305\,x^{39}}$$

input
```
integrate((b*x^3+a)^8/x^40,x, algorithm="maxima")
```

output
```
-1/19305*(1287*b^8*x^24 + 8580*a*b^7*x^21 + 25740*a^2*b^6*x^18 + 45045*a^3
*b^5*x^15 + 50050*a^4*b^4*x^12 + 36036*a^5*b^3*x^9 + 16380*a^6*b^2*x^6 + 4
290*a^7*b*x^3 + 495*a^8)/x^39
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = -\frac{1287\,b^8x^{24}+8580\,ab^7x^{21}+25740\,a^2b^6x^{18}+45045\,a^3b^5x^{15}+50050\,a^4b^4x^{12}+36036\,a^5b^3x^9+16380\,a^6}{19305\,x^{39}}$$

input
```
integrate((b*x^3+a)^8/x^40,x, algorithm="giac")
```

output
```
-1/19305*(1287*b^8*x^24 + 8580*a*b^7*x^21 + 25740*a^2*b^6*x^18 + 45045*a^3
*b^5*x^15 + 50050*a^4*b^4*x^12 + 36036*a^5*b^3*x^9 + 16380*a^6*b^2*x^6 + 4
290*a^7*b*x^3 + 495*a^8)/x^39
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx = -\frac{\frac{a^8}{39}+\frac{2\,a^7\,b\,x^3}{9}+\frac{28\,a^6\,b^2\,x^6}{33}+\frac{28\,a^5\,b^3\,x^9}{15}+\frac{70\,a^4\,b^4\,x^{12}}{27}+\frac{7\,a^3\,b^5\,x^{15}}{3}+\frac{4\,a^2\,b^6\,x^{18}}{3}+\frac{4\,a\,b^7\,x^{21}}{9}+\frac{b^8\,x^{24}}{15}}{x^{39}}$$

input
```
int((a + b*x^3)^8/x^40,x)
```

output
```
-(a^8/39 + (b^8*x^24)/15 + (2*a^7*b*x^3)/9 + (4*a*b^7*x^21)/9 + (28*a^6*b^
2*x^6)/33 + (28*a^5*b^3*x^9)/15 + (70*a^4*b^4*x^12)/27 + (7*a^3*b^5*x^15)/
3 + (4*a^2*b^6*x^18)/3)/x^39
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^3)^8}{x^{40}}\,dx$$
$$= \frac{-1287b^8x^{24} - 8580a\,b^7x^{21} - 25740a^2b^6x^{18} - 45045a^3b^5x^{15} - 50050a^4b^4x^{12} - 36036a^5b^3x^9 - 16380a^6b^2x}{19305x^{39}}$$

input
```
int((b*x^3+a)^8/x^40,x)
```

output
```
( - 495*a**8 - 4290*a**7*b*x**3 - 16380*a**6*b**2*x**6 - 36036*a**5*b**3*x
**9 - 50050*a**4*b**4*x**12 - 45045*a**3*b**5*x**15 - 25740*a**2*b**6*x**1
8 - 8580*a*b**7*x**21 - 1287*b**8*x**24)/(19305*x**39)
```

3.101 $\int \frac{(a+bx^3)^8}{x^{43}} dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = -\frac{a^8}{42x^{42}}-\frac{8a^7b}{39x^{39}}-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8ab^7}{21x^{21}}-\frac{b^8}{18x^{18}}$$

output
```
-1/42*a^8/x^42-8/39*a^7*b/x^39-7/9*a^6*b^2/x^36-56/33*a^5*b^3/x^33-7/3*a^4
*b^4/x^30-56/27*a^3*b^5/x^27-7/6*a^2*b^6/x^24-8/21*a*b^7/x^21-1/18*b^8/x^1
8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = -\frac{a^8}{42x^{42}}-\frac{8a^7b}{39x^{39}}-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8ab^7}{21x^{21}}-\frac{b^8}{18x^{18}}$$

input
```
Integrate[(a + b*x^3)^8/x^43,x]
```

output
```
-1/42*a^8/x^42 - (8*a^7*b)/(39*x^39) - (7*a^6*b^2)/(9*x^36) - (56*a^5*b^3)
/(33*x^33) - (7*a^4*b^4)/(3*x^30) - (56*a^3*b^5)/(27*x^27) - (7*a^2*b^6)/(
6*x^24) - (8*a*b^7)/(21*x^21) - b^8/(18*x^18)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{45}}dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3}\int \left(\frac{a^8}{x^{45}}+\frac{8ba^7}{x^{42}}+\frac{28b^2a^6}{x^{39}}+\frac{56b^3a^5}{x^{36}}+\frac{70b^4a^4}{x^{33}}+\frac{56b^5a^3}{x^{30}}+\frac{28b^6a^2}{x^{27}}+\frac{8b^7a}{x^{24}}+\frac{b^8}{x^{21}}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{a^8}{14x^{42}}-\frac{8a^7b}{13x^{39}}-\frac{7a^6b^2}{3x^{36}}-\frac{56a^5b^3}{11x^{33}}-\frac{7a^4b^4}{x^{30}}-\frac{56a^3b^5}{9x^{27}}-\frac{7a^2b^6}{2x^{24}}-\frac{8ab^7}{7x^{21}}-\frac{b^8}{6x^{18}}\right)$$

input
```
Int[(a + b*x^3)^8/x^43,x]
```

output
```
(-1/14*a^8/x^42 - (8*a^7*b)/(13*x^39) - (7*a^6*b^2)/(3*x^36) - (56*a^5*b^3
)/(11*x^33) - (7*a^4*b^4)/x^30 - (56*a^3*b^5)/(9*x^27) - (7*a^2*b^6)/(2*x^
24) - (8*a*b^7)/(7*x^21) - b^8/(6*x^18))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
default	$-\frac{a^8}{42x^{42}}-\frac{8a^7b}{39x^{39}}-\frac{7a^6b^2}{9x^{36}}-\frac{56a^5b^3}{33x^{33}}-\frac{7a^4b^4}{3x^{30}}-\frac{56a^3b^5}{27x^{27}}-\frac{7a^2b^6}{6x^{24}}-\frac{8a\,b^7}{21x^{21}}-\frac{b^8}{18x^{18}}$
risch	$\frac{-\frac{1}{42}a^8-\frac{7}{6}a^2b^6x^{18}-\frac{7}{9}a^6b^2x^6-\frac{7}{3}a^4b^4x^{12}-\frac{56}{33}a^5b^3x^9-\frac{8}{21}a\,b^7x^{21}-\frac{8}{39}a^7b\,x^3-\frac{1}{18}b^8x^{24}-\frac{56}{27}a^3b^5x^{15}}{x^{42}}$
gosper	$-\frac{3003b^8x^{24}+20592a\,b^7x^{21}+63063a^2b^6x^{18}+112112a^3b^5x^{15}+126126a^4b^4x^{12}+91728a^5b^3x^9+42042a^6b^2x^6+11088a^7b\,x^3+}{54054x^{42}}$
parallelrisch	$\frac{-3003b^8x^{24}-20592a\,b^7x^{21}-63063a^2b^6x^{18}-112112a^3b^5x^{15}-126126a^4b^4x^{12}-91728a^5b^3x^9-42042a^6b^2x^6-11088a^7b\,x^3-}{54054x^{42}}$
orering	$-\frac{3003b^8x^{24}+20592a\,b^7x^{21}+63063a^2b^6x^{18}+112112a^3b^5x^{15}+126126a^4b^4x^{12}+91728a^5b^3x^9+42042a^6b^2x^6+11088a^7b\,x^3+}{54054x^{42}}$

input
```
int((b*x^3+a)^8/x^43,x,method=_RETURNVERBOSE)
```

output
```
-1/42*a^8/x^42-8/39*a^7*b/x^39-7/9*a^6*b^2/x^36-56/33*a^5*b^3/x^33-7/3*a^4
*b^4/x^30-56/27*a^3*b^5/x^27-7/6*a^2*b^6/x^24-8/21*a*b^7/x^21-1/18*b^8/x^1
8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = -\frac{3003\,b^8x^{24} + 20592\,ab^7x^{21} + 63063\,a^2b^6x^{18} + 112112\,a^3b^5x^{15} + 126126\,a^4b^4x^{12} + 91728\,a^5b^3x^9 + 4204}{54054\,x^{42}}$$

input
```
integrate((b*x^3+a)^8/x^43,x, algorithm="fricas")
```

output
```
-1/54054*(3003*b^8*x^24 + 20592*a*b^7*x^21 + 63063*a^2*b^6*x^18 + 112112*a
^3*b^5*x^15 + 126126*a^4*b^4*x^12 + 91728*a^5*b^3*x^9 + 42042*a^6*b^2*x^6
+ 11088*a^7*b*x^3 + 1287*a^8)/x^42
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = \frac{-1287a^8 - 11088a^7bx^3 - 42042a^6b^2x^6 - 91728a^5b^3x^9 - 126126a^4b^4x^{12} - 112112a^3b^5x^{15} - 63063a^2b^6x^1}{54054x^{42}}$$

input
```
integrate((b*x**3+a)**8/x**43,x)
```

output
```
(-1287*a**8 - 11088*a**7*b*x**3 - 42042*a**6*b**2*x**6 - 91728*a**5*b**3*x
**9 - 126126*a**4*b**4*x**12 - 112112*a**3*b**5*x**15 - 63063*a**2*b**6*x*
*18 - 20592*a*b**7*x**21 - 3003*b**8*x**24)/(54054*x**42)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = -\frac{3003\,b^8x^{24} + 20592\,ab^7x^{21} + 63063\,a^2b^6x^{18} + 112112\,a^3b^5x^{15} + 126126\,a^4b^4x^{12} + 91728\,a^5b^3x^9 + 4204}{54054\,x^{42}}$$

input
```
integrate((b*x^3+a)^8/x^43,x, algorithm="maxima")
```

output
```
-1/54054*(3003*b^8*x^24 + 20592*a*b^7*x^21 + 63063*a^2*b^6*x^18 + 112112*a
^3*b^5*x^15 + 126126*a^4*b^4*x^12 + 91728*a^5*b^3*x^9 + 42042*a^6*b^2*x^6
+ 11088*a^7*b*x^3 + 1287*a^8)/x^42
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = -\frac{3003\,b^8x^{24} + 20592\,ab^7x^{21} + 63063\,a^2b^6x^{18} + 112112\,a^3b^5x^{15} + 126126\,a^4b^4x^{12} + 91728\,a^5b^3x^9 + 4204}{54054\,x^{42}}$$

input
```
integrate((b*x^3+a)^8/x^43,x, algorithm="giac")
```

output
```
-1/54054*(3003*b^8*x^24 + 20592*a*b^7*x^21 + 63063*a^2*b^6*x^18 + 112112*a
^3*b^5*x^15 + 126126*a^4*b^4*x^12 + 91728*a^5*b^3*x^9 + 42042*a^6*b^2*x^6
+ 11088*a^7*b*x^3 + 1287*a^8)/x^42
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = -\frac{\frac{a^8}{42}+\frac{8\,a^7\,b\,x^3}{39}+\frac{7\,a^6\,b^2\,x^6}{9}+\frac{56\,a^5\,b^3\,x^9}{33}+\frac{7\,a^4\,b^4\,x^{12}}{3}+\frac{56\,a^3\,b^5\,x^{15}}{27}+\frac{7\,a^2\,b^6\,x^{18}}{6}+\frac{8\,a\,b^7\,x^{21}}{21}+\frac{b^8\,x^{24}}{18}}{x^{42}}$$

input
```
int((a + b*x^3)^8/x^43,x)
```

output
```
-(a^8/42 + (b^8*x^24)/18 + (8*a^7*b*x^3)/39 + (8*a*b^7*x^21)/21 + (7*a^6*b
^2*x^6)/9 + (56*a^5*b^3*x^9)/33 + (7*a^4*b^4*x^12)/3 + (56*a^3*b^5*x^15)/2
7 + (7*a^2*b^6*x^18)/6)/x^42
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{43}}\,dx = \frac{-3003b^8x^{24}-20592a\,b^7x^{21}-63063a^2b^6x^{18}-112112a^3b^5x^{15}-126126a^4b^4x^{12}-91728a^5b^3x^9-42042a}{54054x^{42}}$$

input
```
int((b*x^3+a)^8/x^43,x)
```

output
```
( - 1287*a**8 - 11088*a**7*b*x**3 - 42042*a**6*b**2*x**6 - 91728*a**5*b**3
*x**9 - 126126*a**4*b**4*x**12 - 112112*a**3*b**5*x**15 - 63063*a**2*b**6*
x**18 - 20592*a*b**7*x**21 - 3003*b**8*x**24)/(54054*x**42)
```

3.102 $\int \frac{(a+bx^3)^8}{x^{46}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = -\frac{a^8}{45x^{45}} - \frac{4a^7b}{21x^{42}} - \frac{28a^6b^2}{39x^{39}} - \frac{14a^5b^3}{9x^{36}} - \frac{70a^4b^4}{33x^{33}} - \frac{28a^3b^5}{15x^{30}} - \frac{28a^2b^6}{27x^{27}} - \frac{ab^7}{3x^{24}} - \frac{b^8}{21x^{21}}$$

output
```
-1/45*a^8/x^45-4/21*a^7*b/x^42-28/39*a^6*b^2/x^39-14/9*a^5*b^3/x^36-70/33*
a^4*b^4/x^33-28/15*a^3*b^5/x^30-28/27*a^2*b^6/x^27-1/3*a*b^7/x^24-1/21*b^8
/x^21
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = -\frac{a^8}{45x^{45}} - \frac{4a^7b}{21x^{42}} - \frac{28a^6b^2}{39x^{39}} - \frac{14a^5b^3}{9x^{36}} - \frac{70a^4b^4}{33x^{33}} - \frac{28a^3b^5}{15x^{30}} - \frac{28a^2b^6}{27x^{27}} - \frac{ab^7}{3x^{24}} - \frac{b^8}{21x^{21}}$$

input
```
Integrate[(a + b*x^3)^8/x^46,x]
```

output
```
-1/45*a^8/x^45 - (4*a^7*b)/(21*x^42) - (28*a^6*b^2)/(39*x^39) - (14*a^5*b^
3)/(9*x^36) - (70*a^4*b^4)/(33*x^33) - (28*a^3*b^5)/(15*x^30) - (28*a^2*b^
6)/(27*x^27) - (a*b^7)/(3*x^24) - b^8/(21*x^21)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^8}{x^{48}}dx^3$$

$$\Big\downarrow \ 53$$

$$\frac{1}{3}\int \left(\frac{a^8}{x^{48}}+\frac{8ba^7}{x^{45}}+\frac{28b^2a^6}{x^{42}}+\frac{56b^3a^5}{x^{39}}+\frac{70b^4a^4}{x^{36}}+\frac{56b^5a^3}{x^{33}}+\frac{28b^6a^2}{x^{30}}+\frac{8b^7a}{x^{27}}+\frac{b^8}{x^{24}}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{a^8}{15x^{45}}-\frac{4a^7b}{7x^{42}}-\frac{28a^6b^2}{13x^{39}}-\frac{14a^5b^3}{3x^{36}}-\frac{70a^4b^4}{11x^{33}}-\frac{28a^3b^5}{5x^{30}}-\frac{28a^2b^6}{9x^{27}}-\frac{ab^7}{x^{24}}-\frac{b^8}{7x^{21}}\right)$$

input
```
Int[(a + b*x^3)^8/x^46,x]
```

output
```
(-1/15*a^8/x^45 - (4*a^7*b)/(7*x^42) - (28*a^6*b^2)/(13*x^39) - (14*a^5*b^
3)/(3*x^36) - (70*a^4*b^4)/(11*x^33) - (28*a^3*b^5)/(5*x^30) - (28*a^2*b^6
)/(9*x^27) - (a*b^7)/x^24 - b^8/(7*x^21))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
default	$-\frac{a^{8}}{45 x^{45}}-\frac{4 a^{7} b}{21 x^{42}}-\frac{28 a^{6} b^{2}}{39 x^{39}}-\frac{14 a^{5} b^{3}}{9 x^{36}}-\frac{70 a^{4} b^{4}}{33 x^{33}}-\frac{28 a^{3} b^{5}}{15 x^{30}}-\frac{28 a^{2} b^{6}}{27 x^{27}}-\frac{a \,b^{7}}{3 x^{24}}-\frac{b^{8}}{21 x^{21}}$
risch	$\frac{-\frac{1}{45} a^{8}-\frac{28}{15} a^{3} b^{5} x^{15}-\frac{1}{21} b^{8} x^{24}-\frac{4}{21} a^{7} b \,x^{3}-\frac{14}{9} a^{5} b^{3} x^{9}-\frac{28}{39} a^{6} b^{2} x^{6}-\frac{28}{27} a^{2} b^{6} x^{18}-\frac{1}{3} a \,b^{7} x^{21}-\frac{70}{33} a^{4} b^{4} x^{12}}{x^{45}}$
gosper	$-\frac{6435 b^{8} x^{24}+45045 a \,b^{7} x^{21}+140140 a^{2} b^{6} x^{18}+252252 a^{3} b^{5} x^{15}+286650 a^{4} b^{4} x^{12}+210210 a^{5} b^{3} x^{9}+97020 a^{6} b^{2} x^{6}+25740 a^{7} b \,x^{3}}{135135 x^{45}}$
parallelrisch	$\frac{-6435 b^{8} x^{24}-45045 a \,b^{7} x^{21}-140140 a^{2} b^{6} x^{18}-252252 a^{3} b^{5} x^{15}-286650 a^{4} b^{4} x^{12}-210210 a^{5} b^{3} x^{9}-97020 a^{6} b^{2} x^{6}-25740 a^{7} b \,x^{3}}{135135 x^{45}}$
orering	$-\frac{6435 b^{8} x^{24}+45045 a \,b^{7} x^{21}+140140 a^{2} b^{6} x^{18}+252252 a^{3} b^{5} x^{15}+286650 a^{4} b^{4} x^{12}+210210 a^{5} b^{3} x^{9}+97020 a^{6} b^{2} x^{6}+25740 a^{7} b \,x^{3}}{135135 x^{45}}$

input
```
int((b*x^3+a)^8/x^46,x,method=_RETURNVERBOSE)
```

output
```
-1/45*a^8/x^45-4/21*a^7*b/x^42-28/39*a^6*b^2/x^39-14/9*a^5*b^3/x^36-70/33*
a^4*b^4/x^33-28/15*a^3*b^5/x^30-28/27*a^2*b^6/x^27-1/3*a*b^7/x^24-1/21*b^8
/x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = -\frac{6435\,b^8x^{24} + 45045\,ab^7x^{21} + 140140\,a^2b^6x^{18} + 252252\,a^3b^5x^{15} + 286650\,a^4b^4x^{12} + 210210\,a^5b^3x^9 + 97}{135135\,x^{45}}$$

input
```
integrate((b*x^3+a)^8/x^46,x, algorithm="fricas")
```

output
```
-1/135135*(6435*b^8*x^24 + 45045*a*b^7*x^21 + 140140*a^2*b^6*x^18 + 252252
*a^3*b^5*x^15 + 286650*a^4*b^4*x^12 + 210210*a^5*b^3*x^9 + 97020*a^6*b^2*x
^6 + 25740*a^7*b*x^3 + 3003*a^8)/x^45
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = \frac{-3003a^8 - 25740a^7bx^3 - 97020a^6b^2x^6 - 210210a^5b^3x^9 - 286650a^4b^4x^{12} - 252252a^3b^5x^{15} - 140140a^2b^6}{135135x^{45}}$$

input
```
integrate((b*x**3+a)**8/x**46,x)
```

output
```
(-3003*a**8 - 25740*a**7*b*x**3 - 97020*a**6*b**2*x**6 - 210210*a**5*b**3*
x**9 - 286650*a**4*b**4*x**12 - 252252*a**3*b**5*x**15 - 140140*a**2*b**6*
x**18 - 45045*a*b**7*x**21 - 6435*b**8*x**24)/(135135*x**45)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = -\frac{6435\,b^8x^{24}+45045\,ab^7x^{21}+140140\,a^2b^6x^{18}+252252\,a^3b^5x^{15}+286650\,a^4b^4x^{12}+210210\,a^5b^3x^9+97}{135135\,x^{45}}$$

input
```
integrate((b*x^3+a)^8/x^46,x, algorithm="maxima")
```

output
```
-1/135135*(6435*b^8*x^24 + 45045*a*b^7*x^21 + 140140*a^2*b^6*x^18 + 252252
*a^3*b^5*x^15 + 286650*a^4*b^4*x^12 + 210210*a^5*b^3*x^9 + 97020*a^6*b^2*x
^6 + 25740*a^7*b*x^3 + 3003*a^8)/x^45
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = -\frac{6435\,b^8x^{24}+45045\,ab^7x^{21}+140140\,a^2b^6x^{18}+252252\,a^3b^5x^{15}+286650\,a^4b^4x^{12}+210210\,a^5b^3x^9+97}{135135\,x^{45}}$$

input
```
integrate((b*x^3+a)^8/x^46,x, algorithm="giac")
```

output
```
-1/135135*(6435*b^8*x^24 + 45045*a*b^7*x^21 + 140140*a^2*b^6*x^18 + 252252
*a^3*b^5*x^15 + 286650*a^4*b^4*x^12 + 210210*a^5*b^3*x^9 + 97020*a^6*b^2*x
^6 + 25740*a^7*b*x^3 + 3003*a^8)/x^45
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = -\frac{\frac{a^8}{45}+\frac{4\,a^7\,b\,x^3}{21}+\frac{28\,a^6\,b^2\,x^6}{39}+\frac{14\,a^5\,b^3\,x^9}{9}+\frac{70\,a^4\,b^4\,x^{12}}{33}+\frac{28\,a^3\,b^5\,x^{15}}{15}+\frac{28\,a^2\,b^6\,x^{18}}{27}+\frac{a\,b^7\,x^{21}}{3}+\frac{b^8\,x^{24}}{21}}{x^{45}}$$

input
```
int((a + b*x^3)^8/x^46,x)
```

output
```
-(a^8/45 + (b^8*x^24)/21 + (4*a^7*b*x^3)/21 + (a*b^7*x^21)/3 + (28*a^6*b^2
*x^6)/39 + (14*a^5*b^3*x^9)/9 + (70*a^4*b^4*x^12)/33 + (28*a^3*b^5*x^15)/1
5 + (28*a^2*b^6*x^18)/27)/x^45
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^8}{x^{46}}\,dx = \frac{-6435b^8x^{24}-45045a\,b^7x^{21}-140140a^2b^6x^{18}-252252a^3b^5x^{15}-286650a^4b^4x^{12}-210210a^5b^3x^9-9702}{135135x^{45}}$$

input
```
int((b*x^3+a)^8/x^46,x)
```

output
```
( - 3003*a**8 - 25740*a**7*b*x**3 - 97020*a**6*b**2*x**6 - 210210*a**5*b**
3*x**9 - 286650*a**4*b**4*x**12 - 252252*a**3*b**5*x**15 - 140140*a**2*b**
6*x**18 - 45045*a*b**7*x**21 - 6435*b**8*x**24)/(135135*x**45)
```

3.103 $\int x^4(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 103

$$\int x^4(a+bx^3)^8\,dx=\frac{a^8x^5}{5}+a^7bx^8+\frac{28}{11}a^6b^2x^{11}+4a^5b^3x^{14}+\frac{70}{17}a^4b^4x^{17}+\frac{14}{5}a^3b^5x^{20}+\frac{28}{23}a^2b^6x^{23}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{29}}{29}$$

output
```
1/5*a^8*x^5+a^7*b*x^8+28/11*a^6*b^2*x^11+4*a^5*b^3*x^14+70/17*a^4*b^4*x^17
+14/5*a^3*b^5*x^20+28/23*a^2*b^6*x^23+4/13*a*b^7*x^26+1/29*b^8*x^29
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^3)^8\,dx=\frac{a^8x^5}{5}+a^7bx^8+\frac{28}{11}a^6b^2x^{11}+4a^5b^3x^{14}+\frac{70}{17}a^4b^4x^{17}+\frac{14}{5}a^3b^5x^{20}+\frac{28}{23}a^2b^6x^{23}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{29}}{29}$$

input
```
Integrate[x^4*(a + b*x^3)^8,x]
```

output
```
(a^8*x^5)/5 + a^7*b*x^8 + (28*a^6*b^2*x^11)/11 + 4*a^5*b^3*x^14 + (70*a^4*
b^4*x^17)/17 + (14*a^3*b^5*x^20)/5 + (28*a^2*b^6*x^23)/23 + (4*a*b^7*x^26)
/13 + (b^8*x^29)/29
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^8 \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(a^8x^4+8a^7bx^7+28a^6b^2x^{10}+56a^5b^3x^{13}+70a^4b^4x^{16}+56a^3b^5x^{19}+28a^2b^6x^{22}+8ab^7x^{25}+b^8x^{28}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^8x^5}{5}+a^7bx^8+\frac{28}{11}a^6b^2x^{11}+4a^5b^3x^{14}+\frac{70}{17}a^4b^4x^{17}+\frac{14}{5}a^3b^5x^{20}+\frac{28}{23}a^2b^6x^{23}+\frac{4}{13}ab^7x^{26}+\frac{b^8x^{29}}{29}$$

input
```
Int[x^4*(a + b*x^3)^8,x]
```

output

```
(a^8*x^5)/5 + a^7*b*x^8 + (28*a^6*b^2*x^11)/11 + 4*a^5*b^3*x^14 + (70*a^4*
b^4*x^17)/17 + (14*a^3*b^5*x^20)/5 + (28*a^2*b^6*x^23)/23 + (4*a*b^7*x^26)
/13 + (b^8*x^29)/29
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

method	result
gosper	$\frac{1}{5}a^8x^5 + a^7b\,x^8 + \frac{28}{11}a^6b^2x^{11} + 4a^5b^3x^{14} + \frac{70}{17}a^4b^4x^{17} + \frac{14}{5}a^3b^5x^{20} + \frac{28}{23}a^2b^6x^{23} + \frac{4}{13}a\,b^7x^{26} +$
default	$\frac{1}{5}a^8x^5 + a^7b\,x^8 + \frac{28}{11}a^6b^2x^{11} + 4a^5b^3x^{14} + \frac{70}{17}a^4b^4x^{17} + \frac{14}{5}a^3b^5x^{20} + \frac{28}{23}a^2b^6x^{23} + \frac{4}{13}a\,b^7x^{26} +$
norman	$\frac{1}{5}a^8x^5 + a^7b\,x^8 + \frac{28}{11}a^6b^2x^{11} + 4a^5b^3x^{14} + \frac{70}{17}a^4b^4x^{17} + \frac{14}{5}a^3b^5x^{20} + \frac{28}{23}a^2b^6x^{23} + \frac{4}{13}a\,b^7x^{26} +$
risch	$\frac{1}{5}a^8x^5 + a^7b\,x^8 + \frac{28}{11}a^6b^2x^{11} + 4a^5b^3x^{14} + \frac{70}{17}a^4b^4x^{17} + \frac{14}{5}a^3b^5x^{20} + \frac{28}{23}a^2b^6x^{23} + \frac{4}{13}a\,b^7x^{26} +$
parallelrisch	$\frac{1}{5}a^8x^5 + a^7b\,x^8 + \frac{28}{11}a^6b^2x^{11} + 4a^5b^3x^{14} + \frac{70}{17}a^4b^4x^{17} + \frac{14}{5}a^3b^5x^{20} + \frac{28}{23}a^2b^6x^{23} + \frac{4}{13}a\,b^7x^{26} +$
orering	$\frac{x^5(279565b^8x^{24}+2494580a\,b^7x^{21}+9869860a^2b^6x^{18}+22700678a^3b^5x^{15}+33383350a^4b^4x^{12}+32429540a^5b^3x^9+20636980a^6b}{8107385}$

input
```
int(x^4*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^8*x^5+a^7*b*x^8+28/11*a^6*b^2*x^11+4*a^5*b^3*x^14+70/17*a^4*b^4*x^17
+14/5*a^3*b^5*x^20+28/23*a^2*b^6*x^23+4/13*a*b^7*x^26+1/29*b^8*x^29
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int x^4(a+bx^3)^8\,dx = \frac{1}{29}\,b^8x^{29} + \frac{4}{13}\,ab^7x^{26} + \frac{28}{23}\,a^2b^6x^{23} + \frac{14}{5}\,a^3b^5x^{20} + \frac{70}{17}\,a^4b^4x^{17} + 4\,a^5b^3x^{14} + \frac{28}{11}\,a^6b^2x^{11} + a^7bx^8 + \frac{1}{5}\,a^8x^5$$

input
```
integrate(x^4*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/29*b^8*x^29 + 4/13*a*b^7*x^26 + 28/23*a^2*b^6*x^23 + 14/5*a^3*b^5*x^20 +
 70/17*a^4*b^4*x^17 + 4*a^5*b^3*x^14 + 28/11*a^6*b^2*x^11 + a^7*b*x^8 + 1/
5*a^8*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.99

$$\int x^4(a+bx^3)^8\,dx = \frac{a^8x^5}{5} + a^7bx^8 + \frac{28a^6b^2x^{11}}{11} + 4a^5b^3x^{14} + \frac{70a^4b^4x^{17}}{17} + \frac{14a^3b^5x^{20}}{5} + \frac{28a^2b^6x^{23}}{23} + \frac{4ab^7x^{26}}{13} + \frac{b^8x^{29}}{29}$$

input
```
integrate(x**4*(b*x**3+a)**8,x)
```

output
```
a**8*x**5/5 + a**7*b*x**8 + 28*a**6*b**2*x**11/11 + 4*a**5*b**3*x**14 + 70
*a**4*b**4*x**17/17 + 14*a**3*b**5*x**20/5 + 28*a**2*b**6*x**23/23 + 4*a*b
**7*x**26/13 + b**8*x**29/29
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int x^4(a+bx^3)^8\,dx = \frac{1}{29}\,b^8x^{29} + \frac{4}{13}\,ab^7x^{26} + \frac{28}{23}\,a^2b^6x^{23} + \frac{14}{5}\,a^3b^5x^{20} + \frac{70}{17}\,a^4b^4x^{17} + 4\,a^5b^3x^{14} + \frac{28}{11}\,a^6b^2x^{11} + a^7bx^8 + \frac{1}{5}\,a^8x^5$$

input
```
integrate(x^4*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/29*b^8*x^29 + 4/13*a*b^7*x^26 + 28/23*a^2*b^6*x^23 + 14/5*a^3*b^5*x^20 +
 70/17*a^4*b^4*x^17 + 4*a^5*b^3*x^14 + 28/11*a^6*b^2*x^11 + a^7*b*x^8 + 1/
5*a^8*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int x^4 (a+bx^3)^8 \, dx = \frac{1}{29} b^8 x^{29} + \frac{4}{13} a b^7 x^{26} + \frac{28}{23} a^2 b^6 x^{23} + \frac{14}{5} a^3 b^5 x^{20} + \frac{70}{17} a^4 b^4 x^{17} + 4 a^5 b^3 x^{14} + \frac{28}{11} a^6 b^2 x^{11} + a^7 b x^8 + \frac{1}{5} a^8 x^5$$

input
```
integrate(x^4*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/29*b^8*x^29 + 4/13*a*b^7*x^26 + 28/23*a^2*b^6*x^23 + 14/5*a^3*b^5*x^20 +
 70/17*a^4*b^4*x^17 + 4*a^5*b^3*x^14 + 28/11*a^6*b^2*x^11 + a^7*b*x^8 + 1/
5*a^8*x^5
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int x^4 (a+bx^3)^8 \, dx = \frac{a^8 x^5}{5} + a^7 b x^8 + \frac{28 a^6 b^2 x^{11}}{11} + 4 a^5 b^3 x^{14} + \frac{70 a^4 b^4 x^{17}}{17} + \frac{14 a^3 b^5 x^{20}}{5} + \frac{28 a^2 b^6 x^{23}}{23} + \frac{4 a b^7 x^{26}}{13} + \frac{b^8 x^{29}}{29}$$

input
```
int(x^4*(a + b*x^3)^8,x)
```

output
```
(a^8*x^5)/5 + (b^8*x^29)/29 + a^7*b*x^8 + (4*a*b^7*x^26)/13 + (28*a^6*b^2*
x^11)/11 + 4*a^5*b^3*x^14 + (70*a^4*b^4*x^17)/17 + (14*a^3*b^5*x^20)/5 + (
28*a^2*b^6*x^23)/23
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

$$\int x^4(a+bx^3)^8\,dx$$
$$=\frac{x^5(279565b^8x^{24}+2494580a\,b^7x^{21}+9869860a^2b^6x^{18}+22700678a^3b^5x^{15}+33383350a^4b^4x^{12}+32429540}{8107385}$$

input
```
int(x^4*(b*x^3+a)^8,x)
```

output
```
(x**5*(1621477*a**8 + 8107385*a**7*b*x**3 + 20636980*a**6*b**2*x**6 + 3242
9540*a**5*b**3*x**9 + 33383350*a**4*b**4*x**12 + 22700678*a**3*b**5*x**15
+ 9869860*a**2*b**6*x**18 + 2494580*a*b**7*x**21 + 279565*b**8*x**24))/810
7385
```

3.104 $\int x^3(a+bx^3)^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int x^3(a+bx^3)^8 \, dx = \frac{a^8x^4}{4}+\frac{8}{7}a^7bx^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}ab^7x^{25}+\frac{b^8x^{28}}{28}$$

output
```
1/4*a^8*x^4+8/7*a^7*b*x^7+14/5*a^6*b^2*x^10+56/13*a^5*b^3*x^13+35/8*a^4*b^
4*x^16+56/19*a^3*b^5*x^19+14/11*a^2*b^6*x^22+8/25*a*b^7*x^25+1/28*b^8*x^28
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^3)^8 \, dx = \frac{a^8x^4}{4}+\frac{8}{7}a^7bx^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}ab^7x^{25}+\frac{b^8x^{28}}{28}$$

input
```
Integrate[x^3*(a + b*x^3)^8,x]
```

output
```
(a^8*x^4)/4 + (8*a^7*b*x^7)/7 + (14*a^6*b^2*x^10)/5 + (56*a^5*b^3*x^13)/13
 + (35*a^4*b^4*x^16)/8 + (56*a^3*b^5*x^19)/19 + (14*a^2*b^6*x^22)/11 + (8*
a*b^7*x^25)/25 + (b^8*x^28)/28
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^8 \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(a^8x^3+8a^7bx^6+28a^6b^2x^9+56a^5b^3x^{12}+70a^4b^4x^{15}+56a^3b^5x^{18}+28a^2b^6x^{21}+8ab^7x^{24}+b^8x^{27}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^8x^4}{4}+\frac{8}{7}a^7bx^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}ab^7x^{25}+\frac{b^8x^{28}}{28}$$

input
```
Int[x^3*(a + b*x^3)^8,x]
```

output

```
(a^8*x^4)/4 + (8*a^7*b*x^7)/7 + (14*a^6*b^2*x^10)/5 + (56*a^5*b^3*x^13)/13
 + (35*a^4*b^4*x^16)/8 + (56*a^3*b^5*x^19)/19 + (14*a^2*b^6*x^22)/11 + (8*
a*b^7*x^25)/25 + (b^8*x^28)/28
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{1}{4}a^8x^4+\frac{8}{7}a^7b\,x^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}a\,b^7x^{25}$
default	$\frac{1}{4}a^8x^4+\frac{8}{7}a^7b\,x^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}a\,b^7x^{25}$
norman	$\frac{1}{4}a^8x^4+\frac{8}{7}a^7b\,x^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}a\,b^7x^{25}$
risch	$\frac{1}{4}a^8x^4+\frac{8}{7}a^7b\,x^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}a\,b^7x^{25}$
parallelrisch	$\frac{1}{4}a^8x^4+\frac{8}{7}a^7b\,x^7+\frac{14}{5}a^6b^2x^{10}+\frac{56}{13}a^5b^3x^{13}+\frac{35}{8}a^4b^4x^{16}+\frac{56}{19}a^3b^5x^{19}+\frac{14}{11}a^2b^6x^{22}+\frac{8}{25}a\,b^7x^{25}$
orering	$\frac{x^4(135850b^8x^{24}+1217216a\,b^7x^{21}+4841200a^2b^6x^{18}+11211200a^3b^5x^{15}+16641625a^4b^4x^{12}+16385600a^5b^3x^9+10650640a^6b}{3803800}$

input
```
int(x^3*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^8*x^4+8/7*a^7*b*x^7+14/5*a^6*b^2*x^10+56/13*a^5*b^3*x^13+35/8*a^4*b^
4*x^16+56/19*a^3*b^5*x^19+14/11*a^2*b^6*x^22+8/25*a*b^7*x^25+1/28*b^8*x^28
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3(a+bx^3)^8\,dx=\frac{1}{28}\,b^8x^{28}+\frac{8}{25}\,ab^7x^{25}+\frac{14}{11}\,a^2b^6x^{22}+\frac{56}{19}\,a^3b^5x^{19}+\frac{35}{8}\,a^4b^4x^{16}+\frac{56}{13}\,a^5b^3x^{13}+\frac{14}{5}\,a^6b^2x^{10}+\frac{8}{7}\,a^7bx^7+\frac{1}{4}\,a^8x^4$$

input
```
integrate(x^3*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/28*b^8*x^28 + 8/25*a*b^7*x^25 + 14/11*a^2*b^6*x^22 + 56/19*a^3*b^5*x^19
+ 35/8*a^4*b^4*x^16 + 56/13*a^5*b^3*x^13 + 14/5*a^6*b^2*x^10 + 8/7*a^7*b*x
^7 + 1/4*a^8*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^3)^8\,dx = \frac{a^8x^4}{4} + \frac{8a^7bx^7}{7} + \frac{14a^6b^2x^{10}}{5} + \frac{56a^5b^3x^{13}}{13} + \frac{35a^4b^4x^{16}}{8} + \frac{56a^3b^5x^{19}}{19} + \frac{14a^2b^6x^{22}}{11} + \frac{8ab^7x^{25}}{25} + \frac{b^8x^{28}}{28}$$

input
```
integrate(x**3*(b*x**3+a)**8,x)
```

output
```
a**8*x**4/4 + 8*a**7*b*x**7/7 + 14*a**6*b**2*x**10/5 + 56*a**5*b**3*x**13/
13 + 35*a**4*b**4*x**16/8 + 56*a**3*b**5*x**19/19 + 14*a**2*b**6*x**22/11
+ 8*a*b**7*x**25/25 + b**8*x**28/28
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3(a+bx^3)^8\,dx = \frac{1}{28}\,b^8x^{28} + \frac{8}{25}\,ab^7x^{25} + \frac{14}{11}\,a^2b^6x^{22} + \frac{56}{19}\,a^3b^5x^{19} + \frac{35}{8}\,a^4b^4x^{16} + \frac{56}{13}\,a^5b^3x^{13} + \frac{14}{5}\,a^6b^2x^{10} + \frac{8}{7}\,a^7bx^7 + \frac{1}{4}\,a^8x^4$$

input
```
integrate(x^3*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/28*b^8*x^28 + 8/25*a*b^7*x^25 + 14/11*a^2*b^6*x^22 + 56/19*a^3*b^5*x^19
+ 35/8*a^4*b^4*x^16 + 56/13*a^5*b^3*x^13 + 14/5*a^6*b^2*x^10 + 8/7*a^7*b*x
^7 + 1/4*a^8*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3 \left(a+bx^3\right)^8 \, dx = \frac{1}{28} b^8 x^{28} + \frac{8}{25} a b^7 x^{25} + \frac{14}{11} a^2 b^6 x^{22} + \frac{56}{19} a^3 b^5 x^{19} + \frac{35}{8} a^4 b^4 x^{16} + \frac{56}{13} a^5 b^3 x^{13} + \frac{14}{5} a^6 b^2 x^{10} + \frac{8}{7} a^7 b x^7 + \frac{1}{4} a^8 x^4$$

input
```
integrate(x^3*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/28*b^8*x^28 + 8/25*a*b^7*x^25 + 14/11*a^2*b^6*x^22 + 56/19*a^3*b^5*x^19
+ 35/8*a^4*b^4*x^16 + 56/13*a^5*b^3*x^13 + 14/5*a^6*b^2*x^10 + 8/7*a^7*b*x
^7 + 1/4*a^8*x^4
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^3 \left(a+bx^3\right)^8 \, dx = \frac{a^8\,x^4}{4} + \frac{8\,a^7\,b\,x^7}{7} + \frac{14\,a^6\,b^2\,x^{10}}{5} + \frac{56\,a^5\,b^3\,x^{13}}{13} + \frac{35\,a^4\,b^4\,x^{16}}{8} + \frac{56\,a^3\,b^5\,x^{19}}{19} + \frac{14\,a^2\,b^6\,x^{22}}{11} + \frac{8\,a\,b^7\,x^{25}}{25} + \frac{b^8\,x^{28}}{28}$$

input
```
int(x^3*(a + b*x^3)^8,x)
```

output
```
(a^8*x^4)/4 + (b^8*x^28)/28 + (8*a^7*b*x^7)/7 + (8*a*b^7*x^25)/25 + (14*a^
6*b^2*x^10)/5 + (56*a^5*b^3*x^13)/13 + (35*a^4*b^4*x^16)/8 + (56*a^3*b^5*x
^19)/19 + (14*a^2*b^6*x^22)/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int x^3(a+bx^3)^8\,dx$$
$$=\frac{x^4(135850b^8x^{24}+1217216a\,b^7x^{21}+4841200a^2b^6x^{18}+11211200a^3b^5x^{15}+16641625a^4b^4x^{12}+16385600}{3803800}$$

input
```
int(x^3*(b*x^3+a)^8,x)
```

output
```
(x**4*(950950*a**8 + 4347200*a**7*b*x**3 + 10650640*a**6*b**2*x**6 + 16385
600*a**5*b**3*x**9 + 16641625*a**4*b**4*x**12 + 11211200*a**3*b**5*x**15 +
 4841200*a**2*b**6*x**18 + 1217216*a*b**7*x**21 + 135850*b**8*x**24))/3803
800
```

3.105 $\int x(a+bx^3)^8\,dx$

Optimal result

Integrand size = 11, antiderivative size = 106

$$\int x\left(a+bx^3\right)^8\,dx=\frac{a^8x^2}{2}+\frac{8}{5}a^7bx^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{26}}{26}$$

output
```
1/2*a^8*x^2+8/5*a^7*b*x^5+7/2*a^6*b^2*x^8+56/11*a^5*b^3*x^11+5*a^4*b^4*x^1
4+56/17*a^3*b^5*x^17+7/5*a^2*b^6*x^20+8/23*a*b^7*x^23+1/26*b^8*x^26
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^3\right)^8\,dx=\frac{a^8x^2}{2}+\frac{8}{5}a^7bx^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{26}}{26}$$

input
```
Integrate[x*(a + b*x^3)^8,x]
```

output
```
(a^8*x^2)/2 + (8*a^7*b*x^5)/5 + (7*a^6*b^2*x^8)/2 + (56*a^5*b^3*x^11)/11 +
 5*a^4*b^4*x^14 + (56*a^3*b^5*x^17)/17 + (7*a^2*b^6*x^20)/5 + (8*a*b^7*x^2
3)/23 + (b^8*x^26)/26
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^8\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^8x+8a^7bx^4+28a^6b^2x^7+56a^5b^3x^{10}+70a^4b^4x^{13}+56a^3b^5x^{16}+28a^2b^6x^{19}+8ab^7x^{22}+b^8x^{25}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^2}{2}+\frac{8}{5}a^7bx^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{26}}{26}$$

input
```
Int[x*(a + b*x^3)^8,x]
```

output

```
(a^8*x^2)/2 + (8*a^7*b*x^5)/5 + (7*a^6*b^2*x^8)/2 + (56*a^5*b^3*x^11)/11 +
 5*a^4*b^4*x^14 + (56*a^3*b^5*x^17)/17 + (7*a^2*b^6*x^20)/5 + (8*a*b^7*x^2
3)/23 + (b^8*x^26)/26
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{1}{2}a^8x^2+\frac{8}{5}a^7b\,x^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}a\,b^7x^{23}+$
default	$\frac{1}{2}a^8x^2+\frac{8}{5}a^7b\,x^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}a\,b^7x^{23}+$
norman	$\frac{1}{2}a^8x^2+\frac{8}{5}a^7b\,x^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}a\,b^7x^{23}+$
risch	$\frac{1}{2}a^8x^2+\frac{8}{5}a^7b\,x^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}a\,b^7x^{23}+$
parallelrisch	$\frac{1}{2}a^8x^2+\frac{8}{5}a^7b\,x^5+\frac{7}{2}a^6b^2x^8+\frac{56}{11}a^5b^3x^{11}+5a^4b^4x^{14}+\frac{56}{17}a^3b^5x^{17}+\frac{7}{5}a^2b^6x^{20}+\frac{8}{23}a\,b^7x^{23}+$
orering	$\frac{x^2(21505b^8x^{24}+194480a\,b^7x^{21}+782782a^2b^6x^{18}+1841840a^3b^5x^{15}+2795650a^4b^4x^{12}+2846480a^5b^3x^9+1956955a^6b^2x^6+894}{559130}$

input
```
int(x*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^8*x^2+8/5*a^7*b*x^5+7/2*a^6*b^2*x^8+56/11*a^5*b^3*x^11+5*a^4*b^4*x^1
4+56/17*a^3*b^5*x^17+7/5*a^2*b^6*x^20+8/23*a*b^7*x^23+1/26*b^8*x^26
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)^8\,dx = \frac{1}{26}\,b^8x^{26}+\frac{8}{23}\,ab^7x^{23}+\frac{7}{5}\,a^2b^6x^{20}+\frac{56}{17}\,a^3b^5x^{17}+5\,a^4b^4x^{14}+\frac{56}{11}\,a^5b^3x^{11}+\frac{7}{2}\,a^6b^2x^8+\frac{8}{5}\,a^7bx^5+\frac{1}{2}\,a^8x^2$$

input
```
integrate(x*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/26*b^8*x^26 + 8/23*a*b^7*x^23 + 7/5*a^2*b^6*x^20 + 56/17*a^3*b^5*x^17 +
5*a^4*b^4*x^14 + 56/11*a^5*b^3*x^11 + 7/2*a^6*b^2*x^8 + 8/5*a^7*b*x^5 + 1/
2*a^8*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int x(a+bx^3)^8\,dx = \frac{a^8x^2}{2}+\frac{8a^7bx^5}{5}+\frac{7a^6b^2x^8}{2}+\frac{56a^5b^3x^{11}}{11}+5a^4b^4x^{14}+\frac{56a^3b^5x^{17}}{17}+\frac{7a^2b^6x^{20}}{5}+\frac{8ab^7x^{23}}{23}+\frac{b^8x^{26}}{26}$$

input
```
integrate(x*(b*x**3+a)**8,x)
```

output
```
a**8*x**2/2 + 8*a**7*b*x**5/5 + 7*a**6*b**2*x**8/2 + 56*a**5*b**3*x**11/11
 + 5*a**4*b**4*x**14 + 56*a**3*b**5*x**17/17 + 7*a**2*b**6*x**20/5 + 8*a*b
**7*x**23/23 + b**8*x**26/26
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)^8\,dx = \frac{1}{26}\,b^8x^{26}+\frac{8}{23}\,ab^7x^{23}+\frac{7}{5}\,a^2b^6x^{20}+\frac{56}{17}\,a^3b^5x^{17}+5\,a^4b^4x^{14}+\frac{56}{11}\,a^5b^3x^{11}+\frac{7}{2}\,a^6b^2x^8+\frac{8}{5}\,a^7bx^5+\frac{1}{2}\,a^8x^2$$

input
```
integrate(x*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/26*b^8*x^26 + 8/23*a*b^7*x^23 + 7/5*a^2*b^6*x^20 + 56/17*a^3*b^5*x^17 +
5*a^4*b^4*x^14 + 56/11*a^5*b^3*x^11 + 7/2*a^6*b^2*x^8 + 8/5*a^7*b*x^5 + 1/
2*a^8*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)^8\,dx = \frac{1}{26}b^8x^{26} + \frac{8}{23}ab^7x^{23} + \frac{7}{5}a^2b^6x^{20} + \frac{56}{17}a^3b^5x^{17} + 5a^4b^4x^{14} + \frac{56}{11}a^5b^3x^{11} + \frac{7}{2}a^6b^2x^8 + \frac{8}{5}a^7bx^5 + \frac{1}{2}a^8x^2$$

input
```
integrate(x*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/26*b^8*x^26 + 8/23*a*b^7*x^23 + 7/5*a^2*b^6*x^20 + 56/17*a^3*b^5*x^17 +
5*a^4*b^4*x^14 + 56/11*a^5*b^3*x^11 + 7/2*a^6*b^2*x^8 + 8/5*a^7*b*x^5 + 1/
2*a^8*x^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x(a+bx^3)^8\,dx = \frac{a^8x^2}{2} + \frac{8a^7bx^5}{5} + \frac{7a^6b^2x^8}{2} + \frac{56a^5b^3x^{11}}{11} + 5a^4b^4x^{14} + \frac{56a^3b^5x^{17}}{17} + \frac{7a^2b^6x^{20}}{5} + \frac{8ab^7x^{23}}{23} + \frac{b^8x^{26}}{26}$$

input
```
int(x*(a + b*x^3)^8,x)
```

output
```
(a^8*x^2)/2 + (b^8*x^26)/26 + (8*a^7*b*x^5)/5 + (8*a*b^7*x^23)/23 + (7*a^6
*b^2*x^8)/2 + (56*a^5*b^3*x^11)/11 + 5*a^4*b^4*x^14 + (56*a^3*b^5*x^17)/17
 + (7*a^2*b^6*x^20)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int x(a+bx^3)^8\,dx = \frac{x^2(21505b^8x^{24}+194480a\,b^7x^{21}+782782a^2b^6x^{18}+1841840a^3b^5x^{15}+2795650a^4b^4x^{12}+2846480a^5b^3x^9}{559130}$$

input
```
int(x*(b*x^3+a)^8,x)
```

output
```
(x**2*(279565*a**8 + 894608*a**7*b*x**3 + 1956955*a**6*b**2*x**6 + 2846480
*a**5*b**3*x**9 + 2795650*a**4*b**4*x**12 + 1841840*a**3*b**5*x**15 + 7827
82*a**2*b**6*x**18 + 194480*a*b**7*x**21 + 21505*b**8*x**24))/559130
```

3.106 $\int (a+bx^3)^8\, dx$

Optimal result

Integrand size = 9, antiderivative size = 99

$$\int (a+bx^3)^8\, dx = a^8x+2a^7bx^4+4a^6b^2x^7+\frac{28}{5}a^5b^3x^{10}+\frac{70}{13}a^4b^4x^{13}+\frac{7}{2}a^3b^5x^{16}+\frac{28}{19}a^2b^6x^{19}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{25}}{25}$$

output
```
a^8*x+2*a^7*b*x^4+4*a^6*b^2*x^7+28/5*a^5*b^3*x^10+70/13*a^4*b^4*x^13+7/2*a
^3*b^5*x^16+28/19*a^2*b^6*x^19+4/11*a*b^7*x^22+1/25*b^8*x^25
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int (a+bx^3)^8\, dx = a^8x+2a^7bx^4+4a^6b^2x^7+\frac{28}{5}a^5b^3x^{10}+\frac{70}{13}a^4b^4x^{13}+\frac{7}{2}a^3b^5x^{16}+\frac{28}{19}a^2b^6x^{19}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{25}}{25}$$

input
```
Integrate[(a + b*x^3)^8,x]
```

output
```
a^8*x + 2*a^7*b*x^4 + 4*a^6*b^2*x^7 + (28*a^5*b^3*x^10)/5 + (70*a^4*b^4*x^
13)/13 + (7*a^3*b^5*x^16)/2 + (28*a^2*b^6*x^19)/19 + (4*a*b^7*x^22)/11 + (
b^8*x^25)/25
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^8 \, dx$$

$$\Big\downarrow \text{ 747}$$

$$\int \left(a^8+8a^7bx^3+28a^6b^2x^6+56a^5b^3x^9+70a^4b^4x^{12}+56a^3b^5x^{15}+28a^2b^6x^{18}+8ab^7x^{21}+b^8x^{24}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^8x+2a^7bx^4+4a^6b^2x^7+\frac{28}{5}a^5b^3x^{10}+\frac{70}{13}a^4b^4x^{13}+\frac{7}{2}a^3b^5x^{16}+\frac{28}{19}a^2b^6x^{19}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{25}}{25}$$

input
```
Int[(a + b*x^3)^8,x]
```

output
```
a^8*x + 2*a^7*b*x^4 + 4*a^6*b^2*x^7 + (28*a^5*b^3*x^10)/5 + (70*a^4*b^4*x^
13)/13 + (7*a^3*b^5*x^16)/2 + (28*a^2*b^6*x^19)/19 + (4*a*b^7*x^22)/11 + (
b^8*x^25)/25
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
gosper	$a^{8}x+2a^{7}b\,x^{4}+4a^{6}b^{2}x^{7}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{13}a^{4}b^{4}x^{13}+\frac{7}{2}a^{3}b^{5}x^{16}+\frac{28}{19}a^{2}b^{6}x^{19}+\frac{4}{11}a\,b^{7}x^{22}+\frac{1}{25}$
default	$a^{8}x+2a^{7}b\,x^{4}+4a^{6}b^{2}x^{7}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{13}a^{4}b^{4}x^{13}+\frac{7}{2}a^{3}b^{5}x^{16}+\frac{28}{19}a^{2}b^{6}x^{19}+\frac{4}{11}a\,b^{7}x^{22}+\frac{1}{25}$
norman	$a^{8}x+2a^{7}b\,x^{4}+4a^{6}b^{2}x^{7}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{13}a^{4}b^{4}x^{13}+\frac{7}{2}a^{3}b^{5}x^{16}+\frac{28}{19}a^{2}b^{6}x^{19}+\frac{4}{11}a\,b^{7}x^{22}+\frac{1}{25}$
risch	$a^{8}x+2a^{7}b\,x^{4}+4a^{6}b^{2}x^{7}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{13}a^{4}b^{4}x^{13}+\frac{7}{2}a^{3}b^{5}x^{16}+\frac{28}{19}a^{2}b^{6}x^{19}+\frac{4}{11}a\,b^{7}x^{22}+\frac{1}{25}$
parallelrisch	$a^{8}x+2a^{7}b\,x^{4}+4a^{6}b^{2}x^{7}+\frac{28}{5}a^{5}b^{3}x^{10}+\frac{70}{13}a^{4}b^{4}x^{13}+\frac{7}{2}a^{3}b^{5}x^{16}+\frac{28}{19}a^{2}b^{6}x^{19}+\frac{4}{11}a\,b^{7}x^{22}+\frac{1}{25}$
orering	$\frac{x\left(5434b^{8}x^{24}+49400a\,b^{7}x^{21}+200200a^{2}b^{6}x^{18}+475475a^{3}b^{5}x^{15}+731500a^{4}b^{4}x^{12}+760760a^{5}b^{3}x^{9}+543400a^{6}b^{2}x^{6}+271700a^{7}b\right.}{135850}$

input
```
int((b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
a^8*x+2*a^7*b*x^4+4*a^6*b^2*x^7+28/5*a^5*b^3*x^10+70/13*a^4*b^4*x^13+7/2*a
^3*b^5*x^16+28/19*a^2*b^6*x^19+4/11*a*b^7*x^22+1/25*b^8*x^25
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \left(a+bx^{3}\right)^{8}\,dx=\frac{1}{25}\,b^{8}x^{25}+\frac{4}{11}\,ab^{7}x^{22}+\frac{28}{19}\,a^{2}b^{6}x^{19}+\frac{7}{2}\,a^{3}b^{5}x^{16}+\frac{70}{13}\,a^{4}b^{4}x^{13}+\frac{28}{5}\,a^{5}b^{3}x^{10}+4\,a^{6}b^{2}x^{7}+2\,a^{7}bx^{4}+a^{8}x$$

input
```
integrate((b*x^3+a)^8,x, algorithm="fricas")
```

output
```
1/25*b^8*x^25 + 4/11*a*b^7*x^22 + 28/19*a^2*b^6*x^19 + 7/2*a^3*b^5*x^16 +
70/13*a^4*b^4*x^13 + 28/5*a^5*b^3*x^10 + 4*a^6*b^2*x^7 + 2*a^7*b*x^4 + a^8
*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int (a+bx^3)^8\,dx = a^8x + 2a^7bx^4 + 4a^6b^2x^7 + \frac{28a^5b^3x^{10}}{5} + \frac{70a^4b^4x^{13}}{13} + \frac{7a^3b^5x^{16}}{2} + \frac{28a^2b^6x^{19}}{19} + \frac{4ab^7x^{22}}{11} + \frac{b^8x^{25}}{25}$$

input
```
integrate((b*x**3+a)**8,x)
```

output
```
a**8*x + 2*a**7*b*x**4 + 4*a**6*b**2*x**7 + 28*a**5*b**3*x**10/5 + 70*a**4
*b**4*x**13/13 + 7*a**3*b**5*x**16/2 + 28*a**2*b**6*x**19/19 + 4*a*b**7*x*
*22/11 + b**8*x**25/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int (a+bx^3)^8\,dx = \frac{1}{25}\,b^8x^{25} + \frac{4}{11}\,ab^7x^{22} + \frac{28}{19}\,a^2b^6x^{19} + \frac{7}{2}\,a^3b^5x^{16} + \frac{70}{13}\,a^4b^4x^{13} + \frac{28}{5}\,a^5b^3x^{10} + 4\,a^6b^2x^7 + 2\,a^7bx^4 + a^8x$$

input
```
integrate((b*x^3+a)^8,x, algorithm="maxima")
```

output
```
1/25*b^8*x^25 + 4/11*a*b^7*x^22 + 28/19*a^2*b^6*x^19 + 7/2*a^3*b^5*x^16 +
70/13*a^4*b^4*x^13 + 28/5*a^5*b^3*x^10 + 4*a^6*b^2*x^7 + 2*a^7*b*x^4 + a^8
*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \left(a+bx^3\right)^8\,dx=\frac{1}{25}\,b^8x^{25}+\frac{4}{11}\,ab^7x^{22}+\frac{28}{19}\,a^2b^6x^{19}+\frac{7}{2}\,a^3b^5x^{16}+\frac{70}{13}\,a^4b^4x^{13}+\frac{28}{5}\,a^5b^3x^{10}+4\,a^6b^2x^7+2\,a^7bx^4+a^8x$$

input
```
integrate((b*x^3+a)^8,x, algorithm="giac")
```

output
```
1/25*b^8*x^25 + 4/11*a*b^7*x^22 + 28/19*a^2*b^6*x^19 + 7/2*a^3*b^5*x^16 +
70/13*a^4*b^4*x^13 + 28/5*a^5*b^3*x^10 + 4*a^6*b^2*x^7 + 2*a^7*b*x^4 + a^8
*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.88

$$\int \left(a+bx^3\right)^8\,dx=a^8\,x+2\,a^7\,b\,x^4+4\,a^6\,b^2\,x^7+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{70\,a^4\,b^4\,x^{13}}{13}+\frac{7\,a^3\,b^5\,x^{16}}{2}+\frac{28\,a^2\,b^6\,x^{19}}{19}+\frac{4\,a\,b^7\,x^{22}}{11}+\frac{b^8\,x^{25}}{25}$$

input
```
int((a + b*x^3)^8,x)
```

output
```
a^8*x + (b^8*x^25)/25 + 2*a^7*b*x^4 + (4*a*b^7*x^22)/11 + 4*a^6*b^2*x^7 +
(28*a^5*b^3*x^10)/5 + (70*a^4*b^4*x^13)/13 + (7*a^3*b^5*x^16)/2 + (28*a^2*
b^6*x^19)/19
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \left(a+bx^3\right)^8 \, dx$$
$$= \frac{x(5434b^8x^{24}+49400a\,b^7x^{21}+200200a^2b^6x^{18}+475475a^3b^5x^{15}+731500a^4b^4x^{12}+760760a^5b^3x^9+5434}{135850}$$

input
```
int((b*x^3+a)^8,x)
```

output
```
(x*(135850*a**8 + 271700*a**7*b*x**3 + 543400*a**6*b**2*x**6 + 760760*a**5
*b**3*x**9 + 731500*a**4*b**4*x**12 + 475475*a**3*b**5*x**15 + 200200*a**2
*b**6*x**18 + 49400*a*b**7*x**21 + 5434*b**8*x**24))/135850
```

3.107 $\int \frac{(a+bx^3)^8}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^3)^8}{x^2} dx = -\frac{a^8}{x} + 4a^7bx^2 + \frac{28}{5}a^6b^2x^5 + 7a^5b^3x^8 + \frac{70}{11}a^4b^4x^{11} + 4a^3b^5x^{14} + \frac{28}{17}a^2b^6x^{17} + \frac{2}{5}ab^7x^{20} + \frac{b^8x^{23}}{23}$$

output
```
-a^8/x+4*a^7*b*x^2+28/5*a^6*b^2*x^5+7*a^5*b^3*x^8+70/11*a^4*b^4*x^11+4*a^3
*b^5*x^14+28/17*a^2*b^6*x^17+2/5*a*b^7*x^20+1/23*b^8*x^23
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^2} dx = -\frac{a^8}{x} + 4a^7bx^2 + \frac{28}{5}a^6b^2x^5 + 7a^5b^3x^8 + \frac{70}{11}a^4b^4x^{11} + 4a^3b^5x^{14} + \frac{28}{17}a^2b^6x^{17} + \frac{2}{5}ab^7x^{20} + \frac{b^8x^{23}}{23}$$

input
```
Integrate[(a + b*x^3)^8/x^2,x]
```

output
```
-(a^8/x) + 4*a^7*b*x^2 + (28*a^6*b^2*x^5)/5 + 7*a^5*b^3*x^8 + (70*a^4*b^4*
x^11)/11 + 4*a^3*b^5*x^14 + (28*a^2*b^6*x^17)/17 + (2*a*b^7*x^20)/5 + (b^8
*x^23)/23
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^8}{x^2}+8a^7bx+28a^6b^2x^4+56a^5b^3x^7+70a^4b^4x^{10}+56a^3b^5x^{13}+28a^2b^6x^{16}+8ab^7x^{19}+b^8x^{22}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{x}+4a^7bx^2+\frac{28}{5}a^6b^2x^5+7a^5b^3x^8+\frac{70}{11}a^4b^4x^{11}+4a^3b^5x^{14}+\frac{28}{17}a^2b^6x^{17}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{23}}{23}$$

input
```
Int[(a + b*x^3)^8/x^2,x]
```

output

```
-(a^8/x) + 4*a^7*b*x^2 + (28*a^6*b^2*x^5)/5 + 7*a^5*b^3*x^8 + (70*a^4*b^4*
x^11)/11 + 4*a^3*b^5*x^14 + (28*a^2*b^6*x^17)/17 + (2*a*b^7*x^20)/5 + (b^8
*x^23)/23
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^{8}}{x}+4a^{7}b\,x^{2}+\frac{28a^{6}b^{2}x^{5}}{5}+7a^{5}b^{3}x^{8}+\frac{70a^{4}b^{4}x^{11}}{11}+4a^{3}b^{5}x^{14}+\frac{28a^{2}b^{6}x^{17}}{17}+\frac{2a\,b^{7}x^{20}}{5}+\frac{b^{8}x^{23}}{23}$
risch	$-\frac{a^{8}}{x}+4a^{7}b\,x^{2}+\frac{28a^{6}b^{2}x^{5}}{5}+7a^{5}b^{3}x^{8}+\frac{70a^{4}b^{4}x^{11}}{11}+4a^{3}b^{5}x^{14}+\frac{28a^{2}b^{6}x^{17}}{17}+\frac{2a\,b^{7}x^{20}}{5}+\frac{b^{8}x^{23}}{23}$
norman	$\frac{-a^{8}+\frac{2}{5}a\,b^{7}x^{21}+7a^{5}b^{3}x^{9}+4a^{7}b\,x^{3}+\frac{28}{17}a^{2}b^{6}x^{18}+4a^{3}b^{5}x^{15}+\frac{1}{23}b^{8}x^{24}+\frac{28}{5}a^{6}b^{2}x^{6}+\frac{70}{11}a^{4}b^{4}x^{12}}{x}$
gosper	$-\frac{-935b^{8}x^{24}-8602a\,b^{7}x^{21}-35420a^{2}b^{6}x^{18}-86020a^{3}b^{5}x^{15}-136850a^{4}b^{4}x^{12}-150535a^{5}b^{3}x^{9}-120428a^{6}b^{2}x^{6}-86020a^{7}b\,x^{3}+}{21505x}$
parallelrisch	$\frac{935b^{8}x^{24}+8602a\,b^{7}x^{21}+35420a^{2}b^{6}x^{18}+86020a^{3}b^{5}x^{15}+136850a^{4}b^{4}x^{12}+150535a^{5}b^{3}x^{9}+120428a^{6}b^{2}x^{6}+86020a^{7}b\,x^{3}-2150}{21505x}$
orering	$-\frac{-935b^{8}x^{24}-8602a\,b^{7}x^{21}-35420a^{2}b^{6}x^{18}-86020a^{3}b^{5}x^{15}-136850a^{4}b^{4}x^{12}-150535a^{5}b^{3}x^{9}-120428a^{6}b^{2}x^{6}-86020a^{7}b\,x^{3}+}{21505x}$

input
```
int((b*x^3+a)^8/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^8/x+4*a^7*b*x^2+28/5*a^6*b^2*x^5+7*a^5*b^3*x^8+70/11*a^4*b^4*x^11+4*a^3
*b^5*x^14+28/17*a^2*b^6*x^17+2/5*a*b^7*x^20+1/23*b^8*x^23
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^8}{x^2}\,dx$$
$$=\frac{935\,b^{8}x^{24}+8602\,ab^{7}x^{21}+35420\,a^{2}b^{6}x^{18}+86020\,a^{3}b^{5}x^{15}+136850\,a^{4}b^{4}x^{12}+150535\,a^{5}b^{3}x^{9}+120428\,a^{6}}{21505\,x}$$

input
```
integrate((b*x^3+a)^8/x^2,x, algorithm="fricas")
```

output
```
1/21505*(935*b^8*x^24 + 8602*a*b^7*x^21 + 35420*a^2*b^6*x^18 + 86020*a^3*b
^5*x^15 + 136850*a^4*b^4*x^12 + 150535*a^5*b^3*x^9 + 120428*a^6*b^2*x^6 +
86020*a^7*b*x^3 - 21505*a^8)/x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^8}{x^2}\,dx = -\frac{a^8}{x} + 4a^7bx^2 + \frac{28a^6b^2x^5}{5} + 7a^5b^3x^8 + \frac{70a^4b^4x^{11}}{11} + 4a^3b^5x^{14} + \frac{28a^2b^6x^{17}}{17} + \frac{2ab^7x^{20}}{5} + \frac{b^8x^{23}}{23}$$

input
```
integrate((b*x**3+a)**8/x**2,x)
```

output
```
-a**8/x + 4*a**7*b*x**2 + 28*a**6*b**2*x**5/5 + 7*a**5*b**3*x**8 + 70*a**4
*b**4*x**11/11 + 4*a**3*b**5*x**14 + 28*a**2*b**6*x**17/17 + 2*a*b**7*x**2
0/5 + b**8*x**23/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^2}\,dx = \frac{1}{23}\,b^8x^{23} + \frac{2}{5}\,ab^7x^{20} + \frac{28}{17}\,a^2b^6x^{17} + 4\,a^3b^5x^{14} + \frac{70}{11}\,a^4b^4x^{11} + 7\,a^5b^3x^8 + \frac{28}{5}\,a^6b^2x^5 + 4\,a^7bx^2 - \frac{a^8}{x}$$

input
```
integrate((b*x^3+a)^8/x^2,x, algorithm="maxima")
```

output
```
1/23*b^8*x^23 + 2/5*a*b^7*x^20 + 28/17*a^2*b^6*x^17 + 4*a^3*b^5*x^14 + 70/
11*a^4*b^4*x^11 + 7*a^5*b^3*x^8 + 28/5*a^6*b^2*x^5 + 4*a^7*b*x^2 - a^8/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^2}\,dx = \frac{1}{23}\,b^8x^{23} + \frac{2}{5}\,ab^7x^{20} + \frac{28}{17}\,a^2b^6x^{17} + 4\,a^3b^5x^{14} + \frac{70}{11}\,a^4b^4x^{11} + 7\,a^5b^3x^8 + \frac{28}{5}\,a^6b^2x^5 + 4\,a^7bx^2 - \frac{a^8}{x}$$

input
```
integrate((b*x^3+a)^8/x^2,x, algorithm="giac")
```

output
```
1/23*b^8*x^23 + 2/5*a*b^7*x^20 + 28/17*a^2*b^6*x^17 + 4*a^3*b^5*x^14 + 70/
11*a^4*b^4*x^11 + 7*a^5*b^3*x^8 + 28/5*a^6*b^2*x^5 + 4*a^7*b*x^2 - a^8/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^2}\,dx = \frac{b^8\,x^{23}}{23} - \frac{a^8}{x} + 4\,a^7\,b\,x^2 + \frac{2\,a\,b^7\,x^{20}}{5} + \frac{28\,a^6\,b^2\,x^5}{5} + 7\,a^5\,b^3\,x^8 + \frac{70\,a^4\,b^4\,x^{11}}{11} + 4\,a^3\,b^5\,x^{14} + \frac{28\,a^2\,b^6\,x^{17}}{17}$$

input
```
int((a + b*x^3)^8/x^2,x)
```

output
```
(b^8*x^23)/23 - a^8/x + 4*a^7*b*x^2 + (2*a*b^7*x^20)/5 + (28*a^6*b^2*x^5)/
5 + 7*a^5*b^3*x^8 + (70*a^4*b^4*x^11)/11 + 4*a^3*b^5*x^14 + (28*a^2*b^6*x^
17)/17
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^8}{x^2}\,dx$$
$$= \frac{935b^8x^{24} + 8602a\,b^7x^{21} + 35420a^2b^6x^{18} + 86020a^3b^5x^{15} + 136850a^4b^4x^{12} + 150535a^5b^3x^9 + 120428a^6b^2}{21505x}$$

input
```
int((b*x^3+a)^8/x^2,x)
```

output
```
( - 21505*a**8 + 86020*a**7*b*x**3 + 120428*a**6*b**2*x**6 + 150535*a**5*b
**3*x**9 + 136850*a**4*b**4*x**12 + 86020*a**3*b**5*x**15 + 35420*a**2*b**
6*x**18 + 8602*a*b**7*x**21 + 935*b**8*x**24)/(21505*x)
```

3.108 $\int \frac{(a+bx^3)^8}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{(a+bx^3)^8}{x^3} dx = -\frac{a^8}{2x^2} + 8a^7bx + 7a^6b^2x^4 + 8a^5b^3x^7 + 7a^4b^4x^{10} + \frac{56}{13}a^3b^5x^{13} + \frac{7}{4}a^2b^6x^{16} + \frac{8}{19}ab^7x^{19} + \frac{b^8x^{22}}{22}$$

output
```
-1/2*a^8/x^2+8*a^7*b*x+7*a^6*b^2*x^4+8*a^5*b^3*x^7+7*a^4*b^4*x^10+56/13*a^
3*b^5*x^13+7/4*a^2*b^6*x^16+8/19*a*b^7*x^19+1/22*b^8*x^22
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^3} dx = -\frac{a^8}{2x^2} + 8a^7bx + 7a^6b^2x^4 + 8a^5b^3x^7 + 7a^4b^4x^{10} + \frac{56}{13}a^3b^5x^{13} + \frac{7}{4}a^2b^6x^{16} + \frac{8}{19}ab^7x^{19} + \frac{b^8x^{22}}{22}$$

input
```
Integrate[(a + b*x^3)^8/x^3,x]
```

output
```
-1/2*a^8/x^2 + 8*a^7*b*x + 7*a^6*b^2*x^4 + 8*a^5*b^3*x^7 + 7*a^4*b^4*x^10
+ (56*a^3*b^5*x^13)/13 + (7*a^2*b^6*x^16)/4 + (8*a*b^7*x^19)/19 + (b^8*x^2
2)/22
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^8}{x^3}+8a^7b+28a^6b^2x^3+56a^5b^3x^6+70a^4b^4x^9+56a^3b^5x^{12}+28a^2b^6x^{15}+8ab^7x^{18}+b^8x^{21}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{2x^2}+8a^7bx+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56}{13}a^3b^5x^{13}+\frac{7}{4}a^2b^6x^{16}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{22}}{22}$$

input
```
Int[(a + b*x^3)^8/x^3,x]
```

output

```
-1/2*a^8/x^2 + 8*a^7*b*x + 7*a^6*b^2*x^4 + 8*a^5*b^3*x^7 + 7*a^4*b^4*x^10
+ (56*a^3*b^5*x^13)/13 + (7*a^2*b^6*x^16)/4 + (8*a*b^7*x^19)/19 + (b^8*x^2
2)/22
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^8}{2x^2}+8a^7xb+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56a^3b^5x^{13}}{13}+\frac{7a^2b^6x^{16}}{4}+\frac{8a\,b^7x^{19}}{19}+\frac{b^8x^{22}}{22}$
risch	$-\frac{a^8}{2x^2}+8a^7xb+7a^6b^2x^4+8a^5b^3x^7+7a^4b^4x^{10}+\frac{56a^3b^5x^{13}}{13}+\frac{7a^2b^6x^{16}}{4}+\frac{8a\,b^7x^{19}}{19}+\frac{b^8x^{22}}{22}$
norman	$\frac{-\frac{1}{2}a^8+7a^6b^2x^6+8a^5b^3x^9+\frac{56}{13}a^3b^5x^{15}+\frac{8}{19}a\,b^7x^{21}+\frac{7}{4}a^2b^6x^{18}+7a^4b^4x^{12}+8a^7b\,x^3+\frac{1}{22}b^8x^{24}}{x^2}$
gosper	$-\frac{-494b^8x^{24}-4576a\,b^7x^{21}-19019a^2b^6x^{18}-46816a^3b^5x^{15}-76076a^4b^4x^{12}-86944a^5b^3x^9-76076a^6b^2x^6-86944a^7b\,x^3+543}{10868x^2}$
parallelrisch	$\frac{494b^8x^{24}+4576a\,b^7x^{21}+19019a^2b^6x^{18}+46816a^3b^5x^{15}+76076a^4b^4x^{12}+86944a^5b^3x^9+76076a^6b^2x^6+86944a^7b\,x^3-5434a^8}{10868x^2}$
orering	$-\frac{-494b^8x^{24}-4576a\,b^7x^{21}-19019a^2b^6x^{18}-46816a^3b^5x^{15}-76076a^4b^4x^{12}-86944a^5b^3x^9-76076a^6b^2x^6-86944a^7b\,x^3+543}{10868x^2}$

input
```
int((b*x^3+a)^8/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^8/x^2+8*a^7*x*b+7*a^6*b^2*x^4+8*a^5*b^3*x^7+7*a^4*b^4*x^10+56/13*a^
3*b^5*x^13+7/4*a^2*b^6*x^16+8/19*a*b^7*x^19+1/22*b^8*x^22
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^3}\,dx$$
$$=\frac{494\,b^8x^{24}+4576\,ab^7x^{21}+19019\,a^2b^6x^{18}+46816\,a^3b^5x^{15}+76076\,a^4b^4x^{12}+86944\,a^5b^3x^9+76076\,a^6b^2x^{}}{10868\,x^2}$$

input
```
integrate((b*x^3+a)^8/x^3,x, algorithm="fricas")
```

output
```
1/10868*(494*b^8*x^24 + 4576*a*b^7*x^21 + 19019*a^2*b^6*x^18 + 46816*a^3*b
^5*x^15 + 76076*a^4*b^4*x^12 + 86944*a^5*b^3*x^9 + 76076*a^6*b^2*x^6 + 869
44*a^7*b*x^3 - 5434*a^8)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^8}{x^3}\,dx = -\frac{a^8}{2x^2} + 8a^7bx + 7a^6b^2x^4 + 8a^5b^3x^7 + 7a^4b^4x^{10} + \frac{56a^3b^5x^{13}}{13} + \frac{7a^2b^6x^{16}}{4} + \frac{8ab^7x^{19}}{19} + \frac{b^8x^{22}}{22}$$

input
```
integrate((b*x**3+a)**8/x**3,x)
```

output
```
-a**8/(2*x**2) + 8*a**7*b*x + 7*a**6*b**2*x**4 + 8*a**5*b**3*x**7 + 7*a**4
*b**4*x**10 + 56*a**3*b**5*x**13/13 + 7*a**2*b**6*x**16/4 + 8*a*b**7*x**19
/19 + b**8*x**22/22
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^3}\,dx = \frac{1}{22}\,b^8x^{22} + \frac{8}{19}\,ab^7x^{19} + \frac{7}{4}\,a^2b^6x^{16} + \frac{56}{13}\,a^3b^5x^{13} + 7\,a^4b^4x^{10} + 8\,a^5b^3x^7 + 7\,a^6b^2x^4 + 8\,a^7bx - \frac{a^8}{2\,x^2}$$

input
```
integrate((b*x^3+a)^8/x^3,x, algorithm="maxima")
```

output
```
1/22*b^8*x^22 + 8/19*a*b^7*x^19 + 7/4*a^2*b^6*x^16 + 56/13*a^3*b^5*x^13 +
7*a^4*b^4*x^10 + 8*a^5*b^3*x^7 + 7*a^6*b^2*x^4 + 8*a^7*b*x - 1/2*a^8/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^3}\,dx = \frac{1}{22}\,b^8x^{22} + \frac{8}{19}\,ab^7x^{19} + \frac{7}{4}\,a^2b^6x^{16} + \frac{56}{13}\,a^3b^5x^{13} + 7\,a^4b^4x^{10} + 8\,a^5b^3x^7 + 7\,a^6b^2x^4 + 8\,a^7bx - \frac{a^8}{2\,x^2}$$

input
```
integrate((b*x^3+a)^8/x^3,x, algorithm="giac")
```

output
```
1/22*b^8*x^22 + 8/19*a*b^7*x^19 + 7/4*a^2*b^6*x^16 + 56/13*a^3*b^5*x^13 +
7*a^4*b^4*x^10 + 8*a^5*b^3*x^7 + 7*a^6*b^2*x^4 + 8*a^7*b*x - 1/2*a^8/x^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^3}\,dx = \frac{b^8\,x^{22}}{22} - \frac{a^8}{2\,x^2} + \frac{8\,a\,b^7\,x^{19}}{19} + 7\,a^6\,b^2\,x^4 + 8\,a^5\,b^3\,x^7 + 7\,a^4\,b^4\,x^{10} + \frac{56\,a^3\,b^5\,x^{13}}{13} + \frac{7\,a^2\,b^6\,x^{16}}{4} + 8\,a^7\,b\,x$$

input
```
int((a + b*x^3)^8/x^3,x)
```

output
```
(b^8*x^22)/22 - a^8/(2*x^2) + (8*a*b^7*x^19)/19 + 7*a^6*b^2*x^4 + 8*a^5*b^
3*x^7 + 7*a^4*b^4*x^10 + (56*a^3*b^5*x^13)/13 + (7*a^2*b^6*x^16)/4 + 8*a^7
*b*x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^3}\,dx = \frac{494b^8x^{24}+4576a\,b^7x^{21}+19019a^2b^6x^{18}+46816a^3b^5x^{15}+76076a^4b^4x^{12}+86944a^5b^3x^9+76076a^6b^2x^6+}{10868x^2}$$

input
```
int((b*x^3+a)^8/x^3,x)
```

output
```
( - 5434*a**8 + 86944*a**7*b*x**3 + 76076*a**6*b**2*x**6 + 86944*a**5*b**3
*x**9 + 76076*a**4*b**4*x**12 + 46816*a**3*b**5*x**15 + 19019*a**2*b**6*x*
*18 + 4576*a*b**7*x**21 + 494*b**8*x**24)/(10868*x**2)
```

3.109 $\int \frac{(a+bx^3)^8}{x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \frac{(a+bx^3)^8}{x^5} \, dx = -\frac{a^8}{4x^4} - \frac{8a^7b}{x} + 14a^6b^2x^2 + \frac{56}{5}a^5b^3x^5 + \frac{35}{4}a^4b^4x^8 + \frac{56}{11}a^3b^5x^{11} + 2a^2b^6x^{14} + \frac{8}{17}ab^7x^{17} + \frac{b^8x^{20}}{20}$$

output
```
-1/4*a^8/x^4-8*a^7*b/x+14*a^6*b^2*x^2+56/5*a^5*b^3*x^5+35/4*a^4*b^4*x^8+56
/11*a^3*b^5*x^11+2*a^2*b^6*x^14+8/17*a*b^7*x^17+1/20*b^8*x^20
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^5} \, dx = -\frac{a^8}{4x^4} - \frac{8a^7b}{x} + 14a^6b^2x^2 + \frac{56}{5}a^5b^3x^5 + \frac{35}{4}a^4b^4x^8 + \frac{56}{11}a^3b^5x^{11} + 2a^2b^6x^{14} + \frac{8}{17}ab^7x^{17} + \frac{b^8x^{20}}{20}$$

input
```
Integrate[(a + b*x^3)^8/x^5,x]
```

output
```
-1/4*a^8/x^4 - (8*a^7*b)/x + 14*a^6*b^2*x^2 + (56*a^5*b^3*x^5)/5 + (35*a^4
*b^4*x^8)/4 + (56*a^3*b^5*x^11)/11 + 2*a^2*b^6*x^14 + (8*a*b^7*x^17)/17 +
(b^8*x^20)/20
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^5} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^8}{x^5}+\frac{8a^7b}{x^2}+28a^6b^2x+56a^5b^3x^4+70a^4b^4x^7+56a^3b^5x^{10}+28a^2b^6x^{13}+8ab^7x^{16}+b^8x^{19}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{4x^4}-\frac{8a^7b}{x}+14a^6b^2x^2+\frac{56}{5}a^5b^3x^5+\frac{35}{4}a^4b^4x^8+\frac{56}{11}a^3b^5x^{11}+2a^2b^6x^{14}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{20}}{20}$$

input
```
Int[(a + b*x^3)^8/x^5,x]
```

output

```
-1/4*a^8/x^4 - (8*a^7*b)/x + 14*a^6*b^2*x^2 + (56*a^5*b^3*x^5)/5 + (35*a^4
*b^4*x^8)/4 + (56*a^3*b^5*x^11)/11 + 2*a^2*b^6*x^14 + (8*a*b^7*x^17)/17 +
(b^8*x^20)/20
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{8}}{4x^{4}}-\frac{8a^{7}b}{x}+14a^{6}x^{2}b^{2}+\frac{56a^{5}b^{3}x^{5}}{5}+\frac{35a^{4}b^{4}x^{8}}{4}+\frac{56a^{3}b^{5}x^{11}}{11}+2a^{2}b^{6}x^{14}+\frac{8a\,b^{7}x^{17}}{17}+\frac{b^{8}x^{20}}{20}$
norman	$\frac{-\frac{1}{4}a^{8}+\frac{1}{20}b^{8}x^{24}-8a^{7}b\,x^{3}+14a^{6}b^{2}x^{6}+\frac{35}{4}a^{4}b^{4}x^{12}+2a^{2}b^{6}x^{18}+\frac{56}{11}a^{3}b^{5}x^{15}+\frac{56}{5}a^{5}b^{3}x^{9}+\frac{8}{17}a\,b^{7}x^{21}}{x^{4}}$
gosper	$-\frac{-187b^{8}x^{24}-1760a\,b^{7}x^{21}-7480a^{2}b^{6}x^{18}-19040a^{3}b^{5}x^{15}-32725a^{4}b^{4}x^{12}-41888a^{5}b^{3}x^{9}-52360a^{6}b^{2}x^{6}+29920a^{7}b\,x^{3}+935a}{3740x^{4}}$
risch	$\frac{b^{8}x^{20}}{20}+\frac{8a\,b^{7}x^{17}}{17}+2a^{2}b^{6}x^{14}+\frac{56a^{3}b^{5}x^{11}}{11}+\frac{35a^{4}b^{4}x^{8}}{4}+\frac{56a^{5}b^{3}x^{5}}{5}+14a^{6}x^{2}b^{2}+\frac{-8a^{7}b\,x^{3}-\frac{1}{4}a^{8}}{x^{4}}$
parallelrisch	$\frac{187b^{8}x^{24}+1760a\,b^{7}x^{21}+7480a^{2}b^{6}x^{18}+19040a^{3}b^{5}x^{15}+32725a^{4}b^{4}x^{12}+41888a^{5}b^{3}x^{9}+52360a^{6}b^{2}x^{6}-29920a^{7}b\,x^{3}-935a^{8}}{3740x^{4}}$
orering	$-\frac{-187b^{8}x^{24}-1760a\,b^{7}x^{21}-7480a^{2}b^{6}x^{18}-19040a^{3}b^{5}x^{15}-32725a^{4}b^{4}x^{12}-41888a^{5}b^{3}x^{9}-52360a^{6}b^{2}x^{6}+29920a^{7}b\,x^{3}+935a}{3740x^{4}}$

input
```
int((b*x^3+a)^8/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^8/x^4-8*a^7*b/x+14*a^6*x^2*b^2+56/5*a^5*b^3*x^5+35/4*a^4*b^4*x^8+56
/11*a^3*b^5*x^11+2*a^2*b^6*x^14+8/17*a*b^7*x^17+1/20*b^8*x^20
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^5}\,dx = \frac{187\,b^8x^{24}+1760\,ab^7x^{21}+7480\,a^2b^6x^{18}+19040\,a^3b^5x^{15}+32725\,a^4b^4x^{12}+41888\,a^5b^3x^9+52360\,a^6b^2x^6}{3740\,x^4}$$

input
```
integrate((b*x^3+a)^8/x^5,x, algorithm="fricas")
```

output
```
1/3740*(187*b^8*x^24 + 1760*a*b^7*x^21 + 7480*a^2*b^6*x^18 + 19040*a^3*b^5
*x^15 + 32725*a^4*b^4*x^12 + 41888*a^5*b^3*x^9 + 52360*a^6*b^2*x^6 - 29920
*a^7*b*x^3 - 935*a^8)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^8}{x^5}\,dx = 14a^6b^2x^2+\frac{56a^5b^3x^5}{5}+\frac{35a^4b^4x^8}{4}+\frac{56a^3b^5x^{11}}{11}+2a^2b^6x^{14}+\frac{8ab^7x^{17}}{17}+\frac{b^8x^{20}}{20}+\frac{-a^8-32a^7bx^3}{4x^4}$$

input
```
integrate((b*x**3+a)**8/x**5,x)
```

output
```
14*a**6*b**2*x**2 + 56*a**5*b**3*x**5/5 + 35*a**4*b**4*x**8/4 + 56*a**3*b*
*5*x**11/11 + 2*a**2*b**6*x**14 + 8*a*b**7*x**17/17 + b**8*x**20/20 + (-a*
*8 - 32*a**7*b*x**3)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^8}{x^5}\,dx = \frac{1}{20}\,b^8x^{20} + \frac{8}{17}\,ab^7x^{17} + 2\,a^2b^6x^{14} + \frac{56}{11}\,a^3b^5x^{11} + \frac{35}{4}\,a^4b^4x^8 + \frac{56}{5}\,a^5b^3x^5 + 14\,a^6b^2x^2 - \frac{32\,a^7bx^3 + a^8}{4\,x^4}$$

input
```
integrate((b*x^3+a)^8/x^5,x, algorithm="maxima")
```

output
```
1/20*b^8*x^20 + 8/17*a*b^7*x^17 + 2*a^2*b^6*x^14 + 56/11*a^3*b^5*x^11 + 35
/4*a^4*b^4*x^8 + 56/5*a^5*b^3*x^5 + 14*a^6*b^2*x^2 - 1/4*(32*a^7*b*x^3 + a
^8)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^8}{x^5}\,dx = \frac{1}{20}\,b^8x^{20} + \frac{8}{17}\,ab^7x^{17} + 2\,a^2b^6x^{14} + \frac{56}{11}\,a^3b^5x^{11} + \frac{35}{4}\,a^4b^4x^8 + \frac{56}{5}\,a^5b^3x^5 + 14\,a^6b^2x^2 - \frac{32\,a^7bx^3 + a^8}{4\,x^4}$$

input
```
integrate((b*x^3+a)^8/x^5,x, algorithm="giac")
```

output
```
1/20*b^8*x^20 + 8/17*a*b^7*x^17 + 2*a^2*b^6*x^14 + 56/11*a^3*b^5*x^11 + 35
/4*a^4*b^4*x^8 + 56/5*a^5*b^3*x^5 + 14*a^6*b^2*x^2 - 1/4*(32*a^7*b*x^3 + a
^8)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^8}{x^5}\,dx = \frac{b^8\,x^{20}}{20} - \frac{\frac{a^8}{4} + 8\,b\,a^7\,x^3}{x^4} + \frac{8\,a\,b^7\,x^{17}}{17} + 14\,a^6\,b^2\,x^2 + \frac{56\,a^5\,b^3\,x^5}{5} + \frac{35\,a^4\,b^4\,x^8}{4} + \frac{56\,a^3\,b^5\,x^{11}}{11} + 2\,a^2\,b^6\,x^{14}$$

input
```
int((a + b*x^3)^8/x^5,x)
```

output
```
(b^8*x^20)/20 - (a^8/4 + 8*a^7*b*x^3)/x^4 + (8*a*b^7*x^17)/17 + 14*a^6*b^2
*x^2 + (56*a^5*b^3*x^5)/5 + (35*a^4*b^4*x^8)/4 + (56*a^3*b^5*x^11)/11 + 2*
a^2*b^6*x^14
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^8}{x^5}\,dx = \frac{187b^8x^{24} + 1760a\,b^7x^{21} + 7480a^2b^6x^{18} + 19040a^3b^5x^{15} + 32725a^4b^4x^{12} + 41888a^5b^3x^9 + 52360a^6b^2x^6 -}{3740x^4}$$

input
```
int((b*x^3+a)^8/x^5,x)
```

output
```
( - 935*a**8 - 29920*a**7*b*x**3 + 52360*a**6*b**2*x**6 + 41888*a**5*b**3*
x**9 + 32725*a**4*b**4*x**12 + 19040*a**3*b**5*x**15 + 7480*a**2*b**6*x**1
8 + 1760*a*b**7*x**21 + 187*b**8*x**24)/(3740*x**4)
```

3.110 $\int \frac{(a+bx^3)^8}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{(a+bx^3)^8}{x^6} dx = -\frac{a^8}{5x^5} - \frac{4a^7b}{x^2} + 28a^6b^2x + 14a^5b^3x^4 + 10a^4b^4x^7 + \frac{28}{5}a^3b^5x^{10} + \frac{28}{13}a^2b^6x^{13} + \frac{1}{2}ab^7x^{16} + \frac{b^8x^{19}}{19}$$

output
```
-1/5*a^8/x^5-4*a^7*b/x^2+28*a^6*b^2*x+14*a^5*b^3*x^4+10*a^4*b^4*x^7+28/5*a
^3*b^5*x^10+28/13*a^2*b^6*x^13+1/2*a*b^7*x^16+1/19*b^8*x^19
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^6} dx = -\frac{a^8}{5x^5} - \frac{4a^7b}{x^2} + 28a^6b^2x + 14a^5b^3x^4 + 10a^4b^4x^7 + \frac{28}{5}a^3b^5x^{10} + \frac{28}{13}a^2b^6x^{13} + \frac{1}{2}ab^7x^{16} + \frac{b^8x^{19}}{19}$$

input
```
Integrate[(a + b*x^3)^8/x^6,x]
```

output
```
-1/5*a^8/x^5 - (4*a^7*b)/x^2 + 28*a^6*b^2*x + 14*a^5*b^3*x^4 + 10*a^4*b^4*
x^7 + (28*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^13)/13 + (a*b^7*x^16)/2 + (b^8*x
^19)/19
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^6}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^8}{x^6}+\frac{8a^7b}{x^3}+28a^6b^2+56a^5b^3x^3+70a^4b^4x^6+56a^3b^5x^9+28a^2b^6x^{12}+8ab^7x^{15}+b^8x^{18}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{5x^5}-\frac{4a^7b}{x^2}+28a^6b^2x+14a^5b^3x^4+10a^4b^4x^7+\frac{28}{5}a^3b^5x^{10}+\frac{28}{13}a^2b^6x^{13}+\frac{1}{2}ab^7x^{16}+\frac{b^8x^{19}}{19}$$

input
```
Int[(a + b*x^3)^8/x^6,x]
```

output

```
-1/5*a^8/x^5 - (4*a^7*b)/x^2 + 28*a^6*b^2*x + 14*a^5*b^3*x^4 + 10*a^4*b^4*
x^7 + (28*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^13)/13 + (a*b^7*x^16)/2 + (b^8*x
^19)/19
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^{8}}{5x^{5}}-\frac{4a^{7}b}{x^{2}}+28a^{6}b^{2}x+14a^{5}b^{3}x^{4}+10a^{4}b^{4}x^{7}+\frac{28a^{3}b^{5}x^{10}}{5}+\frac{28a^{2}b^{6}x^{13}}{13}+\frac{a\,b^{7}x^{16}}{2}+\frac{b^{8}x^{19}}{19}$
risch	$\frac{b^{8}x^{19}}{19}+\frac{a\,b^{7}x^{16}}{2}+\frac{28a^{2}b^{6}x^{13}}{13}+\frac{28a^{3}b^{5}x^{10}}{5}+10a^{4}b^{4}x^{7}+14a^{5}b^{3}x^{4}+28a^{6}b^{2}x+\frac{-4a^{7}b\,x^{3}-\frac{1}{5}a^{8}}{x^{5}}$
norman	$\frac{-\frac{1}{5}a^{8}+\frac{28}{13}a^{2}b^{6}x^{18}+14a^{5}b^{3}x^{9}+\frac{28}{5}a^{3}b^{5}x^{15}-4a^{7}b\,x^{3}+28a^{6}b^{2}x^{6}+\frac{1}{19}b^{8}x^{24}+\frac{1}{2}a\,b^{7}x^{21}+10a^{4}b^{4}x^{12}}{x^{5}}$
gosper	$-\frac{-130b^{8}x^{24}-1235a\,b^{7}x^{21}-5320a^{2}b^{6}x^{18}-13832a^{3}b^{5}x^{15}-24700a^{4}b^{4}x^{12}-34580a^{5}b^{3}x^{9}-69160a^{6}b^{2}x^{6}+9880a^{7}b\,x^{3}+494a^{8}}{2470x^{5}}$
parallelrisch	$\frac{130b^{8}x^{24}+1235a\,b^{7}x^{21}+5320a^{2}b^{6}x^{18}+13832a^{3}b^{5}x^{15}+24700a^{4}b^{4}x^{12}+34580a^{5}b^{3}x^{9}+69160a^{6}b^{2}x^{6}-9880a^{7}b\,x^{3}-494a^{8}}{2470x^{5}}$
orering	$-\frac{-130b^{8}x^{24}-1235a\,b^{7}x^{21}-5320a^{2}b^{6}x^{18}-13832a^{3}b^{5}x^{15}-24700a^{4}b^{4}x^{12}-34580a^{5}b^{3}x^{9}-69160a^{6}b^{2}x^{6}+9880a^{7}b\,x^{3}+494a^{8}}{2470x^{5}}$

input
```
int((b*x^3+a)^8/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^8/x^5-4*a^7*b/x^2+28*a^6*b^2*x+14*a^5*b^3*x^4+10*a^4*b^4*x^7+28/5*a
^3*b^5*x^10+28/13*a^2*b^6*x^13+1/2*a*b^7*x^16+1/19*b^8*x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^6}\,dx = \frac{130\,b^8x^{24}+1235\,ab^7x^{21}+5320\,a^2b^6x^{18}+13832\,a^3b^5x^{15}+24700\,a^4b^4x^{12}+34580\,a^5b^3x^9+69160\,a^6b^2x^6}{2470\,x^5}$$

input
```
integrate((b*x^3+a)^8/x^6,x, algorithm="fricas")
```

output
```
1/2470*(130*b^8*x^24 + 1235*a*b^7*x^21 + 5320*a^2*b^6*x^18 + 13832*a^3*b^5
*x^15 + 24700*a^4*b^4*x^12 + 34580*a^5*b^3*x^9 + 69160*a^6*b^2*x^6 - 9880*
a^7*b*x^3 - 494*a^8)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^8}{x^6}\,dx = 28a^6b^2x + 14a^5b^3x^4 + 10a^4b^4x^7 + \frac{28a^3b^5x^{10}}{5} + \frac{28a^2b^6x^{13}}{13} + \frac{ab^7x^{16}}{2} + \frac{b^8x^{19}}{19} + \frac{-a^8-20a^7bx^3}{5x^5}$$

input
```
integrate((b*x**3+a)**8/x**6,x)
```

output
```
28*a**6*b**2*x + 14*a**5*b**3*x**4 + 10*a**4*b**4*x**7 + 28*a**3*b**5*x**1
0/5 + 28*a**2*b**6*x**13/13 + a*b**7*x**16/2 + b**8*x**19/19 + (-a**8 - 20
*a**7*b*x**3)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^8}{x^6}\,dx = \frac{1}{19}\,b^8x^{19} + \frac{1}{2}\,ab^7x^{16} + \frac{28}{13}\,a^2b^6x^{13} + \frac{28}{5}\,a^3b^5x^{10} + 10\,a^4b^4x^7 + 14\,a^5b^3x^4 + 28\,a^6b^2x - \frac{20\,a^7bx^3 + a^8}{5\,x^5}$$

input
```
integrate((b*x^3+a)^8/x^6,x, algorithm="maxima")
```

output
```
1/19*b^8*x^19 + 1/2*a*b^7*x^16 + 28/13*a^2*b^6*x^13 + 28/5*a^3*b^5*x^10 +
10*a^4*b^4*x^7 + 14*a^5*b^3*x^4 + 28*a^6*b^2*x - 1/5*(20*a^7*b*x^3 + a^8)/
x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^8}{x^6}\,dx = \frac{1}{19}\,b^8x^{19} + \frac{1}{2}\,ab^7x^{16} + \frac{28}{13}\,a^2b^6x^{13} + \frac{28}{5}\,a^3b^5x^{10} + 10\,a^4b^4x^7 + 14\,a^5b^3x^4 + 28\,a^6b^2x - \frac{20\,a^7bx^3 + a^8}{5\,x^5}$$

input
```
integrate((b*x^3+a)^8/x^6,x, algorithm="giac")
```

output
```
1/19*b^8*x^19 + 1/2*a*b^7*x^16 + 28/13*a^2*b^6*x^13 + 28/5*a^3*b^5*x^10 +
10*a^4*b^4*x^7 + 14*a^5*b^3*x^4 + 28*a^6*b^2*x - 1/5*(20*a^7*b*x^3 + a^8)/
x^5
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^8}{x^6}\,dx = \frac{b^8\,x^{19}}{19} - \frac{\frac{a^8}{5} + 4\,b\,a^7\,x^3}{x^5} + 28\,a^6\,b^2\,x + \frac{a\,b^7\,x^{16}}{2} + 14\,a^5\,b^3\,x^4 + 10\,a^4\,b^4\,x^7 + \frac{28\,a^3\,b^5\,x^{10}}{5} + \frac{28\,a^2\,b^6\,x^{13}}{13}$$

input
```
int((a + b*x^3)^8/x^6,x)
```

output
```
(b^8*x^19)/19 - (a^8/5 + 4*a^7*b*x^3)/x^5 + 28*a^6*b^2*x + (a*b^7*x^16)/2
+ 14*a^5*b^3*x^4 + 10*a^4*b^4*x^7 + (28*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^13
)/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^6}\,dx = \frac{130b^8x^{24} + 1235a\,b^7x^{21} + 5320a^2b^6x^{18} + 13832a^3b^5x^{15} + 24700a^4b^4x^{12} + 34580a^5b^3x^9 + 69160a^6b^2x^6 -}{2470x^5}$$

input
```
int((b*x^3+a)^8/x^6,x)
```

output
```
( - 494*a**8 - 9880*a**7*b*x**3 + 69160*a**6*b**2*x**6 + 34580*a**5*b**3*x
**9 + 24700*a**4*b**4*x**12 + 13832*a**3*b**5*x**15 + 5320*a**2*b**6*x**18
 + 1235*a*b**7*x**21 + 130*b**8*x**24)/(2470*x**5)
```

3.111 $\int \frac{(a+bx^3)^8}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{(a+bx^3)^8}{x^8} dx = -\frac{a^8}{7x^7} - \frac{2a^7b}{x^4} - \frac{28a^6b^2}{x} + 28a^5b^3x^2 + 14a^4b^4x^5 + 7a^3b^5x^8 + \frac{28}{11}a^2b^6x^{11} + \frac{4}{7}ab^7x^{14} + \frac{b^8x^{17}}{17}$$

output
```
-1/7*a^8/x^7-2*a^7*b/x^4-28*a^6*b^2/x+28*a^5*b^3*x^2+14*a^4*b^4*x^5+7*a^3*
b^5*x^8+28/11*a^2*b^6*x^11+4/7*a*b^7*x^14+1/17*b^8*x^17
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^8} dx = -\frac{a^8}{7x^7} - \frac{2a^7b}{x^4} - \frac{28a^6b^2}{x} + 28a^5b^3x^2 + 14a^4b^4x^5 + 7a^3b^5x^8 + \frac{28}{11}a^2b^6x^{11} + \frac{4}{7}ab^7x^{14} + \frac{b^8x^{17}}{17}$$

input
```
Integrate[(a + b*x^3)^8/x^8,x]
```

output
```
-1/7*a^8/x^7 - (2*a^7*b)/x^4 - (28*a^6*b^2)/x + 28*a^5*b^3*x^2 + 14*a^4*b^
4*x^5 + 7*a^3*b^5*x^8 + (28*a^2*b^6*x^11)/11 + (4*a*b^7*x^14)/7 + (b^8*x^1
7)/17
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^8}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^8}{x^8}+\frac{8a^7b}{x^5}+\frac{28a^6b^2}{x^2}+56a^5b^3x+70a^4b^4x^4+56a^3b^5x^7+28a^2b^6x^{10}+8ab^7x^{13}+b^8x^{16}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{7x^7}-\frac{2a^7b}{x^4}-\frac{28a^6b^2}{x}+28a^5b^3x^2+14a^4b^4x^5+7a^3b^5x^8+\frac{28}{11}a^2b^6x^{11}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{17}}{17}$$

input
```
Int[(a + b*x^3)^8/x^8,x]
```

output
```
-1/7*a^8/x^7 - (2*a^7*b)/x^4 - (28*a^6*b^2)/x + 28*a^5*b^3*x^2 + 14*a^4*b^
4*x^5 + 7*a^3*b^5*x^8 + (28*a^2*b^6*x^11)/11 + (4*a*b^7*x^14)/7 + (b^8*x^1
7)/17
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

method	result
default	$-\frac{a^8}{7x^7}-\frac{2a^7b}{x^4}-\frac{28a^6b^2}{x}+28a^5b^3x^2+14a^4b^4x^5+7a^3b^5x^8+\frac{28a^2b^6x^{11}}{11}+\frac{4a\,b^7x^{14}}{7}+\frac{b^8x^{17}}{17}$
norman	$\frac{-\frac{1}{7}a^8+\frac{28}{11}a^2b^6x^{18}-28a^6b^2x^6+14a^4b^4x^{12}+28a^5b^3x^9+\frac{4}{7}a\,b^7x^{21}-2a^7b\,x^3+\frac{1}{17}b^8x^{24}+7a^3b^5x^{15}}{x^7}$
gosper	$-\frac{-77b^8x^{24}-748a\,b^7x^{21}-3332a^2b^6x^{18}-9163a^3b^5x^{15}-18326a^4b^4x^{12}-36652a^5b^3x^9+36652a^6b^2x^6+2618a^7b\,x^3+187a^8}{1309x^7}$
risch	$\frac{b^8x^{17}}{17}+\frac{4a\,b^7x^{14}}{7}+\frac{28a^2b^6x^{11}}{11}+7a^3b^5x^8+14a^4b^4x^5+28a^5b^3x^2+\frac{-28a^6b^2x^6-2a^7b\,x^3-\frac{1}{7}a^8}{x^7}$
parallelrisch	$\frac{77b^8x^{24}+748a\,b^7x^{21}+3332a^2b^6x^{18}+9163a^3b^5x^{15}+18326a^4b^4x^{12}+36652a^5b^3x^9-36652a^6b^2x^6-2618a^7b\,x^3-187a^8}{1309x^7}$
orering	$-\frac{-77b^8x^{24}-748a\,b^7x^{21}-3332a^2b^6x^{18}-9163a^3b^5x^{15}-18326a^4b^4x^{12}-36652a^5b^3x^9+36652a^6b^2x^6+2618a^7b\,x^3+187a^8}{1309x^7}$

input
```
int((b*x^3+a)^8/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^8/x^7-2*a^7*b/x^4-28*a^6*b^2/x+28*a^5*b^3*x^2+14*a^4*b^4*x^5+7*a^3*
b^5*x^8+28/11*a^2*b^6*x^11+4/7*a*b^7*x^14+1/17*b^8*x^17
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^8}\,dx$$
$$= \frac{77\,b^8x^{24} + 748\,ab^7x^{21} + 3332\,a^2b^6x^{18} + 9163\,a^3b^5x^{15} + 18326\,a^4b^4x^{12} + 36652\,a^5b^3x^9 - 36652\,a^6b^2x^6 - 2}{1309\,x^7}$$

input
```
integrate((b*x^3+a)^8/x^8,x, algorithm="fricas")
```

output
```
1/1309*(77*b^8*x^24 + 748*a*b^7*x^21 + 3332*a^2*b^6*x^18 + 9163*a^3*b^5*x^
15 + 18326*a^4*b^4*x^12 + 36652*a^5*b^3*x^9 - 36652*a^6*b^2*x^6 - 2618*a^7
*b*x^3 - 187*a^8)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^8}{x^8}\,dx = 28a^5b^3x^2 + 14a^4b^4x^5 + 7a^3b^5x^8 + \frac{28a^2b^6x^{11}}{11} + \frac{4ab^7x^{14}}{7} + \frac{b^8x^{17}}{17} + \frac{-a^8 - 14a^7bx^3 - 196a^6b^2x^6}{7x^7}$$

input
```
integrate((b*x**3+a)**8/x**8,x)
```

output
```
28*a**5*b**3*x**2 + 14*a**4*b**4*x**5 + 7*a**3*b**5*x**8 + 28*a**2*b**6*x*
*11/11 + 4*a*b**7*x**14/7 + b**8*x**17/17 + (-a**8 - 14*a**7*b*x**3 - 196*
a**6*b**2*x**6)/(7*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^8}{x^8}\,dx = \frac{1}{17}\,b^8x^{17} + \frac{4}{7}\,ab^7x^{14} + \frac{28}{11}\,a^2b^6x^{11} + 7\,a^3b^5x^8 + 14\,a^4b^4x^5 + 28\,a^5b^3x^2 - \frac{196\,a^6b^2x^6 + 14\,a^7bx^3 + a^8}{7\,x^7}$$

input
```
integrate((b*x^3+a)^8/x^8,x, algorithm="maxima")
```

output
```
1/17*b^8*x^17 + 4/7*a*b^7*x^14 + 28/11*a^2*b^6*x^11 + 7*a^3*b^5*x^8 + 14*a
^4*b^4*x^5 + 28*a^5*b^3*x^2 - 1/7*(196*a^6*b^2*x^6 + 14*a^7*b*x^3 + a^8)/x
^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^3)^8}{x^8}\,dx = \frac{1}{17}\,b^8x^{17} + \frac{4}{7}\,ab^7x^{14} + \frac{28}{11}\,a^2b^6x^{11} + 7\,a^3b^5x^8 + 14\,a^4b^4x^5 + 28\,a^5b^3x^2 - \frac{196\,a^6b^2x^6 + 14\,a^7bx^3 + a^8}{7\,x^7}$$

input
```
integrate((b*x^3+a)^8/x^8,x, algorithm="giac")
```

output
```
1/17*b^8*x^17 + 4/7*a*b^7*x^14 + 28/11*a^2*b^6*x^11 + 7*a^3*b^5*x^8 + 14*a
^4*b^4*x^5 + 28*a^5*b^3*x^2 - 1/7*(196*a^6*b^2*x^6 + 14*a^7*b*x^3 + a^8)/x
^7
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^8}{x^8}\,dx = \frac{b^8\,x^{17}}{17} - \frac{\frac{a^8}{7} + 2\,a^7\,b\,x^3 + 28\,a^6\,b^2\,x^6}{x^7} + \frac{4\,a\,b^7\,x^{14}}{7} + 28\,a^5\,b^3\,x^2 + 14\,a^4\,b^4\,x^5 + 7\,a^3\,b^5\,x^8 + \frac{28\,a^2\,b^6\,x^{11}}{11}$$

input
```
int((a + b*x^3)^8/x^8,x)
```

output
```
(b^8*x^17)/17 - (a^8/7 + 2*a^7*b*x^3 + 28*a^6*b^2*x^6)/x^7 + (4*a*b^7*x^14
)/7 + 28*a^5*b^3*x^2 + 14*a^4*b^4*x^5 + 7*a^3*b^5*x^8 + (28*a^2*b^6*x^11)/
11
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^3)^8}{x^8}\,dx = \frac{77b^8x^{24} + 748a\,b^7x^{21} + 3332a^2b^6x^{18} + 9163a^3b^5x^{15} + 18326a^4b^4x^{12} + 36652a^5b^3x^9 - 36652a^6b^2x^6 - 261}{1309x^7}$$

input
```
int((b*x^3+a)^8/x^8,x)
```

output
```
( - 187*a**8 - 2618*a**7*b*x**3 - 36652*a**6*b**2*x**6 + 36652*a**5*b**3*x
**9 + 18326*a**4*b**4*x**12 + 9163*a**3*b**5*x**15 + 3332*a**2*b**6*x**18
+ 748*a*b**7*x**21 + 77*b**8*x**24)/(1309*x**7)
```

3.112 $\int \frac{(a+bx^3)^8}{x^9}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = -\frac{a^8}{8x^8} - \frac{8a^7b}{5x^5} - \frac{14a^6b^2}{x^2} + 56a^5b^3x + \frac{35}{2}a^4b^4x^4 + 8a^3b^5x^7 + \frac{14}{5}a^2b^6x^{10} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{16}}{16}$$

output
```
-1/8*a^8/x^8-8/5*a^7*b/x^5-14*a^6*b^2/x^2+56*a^5*b^3*x+35/2*a^4*b^4*x^4+8*
a^3*b^5*x^7+14/5*a^2*b^6*x^10+8/13*a*b^7*x^13+1/16*b^8*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = -\frac{a^8}{8x^8} - \frac{8a^7b}{5x^5} - \frac{14a^6b^2}{x^2} + 56a^5b^3x + \frac{35}{2}a^4b^4x^4 + 8a^3b^5x^7 + \frac{14}{5}a^2b^6x^{10} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{16}}{16}$$

input
```
Integrate[(a + b*x^3)^8/x^9,x]
```

output
```
-1/8*a^8/x^8 - (8*a^7*b)/(5*x^5) - (14*a^6*b^2)/x^2 + 56*a^5*b^3*x + (35*a
^4*b^4*x^4)/2 + 8*a^3*b^5*x^7 + (14*a^2*b^6*x^10)/5 + (8*a*b^7*x^13)/13 +
(b^8*x^16)/16
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^8}{x^9}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^8}{x^9}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^3}+56a^5b^3+70a^4b^4x^3+56a^3b^5x^6+28a^2b^6x^9+8ab^7x^{12}+b^8x^{15}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{8x^8}-\frac{8a^7b}{5x^5}-\frac{14a^6b^2}{x^2}+56a^5b^3x+\frac{35}{2}a^4b^4x^4+8a^3b^5x^7+\frac{14}{5}a^2b^6x^{10}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{16}}{16}$$

input
```
Int[(a + b*x^3)^8/x^9,x]
```

output

```
-1/8*a^8/x^8 - (8*a^7*b)/(5*x^5) - (14*a^6*b^2)/x^2 + 56*a^5*b^3*x + (35*a
^4*b^4*x^4)/2 + 8*a^3*b^5*x^7 + (14*a^2*b^6*x^10)/5 + (8*a*b^7*x^13)/13 +
(b^8*x^16)/16
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^8}{8x^8}-\frac{8a^7b}{5x^5}-\frac{14a^6b^2}{x^2}+56a^5b^3x+\frac{35a^4x^4b^4}{2}+8a^3b^5x^7+\frac{14a^2b^6x^{10}}{5}+\frac{8a\,b^7x^{13}}{13}+\frac{b^8x^{16}}{16}$
risch	$\frac{b^8x^{16}}{16}+\frac{8a\,b^7x^{13}}{13}+\frac{14a^2b^6x^{10}}{5}+8a^3b^5x^7+\frac{35a^4x^4b^4}{2}+56a^5b^3x+\frac{-14a^6b^2x^6-\frac{8}{5}a^7b\,x^3-\frac{1}{8}a^8}{x^8}$
norman	$\frac{-\frac{1}{8}a^8+\frac{1}{16}b^8x^{24}+\frac{35}{2}a^4b^4x^{12}-14a^6b^2x^6-\frac{8}{5}a^7b\,x^3+\frac{8}{13}a\,b^7x^{21}+\frac{14}{5}a^2b^6x^{18}+56a^5b^3x^9+8a^3b^5x^{15}}{x^8}$
gosper	$-\frac{-65b^8x^{24}-640a\,b^7x^{21}-2912a^2b^6x^{18}-8320a^3b^5x^{15}-18200a^4b^4x^{12}-58240a^5b^3x^9+14560a^6b^2x^6+1664a^7b\,x^3+130a^8}{1040x^8}$
parallelrisch	$\frac{65b^8x^{24}+640a\,b^7x^{21}+2912a^2b^6x^{18}+8320a^3b^5x^{15}+18200a^4b^4x^{12}+58240a^5b^3x^9-14560a^6b^2x^6-1664a^7b\,x^3-130a^8}{1040x^8}$
orering	$-\frac{-65b^8x^{24}-640a\,b^7x^{21}-2912a^2b^6x^{18}-8320a^3b^5x^{15}-18200a^4b^4x^{12}-58240a^5b^3x^9+14560a^6b^2x^6+1664a^7b\,x^3+130a^8}{1040x^8}$

input
```
int((b*x^3+a)^8/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^8/x^8-8/5*a^7*b/x^5-14*a^6*b^2/x^2+56*a^5*b^3*x+35/2*a^4*x^4*b^4+8*
a^3*b^5*x^7+14/5*a^2*b^6*x^10+8/13*a*b^7*x^13+1/16*b^8*x^16
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^8}{x^9}\,dx$$
$$= \frac{65\,b^8x^{24} + 640\,ab^7x^{21} + 2912\,a^2b^6x^{18} + 8320\,a^3b^5x^{15} + 18200\,a^4b^4x^{12} + 58240\,a^5b^3x^9 - 14560\,a^6b^2x^6 - 1}{1040\,x^8}$$

input
```
integrate((b*x^3+a)^8/x^9,x, algorithm="fricas")
```

output
```
1/1040*(65*b^8*x^24 + 640*a*b^7*x^21 + 2912*a^2*b^6*x^18 + 8320*a^3*b^5*x^
15 + 18200*a^4*b^4*x^12 + 58240*a^5*b^3*x^9 - 14560*a^6*b^2*x^6 - 1664*a^7
*b*x^3 - 130*a^8)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = 56a^5b^3x + \frac{35a^4b^4x^4}{2} + 8a^3b^5x^7 + \frac{14a^2b^6x^{10}}{5} + \frac{8ab^7x^{13}}{13} + \frac{b^8x^{16}}{16} + \frac{-5a^8 - 64a^7bx^3 - 560a^6b^2x^6}{40x^8}$$

input
```
integrate((b*x**3+a)**8/x**9,x)
```

output
```
56*a**5*b**3*x + 35*a**4*b**4*x**4/2 + 8*a**3*b**5*x**7 + 14*a**2*b**6*x**
10/5 + 8*a*b**7*x**13/13 + b**8*x**16/16 + (-5*a**8 - 64*a**7*b*x**3 - 560
*a**6*b**2*x**6)/(40*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = \frac{1}{16}\,b^8x^{16} + \frac{8}{13}\,ab^7x^{13} + \frac{14}{5}\,a^2b^6x^{10} + 8\,a^3b^5x^7 + \frac{35}{2}\,a^4b^4x^4 + 56\,a^5b^3x - \frac{560\,a^6b^2x^6 + 64\,a^7bx^3 + 5\,a^8}{40\,x^8}$$

input
```
integrate((b*x^3+a)^8/x^9,x, algorithm="maxima")
```

output
```
1/16*b^8*x^16 + 8/13*a*b^7*x^13 + 14/5*a^2*b^6*x^10 + 8*a^3*b^5*x^7 + 35/2
*a^4*b^4*x^4 + 56*a^5*b^3*x - 1/40*(560*a^6*b^2*x^6 + 64*a^7*b*x^3 + 5*a^8
)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = \frac{1}{16}\,b^8x^{16} + \frac{8}{13}\,ab^7x^{13} + \frac{14}{5}\,a^2b^6x^{10} + 8\,a^3b^5x^7 + \frac{35}{2}\,a^4b^4x^4 + 56\,a^5b^3x - \frac{560\,a^6b^2x^6 + 64\,a^7bx^3 + 5\,a^8}{40\,x^8}$$

input
```
integrate((b*x^3+a)^8/x^9,x, algorithm="giac")
```

output
```
1/16*b^8*x^16 + 8/13*a*b^7*x^13 + 14/5*a^2*b^6*x^10 + 8*a^3*b^5*x^7 + 35/2
*a^4*b^4*x^4 + 56*a^5*b^3*x - 1/40*(560*a^6*b^2*x^6 + 64*a^7*b*x^3 + 5*a^8
)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = \frac{b^8\,x^{16}}{16} - \frac{\frac{a^8}{8}+\frac{8\,a^7\,b\,x^3}{5}+14\,a^6\,b^2\,x^6}{x^8} + 56\,a^5\,b^3\,x + \frac{8\,a\,b^7\,x^{13}}{13} + \frac{35\,a^4\,b^4\,x^4}{2} + 8\,a^3\,b^5\,x^7 + \frac{14\,a^2\,b^6\,x^{10}}{5}$$

input
```
int((a + b*x^3)^8/x^9,x)
```

output
```
(b^8*x^16)/16 - (a^8/8 + (8*a^7*b*x^3)/5 + 14*a^6*b^2*x^6)/x^8 + 56*a^5*b^
3*x + (8*a*b^7*x^13)/13 + (35*a^4*b^4*x^4)/2 + 8*a^3*b^5*x^7 + (14*a^2*b^6
*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^8}{x^9}\,dx = \frac{65b^8x^{24}+640a\,b^7x^{21}+2912a^2b^6x^{18}+8320a^3b^5x^{15}+18200a^4b^4x^{12}+58240a^5b^3x^9-14560a^6b^2x^6-166}{1040x^8}$$

input
```
int((b*x^3+a)^8/x^9,x)
```

output
```
( - 130*a**8 - 1664*a**7*b*x**3 - 14560*a**6*b**2*x**6 + 58240*a**5*b**3*x
**9 + 18200*a**4*b**4*x**12 + 8320*a**3*b**5*x**15 + 2912*a**2*b**6*x**18
+ 640*a*b**7*x**21 + 65*b**8*x**24)/(1040*x**8)
```

3.113 $\int x(1-x^3)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int x\left(1-x^3\right)^2\,dx=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

output
```
1/2*x^2-2/5*x^5+1/8*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(1-x^3\right)^2\,dx=\frac{x^2}{2}-\frac{2x^5}{5}+\frac{x^8}{8}$$

input
```
Integrate[x*(1 - x^3)^2,x]
```

output
```
x^2/2 - (2*x^5)/5 + x^8/8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(1-x^3\right)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(x^7-2 x^4+x\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^8}{8}-\frac{2 x^5}{5}+\frac{x^2}{2}$$

input
```
Int[x*(1 - x^3)^2,x]
```

output
```
x^2/2 - (2*x^5)/5 + x^8/8
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
norman	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
risch	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
parallelrisch	$\frac{1}{2}x^2-\frac{2}{5}x^5+\frac{1}{8}x^8$	17
gosper	$\frac{x^2(5x^6-16x^3+20)}{40}$	18
orering	$\frac{x^2(5x^6-16x^3+20)(-x^3+1)^2}{40(-1+x)^2(x^2+x+1)^2}$	40

input
```
int(x*(-x^3+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-2/5*x^5+1/8*x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x\left(1-x^3\right)^2\,dx=\frac{1}{8}\,x^8-\frac{2}{5}\,x^5+\frac{1}{2}\,x^2$$

input
```
integrate(x*(-x^3+1)^2,x, algorithm="fricas")
```

output
```
1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int x(1-x^3)^2\,dx = \frac{x^8}{8} - \frac{2x^5}{5} + \frac{x^2}{2}$$

input
```
integrate(x*(-x**3+1)**2,x)
```

output
```
x**8/8 - 2*x**5/5 + x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx = \frac{1}{8}\,x^8 - \frac{2}{5}\,x^5 + \frac{1}{2}\,x^2$$

input
```
integrate(x*(-x^3+1)^2,x, algorithm="maxima")
```

output
```
1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x(1-x^3)^2\,dx = \frac{1}{8}\,x^8 - \frac{2}{5}\,x^5 + \frac{1}{2}\,x^2$$

input
```
integrate(x*(-x^3+1)^2,x, algorithm="giac")
```

output
```
1/8*x^8 - 2/5*x^5 + 1/2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x(1-x^3)^2\,dx=\frac{x^2\,(5\,x^6-16\,x^3+20)}{40}$$

input
```
int(x*(x^3 - 1)^2,x)
```

output
```
(x^2*(5*x^6 - 16*x^3 + 20))/40
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x(1-x^3)^2\,dx=\frac{x^2(5x^6-16x^3+20)}{40}$$

input
```
int(x*(-x^3+1)^2,x)
```

output
```
(x**2*(5*x**6 - 16*x**3 + 20))/40
```

3.114 $\int \frac{x^8}{a+bx^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^8}{a+bx^3} \, dx = -\frac{ax^3}{3b^2} + \frac{x^6}{6b} + \frac{a^2 \log(a+bx^3)}{3b^3}$$

output
```
-1/3*a*x^3/b^2+1/6*x^6/b+1/3*a^2*ln(b*x^3+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{a+bx^3} \, dx = -\frac{ax^3}{3b^2} + \frac{x^6}{6b} + \frac{a^2 \log(a+bx^3)}{3b^3}$$

input
```
Integrate[x^8/(a + b*x^3),x]
```

output
```
-1/3*(a*x^3)/b^2 + x^6/(6*b) + (a^2*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+bx^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{bx^3+a}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{x^3}{b}+\frac{a^2}{b^2 (bx^3+a)}-\frac{a}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^2\log (a+bx^3)}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{2b}\right)$$

input
```
Int[x^8/(a + b*x^3),x]
```

output
```
(-((a*x^3)/b^2) + x^6/(2*b) + (a^2*Log[a + b*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{b^{2} x^{6}-2 a b \,x^{3}+2 a^{2} \ln \left(b \,x^{3}+a \right)}{6 b^{3}}$	34
default	$-\frac{-\frac{1}{2} b \,x^{6}+a \,x^{3}}{3 b^{2}}+\frac{a^{2} \ln \left(b \,x^{3}+a \right)}{3 b^{3}}$	35
norman	$-\frac{a \,x^{3}}{3 b^{2}}+\frac{x^{6}}{6 b}+\frac{a^{2} \ln \left(b \,x^{3}+a \right)}{3 b^{3}}$	35
risch	$\frac{x^{6}}{6 b}-\frac{a \,x^{3}}{3 b^{2}}+\frac{a^{2}}{6 b^{3}}+\frac{a^{2} \ln \left(b \,x^{3}+a \right)}{3 b^{3}}$	43

input
```
int(x^8/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*x^6-2*a*b*x^3+2*a^2*ln(b*x^3+a))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{a+bx^3}\,dx = \frac{b^2x^6 - 2\,abx^3 + 2\,a^2\log\left(bx^3 + a\right)}{6\,b^3}$$

input
```
integrate(x^8/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^6 - 2*a*b*x^3 + 2*a^2*log(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{a+bx^3}\,dx=\frac{a^2\log\left(a+bx^3\right)}{3b^3}-\frac{ax^3}{3b^2}+\frac{x^6}{6b}$$

input
```
integrate(x**8/(b*x**3+a),x)
```

output
```
a**2*log(a + b*x**3)/(3*b**3) - a*x**3/(3*b**2) + x**6/(6*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{a+bx^3}\,dx=\frac{a^2\log\left(bx^3+a\right)}{3\,b^3}+\frac{bx^6-2\,ax^3}{6\,b^2}$$

input
```
integrate(x^8/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*a^2*log(b*x^3 + a)/b^3 + 1/6*(b*x^6 - 2*a*x^3)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{a+bx^3}\,dx=\frac{a^2\log\left(|bx^3+a|\right)}{3\,b^3}+\frac{bx^6-2\,ax^3}{6\,b^2}$$

input
```
integrate(x^8/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*a^2*log(abs(b*x^3 + a))/b^3 + 1/6*(b*x^6 - 2*a*x^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{a+bx^3}\,dx = \frac{2\,a^2\,\ln\left(b\,x^3+a\right)+b^2\,x^6-2\,a\,b\,x^3}{6\,b^3}$$

input
```
int(x^8/(a + b*x^3),x)
```

output
```
(2*a^2*log(a + b*x^3) + b^2*x^6 - 2*a*b*x^3)/(6*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{x^8}{a+bx^3}\,dx = \frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^2-2ab\,x^3+b^2x^6}{6b^3}$$

input
```
int(x^8/(b*x^3+a),x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 2*log(a**(1/
3) + b**(1/3)*x)*a**2 - 2*a*b*x**3 + b**2*x**6)/(6*b**3)
```

3.115 $\int \frac{x^5}{a+bx^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^5}{a+bx^3} dx = \frac{x^3}{3b} - \frac{a \log(a+bx^3)}{3b^2}$$

output
```
1/3*x^3/b-1/3*a*ln(b*x^3+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a+bx^3} dx = \frac{x^3}{3b} - \frac{a \log(a+bx^3)}{3b^2}$$

input
```
Integrate[x^5/(a + b*x^3),x]
```

output
```
x^3/(3*b) - (a*Log[a + b*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^3} \, dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3}\int \frac{x^3}{bx^3+a}dx^3$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{3}\int \left(\frac{1}{b}-\frac{a}{b\left(bx^3+a\right)}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{x^3}{b}-\frac{a\log\left(a+bx^3\right)}{b^2}\right)$$

input
```
Int[x^5/(a + b*x^3),x]
```

output
```
(x^3/b - (a*Log[a + b*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-b\,x^{3}+a\ln\left(b\,x^{3}+a\right)}{3b^{2}}$	23
default	$\frac{x^{3}}{3b}-\frac{a\ln\left(b\,x^{3}+a\right)}{3b^{2}}$	24
norman	$\frac{x^{3}}{3b}-\frac{a\ln\left(b\,x^{3}+a\right)}{3b^{2}}$	24
risch	$\frac{x^{3}}{3b}-\frac{a\ln\left(b\,x^{3}+a\right)}{3b^{2}}$	24

input
```
int(x^5/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^3+a*ln(b*x^3+a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{a+bx^3}\,dx=\frac{bx^3-a\log\left(bx^3+a\right)}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(b*x^3 - a*log(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{a+bx^3}\,dx = -\frac{a\log{\left(a+bx^3\right)}}{3b^2}+\frac{x^3}{3b}$$

input
```
integrate(x**5/(b*x**3+a),x)
```

output
```
-a*log(a + b*x**3)/(3*b**2) + x**3/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{a+bx^3}\,dx = \frac{x^3}{3\,b}-\frac{a\log\left(bx^3+a\right)}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*x^3/b - 1/3*a*log(b*x^3 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{a+bx^3}\,dx = \frac{x^3}{3\,b}-\frac{a\log\left(|bx^3+a|\right)}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*x^3/b - 1/3*a*log(abs(b*x^3 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{a+bx^3}\,dx = -\frac{a\,\ln\left(b\,x^3+a\right)-b\,x^3}{3\,b^2}$$

input
```
int(x^5/(a + b*x^3),x)
```

output
```
-(a*log(a + b*x^3) - b*x^3)/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{x^5}{a+bx^3}\,dx = \frac{-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a-\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a+b\,x^3}{3b^2}$$

input
```
int(x^5/(b*x^3+a),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a - log(a**(1/3) +
 b**(1/3)*x)*a + b*x**3)/(3*b**2)
```

3.116 $\int \frac{x^2}{a+bx^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x^2}{a+bx^3} \, dx = \frac{\log(a+bx^3)}{3b}$$

output
```
1/3*ln(b*x^3+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx^3} \, dx = \frac{\log(a+bx^3)}{3b}$$

input
```
Integrate[x^2/(a + b*x^3),x]
```

output
```
Log[a + b*x^3]/(3*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^3} \, dx$$

$$\Big\downarrow \; 792$$

$$\frac{\log \left(a+bx^3\right)}{3b}$$

input
```
Int[x^2/(a + b*x^3),x]
```

output
```
Log[a + b*x^3]/(3*b)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln(b\,x^3+a)}{3b}$	14
default	$\frac{\ln(b\,x^3+a)}{3b}$	14
norman	$\frac{\ln(b\,x^3+a)}{3b}$	14
risch	$\frac{\ln(b\,x^3+a)}{3b}$	14
parallelrisch	$\frac{\ln(b\,x^3+a)}{3b}$	14

input
```
int(x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(b*x^3+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log(bx^3+a)}{3\,b}$$

input
```
integrate(x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*log(b*x^3 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{a+bx^3}\,dx=\frac{\log\left(a+bx^3\right)}{3b}$$

input
```
integrate(x**2/(b*x**3+a),x)
```

output
```
log(a + b*x**3)/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+bx^3}\,dx=\frac{\log\left(bx^3+a\right)}{3\,b}$$

input
```
integrate(x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*log(b*x^3 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+bx^3}\,dx=\frac{\log\left(|bx^3+a|\right)}{3\,b}$$

input
```
integrate(x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*log(abs(b*x^3 + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\ln\left(b\,x^3+a\right)}{3\,b}$$

input
```
int(x^2/(a + b*x^3),x)
```

output
```
log(a + b*x^3)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.47

$$\int \frac{x^2}{a+bx^3}\,dx = \frac{\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{3b}$$

input
```
int(x^2/(b*x^3+a),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + log(a**(1/3) + b**(
1/3)*x))/(3*b)
```

3.117 $\int \frac{1}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(a+bx^3)}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx^3)}{3a}$$

output
```
ln(x)/a-1/3*ln(b*x^3+a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^3)}\,dx = \frac{\log(x)}{a} - \frac{\log(a+bx^3)}{3a}$$

input
```
Integrate[1/(x*(a + b*x^3)),x]
```

output
```
Log[x]/a - Log[a + b*x^3]/(3*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3)}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3(bx^3+a)}dx^3$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3}dx^3}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3}\left(\frac{\log(x^3)}{a}-\frac{b\int \frac{1}{bx^3+a}dx^3}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{\log(x^3)}{a}-\frac{\log(a+bx^3)}{a}\right)$$

input
```
Int[1/(x*(a + b*x^3)),x]
```

output
```
(Log[x^3]/a - Log[a + b*x^3]/a)/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^3+a)}{3a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^3+a)}{3a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^3+a)}{3a}$	21
parallelrisch	$\frac{3\ln(x)-\ln(b\,x^3+a)}{3a}$	21

input
```
int(1/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/3*ln(b*x^3+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(a+bx^3)}\,dx = -\frac{\log\left(bx^3+a\right)-3\,\log\left(x\right)}{3\,a}$$

input
```
integrate(1/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\,(a+bx^3)}\,dx = \frac{\log\left(x\right)}{a} - \frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

input
```
integrate(1/x/(b*x**3+a),x)
```

output
```
log(x)/a - log(a/b + x**3)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\,(a+bx^3)}\,dx = -\frac{\log\left(bx^3+a\right)}{3\,a} + \frac{\log\left(x^3\right)}{3\,a}$$

input
```
integrate(1/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*log(b*x^3 + a)/a + 1/3*log(x^3)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^3)}\,dx = -\frac{\log(|bx^3+a|)}{3\,a} + \frac{\log(|x|)}{a}$$

input
```
integrate(1/x/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a+bx^3)}\,dx = -\frac{\ln(b\,x^3+a) - 3\,\ln(x)}{3\,a}$$

input
```
int(1/(x*(a + b*x^3)),x)
```

output
```
-(log(a + b*x^3) - 3*log(x))/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \frac{1}{x(a+bx^3)}\,dx = \frac{-\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) - \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right) + 3\log(x)}{3a}$$

input
```
int(1/x/(b*x^3+a),x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - log(a**(1/3) + b
**(1/3)*x) + 3*log(x))/(3*a)
```

3.118 $\int \frac{1}{x^4(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{x^4\left(a+bx^3\right)}\,dx=-\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3\right)}{3a^2}$$

output
```
-1/3/a/x^3-b*ln(x)/a^2+1/3*b*ln(b*x^3+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a+bx^3\right)}\,dx=-\frac{1}{3ax^3}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3\right)}{3a^2}$$

input
```
Integrate[1/(x^4*(a + b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) - (b*Log[x])/a^2 + (b*Log[a + b*x^3])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a)} dx^3$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3} \int \left(\frac{b^2}{a^2 (bx^3+a)} - \frac{b}{a^2 x^3} + \frac{1}{a x^6}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{b \log (x^3)}{a^2} + \frac{b \log (a+bx^3)}{a^2} - \frac{1}{a x^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)),x]
```

output
```
(-(1/(a*x^3)) - (b*Log[x^3])/a^2 + (b*Log[a + b*x^3])/a^2)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{3}+a\right)}{3a^{2}}$	32
norman	$-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{3}+a\right)}{3a^{2}}$	32
parallelrisch	$-\frac{3b\ln\left(x\right)x^{3}-b\ln\left(b\,x^{3}+a\right)x^{3}+a}{3a^{2}x^{3}}$	33
risch	$-\frac{1}{3a\,x^{3}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(-b\,x^{3}-a\right)}{3a^{2}}$	35

input
```
int(1/x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3-b*ln(x)/a^2+1/3*b*ln(b*x^3+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\left(a+bx^3\right)}\,dx=\frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log\left(x\right)-a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 (a+bx^3)}\,dx = -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

input
```
integrate(1/x**4/(b*x**3+a),x)
```

output
```
-1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4 (a+bx^3)}\,dx = \frac{b\log(bx^3+a)}{3\,a^2} - \frac{b\log(x^3)}{3\,a^2} - \frac{1}{3\,ax^3}$$

input
```
integrate(1/x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*b*log(b*x^3 + a)/a^2 - 1/3*b*log(x^3)/a^2 - 1/3/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^4 (a+bx^3)}\,dx = \frac{b\log(|bx^3+a|)}{3\,a^2} - \frac{b\log(|x|)}{a^2} + \frac{bx^3-a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*b*log(abs(b*x^3 + a))/a^2 - b*log(abs(x))/a^2 + 1/3*(b*x^3 - a)/(a^2*x
^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 (a + bx^3)} \, dx = \frac{b \ln (b x^3 + a)}{3 a^2} - \frac{1}{3 a x^3} - \frac{b \ln (x)}{a^2}$$

input
```
int(1/(x^4*(a + b*x^3)),x)
```

output
```
(b*log(a + b*x^3))/(3*a^2) - 1/(3*a*x^3) - (b*log(x))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{1}{x^4 (a + bx^3)} \, dx = \frac{\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) b x^3 + \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) b x^3 - 3 \log(x) b x^3 - a}{3a^2 x^3}$$

input
```
int(1/x^4/(b*x^3+a),x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + log(a**(1/3)
 + b**(1/3)*x)*b*x**3 - 3*log(x)*b*x**3 - a)/(3*a**2*x**3)
```

3.119 $\int \frac{x^4}{a+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2b}+\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}}-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}$$

output
```
1/2*x^2/b+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^
(1/2)/b^(5/3)+1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)-1/6*a^(2/3)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{3b^{2/3}x^2+2\sqrt{3}a^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}$$

input
```
Integrate[x^4/(a + b*x^3),x]
```

output
```
(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt
[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {843, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^3}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^2}{2b}-\frac{a\int \frac{x}{bx^3+a}dx}{b}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \ 27$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \ 1082$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$$\Big\downarrow \ 217$$

$$\frac{x^2}{2b}-\frac{a\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

$\downarrow$ 1103

$$\frac{x^2}{2b}-\frac{a\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}$$

input
```
Int[x^4/(a + b*x^3),x]
```

output
```
x^2/(2*b) - (a*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt
[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{x^2}{2b}-\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3b^2}$	37
default	$\frac{x^2}{2b}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	106

input
```
int(x^4/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b-1/3/b^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{3\,x^2-2\,\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)-\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(\right.}{6\,b}$$

input
```
integrate(x^4/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)
^(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) +
 a*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3b^5 - a^2, \left(t \mapsto t\log\left(\frac{9t^2b^3}{a} + x\right)\right)\right) + \frac{x^2}{2b}$$

input
```
integrate(x**4/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x*
*2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2\,b} - \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{a\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*
(a/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2\,b} + \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}$$

input
```
integrate(x^4/(b*x^3+a),x, algorithm="giac")
```

output
```
1/2*x^2/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/b + 1/3*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 - 1
/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{x^2}{2\,b} + \frac{a^{2/3}\,\ln\left(\frac{a^{7/3}}{b^{4/3}}+\frac{a^2\,x}{b}\right)}{3\,b^{5/3}} - \frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}+\frac{a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{5/3}} + \frac{a^{2/3}\,\ln\left(\frac{a^2\,x}{b}+\frac{9\,a^{7/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{b^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{5/3}}$$

input
```
int(x^4/(a + b*x^3),x)
```

output
```
x^2/(2*b) + (a^(2/3)*log(a^(7/3)/b^(4/3) + (a^2*x)/b))/(3*b^(5/3)) - (a^(2
/3)*log((a^2*x)/b + (a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/b^(4/3))*((3^(1/2)*
1i)/2 + 1/2))/(3*b^(5/3)) + (a^(2/3)*log((a^2*x)/b + (9*a^(7/3)*((3^(1/2)*
1i)/6 - 1/6)^2)/b^(4/3))*((3^(1/2)*1i)/6 - 1/6))/b^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{a+bx^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a+3b^{\frac{2}{3}}a^{\frac{1}{3}}x^2-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a}{6b^{\frac{5}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^4/(b*x^3+a),x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a + 3*b**(2/
3)*a**(1/3)*x**2 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
 2*log(a**(1/3) + b**(1/3)*x)*a)/(6*b**(2/3)*a**(1/3)*b)
```

3.120 $\int \frac{x^3}{a+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{x}{b}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}$$

output
```
x/b+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/
b^(4/3)-1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)+1/6*a^(1/3)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{6\sqrt[3]{b}x+2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}$$

input
```
Integrate[x^3/(a + b*x^3),x]
```

output
```
(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^3}dx$$
$$\Big\downarrow \text{843}$$
$$\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}$$
$$\Big\downarrow \text{750}$$
$$\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}$$
$$\Big\downarrow \text{16}$$
$$\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$
$$\Big\downarrow \text{1142}$$

$$\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$\Big\downarrow$ 25

$$\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$\Big\downarrow$ 27

$$\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$\Big\downarrow$ 1082

$$\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$\Big\downarrow$ 217

$$\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$

$\downarrow$ 1103

$$\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}$$

input
```
Int[x^3/(a + b*x^3),x]
```

output
```
x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{x}{b}-\frac{a\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{3b^{2}}$	34
default	$\frac{x}{b}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	103

input
```
int(x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
x/b-1/3/b^2*a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int\frac{x^3}{a+bx^3}\,dx$$
$$=\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)\right.}{6b}$$

input
```
integrate(x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(
3)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.18

$$\int \frac{x^3}{a+bx^3}\,dx = \text{RootSum}\left(27t^3b^4+a,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)+\frac{x}{b}$$

input
```
integrate(x**3/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{x}{b}-\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b
^2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^
(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b}+\frac{x}{b}-\frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2}$$

input
```
integrate(x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/b + x/b - 1/3*sqrt(3)*(-a*b^2)
^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 - 1/6*(-a
*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{a+bx^3}\,dx = \frac{x}{b}+\frac{(-a)^{1/3}\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{3\,b^{4/3}}-\frac{(-a)^{1/3}\ln\left(3\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-3\,a\,b\,x\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,b^{4/3}}+\frac{(-a)^{1/3}\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+3\,a\,b\,x\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{b^{4/3}}$$

input
```
int(x^3/(a + b*x^3),x)
```

output
```
x/b + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x))/(3*b^(4/3)) - ((-a)^(1/3)
*log(3*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2) - 3*a*b*x)*((3^(1/2)*1i)/
2 + 1/2))/(3*b^(4/3)) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)
/6 - 1/6) + 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/b^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int \frac{x^3}{a+bx^3}\,dx$$
$$= \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)+6b^{\frac{1}{3}}x}{6b^{\frac{4}{3}}}$$

input
```
int(x^3/(b*x^3+a),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + a
**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*a**(1/3)*l
og(a**(1/3) + b**(1/3)*x) + 6*b**(1/3)*x)/(6*b**(1/3)*b)
```

3.121 $\int \frac{x}{a+bx^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{x}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(
2/3)-1/3*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{x}{a+bx^3}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + \log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[x/(a + b*x^3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) +
b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^
(2/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^3}dx$$

$$\Big\downarrow \text{ 821}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \textbf{217}$$

$$\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[x/(a + b*x^3),x]
```

output
```
-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{3b}$	27
default	$-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	91

input
```
int(x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.64

$$\int \frac{x}{a+bx^3}\,dx$$
$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^{2}x^{3}-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)+\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x\right.}{6\,ab^{2}}\right.$$

input
```
integrate(x/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/
3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.21

$$\int \frac{x}{a+bx^3}\,dx = \operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

input
```
integrate(x/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{x}{a+bx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*l
og(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{x}{a+bx^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

input
```
integrate(x/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a - 1/3*sqrt(3)*(-a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*(-a*
b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{x}{a+bx^3}\,dx = \frac{\ln\left(b^{1/3}\,x-(-a)^{1/3}\right)}{3\,(-a)^{1/3}\,b^{2/3}} + \frac{\ln\left(b\,x-\frac{(-a)^{1/3}\,b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}} - \frac{\ln\left(b\,x-\frac{(-a)^{1/3}\,b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}}$$

input
```
int(x/(a + b*x^3),x)
```

output
```
log(b^(1/3)*x - (-a)^(1/3))/(3*(-a)^(1/3)*b^(2/3)) + (log(b*x - ((-a)^(1/3
)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*(-a)^(1/3)*b^(2/3))
- (log(b*x - ((-a)^(1/3)*b^(2/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/
(6*(-a)^(1/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{x}{a+bx^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(x/(b*x^3+a),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*log(a**(1/3) + b**(1/3)*x
))/(6*b**(2/3)*a**(1/3))
```

3.122 $\int \frac{1}{a+bx^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \frac{1}{a+bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(
1/3)+1/3*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)-1/6*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{1}{a+bx^3}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[(a + b*x^3)^(-1),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3
) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(2/3)*
b^(1/3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^3}\,dx$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[(a + b*x^3)^(-1),x]
```

output
```
Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

method	result	size
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{3b}$	27
default	$\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	91

input
```
int(1/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.60

$$\int \frac{1}{a+bx^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-\right.}{6\,a^2b}\right.$$

input
```
integrate(1/(b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1
/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a
)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*
b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
)/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2
/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log
(a*b*x + (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{a+bx^3}\,dx = \text{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

input
```
integrate(1/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+bx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
2/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*l
og(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+bx^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

input
```
integrate(1/(b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a + 1/3*sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/6*(-a*b^
2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+bx^3}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}}+\frac{\ln\left(3\,b^2\,x+\frac{3\,a^{1/3}\,b^{5/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(3\,b^2\,x-\frac{3\,a^{1/3}\,b^{5/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}}$$

input
```
int(1/(a + b*x^3),x)
```

output
```
log(b^(1/3)*x + a^(1/3))/(3*a^(2/3)*b^(1/3)) + (log(3*b^2*x + (3*a^(1/3)*b
^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(1/3)) - (log(3
*b^2*x - (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2
/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.58

$$\int \frac{1}{a+bx^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)}{6a^{\frac{2}{3}}b^{\frac{1}{3}}}$$

input
```
int(1/(b*x^3+a),x)
```

output
```
(a**(1/3)*( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
 - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + 2*log(a**(1/3) +
b**(1/3)*x)))/(6*b**(1/3)*a)
```

3.123 $\int \frac{1}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int \frac{1}{x^2\left(a+bx^3\right)}\,dx=-\frac{1}{ax}+\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}+\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}}-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}}$$

output
```
-1/a/x+1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/
2)/a^(4/3)+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)-1/6*b^(1/3)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a+bx^3\right)}\,dx=\frac{-6\sqrt[3]{a}+2\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}x}$$

input
```
Integrate[1/(x^2*(a + b*x^3)),x]
```

output
```
(-6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] + 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] - b^(1/3)*x*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{821}$$

$$-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{16}$$

$$-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 25

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 27

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 1082

$$-\frac{b\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 217

$$-\frac{b\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x^3)),x]
```

output
```
-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqr
t[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.43

method	result	size
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{4}\_Z^{3}-b\right)}\_R\ln\left(\left(-4\_R^{3}a^{4}+3b\right)x-a^{3}\_R^{2}\right)\right)}{3}$	53
default	$-\frac{1}{ax}-\frac{\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	106

input
```
int(1/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+1/3*sum(_R*ln((-4*_R^3*a^4+3*b)*x-a^3*_R^2),_R=RootOf(_Z^3*a^4-b))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\left(a+bx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\right.}{6\,ax}$$

input
```
integrate(1/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/
a)^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^2(a+bx^3)}\,dx = \operatorname{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t\log\left(\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

input
```
integrate(1/x**2/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2(a+bx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*
log(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2(a+bx^3)}\,dx = \frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b} - \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2b} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 1/6*(
-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2(a+bx^3)}\,dx = \frac{b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{3\,a^{4/3}} - \frac{1}{a\,x} - \frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,a^{4/3}} + \frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{4/3}}$$

input
```
int(1/(x^2*(a + b*x^3)),x)
```

output
```
(b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(3*a^(4/3)) - 1/(a*x) - (b^(1/3)*log(3^
(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(
4/3)) + (b^(1/3)*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/
2)*1i)/6 - 1/6))/a^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 (a + bx^3)} \, dx$$
$$= \frac{2\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}} - 2b^{\frac{1}{3}} x}{a^{\frac{1}{3}} \sqrt{3}}\right) bx - 6b^{\frac{2}{3}} a^{\frac{1}{3}} - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) bx + 2 \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) bx}{6b^{\frac{2}{3}} a^{\frac{4}{3}} x}$$

input
```
int(1/x^2/(b*x^3+a),x)
```

output
```
(2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x - 6*b**(
2/3)*a**(1/3) - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x +
2*log(a**(1/3) + b**(1/3)*x)*b*x)/(6*b**(2/3)*a**(1/3)*a*x)
```

3.124 $\int \frac{1}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{1}{x^3(a+bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}$$

output
```
-1/2/a/x^2+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3
^(1/2)/a^(5/3)-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*ln(a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(a+bx^3)}\,dx = \frac{-3a^{2/3}+2\sqrt{3}b^{2/3}x^2\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x\right.}{6a^{5/3}x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^3)),x]
```

output
```
(-3*a^(2/3) + 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] - 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] + b^(2/3)*x^2*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*x^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{b \int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{750}$$

$$-\frac{b \left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{b \left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\downarrow$ 25

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\downarrow$ 27

$$-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\downarrow$ 1082

$$-\frac{b\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\downarrow$ 217

$$-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 1103

$$-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sq
rt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{1}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{5}-3b^{2}\right)x-a^{2}b\_R\right)\right)}{3}$	54
default	$-\frac{1}{2a\,x^{2}}-\frac{\left(\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}$	106

input
```
int(1/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+1/3*sum(_R*ln((-4*_R^3*a^5-3*b^2)*x-a^2*b*_R),_R=RootOf(_Z^3*a^
5+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)}\,dx$$
$$=\frac{2\sqrt{3}x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+2\,x^{2}}{6\,ax^{2}}$$

input
```
integrate(1/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(
2/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)
^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/
a^2)^(1/3)) - 3)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3(a+bx^3)}\,dx = \operatorname{RootSum}\left(27t^3a^5+b^2,\left(t\mapsto t\log\left(-\frac{3ta^2}{b}+x\right)\right)\right)-\frac{1}{2ax^2}$$

input
```
integrate(1/x**3/(b*x**3+a),x)
```

output
```
RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2
*a*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(a+bx^3)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{1}{2ax^2}$$

input
```
integrate(1/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*
log(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^3\left(a+bx^3\right)}\,dx=\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2}-\frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 1/3*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b
^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 1/2/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\left(a+bx^3\right)}\,dx=\frac{b^{2/3}\ \ln\left(\left(-a\right)^{7/3}-a^2\,b^{1/3}\,x\right)}{3\left(-a\right)^{5/3}}-\frac{1}{2\,a\,x^2}-\frac{b^{2/3}\ \ln\left(3\,a^2\,b^3\,x+3\left(-a\right)^{7/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\left(-a\right)^{5/3}}+\frac{b^{2/3}\ \ln\left(3\,a^2\,b^3\,x-9\left(-a\right)^{7/3}\,b^{8/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{\left(-a\right)^{5/3}}$$

input
```
int(1/(x^3*(a + b*x^3)),x)
```

output
```
(b^(2/3)*log((-a)^(7/3) - a^2*b^(1/3)*x))/(3*(-a)^(5/3)) - 1/(2*a*x^2) - (
b^(2/3)*log(3*a^2*b^3*x + 3*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2))*((3
^(1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)) + (b^(2/3)*log(3*a^2*b^3*x - 9*(-a)^(7
/3)*b^(8/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-a)^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3 (a+bx^3)}\,dx = \frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^2+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^2-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)b\,x^2-3b^{\frac{1}{3}}a}{6b^{\frac{1}{3}}a^2x^2}$$

input
```
int(1/x^3/(b*x^3+a),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x
**2 + a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**2
- 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b*x**2 - 3*b**(1/3)*a)/(6*b**(1/3)
*a**2*x**2)
```

3.125 $\int \frac{x^8}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^8}{(a+bx^3)^2} dx = \frac{x^3}{3b^2} - \frac{a^2}{3b^3(a+bx^3)} - \frac{2a\log(a+bx^3)}{3b^3}$$

output
```
1/3*x^3/b^2-1/3*a^2/b^3/(b*x^3+a)-2/3*a*ln(b*x^3+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(a+bx^3)^2} dx = \frac{bx^3 - \frac{a^2}{a+bx^3} - 2a\log(a+bx^3)}{3b^3}$$

input
```
Integrate[x^8/(a + b*x^3)^2,x]
```

output
```
(b*x^3 - a^2/(a + b*x^3) - 2*a*Log[a + b*x^3])/(3*b^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2 (bx^3+a)^2}-\frac{2a}{b^2 (bx^3+a)}+\frac{1}{b^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{a^2}{b^3 (a+bx^3)}-\frac{2a\log (a+bx^3)}{b^3}+\frac{x^3}{b^2}\right)$$

input
```
Int[x^8/(a + b*x^3)^2,x]
```

output
```
(x^3/b^2 - a^2/(b^3*(a + b*x^3)) - (2*a*Log[a + b*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x^{3}}{3b^{2}}-\frac{a^{2}}{3b^{3}\left(b\,x^{3}+a\right)}-\frac{2a\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	41
norman	$\frac{\frac{x^{6}}{3b}-\frac{2a^{2}}{3b^{3}}}{b\,x^{3}+a}-\frac{2a\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	43
default	$\frac{x^{3}}{3b^{2}}-\frac{a\left(\frac{a}{b\left(b\,x^{3}+a\right)}+\frac{2\ln\left(b\,x^{3}+a\right)}{b}\right)}{3b^{2}}$	44
parallelrisch	$-\frac{-b^{2}x^{6}+2\ln\left(b\,x^{3}+a\right)x^{3}ab+2a^{2}\ln\left(b\,x^{3}+a\right)+2a^{2}}{3b^{3}\left(b\,x^{3}+a\right)}$	57

input
```
int(x^8/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/b^2*x^3-1/3*a^2/b^3/(b*x^3+a)-2/3*a*ln(b*x^3+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{x^8}{\left(a+bx^3\right)^2}\,dx=\frac{b^2x^6+abx^3-a^2-2\left(abx^3+a^2\right)\log\left(bx^3+a\right)}{3\left(b^4x^3+ab^3\right)}$$

input
```
integrate(x^8/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^6 + a*b*x^3 - a^2 - 2*(a*b*x^3 + a^2)*log(b*x^3 + a))/(b^4*x^3
+ a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{(a+bx^3)^2}\,dx = -\frac{a^2}{3ab^3+3b^4x^3} - \frac{2a\log{(a+bx^3)}}{3b^3} + \frac{x^3}{3b^2}$$

input
```
integrate(x**8/(b*x**3+a)**2,x)
```

output
```
-a**2/(3*a*b**3 + 3*b**4*x**3) - 2*a*log(a + b*x**3)/(3*b**3) + x**3/(3*b*
*2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^8}{(a+bx^3)^2}\,dx = -\frac{a^2}{3\,(b^4x^3+ab^3)} + \frac{x^3}{3\,b^2} - \frac{2\,a\log\left(bx^3+a\right)}{3\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*a^2/(b^4*x^3 + a*b^3) + 1/3*x^3/b^2 - 2/3*a*log(b*x^3 + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^8}{(a+bx^3)^2}\,dx = \frac{x^3}{3\,b^2} - \frac{2\,a\log\left(|bx^3+a|\right)}{3\,b^3} + \frac{2\,abx^3+a^2}{3\,(bx^3+a)b^3}$$

input
```
integrate(x^8/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*x^3/b^2 - 2/3*a*log(abs(b*x^3 + a))/b^3 + 1/3*(2*a*b*x^3 + a^2)/((b*x^
3 + a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{(a+bx^3)^2}\,dx = \frac{x^3}{3\,b^2} - \frac{a^2}{3\,\left(b^4\,x^3+a\,b^3\right)} - \frac{2\,a\,\ln\left(b\,x^3+a\right)}{3\,b^3}$$

input
```
int(x^8/(a + b*x^3)^2,x)
```

output
```
x^3/(3*b^2) - a^2/(3*(a*b^3 + b^4*x^3)) - (2*a*log(a + b*x^3))/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.50

$$\int \frac{x^8}{(a+bx^3)^2}\,dx$$
$$= \frac{-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3-2\log\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)a^2-2\log\left(a^{\frac{1}{3}}+b\right.}{3b^3\left(b\,x^3+a\right)}$$

input
```
int(x^8/(b*x^3+a)^2,x)
```

output

```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 2*log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 - 2*log(a**(1/3) + b
**(1/3)*x)*a**2 - 2*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 2*a*b*x**3 + b**
2*x**6)/(3*b**3*(a + b*x**3))
```

3.126 $\int \frac{x^5}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{x^5}{(a+bx^3)^2} dx = \frac{a}{3b^2(a+bx^3)} + \frac{\log(a+bx^3)}{3b^2}$$

output
```
1/3*a/b^2/(b*x^3+a)+1/3*ln(b*x^3+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+bx^3)^2} dx = \frac{\frac{a}{a+bx^3} + \log(a+bx^3)}{3b^2}$$

input
```
Integrate[x^5/(a + b*x^3)^2,x]
```

output
```
(a/(a + b*x^3) + Log[a + b*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3}\int \left(\frac{1}{b\,(bx^3+a)}-\frac{a}{b\,(bx^3+a)^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{a}{b^2\,(a+bx^3)}+\frac{\log\left(a+bx^3\right)}{b^2}\right)$$

input
```
Int[x^5/(a + b*x^3)^2,x]
```

output
```
(a/(b^2*(a + b*x^3)) + Log[a + b*x^3]/b^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{a}{3b^{2}(b\,x^{3}+a)}+\frac{\ln(b\,x^{3}+a)}{3b^{2}}$	30
norman	$\frac{a}{3b^{2}(b\,x^{3}+a)}+\frac{\ln(b\,x^{3}+a)}{3b^{2}}$	30
risch	$\frac{a}{3b^{2}(b\,x^{3}+a)}+\frac{\ln(b\,x^{3}+a)}{3b^{2}}$	30
parallelrisch	$\frac{b\ln(b\,x^{3}+a)x^{3}+a\ln(b\,x^{3}+a)+a}{3b^{2}(b\,x^{3}+a)}$	40

input
```
int(x^5/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a/b^2/(b*x^3+a)+1/3*ln(b*x^3+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{x^5}{(a+bx^3)^2}\,dx=\frac{(bx^3+a)\log(bx^3+a)+a}{3\,(b^3x^3+ab^2)}$$

input
```
integrate(x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/3*((b*x^3 + a)*log(b*x^3 + a) + a)/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(a+bx^3)^2}\,dx = \frac{a}{3ab^2+3b^3x^3} + \frac{\log(a+bx^3)}{3b^2}$$

input
```
integrate(x**5/(b*x**3+a)**2,x)
```

output
```
a/(3*a*b**2 + 3*b**3*x**3) + log(a + b*x**3)/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^3)^2}\,dx = \frac{a}{3\,(b^3x^3+ab^2)} + \frac{\log(bx^3+a)}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*a/(b^3*x^3 + a*b^2) + 1/3*log(b*x^3 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{x^5}{(a+bx^3)^2}\,dx = -\frac{\frac{\log\left(\frac{|bx^3+a|}{(bx^3+a)^2|b|}\right)}{b} - \frac{a}{(bx^3+a)b}}{3\,b}$$

input
```
integrate(x^5/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{(a+bx^3)^2}\,dx = \frac{\ln(b\,x^3+a)}{3\,b^2} + \frac{a}{3\,b^2\,(b\,x^3+a)}$$

input
```
int(x^5/(a + b*x^3)^2,x)
```

output
```
log(a + b*x^3)/(3*b^2) + a/(3*b^2*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.94

$$\int \frac{x^5}{(a+bx^3)^2}\,dx$$
$$= \frac{\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a + \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)b\,x^3 + \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)a + \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)b\,x^3 -}{3b^2\,(b\,x^3+a)}$$

input
```
int(x^5/(b*x^3+a)^2,x)
```

output
```
(log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + log(a**(1/3) + b**(1/3)*x)*a +
 log(a**(1/3) + b**(1/3)*x)*b*x**3 - b*x**3)/(3*b**2*(a + b*x**3))
```

3.127 $\int \frac{x^2}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{(a+bx^3)^2} dx = -\frac{1}{3b(a+bx^3)}$$

output
```
-1/3/b/(b*x^3+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)^2} dx = -\frac{1}{3b(a+bx^3)}$$

input
```
Integrate[x^2/(a + b*x^3)^2,x]
```

output
```
-1/3*1/(b*(a + b*x^3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^2} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{3b\,(a+bx^3)}$$

input
```
Int[x^2/(a + b*x^3)^2,x]
```

output
```
-1/3*1/(b*(a + b*x^3))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{3b(b\,x^3+a)}$	15
derivativedivides	$-\frac{1}{3b(b\,x^3+a)}$	15
default	$-\frac{1}{3b(b\,x^3+a)}$	15
norman	$-\frac{1}{3b(b\,x^3+a)}$	15
risch	$-\frac{1}{3b(b\,x^3+a)}$	15
parallelrisch	$-\frac{1}{3b(b\,x^3+a)}$	15
orering	$-\frac{1}{3b(b\,x^3+a)}$	15

input
```
int(x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\left(a+bx^3\right)^2}\,dx=-\frac{1}{3\left(b^2x^3+ab\right)}$$

input
```
integrate(x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/3/(b^2*x^3 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3ab+3b^2x^3}$$

input
```
integrate(x**2/(b*x**3+a)**2,x)
```

output
```
-1/(3*a*b + 3*b**2*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,(bx^3+a)b}$$

input
```
integrate(x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3/((b*x^3 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,(bx^3+a)b}$$

input
```
integrate(x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3/((b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = -\frac{1}{3\,b\,(b\,x^3+a)}$$

input
```
int(x^2/(a + b*x^3)^2,x)
```

output
```
-1/(3*b*(a + b*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{(a+bx^3)^2}\,dx = \frac{x^3}{3a\,(b\,x^3+a)}$$

input
```
int(x^2/(b*x^3+a)^2,x)
```

output
```
x**3/(3*a*(a + b*x**3))
```

3.128 $\int \frac{1}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{x(a+bx^3)^2} dx = \frac{1}{3a(a+bx^3)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx^3)}{3a^2}$$

output
```
1/3/a/(b*x^3+a)+ln(x)/a^2-1/3*ln(b*x^3+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+bx^3)^2} dx = \frac{\frac{a}{a+bx^3} + 3\log(x) - \log(a+bx^3)}{3a^2}$$

input
```
Integrate[1/(x*(a + b*x^3)^2),x]
```

output
```
(a/(a + b*x^3) + 3*Log[x] - Log[a + b*x^3])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3)^2}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{1}{x^3(bx^3+a)^2}dx^3$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{3}\int \left(-\frac{b}{a^2(bx^3+a)}-\frac{b}{a(bx^3+a)^2}+\frac{1}{a^2x^3}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(-\frac{\log(a+bx^3)}{a^2}+\frac{\log(x^3)}{a^2}+\frac{1}{a(a+bx^3)}\right)$$

input
```
Int[1/(x*(a + b*x^3)^2),x]
```

output
```
(1/(a*(a + b*x^3)) + Log[x^3]/a^2 - Log[a + b*x^3]/a^2)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{3a(b\,x^3+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^3+a)}{3a^2}$	35
norman	$-\frac{b\,x^3}{3a^2(b\,x^3+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^3+a)}{3a^2}$	39
default	$\frac{\ln(x)}{a^2}-\frac{b\left(-\frac{a}{b\left(b\,x^3+a\right)}+\frac{\ln\left(b\,x^3+a\right)}{b}\right)}{3a^2}$	42
parallelrisch	$\frac{3b\ln(x)x^3-b\ln(b\,x^3+a)x^3-b\,x^3+3a\ln(x)-a\ln(b\,x^3+a)}{3a^2(b\,x^3+a)}$	60

input
```
int(1/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a/(b*x^3+a)+ln(x)/a^2-1/3*ln(b*x^3+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx=-\frac{(bx^3+a)\log\left(bx^3+a\right)-3\left(bx^3+a\right)\log\left(x\right)-a}{3\left(a^2bx^3+a^3\right)}$$

input
```
integrate(1/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/3*((b*x^3 + a)*log(b*x^3 + a) - 3*(b*x^3 + a)*log(x) - a)/(a^2*b*x^3 +
a^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3a^2+3abx^3} + \frac{\log{(x)}}{a^2} - \frac{\log{\left(\frac{a}{b}+x^3\right)}}{3a^2}$$

input
```
integrate(1/x/(b*x**3+a)**2,x)
```

output
```
1/(3*a**2 + 3*a*b*x**3) + log(x)/a**2 - log(a/b + x**3)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3\left(abx^3+a^2\right)} - \frac{\log\left(bx^3+a\right)}{3\,a^2} + \frac{\log\left(x^3\right)}{3\,a^2}$$

input
```
integrate(1/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3/(a*b*x^3 + a^2) - 1/3*log(b*x^3 + a)/a^2 + 1/3*log(x^3)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(a+bx^3\right)^2}\,dx = -\frac{\log\left(|bx^3+a|\right)}{3\,a^2} + \frac{\log\left(|x|\right)}{a^2} + \frac{bx^3+2\,a}{3\left(bx^3+a\right)a^2}$$

input
```
integrate(1/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/a^2 + log(abs(x))/a^2 + 1/3*(b*x^3 + 2*a)/((b*x^3
 + a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(a+bx^3)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{1}{3\,a\,\left(b\,x^3+a\right)} - \frac{\ln\left(b\,x^3+a\right)}{3\,a^2}$$

input
```
int(1/(x*(a + b*x^3)^2),x)
```

output
```
log(x)/a^2 + 1/(3*a*(a + b*x^3)) - log(a + b*x^3)/(3*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.00

$$\int \frac{1}{x\,(a+bx^3)^2}\,dx = \frac{-\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a - \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)b\,x^3 - \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)a - \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}}x\right)b\,x^3 +}{3a^2\left(b\,x^3+a\right)}$$

input
```
int(1/x/(b*x^3+a)^2,x)
```

output
```
( - log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a - log(a**(2/3) -
 b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - log(a**(1/3) + b**(1/3)*x)*
a - log(a**(1/3) + b**(1/3)*x)*b*x**3 + 3*log(x)*a + 3*log(x)*b*x**3 - b*x
**3)/(3*a**2*(a + b*x**3))
```

3.129 $\int \frac{1}{x^4(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{1}{3a^2x^3}-\frac{b}{3a^2\left(a+bx^3\right)}-\frac{2b\log(x)}{a^3}+\frac{2b\log\left(a+bx^3\right)}{3a^3}$$

output
```
-1/3/a^2/x^3-1/3*b/a^2/(b*x^3+a)-2*b*ln(x)/a^3+2/3*b*ln(b*x^3+a)/a^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{a\left(\frac{1}{x^3}+\frac{b}{a+bx^3}\right)+6b\log(x)-2b\log\left(a+bx^3\right)}{3a^3}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^2),x]
```

output
```
-1/3*(a*(x^(-3) + b/(a + b*x^3)) + 6*b*Log[x] - 2*b*Log[a + b*x^3])/a^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{1}{x^6 (bx^3+a)^2} dx^3$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3} \int \left(\frac{2b^2}{a^3 (bx^3+a)} + \frac{b^2}{a^2 (bx^3+a)^2} - \frac{2b}{a^3 x^3} + \frac{1}{a^2 x^6}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2b \log (x^3)}{a^3} + \frac{2b \log (a+bx^3)}{a^3} - \frac{b}{a^2 (a+bx^3)} - \frac{1}{a^2 x^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^2),x]
```

output
```
(-(1/(a^2*x^3)) - b/(a^2*(a + b*x^3)) - (2*b*Log[x^3])/a^3 + (2*b*Log[a +
b*x^3])/a^3)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
norman	$\frac{-\frac{1}{3a}+\frac{2b^2x^6}{3a^3}}{x^3(b\,x^3+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(b\,x^3+a)}{3a^3}$	54
default	$-\frac{1}{3a^2x^3}-\frac{2b\ln(x)}{a^3}+\frac{b^2\left(-\frac{a}{b\left(b\,x^3+a\right)}+\frac{2\ln\left(b\,x^3+a\right)}{b}\right)}{3a^3}$	55
risch	$\frac{-\frac{2b\,x^3}{3a^2}-\frac{1}{3a}}{x^3(b\,x^3+a)}-\frac{2b\ln(x)}{a^3}+\frac{2b\ln(-b\,x^3-a)}{3a^3}$	55
parallelrisch	$-\frac{6\ln(x)x^6b^2-2\ln(b\,x^3+a)x^6b^2-2b^2x^6+6\ln(x)x^3ab-2\ln(b\,x^3+a)x^3ab+a^2}{3a^3x^3(b\,x^3+a)}$	80

input
```
int(1/x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-1/3/a+2/3*b^2/a^3*x^6)/x^3/(b*x^3+a)-2*b*ln(x)/a^3+2/3*b*ln(b*x^3+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^4\left(a+bx^3\right)^2}\,dx = -\frac{2\,abx^3+a^2-2\left(b^2x^6+abx^3\right)\log\left(bx^3+a\right)+6\left(b^2x^6+abx^3\right)\log\left(x\right)}{3\left(a^3bx^6+a^4x^3\right)}$$

input
```
integrate(1/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*x^3 + a^2 - 2*(b^2*x^6 + a*b*x^3)*log(b*x^3 + a) + 6*(b^2*x^6
+ a*b*x^3)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4 (a+bx^3)^2} \, dx = \frac{-a-2bx^3}{3a^3x^3+3a^2bx^6} - \frac{2b\log(x)}{a^3} + \frac{2b\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

input
```
integrate(1/x**4/(b*x**3+a)**2,x)
```

output
```
(-a - 2*b*x**3)/(3*a**3*x**3 + 3*a**2*b*x**6) - 2*b*log(x)/a**3 + 2*b*log(
a/b + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4 (a+bx^3)^2} \, dx = -\frac{2\,bx^3+a}{3\,(a^2bx^6+a^3x^3)} + \frac{2\,b\log(bx^3+a)}{3\,a^3} - \frac{2\,b\log(x^3)}{3\,a^3}$$

input
```
integrate(1/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(2*b*x^3 + a)/(a^2*b*x^6 + a^3*x^3) + 2/3*b*log(b*x^3 + a)/a^3 - 2/3*
b*log(x^3)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^4 (a+bx^3)^2} \, dx = \frac{2\, b \log(|bx^3+a|)}{3\, a^3} - \frac{2\, b \log(|x|)}{a^3} - \frac{2\, bx^3+a}{3\, (bx^6+ax^3)a^2}$$

input
```
integrate(1/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
2/3*b*log(abs(b*x^3 + a))/a^3 - 2*b*log(abs(x))/a^3 - 1/3*(2*b*x^3 + a)/((
b*x^6 + a*x^3)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^4 (a+bx^3)^2} \, dx = \frac{2\, b\, \ln(b\, x^3+a)}{3\, a^3} - \frac{\frac{1}{3\, a} + \frac{2\, b\, x^3}{3\, a^2}}{b\, x^6 + a\, x^3} - \frac{2\, b\, \ln(x)}{a^3}$$

input
```
int(1/(x^4*(a + b*x^3)^2),x)
```

output
```
(2*b*log(a + b*x^3))/(3*a^3) - (1/(3*a) + (2*b*x^3)/(3*a^2))/(a*x^3 + b*x^
6) - (2*b*log(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 (a+bx^3)^2} \, dx = \frac{2 \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) ab\, x^3 + 2 \log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}} a^{\frac{1}{3}} x + b^{\frac{2}{3}} x^2\right) b^2 x^6 + 2 \log\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} x\right) ab\, x^3 + 2 \log\left(a^{\frac{1}{3}}\right.}{3a^3 x^3 (b\, x^3 + a)}$$

input
```
int(1/x^4/(b*x^3+a)^2,x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 + 2*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 + 2*log(a**(1/3) +
 b**(1/3)*x)*a*b*x**3 + 2*log(a**(1/3) + b**(1/3)*x)*b**2*x**6 - 6*log(x)*
a*b*x**3 - 6*log(x)*b**2*x**6 - a**2 + 2*b**2*x**6)/(3*a**3*x**3*(a + b*x*
*3))
```

3.130 $\int \frac{x^6}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{x^6}{(a+bx^3)^2} dx = \frac{x}{b^2} + \frac{ax}{3b^2(a+bx^3)} + \frac{4\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}b^{7/3}} - \frac{4\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{7/3}} + \frac{2\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9b^{7/3}}$$

output
```
x/b^2+1/3*a*x/b^2/(b*x^3+a)+4/9*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3
^(1/2)/a^(1/3))*3^(1/2)/b^(7/3)-4/9*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(7/3)+
2/9*a^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{(a+bx^3)^2} dx$$
$$= \frac{9\sqrt[3]{b}x + \frac{3a\sqrt[3]{b}x}{a+bx^3} + 4\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 4\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) + 2\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right.}{9b^{7/3}}$$

input
```
Integrate[x^6/(a + b*x^3)^2,x]
```

output
```
(9*b^(1/3)*x + (3*a*b^(1/3)*x)/(a + b*x^3) + 4*Sqrt[3]*a^(1/3)*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + 2*
a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*b^(7/3))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {817, 843, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^2}dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{4\int \frac{x^3}{bx^3+a}dx}{3b}-\frac{x^4}{3b(a+bx^3)}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{4\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^3+a}dx}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{b}\right)}{3b}-\frac{x^4}{3b(a+bx^3)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

$\Big\downarrow$ 1142

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \textbf{1082}$$

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

$\downarrow$ 217

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{4\left(\frac{x}{b}-\frac{a\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}\right)}{3b}-\frac{x^4}{3b\left(a+bx^3\right)}$$

input
```
Int[x^6/(a + b*x^3)^2,x]
```

output
```
-1/3*x^4/(b*(a + b*x^3)) + (4*(x/b - (a*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/
3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(
1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(
2/3))))/b))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{x}{b^2}+\frac{ax}{3b^2(b\,x^3+a)}-\frac{4a\left(\sum_{\_R=\operatorname{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{9b^3}$	50
default	$\frac{x}{b^2}-\frac{a\left(-\frac{x}{3(b\,x^3+a)}+\frac{4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{b^2}$	115

input
```
int(x^6/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2+1/3*a*x/b^2/(b*x^3+a)-4/9/b^3*a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b
+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{x^6}{(a+bx^3)^2}\,dx$$
$$= \frac{9\,bx^4+4\,\sqrt{3}(bx^3+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3\,a}\right)-2\,(bx^3+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right.}{9\,(b^3x^3+ab^2)}$$

input
```
integrate(x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/9*(9*b*x^4 + 4*sqrt(3)*(b*x^3 + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*
x*(-a/b)^(2/3) - sqrt(3)*a)/a) - 2*(b*x^3 + a)*(-a/b)^(1/3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x^3 + a)*(-a/b)^(1/3)*log(x - (-a/b)^(1/
3)) + 12*a*x)/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{x^6}{(a+bx^3)^2}\,dx = \frac{ax}{3ab^2+3b^3x^3} + \operatorname{RootSum}\left(729t^3b^7+64a,\left(t\mapsto t\log\left(-\frac{9tb^2}{4}+x\right)\right)\right)+\frac{x}{b^2}$$

input
```
integrate(x**6/(b*x**3+a)**2,x)
```

output
```
a*x/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*b**7 + 64*a, Lambda(_t, _
t*log(-9*_t*b**2/4 + x))) + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(a+bx^3)^2}\,dx = \frac{ax}{3\,(b^3x^3+ab^2)} + \frac{x}{b^2} - \frac{4\,\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{4\,a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*a*x/(b^3*x^3 + a*b^2) + x/b^2 - 4/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 2/9*a*log(x^2 - x*(a/b)^(1
/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4/9*a*log(x + (a/b)^(1/3))/(b^3*(a/
b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{(a+bx^3)^2}\,dx = \frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,b^2} + \frac{ax}{3\,(bx^3+a)b^2} + \frac{x}{b^2} - \frac{4\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^3} - \frac{2\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^3}$$

input
```
integrate(x^6/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
4/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/b^2 + 1/3*a*x/((b*x^3 + a)*b^2
) + x/b^2 - 4/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1
/3))/(-a/b)^(1/3))/b^3 - 2/9*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a
/b)^(2/3))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(a+bx^3)^2}\,dx = \frac{x}{b^2} + \frac{4\,(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)}{9\,b^{7/3}} + \frac{a\,x}{3\,(b^3\,x^3+a\,b^2)} - \frac{4\,(-a)^{1/3}\,\ln\left(4\,a\,x-\frac{4\,(-a)^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9\,b^{7/3}} + \frac{(-a)^{1/3}\,\ln\left(4\,a\,x+\frac{9\,(-a)^{4/3}\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2i}{9}\right)}{b^{1/3}}\right)\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2i}{9}\right)}{b^{7/3}}$$

input
```
int(x^6/(a + b*x^3)^2,x)
```

output
```
x/b^2 + (4*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x))/(9*b^(7/3)) + (a*x)/(
3*(a*b^2 + b^3*x^3)) - (4*(-a)^(1/3)*log(4*a*x - (4*(-a)^(4/3)*((3^(1/2)*1
i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*b^(7/3)) + ((-a)^(1/3)*lo
g(4*a*x + (9*(-a)^(4/3)*((3^(1/2)*2i)/9 - 2/9))/b^(1/3))*((3^(1/2)*2i)/9 -
 2/9))/b^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\int \frac{x^6}{(a+bx^3)^2}\,dx = \frac{4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3+2a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)+2a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}\right.}{9b^{\frac{7}{3}}\left(b\,x^3+a\right)}$$

input
```
int(x^6/(b*x^3+a)^2,x)
```

output
```
(4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a +
 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x
**3 + 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + 2
*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - 4*a
**(1/3)*log(a**(1/3) + b**(1/3)*x)*a - 4*a**(1/3)*log(a**(1/3) + b**(1/3)*
x)*b*x**3 + 12*b**(1/3)*a*x + 9*b**(1/3)*b*x**4)/(9*b**(1/3)*b**2*(a + b*x
**3))
```

3.131 $\int \frac{x^4}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = -\frac{x^2}{3b(a+bx^3)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{5/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9\sqrt[3]{a}b^{5/3}}$$

output
```
-1/3*x^2/b/(b*x^3+a)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))
*3^(1/2)/a^(1/3)/b^(5/3)-2/9*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5/3)+1/9*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = \frac{-\frac{3b^{2/3}x^2}{a+bx^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{9b^{5/3}}$$

input
```
Integrate[x^4/(a + b*x^3)^2,x]
```

output
```
((-3*b^(2/3)*x^2)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(1/3) - (2*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(1/3))/(9*b^(5/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{2\int \frac{x}{bx^3+a}dx}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \text{821}$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 16$$

$$\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{2\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$\Big\downarrow$ 217

$$\frac{2\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

$\Big\downarrow$ 1103

$$\frac{2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b\left(a+bx^3\right)}$$

input
```
Int[x^4/(a + b*x^3)^2,x]
```

output
```
-1/3*x^2/(b*(a + b*x^3)) + (2*(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2
/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^
(1/3))))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.33

method	result	size
risch	$-\frac{x^{2}}{3b\left(b\,x^{3}+a\right)}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{9b^{2}}$	45
default	$-\frac{x^{2}}{3b\left(b\,x^{3}+a\right)}+\frac{-\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b}$	114

input
```
int(x^4/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^2/b/(b*x^3+a)+2/9/b^2*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.94

$$\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx$$

$$= \left[-\frac{3\,ab^2x^2 - 3\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}\cdots}{bx^3+a}\right.\cdots}{9\left(ab^4x\cdots\right.}\right.$$

$$-\frac{3\,ab^2x^2 - 6\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)-(bx^3+a)(-ab^2)^{\frac{2}{3}}\,\mathrm{lo}\cdots}{9\left(ab^4x^3+a^2b^3\right)}$$

input
```
integrate(x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/9*(3*a*b^2*x^2 - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b
^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (
b*x^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3
)) + 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^4*x^3 +
a^2*b^3), -1/9*(3*a*b^2*x^2 - 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(-a*b^
2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)
/a)/b) - (b*x^3 + a)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a
*b^2)^(2/3)) + 2*(b*x^3 + a)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*
b^4*x^3 + a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = -\frac{x^2}{3ab+3b^2x^3} + \operatorname{RootSum}\left(729t^3ab^5+8,\left(t\mapsto t\log\left(\frac{81t^2ab^3}{4}+x\right)\right)\right)$$

input
```
integrate(x**4/(b*x**3+a)**2,x)
```

output
```
-x**2/(3*a*b + 3*b**2*x**3) + RootSum(729*_t**3*a*b**5 + 8, Lambda(_t, _t*
log(81*_t**2*a*b**3/4 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = -\frac{x^2}{3\,(b^2x^3+ab)} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*x^2/(b^2*x^3 + a*b) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/9*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(b^2*(a/b)^(1/3)) - 2/9*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = -\frac{x^2}{3\,(bx^3+a)b} - \frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{2\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3}$$

input
```
integrate(x^4/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/3*x^2/((b*x^3 + a)*b) - 2/9*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*
b) - 2/9*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-
a/b)^(1/3))/(a*b^3) + 1/9*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = \frac{2\,\ln\left(\frac{4\,x}{9\,b}-\frac{4\,(-a)^{1/3}}{9\,b^{4/3}}\right)}{9\,(-a)^{1/3}\,b^{5/3}} - \frac{x^2}{3\,b\,\left(b\,x^3+a\right)} + \frac{\ln\left(\frac{4\,x}{9\,b}-\frac{(-a)^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9\,b^{4/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,(-a)^{1/3}\,b^{5/3}} - \frac{\ln\left(\frac{4\,x}{9\,b}-\frac{(-a)^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9\,b^{4/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,(-a)^{1/3}\,b^{5/3}}$$

input
```
int(x^4/(a + b*x^3)^2,x)
```

output
```
(2*log((4*x)/(9*b) - (4*(-a)^(1/3))/(9*b^(4/3))))/(9*(-a)^(1/3)*b^(5/3)) -
 x^2/(3*b*(a + b*x^3)) + (log((4*x)/(9*b) - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2
)/(9*b^(4/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(1/3)*b^(5/3)) - (log((4*x)/(9*b)
 - ((-a)^(1/3)*(3^(1/2)*1i + 1)^2)/(9*b^(4/3)))*(3^(1/2)*1i + 1))/(9*(-a)^
(1/3)*b^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.16

$$\int \frac{x^4}{(a+bx^3)^2}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-3b^{\frac{2}{3}}a^{\frac{1}{3}}x^2+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+\log\left(a^{\frac{2}{3}}-\right.}{9b^{\frac{5}{3}}a^{\frac{1}{3}}\left(b\,x^3+a\right)}$$

input
```
int(x^4/(b*x^3+a)^2,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a - 2*sqr
t(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x**3 - 3*b**(2/3
)*a**(1/3)*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - 2*log(a**(1/3
) + b**(1/3)*x)*a - 2*log(a**(1/3) + b**(1/3)*x)*b*x**3)/(9*b**(2/3)*a**(1
/3)*b*(a + b*x**3))
```

3.132 $\int \frac{x^3}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{x^3}{(a+bx^3)^2} dx = -\frac{x}{3b(a+bx^3)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{4/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{4/3}}$$

output
```
-1/3*x/b/(b*x^3+a)-1/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3
^(1/2)/a^(2/3)/b^(4/3)+1/9*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/18*ln(a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = \frac{-\frac{6\sqrt[3]{b}x}{a+bx^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}}{18b^{4/3}}$$

input
```
Integrate[x^3/(a + b*x^3)^2,x]
```

output
```
((-6*b^(1/3)*x)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(2/3) + (2*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) - Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(2/3))/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{\int \frac{1}{bx^3+a}dx}{3b}-\frac{x}{3b(a+bx^3)}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3b}-\frac{x}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 16$$

$$\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 1142$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b\left(a+bx^3\right)}$$

input
```
Int[x^3/(a + b*x^3)^2,x]
```

output
```
-1/3*x/(b*(a + b*x^3)) + (Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (
-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3)))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

method	result	size
risch	$-\frac{x}{3b(b\,x^3+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{9b^2}$	43
default	$-\frac{x}{3b(b\,x^3+a)}+\frac{\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{3b}$	112

input
```
int(x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*x/b/(b*x^3+a)+1/9/b^2*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.92

$$\int \frac{x^3}{(a+bx^3)^2}\,dx$$

$$= \left[-\frac{6\,a^2bx-3\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)}{18\,(a^2b^3x^3+\ldots}\right.$$

$$-\frac{6\,a^2bx-6\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)+(bx^3+a)(a^2b)^{\frac{2}{3}}\log\left(a\ldots\right.}{18\,(a^2b^3x^3+a^3b^2)}$$

input
```
integrate(x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[-1/18*(6*a^2*b*x - 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (b*
x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^3*x^3 + a^3
*b^2), -1/18*(6*a^2*b*x - 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a^2*b)^(1/
3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^
(1/3)/b)/a^2) + (b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x +
(a^2*b)^(1/3)*a) - 2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))
/(a^2*b^3*x^3 + a^3*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{x}{3ab+3b^2x^3} + \operatorname{RootSum}\left(729t^3a^2b^4 - 1, \left(t \mapsto t\log\left(9tab+x\right)\right)\right)$$

input
```
integrate(x**3/(b*x**3+a)**2,x)
```

output
```
-x/(3*a*b + 3*b**2*x**3) + RootSum(729*_t**3*a**2*b**4 - 1, Lambda(_t, _t*
log(9*_t*a*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{x}{3\,(b^2x^3+ab)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*x/(b^2*x^3 + a*b) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/18*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b^2*(a/b)^(2/3)) + 1/9*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab} - \frac{x}{3\,(bx^3+a)b} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2}$$

input
```
integrate(x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-1/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) - 1/3*x/((b*x^3 + a)*b)
 + 1/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/
b)^(1/3))/(a*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^
(2/3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = \frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)}{9\,a^{2/3}\,b^{4/3}} - \frac{x}{3\,b\,\left(b\,x^3+a\right)} + \frac{\ln\left(b\,x+\frac{a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}} - \frac{\ln\left(b\,x-\frac{a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{18\,a^{2/3}\,b^{4/3}}$$

input
```
int(x^3/(a + b*x^3)^2,x)
```

output
```
log(b^(1/3)*x + a^(1/3))/(9*a^(2/3)*b^(4/3)) - x/(3*b*(a + b*x^3)) + (log(
b*x + (a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(18*a^(2/3)*
b^(4/3)) - (log(b*x - (a^(1/3)*b^(2/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i +
1))/(18*a^(2/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{x^3}{(a+bx^3)^2}\,dx = \frac{-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) - 2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3 - a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right) - a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a\right.}{18b^{\frac{4}{3}}a\,(b\,x^3+a)}$$

input
```
int(x^3/(b*x^3+a)^2,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
b*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a -
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + 2*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*a + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x
)*b*x**3 - 6*b**(1/3)*a*x)/(18*b**(1/3)*a*b*(a + b*x**3))
```

3.133 $\int \frac{x}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 136

$$\int \frac{x}{(a+bx^3)^2} dx = \frac{x^2}{3a(a+bx^3)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{4/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{4/3}b^{2/3}}$$

output
```
1/3*x^2/a/(b*x^3+a)-1/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*
3^(1/2)/a^(4/3)/b^(2/3)-1/9*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)+1/18*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{\frac{6\sqrt[3]{a}x^2}{a+bx^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}}{18a^{4/3}}$$

input
```
Integrate[x/(a + b*x^3)^2,x]
```

output
```
((6*a^(1/3)*x^2)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/b^(2/3) - (2*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(2/3))/(18*a^(4/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a\,(a+bx^3)}$$

$$\Big\downarrow \text{821}$$

$$\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$\Big\downarrow$ 16

$$\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$\Big\downarrow$ 1142

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$\Big\downarrow$ 1082

$$\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$\Big\downarrow$ 217

$$\frac{\frac{-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a\left(a+bx^3\right)}$$

input
```
Int[x/(a + b*x^3)^2,x]
```

output
```
x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) +
 (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
)/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{x^{2}}{3a\left(b\,x^{3}+a\right)}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{9ab}$	48
default	$\frac{x^{2}}{3a\left(b\,x^{3}+a\right)}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{3a}$	114

input
```
int(x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^2/a/(b*x^3+a)+1/9/a/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.84

$$\int \frac{x}{(a+bx^3)^2}\,dx$$
$$= \left[\frac{6\,ab^2x^2+3\,\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\,(ab^2)^{\frac{2}{3}}x^2-(ab^2)^{\frac{1}{3}}a\right)\sqrt{-\frac{(ab^2)^{\frac{1}{3}}}{a}}-3\,(ab^2)^{\frac{2}{3}}x}{bx^3+a}\right)+}{18\,(a^2b^3x^3+a^3b}\right.$$

input
```
integrate(x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/18*(6*a*b^2*x^2 + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2
)^(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + (b*x
^3 + a)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 2
*(b*x^3 + a)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^2*b^3*x^3 + a^3*b^
2), 1/18*(6*a*b^2*x^2 - 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a*b^2)^(1/3)
/a)*arctan(-sqrt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + (
b*x^3 + a)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3))
- 2*(b*x^3 + a)*(a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^2*b^3*x^3 + a^3
*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3a^2+3abx^3} + \operatorname{RootSum}\left(729t^3a^4b^2+1, \left(t \mapsto t\log\left(81t^2a^3b+x\right)\right)\right)$$

input
```
integrate(x/(b*x**3+a)**2,x)
```

output
```
x**2/(3*a**2 + 3*a*b*x**3) + RootSum(729*_t**3*a**4*b**2 + 1, Lambda(_t, _
t*log(81*_t**2*a**3*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3\,(abx^3+a^2)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*x^2/(a*b*x^3 + a^2) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3
))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/18*log(x^2 - x*(a/b)^(1/3) + (a/b)^(
2/3))/(a*b*(a/b)^(1/3)) - 1/9*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3\,(bx^3+a)a} - \frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b^2}$$

input
```
integrate(x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
1/3*x^2/((b*x^3 + a)*a) - 1/9*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2
- 1/9*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b
)^(1/3))/(a^2*b^2) + 1/18*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{x}{(a+bx^3)^2}\,dx = \frac{x^2}{3\,a\,\left(b\,x^3+a\right)} + \frac{(-1)^{1/3}\,\ln\left(\frac{(-1)^{2/3}\,b^{2/3}}{9\,a^{5/3}}+\frac{b\,x}{9\,a^2}\right)}{9\,a^{4/3}\,b^{2/3}} - \frac{(-1)^{1/3}\,\ln\left(\left((-1)^{2/3}\,a^{1/3}-2\,b^{1/3}\,x+(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)}{9\,a^{4/3}\,b^{2/3}} + \frac{(-1)^{1/3}\,\ln\left(\left(2\,b^{1/3}\,x-(-1)^{2/3}\,a^{1/3}+(-1)^{1/6}\,\sqrt{3}\,a^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)}{9\,a^{4/3}\,b^{2/3}}$$

input
```
int(x/(a + b*x^3)^2,x)
```

output
```
x^2/(3*a*(a + b*x^3)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/(9*a^(5/3)) +
 (b*x)/(9*a^2)))/(9*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((-1)^(2/3)*a^(1/3)
- 2*b^(1/3)*x + (-1)^(1/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(
4/3)*b^(2/3)) + ((-1)^(1/3)*log(2*b^(1/3)*x - (-1)^(2/3)*a^(1/3) + (-1)^(1
/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(4/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.16

$$\int \frac{x}{(a+bx^3)^2}\,dx$$
$$= \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3+6b^{\frac{2}{3}}a^{\frac{1}{3}}x^2+\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a+\log\left(a^{\frac{2}{3}}-\right.}{18b^{\frac{2}{3}}a^{\frac{4}{3}}\left(b\,x^3+a\right)}$$

input
```
int(x/(b*x^3+a)^2,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a - 2*sqr
t(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x**3 + 6*b**(2/3
)*a**(1/3)*x**2 + log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a +
log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - 2*log(a**(1/3
) + b**(1/3)*x)*a - 2*log(a**(1/3) + b**(1/3)*x)*b*x**3)/(18*b**(2/3)*a**(
1/3)*a*(a + b*x**3))
```

3.134 $\int \frac{1}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 134

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3a(a+bx^3)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}\sqrt[3]{b}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}\sqrt[3]{b}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}\sqrt[3]{b}}$$

output
```
1/3*x/a/(b*x^3+a)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^
(1/2)/a^(5/3)/b^(1/3)+2/9*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1/3)-1/9*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{\frac{3a^{2/3}x}{a+bx^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{9a^{5/3}}$$

input
```
Integrate[(a + b*x^3)^(-2),x]
```

output
```
((3*a^(2/3)*x)/(a + b*x^3) - (2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]])/b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/b^(1/3))/(9*a^(5/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{749}$$

$$\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}$$

$$\Big\downarrow \text{750}$$

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$\big\downarrow$ 16

$$\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$\big\downarrow$ 1142

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$\big\downarrow$ 25

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$\big\downarrow$ 27

$$\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$\big\downarrow$ 1082

$$\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a\left(a+bx^3\right)}$$

input
```
Int[(a + b*x^3)^(-2),x]
```

output
```
x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (
-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) - Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x}{3a(b\,x^{3}+a)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{9ab}$	46
default	$\frac{x}{3a(b\,x^{3}+a)}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{a}$	112

input
```
int(1/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x/a/(b*x^3+a)+2/9/a/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.90

$$\int \frac{1}{(a+bx^3)^2}\,dx = \left[\frac{3\,a^2bx + 3\,\sqrt{\frac{1}{3}}(ab^2x^3+a^2b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-}{9\,(a^3b^2x^3+a^4}\right.$$

input
```
integrate(1/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
[1/9*(3*a^2*b*x + 3*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-(a^2*b)^(1/3)/b)*l
og((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (b*x^
3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*
(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^2*x^3 + a^4*b
), 1/9*(3*a^2*b*x + 6*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt((a^2*b)^(1/3)/b)*
arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/
b)/a^2) - (b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b
)^(1/3)*a) + 2*(b*x^3 + a)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*
b^2*x^3 + a^4*b)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3a^2+3abx^3} + \operatorname{RootSum}\left(729t^3a^5b-8,\left(t\mapsto t\log\left(\frac{9ta^2}{2}+x\right)\right)\right)$$

input
```
integrate(1/(b*x**3+a)**2,x)
```

output
```
x/(3*a**2 + 3*a*b*x**3) + RootSum(729*_t**3*a**5*b - 8, Lambda(_t, _t*log(
9*_t*a**2/2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3\,(abx^3+a^2)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/3*x/(a*b*x^3 + a^2) + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))
/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/9*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3
))/(a*b*(a/b)^(2/3)) + 2/9*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx^3)^2}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2} + \frac{x}{3\,(bx^3+a)a} + \frac{2\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b}$$

input
```
integrate(1/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
-2/9*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*x/((b*x^3 + a)*a) +
 2/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a^2*b) + 1/9*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/
3))/(a^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{x}{3\,a\,\left(b\,x^3+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,x}{a}\right)}{9\,a^{5/3}\,b^{1/3}}+\frac{\ln\left(\frac{2\,b^2\,x}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(\frac{2\,b^2\,x}{a}-\frac{b^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{a^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}}$$

input
```
int(1/(a + b*x^3)^2,x)
```

output
```
x/(3*a*(a + b*x^3)) + (2*log((2*b^(5/3))/a^(2/3) + (2*b^2*x)/a))/(9*a^(5/3
)*b^(1/3)) + (log((2*b^2*x)/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/
2)*1i - 1))/(9*a^(5/3)*b^(1/3)) - (log((2*b^2*x)/a - (b^(5/3)*(3^(1/2)*1i
+ 1))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a+bx^3)^2}\,dx = \frac{-2a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a \right.}{9b^{\frac{1}{3}}a^2\left(b\,x^3+a\right)}$$

input
```
int(1/(b*x^3+a)^2,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
b*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a -
a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 + 2*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*a + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x
)*b*x**3 + 3*b**(1/3)*a*x)/(9*b**(1/3)*a**2*(a + b*x**3))
```

3.135 $\int \frac{1}{x^2(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 145

$$\int \frac{1}{x^2(a+bx^3)^2}\,dx = -\frac{1}{a^2x}-\frac{bx^2}{3a^2(a+bx^3)}+\frac{4\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}}+\frac{4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}}-\frac{2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{7/3}}$$

output

```
-1/a^2/x-1/3*b*x^2/a^2/(b*x^3+a)+4/9*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)
*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)+4/9*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(
7/3)-2/9*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a+bx^3)^2}\,dx = \frac{-\frac{9\sqrt[3]{a}}{x}-\frac{3\sqrt[3]{a}bx^2}{a+bx^3}+4\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}\right.}{9a^{7/3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^2),x]
```

output
```
((-9*a^(1/3))/x - (3*a^(1/3)*b*x^2)/(a + b*x^3) + 4*Sqrt[3]*b^(1/3)*ArcTan
[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*Log[a^(1/3) + b^(1/3)*x]
 - 2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(7/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{4\int \frac{1}{x^2(bx^3+a)}dx}{3a}+\frac{1}{3ax(a+bx^3)}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{4\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$$\Big\downarrow \ 1142$$

$$\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$\downarrow$ 27

$$\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$\downarrow$ 1082

$$\frac{4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$\downarrow$ 217

$$\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

$\downarrow$ 1103

$$\frac{4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax\left(a+bx^3\right)}$$

input
```
Int[1/(x^2*(a + b*x^3)^2),x]
```

output
```
1/(3*a*x*(a + b*x^3)) + (4*(-(1/(a*x)) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*x]
/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))
/(3*a^(1/3)*b^(1/3))))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

method	result	size
risch	$\frac{-\frac{4b\,x^{3}}{3a^{2}}-\frac{1}{a}}{x(b\,x^{3}+a)}+\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a^{7}\_Z^{3}-b\right)}\_R\ln\left(\left(-4\_R^{3}a^{7}+3b\right)x-a^{5}\_R^{2}\right)\right)}{9}$	73
default	$-\frac{1}{a^{2}x}-\frac{b\left(\frac{x^{2}}{3b\,x^{3}+3a}-\frac{4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}}$	120

input
```
int(1/x^2/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-4/3*b/a^2*x^3-1/a)/x/(b*x^3+a)+4/9*sum(_R*ln((-4*_R^3*a^7+3*b)*x-a^5*_R^
2),_R=RootOf(_Z^3*a^7-b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx = -\frac{12\,bx^3+4\,\sqrt{3}(bx^4+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+2\,(bx^4+ax)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\right.}{9\,(a^2bx^4+a^3x)}$$

input
```
integrate(1/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/9*(12*b*x^3 + 4*sqrt(3)*(b*x^4 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*
(b/a)^(1/3) - 1/3*sqrt(3)) + 2*(b*x^4 + a*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(
b/a)^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^4 + a*x)*(b/a)^(1/3)*log(b*x + a*(b/a
)^(2/3)) + 9*a)/(a^2*b*x^4 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = \frac{-3a-4bx^3}{3a^3x+3a^2bx^4} + \operatorname{RootSum}\left(729t^3a^7-64b,\left(t\mapsto t\log\left(\frac{81t^2a^5}{16b}+x\right)\right)\right)$$

input
```
integrate(1/x**2/(b*x**3+a)**2,x)
```

output
```
(-3*a - 4*b*x**3)/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t**3*a**7 - 64
*b, Lambda(_t, _t*log(81*_t**2*a**5/(16*b) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx = -\frac{4\,bx^3+3\,a}{3\,(a^2bx^4+a^3x)} - \frac{4\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{4\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/3*(4*b*x^3 + 3*a)/(a^2*b*x^4 + a^3*x) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/9*log(x^2 - x*(a/b)
^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/9*log(x + (a/b)^(1/3))/(a^2*(a
/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx = \frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3} + \frac{4\,\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b} - \frac{4\,bx^3+3\,a}{3\left(bx^4+ax\right)a^2} - \frac{2\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b}$$

input
```
integrate(1/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
4/9*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 4/9*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/3*(
4*b*x^3 + 3*a)/((b*x^4 + a*x)*a^2) - 2/9*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\left(a+bx^3\right)^2}\,dx = \frac{4\,b^{1/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{9\,a^{7/3}} - \frac{\frac{1}{a}+\frac{4\,b\,x^3}{3\,a^2}}{b\,x^4+a\,x} - \frac{4\,b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}+\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{7/3}} + \frac{b^{1/3}\,\ln\left(4\,b^{1/3}\,x-2\,a^{1/3}-\sqrt{3}\,a^{1/3}\,2\mathrm{i}\right)\,\left(-\frac{2}{9}+\frac{\sqrt{3}\,2\mathrm{i}}{9}\right)}{a^{7/3}}$$

input
```
int(1/(x^2*(a + b*x^3)^2),x)
```

output
```
(4*b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(9*a^(7/3)) - (1/a + (4*b*x^3)/(3*a^2
))/(a*x + b*x^4) - (4*b^(1/3)*log(3^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(
1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(7/3)) + (b^(1/3)*log(4*b^(1/3)*x - 3^(
1/2)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/2)*2i)/9 - 2/9))/a^(7/3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2 (a+bx^3)^2}\,dx$$
$$= \frac{4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)abx+4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^4-9b^{\frac{2}{3}}a^{\frac{4}{3}}-12b^{\frac{5}{3}}a^{\frac{1}{3}}x^3-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)}{9b^{\frac{2}{3}}a^{\frac{7}{3}}x\,(b\,x^3+a)}$$

input
```
int(1/x^2/(b*x^3+a)^2,x)
```

output
```
(4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x + 4*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**4 - 9*b**
(2/3)*a**(1/3)*a - 12*b**(2/3)*a**(1/3)*b*x**3 - 2*log(a**(2/3) - b**(1/3)
*a**(1/3)*x + b**(2/3)*x**2)*a*b*x - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x
+ b**(2/3)*x**2)*b**2*x**4 + 4*log(a**(1/3) + b**(1/3)*x)*a*b*x + 4*log(a*
*(1/3) + b**(1/3)*x)*b**2*x**4)/(9*b**(2/3)*a**(1/3)*a**2*x*(a + b*x**3))
```

3.136 $\int \frac{1}{x^3(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 145

$$\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx = -\frac{1}{2a^2x^2}-\frac{bx}{3a^2\left(a+bx^3\right)}+\frac{5b^{2/3}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}} -\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}}+\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}}$$

output
```
-1/2/a^2/x^2-1/3*b*x/a^2/(b*x^3+a)+5/9*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)-5/9*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a
^(8/3)+5/18*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx = \frac{-\frac{9a^{2/3}}{x^2}-\frac{6a^{2/3}bx}{a+bx^3}+10\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+\right.}{18a^{8/3}}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^2),x]
```

output
```
((-9*a^(2/3))/x^2 - (6*a^(2/3)*b*x)/(a + b*x^3) + 10*Sqrt[3]*b^(2/3)*ArcTa
n[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*b^(2/3)*Log[a^(1/3) + b^(1/3)*
x] + 5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(8/3)
)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {819, 847, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)^2} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5 \int \frac{1}{x^3 (bx^3+a)}dx}{3a}+\frac{1}{3ax^2 (a+bx^3)}$$

$$\Big\downarrow \text{847}$$

$$\frac{5 \left(-\frac{b \int \frac{1}{bx^3+a}dx}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2 (a+bx^3)}$$

$$\Big\downarrow \text{750}$$

$$\frac{5 \left(-\frac{b \left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2 (a+bx^3)}$$

$$\Big\downarrow \; 16$$

$$\frac{5\left(-\frac{b\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1142$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

$\big\downarrow$ 217

$$\frac{5\left(-\frac{b\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

$\big\downarrow$ 1103

$$\frac{5\left(-\frac{b\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}\right)}{3a}+\frac{1}{3ax^2\left(a+bx^3\right)}$$

input
```
Int[1/(x^3*(a + b*x^3)^2),x]
```

output
```
1/(3*a*x^2*(a + b*x^3)) + (5*(-1/2*1/(a*x^2) - (b*(Log[a^(1/3) + b^(1/3)*x
]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3
)))/(3*a^(2/3))))/a))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{-\frac{5b\,x^{3}}{6a^{2}}-\frac{1}{2a}}{x^{2}\left(b\,x^{3}+a\right)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(a^{8}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(-4\_R^{3}a^{8}-3b^{2}\right)x-a^{3}b\_R\right)\right)}{9}$	74
default	$-\frac{1}{2a^{2}x^{2}}-\frac{b\left(\frac{x}{3b\,x^{3}+3a}+\frac{5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{a^{2}}$	118

input
```
int(1/x^3/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/6*b/a^2*x^3-1/2/a)/x^2/(b*x^3+a)+5/9*sum(_R*ln((-4*_R^3*a^8-3*b^2)*x-a
^3*b*_R),_R=RootOf(_Z^3*a^8+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^3 (a+bx^3)^2}\, dx = -\frac{15\, bx^3 + 10\sqrt{3}(bx^5 + ax^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}} - \sqrt{3}b}{3\,b}\right) - 5\,(bx^5 + ax^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \log\left(b^2x^2 - abx\left(\frac{b}{a}\right.\right.}{18\,(a^2bx^5 + a^3x^2)}$$

input
```
integrate(1/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/18*(15*b*x^3 + 10*sqrt(3)*(b*x^5 + a*x^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2
*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^5 + a*x^2)*(b^2/a^2)
^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(b*
x^5 + a*x^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 9*a)/(a^2*b*x^
5 + a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^3 (a+bx^3)^2}\, dx = \frac{-3a - 5bx^3}{6a^3x^2 + 6a^2bx^5} + \operatorname{RootSum}\left(729t^3a^8 + 125b^2, \left(t \mapsto t\log\left(-\frac{9ta^3}{5b} + x\right)\right)\right)$$

input
```
integrate(1/x**3/(b*x**3+a)**2,x)
```

output
```
(-3*a - 5*b*x**3)/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(729*_t**3*a**8 +
 125*b**2, Lambda(_t, _t*log(-9*_t*a**3/(5*b) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a+bx^3)^2}\, dx = -\frac{5\, bx^3 + 3\, a}{6\, (a^2bx^5 + a^3x^2)} - \frac{5\, \sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2\, x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\, a^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\, \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\, a^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\, \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\, a^2 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(5*b*x^3 + 3*a)/(a^2*b*x^5 + a^3*x^2) - 5/9*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/18*log(x^2 - x*(a
/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/9*log(x + (a/b)^(1/3))/(a^2
*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3 (a+bx^3)^2}\, dx = \frac{5\, b\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\, a^3} - \frac{bx}{3\, (bx^3 + a)a^2} - \frac{5\, \sqrt{3}(-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\, \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\, a^3} - \frac{5\, (-ab^2)^{\frac{1}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\, a^3} - \frac{1}{2\, a^2x^2}$$

input
```
integrate(1/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
5/9*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 - 1/3*b*x/((b*x^3 + a)*a
^2) - 5/9*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(
-a/b)^(1/3))/a^3 - 5/18*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/a^3 - 1/2/(a^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3 (a+bx^3)^2} \, dx = \frac{5 (-1)^{1/3} b^{2/3} \ln\left((-1)^{1/3} a^{1/3} - b^{1/3} x\right)}{9 a^{8/3}} - \frac{\frac{1}{2a} + \frac{5 b x^3}{6 a^2}}{b x^5 + a x^2} - \frac{5 (-1)^{1/3} b^{2/3} \ln\left((-1)^{1/3} a^{1/3} + 2 b^{1/3} x + (-1)^{5/6} \sqrt{3} a^{1/3}\right) \left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9 a^{8/3}} + \frac{5 (-1)^{1/3} b^{2/3} \ln\left((-1)^{1/3} a^{1/3} + 2 b^{1/3} x - (-1)^{5/6} \sqrt{3} a^{1/3}\right) \left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9 a^{8/3}}$$

input
```
int(1/(x^3*(a + b*x^3)^2),x)
```

output
```
(5*(-1)^(1/3)*b^(2/3)*log((-1)^(1/3)*a^(1/3) - b^(1/3)*x))/(9*a^(8/3)) - (
1/(2*a) + (5*b*x^3)/(6*a^2))/(a*x^2 + b*x^5) - (5*(-1)^(1/3)*b^(2/3)*log((
-1)^(1/3)*a^(1/3) + 2*b^(1/3)*x + (-1)^(5/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i
)/2 + 1/2))/(9*a^(8/3)) + (5*(-1)^(1/3)*b^(2/3)*log((-1)^(1/3)*a^(1/3) + 2
*b^(1/3)*x - (-1)^(5/6)*3^(1/2)*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3
))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^3 (a+bx^3)^2} \, dx$$
$$= \frac{10a^{\frac{4}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b\, x^2 + 10a^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2x^5 + 5a^{\frac{4}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) b\, x^2 + 5a^{\frac{1}{3}}\mathrm{lo}}{18b^{\frac{1}{3}}a^3x^2}$$

input
```
int(1/x^3/(b*x^3+a)^2,x)
```

output
```
(10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*
b*x**2 + 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt
(3)))*b**2*x**5 + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*a*b*x**2 + 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*b**2*x**5 - 10*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b*x**2 - 10*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*b**2*x**5 - 9*b**(1/3)*a**2 - 15*b**(1/3
)*a*b*x**3)/(18*b**(1/3)*a**3*x**2*(a + b*x**3))
```

3.137 $\int \frac{x^{11}}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{x^{11}}{(a+bx^3)^3} dx = \frac{x^3}{3b^3} + \frac{a^3}{6b^4(a+bx^3)^2} - \frac{a^2}{b^4(a+bx^3)} - \frac{a\log(a+bx^3)}{b^4}$$

output
```
1/3*x^3/b^3+1/6*a^3/b^4/(b*x^3+a)^2-a^2/b^4/(b*x^3+a)-a*ln(b*x^3+a)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int \frac{x^{11}}{(a+bx^3)^3} dx = -\frac{-2bx^3 + \frac{a^2(5a+6bx^3)}{(a+bx^3)^2} + 6a\log(a+bx^3)}{6b^4}$$

input
```
Integrate[x^11/(a + b*x^3)^3,x]
```

output
```
-1/6*(-2*b*x^3 + (a^2*(5*a + 6*b*x^3))/(a + b*x^3)^2 + 6*a*Log[a + b*x^3])
/b^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^3(bx^3+a)^3}+\frac{3a^2}{b^3(bx^3+a)^2}-\frac{3a}{b^3(bx^3+a)}+\frac{1}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{a^3}{2b^4(a+bx^3)^2}-\frac{3a^2}{b^4(a+bx^3)}-\frac{3a\log(a+bx^3)}{b^4}+\frac{x^3}{b^3}\right)$$

input
```
Int[x^11/(a + b*x^3)^3,x]
```

output
```
(x^3/b^3 + a^3/(2*b^4*(a + b*x^3)^2) - (3*a^2)/(b^4*(a + b*x^3)) - (3*a*Lo
g[a + b*x^3])/b^4)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

method	result	size
norman	$\frac{\frac{x^9}{3b}-\frac{3a^3}{2b^4}-\frac{2a^2x^3}{b^3}}{(b\,x^3+a)^2}-\frac{a\ln(b\,x^3+a)}{b^4}$	54
risch	$\frac{x^3}{3b^3}+\frac{-a^2x^3-\frac{5a^3}{6b}}{b^3(b\,x^3+a)^2}-\frac{a\ln(b\,x^3+a)}{b^4}$	54
default	$\frac{x^3}{3b^3}-\frac{a\left(-\frac{a^2}{2b(b\,x^3+a)^2}+\frac{3a}{b(b\,x^3+a)}+\frac{3\ln(b\,x^3+a)}{b}\right)}{3b^3}$	62
parallelrisch	$-\frac{-2b^3x^9+6\ln(b\,x^3+a)x^6a\,b^2+12\ln(b\,x^3+a)x^3a^2b+12a^2b\,x^3+6\ln(b\,x^3+a)a^3+9a^3}{6b^4(b\,x^3+a)^2}$	85

input
```
int(x^11/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^9/b-3/2*a^3/b^4-2*a^2*x^3/b^3)/(b*x^3+a)^2-a*ln(b*x^3+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx = \frac{2\,b^3x^9+4\,ab^2x^6-4\,a^2bx^3-5\,a^3-6\,(ab^2x^6+2\,a^2bx^3+a^3)\log(bx^3+a)}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)}$$

input
```
integrate(x^11/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^9 + 4*a*b^2*x^6 - 4*a^2*b*x^3 - 5*a^3 - 6*(a*b^2*x^6 + 2*a^2*
b*x^3 + a^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx = -\frac{a\log(a+bx^3)}{b^4} + \frac{-5a^3-6a^2bx^3}{6a^2b^4+12ab^5x^3+6b^6x^6} + \frac{x^3}{3b^3}$$

input
```
integrate(x**11/(b*x**3+a)**3,x)
```

output
```
-a*log(a + b*x**3)/b**4 + (-5*a**3 - 6*a**2*b*x**3)/(6*a**2*b**4 + 12*a*b*
*5*x**3 + 6*b**6*x**6) + x**3/(3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx = -\frac{6\,a^2bx^3+5\,a^3}{6\,(b^6x^6+2\,ab^5x^3+a^2b^4)} + \frac{x^3}{3\,b^3} - \frac{a\log(bx^3+a)}{b^4}$$

input
```
integrate(x^11/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(6*a^2*b*x^3 + 5*a^3)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) + 1/3*x^3/b^3
 - a*log(b*x^3 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx = \frac{x^3}{3\,b^3} - \frac{a\log\left(|bx^3+a|\right)}{b^4} + \frac{9\,ab^2x^6+12\,a^2bx^3+4\,a^3}{6\,(bx^3+a)^2b^4}$$

input
```
integrate(x^11/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*x^3/b^3 - a*log(abs(b*x^3 + a))/b^4 + 1/6*(9*a*b^2*x^6 + 12*a^2*b*x^3
+ 4*a^3)/((b*x^3 + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx = \frac{x^3}{3\,b^3} - \frac{\frac{5\,a^3}{6\,b}+a^2\,x^3}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6} - \frac{a\,\ln\left(b\,x^3+a\right)}{b^4}$$

input
```
int(x^11/(a + b*x^3)^3,x)
```

output
```
x^3/(3*b^3) - ((5*a^3)/(6*b) + a^2*x^3)/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3)
- (a*log(a + b*x^3))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.07

$$\int \frac{x^{11}}{(a+bx^3)^3}\,dx$$
$$= \frac{-6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^3-12\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2b\,x^3-6\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a\,b^2x}{6b^4\left(b^2x^6+2a\right.}$$

input
```
int(x^11/(b*x^3+a)^3,x)
```

output
```
( - 6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**3 - 12*log(a*
*(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*x**3 - 6*log(a**(2/3)
 - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*x**6 - 6*log(a**(1/3) + b**
(1/3)*x)*a**3 - 12*log(a**(1/3) + b**(1/3)*x)*a**2*b*x**3 - 6*log(a**(1/3)
 + b**(1/3)*x)*a*b**2*x**6 - 3*a**3 + 6*a*b**2*x**6 + 2*b**3*x**9)/(6*b**4
*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.138 $\int \frac{x^8}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{x^8}{(a+bx^3)^3} dx = -\frac{a^2}{6b^3(a+bx^3)^2} + \frac{2a}{3b^3(a+bx^3)} + \frac{\log(a+bx^3)}{3b^3}$$

output
```
-1/6*a^2/b^3/(b*x^3+a)^2+2/3*a/b^3/(b*x^3+a)+1/3*ln(b*x^3+a)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{(a+bx^3)^3} dx = \frac{\frac{a(3a+4bx^3)}{(a+bx^3)^2} + 2\log(a+bx^3)}{6b^3}$$

input
```
Integrate[x^8/(a + b*x^3)^3,x]
```

output
```
((a*(3*a + 4*b*x^3))/(a + b*x^3)^2 + 2*Log[a + b*x^3])/(6*b^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2\,(bx^3+a)^3}-\frac{2a}{b^2\,(bx^3+a)^2}+\frac{1}{b^2\,(bx^3+a)}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^2}{2b^3\,(a+bx^3)^2}+\frac{2a}{b^3\,(a+bx^3)}+\frac{\log(a+bx^3)}{b^3}\right)$$

input
```
Int[x^8/(a + b*x^3)^3,x]
```

output
```
(-1/2*a^2/(b^3*(a + b*x^3)^2) + (2*a)/(b^3*(a + b*x^3)) + Log[a + b*x^3]/b
^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{\frac{a^{2}}{2b^{3}}+\frac{2a\,x^{3}}{3b^{2}}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	43
risch	$\frac{\frac{a^{2}}{2b^{3}}+\frac{2a\,x^{3}}{3b^{2}}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	43
default	$-\frac{a^{2}}{6b^{3}\left(b\,x^{3}+a\right)^{2}}+\frac{2a}{3b^{3}\left(b\,x^{3}+a\right)}+\frac{\ln\left(b\,x^{3}+a\right)}{3b^{3}}$	47
parallelrisch	$\frac{2\ln\left(b\,x^{3}+a\right)x^{6}b^{2}+4\ln\left(b\,x^{3}+a\right)x^{3}ab+4ab\,x^{3}+2a^{2}\ln\left(b\,x^{3}+a\right)+3a^{2}}{6b^{3}\left(b\,x^{3}+a\right)^{2}}$	72

input
```
int(x^8/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*a^2/b^3+2/3*a*x^3/b^2)/(b*x^3+a)^2+1/3*ln(b*x^3+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{x^8}{(a+bx^3)^3}\,dx=\frac{4\,abx^3+3\,a^2+2\,(b^2x^6+2\,abx^3+a^2)\log\left(bx^3+a\right)}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)}$$

input
```
integrate(x^8/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(4*a*b*x^3 + 3*a^2 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a))/(b^
5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(a+bx^3)^3}\,dx = \frac{3a^2+4abx^3}{6a^2b^3+12ab^4x^3+6b^5x^6} + \frac{\log(a+bx^3)}{3b^3}$$

input
```
integrate(x**8/(b*x**3+a)**3,x)
```

output
```
(3*a**2 + 4*a*b*x**3)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + log(a
 + b*x**3)/(3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{x^8}{(a+bx^3)^3}\,dx = \frac{4\,abx^3+3\,a^2}{6\,(b^5x^6+2\,ab^4x^3+a^2b^3)} + \frac{\log(bx^3+a)}{3\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(4*a*b*x^3 + 3*a^2)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3) + 1/3*log(b*x^3
+ a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{x^8}{(a+bx^3)^3}\,dx = \frac{\log(|bx^3+a|)}{3\,b^3} - \frac{3\,bx^6+2\,ax^3}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^8/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
1/3*log(abs(b*x^3 + a))/b^3 - 1/6*(3*b*x^6 + 2*a*x^3)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{(a+bx^3)^3}\,dx = \frac{\frac{a^2}{2\,b^3}+\frac{2\,a\,x^3}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(b\,x^3+a\right)}{3\,b^3}$$

input
```
int(x^8/(a + b*x^3)^3,x)
```

output
```
(a^2/(2*b^3) + (2*a*x^3)/(3*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + log(a + b*
x^3)/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.27

$$\int \frac{x^8}{(a+bx^3)^3}\,dx = \frac{2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2+4\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3+2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^6+2\,\mathrm{l}}{6b^3\left(b^2x^6+2ab\,x^3+a^2\right)}$$

input
```
int(x^8/(b*x^3+a)^3,x)
```

output
```
(2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 4*log(a**(2/
3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 + 2*log(a**(2/3) - b**(
1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 + 2*log(a**(1/3) + b**(1/3)*x)*
a**2 + 4*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 2*log(a**(1/3) + b**(1/3)*x
)*b**2*x**6 + a**2 - 2*b**2*x**6)/(6*b**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.139 $\int \frac{x^5}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = \frac{x^6}{6a(a+bx^3)^2}$$

output
```
1/6*x^6/a/(b*x^3+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = -\frac{a+2bx^3}{6b^2(a+bx^3)^2}$$

input
```
Integrate[x^5/(a + b*x^3)^3,x]
```

output
```
-1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{ 796}$$
$$\frac{x^6}{6a\left(a+bx^3\right)^2}$$

input
```
Int[x^5/(a + b*x^3)^3,x]
```

output
```
x^6/(6*a*(a + b*x^3)^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2b\,x^3+a}{6(b\,x^3+a)^2 b^2}$	23
orering	$-\frac{2b\,x^3+a}{6(b\,x^3+a)^2 b^2}$	23
parallelrisch	$\frac{-2b\,x^3-a}{6b^2(b\,x^3+a)^2}$	25
norman	$\frac{-\frac{x^3}{3b}-\frac{a}{6b^2}}{(b\,x^3+a)^2}$	26
risch	$\frac{-\frac{x^3}{3b}-\frac{a}{6b^2}}{(b\,x^3+a)^2}$	26
default	$\frac{a}{6b^2(b\,x^3+a)^2}-\frac{1}{3b^2(b\,x^3+a)}$	31

input
```
int(x^5/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*b*x^3+a)/(b*x^3+a)^2/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = \frac{-a-2bx^3}{6a^2b^2+12ab^3x^3+6b^4x^6}$$

input
```
integrate(x**5/(b*x**3+a)**3,x)
```

output
```
(-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = -\frac{2\,bx^3+a}{6\,(b^4x^6+2\,ab^3x^3+a^2b^2)}$$

input
```
integrate(x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = -\frac{2\,bx^3+a}{6\,(bx^3+a)^2b^2}$$

input
```
integrate(x^5/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = -\frac{\frac{a}{6\,b^2}+\frac{x^3}{3\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

input
```
int(x^5/(a + b*x^3)^3,x)
```

output
```
-(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{x^5}{(a+bx^3)^3}\,dx = \frac{x^6}{6a\,(b^2x^6+2ab\,x^3+a^2)}$$

input
```
int(x^5/(b*x^3+a)^3,x)
```

output
```
x**6/(6*a*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.140 $\int \frac{x^2}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^2}{(a+bx^3)^3} \, dx = -\frac{1}{6b(a+bx^3)^2}$$

output
```
-1/6/b/(b*x^3+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)^3} \, dx = -\frac{1}{6b(a+bx^3)^2}$$

input
```
Integrate[x^2/(a + b*x^3)^3,x]
```

output
```
-1/6*1/(b*(a + b*x^3)^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^3} \, dx$$
$$\Big\downarrow \; 793$$
$$-\frac{1}{6b(a+bx^3)^2}$$

input
```
Int[x^2/(a + b*x^3)^3,x]
```

output
```
-1/6*1/(b*(a + b*x^3)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{6b(b\,x^3+a)^2}$	15
derivativedivides	$-\frac{1}{6b(b\,x^3+a)^2}$	15
default	$-\frac{1}{6b(b\,x^3+a)^2}$	15
norman	$-\frac{1}{6b(b\,x^3+a)^2}$	15
risch	$-\frac{1}{6b(b\,x^3+a)^2}$	15
parallelrisch	$-\frac{1}{6b(b\,x^3+a)^2}$	15
orering	$-\frac{1}{6b(b\,x^3+a)^2}$	15

input
```
int(x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/b/(b*x^3+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{(a+bx^3)^3}\,dx = -\frac{1}{6\,(b^3x^6+2\,ab^2x^3+a^2b)}$$

input
```
integrate(x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(a+bx^3)^3}\,dx = -\frac{1}{6a^2b+12ab^2x^3+6b^3x^6}$$

input
```
integrate(x**2/(b*x**3+a)**3,x)
```

output
```
-1/(6*a**2*b + 12*a*b**2*x**3 + 6*b**3*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^3}\,dx = -\frac{1}{6\,(bx^3+a)^2b}$$

input
```
integrate(x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6/((b*x^3 + a)^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx^3)^3}\,dx = -\frac{1}{6\,(bx^3+a)^2b}$$

input
```
integrate(x^2/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/6/((b*x^3 + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{x^2}{(a+bx^3)^3}\,dx = -\frac{1}{6\,a^2\,b+12\,a\,b^2\,x^3+6\,b^3\,x^6}$$

input
```
int(x^2/(a + b*x^3)^3,x)
```

output
```
-1/(6*a^2*b + 6*b^3*x^6 + 12*a*b^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x^2}{(a+bx^3)^3}\,dx = -\frac{1}{6b\,(b^2x^6+2ab\,x^3+a^2)}$$

input
```
int(x^2/(b*x^3+a)^3,x)
```

output
```
( - 1)/(6*b*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.141 $\int \frac{1}{x(a+bx^3)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a+bx^3)^3} dx = \frac{1}{6a(a+bx^3)^2} + \frac{1}{3a^2(a+bx^3)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx^3)}{3a^3}$$

output
```
1/6/a/(b*x^3+a)^2+1/3/a^2/(b*x^3+a)+ln(x)/a^3-1/3*ln(b*x^3+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(a+bx^3)^3} dx = \frac{\frac{a(3a+2bx^3)}{(a+bx^3)^2} + 6\log(x) - 2\log(a+bx^3)}{6a^3}$$

input
```
Integrate[1/(x*(a + b*x^3)^3),x]
```

output
```
((a*(3*a + 2*b*x^3))/(a + b*x^3)^2 + 6*Log[x] - 2*Log[a + b*x^3])/(6*a^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3)^3}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3(bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}\int \left(-\frac{b}{a^3(bx^3+a)}-\frac{b}{a^2(bx^3+a)^2}-\frac{b}{a(bx^3+a)^3}+\frac{1}{a^3x^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{\log(a+bx^3)}{a^3}+\frac{\log(x^3)}{a^3}+\frac{1}{a^2(a+bx^3)}+\frac{1}{2a(a+bx^3)^2}\right)$$

input
```
Int[1/(x*(a + b*x^3)^3),x]
```

output
```
(1/(2*a*(a + b*x^3)^2) + 1/(a^2*(a + b*x^3)) + Log[x^3]/a^3 - Log[a + b*x^
3]/a^3)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\frac{b\,x^{3}}{3a^{2}}+\frac{1}{2a}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(b\,x^{3}+a\right)}{3a^{3}}$	46
norman	$\frac{-\frac{2b\,x^{3}}{3a^{2}}-\frac{b^{2}x^{6}}{2a^{3}}}{\left(b\,x^{3}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(b\,x^{3}+a\right)}{3a^{3}}$	52
default	$\frac{\ln\left(x\right)}{a^{3}}-\frac{b\left(-\frac{a^{2}}{2b\left(b\,x^{3}+a\right)^{2}}-\frac{a}{b\left(b\,x^{3}+a\right)}+\frac{\ln\left(b\,x^{3}+a\right)}{b}\right)}{3a^{3}}$	59
parallelrisch	$\frac{6\ln\left(x\right)x^{6}b^{2}-2\ln\left(b\,x^{3}+a\right)x^{6}b^{2}-3b^{2}x^{6}+12\ln\left(x\right)x^{3}ab-4\ln\left(b\,x^{3}+a\right)x^{3}ab-4ab\,x^{3}+6a^{2}\ln\left(x\right)-2a^{2}\ln\left(b\,x^{3}+a\right)}{6a^{3}\left(b\,x^{3}+a\right)^{2}}$	101

input
```
int(1/x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/3*b/a^2*x^3+1/2/a)/(b*x^3+a)^2+ln(x)/a^3-1/3*ln(b*x^3+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(a+bx^3\right)^3}\,dx = \frac{2\,abx^{3}+3\,a^{2}-2\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)\log\left(bx^{3}+a\right)+6\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)\log\left(x\right)}{6\,\left(a^{3}b^{2}x^{6}+2\,a^{4}bx^{3}+a^{5}\right)}$$

input
```
integrate(1/x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*
(b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+bx^3\right)^3}\,dx = \frac{3a+2bx^3}{6a^4+12a^3bx^3+6a^2b^2x^6}+\frac{\log(x)}{a^3}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a^3}$$

input
```
integrate(1/x/(b*x**3+a)**3,x)
```

output
```
(3*a + 2*b*x**3)/(6*a**4 + 12*a**3*b*x**3 + 6*a**2*b**2*x**6) + log(x)/a**
3 - log(a/b + x**3)/(3*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+bx^3\right)^3}\,dx = \frac{2\,bx^3+3\,a}{6\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}-\frac{\log\left(bx^3+a\right)}{3\,a^3}+\frac{\log\left(x^3\right)}{3\,a^3}$$

input
```
integrate(1/x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/6*(2*b*x^3 + 3*a)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) - 1/3*log(b*x^3 + a)
/a^3 + 1/3*log(x^3)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+bx^3\right)^3}\,dx=-\frac{\log\left(\left|bx^3+a\right|\right)}{3\,a^3}+\frac{\log\left(\left|x\right|\right)}{a^3}+\frac{3\,b^2x^6+8\,abx^3+6\,a^2}{6\left(bx^3+a\right)^2a^3}$$

input
```
integrate(1/x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 + a))/a^3 + log(abs(x))/a^3 + 1/6*(3*b^2*x^6 + 8*a*b*x^
3 + 6*a^2)/((b*x^3 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+bx^3\right)^3}\,dx=\frac{\ln\left(x\right)}{a^3}+\frac{\frac{1}{2\,a}+\frac{b\,x^3}{3\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}-\frac{\ln\left(b\,x^3+a\right)}{3\,a^3}$$

input
```
int(1/(x*(a + b*x^3)^3),x)
```

output
```
log(x)/a^3 + (1/(2*a) + (b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) - log
(a + b*x^3)/(3*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.67

$$\int \frac{1}{x\left(a+bx^3\right)^3}\,dx$$
$$=\frac{-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2-4\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)ab\,x^3-2\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b^2x^6-}{}$$

input
```
int(1/x/(b*x^3+a)^3,x)
```

output
```
( - 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 4*log(a**
(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b*x**3 - 2*log(a**(2/3) - b
**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**2*x**6 - 2*log(a**(1/3) + b**(1/3)*
x)*a**2 - 4*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 - 2*log(a**(1/3) + b**(1/3
)*x)*b**2*x**6 + 6*log(x)*a**2 + 12*log(x)*a*b*x**3 + 6*log(x)*b**2*x**6 +
 2*a**2 - b**2*x**6)/(6*a**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.142 $\int \frac{1}{x^4(a+bx^3)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = -\frac{1}{3a^3x^3} - \frac{b}{6a^2 (a+bx^3)^2} - \frac{2b}{3a^3 (a+bx^3)} - \frac{3b\log(x)}{a^4} + \frac{b\log(a+bx^3)}{a^4}$$

output
```
-1/3/a^3/x^3-1/6*b/a^2/(b*x^3+a)^2-2/3*b/a^3/(b*x^3+a)-3*b*ln(x)/a^4+b*ln(
b*x^3+a)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = -\frac{\frac{a(2a^2+9abx^3+6b^2x^6)}{x^3(a+bx^3)^2} + 18b\log(x) - 6b\log(a+bx^3)}{6a^4}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^3),x]
```

output
```
-1/6*((a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6))/(x^3*(a + b*x^3)^2) + 18*b*Log[x
] - 6*b*Log[a + b*x^3])/a^4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^3+a)^3}dx^3$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{3}\int \left(\frac{3b^2}{a^4 (bx^3+a)}+\frac{2b^2}{a^3 (bx^3+a)^2}+\frac{b^2}{a^2 (bx^3+a)^3}-\frac{3b}{a^4x^3}+\frac{1}{a^3x^6}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3b\log \left(x^3\right)}{a^4}+\frac{3b\log \left(a+bx^3\right)}{a^4}-\frac{2b}{a^3 (a+bx^3)}-\frac{1}{a^3x^3}-\frac{b}{2a^2 (a+bx^3)^2}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^3),x]
```

output
```
(-(1/(a^3*x^3)) - b/(2*a^2*(a + b*x^3)^2) - (2*b)/(a^3*(a + b*x^3)) - (3*b
*Log[x^3])/a^4 + (3*b*Log[a + b*x^3])/a^4)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

method	result
norman	$\frac{-\frac{1}{3a}+\frac{2b^2x^6}{a^3}+\frac{3b^3x^9}{2a^4}}{x^3\left(b\,x^3+a\right)^2}+\frac{b\ln\left(b\,x^3+a\right)}{a^4}-\frac{3b\ln\left(x\right)}{a^4}$
risch	$\frac{-\frac{b^2x^6}{a^3}-\frac{3b\,x^3}{2a^2}-\frac{1}{3a}}{x^3\left(b\,x^3+a\right)^2}-\frac{3b\ln\left(x\right)}{a^4}+\frac{b\ln\left(-b\,x^3-a\right)}{a^4}$
default	$-\frac{1}{3a^3x^3}-\frac{3b\ln\left(x\right)}{a^4}+\frac{b^2\left(-\frac{a^2}{2b\left(b\,x^3+a\right)^2}-\frac{2a}{b\left(b\,x^3+a\right)}+\frac{3\ln\left(b\,x^3+a\right)}{b}\right)}{3a^4}$
parallelrisch	$-\frac{18b^3\ln\left(x\right)x^9-6\ln\left(b\,x^3+a\right)x^9b^3-9b^3x^9+36a\,b^2\ln\left(x\right)x^6-12\ln\left(b\,x^3+a\right)x^6a\,b^2-12a\,b^2x^6+18\ln\left(x\right)x^3a^2b-6\ln\left(b\,x^3+a\right)x^3a}{6a^4x^3\left(b\,x^3+a\right)^2}$

input
```
int(1/x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/3/a+2*b^2/a^3*x^6+3/2*b^3/a^4*x^9)/x^3/(b*x^3+a)^2+b*ln(b*x^3+a)/a^4-3
*b*ln(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = -\frac{6 ab^2x^6 + 9 a^2bx^3 + 2 a^3 - 6 (b^3x^9 + 2 ab^2x^6 + a^2bx^3) \log (bx^3 + a) + 18 (b^3x^9 + 2 ab^2x^6 + a^2bx^3) \log (}{6 (a^4b^2x^9 + 2 a^5bx^6 + a^6x^3)}$$

input
```
integrate(1/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b
*x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4
*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = \frac{-2a^2 - 9abx^3 - 6b^2x^6}{6a^5x^3 + 12a^4bx^6 + 6a^3b^2x^9} - \frac{3b \log (x)}{a^4} + \frac{b \log \left(\frac{a}{b} + x^3\right)}{a^4}$$

input
```
integrate(1/x**4/(b*x**3+a)**3,x)
```

output
```
(-2*a**2 - 9*a*b*x**3 - 6*b**2*x**6)/(6*a**5*x**3 + 12*a**4*b*x**6 + 6*a**
3*b**2*x**9) - 3*b*log(x)/a**4 + b*log(a/b + x**3)/a**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = -\frac{6 b^2x^6 + 9 abx^3 + 2 a^2}{6 (a^3b^2x^9 + 2 a^4bx^6 + a^5x^3)} + \frac{b \log (bx^3 + a)}{a^4} - \frac{b \log (x^3)}{a^4}$$

input
```
integrate(1/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/6*(6*b^2*x^6 + 9*a*b*x^3 + 2*a^2)/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)
 + b*log(b*x^3 + a)/a^4 - b*log(x^3)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = \frac{b \log(|bx^3+a|)}{a^4} - \frac{3 b \log(|x|)}{a^4} - \frac{9 b^3 x^6 + 22 ab^2 x^3 + 14 a^2 b}{6 (bx^3+a)^2 a^4} + \frac{3 bx^3 - a}{3 a^4 x^3}$$

input
```
integrate(1/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
b*log(abs(b*x^3 + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/6*(9*b^3*x^6 + 22*a*b^
2*x^3 + 14*a^2*b)/((b*x^3 + a)^2*a^4) + 1/3*(3*b*x^3 - a)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^4 (a+bx^3)^3} dx = \frac{b \ln(b x^3 + a)}{a^4} - \frac{\frac{1}{3 a} + \frac{3 b x^3}{2 a^2} + \frac{b^2 x^6}{a^3}}{a^2 x^3 + 2 a b x^6 + b^2 x^9} - \frac{3 b \ln(x)}{a^4}$$

input
```
int(1/(x^4*(a + b*x^3)^3),x)
```

output
```
(b*log(a + b*x^3))/a^4 - (1/(3*a) + (3*b*x^3)/(2*a^2) + (b^2*x^6)/a^3)/(a^
2*x^3 + b^2*x^9 + 2*a*b*x^6) - (3*b*log(x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 228, normalized size of antiderivative = 3.45

$$\int \frac{1}{x^4 (a+bx^3)^3}\,dx = \frac{6\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a^2b\,x^3 + 12\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)a\,b^2x^6 + 6\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right)b^3x}{}$$

input
```
int(1/x^4/(b*x^3+a)^3,x)
```

output
```
(6*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2*b*x**3 + 12*lo
g(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**2*x**6 + 6*log(a**(
2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**9 + 6*log(a**(1/3) + b
**(1/3)*x)*a**2*b*x**3 + 12*log(a**(1/3) + b**(1/3)*x)*a*b**2*x**6 + 6*log
(a**(1/3) + b**(1/3)*x)*b**3*x**9 - 18*log(x)*a**2*b*x**3 - 36*log(x)*a*b*
*2*x**6 - 18*log(x)*b**3*x**9 - 2*a**3 - 6*a**2*b*x**3 + 3*b**3*x**9)/(6*a
**4*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.143 $\int \frac{x^7}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 155

$$\int \frac{x^7}{(a+bx^3)^3}\,dx = -\frac{x^5}{6b(a+bx^3)^2} - \frac{5x^2}{18b^2(a+bx^3)} - \frac{5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}b^{8/3}} - \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}b^{8/3}} + \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}b^{8/3}}$$

output

```
-1/6*x^5/b/(b*x^3+a)^2-5/18*x^2/b^2/(b*x^3+a)-5/27*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(8/3)-5/27*ln(a^(1/3)+b^(1/3)
*x)/a^(1/3)/b^(8/3)+5/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)
/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int \frac{x^7}{(a+bx^3)^3}\,dx$$

$$=\frac{\frac{9ab^{2/3}x^2}{(a+bx^3)^2}-\frac{24b^{2/3}x^2}{a+bx^3}-\frac{10\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}+\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}}}{54b^{8/3}}$$

input
```
Integrate[x^7/(a + b*x^3)^3,x]
```

output
```
((9*a*b^(2/3)*x^2)/(a + b*x^3)^2 - (24*b^(2/3)*x^2)/(a + b*x^3) - (10*Sqrt
[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) - (10*Log[a^(1/3)
 + b^(1/3)*x])/a^(1/3) + (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/a^(1/3))/(54*b^(8/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {817, 817, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5\int \frac{x^4}{(bx^3+a)^2}dx}{6b}-\frac{x^5}{6b(a+bx^3)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{5\left(\frac{2\int \frac{x}{bx^3+a}dx}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 821

$$\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{5\left(\frac{2\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 25

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{5\left(\frac{2\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{5\left(\frac{2\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{2\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{3b}-\frac{x^2}{3b(a+bx^3)}\right)}{6b}-\frac{x^5}{6b\left(a+bx^3\right)^2}$$

input
```
Int[x^7/(a + b*x^3)^3,x]
```

output
```
-1/6*x^5/(b*(a + b*x^3)^2) + (5*(-1/3*x^2/(b*(a + b*x^3)) + (2*(-1/3*Log[a
^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/(3*b)))/(6*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{-\frac{4x^5}{9b}-\frac{5a\,x^2}{18b^2}}{(b\,x^3+a)^2}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{27b^3}$	56
default	$\frac{-\frac{4x^5}{9b}-\frac{5a\,x^2}{18b^2}}{(b\,x^3+a)^2}+\frac{-\frac{5\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{b^2}$	125

input

```
int(x^7/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output

```
(-4/9*x^5/b-5/18*a*x^2/b^2)/(b*x^3+a)^2+5/27/b^3*sum(1/_R*ln(x-_R),_R=Root
Of(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(114) = 228.

Time = 0.14 (sec) , antiderivative size = 510, normalized size of antiderivative = 3.29

$$\int \frac{x^7}{(a+bx^3)^3}\,dx$$

$$= \left[-\frac{24\,ab^3x^5 + 15\,a^2b^2x^2 - 15\,\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\,(-ab^2)^{\frac{2}{3}}x^2 + (-\cdots\right.}{bx^3 + a\cdots}\right.}{\cdots}\right.$$

$$-\frac{24\,ab^3x^5 + 15\,a^2b^2x^2 - 30\,\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx + (-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{b}\cdots\right.}{5\cdots}$$

input
```
integrate(x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(24*a*b^3*x^5 + 15*a^2*b^2*x^2 - 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^
2*x^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(
a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) -
3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^
(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*(b^2*x^6 + 2
*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^6*x^6 + 2*a
^2*b^5*x^3 + a^3*b^4), -1/54*(24*a*b^3*x^5 + 15*a^2*b^2*x^2 - 30*sqrt(1/3)
*(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1
/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*(b^2*x^6 + 2*a
*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(
2/3)) + 10*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(
1/3)))/(a*b^6*x^6 + 2*a^2*b^5*x^3 + a^3*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.45

$$\int \frac{x^7}{(a+bx^3)^3}\,dx = \frac{-5ax^2-8bx^5}{18a^2b^2+36ab^3x^3+18b^4x^6} + \operatorname{RootSum}\left(19683t^3ab^8+125, \left(t \mapsto t\log\left(\frac{729t^2ab^5}{25}+x\right)\right)\right)$$

input
```
integrate(x**7/(b*x**3+a)**3,x)
```

output
```
(-5*a*x**2 - 8*b*x**5)/(18*a**2*b**2 + 36*a*b**3*x**3 + 18*b**4*x**6) + Ro
otSum(19683*_t**3*a*b**8 + 125, Lambda(_t, _t*log(729*_t**2*a*b**5/25 + x)
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{x^7}{(a+bx^3)^3}\,dx = -\frac{8\,bx^5+5\,ax^2}{18\,(b^4x^6+2\,ab^3x^3+a^2b^2)} + \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(8*b*x^5 + 5*a*x^2)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2) + 5/27*sqrt(3)
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 5
/54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 5/27*log(x
+ (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx^3)^3}\,dx = -\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^2} - \frac{5\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^4} - \frac{8\,bx^5+5\,ax^2}{18\,(bx^3+a)^2b^2} + \frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^4}$$

input
```
integrate(x^7/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-5/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) - 5/27*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) -
1/18*(8*b*x^5 + 5*a*x^2)/((b*x^3 + a)^2*b^2) + 5/54*(-a*b^2)^(2/3)*log(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04

$$\int \frac{x^7}{(a+bx^3)^3}\,dx = \frac{5\,\ln\left(\frac{25\,x}{81\,b^3}-\frac{25\,(-a)^{1/3}}{81\,b^{10/3}}\right)}{27\,(-a)^{1/3}\,b^{8/3}} - \frac{\frac{4\,x^5}{9\,b}+\frac{5\,a\,x^2}{18\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\ln\left(\frac{25\,x}{81\,b^3}-\frac{(-a)^{1/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)^2}{324\,b^{10/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{54\,(-a)^{1/3}\,b^{8/3}} - \frac{\ln\left(\frac{25\,x}{81\,b^3}-\frac{(-a)^{1/3}\left(5+\sqrt{3}\,5\mathrm{i}\right)^2}{324\,b^{10/3}}\right)\left(5+\sqrt{3}\,5\mathrm{i}\right)}{54\,(-a)^{1/3}\,b^{8/3}}$$

input
```
int(x^7/(a + b*x^3)^3,x)
```

output
```
(5*log((25*x)/(81*b^3) - (25*(-a)^(1/3))/(81*b^(10/3))))/(27*(-a)^(1/3)*b^
(8/3)) - ((4*x^5)/(9*b) + (5*a*x^2)/(18*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3)
+ (log((25*x)/(81*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(324*b^(10/3)))*(
3^(1/2)*5i - 5))/(54*(-a)^(1/3)*b^(8/3)) - (log((25*x)/(81*b^3) - ((-a)^(1
/3)*(3^(1/2)*5i + 5)^2)/(324*b^(10/3)))*(3^(1/2)*5i + 5))/(54*(-a)^(1/3)*b
^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.72

$$\int \frac{x^7}{(a+bx^3)^3}\,dx$$
$$= \frac{-10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2 - 20\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ab\,x^3 - 10\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^6 - 15b^{\frac{2}{3}}a^{\frac{4}{3}}x^2 - 24b}{}$$

input
```
int(x^7/(b*x^3+a)^3,x)
```

output
```
( - 10*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 2
0*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 - 10
*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**6 - 15
*b**(2/3)*a**(1/3)*a*x**2 - 24*b**(2/3)*a**(1/3)*b*x**5 + 5*log(a**(2/3) -
 b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 10*log(a**(2/3) - b**(1/3)*a*
*(1/3)*x + b**(2/3)*x**2)*a*b*x**3 + 5*log(a**(2/3) - b**(1/3)*a**(1/3)*x
+ b**(2/3)*x**2)*b**2*x**6 - 10*log(a**(1/3) + b**(1/3)*x)*a**2 - 20*log(a
**(1/3) + b**(1/3)*x)*a*b*x**3 - 10*log(a**(1/3) + b**(1/3)*x)*b**2*x**6)/
(54*b**(2/3)*a**(1/3)*b**2*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.144 $\int \frac{x^6}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \frac{x^6}{(a+bx^3)^3} \, dx = -\frac{x^4}{6b(a+bx^3)^2} - \frac{2x}{9b^2(a+bx^3)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{7/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{7/3}} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{7/3}}$$

output

```
-1/6*x^4/b/(b*x^3+a)^2-2/9*x/b^2/(b*x^3+a)-2/27*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(7/3)+2/27*ln(a^(1/3)+b^(1/3)*x)
/a^(2/3)/b^(7/3)-1/27*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^
(7/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(a+bx^3)^3} \, dx$$
$$= \frac{\frac{9a\sqrt[3]{b}x}{(a+bx^3)^2} - \frac{21\sqrt[3]{b}x}{a+bx^3} - \frac{4\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} + \frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} - \frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}}{54b^{7/3}}$$

input
```
Integrate[x^6/(a + b*x^3)^3,x]
```

output
```
((9*a*b^(1/3)*x)/(a + b*x^3)^2 - (21*b^(1/3)*x)/(a + b*x^3) - (4*Sqrt[3]*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*Log[a^(1/3) + b^(
1/3)*x])/a^(2/3) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2
/3))/(54*b^(7/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 817, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^3} \, dx$$
$$\Big\downarrow \text{817}$$
$$\frac{2\int \frac{x^3}{(bx^3+a)^2}dx}{3b} - \frac{x^4}{6b(a+bx^3)^2}$$
$$\Big\downarrow \text{817}$$

$$\frac{2\left(\frac{\int \frac{1}{bx^3+a}dx}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 750$$

$$\frac{2\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{2\left(\frac{\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}}{3b}-\frac{x}{3b(a+bx^3)}\right)}{3b}-\frac{x^4}{6b\left(a+bx^3\right)^2}$$

input
```
Int[x^6/(a + b*x^3)^3,x]
```

output
```
-1/6*x^4/(b*(a + b*x^3)^2) + (2*(-1/3*x/(b*(a + b*x^3)) + (Log[a^(1/3) + b
^(1/3)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(
2*b^(1/3)))/(3*a^(2/3)))/(3*b)))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.35

method	result	size
risch	$\frac{-\frac{7x^4}{18b}-\frac{2ax}{9b^2}}{(b\,x^3+a)^2}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{27b^3}$	54
default	$\frac{-\frac{7x^4}{18b}-\frac{2ax}{9b^2}}{(b\,x^3+a)^2}+\frac{\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{b^2}$	123

input
```
int(x^6/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-7/18*x^4/b-2/9*a*x/b^2)/(b*x^3+a)^2+2/27/b^3*sum(1/_R^2*ln(x-_R),_R=Root
Of(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(112) = 224$.

Time = 0.09 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.29

$$\int \frac{x^6}{(a+bx^3)^3}\,dx$$

$$= \left[-\frac{21\,a^2b^2x^4 + 12\,a^3bx - 6\,\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3 - 3\,(a^2b)^{\frac{1}{3}}\,ax - a^2 + 3\,\sqrt{\frac{1}{3}}\Big(2\,abx^2 + (a^{\cdots}}{bx^3+a}\right.}{\cdots}\right.$$

$$-\frac{21\,a^2b^2x^4 + 12\,a^3bx - 12\,\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x - (a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)\cdots}{54\,(a^2b^{\cdots}}$$

input
```
integrate(x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[-1/54*(21*a^2*b^2*x^4 + 12*a^3*b*x - 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^5*x^6 + 2*a^3*b^4
*x^3 + a^4*b^3), -1/54*(21*a^2*b^2*x^4 + 12*a^3*b*x - 12*sqrt(1/3)*(a*b^3*
x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/
3)*a) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.44

$$\int \frac{x^6}{(a+bx^3)^3}\,dx = \frac{-4ax-7bx^4}{18a^2b^2+36ab^3x^3+18b^4x^6} + \operatorname{RootSum}\left(19683t^3a^2b^7-8,\left(t\mapsto t\log\left(\frac{27tab^2}{2}+x\right)\right)\right)$$

input
```
integrate(x**6/(b*x**3+a)**3,x)
```

output
```
(-4*a*x - 7*b*x**4)/(18*a**2*b**2 + 36*a*b**3*x**3 + 18*b**4*x**6) + RootS
um(19683*_t**3*a**2*b**7 - 8, Lambda(_t, _t*log(27*_t*a*b**2/2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

$$\int \frac{x^6}{(a+bx^3)^3}\,dx = -\frac{7\,bx^4+4\,ax}{18\,(b^4x^6+2\,ab^3x^3+a^2b^2)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(7*b*x^4 + 4*a*x)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/2
7*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 2/27*log(x +
(a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(a+bx^3)^3}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^2} + \frac{2\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^3} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,ab^3} - \frac{7\,bx^4+4\,ax}{18\,(bx^3+a)^2b^2}$$

input
```
integrate(x^6/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-2/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2) + 2/27*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3) +
1/27*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3) - 1/1
8*(7*b*x^4 + 4*a*x)/((b*x^3 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.83

$$\int \frac{x^6}{(a+bx^3)^3}\,dx = \frac{2\,\ln\left(x+\frac{a^{1/3}}{b^{1/3}}\right)}{27\,a^{2/3}\,b^{7/3}} - \frac{\frac{7\,x^4}{18\,b}+\frac{2\,a\,x}{9\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\ln\left(x+\frac{a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{1/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{27\,a^{2/3}\,b^{7/3}} - \frac{\ln\left(x-\frac{a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{1/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{27\,a^{2/3}\,b^{7/3}}$$

input
```
int(x^6/(a + b*x^3)^3,x)
```

output
```
(2*log(x + a^(1/3)/b^(1/3)))/(27*a^(2/3)*b^(7/3)) - ((7*x^4)/(18*b) + (2*a
*x)/(9*b^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (log(x + (a^(1/3)*(3^(1/2)*1i -
 1))/(2*b^(1/3)))*(3^(1/2)*1i - 1))/(27*a^(2/3)*b^(7/3)) - (log(x - (a^(1/
3)*(3^(1/2)*1i + 1))/(2*b^(1/3)))*(3^(1/2)*1i + 1))/(27*a^(2/3)*b^(7/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.82

$$\int \frac{x^6}{(a+bx^3)^3}\,dx = \frac{-4a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-8a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-4a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^6-2a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}-b^{\frac{1}{3}}\right.}{}$$

input
```
int(x^6/(b*x^3+a)^3,x)
```

output
```
( - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2 - 8*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*a*b*x**3 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*b**2*x**6 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*a**2 - 4*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*a*b*x**3 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*b**2*x**6 + 4*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 8*a**(1/3)
*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 4*a**(1/3)*log(a**(1/3) + b**(1/3)*
x)*b**2*x**6 - 12*b**(1/3)*a**2*x - 21*b**(1/3)*a*b*x**4)/(54*b**(1/3)*a*b
**2*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.145 $\int \frac{x^4}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 158

$$\int \frac{x^4}{(a+bx^3)^3} dx = -\frac{x^2}{6b(a+bx^3)^2} + \frac{x^2}{9ab(a+bx^3)} - \frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{5/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}}$$

output
```
-1/6*x^2/b/(b*x^3+a)^2+1/9*x^2/a/b/(b*x^3+a)-1/27*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)/b^(5/3)-1/27*ln(a^(1/3)+b^(1/3)*
x)/a^(4/3)/b^(5/3)+1/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/
b^(5/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{(a+bx^3)^3}\,dx$$
$$= \frac{-\frac{9b^{2/3}x^2}{(a+bx^3)^2}+\frac{6b^{2/3}x^2}{a^2+abx^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}+\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}}}{54b^{5/3}}$$

input
```
Integrate[x^4/(a + b*x^3)^3,x]
```

output
```
((-9*b^(2/3)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*x^2)/(a^2 + a*b*x^3) - (2*Sqr
t[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(4/3) - (2*Log[a^(1/3)
 + b^(1/3)*x])/a^(4/3) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^
(4/3))/(54*b^(5/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{ 817}$$
$$\frac{\int \frac{x}{(bx^3+a)^2}dx}{3b}-\frac{x^2}{6b\,(a+bx^3)^2}$$
$$\Big\downarrow \text{ 819}$$

$$\frac{\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 821

$$\frac{\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}}{3b}-\frac{x^2}{6b\left(a+bx^3\right)^2}$$

input
```
Int[x^4/(a + b*x^3)^3,x]
```

output
```
-1/6*x^2/(b*(a + b*x^3)^2) + (x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) +
b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2
*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a))/(3*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.37

method	result	size
risch	$\frac{\frac{x^5}{9a}-\frac{x^2}{18b}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{27a\,b^2}$	58
default	$\frac{\frac{x^5}{9a}-\frac{x^2}{18b}}{(b\,x^3+a)^2}+\frac{-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}}{9ab}$	127

input
```
int(x^4/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/9/a*x^5-1/18*x^2/b)/(b*x^3+a)^2+1/27/a/b^2*sum(1/_R*ln(x-_R),_R=RootOf(
_Z^3*b+a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.24

$$\int \frac{x^4}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{6\,ab^3x^5 - 3\,a^2b^2x^2 + 3\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\sqrt{\frac{1}{3}}\left(abx+2\,(-ab^2)^{\frac{2}{3}}x^2+(-ab^2)^{\frac{1}{3}}\right.}{bx^3+a}\right.}{}\right.$$

input
```
integrate(x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-
a*b^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^
3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^
4*x^3 + a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*
x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*
b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{(a+bx^3)^3}\,dx = \frac{-ax^2+2bx^5}{18a^3b+36a^2b^2x^3+18ab^3x^6} + \operatorname{RootSum}\left(19683t^3a^4b^5+1,\left(t\mapsto t\log\left(729t^2a^3b^3+x\right)\right)\right)$$

input
```
integrate(x**4/(b*x**3+a)**3,x)
```

output
```
(-a*x**2 + 2*b*x**5)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6) + Ro
otSum(19683*_t**3*a**4*b**5 + 1, Lambda(_t, _t*log(729*_t**2*a**3*b**3 + x
)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a+bx^3)^3}\,dx = \frac{2\,bx^5-ax^2}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) +
1/54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log
(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(a+bx^3)^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b} - \frac{\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^3} + \frac{2\,bx^5-ax^2}{18\,(bx^3+a)^2ab} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^3}$$

input
```
integrate(x^4/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/27*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^3)
+ 1/18*(2*b*x^5 - a*x^2)/((b*x^3 + a)^2*a*b) + 1/54*(-a*b^2)^(2/3)*log(x^2
 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int \frac{x^4}{(a+bx^3)^3}\,dx = \frac{\frac{x^5}{9\,a}-\frac{x^2}{18\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\ln\left(\frac{1}{81\,a^{5/3}\,(-b)^{4/3}}+\frac{x}{81\,a^2\,b}\right)}{27\,a^{4/3}\,(-b)^{5/3}} + \frac{\ln\left(\frac{x}{81\,a^2\,b}+\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{324\,a^{5/3}\,(-b)^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{4/3}\,(-b)^{5/3}} - \frac{\ln\left(\frac{x}{81\,a^2\,b}+\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{324\,a^{5/3}\,(-b)^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{4/3}\,(-b)^{5/3}}$$

input
```
int(x^4/(a + b*x^3)^3,x)
```

output
```
(x^5/(9*a) - x^2/(18*b))/(a^2 + b^2*x^6 + 2*a*b*x^3) + log(1/(81*a^(5/3)*(
-b)^(4/3)) + x/(81*a^2*b))/(27*a^(4/3)*(-b)^(5/3)) + (log(x/(81*a^2*b) + (
3^(1/2)*1i - 1)^2/(324*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/(54*a^(4/3)*
(-b)^(5/3)) - (log(x/(81*a^2*b) + (3^(1/2)*1i + 1)^2/(324*a^(5/3)*(-b)^(4/
3)))*(3^(1/2)*1i + 1))/(54*a^(4/3)*(-b)^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(a+bx^3)^3} \, dx$$
$$= \frac{-2\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) a^2 - 4\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) ab\, x^3 - 2\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^2x^6 - 3b^{\frac{2}{3}}a^{\frac{4}{3}}x^2 + 6b^{\frac{5}{3}}a^{\frac{1}{3}}x^5}{}$$

input
```
int(x^4/(b*x^3+a)^3,x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 4*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 - 2*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**6 - 3*b**
(2/3)*a**(1/3)*a*x**2 + 6*b**(2/3)*a**(1/3)*b*x**5 + log(a**(2/3) - b**(1/
3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x
 + b**(2/3)*x**2)*a*b*x**3 + log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)
*x**2)*b**2*x**6 - 2*log(a**(1/3) + b**(1/3)*x)*a**2 - 4*log(a**(1/3) + b*
*(1/3)*x)*a*b*x**3 - 2*log(a**(1/3) + b**(1/3)*x)*b**2*x**6)/(54*b**(2/3)*
a**(1/3)*a*b*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.146 $\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 154

$$\int \frac{x^3}{\left(a+bx^3\right)^3}\,dx = -\frac{x}{6b\left(a+bx^3\right)^2}+\frac{x}{18ab\left(a+bx^3\right)}-\frac{\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{4/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{4/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}}$$

output
```
-1/6*x/b/(b*x^3+a)^2+1/18*x/a/b/(b*x^3+a)-1/27*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)/b^(4/3)+1/27*ln(a^(1/3)+b^(1/3)*x)/
a^(5/3)/b^(4/3)-1/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(
4/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{(a+bx^3)^3}\,dx$$

$$=\frac{-\frac{9\sqrt[3]{b}x}{(a+bx^3)^2}+\frac{3\sqrt[3]{b}x}{a^2+abx^3}-\frac{2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}}{54b^{4/3}}$$

input
```
Integrate[x^3/(a + b*x^3)^3,x]
```

output
```
((-9*b^(1/3)*x)/(a + b*x^3)^2 + (3*b^(1/3)*x)/(a^2 + a*b*x^3) - (2*Sqrt[3]
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(5/3) + (2*Log[a^(1/3) + b
^(1/3)*x])/a^(5/3) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/a^(5/3
))/(54*b^(4/3))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{\int \frac{1}{(bx^3+a)^2}dx}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{749}$$

$$\frac{\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 1082

$$\frac{\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 217

$$\frac{\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}}{6b}-\frac{x}{6b\left(a+bx^3\right)^2}$$

input
```
Int[x^3/(a + b*x^3)^3,x]
```

output
```
-1/6*x/(b*(a + b*x^3)^2) + (x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3
)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(
1/3)))/(3*a^(2/3))))/(3*a))/(6*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{\frac{x^4}{18a}-\frac{x}{9b}}{(b\,x^3+a)^2}+\frac{\sum_{\_R=\operatorname{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R^2}}{27a\,b^2}$	56
default	$\frac{\frac{x^4}{18a}-\frac{x}{9b}}{(b\,x^3+a)^2}+\frac{\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9ab}$	125

input
```
int(x^3/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/18/a*x^4-1/9*x/b)/(b*x^3+a)^2+1/27/a/b^2*sum(1/_R^2*ln(x-_R),_R=RootOf(
_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(113) = 226.

Time = 0.08 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.27

$$\int \frac{x^3}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{3\,a^2b^2x^4-6\,a^3bx+3\,\sqrt{\frac{1}{3}}(ab^3x^6+2\,a^2b^2x^3+a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x\right.}{bx^3+a}\right.}{} \right.$$

input
```
integrate(x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
+ a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2
 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3
+ a^5*b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*
b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.42

$$\int \frac{x^3}{(a+bx^3)^3}\,dx = \frac{-2ax+bx^4}{18a^3b+36a^2b^2x^3+18ab^3x^6} + \operatorname{RootSum}\left(19683t^3a^5b^4-1,\left(t\mapsto t\log\left(27ta^2b+x\right)\right)\right)$$

input
```
integrate(x**3/(b*x**3+a)**3,x)
```

output
```
(-2*a*x + b*x**4)/(18*a**3*b + 36*a**2*b**2*x**3 + 18*a*b**3*x**6) + RootS
um(19683*_t**3*a**5*b**4 - 1, Lambda(_t, _t*log(27*_t*a**2*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{(a+bx^3)^3}\,dx = \frac{bx^4-2\,ax}{18\,(ab^3x^6+2\,a^2b^2x^3+a^3b)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x
 + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx^3)^3}\,dx = -\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b} + \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2} + \frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b^2} + \frac{bx^4-2\,ax}{18\,(bx^3+a)^2ab}$$

input
```
integrate(x^3/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-1/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/27*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2)
+ 1/54*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2) +
 1/18*(b*x^4 - 2*a*x)/((b*x^3 + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^3)^3}\,dx = \frac{\ln\left(\frac{b^{2/3}}{3\,a^{2/3}}+\frac{b\,x}{3\,a}\right)}{27\,a^{5/3}\,b^{4/3}} - \frac{\frac{x}{9\,b}-\frac{x^4}{18\,a}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{\ln\left(\frac{b\,x}{3\,a}+\frac{b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(\frac{b\,x}{3\,a}-\frac{b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{54\,a^{5/3}\,b^{4/3}}$$

input
```
int(x^3/(a + b*x^3)^3,x)
```

output
```
log(b^(2/3)/(3*a^(2/3)) + (b*x)/(3*a))/(27*a^(5/3)*b^(4/3)) - (x/(9*b) - x
^4/(18*a))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (log((b*x)/(3*a) + (b^(2/3)*(3^(1
/2)*1i - 1))/(6*a^(2/3)))*(3^(1/2)*1i - 1))/(54*a^(5/3)*b^(4/3)) - (log((b
*x)/(3*a) - (b^(2/3)*(3^(1/2)*1i + 1))/(6*a^(2/3)))*(3^(1/2)*1i + 1))/(54*
a^(5/3)*b^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.81

$$\int \frac{x^3}{(a+bx^3)^3}\,dx = \frac{-2a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) - 4a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3 - 2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^6 - a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}} - b^{\frac{1}{3}}a\right.}{}$$

input
```
int(x^3/(b*x^3+a)^3,x)
```

output
```
( - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*
a**2 - 4*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)
))*a*b*x**3 - 2*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*
sqrt(3)))*b**2*x**6 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/
3)*x**2)*a**2 - 2*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x
**2)*a*b*x**3 - a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*b**2*x**6 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 4*a**(1/3)*log
(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 2*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*b
**2*x**6 - 6*b**(1/3)*a**2*x + 3*b**(1/3)*a*b*x**4)/(54*b**(1/3)*a**2*b*(a
**2 + 2*a*b*x**3 + b**2*x**6))
```

3.147 $\int \frac{x}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 155

$$\int \frac{x}{(a+bx^3)^3} \, dx = \frac{x^2}{6a(a+bx^3)^2} + \frac{2x^2}{9a^2(a+bx^3)} - \frac{2\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{7/3}b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{7/3}b^{2/3}} + \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}}$$

output
```
1/6*x^2/a/(b*x^3+a)^2+2/9*x^2/a^2/(b*x^3+a)-2/27*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)/b^(2/3)-2/27*ln(a^(1/3)+b^(1/3)*x
)/a^(7/3)/b^(2/3)+1/27*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b
^(2/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{x}{(a+bx^3)^3}\,dx$$

$$= \frac{\frac{9a^{4/3}x^2}{(a+bx^3)^2}+\frac{12\sqrt[3]{a}x^2}{a+bx^3}-\frac{4\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}-\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}}{54a^{7/3}}$$

input
```
Integrate[x/(a + b*x^3)^3,x]
```

output
```
((9*a^(4/3)*x^2)/(a + b*x^3)^2 + (12*a^(1/3)*x^2)/(a + b*x^3) - (4*Sqrt[3]
*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) - (4*Log[a^(1/3) + b
^(1/3)*x])/b^(2/3) + (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^
(2/3))/(54*a^(7/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {819, 819, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{2\int \frac{x}{(bx^3+a)^2}dx}{3a}+\frac{x^2}{6a(a+bx^3)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{2\left(\frac{\int \frac{x}{bx^3+a}dx}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 821

$$\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 16

$$\frac{2\left(\frac{\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1142

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 25

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}}{3a}+\frac{x^2}{3a(a+bx^3)}\right)}{3a}+\frac{x^2}{6a\left(a+bx^3\right)^2}$$

input
```
Int[x/(a + b*x^3)^3,x]
```

output
```
x^2/(6*a*(a + b*x^3)^2) + (2*(x^2/(3*a*(a + b*x^3)) + (-1/3*Log[a^(1/3) +
b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2
*b^(1/3)))/(3*a^(1/3)*b^(1/3)))/(3*a)))/(3*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{\frac{2b\,x^5}{9a^2}+\frac{7x^2}{18a}}{\left(b\,x^3+a\right)^2}+\frac{2\left(\sum_{\_R=\operatorname{RootOf}\left(b\,\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{27a^2b}$	59
default	$\frac{x^2}{6a\left(b\,x^3+a\right)^2}+\frac{\frac{2x^2}{9a\left(b\,x^3+a\right)}+\frac{2\left(-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a}}{a}$	137

input
```
int(x/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2/9*b/a^2*x^5+7/18/a*x^2)/(b*x^3+a)^2+2/27/a^2/b*sum(1/_R*ln(x-_R),_R=Roo
tOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(114) = 228$.

Time = 0.09 (sec) , antiderivative size = 514, normalized size of antiderivative = 3.32

$$\int \frac{x}{(a+bx^3)^3}\,dx$$

$$= \left[\frac{12\,ab^3x^5 + 21\,a^2b^2x^2 + 6\,\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3 - ab + 3\,\sqrt{\frac{1}{3}}\left(abx + 2\,(-ab^2)^{\frac{2}{3}}x^2 + (-ab^2\right.}{bx^3+a}\right.}{} \right.$$

input
```
integrate(x/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*
b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*
(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*
b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^
4*b^3*x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(
a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3
)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b
*x^3 + a^2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/
3)) - 4*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3
)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.45

$$\int \frac{x}{(a+bx^3)^3}\,dx = \frac{7ax^2+4bx^5}{18a^4+36a^3bx^3+18a^2b^2x^6} + \operatorname{RootSum}\left(19683t^3a^7b^2+8, \left(t\mapsto t\log\left(\frac{729t^2a^5b}{4}+x\right)\right)\right)$$

input
```
integrate(x/(b*x**3+a)**3,x)
```

output
```
(7*a*x**2 + 4*b*x**5)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6) + Roo
tSum(19683*_t**3*a**7*b**2 + 8, Lambda(_t, _t*log(729*_t**2*a**5*b/4 + x))
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

$$\int \frac{x}{(a+bx^3)^3}\,dx = \frac{4\,bx^5+7\,ax^2}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*
arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) +
1/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log
(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{x}{(a+bx^3)^3}\,dx = -\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} - \frac{2\,\sqrt{3}(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b^2} + \frac{4\,bx^5+7\,ax^2}{18\,(bx^3+a)^2a^2} + \frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3b^2}$$

input
```
integrate(x/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-2/27*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^3 - 2/27*sqrt(3)*(-a*b^2)^
(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b^2) + 1/
18*(4*b*x^5 + 7*a*x^2)/((b*x^3 + a)^2*a^2) + 1/27*(-a*b^2)^(2/3)*log(x^2 +
 x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05

$$\int \frac{x}{(a+bx^3)^3}\,dx = \frac{\frac{7\,x^2}{18\,a}+\frac{2\,b\,x^5}{9\,a^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6} + \frac{2\,\ln\left(\frac{4\,b\,x}{81\,a^4}-\frac{4\,b^{2/3}}{81\,(-a)^{11/3}}\right)}{27\,(-a)^{7/3}\,b^{2/3}} + \frac{\ln\left(\frac{4\,b\,x}{81\,a^4}-\frac{b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{81\,(-a)^{11/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{27\,(-a)^{7/3}\,b^{2/3}} - \frac{\ln\left(\frac{4\,b\,x}{81\,a^4}-\frac{b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{81\,(-a)^{11/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{27\,(-a)^{7/3}\,b^{2/3}}$$

input
```
int(x/(a + b*x^3)^3,x)
```

output
```
((7*x^2)/(18*a) + (2*b*x^5)/(9*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (2*log(
(4*b*x)/(81*a^4) - (4*b^(2/3))/(81*(-a)^(11/3))))/(27*(-a)^(7/3)*b^(2/3))
+ (log((4*b*x)/(81*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(81*(-a)^(11/3)))*(
3^(1/2)*1i - 1))/(27*(-a)^(7/3)*b^(2/3)) - (log((4*b*x)/(81*a^4) - (b^(2/3
)*(3^(1/2)*1i + 1)^2)/(81*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(27*(-a)^(7/3)*b
^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.72

$$\int \frac{x}{(a+bx^3)^3}\,dx$$
$$= \frac{-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a^2-8\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)ab\,x^3-4\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^6+21b^{\frac{2}{3}}a^{\frac{4}{3}}x^2+12b^{\frac{5}{3}}a^{\frac{1}{3}}}{}$$

input
```
int(x/(b*x^3+a)^3,x)
```

output
```
( - 4*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2 - 8*
sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b*x**3 - 4*sq
rt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**2*x**6 + 21*b*
*(2/3)*a**(1/3)*a*x**2 + 12*b**(2/3)*a**(1/3)*b*x**5 + 2*log(a**(2/3) - b*
*(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 + 4*log(a**(2/3) - b**(1/3)*a**(1/
3)*x + b**(2/3)*x**2)*a*b*x**3 + 2*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b*
*(2/3)*x**2)*b**2*x**6 - 4*log(a**(1/3) + b**(1/3)*x)*a**2 - 8*log(a**(1/3
) + b**(1/3)*x)*a*b*x**3 - 4*log(a**(1/3) + b**(1/3)*x)*b**2*x**6)/(54*b**
(2/3)*a**(1/3)*a**2*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.148 $\int \frac{1}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 151

$$\int \frac{1}{(a+bx^3)^3} \, dx = \frac{x}{6a(a+bx^3)^2} + \frac{5x}{18a^2(a+bx^3)} - \frac{5\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}\sqrt[3]{b}} + \frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}\sqrt[3]{b}} - \frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}}$$

output
```
1/6*x/a/(b*x^3+a)^2+5/18*x/a^2/(b*x^3+a)-5/27*arctan(1/3*(a^(1/3)-2*b^(1/3
)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)/b^(1/3)+5/27*ln(a^(1/3)+b^(1/3)*x)/a
^(8/3)/b^(1/3)-5/54*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1
/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^3)^3}\,dx$$
$$=\frac{\frac{9a^{5/3}x}{(a+bx^3)^2}+\frac{15a^{2/3}x}{a+bx^3}-\frac{10\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}+\frac{10\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}}{54a^{8/3}}$$

input
```
Integrate[(a + b*x^3)^(-3),x]
```

output
```
((9*a^(5/3)*x)/(a + b*x^3)^2 + (15*a^(2/3)*x)/(a + b*x^3) - (10*Sqrt[3]*Ar
cTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*Log[a^(1/3) + b^(
1/3)*x])/b^(1/3) - (5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1
/3))/(54*a^(8/3))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {749, 749, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^3}\,dx$$
$$\Big\downarrow \text{749}$$
$$\frac{5\int \frac{1}{(bx^3+a)^2}dx}{6a}+\frac{x}{6a\,(a+bx^3)^2}$$
$$\Big\downarrow \text{749}$$

$$\frac{5\left(\frac{2\int \frac{1}{bx^3+a}dx}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 750$$

$$\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3a^{2/3}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 16$$

$$\frac{5\left(\frac{2\left(\frac{\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{2\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 217

$$\frac{5\left(\frac{2\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{2\left(\frac{-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{3a}+\frac{x}{3a(a+bx^3)}\right)}{6a}+\frac{x}{6a\left(a+bx^3\right)^2}$$

input
```
Int[(a + b*x^3)^(-3),x]
```

output
```
x/(6*a*(a + b*x^3)^2) + (5*(x/(3*a*(a + b*x^3)) + (2*(Log[a^(1/3) + b^(1/3
)*x]/(3*a^(2/3)*b^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(
1/3)))/(3*a^(2/3))))/(3*a)))/(6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

method	result	size
risch	$\frac{\frac{5b\,x^{4}}{18a^{2}}+\frac{4x}{9a}}{\left(b\,x^{3}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27a^{2}b}$	57
default	$\frac{x}{6a\left(b\,x^{3}+a\right)^{2}}+\frac{\frac{5x}{18a\left(b\,x^{3}+a\right)}+\frac{5\left(\frac{2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)}{6a}}{a}$	133

input

```
int(1/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output

```
(5/18*b/a^2*x^4+4/9*x/a)/(b*x^3+a)^2+5/27/a^2/b*sum(1/_R^2*ln(x-_R),_R=Roo
tOf(_Z^3*b+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(110) = 220$.

Time = 0.09 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.30

$$\int \frac{1}{(a+bx^3)^3}\,dx$$
$$= \left[\frac{15\,a^2b^2x^4 + 24\,a^3bx + 15\,\sqrt{\frac{1}{3}}(ab^3x^6 + 2\,a^2b^2x^3 + a^3b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2\right.}{bx^3+a}\right.}{}\right.$$

input
```
integrate(1/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
[1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^
3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^
2*x^3 + a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^
6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*
a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/
3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.42

$$\int \frac{1}{(a+bx^3)^3}\,dx = \frac{8ax+5bx^4}{18a^4+36a^3bx^3+18a^2b^2x^6} + \operatorname{RootSum}\left(19683t^3a^8b-125,\left(t\mapsto t\log\left(\frac{27ta^3}{5}+x\right)\right)\right)$$

input
```
integrate(1/(b*x**3+a)**3,x)
```

output
```
(8*a*x + 5*b*x**4)/(18*a**4 + 36*a**3*b*x**3 + 18*a**2*b**2*x**6) + RootSu
m(19683*_t**3*a**8*b - 125, Lambda(_t, _t*log(27*_t*a**3/5 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx^3)^3}\,dx = \frac{5\,bx^4+8\,ax}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)} + \frac{5\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x
 + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+bx^3)^3}\,dx = -\frac{5\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3} + \frac{5\,\sqrt{3}(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b} + \frac{5\,(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b} + \frac{5\,bx^4+8\,ax}{18\,(bx^3+a)^2a^2}$$

input
```
integrate(1/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
-5/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 5/27*sqrt(3)*(-a*b^2)^
(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 5/54
*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b) + 1/18*(5
*b*x^4 + 8*a*x)/((b*x^3 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^3)^3}\,dx = \frac{\frac{4x}{9a}+\frac{5bx^4}{18a^2}}{a^2+2abx^3+b^2x^6}+\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)}{27\,a^{8/3}\,b^{1/3}} + \frac{\ln\left(\frac{5b^2x}{3a^2}+\frac{b^{5/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{6\,a^{5/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{54\,a^{8/3}\,b^{1/3}} - \frac{\ln\left(\frac{5b^2x}{3a^2}-\frac{b^{5/3}\left(5+\sqrt{3}\,5\mathrm{i}\right)}{6\,a^{5/3}}\right)\left(5+\sqrt{3}\,5\mathrm{i}\right)}{54\,a^{8/3}\,b^{1/3}}$$

input
```
int(1/(a + b*x^3)^3,x)
```

output
```
((4*x)/(9*a) + (5*b*x^4)/(18*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) + (5*log(b^
(1/3)*x + a^(1/3)))/(27*a^(8/3)*b^(1/3)) + (log((5*b^2*x)/(3*a^2) + (b^(5/
3)*(3^(1/2)*5i - 5))/(6*a^(5/3)))*(3^(1/2)*5i - 5))/(54*a^(8/3)*b^(1/3)) -
 (log((5*b^2*x)/(3*a^2) - (b^(5/3)*(3^(1/2)*5i + 5))/(6*a^(5/3)))*(3^(1/2)
*5i + 5))/(54*a^(8/3)*b^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.84

$$\int \frac{1}{(a+bx^3)^3}\,dx$$
$$= \frac{-10a^{\frac{7}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)-20a^{\frac{4}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b\,x^3-10a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)b^2x^6-5a^{\frac{7}{3}}\log\left(a^{\frac{2}{3}}\right.}{}$$

input
```
int(1/(b*x^3+a)^3,x)
```

output
```
( - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))
*a**2 - 20*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(
3)))*a*b*x**3 - 10*a**(1/3)*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/
3)*sqrt(3)))*b**2*x**6 - 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b
**(2/3)*x**2)*a**2 - 10*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*a*b*x**3 - 5*a**(1/3)*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(
2/3)*x**2)*b**2*x**6 + 10*a**(1/3)*log(a**(1/3) + b**(1/3)*x)*a**2 + 20*a*
*(1/3)*log(a**(1/3) + b**(1/3)*x)*a*b*x**3 + 10*a**(1/3)*log(a**(1/3) + b*
*(1/3)*x)*b**2*x**6 + 24*b**(1/3)*a**2*x + 15*b**(1/3)*a*b*x**4)/(54*b**(1
/3)*a**3*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.149 $\int \frac{1}{x^2(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 165

$$\int \frac{1}{x^2(a+bx^3)^3}\,dx = -\frac{1}{a^3x} - \frac{bx^2}{6a^2(a+bx^3)^2} - \frac{5bx^2}{9a^3(a+bx^3)} + \frac{14\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}} + \frac{14\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}} - \frac{7\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{10/3}}$$

output
```
-1/a^3/x-1/6*b*x^2/a^2/(b*x^3+a)^2-5/9*b*x^2/a^3/(b*x^3+a)+14/27*b^(1/3)*a
rctan(1/3*(a^(1/3)-2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(10/3)+14/27*b^
(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)-7/27*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2(a+bx^3)^3}\,dx = \frac{-\frac{54\sqrt[3]{a}}{x}-\frac{9a^{4/3}bx^2}{(a+bx^3)^2}-\frac{30\sqrt[3]{a}bx^2}{a+bx^3}+28\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-14\sqrt[3]{b}\log\left(a^{2/3}\right.}{54a^{10/3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^3),x]
```

output
```
((-54*a^(1/3))/x - (9*a^(4/3)*b*x^2)/(a + b*x^3)^2 - (30*a^(1/3)*b*x^2)/(a
 + b*x^3) + 28*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]]
 + 28*b^(1/3)*Log[a^(1/3) + b^(1/3)*x] - 14*b^(1/3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2])/(54*a^(10/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {819, 819, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7\int \frac{1}{x^2(bx^3+a)^2}dx}{6a}+\frac{1}{6ax(a+bx^3)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{7\left(\frac{4\int \frac{1}{x^2\left(bx^3+a\right)}dx}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{7\left(\frac{4\left(-\frac{b\int \frac{x}{bx^3+a}dx}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}x+\sqrt[3]{a}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$\downarrow$ 27

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int\frac{1}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$$\downarrow \text{217}$$

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{b^{2/3}x^2-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

$\downarrow$ 1103

$$\frac{7\left(\frac{4\left(-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}\right)}{3a}+\frac{1}{3ax(a+bx^3)}\right)}{6a}+\frac{1}{6ax\left(a+bx^3\right)^2}$$

input
```
Int[1/(x^2*(a + b*x^3)^3),x]
```

output
```
1/(6*a*x*(a + b*x^3)^2) + (7*(1/(3*a*x*(a + b*x^3)) + (4*(-(1/(a*x)) - (b*
(-1/3*Log[a^(1/3) + b^(1/3)*x]/(a^(1/3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 -
 (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a))/(3*a)))/(6*a)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{-\frac{14b^2x^6}{9a^3}-\frac{49b\,x^3}{18a^2}-\frac{1}{a}}{x(b\,x^3+a)^2}+\frac{14\left(\sum_{\_R=\mathrm{RootOf}(a^{10}\_Z^3-b)}\_R\ln\left(\left(-4\_R^3a^{10}+3b\right)x-a^7\_R^2\right)\right)}{27}$	84
default	$-\frac{1}{a^3x}-\frac{b\left(\frac{\frac{5}{9}b\,x^5+\frac{13}{18}a\,x^2}{(b\,x^3+a)^2}-\frac{14\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3}$	129

input
```
int(1/x^2/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-14/9*b^2/a^3*x^6-49/18*b/a^2*x^3-1/a)/x/(b*x^3+a)^2+14/27*sum(_R*ln((-4*
_R^3*a^10+3*b)*x-a^7*_R^2),_R=RootOf(_Z^3*a^10-b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2\,(a+bx^3)^3}\,dx = -\frac{84\,b^2x^6+147\,abx^3+28\,\sqrt{3}(b^2x^7+2\,abx^4+a^2x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\,(b^2x^7+2\,ab}{54\,(a^3b^2x^7+2}$$

input
```
integrate(1/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)
*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7
 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1
/3)) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3
)) + 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2(a+bx^3)^3}\,dx = \frac{-18a^2-49abx^3-28b^2x^6}{18a^5x+36a^4bx^4+18a^3b^2x^7} + \operatorname{RootSum}\left(19683t^3a^{10}-2744b, \left(t\mapsto t\log\left(\frac{729t^2a^7}{196b}+x\right)\right)\right)$$

input
```
integrate(1/x**2/(b*x**3+a)**3,x)
```

output
```
(-18*a**2 - 49*a*b*x**3 - 28*b**2*x**6)/(18*a**5*x + 36*a**4*b*x**4 + 18*a
**3*b**2*x**7) + RootSum(19683*_t**3*a**10 - 2744*b, Lambda(_t, _t*log(729
*_t**2*a**7/(196*b) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2(a+bx^3)^3}\,dx = -\frac{28\,b^2x^6+49\,abx^3+18\,a^2}{18\,(a^3b^2x^7+2\,a^4bx^4+a^5x)} - \frac{14\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{7\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{14\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(1/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
-1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*
x) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^
3*(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1
/3)) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a+bx^3)^3} \, dx = \frac{14 \, b \left(-\frac{a}{b}\right)^{\frac{2}{3}} \log \left(\left| x - \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right| \right)}{27 \, a^4} + \frac{14 \, \sqrt{3} (-ab^2)^{\frac{2}{3}} \arctan \left(\frac{\sqrt{3} \left(2 \, x + \left(-\frac{a}{b}\right)^{\frac{1}{3}} \right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}} \right)}{27 \, a^4 b} - \frac{7 \, (-ab^2)^{\frac{2}{3}} \log \left(x^2 + x \left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}} \right)}{27 \, a^4 b} - \frac{10 \, b^2 x^5 + 13 \, abx^2}{18 \, (bx^3 + a)^2 a^3} - \frac{1}{a^3 x}$$

input
```
integrate(1/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
14/27*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 14/27*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 7
/27*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/18
*(10*b^2*x^5 + 13*a*b*x^2)/((b*x^3 + a)^2*a^3) - 1/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (a+bx^3)^3} \, dx = \frac{14 \, b^{1/3} \, \ln \left(b^{1/3} \, x + a^{1/3} \right)}{27 \, a^{10/3}} - \frac{\frac{1}{a} + \frac{49 \, b \, x^3}{18 \, a^2} + \frac{14 \, b^2 \, x^6}{9 \, a^3}}{a^2 \, x + 2 \, a \, b \, x^4 + b^2 \, x^7} + \frac{14 \, b^{1/3} \, \ln \left(4 \, b^{1/3} \, x - 2 \, a^{1/3} - \sqrt{3} \, a^{1/3} \, 2\mathrm{i} \right) \left(-\frac{1}{2} + \frac{\sqrt{3} \, 1\mathrm{i}}{2} \right)}{27 \, a^{10/3}} - \frac{14 \, b^{1/3} \, \ln \left(4 \, b^{1/3} \, x - 2 \, a^{1/3} + \sqrt{3} \, a^{1/3} \, 2\mathrm{i} \right) \left(\frac{1}{2} + \frac{\sqrt{3} \, 1\mathrm{i}}{2} \right)}{27 \, a^{10/3}}$$

input
```
int(1/(x^2*(a + b*x^3)^3),x)
```

output
```
(14*b^(1/3)*log(b^(1/3)*x + a^(1/3)))/(27*a^(10/3)) - (1/a + (49*b*x^3)/(1
8*a^2) + (14*b^2*x^6)/(9*a^3))/(a^2*x + b^2*x^7 + 2*a*b*x^4) + (14*b^(1/3)
*log(4*b^(1/3)*x - 3^(1/2)*a^(1/3)*2i - 2*a^(1/3))*((3^(1/2)*1i)/2 - 1/2))
/(27*a^(10/3)) - (14*b^(1/3)*log(3^(1/2)*a^(1/3)*2i + 4*b^(1/3)*x - 2*a^(1
/3))*((3^(1/2)*1i)/2 + 1/2))/(27*a^(10/3))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2 (a+bx^3)^3} \, dx$$
$$= \frac{28\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) a^2bx + 56\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) a\, b^2x^4 + 28\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b^3x^7 - 54b^{\frac{2}{3}}a^{\frac{7}{3}} - 147b}{}$$

input
```
int(1/x^2/(b*x^3+a)^3,x)
```

output
```
(28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a**2*b*x +
56*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a*b**2*x**4
+ 28*sqrt(3)*atan((a**(1/3) - 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b**3*x**7
- 54*b**(2/3)*a**(1/3)*a**2 - 147*b**(2/3)*a**(1/3)*a*b*x**3 - 84*b**(2/3)
*a**(1/3)*b**2*x**6 - 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**
2)*a**2*b*x - 28*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a*b**
2*x**4 - 14*log(a**(2/3) - b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b**3*x**7
+ 28*log(a**(1/3) + b**(1/3)*x)*a**2*b*x + 56*log(a**(1/3) + b**(1/3)*x)*a
*b**2*x**4 + 28*log(a**(1/3) + b**(1/3)*x)*b**3*x**7)/(54*b**(2/3)*a**(1/3
)*a**3*x*(a**2 + 2*a*b*x**3 + b**2*x**6))
```

3.150 $\int \frac{x^8}{a-bx^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{x^8}{a-bx^3} dx = -\frac{ax^3}{3b^2} - \frac{x^6}{6b} - \frac{a^2 \log(a-bx^3)}{3b^3}$$

output
```
-1/3*a*x^3/b^2-1/6*x^6/b-1/3*a^2*ln(-b*x^3+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{a-bx^3} dx = -\frac{ax^3}{3b^2} - \frac{x^6}{6b} - \frac{a^2 \log(a-bx^3)}{3b^3}$$

input
```
Integrate[x^8/(a - b*x^3),x]
```

output
```
-1/3*(a*x^3)/b^2 - x^6/(6*b) - (a^2*Log[a - b*x^3])/(3*b^3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a-bx^3} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3} \int \frac{x^6}{a-bx^3}dx^3$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{3} \int \left(-\frac{x^3}{b}-\frac{a^2}{b^2 \left(bx^3-a\right)}-\frac{a}{b^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(-\frac{a^2 \log \left(a-bx^3\right)}{b^3}-\frac{ax^3}{b^2}-\frac{x^6}{2b}\right)$$

input
```
Int[x^8/(a - b*x^3),x]
```

output
```
(-((a*x^3)/b^2) - x^6/(2*b) - (a^2*Log[a - b*x^3])/b^3)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\frac{1}{2}b\,x^{6}+a\,x^{3}}{3b^{2}}-\frac{a^{2}\ln\left(-b\,x^{3}+a\right)}{3b^{3}}$	36
norman	$-\frac{a\,x^{3}}{3b^{2}}-\frac{x^{6}}{6b}-\frac{a^{2}\ln\left(-b\,x^{3}+a\right)}{3b^{3}}$	36
parallelrisch	$-\frac{b^{2}x^{6}+2ab\,x^{3}+2a^{2}\ln\left(b\,x^{3}-a\right)}{6b^{3}}$	36
risch	$-\frac{x^{6}}{6b}-\frac{a\,x^{3}}{3b^{2}}-\frac{a^{2}}{6b^{3}}-\frac{a^{2}\ln\left(-b\,x^{3}+a\right)}{3b^{3}}$	44

input
```
int(x^8/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2*(1/2*b*x^6+a*x^3)-1/3*a^2*ln(-b*x^3+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{a-bx^3}\,dx=-\frac{b^2x^6+2\,abx^3+2\,a^2\log\left(bx^3-a\right)}{6\,b^3}$$

input
```
integrate(x^8/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(b^2*x^6 + 2*a*b*x^3 + 2*a^2*log(b*x^3 - a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{a-bx^3}\,dx = -\frac{a^2\log\left(-a+bx^3\right)}{3b^3} - \frac{ax^3}{3b^2} - \frac{x^6}{6b}$$

input
```
integrate(x**8/(-b*x**3+a),x)
```

output
```
-a**2*log(-a + b*x**3)/(3*b**3) - a*x**3/(3*b**2) - x**6/(6*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{a-bx^3}\,dx = -\frac{a^2\log\left(bx^3-a\right)}{3\,b^3} - \frac{bx^6+2\,ax^3}{6\,b^2}$$

input
```
integrate(x^8/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*a^2*log(b*x^3 - a)/b^3 - 1/6*(b*x^6 + 2*a*x^3)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{x^8}{a-bx^3}\,dx = -\frac{a^2\log\left(|bx^3-a|\right)}{3\,b^3} - \frac{bx^6+2\,ax^3}{6\,b^2}$$

input
```
integrate(x^8/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*a^2*log(abs(b*x^3 - a))/b^3 - 1/6*(b*x^6 + 2*a*x^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{a-bx^3}\,dx = -\frac{2\,a^2\,\ln\left(b\,x^3-a\right)+b^2\,x^6+2\,a\,b\,x^3}{6\,b^3}$$

input
```
int(x^8/(a - b*x^3),x)
```

output
```
-(2*a^2*log(b*x^3 - a) + b^2*x^6 + 2*a*b*x^3)/(6*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{x^8}{a-bx^3}\,dx = \frac{-2\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a^2-2\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)a^2-2ab\,x^3-b^2x^6}{6b^3}$$

input
```
int(x^8/(-b*x^3+a),x)
```

output
```
( - 2*log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a**2 - 2*log(a**
(1/3) - b**(1/3)*x)*a**2 - 2*a*b*x**3 - b**2*x**6)/(6*b**3)
```

3.151 $\int \frac{x^5}{a-bx^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{x^5}{a-bx^3}\,dx=-\frac{x^3}{3b}-\frac{a\log\left(a-bx^3\right)}{3b^2}$$

output
```
-1/3*x^3/b-1/3*a*ln(-b*x^3+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a-bx^3}\,dx=-\frac{x^3}{3b}-\frac{a\log\left(a-bx^3\right)}{3b^2}$$

input
```
Integrate[x^5/(a - b*x^3),x]
```

output
```
-1/3*x^3/b - (a*Log[a - b*x^3])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a-bx^3}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{a-bx^3}dx^3$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}\int \left(-\frac{a}{b(bx^3-a)}-\frac{1}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a\log(a-bx^3)}{b^2}-\frac{x^3}{b}\right)$$

input
```
Int[x^5/(a - b*x^3),x]
```

output
```
(-(x^3/b) - (a*Log[a - b*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$-\frac{b\,x^{3}+a\ln\left(b\,x^{3}-a\right)}{3b^{2}}$	24
default	$-\frac{x^{3}}{3b}-\frac{a\ln\left(-b\,x^{3}+a\right)}{3b^{2}}$	25
norman	$-\frac{x^{3}}{3b}-\frac{a\ln\left(-b\,x^{3}+a\right)}{3b^{2}}$	25
risch	$-\frac{x^{3}}{3b}-\frac{a\ln\left(-b\,x^{3}+a\right)}{3b^{2}}$	25

input
```
int(x^5/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^3+a*ln(b*x^3-a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a-bx^3}\,dx=-\frac{bx^3+a\log\left(bx^3-a\right)}{3\,b^2}$$

input
```
integrate(x^5/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(b*x^3 + a*log(b*x^3 - a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{a-bx^3}\,dx = -\frac{a\log\left(-a+bx^3\right)}{3b^2} - \frac{x^3}{3b}$$

input
```
integrate(x**5/(-b*x**3+a),x)
```

output
```
-a*log(-a + b*x**3)/(3*b**2) - x**3/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{a-bx^3}\,dx = -\frac{x^3}{3\,b} - \frac{a\log\left(bx^3-a\right)}{3\,b^2}$$

input
```
integrate(x^5/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*x^3/b - 1/3*a*log(b*x^3 - a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{a-bx^3}\,dx = -\frac{x^3}{3\,b} - \frac{a\log\left(|bx^3-a|\right)}{3\,b^2}$$

input
```
integrate(x^5/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*x^3/b - 1/3*a*log(abs(b*x^3 - a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a-bx^3}\,dx = -\frac{b\,x^3+a\,\ln\left(b\,x^3-a\right)}{3\,b^2}$$

input
```
int(x^5/(a - b*x^3),x)
```

output
```
-(b*x^3 + a*log(b*x^3 - a))/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$\int \frac{x^5}{a-bx^3}\,dx = \frac{-\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a-\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)a-b\,x^3}{3b^2}$$

input
```
int(x^5/(-b*x^3+a),x)
```

output
```
( - (log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*a + log(a**(1/3)
- b**(1/3)*x)*a + b*x**3))/(3*b**2)
```

3.152 $\int \frac{x^2}{a-bx^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{x^2}{a-bx^3} dx = -\frac{\log(a-bx^3)}{3b}$$

output
```
-1/3*ln(-b*x^3+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a-bx^3} dx = -\frac{\log(a-bx^3)}{3b}$$

input
```
Integrate[x^2/(a - b*x^3),x]
```

output
```
-1/3*Log[a - b*x^3]/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a-bx^3} \, dx$$

$$\Big\downarrow \; 792$$

$$-\frac{\log \left(a-bx^3\right)}{3b}$$

input
```
Int[x^2/(a - b*x^3),x]
```

output
```
-1/3*Log[a - b*x^3]/b
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{\ln(-b\,x^3+a)}{3b}$	15
default	$-\frac{\ln(-b\,x^3+a)}{3b}$	15
norman	$-\frac{\ln(-b\,x^3+a)}{3b}$	15
risch	$-\frac{\ln(-b\,x^3+a)}{3b}$	15
parallelrisch	$-\frac{\ln(b\,x^3-a)}{3b}$	16

input
```
int(x^2/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(-b*x^3+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a-bx^3}\,dx = -\frac{\log\left(bx^3-a\right)}{3\,b}$$

input
```
integrate(x^2/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*log(b*x^3 - a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a - bx^3}\, dx = -\frac{\log\left(-a + bx^3\right)}{3b}$$

input
```
integrate(x**2/(-b*x**3+a),x)
```

output
```
-log(-a + b*x**3)/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a - bx^3}\, dx = -\frac{\log\left(bx^3 - a\right)}{3\,b}$$

input
```
integrate(x^2/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*log(b*x^3 - a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a - bx^3}\, dx = -\frac{\log\left(|bx^3 - a|\right)}{3\,b}$$

input
```
integrate(x^2/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a-bx^3}\,dx = -\frac{\ln\left(b\,x^3-a\right)}{3\,b}$$

input
```
int(x^2/(a - b*x^3),x)
```

output
```
-log(b*x^3 - a)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{x^2}{a-bx^3}\,dx = \frac{-\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{3b}$$

input
```
int(x^2/(-b*x^3+a),x)
```

output
```
( - (log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) + log(a**(1/3) -
b**(1/3)*x)))/(3*b)
```

3.153 $\int \frac{1}{x(a-bx^3)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{1}{x(a-bx^3)}\,dx = \frac{\log(x)}{a} - \frac{\log(a-bx^3)}{3a}$$

output
```
ln(x)/a-1/3*ln(-b*x^3+a)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a-bx^3)}\,dx = \frac{\log(x)}{a} - \frac{\log(a-bx^3)}{3a}$$

input
```
Integrate[1/(x*(a - b*x^3)),x]
```

output
```
Log[x]/a - Log[a - b*x^3]/(3*a)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a-b x^3)} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{1}{x^3 (a-b x^3)} dx^3$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{3} \left(\frac{b \int \frac{1}{a-b x^3} dx^3}{a}+\frac{\int \frac{1}{x^3} dx^3}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{3} \left(\frac{b \int \frac{1}{a-b x^3} dx^3}{a}+\frac{\log (x^3)}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3} \left(\frac{\log (x^3)}{a}-\frac{\log (a-b x^3)}{a}\right)$$

input
```
Int[1/(x*(a - b*x^3)),x]
```

output
```
(Log[x^3]/a - Log[a - b*x^3]/a)/3
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(-b\,x^3+a)}{3a}$	22
norman	$\frac{\ln(x)}{a}-\frac{\ln(-b\,x^3+a)}{3a}$	22
risch	$\frac{\ln(x)}{a}-\frac{\ln(-b\,x^3+a)}{3a}$	22
parallelrisch	$\frac{3\ln(x)-\ln(b\,x^3-a)}{3a}$	23

input
```
int(1/x/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/3*ln(-b*x^3+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a-bx^3\right)}\,dx = -\frac{\log\left(bx^3-a\right)-3\,\log\left(x\right)}{3\,a}$$

input
```
integrate(1/x/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(log(b*x^3 - a) - 3*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\left(a-bx^3\right)}\,dx = \frac{\log\left(x\right)}{a}-\frac{\log\left(-\frac{a}{b}+x^3\right)}{3a}$$

input
```
integrate(1/x/(-b*x**3+a),x)
```

output
```
log(x)/a - log(-a/b + x**3)/(3*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a-bx^3\right)}\,dx = -\frac{\log\left(bx^3-a\right)}{3\,a}+\frac{\log\left(x^3\right)}{3\,a}$$

input
```
integrate(1/x/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*log(b*x^3 - a)/a + 1/3*log(x^3)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x(a-bx^3)}\,dx = -\frac{\log(|bx^3-a|)}{3\,a} + \frac{\log(|x|)}{a}$$

input
```
integrate(1/x/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*log(abs(b*x^3 - a))/a + log(abs(x))/a
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(a-bx^3)}\,dx = \frac{\ln(x)}{a} - \frac{\ln(a-b\,x^3)}{3\,a}$$

input
```
int(1/(x*(a - b*x^3)),x)
```

output
```
log(x)/a - log(a - b*x^3)/(3*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{1}{x(a-bx^3)}\,dx = \frac{-\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)+3\log(x)}{3a}$$

input
```
int(1/x/(-b*x^3+a),x)
```

output
```
( - log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - log(a**(1/3) - b
**(1/3)*x) + 3*log(x))/(3*a)
```

3.154 $\int \frac{1}{x^4(a-bx^3)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{1}{x^4\left(a-bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^3\right)}{3a^2}$$

output
```
-1/3/a/x^3+b*ln(x)/a^2-1/3*b*ln(-b*x^3+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a-bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b\log(x)}{a^2}-\frac{b\log\left(a-bx^3\right)}{3a^2}$$

input
```
Integrate[1/(x^4*(a - b*x^3)),x]
```

output
```
-1/3*1/(a*x^3) + (b*Log[x])/a^2 - (b*Log[a - b*x^3])/(3*a^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^3)} \, dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3} \int \frac{1}{x^6 (a-bx^3)}dx^3$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{3} \int \left(\frac{b^2}{a^2 (a-bx^3)}+\frac{b}{a^2 x^3}+\frac{1}{a x^6}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3} \left(\frac{b \log \left(x^3\right)}{a^2}-\frac{b \log \left(a-bx^3\right)}{a^2}-\frac{1}{a x^3}\right)$$

input
```
Int[1/(x^4*(a - b*x^3)),x]
```

output
```
(-(1/(a*x^3)) + (b*Log[x^3])/a^2 - (b*Log[a - b*x^3])/a^2)/3
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{3a\,x^{3}}+\frac{b\ln\left(x\right)}{a^{2}}-\frac{b\ln\left(-b\,x^{3}+a\right)}{3a^{2}}$	32
norman	$-\frac{1}{3a\,x^{3}}+\frac{b\ln\left(x\right)}{a^{2}}-\frac{b\ln\left(-b\,x^{3}+a\right)}{3a^{2}}$	32
risch	$-\frac{1}{3a\,x^{3}}+\frac{b\ln\left(x\right)}{a^{2}}-\frac{b\ln\left(-b\,x^{3}+a\right)}{3a^{2}}$	32
parallelrisch	$\frac{3b\ln\left(x\right)x^{3}-b\ln\left(b\,x^{3}-a\right)x^{3}-a}{3a^{2}x^{3}}$	37

input
```
int(1/x^4/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+b*ln(x)/a^2-1/3*b*ln(-b*x^3+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^4\left(a-bx^3\right)}\,dx=-\frac{bx^3\log\left(bx^3-a\right)-3\,bx^3\log\left(x\right)+a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/3*(b*x^3*log(b*x^3 - a) - 3*b*x^3*log(x) + a)/(a^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4\left(a-bx^3\right)}\,dx=-\frac{1}{3ax^3}+\frac{b\log\left(x\right)}{a^2}-\frac{b\log\left(-\frac{a}{b}+x^3\right)}{3a^2}$$

input
```
integrate(1/x**4/(-b*x**3+a),x)
```

output
```
-1/(3*a*x**3) + b*log(x)/a**2 - b*log(-a/b + x**3)/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a-bx^3\right)}\,dx=-\frac{b\log\left(bx^3-a\right)}{3\,a^2}+\frac{b\log\left(x^3\right)}{3\,a^2}-\frac{1}{3\,ax^3}$$

input
```
integrate(1/x^4/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*b*log(b*x^3 - a)/a^2 + 1/3*b*log(x^3)/a^2 - 1/3/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^4\left(a-bx^3\right)}\,dx=-\frac{b\log\left(|bx^3-a|\right)}{3\,a^2}+\frac{b\log\left(|x|\right)}{a^2}-\frac{bx^3+a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*log(abs(b*x^3 - a))/a^2 + b*log(abs(x))/a^2 - 1/3*(b*x^3 + a)/(a^2*
x^3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4\,(a-bx^3)}\,dx = \frac{b\,\ln\left(x\right)}{a^2}-\frac{b\,\ln\left(a-b\,x^3\right)}{3\,a^2}-\frac{1}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a - b*x^3)),x)
```

output
```
(b*log(x))/a^2 - (b*log(a - b*x^3))/(3*a^2) - 1/(3*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^4\,(a-bx^3)}\,dx$$
$$= \frac{-\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)b\,x^3-\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)b\,x^3+3\log(x)\,b\,x^3-a}{3a^2x^3}$$

input
```
int(1/x^4/(-b*x^3+a),x)
```

output
```
( - log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**3 - log(a**(1
/3) - b**(1/3)*x)*b*x**3 + 3*log(x)*b*x**3 - a)/(3*a**2*x**3)
```

3.155 $\int \frac{x^4}{a-bx^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \frac{x^4}{a-bx^3}\,dx=-\frac{x^2}{2b}-\frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{5/3}}-\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3b^{5/3}}+\frac{a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}$$

output
```
-1/2*x^2/b-1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3
^(1/2)/b^(5/3)-1/3*a^(2/3)*ln(a^(1/3)-b^(1/3)*x)/b^(5/3)+1/6*a^(2/3)*ln(a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{a-bx^3}\,dx=-\frac{3b^{2/3}x^2+2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{5/3}}$$

input
```
Integrate[x^4/(a - b*x^3),x]
```

output
```
-1/6*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) - b^(1/3)*x] - a^(2/3)*Log[a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {843, 821, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a-bx^3}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{a\int \frac{x}{a-bx^3}dx}{b}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{821}$$

$$\frac{a\left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{b}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\left(-\frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{a\left(-\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{b}x+\sqrt[3]{a}\right)}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{x^2}{2b}$$

$\Big\downarrow$ 27

$$\frac{a\left(-\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{x^2}{2b}$$

$\Big\downarrow$ 1082

$$\frac{a\left(-\frac{-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{x^2}{2b}$$

$\Big\downarrow$ 217

$$\frac{a\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{x^2}{2b}$$

$\Big\downarrow$ 1103

$$\frac{a\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{b}-\frac{x^2}{2b}$$

input
```
Int[x^4/(a - b*x^3),x]
```

output
```
-1/2*x^2/b + (a*(-1/3*Log[a^(1/3) - b^(1/3)*x]/(a^(1/3)*b^(2/3)) - ((Sqrt[
3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - Log[a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

method	result	size
risch	$-\frac{x^2}{2b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3-a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3b^2}$	39
default	$-\frac{x^2}{2b}-\frac{\left(\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)a}{b}$	107

input
```
int(x^4/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2/b-1/3/b^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b-a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int\frac{x^4}{a-bx^3}\,dx = -\frac{3\,x^2+2\,\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3\,a}\right)+\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2+bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}-a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-2}{6\,b}$$

input
```
integrate(x^4/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(3*x^2 + 2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b
^2)^(1/3) - sqrt(3)*a)/a) + (-a^2/b^2)^(1/3)*log(a*x^2 + b*x*(-a^2/b^2)^(2
/3) - a*(-a^2/b^2)^(1/3)) - 2*(-a^2/b^2)^(1/3)*log(a*x - b*(-a^2/b^2)^(2/3
)))/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{a-bx^3}\,dx = -\operatorname{RootSum}\left(27t^3b^5-a^2,\left(t\mapsto t\log\left(-\frac{9t^2b^3}{a}+x\right)\right)\right)-\frac{x^2}{2b}$$

input
```
integrate(x**4/(-b*x**3+a),x)
```

output
```
-RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(-9*_t**2*b**3/a + x))) -
x**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{a-bx^3}\,dx = -\frac{x^2}{2\,b}-\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x^4/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1
/3))/(b^2*(a/b)^(1/3)) + 1/6*a*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2
*(a/b)^(1/3)) - 1/3*a*log(x - (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{a-bx^3}\,dx = -\frac{x^2}{2\,b} - \frac{\left(\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b} - \frac{\sqrt{3}(ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3} + \frac{(ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3}$$

input
```
integrate(x^4/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/2*x^2/b - 1/3*(a/b)^(2/3)*log(abs(x - (a/b)^(1/3)))/b - 1/3*sqrt(3)*(a*
b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b^3 + 1/6*(
a*b^2)^(2/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{a-bx^3}\,dx = \frac{a^{2/3}\ln\left(\frac{a^2\,x}{b}-\frac{a^{7/3}}{(-b)^{4/3}}\right)}{3\,(-b)^{5/3}} - \frac{x^2}{2\,b} - \frac{a^{2/3}\ln\left(\frac{a^2\,x}{b}-\frac{a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{(-b)^{4/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(-b)^{5/3}} + \frac{a^{2/3}\ln\left(\frac{a^2\,x}{b}-\frac{9\,a^{7/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2}{(-b)^{4/3}}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{(-b)^{5/3}}$$

input
```
int(x^4/(a - b*x^3),x)
```

output
```
(a^(2/3)*log((a^2*x)/b - a^(7/3)/(-b)^(4/3)))/(3*(-b)^(5/3)) - x^2/(2*b) -
 (a^(2/3)*log((a^2*x)/b - (a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(-b)^(4/3))*(
(3^(1/2)*1i)/2 + 1/2))/(3*(-b)^(5/3)) + (a^(2/3)*log((a^2*x)/b - (9*a^(7/3
)*((3^(1/2)*1i)/6 - 1/6)^2)/(-b)^(4/3))*((3^(1/2)*1i)/6 - 1/6))/(-b)^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{a-bx^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)a-3b^{\frac{2}{3}}a^{\frac{1}{3}}x^2+\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)a-2\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)a}{6b^{\frac{5}{3}}a^{\frac{1}{3}}}$$

input
```
int(x^4/(-b*x^3+a),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) + 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*a - 3*b**
(2/3)*a**(1/3)*x**2 + log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*
a - 2*log(a**(1/3) - b**(1/3)*x)*a)/(6*b**(2/3)*a**(1/3)*b)
```

3.156 $\int \frac{x^3}{a-bx^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 120

$$\int \frac{x^3}{a-bx^3}\,dx=-\frac{x}{b}+\frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{4/3}}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3b^{4/3}}+\frac{\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}$$

output

```
-x/b+1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)
/b^(4/3)-1/3*a^(1/3)*ln(a^(1/3)-b^(1/3)*x)/b^(4/3)+1/6*a^(1/3)*ln(a^(2/3)+
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{a-bx^3}\,dx=\frac{-6\sqrt[3]{b}x+2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}}$$

input
```
Integrate[x^3/(a - b*x^3),x]
```

output
```
(-6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[
3]] - 2*a^(1/3)*Log[a^(1/3) - b^(1/3)*x] + a^(1/3)*Log[a^(2/3) + a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {843, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a-bx^3}dx$$
$$\Big\downarrow \text{843}$$
$$\frac{a\int \frac{1}{a-bx^3}dx}{b}-\frac{x}{b}$$
$$\Big\downarrow \text{750}$$
$$\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+2\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3a^{2/3}}\right)}{b}-\frac{x}{b}$$
$$\Big\downarrow \text{16}$$
$$\frac{a\left(\frac{\int \frac{\sqrt[3]{b}x+2\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{x}{b}$$
$$\Big\downarrow \text{1142}$$

$$\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{b}x+\sqrt[3]{a}\right)}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{x}{b}$$

$\big\downarrow$ 27

$$\frac{a\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{x}{b}$$

$\big\downarrow$ 1082

$$\frac{a\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{x}{b}$$

$\big\downarrow$ 217

$$\frac{a\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{x}{b}$$

$\big\downarrow$ 1103

$$\frac{a\left(\frac{\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{b}-\frac{x}{b}$$

input
```
Int[x^3/(a - b*x^3),x]
```

output
```
-(x/b) + (a*(-1/3*Log[a^(1/3) - b^(1/3)*x]/(a^(2/3)*b^(1/3)) + ((Sqrt[3]*A
rcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + Log[a^(2/3) + a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.31

method	result	size
risch	$-\frac{x}{b}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3-a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{3b^2}$	37
default	$-\frac{x}{b}-\frac{\left(\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)a}{b}$	105

input
```
int(x^3/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-x/b-1/3/b^2*a*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b-a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{a-bx^3}\,dx = -\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x+\left(-\right.\right.}{6b}$$

input
```
integrate(x^3/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) + sqrt
(3)*a)/a) + (-a/b)^(1/3)*log(x^2 - x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 2*(-a/
b)^(1/3)*log(x + (-a/b)^(1/3)) + 6*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.20

$$\int \frac{x^3}{a-bx^3}\,dx = -\operatorname{RootSum}\left(27t^3b^4-a,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)-\frac{x}{b}$$

input
```
integrate(x**3/(-b*x**3+a),x)
```

output
```
-RootSum(27*_t**3*b**4 - a, Lambda(_t, _t*log(-3*_t*b + x))) - x/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{a-bx^3}\,dx = -\frac{x}{b}+\frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(x^3/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-x/b + 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(
b^2*(a/b)^(2/3)) + 1/6*a*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)
^(2/3)) - 1/3*a*log(x - (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a-bx^3}\,dx = -\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{x}{b}+\frac{\sqrt{3}(ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2}+\frac{(ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2}$$

input
```
integrate(x^3/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/b - x/b + 1/3*sqrt(3)*(a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/b^2 + 1/6*(a*b^2)
^(1/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{a-bx^3}\,dx = \frac{(-a)^{1/3}\ln\left((-a)^{4/3}-a\,b^{1/3}\,x\right)}{3\,b^{4/3}}-\frac{x}{b}-\frac{(-a)^{1/3}\ln\left(3\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)+3\,a\,b\,x\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{3\,b^{4/3}}+\frac{(-a)^{1/3}\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)-3\,a\,b\,x\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)}{b^{4/3}}$$

input
```
int(x^3/(a - b*x^3),x)
```

output
```
((-a)^(1/3)*log((-a)^(4/3) - a*b^(1/3)*x))/(3*b^(4/3)) - x/b - ((-a)^(1/3)
*log(3*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 3*a*b*x)*((3^(1/2)*1i)/
2 + 1/2))/(3*b^(4/3)) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)
/6 - 1/6) - 3*a*b*x)*((3^(1/2)*1i)/6 - 1/6))/b^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{a-bx^3}\,dx$$
$$=\frac{2a^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)-6b^{\frac{1}{3}}x}{6b^{\frac{4}{3}}}$$

input
```
int(x^3/(-b*x^3+a),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) + 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + a
**(1/3)*log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*a**(1/3)*l
og(a**(1/3) - b**(1/3)*x) - 6*b**(1/3)*x)/(6*b**(1/3)*b)
```

3.157 $\int \frac{x}{a-bx^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 115

$$\int \frac{x}{a-bx^3}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

output
```
-1/3*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3)/b^(
2/3)-1/3*ln(a^(1/3)-b^(1/3)*x)/a^(1/3)/b^(2/3)+1/6*ln(a^(2/3)+a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{x}{a-bx^3}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right) + \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}$$

input
```
Integrate[x/(a - b*x^3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) -
b^(1/3)*x] + Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^
(2/3))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {821, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a-bx^3}dx$$

$$\Big\downarrow \text{821}$$

$$\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{b}x+\sqrt[3]{a}\right)}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[x/(a - b*x^3),x]
```

output
```
-1/3*Log[a^(1/3) - b^(1/3)*x]/(a^(1/3)*b^(2/3)) - ((Sqrt[3]*ArcTan[(1 + (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - Log[a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.25

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3-a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{3b}$	29
default	$-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	92

input
```
int(x/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*sum(1/_R*ln(x-_R),_R=RootOf(_Z^3*b-a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.67

$$\int\frac{x}{a-bx^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3+ab+3\sqrt{\frac{1}{3}}\left(abx-2\left(-ab^2\right)^{\frac{2}{3}}x^2-\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3-a}\right)+\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x\right.}{6\,ab^2}\right.$$

input
```
integrate(x/(-b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 + a*b + 3*sqrt
(1/3)*(a*b*x - 2*(-a*b^2)^(2/3)*x^2 - (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 - a)) + (-a*b^2)^(2/3)*log(b^2*x^2 - (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x + (-a*b^2)^(1
/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(-sqrt(1
/3)*(2*b*x - (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*l
og(b^2*x^2 - (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b
*x + (-a*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.23

$$\int \frac{x}{a-bx^3}\,dx = -\operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(-9t^2ab+x\right)\right)\right)$$

input
```
integrate(x/(-b*x**3+a),x)
```

output
```
-RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-9*_t**2*a*b + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{x}{a-bx^3}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

input
```
integrate(x/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(1/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*
log(x - (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{x}{a-bx^3}\,dx = -\frac{\left(\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} - \frac{\sqrt{3}(ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{(ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^2}$$

input
```
integrate(x/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(a/b)^(2/3)*log(abs(x - (a/b)^(1/3)))/a - 1/3*sqrt(3)*(a*b^2)^(2/3)*a
rctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2) + 1/6*(a*b^2)^(
2/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \frac{x}{a-bx^3}\,dx = \frac{\ln\left((-a)^{1/3}+b^{1/3}\,x\right)}{3\,(-a)^{1/3}\,b^{2/3}} + \frac{\ln\left(b\,x+\frac{(-a)^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}} - \frac{\ln\left(b\,x+\frac{(-a)^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,(-a)^{1/3}\,b^{2/3}}$$

input
```
int(x/(a - b*x^3),x)
```

output
```
log((-a)^(1/3) + b^(1/3)*x)/(3*(-a)^(1/3)*b^(2/3)) + (log(b*x + ((-a)^(1/3
)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*(-a)^(1/3)*b^(2/3))
- (log(b*x + ((-a)^(1/3)*b^(2/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/
(6*(-a)^(1/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{x}{a-bx^3}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{6b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int(x/(-b*x^3+a),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) + 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) + log(a**
(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*log(a**(1/3) - b**(1/3)*x
))/(6*b**(2/3)*a**(1/3))
```

3.158 $\int \frac{1}{a-bx^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 114

$$\int \frac{1}{a-bx^3} dx = \frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

output
```
1/3*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)/b^(1
/3)-1/3*ln(a^(1/3)-b^(1/3)*x)/a^(2/3)/b^(1/3)+1/6*ln(a^(2/3)+a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.78

$$\int \frac{1}{a-bx^3} dx = \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right) + \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}}$$

input
```
Integrate[(a - b*x^3)^(-1),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - b
^(1/3)*x] + Log[a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(
1/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-bx^3} \, dx$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{\sqrt[3]{b}x+2\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3a^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{\sqrt[3]{b}x+2\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{b}x+\sqrt[3]{a}\right)}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[(a - b*x^3)^(-1),x]
```

output
```
-1/3*Log[a^(1/3) - b^(1/3)*x]/(a^(2/3)*b^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + Log[a^(2/3) + a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.25

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3-a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}}{3b}$	29
default	$-\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	92

input
```
int(1/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b*sum(1/_R^2*ln(x-_R),_R=RootOf(_Z^3*b-a))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.81

$$\int \frac{1}{a-bx^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax+a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2-\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3-a}\right)+\left(-a^2b\right)^{\frac{2}{3}}\log\left(ab\right.}{6\,a^2b}\right.$$

input
```
integrate(1/(-b*x^3+a),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(
1/3)*a*x + a^2 + 3*sqrt(1/3)*(2*a*b*x^2 - (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3
)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 - a)) + (-a^2*b)^(2/3)*log(a*b*x^2 + (
-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(-a^2*b)^(2/3)*log(a*b*x - (-a^2*b
)^(2/3)))/(a^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqr
t(1/3)*(2*(-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2
) + (-a^2*b)^(2/3)*log(a*b*x^2 + (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*
(-a^2*b)^(2/3)*log(a*b*x - (-a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

$$\int \frac{1}{a-bx^3}\,dx = -\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(-3ta+x\right)\right)\right)$$

input
```
integrate(1/(-b*x**3+a),x)
```

output
```
-RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(-3*_t*a + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{1}{a-bx^3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

input
```
integrate(1/(-b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(
2/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 1/3*l
og(x - (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{1}{a-bx^3}\,dx = -\frac{\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a} + \frac{\sqrt{3}(ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab} + \frac{(ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

input
```
integrate(1/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/a + 1/3*sqrt(3)*(a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*b) + 1/6*(a*b^2)^(1/
3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{1}{a-bx^3}\,dx = \frac{\ln\left(a^{1/3}\,(-b)^{5/3}+b^2\,x\right)}{3\,a^{2/3}\,(-b)^{1/3}} + \frac{\ln\left(3\,b^2\,x+\frac{3\,a^{1/3}\,(-b)^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}} - \frac{\ln\left(3\,b^2\,x-\frac{3\,a^{1/3}\,(-b)^{5/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{2/3}\,(-b)^{1/3}}$$

input
```
int(1/(a - b*x^3),x)
```

output
```
log(a^(1/3)*(-b)^(5/3) + b^2*x)/(3*a^(2/3)*(-b)^(1/3)) + (log(3*b^2*x + (3
*a^(1/3)*(-b)^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*(-b)
^(1/3)) - (log(3*b^2*x - (3*a^(1/3)*(-b)^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(1/
2)*1i + 1))/(6*a^(2/3)*(-b)^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{1}{a-bx^3}\,dx = \frac{2\sqrt{3}\,atan\left(\frac{a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right)+\log\left(a^{\frac{2}{3}}+b^{\frac{1}{3}}a^{\frac{1}{3}}x+b^{\frac{2}{3}}x^2\right)-2\log\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)}{6a^{\frac{2}{3}}b^{\frac{1}{3}}}$$

input
```
int(1/(-b*x^3+a),x)
```

output
```
(a**(1/3)*(2*sqrt(3)*atan((a**(1/3) + 2*b**(1/3)*x)/(a**(1/3)*sqrt(3))) +
log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2) - 2*log(a**(1/3) - b**
(1/3)*x)))/(6*b**(1/3)*a)
```

3.159 $\int \frac{1}{x^2(a-bx^3)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 123

$$\int \frac{1}{x^2(a-bx^3)}\,dx = -\frac{1}{ax}-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}}$$

output
```
-1/a/x-1/3*b^(1/3)*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3^(1/
2)/a^(4/3)-1/3*b^(1/3)*ln(a^(1/3)-b^(1/3)*x)/a^(4/3)+1/6*b^(1/3)*ln(a^(2/3
)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2(a-bx^3)}\,dx = -\frac{6\sqrt[3]{a}+2\sqrt{3}\sqrt[3]{b}x\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}x\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)-\sqrt[3]{b}x\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}x}$$

input
```
Integrate[1/(x^2*(a - b*x^3)),x]
```

output
```
-1/6*(6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/S
qrt[3]] + 2*b^(1/3)*x*Log[a^(1/3) - b^(1/3)*x] - b^(1/3)*x*Log[a^(2/3) + a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(a^(4/3)*x)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {847, 821, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^3)} \, dx$$

$$\Big\downarrow \text{847}$$

$$\frac{b \int \frac{x}{a-bx^3}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{821}$$

$$\frac{b \left(\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3 \sqrt[3]{a} \sqrt[3]{b}}-\frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2+\sqrt[3]{a} \sqrt[3]{b}x+a^{2/3}}dx}{3 \sqrt[3]{a} \sqrt[3]{b}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{16}$$

$$\frac{b \left(-\frac{\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{b^{2/3}x^2+\sqrt[3]{a} \sqrt[3]{b}x+a^{2/3}}dx}{3 \sqrt[3]{a} \sqrt[3]{b}}-\frac{\log \left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3 \sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1142}$$

$$\frac{b\left(-\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{b}x+\sqrt[3]{a}\right)}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 27

$$\frac{b\left(-\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 1082

$$\frac{b\left(-\frac{-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 217

$$\frac{b\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 1103

$$\frac{b\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a - b*x^3)),x]
```

output
```
-(1/(a*x)) + (b*(-1/3*Log[a^(1/3) - b^(1/3)*x]/(a^(1/3)*b^(2/3)) - ((Sqrt[
3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) - Log[a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.41

method	result	size
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{4}\_Z^{3}+b\right)}\_R\ln\left(\left(4\_R^{3}a^{4}+3b\right)x+a^{3}\_R^{2}\right)\right)}{3}$	50
default	$-\frac{1}{ax}-\frac{\left(\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b}{a}$	107

input
```
int(1/x^2/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+1/3*sum(_R*ln((4*_R^3*a^4+3*b)*x+a^3*_R^2),_R=RootOf(_Z^3*a^4+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\left(a-bx^3\right)}\,dx = -\frac{2\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2+ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}}{6\,ax}$$

input
```
integrate(1/x^2/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) - 1/3*sqr
t(3)) + x*(-b/a)^(1/3)*log(b*x^2 + a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) - 2*
x*(-b/a)^(1/3)*log(b*x - a*(-b/a)^(2/3)) + 6)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2 (a - bx^3)} \, dx = -\operatorname{RootSum}\left(27t^3a^4 - b, \left(t \mapsto t\log\left(-\frac{9t^2a^3}{b} + x\right)\right)\right) - \frac{1}{ax}$$

input
```
integrate(1/x**2/(-b*x**3+a),x)
```

output
```
-RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(-9*_t**2*a**3/b + x))) - 1/(
a*x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a - bx^3)} \, dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(-b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^
(1/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) - 1/3*
log(x - (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a - bx^3)} dx = -\frac{b\left(\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3 a^2} - \frac{\sqrt{3}(ab^2)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 a^2 b} + \frac{(ab^2)^{\frac{2}{3}} \log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 a^2 b} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*(a/b)^(2/3)*log(abs(x - (a/b)^(1/3)))/a^2 - 1/3*sqrt(3)*(a*b^2)^(2/
3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b) + 1/6*(a*b^
2)^(2/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^2 (a - bx^3)} dx = \frac{(-b)^{1/3} \ln\left(b^3 x - a^{1/3} (-b)^{8/3}\right)}{3 a^{4/3}} - \frac{1}{a x} - \frac{(-b)^{1/3} \ln\left(a b^3 x - a^{4/3} (-b)^{8/3} \left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right) \left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3 a^{4/3}} + \frac{(-b)^{1/3} \ln\left(a b^3 x - 9 a^{4/3} (-b)^{8/3} \left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right) \left(-\frac{1}{6} + \frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{a^{4/3}}$$

input
```
int(1/(x^2*(a - b*x^3)),x)
```

output
```
((-b)^(1/3)*log(b^3*x - a^(1/3)*(-b)^(8/3)))/(3*a^(4/3)) - 1/(a*x) - ((-b)
^(1/3)*log(a*b^3*x - a^(4/3)*(-b)^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2
)*1i)/2 + 1/2))/(3*a^(4/3)) + ((-b)^(1/3)*log(a*b^3*x - 9*a^(4/3)*(-b)^(8/
3)*((3^(1/2)*1i)/6 - 1/6)^2)*((3^(1/2)*1i)/6 - 1/6))/a^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2 (a - bx^3)} \, dx$$
$$= \frac{-2\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) bx - 6b^{\frac{2}{3}}a^{\frac{1}{3}} + \log\left(a^{\frac{2}{3}} + b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) bx - 2\log\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) bx}{6b^{\frac{2}{3}}a^{\frac{4}{3}}x}$$

input
```
int(1/x^2/(-b*x^3+a),x)
```

output
```
( - 2*sqrt(3)*atan((a**(1/3) + 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x - 6*b
**(2/3)*a**(1/3) + log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x
 - 2*log(a**(1/3) - b**(1/3)*x)*b*x)/(6*b**(2/3)*a**(1/3)*a*x)
```

3.160 $\int \frac{1}{x^3(a-bx^3)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int \frac{1}{x^3(a-bx^3)}\,dx = -\frac{1}{2ax^2}+\frac{b^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}}$$

output
```
-1/2/a/x^2+1/3*b^(2/3)*arctan(1/3*(a^(1/3)+2*b^(1/3)*x)*3^(1/2)/a^(1/3))*3
^(1/2)/a^(5/3)-1/3*b^(2/3)*ln(a^(1/3)-b^(1/3)*x)/a^(5/3)+1/6*b^(2/3)*ln(a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3(a-bx^3)}\,dx$$

$$= \frac{-3a^{2/3}+2\sqrt{3}b^{2/3}x^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2b^{2/3}x^2\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)+b^{2/3}x^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x\right.}{6a^{5/3}x^2}$$

input
```
Integrate[1/(x^3*(a - b*x^3)),x]
```

output
```
(-3*a^(2/3) + 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqr
t[3]] - 2*b^(2/3)*x^2*Log[a^(1/3) - b^(1/3)*x] + b^(2/3)*x^2*Log[a^(2/3) +
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*x^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {847, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a-bx^3)} dx$$

$$\Big\downarrow \text{847}$$

$$\frac{b\int \frac{1}{a-bx^3}dx}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{750}$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+2\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b}x}dx}{3a^{2/3}}\right)}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+2\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\int \frac{\sqrt[3]{b}\left(2\sqrt[3]{b}x+\sqrt[3]{a}\right)}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{2\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 27

$$\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 1082

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx-\frac{3\int \frac{1}{-\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)^2-3}d\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 217

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{2\sqrt[3]{b}x+\sqrt[3]{a}}{b^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}}dx+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

$\Big\downarrow$ 1103

$$\frac{b\left(\frac{\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}}-\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{3a^{2/3}\sqrt[3]{b}}\right)}{a}-\frac{1}{2ax^2}$$

input
```
Int[1/(x^3*(a - b*x^3)),x]
```

output
```
-1/2*1/(a*x^2) + (b*(-1/3*Log[a^(1/3) - b^(1/3)*x]/(a^(2/3)*b^(1/3)) + ((S
qrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + Log[a^(2/3)
+ a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(2*b^(1/3)))/(3*a^(2/3))))/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{1}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(4\_R^{3}a^{5}+3b^{2}\right)x-a^{2}b\_R\right)\right)}{3}$	54
default	$-\frac{1}{2a\,x^{2}}-\frac{\left(\frac{\ln\left(x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{3b\left(\frac{a}{b}\right)^{\frac{2}{3}}}\right)b}{a}$	107

input
```
int(1/x^3/(-b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+1/3*sum(_R*ln((4*_R^3*a^5+3*b^2)*x-a^2*b*_R),_R=RootOf(_Z^3*a^5
+b^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3\left(a-bx^3\right)}\,dx =$$

$$-\frac{2\sqrt{3}x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}+\sqrt{3}b}{3\,b}\right)+x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}-abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-2}{6\,ax^{2}}$$

input
```
integrate(1/x^3/(-b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^
(2/3) + sqrt(3)*b)/b) + x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(-b^2/a^2
)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 2*x^2*(-b^2/a^2)^(1/3)*log(b*x + a*(-b^2
/a^2)^(1/3)) + 3)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^3 (a - bx^3)} \, dx = -\operatorname{RootSum}\left(27t^3a^5 - b^2, \left(t \mapsto t \log\left(-\frac{3ta^2}{b} + x\right)\right)\right) - \frac{1}{2ax^2}$$

input
```
integrate(1/x**3/(-b*x**3+a),x)
```

output
```
-RootSum(27*_t**3*a**5 - b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(
2*a*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (a - bx^3)} \, dx = \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^2 + x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{1}{2ax^2}$$

input
```
integrate(1/x^3/(-b*x^3+a),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*l
og(x - (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3(a-bx^3)}\,dx = -\frac{b\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2} + \frac{\sqrt{3}(ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2} + \frac{(ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2} - \frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/(-b*x^3+a),x, algorithm="giac")
```

output
```
-1/3*b*(a/b)^(1/3)*log(abs(x - (a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*(a*b^2)^(1/
3)*arctan(1/3*sqrt(3)*(2*x + (a/b)^(1/3))/(a/b)^(1/3))/a^2 + 1/6*(a*b^2)^(
1/3)*log(x^2 + x*(a/b)^(1/3) + (a/b)^(2/3))/a^2 - 1/2/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3(a-bx^3)}\,dx = \frac{b^{2/3}\ln\left((-a)^{7/3}+a^2\,b^{1/3}\,x\right)}{3\,(-a)^{5/3}} - \frac{1}{2\,a\,x^2} - \frac{b^{2/3}\ln\left(3\,a^2\,b^3\,x-3\,(-a)^{7/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3\,(-a)^{5/3}} + \frac{b^{2/3}\ln\left(3\,a^2\,b^3\,x+9\,(-a)^{7/3}\,b^{8/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)}{(-a)^{5/3}}$$

input
```
int(1/(x^3*(a - b*x^3)),x)
```

output
```
(b^(2/3)*log((-a)^(7/3) + a^2*b^(1/3)*x))/(3*(-a)^(5/3)) - 1/(2*a*x^2) - (
b^(2/3)*log(3*a^2*b^3*x - 3*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2))*((3
^(1/2)*1i)/2 + 1/2))/(3*(-a)^(5/3)) + (b^(2/3)*log(3*a^2*b^3*x + 9*(-a)^(7
/3)*b^(8/3)*((3^(1/2)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6))/(-a)^(5/3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3 (a - b x^3)} \, dx$$
$$= \frac{2a^{\frac{1}{3}}\sqrt{3}\, atan\left(\frac{a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{a^{\frac{1}{3}}\sqrt{3}}\right) b\, x^2 + a^{\frac{1}{3}}\log\left(a^{\frac{2}{3}} + b^{\frac{1}{3}}a^{\frac{1}{3}}x + b^{\frac{2}{3}}x^2\right) b\, x^2 - 2a^{\frac{1}{3}}\log\left(a^{\frac{1}{3}} - b^{\frac{1}{3}}x\right) b\, x^2 - 3b^{\frac{1}{3}}a}{6b^{\frac{1}{3}}a^2x^2}$$

input
```
int(1/x^3/(-b*x^3+a),x)
```

output
```
(2*a**(1/3)*sqrt(3)*atan((a**(1/3) + 2*b**(1/3)*x)/(a**(1/3)*sqrt(3)))*b*x
**2 + a**(1/3)*log(a**(2/3) + b**(1/3)*a**(1/3)*x + b**(2/3)*x**2)*b*x**2
- 2*a**(1/3)*log(a**(1/3) - b**(1/3)*x)*b*x**2 - 3*b**(1/3)*a)/(6*b**(1/3)
*a**2*x**2)
```

3.161 $\int \frac{\sqrt{x}}{1+x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{\sqrt{x}}{1+x^3} \, dx = \frac{2}{3} \arctan \left(x^{3/2}\right)$$

output
```
2/3*arctan(x^(3/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+x^3} \, dx = \frac{2}{3} \arctan \left(x^{3/2}\right)$$

input
```
Integrate[Sqrt[x]/(1 + x^3),x]
```

output
```
(2*ArcTan[x^(3/2)])/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {851, 807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x^3+1}\,dx$$
$$\Big\downarrow \text{ 851}$$
$$2\int \frac{x}{x^3+1}d\sqrt{x}$$
$$\Big\downarrow \text{ 807}$$
$$\frac{2}{3}\int \frac{1}{x+1}dx^{3/2}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{2}{3}\arctan\left(x^{3/2}\right)$$

input
```
Int[Sqrt[x]/(1 + x^3),x]
```

output
```
(2*ArcTan[x^(3/2)])/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{2\arctan\left(x^{\frac{3}{2}}\right)}{3}$	7
default	$\frac{2\arctan\left(x^{\frac{3}{2}}\right)}{3}$	7
meijerg	$\frac{2\arctan\left(x^{\frac{3}{2}}\right)}{3}$	7
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2x^{\frac{3}{2}}-\mathrm{RootOf}\left(\_Z^2+1\right)}{(1+x)\left(x^2-x+1\right)}\right)}{3}$	50

input
```
int(x^(1/2)/(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(x^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{1+x^3}\,dx = \frac{2}{3}\arctan\left(x^{\frac{3}{2}}\right)$$

input
```
integrate(x^(1/2)/(x^3+1),x, algorithm="fricas")
```

output
```
2/3*arctan(x^(3/2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(8) = 16$.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{x}}{1+x^3}\,dx = -\frac{2\operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{2\operatorname{atan}\left(2\sqrt{x}-\sqrt{3}\right)}{3} + \frac{2\operatorname{atan}\left(2\sqrt{x}+\sqrt{3}\right)}{3}$$

input
```
integrate(x**(1/2)/(x**3+1),x)
```

output
```
-2*atan(sqrt(x))/3 + 2*atan(2*sqrt(x) - sqrt(3))/3 + 2*atan(2*sqrt(x) + sq
rt(3))/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{1+x^3}\,dx = \frac{2}{3}\arctan\left(x^{\frac{3}{2}}\right)$$

input
```
integrate(x^(1/2)/(x^3+1),x, algorithm="maxima")
```

output
```
2/3*arctan(x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{1+x^3}\,dx = \frac{2}{3}\arctan\left(x^{\frac{3}{2}}\right)$$

input
```
integrate(x^(1/2)/(x^3+1),x, algorithm="giac")
```

output
```
2/3*arctan(x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{1+x^3}\,dx = \frac{2\,\mathrm{atan}\left(x^{3/2}\right)}{3}$$

input
```
int(x^(1/2)/(x^3 + 1),x)
```

output
```
(2*atan(x^(3/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{x}}{1+x^3}\,dx = \frac{2atan\left(2\sqrt{x}-\sqrt{3}\right)}{3} + \frac{2atan\left(2\sqrt{x}+\sqrt{3}\right)}{3} - \frac{2atan\left(\sqrt{x}\right)}{3}$$

input
```
int(x^(1/2)/(x^3+1),x)
```

output
```
(2*(atan(2*sqrt(x) - sqrt(3)) + atan(2*sqrt(x) + sqrt(3)) - atan(sqrt(x)))
)/3
```

3.162 $\int x^{11}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{11}\sqrt{a+bx^3}\,dx=-\frac{2a^3(a+bx^3)^{3/2}}{9b^4}+\frac{2a^2(a+bx^3)^{5/2}}{5b^4}-\frac{2a(a+bx^3)^{7/2}}{7b^4}+\frac{2(a+bx^3)^{9/2}}{27b^4}$$

output
```
-2/9*a^3*(b*x^3+a)^(3/2)/b^4+2/5*a^2*(b*x^3+a)^(5/2)/b^4-2/7*a*(b*x^3+a)^(
7/2)/b^4+2/27*(b*x^3+a)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^{11}\sqrt{a+bx^3}\,dx=\frac{2\sqrt{a+bx^3}(-16a^4+8a^3bx^3-6a^2b^2x^6+5ab^3x^9+35b^4x^{12})}{945b^4}$$

input
```
Integrate[x^11*Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(-16*a^4 + 8*a^3*b*x^3 - 6*a^2*b^2*x^6 + 5*a*b^3*x^9 +
35*b^4*x^12))/(945*b^4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int x^9\sqrt{bx^3+a}dx^3$$

$$\Big\downarrow \ 53$$

$$\frac{1}{3}\int \left(\frac{(bx^3+a)^{7/2}}{b^3}-\frac{3a(bx^3+a)^{5/2}}{b^3}+\frac{3a^2(bx^3+a)^{3/2}}{b^3}-\frac{a^3\sqrt{bx^3+a}}{b^3}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(-\frac{2a^3(a+bx^3)^{3/2}}{3b^4}+\frac{6a^2(a+bx^3)^{5/2}}{5b^4}+\frac{2(a+bx^3)^{9/2}}{9b^4}-\frac{6a(a+bx^3)^{7/2}}{7b^4}\right)$$

input
```
Int[x^11*Sqrt[a + b*x^3],x]
```

output
```
((-2*a^3*(a + b*x^3)^(3/2))/(3*b^4) + (6*a^2*(a + b*x^3)^(5/2))/(5*b^4) -
(6*a*(a + b*x^3)^(7/2))/(7*b^4) + (2*(a + b*x^3)^(9/2))/(9*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2(b\,x^{3}+a)^{\frac{3}{2}}\left(-35b^{3}x^{9}+30a\,b^{2}x^{6}-24a^{2}b\,x^{3}+16a^{3}\right)}{945b^{4}}$	47
pseudoelliptic	$-\frac{2(b\,x^{3}+a)^{\frac{3}{2}}\left(-35b^{3}x^{9}+30a\,b^{2}x^{6}-24a^{2}b\,x^{3}+16a^{3}\right)}{945b^{4}}$	47
orering	$-\frac{2(b\,x^{3}+a)^{\frac{3}{2}}\left(-35b^{3}x^{9}+30a\,b^{2}x^{6}-24a^{2}b\,x^{3}+16a^{3}\right)}{945b^{4}}$	47
trager	$-\frac{2\left(-35b^{4}x^{12}-5a\,b^{3}x^{9}+6a^{2}b^{2}x^{6}-8a^{3}b\,x^{3}+16a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$	58
risch	$-\frac{2\left(-35b^{4}x^{12}-5a\,b^{3}x^{9}+6a^{2}b^{2}x^{6}-8a^{3}b\,x^{3}+16a^{4}\right)\sqrt{b\,x^{3}+a}}{945b^{4}}$	58
default	$\frac{2x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2a\,x^{9}\sqrt{b\,x^{3}+a}}{189b}-\frac{4a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{3}}-\frac{32a^{4}\sqrt{b\,x^{3}+a}}{945b^{4}}$	91
elliptic	$\frac{2x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{2a\,x^{9}\sqrt{b\,x^{3}+a}}{189b}-\frac{4a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{3}}-\frac{32a^{4}\sqrt{b\,x^{3}+a}}{945b^{4}}$	91

input
```
int(x^11*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/945*(b*x^3+a)^(3/2)*(-35*b^3*x^9+30*a*b^2*x^6-24*a^2*b*x^3+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\sqrt{a+bx^3}\,dx=\frac{2\left(35\,b^4x^{12}+5\,ab^3x^9-6\,a^2b^2x^6+8\,a^3bx^3-16\,a^4\right)\sqrt{bx^3+a}}{945\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/945*(35*b^4*x^12 + 5*a*b^3*x^9 - 6*a^2*b^2*x^6 + 8*a^3*b*x^3 - 16*a^4)*s
qrt(b*x^3 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int x^{11}\sqrt{a+bx^3}\,dx$$
$$=\begin{cases}-\frac{32a^4\sqrt{a+bx^3}}{945b^4}+\frac{16a^3x^3\sqrt{a+bx^3}}{945b^3}-\frac{4a^2x^6\sqrt{a+bx^3}}{315b^2}+\frac{2ax^9\sqrt{a+bx^3}}{189b}+\frac{2x^{12}\sqrt{a+bx^3}}{27} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-32*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*a**3*x**3*sqrt(a + b*
x**3)/(945*b**3) - 4*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*a*x**9*sqrt
(a + b*x**3)/(189*b) + 2*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (sqrt(a)*x*
*12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{11}\sqrt{a+bx^3}\,dx = \frac{2\,(bx^3+a)^{\frac{9}{2}}}{27\,b^4} - \frac{2\,(bx^3+a)^{\frac{7}{2}}a}{7\,b^4} + \frac{2\,(bx^3+a)^{\frac{5}{2}}a^2}{5\,b^4} - \frac{2\,(bx^3+a)^{\frac{3}{2}}a^3}{9\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/27*(b*x^3 + a)^(9/2)/b^4 - 2/7*(b*x^3 + a)^(7/2)*a/b^4 + 2/5*(b*x^3 + a)
^(5/2)*a^2/b^4 - 2/9*(b*x^3 + a)^(3/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\sqrt{a+bx^3}\,dx = \frac{2\left(35\,(bx^3+a)^{\frac{9}{2}} - 135\,(bx^3+a)^{\frac{7}{2}}a + 189\,(bx^3+a)^{\frac{5}{2}}a^2 - 105\,(bx^3+a)^{\frac{3}{2}}a^3\right)}{945\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2) - 135*(b*x^3 + a)^(7/2)*a + 189*(b*x^3 + a)^(5
/2)*a^2 - 105*(b*x^3 + a)^(3/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int x^{11}\sqrt{a+bx^3}\,dx = \frac{2\,x^{12}\,\sqrt{b\,x^3+a}}{27} - \frac{32\,a^4\,\sqrt{b\,x^3+a}}{945\,b^4} + \frac{2\,a\,x^9\,\sqrt{b\,x^3+a}}{189\,b} + \frac{16\,a^3\,x^3\,\sqrt{b\,x^3+a}}{945\,b^3} - \frac{4\,a^2\,x^6\,\sqrt{b\,x^3+a}}{315\,b^2}$$

input
```
int(x^11*(a + b*x^3)^(1/2),x)
```

output
```
(2*x^12*(a + b*x^3)^(1/2))/27 - (32*a^4*(a + b*x^3)^(1/2))/(945*b^4) + (2*
a*x^9*(a + b*x^3)^(1/2))/(189*b) + (16*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^3
) - (4*a^2*x^6*(a + b*x^3)^(1/2))/(315*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int x^{11}\sqrt{a+bx^3}\,dx = \frac{2\sqrt{b\,x^3+a}\,(35b^4x^{12}+5a\,b^3x^9-6a^2b^2x^6+8a^3b\,x^3-16a^4)}{945b^4}$$

input
```
int(x^11*(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 16*a**4 + 8*a**3*b*x**3 - 6*a**2*b**2*x**6 + 5*a*b
**3*x**9 + 35*b**4*x**12))/(945*b**4)
```

3.163 $\int x^8\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^8\sqrt{a+bx^3}\,dx=\frac{2a^2(a+bx^3)^{3/2}}{9b^3}-\frac{4a(a+bx^3)^{5/2}}{15b^3}+\frac{2(a+bx^3)^{7/2}}{21b^3}$$

output
```
2/9*a^2*(b*x^3+a)^(3/2)/b^3-4/15*a*(b*x^3+a)^(5/2)/b^3+2/21*(b*x^3+a)^(7/2
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x^8\sqrt{a+bx^3}\,dx=\frac{2\sqrt{a+bx^3}(8a^3-4a^2bx^3+3ab^2x^6+15b^3x^9)}{315b^3}$$

input
```
Integrate[x^8*Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(8*a^3 - 4*a^2*b*x^3 + 3*a*b^2*x^6 + 15*b^3*x^9))/(315*
b^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^6\sqrt{bx^3+a}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{5/2}}{b^2}-\frac{2a\left(bx^3+a\right)^{3/2}}{b^2}+\frac{a^2\sqrt{bx^3+a}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^2\left(a+bx^3\right)^{3/2}}{3b^3}+\frac{2\left(a+bx^3\right)^{7/2}}{7b^3}-\frac{4a\left(a+bx^3\right)^{5/2}}{5b^3}\right)$$

input
```
Int[x^8*Sqrt[a + b*x^3],x]
```

output
```
((2*a^2*(a + b*x^3)^(3/2))/(3*b^3) - (4*a*(a + b*x^3)^(5/2))/(5*b^3) + (2*
(a + b*x^3)^(7/2))/(7*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(15b^{2}x^{6}-12ab\,x^{3}+8a^{2}\right)}{315b^{3}}$	36
pseudoelliptic	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(15b^{2}x^{6}-12ab\,x^{3}+8a^{2}\right)}{315b^{3}}$	36
orering	$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(15b^{2}x^{6}-12ab\,x^{3}+8a^{2}\right)}{315b^{3}}$	36
trager	$\frac{2\left(15b^{3}x^{9}+3a\,b^{2}x^{6}-4a^{2}b\,x^{3}+8a^{3}\right)\sqrt{b\,x^{3}+a}}{315b^{3}}$	47
risch	$\frac{2\left(15b^{3}x^{9}+3a\,b^{2}x^{6}-4a^{2}b\,x^{3}+8a^{3}\right)\sqrt{b\,x^{3}+a}}{315b^{3}}$	47
default	$\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2a\,x^{6}\sqrt{b\,x^{3}+a}}{105b}-\frac{8a^{2}x^{3}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}\sqrt{b\,x^{3}+a}}{315b^{3}}$	71
elliptic	$\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21}+\frac{2a\,x^{6}\sqrt{b\,x^{3}+a}}{105b}-\frac{8a^{2}x^{3}\sqrt{b\,x^{3}+a}}{315b^{2}}+\frac{16a^{3}\sqrt{b\,x^{3}+a}}{315b^{3}}$	71

input
```
int(x^8*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(b*x^3+a)^(3/2)*(15*b^2*x^6-12*a*b*x^3+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^8\sqrt{a+bx^3}\,dx = \frac{2\,(15\,b^3x^9+3\,ab^2x^6-4\,a^2bx^3+8\,a^3)\sqrt{bx^3+a}}{315\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(15*b^3*x^9 + 3*a*b^2*x^6 - 4*a^2*b*x^3 + 8*a^3)*sqrt(b*x^3 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int x^8\sqrt{a+bx^3}\,dx = \begin{cases} \frac{16a^3\sqrt{a+bx^3}}{315b^3} - \frac{8a^2x^3\sqrt{a+bx^3}}{315b^2} + \frac{2ax^6\sqrt{a+bx^3}}{105b} + \frac{2x^9\sqrt{a+bx^3}}{21} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^9}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((16*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*a**2*x**3*sqrt(a + b*x*
*3)/(315*b**2) + 2*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*x**9*sqrt(a + b*x**
3)/21, Ne(b, 0)), (sqrt(a)*x**9/9, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^8\sqrt{a+bx^3}\,dx = \frac{2\,(bx^3+a)^{\frac{7}{2}}}{21\,b^3} - \frac{4\,(bx^3+a)^{\frac{5}{2}}a}{15\,b^3} + \frac{2\,(bx^3+a)^{\frac{3}{2}}a^2}{9\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(b*x^3 + a)^(7/2)/b^3 - 4/15*(b*x^3 + a)^(5/2)*a/b^3 + 2/9*(b*x^3 + a
)^(3/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^8\sqrt{a+bx^3}\,dx = \frac{2\left(15\,(bx^3+a)^{\frac{7}{2}}-42\,(bx^3+a)^{\frac{5}{2}}a+35\,(bx^3+a)^{\frac{3}{2}}a^2\right)}{315\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(15*(b*x^3 + a)^(7/2) - 42*(b*x^3 + a)^(5/2)*a + 35*(b*x^3 + a)^(3/2
)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int x^8\sqrt{a+bx^3}\,dx = \frac{2\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{16\,a^3\,\sqrt{b\,x^3+a}}{315\,b^3} + \frac{2\,a\,x^6\,\sqrt{b\,x^3+a}}{105\,b}-\frac{8\,a^2\,x^3\,\sqrt{b\,x^3+a}}{315\,b^2}$$

input
```
int(x^8*(a + b*x^3)^(1/2),x)
```

output
```
(2*x^9*(a + b*x^3)^(1/2))/21 + (16*a^3*(a + b*x^3)^(1/2))/(315*b^3) + (2*a
*x^6*(a + b*x^3)^(1/2))/(105*b) - (8*a^2*x^3*(a + b*x^3)^(1/2))/(315*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int x^8\sqrt{a+bx^3}\,dx=\frac{2\sqrt{b\,x^3+a}\,(15b^3x^9+3a\,b^2x^6-4a^2b\,x^3+8a^3)}{315b^3}$$

input
```
int(x^8*(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**3 - 4*a**2*b*x**3 + 3*a*b**2*x**6 + 15*b**3*x**9
))/(315*b**3)
```

3.164 $\int x^5\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^5\sqrt{a+bx^3}\,dx=-\frac{2a(a+bx^3)^{3/2}}{9b^2}+\frac{2(a+bx^3)^{5/2}}{15b^2}$$

output
```
-2/9*a*(b*x^3+a)^(3/2)/b^2+2/15*(b*x^3+a)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^5\sqrt{a+bx^3}\,dx=\frac{2\sqrt{a+bx^3}(-2a^2+abx^3+3b^2x^6)}{45b^2}$$

input
```
Integrate[x^5*Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(-2*a^2 + a*b*x^3 + 3*b^2*x^6))/(45*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3} \int x^3 \sqrt{bx^3+a}dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{3/2}}{b}-\frac{a\sqrt{bx^3+a}}{b}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{5/2}}{5b^2}-\frac{2a\left(a+bx^3\right)^{3/2}}{3b^2}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^3],x]
```

output
```
((-2*a*(a + b*x^3)^(3/2))/(3*b^2) + (2*(a + b*x^3)^(5/2))/(5*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-3b\,x^{3}+2a\right)}{45b^{2}}$	25
pseudoelliptic	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-3b\,x^{3}+2a\right)}{45b^{2}}$	25
orering	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-3b\,x^{3}+2a\right)}{45b^{2}}$	25
trager	$-\frac{2\left(-3b^{2}x^{6}-ab\,x^{3}+2a^{2}\right)\sqrt{b\,x^{3}+a}}{45b^{2}}$	36
risch	$-\frac{2\left(-3b^{2}x^{6}-ab\,x^{3}+2a^{2}\right)\sqrt{b\,x^{3}+a}}{45b^{2}}$	36
default	$\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{45b}-\frac{4a^{2}\sqrt{b\,x^{3}+a}}{45b^{2}}$	51
elliptic	$\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15}+\frac{2a\,x^{3}\sqrt{b\,x^{3}+a}}{45b}-\frac{4a^{2}\sqrt{b\,x^{3}+a}}{45b^{2}}$	51

input
```
int(x^5*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*(b*x^3+a)^(3/2)*(-3*b*x^3+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^5\sqrt{a+bx^3}\,dx = \frac{2\,(3\,b^2x^6+abx^3-2\,a^2)\sqrt{bx^3+a}}{45\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*b^2*x^6 + a*b*x^3 - 2*a^2)*sqrt(b*x^3 + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int x^5\sqrt{a+bx^3}\,dx = \begin{cases} -\frac{4a^2\sqrt{a+bx^3}}{45b^2}+\frac{2ax^3\sqrt{a+bx^3}}{45b}+\frac{2x^6\sqrt{a+bx^3}}{15} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-4*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*a*x**3*sqrt(a + b*x**3)/
(45*b) + 2*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^5\sqrt{a+bx^3}\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}}{15\,b^2} - \frac{2\,(bx^3+a)^{\frac{3}{2}}a}{9\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)/b^2 - 2/9*(b*x^3 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^5\sqrt{a+bx^3}\,dx=\frac{2\left(3\,(bx^3+a)^{\frac{5}{2}}-5\,(bx^3+a)^{\frac{3}{2}}a\right)}{45\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/45*(3*(b*x^3 + a)^(5/2) - 5*(b*x^3 + a)^(3/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^5\sqrt{a+bx^3}\,dx=-\frac{10\,a\,(b\,x^3+a)^{3/2}-6\,(b\,x^3+a)^{5/2}}{45\,b^2}$$

input
```
int(x^5*(a + b*x^3)^(1/2),x)
```

output
```
-(10*a*(a + b*x^3)^(3/2) - 6*(a + b*x^3)^(5/2))/(45*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^5\sqrt{a+bx^3}\,dx=\frac{2\sqrt{b\,x^3+a}\,(3b^2x^6+ab\,x^3-2a^2)}{45b^2}$$

input
```
int(x^5*(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a**2 + a*b*x**3 + 3*b**2*x**6))/(45*b**2)
```

3.165 $\int x^2\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^2\sqrt{a+bx^3}\,dx=\frac{2(a+bx^3)^{3/2}}{9b}$$

output
```
2/9*(b*x^3+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{a+bx^3}\,dx=\frac{2(a+bx^3)^{3/2}}{9b}$$

input
```
Integrate[x^2*Sqrt[a + b*x^3],x]
```

output
```
(2*(a + b*x^3)^(3/2))/(9*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{2(a+bx^3)^{3/2}}{9b}$$

input
```
Int[x^2*Sqrt[a + b*x^3],x]
```

output
```
(2*(a + b*x^3)^(3/2))/(9*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15
derivativedivides	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15
default	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15
trager	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15
risch	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15
pseudoelliptic	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15
orering	$\frac{2(b\,x^3+a)^{\frac{3}{2}}}{9b}$	15

input
```
int(x^2*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b*x^3+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{a+bx^3}\,dx=\frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2\sqrt{a+bx^3}\,dx = \begin{cases} \frac{2a\sqrt{a+bx^3}}{9b} + \frac{2x^3\sqrt{a+bx^3}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*a*sqrt(a + b*x**3)/(9*b) + 2*x**3*sqrt(a + b*x**3)/9, Ne(b, 0
)), (sqrt(a)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{a+bx^3}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{a+bx^3}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{a+bx^3}\,dx=\frac{2\left(b\,x^3+a\right)^{3/2}}{9\,b}$$

input
```
int(x^2*(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(3/2))/(9*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2\sqrt{a+bx^3}\,dx=\frac{2\sqrt{b\,x^3+a}\,\left(b\,x^3+a\right)}{9b}$$

input
```
int(x^2*(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(a + b*x**3))/(9*b)
```

3.166 $\int \frac{\sqrt{a+bx^3}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \frac{2}{3}\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \frac{2}{3}\sqrt{a+bx^3} - \frac{2}{3}\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*x^3]/x,x]
```

output
```
(2*Sqrt[a + b*x^3])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}}{x^3}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*x^3]/x,x]
```

output
```
(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{2\sqrt{b\,x^{3}+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}$	32
elliptic	$\frac{2\sqrt{b\,x^{3}+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}$	32
pseudoelliptic	$\frac{2\sqrt{b\,x^{3}+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}$	32

input
```
int((b*x^3+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \left[\frac{1}{3}\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right) + \frac{2}{3}\sqrt{bx^3+a}, \frac{2}{3}\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+\frac{2}{3}\sqrt{bx^3+a}\right]$$

input
```
integrate((b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/3*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2/3*sqrt
(b*x^3 + a), 2/3*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + 2/3*sqrt(b*x^
3 + a)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.85 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = -\frac{2\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}+\frac{2a}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((b*x**3+a)**(1/2)/x,x)
```

output
```
-2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2*a/(3*sqrt(b)*x**(3/2)*s
qrt(a/(b*x**3) + 1)) + 2*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+\frac{2}{3}\sqrt{bx^3+a}$$

input
```
integrate((b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/3*sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
 2/3*sqrt(b*x^3 + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \frac{2\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2}{3}\sqrt{bx^3+a}$$

input
```
integrate((b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

output
```
2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \frac{\sqrt{a}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}+\frac{2\sqrt{b\,x^3+a}}{3}$$

input
```
int((a + b*x^3)^(1/2)/x,x)
```

output
```
(a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)
))/x^6))/3 + (2*(a + b*x^3)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+bx^3}}{x}\,dx = \frac{2\sqrt{b\,x^3+a}}{3} + \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{3} - \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{3}$$

input
```
int((b*x^3+a)^(1/2)/x,x)
```

output
```
(2*sqrt(a + b*x**3) + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a)) - sqrt(a)*lo
g(sqrt(a + b*x**3) + sqrt(a)))/3
```

3.167 $\int \frac{\sqrt{a+bx^3}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\sqrt{a+bx^3}}{x^4} dx = -\frac{\sqrt{a+bx^3}}{3x^3} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^3}}{x^4} dx = -\frac{\sqrt{a+bx^3}}{3x^3} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^4,x]
```

output
```
-1/3*Sqrt[a + b*x^3]/x^3 - (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]
)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}}{x^6}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{1}{2}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3-\frac{\sqrt{a+bx^3}}{x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}-\frac{\sqrt{a+bx^3}}{x^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^3}}{x^3}\right)$$

input
```
Int[Sqrt[a + b*x^3]/x^4,x]
```

output
```
(-(Sqrt[a + b*x^3]/x^3) - (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a])/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	36
risch	$-\frac{\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	36
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{3x^{3}}-\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	36
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{b\,x^{3}+a}}{x^{3}b}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{3}$	41

input
```
int((b*x^3+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a+bx^3}}{x^4}\,dx = \left[\frac{\sqrt{a}bx^3\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)-2\sqrt{bx^3+a}a}{6\,ax^3}, \frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)-\sqrt{bx^3+a}a}{3\,ax^3}\right]$$

input
```
integrate((b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*b*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2
*sqrt(b*x^3 + a)*a)/(a*x^3), 1/3*(sqrt(-a)*b*x^3*arctan(sqrt(-a)/sqrt(b*x^
3 + a)) - sqrt(b*x^3 + a)*a)/(a*x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{a+bx^3}}{x^4}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate((b*x**3+a)**(1/2)/x**4,x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - b*asinh(sqrt(a)/(sqrt(b)*x**(
3/2)))/(3*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+bx^3}}{x^4}\,dx = \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{6\sqrt{a}} - \frac{\sqrt{bx^3+a}}{3\,x^3}$$

input
```
integrate((b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/6*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 - 1/3*sqrt(b*x^3 + a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx^3}}{x^4}\,dx = \frac{1}{3}\,b\left(\frac{\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^3+a}}{bx^3}\right)$$

input
```
integrate((b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
1/3*b*(arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^3 + a)/(b*x^3)
)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+bx^3}}{x^4}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{6\sqrt{a}} - \frac{\sqrt{b\,x^3+a}}{3\,x^3}$$

input
```
int((a + b*x^3)^(1/2)/x^4,x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6
))/(6*a^(1/2)) - (a + b*x^3)^(1/2)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx^3}}{x^4}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a+\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3-\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b\,x^3}{6a\,x^3}$$

input
```
int((b*x^3+a)^(1/2)/x^4,x)
```

output
```
( - 2*sqrt(a + b*x**3)*a + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3
- sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(6*a*x**3)
```

3.168 $\int \frac{\sqrt{a+bx^3}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\sqrt{a+bx^3}}{x^7} dx = -\frac{\sqrt{a+bx^3}}{6x^6} - \frac{b\sqrt{a+bx^3}}{12ax^3} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

output
```
-1/6*(b*x^3+a)^(1/2)/x^6-1/12*b*(b*x^3+a)^(1/2)/a/x^3+1/12*b^2*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+bx^3}}{x^7} dx = \frac{(-2a-bx^3)\sqrt{a+bx^3}}{12ax^6} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^7,x]
```

output
```
((-2*a - b*x^3)*Sqrt[a + b*x^3])/(12*a*x^6) + (b^2*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]])/(12*a^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^7}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}}{x^9}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{1}{4}b\int \frac{1}{x^6\sqrt{bx^3+a}}dx^3-\frac{\sqrt{a+bx^3}}{2x^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)-\frac{\sqrt{a+bx^3}}{2x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)-\frac{\sqrt{a+bx^3}}{2x^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^3}}{ax^3}\right)-\frac{\sqrt{a+bx^3}}{2x^6}\right)$$

input
```
Int[Sqrt[a + b*x^3]/x^7,x]
```

output
```
(-1/2*Sqrt[a + b*x^3]/x^6 + (b*(-(Sqrt[a + b*x^3]/(a*x^3)) + (b*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/4)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\sqrt{b\,x^3+a}\,(b\,x^3+2a)}{12x^6a}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}$	50
default	$-\frac{\sqrt{b\,x^3+a}}{6x^6}-\frac{b\sqrt{b\,x^3+a}}{12a\,x^3}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}$	56
elliptic	$-\frac{\sqrt{b\,x^3+a}}{6x^6}-\frac{b\sqrt{b\,x^3+a}}{12a\,x^3}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{12a^{\frac{3}{2}}}$	56
pseudoelliptic	$\frac{\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)b^2x^6-\sqrt{b\,x^3+a}\left(b\,x^3\sqrt{a}+2a^{\frac{3}{2}}\right)}{12a^{\frac{3}{2}}x^6}$	56

input
```
int((b*x^3+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(b*x^3+2*a)/x^6/a+1/12*b^2*arctanh((b*x^3+a)^(1/2)/a
^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+bx^3}}{x^7}\,dx=\left[\frac{\sqrt{a}b^2x^6\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\left(abx^3+2a^2\right)\sqrt{bx^3+a}}{24a^2x^6},\ -\frac{\sqrt{-a}b^2x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+\left(abx^3+2a^2\right)\sqrt{bx^3+a}}{12a^2x^6}\right]$$

input
```
integrate((b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[1/24*(sqrt(a)*b^2*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3)
- 2*(a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/(a^2*x^6), -1/12*(sqrt(-a)*b^2*x^6*
arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/(a^2
*x^6)]
```

Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^3}}{x^7}\,dx = -\frac{a}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}$$

input
```
integrate((b*x**3+a)**(1/2)/x**7,x)
```

output
```
-a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(4*x**(9/2)*sqrt(a
/(b*x**3) + 1)) - b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + b**2*asi
nh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^3}}{x^7}\,dx = -\frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{24\,a^{\frac{3}{2}}} - \frac{(bx^3+a)^{\frac{3}{2}}b^2+\sqrt{bx^3+a}\,ab^2}{12\left((bx^3+a)^2a-2\,(bx^3+a)a^2+a^3\right)}$$

input
```
integrate((b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/24*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) - 1/12*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2
*a - 2*(b*x^3 + a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^3}}{x^7}\,dx = -\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{(bx^3+a)^{\frac{3}{2}}b^3+\sqrt{bx^3+a}ab^3}{ab^2x^6}}{12\,b}$$

input
```
integrate((b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/12*(b^3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^3 + a)^(3
/2)*b^3 + sqrt(b*x^3 + a)*a*b^3)/(a*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^3}}{x^7}\,dx = \frac{b^2\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{24\,a^{3/2}} - \frac{\sqrt{b\,x^3+a}}{6\,x^6} - \frac{b\,\sqrt{b\,x^3+a}}{12\,a\,x^3}$$

input
```
int((a + b*x^3)^(1/2)/x^7,x)
```

output
```
(b^2*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x
^6))/(24*a^(3/2)) - (a + b*x^3)^(1/2)/(6*x^6) - (b*(a + b*x^3)^(1/2))/(12*
a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{a+bx^3}}{x^7}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^2-2\sqrt{b\,x^3+a}\,ab\,x^3-\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^2x^6+\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b^2x^6}{24a^2x^6}$$

input
```
int((b*x^3+a)^(1/2)/x^7,x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 - 2*sqrt(a + b*x**3)*a*b*x**3 - sqrt(a)*log(sq
rt(a + b*x**3) - sqrt(a))*b**2*x**6 + sqrt(a)*log(sqrt(a + b*x**3) + sqrt(
a))*b**2*x**6)/(24*a**2*x**6)
```

3.169 $\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = -\frac{\sqrt{a+bx^3}}{9x^9} - \frac{b\sqrt{a+bx^3}}{36ax^6} + \frac{b^2\sqrt{a+bx^3}}{24a^2x^3} - \frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{5/2}}$$

output
```
-1/9*(b*x^3+a)^(1/2)/x^9-1/36*b*(b*x^3+a)^(1/2)/a/x^6+1/24*b^2*(b*x^3+a)^(
1/2)/a^2/x^3-1/24*b^3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = \frac{\sqrt{a+bx^3}(-8a^2-2abx^3+3b^2x^6)}{72a^2x^9} - \frac{b^3\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{24a^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^10,x]
```

output
```
(Sqrt[a + b*x^3]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6))/(72*a^2*x^9) - (b^3*Arc
Tanh[Sqrt[a + b*x^3]/Sqrt[a]])/(24*a^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sqrt{bx^3+a}}{x^{12}}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{1}{6}b\int \frac{1}{x^9\sqrt{bx^3+a}}dx^3-\frac{\sqrt{a+bx^3}}{3x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^6\sqrt{bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)-\frac{\sqrt{a+bx^3}}{3x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)-\frac{\sqrt{a+bx^3}}{3x^9}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)-\frac{\sqrt{a+bx^3}}{3x^9}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)-\frac{\sqrt{a+bx^3}}{3x^9}\right)$$

input
```
Int[Sqrt[a + b*x^3]/x^10,x]
```

output
```
(-1/3*Sqrt[a + b*x^3]/x^9 + (b*(-1/2*Sqrt[a + b*x^3]/(a*x^6) - (3*b*(-(Sqr
t[a + b*x^3]/(a*x^3)) + (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(4*
a)))/6)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-3 b^{2} x^{6}+2 a b\,x^{3}+8 a^{2}\right)}{72 x^{9} a^{2}}-\frac{b^{3} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{24 a^{\frac{5}{2}}}$	62
default	$-\frac{\sqrt{b\,x^{3}+a}}{9 x^{9}}-\frac{b\sqrt{b\,x^{3}+a}}{36 a\,x^{6}}+\frac{b^{2} \sqrt{b\,x^{3}+a}}{24 a^{2} x^{3}}-\frac{b^{3} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{24 a^{\frac{5}{2}}}$	76
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{9 x^{9}}-\frac{b\sqrt{b\,x^{3}+a}}{36 a\,x^{6}}+\frac{b^{2} \sqrt{b\,x^{3}+a}}{24 a^{2} x^{3}}-\frac{b^{3} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{24 a^{\frac{5}{2}}}$	76
pseudoelliptic	$\frac{-3 \,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right) b^{3} x^{9}+3 b^{2} x^{6} \sqrt{b\,x^{3}+a}\,\sqrt{a}-2 a^{\frac{3}{2}} b\,x^{3} \sqrt{b\,x^{3}+a}-8 a^{\frac{5}{2}} \sqrt{b\,x^{3}+a}}{72 a^{\frac{5}{2}} x^{9}}$	84

input
```
int((b*x^3+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/72*(b*x^3+a)^(1/2)*(-3*b^2*x^6+2*a*b*x^3+8*a^2)/x^9/a^2-1/24*b^3*arctan
h((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = \left[\frac{3\,\sqrt{a}b^3x^9\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,ab^2x^6-2\,a^2bx^3-8\,a^3)\sqrt{bx^3+a}}{144\,a^3x^9}, \frac{3\,\sqrt{-a}b^3x^9\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)}{}\right.$$

input
```
integrate((b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
[1/144*(3*sqrt(a)*b^3*x^9*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^
3) + 2*(3*a*b^2*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(b*x^3 + a))/(a^3*x^9), 1/7
2*(3*sqrt(-a)*b^3*x^9*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (3*a*b^2*x^6 - 2*
a^2*b*x^3 - 8*a^3)*sqrt(b*x^3 + a))/(a^3*x^9)]
```

Sympy [A] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = -\frac{a}{9\sqrt{b}x^{\frac{21}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{5\sqrt{b}}{36x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^{\frac{3}{2}}}{72ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^{\frac{5}{2}}}{24a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{24a^{\frac{5}{2}}}$$

input
```
integrate((b*x**3+a)**(1/2)/x**10,x)
```

output
```
-a/(9*sqrt(b)*x**(21/2)*sqrt(a/(b*x**3) + 1)) - 5*sqrt(b)/(36*x**(15/2)*sq
rt(a/(b*x**3) + 1)) + b**(3/2)/(72*a*x**(9/2)*sqrt(a/(b*x**3) + 1)) + b**(
5/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(24*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = \frac{b^3\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{48\,a^{\frac{5}{2}}} + \frac{3\,(bx^3+a)^{\frac{5}{2}}b^3 - 8\,(bx^3+a)^{\frac{3}{2}}ab^3 - 3\,\sqrt{bx^3+a}a^2b^3}{72\,\left((bx^3+a)^3a^2 - 3\,(bx^3+a)^2a^3 + 3\,(bx^3+a)a^4 - a^5\right)}$$

input
```
integrate((b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
1/48*b^3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5
/2) + 1/72*(3*(b*x^3 + a)^(5/2)*b^3 - 8*(b*x^3 + a)^(3/2)*a*b^3 - 3*sqrt(b
*x^3 + a)*a^2*b^3)/((b*x^3 + a)^3*a^2 - 3*(b*x^3 + a)^2*a^3 + 3*(b*x^3 + a
)*a^4 - a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = \frac{1}{72}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx^3+a)^{\frac{5}{2}} - 8\,(bx^3+a)^{\frac{3}{2}}a - 3\,\sqrt{bx^3+a}a^2}{a^2b^3x^9}\right)$$

input
```
integrate((b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

output
```
1/72*b^3*(3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^3 +
a)^(5/2) - 8*(b*x^3 + a)^(3/2)*a - 3*sqrt(b*x^3 + a)*a^2)/(a^2*b^3*x^9))
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx = \frac{b^3\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{48\,a^{5/2}} - \frac{\sqrt{b\,x^3+a}}{9\,x^9} - \frac{b\,\sqrt{b\,x^3+a}}{36\,a\,x^6} + \frac{b^2\,\sqrt{b\,x^3+a}}{24\,a^2\,x^3}$$

input
```
int((a + b*x^3)^(1/2)/x^10,x)
```

output
```
(b^3*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x
^6))/(48*a^(5/2)) - (a + b*x^3)^(1/2)/(9*x^9) - (b*(a + b*x^3)^(1/2))/(36*
a*x^6) + (b^2*(a + b*x^3)^(1/2))/(24*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^3}}{x^{10}}\,dx$$
$$= \frac{-16\sqrt{b\,x^3+a}\,a^3-4\sqrt{b\,x^3+a}\,a^2b\,x^3+6\sqrt{b\,x^3+a}\,a\,b^2x^6+3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^3x^9-3\sqrt{a}\log\left(\sqrt{}\right.}{144a^3x^9}$$

input
```
int((b*x^3+a)^(1/2)/x^10,x)
```

output
```
( - 16*sqrt(a + b*x**3)*a**3 - 4*sqrt(a + b*x**3)*a**2*b*x**3 + 6*sqrt(a +
 b*x**3)*a*b**2*x**6 + 3*sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b**3*x**9
 - 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b**3*x**9)/(144*a**3*x**9)
```

3.170 $\int x^6\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 275

$$\int x^6\sqrt{a+bx^3}\,dx = -\frac{48a^2x\sqrt{a+bx^3}}{935b^2}+\frac{6ax^4\sqrt{a+bx^3}}{187b}+\frac{2}{17}x^7\sqrt{a+bx^3} + \frac{32\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\right.}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-48/935*a^2*x*(b*x^3+a)^(1/2)/b^2+6/187*a*x^4*(b*x^3+a)^(1/2)/b+2/17*x^7*(
b*x^3+a)^(1/2)+32/935*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^3*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.34

$$\int x^6\sqrt{a+bx^3}\,dx = \frac{2x\sqrt{a+bx^3}\left(\sqrt{1+\frac{bx^3}{a}}(-8a^2+3abx^3+11b^2x^6)+8a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{187b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^6*Sqrt[a + b*x^3],x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(Sqrt[1 + (b*x^3)/a]*(-8*a^2 + 3*a*b*x^3 + 11*b^2*x^6
) + 8*a^2*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)]))/(187*b^2*Sqrt[
1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {811, 843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{17}a\int \frac{x^6}{\sqrt{bx^3+a}}dx+\frac{2}{17}x^7\sqrt{a+bx^3}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{17}a\left(\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\int \frac{x^3}{\sqrt{bx^3+a}}dx}{11b}\right)+\frac{2}{17}x^7\sqrt{a+bx^3}$$

$$\Big\downarrow \; 843$$

$$\frac{3}{17}a\left(\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{11b}\right)+\frac{2}{17}x^7\sqrt{a+bx^3}$$

$$\Big\downarrow \; 759$$

$$\frac{3}{17}a\left(\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{11b}\right.$$

$$\frac{2}{17}x^7\sqrt{a+bx^3}$$

input
```
Int[x^6*Sqrt[a + b*x^3],x]
```

output
```
(2*x^7*Sqrt[a + b*x^3])/17 + (3*a*((2*x^4*Sqrt[a + b*x^3])/(11*b) - (8*a*(
(2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])))/(11*b)))/17
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{2x(-55b^2x^6-15abx^3+24a^2)\sqrt{bx^3+a}}{935b^2}-\frac{32ia^3\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab}{2b}}}}{}$
default	$\frac{2x^7\sqrt{bx^3+a}}{17}+\frac{6a\,x^4\sqrt{bx^3+a}}{187b}-\frac{48a^2x\sqrt{bx^3+a}}{935b^2}-\frac{32ia^3\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-(-}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}}}}{}$
elliptic	$\frac{2x^7\sqrt{bx^3+a}}{17}+\frac{6a\,x^4\sqrt{bx^3+a}}{187b}-\frac{48a^2x\sqrt{bx^3+a}}{935b^2}-\frac{32ia^3\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-(-}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}}}}{}$

input

```
int(x^6*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/935*x*(-55*b^2*x^6-15*a*b*x^3+24*a^2)*(b*x^3+a)^(1/2)/b^2-32/935*I*a^3/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.21

$$\int x^6\sqrt{a+bx^3}\,dx = \frac{2\left(48\,a^3\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(55\,b^3x^7+15\,ab^2x^4-24\,a^2bx\right)\sqrt{bx^3+a}\right)}{935\,b^3}$$

input
```
integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/935*(48*a^3*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (55*b^3*x^7 + 15
*a*b^2*x^4 - 24*a^2*b*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.14

$$\int x^6\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int x^6\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}x^6\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*x^6, x)
```

Giac [F]

$$\int x^6\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}x^6\,dx$$

input
```
integrate(x^6*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6\sqrt{a+bx^3}\,dx=\int x^6\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^6*(a + b*x^3)^(1/2),x)
```

output
```
int(x^6*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^6\sqrt{a+bx^3}\,dx = \frac{-\frac{48\sqrt{b\,x^3+a}\,a^2x}{935}+\frac{6\sqrt{b\,x^3+a}\,ab\,x^4}{187}+\frac{2\sqrt{b\,x^3+a}\,b^2x^7}{17}+\frac{48\left(\int\frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^3}{935}}{b^2}$$

input
```
int(x^6*(b*x^3+a)^(1/2),x)
```

output
```
(2*( - 24*sqrt(a + b*x**3)*a**2*x + 15*sqrt(a + b*x**3)*a*b*x**4 + 55*sqrt
(a + b*x**3)*b**2*x**7 + 24*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**3))/(9
35*b**2)
```

3.171 $\int x^3\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 251

$$\int x^3\sqrt{a+bx^3}\,dx=\frac{6ax\sqrt{a+bx^3}}{55b}+\frac{2}{11}x^4\sqrt{a+bx^3}-\frac{4\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
6/55*a*x*(b*x^3+a)^(1/2)/b+2/11*x^4*(b*x^3+a)^(1/2)-4/55*3^(3/4)*(1/2*6^(1
/2)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(4/3)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.94 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.25

$$\int x^3\sqrt{a+bx^3}\,dx=\frac{2x\sqrt{a+bx^3}\left(a+bx^3-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{11b}$$

input
```
Integrate[x^3*Sqrt[a + b*x^3],x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(a + b*x^3 - (a*Hypergeometric2F1[-1/2, 1/3, 4/3, -((
b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{3}{11}a\int \frac{x^3}{\sqrt{bx^3+a}}dx+\frac{2}{11}x^4\sqrt{a+bx^3}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)+\frac{2}{11}x^4\sqrt{a+bx^3}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{2}{11}x^4\sqrt{a+bx^3}$$

input
```
Int[x^3*Sqrt[a + b*x^3],x]
```

output
```
(2*x^4*Sqrt[a + b*x^3])/11 + (3*a*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2
 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - S
qrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/11
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.24

method	result
risch	$\frac{2x(5bx^3+3a)\sqrt{bx^3+a}}{55b}+\frac{4ia^2\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a}{}\right.}{}}}{}$
default	$\frac{2x^4\sqrt{bx^3+a}}{11}+\frac{6ax\sqrt{bx^3+a}}{55b}+\frac{4ia^2\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i}{}}}{}$
elliptic	$\frac{2x^4\sqrt{bx^3+a}}{11}+\frac{6ax\sqrt{bx^3+a}}{55b}+\frac{4ia^2\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-ab^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-ab^2)^{\frac{1}{3}}}{b}}{-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i}{}}}{}$

input
```
int(x^3*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*b*x^3+3*a)/b*(b*x^3+a)^(1/2)+4/55*I/b^2*a^2*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.19

$$\int x^3\sqrt{a+bx^3}\,dx = -\frac{2\left(6\,a^2\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-\left(5\,b^2x^4+3\,abx\right)\sqrt{bx^3+a}\right)}{55\,b^2}$$

input
```
integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(6*a^2*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (5*b^2*x^4 + 3*a*
b*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int x^3\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(7/3))
```

Maxima [F]

$$\int x^3\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+bx^3}\,dx=\int x^3\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^3*(a + b*x^3)^(1/2),x)
```

output
```
int(x^3*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^3\sqrt{a+bx^3}\,dx = \frac{\frac{6\sqrt{b\,x^3+a}\,ax}{55}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{11}-\frac{6\left(\int\frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2}{55}}{b}$$

input
```
int(x^3*(b*x^3+a)^(1/2),x)
```

output
```
(2*(3*sqrt(a + b*x**3)*a*x + 5*sqrt(a + b*x**3)*b*x**4 - 3*int(sqrt(a + b*
x**3)/(a + b*x**3),x)*a**2))/(55*b)
```

3.172 $\int \sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 227

$$\int \sqrt{a+bx^3}\,dx = \frac{2}{5}x\sqrt{a+bx^3} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{\ }\right.}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/5*x*(b*x^3+a)^(1/2)+2/5*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a*(a^(1/3)+b^(
1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.20

$$\int \sqrt{a+bx^3}\,dx=\frac{x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{748}$$

$$\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2}{5}x\sqrt{a+bx^3}$$

input
```
Int[Sqrt[a + b*x^3],x]
```

output
```
(2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.31

method	result
default	$\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\cdots}}\cdots}{5b\sqrt{b\,x^{3}+\cdots}}$
risch	$\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\cdots}}\cdots}{5b\sqrt{b\,x^{3}+\cdots}}$
elliptic	$\frac{2x\sqrt{b\,x^{3}+a}}{5}-\frac{2ia\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(\cdots}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\cdots}}\cdots}{5b\sqrt{b\,x^{3}+\cdots}}$

input
```
int((b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^3+a)^(1/2)-2/5*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((
x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1
/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3
))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.15

$$\int \sqrt{a+bx^3}\,dx = \frac{2\left(\sqrt{bx^3+a}bx+3\,a\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b}$$

input
```
integrate((b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a)*b*x + 3*a*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/
b
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \sqrt{a+bx^3}\,dx = \frac{\sqrt{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3))
```

Maxima [F]

$$\int \sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}\,dx$$

input
```
integrate((b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \sqrt{a+bx^3}\,dx=\frac{x\,\sqrt{b\,x^3+a}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\sqrt{\frac{b\,x^3}{a}+1}}$$

input
```
int((a + b*x^3)^(1/2),x)
```

output
```
(x*(a + b*x^3)^(1/2)*hypergeom([-1/2, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
 1)^(1/2)
```

Reduce [F]

$$\int \sqrt{a+bx^3}\,dx=\frac{2\sqrt{b\,x^3+a}\,x}{5}+\frac{3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a}{5}$$

input
```
int((b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*x + 3*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a)/5
```

3.173 $\int \frac{\sqrt{a+bx^3}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 228

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx = -\frac{\sqrt{a+bx^3}}{2x^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{}\right.}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/2*(b*x^3+a)^(1/2)/x^2+1/2*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx = -\frac{\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^3,x]
```

output
```
-1/2*(Sqrt[a + b*x^3]*Hypergeometric2F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/(x
^2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3}{4}b\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{2x^2}$$

$$\Big\downarrow \text{759}$$

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2x^2}$$

input
```
Int[Sqrt[a + b*x^3]/x^3,x]
```

output
```
-1/2*Sqrt[a + b*x^3]/x^2 + (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)
*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{b\,x^3+a}}{2x^2}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{2\sqrt{b\,x^3+a}}$
risch	$-\frac{\sqrt{b\,x^3+a}}{2x^2}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{2\sqrt{b\,x^3+a}}$
elliptic	$-\frac{\sqrt{b\,x^3+a}}{2x^2}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{2\sqrt{b\,x^3+a}}$

input
```
int((b*x^3+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx = \frac{3\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)-\sqrt{bx^3+a}}{2x^2}$$

input
```
integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/2*(3*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) - sqrt(b*x^3 + a))/x^
2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/x**3,x)
```

output
```
sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx=\int \frac{\sqrt{bx^3+a}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx=\int \frac{\sqrt{b\,x^3+a}}{x^3}\,dx$$

input
```
int((a + b*x^3)^(1/2)/x^3,x)
```

output
```
int((a + b*x^3)^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{x^3}\,dx=\frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx\right)a\,x^2}{x^2}$$

input
```
int((b*x^3+a)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**3 + b*x**6),x)*a*x**
2)/x**2
```

3.174 $\int \frac{\sqrt{a+bx^3}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 253

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = -\frac{\sqrt{a+bx^3}}{5x^5} - \frac{3b\sqrt{a+bx^3}}{20ax^2} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{\ }\right.}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*(b*x^3+a)^(1/2)/x^5-3/20*b*(b*x^3+a)^(1/2)/a/x^2-1/20*3^(3/4)*(1/2*6^
(1/2)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/
2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = -\frac{\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^6,x]
```

output
```
-1/5*(Sqrt[a + b*x^3]*Hypergeometric2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/(
x^5*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {809, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3}{10}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{10}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)-\frac{\sqrt{a+bx^3}}{5x^5}$$

$$\Big\downarrow \text{759}$$

$$\frac{3}{10}b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \right.$$

$$\frac{\sqrt{a+bx^3}}{5x^5}$$

input
```
Int[Sqrt[a + b*x^3]/x^6,x]
```

output
```
-1/5*Sqrt[a + b*x^3]/x^5 + (3*b*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 +
Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
- Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/10
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(3b\,x^{3}+4a)}{20x^{5}a}+\frac{i b\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^{2})^{\cdots}}{2b}\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{3b\sqrt{b\,x^{3}+a}}{20a\,x^{2}}+\frac{i b\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(\cdots}{\cdots}\cdots\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{3b\sqrt{b\,x^{3}+a}}{20a\,x^{2}}+\frac{i b\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(\cdots}{\cdots}\cdots\right.}{\cdots}}\cdots}{\cdots}$

input
```
int((b*x^3+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(3*b*x^3+4*a)/x^5/a+1/20*I/a*b*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a
*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = -\frac{3\,b^{\frac{3}{2}}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(3\,bx^3+4\,a\right)\sqrt{bx^3+a}}{20\,ax^5}$$

input
```
integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/20*(3*b^(3/2)*x^5*weierstrassPInverse(0, -4*a/b, x) + (3*b*x^3 + 4*a)*s
qrt(b*x^3 + a))/(a*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/x**6,x)
```

output
```
sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = \int \frac{\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = \int \frac{\sqrt{bx^3+a}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = \int \frac{\sqrt{b\,x^3+a}}{x^6}\,dx$$

input
```
int((a + b*x^3)^(1/2)/x^6,x)
```

output
```
int((a + b*x^3)^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{x^6}\,dx = \frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}dx\right)a\,x^5}{7x^5}$$

input
```
int((b*x^3+a)^(1/2)/x^6,x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**6 + b*x**9),x)*a*x**
5)/(7*x**5)
```

3.175 $\int \frac{\sqrt{a+bx^3}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 277

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = -\frac{\sqrt{a+bx^3}}{8x^8}-\frac{3b\sqrt{a+bx^3}}{80ax^5}+\frac{21b^2\sqrt{a+bx^3}}{320a^2x^2} + \frac{7\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{8/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\ldots\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/8*(b*x^3+a)^(1/2)/x^8-3/80*b*(b*x^3+a)^(1/2)/a/x^5+21/320*b^2*(b*x^3+a)
^(1/2)/a^2/x^2+7/320*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*b^(8/3)*(a^(1/3)+b^
(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^
(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^2/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = -\frac{\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{8}{3},-\frac{1}{2},-\frac{5}{3},-\frac{bx^3}{a}\right)}{8x^8\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^9,x]
```

output
```
-1/8*(Sqrt[a + b*x^3]*Hypergeometric2F1[-8/3, -1/2, -5/3, -((b*x^3)/a)])/(
x^8*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {809, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3}{16}b\int \frac{1}{x^6\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{8x^8}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{16}b\left(-\frac{7b\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}\right)-\frac{\sqrt{a+bx^3}}{8x^8}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{16}b\left(-\frac{7b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}\right)-\frac{\sqrt{a+bx^3}}{8x^8}$$

$\downarrow$ 759

$$\frac{3}{16}b\left(-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\right.$$

$$\frac{\sqrt{a+bx^3}}{8x^8}$$

input
```
Int[Sqrt[a + b*x^3]/x^9,x]
```

output
```
-1/8*Sqrt[a + b*x^3]/x^8 + (3*b*(-1/5*Sqrt[a + b*x^3]/(a*x^5) - (7*b*(-1/2
*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/
3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
b*x^3])))/(10*a)))/16
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-21b^2x^6+12ab\,x^3+40a^2\right)}{320x^8a^2}-\frac{7ib^2\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}$
default	$-\frac{\sqrt{b\,x^3+a}}{8x^8}-\frac{3b\sqrt{b\,x^3+a}}{80a\,x^5}+\frac{21b^2\sqrt{b\,x^3+a}}{320a^2x^2}-\frac{7ib^2\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{}}}}{}$
elliptic	$-\frac{\sqrt{b\,x^3+a}}{8x^8}-\frac{3b\sqrt{b\,x^3+a}}{80a\,x^5}+\frac{21b^2\sqrt{b\,x^3+a}}{320a^2x^2}-\frac{7ib^2\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-\right.}{}}}}{}$

input

```
int((b*x^3+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output

```
-1/320*(b*x^3+a)^(1/2)*(-21*b^2*x^6+12*a*b*x^3+40*a^2)/x^8/a^2-7/320*I/a^2
*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b
^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = \frac{21\,b^{\frac{5}{2}}x^8\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(21\,b^2x^6-12\,abx^3-40\,a^2\right)\sqrt{bx^3+a}}{320\,a^2x^8}$$

input
```
integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
1/320*(21*b^(5/2)*x^8*weierstrassPInverse(0, -4*a/b, x) + (21*b^2*x^6 - 12
*a*b*x^3 - 40*a^2)*sqrt(b*x^3 + a))/(a^2*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/x**9,x)
```

output
```
sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**8*gamma(-5/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = \int \frac{\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/x^9, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = \int \frac{\sqrt{bx^3+a}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = \int \frac{\sqrt{b\,x^3+a}}{x^9}\,dx$$

input
```
int((a + b*x^3)^(1/2)/x^9,x)
```

output
```
int((a + b*x^3)^(1/2)/x^9, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{x^9}\,dx = \frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^{12}+a\,x^9}dx\right)a\,x^8}{13x^8}$$

input
```
int((b*x^3+a)^(1/2)/x^9,x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**9 + b*x**12),x)*a*x*
*8)/(13*x**8)
```

3.176 $\int x^7\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 535

$$\int x^7\sqrt{a+bx^3}\,dx=-\frac{60a^2x^2\sqrt{a+bx^3}}{1729b^2}+\frac{6ax^5\sqrt{a+bx^3}}{247b}+\frac{2}{19}x^8\sqrt{a+bx^3}+\frac{240a^3\sqrt{a+bx^3}}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{120\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{80\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-60/1729*a^2*x^2*(b*x^3+a)^(1/2)/b^2+6/247*a*x^5*(b*x^3+a)^(1/2)/b+2/19*x^
8*(b*x^3+a)^(1/2)+240/1729*a^3*(b*x^3+a)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)-120/1729*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(10/3)*(a^(1/3)+
b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+80/1729*2^(1/2)*3^(3
/4)*a^(10/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^
(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.18

$$\int x^7\sqrt{a+bx^3}\,dx = \frac{2x^2\sqrt{a+bx^3}\left(\sqrt{1+\frac{bx^3}{a}}(-10a^2+3abx^3+13b^2x^6)+10a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{247b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^7*Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(Sqrt[1 + (b*x^3)/a]*(-10*a^2 + 3*a*b*x^3 + 13*b^2*
x^6) + 10*a^2*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)]))/(247*b^2*S
qrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {811, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{19}a\int \frac{x^7}{\sqrt{bx^3+a}}dx+\frac{2}{19}x^8\sqrt{a+bx^3}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\int \frac{x^4}{\sqrt{bx^3+a}}dx}{13b}\right)+\frac{2}{19}x^8\sqrt{a+bx^3}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)}{13b}\right)+\frac{2}{19}x^8\sqrt{a+bx^3}$$

$$\Big\downarrow \text{832}$$

$$\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}\right)+$$

$$\frac{2}{19}x^8\sqrt{a+bx^3}$$

$\Big\downarrow$ 759

$$\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x \cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^{2}\cdots}}\cdots}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}\cdots}\right)}{7b}\cdots\right)}{13b}\cdots\right)$$

$$\frac{2}{19}x^8\sqrt{a+bx^3}$$

$\Big\downarrow$ 2416

$$\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}}}}{\sqrt[3]{b}}}{}}{}\right.$$

$$\frac{2}{19}x^8\sqrt{a+bx^3}$$

input
```
Int[x^7*Sqrt[a + b*x^3],x]
```

output
```
(2*x^8*Sqrt[a + b*x^3])/19 + (3*a*((2*x^5*Sqrt[a + b*x^3])/(13*b) - (10*a*
((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
 + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1
/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)))/(13*b)))/19
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{2x^2\left(-91b^2x^6-21ab\,x^3+30a^2\right)\sqrt{b\,x^3+a}}{1729b^2}-\frac{80ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\right.}{2b}}}}{}$
default	$\frac{2x^8\sqrt{b\,x^3+a}}{19}+\frac{6a\,x^5\sqrt{b\,x^3+a}}{247b}-\frac{60a^2x^2\sqrt{b\,x^3+a}}{1729b^2}-\frac{80ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}$
elliptic	$\frac{2x^8\sqrt{b\,x^3+a}}{19}+\frac{6a\,x^5\sqrt{b\,x^3+a}}{247b}-\frac{60a^2x^2\sqrt{b\,x^3+a}}{1729b^2}-\frac{80ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}$

input
```
int(x^7*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1729*x^2*(-91*b^2*x^6-21*a*b*x^3+30*a^2)/b^2*(b*x^3+a)^(1/2)-80/1729*I*
a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.13

$$\int x^7\sqrt{a+bx^3}\,dx =$$

$$-\frac{2\left(120\,a^3\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(91\,b^3x^8+21\,ab^2x^5-30\,a^2bx\right.\right.}{1729\,b^3}$$

input
```
integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/1729*(120*a^3*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
 -4*a/b, x)) - (91*b^3*x^8 + 21*a*b^2*x^5 - 30*a^2*b*x^2)*sqrt(b*x^3 + a))
/b^3
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int x^7\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int x^7\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x^7\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*x^7, x)
```

Giac [F]

$$\int x^7\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x^7\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7\sqrt{a+bx^3}\,dx=\int x^7\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^7*(a + b*x^3)^(1/2),x)
```

output
```
int(x^7*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^7\sqrt{a+bx^3}\,dx=\frac{-\frac{60\sqrt{b\,x^3+a}\,a^2x^2}{1729}+\frac{6\sqrt{b\,x^3+a}\,ab\,x^5}{247}+\frac{2\sqrt{b\,x^3+a}\,b^2x^8}{19}+\frac{120\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^3}{1729}}{b^2}$$

input
```
int(x^7*(b*x^3+a)^(1/2),x)
```

output
```
(2*( - 30*sqrt(a + b*x**3)*a**2*x**2 + 21*sqrt(a + b*x**3)*a*b*x**5 + 91*s
qrt(a + b*x**3)*b**2*x**8 + 60*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a*
*3))/(1729*b**2)
```

3.177 $\int x^4\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 511

$$\int x^4\sqrt{a+bx^3}\,dx=\frac{6ax^2\sqrt{a+bx^3}}{91b}+\frac{2}{13}x^5\sqrt{a+bx^3}-\frac{24a^2\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{8\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
6/91*a*x^2*(b*x^3+a)^(1/2)/b+2/13*x^5*(b*x^3+a)^(1/2)-24/91*a^2*(b*x^3+a)^
(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+12/91*3^(1/4)*(1/2*6^(1/2)-1
/2*2^(1/2))*a^(7/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(
1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+
a)^(1/2)-8/91*2^(1/2)*3^(3/4)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2
)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.13

$$\int x^4\sqrt{a+bx^3}\,dx=\frac{2x^2\sqrt{a+bx^3}\left(a+bx^3-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{13b}$$

input
```
Integrate[x^4*Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(a + b*x^3 - (a*Hypergeometric2F1[-1/2, 2/3, 5/3, -
((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(13*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{3}{13}a\int \frac{x^4}{\sqrt{bx^3+a}}dx+\frac{2}{13}x^5\sqrt{a+bx^3}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arc}\ldots\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{7b}\right)$$

$$\frac{2}{13}x^5\sqrt{a+bx^3}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\cdots}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\cdots}\right.}\cdots\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\sqrt[3]{b}}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{2}{13}x^5\sqrt{a+bx^3}$$

input

```
Int[x^4*Sqrt[a + b*x^3],x]
```

output
```
(2*x^5*Sqrt[a + b*x^3])/13 + (3*a*((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a*((
(2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sq
rt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)))
/13
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.91

method	result
risch	$\frac{2x^{2}\left(7b\,x^{3}+3a\right)\sqrt{b\,x^{3}+a}}{91b}+\frac{8ia^{2}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
default	$\frac{2x^{5}\sqrt{b\,x^{3}+a}}{13}+\frac{6a\,x^{2}\sqrt{b\,x^{3}+a}}{91b}+\frac{8ia^{2}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\cdots}{-\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2x^{5}\sqrt{b\,x^{3}+a}}{13}+\frac{6a\,x^{2}\sqrt{b\,x^{3}+a}}{91b}+\frac{8ia^{2}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{\frac{i\cdots}{-\cdots}}\cdots}{\cdots}$

input
```
int(x^4*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*b*x^3+3*a)/b*(b*x^3+a)^(1/2)+8/91*I/b^2*a^2*3^(1/2)*(-a*b^2)^(
1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.11

$$\int x^4\sqrt{a+bx^3}\,dx = \frac{2\left(12\,a^2\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(7\,b^2x^5+3\,abx^2\right)\sqrt{bx^3+a}\right)}{91\,b^2}$$

input
```
integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/91*(12*a^2*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (7*b^2*x^5 + 3*a*b*x^2)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int x^4\sqrt{a+bx^3}\,dx=\frac{\sqrt{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x^4\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^3}\,dx = \int x^4\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^4*(a + b*x^3)^(1/2),x)
```

output
```
int(x^4*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+bx^3}\,dx = \frac{\frac{6\sqrt{b\,x^3+a}\,a\,x^2}{91}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{13}-\frac{12\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2}{91}}{b}$$

input
```
int(x^4*(b*x^3+a)^(1/2),x)
```

output
```
(2*(3*sqrt(a + b*x**3)*a*x**2 + 7*sqrt(a + b*x**3)*b*x**5 - 6*int((sqrt(a
+ b*x**3)*x)/(a + b*x**3),x)*a**2))/(91*b)
```

3.178 $\int x\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 487

$$\int x\sqrt{a+bx^3}\,dx = \frac{2}{7}x^2\sqrt{a+bx^3}+\frac{6a\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/7*x^2*(b*x^3+a)^(1/2)+6/7*a*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x)-3/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/3)
*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2/7*2^(1/2)*3^(3/4)*a^(4/3)
*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1
/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int x\sqrt{a+bx^3}\,dx=\frac{x^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x*Sqrt[a + b*x^3],x]
```

output
```
(x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*S
qrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \ 811$$

$$\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}$$

$$\Big\downarrow \ 832$$

$$\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}$$

$$\Big\downarrow \ 759$$

$$\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{\ }}{\sqrt[3]{\ }}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{2}{7}x^2\sqrt{a+bx^3}$$

$$\Big\downarrow \ 2416$$

$$\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.$$

$$\frac{2}{7}x^2\sqrt{a+bx^3}$$

input
```
Int[x*Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqr
t[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3])))/7
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.93

method	result
default	$\frac{2x^2\sqrt{b\,x^3+a}}{7}-\frac{2ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{}$
risch	$\frac{2x^2\sqrt{b\,x^3+a}}{7}-\frac{2ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{}$
elliptic	$\frac{2x^2\sqrt{b\,x^3+a}}{7}-\frac{2ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{}$

input
```
int(x*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(b*x^3+a)^(1/2)-2/7*I*a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int x\sqrt{a+bx^3}\,dx = \frac{2\left(\sqrt{bx^3+a}bx^2-3\,a\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b}$$

input
```
integrate(x*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(b*x^3 + a)*b*x^2 - 3*a*sqrt(b)*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)))/b
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int x\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(5/3))
```

Maxima [F]

$$\int x\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+bx^3}\,dx=\int x\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x*(a + b*x^3)^(1/2),x)
```

output
```
int(x*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x\sqrt{a+bx^3}\,dx=\frac{2\sqrt{b\,x^3+a}\,x^2}{7}+\frac{3\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{7}$$

input
```
int(x*(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*x**2 + 3*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a)/7
```

3.179 $\int \frac{\sqrt{a+bx^3}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 479

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = -\frac{\sqrt{a+bx^3}}{x}+\frac{3\sqrt[3]{b}\sqrt{a+bx^3}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-(b*x^3+a)^(1/2)/x+3*b^(1/3)*(b*x^3+a)^(1/2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*
x)-3/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*b^(1/3)*(a^(1/3)+b^(1/3)*
x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+2^(1/2)*3^(3/4)*a^(1/3)*b^(1/3)*(a^(
1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^
(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx=-\frac{\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^2,x]
```

output
```
-((Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/3, 2/3, -((b*x^3)/a)])/(x*Sq
rt[1 + (b*x^3)/a]))
```

Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx$$

$$\Big\downarrow \; 809$$

$$\frac{3}{2}b\int \frac{x}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{x}$$

$$\Big\downarrow \; 832$$

$$\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+bx^3}}{x}$$

$$\Big\downarrow \; 759$$

$$\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right.$$

$$\frac{\sqrt{a+bx^3}}{x}$$

$$\Big\downarrow \; 2416$$

$$\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\sqrt[3]{b}}\cdots\right.$$

$$\frac{\sqrt{a+bx^3}}{x}$$

input
```
Int[Sqrt[a + b*x^3]/x^2,x]
```

output
```
-(Sqrt[a + b*x^3]/x) + (3*b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])))/2
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{b\,x^3+a}}{x}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
risch	$-\frac{\sqrt{b\,x^3+a}}{x}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{\sqrt{b\,x^3+a}}{x}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{(-a\,b^2)^{\frac{1}{3}}}}}{}$

input
```
int((b*x^3+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b
^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(
-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2
)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b
*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^
2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = -\frac{3\,\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\sqrt{bx^3+a}}{x}$$

input
```
integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(3*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x)
) + sqrt(b*x^3 + a))/x
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/x**2,x)
```

output
```
sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/
(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+a}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = \frac{2\sqrt{b\,x^3+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{6};\ \frac{5}{6};\ -\frac{a}{b\,x^3}\right)}{x\,\sqrt{\frac{a}{b\,x^3}+1}}$$

input
```
int((a + b*x^3)^(1/2)/x^2,x)
```

output
```
(2*(a + b*x^3)^(1/2)*hypergeom([-1/2, -1/6], 5/6, -a/(b*x^3)))/(x*(a/(b*x^
3) + 1)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{x^2}\,dx = \frac{2\sqrt{b\,x^3+a}+3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)ax}{x}$$

input
```
int((b*x^3+a)^(1/2)/x^2,x)
```

output
```
(2*sqrt(a + b*x**3) + 3*int(sqrt(a + b*x**3)/(a*x**2 + b*x**5),x)*a*x)/x
```

3.180 $\int \frac{\sqrt{a+bx^3}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 511

$$\int \frac{\sqrt{a+bx^3}}{x^5} \, dx = -\frac{\sqrt{a+bx^3}}{4x^4} - \frac{3b\sqrt{a+bx^3}}{8ax} + \frac{3b^{4/3}\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$- \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{16a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+ \frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*(b*x^3+a)^(1/2)/x^4-3/8*b*(b*x^3+a)^(1/2)/a/x+3/8*b^(4/3)*(b*x^3+a)^(
1/2)/a/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/
2))*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(2/3)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
+1/8*3^(3/4)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)/a^
(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx = -\frac{\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/x^5,x]
```

output
```
-1/4*(Sqrt[a + b*x^3]*Hypergeometric2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/(
x^4*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {809, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3}{8}b\int \frac{1}{x^2\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{8}b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}$$

$$\Big\downarrow \text{832}$$

$$\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)-\frac{\sqrt{a+bx^3}}{4x^4}$$

$$\Big\downarrow \text{759}$$

$$\frac{3}{8}b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.$$

$$\frac{\sqrt{a+bx^3}}{4x^4}$$

$$\Big\downarrow \text{2416}$$

$$\frac{3}{8}b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}-\cdots}{2a}\right.$$

$$\frac{\sqrt{a+bx^3}}{4x^4}$$

input
```
Int[Sqrt[a + b*x^3]/x^5,x]
```

output
```
-1/4*Sqrt[a + b*x^3]/x^4 + (3*b*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a
 + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2
 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sq
rt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/8
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(3b\,x^{3}+2a\right)}{8x^{4}a}-\frac{i b\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{3b\sqrt{b\,x^{3}+a}}{8ax}-\frac{i b\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\cdots\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{3b\sqrt{b\,x^{3}+a}}{8ax}-\frac{i b\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\cdots\right.}{\cdots}}\cdots}{\cdots}$

input
```
int((b*x^3+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(1/2)*(3*b*x^3+2*a)/x^4/a-1/8*I/a*b*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1
/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx = -\frac{3\,b^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(3\,bx^3+2\,a\right)\sqrt{bx^3+a}}{8\,ax^4}$$

input
```
integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(3*b^(3/2)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a
/b, x)) + (3*b*x^3 + 2*a)*sqrt(b*x^3 + a))/(a*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/x**5,x)
```

output
```
sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx = \int \frac{\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/x^5, x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx = \int \frac{\sqrt{bx^3+a}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx=\int \frac{\sqrt{b\,x^3+a}}{x^5}\,dx$$

input
```
int((a + b*x^3)^(1/2)/x^5,x)
```

output
```
int((a + b*x^3)^(1/2)/x^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{x^5}\,dx=\frac{-2\sqrt{b\,x^3+a}-3\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx\right)a\,x^4}{5x^4}$$

input
```
int((b*x^3+a)^(1/2)/x^5,x)
```

output
```
( - 2*sqrt(a + b*x**3) - 3*int(sqrt(a + b*x**3)/(a*x**5 + b*x**8),x)*a*x**
4)/(5*x**4)
```

3.181 $\int x^{11}(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{11}(a+bx^3)^{3/2}\,dx = -\frac{2a^3(a+bx^3)^{5/2}}{15b^4} + \frac{2a^2(a+bx^3)^{7/2}}{7b^4} - \frac{2a(a+bx^3)^{9/2}}{9b^4} + \frac{2(a+bx^3)^{11/2}}{33b^4}$$

output
```
-2/15*a^3*(b*x^3+a)^(5/2)/b^4+2/7*a^2*(b*x^3+a)^(7/2)/b^4-2/9*a*(b*x^3+a)^
(9/2)/b^4+2/33*(b*x^3+a)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^{11}(a+bx^3)^{3/2}\,dx = \frac{2(a+bx^3)^{5/2}(-16a^3+40a^2bx^3-70ab^2x^6+105b^3x^9)}{3465b^4}$$

input
```
Integrate[x^11*(a + b*x^3)^(3/2),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(-16*a^3 + 40*a^2*b*x^3 - 70*a*b^2*x^6 + 105*b^3*x^9)
)/(3465*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \left(a+b x^3\right)^{3/2} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^9 \left(b x^3+a\right)^{3/2}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{\left(b x^3+a\right)^{9/2}}{b^3}-\frac{3 a \left(b x^3+a\right)^{7/2}}{b^3}+\frac{3 a^2 \left(b x^3+a\right)^{5/2}}{b^3}-\frac{a^3 \left(b x^3+a\right)^{3/2}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{2 a^3 \left(a+b x^3\right)^{5/2}}{5 b^4}+\frac{6 a^2 \left(a+b x^3\right)^{7/2}}{7 b^4}+\frac{2 \left(a+b x^3\right)^{11/2}}{11 b^4}-\frac{2 a \left(a+b x^3\right)^{9/2}}{3 b^4}\right)$$

input
```
Int[x^11*(a + b*x^3)^(3/2),x]
```

output
```
((-2*a^3*(a + b*x^3)^(5/2))/(5*b^4) + (6*a^2*(a + b*x^3)^(7/2))/(7*b^4) -
(2*a*(a + b*x^3)^(9/2))/(3*b^4) + (2*(a + b*x^3)^(11/2))/(11*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}(-105b^3x^9+70a\,b^2x^6-40a^2b\,x^3+16a^3)}{3465b^4}$	47
pseudoelliptic	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}(-105b^3x^9+70a\,b^2x^6-40a^2b\,x^3+16a^3)}{3465b^4}$	47
orering	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}(-105b^3x^9+70a\,b^2x^6-40a^2b\,x^3+16a^3)}{3465b^4}$	47
trager	$-\frac{2(-105b^5x^{15}-140a\,b^4x^{12}-5a^2b^3x^9+6a^3b^2x^6-8a^4b\,x^3+16a^5)\sqrt{b\,x^3+a}}{3465b^4}$	69
risch	$-\frac{2(-105b^5x^{15}-140a\,b^4x^{12}-5a^2b^3x^9+6a^3b^2x^6-8a^4b\,x^3+16a^5)\sqrt{b\,x^3+a}}{3465b^4}$	69
default	$\frac{2b\,x^{15}\sqrt{b\,x^3+a}}{33}+\frac{8a\,x^{12}\sqrt{b\,x^3+a}}{99}+\frac{2a^2x^9\sqrt{b\,x^3+a}}{693b}-\frac{4a^3x^6\sqrt{b\,x^3+a}}{1155b^2}+\frac{16a^4x^3\sqrt{b\,x^3+a}}{3465b^3}-\frac{32a^5\sqrt{b\,x^3+a}}{3465b^4}$	109
elliptic	$\frac{2b\,x^{15}\sqrt{b\,x^3+a}}{33}+\frac{8a\,x^{12}\sqrt{b\,x^3+a}}{99}+\frac{2a^2x^9\sqrt{b\,x^3+a}}{693b}-\frac{4a^3x^6\sqrt{b\,x^3+a}}{1155b^2}+\frac{16a^4x^3\sqrt{b\,x^3+a}}{3465b^3}-\frac{32a^5\sqrt{b\,x^3+a}}{3465b^4}$	109

input
```
int(x^11*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*(b*x^3+a)^(5/2)*(-105*b^3*x^9+70*a*b^2*x^6-40*a^2*b*x^3+16*a^3)/b^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x^{11}(a+bx^3)^{3/2}\,dx = \frac{2\,(105\,b^5x^{15}+140\,ab^4x^{12}+5\,a^2b^3x^9-6\,a^3b^2x^6+8\,a^4bx^3-16\,a^5)\sqrt{bx^3+a}}{3465\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3465*(105*b^5*x^15 + 140*a*b^4*x^12 + 5*a^2*b^3*x^9 - 6*a^3*b^2*x^6 + 8*
a^4*b*x^3 - 16*a^5)*sqrt(b*x^3 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

$$\int x^{11}(a+bx^3)^{3/2}\,dx = \begin{cases} -\frac{32a^5\sqrt{a+bx^3}}{3465b^4}+\frac{16a^4x^3\sqrt{a+bx^3}}{3465b^3}-\frac{4a^3x^6\sqrt{a+bx^3}}{1155b^2}+\frac{2a^2x^9\sqrt{a+bx^3}}{693b}+\frac{8ax^{12}\sqrt{a+bx^3}}{99}+\frac{2bx^{15}\sqrt{a+bx^3}}{33} & \text{for } b \\ \frac{a^{\frac{3}{2}}x^{12}}{12} & \text{other} \end{cases}$$

input
```
integrate(x**11*(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-32*a**5*sqrt(a + b*x**3)/(3465*b**4) + 16*a**4*x**3*sqrt(a + b
*x**3)/(3465*b**3) - 4*a**3*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*a**2*x**
9*sqrt(a + b*x**3)/(693*b) + 8*a*x**12*sqrt(a + b*x**3)/99 + 2*b*x**15*sqr
t(a + b*x**3)/33, Ne(b, 0)), (a**(3/2)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{11}\left(a+bx^3\right)^{3/2}\,dx=\frac{2\left(bx^3+a\right)^{\frac{11}{2}}}{33\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{9}{2}}a}{9\,b^4}+\frac{2\left(bx^3+a\right)^{\frac{7}{2}}a^2}{7\,b^4}-\frac{2\left(bx^3+a\right)^{\frac{5}{2}}a^3}{15\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/33*(b*x^3 + a)^(11/2)/b^4 - 2/9*(b*x^3 + a)^(9/2)*a/b^4 + 2/7*(b*x^3 + a
)^(7/2)*a^2/b^4 - 2/15*(b*x^3 + a)^(5/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\left(a+bx^3\right)^{3/2}\,dx=\frac{2\left(105\left(bx^3+a\right)^{\frac{11}{2}}-385\left(bx^3+a\right)^{\frac{9}{2}}a+495\left(bx^3+a\right)^{\frac{7}{2}}a^2-231\left(bx^3+a\right)^{\frac{5}{2}}a^3\right)}{3465\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3465*(105*(b*x^3 + a)^(11/2) - 385*(b*x^3 + a)^(9/2)*a + 495*(b*x^3 + a)
^(7/2)*a^2 - 231*(b*x^3 + a)^(5/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int x^{11}\left(a+bx^3\right)^{3/2}\,dx=\frac{8\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}-\frac{32\,a^5\,\sqrt{b\,x^3+a}}{3465\,b^4}+\frac{16\,a^4\,x^3\,\sqrt{b\,x^3+a}}{3465\,b^3}-\frac{4\,a^3\,x^6\,\sqrt{b\,x^3+a}}{1155\,b^2}+\frac{2\,a^2\,x^9\,\sqrt{b\,x^3+a}}{693\,b}$$

input
```
int(x^11*(a + b*x^3)^(3/2),x)
```

output
```
(8*a*x^12*(a + b*x^3)^(1/2))/99 + (2*b*x^15*(a + b*x^3)^(1/2))/33 - (32*a^
5*(a + b*x^3)^(1/2))/(3465*b^4) + (16*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3
) - (4*a^3*x^6*(a + b*x^3)^(1/2))/(1155*b^2) + (2*a^2*x^9*(a + b*x^3)^(1/2
))/(693*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int x^{11}(a+bx^3)^{3/2}\,dx = \frac{2\sqrt{b\,x^3+a}\,(105b^5x^{15}+140a\,b^4x^{12}+5a^2b^3x^9-6a^3b^2x^6+8a^4b\,x^3-16a^5)}{3465b^4}$$

input
```
int(x^11*(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 16*a**5 + 8*a**4*b*x**3 - 6*a**3*b**2*x**6 + 5*a**
2*b**3*x**9 + 140*a*b**4*x**12 + 105*b**5*x**15))/(3465*b**4)
```

3.182 $\int x^8(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^8\left(a+bx^3\right)^{3/2}\,dx=\frac{2a^2(a+bx^3)^{5/2}}{15b^3}-\frac{4a(a+bx^3)^{7/2}}{21b^3}+\frac{2(a+bx^3)^{9/2}}{27b^3}$$

output
```
2/15*a^2*(b*x^3+a)^(5/2)/b^3-4/21*a*(b*x^3+a)^(7/2)/b^3+2/27*(b*x^3+a)^(9/
2)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^8\left(a+bx^3\right)^{3/2}\,dx=\frac{2(a+bx^3)^{5/2}\left(8a^2-20abx^3+35b^2x^6\right)}{945b^3}$$

input
```
Integrate[x^8*(a + b*x^3)^(3/2),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(8*a^2 - 20*a*b*x^3 + 35*b^2*x^6))/(945*b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^6 \left(bx^3+a\right)^{3/2}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{7/2}}{b^2}-\frac{2a\left(bx^3+a\right)^{5/2}}{b^2}+\frac{a^2\left(bx^3+a\right)^{3/2}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^2\left(a+bx^3\right)^{5/2}}{5b^3}+\frac{2\left(a+bx^3\right)^{9/2}}{9b^3}-\frac{4a\left(a+bx^3\right)^{7/2}}{7b^3}\right)$$

input
```
Int[x^8*(a + b*x^3)^(3/2),x]
```

output
```
((2*a^2*(a + b*x^3)^(5/2))/(5*b^3) - (4*a*(a + b*x^3)^(7/2))/(7*b^3) + (2*
(a + b*x^3)^(9/2))/(9*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2(b\,x^{3}+a)^{\frac{5}{2}}(35b^{2}x^{6}-20ab\,x^{3}+8a^{2})}{945b^{3}}$	36
pseudoelliptic	$\frac{2(b\,x^{3}+a)^{\frac{5}{2}}(35b^{2}x^{6}-20ab\,x^{3}+8a^{2})}{945b^{3}}$	36
orering	$\frac{2(b\,x^{3}+a)^{\frac{5}{2}}(35b^{2}x^{6}-20ab\,x^{3}+8a^{2})}{945b^{3}}$	36
trager	$\frac{2(35b^{4}x^{12}+50a\,b^{3}x^{9}+3a^{2}b^{2}x^{6}-4a^{3}b\,x^{3}+8a^{4})\sqrt{b\,x^{3}+a}}{945b^{3}}$	58
risch	$\frac{2(35b^{4}x^{12}+50a\,b^{3}x^{9}+3a^{2}b^{2}x^{6}-4a^{3}b\,x^{3}+8a^{4})\sqrt{b\,x^{3}+a}}{945b^{3}}$	58
default	$\frac{2b\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{20a\,x^{9}\sqrt{b\,x^{3}+a}}{189}+\frac{2a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b}-\frac{8a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{2}}+\frac{16a^{4}\sqrt{b\,x^{3}+a}}{945b^{3}}$	89
elliptic	$\frac{2b\,x^{12}\sqrt{b\,x^{3}+a}}{27}+\frac{20a\,x^{9}\sqrt{b\,x^{3}+a}}{189}+\frac{2a^{2}x^{6}\sqrt{b\,x^{3}+a}}{315b}-\frac{8a^{3}x^{3}\sqrt{b\,x^{3}+a}}{945b^{2}}+\frac{16a^{4}\sqrt{b\,x^{3}+a}}{945b^{3}}$	89

input
```
int(x^8*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/945*(b*x^3+a)^(5/2)*(35*b^2*x^6-20*a*b*x^3+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^8 (a+bx^3)^{3/2}\,dx = \frac{2\,(35\,b^4x^{12}+50\,ab^3x^9+3\,a^2b^2x^6-4\,a^3bx^3+8\,a^4)\sqrt{bx^3+a}}{945\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/945*(35*b^4*x^12 + 50*a*b^3*x^9 + 3*a^2*b^2*x^6 - 4*a^3*b*x^3 + 8*a^4)*s
qrt(b*x^3 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.90

$$\int x^8 (a+bx^3)^{3/2}\,dx = \begin{cases} \frac{16a^4\sqrt{a+bx^3}}{945b^3} - \frac{8a^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2a^2x^6\sqrt{a+bx^3}}{315b} + \frac{20ax^9\sqrt{a+bx^3}}{189} + \frac{2bx^{12}\sqrt{a+bx^3}}{27} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^9}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((16*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*a**3*x**3*sqrt(a + b*x*
*3)/(945*b**2) + 2*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*a*x**9*sqrt(a +
 b*x**3)/189 + 2*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*x**9/9,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^8 (a + bx^3)^{3/2} \, dx = \frac{2 (bx^3 + a)^{\frac{9}{2}}}{27 \, b^3} - \frac{4 (bx^3 + a)^{\frac{7}{2}} a}{21 \, b^3} + \frac{2 (bx^3 + a)^{\frac{5}{2}} a^2}{15 \, b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/27*(b*x^3 + a)^(9/2)/b^3 - 4/21*(b*x^3 + a)^(7/2)*a/b^3 + 2/15*(b*x^3 +
a)^(5/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^8 (a + bx^3)^{3/2} \, dx = \frac{2 \left(35 (bx^3 + a)^{\frac{9}{2}} - 90 (bx^3 + a)^{\frac{7}{2}} a + 63 (bx^3 + a)^{\frac{5}{2}} a^2\right)}{945 \, b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/945*(35*(b*x^3 + a)^(9/2) - 90*(b*x^3 + a)^(7/2)*a + 63*(b*x^3 + a)^(5/2
)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int x^8 (a + bx^3)^{3/2} \, dx = \frac{20 \, a \, x^9 \, \sqrt{b \, x^3 + a}}{189} + \frac{2 \, b \, x^{12} \, \sqrt{b \, x^3 + a}}{27} + \frac{16 \, a^4 \, \sqrt{b \, x^3 + a}}{945 \, b^3} - \frac{8 \, a^3 \, x^3 \, \sqrt{b \, x^3 + a}}{945 \, b^2} + \frac{2 \, a^2 \, x^6 \, \sqrt{b \, x^3 + a}}{315 \, b}$$

input
```
int(x^8*(a + b*x^3)^(3/2),x)
```

output
```
(20*a*x^9*(a + b*x^3)^(1/2))/189 + (2*b*x^12*(a + b*x^3)^(1/2))/27 + (16*a
^4*(a + b*x^3)^(1/2))/(945*b^3) - (8*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2)
+ (2*a^2*x^6*(a + b*x^3)^(1/2))/(315*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int x^8(a+bx^3)^{3/2}\,dx=\frac{2\sqrt{b\,x^3+a}\,(35b^4x^{12}+50a\,b^3x^9+3a^2b^2x^6-4a^3b\,x^3+8a^4)}{945b^3}$$

input
```
int(x^8*(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**4 - 4*a**3*b*x**3 + 3*a**2*b**2*x**6 + 50*a*b**3
*x**9 + 35*b**4*x**12))/(945*b**3)
```

3.183 $\int x^5(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^5\left(a+bx^3\right)^{3/2}\,dx=-\frac{2a(a+bx^3)^{5/2}}{15b^2}+\frac{2(a+bx^3)^{7/2}}{21b^2}$$

output
```
-2/15*a*(b*x^3+a)^(5/2)/b^2+2/21*(b*x^3+a)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^5\left(a+bx^3\right)^{3/2}\,dx=\frac{2(a+bx^3)^{5/2}\,(-2a+5bx^3)}{105b^2}$$

input
```
Integrate[x^5*(a + b*x^3)^(3/2),x]
```

output
```
(2*(a + b*x^3)^(5/2)*(-2*a + 5*b*x^3))/(105*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^3 \left(bx^3+a\right)^{3/2} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{5/2}}{b}-\frac{a\left(bx^3+a\right)^{3/2}}{b}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{7/2}}{7b^2}-\frac{2a\left(a+bx^3\right)^{5/2}}{5b^2}\right)$$

input
```
Int[x^5*(a + b*x^3)^(3/2),x]
```

output
```
((-2*a*(a + b*x^3)^(5/2))/(5*b^2) + (2*(a + b*x^3)^(7/2))/(7*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}(-5b\,x^3+2a)}{105b^2}$	25
pseudoelliptic	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}(-5b\,x^3+2a)}{105b^2}$	25
orering	$-\frac{2(b\,x^3+a)^{\frac{5}{2}}(-5b\,x^3+2a)}{105b^2}$	25
trager	$-\frac{2(-5b^3x^9-8a\,b^2x^6-a^2b\,x^3+2a^3)\sqrt{b\,x^3+a}}{105b^2}$	47
risch	$-\frac{2(-5b^3x^9-8a\,b^2x^6-a^2b\,x^3+2a^3)\sqrt{b\,x^3+a}}{105b^2}$	47
default	$\frac{2b\,x^9\sqrt{b\,x^3+a}}{21}+\frac{16a\,x^6\sqrt{b\,x^3+a}}{105}+\frac{2a^2x^3\sqrt{b\,x^3+a}}{105b}-\frac{4a^3\sqrt{b\,x^3+a}}{105b^2}$	69
elliptic	$\frac{2b\,x^9\sqrt{b\,x^3+a}}{21}+\frac{16a\,x^6\sqrt{b\,x^3+a}}{105}+\frac{2a^2x^3\sqrt{b\,x^3+a}}{105b}-\frac{4a^3\sqrt{b\,x^3+a}}{105b^2}$	69

input
```
int(x^5*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x^3+a)^(5/2)*(-5*b*x^3+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^5 \left(a+bx^3\right)^{3/2} \, dx = \frac{2 \left(5 \, b^3x^9 + 8 \, ab^2x^6 + a^2bx^3 - 2 \, a^3\right)\sqrt{bx^3+a}}{105 \, b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(5*b^3*x^9 + 8*a*b^2*x^6 + a^2*b*x^3 - 2*a^3)*sqrt(b*x^3 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int x^5 \left(a+bx^3\right)^{3/2} \, dx = \begin{cases} -\frac{4a^3\sqrt{a+bx^3}}{105b^2} + \frac{2a^2x^3\sqrt{a+bx^3}}{105b} + \frac{16ax^6\sqrt{a+bx^3}}{105} + \frac{2bx^9\sqrt{a+bx^3}}{21} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-4*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*a**2*x**3*sqrt(a + b*x*
*3)/(105*b) + 16*a*x**6*sqrt(a + b*x**3)/105 + 2*b*x**9*sqrt(a + b*x**3)/2
1, Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^5 \left(a+bx^3\right)^{3/2} \, dx = \frac{2 \left(bx^3+a\right)^{\frac{7}{2}}}{21 \, b^2} - \frac{2 \left(bx^3+a\right)^{\frac{5}{2}}a}{15 \, b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/21*(b*x^3 + a)^(7/2)/b^2 - 2/15*(b*x^3 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^5 \left(a+bx^3\right)^{3/2} \, dx = \frac{2 \left(5 \left(bx^3+a\right)^{\frac{7}{2}} - 7 \left(bx^3+a\right)^{\frac{5}{2}} a\right)}{105 \, b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/105*(5*(b*x^3 + a)^(7/2) - 7*(b*x^3 + a)^(5/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int x^5 \left(a+bx^3\right)^{3/2} \, dx = \frac{16 \, a \, x^6 \, \sqrt{b \, x^3+a}}{105} + \frac{2 \, b \, x^9 \, \sqrt{b \, x^3+a}}{21} - \frac{4 \, a^3 \, \sqrt{b \, x^3+a}}{105 \, b^2} + \frac{2 \, a^2 \, x^3 \, \sqrt{b \, x^3+a}}{105 \, b}$$

input
```
int(x^5*(a + b*x^3)^(3/2),x)
```

output
```
(16*a*x^6*(a + b*x^3)^(1/2))/105 + (2*b*x^9*(a + b*x^3)^(1/2))/21 - (4*a^3
*(a + b*x^3)^(1/2))/(105*b^2) + (2*a^2*x^3*(a + b*x^3)^(1/2))/(105*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^5(a+bx^3)^{3/2}\,dx=\frac{2\sqrt{b\,x^3+a}\,(5b^3x^9+8a\,b^2x^6+a^2b\,x^3-2a^3)}{105b^2}$$

input
```
int(x^5*(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a**3 + a**2*b*x**3 + 8*a*b**2*x**6 + 5*b**3*x**9
))/(105*b**2)
```

3.184 $\int x^2(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^2\left(a+bx^3\right)^{3/2}\,dx=\frac{2(a+bx^3)^{5/2}}{15b}$$

output
```
2/15*(b*x^3+a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^3\right)^{3/2}\,dx=\frac{2(a+bx^3)^{5/2}}{15b}$$

input
```
Integrate[x^2*(a + b*x^3)^(3/2),x]
```

output
```
(2*(a + b*x^3)^(5/2))/(15*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \; 793$$

$$\frac{2\left(a+bx^3\right)^{5/2}}{15b}$$

input
```
Int[x^2*(a + b*x^3)^(3/2),x]
```

output
```
(2*(a + b*x^3)^(5/2))/(15*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2(b x^3+a)^{\frac{5}{2}}}{15b}$	15
derivativedivides	$\frac{2(b x^3+a)^{\frac{5}{2}}}{15b}$	15
default	$\frac{2(b x^3+a)^{\frac{5}{2}}}{15b}$	15
pseudoelliptic	$\frac{2(b x^3+a)^{\frac{5}{2}}}{15b}$	15
orering	$\frac{2(b x^3+a)^{\frac{5}{2}}}{15b}$	15
trager	$\frac{2(b^2x^6+2ab x^3+a^2)\sqrt{b x^3+a}}{15b}$	33
risch	$\frac{2(b^2x^6+2ab x^3+a^2)\sqrt{b x^3+a}}{15b}$	33

input
```
int(x^2*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x^3+a)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^2 \left(a+bx^3\right)^{3/2} \, dx = \frac{2 \left(b^2x^6 + 2 \, abx^3 + a^2\right)\sqrt{bx^3+a}}{15 \, b}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(b^2*x^6 + 2*a*b*x^3 + a^2)*sqrt(b*x^3 + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(14) = 28$.

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.61

$$\int x^2 \left(a+bx^3\right)^{3/2}\,dx = \begin{cases} \frac{2a^2\sqrt{a+bx^3}}{15b} + \frac{4ax^3\sqrt{a+bx^3}}{15} + \frac{2bx^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((2*a**2*sqrt(a + b*x**3)/(15*b) + 4*a*x**3*sqrt(a + b*x**3)/15 +
 2*b*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (a**(3/2)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2 \left(a+bx^3\right)^{3/2}\,dx = \frac{2\,\left(bx^3+a\right)^{\frac{5}{2}}}{15\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2 \left(a+bx^3\right)^{3/2}\,dx = \frac{2\,\left(bx^3+a\right)^{\frac{5}{2}}}{15\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/15*(b*x^3 + a)^(5/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2(a+bx^3)^{3/2}\,dx=\frac{2\,{\left(b\,x^3+a\right)}^{5/2}}{15\,b}$$

input
```
int(x^2*(a + b*x^3)^(3/2),x)
```

output
```
(2*(a + b*x^3)^(5/2))/(15*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int x^2(a+bx^3)^{3/2}\,dx=\frac{2\sqrt{b\,x^3+a}\,(b^2x^6+2ab\,x^3+a^2)}{15b}$$

input
```
int(x^2*(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(a**2 + 2*a*b*x**3 + b**2*x**6))/(15*b)
```

3.185 $\int \frac{(a+bx^3)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{(a+bx^3)^{3/2}}{x} \, dx = \frac{2}{3}a\sqrt{a+bx^3} + \frac{2}{9}(a+bx^3)^{3/2} - \frac{2}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/3*a*(b*x^3+a)^(1/2)+2/9*(b*x^3+a)^(3/2)-2/3*a^(3/2)*arctanh((b*x^3+a)^(1
/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^{3/2}}{x} \, dx = \frac{2}{9}\sqrt{a+bx^3}(4a+bx^3) - \frac{2}{3}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^3)^(3/2)/x,x]
```

output
```
(2*Sqrt[a + b*x^3]*(4*a + b*x^3))/9 - (2*a^(3/2)*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}}{x^3}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\int \frac{\sqrt{bx^3+a}}{x^3}dx^3+\frac{2}{3}(a+bx^3)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(a\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(a\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^3)^{3/2}\right)$$

input
```
Int[(a + b*x^3)^(3/2)/x,x]
```

output
```
((2*(a + b*x^3)^(3/2))/3 + a*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]]))/3
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}+\frac{2\sqrt{b\,x^{3}+a}\,(b\,x^{3}+4a)}{9}$	41
default	$\frac{2x^{3}b\sqrt{b\,x^{3}+a}}{9}+\frac{8a\sqrt{b\,x^{3}+a}}{9}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}$	48
elliptic	$\frac{2x^{3}b\sqrt{b\,x^{3}+a}}{9}+\frac{8a\sqrt{b\,x^{3}+a}}{9}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3}$	48

input
```
int((b*x^3+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))+2/9*(b*x^3+a)^(1/2)*(b*x^3+4
*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^3)^{3/2}}{x}\,dx = \left[\frac{1}{3}\,a^{\frac{3}{2}}\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+\frac{2}{9}\left(bx^3+4\,a\right)\sqrt{bx^3+a},\ \frac{2}{3}\,\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+\frac{2}{9}\left(bx^3+4\,a\right)\sqrt{bx^3+a}\right]$$

input
```
integrate((b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*a^(3/2)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2/9*(b*x
^3 + 4*a)*sqrt(b*x^3 + a), 2/3*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(b*x^3 + a))
 + 2/9*(b*x^3 + 4*a)*sqrt(b*x^3 + a)]
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx^3)^{3/2}}{x}\,dx = \frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{9}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^3}{a}\right)}{3}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3}+\frac{2\sqrt{a}bx^3\sqrt{1+\frac{bx^3}{a}}}{9}$$

input
```
integrate((b*x**3+a)**(3/2)/x,x)
```

output
```
8*a**(3/2)*sqrt(1 + b*x**3/a)/9 + a**(3/2)*log(b*x**3/a)/3 - 2*a**(3/2)*lo
g(sqrt(1 + b*x**3/a) + 1)/3 + 2*sqrt(a)*b*x**3*sqrt(1 + b*x**3/a)/9
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^3)^{3/2}}{x}\,dx = \frac{1}{3}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+\frac{2}{9}\,(bx^3+a)^{\frac{3}{2}}+\frac{2}{3}\,\sqrt{bx^3+a}a$$

input
```
integrate((b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
 2/9*(b*x^3 + a)^(3/2) + 2/3*sqrt(b*x^3 + a)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^3)^{3/2}}{x}\,dx = \frac{2\,a^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2}{9}\,(bx^3+a)^{\frac{3}{2}}+\frac{2}{3}\,\sqrt{bx^3+a}a$$

input
```
integrate((b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

output
```
2/3*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x^3 + a)^(3/2)
+ 2/3*sqrt(b*x^3 + a)*a
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^3)^{3/2}}{x}\,dx = \frac{a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3} + \frac{8\,a\,\sqrt{b\,x^3+a}}{9}+\frac{2\,b\,x^3\,\sqrt{b\,x^3+a}}{9}$$

input
```
int((a + b*x^3)^(3/2)/x,x)
```

output
```
(a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)
))/x^6))/3 + (8*a*(a + b*x^3)^(1/2))/9 + (2*b*x^3*(a + b*x^3)^(1/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^{3/2}}{x}\,dx = \frac{8\sqrt{b\,x^3+a}\,a}{9} + \frac{2\sqrt{b\,x^3+a}\,b\,x^3}{9} + \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)a}{3} - \frac{\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)a}{3}$$

input
```
int((b*x^3+a)^(3/2)/x,x)
```

output
```
(8*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 + 3*sqrt(a)*log(sqrt(a +
 b*x**3) - sqrt(a))*a - 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*a)/9
```

3.186 $\int \frac{(a+bx^3)^{3/2}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{(a+bx^3)^{3/2}}{x^4} \, dx = \frac{2}{3}b\sqrt{a+bx^3} - \frac{a\sqrt{a+bx^3}}{3x^3} - \sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

output
```
2/3*b*(b*x^3+a)^(1/2)-1/3*a*(b*x^3+a)^(1/2)/x^3-a^(1/2)*b*arctanh((b*x^3+a
)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^3)^{3/2}}{x^4} \, dx = \frac{\sqrt{a+bx^3}(-a+2bx^3)}{3x^3} - \sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^3)^(3/2)/x^4,x]
```

output
```
(Sqrt[a + b*x^3]*(-a + 2*b*x^3))/(3*x^3) - Sqrt[a]*b*ArcTanh[Sqrt[a + b*x^
3]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}}{x^6}dx^3$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{3}\left(\frac{3}{2}b\int \frac{\sqrt{bx^3+a}}{x^3}dx^3-\frac{(a+bx^3)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\left(\frac{3}{2}b\left(a\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3+2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{3}\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{b}+2\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{3}\left(\frac{3}{2}b\left(2\sqrt{a+bx^3}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)-\frac{(a+bx^3)^{3/2}}{x^3}\right)$$

input
```
Int[(a + b*x^3)^(3/2)/x^4,x]
```

output
```
(-((a + b*x^3)^(3/2)/x^3) + (3*b*(2*Sqrt[a + b*x^3] - 2*Sqrt[a]*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]]))/2)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2b\sqrt{b\,x^{3}+a}}{3}-\frac{a\sqrt{b\,x^{3}+a}}{3x^{3}}-\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)$	49
elliptic	$\frac{2b\sqrt{b\,x^{3}+a}}{3}-\frac{a\sqrt{b\,x^{3}+a}}{3x^{3}}-\sqrt{a}\,b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)$	49
risch	$-\frac{a\sqrt{b\,x^{3}+a}}{3x^{3}}+\frac{b\left(\frac{4\sqrt{b\,x^{3}+a}}{3}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\right)}{2}$	51
pseudoelliptic	$-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)ab\,x^{3}+\sqrt{a}\,(-2b\,x^{3}+a)\sqrt{b\,x^{3}+a}}{3\sqrt{a}\,x^{3}}$	52

input
```
int((b*x^3+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*b*(b*x^3+a)^(1/2)-1/3*a*(b*x^3+a)^(1/2)/x^3-a^(1/2)*b*arctanh((b*x^3+a
)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^3)^{3/2}}{x^4}\,dx=\left[\frac{3\sqrt{a}bx^3\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\left(2bx^3-a\right)\sqrt{bx^3+a}}{6x^3},\ \frac{3\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)\cdots}{3\cdots}\right.$$

input
```
integrate((b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(a)*b*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
 2*(2*b*x^3 - a)*sqrt(b*x^3 + a))/x^3, 1/3*(3*sqrt(-a)*b*x^3*arctan(sqrt(-
a)/sqrt(b*x^3 + a)) + (2*b*x^3 - a)*sqrt(b*x^3 + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^3)^{3/2}}{x^4}\,dx = -\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right) - \frac{a^2}{3\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{a\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{2b^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate((b*x**3+a)**(3/2)/x**4,x)
```

output
```
-sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - a**2/(3*sqrt(b)*x**(9/2)*sq
rt(a/(b*x**3) + 1)) + a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*b**(
3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^3)^{3/2}}{x^4}\,dx = \frac{1}{2}\sqrt{a}b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + \frac{2}{3}\sqrt{bx^3+a}b - \frac{\sqrt{bx^3+a}a}{3\,x^3}$$

input
```
integrate((b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))
 + 2/3*sqrt(b*x^3 + a)*b - 1/3*sqrt(b*x^3 + a)*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^{3/2}}{x^4}\,dx = \frac{1}{3}\left(\frac{3\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{bx^3+a} - \frac{\sqrt{bx^3+a}a}{bx^3}\right)b$$

input
```
integrate((b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
1/3*(3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^3 + a) - s
qrt(b*x^3 + a)*a/(b*x^3))*b
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^3)^{3/2}}{x^4}\,dx = \frac{2\,b\sqrt{b\,x^3+a}}{3} - \frac{a\sqrt{b\,x^3+a}}{3\,x^3} + \frac{\sqrt{a}\,b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{2}$$

input
```
int((a + b*x^3)^(3/2)/x^4,x)
```

output
```
(2*b*(a + b*x^3)^(1/2))/3 - (a*(a + b*x^3)^(1/2))/(3*x^3) + (a^(1/2)*b*log
((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)^{3/2}}{x^4}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a + 4\sqrt{b\,x^3+a}\,b\,x^3 + 3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3 - 3\sqrt{a}\log\left(\sqrt{b\,x^3+\right.}{6x^3}$$

input
```
int((b*x^3+a)^(3/2)/x^4,x)
```

output
```
( - 2*sqrt(a + b*x**3)*a + 4*sqrt(a + b*x**3)*b*x**3 + 3*sqrt(a)*log(sqrt(
a + b*x**3) - sqrt(a))*b*x**3 - 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*
b*x**3)/(6*x**3)
```

3.187 $\int \frac{(a+bx^3)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{(a+bx^3)^{3/2}}{x^7} dx = -\frac{a\sqrt{a+bx^3}}{6x^6} - \frac{5b\sqrt{a+bx^3}}{12x^3} - \frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*b*(b*x^3+a)^(1/2)/x^3-1/4*b^2*arctanh((b*x
^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^{3/2}}{x^7} dx = \frac{(-2a-5bx^3)\sqrt{a+bx^3}}{12x^6} - \frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[(a + b*x^3)^(3/2)/x^7,x]
```

output
```
((-2*a - 5*b*x^3)*Sqrt[a + b*x^3])/(12*x^6) - (b^2*ArcTanh[Sqrt[a + b*x^3]
/Sqrt[a]])/(4*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}}{x^7} \, dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}\int \frac{(bx^3+a)^{3/2}}{x^9}dx^3$$

$$\Big\downarrow \; 51$$

$$\frac{1}{3}\left(\frac{3}{4}b\int \frac{\sqrt{bx^3+a}}{x^6}dx^3-\frac{(a+bx^3)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{3}\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3-\frac{\sqrt{a+bx^3}}{x^3}\right)-\frac{(a+bx^3)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{3}\left(\frac{3}{4}b\left(\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}-\frac{\sqrt{a+bx^3}}{x^3}\right)-\frac{(a+bx^3)^{3/2}}{2x^6}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{3}\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^3}}{x^3}\right)-\frac{(a+bx^3)^{3/2}}{2x^6}\right)$$

input
```
Int[(a + b*x^3)^(3/2)/x^7,x]
```

output
```
(-1/2*(a + b*x^3)^(3/2)/x^6 + (3*b*(-(Sqrt[a + b*x^3]/x^3) - (b*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/Sqrt[a]))/4)/3
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(5 b\,x^{3}+2 a\right)}{12 x^{6}}-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 \sqrt{a}}$	48
default	$-\frac{a\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{5 b\sqrt{b\,x^{3}+a}}{12 x^{3}}-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 \sqrt{a}}$	54
elliptic	$-\frac{a\sqrt{b\,x^{3}+a}}{6 x^{6}}-\frac{5 b\sqrt{b\,x^{3}+a}}{12 x^{3}}-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 \sqrt{a}}$	54
pseudoelliptic	$\frac{-3 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right) b^{2} x^{6}-5 b\,x^{3} \sqrt{b\,x^{3}+a}\,\sqrt{a}-2 a^{\frac{3}{2}} \sqrt{b\,x^{3}+a}}{12 x^{6} \sqrt{a}}$	64

input
```
int((b*x^3+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(5*b*x^3+2*a)/x^6-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^
(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx^3)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{a}b^2x^6\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\left(5abx^3+2a^2\right)\sqrt{bx^3+a}}{24ax^6}, \frac{3\sqrt{-a}b^2x^6\arctan\left(\right.}{}\right.$$

input
```
integrate((b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(a)*b^2*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3
) - 2*(5*a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))/(a*x^6), 1/12*(3*sqrt(-a)*b^2*x
^6*arctan(sqrt(-a)/sqrt(b*x^3 + a)) - (5*a*b*x^3 + 2*a^2)*sqrt(b*x^3 + a))
/(a*x^6)]
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^3)^{3/2}}{x^7}\,dx = -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{6x^{\frac{9}{2}}} - \frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{12x^{\frac{3}{2}}} - \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4\sqrt{a}}$$

input
```
integrate((b*x**3+a)**(3/2)/x**7,x)
```

output
```
-a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(6*x**(9/2)) - 5*b**(3/2)*sqrt(a/(b*x**3)
+ 1)/(12*x**(3/2)) - b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^3)^{3/2}}{x^7}\,dx = \frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{8\sqrt{a}} - \frac{5\,(bx^3+a)^{\frac{3}{2}}b^2 - 3\sqrt{bx^3+a}ab^2}{12\left((bx^3+a)^2 - 2\,(bx^3+a)a + a^2\right)}$$

input
```
integrate((b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/8*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(
a) - 1/12*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)
^2 - 2*(b*x^3 + a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{3/2}}{x^7}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5\,(bx^3+a)^{\frac{3}{2}}b^3 - 3\sqrt{bx^3+a}ab^3}{b^2x^6}}{12\,b}$$

input
```
integrate((b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/12*(3*b^3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^3 + a)^(3/
2)*b^3 - 3*sqrt(b*x^3 + a)*a*b^3)/(b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^{3/2}}{x^7}\,dx = \frac{b^2\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{8\sqrt{a}} - \frac{5\,b\sqrt{b\,x^3+a}}{12\,x^3} - \frac{a\sqrt{b\,x^3+a}}{6\,x^6}$$

input
```
int((a + b*x^3)^(3/2)/x^7,x)
```

output
```
(b^2*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x
^6))/(8*a^(1/2)) - (5*b*(a + b*x^3)^(1/2))/(12*x^3) - (a*(a + b*x^3)^(1/2)
)/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^3)^{3/2}}{x^7}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^2-10\sqrt{b\,x^3+a}\,ab\,x^3+3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^2x^6-3\sqrt{a}\log\left(\sqrt{b}\right.}{24a\,x^6}$$

input
```
int((b*x^3+a)^(3/2)/x^7,x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 - 10*sqrt(a + b*x**3)*a*b*x**3 + 3*sqrt(a)*log
(sqrt(a + b*x**3) - sqrt(a))*b**2*x**6 - 3*sqrt(a)*log(sqrt(a + b*x**3) +
sqrt(a))*b**2*x**6)/(24*a*x**6)
```

3.188 $\int x^6(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 296

$$\int x^6\left(a+bx^3\right)^{3/2}\,dx=-\frac{432a^3x\sqrt{a+bx^3}}{21505b^2}+\frac{54a^2x^4\sqrt{a+bx^3}}{4301b}+\frac{18}{391}ax^7\sqrt{a+bx^3}+\frac{2}{23}x^7\left(a+bx^3\right)^{3/2}+\frac{288\,3^{3/4}\sqrt{2+\sqrt{3}}a^4\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-432/21505*a^3*x*(b*x^3+a)^(1/2)/b^2+54/4301*a^2*x^4*(b*x^3+a)^(1/2)/b+18/
391*a*x^7*(b*x^3+a)^(1/2)+2/23*x^7*(b*x^3+a)^(3/2)+288/21505*3^(3/4)*(1/2*
6^(1/2)+1/2*2^(1/2))*a^4*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2)
)*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(7/3
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b
*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.27

$$\int x^6 (a+bx^3)^{3/2}\, dx = \frac{2x\sqrt{a+bx^3}\left(-\left((8a-17bx^3)(a+bx^3)^2\right)+\frac{8a^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{391b^2}$$

input
```
Integrate[x^6*(a + b*x^3)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*(-((8*a - 17*b*x^3)*(a + b*x^3)^2) + (8*a^3*Hypergeom
etric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(391*b^2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {811, 811, 843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 (a+bx^3)^{3/2}\, dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{9}{23}a\int x^6\sqrt{bx^3+a}dx+\frac{2}{23}x^7(a+bx^3)^{3/2}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{9}{23}a\left(\frac{3}{17}a\int \frac{x^6}{\sqrt{bx^3+a}}dx+\frac{2}{17}x^7\sqrt{a+bx^3}\right)+\frac{2}{23}x^7(a+bx^3)^{3/2}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{9}{23}a\left(\frac{3}{17}a\left(\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\int\frac{x^3}{\sqrt{bx^3+a}}dx}{11b}\right)+\frac{2}{17}x^7\sqrt{a+bx^3}\right)+\frac{2}{23}x^7\left(a+bx^3\right)^{3/2}$$

$\Big\downarrow$ 843

$$\frac{9}{23}a\left(\frac{3}{17}a\left(\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{11b}\right)+\frac{2}{17}x^7\sqrt{a+bx^3}\right)+\frac{2}{23}x^7\left(a+bx^3\right)^{3/2}$$

$\Big\downarrow$ 759

$$\frac{9}{23}a\left(\frac{3}{17}a\left(\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{11b}\right)+\frac{2}{17}x^7\sqrt{a+bx^3}\right)+\frac{2}{23}x^7\left(a+bx^3\right)^{3/2}$$

input

```
Int[x^6*(a + b*x^3)^(3/2),x]
```

output

```
(2*x^7*(a + b*x^3)^(3/2))/23 + (9*a*((2*x^7*Sqrt[a + b*x^3])/17 + (3*a*((2
*x^4*Sqrt[a + b*x^3])/(11*b) - (8*a*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt
[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b)))/17))/23
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{2x\left(-935b^3x^9-1430a\,b^2x^6-135a^2b\,x^3+216a^3\right)\sqrt{b\,x^3+a}}{21505b^2}-\frac{288ia^4\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\cdots}{-\frac{3\left(\cdots\right.}{}}}\cdots}{\cdots}$
default	$\frac{2x^{10}\sqrt{b\,x^3+a}\,b}{23}+\frac{52a\,x^7\sqrt{b\,x^3+a}}{391}+\frac{54a^2x^4\sqrt{b\,x^3+a}}{4301b}-\frac{432a^3x\sqrt{b\,x^3+a}}{21505b^2}-\frac{288ia^4\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b\cdots\right.}{2b}\cdots\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$
elliptic	$\frac{2x^{10}\sqrt{b\,x^3+a}\,b}{23}+\frac{52a\,x^7\sqrt{b\,x^3+a}}{391}+\frac{54a^2x^4\sqrt{b\,x^3+a}}{4301b}-\frac{432a^3x\sqrt{b\,x^3+a}}{21505b^2}-\frac{288ia^4\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b\cdots\right.}{2b}\cdots\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$

input
```
int(x^6*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21505*x*(-935*b^3*x^9-1430*a*b^2*x^6-135*a^2*b*x^3+216*a^3)*(b*x^3+a)^(
1/2)/b^2-288/21505*I/b^3*a^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.23

$$\int x^6(a+bx^3)^{3/2}\,dx = \frac{2\left(432\,a^4\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(935\,b^4x^{10}+1430\,ab^3x^7+135\,a^2b^2x^4-216\,a\right.}{21505\,b^3}$$

input
```
integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/21505*(432*a^4*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (935*b^4*x^10
 + 1430*a*b^3*x^7 + 135*a^2*b^2*x^4 - 216*a^3*b*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.13

$$\int x^6\left(a+bx^3\right)^{3/2}\,dx = \frac{a^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6*(b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x**7*gamma(7/3)*hyper((-3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(10/3))
```

Maxima [F]

$$\int x^6 (a + bx^3)^{3/2} \, dx = \int (bx^3 + a)^{\frac{3}{2}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^6, x)
```

Giac [F]

$$\int x^6 (a + bx^3)^{3/2} \, dx = \int (bx^3 + a)^{\frac{3}{2}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + bx^3)^{3/2} \, dx = \int x^6 \left(b x^3 + a\right)^{3/2} \, dx$$

input
```
int(x^6*(a + b*x^3)^(3/2),x)
```

output
```
int(x^6*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^6 (a+bx^3)^{3/2}\,dx = \frac{-\frac{432\sqrt{b x^3+a}\,a^3 x}{21505}+\frac{54\sqrt{b x^3+a}\,a^2 b x^4}{4301}+\frac{52\sqrt{b x^3+a}\,a b^2 x^7}{391}+\frac{2\sqrt{b x^3+a}\,b^3 x^{10}}{23}+\frac{432\left(\int \frac{\sqrt{b x^3+a}}{b x^3+a}dx\right)a^4}{21505}}{b^2}$$

input
```
int(x^6*(b*x^3+a)^(3/2),x)
```

output
```
(2*( - 216*sqrt(a + b*x**3)*a**3*x + 135*sqrt(a + b*x**3)*a**2*b*x**4 + 14
30*sqrt(a + b*x**3)*a*b**2*x**7 + 935*sqrt(a + b*x**3)*b**3*x**10 + 216*in
t(sqrt(a + b*x**3)/(a + b*x**3),x)*a**4))/(21505*b**2)
```

3.189 $\int x^3(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 272

$$\int x^3\left(a+bx^3\right)^{3/2}\,dx=\frac{54a^2x\sqrt{a+bx^3}}{935b}+\frac{18}{187}ax^4\sqrt{a+bx^3}+\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}$$
$$-\frac{36\ 3^{3/4}\sqrt{2+\sqrt{3}}a^3\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
54/935*a^2*x*(b*x^3+a)^(1/2)/b+18/187*a*x^4*(b*x^3+a)^(1/2)+2/17*x^4*(b*x^
3+a)^(3/2)-36/935*3^(3/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^3*(a^(1/3)+b^(1/3)*x
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)
^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x),I*3^(1/2)+2*I)/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.25

$$\int x^3(a+bx^3)^{3/2}\,dx=\frac{2x\sqrt{a+bx^3}\left((a+bx^3)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{17b}$$

input
```
Integrate[x^3*(a + b*x^3)^(3/2),x]
```

output
```
(2*x*Sqrt[a + b*x^3]*((a + b*x^3)^2 - (a^2*Hypergeometric2F1[-3/2, 1/3, 4/
3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {811, 811, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^3)^{3/2}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{9}{17}a\int x^3\sqrt{bx^3+a}dx+\frac{2}{17}x^4(a+bx^3)^{3/2}$$

$$\Big\downarrow \text{811}$$

$$\frac{9}{17}a\left(\frac{3}{11}a\int\frac{x^3}{\sqrt{bx^3+a}}dx+\frac{2}{11}x^4\sqrt{a+bx^3}\right)+\frac{2}{17}x^4(a+bx^3)^{3/2}$$

$$\Big\downarrow \text{843}$$

$$\frac{9}{17}a\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int\frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)+\frac{2}{11}x^4\sqrt{a+bx^3}\right)+\frac{2}{17}x^4(a+bx^3)^{3/2}$$

$$\Big\downarrow \ 759$$

$$\frac{9}{17}a\left(\frac{3}{11}a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$

$$\frac{2}{17}x^4\left(a+bx^3\right)^{3/2}$$

input
```
Int[x^3*(a + b*x^3)^(3/2),x]
```

output
```
(2*x^4*(a + b*x^3)^(3/2))/17 + (9*a*((2*x^4*Sqrt[a + b*x^3])/11 + (3*a*((2
*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3])))/11))/17
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.19

method	result
risch	$\frac{2x\left(55b^2x^6+100ab\,x^3+27a^2\right)\sqrt{b\,x^3+a}}{935b}+\frac{36ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}}{}$
default	$\frac{2x^7\sqrt{b\,x^3+a}\,b}{17}+\frac{40a\,x^4\sqrt{b\,x^3+a}}{187}+\frac{54a^2x\sqrt{b\,x^3+a}}{935b}+\frac{36ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right)}{2b}}}}{}$
elliptic	$\frac{2x^7\sqrt{b\,x^3+a}\,b}{17}+\frac{40a\,x^4\sqrt{b\,x^3+a}}{187}+\frac{54a^2x\sqrt{b\,x^3+a}}{935b}+\frac{36ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right)}{2b}}}}{}$

input
```
int(x^3*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/935*x*(55*b^2*x^6+100*a*b*x^3+27*a^2)/b*(b*x^3+a)^(1/2)+36/935*I/b^2*a^3
*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^
(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.22

$$\int x^3 \left(a+bx^3\right)^{3/2} \, dx = -\frac{2 \left(54 \, a^3 \sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4 \, a}{b}, x\right) - \left(55 \, b^3 x^7 + 100 \, a b^2 x^4 + 27 \, a^2 b x\right) \sqrt{b x^3 + a}\right)}{935 \, b^2}$$

input
```
integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/935*(54*a^3*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (55*b^3*x^7 + 1
00*a*b^2*x^4 + 27*a^2*b*x)*sqrt(b*x^3 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.14

$$\int x^3 \left(a+bx^3\right)^{3/2} \, dx = \frac{a^{\frac{3}{2}} x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{3}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\right| \frac{b x^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x**4*gamma(4/3)*hyper((-3/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int x^3 (a+bx^3)^{3/2}\, dx = \int (bx^3+a)^{\frac{3}{2}} x^3\, dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^3, x)
```

Giac [F]

$$\int x^3 (a+bx^3)^{3/2}\, dx = \int (bx^3+a)^{\frac{3}{2}} x^3\, dx$$

input
```
integrate(x^3*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 (a+bx^3)^{3/2}\, dx = \int x^3 \left(b\, x^3+a\right)^{3/2} dx$$

input
```
int(x^3*(a + b*x^3)^(3/2),x)
```

output
```
int(x^3*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^3 \left(a+bx^3\right)^{3/2} \, dx = \frac{\frac{54\sqrt{b\,x^3+a}\,a^2x}{935} + \frac{40\sqrt{b\,x^3+a}\,ab\,x^4}{187} + \frac{2\sqrt{b\,x^3+a}\,b^2x^7}{17} - \frac{54\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^3}{935}}{b}$$

input
```
int(x^3*(b*x^3+a)^(3/2),x)
```

output
```
(2*(27*sqrt(a + b*x**3)*a**2*x + 100*sqrt(a + b*x**3)*a*b*x**4 + 55*sqrt(a
 + b*x**3)*b**2*x**7 - 27*int(sqrt(a + b*x**3)/(a + b*x**3),x)*a**3))/(935
*b)
```

3.190 $\int (a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 246

$$\int (a+bx^3)^{3/2}\,dx = \frac{18}{55}ax\sqrt{a+bx^3}+\frac{2}{11}x(a+bx^3)^{3/2} + \frac{18\,3^{3/4}\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{55\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
18/55*a*x*(b*x^3+a)^(1/2)+2/11*x*(b*x^3+a)^(3/2)+18/55*3^(3/4)*(1/2*6^(1/2
)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1
/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(1/3)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

$$\int \left(a+bx^3\right)^{3/2} dx = \frac{ax\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2),x]
```

output
```
(a*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt
[1 + (b*x^3)/a]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {748, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{748}$$

$$\frac{9}{11}a\int \sqrt{bx^3+a}dx+\frac{2}{11}x\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{748}$$

$$\frac{9}{11}a\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)+\frac{2}{11}x\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{759}$$

$$\frac{9}{11}a\left(\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{2}{11}x\left(a+bx^3\right)^{3/2}$$

input
```
Int[(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x^3)^(3/2))/11 + (9*a*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sq
rt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/11
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.26

method	result
risch	$\frac{2x(5b\,x^3+14a)\sqrt{b\,x^3+a}}{55}-\frac{18ia^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\right.}{}}}{}$
default	$\frac{2b\,x^4\sqrt{b\,x^3+a}}{11}+\frac{28ax\sqrt{b\,x^3+a}}{55}-\frac{18ia^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{}}{}$
elliptic	$\frac{2b\,x^4\sqrt{b\,x^3+a}}{11}+\frac{28ax\sqrt{b\,x^3+a}}{55}-\frac{18ia^2\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}}}\sqrt{}}{}$

input
```
int((b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/55*x*(5*b*x^3+14*a)*(b*x^3+a)^(1/2)-18/55*I*a^2*3^(1/2)/b*(-a*b^2)^(1/3)
*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

$$\int (a + bx^3)^{3/2} \, dx = \frac{2 \left(27 \, a^2 \sqrt{b} \text{weierstrassPInverse}\left(0, -\frac{4 \, a}{b}, x\right) + (5 \, b^2 x^4 + 14 \, abx) \sqrt{bx^3 + a}\right)}{55 \, b}$$

input
```
integrate((b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/55*(27*a^2*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (5*b^2*x^4 + 14*a
*b*x)*sqrt(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int (a + bx^3)^{3/2} \, dx = \frac{a^{\frac{3}{2}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{3}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x*gamma(1/3)*hyper((-3/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{3/2} dx = \int \left(bx^3+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{3/2} dx = \int \left(bx^3+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \left(a+bx^3\right)^{3/2} dx = \frac{x\left(b\,x^3+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^3)^(3/2),x)
```

output
```
(x*(a + b*x^3)^(3/2)*hypergeom([-3/2, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
 1)^(3/2)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{3/2}\,dx=\frac{28\sqrt{b\,x^3+a}\,ax}{55}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{11}+\frac{27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2}{55}$$

input
```
int((b*x^3+a)^(3/2),x)
```

output
```
(28*sqrt(a + b*x**3)*a*x + 10*sqrt(a + b*x**3)*b*x**4 + 27*int(sqrt(a + b*
x**3)/(a + b*x**3),x)*a**2)/55
```

3.191 $\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 247

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^3}}{2x^2}+\frac{2}{5}bx\sqrt{a+bx^3}$$
$$+\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,ab^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{}\right.}{10\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/2*a*(b*x^3+a)^(1/2)/x^2+2/5*b*x*(b*x^3+a)^(1/2)+9/10*3^(3/4)*(1/2*6^(1/
2)+1/2*2^(1/2))*a*b^(2/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = -\frac{a\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2)/x^3,x]
```

output
```
-1/2*(a*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/
(x^2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {809, 748, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{ 809}$$

$$\frac{9}{4}b\int \sqrt{bx^3+a}dx-\frac{(a+bx^3)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 748}$$

$$\frac{9}{4}b\left(\frac{3}{5}a\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{2}{5}x\sqrt{a+bx^3}\right)-\frac{(a+bx^3)^{3/2}}{2x^2}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{9}{4}b\left(\frac{2\ 3^{3/4}\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}}{2x^2}$$

input
```
Int[(a + b*x^3)^(3/2)/x^3,x]
```

output
```
-1/2*(a + b*x^3)^(3/2)/x^2 + (9*b*((2*x*Sqrt[a + b*x^3])/5 + (2*3^(3/4)*Sq
rt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(5*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/4
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-4b\,x^3+5a\right)}{10x^2}-\frac{9ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{a\sqrt{b\,x^3+a}}{2x^2}+\frac{2bx\sqrt{b\,x^3+a}}{5}-\frac{9ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{a\sqrt{b\,x^3+a}}{2x^2}+\frac{2bx\sqrt{b\,x^3+a}}{5}-\frac{9ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}\cdots}{\cdots}$

input
```
int((b*x^3+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(1/2)*(-4*b*x^3+5*a)/x^2-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = \frac{27\,a\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+(4\,bx^3-5\,a)\sqrt{bx^3+a}}{10\,x^2}$$

input
```
integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/10*(27*a*sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) + (4*b*x^3 - 5*a)
*sqrt(b*x^3 + a))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)/x**3,x)
```

output
```
a**(3/2)*gamma(-2/3)*hyper((-3/2, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = \int \frac{(b\,x^3+a)^{3/2}}{x^3}\,dx$$

input
```
int((a + b*x^3)^(3/2)/x^3,x)
```

output
```
int((a + b*x^3)^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^3}\,dx = \frac{-16\sqrt{b\,x^3+a}\,a+2\sqrt{b\,x^3+a}\,b\,x^3-27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx\right)a^2x^2}{5x^2}$$

input
```
int((b*x^3+a)^(3/2)/x^3,x)
```

output
```
( - 16*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 - 27*int(sqrt(a + b*
x**3)/(a*x**3 + b*x**6),x)*a**2*x**2)/(5*x**2)
```

3.192 $\int \frac{(a+bx^3)^{3/2}}{x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 248

$$\int \frac{(a+bx^3)^{3/2}}{x^6} \, dx = -\frac{a\sqrt{a+bx^3}}{5x^5} - \frac{13b\sqrt{a+bx^3}}{20x^2} + \frac{9\, 3^{3/4}\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right), -7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*b*(b*x^3+a)^(1/2)/x^2+9/20*3^(3/4)*(1/2*6
^(1/2)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1
/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = -\frac{a\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2)/x^6,x]
```

output
```
-1/5*(a*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])
/(x^5*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {809, 809, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{9}{10}b\int \frac{\sqrt{bx^3+a}}{x^3}dx-\frac{(a+bx^3)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{809}$$

$$\frac{9}{10}b\left(\frac{3}{4}b\int \frac{1}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{2x^2}\right)-\frac{(a+bx^3)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{759}$$

$$\frac{9}{10}b\left(\frac{3^{3/4}\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}}{5x^5}$$

input
```
Int[(a + b*x^3)^(3/2)/x^6,x]
```

output
```
-1/5*(a + b*x^3)^(3/2)/x^5 + (9*b*(-1/2*Sqrt[a + b*x^3]/x^2 + (3^(3/4)*Sqr
t[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/10
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(13 b\,x^{3}+4 a)}{20 x^{5}}-\frac{9 i b\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{a\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{13 b\sqrt{b\,x^{3}+a}}{20 x^{2}}-\frac{9 i b\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{a\sqrt{b\,x^{3}+a}}{5x^{5}}-\frac{13 b\sqrt{b\,x^{3}+a}}{20 x^{2}}-\frac{9 i b\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}\cdots}{\cdots}$

input
```
int((b*x^3+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(13*b*x^3+4*a)/x^5-9/20*I*b*3^(1/2)*(-a*b^2)^(1/3)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = \frac{27\,b^{\frac{3}{2}}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-(13\,bx^3+4\,a)\sqrt{bx^3+a}}{20\,x^5}$$

input
```
integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/20*(27*b^(3/2)*x^5*weierstrassPInverse(0, -4*a/b, x) - (13*b*x^3 + 4*a)*
sqrt(b*x^3 + a))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.19

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)/x**6,x)
```

output
```
a**(3/2)*gamma(-5/3)*hyper((-5/3, -3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = \int \frac{(b\,x^3+a)^{3/2}}{x^6}\,dx$$

input
```
int((a + b*x^3)^(3/2)/x^6,x)
```

output
```
int((a + b*x^3)^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^6}\,dx = \frac{4\sqrt{b\,x^3+a}\,a-14\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}dx\right)a^2x^5}{7x^5}$$

input
```
int((b*x^3+a)^(3/2)/x^6,x)
```

output
```
(4*sqrt(a + b*x**3)*a - 14*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b*x**
3)/(a*x**6 + b*x**9),x)*a**2*x**5)/(7*x**5)
```

3.193 $\int x^7(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 556

$$\int x^7\left(a+bx^3\right)^{3/2}\,dx=-\frac{108a^3x^2\sqrt{a+bx^3}}{8645b^2}+\frac{54a^2x^5\sqrt{a+bx^3}}{6175b}+\frac{18}{475}ax^8\sqrt{a+bx^3}+\frac{432a^4\sqrt{a+bx^3}}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{25}x^8\left(a+bx^3\right)^{3/2}-\frac{216\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{144\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-108/8645*a^3*x^2*(b*x^3+a)^(1/2)/b^2+54/6175*a^2*x^5*(b*x^3+a)^(1/2)/b+18
/475*a*x^8*(b*x^3+a)^(1/2)+432/8645*a^4*(b*x^3+a)^(1/2)/b^(8/3)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x)+2/25*x^8*(b*x^3+a)^(3/2)-216/8645*3^(1/4)*(1/2*6^(1/
2)-1/2*2^(1/2))*a^(13/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2)
)*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/3
)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b
*x^3+a)^(1/2)+144/8645*2^(1/2)*3^(3/4)*a^(13/3)*(a^(1/3)+b^(1/3)*x)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
),I*3^(1/2)+2*I)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)
+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.57 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.15

$$\int x^7(a+bx^3)^{3/2}\,dx=\frac{2x^2\sqrt{a+bx^3}\left(-\left((10a-19bx^3)\left(a+bx^3\right)^2\right)+\frac{10a^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{475b^2}$$

input
```
Integrate[x^7*(a + b*x^3)^(3/2),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*(-((10*a - 19*b*x^3)*(a + b*x^3)^2) + (10*a^3*Hyper
geometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(475*b^2
)
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {811, 811, 843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^3\right)^{3/2} \, dx$$

$$\Big\downarrow \text{811}$$

$$\frac{9}{25} a \int x^7 \sqrt{bx^3+a}dx+\frac{2}{25} x^8 \left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{811}$$

$$\frac{9}{25} a \left(\frac{3}{19} a \int \frac{x^7}{\sqrt{bx^3+a}}dx+\frac{2}{19} x^8 \sqrt{a+bx^3}\right)+\frac{2}{25} x^8 \left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{843}$$

$$\frac{9}{25} a \left(\frac{3}{19} a \left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\int \frac{x^4}{\sqrt{bx^3+a}}dx}{13b}\right)+\frac{2}{19} x^8 \sqrt{a+bx^3}\right)+\frac{2}{25} x^8 \left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{843}$$

$$\frac{9}{25} a \left(\frac{3}{19} a \left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)}{13b}\right)+\frac{2}{19} x^8 \sqrt{a+bx^3}\right)+ \frac{2}{25} x^8 \left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{832}$$

$$\frac{9}{25}a\left(\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}\right)+\frac{2}{19}x^8\sqrt{a+}\right.$$

$$\frac{2}{25}x^8\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow 759$$

$$\frac{9}{25}a\left(\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\cdots\right.}{\left(\left(1+\sqrt{3}\cdots\right.\right.}}}\right.}{7b}\cdots\right.}{13b}\cdots\right.\right.$$

$$\frac{2}{25}x^8\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{2416}$$

$$\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}$$

$$4a\left(\cdots\right) \Big/ \sqrt[3]{b}$$

$$10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\cdots\right)$$

$$\frac{9}{25}a\left(\frac{3}{19}a\left(\frac{2x^5\sqrt{a+bx^3}}{13b}-\cdots\right)\right)$$

$$\frac{2}{25}x^8\left(a+bx^3\right)^{3/2}$$

input
```
Int[x^7*(a + b*x^3)^(3/2),x]
```

output
```
(2*x^8*(a + b*x^3)^(3/2))/25 + (9*a*((2*x^8*Sqrt[a + b*x^3])/19 + (3*a*((2
*x^5*Sqrt[a + b*x^3])/(13*b) - (10*a*((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a
*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 -
 Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b
)))/(13*b)))/19))/25
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 488, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{2x^{2}\left(-1729b^{3}x^{9}-2548a\,b^{2}x^{6}-189a^{2}b\,x^{3}+270a^{3}\right)\sqrt{b\,x^{3}+a}}{43225b^{2}}-\frac{144ia^{4}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\ldots}}{\ldots}$
default	$\frac{2b\,x^{11}\sqrt{b\,x^{3}+a}}{25}+\frac{56a\,x^{8}\sqrt{b\,x^{3}+a}}{475}+\frac{54a^{2}x^{5}\sqrt{b\,x^{3}+a}}{6175b}-\frac{108a^{3}x^{2}\sqrt{b\,x^{3}+a}}{8645b^{2}}-\frac{144ia^{4}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\ldots\right.}{2b}\ldots\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\ldots}{\ldots}$
elliptic	$\frac{2b\,x^{11}\sqrt{b\,x^{3}+a}}{25}+\frac{56a\,x^{8}\sqrt{b\,x^{3}+a}}{475}+\frac{54a^{2}x^{5}\sqrt{b\,x^{3}+a}}{6175b}-\frac{108a^{3}x^{2}\sqrt{b\,x^{3}+a}}{8645b^{2}}-\frac{144ia^{4}\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\ldots\right.}{2b}\ldots\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\ldots}{\ldots}$

input
```
int(x^7*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/43225*x^2*(-1729*b^3*x^9-2548*a*b^2*x^6-189*a^2*b*x^3+270*a^3)/b^2*(b*x
^3+a)^(1/2)-144/8645*I/b^3*a^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x
-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)
/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.14

$$\int x^7 \left(a+bx^3\right)^{3/2} \, dx = -\frac{2 \left(1080\, a^4 \sqrt{b} \text{weierstrassZeta}\left(0, -\frac{4\,a}{b}, \text{weierstrassPInverse}\left(0, -\frac{4\,a}{b}, x\right)\right) - \left(1729\, b^4 x^{11} + 2548\, ab^3 x^8 + 189\right.\right.}{43225\, b^3}$$

input
```
integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/43225*(1080*a^4*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(
0, -4*a/b, x)) - (1729*b^4*x^11 + 2548*a*b^3*x^8 + 189*a^2*b^2*x^5 - 270*a
^3*b*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int x^7 (a+bx^3)^{3/2}\,dx = \frac{a^{\frac{3}{2}} x^8 \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7*(b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x**8*gamma(8/3)*hyper((-3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(11/3))
```

Maxima [F]

$$\int x^7 (a+bx^3)^{3/2}\,dx = \int (bx^3+a)^{\frac{3}{2}} x^7\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^7, x)
```

Giac [F]

$$\int x^7 (a+bx^3)^{3/2}\,dx = \int (bx^3+a)^{\frac{3}{2}} x^7\,dx$$

input
```
integrate(x^7*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^7 \left(a+bx^3\right)^{3/2} \, dx = \int x^7 \, \left(b\,x^3+a\right)^{3/2} \,dx$$

input
```
int(x^7*(a + b*x^3)^(3/2),x)
```

output
```
int(x^7*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^7 \left(a+bx^3\right)^{3/2} \, dx = \frac{-\frac{108\sqrt{b\,x^3+a}\,a^3x^2}{8645}+\frac{54\sqrt{b\,x^3+a}\,a^2b\,x^5}{6175}+\frac{56\sqrt{b\,x^3+a}\,a\,b^2x^8}{475}+\frac{2\sqrt{b\,x^3+a}\,b^3x^{11}}{25}+\frac{216\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^4}{8645}}{b^2}$$

input
```
int(x^7*(b*x^3+a)^(3/2),x)
```

output
```
(2*( - 270*sqrt(a + b*x**3)*a**3*x**2 + 189*sqrt(a + b*x**3)*a**2*b*x**5 +
 2548*sqrt(a + b*x**3)*a*b**2*x**8 + 1729*sqrt(a + b*x**3)*b**3*x**11 + 54
0*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**4))/(43225*b**2)
```

3.194 $\int x^4(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 532

$$\int x^4\left(a+bx^3\right)^{3/2}\,dx=\frac{54a^2x^2\sqrt{a+bx^3}}{1729b}+\frac{18}{247}ax^5\sqrt{a+bx^3}-\frac{216a^3\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$
$$+\frac{108\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{72\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
54/1729*a^2*x^2*(b*x^3+a)^(1/2)/b+18/247*a*x^5*(b*x^3+a)^(1/2)-216/1729*a^
3*(b*x^3+a)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+2/19*x^5*(b*x^3+
a)^(3/2)+108/1729*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(10/3)*(a^(1/3)+b^(1
/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/
3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(
1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-72/1729*2^(1/2)*3^(3/4)*
a^(10/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3
)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.63 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.13

$$\int x^4 \left(a+bx^3\right)^{3/2} dx = \frac{2x^2\sqrt{a+bx^3}\left(\left(a+bx^3\right)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt{1+\frac{bx^3}{a}}}\right)}{19b}$$

input
```
Integrate[x^4*(a + b*x^3)^(3/2),x]
```

output
```
(2*x^2*Sqrt[a + b*x^3]*((a + b*x^3)^2 - (a^2*Hypergeometric2F1[-3/2, 2/3,
5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(19*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {811, 811, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^{3/2} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{9}{19}a\int x^4\sqrt{bx^3+a}dx+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{811}$$

$$\frac{9}{19}a\left(\frac{3}{13}a\int \frac{x^4}{\sqrt{bx^3+a}}dx+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{843}$$

$$\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{832}$$

$$\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)+\frac{2}{13}x^5\sqrt{a+bx^3}\right)+\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \text{759}$$

$$\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{Ellipti}}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}}\right)}{7b}\right)\right.$$

$$\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$

$\downarrow$ 2416

$$\frac{9}{19}a\left(\frac{3}{13}a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}}{\sqrt[3]{b}}\right.\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.}{}\right.\right.$$

$$\frac{2}{19}x^5\left(a+bx^3\right)^{3/2}$$

input

```
Int[x^4*(a + b*x^3)^(3/2),x]
```

output
```
(2*x^5*(a + b*x^3)^(3/2))/19 + (9*a*((2*x^5*Sqrt[a + b*x^3])/13 + (3*a*((2
*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)
*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)))/13))/19
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.90

method	result
risch	$72ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)}{2b}}}$ $\frac{2x^2\left(91b^2x^6+154ab\,x^3+27a^2\right)\sqrt{b\,x^3+a}}{1729b}+$ ———
default	$72ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right.}{2b}}}$ $\frac{2x^8\sqrt{b\,x^3+a}\,b}{19}+\frac{44a\,x^5\sqrt{b\,x^3+a}}{247}+\frac{54a^2x^2\sqrt{b\,x^3+a}}{1729b}+$ ———
elliptic	$72ia^3\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-}{-\frac{3\left(-a\,b^2\right.}{2b}}}$ $\frac{2x^8\sqrt{b\,x^3+a}\,b}{19}+\frac{44a\,x^5\sqrt{b\,x^3+a}}{247}+\frac{54a^2x^2\sqrt{b\,x^3+a}}{1729b}+$ ———

input
```
int(x^4*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1729*x^2*(91*b^2*x^6+154*a*b*x^3+27*a^2)/b*(b*x^3+a)^(1/2)+72/1729*I/b^2
*a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*E
llipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.13

$$\int x^4\left(a+bx^3\right)^{3/2}\,dx=\frac{2\left(108\,a^3\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(91\,b^3x^8+154\,ab^2x\right.\right.}{1729\,b^2}$$

input
```
integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/1729*(108*a^3*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0,
-4*a/b, x)) + (91*b^3*x^8 + 154*a*b^2*x^5 + 27*a^2*b*x^2)*sqrt(b*x^3 + a))
/b^2
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int x^4 \left(a+bx^3\right)^{3/2}\,dx=\frac{a^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x**5*gamma(5/3)*hyper((-3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x^4 \left(a+bx^3\right)^{3/2}\,dx=\int \left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a+bx^3\right)^{3/2}\,dx=\int \left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a+bx^3)^{3/2} \, dx = \int x^4 \left(b x^3 + a\right)^{3/2} dx$$

input
```
int(x^4*(a + b*x^3)^(3/2),x)
```

output
```
int(x^4*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^4 (a+bx^3)^{3/2} \, dx = \frac{\frac{54\sqrt{b x^3+a}\, a^2 x^2}{1729} + \frac{44\sqrt{b x^3+a}\, a b x^5}{247} + \frac{2\sqrt{b x^3+a}\, b^2 x^8}{19} - \frac{108 \left(\int \frac{\sqrt{b x^3+a}\, x}{b x^3+a} dx\right) a^3}{1729}}{b}$$

input
```
int(x^4*(b*x^3+a)^(3/2),x)
```

output
```
(2*(27*sqrt(a + b*x**3)*a**2*x**2 + 154*sqrt(a + b*x**3)*a*b*x**5 + 91*sqr
t(a + b*x**3)*b**2*x**8 - 54*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**3
))/(1729*b)
```

3.195 $\int x(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 508

$$\int x\left(a+bx^3\right)^{3/2}\,dx=\frac{18}{91}ax^2\sqrt{a+bx^3}+\frac{54a^2\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)\middle|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{18\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
18/91*a*x^2*(b*x^3+a)^(1/2)+54/91*a^2*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))
*a^(1/3)+b^(1/3)*x)+2/13*x^2*(b*x^3+a)^(3/2)-27/91*3^(1/4)*(1/2*6^(1/2)-1/
2*2^(1/2))*a^(7/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1
/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/3)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a
)^(1/2)+18/91*2^(1/2)*3^(3/4)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2
)+2*I)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.83 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.10

$$\int x(a+bx^3)^{3/2}\,dx=\frac{ax^2\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x*(a + b*x^3)^(3/2),x]
```

output
```
(a*x^2*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {811, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^{3/2}\,dx$$

$$\Big\downarrow \; 811$$

$$\frac{9}{13}a\int x\sqrt{bx^3+a}dx+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 811$$

$$\frac{9}{13}a\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 832$$

$$\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)+$$
$$\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 759$$

$$\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcs}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$
$$\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}$$

$$\Big\downarrow \; 2416$$

$$\frac{9}{13}a\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.$$

$$\frac{2}{13}x^2\left(a+bx^3\right)^{3/2}$$

input
```
Int[x*(a + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(a + b*x^3)^(3/2))/13 + (9*a*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2
*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)
*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt
[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/7))/13
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.91

method	result
risch	$\frac{2x^2(7bx^3+16a)\sqrt{bx^3+a}}{91} - \frac{18ia^2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\right.}{}}}{}$
default	$\frac{2x^5\sqrt{bx^3+a}\,b}{13}+\frac{32a\,x^2\sqrt{bx^3+a}}{91} - \frac{18ia^2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}}{}$
elliptic	$\frac{2x^5\sqrt{bx^3+a}\,b}{13}+\frac{32a\,x^2\sqrt{bx^3+a}}{91} - \frac{18ia^2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-ab^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-ab^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{2b}}}}{}$

input
```
int(x*(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/91*x^2*(7*b*x^3+16*a)*(b*x^3+a)^(1/2)-18/91*I*a^2*3^(1/2)/b*(-a*b^2)^(1/
3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.11

$$\int x(a+bx^3)^{3/2}\,dx = -\frac{2\left(27\,a^2\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(7\,b^2x^5+16\,abx^2\right)\sqrt{bx^3+a}\right)}{91\,b}$$

input
```
integrate(x*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/91*(27*a^2*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) - (7*b^2*x^5 + 16*a*b*x^2)*sqrt(b*x^3 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int x(a+bx^3)^{3/2}\,dx=\frac{a^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x**2*gamma(2/3)*hyper((-3/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int x(a+bx^3)^{3/2}\,dx=\int (bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+bx^3)^{3/2}\,dx=\int (bx^3+a)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^3\right)^{3/2}\,dx=\int x\,\left(b\,x^3+a\right)^{3/2}\,dx$$

input
```
int(x*(a + b*x^3)^(3/2),x)
```

output
```
int(x*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x\left(a+bx^3\right)^{3/2}\,dx=\frac{32\sqrt{b\,x^3+a}\,a\,x^2}{91}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{13}+\frac{27\left(\int\frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2}{91}$$

input
```
int(x*(b*x^3+a)^(3/2),x)
```

output
```
(32*sqrt(a + b*x**3)*a*x**2 + 14*sqrt(a + b*x**3)*b*x**5 + 27*int((sqrt(a
+ b*x**3)*x)/(a + b*x**3),x)*a**2)/91
```

3.196 $\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 505

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+bx^3}}{x}+\frac{2}{7}bx^2\sqrt{a+bx^3}+\frac{27a\sqrt[3]{b}\sqrt{a+bx^3}}{7\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{14\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{9\sqrt{2}3^{3/4}a^{4/3}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-a*(b*x^3+a)^(1/2)/x+2/7*b*x^2*(b*x^3+a)^(1/2)+27*a*b^(1/3)*(b*x^3+a)^(1/2
)/(7*(1+3^(1/2))*a^(1/3)+7*b^(1/3)*x)-27/14*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/
2))*a^(4/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(
1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
+9/7*2^(1/2)*3^(3/4)*a^(4/3)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF
(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)
+2*I)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(x*
Sqrt[1 + (b*x^3)/a]))
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {809, 811, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \; 809$$

$$\frac{9}{2}b\int x\sqrt{bx^3+a}dx-\frac{\left(a+bx^3\right)^{3/2}}{x}$$

$$\Big\downarrow \; 811$$

$$\frac{9}{2}b\left(\frac{3}{7}a\int \frac{x}{\sqrt{bx^3+a}}dx+\frac{2}{7}x^2\sqrt{a+bx^3}\right)-\frac{\left(a+bx^3\right)^{3/2}}{x}$$

$$\Big\downarrow \; 832$$

$$\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)+\frac{2}{7}x^2\sqrt{a+bx^3}\right)-\frac{\left(a+bx^3\right)^{3/2}}{x}$$

$$\Big\downarrow \; 759$$

$$\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}}{x}$$

$$\Big\downarrow \; 2416$$

$$\frac{9}{2}b\left(\frac{3}{7}a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}}{x}$$

input
```
Int[(a + b*x^3)^(3/2)/x^2,x]
```

output
```
-((a + b*x^3)^(3/2)/x) + (9*b*((2*x^2*Sqrt[a + b*x^3])/7 + (3*a*(((2*Sqrt[
a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[
2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*S
qrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/7))/2
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-2b\,x^3+7a\right)}{7x}-\frac{9ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{a\sqrt{b\,x^3+a}}{x}+\frac{2b\,x^2\sqrt{b\,x^3+a}}{7}-\frac{9ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\cdots\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{a\sqrt{b\,x^3+a}}{x}+\frac{2b\,x^2\sqrt{b\,x^3+a}}{7}-\frac{9ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(\cdots\right.}{\cdots}}\cdots}{\cdots}$

input
```
int((b*x^3+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^3+a)^(1/2)*(-2*b*x^3+7*a)/x-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = -\frac{27\,a\sqrt{b}x\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(2\,bx^3-7\,a\right)\sqrt{bx^3+a}}{7\,x}$$

input
```
integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-1/7*(27*a*sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) - (2*b*x^3 - 7*a)*sqrt(b*x^3 + a))/x
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)/x**2,x)
```

output
```
a**(3/2)*gamma(-1/3)*hyper((-3/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = \frac{2\,(b\,x^3+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{7}{6};\ -\frac{1}{6};\ -\frac{a}{b\,x^3}\right)}{7\,x\,\left(\frac{a}{b\,x^3}+1\right)^{3/2}}$$

input
```
int((a + b*x^3)^(3/2)/x^2,x)
```

output
```
(2*(a + b*x^3)^(3/2)*hypergeom([-3/2, -7/6], -1/6, -a/(b*x^3)))/(7*x*(a/(b
*x^3) + 1)^(3/2))
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^2}\,dx = \frac{20\sqrt{b\,x^3+a}\,a+2\sqrt{b\,x^3+a}\,b\,x^3+27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)a^2x}{7x}$$

input
```
int((b*x^3+a)^(3/2)/x^2,x)
```

output
```
(20*sqrt(a + b*x**3)*a + 2*sqrt(a + b*x**3)*b*x**3 + 27*int(sqrt(a + b*x**
3)/(a*x**2 + b*x**5),x)*a**2*x)/(7*x)
```

3.197 $\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 506

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = -\frac{a\sqrt{a+bx^3}}{4x^4} - \frac{11b\sqrt{a+bx^3}}{8x} + \frac{27b^{4/3}\sqrt{a+bx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{9\,3^{3/4}\sqrt[3]{a}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*b*(b*x^3+a)^(1/2)/x+27*b^(4/3)*(b*x^3+a)^(
1/2)/(8*(1+3^(1/2))*a^(1/3)+8*b^(1/3)*x)-27/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^
(1/2))*a^(1/3)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1
/2)+9/8*3^(3/4)*a^(1/3)*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-
3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)
*2^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx=-\frac{a\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(3/2)/x^5,x]
```

output
```
-1/4*(a*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])
/(x^4*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {809, 809, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{3/2}}{x^5}\,dx$$

$$\Big\downarrow \text{ 809}$$

$$\frac{9}{8}b\int \frac{\sqrt{bx^3+a}}{x^2}dx-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{9}{8}b\left(\frac{3}{2}b\int \frac{x}{\sqrt{bx^3+a}}dx-\frac{\sqrt{a+bx^3}}{x}\right)-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)-\frac{\sqrt{a+bx^3}}{x}\right)-\frac{\left(a+bx^3\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\text{arcsin}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}}{4x^4}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{9}{8}b\left(\frac{3}{2}b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\right.\right.$$

$$\frac{\left(a+bx^3\right)^{3/2}}{4x^4}$$

input
```
Int[(a + b*x^3)^(3/2)/x^5,x]
```

output
```
-1/4*(a + b*x^3)^(3/2)/x^4 + (9*b*(-(Sqrt[a + b*x^3]/x) + (3*b*(((2*Sqrt[a
 + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2
 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[(
(1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sq
rt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/2))/8
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,(11b\,x^{3}+2a)}{8x^{4}}-\frac{9ib\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-a}{}\right.}{}}}{}$
default	$-\frac{a\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{11b\sqrt{b\,x^{3}+a}}{8x}-\frac{9ib\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\right.}{}}}{}$
elliptic	$-\frac{a\sqrt{b\,x^{3}+a}}{4x^{4}}-\frac{11b\sqrt{b\,x^{3}+a}}{8x}-\frac{9ib\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\right.}{}}}{}$

input
```
int((b*x^3+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(1/2)*(11*b*x^3+2*a)/x^4-9/8*I*b*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = -\frac{27\,b^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(11\,bx^3+2\,a\right)\sqrt{bx^3+a}}{8\,x^4}$$

input
```
integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
-1/8*(27*b^(3/2)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*
a/b, x)) + (11*b*x^3 + 2*a)*sqrt(b*x^3 + a))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{4}{3} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(3/2)/x**5,x)
```

output
```
a**(3/2)*gamma(-4/3)*hyper((-3/2, -4/3), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(bx^3+a)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = \int \frac{(b\,x^3+a)^{3/2}}{x^5}\,dx$$

input
```
int((a + b*x^3)^(3/2)/x^5,x)
```

output
```
int((a + b*x^3)^(3/2)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{3/2}}{x^5}\,dx = \frac{-8\sqrt{b\,x^3+a}\,a+10\sqrt{b\,x^3+a}\,b\,x^3-27\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx\right)a^2x^4}{5x^4}$$

input
```
int((b*x^3+a)^(3/2)/x^5,x)
```

output
```
( - 8*sqrt(a + b*x**3)*a + 10*sqrt(a + b*x**3)*b*x**3 - 27*int(sqrt(a + b*
x**3)/(a*x**5 + b*x**8),x)*a**2*x**4)/(5*x**4)
```

3.198 $\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = -\frac{2a^3\sqrt{a+bx^3}}{3b^4}+\frac{2a^2(a+bx^3)^{3/2}}{3b^4}-\frac{2a(a+bx^3)^{5/2}}{5b^4}+\frac{2(a+bx^3)^{7/2}}{21b^4}$$

output
```
-2/3*a^3*(b*x^3+a)^(1/2)/b^4+2/3*a^2*(b*x^3+a)^(3/2)/b^4-2/5*a*(b*x^3+a)^(
5/2)/b^4+2/21*(b*x^3+a)^(7/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(-16a^3+8a^2bx^3-6ab^2x^6+5b^3x^9)}{105b^4}$$

input
```
Integrate[x^11/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(-16*a^3 + 8*a^2*b*x^3 - 6*a*b^2*x^6 + 5*b^3*x^9))/(105
*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{x^9}{\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(-\frac{a^3}{b^3\sqrt{bx^3+a}}+\frac{3\sqrt{bx^3+a}a^2}{b^3}-\frac{3\left(bx^3+a\right)^{3/2}a}{b^3}+\frac{\left(bx^3+a\right)^{5/2}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^3\sqrt{a+bx^3}}{b^4}+\frac{2a^2\left(a+bx^3\right)^{3/2}}{b^4}+\frac{2\left(a+bx^3\right)^{7/2}}{7b^4}-\frac{6a\left(a+bx^3\right)^{5/2}}{5b^4}\right)$$

input
```
Int[x^11/Sqrt[a + b*x^3],x]
```

output
```
((-2*a^3*Sqrt[a + b*x^3])/b^4 + (2*a^2*(a + b*x^3)^(3/2))/b^4 - (6*a*(a +
b*x^3)^(5/2))/(5*b^4) + (2*(a + b*x^3)^(7/2))/(7*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-5b^{3}x^{9}+6a\,b^{2}x^{6}-8a^{2}b\,x^{3}+16a^{3}\right)}{105b^{4}}$	47
trager	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-5b^{3}x^{9}+6a\,b^{2}x^{6}-8a^{2}b\,x^{3}+16a^{3}\right)}{105b^{4}}$	47
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-5b^{3}x^{9}+6a\,b^{2}x^{6}-8a^{2}b\,x^{3}+16a^{3}\right)}{105b^{4}}$	47
pseudoelliptic	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-5b^{3}x^{9}+6a\,b^{2}x^{6}-8a^{2}b\,x^{3}+16a^{3}\right)}{105b^{4}}$	47
orering	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-5b^{3}x^{9}+6a\,b^{2}x^{6}-8a^{2}b\,x^{3}+16a^{3}\right)}{105b^{4}}$	47
default	$\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21b}-\frac{4a\,x^{6}\sqrt{b\,x^{3}+a}}{35b^{2}}+\frac{16a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b^{3}}-\frac{32a^{3}\sqrt{b\,x^{3}+a}}{105b^{4}}$	74
elliptic	$\frac{2x^{9}\sqrt{b\,x^{3}+a}}{21b}-\frac{4a\,x^{6}\sqrt{b\,x^{3}+a}}{35b^{2}}+\frac{16a^{2}x^{3}\sqrt{b\,x^{3}+a}}{105b^{3}}-\frac{32a^{3}\sqrt{b\,x^{3}+a}}{105b^{4}}$	74

input
```
int(x^11/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(b*x^3+a)^(1/2)*(-5*b^3*x^9+6*a*b^2*x^6-8*a^2*b*x^3+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = \frac{2\,(5\,b^3x^9-6\,ab^2x^6+8\,a^2bx^3-16\,a^3)\sqrt{bx^3+a}}{105\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(5*b^3*x^9 - 6*a*b^2*x^6 + 8*a^2*b*x^3 - 16*a^3)*sqrt(b*x^3 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = \begin{cases} -\frac{32a^3\sqrt{a+bx^3}}{105b^4} + \frac{16a^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4ax^6\sqrt{a+bx^3}}{35b^2} + \frac{2x^9\sqrt{a+bx^3}}{21b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-32*a**3*sqrt(a + b*x**3)/(105*b**4) + 16*a**2*x**3*sqrt(a + b*
x**3)/(105*b**3) - 4*a*x**6*sqrt(a + b*x**3)/(35*b**2) + 2*x**9*sqrt(a + b
*x**3)/(21*b), Ne(b, 0)), (x**12/(12*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{7}{2}}}{21\,b^4} - \frac{2\,(bx^3+a)^{\frac{5}{2}}a}{5\,b^4} + \frac{2\,(bx^3+a)^{\frac{3}{2}}a^2}{3\,b^4} - \frac{2\,\sqrt{bx^3+a}a^3}{3\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(b*x^3 + a)^(7/2)/b^4 - 2/5*(b*x^3 + a)^(5/2)*a/b^4 + 2/3*(b*x^3 + a)
^(3/2)*a^2/b^4 - 2/3*sqrt(b*x^3 + a)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{bx^3+a}a^3}{3\,b^4} + \frac{2\left(5\,(bx^3+a)^{\frac{7}{2}} - 21\,(bx^3+a)^{\frac{5}{2}}a + 35\,(bx^3+a)^{\frac{3}{2}}a^2\right)}{105\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x^3 + a)*a^3/b^4 + 2/105*(5*(b*x^3 + a)^(7/2) - 21*(b*x^3 + a)
^(5/2)*a + 35*(b*x^3 + a)^(3/2)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = \frac{2\,x^9\,\sqrt{b\,x^3+a}}{21\,b} - \frac{32\,a^3\,\sqrt{b\,x^3+a}}{105\,b^4} - \frac{4\,a\,x^6\,\sqrt{b\,x^3+a}}{35\,b^2} + \frac{16\,a^2\,x^3\,\sqrt{b\,x^3+a}}{105\,b^3}$$

input
```
int(x^11/(a + b*x^3)^(1/2),x)
```

output
```
(2*x^9*(a + b*x^3)^(1/2))/(21*b) - (32*a^3*(a + b*x^3)^(1/2))/(105*b^4) -
(4*a*x^6*(a + b*x^3)^(1/2))/(35*b^2) + (16*a^2*x^3*(a + b*x^3)^(1/2))/(105
*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.56

$$\int \frac{x^{11}}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(5b^3x^9-6a\,b^2x^6+8a^2b\,x^3-16a^3)}{105b^4}$$

input
```
int(x^11/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 16*a**3 + 8*a**2*b*x**3 - 6*a*b**2*x**6 + 5*b**3*x
**9))/(105*b**4)
```

3.199 $\int \frac{x^8}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2a^2\sqrt{a+bx^3}}{3b^3} - \frac{4a(a+bx^3)^{3/2}}{9b^3} + \frac{2(a+bx^3)^{5/2}}{15b^3}$$

output
```
2/3*a^2*(b*x^3+a)^(1/2)/b^3-4/9*a*(b*x^3+a)^(3/2)/b^3+2/15*(b*x^3+a)^(5/2)
/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}(8a^2-4abx^3+3b^2x^6)}{45b^3}$$

input
```
Integrate[x^8/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \ 53$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2\sqrt{bx^3+a}}-\frac{2\sqrt{bx^3+a}a}{b^2}+\frac{\left(bx^3+a\right)^{3/2}}{b^2}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{2a^2\sqrt{a+bx^3}}{b^3}+\frac{2\left(a+bx^3\right)^{5/2}}{5b^3}-\frac{4a\left(a+bx^3\right)^{3/2}}{3b^3}\right)$$

input
```
Int[x^8/Sqrt[a + b*x^3],x]
```

output
```
((2*a^2*Sqrt[a + b*x^3])/b^3 - (4*a*(a + b*x^3)^(3/2))/(3*b^3) + (2*(a + b
*x^3)^(5/2))/(5*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{2\sqrt{b\,x^{3}+a}\,\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45b^{3}}$	36
trager	$\frac{2\sqrt{b\,x^{3}+a}\,\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45b^{3}}$	36
risch	$\frac{2\sqrt{b\,x^{3}+a}\,\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45b^{3}}$	36
pseudoelliptic	$\frac{2\sqrt{b\,x^{3}+a}\,\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45b^{3}}$	36
orering	$\frac{2\sqrt{b\,x^{3}+a}\,\left(3b^{2}x^{6}-4ab\,x^{3}+8a^{2}\right)}{45b^{3}}$	36
default	$\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}$	54
elliptic	$\frac{2x^{6}\sqrt{b\,x^{3}+a}}{15b}-\frac{8a\,x^{3}\sqrt{b\,x^{3}+a}}{45b^{2}}+\frac{16a^{2}\sqrt{b\,x^{3}+a}}{45b^{3}}$	54

input
```
int(x^8/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(b*x^3+a)^(1/2)*(3*b^2*x^6-4*a*b*x^3+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2\,(3\,b^2x^6-4\,abx^3+8\,a^2)\sqrt{bx^3+a}}{45\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^3 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{16a^2\sqrt{a+bx^3}}{45b^3} - \frac{8ax^3\sqrt{a+bx^3}}{45b^2} + \frac{2x^6\sqrt{a+bx^3}}{15b} & \text{for } b \neq 0 \\ \frac{x^9}{9\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((16*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*a*x**3*sqrt(a + b*x**3)/
(45*b**2) + 2*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), (x**9/(9*sqrt(a)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}}{15\,b^3} - \frac{4\,(bx^3+a)^{\frac{3}{2}}a}{9\,b^3} + \frac{2\,\sqrt{bx^3+a}a^2}{3\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)/b^3 - 4/9*(b*x^3 + a)^(3/2)*a/b^3 + 2/3*sqrt(b*x^3
+ a)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{bx^3+a}a^2}{3\,b^3} + \frac{2\left(3\,(bx^3+a)^{\frac{5}{2}} - 10\,(bx^3+a)^{\frac{3}{2}}a\right)}{45\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)*a^2/b^3 + 2/45*(3*(b*x^3 + a)^(5/2) - 10*(b*x^3 + a)^(
3/2)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(8\,a^2-4\,a\,b\,x^3+3\,b^2\,x^6)}{45\,b^3}$$

input
```
int(x^8/(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.58

$$\int \frac{x^8}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(3b^2x^6-4ab\,x^3+8a^2)}{45b^3}$$

input
```
int(x^8/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(8*a**2 - 4*a*b*x**3 + 3*b**2*x**6))/(45*b**3)
```

3.200 $\int \frac{x^5}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = -\frac{2a\sqrt{a+bx^3}}{3b^2} + \frac{2(a+bx^3)^{3/2}}{9b^2}$$

output
```
-2/3*a*(b*x^3+a)^(1/2)/b^2+2/9*(b*x^3+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = \frac{2(-2a+bx^3)\sqrt{a+bx^3}}{9b^2}$$

input
```
Integrate[x^5/Sqrt[a + b*x^3],x]
```

output
```
(2*(-2*a + b*x^3)*Sqrt[a + b*x^3])/(9*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{\sqrt{bx^3+a}}{b}-\frac{a}{b\sqrt{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2\left(a+bx^3\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx^3}}{b^2}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^3],x]
```

output
```
((-2*a*Sqrt[a + b*x^3])/b^2 + (2*(a + b*x^3)^(3/2))/(3*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$	25
trager	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$	25
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$	25
pseudoelliptic	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$	25
orering	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-b\,x^{3}+2a\right)}{9b^{2}}$	25
default	$\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}$	34
elliptic	$\frac{2x^{3}\sqrt{b\,x^{3}+a}}{9b}-\frac{4a\sqrt{b\,x^{3}+a}}{9b^{2}}$	34

input
```
int(x^5/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(b*x^3+a)^(1/2)*(-b*x^3+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+a}(bx^3-2\,a)}{9\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(b*x^3 + a)*(b*x^3 - 2*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = \begin{cases} -\frac{4a\sqrt{a+bx^3}}{9b^2} + \frac{2x^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((-4*a*sqrt(a + b*x**3)/(9*b**2) + 2*x**3*sqrt(a + b*x**3)/(9*b),
 Ne(b, 0)), (x**6/(6*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b^2} - \frac{2\,\sqrt{bx^3+a}a}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b^2 - 2/3*sqrt(b*x^3 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b^2} - \frac{2\,\sqrt{bx^3+a}a}{3\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b^2 - 2/3*sqrt(b*x^3 + a)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = -\frac{2\,\sqrt{b\,x^3+a}\,(2\,a-b\,x^3)}{9\,b^2}$$

input
```
int(x^5/(a + b*x^3)^(1/2),x)
```

output
```
-(2*(a + b*x^3)^(1/2)*(2*a - b*x^3))/(9*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{x^5}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b\,x^3-2a)}{9b^2}$$

input
```
int(x^5/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 2*a + b*x**3))/(9*b**2)
```

3.201 $\int \frac{x^2}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{3b}$$

output
```
2/3*(b*x^3+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{3b}$$

input
```
Integrate[x^2/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3])/(3*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^3}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{2\sqrt{a+bx^3}}{3b}$$

input
```
Int[x^2/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[a + b*x^3])/(3*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
derivativedivides	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
default	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
trager	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
risch	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
elliptic	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
pseudoelliptic	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15
orering	$\frac{2\sqrt{b\,x^{3}+a}}{3b}$	15

input
```
int(x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x^3+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+a}}{3\,b}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\sqrt{a+bx^3}}{3b} & \text{for } b \neq 0 \\ \frac{x^3}{3\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)**(1/2),x)
```

output
```
Piecewise((2*sqrt(a + b*x**3)/(3*b), Ne(b, 0)), (x**3/(3*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{bx^3+a}}{3b}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^3 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{bx^3+a}}{3b}$$

input
```
integrate(x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x^3 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}}{3\,b}$$

input
```
int(x^2/(a + b*x^3)^(1/2),x)
```

output
```
(2*(a + b*x^3)^(1/2))/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{b\,x^3+a}}{3b}$$

input
```
int(x^2/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x**3))/(3*b)
```

3.202 $\int \frac{1}{x\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

output
```
-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^3}}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3}\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3$$
$$\Big\downarrow \text{73}$$
$$\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{3b}$$
$$\Big\downarrow \text{221}$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	20
elliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	20
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}$	20

input
```
int(1/x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = \left[\frac{\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)}{3\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)}{3a}\right]$$

input
```
integrate(1/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3)/sqrt(a), 2/3*sqrt(
-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

input
```
integrate(1/x/(b*x**3+a)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = \frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

input
```
integrate(1/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}}$$

input
```
integrate(1/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^3)^(1/2)),x)
```

output
```
log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)/(
3*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a}\left(\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)-\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\right)}{3a}$$

input
```
int(1/x/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x**3) - sqrt(a)) - log(sqrt(a + b*x**3) + sqrt(a)
)))/(3*a)
```

3.203 $\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{3ax^3} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
-1/3*(b*x^3+a)^(1/2)/a/x^3+1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{3ax^3} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^3]),x]
```

output
```
-1/3*Sqrt[a + b*x^3]/(a*x^3) + (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(
3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^3}}{ax^3}\right)$$

input
```
Int[1/(x^4*Sqrt[a + b*x^3]),x]
```

output
```
(-(Sqrt[a + b*x^3]/(a*x^3)) + (b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)
)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\sqrt{b\,x^{3}+a}}{3 a\,x^{3}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}$	39
risch	$-\frac{\sqrt{b\,x^{3}+a}}{3 a\,x^{3}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}$	39
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{3 a\,x^{3}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}$	39
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right) b\,x^{3}-\sqrt{b\,x^{3}+a}\,\sqrt{a}}{3 a^{\frac{3}{2}} x^{3}}$	43

input
```
int(1/x^4/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^3+a)^(1/2)/a/x^3+1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = \left[\frac{\sqrt{a}bx^3\log\left(\frac{bx^3+2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)-2\sqrt{bx^3+a}a}{6a^2x^3}, -\frac{\sqrt{-a}bx^3\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+\sqrt{bx^3+a}a}{3a^2x^3}\right]$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*(sqrt(a)*b*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2
*sqrt(b*x^3 + a)*a)/(a^2*x^3), -1/3*(sqrt(-a)*b*x^3*arctan(sqrt(-a)/sqrt(b
*x^3 + a)) + sqrt(b*x^3 + a)*a)/(a^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

input
```
integrate(1/x**4/(b*x**3+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + b*asinh(sqrt(a)/(sqrt(b)*x*
*(3/2)))/(3*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{bx^3+a}b}{3\left((bx^3+a)a-a^2\right)} - \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{6\,a^{\frac{3}{2}}}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) - 1/6*b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = -\frac{1}{3}\,b\left(\frac{\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx^3+a}}{abx^3}\right)$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-1/3*b*(arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^3 + a)/(a
*b*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}^3}{x^6}\right)}{6\,a^{3/2}} - \frac{\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a + b*x^3)^(1/2)),x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6
))/(6*a^(3/2)) - (a + b*x^3)^(1/2)/(3*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4\sqrt{a+bx^3}}\,dx$$
$$= \frac{-2\sqrt{b\,x^3+a}\,a-\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3+\sqrt{a}\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b\,x^3}{6a^2x^3}$$

input
```
int(1/x^4/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*a - sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3
+ sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(6*a**2*x**3)
```

3.204 $\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{6ax^6}+\frac{b\sqrt{a+bx^3}}{4a^2x^3}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-1/6*(b*x^3+a)^(1/2)/a/x^6+1/4*b*(b*x^3+a)^(1/2)/a^2/x^3-1/4*b^2*arctanh((
b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a+bx^3}(-2a+3bx^3)}{12a^2x^6}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[1/(x^7*Sqrt[a + b*x^3]),x]
```

output
```
(Sqrt[a + b*x^3]*(-2*a + 3*b*x^3))/(12*a^2*x^6) - (b^2*ArcTanh[Sqrt[a + b*
x^3]/Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^9\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{3b\int \frac{1}{x^6\sqrt{bx^3+a}}dx^3}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{2a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{a}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^3}}{ax^3}\right)}{4a}-\frac{\sqrt{a+bx^3}}{2ax^6}\right)$$

input
```
Int[1/(x^7*Sqrt[a + b*x^3]),x]
```

output
```
(-1/2*Sqrt[a + b*x^3]/(a*x^6) - (3*b*(-(Sqrt[a + b*x^3]/(a*x^3)) + (b*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(4*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-3b\,x^{3}+2a\right)}{12a^{2}x^{6}}-\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	51
default	$-\frac{\sqrt{b\,x^{3}+a}}{6a\,x^{6}}+\frac{b\sqrt{b\,x^{3}+a}}{4a^{2}x^{3}}-\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	59
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{6a\,x^{6}}+\frac{b\sqrt{b\,x^{3}+a}}{4a^{2}x^{3}}-\frac{b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}$	59
pseudoelliptic	$\frac{-3\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)b^{2}x^{6}+3b\,x^{3}\sqrt{b\,x^{3}+a}\,\sqrt{a}-2a^{\frac{3}{2}}\sqrt{b\,x^{3}+a}}{12a^{\frac{5}{2}}x^{6}}$	64

input
```
int(1/x^7/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(b*x^3+a)^(1/2)*(-3*b*x^3+2*a)/a^2/x^6-1/4*b^2*arctanh((b*x^3+a)^(1/
2)/a^(1/2))/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = \left[\frac{3\,\sqrt{a}b^2x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)+2\,(3\,abx^3-2\,a^2)\sqrt{bx^3+a}}{24\,a^3x^6}, \frac{3\,\sqrt{-a}b^2x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+(3\,abx^3}{12\,a^3x^6}\right.$$

input
```
integrate(1/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(3*sqrt(a)*b^2*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3
) + 2*(3*a*b*x^3 - 2*a^2)*sqrt(b*x^3 + a))/(a^3*x^6), 1/12*(3*sqrt(-a)*b^2
*x^6*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (3*a*b*x^3 - 2*a^2)*sqrt(b*x^3 + a
))/(a^3*x^6)]
```

Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = -\frac{1}{6\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b^{\frac{3}{2}}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate(1/x**7/(b*x**3+a)**(1/2),x)
```

output
```
-1/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + sqrt(b)/(12*a*x**(9/2)*sqr
t(a/(b*x**3) + 1)) + b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - b**
2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = \frac{b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}} + \frac{3\,(bx^3+a)^{\frac{3}{2}}b^2 - 5\,\sqrt{bx^3+a}ab^2}{12\,\left((bx^3+a)^2a^2 - 2\,(bx^3+a)a^3 + a^4\right)}$$

input
```
integrate(1/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/
2) + 1/12*(3*(b*x^3 + a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)
^2*a^2 - 2*(b*x^3 + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx^3+a)^{\frac{3}{2}}b^3-5\,\sqrt{bx^3+a}ab^3}{a^2b^2x^6}}{12\,b}$$

input
```
integrate(1/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*b^3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^3 +
a)^(3/2)*b^3 - 5*sqrt(b*x^3 + a)*a*b^3)/(a^2*b^2*x^6))/b
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = \frac{b^2\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{8\,a^{5/2}}-\frac{\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\sqrt{b\,x^3+a}}{4\,a^2\,x^3}$$

input
```
int(1/(x^7*(a + b*x^3)^(1/2)),x)
```

output
```
(b^2*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x
^6))/(8*a^(5/2)) - (a + b*x^3)^(1/2)/(6*a*x^6) + (b*(a + b*x^3)^(1/2))/(4*
a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^7\sqrt{a+bx^3}}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^2+6\sqrt{b\,x^3+a}\,ab\,x^3+3\sqrt{a}\,\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b^2x^6-3\sqrt{a}\,\log\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)b^2x^6}{24a^3x^6}$$

input
```
int(1/x^7/(b*x^3+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**2 + 6*sqrt(a + b*x**3)*a*b*x**3 + 3*sqrt(a)*log(
sqrt(a + b*x**3) - sqrt(a))*b**2*x**6 - 3*sqrt(a)*log(sqrt(a + b*x**3) + s
qrt(a))*b**2*x**6)/(24*a**3*x**6)
```

3.205 $\int \frac{x^6}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 254

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = -\frac{16ax\sqrt{a+bx^3}}{55b^2}+\frac{2x^4\sqrt{a+bx^3}}{11b} + \frac{32\sqrt{2+\sqrt{3}}a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-16/55*a*x*(b*x^3+a)^(1/2)/b^2+2/11*x^4*(b*x^3+a)^(1/2)/b+32/165*(1/2*6^(1
/2)+1/2*2^(1/2))*a^2*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^
(7/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.31

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \frac{2\left(-8a^2x-3abx^4+5b^2x^7+8a^2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{55b^2\sqrt{a+bx^3}}$$

input
```
Integrate[x^6/Sqrt[a + b*x^3],x]
```

output
```
(2*(-8*a^2*x - 3*a*b*x^4 + 5*b^2*x^7 + 8*a^2*x*Sqrt[1 + (b*x^3)/a]*Hyperge
ometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\int \frac{x^3}{\sqrt{bx^3+a}}dx}{11b}$$

$$\Big\downarrow \text{843}$$

$$\frac{2x^4\sqrt{a+bx^3}}{11b}-\frac{8a\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{11b}$$

$$\Big\downarrow \text{759}$$

$$\frac{\frac{2x^4\sqrt{a+bx^3}}{11b} - 8a\left(\frac{2x\sqrt{a+bx^3}}{5b} - \frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{11b}$$

input
```
Int[x^6/Sqrt[a + b*x^3],x]
```

output
```
(2*x^4*Sqrt[a + b*x^3])/(11*b) - (8*a*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sq
rt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(11*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{2x(-5b\,x^{3}+8a)\sqrt{b\,x^{3}+a}}{55b^{2}}-\frac{32ia^{2}\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x\cdots\right.}{\cdots}}\cdots}{\cdots}$
default	$\frac{2x^{4}\sqrt{b\,x^{3}+a}}{11b}-\frac{16ax\sqrt{b\,x^{3}+a}}{55b^{2}}-\frac{32ia^{2}\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
elliptic	$\frac{2x^{4}\sqrt{b\,x^{3}+a}}{11b}-\frac{16ax\sqrt{b\,x^{3}+a}}{55b^{2}}-\frac{32ia^{2}\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a\,b^{2})^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a\,b^{2})^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a\,b^{2})^{\frac{1}{3}}}{b}}{-\frac{3(-a\,b^{2})^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^{2})^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$

input
```
int(x^6/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*x*(-5*b*x^3+8*a)*(b*x^3+a)^(1/2)/b^2-32/165*I*a^2/b^3*3^(1/2)*(-a*b^
2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \frac{2\left(16\,a^2\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(5\,b^2x^4-8\,abx\right)\sqrt{bx^3+a}\right)}{55\,b^3}$$

input
```
integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(16*a^2*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (5*b^2*x^4 - 8*a*
b*x)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \frac{x^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(b*x**3+a)**(1/2),x)
```

output
```
x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \int \frac{x^6}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \int \frac{x^6}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \int \frac{x^6}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(x^6/(a + b*x^3)^(1/2),x)
```

output
```
int(x^6/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^3}}\,dx = \frac{-\frac{16\sqrt{b\,x^3+a}\,ax}{55}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{11}+\frac{16\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a^2}{55}}{b^2}$$

input
```
int(x^6/(b*x^3+a)^(1/2),x)
```

output
```
(2*( - 8*sqrt(a + b*x**3)*a*x + 5*sqrt(a + b*x**3)*b*x**4 + 8*int(sqrt(a +
 b*x**3)/(a + b*x**3),x)*a**2))/(55*b**2)
```

3.206 $\int \frac{x^3}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 230

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx=\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/5*x*(b*x^3+a)^(1/2)/b-4/15*(1/2*6^(1/2)+1/2*2^(1/2))*a*(a^(1/3)+b^(1/3)*
x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+
b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1
+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.27

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \frac{2x\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{a+bx^3}}$$

input
```
Integrate[x^3/Sqrt[a + b*x^3],x]
```

output
```
(2*x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -
((b*x^3)/a)]))/(5*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2x\sqrt{a+bx^3}}{5b} - \frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[x^3/Sqrt[a + b*x^3],x]
```

output
```
(2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.30

method	result
default	$\frac{2x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(\cdots\right.}{}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{15b^2\sqrt{b\,x^3\cdots}}$
risch	$\frac{2x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(\cdots\right.}{}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{15b^2\sqrt{b\,x^3\cdots}}$
elliptic	$\frac{2x\sqrt{b\,x^3+a}}{5b}+\frac{4ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(\cdots\right.}{}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{15b^2\sqrt{b\,x^3\cdots}}$

input
```
int(x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x*(b*x^3+a)^(1/2)/b+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)
^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}bx - 2\,a\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{5\,b^2}$$

input
```
integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^3 + a)*b*x - 2*a*sqrt(b)*weierstrassPInverse(0, -4*a/b, x))/
b^2
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(b*x**3+a)**(1/2),x)
```

output
```
x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \int \frac{x^3}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(x^3/(a + b*x^3)^(1/2),x)
```

output
```
int(x^3/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+bx^3}}\,dx = \frac{\frac{2\sqrt{b\,x^3+a}\,x}{5}-\frac{2\left(\int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx\right)a}{5}}{b}$$

input
```
int(x^3/(b*x^3+a)^(1/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x - int(sqrt(a + b*x**3)/(a + b*x**3),x)*a))/(5*b)
```

3.207 $\int \frac{1}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 207

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^
(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3
^(3/4)/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x
)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx=\frac{x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt{a+bx^3}}$$

input
```
Integrate[1/Sqrt[a + b*x^3],x]
```

output
```
(x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sqr
t[a + b*x^3]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[1/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.37

method	result
default	$-\frac{2 i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}}{3b\sqrt{b x^3+a}}$
elliptic	$-\frac{2 i \sqrt{3}\left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}}{3b\sqrt{b x^3+a}}$

input
```
int(1/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(
-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{2\,\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4*a/b, x)/sqrt(b)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(b*x**3+a)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^3 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \frac{x\,\sqrt{\frac{b\,x^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^3+a}}$$

input
```
int(1/(a + b*x^3)^(1/2),x)
```

output
```
(x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/3, 1/2], 4/3, -(b*x^3)/a))/(a + b*x^
3)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^3+a}dx$$

input
```
int(1/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a + b*x**3),x)
```

3.208 $\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 234

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/2*(b*x^3+a)^(1/2)/a/x^2-1/6*(1/2*6^(1/2)+1/2*2^(1/2))*b^(2/3)*(a^(1/3)+
b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*
a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
(1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])
/(x^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}$$

$$\Big\downarrow \text{759}$$

$$-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^3]),x]
```

output
```
-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\sqrt{b\,x^{3}+a}}{2a\,x^{2}}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{6a\sqrt{b\,x^{3}+a}}$
risch	$-\frac{\sqrt{b\,x^{3}+a}}{2a\,x^{2}}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{6a\sqrt{b\,x^{3}+a}}$
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{2a\,x^{2}}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{6a\sqrt{b\,x^{3}+a}}$

input
```
int(1/x^3/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^3+a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3
^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(
1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{b}x^2\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\sqrt{bx^3+a}}{2ax^2}$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(b)*x^2*weierstrassPInverse(0, -4*a/b, x) + sqrt(b*x^3 + a))/(a*
x^2)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{1}{x^3\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^6+a\,x^3}dx$$

input
```
int(1/x^3/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**3 + b*x**6),x)
```

3.209 $\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 256

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{5ax^5}+\frac{7b\sqrt{a+bx^3}}{20a^2x^2} + \frac{7\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{20\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-1/5*(b*x^3+a)^(1/2)/a/x^5+7/20*b*(b*x^3+a)^(1/2)/a^2/x^2+7/60*(1/2*6^(1/2
)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*
a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/
a^2/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/
(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^6*Sqrt[a + b*x^3]),x]
```

output
```
-1/5*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/3, 1/2, -2/3, -((b*x^3)/a)]
)/(x^5*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{7b\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{7b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}$$

$$\Big\downarrow \text{759}$$

$$-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}$$

input
```
Int[1/(x^6*Sqrt[a + b*x^3]),x]
```

output
```
-1/5*Sqrt[a + b*x^3]/(a*x^5) - (7*b*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[
2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
-7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(10*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{b x^3+a}\,(-7 b x^3+4 a)}{20 a^2 x^5} - \frac{7 i b\sqrt{3}\,(-a b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(-\cdots}{}\right)}{}}}{}$
default	$-\frac{\sqrt{b x^3+a}}{5 a x^5}+\frac{7 b\sqrt{b x^3+a}}{20 a^2 x^2} - \frac{7 i b\sqrt{3}\,(-a b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(\cdots}{}\right)}{}}}{}$
elliptic	$-\frac{\sqrt{b x^3+a}}{5 a x^5}+\frac{7 b\sqrt{b x^3+a}}{20 a^2 x^2} - \frac{7 i b\sqrt{3}\,(-a b^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{(-a b^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-a b^2)^{\frac{1}{3}}}{b}}{-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{(\cdots}{}\right)}{}}}{}$

input
```
int(1/x^6/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(b*x^3+a)^(1/2)*(-7*b*x^3+4*a)/a^2/x^5-7/60*I*b/a^2*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*
b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = \frac{7\,b^{\frac{3}{2}}x^5\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)+\left(7\,bx^3-4\,a\right)\sqrt{bx^3+a}}{20\,a^2x^5}$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(7*b^(3/2)*x^5*weierstrassPInverse(0, -4*a/b, x) + (7*b*x^3 - 4*a)*sq
rt(b*x^3 + a))/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^6}}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^6}}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{1}{x^6\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/(x^6*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^9+a\,x^6}dx$$

input
```
int(1/x^6/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**6 + b*x**9),x)
```

3.210 $\int \frac{x^7}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 514

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = -\frac{20ax^2\sqrt{a+bx^3}}{91b^2}+\frac{2x^5\sqrt{a+bx^3}}{13b}+\frac{80a^2\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$-\frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{91b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{80\sqrt{2}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-20/91*a*x^2*(b*x^3+a)^(1/2)/b^2+2/13*x^5*(b*x^3+a)^(1/2)/b+80/91*a^2*(b*x
^3+a)^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-40/91*3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*a^(7/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(8/
3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(
b*x^3+a)^(1/2)+80/273*2^(1/2)*a^(7/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*Elliptic
F(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2
)+2*I)*3^(3/4)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b
^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.16

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = \frac{2x^2\left(-10a^2-3abx^3+7b^2x^6+10a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{91b^2\sqrt{a+bx^3}}$$

input
```
Integrate[x^7/Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*(-10*a^2 - 3*a*b*x^3 + 7*b^2*x^6 + 10*a^2*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(91*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\int \frac{x^4}{\sqrt{bx^3+a}}dx}{13b}$$

$$\Big\downarrow \text{843}$$

$$\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)}{13b}$$

$$\Big\downarrow \text{832}$$

$$\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)}{13b}$$

$$\Big\downarrow \text{759}$$

$$\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\cdots\right)}{\sqrt[3]{b}x+\left(1+\cdots\right)}\right)\cdots\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots\right)}{7b}\cdots\right)}{13b}$$

$\downarrow$ 2416

$$\frac{2x^5\sqrt{a+bx^3}}{13b}-\frac{10a\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}\right)\cdots\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}\cdots\right)}{\cdots}\cdots\right)}{13b}$$

input
```
Int[x^7/Sqrt[a + b*x^3],x]
```

output
```
(2*x^5*Sqrt[a + b*x^3])/(13*b) - (10*a*((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4
*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x)
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1
 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7
*b)))/(13*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{2x^{2}\left(-7b\,x^{3}+10a\right)\sqrt{b\,x^{3}+a}}{91b^{2}}-\frac{80ia^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\frac{i\cdots}{-\cdots}}\cdots}{\cdots}$
default	$\frac{2x^{5}\sqrt{b\,x^{3}+a}}{13b}-\frac{20a\,x^{2}\sqrt{b\,x^{3}+a}}{91b^{2}}-\frac{80ia^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2x^{5}\sqrt{b\,x^{3}+a}}{13b}-\frac{20a\,x^{2}\sqrt{b\,x^{3}+a}}{91b^{2}}-\frac{80ia^{2}\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{\cdots}}{\cdots}$

input
```
int(x^7/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/91*x^2*(-7*b*x^3+10*a)/b^2*(b*x^3+a)^(1/2)-80/273*I*a^2/b^3*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/
(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = -\frac{2\left(40\,a^2\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)-\left(7\,b^2x^5-10\,abx^2\right)\sqrt{bx^3+a}\right)}{91\,b^3}$$

input
```
integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/91*(40*a^2*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4
*a/b, x)) - (7*b^2*x^5 - 10*a*b*x^2)*sqrt(b*x^3 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(b*x**3+a)**(1/2),x)
```

output
```
x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = \int \frac{x^7}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx = \int \frac{x^7}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx=\int \frac{x^7}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(x^7/(a + b*x^3)^(1/2),x)
```

output
```
int(x^7/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{a+bx^3}}\,dx=\frac{-\frac{20\sqrt{b\,x^3+a}\,a\,x^2}{91}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{13}+\frac{40\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2}{91}}{b^2}$$

input
```
int(x^7/(b*x^3+a)^(1/2),x)
```

output
```
(2*( - 10*sqrt(a + b*x**3)*a*x**2 + 7*sqrt(a + b*x**3)*b*x**5 + 20*int((sq
rt(a + b*x**3)*x)/(a + b*x**3),x)*a**2))/(91*b**2)
```

3.211 $\int \frac{x^4}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 490

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \frac{2x^2\sqrt{a+bx^3}}{7b} - \frac{8a\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{8\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/7*x^2*(b*x^3+a)^(1/2)/b-8/7*a*(b*x^3+a)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)+4/7*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^(4/3)*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-8/21*2^(1/2)*a^(4/3)*(a^(
1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(5/3)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx=\frac{2x^2\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{7b\sqrt{a+bx^3}}$$

input
```
Integrate[x^4/Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
 -((b*x^3)/a)]))/(7*b*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{7b}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}-\frac{2\left(1-\right.}{}\right.}{7b}$$

input
```
Int[x^4/Sqrt[a + b*x^3],x]
```

output
```
(2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.93

method	result
default	$\frac{2x^2\sqrt{b\,x^3+a}}{7b}+\frac{8ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}\right.}{\left(-a\,b^2\right)\cdots}}\cdots}{\cdots}$
risch	$\frac{2x^2\sqrt{b\,x^3+a}}{7b}+\frac{8ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}\right.}{\left(-a\,b^2\right)\cdots}}\cdots}{\cdots}$
elliptic	$\frac{2x^2\sqrt{b\,x^3+a}}{7b}+\frac{8ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}\right.}{\left(-a\,b^2\right)\cdots}}\cdots}{\cdots}$

input
```
int(x^4/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*x^2*(b*x^3+a)^(1/2)/b+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I
*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt{bx^3+a}bx^2+4\,a\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)\right)}{7\,b^2}$$

input
```
integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(sqrt(b*x^3 + a)*b*x^2 + 4*a*sqrt(b)*weierstrassZeta(0, -4*a/b, weiers
trassPInverse(0, -4*a/b, x)))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.08

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(b*x**3+a)**(1/2),x)
```

output
```
x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \int \frac{x^4}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \int \frac{x^4}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(x^4/(a + b*x^3)^(1/2),x)
```

output
```
int(x^4/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^3}}\,dx = \frac{\frac{2\sqrt{b\,x^3+a}\,x^2}{7} - \frac{4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a}{7}}{b}$$

input
```
int(x^4/(b*x^3+a)^(1/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x**2 - 2*int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a))
/(7*b)
```

3.212 $\int \frac{x}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 462

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2*(b*x^3+a)^(1/2)/b^(2/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-3^(1/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2
))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/b^(2/
3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(
b*x^3+a)^(1/2)+2/3*2^(1/2)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF((
(1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2
*I)*3^(3/4)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1
/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx=\frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt{a+bx^3}}$$

input
```
Integrate[x/Sqrt[a + b*x^3],x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(
2*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{832}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2416}$$

$$\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[x/Sqrt[a + b*x^3],x]
```

output
```
((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - S
qrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$
elliptic	$-\frac{2i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{}$

input
```
int(x/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1
/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.05

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = -\frac{2\,\mathrm{weierstrassZeta}\left(0,-\frac{4\,a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{\sqrt{b}}$$

input
```
integrate(x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))/sqrt(b)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.08

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(b*x**3+a)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqr
t(a)*gamma(5/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \int \frac{x}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \int \frac{x}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(x/(a + b*x^3)^(1/2),x)
```

output
```
int(x/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx$$

input
```
int(x/(b*x^3+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**3)*x)/(a + b*x**3),x)
```

3.213 $\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 484

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{ax} + \frac{\sqrt[3]{b}\sqrt{a+bx^3}}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{\sqrt{2}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-(b*x^3+a)^(1/2)/a/x+b^(1/3)*(b*x^3+a)^(1/2)/a/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1/3)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+1/3*2^(1/2)*b^(1/3)*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx=-\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
-((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(x
*Sqrt[a + b*x^3]))
```

Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx$$

$$\downarrow 847$$

$$\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}$$

$$\downarrow 832$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}$$

$$\downarrow 759$$

$$\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\ldots}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\ldots\right)}{2a}$$

$$\frac{\sqrt{a+bx^3}}{ax}$$

$$\downarrow 2416$$

$$\frac{b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}-\frac{2\left(1-\ldots\right)}{\ldots}\right)}{2a}$$

$$\frac{\sqrt{a+bx^3}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^3]),x]
```

output
```
-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^
(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])))/(2*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\sqrt{b\,x^3+a}}{a x}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a \cdots\right.}{2b \cdots}\cdots\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$
risch	$-\frac{\sqrt{b\,x^3+a}}{a x}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a \cdots\right.}{2b \cdots}\cdots\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^3+a}}{a x}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a \cdots\right.}{2b \cdots}\cdots\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}$

input
```
int(1/x^2/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^3+a)^(1/2)/a/x-1/3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1
/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))
^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)
*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b
*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/
b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{b}x\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+\sqrt{bx^3+a}}{ax}$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(b)*x*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/b, x))
+ sqrt(b*x^3 + a))/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle|\, \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{\frac{a}{b\,x^3}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b\,x^3}\right)}{5\,x\,\sqrt{b\,x^3+a}}$$

input
```
int(1/(x^2*(a + b*x^3)^(1/2)),x)
```

output
```
-(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 5/6], 11/6, -a/(b*x^3)))/(5*x*(a
 + b*x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**2 + b*x**5),x)
```

3.214 $\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 514

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{a+bx^3}}{4ax^4}+\frac{5b\sqrt{a+bx^3}}{8a^2x}-\frac{5b^{4/3}\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*b*(b*x^3+a)^(1/2)/a^2/x-5/8*b^(4/3)*(b*x^3+
a)^(1/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+5/16*3^(1/4)*(1/2*6^(1/2)-1/2
*2^(1/2))*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/
3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)/a^(5/3)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)
^(1/2)-5/24*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(
1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(
3/4)/a^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx=-\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^5*Sqrt[a + b*x^3]),x]
```

output
```
-1/4*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)]
)/(x^4*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{847}$$

$$-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{832}$$

$$-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{759}$$

$$-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{\ldots}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\ldots\right)}{2a}\ldots\right)}{8a}$$

$$\frac{\sqrt{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{5b\left(\frac{b\left(\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}} - - \cdots 2(\cdots}{2a} \cdots}{8a} \cdots$$

$$\frac{\sqrt{a+bx^3}}{4ax^4}$$

input

```
Int[1/(x^5*Sqrt[a + b*x^3]),x]
```

output
```
-1/4*Sqrt[a + b*x^3]/(a*x^4) - (5*b*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sq
rt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sq
rt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3]
)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*
x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/(8*a
)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-5b\,x^{3}+2a\right)}{8a^{2}x^{4}}+\frac{5ib\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
default	$-\frac{\sqrt{b\,x^{3}+a}}{4a\,x^{4}}+\frac{5b\sqrt{b\,x^{3}+a}}{8a^{2}x}+\frac{5ib\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{4a\,x^{4}}+\frac{5b\sqrt{b\,x^{3}+a}}{8a^{2}x}+\frac{5ib\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(\cdots\right.}{\cdots}\right.}{\cdots}}\cdots}{\cdots}$

input
```
int(1/x^5/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^3+a)^(1/2)*(-5*b*x^3+2*a)/a^2/x^4+5/24*I*b/a^2*3^(1/2)*(-a*b^2)^
(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b
/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = \frac{5\,b^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+\left(5\,bx^3-2\,a\right)\sqrt{bx^3+a}}{8\,a^2x^4}$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(5*b^(3/2)*x^4*weierstrassZeta(0, -4*a/b, weierstrassPInverse(0, -4*a/
b, x)) + (5*b*x^3 - 2*a)*sqrt(b*x^3 + a))/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{1}{x^5\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b\,x^8+a\,x^5}dx$$

input
```
int(1/x^5/(b*x^3+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a*x**5 + b*x**8),x)
```

3.215 $\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{2a^3}{3b^4\sqrt{a+bx^3}} + \frac{2a^2\sqrt{a+bx^3}}{b^4} - \frac{2a(a+bx^3)^{3/2}}{3b^4} + \frac{2(a+bx^3)^{5/2}}{15b^4}$$

output
```
2/3*a^3/b^4/(b*x^3+a)^(1/2)+2*a^2*(b*x^3+a)^(1/2)/b^4-2/3*a*(b*x^3+a)^(3/2
)/b^4+2/15*(b*x^3+a)^(5/2)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.63

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{2(16a^3+8a^2bx^3-2ab^2x^6+b^3x^9)}{15b^4\sqrt{a+bx^3}}$$

input
```
Integrate[x^11/(a + b*x^3)^(3/2),x]
```

output
```
(2*(16*a^3 + 8*a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9))/(15*b^4*Sqrt[a + b*x^3]
)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^3\,(bx^3+a)^{3/2}}+\frac{3a^2}{b^3\sqrt{bx^3+a}}-\frac{3\sqrt{bx^3+a}a}{b^3}+\frac{(bx^3+a)^{3/2}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a^3}{b^4\sqrt{a+bx^3}}+\frac{6a^2\sqrt{a+bx^3}}{b^4}-\frac{2a\left(a+bx^3\right)^{3/2}}{b^4}+\frac{2\left(a+bx^3\right)^{5/2}}{5b^4}\right)$$

input
```
Int[x^11/(a + b*x^3)^(3/2),x]
```

output
```
((2*a^3)/(b^4*Sqrt[a + b*x^3]) + (6*a^2*Sqrt[a + b*x^3])/b^4 - (2*a*(a + b
*x^3)^(3/2))/b^4 + (2*(a + b*x^3)^(5/2))/(5*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{\frac{2}{15}b^3x^9-\frac{4}{15}a\,b^2x^6+\frac{16}{15}a^2b\,x^3+\frac{32}{15}a^3}{\sqrt{b\,x^3+a}\,b^4}$	46
trager	$\frac{\frac{2}{15}b^3x^9-\frac{4}{15}a\,b^2x^6+\frac{16}{15}a^2b\,x^3+\frac{32}{15}a^3}{\sqrt{b\,x^3+a}\,b^4}$	46
pseudoelliptic	$\frac{\frac{2}{15}b^3x^9-\frac{4}{15}a\,b^2x^6+\frac{16}{15}a^2b\,x^3+\frac{32}{15}a^3}{\sqrt{b\,x^3+a}\,b^4}$	46
orering	$\frac{\frac{2}{15}b^3x^9-\frac{4}{15}a\,b^2x^6+\frac{16}{15}a^2b\,x^3+\frac{32}{15}a^3}{\sqrt{b\,x^3+a}\,b^4}$	46
risch	$\frac{2(b^2x^6-3ab\,x^3+11a^2)\sqrt{b\,x^3+a}}{15b^4}+\frac{2a^3}{3b^4\sqrt{b\,x^3+a}}$	53
default	$\frac{2a^3}{3b^4\sqrt{(x^3+\frac{a}{b})b}}+\frac{2x^6\sqrt{b\,x^3+a}}{15b^2}-\frac{2a\,x^3\sqrt{b\,x^3+a}}{5b^3}+\frac{22a^2\sqrt{b\,x^3+a}}{15b^4}$	75
elliptic	$\frac{2a^3}{3b^4\sqrt{(x^3+\frac{a}{b})b}}+\frac{2x^6\sqrt{b\,x^3+a}}{15b^2}-\frac{2a\,x^3\sqrt{b\,x^3+a}}{5b^3}+\frac{22a^2\sqrt{b\,x^3+a}}{15b^4}$	75

input
```
int(x^11/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15/(b*x^3+a)^(1/2)*(b^3*x^9-2*a*b^2*x^6+8*a^2*b*x^3+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.73

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^9 - 2\,ab^2x^6 + 8\,a^2bx^3 + 16\,a^3)\sqrt{bx^3+a}}{15\,(b^5x^3 + ab^4)}$$

input
```
integrate(x^11/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(b^3*x^9 - 2*a*b^2*x^6 + 8*a^2*b*x^3 + 16*a^3)*sqrt(b*x^3 + a)/(b^5*x
^3 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \begin{cases} \frac{32a^3}{15b^4\sqrt{a+bx^3}} + \frac{16a^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4ax^6}{15b^2\sqrt{a+bx^3}} + \frac{2x^9}{15b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((32*a**3/(15*b**4*sqrt(a + b*x**3)) + 16*a**2*x**3/(15*b**3*sqrt
(a + b*x**3)) - 4*a*x**6/(15*b**2*sqrt(a + b*x**3)) + 2*x**9/(15*b*sqrt(a
+ b*x**3)), Ne(b, 0)), (x**12/(12*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(bx^3+a)^{\frac{5}{2}}}{15\,b^4} - \frac{2\,(bx^3+a)^{\frac{3}{2}}a}{3\,b^4} + \frac{2\,\sqrt{bx^3+a}a^2}{b^4} + \frac{2\,a^3}{3\,\sqrt{bx^3+a}b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/15*(b*x^3 + a)^(5/2)/b^4 - 2/3*(b*x^3 + a)^(3/2)*a/b^4 + 2*sqrt(b*x^3 +
a)*a^2/b^4 + 2/3*a^3/(sqrt(b*x^3 + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{2\,a^3}{3\,\sqrt{bx^3+a}b^4} + \frac{2\left((bx^3+a)^{\frac{5}{2}}b^{16} - 5\,(bx^3+a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx^3+a}a^2b^{16}\right)}{15\,b^{20}}$$

input
```
integrate(x^11/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*a^3/(sqrt(b*x^3 + a)*b^4) + 2/15*((b*x^3 + a)^(5/2)*b^16 - 5*(b*x^3 +
a)^(3/2)*a*b^16 + 15*sqrt(b*x^3 + a)*a^2*b^16)/b^20
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{22\,a^2\,\sqrt{b\,x^3+a}}{15\,b^4} + \frac{2\,a^3}{3\,b^4\,\sqrt{b\,x^3+a}} + \frac{2\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2} - \frac{2\,a\,x^3\,\sqrt{b\,x^3+a}}{5\,b^3}$$

input
```
int(x^11/(a + b*x^3)^(3/2),x)
```

output
```
(22*a^2*(a + b*x^3)^(1/2))/(15*b^4) + (2*a^3)/(3*b^4*(a + b*x^3)^(1/2)) +
(2*x^6*(a + b*x^3)^(1/2))/(15*b^2) - (2*a*x^3*(a + b*x^3)^(1/2))/(5*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

$$\int \frac{x^{11}}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b^3x^9-2a\,b^2x^6+8a^2b\,x^3+16a^3)}{15b^4\,(b\,x^3+a)}$$

input
```
int(x^11/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(16*a**3 + 8*a**2*b*x**3 - 2*a*b**2*x**6 + b**3*x**9))
/(15*b**4*(a + b*x**3))
```

3.216 $\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = -\frac{2a^2}{3b^3\sqrt{a+bx^3}} - \frac{4a\sqrt{a+bx^3}}{3b^3} + \frac{2(a+bx^3)^{3/2}}{9b^3}$$

output
```
-2/3*a^2/b^3/(b*x^3+a)^(1/2)-4/3*a*(b*x^3+a)^(1/2)/b^3+2/9*(b*x^3+a)^(3/2)
/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = \frac{2(-8a^2-4abx^3+b^2x^6)}{9b^3\sqrt{a+bx^3}}$$

input
```
Integrate[x^8/(a + b*x^3)^(3/2),x]
```

output
```
(2*(-8*a^2 - 4*a*b*x^3 + b^2*x^6))/(9*b^3*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2\left(bx^3+a\right)^{3/2}}-\frac{2a}{b^2\sqrt{bx^3+a}}+\frac{\sqrt{bx^3+a}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2a^2}{b^3\sqrt{a+bx^3}}-\frac{4a\sqrt{a+bx^3}}{b^3}+\frac{2\left(a+bx^3\right)^{3/2}}{3b^3}\right)$$

input
```
Int[x^8/(a + b*x^3)^(3/2),x]
```

output
```
((-2*a^2)/(b^3*Sqrt[a + b*x^3]) - (4*a*Sqrt[a + b*x^3])/b^3 + (2*(a + b*x^
3)^(3/2))/(3*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\frac{2}{9}b^2x^6-\frac{8}{9}ab\,x^3-\frac{16}{9}a^2}{\sqrt{b\,x^3+a}\,b^3}$	35
gosper	$-\frac{2(-b^2x^6+4ab\,x^3+8a^2)}{9\sqrt{b\,x^3+a}\,b^3}$	36
trager	$-\frac{2(-b^2x^6+4ab\,x^3+8a^2)}{9\sqrt{b\,x^3+a}\,b^3}$	36
orering	$-\frac{2(-b^2x^6+4ab\,x^3+8a^2)}{9\sqrt{b\,x^3+a}\,b^3}$	36
risch	$-\frac{2(-b\,x^3+5a)\sqrt{b\,x^3+a}}{9b^3}-\frac{2a^2}{3b^3\sqrt{b\,x^3+a}}$	43
default	$-\frac{2a^2}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^3\sqrt{b\,x^3+a}}{9b^2}-\frac{10a\sqrt{b\,x^3+a}}{9b^3}$	55
elliptic	$-\frac{2a^2}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^3\sqrt{b\,x^3+a}}{9b^2}-\frac{10a\sqrt{b\,x^3+a}}{9b^3}$	55

input
```
int(x^8/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(b^2*x^6-4*a*b*x^3-8*a^2)/(b*x^3+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^6-4\,abx^3-8\,a^2)\sqrt{bx^3+a}}{9\,(b^4x^3+ab^3)}$$

input
```
integrate(x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(b^2*x^6 - 4*a*b*x^3 - 8*a^2)*sqrt(b*x^3 + a)/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{16a^2}{9b^3\sqrt{a+bx^3}} - \frac{8ax^3}{9b^2\sqrt{a+bx^3}} + \frac{2x^6}{9b\sqrt{a+bx^3}} & \text{for } b\neq 0 \\ \frac{x^9}{9a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-16*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*a*x**3/(9*b**2*sqrt(a +
b*x**3)) + 2*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**9/(9*a**(3/2)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(bx^3+a)^{\frac{3}{2}}}{9\,b^3} - \frac{4\,\sqrt{bx^3+a}a}{3\,b^3} - \frac{2\,a^2}{3\,\sqrt{bx^3+a}b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/9*(b*x^3 + a)^(3/2)/b^3 - 4/3*sqrt(b*x^3 + a)*a/b^3 - 2/3*a^2/(sqrt(b*x^
3 + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\frac{3\,a^2}{\sqrt{bx^3+a}b} - \frac{(bx^3+a)^{\frac{3}{2}}b^2 - 6\,\sqrt{bx^3+a}ab^2}{b^3}\right)}{9\,b^2}$$

input
```
integrate(x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/9*(3*a^2/(sqrt(b*x^3 + a)*b) - ((b*x^3 + a)^(3/2)*b^2 - 6*sqrt(b*x^3 +
a)*a*b^2)/b^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = -\frac{12\,a\,(b\,x^3+a) - 2\,(b\,x^3+a)^2 + 6\,a^2}{9\,b^3\,\sqrt{b\,x^3+a}}$$

input
```
int(x^8/(a + b*x^3)^(3/2),x)
```

output
```
-(12*a*(a + b*x^3) - 2*(a + b*x^3)^2 + 6*a^2)/(9*b^3*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x^8}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b^2x^6-4ab\,x^3-8a^2)}{9b^3\,(b\,x^3+a)}$$

input
```
int(x^8/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*( - 8*a**2 - 4*a*b*x**3 + b**2*x**6))/(9*b**3*(a + b*x
**3))
```

3.217 $\int \frac{x^5}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^5}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2a}{3b^2\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{3b^2}$$

output
```
2/3*a/b^2/(b*x^3+a)^(1/2)+2/3*(b*x^3+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(2a+bx^3)}{3b^2\sqrt{a+bx^3}}$$

input
```
Integrate[x^5/(a + b*x^3)^(3/2),x]
```

output
```
(2*(2*a + b*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{1}{b\sqrt{bx^3+a}}-\frac{a}{b\,(bx^3+a)^{3/2}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2a}{b^2\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{b^2}\right)$$

input
```
Int[x^5/(a + b*x^3)^(3/2),x]
```

output
```
((2*a)/(b^2*Sqrt[a + b*x^3]) + (2*Sqrt[a + b*x^3])/b^2)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\frac{2b\,x^3}{3}+\frac{4a}{3}}{\sqrt{b\,x^3+a}\,b^2}$	24
trager	$\frac{\frac{2b\,x^3}{3}+\frac{4a}{3}}{\sqrt{b\,x^3+a}\,b^2}$	24
orering	$\frac{\frac{2b\,x^3}{3}+\frac{4a}{3}}{\sqrt{b\,x^3+a}\,b^2}$	24
pseudoelliptic	$\frac{2b\,x^3+4a}{3\sqrt{b\,x^3+a}\,b^2}$	25
risch	$\frac{2a}{3b^2\sqrt{b\,x^3+a}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}$	31
default	$\frac{2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}$	35
elliptic	$\frac{2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2\sqrt{b\,x^3+a}}{3b^2}$	35

input
```
int(x^5/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^3+a)^(1/2)*(b*x^3+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx = \frac{2\,(bx^3+2\,a)\sqrt{bx^3+a}}{3\,(b^3x^3+ab^2)}$$

input
```
integrate(x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b*x^3 + 2*a)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx = \begin{cases} \frac{4a}{3b^2\sqrt{a+bx^3}} + \frac{2x^3}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((4*a/(3*b**2*sqrt(a + b*x**3)) + 2*x**3/(3*b*sqrt(a + b*x**3)),
Ne(b, 0)), (x**6/(6*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{bx^3+a}}{3\,b^2} + \frac{2\,a}{3\,\sqrt{bx^3+a}b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^3 + a)/b^2 + 2/3*a/(sqrt(b*x^3 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\frac{\sqrt{bx^3+a}}{b} + \frac{a}{\sqrt{bx^3+a}b}\right)}{3\,b}$$

input
```
integrate(x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*(sqrt(b*x^3 + a)/b + a/(sqrt(b*x^3 + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx = \frac{2\,b\,x^3+4\,a}{3\,b^2\,\sqrt{b\,x^3+a}}$$

input
```
int(x^5/(a + b*x^3)^(3/2),x)
```

output
```
(4*a + 2*b*x^3)/(3*b^2*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,(b\,x^3+2a)}{3b^2\,(b\,x^3+a)}$$

input
```
int(x^5/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*(2*a + b*x**3))/(3*b**2*(a + b*x**3))
```

3.218 $\int \frac{x^2}{(a+bx^3)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x^2}{(a+bx^3)^{3/2}} dx = -\frac{2}{3b\sqrt{a+bx^3}}$$

output
```
-2/3/b/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)^{3/2}} dx = -\frac{2}{3b\sqrt{a+bx^3}}$$

input
```
Integrate[x^2/(a + b*x^3)^(3/2),x]
```

output
```
-2/(3*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 793$$

$$-\frac{2}{3b\sqrt{a+bx^3}}$$

input
```
Int[x^2/(a + b*x^3)^(3/2),x]
```

output
```
-2/(3*b*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{2}{3b\sqrt{b\,x^3+a}}$	15
derivativedivides	$-\frac{2}{3b\sqrt{b\,x^3+a}}$	15
default	$-\frac{2}{3b\sqrt{b\,x^3+a}}$	15
trager	$-\frac{2}{3b\sqrt{b\,x^3+a}}$	15
pseudoelliptic	$-\frac{2}{3b\sqrt{b\,x^3+a}}$	15
orering	$-\frac{2}{3b\sqrt{b\,x^3+a}}$	15
elliptic	$-\frac{2}{3b\sqrt{\left(x^3+\frac{a}{b}\right)b}}$	19

input
```
int(x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{bx^3+a}}{3\,(b^2x^3+ab)}$$

input
```
integrate(x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^3 + a)/(b^2*x^3 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx = \begin{cases} -\frac{2}{3b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)**(3/2),x)
```

output
```
Piecewise((-2/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), (x**3/(3*a**(3/2)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2}{3\sqrt{bx^3+a}b}$$

input
```
integrate(x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
-2/3/(sqrt(b*x^3 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2}{3\sqrt{bx^3+a}b}$$

input
```
integrate(x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3/(sqrt(b*x^3 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2}{3\,b\,\sqrt{b\,x^3+a}}$$

input
```
int(x^2/(a + b*x^3)^(3/2),x)
```

output
```
-2/(3*b*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{b\,x^3+a}}{3b\,(b\,x^3+a)}$$

input
```
int(x^2/(b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3))/(3*b*(a + b*x**3))
```

3.219 $\int \frac{1}{x(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{x(a+bx^3)^{3/2}}\,dx = \frac{2}{3a\sqrt{a+bx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

output
```
2/3/a/(b*x^3+a)^(1/2)-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^3)^{3/2}}\,dx = \frac{2}{3a\sqrt{a+bx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx^3$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^3)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2))/3
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{2}{3a\sqrt{b\,x^{3}+a}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	35
default	$\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	39
elliptic	$\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}$	39

input
```
int(1/x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/a/(b*x^3+a)^(1/2)-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{x\left(a+bx^3\right)^{3/2}}\,dx = \left[\frac{(bx^3+a)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\sqrt{bx^3+a}a}{3\left(a^2bx^3+a^3\right)}, \frac{2\left((bx^3+a)\sqrt{-a}\arctan\left(\ldots\right.\right.}{3\left(a^2bx^3+\ldots\right.}\right.$$

input
```
integrate(1/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*((b*x^3 + a)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^
3) + 2*sqrt(b*x^3 + a)*a)/(a^2*b*x^3 + a^3), 2/3*((b*x^3 + a)*sqrt(-a)*arc
tan(sqrt(-a)/sqrt(b*x^3 + a)) + sqrt(b*x^3 + a)*a)/(a^2*b*x^3 + a^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(39) = 78.

Time = 0.91 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{1}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} + \frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} + \frac{a^2bx^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3} - \frac{2a^2bx^3\log\left(\sqrt{1+\frac{bx^3}{a}}+1\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}$$

input
```
integrate(1/x/(b*x**3+a)**(3/2),x)
```

output
```
2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x*
*3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1
)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2)
 + 3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9
/2) + 3*a**(7/2)*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\,(a+bx^3)^{3/2}}\,dx = \frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\,a^{\frac{3}{2}}} + \frac{2}{3\,\sqrt{bx^3+a}a}$$

input
```
integrate(1/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2) +
 2/3/(sqrt(b*x^3 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(a+bx^3)^{3/2}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}a} + \frac{2}{3\,\sqrt{bx^3+a}a}$$

input
```
integrate(1/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) + 2/3/(sqrt(b*x^3 + a)*a
)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\,(a+bx^3)^{3/2}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{3/2}} + \frac{2}{3\,a\,\sqrt{b\,x^3+a}}$$

input
```
int(1/(x*(a + b*x^3)^(3/2)),x)
```

output
```
log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)/(
3*a^(3/2)) + 2/(3*a*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\,(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a}\,a+\sqrt{a}\,\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)a+\sqrt{a}\,\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)b\,x^3-\sqrt{a}\,\log}{3a^2\,(b\,x^3+a)}$$

input
```
int(1/x/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**3)*a + sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*a + sqrt(a
)*log(sqrt(a + b*x**3) - sqrt(a))*b*x**3 - sqrt(a)*log(sqrt(a + b*x**3) +
sqrt(a))*a - sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b*x**3)/(3*a**2*(a +
b*x**3))
```

3.220 $\int \frac{1}{x^4(a+bx^3)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{1}{x^4(a+bx^3)^{3/2}}\,dx=-\frac{b}{a^2\sqrt{a+bx^3}}-\frac{1}{3ax^3\sqrt{a+bx^3}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-b/a^2/(b*x^3+a)^(1/2)-1/3/a/x^3/(b*x^3+a)^(1/2)+b*arctanh((b*x^3+a)^(1/2)
/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4(a+bx^3)^{3/2}}\,dx=\frac{-a-3bx^3}{3a^2x^3\sqrt{a+bx^3}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(3/2)),x]
```

output
```
(-a - 3*b*x^3)/(3*a^2*x^3*Sqrt[a + b*x^3]) + (b*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^6 (bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{3b\int \frac{1}{x^3(bx^3+a)^{3/2}}dx^3}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{3b\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^(3/2)),x]
```

output
```
(-(1/(a*x^3*Sqrt[a + b*x^3])) - (3*b*(2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/(2*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{b\,x^{3}+a}}{x^{3} b}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2}{\sqrt{b\,x^{3}+a}}\right)}{3a^{2}}$	55
default	$-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2b}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	57
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{2b}{3a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$	57
risch	$-\frac{\sqrt{b\,x^{3}+a}}{3a^{2}x^{3}}-\frac{b\left(-\frac{2}{3\sqrt{b\,x^{3}+a}}+3a\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)\right)}{2a^{2}}$	78

input
```
int(1/x^4/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*b/a^2*(-(b*x^3+a)^(1/2)/x^3/b+3*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/
2)-2/(b*x^3+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}}\,dx = \left[\frac{3\,(b^2x^6+abx^3)\sqrt{a}\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\sqrt{a}+2\,a}{x^3}\right)-2\,(3\,abx^3+a^2)\sqrt{bx^3+a}}{6\,(a^3bx^6+a^4x^3)},\right.$$
$$\left.-\frac{3\,(b^2x^6+abx^3)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^3+a}}\right)+(3\,abx^3+a^2)\sqrt{bx^3+a}}{3\,(a^3bx^6+a^4x^3)}\right]$$

input
```
integrate(1/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*x^6 + a*b*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a)
 + 2*a)/x^3) - 2*(3*a*b*x^3 + a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3),
 -1/3*(3*(b^2*x^6 + a*b*x^3)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^3 + a)) + (
3*a*b*x^3 + a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}} \, dx = -\frac{1}{3a\sqrt{b}x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}$$

input
```
integrate(1/x**4/(b*x**3+a)**(3/2),x)
```

output
```
-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}} \, dx = -\frac{3\,(bx^3+a)b - 2\,ab}{3\left((bx^3+a)^{\frac{3}{2}}a^2 - \sqrt{bx^3+a}a^3\right)} - \frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

input
```
integrate(1/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/3*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)*a^
3) - 1/2*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^
(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}}\,dx = -\frac{b\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{3\,(bx^3+a)b-2\,ab}{3\left((bx^3+a)^{\frac{3}{2}}-\sqrt{bx^3+a}a\right)a^2}$$

input
```
integrate(1/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-b*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/3*(3*(b*x^3 + a)*b
- 2*a*b)/(((b*x^3 + a)^(3/2) - sqrt(b*x^3 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}}\,dx = \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{2\,a^{5/2}} - \frac{2\,b}{3\,a^2\,\sqrt{b\,x^3+a}} - \frac{\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

input
```
int(1/(x^4*(a + b*x^3)^(3/2)),x)
```

output
```
(b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6
))/(2*a^(5/2)) - (2*b)/(3*a^2*(a + b*x^3)^(1/2)) - (a + b*x^3)^(1/2)/(3*a^
2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^4 (a+bx^3)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,a^2 - 6\sqrt{b\,x^3+a}\,ab\,x^3 - 3\sqrt{a}\log\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)ab\,x^3 - 3\sqrt{a}\log\left(\sqrt{\right.}{6a^3x}$$

input
```
int(1/x^4/(b*x^3+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**3)*a**2 - 6*sqrt(a + b*x**3)*a*b*x**3 - 3*sqrt(a)*log(
sqrt(a + b*x**3) - sqrt(a))*a*b*x**3 - 3*sqrt(a)*log(sqrt(a + b*x**3) - sq
rt(a))*b**2*x**6 + 3*sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*a*b*x**3 + 3*
sqrt(a)*log(sqrt(a + b*x**3) + sqrt(a))*b**2*x**6)/(6*a**3*x**3*(a + b*x**
3))
```

3.221 $\int \frac{1}{x^7(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{5b^2}{4a^3\sqrt{a+bx^3}} - \frac{1}{6ax^6\sqrt{a+bx^3}} + \frac{5b}{12a^2x^3\sqrt{a+bx^3}} - \frac{5b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
5/4*b^2/a^3/(b*x^3+a)^(1/2)-1/6/a/x^6/(b*x^3+a)^(1/2)+5/12*b/a^2/x^3/(b*x^
3+a)^(1/2)-5/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{-2a^2+5abx^3+15b^2x^6}{12a^3x^6\sqrt{a+bx^3}} - \frac{5b^2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}$$

input
```
Integrate[1/(x^7*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*a^2 + 5*a*b*x^3 + 15*b^2*x^6)/(12*a^3*x^6*Sqrt[a + b*x^3]) - (5*b^2*Ar
cTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 52, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^9 (bx^3+a)^{3/2}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{5b\int \frac{1}{x^6(bx^3+a)^{3/2}}dx^3}{4a}-\frac{1}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x^3\left(bx^3+a\right)^{3/2}}dx^3}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{1}{2ax^6\sqrt{a+bx^3}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{3}\left(-\frac{5b\left(-\frac{3b\left(\frac{\int \frac{1}{x^3\sqrt{bx^3+a}}dx^3}{a}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{1}{2ax^6\sqrt{a+bx^3}}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{3}\left(-\frac{5b\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^6}{b}-\frac{a}{b}}d\sqrt{bx^3+a}}{ab}+\frac{2}{a\sqrt{a+bx^3}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{1}{2ax^6\sqrt{a+bx^3}}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{3}\left(-\frac{5b\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^3\sqrt{a+bx^3}}\right)}{4a}-\frac{1}{2ax^6\sqrt{a+bx^3}}\right)$$

input
```
Int[1/(x^7*(a + b*x^3)^(3/2)),x]
```

output
```
(-1/2*1/(a*x^6*Sqrt[a + b*x^3]) - (5*b*(-(1/(a*x^3*Sqrt[a + b*x^3])) - (3*
b*(2/(a*Sqrt[a + b*x^3]) - (2*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/a^(3/2)))/
(2*a)))/(4*a))/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$-\frac{5\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)\sqrt{b\,x^{3}+a}\,b^{2}x^{6}-b^{2}x^{6}\sqrt{a}-\frac{a^{\frac{3}{2}}b\,x^{3}}{3}+\frac{2a^{\frac{5}{2}}}{15}\right)}{4\sqrt{b\,x^{3}+a}\,a^{\frac{7}{2}}x^{6}}$	74
default	$-\frac{\sqrt{b\,x^{3}+a}}{6a^{2}x^{6}}+\frac{7b\sqrt{b\,x^{3}+a}}{12a^{3}x^{3}}+\frac{2b^{2}}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{5b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$	80
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{6a^{2}x^{6}}+\frac{7b\sqrt{b\,x^{3}+a}}{12a^{3}x^{3}}+\frac{2b^{2}}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{5b^{2}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}$	80
risch	$-\frac{\sqrt{b\,x^{3}+a}\,\left(-7b\,x^{3}+2a\right)}{12a^{3}x^{6}}+\frac{b^{2}\left(-\frac{14}{3\sqrt{b\,x^{3}+a}}+15a\left(\frac{2}{3a\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}\right)\right)}{8a^{3}}$	90

input
```
int(1/x^7/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4/(b*x^3+a)^(1/2)/a^(7/2)*(arctanh((b*x^3+a)^(1/2)/a^(1/2))*(b*x^3+a)^(
1/2)*b^2*x^6-b^2*x^6*a^(1/2)-1/3*a^(3/2)*b*x^3+2/15*a^(5/2))/x^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.11

$$\int \frac{1}{x^7\left(a+bx^3\right)^{3/2}}\,dx=\left[\frac{15\left(b^3x^9+ab^2x^6\right)\sqrt{a}\log\left(\frac{bx^3-2\sqrt{bx^3+a}\sqrt{a}+2a}{x^3}\right)+2\left(15ab^2x^6+5a^2bx^3-2a^3\right)\sqrt{bx^3}}{24\left(a^4bx^9+a^5x^6\right)}\right.$$

input
```
integrate(1/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*(15*(b^3*x^9 + a*b^2*x^6)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqr
t(a) + 2*a)/x^3) + 2*(15*a*b^2*x^6 + 5*a^2*b*x^3 - 2*a^3)*sqrt(b*x^3 + a))
/(a^4*b*x^9 + a^5*x^6), 1/12*(15*(b^3*x^9 + a*b^2*x^6)*sqrt(-a)*arctan(sqr
t(-a)/sqrt(b*x^3 + a)) + (15*a*b^2*x^6 + 5*a^2*b*x^3 - 2*a^3)*sqrt(b*x^3 +
 a))/(a^4*b*x^9 + a^5*x^6)]
```

Sympy [A] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^7 (a+bx^3)^{3/2}}\,dx = -\frac{1}{6a\sqrt{b}x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}} + \frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}} - \frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}$$

input
```
integrate(1/x**7/(b*x**3+a)**(3/2),x)
```

output
```
-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x**(9
/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3) +
1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^7 (a+bx^3)^{3/2}}\,dx = \frac{15\,(bx^3+a)^2b^2 - 25\,(bx^3+a)ab^2 + 8\,a^2b^2}{12\left((bx^3+a)^{\frac{5}{2}}a^3 - 2\,(bx^3+a)^{\frac{3}{2}}a^4 + \sqrt{bx^3+a}a^5\right)} + \frac{5\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

input
```
integrate(1/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
1/12*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3 + a
)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 5/8*b^2*log
((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^7 (a+bx^3)^{3/2}}\,dx = \frac{5\,b^2 \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3} + \frac{2\,b^2}{3\,\sqrt{bx^3+a}a^3} + \frac{7\,(bx^3+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx^3+a}ab^2}{12\,a^3b^2x^6}$$

input
```
integrate(1/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
5/4*b^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/3*b^2/(sqrt(b*
x^3 + a)*a^3) + 1/12*(7*(b*x^3 + a)^(3/2)*b^2 - 9*sqrt(b*x^3 + a)*a*b^2)/(
a^3*b^2*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^7 (a+bx^3)^{3/2}}\,dx = \frac{2\,b^2}{3\,a^3\,\sqrt{b\,x^3+a}} - \frac{\sqrt{b\,x^3+a}}{6\,a^2\,x^6} + \frac{5\,b^2\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{8\,a^{7/2}} + \frac{7\,b\,\sqrt{b\,x^3+a}}{12\,a^3\,x^3}$$

input
```
int(1/(x^7*(a + b*x^3)^(3/2)),x)
```

output
```
(2*b^2)/(3*a^3*(a + b*x^3)^(1/2)) - (a + b*x^3)^(1/2)/(6*a^2*x^6) + (5*b^2
*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))
/(8*a^(7/2)) + (7*b*(a + b*x^3)^(1/2))/(12*a^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^7 \left(a+bx^3\right)^{3/2}}\,dx = \frac{-4\sqrt{b\,x^3+a}\,a^3+10\sqrt{b\,x^3+a}\,a^2b\,x^3+30\sqrt{b\,x^3+a}\,a\,b^2x^6+15\sqrt{a}\log\left(\sqrt{b\,x^3+a}\right.}{}$$

input
```
int(1/x^7/(b*x^3+a)^(3/2),x)
```

output
```
( - 4*sqrt(a + b*x**3)*a**3 + 10*sqrt(a + b*x**3)*a**2*b*x**3 + 30*sqrt(a
+ b*x**3)*a*b**2*x**6 + 15*sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*a*b**2*
x**6 + 15*sqrt(a)*log(sqrt(a + b*x**3) - sqrt(a))*b**3*x**9 - 15*sqrt(a)*l
og(sqrt(a + b*x**3) + sqrt(a))*a*b**2*x**6 - 15*sqrt(a)*log(sqrt(a + b*x**
3) + sqrt(a))*b**3*x**9)/(24*a**4*x**6*(a + b*x**3))
```

3.222 $\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 251

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = -\frac{2x^4}{3b\sqrt{a+bx^3}}+\frac{16x\sqrt{a+bx^3}}{15b^2} - \frac{32\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*x^4/b/(b*x^3+a)^(1/2)+16/15*x*(b*x^3+a)^(1/2)/b^2-32/45*(1/2*6^(1/2)+
1/2*2^(1/2))*a*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+
b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(7/3)/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x
^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.98 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.26

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(8a+3bx^3-8a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{15b^2\sqrt{a+bx^3}}$$

input
```
Integrate[x^6/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(8*a + 3*b*x^3 - 8*a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2,
4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {817, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{8\int \frac{x^3}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{8\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{2a\int \frac{1}{\sqrt{bx^3+a}}dx}{5b}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{8\left(\frac{2x\sqrt{a+bx^3}}{5b}-\frac{4\sqrt{2+\sqrt{3}}a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{3b}-\frac{2x^4}{3b\sqrt{a+bx^3}}$$

input
```
Int[x^6/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x^4)/(3*b*Sqrt[a + b*x^3]) + (8*((2*x*Sqrt[a + b*x^3])/(5*b) - (4*Sqrt
[2 + Sqrt[3]]*a*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.27

method	result
default	$\frac{2xa}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^2}+\frac{32ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
elliptic	$\frac{2xa}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x\sqrt{b\,x^3+a}}{5b^2}+\frac{32ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
risch	$\frac{2x\sqrt{b\,x^3+a}}{5b^2}-\frac{a\left(2a\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}\cdots\right.\right.}{\cdots}$

input
```
int(x^6/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b^2*x*a/((x^3+a/b)*b)^(1/2)+2/5*x*(b*x^3+a)^(1/2)/b^2+32/45*I*a/b^3*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(16\,(abx^3+a^2)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)-(3\,b^2x^4+8\,abx)\sqrt{bx^3+a}\right)}{15\,(b^4x^3+ab^3)}$$

input
```
integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(16*(a*b*x^3 + a^2)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) - (3*b
^2*x^4 + 8*a*b*x)*sqrt(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = \frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(b*x**3+a)**(3/2),x)
```

output
```
x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^6}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^6/(a + b*x^3)^(3/2),x)
```

output
```
int(x^6/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^3)^{3/2}}\,dx = \frac{\frac{16\sqrt{b\,x^3+a}\,ax}{5}+\frac{2\sqrt{b\,x^3+a}\,b\,x^4}{5}-\frac{16\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^3}{5}-\frac{16\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2b\,x^3}{5}}{b^2\left(b\,x^3+a\right)}$$

input
```
int(x^6/(b*x^3+a)^(3/2),x)
```

output
```
(2*(8*sqrt(a + b*x**3)*a*x + sqrt(a + b*x**3)*b*x**4 - 8*int(sqrt(a + b*x*
*3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**3 - 8*int(sqrt(a + b*x**3)/(a**2
 + 2*a*b*x**3 + b**2*x**6),x)*a**2*b*x**3))/(5*b**2*(a + b*x**3))
```

3.223 $\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = -\frac{2x}{3b\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/3*x/b/(b*x^3+a)^(1/2)+4/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(4/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3
^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.95 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.24

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = \frac{2x\left(-1+\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3b\sqrt{a+bx^3}}$$

input
```
Integrate[x^3/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(-1 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/
a)]))/(3*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {817, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \; 817$$

$$\frac{2\int \frac{1}{\sqrt{bx^3+a}}dx}{3b} - \frac{2x}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 759$$

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x}{3b\sqrt{a+bx^3}}$$

input
```
Int[x^3/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x)/(3*b*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(4/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.32

method	result
default	$-\frac{2x}{3b\sqrt{(x^3+\frac{a}{b})b}}-\frac{4i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right)}{\left(-a\,b\cdots\right.}}\cdots}{9b^2\sqrt{\cdots}}$
elliptic	$-\frac{2x}{3b\sqrt{(x^3+\frac{a}{b})b}}-\frac{4i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\cdots\right)}{\left(-a\,b\cdots\right.}}\cdots}{9b^2\sqrt{\cdots}}$

input
```
int(x^3/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*x/((x^3+a/b)*b)^(1/2)-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(
1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(
I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{bx^3+a}bx - 2\,(bx^3+a)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)}{3\,(b^3x^3+ab^2)}$$

input
```
integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*b*x - 2*(b*x^3 + a)*sqrt(b)*weierstrassPInverse(0, -
4*a/b, x))/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(b*x**3+a)**(3/2),x)
```

output
```
x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^3}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^3/(a + b*x^3)^(3/2),x)
```

output
```
int(x^3/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)^{3/2}}\,dx = \frac{-2\sqrt{b\,x^3+a}\,x+2\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2+2\left(\int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx\right)ab\,x^3}{b\,(b\,x^3+a)}$$

input
```
int(x^3/(b*x^3+a)^(3/2),x)
```

output
```
(2*( - sqrt(a + b*x**3)*x + int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2
*x**6),x)*a**2 + int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a
*b*x**3))/(b*(a + b*x**3))
```

3.224 $\int \frac{1}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 232

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{2x}{3a\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x/a/(b*x^3+a)^(1/2)+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2
)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(
1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a/b^(1/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+
3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.24

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(2+\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{bx^3}{a}\right)\right)}{3a\sqrt{a+bx^3}}$$

input
```
Integrate[(a + b*x^3)^(-3/2),x]
```

output
```
(x*(2 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]
))/(3*a*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {749, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3a}+\frac{2x}{3a\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{3a\sqrt{a+bx^3}}$$

input
```
Int[(a + b*x^3)^(-3/2),x]
```

output
```
(2*x)/(3*a*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a*b^(1/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.32

method	result
default	$\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}} - \frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{9ab\sqrt{b\,x}}$
elliptic	$\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}} - \frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{9ab\sqrt{b\,x}}$

input
```
int(1/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x/a/((x^3+a/b)*b)^(1/2)-2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2
)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{bx^3+a}bx+(bx^3+a)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)}{3\left(ab^2x^3+a^2b\right)}$$

input
```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*x^3 + a)*b*x + (b*x^3 + a)*sqrt(b)*weierstrassPInverse(0, -4*a
/b, x))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(b*x**3+a)**(3/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \frac{x\left(\frac{b\,x^3}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{3}{2};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{(b\,x^3+a)^{3/2}}$$

input
```
int(1/(a + b*x^3)^(3/2),x)
```

output
```
(x*((b*x^3)/a + 1)^(3/2)*hypergeom([1/3, 3/2], 4/3, -(b*x^3)/a))/(a + b*x^
3)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(1/(b*x^3+a)^(3/2),x)
```

output

```
int(sqrt(a + b*x**3)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.225 $\int \frac{1}{x^3(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 255

$$\int \frac{1}{x^3(a+bx^3)^{3/2}}\,dx = \frac{2}{3ax^2\sqrt{a+bx^3}} - \frac{7\sqrt{a+bx^3}}{6a^2x^2} - \frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{6\sqrt[4]{3}a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/x^2/(b*x^3+a)^(1/2)-7/6*(b*x^3+a)^(1/2)/a^2/x^2-7/18*(1/2*6^(1/2)+1/
2*2^(1/2))*b^(2/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1
/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^2/
(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x
^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{3}{2},\frac{1}{3},-\frac{bx^3}{a}\right)}{2ax^2\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(3/2)),x]
```

output
```
-1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 3/2, 1/3, -((b*x^3)/a)])
/(a*x^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {819, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7\left(-\frac{b\int \frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{7\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{3a}+\frac{2}{3ax^2\sqrt{a+bx^3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*x^2*Sqrt[a + b*x^3]) + (7*(-1/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 +
 Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{b\,x^3+a}}{2a^2x^2}-\frac{2bx}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{7i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^3+a}}{2a^2x^2}-\frac{2bx}{3a^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{7i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
risch	$-\frac{\sqrt{b\,x^3+a}}{2a^2x^2}-\frac{b\left(b\left(-\frac{2x}{3b\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{4i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}\cdots\right.\right.}{\cdots}$

input
```
int(1/x^3/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^3+a)^(1/2)/a^2/x^2-2/3*b/a^2*x/((x^3+a/b)*b)^(1/2)+7/18*I/a^2*3^
(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/
3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}} \, dx = -\frac{7 (bx^5+ax^2)\sqrt{b}\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+(7bx^3+3a)\sqrt{bx^3+a}}{6(a^2bx^5+a^3x^2)}$$

input
```
integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(7*(b*x^5 + a*x^2)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (7*b*x
^3 + 3*a)*sqrt(b*x^3 + a))/(a^2*b*x^5 + a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}} \, dx = \frac{\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{x^3\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/(x^3*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^9+2ab\,x^6+a^2x^3}dx$$

input
```
int(1/x^3/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**3 + 2*a*b*x**6 + b**2*x**9),x)
```

3.226 $\int \frac{1}{x^6(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 277

$$\int \frac{1}{x^6\left(a+bx^3\right)^{3/2}}\,dx=\frac{2}{3ax^5\sqrt{a+bx^3}}-\frac{13\sqrt{a+bx^3}}{15a^2x^5}+\frac{91b\sqrt{a+bx^3}}{60a^3x^2}$$
$$+\frac{91\sqrt{2+\sqrt{3}}b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{60\sqrt[4]{3}a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/x^5/(b*x^3+a)^(1/2)-13/15*(b*x^3+a)^(1/2)/a^2/x^5+91/60*b*(b*x^3+a)^
(1/2)/a^3/x^2+91/180*(1/2*6^(1/2)+1/2*2^(1/2))*b^(5/3)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^3/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^6 (a+bx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\ \text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{3}{2},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5ax^5\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(3/2)),x]
```

output
```
-1/5*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/3, 3/2, -2/3, -((b*x^3)/a)]
)/(a*x^5*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {819, 847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{13\int \frac{1}{x^6\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$\frac{13\left(-\frac{7b\int \frac{1}{x^3\sqrt{bx^3+a}}dx}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$\frac{13\left(-\frac{7b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+a}}dx}{4a}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5\sqrt{a+bx^3}}$$

$\big\downarrow$ 759

$$\frac{13\left(-\frac{7b\left(-\frac{\sqrt{2+\sqrt{3}}b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{a+bx^3}}{2ax^2}\right)}{10a}-\frac{\sqrt{a+bx^3}}{5ax^5}\right)}{3a}$$

$$\frac{2}{3ax^5\sqrt{a+bx^3}}$$

input
```
Int[1/(x^6*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*x^5*Sqrt[a + b*x^3]) + (13*(-1/5*Sqrt[a + b*x^3]/(a*x^5) - (7*b*(-1
/2*Sqrt[a + b*x^3]/(a*x^2) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a^(
1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])))/(10*a)))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{17b\sqrt{b\,x^3+a}}{20a^3x^2}+\frac{2b^2x}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{91ib\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)\cdots}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}}}\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^3+a}}{5a^2x^5}+\frac{17b\sqrt{b\,x^3+a}}{20a^3x^2}+\frac{2b^2x}{3a^3\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{91ib\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)\cdots}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}}}\cdots}{\cdots}$
risch	$-\frac{\sqrt{b\,x^3+a}\,\left(-17b\,x^3+4a\right)}{20a^3x^5}+\frac{b^2\left(57a\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}-\frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)\cdots}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\cdots}{\cdots}}}\cdots}{\cdots}\right.\right.\cdots}{\cdots}$

input

```
int(1/x^6/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^3+a)^(1/2)/a^2/x^5+17/20*b*(b*x^3+a)^(1/2)/a^3/x^2+2/3*b^2/a^3*x
/((x^3+a/b)*b)^(1/2)-91/180*I*b/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2
)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{3/2}}\,dx = \frac{91\left(b^2x^8+abx^5\right)\sqrt{b}\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)+\left(91\,b^2x^6+39\,abx^3-12\,a^2\right)\sqrt{}}{60\left(a^3bx^8+a^4x^5\right)}$$

input
```
integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/60*(91*(b^2*x^8 + a*b*x^5)*sqrt(b)*weierstrassPInverse(0, -4*a/b, x) + (
91*b^2*x^6 + 39*a*b*x^3 - 12*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^8 + a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^6 \left(a+bx^3\right)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{x^6\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/(x^6*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^{12}+2ab\,x^9+a^2x^6}dx$$

input
```
int(1/x^6/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**6 + 2*a*b*x**9 + b**2*x**12),x)
```

3.227 $\int \frac{x^7}{\left(a+bx^3\right)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 511

$$\int \frac{x^7}{\left(a+bx^3\right)^{3/2}} dx = -\frac{2x^5}{3b\sqrt{a+bx^3}} + \frac{20x^2\sqrt{a+bx^3}}{21b^2} - \frac{80a\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}$$
$$+ \frac{40\sqrt{2-\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{7\ 3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$- \frac{80\sqrt{2}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{21\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*x^5/b/(b*x^3+a)^(1/2)+20/21*x^2*(b*x^3+a)^(1/2)/b^2-80/21*a*(b*x^3+a)
^(1/2)/b^(8/3)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)+40/21*(1/2*6^(1/2)-1/2*2^(1
/2))*a^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^
(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/b^(8/3)/(a
^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)-80/63*2^(1/2)*a^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-
3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)
*3^(3/4)/b^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.97 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.13

$$\int \frac{x^7}{(a+bx^3)^{3/2}}\,dx = \frac{2x^2\left(-10a+bx^3+10a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{7b^2\sqrt{a+bx^3}}$$

input
```
Integrate[x^7/(a + b*x^3)^(3/2),x]
```

output
```
(2*x^2*(-10*a + b*x^3 + 10*a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/
2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {817, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{10\int \frac{x^4}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^5}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{10\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\int \frac{x}{\sqrt{bx^3+a}}dx}{7b}\right)}{3b}-\frac{2x^5}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{10\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{7b}\right)}{3b}-\frac{2x^5}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{10\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{\ldots}\right.}{\sqrt[3]{b}x+\left(1+\sqrt{\ldots}\right.}\right.\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{7b}\right)}{3b}$$

$$\frac{2x^5}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{10\left(\frac{2x^2\sqrt{a+bx^3}}{7b}-\frac{4a\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\cdots\right.}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\cdots}{\sqrt[3]{b}}\cdots\right.}{\cdots}\cdots\right.}{3b}$$

$$\frac{2x^5}{3b\sqrt{a+bx^3}}$$

input
```
Int[x^7/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x^5)/(3*b*Sqrt[a + b*x^3]) + (10*((2*x^2*Sqrt[a + b*x^3])/(7*b) - (4*a
*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 -
 Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*El
lipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(7*b
)))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 476, normalized size of antiderivative = 0.93

method	result
default	$\frac{2x^2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^2\sqrt{b\,x^3+a}}{7b^2}+\frac{80ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
elliptic	$\frac{2x^2a}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2x^2\sqrt{b\,x^3+a}}{7b^2}+\frac{80ia\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\cdots}}{\cdots}$
risch	Expression too large to display

input
```
int(x^7/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/b^2*x^2*a/((x^3+a/b)*b)^(1/2)+2/7*x^2*(b*x^3+a)^(1/2)/b^2+80/63*I*a/b^
3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b
*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.15

$$\int \frac{x^7}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2\left(40\left(abx^3+a^2\right)\sqrt{b}\mathrm{weierstrassZeta}\left(0,-\frac{4a}{b},\mathrm{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(3b^2\right.}{21\left(b^4x^3+ab^3\right)}$$

input
```
integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/21*(40*(a*b*x^3 + a^2)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPIn
verse(0, -4*a/b, x)) + (3*b^2*x^5 + 10*a*b*x^2)*sqrt(b*x^3 + a))/(b^4*x^3
+ a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{x^7}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(b*x**3+a)**(3/2),x)
```

output
```
x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^7/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^7/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^7/(a + b*x^3)^(3/2),x)
```

output
```
int(x^7/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^3)^{3/2}}\,dx = \frac{-\frac{20\sqrt{b\,x^3+a}\,a\,x^2}{7}+\frac{2\sqrt{b\,x^3+a}\,b\,x^5}{7}+\frac{40\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^3}{7}+\frac{40\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2b\,x^3}{7}}{b^2\left(b\,x^3+a\right)}$$

input

```
int(x^7/(b*x^3+a)^(3/2),x)
```

output

```
(2*( - 10*sqrt(a + b*x**3)*a*x**2 + sqrt(a + b*x**3)*b*x**5 + 20*int((sqrt
(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**3 + 20*int((sqrt(a +
 b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**2*b*x**3))/(7*b**2*(a +
b*x**3))
```

3.228 $\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 487

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx=-\frac{2x^2}{3b\sqrt{a+bx^3}}+\frac{8\sqrt{a+bx^3}}{3b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{4\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*x^2/b/(b*x^3+a)^(1/2)+8/3*(b*x^3+a)^(1/2)/b^(5/3)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)-4/3*(1/2*6^(1/2)-1/2*2^(1/2))*a^(1/3)*(a^(1/3)+b^(1/3)*x)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(
1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3
)*x),I*3^(1/2)+2*I)*3^(1/4)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+8/9*2^(1/2)*a^(1/3)*(a^(1/
3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3
)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2
))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/b^(5/3)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.83 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2x^2\left(-1+\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{b\sqrt{a+bx^3}}$$

input
```
Integrate[x^4/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x^2*(-1 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^
3)/a)]))/(b*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.82 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {817, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx$$

$\Big\downarrow$ 817

$$\frac{4\int \frac{x}{\sqrt{bx^3+a}}dx}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}$$

$\Big\downarrow$ 832

$$\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{3b}-\frac{2x^2}{3b\sqrt{a+bx^3}}$$

$\Big\downarrow$ 759

$$\frac{4\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{3b}$$

$$\frac{2x^2}{3b\sqrt{a+bx^3}}$$

$\Big\downarrow$ 2416

$$\frac{4\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}-\frac{2\left(1-\right.}{}\right.}{3b}$$

$$\frac{2x^2}{3b\sqrt{a+bx^3}}$$

input
```
Int[x^4/(a + b*x^3)^(3/2),x]
```

output
```
(-2*x^2)/(3*b*Sqrt[a + b*x^3]) + (4*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])))/(3*b)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2x^{2}}{3b\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{8i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\,b\cdots\right.}}\cdots}{\cdots}$
elliptic	$-\frac{2x^{2}}{3b\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}-\frac{8i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\cdots\right.}{\left(-a\,b\cdots\right.}}\cdots}{\cdots}$

input
```
int(x^4/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/b*x^2/((x^3+a/b)*b)^(1/2)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))
^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)
,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{bx^3+a}bx^2+4\left(bx^3+a\right)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)\right)}{3\left(b^3x^3+ab^2\right)}$$

input
```
integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*b*x^2 + 4*(b*x^3 + a)*sqrt(b)*weierstrassZeta(0, -4*
a/b, weierstrassPInverse(0, -4*a/b, x)))/(b^3*x^3 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.08

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(b*x**3+a)**(3/2),x)
```

output
```
x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx=\int \frac{x^4}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^4/(a + b*x^3)^(3/2),x)
```

output
```
int(x^4/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^3)^{3/2}}\,dx=\frac{2\sqrt{b\,x^3+a}\,x^2-4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2-4\left(\int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)ab\,x^3}{b\,(b\,x^3+a)}$$

input
```
int(x^4/(b*x^3+a)^(3/2),x)
```

output
```
(2*(sqrt(a + b*x**3)*x**2 - 2*int((sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3
+ b**2*x**6),x)*a**2 - 2*int((sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**
2*x**6),x)*a*b*x**3))/(b*(a + b*x**3))
```

3.229 $\int \frac{x}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 489

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx=\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{2\sqrt{a+bx^3}}{3ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*x^2/a/(b*x^3+a)^(1/2)-2/3*(b*x^3+a)^(1/2)/a/b^(2/3)/((1+3^(1/2))*a^(1/
3)+b^(1/3)*x)+1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*
3^(1/2)+2*I)*3^(1/4)/a^(2/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1
/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-2/9*2^(1/2)*(a^(1/3)+b^(1
/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/
3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1
/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^(2/3)/b^(2/3)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.99 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{3}{2},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt{a+bx^3}}$$

input
```
Integrate[x/(a + b*x^3)^(3/2),x]
```

output
```
(x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)])/(
2*a*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.85 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {819, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 819$$

$$\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\int \frac{x}{\sqrt{bx^3+a}}dx}{3a}$$

$$\Big\downarrow \; 832$$

$$\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}}{3a}$$

$$\Big\downarrow \; 759$$

$$\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{\cdots}\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{3a}$$

$$\Big\downarrow \; 2416$$

$$\frac{2x^2}{3a\sqrt{a+bx^3}}-\frac{\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\cdots}{\cdots}}{3a}$$

input
```
Int[x/(a + b*x^3)^(3/2),x]
```

output
```
(2*x^2)/(3*a*Sqrt[a + b*x^3]) - (((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^
(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a + b*x^3]))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.94

method	result
default	$\frac{2x^2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{}$
elliptic	$\frac{2x^2}{3a\sqrt{\left(x^3+\frac{a}{b}\right)b}}+\frac{2i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}\right.}{\left(-a\,b^2\right)}}}{}$

input
```
int(x/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2/a/((x^3+a/b)*b)^(1/2)+2/9*I/a*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b
*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^
2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(
I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),
(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.12

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \frac{2\left(\sqrt{bx^3+a}abx^2 + (bx^3+a)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right.\right.}{3\,(ab^2x^3+a^2b)}$$

input
```
integrate(x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(sqrt(b*x^3 + a)*b*x^2 + (b*x^3 + a)*sqrt(b)*weierstrassZeta(0, -4*a/b
, weierstrassPInverse(0, -4*a/b, x)))/(a*b^2*x^3 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.08

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(b*x**3+a)**(3/2),x)
```

output
```
x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(3/2)*gamma(5/3))
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \int \frac{x}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x/(a + b*x^3)^(3/2),x)
```

output
```
int(x/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x/(b*x^3+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.230 $\int \frac{1}{x^2(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 513

$$\int \frac{1}{x^2(a+bx^3)^{3/2}}\,dx = \frac{2}{3ax\sqrt{a+bx^3}} - \frac{5\sqrt{a+bx^3}}{3a^2x} + \frac{5\sqrt[3]{b}\sqrt{a+bx^3}}{3a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{2\,3^{3/4}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} + \frac{5\sqrt{2}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/x/(b*x^3+a)^(1/2)-5/3*(b*x^3+a)^(1/2)/a^2/x+5/3*b^(1/3)*(b*x^3+a)^(1
/2)/a^2/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)-5/6*(1/2*6^(1/2)-1/2*2^(1/2))*b^(1
/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/
2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/
((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(1/4)/a^(5/3)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
+5/9*2^(1/2)*b^(1/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*EllipticF(((1-3^(1/2))*a^
(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*3^(1/2)+2*I)*3^(3/4)/a^
(5/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2
)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{3}{2},\frac{2}{3},-\frac{bx^3}{a}\right)}{ax\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
-((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 3/2, 2/3, -((b*x^3)/a)])/(a
*x*Sqrt[a + b*x^3]))
```

Rubi [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {819, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx$$

$\Big\downarrow$ 819

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}$$

$\Big\downarrow$ 847

$$\frac{5\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}$$

$\Big\downarrow$ 832

$$\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{3a}+\frac{2}{3ax\sqrt{a+bx^3}}$$

$\Big\downarrow$ 759

$$\frac{5\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right)}{2a}\right.}{3a}$$

$$\frac{2}{3ax\sqrt{a+bx^3}}$$

$\Big\downarrow$ 2416

$$
\frac{5\left(\frac{b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}}-\frac{2\left(1-\cdots\right.}{\cdots}\right.}{2a}\cdots\right.}{3a}\cdots
$$

$$
\frac{2}{3ax\sqrt{a+bx^3}}
$$

input
```
Int[1/(x^2*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*x*Sqrt[a + b*x^3]) + (5*(-(Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a
+ b*x^3])/(b^(1/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2
- Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqr
t[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/
3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])))/(2*a)))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{b\,x^{3}+a}}{a^{2} x}-\frac{2 b\,x^{2}}{3 a^{2} \sqrt{\left(x^{3}+\frac{a}{b}\right) b}}-\frac{5 i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x \cdots\right.}{\cdots}}\cdots}{\cdots}$
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{a^{2} x}-\frac{2 b\,x^{2}}{3 a^{2} \sqrt{\left(x^{3}+\frac{a}{b}\right) b}}-\frac{5 i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x \cdots\right.}{\cdots}}\cdots}{\cdots}$
risch	Expression too large to display

input
```
int(1/x^2/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^3+a)^(1/2)/a^2/x-2/3*b*x^2/a^2/((x^3+a/b)*b)^(1/2)-5/9*I/a^2*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(
1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3
*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2
)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/
2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = -\frac{5\,(bx^4+ax)\sqrt{b}\text{weierstrassZeta}\left(0,-\frac{4\,a}{b},\text{weierstrassPInverse}\left(0,-\frac{4\,a}{b},x\right)\right)+(5\,bx^3+3\,a)\sqrt{bx^3+a}}{3\,(a^2bx^4+a^3x)}$$

input
```
integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(b*x^4 + a*x)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstrassPInver
se(0, -4*a/b, x)) + (5*b*x^3 + 3*a)*sqrt(b*x^3 + a))/(a^2*b*x^4 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{3}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = -\frac{2\left(\frac{a}{b\,x^3}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^3}\right)}{11\,x\,(b\,x^3+a)^{3/2}}$$

input
```
int(1/(x^2*(a + b*x^3)^(3/2)),x)
```

output
```
-(2*(a/(b*x^3) + 1)^(3/2)*hypergeom([3/2, 11/6], 17/6, -a/(b*x^3)))/(11*x*
(a + b*x^3)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^8+2ab\,x^5+a^2x^2}dx$$

input
```
int(1/x^2/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**2 + 2*a*b*x**5 + b**2*x**8),x)
```

3.231 $\int \frac{1}{x^5 (a+bx^3)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 535

$$\begin{aligned}
\int \frac{1}{x^5 (a+bx^3)^{3/2}} \, dx &= \frac{2}{3ax^4\sqrt{a+bx^3}} - \frac{11\sqrt{a+bx^3}}{12a^2x^4} \\
&+ \frac{55b\sqrt{a+bx^3}}{24a^3x} - \frac{55b^{4/3}\sqrt{a+bx^3}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} \\
&+ \frac{55\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{16\ 3^{3/4}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} \\
&- \frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}
\end{aligned}$$

output
```
2/3/a/x^4/(b*x^3+a)^(1/2)-11/12*(b*x^3+a)^(1/2)/a^2/x^4+55/24*b*(b*x^3+a)^
(1/2)/a^3/x-55/24*b^(4/3)*(b*x^3+a)^(1/2)/a^3/((1+3^(1/2))*a^(1/3)+b^(1/3)
*x)+55/48*(1/2*6^(1/2)-1/2*2^(1/2))*b^(4/3)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)*El
lipticE(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x),I*
3^(1/2)+2*I)*3^(1/4)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(
1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-55/72*b^(4/3)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^
2)^(1/2)*EllipticF(((1-3^(1/2))*a^(1/3)+b^(1/3)*x)/((1+3^(1/2))*a^(1/3)+b^
(1/3)*x),I*3^(1/2)+2*I)*2^(1/2)*3^(3/4)/a^(8/3)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/((1+3^(1/2))*a^(1/3)+b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^5 (a+bx^3)^{3/2}} \, dx = -\frac{\sqrt{1+\frac{bx^3}{a}} \text{Hypergeometric2F1}\left(-\frac{4}{3}, \frac{3}{2}, -\frac{1}{3}, -\frac{bx^3}{a}\right)}{4ax^4\sqrt{a+bx^3}}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(3/2)),x]
```

output
```
-1/4*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3, 3/2, -1/3, -((b*x^3)/a)]
)/(a*x^4*Sqrt[a + b*x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {819, 847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{3/2}}\,dx$$

$\Big\downarrow$ 819

$$\frac{11\int \frac{1}{x^5\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}$$

$\Big\downarrow$ 847

$$\frac{11\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+a}}dx}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}$$

$\Big\downarrow$ 847

$$\frac{11\left(-\frac{5b\left(\frac{b\int \frac{x}{\sqrt{bx^3+a}}dx}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}$$

$\Big\downarrow$ 832

$$\frac{11\left(-\frac{5b\left(\frac{b\left(\frac{\int \frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}\right)}{2a}-\frac{\sqrt{a+bx^3}}{ax}\right)}{8a}-\frac{\sqrt{a+bx^3}}{4ax^4}\right)}{3a}+\frac{2}{3ax^4\sqrt{a+bx^3}}$$

$\Big\downarrow$ 759

$$11\left(-\frac{5b\left(\frac{b\left(\frac{\int\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{bx^3+a}}dx}{\sqrt[3]{b}}-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7\right.}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}\right.}{2a}\right.}{8a}\right.}{3a}$$

$$\frac{2}{3ax^4\sqrt{a+bx^3}}$$

$\downarrow$ 2416

$$11 \quad \left(-\frac{5b\left(\frac{b\left(\frac{\frac{2\sqrt{a+bx^3}}{\sqrt[3]{b}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,E\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}}{\sqrt[3]{b}} \right.}{2a}}{8a} \right.$$

$$3a$$

$$\frac{2}{3ax^4\sqrt{a+bx^3}}$$

input
```
Int[1/(x^5*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*x^4*Sqrt[a + b*x^3]) + (11*(-1/4*Sqrt[a + b*x^3]/(a*x^4) - (5*b*(-(
Sqrt[a + b*x^3]/(a*x)) + (b*(((2*Sqrt[a + b*x^3])/(b^(1/3)*((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(1/3)*Sqrt[(a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a +
 b*x^3]))/b^(1/3) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*Sqrt[a + b*x^3])))/(2*a)))/(8*a)))/(3*a)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\sqrt{b\,x^{3}+a}}{4a^{2}x^{4}}+\frac{13b\sqrt{b\,x^{3}+a}}{8a^{3}x}+\frac{2b^{2}x^{2}}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+\frac{55ib\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}}{}$
	$\frac{55ib\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}}{}}}}{}$
elliptic	$-\frac{\sqrt{b\,x^{3}+a}}{4a^{2}x^{4}}+\frac{13b\sqrt{b\,x^{3}+a}}{8a^{3}x}+\frac{2b^{2}x^{2}}{3a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)b}}+$
risch	Expression too large to display

input
```
int(1/x^5/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^3+a)^(1/2)/a^2/x^4+13/8*b*(b*x^3+a)^(1/2)/a^3/x+2/3*b^2/a^3*x^2/
((x^3+a/b)*b)^(1/2)+55/72*I*b/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*
((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1
/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(
1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx = \frac{55\left(b^2x^7+abx^4\right)\sqrt{b}\,\text{weierstrassZeta}\left(0,-\frac{4a}{b},\text{weierstrassPInverse}\left(0,-\frac{4a}{b},x\right)\right)+(55}{24\left(a^3bx^7+a^4x^4\right)}$$

input
```
integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(55*(b^2*x^7 + a*b*x^4)*sqrt(b)*weierstrassZeta(0, -4*a/b, weierstras
sPInverse(0, -4*a/b, x)) + (55*b^2*x^6 + 33*a*b*x^3 - 6*a^2)*sqrt(b*x^3 +
a))/(a^3*b*x^7 + a^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5 (a + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + a)^{\frac{3}{2}} x^5} \, dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a + bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3 + a)^{\frac{3}{2}} x^5} \, dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)^{3/2}}\,dx = \int \frac{1}{x^5 \left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{b^2x^{11}+2ab\,x^8+a^2x^5}dx$$

input
```
int(1/x^5/(b*x^3+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**3)/(a**2*x**5 + 2*a*b*x**8 + b**2*x**11),x)
```

3.232 $\int \frac{x^{11}}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^{11}}{\sqrt{1+x^3}} \, dx = -\frac{2}{3}\sqrt{1+x^3} + \frac{2}{3}\left(1+x^3\right)^{3/2} - \frac{2}{5}\left(1+x^3\right)^{5/2} + \frac{2}{21}\left(1+x^3\right)^{7/2}$$

output
```
-2/3*(x^3+1)^(1/2)+2/3*(x^3+1)^(3/2)-2/5*(x^3+1)^(5/2)+2/21*(x^3+1)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{x^{11}}{\sqrt{1+x^3}} \, dx = \frac{2}{105}\sqrt{1+x^3}\left(-16+8x^3-6x^6+5x^9\right)$$

input
```
Integrate[x^11/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3]*(-16 + 8*x^3 - 6*x^6 + 5*x^9))/105
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\left(x^3+1\right)^{5/2}-3\left(x^3+1\right)^{3/2}+3\sqrt{x^3+1}-\frac{1}{\sqrt{x^3+1}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{7}\left(x^3+1\right)^{7/2}-\frac{6}{5}\left(x^3+1\right)^{5/2}+2\left(x^3+1\right)^{3/2}-2\sqrt{x^3+1}\right)$$

input
```
Int[x^11/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[1 + x^3] + 2*(1 + x^3)^(3/2) - (6*(1 + x^3)^(5/2))/5 + (2*(1 + x^
3)^(7/2))/7)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.49

method	result	size
trager	$\left(\frac{2}{21}x^{9}-\frac{4}{35}x^{6}+\frac{16}{105}x^{3}-\frac{32}{105}\right)\sqrt{x^{3}+1}$	26
risch	$\frac{2\left(5x^{9}-6x^{6}+8x^{3}-16\right)\sqrt{x^{3}+1}}{105}$	27
pseudoelliptic	$\frac{2\left(5x^{9}-6x^{6}+8x^{3}-16\right)\sqrt{x^{3}+1}}{105}$	27
gosper	$\frac{2(1+x)\left(x^{2}-x+1\right)\left(5x^{9}-6x^{6}+8x^{3}-16\right)}{105\sqrt{x^{3}+1}}$	38
orering	$\frac{2(1+x)\left(x^{2}-x+1\right)\left(5x^{9}-6x^{6}+8x^{3}-16\right)}{105\sqrt{x^{3}+1}}$	38
meijerg	$\frac{\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\left(-40x^{9}+48x^{6}-64x^{3}+128\right)\sqrt{x^{3}+1}}{140}}{3\sqrt{\pi}}$	41
default	$\frac{2x^{9}\sqrt{x^{3}+1}}{21}-\frac{4x^{6}\sqrt{x^{3}+1}}{35}+\frac{16x^{3}\sqrt{x^{3}+1}}{105}-\frac{32\sqrt{x^{3}+1}}{105}$	47
elliptic	$\frac{2x^{9}\sqrt{x^{3}+1}}{21}-\frac{4x^{6}\sqrt{x^{3}+1}}{35}+\frac{16x^{3}\sqrt{x^{3}+1}}{105}-\frac{32\sqrt{x^{3}+1}}{105}$	47

input
```
int(x^11/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/21*x^9-4/35*x^6+16/105*x^3-32/105)*(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.49

$$\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx = \frac{2}{105}\left(5\,x^9-6\,x^6+8\,x^3-16\right)\sqrt{x^3+1}$$

input
```
integrate(x^11/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(5*x^9 - 6*x^6 + 8*x^3 - 16)*sqrt(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx = \frac{2x^9\sqrt{x^3+1}}{21}-\frac{4x^6\sqrt{x^3+1}}{35}+\frac{16x^3\sqrt{x^3+1}}{105}-\frac{32\sqrt{x^3+1}}{105}$$

input
```
integrate(x**11/(x**3+1)**(1/2),x)
```

output
```
2*x**9*sqrt(x**3 + 1)/21 - 4*x**6*sqrt(x**3 + 1)/35 + 16*x**3*sqrt(x**3 +
1)/105 - 32*sqrt(x**3 + 1)/105
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx = \frac{2}{21}\left(x^3+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^3+1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^11/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(x^3 + 1)^(7/2) - 2/5*(x^3 + 1)^(5/2) + 2/3*(x^3 + 1)^(3/2) - 2/3*sqr
t(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx = \frac{2}{21}\left(x^3+1\right)^{\frac{7}{2}} - \frac{2}{5}\left(x^3+1\right)^{\frac{5}{2}} + \frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^11/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/21*(x^3 + 1)^(7/2) - 2/5*(x^3 + 1)^(5/2) + 2/3*(x^3 + 1)^(3/2) - 2/3*sqr
t(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx = \frac{16\,x^3\,\sqrt{x^3+1}}{105} - \frac{32\,\sqrt{x^3+1}}{105} - \frac{4\,x^6\,\sqrt{x^3+1}}{35} + \frac{2\,x^9\,\sqrt{x^3+1}}{21}$$

input
```
int(x^11/(x^3 + 1)^(1/2),x)
```

output
```
(16*x^3*(x^3 + 1)^(1/2))/105 - (32*(x^3 + 1)^(1/2))/105 - (4*x^6*(x^3 + 1)
^(1/2))/35 + (2*x^9*(x^3 + 1)^(1/2))/21
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.47

$$\int \frac{x^{11}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,(5x^9-6x^6+8x^3-16)}{105}$$

input
```
int(x^11/(x^3+1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*(5*x**9 - 6*x**6 + 8*x**3 - 16))/105
```

3.233 $\int \frac{x^8}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^8}{\sqrt{1+x^3}} \, dx = \frac{2\sqrt{1+x^3}}{3} - \frac{4}{9}(1+x^3)^{3/2} + \frac{2}{15}(1+x^3)^{5/2}$$

output
```
2/3*(x^3+1)^(1/2)-4/9*(x^3+1)^(3/2)+2/15*(x^3+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{x^8}{\sqrt{1+x^3}} \, dx = \frac{2}{45}\sqrt{1+x^3}(8-4x^3+3x^6)$$

input
```
Integrate[x^8/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3]*(8 - 4*x^3 + 3*x^6))/45
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{x^3+1}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{x^6}{\sqrt{x^3+1}} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\left(x^3+1\right)^{3/2} - 2\sqrt{x^3+1} + \frac{1}{\sqrt{x^3+1}} \right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2}{5} \left(x^3+1\right)^{5/2} - \frac{4}{3} \left(x^3+1\right)^{3/2} + 2\sqrt{x^3+1} \right)$$

input
```
Int[x^8/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3] - (4*(1 + x^3)^(3/2))/3 + (2*(1 + x^3)^(5/2))/5)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
trager	$\left(\frac{2}{15}x^{6}-\frac{8}{45}x^{3}+\frac{16}{45}\right)\sqrt{x^{3}+1}$	21
risch	$\frac{2\left(3x^{6}-4x^{3}+8\right)\sqrt{x^{3}+1}}{45}$	22
pseudoelliptic	$\frac{2\left(3x^{6}-4x^{3}+8\right)\sqrt{x^{3}+1}}{45}$	22
gosper	$\frac{2(1+x)\left(x^{2}-x+1\right)\left(3x^{6}-4x^{3}+8\right)}{45\sqrt{x^{3}+1}}$	33
orering	$\frac{2(1+x)\left(x^{2}-x+1\right)\left(3x^{6}-4x^{3}+8\right)}{45\sqrt{x^{3}+1}}$	33
default	$\frac{2x^{6}\sqrt{x^{3}+1}}{15}-\frac{8x^{3}\sqrt{x^{3}+1}}{45}+\frac{16\sqrt{x^{3}+1}}{45}$	35
elliptic	$\frac{2x^{6}\sqrt{x^{3}+1}}{15}-\frac{8x^{3}\sqrt{x^{3}+1}}{45}+\frac{16\sqrt{x^{3}+1}}{45}$	35
meijerg	$\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(6x^{6}-8x^{3}+16\right)\sqrt{x^{3}+1}}{15}}{3\sqrt{\pi}}$	36

input
```
int(x^8/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/15*x^6-8/45*x^3+16/45)*(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{x^8}{\sqrt{1+x^3}}\,dx = \frac{2}{45}\left(3\,x^6-4\,x^3+8\right)\sqrt{x^3+1}$$

input
```
integrate(x^8/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*x^6 - 4*x^3 + 8)*sqrt(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{\sqrt{1+x^3}}\,dx = \frac{2x^6\sqrt{x^3+1}}{15} - \frac{8x^3\sqrt{x^3+1}}{45} + \frac{16\sqrt{x^3+1}}{45}$$

input
```
integrate(x**8/(x**3+1)**(1/2),x)
```

output
```
2*x**6*sqrt(x**3 + 1)/15 - 8*x**3*sqrt(x**3 + 1)/45 + 16*sqrt(x**3 + 1)/45
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{\sqrt{1+x^3}}\,dx = \frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}} - \frac{4}{9}\left(x^3+1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^8/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(x^3 + 1)^(5/2) - 4/9*(x^3 + 1)^(3/2) + 2/3*sqrt(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{\sqrt{1+x^3}}\,dx = \frac{2}{15}\left(x^3+1\right)^{\frac{5}{2}} - \frac{4}{9}\left(x^3+1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^8/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/15*(x^3 + 1)^(5/2) - 4/9*(x^3 + 1)^(3/2) + 2/3*sqrt(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{x^8}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,(3\,x^6-4\,x^3+8)}{45}$$

input
```
int(x^8/(x^3 + 1)^(1/2),x)
```

output
```
(2*(x^3 + 1)^(1/2)*(3*x^6 - 4*x^3 + 8))/45
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,(3x^6-4x^3+8)}{45}$$

input
```
int(x^8/(x^3+1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*(3*x**6 - 4*x**3 + 8))/45
```

3.234 $\int \frac{x^5}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^5}{\sqrt{1+x^3}} \, dx = -\frac{2}{3}\sqrt{1+x^3} + \frac{2}{9}\left(1+x^3\right)^{3/2}$$

output
```
-2/3*(x^3+1)^(1/2)+2/9*(x^3+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^5}{\sqrt{1+x^3}} \, dx = \frac{2}{9}\left(-2+x^3\right)\sqrt{1+x^3}$$

input
```
Integrate[x^5/Sqrt[1 + x^3],x]
```

output
```
(2*(-2 + x^3)*Sqrt[1 + x^3])/9
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{x^3+1}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{x^3}{\sqrt{x^3+1}} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\sqrt{x^3+1} - \frac{1}{\sqrt{x^3+1}}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{2}{3} \left(x^3+1\right)^{3/2} - 2\sqrt{x^3+1}\right)$$

input
```
Int[x^5/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[1 + x^3] + (2*(1 + x^3)^(3/2))/3)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{2(x^3-2)\sqrt{x^3+1}}{9}$	15
pseudoelliptic	$\frac{2(x^3-2)\sqrt{x^3+1}}{9}$	15
trager	$\left(\frac{2x^3}{9}-\frac{4}{9}\right)\sqrt{x^3+1}$	16
default	$\frac{2x^3\sqrt{x^3+1}}{9}-\frac{4\sqrt{x^3+1}}{9}$	23
elliptic	$\frac{2x^3\sqrt{x^3+1}}{9}-\frac{4\sqrt{x^3+1}}{9}$	23
gosper	$\frac{2(1+x)(x^2-x+1)(x^3-2)}{9\sqrt{x^3+1}}$	26
orering	$\frac{2(1+x)(x^2-x+1)(x^3-2)}{9\sqrt{x^3+1}}$	26
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4x^3+8\right)\sqrt{x^3+1}}{6}}{3\sqrt{\pi}}$	31

input
```
int(x^5/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(x^3-2)*(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \frac{2}{9}\sqrt{x^3+1}\left(x^3-2\right)$$

input
```
integrate(x^5/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*sqrt(x^3 + 1)*(x^3 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \frac{2x^3\sqrt{x^3+1}}{9} - \frac{4\sqrt{x^3+1}}{9}$$

input
```
integrate(x**5/(x**3+1)**(1/2),x)
```

output
```
2*x**3*sqrt(x**3 + 1)/9 - 4*sqrt(x**3 + 1)/9
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^5/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(x^3 + 1)^(3/2) - 2/3*sqrt(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \frac{2}{9}\left(x^3+1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^5/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/9*(x^3 + 1)^(3/2) - 2/3*sqrt(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,(x^3-2)}{9}$$

input
```
int(x^5/(x^3 + 1)^(1/2),x)
```

output
```
(2*(x^3 + 1)^(1/2)*(x^3 - 2))/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,(x^3-2)}{9}$$

input
```
int(x^5/(x^3+1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*(x**3 - 2))/9
```

3.235 $\int \frac{x^2}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^2}{\sqrt{1+x^3}} \, dx = \frac{2\sqrt{1+x^3}}{3}$$

output
```
2/3*(x^3+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{1+x^3}} \, dx = \frac{2\sqrt{1+x^3}}{3}$$

input
```
Integrate[x^2/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{2\sqrt{x^3+1}}{3}$$

input
```
Int[x^2/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2\sqrt{x^3+1}}{3}$	10
default	$\frac{2\sqrt{x^3+1}}{3}$	10
trager	$\frac{2\sqrt{x^3+1}}{3}$	10
risch	$\frac{2\sqrt{x^3+1}}{3}$	10
elliptic	$\frac{2\sqrt{x^3+1}}{3}$	10
pseudoelliptic	$\frac{2\sqrt{x^3+1}}{3}$	10
gosper	$\frac{2(1+x)(x^2-x+1)}{3\sqrt{x^3+1}}$	21
orering	$\frac{2(1+x)(x^2-x+1)}{3\sqrt{x^3+1}}$	21
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^3+1}}{3\sqrt{\pi}}$	24

input
```
int(x^2/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\sqrt{x^3+1}$$

input
```
integrate(x^2/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}}{3}$$

input
```
integrate(x**2/(x**3+1)**(1/2),x)
```

output
```
2*sqrt(x**3 + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\,\sqrt{x^3+1}$$

input
```
integrate(x^2/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\,\sqrt{x^3+1}$$

input
```
integrate(x^2/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2\,\sqrt{x^3+1}}{3}$$

input
```
int(x^2/(x^3 + 1)^(1/2),x)
```

output
```
(2*(x^3 + 1)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}}{3}$$

input
```
int(x^2/(x^3+1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1))/3
```

3.236 $\int \frac{1}{x\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

output
```
-2/3*arctanh((x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

input
```
Integrate[1/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[1 + x^3]])/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}\int \frac{1}{x^6-1}d\sqrt{x^3+1}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2}{3}\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[1/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[1 + x^3]])/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}$	11
elliptic	$-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}$	11
pseudoelliptic	$-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^{3}+1}\right)}{3}$	11
trager	$\frac{\ln\left(-\frac{-x^{3}+2\sqrt{x^{3}+1}-2}{x^{3}}\right)}{3}$	25
meijerg	$\frac{\left(-2\ln\left(2\right)+3\ln\left(x\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{3}+1}}{2}\right)}{3\sqrt{\pi}}$	37

input
```
int(1/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((x^3+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

input
```
integrate(1/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate(1/x/(x**3+1)**(1/2),x)
```

output
```
-2*asinh(x**(-3/2))/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^3+1}-1\right)$$

input
```
integrate(1/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(sqrt(x^3 + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{1}{3}\log\left(\sqrt{x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

input
```
integrate(1/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*log(sqrt(x^3 + 1) + 1) + 1/3*log(abs(sqrt(x^3 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 11.71

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x*(x^3 + 1)^(1/2)),x)
```

output
```
-((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{1}{x\sqrt{1+x^3}}\,dx = \frac{\log\left(\sqrt{x^3+1}-1\right)}{3} - \frac{\log\left(\sqrt{x^3+1}+1\right)}{3}$$

input
```
int(1/x/(x^3+1)^(1/2),x)
```

output
```
(log(sqrt(x**3 + 1) - 1) - log(sqrt(x**3 + 1) + 1))/3
```

3.237 $\int \frac{1}{x^4\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{3x^3}+\frac{1}{3}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

output
```
-1/3*(x^3+1)^(1/2)/x^3+1/3*arctanh((x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{3x^3}+\frac{1}{3}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

input
```
Integrate[1/(x^4*Sqrt[1 + x^3]),x]
```

output
```
-1/3*Sqrt[1 + x^3]/x^3 + ArcTanh[Sqrt[1 + x^3]]/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3-\frac{\sqrt{x^3+1}}{x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\int \frac{1}{x^6-1}d\sqrt{x^3+1}-\frac{\sqrt{x^3+1}}{x^3}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{3}\left(\text{arctanh}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{x^3}\right)$$

input
```
Int[1/(x^4*Sqrt[1 + x^3]),x]
```

output
```
(-(Sqrt[1 + x^3]/x^3) + ArcTanh[Sqrt[1 + x^3]])/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	24
risch	$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	24
elliptic	$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	24
trager	$-\frac{\sqrt{x^3+1}}{3x^3}+\frac{\ln\left(-\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)}{6}$	36
pseudoelliptic	$\frac{-\ln\left(-1+\sqrt{x^3+1}\right)x^3+\ln\left(1+\sqrt{x^3+1}\right)x^3-2\sqrt{x^3+1}}{6x^3}$	45
meijerg	$\frac{-\frac{\sqrt{\pi}}{x^3}-\frac{(1-2\ln(2)+3\ln(x))\sqrt{\pi}}{2}+\frac{\sqrt{\pi}\left(4x^3+8\right)}{8x^3}-\frac{\sqrt{\pi}\,\sqrt{x^3+1}}{x^3}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)}{3\sqrt{\pi}}$	76

input
```
int(1/x^4/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x^3+1)^(1/2)/x^3+1/3*arctanh((x^3+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx=\frac{x^3\log\left(\sqrt{x^3+1}+1\right)-x^3\log\left(\sqrt{x^3+1}-1\right)-2\sqrt{x^3+1}}{6\,x^3}$$

input
```
integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(x^3*log(sqrt(x^3 + 1) + 1) - x^3*log(sqrt(x^3 + 1) - 1) - 2*sqrt(x^3
+ 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx=\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{\sqrt{1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/x**4/(x**3+1)**(1/2),x)
```

output
```
asinh(x**(-3/2))/3 - sqrt(1 + x**(-3))/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx=-\frac{\sqrt{x^3+1}}{3\,x^3}+\frac{1}{6}\,\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{6}\,\log\left(\sqrt{x^3+1}-1\right)$$

input
```
integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(x^3 + 1)/x^3 + 1/6*log(sqrt(x^3 + 1) + 1) - 1/6*log(sqrt(x^3 + 1
) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx=-\frac{\sqrt{x^3+1}}{3\,x^3}+\frac{1}{6}\,\log\left(\sqrt{x^3+1}+1\right)-\frac{1}{6}\,\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

input
```
integrate(1/x^4/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(x^3 + 1)/x^3 + 1/6*log(sqrt(x^3 + 1) + 1) - 1/6*log(abs(sqrt(x^3
 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 5.68

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx = -\frac{\sqrt{x^3+1}}{3\,x^3} + \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^4*(x^3 + 1)^(1/2)),x)
```

output
```
(((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x +
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (x^3 + 1)^(1/2)/(3*
x^3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^4\sqrt{1+x^3}}\,dx = \frac{-2\sqrt{x^3+1}-\log\left(\sqrt{x^3+1}-1\right)x^3+\log\left(\sqrt{x^3+1}+1\right)x^3}{6x^3}$$

input
```
int(1/x^4/(x^3+1)^(1/2),x)
```

output
```
( - 2*sqrt(x**3 + 1) - log(sqrt(x**3 + 1) - 1)*x**3 + log(sqrt(x**3 + 1) +
 1)*x**3)/(6*x**3)
```

3.238 $\int \frac{1}{x^7\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{6x^6}+\frac{\sqrt{1+x^3}}{4x^3}-\frac{1}{4}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

output
```
-1/6*(x^3+1)^(1/2)/x^6+1/4*(x^3+1)^(1/2)/x^3-1/4*arctanh((x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = \frac{\sqrt{1+x^3}(-2+3x^3)}{12x^6}-\frac{1}{4}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

input
```
Integrate[1/(x^7*Sqrt[1 + x^3]),x]
```

output
```
(Sqrt[1 + x^3]*(-2 + 3*x^3))/(12*x^6) - ArcTanh[Sqrt[1 + x^3]]/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 52, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^9\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{3}{4}\int \frac{1}{x^6\sqrt{x^3+1}}dx^3-\frac{\sqrt{x^3+1}}{2x^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3-\frac{\sqrt{x^3+1}}{x^3}\right)-\frac{\sqrt{x^3+1}}{2x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{3}{4}\left(-\int \frac{1}{x^6-1}d\sqrt{x^3+1}-\frac{\sqrt{x^3+1}}{x^3}\right)-\frac{\sqrt{x^3+1}}{2x^6}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{3}\left(-\frac{3}{4}\left(\text{arctanh}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{x^3}\right)-\frac{\sqrt{x^3+1}}{2x^6}\right)$$

input
```
Int[1/(x^7*Sqrt[1 + x^3]),x]
```

output
```
(-1/2*Sqrt[1 + x^3]/x^6 - (3*(-(Sqrt[1 + x^3]/x^3) + ArcTanh[Sqrt[1 + x^3]
]))/4)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{3x^6+x^3-2}{12x^6\sqrt{x^3+1}}-\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{4}$	34
default	$-\frac{\sqrt{x^3+1}}{6x^6}+\frac{\sqrt{x^3+1}}{4x^3}-\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{4}$	36
elliptic	$-\frac{\sqrt{x^3+1}}{6x^6}+\frac{\sqrt{x^3+1}}{4x^3}-\frac{\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{4}$	36
trager	$\frac{(3x^3-2)\sqrt{x^3+1}}{12x^6}+\frac{\ln\left(-\frac{-x^3+2\sqrt{x^3+1}-2}{x^3}\right)}{8}$	45
pseudoelliptic	$\frac{3\ln\left(-1+\sqrt{x^3+1}\right)x^6-3\ln\left(1+\sqrt{x^3+1}\right)x^6+6x^3\sqrt{x^3+1}-4\sqrt{x^3+1}}{24\left(-1+\sqrt{x^3+1}\right)^2\left(1+\sqrt{x^3+1}\right)^2}$	77
meijerg	$\frac{-\frac{\sqrt{\pi}}{2x^6}+\frac{\sqrt{\pi}}{2x^3}+\frac{3\left(\frac{7}{6}-2\ln(2)+3\ln(x)\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^6-8x^3+8\right)}{16x^6}-\frac{\sqrt{\pi}\left(-12x^3+8\right)\sqrt{x^3+1}}{16x^6}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)}{4}}{3\sqrt{\pi}}$	97

input
```
int(1/x^7/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^6+x^3-2)/x^6/(x^3+1)^(1/2)-1/4*arctanh((x^3+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx$$
$$= -\frac{3\,x^6\log\left(\sqrt{x^3+1}+1\right)-3\,x^6\log\left(\sqrt{x^3+1}-1\right)-2\,(3\,x^3-2)\sqrt{x^3+1}}{24\,x^6}$$

input
```
integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(3*x^6*log(sqrt(x^3 + 1) + 1) - 3*x^6*log(sqrt(x^3 + 1) - 1) - 2*(3*
x^3 - 2)*sqrt(x^3 + 1))/x^6
```

Sympy [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4} + \frac{1}{4x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}} + \frac{1}{12x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}} - \frac{1}{6x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}$$

input
```
integrate(1/x**7/(x**3+1)**(1/2),x)
```

output
```
-asinh(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(1 + x**(-3))) + 1/(12*x**(9/2)*sq
rt(1 + x**(-3))) - 1/(6*x**(15/2)*sqrt(1 + x**(-3)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = -\frac{3\,(x^3+1)^{\frac{3}{2}} - 5\sqrt{x^3+1}}{12\,(2\,x^3 - (x^3+1)^2 + 1)} - \frac{1}{8}\log\left(\sqrt{x^3+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^3+1}-1\right)$$

input
```
integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/(2*x^3 - (x^3 + 1)^2 + 1) - 1/
8*log(sqrt(x^3 + 1) + 1) + 1/8*log(sqrt(x^3 + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = \frac{3\,(x^3+1)^{\frac{3}{2}} - 5\sqrt{x^3+1}}{12\,x^6} - \frac{1}{8}\log\left(\sqrt{x^3+1}+1\right) + \frac{1}{8}\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

input
```
integrate(1/x^7/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*(x^3 + 1)^(3/2) - 5*sqrt(x^3 + 1))/x^6 - 1/8*log(sqrt(x^3 + 1) + 1
) + 1/8*log(abs(sqrt(x^3 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.02

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = \frac{\sqrt{x^3+1}}{4\,x^3}-\frac{\sqrt{x^3+1}}{6\,x^6} -\frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^7*(x^3 + 1)^(1/2)),x)
```

output
```
(x^3 + 1)^(1/2)/(4*x^3) - (x^3 + 1)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 + 3
/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(x^3 -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^7\sqrt{1+x^3}}\,dx = \frac{6\sqrt{x^3+1}\,x^3-4\sqrt{x^3+1}+3\log\left(\sqrt{x^3+1}-1\right)x^6-3\log\left(\sqrt{x^3+1}+1\right)x^6}{24x^6}$$

input
```
int(1/x^7/(x^3+1)^(1/2),x)
```

output
```
(6*sqrt(x**3 + 1)*x**3 - 4*sqrt(x**3 + 1) + 3*log(sqrt(x**3 + 1) - 1)*x**6
 - 3*log(sqrt(x**3 + 1) + 1)*x**6)/(24*x**6)
```

3.239 $\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{9x^9}+\frac{5\sqrt{1+x^3}}{36x^6}-\frac{5\sqrt{1+x^3}}{24x^3}+\frac{5}{24}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

output
```
-1/9*(x^3+1)^(1/2)/x^9+5/36*(x^3+1)^(1/2)/x^6-5/24*(x^3+1)^(1/2)/x^3+5/24*
arctanh((x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = \frac{\sqrt{1+x^3}(-8+10x^3-15x^6)}{72x^9}+\frac{5}{24}\text{arctanh}\left(\sqrt{1+x^3}\right)$$

input
```
Integrate[1/(x^10*Sqrt[1 + x^3]),x]
```

output
```
(Sqrt[1 + x^3]*(-8 + 10*x^3 - 15*x^6))/(72*x^9) + (5*ArcTanh[Sqrt[1 + x^3]
])/24
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {798, 52, 52, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^{12}\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{5}{6}\int \frac{1}{x^9\sqrt{x^3+1}}dx^3-\frac{\sqrt{x^3+1}}{3x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{5}{6}\left(-\frac{3}{4}\int \frac{1}{x^6\sqrt{x^3+1}}dx^3-\frac{\sqrt{x^3+1}}{2x^6}\right)-\frac{\sqrt{x^3+1}}{3x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(-\frac{5}{6}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3-\frac{\sqrt{x^3+1}}{x^3}\right)-\frac{\sqrt{x^3+1}}{2x^6}\right)-\frac{\sqrt{x^3+1}}{3x^9}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\frac{5}{6}\left(-\frac{3}{4}\left(-\int \frac{1}{x^6-1}d\sqrt{x^3+1}-\frac{\sqrt{x^3+1}}{x^3}\right)-\frac{\sqrt{x^3+1}}{2x^6}\right)-\frac{\sqrt{x^3+1}}{3x^9}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{3}\left(-\frac{5}{6}\left(-\frac{3}{4}\left(\text{arctanh}\left(\sqrt{x^3+1}\right)-\frac{\sqrt{x^3+1}}{x^3}\right)-\frac{\sqrt{x^3+1}}{2x^6}\right)-\frac{\sqrt{x^3+1}}{3x^9}\right)$$

input
```
Int[1/(x^10*Sqrt[1 + x^3]),x]
```

output
```
(-1/3*Sqrt[1 + x^3]/x^9 - (5*(-1/2*Sqrt[1 + x^3]/x^6 - (3*(-(Sqrt[1 + x^3]
/x^3) + ArcTanh[Sqrt[1 + x^3]]))/4))/6)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{15x^9+5x^6-2x^3+8}{72x^9\sqrt{x^3+1}}+\frac{5\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{24}$
default	$-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5\sqrt{x^3+1}}{36x^6}-\frac{5\sqrt{x^3+1}}{24x^3}+\frac{5\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{24}$
trager	$-\frac{(15x^6-10x^3+8)\sqrt{x^3+1}}{72x^9}+\frac{5\ln\left(-\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)}{48}$
elliptic	$-\frac{\sqrt{x^3+1}}{9x^9}+\frac{5\sqrt{x^3+1}}{36x^6}-\frac{5\sqrt{x^3+1}}{24x^3}+\frac{5\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{24}$
pseudoelliptic	$\frac{-15\ln\left(-1+\sqrt{x^3+1}\right)x^9+15\ln\left(1+\sqrt{x^3+1}\right)x^9+(-30x^6+20x^3-16)\sqrt{x^3+1}}{144\left(-1+\sqrt{x^3+1}\right)^3\left(1+\sqrt{x^3+1}\right)^3}$
meijerg	$\frac{-\frac{\sqrt{\pi}}{3x^9}+\frac{\sqrt{\pi}}{4x^6}-\frac{3\sqrt{\pi}}{8x^3}-\frac{5\left(\frac{37}{30}-2\ln(2)+3\ln(x)\right)\sqrt{\pi}}{16}+\frac{\sqrt{\pi}\left(148x^9+144x^6-96x^3+128\right)}{384x^9}-\frac{\sqrt{\pi}\left(240x^6-160x^3+128\right)\sqrt{x^3+1}}{384x^9}+\frac{5\sqrt{\pi}\,\ln\left(\frac{1}{2}+\cdots\right.}{8}}{3\sqrt{\pi}}$

input
```
int(1/x^10/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(15*x^9+5*x^6-2*x^3+8)/x^9/(x^3+1)^(1/2)+5/24*arctanh((x^3+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = \frac{15\,x^9\log\left(\sqrt{x^3+1}+1\right)-15\,x^9\log\left(\sqrt{x^3+1}-1\right)-2\,(15\,x^6-10\,x^3+8)\sqrt{x^3+1}}{144\,x^9}$$

input
```
integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/144*(15*x^9*log(sqrt(x^3 + 1) + 1) - 15*x^9*log(sqrt(x^3 + 1) - 1) - 2*(
15*x^6 - 10*x^3 + 8)*sqrt(x^3 + 1))/x^9
```

Sympy [A] (verification not implemented)

Time = 7.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = \frac{5\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24} - \frac{5}{24x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}} - \frac{5}{72x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}} + \frac{1}{36x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}} - \frac{1}{9x^{\frac{21}{2}}\sqrt{1+\frac{1}{x^3}}}$$

input
```
integrate(1/x**10/(x**3+1)**(1/2),x)
```

output
```
5*asinh(x**(-3/2))/24 - 5/(24*x**(3/2)*sqrt(1 + x**(-3))) - 5/(72*x**(9/2)
*sqrt(1 + x**(-3))) + 1/(36*x**(15/2)*sqrt(1 + x**(-3))) - 1/(9*x**(21/2)*
sqrt(1 + x**(-3)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = -\frac{15\,(x^3+1)^{\frac{5}{2}} - 40\,(x^3+1)^{\frac{3}{2}} + 33\,\sqrt{x^3+1}}{72\,\left((x^3+1)^3 + 3\,x^3 - 3\,(x^3+1)^2 + 2\right)} + \frac{5}{48}\,\log\left(\sqrt{x^3+1}+1\right) - \frac{5}{48}\,\log\left(\sqrt{x^3+1}-1\right)$$

input
```
integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/72*(15*(x^3 + 1)^(5/2) - 40*(x^3 + 1)^(3/2) + 33*sqrt(x^3 + 1))/((x^3 +
 1)^3 + 3*x^3 - 3*(x^3 + 1)^2 + 2) + 5/48*log(sqrt(x^3 + 1) + 1) - 5/48*lo
g(sqrt(x^3 + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = -\frac{15\,(x^3+1)^{\frac{5}{2}} - 40\,(x^3+1)^{\frac{3}{2}} + 33\,\sqrt{x^3+1}}{72\,x^9} + \frac{5}{48}\,\log\left(\sqrt{x^3+1}+1\right) - \frac{5}{48}\,\log\left(\left|\sqrt{x^3+1}-1\right|\right)$$

input
```
integrate(1/x^10/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/72*(15*(x^3 + 1)^(5/2) - 40*(x^3 + 1)^(3/2) + 33*sqrt(x^3 + 1))/x^9 + 5
/48*log(sqrt(x^3 + 1) + 1) - 5/48*log(abs(sqrt(x^3 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.19

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx = \frac{5\,\sqrt{x^3+1}}{36\,x^6} - \frac{5\,\sqrt{x^3+1}}{24\,x^3} - \frac{\sqrt{x^3+1}}{9\,x^9} + \frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^10*(x^3 + 1)^(1/2)),x)
```

output
```
(5*(x^3 + 1)^(1/2))/(36*x^6) - (5*(x^3 + 1)^(1/2))/(24*x^3) - (x^3 + 1)^(1
/2)/(9*x^9) + (5*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*
1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 +
 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(8*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^{10}\sqrt{1+x^3}}\,dx$$
$$= \frac{-30\sqrt{x^3+1}\,x^6+20\sqrt{x^3+1}\,x^3-16\sqrt{x^3+1}-15\log\left(\sqrt{x^3+1}-1\right)x^9+15\log\left(\sqrt{x^3+1}+1\right)x^9}{144x^9}$$

input
```
int(1/x^10/(x^3+1)^(1/2),x)
```

output
```
( - 30*sqrt(x**3 + 1)*x**6 + 20*sqrt(x**3 + 1)*x**3 - 16*sqrt(x**3 + 1) -
15*log(sqrt(x**3 + 1) - 1)*x**9 + 15*log(sqrt(x**3 + 1) + 1)*x**9)/(144*x*
*9)
```

3.240 $\int \frac{x^6}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = -\frac{16}{55}x\sqrt{1+x^3}+\frac{2}{11}x^4\sqrt{1+x^3} + \frac{32\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-16/55*x*(x^3+1)^(1/2)+2/11*x^4*(x^3+1)^(1/2)+32/165*(1/2*6^(1/2)+1/2*2^(1
/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+
3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \frac{2}{55}x\left(\sqrt{1+x^3}\left(-8+5x^3\right)+8\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)\right)$$

input
```
Integrate[x^6/Sqrt[1 + x^3],x]
```

output
```
(2*x*(Sqrt[1 + x^3]*(-8 + 5*x^3) + 8*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3
]))/55
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{11}x^4\sqrt{x^3+1}-\frac{8}{11}\int \frac{x^3}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{11}x^4\sqrt{x^3+1}-\frac{8}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{2}{5}\int \frac{1}{\sqrt{x^3+1}}dx\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{11}x^4\sqrt{x^3+1}-\frac{8}{11}\left(\frac{2}{5}x\sqrt{x^3+1}-\frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)$$

input
```
Int[x^6/Sqrt[1 + x^3],x]
```

output
```
(2*x^4*Sqrt[1 + x^3])/11 - (8*((2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]
]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sq
rt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/11
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

method	result	size
meijerg	$\frac{x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{7}{3}\right],\left[\frac{10}{3}\right],-x^{3}\right)}{7}$	17
risch	$\frac{2 x\left(5 x^{3}-8\right) \sqrt{x^{3}+1}}{55}+\frac{32\left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{55 \sqrt{x^{3}+1}}$	134
default	$\frac{2 x^{4} \sqrt{x^{3}+1}}{11}-\frac{16 x \sqrt{x^{3}+1}}{55}+\frac{32\left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{55 \sqrt{x^{3}+1}}$	139
elliptic	$\frac{2 x^{4} \sqrt{x^{3}+1}}{11}-\frac{16 x \sqrt{x^{3}+1}}{55}+\frac{32\left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{55 \sqrt{x^{3}+1}}$	139

input
```
int(x^6/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*hypergeom([1/2,7/3],[10/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.18

$$\int \frac{x^6}{\sqrt{1+x^3}}\, dx = \frac{2}{55}\left(5\, x^{4} - 8\, x\right)\sqrt{x^{3} + 1} + \frac{32}{55}\, \mathrm{weierstrassPInverse}(0, -4, x)$$

input
```
integrate(x^6/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(5*x^4 - 8*x)*sqrt(x^3 + 1) + 32/55*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.21

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \frac{x^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(x**3+1)**(1/2),x)
```

output
```
x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3*exp_polar(I*pi))/(3*gamma(
10/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \int \frac{x^6}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^6/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \int \frac{x^6}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^6/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \frac{2\,x^4\,\sqrt{x^3+1}}{11}-\frac{16\,x\,\sqrt{x^3+1}}{55} + \frac{32\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{55\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x^6/(x^3 + 1)^(1/2),x)
```

output
```
(2*x^4*(x^3 + 1)^(1/2))/11 - (16*x*(x^3 + 1)^(1/2))/55 + (32*((3^(1/2)*1i)
/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(x^3 - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,x^4}{11}-\frac{16\sqrt{x^3+1}\,x}{55}+\frac{16\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)}{55}$$

input
```
int(x^6/(x^3+1)^(1/2),x)
```

output
```
(2*(5*sqrt(x**3 + 1)*x***4 - 8*sqrt(x**3 + 1)*x + 8*int(sqrt(x**3 + 1)/(x**
3 + 1),x)))/55
```

3.241 $\int \frac{x^3}{\sqrt{1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 120

$$\int \frac{x^3}{\sqrt{1+x^3}} \, dx = \frac{2}{5}x\sqrt{1+x^3} - \frac{4\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/5*x*(x^3+1)^(1/2)-4/15*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3
^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3
/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.27

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx = \frac{2}{5}x\left(\sqrt{1+x^3} - \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)\right)$$

input
```
Integrate[x^3/Sqrt[1 + x^3],x]
```

output
```
(2*x*(Sqrt[1 + x^3] - Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]))/5
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{5}x\sqrt{x^3+1} - \frac{2}{5}\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{5}x\sqrt{x^3+1} - \frac{4\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[x^3/Sqrt[1 + x^3],x]
```

output
```
(2*x*Sqrt[1 + x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)],
-7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3
])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.14

method	result	size
meijerg	$\frac{x^{4} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{4}{3}\right], \left[\frac{7}{3}\right], -x^{3}\right)}{4}$	17
default	$\frac{2 x \sqrt{x^{3}+1}}{5}-\frac{4 \left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{5 \sqrt{x^{3}+1}}$	127
risch	$\frac{2 x \sqrt{x^{3}+1}}{5}-\frac{4 \left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{5 \sqrt{x^{3}+1}}$	127
elliptic	$\frac{2 x \sqrt{x^{3}+1}}{5}-\frac{4 \left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}-\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x-\frac{1}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{5 \sqrt{x^{3}+1}}$	127

input
```
int(x^3/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*hypergeom([1/2,4/3],[7/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx = \frac{2}{5}\sqrt{x^3+1}x - \frac{4}{5}\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(x^3 + 1)*x - 4/5*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(x**3+1)**(1/2),x)
```

output
```
x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3*exp_polar(I*pi))/(3*gamma(7
/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx=\int \frac{x^3}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^3/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx=\int \frac{x^3}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^3/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx = \frac{2\,x\,\sqrt{x^3+1}}{5} - \frac{4\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{5\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$

input
```
int(x^3/(x^3 + 1)^(1/2),x)
```

output
```
(2*x*(x^3 + 1)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 -
 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)
*1i)/2 - 3/2)))/(5*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,x}{5} - \frac{2\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)}{5}$$

input
```
int(x^3/(x^3+1)^(1/2),x)
```

output
```
(2*(sqrt(x**3 + 1)*x - int(sqrt(x**3 + 1)/(x**3 + 1),x)))/5
```

3.242 $\int \frac{1}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 103

$$\int \frac{1}{\sqrt{1+x^3}}\,dx=\frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*Elli
pticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/
2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt{1+x^3}}\,dx=x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[1/Sqrt[1 + x^3],x]
```

output
```
x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \ 759$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[1/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.14

method	result	size
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^{3}\right)$	14
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}$	116
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}}$	116

input
```
int(1/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = 2\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(x**3+1)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(1/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(1/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = \frac{(3+\sqrt{3}\,1\mathrm{i})\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^3 + 1)^(1/2),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{x^3+1}}{x^3+1}dx$$

input
```
int(1/(x^3+1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**3 + 1),x)
```

3.243 $\int \frac{1}{x^3\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 122

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{2x^2} - \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-1/2*(x^3+1)^(1/2)/x^2-1/6*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x
+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^
(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-x^3\right)}{2x^2}$$

input
```
Integrate[1/(x^3*Sqrt[1 + x^3]),x]
```

output
```
-1/2*Hypergeometric2F1[-2/3, 1/2, 1/3, -x^3]/x^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^3+1}}dx$$
$$\Big\downarrow \text{847}$$
$$-\frac{1}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}$$
$$\Big\downarrow \text{759}$$
$$-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 + x^3]),x]
```

output
```
-1/2*Sqrt[1 + x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
+ Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -
7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]
)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.14

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{2}\right],\left[\frac{1}{3}\right],-x^{3}\right)}{2x^{2}}$	17
default	$-\frac{\sqrt{x^{3}+1}}{2x^{2}}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}+1}}$	129
risch	$-\frac{\sqrt{x^{3}+1}}{2x^{2}}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}+1}}$	129
elliptic	$-\frac{\sqrt{x^{3}+1}}{2x^{2}}-\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}+1}}$	129

input
```
int(1/x^3/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*hypergeom([-2/3,1/2],[1/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = -\frac{x^2\text{weierstrassPInverse}(0,-4,x)+\sqrt{x^3+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^2*weierstrassPInverse(0, -4, x) + sqrt(x^3 + 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = \frac{\Gamma\left(-\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\,x^3e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(x**3+1)**(1/2),x)
```

output
```
gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3*exp_polar(I*pi))/(3*x**2*gamma
(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^3}\,dx$$

input
```
integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^3}\,dx$$

input
```
integrate(1/x^3/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = -\frac{\sqrt{x^3+1}}{2\,x^2} - \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^3*(x^3 + 1)^(1/2)),x)
```

output
```
- (x^3 + 1)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 -
 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)
*1i)/2 - 3/2)))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{x^3+1}}{x^6+x^3}dx$$

input
```
int(1/x^3/(x^3+1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**6 + x**3),x)
```

3.244 $\int \frac{1}{x^6\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 138

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{5x^5}+\frac{7\sqrt{1+x^3}}{20x^2} + \frac{7\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-1/5*(x^3+1)^(1/2)/x^5+7/20*(x^3+1)^(1/2)/x^2+7/60*(1/2*6^(1/2)+1/2*2^(1/2
))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^
(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},-x^3\right)}{5x^5}$$

input
```
Integrate[1/(x^6*Sqrt[1 + x^3]),x]
```

output
```
-1/5*Hypergeometric2F1[-5/3, 1/2, -2/3, -x^3]/x^5
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{7}{10}\int \frac{1}{x^3\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{7}{10}\left(-\frac{1}{4}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{2x^2}\right)-\frac{\sqrt{x^3+1}}{5x^5}$$

$$\Big\downarrow \text{759}$$

$$-\frac{7}{10}\left(-\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{x^3+1}}{2x^2}\right)-\frac{\sqrt{x^3+1}}{5x^5}$$

input
```
Int[1/(x^6*Sqrt[1 + x^3]),x]
```

output
```
-1/5*Sqrt[1 + x^3]/x^5 - (7*(-1/2*Sqrt[1 + x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(
1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 + x)/(1 +
Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/10
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{5}{3},\frac{1}{2}\right],\left[-\frac{2}{3}\right],-x^{3}\right)}{5x^{5}}$	17
default	$-\frac{\sqrt{x^{3}+1}}{5x^{5}}+\frac{7\sqrt{x^{3}+1}}{20x^{2}}+\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^{3}+1}}$	141
risch	$\frac{7x^{6}+3x^{3}-4}{20x^{5}\sqrt{x^{3}+1}}+\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^{3}+1}}$	141
elliptic	$-\frac{\sqrt{x^{3}+1}}{5x^{5}}+\frac{7\sqrt{x^{3}+1}}{20x^{2}}+\frac{7\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^{3}+1}}$	141

input
```
int(1/x^6/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5*hypergeom([-5/3,1/2],[-2/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx=\frac{7\,x^5\text{weierstrassPInverse}(0,-4,x)+(7\,x^3-4)\sqrt{x^3+1}}{20\,x^5}$$

input
```
integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(7*x^5*weierstrassPInverse(0, -4, x) + (7*x^3 - 4)*sqrt(x^3 + 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = \frac{\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, \frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(x**3+1)**(1/2),x)
```

output
```
gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3*exp_polar(I*pi))/(3*x**5*gamm
a(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = \frac{7\sqrt{x^3+1}}{20\,x^2} - \frac{\sqrt{x^3+1}}{5\,x^5} + \frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{20\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^6*(x^3 + 1)^(1/2)),x)
```

output
```
(7*(x^3 + 1)^(1/2))/(20*x^2) - (x^3 + 1)^(1/2)/(5*x^5) + (7*((3^(1/2)*1i)/
2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(x^3 - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{x^3+1}}{x^9+x^6}dx$$

input
```
int(1/x^6/(x^3+1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**9 + x**6),x)
```

3.245 $\int \frac{x^7}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 262

$$\begin{aligned}\int \frac{x^7}{\sqrt{1+x^3}}\,dx = &-\frac{20}{91}x^2\sqrt{1+x^3}+\frac{2}{13}x^5\sqrt{1+x^3}+\frac{80\sqrt{1+x^3}}{91\left(1+\sqrt{3}+x\right)} \\ &-\frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} \\ &+\frac{80\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
-20/91*x^2*(x^3+1)^(1/2)+2/13*x^5*(x^3+1)^(1/2)+80*(x^3+1)^(1/2)/(91+91*3^
(1/2)+91*x)-40/91*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+
3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+80/273*2^(1/2)*(1+x)*((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)
*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.16

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx = \frac{2}{91}x^2\left(\sqrt{1+x^3}(-10+7x^3)+10\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)$$

input
```
Integrate[x^7/Sqrt[1 + x^3],x]
```

output
```
(2*x^2*(Sqrt[1 + x^3]*(-10 + 7*x^3) + 10*Hypergeometric2F1[1/2, 2/3, 5/3,
-x^3]))/91
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{13}x^5\sqrt{x^3+1}-\frac{10}{13}\int \frac{x^4}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{13}x^5\sqrt{x^3+1}-\frac{10}{13}\left(\frac{2}{7}x^2\sqrt{x^3+1}-\frac{4}{7}\int \frac{x}{\sqrt{x^3+1}}dx\right)$$

$$\Big\downarrow \text{832}$$

$$\frac{2}{13}x^5\sqrt{x^3+1}-\frac{10}{13}\left(\frac{2}{7}x^2\sqrt{x^3+1}-\frac{4}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{13}x^5\sqrt{x^3+1}-\frac{10}{13}\left(\frac{2}{7}x^2\sqrt{x^3+1}-\frac{4}{7}\left(\int\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right.}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right.\right.$$

$$\Big\downarrow \text{2416}$$

$$\frac{2}{13}x^5\sqrt{x^3+1}-\frac{10}{13}\left(\frac{2}{7}x^2\sqrt{x^3+1}-\frac{4}{7}\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\right.\right.$$

input
```
Int[x^7/Sqrt[1 + x^3],x]
```

output
```
(2*x^5*Sqrt[1 + x^3])/13 - (10*((2*x^2*Sqrt[1 + x^3])/7 - (4*((2*Sqrt[1 +
x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
 + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
 - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt
[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*
Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/7))/
13
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.06

method	result
meijerg	$\frac{x^8 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{8}{3}\right],\left[\frac{11}{3}\right],-x^3\right)}{8}$
risch	$\frac{2x^2(7x^3-10)\sqrt{x^3+1}}{91}+\frac{80\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\ldots\right.\right.}{91\sqrt{x^3+1}}$
default	$\frac{2x^5\sqrt{x^3+1}}{13}-\frac{20x^2\sqrt{x^3+1}}{91}+\frac{80\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\ldots}{-\frac{3}{2}-\ldots}}\right.\right.}{91\sqrt{x^3+1}}$
elliptic	$\frac{2x^5\sqrt{x^3+1}}{13}-\frac{20x^2\sqrt{x^3+1}}{91}+\frac{80\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\ldots}{-\frac{3}{2}-\ldots}}\right.\right.}{91\sqrt{x^3+1}}$

input
```
int(x^7/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*hypergeom([1/2,8/3],[11/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx = \frac{2}{91}\left(7\,x^5-10\,x^2\right)\sqrt{x^3+1} - \frac{80}{91}\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x^7/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/91*(7*x^5 - 10*x^2)*sqrt(x^3 + 1) - 80/91*weierstrassZeta(0, -4, weierst
rassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(x**3+1)**(1/2),x)
```

output
```
x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3*exp_polar(I*pi))/(3*gamma(
11/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx=\int \frac{x^7}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^7/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx=\int \frac{x^7}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^7/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.91

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx=\frac{2\,x^5\,\sqrt{x^3+1}}{13}-\frac{20\,x^2\,\sqrt{x^3+1}}{91} -\frac{80\,\Big(\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\mathrm{F}\Big(\mathrm{asin}\Big(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Big)\Big|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\Big)-\Big(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\mathrm{E}\Big(\mathrm{asin}\Big(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\Big)\Big|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\Big)}{91\,\sqrt{x^3+\Big(-\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)\,\Big(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)-1\Big)\,x-\Big(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\Big)}}$$

input
```
int(x^7/(x^3 + 1)^(1/2),x)
```

output
```
(2*x^5*(x^3 + 1)^(1/2))/13 - (20*x^2*(x^3 + 1)^(1/2))/91 - (80*(((3^(1/2)*
1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipt
icE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(91*(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,x^5}{13} - \frac{20\sqrt{x^3+1}\,x^2}{91} + \frac{40\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)}{91}$$

input
```
int(x^7/(x^3+1)^(1/2),x)
```

output
```
(2*(7*sqrt(x**3 + 1)*x**5 - 10*sqrt(x**3 + 1)*x**2 + 20*int((sqrt(x**3 + 1
)*x)/(x**3 + 1),x)))/91
```

3.246 $\int \frac{x^4}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 246

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{2}{7}x^2\sqrt{1+x^3}-\frac{8\sqrt{1+x^3}}{7\left(1+\sqrt{3}+x\right)} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{8\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/7*x^2*(x^3+1)^(1/2)-8*(x^3+1)^(1/2)/(7+7*3^(1/2)+7*x)+4/7*3^(1/4)*(1/2*6
^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x
-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+
1)^(1/2)-8/21*2^(1/2)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1
+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(
1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{2}{7}x^2\left(\sqrt{1+x^3} - \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)$$

input
```
Integrate[x^4/Sqrt[1 + x^3],x]
```

output
```
(2*x^2*(Sqrt[1 + x^3] - Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/7
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{7}x^2\sqrt{x^3+1} - \frac{4}{7}\int \frac{x}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{832}$$

$$\frac{2}{7}x^2\sqrt{x^3+1} - \frac{4}{7}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx - \left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)$$

$$\Big\downarrow \text{759}$$

$$\frac{2}{7}x^2\sqrt{x^3+1}-\frac{4}{7}\left(\int\frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)$$

$\Big\downarrow$ 2416

$$\frac{2}{7}x^2\sqrt{x^3+1}-\frac{4}{7}\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)}{}\right.$$

input
```
Int[x^4/Sqrt[1 + x^3],x]
```

output
```
(2*x^2*Sqrt[1 + x^3])/7 - (4*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/
4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(
1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1
- Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])))/7
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.07

method	result
meijerg	$\frac{x^5 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{3}\right],\left[\frac{8}{3}\right],-x^3\right)}{5}$
default	$\frac{2x^2\sqrt{x^3+1}}{7} - \frac{8\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{E}\right.}{7\sqrt{x^3+1}}$
risch	$\frac{2x^2\sqrt{x^3+1}}{7} - \frac{8\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{E}\right.}{7\sqrt{x^3+1}}$
elliptic	$\frac{2x^2\sqrt{x^3+1}}{7} - \frac{8\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{E}\right.}{7\sqrt{x^3+1}}$

input
```
int(x^4/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1/2,5/3],[8/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{2}{7}\sqrt{x^3+1}x^2 + \frac{8}{7}\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x^4/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(x^3 + 1)*x^2 + 8/7*weierstrassZeta(0, -4, weierstrassPInverse(0,
-4, x))
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(x**3+1)**(1/2),x)
```

output
```
x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3*exp_polar(I*pi))/(3*gamma(8
/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \int \frac{x^4}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^4/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \int \frac{x^4}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x^4/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{2\,x^2\,\sqrt{x^3+1}}{7} + \frac{8\,\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)\right)}{7\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(}}$$

input
```
int(x^4/(x^3 + 1)^(1/2),x)
```

output
```
(2*x^2*(x^3 + 1)^(1/2))/7 + (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/
2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2))/(7*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{x^3+1}\,x^2}{7} - \frac{4\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)}{7}$$

input
```
int(x^4/(x^3+1)^(1/2),x)
```

output
```
(2*(sqrt(x**3 + 1)*x**2 - 2*int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)))/7
```

3.247 $\int \frac{x}{\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 224

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*(x^3+1)^(1/2)/(1+x+3^(1/2))-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1
/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+2/3*2^(1/2)*(1+x)*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(
1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.10

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \frac{1}{2}x^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[x/Sqrt[1 + x^3],x]
```

output
```
(x^2*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \; 832$$

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \; 759$$

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \; 2416$$

$$-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}}{x+\sqrt{3}+1}$$

input
```
Int[x/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^
2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.08

method	result
meijerg	$\frac{x^2 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticF}\left(\sqrt{\frac{1+}{\frac{3}{2}-}}\right.\right.}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticF}\left(\sqrt{\frac{1+}{\frac{3}{2}-}}\right.\right.}{\sqrt{x^3+1}}$

input
```
int(x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = -2\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.13

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamma(5
/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(x^3 + 1), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = -\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\right.}}$$

input
```
int(x/(x^3 + 1)^(1/2),x)
```

output
```
-(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/
2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2
)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1
/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^
3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{x}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx$$

input
```
int(x/(x^3+1)^(1/2),x)
```

output
```
int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.248 $\int \frac{1}{x^2\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 238

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{x}+\frac{\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-(x^3+1)^(1/2)/x+(x^3+1)^(1/2)/(1+x+3^(1/2))-1/2*3^(1/4)*(1/2*6^(1/2)-1/2*
2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticE((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)+1/
3*2^(1/2)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*EllipticF((1+x-3^(1/2))/
(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-x^3\right)}{x}$$

input
```
Integrate[1/(x^2*Sqrt[1 + x^3]),x]
```

output
```
-(Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3]/x)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 847$$

$$\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}$$

$$\Big\downarrow \ 832$$

$$\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}$$

$$\Big\downarrow \ 759$$

$$\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\sqrt{x^3+1}}{x}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{1}{2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{}}{}\right.$$
$$\frac{\sqrt{x^3+1}}{x}$$

input
```
Int[1/(x^2*Sqrt[1 + x^3]),x]
```

output
```
-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[
2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[Arc
Sin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(
1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1
 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqr
t[3] + x)^2]*Sqrt[1 + x^3]))/2
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.95 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.07

method	result
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{1}{3},\frac{1}{2}\right],\left[\frac{2}{3}\right],-x^{3}\right)}{x}$
default	$-\frac{\sqrt{x^{3}+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{Elli}\right.}{\sqrt{x^{3}+1}}$
risch	$-\frac{\sqrt{x^{3}+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{Elli}\right.}{\sqrt{x^{3}+1}}$
elliptic	$-\frac{\sqrt{x^{3}+1}}{x}+\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{Elli}\right.}{\sqrt{x^{3}+1}}$

input
```
int(1/x^2/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x*hypergeom([-1/3,1/2],[2/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = -\frac{x\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))+\sqrt{x^3+1}}{x}$$

input
```
integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-(x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(x^3 + 1))
/x
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(x**3+1)**(1/2),x)
```

output
```
gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma(2/
3))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = -\frac{\sqrt{x^3+1}}{x} - \frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

input
```
int(1/(x^2*(x^3 + 1)^(1/2)),x)
```

output
```
- (x^3 + 1)^(1/2)/x - ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2
 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{x^3+1}}{x^5+x^2}dx$$

input
```
int(1/x^2/(x^3+1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**5 + x**2),x)
```

3.249 $\int \frac{1}{x^5\sqrt{1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 262

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = -\frac{\sqrt{1+x^3}}{4x^4}+\frac{5\sqrt{1+x^3}}{8x}-\frac{5\sqrt{1+x^3}}{8\left(1+\sqrt{3}+x\right)} + \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{5(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-1/4*(x^3+1)^(1/2)/x^4+5/8*(x^3+1)^(1/2)/x-5*(x^3+1)^(1/2)/(8+8*3^(1/2)+8*
x)+5/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)/((1+x)/(1+x+3
^(1/2))^2)^(1/2)/(x^3+1)^(1/2)-5/24*2^(1/2)*(1+x)*((x^2-x+1)/(1+x+3^(1/2))
^2)^(1/2)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*3^(3/4)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},-x^3\right)}{4x^4}$$

input
```
Integrate[1/(x^5*Sqrt[1 + x^3]),x]
```

output
```
-1/4*Hypergeometric2F1[-4/3, 1/2, -1/3, -x^3]/x^4
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{5}{8}\int \frac{1}{x^2\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$-\frac{5}{8}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)-\frac{\sqrt{x^3+1}}{4x^4}$$

$$\Big\downarrow \text{832}$$

$$-\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)-\frac{\sqrt{x^3+1}}{4x^4}$$

$$\Big\downarrow \text{759}$$

$$-\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right.\right.$$
$$\frac{\sqrt{x^3+1}}{4x^4}$$

$\downarrow$ 2416

$$-\frac{5}{8}\left(\frac{1}{2}\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x}{}\right.\right.$$
$$\frac{\sqrt{x^3+1}}{4x^4}$$

input
```
Int[1/(x^5*Sqrt[1 + x^3]),x]
```

output
```
-1/4*Sqrt[1 + x^3]/x^4 - (5*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 +
Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 -
Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]
*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2))/8
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.06

method	result
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{4}{3},\frac{1}{2}\right],\left[-\frac{1}{3}\right],-x^{3}\right)}{4x^{4}}$
default	$-\frac{\sqrt{x^{3}+1}}{4x^{4}}+\frac{5\sqrt{x^{3}+1}}{8x}-\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\cdots\right.}{8\sqrt{x^{3}+1}}$
risch	$\frac{5x^{6}+3x^{3}-2}{8x^{4}\sqrt{x^{3}+1}}-\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\cdots\right.}{8\sqrt{x^{3}+1}}$
elliptic	$-\frac{\sqrt{x^{3}+1}}{4x^{4}}+\frac{5\sqrt{x^{3}+1}}{8x}-\frac{5\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\cdots\right.}{8\sqrt{x^{3}+1}}$

input
```
int(1/x^5/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4*hypergeom([-4/3,1/2],[-1/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = \frac{5\,x^4\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x)) + (5\,x^3-2)\sqrt{x^3+1}}{8\,x^4}$$

input
```
integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(5*x^4*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + (5*x^3
- 2)*sqrt(x^3 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = \frac{\Gamma\left(-\frac{4}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{4}{3},\frac{1}{2}\\-\frac{1}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(x**3+1)**(1/2),x)
```

output
```
gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3*exp_polar(I*pi))/(3*x**4*gamm
a(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^5}\,dx$$

input
```
integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}x^5}\,dx$$

input
```
integrate(1/x^5/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = \frac{5\sqrt{x^3+1}}{8\,x} - \frac{\sqrt{x^3+1}}{4\,x^4} + \frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right) - \left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right.}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\right.}}$$

input
```
int(1/(x^5*(x^3 + 1)^(1/2)),x)
```

output
```
(5*(x^3 + 1)^(1/2))/(8*x) - (x^3 + 1)^(1/2)/(4*x^4) + (5*(((3^(1/2)*1i)/2
- 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(as
in(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(8*(x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{x^3+1}}{x^8+x^5}dx$$

input
```
int(1/x^5/(x^3+1)^(1/2),x)
```

output
```
int(sqrt(x**3 + 1)/(x**8 + x**5),x)
```

3.250 $\int \frac{x^{11}}{\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{x^{11}}{\sqrt{1-x^3}} \, dx = -\frac{2}{3}\sqrt{1-x^3}+\frac{2}{3}\left(1-x^3\right)^{3/2}-\frac{2}{5}\left(1-x^3\right)^{5/2}+\frac{2}{21}\left(1-x^3\right)^{7/2}$$

output
```
-2/3*(-x^3+1)^(1/2)+2/3*(-x^3+1)^(3/2)-2/5*(-x^3+1)^(5/2)+2/21*(-x^3+1)^(7
/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{x^{11}}{\sqrt{1-x^3}} \, dx = -\frac{2}{105}\sqrt{1-x^3}\left(16+8x^3+6x^6+5x^9\right)$$

input
```
Integrate[x^11/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3]*(16 + 8*x^3 + 6*x^6 + 5*x^9))/105
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt{1-x^3}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(-\left(1-x^3\right)^{5/2}+3\left(1-x^3\right)^{3/2}-3\sqrt{1-x^3}+\frac{1}{\sqrt{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{7}\left(1-x^3\right)^{7/2}-\frac{6}{5}\left(1-x^3\right)^{5/2}+2\left(1-x^3\right)^{3/2}-2\sqrt{1-x^3}\right)$$

input

```
Int[x^11/Sqrt[1 - x^3],x]
```

output

```
(-2*Sqrt[1 - x^3] + 2*(1 - x^3)^(3/2) - (6*(1 - x^3)^(5/2))/5 + (2*(1 - x^
3)^(7/2))/7)/3
```

Defintions of rubi rules used

rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

method	result	size
trager	$\left(-\frac{2}{21}x^{9}-\frac{4}{35}x^{6}-\frac{16}{105}x^{3}-\frac{32}{105}\right)\sqrt{-x^{3}+1}$	28
pseudoelliptic	$-\frac{2\sqrt{-x^{3}+1}\,\left(5x^{9}+6x^{6}+8x^{3}+16\right)}{105}$	29
risch	$\frac{2\left(5x^{9}+6x^{6}+8x^{3}+16\right)\left(x^{3}-1\right)}{105\sqrt{-x^{3}+1}}$	34
gosper	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(5x^{9}+6x^{6}+8x^{3}+16\right)}{105\sqrt{-x^{3}+1}}$	38
orering	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(5x^{9}+6x^{6}+8x^{3}+16\right)}{105\sqrt{-x^{3}+1}}$	38
meijerg	$\frac{\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\,\left(40x^{9}+48x^{6}+64x^{3}+128\right)\sqrt{-x^{3}+1}}{140}}{3\sqrt{\pi}}$	43
default	$-\frac{2x^{9}\sqrt{-x^{3}+1}}{21}-\frac{4x^{6}\sqrt{-x^{3}+1}}{35}-\frac{16x^{3}\sqrt{-x^{3}+1}}{105}-\frac{32\sqrt{-x^{3}+1}}{105}$	55
elliptic	$-\frac{2x^{9}\sqrt{-x^{3}+1}}{21}-\frac{4x^{6}\sqrt{-x^{3}+1}}{35}-\frac{16x^{3}\sqrt{-x^{3}+1}}{105}-\frac{32\sqrt{-x^{3}+1}}{105}$	55

input
```
int(x^11/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/21*x^9-4/35*x^6-16/105*x^3-32/105)*(-x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx = -\frac{2}{105}\left(5\,x^9+6\,x^6+8\,x^3+16\right)\sqrt{-x^3+1}$$

input
```
integrate(x^11/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(5*x^9 + 6*x^6 + 8*x^3 + 16)*sqrt(-x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx = -\frac{2x^9\sqrt{1-x^3}}{21}-\frac{4x^6\sqrt{1-x^3}}{35}-\frac{16x^3\sqrt{1-x^3}}{105}-\frac{32\sqrt{1-x^3}}{105}$$

input
```
integrate(x**11/(-x**3+1)**(1/2),x)
```

output
```
-2*x**9*sqrt(1 - x**3)/21 - 4*x**6*sqrt(1 - x**3)/35 - 16*x**3*sqrt(1 - x*
*3)/105 - 32*sqrt(1 - x**3)/105
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx = \frac{2}{21}\left(-x^3+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{2}}+\frac{2}{3}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^11/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(-x^3 + 1)^(7/2) - 2/5*(-x^3 + 1)^(5/2) + 2/3*(-x^3 + 1)^(3/2) - 2/3*
sqrt(-x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx = -\frac{2}{21}\left(x^3-1\right)^3\sqrt{-x^3+1} - \frac{2}{5}\left(x^3-1\right)^2\sqrt{-x^3+1} + \frac{2}{3}\left(-x^3+1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^11/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-2/21*(x^3 - 1)^3*sqrt(-x^3 + 1) - 2/5*(x^3 - 1)^2*sqrt(-x^3 + 1) + 2/3*(-
x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx = -\frac{16\,x^3\,\sqrt{1-x^3}}{105} - \frac{4\,x^6\,\sqrt{1-x^3}}{35} - \frac{2\,x^9\,\sqrt{1-x^3}}{21} - \frac{32\,\sqrt{1-x^3}}{105}$$

input
```
int(x^11/(1 - x^3)^(1/2),x)
```

output
```
- (16*x^3*(1 - x^3)^(1/2))/105 - (4*x^6*(1 - x^3)^(1/2))/35 - (2*x^9*(1 -
x^3)^(1/2))/21 - (32*(1 - x^3)^(1/2))/105
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.44

$$\int \frac{x^{11}}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{-x^3+1}\,(-5x^9-6x^6-8x^3-16)}{105}$$

input
```
int(x^11/(-x^3+1)^(1/2),x)
```

output
```
(2*sqrt( - x**3 + 1)*( - 5*x**9 - 6*x**6 - 8*x**3 - 16))/105
```

3.251 $\int \frac{x^8}{\sqrt{1-x^3}} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^8}{\sqrt{1-x^3}} dx = -\frac{2}{3}\sqrt{1-x^3} + \frac{4}{9}\left(1-x^3\right)^{3/2} - \frac{2}{15}\left(1-x^3\right)^{5/2}$$

output
```
-2/3*(-x^3+1)^(1/2)+4/9*(-x^3+1)^(3/2)-2/15*(-x^3+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{\sqrt{1-x^3}} dx = -\frac{2}{45}\sqrt{1-x^3}\left(8+4x^3+3x^6\right)$$

input
```
Integrate[x^8/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3]*(8 + 4*x^3 + 3*x^6))/45
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{1-x^3}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\left(1-x^3\right)^{3/2}-2\sqrt{1-x^3}+\frac{1}{\sqrt{1-x^3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2}{5}\left(1-x^3\right)^{5/2}+\frac{4}{3}\left(1-x^3\right)^{3/2}-2\sqrt{1-x^3}\right)$$

input
```
Int[x^8/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3] + (4*(1 - x^3)^(3/2))/3 - (2*(1 - x^3)^(5/2))/5)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\left(-\frac{2}{15}x^6-\frac{8}{45}x^3-\frac{16}{45}\right)\sqrt{-x^3+1}$	23
pseudoelliptic	$-\frac{2\sqrt{-x^3+1}\,(3x^6+4x^3+8)}{45}$	24
risch	$\frac{2(3x^6+4x^3+8)(x^3-1)}{45\sqrt{-x^3+1}}$	29
gosper	$\frac{2(-1+x)(x^2+x+1)(3x^6+4x^3+8)}{45\sqrt{-x^3+1}}$	33
orering	$\frac{2(-1+x)(x^2+x+1)(3x^6+4x^3+8)}{45\sqrt{-x^3+1}}$	33
meijerg	$-\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(6x^6+8x^3+16\right)\sqrt{-x^3+1}}{15}}{3\sqrt{\pi}}$	38
default	$-\frac{2x^6\sqrt{-x^3+1}}{15}-\frac{8x^3\sqrt{-x^3+1}}{45}-\frac{16\sqrt{-x^3+1}}{45}$	41
elliptic	$-\frac{2x^6\sqrt{-x^3+1}}{15}-\frac{8x^3\sqrt{-x^3+1}}{45}-\frac{16\sqrt{-x^3+1}}{45}$	41

input
```
int(x^8/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/15*x^6-8/45*x^3-16/45)*(-x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{1-x^3}}\,dx = -\frac{2}{45}\left(3\,x^6+4\,x^3+8\right)\sqrt{-x^3+1}$$

input
```
integrate(x^8/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*x^6 + 4*x^3 + 8)*sqrt(-x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{\sqrt{1-x^3}}\,dx = -\frac{2x^6\sqrt{1-x^3}}{15}-\frac{8x^3\sqrt{1-x^3}}{45}-\frac{16\sqrt{1-x^3}}{45}$$

input
```
integrate(x**8/(-x**3+1)**(1/2),x)
```

output
```
-2*x**6*sqrt(1 - x**3)/15 - 8*x**3*sqrt(1 - x**3)/45 - 16*sqrt(1 - x**3)/4
5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{\sqrt{1-x^3}}\,dx = -\frac{2}{15}\left(-x^3+1\right)^{\frac{5}{2}}+\frac{4}{9}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^8/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(-x^3 + 1)^(5/2) + 4/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\sqrt{1-x^3}}\,dx = -\frac{2}{15}\left(x^3-1\right)^2\sqrt{-x^3+1}+\frac{4}{9}\left(-x^3+1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^8/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(x^3 - 1)^2*sqrt(-x^3 + 1) + 4/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 +
1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}\,(3\,x^6+4\,x^3+8)}{45}$$

input
```
int(x^8/(1 - x^3)^(1/2),x)
```

output
```
-(2*(1 - x^3)^(1/2)*(4*x^3 + 3*x^6 + 8))/45
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^8}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{-x^3+1}\,(-3x^6-4x^3-8)}{45}$$

input
```
int(x^8/(-x^3+1)^(1/2),x)
```

output
```
(2*sqrt( - x**3 + 1)*( - 3*x**6 - 4*x**3 - 8))/45
```

3.252 $\int \frac{x^5}{\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^5}{\sqrt{1-x^3}} \, dx = -\frac{2}{3}\sqrt{1-x^3} + \frac{2}{9}\left(1-x^3\right)^{3/2}$$

output
```
-2/3*(-x^3+1)^(1/2)+2/9*(-x^3+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{\sqrt{1-x^3}} \, dx = -\frac{2}{9}\sqrt{1-x^3}\left(2+x^3\right)$$

input
```
Integrate[x^5/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3]*(2 + x^3))/9
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3}\int \frac{x^3}{\sqrt{1-x^3}}dx^3$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{3}\int \left(\frac{1}{\sqrt{1-x^3}}-\sqrt{1-x^3}\right)dx^3$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{3}\left(\frac{2}{3}\left(1-x^3\right)^{3/2}-2\sqrt{1-x^3}\right)$$

input
```
Int[x^5/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3] + (2*(1 - x^3)^(3/2))/3)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$-\frac{2\left(x^{3}+2\right)\sqrt{-x^{3}+1}}{9}$	17
trager	$\left(-\frac{2x^{3}}{9}-\frac{4}{9}\right)\sqrt{-x^{3}+1}$	18
risch	$\frac{2\left(x^{3}+2\right)\left(x^{3}-1\right)}{9\sqrt{-x^{3}+1}}$	22
gosper	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(x^{3}+2\right)}{9\sqrt{-x^{3}+1}}$	26
orering	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(x^{3}+2\right)}{9\sqrt{-x^{3}+1}}$	26
default	$-\frac{2x^{3}\sqrt{-x^{3}+1}}{9}-\frac{4\sqrt{-x^{3}+1}}{9}$	27
elliptic	$-\frac{2x^{3}\sqrt{-x^{3}+1}}{9}-\frac{4\sqrt{-x^{3}+1}}{9}$	27
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(4x^{3}+8\right)\sqrt{-x^{3}+1}}{6}}{3\sqrt{\pi}}$	33

input
```
int(x^5/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x^3+2)*(-x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx = -\frac{2}{9}\left(x^3+2\right)\sqrt{-x^3+1}$$

input
```
integrate(x^5/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(x^3 + 2)*sqrt(-x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx = -\frac{2x^3\sqrt{1-x^3}}{9} - \frac{4\sqrt{1-x^3}}{9}$$

input
```
integrate(x**5/(-x**3+1)**(1/2),x)
```

output
```
-2*x**3*sqrt(1 - x**3)/9 - 4*sqrt(1 - x**3)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx = \frac{2}{9}\left(-x^3+1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^5/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx = \frac{2}{9}\left(-x^3+1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^5/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/9*(-x^3 + 1)^(3/2) - 2/3*sqrt(-x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}\,(x^3+2)}{9}$$

input
```
int(x^5/(1 - x^3)^(1/2),x)
```

output
```
-(2*(1 - x^3)^(1/2)*(x^3 + 2))/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x^5}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{-x^3+1}\,(-x^3-2)}{9}$$

input
```
int(x^5/(-x^3+1)^(1/2),x)
```

output
```
(2*sqrt( - x**3 + 1)*( - x**3 - 2))/9
```

3.253 $\int \frac{x^2}{\sqrt{1-x^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^2}{\sqrt{1-x^3}} \, dx = -\frac{2}{3}\sqrt{1-x^3}$$

output
```
-2/3*(-x^3+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{1-x^3}} \, dx = -\frac{2}{3}\sqrt{1-x^3}$$

input
```
Integrate[x^2/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3])/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{2}{3}\sqrt{1-x^3}$$

input
```
Int[x^2/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[1 - x^3])/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{2\sqrt{-x^{3}+1}}{3}$	12
default	$-\frac{2\sqrt{-x^{3}+1}}{3}$	12
trager	$-\frac{2\sqrt{-x^{3}+1}}{3}$	12
elliptic	$-\frac{2\sqrt{-x^{3}+1}}{3}$	12
pseudoelliptic	$-\frac{2\sqrt{-x^{3}+1}}{3}$	12
risch	$\frac{\frac{2x^{3}}{3}-\frac{2}{3}}{\sqrt{-x^{3}+1}}$	17
gosper	$\frac{2(-1+x)\left(x^{2}+x+1\right)}{3\sqrt{-x^{3}+1}}$	21
orering	$\frac{2(-1+x)\left(x^{2}+x+1\right)}{3\sqrt{-x^{3}+1}}$	21
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{3}+1}}{3\sqrt{\pi}}$	26

input
```
int(x^2/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-x^3+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2}{3}\,\sqrt{-x^3+1}$$

input
```
integrate(x^2/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}}{3}$$

input
```
integrate(x**2/(-x**3+1)**(1/2),x)
```

output
```
-2*sqrt(1 - x**3)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^2/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(-x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2}{3}\sqrt{-x^3+1}$$

input
```
integrate(x^2/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(-x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{1-x^3}}{3}$$

input
```
int(x^2/(1 - x^3)^(1/2),x)
```

output
```
-(2*(1 - x^3)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{-x^3+1}}{3}$$

input
```
int(x^2/(-x^3+1)^(1/2),x)
```

output
```
( - 2*sqrt( - x**3 + 1))/3
```

3.254 $\int \frac{1}{x\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

output
```
-2/3*arctanh((-x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Integrate[1/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[1 - x^3]])/3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{1}{3}\int \frac{1}{x^3\sqrt{1-x^3}}dx^3$$
$$\Big\downarrow \text{ 73}$$
$$-\frac{2}{3}\int \frac{1}{1-x^6}d\sqrt{1-x^3}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{2}{3}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[1/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[Sqrt[1 - x^3]])/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{-x^{3}+1}\right)}{3}$	13
elliptic	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{-x^{3}+1}\right)}{3}$	13
pseudoelliptic	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{-x^{3}+1}\right)}{3}$	13
trager	$-\frac{\ln\left(-\frac{-x^{3}+2\sqrt{-x^{3}+1}+2}{x^{3}}\right)}{3}$	27
meijerg	$\frac{(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{3}+1}}{2}\right)}{3\sqrt{\pi}}$	43

input
```
int(1/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*arctanh((-x^3+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = -\frac{1}{3}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{-x^3+1}-1\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(sqrt(-x^3 + 1) + 1) + 1/3*log(sqrt(-x^3 + 1) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = \begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-x**3+1)**(1/2),x)
```

output
```
Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/3
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = -\frac{1}{3}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{3}\log\left(\sqrt{-x^3+1}-1\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*log(sqrt(-x^3 + 1) + 1) + 1/3*log(sqrt(-x^3 + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = -\frac{1}{3}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{3}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

input
```
integrate(1/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*log(sqrt(-x^3 + 1) + 1) + 1/3*log(abs(sqrt(-x^3 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 11.25

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = -\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{2}}\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x*(1 - x^3)^(1/2)),x)
```

output
```
-(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i
)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{x\sqrt{1-x^3}}\,dx = \frac{\log\left(\sqrt{-x^3+1}-1\right)}{3} - \frac{\log\left(\sqrt{-x^3+1}+1\right)}{3}$$

input
```
int(1/x/(-x^3+1)^(1/2),x)
```

output
```
(log(sqrt( - x**3 + 1) - 1) - log(sqrt( - x**3 + 1) + 1))/3
```

3.255 $\int \frac{1}{x^4\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{3x^3} - \frac{1}{3}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

output
```
-1/3*(-x^3+1)^(1/2)/x^3-1/3*arctanh((-x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{3x^3} - \frac{1}{3}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Integrate[1/(x^4*Sqrt[1 - x^3]),x]
```

output
```
-1/3*Sqrt[1 - x^3]/x^3 - ArcTanh[Sqrt[1 - x^3]]/3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{1-x^3}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{1-x^3}}dx^3-\frac{\sqrt{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(-\int \frac{1}{1-x^6}d\sqrt{1-x^3}-\frac{\sqrt{1-x^3}}{x^3}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(-\text{arctanh}\left(\sqrt{1-x^3}\right)-\frac{\sqrt{1-x^3}}{x^3}\right)$$

input
```
Int[1/(x^4*Sqrt[1 - x^3]),x]
```

output
```
(-(Sqrt[1 - x^3]/x^3) - ArcTanh[Sqrt[1 - x^3]])/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\sqrt{-x^3+1}}{3x^3}-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$	28
elliptic	$-\frac{\sqrt{-x^3+1}}{3x^3}-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$	28
risch	$\frac{x^3-1}{3x^3\sqrt{-x^3+1}}-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$	33
trager	$-\frac{\sqrt{-x^3+1}}{3x^3}-\frac{\ln\left(-\frac{-x^3+2\sqrt{-x^3+1}+2}{x^3}\right)}{6}$	42
pseudoelliptic	$\frac{\ln\left(-1+\sqrt{-x^3+1}\right)x^3-\ln\left(1+\sqrt{-x^3+1}\right)x^3-2\sqrt{-x^3+1}}{6x^3}$	51
meijerg	$-\frac{\frac{\sqrt{\pi}}{x^3}-\frac{(1-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{2}-\frac{\sqrt{\pi}\left(-4x^3+8\right)}{8x^3}+\frac{\sqrt{\pi}\sqrt{-x^3+1}}{x^3}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)}{3\sqrt{\pi}}$	82

input
```
int(1/x^4/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-x^3+1)^(1/2)/x^3-1/3*arctanh((-x^3+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = -\frac{x^3\log\left(\sqrt{-x^3+1}+1\right)-x^3\log\left(\sqrt{-x^3+1}-1\right)+2\sqrt{-x^3+1}}{6\,x^3}$$

input
```
integrate(1/x^4/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(x^3*log(sqrt(-x^3 + 1) + 1) - x^3*log(sqrt(-x^3 + 1) - 1) + 2*sqrt(-
x^3 + 1))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.34

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{1}{3x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}} - \frac{1}{3x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} - \frac{i\sqrt{1-\frac{1}{x^3}}}{3x^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(-x**3+1)**(1/2),x)
```

output
```
Piecewise((-acosh(x**(-3/2))/3 + 1/(3*x**(3/2)*sqrt(-1 + x**(-3))) - 1/(3*
x**(9/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (I*asin(x**(-3/2))/3 - I*s
qrt(1 - 1/x**3)/(3*x**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = -\frac{\sqrt{-x^3+1}}{3\,x^3} - \frac{1}{6}\,\log\left(\sqrt{-x^3+1}+1\right) + \frac{1}{6}\,\log\left(\sqrt{-x^3+1}-1\right)$$

input
```
integrate(1/x^4/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-x^3 + 1)/x^3 - 1/6*log(sqrt(-x^3 + 1) + 1) + 1/6*log(sqrt(-x^3
+ 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = -\frac{\sqrt{-x^3+1}}{3\,x^3} - \frac{1}{6}\log\left(\sqrt{-x^3+1}+1\right) + \frac{1}{6}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

input
```
integrate(1/x^4/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(-x^3 + 1)/x^3 - 1/6*log(sqrt(-x^3 + 1) + 1) + 1/6*log(abs(sqrt(-
x^3 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 5.57

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{3\,x^3} - \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^4*(1 - x^3)^(1/2)),x)
```

output
```
- (1 - x^3)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x -
 (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((1 - x^3)^(1/2)
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^4\sqrt{1-x^3}}\,dx = \frac{-2\sqrt{-x^3+1}+\log\left(\sqrt{-x^3+1}-1\right)x^3-\log\left(\sqrt{-x^3+1}+1\right)x^3}{6x^3}$$

input
```
int(1/x^4/(-x^3+1)^(1/2),x)
```

output
```
( - 2*sqrt( - x**3 + 1) + log(sqrt( - x**3 + 1) - 1)*x**3 - log(sqrt( - x*
*3 + 1) + 1)*x**3)/(6*x**3)
```

3.256 $\int \frac{1}{x^7\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{6x^6} - \frac{\sqrt{1-x^3}}{4x^3} - \frac{1}{4}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

output
```
-1/6*(-x^3+1)^(1/2)/x^6-1/4*(-x^3+1)^(1/2)/x^3-1/4*arctanh((-x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = \frac{(-2-3x^3)\sqrt{1-x^3}}{12x^6} - \frac{1}{4}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Integrate[1/(x^7*Sqrt[1 - x^3]),x]
```

output
```
((-2 - 3*x^3)*Sqrt[1 - x^3])/(12*x^6) - ArcTanh[Sqrt[1 - x^3]]/4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^9\sqrt{1-x^3}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3}{4}\int \frac{1}{x^6\sqrt{1-x^3}}dx^3-\frac{\sqrt{1-x^3}}{2x^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{1-x^3}}dx^3-\frac{\sqrt{1-x^3}}{x^3}\right)-\frac{\sqrt{1-x^3}}{2x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(-\int \frac{1}{1-x^6}d\sqrt{1-x^3}-\frac{\sqrt{1-x^3}}{x^3}\right)-\frac{\sqrt{1-x^3}}{2x^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(-\text{arctanh}\left(\sqrt{1-x^3}\right)-\frac{\sqrt{1-x^3}}{x^3}\right)-\frac{\sqrt{1-x^3}}{2x^6}\right)$$

input
```
Int[1/(x^7*Sqrt[1 - x^3]),x]
```

output
```
(-1/2*Sqrt[1 - x^3]/x^6 + (3*(-(Sqrt[1 - x^3]/x^3) - ArcTanh[Sqrt[1 - x^3]
]))/4)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{3x^6-x^3-2}{12x^6\sqrt{-x^3+1}}-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{4}$	40
default	$-\frac{\sqrt{-x^3+1}}{6x^6}-\frac{\sqrt{-x^3+1}}{4x^3}-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{4}$	42
elliptic	$-\frac{\sqrt{-x^3+1}}{6x^6}-\frac{\sqrt{-x^3+1}}{4x^3}-\frac{\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{4}$	42
trager	$-\frac{(3x^3+2)\sqrt{-x^3+1}}{12x^6}+\frac{\ln\left(-\frac{x^3+2\sqrt{-x^3+1}-2}{x^3}\right)}{8}$	47
pseudoelliptic	$\frac{3\ln\left(-1+\sqrt{-x^3+1}\right)x^6-3\ln\left(1+\sqrt{-x^3+1}\right)x^6-6x^3\sqrt{-x^3+1}-4\sqrt{-x^3+1}}{24\left(-1+\sqrt{-x^3+1}\right)^2\left(1+\sqrt{-x^3+1}\right)^2}$	89
meijerg	$\frac{-\frac{\sqrt{\pi}}{2x^6}-\frac{\sqrt{\pi}}{2x^3}+\frac{3\left(\frac{7}{6}-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^6+8x^3+8\right)}{16x^6}-\frac{\sqrt{\pi}\left(12x^3+8\right)\sqrt{-x^3+1}}{16x^6}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)}{4}}{3\sqrt{\pi}}$	105

input
```
int(1/x^7/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^6-x^3-2)/x^6/(-x^3+1)^(1/2)-1/4*arctanh((-x^3+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = -\frac{3\,x^6\log\left(\sqrt{-x^3+1}+1\right)-3\,x^6\log\left(\sqrt{-x^3+1}-1\right)+2\,(3\,x^3+2)\sqrt{-x^3+1}}{24\,x^6}$$

input
```
integrate(1/x^7/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(3*x^6*log(sqrt(-x^3 + 1) + 1) - 3*x^6*log(sqrt(-x^3 + 1) - 1) + 2*(
3*x^3 + 2)*sqrt(-x^3 + 1))/x^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4} + \frac{1}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}} - \frac{1}{12x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} - \frac{1}{6x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4} - \frac{i}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}} + \frac{i}{12x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} + \frac{i}{6x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**7/(-x**3+1)**(1/2),x)
```

output
```
Piecewise((-acosh(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(-1 + x**(-3))) - 1/(12
*x**(9/2)*sqrt(-1 + x**(-3))) - 1/(6*x**(15/2)*sqrt(-1 + x**(-3))), 1/Abs(
x**3) > 1), (I*asin(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(1 - 1/x**3)) + I/(12
*x**(9/2)*sqrt(1 - 1/x**3)) + I/(6*x**(15/2)*sqrt(1 - 1/x**3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = \frac{3\,(-x^3+1)^{\frac{3}{2}} - 5\sqrt{-x^3+1}}{12\,\left(2\,x^3+(x^3-1)^2-1\right)} - \frac{1}{8}\log\left(\sqrt{-x^3+1}+1\right) + \frac{1}{8}\log\left(\sqrt{-x^3+1}-1\right)$$

input
```
integrate(1/x^7/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*(-x^3 + 1)^(3/2) - 5*sqrt(-x^3 + 1))/(2*x^3 + (x^3 - 1)^2 - 1) - 1
/8*log(sqrt(-x^3 + 1) + 1) + 1/8*log(sqrt(-x^3 + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = \frac{3\left(-x^3+1\right)^{\frac{3}{2}}-5\sqrt{-x^3+1}}{12\,x^6} - \frac{1}{8}\log\left(\sqrt{-x^3+1}+1\right)+\frac{1}{8}\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

input
```
integrate(1/x^7/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*(-x^3 + 1)^(3/2) - 5*sqrt(-x^3 + 1))/x^6 - 1/8*log(sqrt(-x^3 + 1)
+ 1) + 1/8*log(abs(sqrt(-x^3 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{4\,x^3}-\frac{\sqrt{1-x^3}}{6\,x^6} - \frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{2}}\right.}{4\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(1/(x^7*(1 - x^3)^(1/2)),x)
```

output
```
- (1 - x^3)^(1/2)/(4*x^3) - (1 - x^3)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 +
 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))
^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(4*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^7\sqrt{1-x^3}}\,dx = \frac{-6\sqrt{-x^3+1}\,x^3-4\sqrt{-x^3+1}+3\log\left(\sqrt{-x^3+1}-1\right)x^6-3\log\left(\sqrt{-x^3+1}+1\right)x^6}{24x^6}$$

input
```
int(1/x^7/(-x^3+1)^(1/2),x)
```

output
```
( - 6*sqrt( - x**3 + 1)*x**3 - 4*sqrt( - x**3 + 1) + 3*log(sqrt( - x**3 +
1) - 1)*x**6 - 3*log(sqrt( - x**3 + 1) + 1)*x**6)/(24*x**6)
```

3.257 $\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx=-\frac{\sqrt{1-x^3}}{9x^9}-\frac{5\sqrt{1-x^3}}{36x^6}-\frac{5\sqrt{1-x^3}}{24x^3}-\frac{5}{24}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

output
```
-1/9*(-x^3+1)^(1/2)/x^9-5/36*(-x^3+1)^(1/2)/x^6-5/24*(-x^3+1)^(1/2)/x^3-5/
24*arctanh((-x^3+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx=\frac{\sqrt{1-x^3}(-8-10x^3-15x^6)}{72x^9}-\frac{5}{24}\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Integrate[1/(x^10*Sqrt[1 - x^3]),x]
```

output
```
(Sqrt[1 - x^3]*(-8 - 10*x^3 - 15*x^6))/(72*x^9) - (5*ArcTanh[Sqrt[1 - x^3]
])/24
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 52, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{1}{x^{12}\sqrt{1-x^3}}dx^3$$

$$\Big\downarrow \ 52$$

$$\frac{1}{3}\left(\frac{5}{6}\int \frac{1}{x^9\sqrt{1-x^3}}dx^3-\frac{\sqrt{1-x^3}}{3x^9}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{1}{x^6\sqrt{1-x^3}}dx^3-\frac{\sqrt{1-x^3}}{2x^6}\right)-\frac{\sqrt{1-x^3}}{3x^9}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{1-x^3}}dx^3-\frac{\sqrt{1-x^3}}{x^3}\right)-\frac{\sqrt{1-x^3}}{2x^6}\right)-\frac{\sqrt{1-x^3}}{3x^9}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\int \frac{1}{1-x^6}d\sqrt{1-x^3}-\frac{\sqrt{1-x^3}}{x^3}\right)-\frac{\sqrt{1-x^3}}{2x^6}\right)-\frac{\sqrt{1-x^3}}{3x^9}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\text{arctanh}\left(\sqrt{1-x^3}\right)-\frac{\sqrt{1-x^3}}{x^3}\right)-\frac{\sqrt{1-x^3}}{2x^6}\right)-\frac{\sqrt{1-x^3}}{3x^9}\right)$$

input
```
Int[1/(x^10*Sqrt[1 - x^3]),x]
```

output
```
(-1/3*Sqrt[1 - x^3]/x^9 + (5*(-1/2*Sqrt[1 - x^3]/x^6 + (3*(-(Sqrt[1 - x^3]
/x^3) - ArcTanh[Sqrt[1 - x^3]]))/4))/6)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.63

method	result
risch	$\frac{15x^9-5x^6-2x^3-8}{72x^9\sqrt{-x^3+1}}-\frac{5\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)}{24}$
trager	$-\frac{(15x^6+10x^3+8)\sqrt{-x^3+1}}{72x^9}-\frac{5\ln\left(-\frac{-x^3+2\sqrt{-x^3+1}+2}{x^3}\right)}{48}$
default	$-\frac{\sqrt{-x^3+1}}{9x^9}-\frac{5\sqrt{-x^3+1}}{36x^6}-\frac{5\sqrt{-x^3+1}}{24x^3}-\frac{5\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)}{24}$
elliptic	$-\frac{\sqrt{-x^3+1}}{9x^9}-\frac{5\sqrt{-x^3+1}}{36x^6}-\frac{5\sqrt{-x^3+1}}{24x^3}-\frac{5\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)}{24}$
pseudoelliptic	$\frac{-15\ln\left(-1+\sqrt{-x^3+1}\right)x^9+15\ln\left(1+\sqrt{-x^3+1}\right)x^9+(30x^6+20x^3+16)\sqrt{-x^3+1}}{144\left(-1+\sqrt{-x^3+1}\right)^3\left(1+\sqrt{-x^3+1}\right)^3}$
meijerg	$-\frac{\frac{\sqrt{\pi}}{3x^9}+\frac{\sqrt{\pi}}{4x^6}+\frac{3\sqrt{\pi}}{8x^3}-\frac{5\left(\frac{37}{30}-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}}{16}-\frac{\sqrt{\pi}\left(-148x^9+144x^6+96x^3+128\right)}{384x^9}+\frac{\sqrt{\pi}\left(240x^6+160x^3+128\right)\sqrt{-x^3+1}}{384x^9}+\frac{5\sqrt{\pi}}{}}{3\sqrt{\pi}}$

input
```
int(1/x^10/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*(15*x^9-5*x^6-2*x^3-8)/x^9/(-x^3+1)^(1/2)-5/24*arctanh((-x^3+1)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx = -\frac{15\,x^9\log\left(\sqrt{-x^3+1}+1\right)-15\,x^9\log\left(\sqrt{-x^3+1}-1\right)+2\left(15\,x^6+10\,x^3+8\right)\sqrt{-x^3+1}}{144\,x^9}$$

input
```
integrate(1/x^10/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/144*(15*x^9*log(sqrt(-x^3 + 1) + 1) - 15*x^9*log(sqrt(-x^3 + 1) - 1) +
2*(15*x^6 + 10*x^3 + 8)*sqrt(-x^3 + 1))/x^9
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx = \begin{cases} -\frac{5\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24} + \frac{5}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}} - \frac{5}{72x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} - \frac{1}{36x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} - \frac{1}{9x^{\frac{21}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{5i\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24} - \frac{5i}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}} + \frac{5i}{72x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} + \frac{i}{36x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} + \frac{i}{9x^{\frac{21}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**10/(-x**3+1)**(1/2),x)
```

output
```
Piecewise((-5*acosh(x**(-3/2))/24 + 5/(24*x**(3/2)*sqrt(-1 + x**(-3))) - 5
/(72*x**(9/2)*sqrt(-1 + x**(-3))) - 1/(36*x**(15/2)*sqrt(-1 + x**(-3))) -
1/(9*x**(21/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (5*I*asin(x**(-3/2))
/24 - 5*I/(24*x**(3/2)*sqrt(1 - 1/x**3)) + 5*I/(72*x**(9/2)*sqrt(1 - 1/x**
3)) + I/(36*x**(15/2)*sqrt(1 - 1/x**3)) + I/(9*x**(21/2)*sqrt(1 - 1/x**3))
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx = -\frac{15\,(-x^3+1)^{\frac{5}{2}} - 40\,(-x^3+1)^{\frac{3}{2}} + 33\,\sqrt{-x^3+1}}{72\,\left((x^3-1)^3 + 3\,x^3 + 3\,(x^3-1)^2 - 2\right)} - \frac{5}{48}\,\log\left(\sqrt{-x^3+1}+1\right) + \frac{5}{48}\,\log\left(\sqrt{-x^3+1}-1\right)$$

input
```
integrate(1/x^10/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/72*(15*(-x^3 + 1)^(5/2) - 40*(-x^3 + 1)^(3/2) + 33*sqrt(-x^3 + 1))/((x^
3 - 1)^3 + 3*x^3 + 3*(x^3 - 1)^2 - 2) - 5/48*log(sqrt(-x^3 + 1) + 1) + 5/4
8*log(sqrt(-x^3 + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx = -\frac{15\,(x^3-1)^2\sqrt{-x^3+1}-40\,(-x^3+1)^{\frac{3}{2}}+33\,\sqrt{-x^3+1}}{72\,x^9} - \frac{5}{48}\,\log\left(\sqrt{-x^3+1}+1\right)+\frac{5}{48}\,\log\left(\left|\sqrt{-x^3+1}-1\right|\right)$$

input
```
integrate(1/x^10/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/72*(15*(x^3 - 1)^2*sqrt(-x^3 + 1) - 40*(-x^3 + 1)^(3/2) + 33*sqrt(-x^3
+ 1))/x^9 - 5/48*log(sqrt(-x^3 + 1) + 1) + 5/48*log(abs(sqrt(-x^3 + 1) - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.14

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx = -\frac{5\,\sqrt{1-x^3}}{24\,x^3}-\frac{5\,\sqrt{1-x^3}}{36\,x^6}-\frac{\sqrt{1-x^3}}{9\,x^9} - \frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{8\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^10*(1 - x^3)^(1/2)),x)
```

output
```
- (5*(1 - x^3)^(1/2))/(24*x^3) - (5*(1 - x^3)^(1/2))/(36*x^6) - (1 - x^3)^
(1/2)/(9*x^9) - (5*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*
1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipti
cPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(8*(1 - x^3)^(1/2)*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{10}\sqrt{1-x^3}}\,dx$$
$$= \frac{-30\sqrt{-x^3+1}\,x^6-20\sqrt{-x^3+1}\,x^3-16\sqrt{-x^3+1}+15\log\left(\sqrt{-x^3+1}-1\right)x^9-15\log\left(\sqrt{-x^3+1}+1\right)x^9}{144x^9}$$

input
```
int(1/x^10/(-x^3+1)^(1/2),x)
```

output
```
( - 30*sqrt( - x**3 + 1)*x**6 - 20*sqrt( - x**3 + 1)*x**3 - 16*sqrt( - x**
3 + 1) + 15*log(sqrt( - x**3 + 1) - 1)*x**9 - 15*log(sqrt( - x**3 + 1) + 1
)*x**9)/(144*x**9)
```

3.258 $\int \frac{x^6}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = -\frac{16}{55}x\sqrt{1-x^3}-\frac{2}{11}x^4\sqrt{1-x^3} - \frac{32\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-16/55*x*(-x^3+1)^(1/2)-2/11*x^4*(-x^3+1)^(1/2)-32/165*(1/2*6^(1/2)+1/2*2^
(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+
3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.26

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = -\frac{2}{55}x\left(\sqrt{1-x^3}(8+5x^3) - 8\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)\right)$$

input
```
Integrate[x^6/Sqrt[1 - x^3],x]
```

output
```
(-2*x*(Sqrt[1 - x^3]*(8 + 5*x^3) - 8*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]
))/55
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {843, 843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{8}{11}\int \frac{x^3}{\sqrt{1-x^3}}dx - \frac{2}{11}x^4\sqrt{1-x^3}$$

$$\Big\downarrow \text{843}$$

$$\frac{8}{11}\left(\frac{2}{5}\int \frac{1}{\sqrt{1-x^3}}dx - \frac{2}{5}x\sqrt{1-x^3}\right) - \frac{2}{11}x^4\sqrt{1-x^3}$$

$$\Big\downarrow \text{759}$$

$$\frac{8}{11}\left(-\frac{4\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{5}\sqrt{1-x^3}x\right)-\frac{2}{11}x^4\sqrt{1-x^3}$$

input
```
Int[x^6/Sqrt[1 - x^3],x]
```

output
```
(-2*x^4*Sqrt[1 - x^3])/11 + (8*((-2*x*Sqrt[1 - x^3])/5 - (4*Sqrt[2 + Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 -
Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 - x)/
(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])))/11
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.10

method	result
meijerg	$\frac{x^7\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{7}{3}\right],\left[\frac{10}{3}\right],x^3\right)}{7}$
risch	$\frac{2x(5x^3+8)(x^3-1)}{55\sqrt{-x^3+1}}-\frac{32i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right.}{165\sqrt{-x^3+1}}$
default	$-\frac{2x^4\sqrt{-x^3+1}}{11}-\frac{16x\sqrt{-x^3+1}}{55}-\frac{32i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{3}\right.}{165\sqrt{-x^3+1}}$
elliptic	$-\frac{2x^4\sqrt{-x^3+1}}{11}-\frac{16x\sqrt{-x^3+1}}{55}-\frac{32i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{3}\right.}{165\sqrt{-x^3+1}}$

input
```
int(x^6/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*hypergeom([1/2,7/3],[10/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx=-\frac{2}{55}\left(5\,x^4+8\,x\right)\sqrt{-x^3+1}-\frac{32}{55}i\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x^6/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(5*x^4 + 8*x)*sqrt(-x^3 + 1) - 32/55*I*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.20

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = \frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(-x**3+1)**(1/2),x)
```

output
```
x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(10/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = \int \frac{x^6}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = \int \frac{x^6}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^6/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.31

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = -\frac{2\,x^4\,\sqrt{1-x^3}}{11}-\frac{16\,x\,\sqrt{1-x^3}}{55} - \frac{32\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{55\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x^6/(1 - x^3)^(1/2),x)
```

output
```
- (2*x^4*(1 - x^3)^(1/2))/11 - (16*x*(1 - x^3)^(1/2))/55 - (32*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(
1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{-x^3+1}\,x^4}{11}-\frac{16\sqrt{-x^3+1}\,x}{55}-\frac{16\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)}{55}$$

input
```
int(x^6/(-x^3+1)^(1/2),x)
```

output
```
(2*( - 5*sqrt( - x**3 + 1)*x**4 - 8*sqrt( - x**3 + 1)*x - 8*int(sqrt( - x*
*3 + 1)/(x**3 - 1),x)))/55
```

3.259 $\int \frac{x^3}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = -\frac{2}{5}x\sqrt{1-x^3} - \frac{4\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/5*x*(-x^3+1)^(1/2)-4/15*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3
^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^
(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.24

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = \frac{2}{5}x\left(-\sqrt{1-x^3}+\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)\right)$$

input
```
Integrate[x^3/Sqrt[1 - x^3],x]
```

output
```
(2*x*(-Sqrt[1 - x^3] + Hypergeometric2F1[1/3, 1/2, 4/3, x^3]))/5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {843, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{2}{5}x\sqrt{1-x^3}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{4\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{5}\sqrt{1-x^3}x$$

input
```
Int[x^3/Sqrt[1 - x^3],x]
```

output
```
(-2*x*Sqrt[1 - x^3])/5 - (4*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)],
 -7 - 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^
3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.11

method	result
meijerg	$\frac{x^{4} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{4}{3}\right],\left[\frac{7}{3}\right],x^{3}\right)}{4}$
default	$-\frac{2x\sqrt{-x^{3}+1}}{5}-\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{15\sqrt{-x^{3}+1}}$
elliptic	$-\frac{2x\sqrt{-x^{3}+1}}{5}-\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{15\sqrt{-x^{3}+1}}$
risch	$\frac{2x\left(x^{3}-1\right)}{5\sqrt{-x^{3}+1}}-\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{15\sqrt{-x^{3}+1}}$

input
```
int(x^3/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*hypergeom([1/2,4/3],[7/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.14

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = -\frac{2}{5}\sqrt{-x^3+1}x - \frac{4}{5}i \operatorname{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x^3/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(-x^3 + 1)*x - 4/5*I*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.23

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(-x**3+1)**(1/2),x)
```

output
```
x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(7/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = \int \frac{x^3}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = \int \frac{x^3}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^3/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = -\frac{2\,x\,\sqrt{1-x^3}}{5} - \frac{4\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{5\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$

input
```
int(x^3/(1 - x^3)^(1/2),x)
```

output
```
- (2*x*(1 - x^3)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x
 - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(5*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{-x^3+1}\,x}{5} - \frac{2\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)}{5}$$

input
```
int(x^3/(-x^3+1)^(1/2),x)
```

output
```
( - 2*(sqrt( - x**3 + 1)*x + int(sqrt( - x**3 + 1)/(x**3 - 1),x)))/5
```

3.260 $\int \frac{1}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 115

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/3*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*Ell
ipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2
)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.13

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[1/Sqrt[1 - x^3],x]
```

output
```
x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 759$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[1/Sqrt[1 - x^3],x]
```

output
```
(-2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.10

method	result	size
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], x^{3}\right)$	12
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}$	107
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}$	107

input
```
int(1/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/3,1/2],[4/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = -2i\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(-x**3+1)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(1/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = -\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(1 - x^3)^(1/2),x)
```

output
```
-(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^3-1}dx\right)$$

input
```
int(1/(-x^3+1)^(1/2),x)
```

output
```
- int(sqrt( - x**3 + 1)/(x**3 - 1),x)
```

3.261 $\int \frac{1}{x^3\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{2x^2} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/2*(-x^3+1)^(1/2)/x^2-1/6*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+
3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3
^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},x^3\right)}{2x^2}$$

input
```
Integrate[1/(x^3*Sqrt[1 - x^3]),x]
```

output
```
-1/2*Hypergeometric2F1[-2/3, 1/2, 1/3, x^3]/x^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{\sqrt{1-x^3}}{2x^2}$$

$$\Big\downarrow \text{759}$$

$$-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{1-x^3}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 - x^3]),x]
```

output
```
-1/2*Sqrt[1 - x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -
7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
)
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.11

method	result	s
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{2}\right],\left[\frac{1}{3}\right],x^{3}\right)}{2x^{2}}$	1
default	$-\frac{\sqrt{-x^{3}+1}}{2x^{2}}-\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^{3}+1}}$	1
elliptic	$-\frac{\sqrt{-x^{3}+1}}{2x^{2}}-\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^{3}+1}}$	1
risch	$\frac{x^{3}-1}{2x^{2}\sqrt{-x^{3}+1}}-\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^{3}+1}}$	1

input
```
int(1/x^3/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2*hypergeom([-2/3,1/2],[1/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = \frac{-i\,x^2\text{weierstrassPInverse}(0,4,x)-\sqrt{-x^3+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(-I*x^2*weierstrassPInverse(0, 4, x) - sqrt(-x^3 + 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = \frac{\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(-x**3+1)**(1/2),x)
```

output
```
gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3*exp_polar(2*I*pi))/(3*x**2*gam
ma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{2\,x^2} - \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^3*(1 - x^3)^(1/2)),x)
```

output
```
- (1 - x^3)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x -
 (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^6-x^3}dx\right)$$

input
```
int(1/x^3/(-x^3+1)^(1/2),x)
```

output
```
 - int(sqrt( - x**3 + 1)/(x**6 - x**3),x)
```

3.262 $\int \frac{1}{x^6\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{5x^5}-\frac{7\sqrt{1-x^3}}{20x^2} - \frac{7\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/5*(-x^3+1)^(1/2)/x^5-7/20*(-x^3+1)^(1/2)/x^2-7/60*(1/2*6^(1/2)+1/2*2^(1
/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^
(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},x^3\right)}{5x^5}$$

input
```
Integrate[1/(x^6*Sqrt[1 - x^3]),x]
```

output
```
-1/5*Hypergeometric2F1[-5/3, 1/2, -2/3, x^3]/x^5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {847, 847, 759}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 847}$$

$$\frac{7}{10}\int \frac{1}{x^3\sqrt{1-x^3}}dx-\frac{\sqrt{1-x^3}}{5x^5}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{7}{10}\left(\frac{1}{4}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{\sqrt{1-x^3}}{2x^2}\right)-\frac{\sqrt{1-x^3}}{5x^5}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{7}{10}\left(-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{1-x^3}}{2x^2}\right)-\frac{\sqrt{1-x^3}}{5x^5}$$

input
```
Int[1/(x^6*Sqrt[1 - x^3]),x]
```

output
```
-1/5*Sqrt[1 - x^3]/x^5 + (7*(-1/2*Sqrt[1 - x^3]/x^2 - (Sqrt[2 + Sqrt[3]]*(
1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[(1 - x)/(1 +
Sqrt[3] - x)^2]*Sqrt[1 - x^3])))/10
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.10

method	result
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{5}{3},\frac{1}{2}\right],\left[-\frac{2}{3}\right],x^{3}\right)}{5x^{5}}$
risch	$\frac{7x^{6}-3x^{3}-4}{20x^{5}\sqrt{-x^{3}+1}}-\frac{7i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{60\sqrt{-x^{3}+1}}$
default	$-\frac{\sqrt{-x^{3}+1}}{5x^{5}}-\frac{7\sqrt{-x^{3}+1}}{20x^{2}}-\frac{7i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\cdots}\right.}{60\sqrt{-x^{3}+1}}$
elliptic	$-\frac{\sqrt{-x^{3}+1}}{5x^{5}}-\frac{7\sqrt{-x^{3}+1}}{20x^{2}}-\frac{7i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\cdots}\right.}{60\sqrt{-x^{3}+1}}$

input
```
int(1/x^6/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5*hypergeom([-5/3,1/2],[-2/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^{6}\sqrt{1-x^{3}}}\,dx=\frac{-7i\,x^{5}\mathrm{weierstrassPInverse}(0,4,x)-(7\,x^{3}+4)\sqrt{-x^{3}+1}}{20\,x^{5}}$$

input
```
integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(-7*I*x^5*weierstrassPInverse(0, 4, x) - (7*x^3 + 4)*sqrt(-x^3 + 1))/
x^5
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = \frac{\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, \frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| x^3 e^{2i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(-x**3+1)**(1/2),x)
```

output
```
gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3*exp_polar(2*I*pi))/(3*x**5*ga
mma(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = -\frac{7\sqrt{1-x^3}}{20\,x^2} - \frac{\sqrt{1-x^3}}{5\,x^5} - \frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{20\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^6*(1 - x^3)^(1/2)),x)
```

output
```
- (7*(1 - x^3)^(1/2))/(20*x^2) - (1 - x^3)^(1/2)/(5*x^5) - (7*((3^(1/2)*1i
)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(1
 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^9-x^6}dx\right)$$

input
```
int(1/x^6/(-x^3+1)^(1/2),x)
```

output
```
- int(sqrt( - x**3 + 1)/(x**9 - x**6),x)
```

3.263 $\int \frac{x^7}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 294

$$\begin{aligned}\int \frac{x^7}{\sqrt{1-x^3}}\,dx &= \frac{80\sqrt{1-x^3}}{91\left(1+\sqrt{3}-x\right)}-\frac{20}{91}x^2\sqrt{1-x^3}-\frac{2}{13}x^5\sqrt{1-x^3}\\ &\quad-\frac{40\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{91\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\\ &\quad+\frac{80\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}\end{aligned}$$

output
```
80*(-x^3+1)^(1/2)/(91+91*3^(1/2)-91*x)-20/91*x^2*(-x^3+1)^(1/2)-2/13*x^5*(
-x^3+1)^(1/2)-40/91*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+
3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)/(
(1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)+80/273*2^(1/2)*(1-x)*((x^2+x+1
)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2
*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.14

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = -\frac{2}{91}x^2\left(\sqrt{1-x^3}(10+7x^3)-10\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)$$

input
```
Integrate[x^7/Sqrt[1 - x^3],x]
```

output
```
(-2*x^2*(Sqrt[1 - x^3]*(10 + 7*x^3) - 10*Hypergeometric2F1[1/2, 2/3, 5/3,
x^3]))/91
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {843, 843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{10}{13}\int \frac{x^4}{\sqrt{1-x^3}}dx-\frac{2}{13}x^5\sqrt{1-x^3}$$

$$\Big\downarrow \text{843}$$

$$\frac{10}{13}\left(\frac{4}{7}\int \frac{x}{\sqrt{1-x^3}}dx-\frac{2}{7}x^2\sqrt{1-x^3}\right)-\frac{2}{13}x^5\sqrt{1-x^3}$$

$$\Big\downarrow \text{832}$$

$$\frac{10}{13}\left(\frac{4}{7}\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx-\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx\right)-\frac{2}{7}x^2\sqrt{1-x^3}\right)-\frac{2}{13}x^5\sqrt{1-x^3}$$

$$\Big\downarrow \text{759}$$

$$\frac{10}{13}\left(\frac{4}{7}\left(\left(-\int\frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-\right.}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}\right.\right.\right.$$

$$\frac{2}{13}x^5\sqrt{1-x^3}$$

$$\Big\downarrow \text{2416}$$

$$\frac{10}{13}\left(\frac{4}{7}\left(\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}\right.\right.\right.$$

$$\frac{2}{13}x^5\sqrt{1-x^3}$$

input
```
Int[x^7/Sqrt[1 - x^3],x]
```

output
```
(-2*x^5*Sqrt[1 - x^3])/13 + (10*((-2*x^2*Sqrt[1 - x^3])/7 + (4*((2*Sqrt[1
- x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x
 + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3
]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sq
rt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 -
4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])))/7)
)/13
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.05

method	result
meijerg	$\frac{x^8 \operatorname{hypergeom}([\frac{1}{2},\frac{8}{3}],[\frac{11}{3}],x^3)}{8}$
risch	$\frac{2x^2(7x^3+10)(x^3-1)}{91\sqrt{-x^3+1}} - \frac{80i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{3}\right.\right.}{273\sqrt{-x^3+1}}$
default	$-\frac{2x^5\sqrt{-x^3+1}}{13} - \frac{20x^2\sqrt{-x^3+1}}{91} - \frac{80i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}}{}\right.\right.}{273\sqrt{-x^3}}$
elliptic	$-\frac{2x^5\sqrt{-x^3+1}}{13} - \frac{20x^2\sqrt{-x^3+1}}{91} - \frac{80i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}}{}\right.\right.}{273\sqrt{-x^3}}$

input
```
int(x^7/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*hypergeom([1/2,8/3],[11/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = -\frac{2}{91}\left(7\,x^5+10\,x^2\right)\sqrt{-x^3+1} + \frac{80}{91}i\,\operatorname{weierstrassZeta}(0,4,\operatorname{weierstrassPInverse}(0,4,x))$$

input
```
integrate(x^7/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/91*(7*x^5 + 10*x^2)*sqrt(-x^3 + 1) + 80/91*I*weierstrassZeta(0, 4, weie
rstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(-x**3+1)**(1/2),x)
```

output
```
x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3*exp_polar(2*I*pi))/(3*gamm
a(11/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = \int \frac{x^7}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = \int \frac{x^7}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^7/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = -\frac{20\,x^2\,\sqrt{1-x^3}}{91} - \frac{2\,x^5\,\sqrt{1-x^3}}{13} - \frac{80\left(\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\sqrt{3}\cdots}{-\frac{3}{2}+\sqrt{\cdots}}\right.\right.\cdots}{91\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(\cdots\right.}}$$

input
```
int(x^7/(1 - x^3)^(1/2),x)
```

output
```
- (20*x^2*(1 - x^3)^(1/2))/91 - (2*x^5*(1 - x^3)^(1/2))/13 - (80*(((3^(1/2
)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ell
ipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x -
 (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2))/(91*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{-x^3+1}\,x^5}{13} - \frac{20\sqrt{-x^3+1}\,x^2}{91} - \frac{40\left(\int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx\right)}{91}$$

input
```
int(x^7/(-x^3+1)^(1/2),x)
```

output
```
(2*( - 7*sqrt( - x**3 + 1)*x**5 - 10*sqrt( - x**3 + 1)*x**2 - 20*int((sqrt
( - x**3 + 1)*x)/(x**3 - 1),x)))/91
```

3.264 $\int \frac{x^4}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 276

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = \frac{8\sqrt{1-x^3}}{7\left(1+\sqrt{3}-x\right)} - \frac{2}{7}x^2\sqrt{1-x^3} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{7\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{8\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
8*(-x^3+1)^(1/2)/(7+7*3^(1/2)-7*x)-2/7*x^2*(-x^3+1)^(1/2)-4/7*3^(1/4)*(1/2
*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1
-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x
^3+1)^(1/2)+8/21*2^(1/2)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF
((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2
)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = \frac{2}{7}x^2\left(-\sqrt{1-x^3}+\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)$$

input
```
Integrate[x^4/Sqrt[1 - x^3],x]
```

output
```
(2*x^2*(-Sqrt[1 - x^3] + Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/7
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {843, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{4}{7}\int \frac{x}{\sqrt{1-x^3}}dx-\frac{2}{7}x^2\sqrt{1-x^3}$$

$$\Big\downarrow \text{832}$$

$$\frac{4}{7}\left(\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx-\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx\right)-\frac{2}{7}x^2\sqrt{1-x^3}$$

$$\Big\downarrow \text{759}$$

$$\frac{4}{7}\left(-\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}\right.$$

$$\frac{2}{7}x^2\sqrt{1-x^3}$$

$$\Big\downarrow \text{2416}$$

$$\frac{4}{7}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-\right.$$

$$\frac{2}{7}x^2\sqrt{1-x^3}$$

input
```
Int[x^4/Sqrt[1 - x^3],x]
```

output
```
(-2*x^2*Sqrt[1 - x^3])/7 + (4*((2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1
/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[
(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sq
rt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1
 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])))/7
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.05

method	result
meijerg	$\frac{x^{5}\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{3}\right],\left[\frac{8}{3}\right],x^{3}\right)}{5}$
default	$-\frac{2x^{2}\sqrt{-x^{3}+1}}{7}-\frac{8i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\right.\right.}{21\sqrt{-x^{3}+1}}$
elliptic	$-\frac{2x^{2}\sqrt{-x^{3}+1}}{7}-\frac{8i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\right.\right.}{21\sqrt{-x^{3}+1}}$
risch	$\frac{2x^{2}\left(x^{3}-1\right)}{7\sqrt{-x^{3}+1}}-\frac{8i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\ldots}\right.\right.}{21\sqrt{-x^{3}+1}}$

input
```
int(x^4/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1/2,5/3],[8/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx=-\frac{2}{7}\sqrt{-x^3+1}x^2+\frac{8}{7}i\operatorname{weierstrassZeta}(0,4,\operatorname{weierstrassPInverse}(0,4,x))$$

input
```
integrate(x^4/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/7*sqrt(-x^3 + 1)*x^2 + 8/7*I*weierstrassZeta(0, 4, weierstrassPInverse(
0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(-x**3+1)**(1/2),x)
```

output
```
x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(8/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = \int \frac{x^4}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = \int \frac{x^4}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x^4/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = -\frac{2\,x^2\,\sqrt{1-x^3}}{7} - \frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.\right.}{7\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\right.}}$$

input
```
int(x^4/(1 - x^3)^(1/2),x)
```

output
```
- (2*x^2*(1 - x^3)^(1/2))/7 - (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-
(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3
^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)
*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(7*(1 - x^3)^(1/2)*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-x^3}}\,dx = -\frac{2\sqrt{-x^3+1}\,x^2}{7} - \frac{4\left(\int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx\right)}{7}$$

input
```
int(x^4/(-x^3+1)^(1/2),x)
```

output
```
(2*( - sqrt( - x**3 + 1)*x**2 - 2*int((sqrt( - x**3 + 1)*x)/(x**3 - 1),x))
)/7
```

3.265 $\int \frac{x}{\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 252

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = \frac{2\sqrt{1-x^3}}{1+\sqrt{3}-x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
2*(-x^3+1)^(1/2)/(1+3^(1/2)-x)-3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x
^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(
1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)+2/3*2^(1/2)*(1-x)*(
(x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3
^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.08

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = \frac{1}{2}x^2 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)$$

input
```
Integrate[x/Sqrt[1 - x^3],x]
```

output
```
(x^2*Hypergeometric2F1[1/2, 2/3, 5/3, x^3])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{832}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx-\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$-\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2416}$$

$$-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{2\sqrt{1-x^3}}{-x+\sqrt{3}+1}$$

input
```
Int[x/Sqrt[1 - x^3],x]
```

output
```
(2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^
2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] -
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 -
x^3])
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.06

method	result
meijerg	$\frac{x^2 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)+\text{Ell}}{3\sqrt{-x^3+1}}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)+\text{Ell}}{3\sqrt{-x^3+1}}$

input
```
int(x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*hypergeom([1/2,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = 2i\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate(x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*I*weierstrassZeta(0, 4, weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.12

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(-x**3+1)**(1/2),x)
```

output
```
x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(-x^3 + 1), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}}\,dx$$

input
```
integrate(x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-x^3 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = -\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{}}\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\right.}}$$

input
```
int(x/(1 - x^3)^(1/2),x)
```

output
```
-(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)
/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)
^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
))
```

Reduce [F]

$$\int \frac{x}{\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}\,x}{x^3-1}dx\right)$$

input
```
int(x/(-x^3+1)^(1/2),x)
```

output
```
- int((sqrt( - x**3 + 1)*x)/(x**3 - 1),x)
```

3.266 $\int \frac{1}{x^2\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{1+\sqrt{3}-x}-\frac{\sqrt{1-x^3}}{x}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-(-x^3+1)^(1/2)/(1+3^(1/2)-x)-(-x^3+1)^(1/2)/x+1/2*3^(1/4)*(1/2*6^(1/2)-1/
2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticE((1-3^(1/2)-x)
/(1+3^(1/2)-x),I*3^(1/2)+2*I)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
-1/3*2^(1/2)*(1-x)*((x^2+x+1)/(1+3^(1/2)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-
x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^
3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},x^3\right)}{x}$$

input
```
Integrate[1/(x^2*Sqrt[1 - x^3]),x]
```

output
```
-(Hypergeometric2F1[-1/3, 1/2, 2/3, x^3]/x)
```

Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{1}{2}\int \frac{x}{\sqrt{1-x^3}}dx-\frac{\sqrt{1-x^3}}{x}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{1}{2}\left(\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx\right)-\frac{\sqrt{1-x^3}}{x}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{1}{2}\left(\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx+\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}\right)$$

$$\frac{\sqrt{1-x^3}}{x}$$

$\big\downarrow$ 2416

$$\frac{1}{2}\left(\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)}{}\right.$$

$$\frac{\sqrt{1-x^3}}{x}$$

input

```
Int[1/(x^2*Sqrt[1 - x^3]),x]
```

output

```
-(Sqrt[1 - x^3]/x) + ((-2*Sqrt[1 - x^3])/(1 + Sqrt[3] - x) + (3^(1/4)*Sqrt
[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticE[Ar
cSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 - x)/
(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(
1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[
3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sq
rt[3] - x)^2]*Sqrt[1 - x^3]))/2
```

Defintions of rubi rules used

rule 759

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.06

method	result
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{1}{3},\frac{1}{2}\right],\left[\frac{2}{3}\right],x^{3}\right)}{x}$
default	$-\frac{\sqrt{-x^{3}+1}}{x}+\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i}{-\frac{3}{2}}}\right.\right.}{3\sqrt{-x^{3}+1}}$
elliptic	$-\frac{\sqrt{-x^{3}+1}}{x}+\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i}{-\frac{3}{2}}}\right.\right.}{3\sqrt{-x^{3}+1}}$
risch	$\frac{x^{3}-1}{x\sqrt{-x^{3}+1}}+\frac{i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{iv}{-\frac{3}{2}+}}\right.\right.}{3\sqrt{-x^{3}+1}}$

input
```
int(1/x^2/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x*hypergeom([-1/3,1/2],[2/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = \frac{-i\,x\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))-\sqrt{-x^3+1}}{x}$$

input
```
integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*x*weierstrassZeta(0, 4, weierstrassPInverse(0, 4, x)) - sqrt(-x^3 + 1)
)/x
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| x^3 e^{2i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(-x**3+1)**(1/2),x)
```

output
```
gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3*exp_polar(2*I*pi))/(3*x*gamma(
2/3))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = -\frac{\sqrt{1-x^3}}{x} + \frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}\right.}}$$

input
```
int(1/(x^2*(1 - x^3)^(1/2)),x)
```

output
```
((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
- 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
- (1 - x^3)^(1/2)/x
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^5-x^2}dx\right)$$

input
```
int(1/x^2/(-x^3+1)^(1/2),x)
```

output
```
 - int(sqrt( - x**3 + 1)/(x**5 - x**2),x)
```

3.267 $\int \frac{1}{x^5\sqrt{1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 294

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = -\frac{5\sqrt{1-x^3}}{8\left(1+\sqrt{3}-x\right)}-\frac{\sqrt{1-x^3}}{4x^4}-\frac{5\sqrt{1-x^3}}{8x} + \frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{16\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{5(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-5*(-x^3+1)^(1/2)/(8+8*3^(1/2)-8*x)-1/4*(-x^3+1)^(1/2)/x^4-5/8*(-x^3+1)^(1
/2)/x+5/16*3^(1/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1+3^(1/2)-x
)^2)^(1/2)*EllipticE((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)/((1-x)/(1+
3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)-5/24*2^(1/2)*(1-x)*((x^2+x+1)/(1+3^(1/2
)-x)^2)^(1/2)*EllipticF((1-3^(1/2)-x)/(1+3^(1/2)-x),I*3^(1/2)+2*I)*3^(3/4)
/((1-x)/(1+3^(1/2)-x)^2)^(1/2)/(-x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},x^3\right)}{4x^4}$$

input
```
Integrate[1/(x^5*Sqrt[1 - x^3]),x]
```

output
```
-1/4*Hypergeometric2F1[-4/3, 1/2, -1/3, x^3]/x^4
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {847, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{8}\int \frac{1}{x^2\sqrt{1-x^3}}dx-\frac{\sqrt{1-x^3}}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{8}\left(-\frac{1}{2}\int \frac{x}{\sqrt{1-x^3}}dx-\frac{\sqrt{1-x^3}}{x}\right)-\frac{\sqrt{1-x^3}}{4x^4}$$

$$\Big\downarrow \text{832}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx\right)-\frac{\sqrt{1-x^3}}{x}\right)-\frac{\sqrt{1-x^3}}{4x^4}$$

$$\Big\downarrow \text{759}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}}dx+\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{\right.}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}\right.\right.$$

$$\frac{\sqrt{1-x^3}}{4x^4}$$

$$\Big\downarrow \; 2416$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-}{}\right.\right.$$

$$\frac{\sqrt{1-x^3}}{4x^4}$$

input
```
Int[1/(x^5*Sqrt[1 - x^3]),x]
```

output
```
-1/4*Sqrt[1 - x^3]/x^4 + (5*(-(Sqrt[1 - x^3]/x) + ((-2*Sqrt[1 - x^3])/(1 +
 Sqrt[3] - x) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 +
 Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (2*(1 -
 Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/
(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]))/2))/8
```

Defintions of rubi rules used

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.05

method	result
meijerg	$-\frac{\text{hypergeom}([-\frac{4}{3},\frac{1}{2}],[-\frac{1}{3}],x^3)}{4x^4}$
risch	$\frac{5x^6-3x^3-2}{8x^4\sqrt{-x^3+1}}+\frac{5i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{-}\right.\right.}{24\sqrt{-x^3+1}}$
default	$-\frac{\sqrt{-x^3+1}}{4x^4}-\frac{5\sqrt{-x^3+1}}{8x}+\frac{5i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{3}\right.\right.}{24\sqrt{-x^3+1}}$
elliptic	$-\frac{\sqrt{-x^3+1}}{4x^4}-\frac{5\sqrt{-x^3+1}}{8x}+\frac{5i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{3}\right.\right.}{24\sqrt{-x^3+1}}$

input
```
int(1/x^5/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/x^4*hypergeom([-4/3,1/2],[-1/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = \frac{-5i\,x^4\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))-(5\,x^3+2)\sqrt{-x^3+1}}{8\,x^4}$$

input
```
integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(-5*I*x^4*weierstrassZeta(0, 4, weierstrassPInverse(0, 4, x)) - (5*x^3
 + 2)*sqrt(-x^3 + 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = \frac{\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3} \end{matrix}\right| x^3 e^{2i\pi}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(-x**3+1)**(1/2),x)
```

output
```
gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3*exp_polar(2*I*pi))/(3*x**4*ga
mma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = -\frac{5\sqrt{1-x^3}}{8\,x} - \frac{\sqrt{1-x^3}}{4\,x^4} + \frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right.}{8\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\right.}}$$

input
```
int(1/(x^5*(1 - x^3)^(1/2)),x)
```

output
```
(5*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/
2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^
(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^
(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2))/(8*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/
2)) - (1 - x^3)^(1/2)/(4*x^4) - (5*(1 - x^3)^(1/2))/(8*x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{1-x^3}}\,dx = -\left(\int \frac{\sqrt{-x^3+1}}{x^8-x^5}dx\right)$$

input
```
int(1/x^5/(-x^3+1)^(1/2),x)
```

output
```
- int(sqrt( - x**3 + 1)/(x**8 - x**5),x)
```

3.268 $\int \frac{x^{11}}{\sqrt{-1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^{11}}{\sqrt{-1+x^3}} \, dx = \frac{2}{3}\sqrt{-1+x^3} + \frac{2}{3}\left(-1+x^3\right)^{3/2} + \frac{2}{5}\left(-1+x^3\right)^{5/2} + \frac{2}{21}\left(-1+x^3\right)^{7/2}$$

output
```
2/3*(x^3-1)^(1/2)+2/3*(x^3-1)^(3/2)+2/5*(x^3-1)^(5/2)+2/21*(x^3-1)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int \frac{x^{11}}{\sqrt{-1+x^3}} \, dx = \frac{2}{105}\sqrt{-1+x^3}\left(16+8x^3+6x^6+5x^9\right)$$

input
```
Integrate[x^11/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3]*(16 + 8*x^3 + 6*x^6 + 5*x^9))/105
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt{x^3-1}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\left(x^3-1\right)^{5/2}+3\left(x^3-1\right)^{3/2}+3\sqrt{x^3-1}+\frac{1}{\sqrt{x^3-1}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{7}\left(x^3-1\right)^{7/2}+\frac{6}{5}\left(x^3-1\right)^{5/2}+2\left(x^3-1\right)^{3/2}+2\sqrt{x^3-1}\right)$$

input
```
Int[x^11/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3] + 2*(-1 + x^3)^(3/2) + (6*(-1 + x^3)^(5/2))/5 + (2*(-1 +
 x^3)^(7/2))/7)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.49

method	result	size
trager	$\left(\frac{2}{21}x^{9}+\frac{4}{35}x^{6}+\frac{16}{105}x^{3}+\frac{32}{105}\right)\sqrt{x^{3}-1}$	26
risch	$\frac{2\left(5x^{9}+6x^{6}+8x^{3}+16\right)\sqrt{x^{3}-1}}{105}$	27
pseudoelliptic	$\frac{2\left(5x^{9}+6x^{6}+8x^{3}+16\right)\sqrt{x^{3}-1}}{105}$	27
gosper	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(5x^{9}+6x^{6}+8x^{3}+16\right)}{105\sqrt{x^{3}-1}}$	36
orering	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(5x^{9}+6x^{6}+8x^{3}+16\right)}{105\sqrt{x^{3}-1}}$	36
default	$\frac{2x^{9}\sqrt{x^{3}-1}}{21}+\frac{4x^{6}\sqrt{x^{3}-1}}{35}+\frac{16x^{3}\sqrt{x^{3}-1}}{105}+\frac{32\sqrt{x^{3}-1}}{105}$	47
elliptic	$\frac{2x^{9}\sqrt{x^{3}-1}}{21}+\frac{4x^{6}\sqrt{x^{3}-1}}{35}+\frac{16x^{3}\sqrt{x^{3}-1}}{105}+\frac{32\sqrt{x^{3}-1}}{105}$	47
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\left(\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\left(40x^{9}+48x^{6}+64x^{3}+128\right)\sqrt{-x^{3}+1}}{140}\right)}{3\sqrt{\pi}\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}$	61

input
```
int(x^11/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/21*x^9+4/35*x^6+16/105*x^3+32/105)*(x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.49

$$\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx = \frac{2}{105}\left(5\,x^9+6\,x^6+8\,x^3+16\right)\sqrt{x^3-1}$$

input
```
integrate(x^11/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(5*x^9 + 6*x^6 + 8*x^3 + 16)*sqrt(x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx = \frac{2x^9\sqrt{x^3-1}}{21}+\frac{4x^6\sqrt{x^3-1}}{35}+\frac{16x^3\sqrt{x^3-1}}{105}+\frac{32\sqrt{x^3-1}}{105}$$

input
```
integrate(x**11/(x**3-1)**(1/2),x)
```

output
```
2*x**9*sqrt(x**3 - 1)/21 + 4*x**6*sqrt(x**3 - 1)/35 + 16*x**3*sqrt(x**3 -
1)/105 + 32*sqrt(x**3 - 1)/105
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx = \frac{2}{21}\left(x^3-1\right)^{\frac{7}{2}}+\frac{2}{5}\left(x^3-1\right)^{\frac{5}{2}}+\frac{2}{3}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^11/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(x^3 - 1)^(7/2) + 2/5*(x^3 - 1)^(5/2) + 2/3*(x^3 - 1)^(3/2) + 2/3*sqr
t(x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx = \frac{2}{21}\left(x^3-1\right)^{\frac{7}{2}} + \frac{2}{5}\left(x^3-1\right)^{\frac{5}{2}} + \frac{2}{3}\left(x^3-1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^11/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/21*(x^3 - 1)^(7/2) + 2/5*(x^3 - 1)^(5/2) + 2/3*(x^3 - 1)^(3/2) + 2/3*sqr
t(x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx = \frac{32\sqrt{x^3-1}}{105} + \frac{16\,x^3\sqrt{x^3-1}}{105} + \frac{4\,x^6\sqrt{x^3-1}}{35} + \frac{2\,x^9\sqrt{x^3-1}}{21}$$

input
```
int(x^11/(x^3 - 1)^(1/2),x)
```

output
```
(32*(x^3 - 1)^(1/2))/105 + (16*x^3*(x^3 - 1)^(1/2))/105 + (4*x^6*(x^3 - 1)
^(1/2))/35 + (2*x^9*(x^3 - 1)^(1/2))/21
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.47

$$\int \frac{x^{11}}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,(5x^9+6x^6+8x^3+16)}{105}$$

input
```
int(x^11/(x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 - 1)*(5*x**9 + 6*x**6 + 8*x**3 + 16))/105
```

3.269 $\int \frac{x^8}{\sqrt{-1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^8}{\sqrt{-1+x^3}} \, dx = \frac{2}{3}\sqrt{-1+x^3} + \frac{4}{9}\left(-1+x^3\right)^{3/2} + \frac{2}{15}\left(-1+x^3\right)^{5/2}$$

output
```
2/3*(x^3-1)^(1/2)+4/9*(x^3-1)^(3/2)+2/15*(x^3-1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{x^8}{\sqrt{-1+x^3}} \, dx = \frac{2}{45}\sqrt{-1+x^3}\left(8+4x^3+3x^6\right)$$

input
```
Integrate[x^8/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3]*(8 + 4*x^3 + 3*x^6))/45
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{x^3-1}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\left(x^3-1\right)^{3/2}+2\sqrt{x^3-1}+\frac{1}{\sqrt{x^3-1}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{5}\left(x^3-1\right)^{5/2}+\frac{4}{3}\left(x^3-1\right)^{3/2}+2\sqrt{x^3-1}\right)$$

input
```
Int[x^8/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3] + (4*(-1 + x^3)^(3/2))/3 + (2*(-1 + x^3)^(5/2))/5)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
trager	$\left(\frac{2}{15}x^{6}+\frac{8}{45}x^{3}+\frac{16}{45}\right)\sqrt{x^{3}-1}$	21
risch	$\frac{2\left(3x^{6}+4x^{3}+8\right)\sqrt{x^{3}-1}}{45}$	22
pseudoelliptic	$\frac{2\left(3x^{6}+4x^{3}+8\right)\sqrt{x^{3}-1}}{45}$	22
gosper	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(3x^{6}+4x^{3}+8\right)}{45\sqrt{x^{3}-1}}$	31
orering	$\frac{2(-1+x)\left(x^{2}+x+1\right)\left(3x^{6}+4x^{3}+8\right)}{45\sqrt{x^{3}-1}}$	31
default	$\frac{2x^{6}\sqrt{x^{3}-1}}{15}+\frac{8x^{3}\sqrt{x^{3}-1}}{45}+\frac{16\sqrt{x^{3}-1}}{45}$	35
elliptic	$\frac{2x^{6}\sqrt{x^{3}-1}}{15}+\frac{8x^{3}\sqrt{x^{3}-1}}{45}+\frac{16\sqrt{x^{3}-1}}{45}$	35
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(6x^{6}+8x^{3}+16\right)\sqrt{-x^{3}+1}}{15}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}$	56

input
```
int(x^8/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2/15*x^6+8/45*x^3+16/45)*(x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{x^8}{\sqrt{-1+x^3}}\,dx = \frac{2}{45}\left(3\,x^6+4\,x^3+8\right)\sqrt{x^3-1}$$

input
```
integrate(x^8/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*x^6 + 4*x^3 + 8)*sqrt(x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^8}{\sqrt{-1+x^3}}\,dx = \frac{2x^6\sqrt{x^3-1}}{15}+\frac{8x^3\sqrt{x^3-1}}{45}+\frac{16\sqrt{x^3-1}}{45}$$

input
```
integrate(x**8/(x**3-1)**(1/2),x)
```

output
```
2*x**6*sqrt(x**3 - 1)/15 + 8*x**3*sqrt(x**3 - 1)/45 + 16*sqrt(x**3 - 1)/45
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{\sqrt{-1+x^3}}\,dx = \frac{2}{15}\left(x^3-1\right)^{\frac{5}{2}}+\frac{4}{9}\left(x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^8/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(x^3 - 1)^(5/2) + 4/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{\sqrt{-1+x^3}}\,dx = \frac{2}{15}\left(x^3-1\right)^{\frac{5}{2}} + \frac{4}{9}\left(x^3-1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^8/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/15*(x^3 - 1)^(5/2) + 4/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{x^8}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,(3\,x^6+4\,x^3+8)}{45}$$

input
```
int(x^8/(x^3 - 1)^(1/2),x)
```

output
```
(2*(x^3 - 1)^(1/2)*(4*x^3 + 3*x^6 + 8))/45
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,(3x^6+4x^3+8)}{45}$$

input
```
int(x^8/(x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 - 1)*(3*x**6 + 4*x**3 + 8))/45
```

3.270 $\int \frac{x^5}{\sqrt{-1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^5}{\sqrt{-1+x^3}} \, dx = \frac{2}{3}\sqrt{-1+x^3} + \frac{2}{9}\left(-1+x^3\right)^{3/2}$$

output
```
2/3*(x^3-1)^(1/2)+2/9*(x^3-1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^5}{\sqrt{-1+x^3}} \, dx = \frac{2}{9}\sqrt{-1+x^3}\left(2+x^3\right)$$

input
```
Integrate[x^5/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3]*(2 + x^3))/9
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{x^3-1}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\sqrt{x^3-1}+\frac{1}{\sqrt{x^3-1}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(x^3-1\right)^{3/2}+2\sqrt{x^3-1}\right)$$

input
```
Int[x^5/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3] + (2*(-1 + x^3)^(3/2))/3)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{2(x^3+2)\sqrt{x^3-1}}{9}$	15
pseudoelliptic	$\frac{2(x^3+2)\sqrt{x^3-1}}{9}$	15
trager	$\left(\frac{2x^3}{9}+\frac{4}{9}\right)\sqrt{x^3-1}$	16
default	$\frac{2x^3\sqrt{x^3-1}}{9}+\frac{4\sqrt{x^3-1}}{9}$	23
elliptic	$\frac{2x^3\sqrt{x^3-1}}{9}+\frac{4\sqrt{x^3-1}}{9}$	23
gosper	$\frac{2(-1+x)(x^2+x+1)(x^3+2)}{9\sqrt{x^3-1}}$	24
orering	$\frac{2(-1+x)(x^2+x+1)(x^3+2)}{9\sqrt{x^3-1}}$	24
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(4x^3+8\right)\sqrt{-x^3+1}}{6}\right)}{3\sqrt{\pi}\sqrt{\operatorname{signum}(x^3-1)}}$	51

input
```
int(x^5/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(x^3+2)*(x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{-1+x^3}}\,dx = \frac{2}{9}\left(x^3+2\right)\sqrt{x^3-1}$$

input
```
integrate(x^5/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(x^3 + 2)*sqrt(x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\sqrt{-1+x^3}}\,dx = \frac{2x^3\sqrt{x^3-1}}{9} + \frac{4\sqrt{x^3-1}}{9}$$

input
```
integrate(x**5/(x**3-1)**(1/2),x)
```

output
```
2*x**3*sqrt(x**3 - 1)/9 + 4*sqrt(x**3 - 1)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\sqrt{-1+x^3}}\,dx = \frac{2}{9}\left(x^3-1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^5/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\sqrt{-1+x^3}}\,dx = \frac{2}{9}\left(x^3-1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^5/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/9*(x^3 - 1)^(3/2) + 2/3*sqrt(x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,(x^3+2)}{9}$$

input
```
int(x^5/(x^3 - 1)^(1/2),x)
```

output
```
(2*(x^3 - 1)^(1/2)*(x^3 + 2))/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,(x^3+2)}{9}$$

input
```
int(x^5/(x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 - 1)*(x**3 + 2))/9
```

3.271 $\int \frac{x^2}{\sqrt{-1+x^3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^2}{\sqrt{-1+x^3}} \, dx = \frac{2}{3}\sqrt{-1+x^3}$$

output
```
2/3*(x^3-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{-1+x^3}} \, dx = \frac{2}{3}\sqrt{-1+x^3}$$

input
```
Integrate[x^2/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3])/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^3-1}}\,dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{2\sqrt{x^3-1}}{3}$$

input
```
Int[x^2/Sqrt[-1 + x^3],x]
```

output
```
(2*Sqrt[-1 + x^3])/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2\sqrt{x^3-1}}{3}$	10
default	$\frac{2\sqrt{x^3-1}}{3}$	10
trager	$\frac{2\sqrt{x^3-1}}{3}$	10
risch	$\frac{2\sqrt{x^3-1}}{3}$	10
elliptic	$\frac{2\sqrt{x^3-1}}{3}$	10
pseudoelliptic	$\frac{2\sqrt{x^3-1}}{3}$	10
gosper	$\frac{2(-1+x)(x^2+x+1)}{3\sqrt{x^3-1}}$	19
orering	$\frac{2(-1+x)(x^2+x+1)}{3\sqrt{x^3-1}}$	19
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-x^3+1}\right)}{3\sqrt{\pi}\sqrt{\operatorname{signum}(x^3-1)}}$	44

input
```
int(x^2/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx = \frac{2}{3}\,\sqrt{x^3-1}$$

input
```
integrate(x^2/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}}{3}$$

input
```
integrate(x**2/(x**3-1)**(1/2),x)
```

output
```
2*sqrt(x**3 - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx = \frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^2/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx = \frac{2}{3}\sqrt{x^3-1}$$

input
```
integrate(x^2/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx=\frac{2\,\sqrt{x^3-1}}{3}$$

input
```
int(x^2/(x^3 - 1)^(1/2),x)
```

output
```
(2*(x^3 - 1)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\sqrt{-1+x^3}}\,dx=\frac{2\sqrt{x^3-1}}{3}$$

input
```
int(x^2/(x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 - 1))/3
```

3.272 $\int \frac{1}{x\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-1+x^3}\right)$$

output
```
2/3*arctan((x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-1+x^3}\right)$$

input
```
Integrate[1/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[Sqrt[-1 + x^3]])/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{x^3-1}}dx^3$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}\int \frac{1}{x^6+1}d\sqrt{x^3-1}$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

input
```
Int[1/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[Sqrt[-1 + x^3]])/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\arctan\left(\sqrt{x^3-1}\right)}{3}$	11
elliptic	$\frac{2\arctan\left(\sqrt{x^3-1}\right)}{3}$	11
pseudoelliptic	$\frac{2\arctan\left(\sqrt{x^3-1}\right)}{3}$	11
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{x^3-1}+2\operatorname{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{3}$	44
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left((-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}$	61

input
```
int(1/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan((x^3-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*arctan(sqrt(x^3 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \begin{cases} \frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(x**3-1)**(1/2),x)
```

output
```
Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3
, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*arctan(sqrt(x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 11.71

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x*(x^3 - 1)^(1/2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1}{x\sqrt{-1+x^3}}\,dx = \frac{atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)}{3}$$

input
```
int(1/x/(x^3-1)^(1/2),x)
```

output
```
atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2))/3
```

3.273 $\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}}{3x^3} + \frac{1}{3}\arctan\left(\sqrt{-1+x^3}\right)$$

output
```
1/3*(x^3-1)^(1/2)/x^3+1/3*arctan((x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \frac{1}{3}\left(\frac{\sqrt{-1+x^3}}{x^3} + \arctan\left(\sqrt{-1+x^3}\right)\right)$$

input
```
Integrate[1/(x^4*Sqrt[-1 + x^3]),x]
```

output
```
(Sqrt[-1 + x^3]/x^3 + ArcTan[Sqrt[-1 + x^3]])/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{x^3-1}}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3}\int \frac{1}{x^6\sqrt{x^3-1}}dx^3$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{3}\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+\frac{\sqrt{x^3-1}}{x^3}\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{3}\left(\int \frac{1}{x^6+1}d\sqrt{x^3-1}+\frac{\sqrt{x^3-1}}{x^3}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{3}\left(\arctan\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}}{x^3}\right)$$

input
```
Int[1/(x^4*Sqrt[-1 + x^3]),x]
```

output
```
(Sqrt[-1 + x^3]/x^3 + ArcTan[Sqrt[-1 + x^3]])/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\sqrt{x^3-1}}{3x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{3}$	24
risch	$\frac{\sqrt{x^3-1}}{3x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{3}$	24
elliptic	$\frac{\sqrt{x^3-1}}{3x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{3}$	24
pseudoelliptic	$\frac{\arctan\left(\sqrt{x^3-1}\right)x^3+\sqrt{x^3-1}}{3x^3}$	26
trager	$\frac{\sqrt{x^3-1}}{3x^3}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{x^3-1}-2\operatorname{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{6}$	56
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(\frac{\sqrt{\pi}}{x^3}-\frac{(1-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{2}-\frac{\sqrt{\pi}\left(-4x^3+8\right)}{8x^3}+\frac{\sqrt{\pi}\sqrt{-x^3+1}}{x^3}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)\right)}{3\sqrt{\pi}\sqrt{\operatorname{signum}(x^3-1)}}$	100

input
```
int(1/x^4/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^3-1)^(1/2)/x^3+1/3*arctan((x^3-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx=\frac{x^3\arctan\left(\sqrt{x^3-1}\right)+\sqrt{x^3-1}}{3\,x^3}$$

input
```
integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^3*arctan(sqrt(x^3 - 1)) + sqrt(x^3 - 1))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} - \frac{i}{3x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}} + \frac{i}{3x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for}\ \frac{1}{|x^3|} > 1 \\ -\frac{\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{\sqrt{1-\frac{1}{x^3}}}{3x^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(x**3-1)**(1/2),x)
```

output
```
Piecewise((I*acosh(x**(-3/2))/3 - I/(3*x**(3/2)*sqrt(-1 + x**(-3))) + I/(3
*x**(9/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (-asin(x**(-3/2))/3 + sqr
t(1 - 1/x**3)/(3*x**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \frac{\sqrt{x^3-1}}{3\,x^3} + \frac{1}{3}\,\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \frac{\sqrt{x^3-1}}{3\,x^3} + \frac{1}{3}\,\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x^4/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(x^3 - 1)/x^3 + 1/3*arctan(sqrt(x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 5.71

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \frac{\sqrt{x^3-1}}{3\,x^3} - \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^4*(x^3 - 1)^(1/2)),x)
```

output
```
(x^3 - 1)^(1/2)/(3*x^3) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(
1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2
)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3
)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^4\sqrt{-1+x^3}}\,dx = \frac{atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)x^3+2\sqrt{x^3-1}}{6x^3}$$

input
```
int(1/x^4/(x^3-1)^(1/2),x)
```

output
```
(atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2))*x**3 + 2*sqrt
(x**3 - 1))/(6*x**3)
```

3.274 $\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}}{6x^6} + \frac{\sqrt{-1+x^3}}{4x^3} + \frac{1}{4}\arctan\left(\sqrt{-1+x^3}\right)$$

output
```
1/6*(x^3-1)^(1/2)/x^6+1/4*(x^3-1)^(1/2)/x^3+1/4*arctan((x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}(2+3x^3)}{12x^6} + \frac{1}{4}\arctan\left(\sqrt{-1+x^3}\right)$$

input
```
Integrate[1/(x^7*Sqrt[-1 + x^3]),x]
```

output
```
(Sqrt[-1 + x^3]*(2 + 3*x^3))/(12*x^6) + ArcTan[Sqrt[-1 + x^3]]/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {798, 52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^9\sqrt{x^3-1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3}{4}\int \frac{1}{x^6\sqrt{x^3-1}}dx^3+\frac{\sqrt{x^3-1}}{2x^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+\frac{\sqrt{x^3-1}}{x^3}\right)+\frac{\sqrt{x^3-1}}{2x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\int \frac{1}{x^6+1}d\sqrt{x^3-1}+\frac{\sqrt{x^3-1}}{x^3}\right)+\frac{\sqrt{x^3-1}}{2x^6}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{3}{4}\left(\arctan\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}}{x^3}\right)+\frac{\sqrt{x^3-1}}{2x^6}\right)$$

input
```
Int[1/(x^7*Sqrt[-1 + x^3]),x]
```

output
```
(Sqrt[-1 + x^3]/(2*x^6) + (3*(Sqrt[-1 + x^3]/x^3 + ArcTan[Sqrt[-1 + x^3]])
)/4)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
default	$\frac{\sqrt{x^3-1}}{6x^6}+\frac{\sqrt{x^3-1}}{4x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{4}$
risch	$\frac{3x^6-x^3-2}{12x^6\sqrt{x^3-1}}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{4}$
elliptic	$\frac{\sqrt{x^3-1}}{6x^6}+\frac{\sqrt{x^3-1}}{4x^3}+\frac{\arctan\left(\sqrt{x^3-1}\right)}{4}$
pseudoelliptic	$\frac{3\arctan\left(\sqrt{x^3-1}\right)x^6+3x^3\sqrt{x^3-1}+2\sqrt{x^3-1}}{12x^6}$
trager	$\frac{(3x^3+2)\sqrt{x^3-1}}{12x^6}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{x^3-1}-2\operatorname{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{8}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(-\frac{\sqrt{\pi}}{2x^6}-\frac{\sqrt{\pi}}{2x^3}+\frac{3\left(\frac{7}{6}-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^6+8x^3+8\right)}{16x^6}-\frac{\sqrt{\pi}\left(12x^3+8\right)\sqrt{-x^3+1}}{16x^6}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\sqrt{\cdots}\right.}{4}\right.}{3\sqrt{\pi}\sqrt{\operatorname{signum}(x^3-1)}}$

input
```
int(1/x^7/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^3-1)^(1/2)/x^6+1/4*(x^3-1)^(1/2)/x^3+1/4*arctan((x^3-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx=\frac{3\,x^6\arctan\left(\sqrt{x^3-1}\right)+(3\,x^3+2)\sqrt{x^3-1}}{12\,x^6}$$

input
```
integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*x^6*arctan(sqrt(x^3 - 1)) + (3*x^3 + 2)*sqrt(x^3 - 1))/x^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.94

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \begin{cases} \frac{i\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4} - \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}} + \frac{i}{12x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} + \frac{i}{6x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|} > 1 \\ -\frac{\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4} + \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}} - \frac{1}{12x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} - \frac{1}{6x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**7/(x**3-1)**(1/2),x)
```

output
```
Piecewise((I*acosh(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(-1 + x**(-3))) + I/(1
2*x**(9/2)*sqrt(-1 + x**(-3))) + I/(6*x**(15/2)*sqrt(-1 + x**(-3))), 1/Abs
(x**3) > 1), (-asin(x**(-3/2))/4 + 1/(4*x**(3/2)*sqrt(1 - 1/x**3)) - 1/(12
*x**(9/2)*sqrt(1 - 1/x**3)) - 1/(6*x**(15/2)*sqrt(1 - 1/x**3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{3\,(x^3-1)^{\frac{3}{2}} + 5\,\sqrt{x^3-1}}{12\,\left(2\,x^3 + (x^3-1)^2 - 1\right)} + \frac{1}{4}\,\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/(2*x^3 + (x^3 - 1)^2 - 1) + 1/4
*arctan(sqrt(x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{3\,(x^3-1)^{\frac{3}{2}}+5\,\sqrt{x^3-1}}{12\,x^6}+\frac{1}{4}\,\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x^7/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*(x^3 - 1)^(3/2) + 5*sqrt(x^3 - 1))/x^6 + 1/4*arctan(sqrt(x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.02

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{\sqrt{x^3-1}}{4\,x^3}+\frac{\sqrt{x^3-1}}{6\,x^6} - \frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^7*(x^3 - 1)^(1/2)),x)
```

output
```
(x^3 - 1)^(1/2)/(4*x^3) + (x^3 - 1)^(1/2)/(6*x^6) - (3*((3^(1/2)*1i)/2 + 3
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(4*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^7\sqrt{-1+x^3}}\,dx = \frac{3atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)x^6+6\sqrt{x^3-1}\,x^3+4\sqrt{x^3-1}}{24x^6}$$

input
```
int(1/x^7/(x^3-1)^(1/2),x)
```

output
```
(3*atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2))*x**6 + 6*sq
rt(x**3 - 1)*x**3 + 4*sqrt(x**3 - 1))/(24*x**6)
```

3.275 $\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx=\frac{\sqrt{-1+x^3}}{9x^9}+\frac{5\sqrt{-1+x^3}}{36x^6}+\frac{5\sqrt{-1+x^3}}{24x^3}+\frac{5}{24}\arctan\left(\sqrt{-1+x^3}\right)$$

output
```
1/9*(x^3-1)^(1/2)/x^9+5/36*(x^3-1)^(1/2)/x^6+5/24*(x^3-1)^(1/2)/x^3+5/24*a
rctan((x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx=\frac{\sqrt{-1+x^3}(8+10x^3+15x^6)}{72x^9}+\frac{5}{24}\arctan\left(\sqrt{-1+x^3}\right)$$

input
```
Integrate[1/(x^10*Sqrt[-1 + x^3]),x]
```

output
```
(Sqrt[-1 + x^3]*(8 + 10*x^3 + 15*x^6))/(72*x^9) + (5*ArcTan[Sqrt[-1 + x^3]
])/24
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {798, 52, 52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^{12}\sqrt{x^3-1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{5}{6}\int \frac{1}{x^9\sqrt{x^3-1}}dx^3+\frac{\sqrt{x^3-1}}{3x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{1}{x^6\sqrt{x^3-1}}dx^3+\frac{\sqrt{x^3-1}}{2x^6}\right)+\frac{\sqrt{x^3-1}}{3x^9}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+\frac{\sqrt{x^3-1}}{x^3}\right)+\frac{\sqrt{x^3-1}}{2x^6}\right)+\frac{\sqrt{x^3-1}}{3x^9}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\left(\int \frac{1}{x^6+1}d\sqrt{x^3-1}+\frac{\sqrt{x^3-1}}{x^3}\right)+\frac{\sqrt{x^3-1}}{2x^6}\right)+\frac{\sqrt{x^3-1}}{3x^9}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\left(\frac{5}{6}\left(\frac{3}{4}\left(\arctan\left(\sqrt{x^3-1}\right)+\frac{\sqrt{x^3-1}}{x^3}\right)+\frac{\sqrt{x^3-1}}{2x^6}\right)+\frac{\sqrt{x^3-1}}{3x^9}\right)$$

input
```
Int[1/(x^10*Sqrt[-1 + x^3]),x]
```

output
```
(Sqrt[-1 + x^3]/(3*x^9) + (5*(Sqrt[-1 + x^3]/(2*x^6) + (3*(Sqrt[-1 + x^3]/
x^3 + ArcTan[Sqrt[-1 + x^3]]))/4))/6)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{(15x^6+10x^3+8)\sqrt{x^3-1}+15\arctan\left(\sqrt{x^3-1}\right)x^9}{72x^9}$
risch	$\frac{15x^9-5x^6-2x^3-8}{72x^9\sqrt{x^3-1}}+\frac{5\arctan\left(\sqrt{x^3-1}\right)}{24}$
default	$\frac{\sqrt{x^3-1}}{9x^9}+\frac{5\sqrt{x^3-1}}{36x^6}+\frac{5\sqrt{x^3-1}}{24x^3}+\frac{5\arctan\left(\sqrt{x^3-1}\right)}{24}$
elliptic	$\frac{\sqrt{x^3-1}}{9x^9}+\frac{5\sqrt{x^3-1}}{36x^6}+\frac{5\sqrt{x^3-1}}{24x^3}+\frac{5\arctan\left(\sqrt{x^3-1}\right)}{24}$
trager	$\frac{(15x^6+10x^3+8)\sqrt{x^3-1}}{72x^9}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{x^3-1}-2\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{48}$
meijerg	$-\frac{\sqrt{-\mathrm{signum}(x^3-1)}\left(\frac{\sqrt{\pi}}{3x^9}+\frac{\sqrt{\pi}}{4x^6}+\frac{3\sqrt{\pi}}{8x^3}-\frac{5\left(\frac{37}{30}-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}}{16}-\frac{\sqrt{\pi}\left(-148x^9+144x^6+96x^3+128\right)}{384x^9}+\frac{\sqrt{\pi}\left(240x^6+160x^3 \right.}{384x}\right.}{3\sqrt{\pi}\sqrt{\mathrm{signum}(x^3-1)}}$

input
```
int(1/x^10/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*((15*x^6+10*x^3+8)*(x^3-1)^(1/2)+15*arctan((x^3-1)^(1/2))*x^9)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx=\frac{15\,x^9\arctan\left(\sqrt{x^3-1}\right)+(15\,x^6+10\,x^3+8)\sqrt{x^3-1}}{72\,x^9}$$

input
```
integrate(1/x^10/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/72*(15*x^9*arctan(sqrt(x^3 - 1)) + (15*x^6 + 10*x^3 + 8)*sqrt(x^3 - 1))/
x^9
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.70 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx = \begin{cases} \frac{5i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24} - \frac{5i}{24x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x^3}}} + \frac{5i}{72x^{\frac{9}{2}}\sqrt{-1+\frac{1}{x^3}}} + \frac{i}{36x^{\frac{15}{2}}\sqrt{-1+\frac{1}{x^3}}} + \frac{i}{9x^{\frac{21}{2}}\sqrt{-1+\frac{1}{x^3}}} & \text{for } \frac{1}{|x^3|} > 1 \\ -\frac{5\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24} + \frac{5}{24x^{\frac{3}{2}}\sqrt{1-\frac{1}{x^3}}} - \frac{5}{72x^{\frac{9}{2}}\sqrt{1-\frac{1}{x^3}}} - \frac{1}{36x^{\frac{15}{2}}\sqrt{1-\frac{1}{x^3}}} - \frac{1}{9x^{\frac{21}{2}}\sqrt{1-\frac{1}{x^3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**10/(x**3-1)**(1/2),x)
```

output
```
Piecewise((5*I*acosh(x**(-3/2))/24 - 5*I/(24*x**(3/2)*sqrt(-1 + x**(-3)))
+ 5*I/(72*x**(9/2)*sqrt(-1 + x**(-3))) + I/(36*x**(15/2)*sqrt(-1 + x**(-3)
)) + I/(9*x**(21/2)*sqrt(-1 + x**(-3))), 1/Abs(x**3) > 1), (-5*asin(x**(-3
/2))/24 + 5/(24*x**(3/2)*sqrt(1 - 1/x**3)) - 5/(72*x**(9/2)*sqrt(1 - 1/x**
3)) - 1/(36*x**(15/2)*sqrt(1 - 1/x**3)) - 1/(9*x**(21/2)*sqrt(1 - 1/x**3))
, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx = \frac{15\,(x^3-1)^{\frac{5}{2}} + 40\,(x^3-1)^{\frac{3}{2}} + 33\sqrt{x^3-1}}{72\left((x^3-1)^3 + 3\,x^3 + 3\,(x^3-1)^2 - 2\right)} + \frac{5}{24}\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x^10/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
1/72*(15*(x^3 - 1)^(5/2) + 40*(x^3 - 1)^(3/2) + 33*sqrt(x^3 - 1))/((x^3 -
1)^3 + 3*x^3 + 3*(x^3 - 1)^2 - 2) + 5/24*arctan(sqrt(x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx = \frac{15\,(x^3-1)^{\frac{5}{2}} + 40\,(x^3-1)^{\frac{3}{2}} + 33\sqrt{x^3-1}}{72\,x^9} + \frac{5}{24}\arctan\left(\sqrt{x^3-1}\right)$$

input
```
integrate(1/x^10/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
1/72*(15*(x^3 - 1)^(5/2) + 40*(x^3 - 1)^(3/2) + 33*sqrt(x^3 - 1))/x^9 + 5/
24*arctan(sqrt(x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.19

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx = \frac{5\sqrt{x^3-1}}{24\,x^3} + \frac{5\sqrt{x^3-1}}{36\,x^6} + \frac{\sqrt{x^3-1}}{9\,x^9} - \frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^10*(x^3 - 1)^(1/2)),x)
```

output
```
(5*(x^3 - 1)^(1/2))/(24*x^3) + (5*(x^3 - 1)^(1/2))/(36*x^6) + (x^3 - 1)^(1
/2)/(9*x^9) - (5*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2
 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/(8*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{10}\sqrt{-1+x^3}}\,dx$$
$$= \frac{15atan\left(\frac{\sqrt{x^3-1}\,x^3-2\sqrt{x^3-1}}{2x^3-2}\right)x^9+30\sqrt{x^3-1}\,x^6+20\sqrt{x^3-1}\,x^3+16\sqrt{x^3-1}}{144x^9}$$

input
```
int(1/x^10/(x^3-1)^(1/2),x)
```

output
```
(15*atan((sqrt(x**3 - 1)*x**3 - 2*sqrt(x**3 - 1))/(2*x**3 - 2))*x**9 + 30*
sqrt(x**3 - 1)*x**6 + 20*sqrt(x**3 - 1)*x**3 + 16*sqrt(x**3 - 1))/(144*x**
9)
```

3.276 $\int \frac{x^6}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \frac{16}{55}x\sqrt{-1+x^3}+\frac{2}{11}x^4\sqrt{-1+x^3} - \frac{32\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
16/55*x*(x^3-1)^(1/2)+2/11*x^4*(x^3-1)^(1/2)-32/165*(1/2*6^(1/2)-1/2*2^(1/
2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(
1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \frac{2x\left(-8+3x^3+5x^6+8\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)\right)}{55\sqrt{-1+x^3}}$$

input
```
Integrate[x^6/Sqrt[-1 + x^3],x]
```

output
```
(2*x*(-8 + 3*x^3 + 5*x^6 + 8*Sqrt[1 - x^3]*Hypergeometric2F1[1/3, 1/2, 4/3
, x^3]))/(55*Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {843, 843, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{8}{11}\int \frac{x^3}{\sqrt{x^3-1}}dx+\frac{2}{11}\sqrt{x^3-1}x^4$$

$$\Big\downarrow \text{843}$$

$$\frac{8}{11}\left(\frac{2}{5}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{2}{5}\sqrt{x^3-1}x\right)+\frac{2}{11}\sqrt{x^3-1}x^4$$

$$\Big\downarrow \text{760}$$

$$\frac{8}{11}\left(\frac{2}{5}x\sqrt{x^3-1}-\frac{4\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right)+\frac{2}{11}\sqrt{x^3-1}x^4$$

input
```
Int[x^6/Sqrt[-1 + x^3],x]
```

output
```
(2*x^4*Sqrt[-1 + x^3])/11 + (8*((2*x*Sqrt[-1 + x^3])/5 - (4*Sqrt[2 - Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 +
Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[-((1 - x
)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])))/11
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.22

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^7 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{7}{3}\right],\left[\frac{10}{3}\right],x^3\right)}{7\sqrt{\operatorname{signum}(x^3-1)}}$	33
risch	$\frac{2x(5x^3+8)\sqrt{x^3-1}}{55}+\frac{32\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3-1}}$	134
default	$\frac{2x^4\sqrt{x^3-1}}{11}+\frac{16x\sqrt{x^3-1}}{55}+\frac{32\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3-1}}$	139
elliptic	$\frac{2x^4\sqrt{x^3-1}}{11}+\frac{16x\sqrt{x^3-1}}{55}+\frac{32\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{55\sqrt{x^3-1}}$	139

input
```
int(x^6/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^7*hypergeom([1/2,7/3],[10
/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.16

$$\int \frac{x^6}{\sqrt{-1+x^3}}\, dx = \frac{2}{55}\left(5\,x^4+8\,x\right)\sqrt{x^3-1}+\frac{32}{55}\operatorname{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x^6/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/55*(5*x^4 + 8*x)*sqrt(x^3 - 1) + 32/55*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = -\frac{ix^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(x**3-1)**(1/2),x)
```

output
```
-I*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3)/(3*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \int \frac{x^6}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^6/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \int \frac{x^6}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^6/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.17

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \frac{16\,x\,\sqrt{x^3-1}}{55} + \frac{2\,x^4\,\sqrt{x^3-1}}{11} - \frac{32\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{55\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x^6/(x^3 - 1)^(1/2),x)
```

output
```
(16*x*(x^3 - 1)^(1/2))/55 + (2*x^4*(x^3 - 1)^(1/2))/11 - (32*((3^(1/2)*1i)
/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
 (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,x^4}{11} + \frac{16\sqrt{x^3-1}\,x}{55} + \frac{16\left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)}{55}$$

input
```
int(x^6/(x^3-1)^(1/2),x)
```

output
```
(2*(5*sqrt(x**3 - 1)*x***4 + 8*sqrt(x**3 - 1)*x + 8*int(sqrt(x**3 - 1)/(x**
3 - 1),x)))/55
```

3.277 $\int \frac{x^3}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 137

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx = \frac{2}{5}x\sqrt{-1+x^3} - \frac{4\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/5*x*(x^3-1)^(1/2)-4/15*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(
1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3
/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.32

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx = \frac{2x\left(-1+x^3+\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)\right)}{5\sqrt{-1+x^3}}$$

input
```
Integrate[x^3/Sqrt[-1 + x^3],x]
```

output
```
(2*x*(-1 + x^3 + Sqrt[1 - x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, x^3]))/(5*
Sqrt[-1 + x^3])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {843, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{5}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{2}{5}\sqrt{x^3-1}x$$

$$\Big\downarrow \text{760}$$

$$\frac{2}{5}x\sqrt{x^3-1}-\frac{4\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[x^3/Sqrt[-1 + x^3],x]
```

output
```
(2*x*Sqrt[-1 + x^3])/5 - (4*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1
+ x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.24

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, x^{4} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{4}{3}\right], \left[\frac{7}{3}\right], x^{3}\right)}{4 \sqrt{\operatorname{signum}\left(x^{3}-1\right)}}$	33
default	$\frac{2 x \sqrt{x^{3}-1}}{5}+\frac{4 \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{5 \sqrt{x^{3}-1}}$	127
risch	$\frac{2 x \sqrt{x^{3}-1}}{5}+\frac{4 \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{5 \sqrt{x^{3}-1}}$	127
elliptic	$\frac{2 x \sqrt{x^{3}-1}}{5}+\frac{4 \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{5 \sqrt{x^{3}-1}}$	127

input
```
int(x^3/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^4*hypergeom([1/2,4/3],[7/
3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx = \frac{2}{5}\sqrt{x^3-1}x + \frac{4}{5}\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x^3/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(x^3 - 1)*x + 4/5*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx = -\frac{ix^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(x**3-1)**(1/2),x)
```

output
```
-I*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3)/(3*gamma(7/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx=\int \frac{x^3}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^3/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx=\int \frac{x^3}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^3/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx=\frac{2\,x\,\sqrt{x^3-1}}{5}-\frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{5\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$

input
```
int(x^3/(x^3 - 1)^(1/2),x)
```

output
```
(2*x*(x^3 - 1)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2
+ 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asi
n((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))/(5*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,x}{5} + \frac{2\left(\int \frac{\sqrt{x^3-1}}{x^3-1}dx\right)}{5}$$

input
```
int(x^3/(x^3-1)^(1/2),x)
```

output
```
(2*(sqrt(x**3 - 1)*x + int(sqrt(x**3 - 1)/(x**3 - 1),x)))/5
```

3.278 $\int \frac{1}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 120

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*Ell
ipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/
2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.29

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[1/Sqrt[-1 + x^3],x]
```

output
```
(x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{760\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \; 760$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[1/Sqrt[-1 + x^3],x]
```

output
```
(-2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/
4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x \operatorname{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}$	30
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$	116
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$	116

input
```
int(1/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = 2\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(0, 4, x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(x**3-1)**(1/2),x)
```

output
```
-I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(1/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(1/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^3 - 1)^(1/2),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}}{x^3-1}dx$$

input
```
int(1/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**3 - 1),x)
```

3.279 $\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
1/2*(x^3-1)^(1/2)/x^2-1/6*(1/2*6^(1/2)-1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^
(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(
3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},x^3\right)}{2x^2\sqrt{-1+x^3}}$$

input
```
Integrate[1/(x^3*Sqrt[-1 + x^3]),x]
```

output
```
-1/2*(Sqrt[1 - x^3]*Hypergeometric2F1[-2/3, 1/2, 1/3, x^3])/(x^2*Sqrt[-1 +
 x^3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {847, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{\sqrt{x^3-1}}{2x^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[1/(x^3*Sqrt[-1 + x^3]),x]
```

output
```
Sqrt[-1 + x^3]/(2*x^2) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
- Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -
7 + 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, \operatorname{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{2}\right],\left[\frac{1}{3}\right],x^{3}\right)}{2 \sqrt{\operatorname{signum}\left(x^{3}-1\right)}\, x^{2}}$	33
default	$\frac{\sqrt{x^{3}-1}}{2 x^{2}}+\frac{\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{2 \sqrt{x^{3}-1}}$	129
risch	$\frac{\sqrt{x^{3}-1}}{2 x^{2}}+\frac{\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{2 \sqrt{x^{3}-1}}$	129
elliptic	$\frac{\sqrt{x^{3}-1}}{2 x^{2}}+\frac{\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{2 \sqrt{x^{3}-1}}$	129

input
```
int(1/x^3/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)/x^2*hypergeom([-2/3,1/2],[
1/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \frac{x^2\text{weierstrassPInverse}(0,4,x)+\sqrt{x^3-1}}{2\,x^2}$$

input
```
integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^2*weierstrassPInverse(0, 4, x) + sqrt(x^3 - 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = -\frac{i\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|x^3\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(x**3-1)**(1/2),x)
```

output
```
-I*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3)/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^3}\,dx$$

input
```
integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^3}\,dx$$

input
```
integrate(1/x^3/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \frac{\sqrt{x^3-1}}{2\,x^2} - \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^3*(x^3 - 1)^(1/2)),x)
```

output
```
(x^3 - 1)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))/(2*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}}{x^6-x^3}dx$$

input
```
int(1/x^3/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**6 - x**3),x)
```

3.280 $\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 155

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = \frac{\sqrt{-1+x^3}}{5x^5}+\frac{7\sqrt{-1+x^3}}{20x^2} - \frac{7\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
1/5*(x^3-1)^(1/2)/x^5+7/20*(x^3-1)^(1/2)/x^2-7/60*(1/2*6^(1/2)-1/2*2^(1/2)
)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/
2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},x^3\right)}{5x^5\sqrt{-1+x^3}}$$

input
```
Integrate[1/(x^6*Sqrt[-1 + x^3]),x]
```

output
```
-1/5*(Sqrt[1 - x^3]*Hypergeometric2F1[-5/3, 1/2, -2/3, x^3])/(x^5*Sqrt[-1
+ x^3])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {847, 847, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{10}\int \frac{1}{x^3\sqrt{x^3-1}}dx+\frac{\sqrt{x^3-1}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{10}\left(\frac{1}{4}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{\sqrt{x^3-1}}{2x^2}\right)+\frac{\sqrt{x^3-1}}{5x^5}$$

$$\Big\downarrow \text{760}$$

$$\frac{7}{10}\left(\frac{\sqrt{x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right)+\frac{\sqrt{x^3-1}}{5x^5}$$

input
```
Int[1/(x^6*Sqrt[-1 + x^3]),x]
```

output
```
Sqrt[-1 + x^3]/(5*x^5) + (7*(Sqrt[-1 + x^3]/(2*x^2) - (Sqrt[2 - Sqrt[3]]*(
1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[
3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[-((1 - x)/(1
- Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])))/10
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.21

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\,\operatorname{hypergeom}\left(\left[-\frac{5}{3},\frac{1}{2}\right],\left[-\frac{2}{3}\right],x^3\right)}{5\sqrt{\operatorname{signum}(x^3-1)}\,x^5}$	33
default	$\frac{\sqrt{x^3-1}}{5x^5}+\frac{7\sqrt{x^3-1}}{20x^2}+\frac{7\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^3-1}}$	141
risch	$\frac{7x^6-3x^3-4}{20x^5\sqrt{x^3-1}}+\frac{7\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^3-1}}$	141
elliptic	$\frac{\sqrt{x^3-1}}{5x^5}+\frac{7\sqrt{x^3-1}}{20x^2}+\frac{7\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{20\sqrt{x^3-1}}$	141

input
```
int(1/x^6/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)/x^5*hypergeom([-5/3,1/2],[
-2/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx=\frac{7\,x^5\text{weierstrassPInverse}(0,4,x)+(7\,x^3+4)\sqrt{x^3-1}}{20\,x^5}$$

input
```
integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(7*x^5*weierstrassPInverse(0, 4, x) + (7*x^3 + 4)*sqrt(x^3 - 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = -\frac{i\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|x^3\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(x**3-1)**(1/2),x)
```

output
```
-I*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3)/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = \frac{7\sqrt{x^3-1}}{20\,x^2}+\frac{\sqrt{x^3-1}}{5\,x^5} - \frac{7\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{20\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^6*(x^3 - 1)^(1/2)),x)
```

output
```
(7*(x^3 - 1)^(1/2))/(20*x^2) + (x^3 - 1)^(1/2)/(5*x^5) - (7*((3^(1/2)*1i)/
2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
 -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}}{x^9-x^6}dx$$

input
```
int(1/x^6/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**9 - x**6),x)
```

3.281 $\int \frac{x^7}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 294

$$\begin{aligned}\int \frac{x^7}{\sqrt{-1+x^3}}\,dx &= -\frac{80\sqrt{-1+x^3}}{91\left(1-\sqrt{3}-x\right)}+\frac{20}{91}x^2\sqrt{-1+x^3}+\frac{2}{13}x^5\sqrt{-1+x^3}\\ &\quad+\frac{40\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{91\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\ &\quad-\frac{80\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

output

```
-80*(x^3-1)^(1/2)/(91-91*3^(1/2)-91*x)+20/91*x^2*(x^3-1)^(1/2)+2/13*x^5*(x
^3-1)^(1/2)+40/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^
(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))/(-(
1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-80/273*2^(1/2)*(1-x)*((x^2+x+1)/
(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2)
)*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.18

$$\int \frac{x^7}{\sqrt{-1+x^3}} \, dx = \frac{2x^2\left(-10+3x^3+7x^6+10\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{91\sqrt{-1+x^3}}$$

input
```
Integrate[x^7/Sqrt[-1 + x^3],x]
```

output
```
(2*x^2*(-10 + 3*x^3 + 7*x^6 + 10*Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 2/3,
 5/3, x^3]))/(91*Sqrt[-1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {843, 843, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{x^3-1}} \, dx$$

$$\Big\downarrow \text{843}$$

$$\frac{10}{13}\int \frac{x^4}{\sqrt{x^3-1}}dx+\frac{2}{13}\sqrt{x^3-1}x^5$$

$$\Big\downarrow \text{843}$$

$$\frac{10}{13}\left(\frac{4}{7}\int \frac{x}{\sqrt{x^3-1}}dx+\frac{2}{7}\sqrt{x^3-1}x^2\right)+\frac{2}{13}\sqrt{x^3-1}x^5$$

$$\Big\downarrow \text{833}$$

$$\frac{10}{13}\left(\frac{4}{7}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx\right)+\frac{2}{7}\sqrt{x^3-1}x^2\right)+\frac{2}{13}\sqrt{x^3-1}x^5$$

$$\Big\downarrow \text{760}$$

$$\frac{10}{13}\left(\frac{4}{7}\left(\left(-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right.\right.\right.$$

$$\frac{2}{13}\sqrt{x^3-1}x^5$$

$$\Big\downarrow \text{2418}$$

$$\frac{10}{13}\left(\frac{4}{7}\left(\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{}\right.\right.\right.$$

$$\frac{2}{13}\sqrt{x^3-1}x^5$$

input
```
Int[x^7/Sqrt[-1 + x^3],x]
```

output
```
(2*x^5*Sqrt[-1 + x^3])/13 + (10*((2*x^2*Sqrt[-1 + x^3])/7 + (4*((-2*Sqrt[-
1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 +
 x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqr
t[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1
 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])))/7))/13
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.11

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^8 \operatorname{hypergeom}\left([\frac{1}{2},\frac{8}{3}],[\frac{11}{3}],x^3\right)}{8\sqrt{\operatorname{signum}(x^3-1)}}$
risch	$\frac{2x^2(7x^3+10)\sqrt{x^3-1}}{91}+\frac{80\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\right.}{91\sqrt{x^3-1}}$
default	$\frac{2x^5\sqrt{x^3-1}}{13}+\frac{20x^2\sqrt{x^3-1}}{91}+\frac{80\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\right.\right.}{91\sqrt{x^3-1}}$
elliptic	$\frac{2x^5\sqrt{x^3-1}}{13}+\frac{20x^2\sqrt{x^3-1}}{91}+\frac{80\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\right.\right.}{91\sqrt{x^3-1}}$

input
```
int(x^7/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^8*hypergeom([1/2,8/3],[11
/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.10

$$\int \frac{x^7}{\sqrt{-1+x^3}}\,dx = \frac{2}{91}\left(7\,x^5+10\,x^2\right)\sqrt{x^3-1} - \frac{80}{91}\,\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x))$$

input
```
integrate(x^7/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/91*(7*x^5 + 10*x^2)*sqrt(x^3 - 1) - 80/91*weierstrassZeta(0, 4, weierstr
assPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{x^7}{\sqrt{-1+x^3}}\,dx = -\frac{ix^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(x**3-1)**(1/2),x)
```

output
```
-I*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3)/(3*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{-1+x^3}}\,dx = \int \frac{x^7}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^7/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{-1+x^3}}\,dx = \int \frac{x^7}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^7/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{\sqrt{-1+x^3}}\,dx = \frac{20\,x^2\,\sqrt{x^3-1}}{91} + \frac{2\,x^5\,\sqrt{x^3-1}}{13} - \frac{80\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right) - \left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\sqrt{3}}{-\frac{3}{2}+\sqrt{3}}\right.\right.}{91\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right.}}$$

input
```
int(x^7/(x^3 - 1)^(1/2),x)
```

output
```
(20*x^2*(x^3 - 1)^(1/2))/91 + (2*x^5*(x^3 - 1)^(1/2))/13 - (80*(((3^(1/2)*
1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellip
ticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 +
1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(91*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,x^5}{13} + \frac{20\sqrt{x^3-1}\,x^2}{91} + \frac{40\left(\int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx\right)}{91}$$

input
```
int(x^7/(x^3-1)^(1/2),x)
```

output
```
(2*(7*sqrt(x**3 - 1)*x**5 + 10*sqrt(x**3 - 1)*x**2 + 20*int((sqrt(x**3 - 1
)*x)/(x**3 - 1),x)))/91
```

3.282 $\int \frac{x^4}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 278

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = -\frac{8\sqrt{-1+x^3}}{7\left(1-\sqrt{3}-x\right)}+\frac{2}{7}x^2\sqrt{-1+x^3} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{8\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output

```
-8*(x^3-1)^(1/2)/(7-7*3^(1/2)-7*x)+2/7*x^2*(x^3-1)^(1/2)+4/7*3^(1/4)*(1/2*
6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+
3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^
3-1)^(1/2)-8/21*2^(1/2)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF(
(1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2
)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \frac{2x^2\left(-1+x^3+\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)\right)}{7\sqrt{-1+x^3}}$$

input
```
Integrate[x^4/Sqrt[-1 + x^3],x]
```

output
```
(2*x^2*(-1 + x^3 + Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, x^3]))/(
7*Sqrt[-1 + x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {843, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{4}{7}\int \frac{x}{\sqrt{x^3-1}}dx+\frac{2}{7}\sqrt{x^3-1}x^2$$

$$\Big\downarrow \text{833}$$

$$\frac{4}{7}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx\right)+\frac{2}{7}\sqrt{x^3-1}x^2$$

$$\Big\downarrow \text{760}$$

$$\frac{4}{7}\left(-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right.$$
$$\frac{2}{7}\sqrt{x^3-1}x^2$$

$$\Big\downarrow \; 2418$$

$$\frac{4}{7}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-\right.$$
$$\frac{2}{7}\sqrt{x^3-1}x^2$$

input
```
Int[x^4/Sqrt[-1 + x^3],x]
```

output
```
(2*x^2*Sqrt[-1 + x^3])/7 + (4*((-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*El
lipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqr
t[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(
1 + Sqrt[3])*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[Arc
Sin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])))/7
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.12

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^5 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{3}\right],\left[\frac{8}{3}\right],x^3\right)}{5\sqrt{\operatorname{signum}(x^3-1)}}$
default	$\frac{2x^2\sqrt{x^3-1}}{7}+\frac{8\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{7\sqrt{x^3-1}}$
risch	$\frac{2x^2\sqrt{x^3-1}}{7}+\frac{8\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{7\sqrt{x^3-1}}$
elliptic	$\frac{2x^2\sqrt{x^3-1}}{7}+\frac{8\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{7\sqrt{x^3-1}}$

input
```
int(x^4/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^5*hypergeom([1/2,5/3],[8/
3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \frac{2}{7}\sqrt{x^3-1}x^2 - \frac{8}{7}\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate(x^4/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/7*sqrt(x^3 - 1)*x^2 - 8/7*weierstrassZeta(0, 4, weierstrassPInverse(0, 4
, x))
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = -\frac{ix^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\,x^3\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(x**3-1)**(1/2),x)
```

output
```
-I*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3)/(3*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \int \frac{x^4}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^4/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \int \frac{x^4}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x^4/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \frac{2\,x^2\,\sqrt{x^3-1}}{7} - \frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\,\Bigg|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\,\Bigg|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.\right.}{7\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int(x^4/(x^3 - 1)^(1/2),x)
```

output
```
(2*x^2*(x^3 - 1)^(1/2))/7 - (8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(
1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2))/(7*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{x^3-1}\,x^2}{7} + \frac{4\left(\int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx\right)}{7}$$

input
```
int(x^4/(x^3-1)^(1/2),x)
```

output
```
(2*(sqrt(x**3 - 1)*x**2 + 2*int((sqrt(x**3 - 1)*x)/(x**3 - 1),x)))/7
```

3.283 $\int \frac{x}{\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 255

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = -\frac{2\sqrt{-1+x^3}}{1-\sqrt{3}-x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output

```
-2*(x^3-1)^(1/2)/(1-3^(1/2)-x)+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x
^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I
*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)-2/3*2^(1/2)*(1-x)*(
(x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I
-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx=\frac{x^2\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},x^3\right)}{2\sqrt{-1+x^3}}$$

input
```
Integrate[x/Sqrt[-1 + x^3],x]
```

output
```
(x^2*Sqrt[1 - x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, x^3])/(2*Sqrt[-1 + x^3
])
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{833}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$-\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\sqrt{x^3-1}}{-x-\sqrt{3}+1}$$

input
```
Int[x/Sqrt[-1 + x^3],x]
```

output
```
(-2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x
)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2
)]*Sqrt[-1 + x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - x)*Sqrt[(1 +
x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt
[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.13

method	result
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\, x^2 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],x^3\right)}{2\sqrt{\operatorname{signum}(x^3-1)}}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticF}\left(\sqrt{\cdots}\right.\right.}{\sqrt{x^3-1}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticF}\left(\sqrt{\cdots}\right.\right.}{\sqrt{x^3-1}}$

input
```
int(x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x^2*hypergeom([1/2,2/3],[5/
3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = -2\,\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x))$$

input
```
integrate(x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*weierstrassZeta(0, 4, weierstrassPInverse(0, 4, x))
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.11

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3)/(3*gamma(5/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(x^3 - 1), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.84

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = -\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\cdots}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\cdots}}$$

input
```
int(x/(x^3 - 1)^(1/2),x)
```

output
```
-(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)
/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3
^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{x}{\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}\,x}{x^3-1}dx$$

input
```
int(x/(x^3-1)^(1/2),x)
```

output
```
int((sqrt(x**3 - 1)*x)/(x**3 - 1),x)
```

3.284 $\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx &= \frac{\sqrt{-1+x^3}}{1-\sqrt{3}-x}+\frac{\sqrt{-1+x^3}}{x}\\
&\quad-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{2\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\
&\quad+\frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

output
```
(x^3-1)^(1/2)/(1-3^(1/2)-x)+(x^3-1)^(1/2)/x-1/2*3^(1/4)*(1/2*6^(1/2)+1/2*2
^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticE((1+3^(1/2)-x)/(1
-3^(1/2)-x),2*I-I*3^(1/2))/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)+1/
3*2^(1/2)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)^(1/2)*EllipticF((1+3^(1/2)-x)/
(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},x^3\right)}{x\sqrt{-1+x^3}}$$

input
```
Integrate[1/(x^2*Sqrt[-1 + x^3]),x]
```

output
```
-((Sqrt[1 - x^3]*Hypergeometric2F1[-1/3, 1/2, 2/3, x^3])/(x*Sqrt[-1 + x^3]
))
```

Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {847, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{x^3-1}}{x}-\frac{1}{2}\int \frac{x}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{833}$$

$$\frac{1}{2}\left(\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx\right)+\frac{\sqrt{x^3-1}}{x}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{2}\left(\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right.$$
$$\frac{\sqrt{x^3-1}}{x}$$

$\downarrow$ 2418

$$\frac{1}{2}\left(\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)}{}\right.$$
$$\frac{\sqrt{x^3-1}}{x}$$

input
```
Int[1/(x^2*Sqrt[-1 + x^3]),x]
```

output
```
Sqrt[-1 + x^3]/x + ((2*Sqrt[-1 + x^3])/(1 - Sqrt[3] - x) - (3^(1/4)*Sqrt[2
 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticE[ArcS
in[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 - x)/
(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])
*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1
- Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]))/2
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.12

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\, \operatorname{hypergeom}\left(\left[-\frac{1}{3}, \frac{1}{2}\right], \left[\frac{2}{3}\right], x^{3}\right)}{\sqrt{\operatorname{signum}\left(x^{3}-1\right)}\, x}$
default	$\frac{\sqrt{x^{3}-1}}{x}-\frac{\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \left(\left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \text{Elli}\right.}{\sqrt{x^{3}-1}}$
risch	$\frac{\sqrt{x^{3}-1}}{x}-\frac{\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \left(\left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \text{Elli}\right.}{\sqrt{x^{3}-1}}$
elliptic	$\frac{\sqrt{x^{3}-1}}{x}-\frac{\left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x+\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \left(\left(\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \text{Elli}\right.}{\sqrt{x^{3}-1}}$

input
```
int(1/x^2/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)/x*hypergeom([-1/3,1/2],[2/3]
,x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = \frac{x\text{weierstrassZeta}(0,4,\text{weierstrassPInverse}(0,4,x)) + \sqrt{x^3-1}}{x}$$

input
```
integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
(x*weierstrassZeta(0, 4, weierstrassPInverse(0, 4, x)) + sqrt(x^3 - 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = -\frac{i\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle| x^3\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(x**3-1)**(1/2),x)
```

output
```
-I*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3)/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = \frac{\sqrt{x^3-1}}{x} + \frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^2*(x^3 - 1)^(1/2)),x)
```

output
```
(x^3 - 1)^(1/2)/x + ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin((-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2
 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (
3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}}{x^5-x^2}dx$$

input
```
int(1/x^2/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**5 - x**2),x)
```

3.285 $\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 294

$$\begin{aligned}\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx &= \frac{5\sqrt{-1+x^3}}{8\left(1-\sqrt{3}-x\right)}+\frac{\sqrt{-1+x^3}}{4x^4}+\frac{5\sqrt{-1+x^3}}{8x}\\&\quad-\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\\&\quad+\frac{5(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}\end{aligned}$$

output

```
5*(x^3-1)^(1/2)/(8-8*3^(1/2)-8*x)+1/4*(x^3-1)^(1/2)/x^4+5/8*(x^3-1)^(1/2)/
x-5/16*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)^2)
^(1/2)*EllipticE((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))/(-(1-x)/(1-3^(
1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)+5/24*2^(1/2)*(1-x)*((x^2+x+1)/(1-3^(1/2)-x)
^2)^(1/2)*EllipticF((1+3^(1/2)-x)/(1-3^(1/2)-x),2*I-I*3^(1/2))*3^(3/4)/(-(
1-x)/(1-3^(1/2)-x)^2)^(1/2)/(x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},x^3\right)}{4x^4\sqrt{-1+x^3}}$$

input
```
Integrate[1/(x^5*Sqrt[-1 + x^3]),x]
```

output
```
-1/4*(Sqrt[1 - x^3]*Hypergeometric2F1[-4/3, 1/2, -1/3, x^3])/(x^4*Sqrt[-1
+ x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {847, 847, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{8}\int \frac{1}{x^2\sqrt{x^3-1}}dx+\frac{\sqrt{x^3-1}}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{8}\left(\frac{\sqrt{x^3-1}}{x}-\frac{1}{2}\int \frac{x}{\sqrt{x^3-1}}dx\right)+\frac{\sqrt{x^3-1}}{4x^4}$$

$$\Big\downarrow \text{833}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx\right)+\frac{\sqrt{x^3-1}}{x}\right)+\frac{\sqrt{x^3-1}}{4x^4}$$

$$\Big\downarrow 760$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{}\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}\right.\right.$$

$$\frac{\sqrt{x^3-1}}{4x^4}$$

$$\Big\downarrow 2418$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-\cdots}{\cdots}\right.\right.$$

$$\frac{\sqrt{x^3-1}}{4x^4}$$

input
```
Int[1/(x^5*Sqrt[-1 + x^3]),x]
```

output
```
Sqrt[-1 + x^3]/(4*x^4) + (5*(Sqrt[-1 + x^3]/x + ((2*Sqrt[-1 + x^3])/(1 - S
qrt[3] - x) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - S
qrt[3] - x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 +
 4*Sqrt[3]])/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (2*Sq
rt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]
)/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]))/2))/8
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.11

method	result
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\ \operatorname{hypergeom}\left(\left[-\frac{4}{3},\frac{1}{2}\right],\left[-\frac{1}{3}\right],x^3\right)}{4\sqrt{\operatorname{signum}(x^3-1)}\,x^4}$
default	$\frac{\sqrt{x^3-1}}{4x^4}+\frac{5\sqrt{x^3-1}}{8x}-\frac{5\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+(\right.}{8\sqrt{x^3-1}}$
risch	$\frac{5x^6-3x^3-2}{8x^4\sqrt{x^3-1}}-\frac{5\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right.\right.}{8\sqrt{x^3-1}}$
elliptic	$\frac{\sqrt{x^3-1}}{4x^4}+\frac{5\sqrt{x^3-1}}{8x}-\frac{5\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\left(\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+(\right.}{8\sqrt{x^3-1}}$

input
```
int(1/x^5/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)/x^4*hypergeom([-4/3,1/2],[
-1/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = \frac{5\,x^4\mathrm{weierstrassZeta}(0,4,\mathrm{weierstrassPInverse}(0,4,x)) + (5\,x^3+2)\sqrt{x^3-1}}{8\,x^4}$$

input
```
integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(5*x^4*weierstrassZeta(0, 4, weierstrassPInverse(0, 4, x)) + (5*x^3 +
2)*sqrt(x^3 - 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = -\frac{i\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| x^3\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(x**3-1)**(1/2),x)
```

output
```
-I*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^5}\,dx$$

input
```
integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}x^5}\,dx$$

input
```
integrate(1/x^5/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 - 1)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = \frac{5\sqrt{x^3-1}}{8\,x} + \frac{\sqrt{x^3-1}}{4\,x^4} + \frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1}{2}}\right.\right.}{8\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right.}}$$

input
```
int(1/(x^5*(x^3 - 1)^(1/2)),x)
```

output
```
(5*(x^3 - 1)^(1/2))/(8*x) + (x^3 - 1)^(1/2)/(4*x^4) + (5*(((3^(1/2)*1i)/2
- 1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(a
sin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/(
(3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(8*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) + x^3)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{-1+x^3}}\,dx = \int \frac{\sqrt{x^3-1}}{x^8-x^5}dx$$

input
```
int(1/x^5/(x^3-1)^(1/2),x)
```

output
```
int(sqrt(x**3 - 1)/(x**8 - x**5),x)
```

3.286 $\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = \frac{2}{3}\sqrt{-1-x^3}+\frac{2}{3}\left(-1-x^3\right)^{3/2}+\frac{2}{5}\left(-1-x^3\right)^{5/2}+\frac{2}{21}\left(-1-x^3\right)^{7/2}$$

output
```
2/3*(-x^3-1)^(1/2)+2/3*(-x^3-1)^(3/2)+2/5*(-x^3-1)^(5/2)+2/21*(-x^3-1)^(7/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = -\frac{2}{105}\sqrt{-1-x^3}\left(-16+8x^3-6x^6+5x^9\right)$$

input
```
Integrate[x^11/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3]*(-16 + 8*x^3 - 6*x^6 + 5*x^9))/105
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt{-x^3-1}}dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}\int \left(-\left(-x^3-1\right)^{5/2}-3\left(-x^3-1\right)^{3/2}-3\sqrt{-x^3-1}-\frac{1}{\sqrt{-x^3-1}}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{2}{7}\left(-x^3-1\right)^{7/2}+\frac{6}{5}\left(-x^3-1\right)^{5/2}+2\left(-x^3-1\right)^{3/2}+2\sqrt{-x^3-1}\right)$$

input
```
Int[x^11/Sqrt[-1 - x^3],x]
```

output
```
(2*Sqrt[-1 - x^3] + 2*(-1 - x^3)^(3/2) + (6*(-1 - x^3)^(5/2))/5 + (2*(-1 -
 x^3)^(7/2))/7)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

method	result	size
trager	$\left(-\frac{2}{21}x^{9}+\frac{4}{35}x^{6}-\frac{16}{105}x^{3}+\frac{32}{105}\right)\sqrt{-x^{3}-1}$	28
pseudoelliptic	$-\frac{2\sqrt{-x^{3}-1}\,\left(5x^{9}-6x^{6}+8x^{3}-16\right)}{105}$	29
risch	$\frac{2\left(5x^{9}-6x^{6}+8x^{3}-16\right)\left(x^{3}+1\right)}{105\sqrt{-x^{3}-1}}$	34
gosper	$\frac{2(1+x)\left(x^{2}-x+1\right)\left(5x^{9}-6x^{6}+8x^{3}-16\right)}{105\sqrt{-x^{3}-1}}$	40
orering	$\frac{2(1+x)\left(x^{2}-x+1\right)\left(5x^{9}-6x^{6}+8x^{3}-16\right)}{105\sqrt{-x^{3}-1}}$	40
meijerg	$-\frac{i\left(\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\,\left(-40x^{9}+48x^{6}-64x^{3}+128\right)\sqrt{x^{3}+1}}{140}\right)}{3\sqrt{\pi}}$	42
default	$-\frac{2x^{9}\sqrt{-x^{3}-1}}{21}+\frac{4x^{6}\sqrt{-x^{3}-1}}{35}-\frac{16x^{3}\sqrt{-x^{3}-1}}{105}+\frac{32\sqrt{-x^{3}-1}}{105}$	55
elliptic	$-\frac{2x^{9}\sqrt{-x^{3}-1}}{21}+\frac{4x^{6}\sqrt{-x^{3}-1}}{35}-\frac{16x^{3}\sqrt{-x^{3}-1}}{105}+\frac{32\sqrt{-x^{3}-1}}{105}$	55

input
```
int(x^11/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/21*x^9+4/35*x^6-16/105*x^3+32/105)*(-x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = -\frac{2}{105}\left(5\,x^9-6\,x^6+8\,x^3-16\right)\sqrt{-x^3-1}$$

input
```
integrate(x^11/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(5*x^9 - 6*x^6 + 8*x^3 - 16)*sqrt(-x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = -\frac{2x^9\sqrt{-x^3-1}}{21}+\frac{4x^6\sqrt{-x^3-1}}{35}-\frac{16x^3\sqrt{-x^3-1}}{105}+\frac{32\sqrt{-x^3-1}}{105}$$

input
```
integrate(x**11/(-x**3-1)**(1/2),x)
```

output
```
-2*x**9*sqrt(-x**3 - 1)/21 + 4*x**6*sqrt(-x**3 - 1)/35 - 16*x**3*sqrt(-x**
3 - 1)/105 + 32*sqrt(-x**3 - 1)/105
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = \frac{2}{21}\left(-x^3-1\right)^{\frac{7}{2}}+\frac{2}{5}\left(-x^3-1\right)^{\frac{5}{2}}+\frac{2}{3}\left(-x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^11/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/21*(-x^3 - 1)^(7/2) + 2/5*(-x^3 - 1)^(5/2) + 2/3*(-x^3 - 1)^(3/2) + 2/3*
sqrt(-x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = -\frac{2}{21}\left(x^3+1\right)^3\sqrt{-x^3-1}+\frac{2}{5}\left(x^3+1\right)^2\sqrt{-x^3-1} + \frac{2}{3}\left(-x^3-1\right)^{\frac{3}{2}}+\frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^11/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
-2/21*(x^3 + 1)^3*sqrt(-x^3 - 1) + 2/5*(x^3 + 1)^2*sqrt(-x^3 - 1) + 2/3*(-
x^3 - 1)^(3/2) + 2/3*sqrt(-x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = \frac{4\,x^6\,\sqrt{-x^3-1}}{35}-\frac{16\,x^3\,\sqrt{-x^3-1}}{105}-\frac{2\,x^9\,\sqrt{-x^3-1}}{21}+\frac{32\,\sqrt{-x^3-1}}{105}$$

input
```
int(x^11/(- x^3 - 1)^(1/2),x)
```

output
```
(4*x^6*(- x^3 - 1)^(1/2))/35 - (16*x^3*(- x^3 - 1)^(1/2))/105 - (2*x^9*(-
x^3 - 1)^(1/2))/21 + (32*(- x^3 - 1)^(1/2))/105
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.43

$$\int \frac{x^{11}}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{x^3+1}\,i(-5x^9+6x^6-8x^3+16)}{105}$$

input
```
int(x^11/(-x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*i*( - 5*x**9 + 6*x**6 - 8*x**3 + 16))/105
```

3.287 $\int \frac{x^8}{\sqrt{-1-x^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^8}{\sqrt{-1-x^3}} \, dx = -\frac{2}{3}\sqrt{-1-x^3} - \frac{4}{9}\left(-1-x^3\right)^{3/2} - \frac{2}{15}\left(-1-x^3\right)^{5/2}$$

output
```
-2/3*(-x^3-1)^(1/2)-4/9*(-x^3-1)^(3/2)-2/15*(-x^3-1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{\sqrt{-1-x^3}} \, dx = -\frac{2}{45}\sqrt{-1-x^3}\left(8-4x^3+3x^6\right)$$

input
```
Integrate[x^8/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3]*(8 - 4*x^3 + 3*x^6))/45
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt{-x^3-1}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\left(-x^3-1\right)^{3/2}+2\sqrt{-x^3-1}+\frac{1}{\sqrt{-x^3-1}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{2}{5}\left(-x^3-1\right)^{5/2}-\frac{4}{3}\left(-x^3-1\right)^{3/2}-2\sqrt{-x^3-1}\right)$$

input
```
Int[x^8/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3] - (4*(-1 - x^3)^(3/2))/3 - (2*(-1 - x^3)^(5/2))/5)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\left(-\frac{2}{15}x^{6}+\frac{8}{45}x^{3}-\frac{16}{45}\right)\sqrt{-x^{3}-1}$	23
pseudoelliptic	$-\frac{2\sqrt{-x^{3}-1}\,(3x^{6}-4x^{3}+8)}{45}$	24
risch	$\frac{2(3x^{6}-4x^{3}+8)(x^{3}+1)}{45\sqrt{-x^{3}-1}}$	29
gosper	$\frac{2(1+x)(x^{2}-x+1)(3x^{6}-4x^{3}+8)}{45\sqrt{-x^{3}-1}}$	35
orering	$\frac{2(1+x)(x^{2}-x+1)(3x^{6}-4x^{3}+8)}{45\sqrt{-x^{3}-1}}$	35
meijerg	$-\frac{i\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,(6x^{6}-8x^{3}+16)\sqrt{x^{3}+1}}{15}\right)}{3\sqrt{\pi}}$	37
default	$-\frac{2x^{6}\sqrt{-x^{3}-1}}{15}+\frac{8x^{3}\sqrt{-x^{3}-1}}{45}-\frac{16\sqrt{-x^{3}-1}}{45}$	41
elliptic	$-\frac{2x^{6}\sqrt{-x^{3}-1}}{15}+\frac{8x^{3}\sqrt{-x^{3}-1}}{45}-\frac{16\sqrt{-x^{3}-1}}{45}$	41

input
```
int(x^8/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-2/15*x^6+8/45*x^3-16/45)*(-x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{-1-x^3}}\,dx = -\frac{2}{45}\left(3\,x^6-4\,x^3+8\right)\sqrt{-x^3-1}$$

input
```
integrate(x^8/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(3*x^6 - 4*x^3 + 8)*sqrt(-x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{\sqrt{-1-x^3}}\,dx = -\frac{2x^6\sqrt{-x^3-1}}{15}+\frac{8x^3\sqrt{-x^3-1}}{45}-\frac{16\sqrt{-x^3-1}}{45}$$

input
```
integrate(x**8/(-x**3-1)**(1/2),x)
```

output
```
-2*x**6*sqrt(-x**3 - 1)/15 + 8*x**3*sqrt(-x**3 - 1)/45 - 16*sqrt(-x**3 - 1
)/45
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{\sqrt{-1-x^3}}\,dx = -\frac{2}{15}\left(-x^3-1\right)^{\frac{5}{2}}-\frac{4}{9}\left(-x^3-1\right)^{\frac{3}{2}}-\frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^8/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-2/15*(-x^3 - 1)^(5/2) - 4/9*(-x^3 - 1)^(3/2) - 2/3*sqrt(-x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\sqrt{-1-x^3}}\,dx = -\frac{2}{15}\left(x^3+1\right)^2\sqrt{-x^3-1} - \frac{4}{9}\left(-x^3-1\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^8/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
-2/15*(x^3 + 1)^2*sqrt(-x^3 - 1) - 4/9*(-x^3 - 1)^(3/2) - 2/3*sqrt(-x^3 -
1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^8}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-x^3-1}\,(3\,x^6-4\,x^3+8)}{45}$$

input
```
int(x^8/(- x^3 - 1)^(1/2),x)
```

output
```
-(2*(- x^3 - 1)^(1/2)*(3*x^6 - 4*x^3 + 8))/45
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \frac{x^8}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{x^3+1}\,i(-3x^6+4x^3-8)}{45}$$

input
```
int(x^8/(-x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*i*( - 3*x**6 + 4*x**3 - 8))/45
```

3.288 $\int \frac{x^5}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = \frac{2}{3}\sqrt{-1-x^3}+\frac{2}{9}\left(-1-x^3\right)^{3/2}$$

output
```
2/3*(-x^3-1)^(1/2)+2/9*(-x^3-1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = -\frac{2}{9}\sqrt{-1-x^3}\left(-2+x^3\right)$$

input
```
Integrate[x^5/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3]*(-2 + x^3))/9
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt{-x^3-1}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(-\sqrt{-x^3-1}-\frac{1}{\sqrt{-x^3-1}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{3}\left(-x^3-1\right)^{3/2}+2\sqrt{-x^3-1}\right)$$

input
```
Int[x^5/Sqrt[-1 - x^3],x]
```

output
```
(2*Sqrt[-1 - x^3] + (2*(-1 - x^3)^(3/2))/3)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$-\frac{2(x^3-2)\sqrt{-x^3-1}}{9}$	17
trager	$\left(-\frac{2x^3}{9}+\frac{4}{9}\right)\sqrt{-x^3-1}$	18
risch	$\frac{2(x^3-2)(x^3+1)}{9\sqrt{-x^3-1}}$	22
default	$-\frac{2x^3\sqrt{-x^3-1}}{9}+\frac{4\sqrt{-x^3-1}}{9}$	27
elliptic	$-\frac{2x^3\sqrt{-x^3-1}}{9}+\frac{4\sqrt{-x^3-1}}{9}$	27
gosper	$\frac{2(1+x)(x^2-x+1)(x^3-2)}{9\sqrt{-x^3-1}}$	28
orering	$\frac{2(1+x)(x^2-x+1)(x^3-2)}{9\sqrt{-x^3-1}}$	28
meijerg	$-\frac{i\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-4x^3+8\right)\sqrt{x^3+1}}{6}\right)}{3\sqrt{\pi}}$	32

input
```
int(x^5/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x^3-2)*(-x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = -\frac{2}{9}\left(x^3-2\right)\sqrt{-x^3-1}$$

input
```
integrate(x^5/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(x^3 - 2)*sqrt(-x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = -\frac{2x^3\sqrt{-x^3-1}}{9} + \frac{4\sqrt{-x^3-1}}{9}$$

input
```
integrate(x**5/(-x**3-1)**(1/2),x)
```

output
```
-2*x**3*sqrt(-x**3 - 1)/9 + 4*sqrt(-x**3 - 1)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = \frac{2}{9}\left(-x^3-1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^5/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/9*(-x^3 - 1)^(3/2) + 2/3*sqrt(-x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = \frac{2}{9}\left(-x^3-1\right)^{\frac{3}{2}} + \frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^5/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/9*(-x^3 - 1)^(3/2) + 2/3*sqrt(-x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-x^3-1}\,(x^3-2)}{9}$$

input
```
int(x^5/(- x^3 - 1)^(1/2),x)
```

output
```
-(2*(- x^3 - 1)^(1/2)*(x^3 - 2))/9
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^5}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{x^3+1}\,i(-x^3+2)}{9}$$

input
```
int(x^5/(-x^3-1)^(1/2),x)
```

output
```
(2*sqrt(x**3 + 1)*i*( - x**3 + 2))/9
```

3.289 $\int \frac{x^2}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\sqrt{-1-x^3}$$

output
```
-2/3*(-x^3-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\sqrt{-1-x^3}$$

input
```
Integrate[x^2/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3])/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{-x^3-1}}dx$$
$$\Big\downarrow \; 793$$
$$-\frac{2}{3}\sqrt{-x^3-1}$$

input
```
Int[x^2/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3])/3
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{2\sqrt{-x^{3}-1}}{3}$	12
default	$-\frac{2\sqrt{-x^{3}-1}}{3}$	12
trager	$-\frac{2\sqrt{-x^{3}-1}}{3}$	12
elliptic	$-\frac{2\sqrt{-x^{3}-1}}{3}$	12
pseudoelliptic	$-\frac{2\sqrt{-x^{3}-1}}{3}$	12
risch	$\frac{\frac{2x^{3}}{3}+\frac{2}{3}}{\sqrt{-x^{3}-1}}$	17
gosper	$\frac{2(1+x)(x^{2}-x+1)}{3\sqrt{-x^{3}-1}}$	23
orering	$\frac{2(1+x)(x^{2}-x+1)}{3\sqrt{-x^{3}-1}}$	23
meijerg	$-\frac{i\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{3}+1}\right)}{3\sqrt{\pi}}$	25

input
```
int(x^2/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-x^3-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx=-\frac{2}{3}\,\sqrt{-x^{3}-1}$$

input
```
integrate(x^2/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-x^3 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-x^3-1}}{3}$$

input
```
integrate(x**2/(-x**3-1)**(1/2),x)
```

output
```
-2*sqrt(-x**3 - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^2/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(-x^3 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\sqrt{-x^3-1}$$

input
```
integrate(x^2/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(-x^3 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-x^3-1}}{3}$$

input
```
int(x^2/(- x^3 - 1)^(1/2),x)
```

output
```
-(2*(- x^3 - 1)^(1/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{x^3+1}\,i}{3}$$

input
```
int(x^2/(-x^3-1)^(1/2),x)
```

output
```
( - 2*sqrt(x**3 + 1)*i)/3
```

3.290 $\int \frac{1}{x\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-1-x^3}\right)$$

output
```
2/3*arctan((-x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-1-x^3}\right)$$

input
```
Integrate[1/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTan[Sqrt[-1 - x^3]])/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2}{3}\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[1/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTan[Sqrt[-1 - x^3]])/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{2 \arctan\left(\sqrt{-x^{3}-1}\right)}{3}$	13
elliptic	$\frac{2 \arctan\left(\sqrt{-x^{3}-1}\right)}{3}$	13
pseudoelliptic	$\frac{2 \arctan\left(\sqrt{-x^{3}-1}\right)}{3}$	13
meijerg	$-\frac{i\left(\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{3}+1}}{2}\right)\right)}{3\sqrt{\pi}}$	38
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{3}+2\sqrt{-x^{3}-1}+2\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x^{3}}\right)}{3}$	45

input
```
int(1/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan((-x^3-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*arctan(sqrt(-x^3 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate(1/x/(-x**3-1)**(1/2),x)
```

output
```
2*I*asinh(x**(-3/2))/3
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*arctan(sqrt(-x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
2/3*arctan(sqrt(-x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 11.25

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = -\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(1/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
-(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/
2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/
2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\sqrt{-1-x^3}}\,dx = \frac{i\left(-\log\left(\sqrt{x^3+1}-1\right)+\log\left(\sqrt{x^3+1}+1\right)\right)}{3}$$

input
```
int(1/x/(-x^3-1)^(1/2),x)
```

output
```
(i*( - log(sqrt(x**3 + 1) - 1) + log(sqrt(x**3 + 1) + 1)))/3
```

3.291 $\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{3x^3} - \frac{1}{3}\arctan\left(\sqrt{-1-x^3}\right)$$

output
```
1/3*(-x^3-1)^(1/2)/x^3-1/3*arctan((-x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{3x^3} - \frac{1}{3}\arctan\left(\sqrt{-1-x^3}\right)$$

input
```
Integrate[1/(x^4*Sqrt[-1 - x^3]),x]
```

output
```
Sqrt[-1 - x^3]/(3*x^3) - ArcTan[Sqrt[-1 - x^3]]/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^6\sqrt{-x^3-1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{x^3}-\frac{1}{2}\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}+\frac{\sqrt{-x^3-1}}{x^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{x^3}-\arctan\left(\sqrt{-x^3-1}\right)\right)$$

input
```
Int[1/(x^4*Sqrt[-1 - x^3]),x]
```

output
```
(Sqrt[-1 - x^3]/x^3 - ArcTan[Sqrt[-1 - x^3]])/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sqrt{-x^{3}-1}}{3x^{3}}-\frac{\arctan\left(\sqrt{-x^{3}-1}\right)}{3}$	28
elliptic	$\frac{\sqrt{-x^{3}-1}}{3x^{3}}-\frac{\arctan\left(\sqrt{-x^{3}-1}\right)}{3}$	28
pseudoelliptic	$\frac{-\arctan\left(\sqrt{-x^{3}-1}\right)x^{3}+\sqrt{-x^{3}-1}}{3x^{3}}$	31
risch	$-\frac{x^{3}+1}{3x^{3}\sqrt{-x^{3}-1}}-\frac{\arctan\left(\sqrt{-x^{3}-1}\right)}{3}$	33
trager	$\frac{\sqrt{-x^{3}-1}}{3x^{3}}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{3}+2\operatorname{RootOf}\left(\_Z^{2}+1\right)-2\sqrt{-x^{3}-1}}{x^{3}}\right)}{6}$	59
meijerg	$-\frac{i\left(-\frac{\sqrt{\pi}}{x^{3}}-\frac{(1-2\ln(2)+3\ln(x))\sqrt{\pi}}{2}+\frac{\sqrt{\pi}\left(4x^{3}+8\right)}{8x^{3}}-\frac{\sqrt{\pi}\sqrt{x^{3}+1}}{x^{3}}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^{3}+1}}{2}\right)\right)}{3\sqrt{\pi}}$	77

input
```
int(1/x^4/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^3-1)^(1/2)/x^3-1/3*arctan((-x^3-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx=-\frac{x^3\arctan\left(\sqrt{-x^3-1}\right)-\sqrt{-x^3-1}}{3\,x^3}$$

input
```
integrate(1/x^4/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(x^3*arctan(sqrt(-x^3 - 1)) - sqrt(-x^3 - 1))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = -\frac{i \operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{i\sqrt{1+\frac{1}{x^3}}}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/x**4/(-x**3-1)**(1/2),x)
```

output
```
-I*asinh(x**(-3/2))/3 + I*sqrt(1 + x**(-3))/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{3\,x^3} - \frac{1}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x^4/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(-x^3 - 1)/x^3 - 1/3*arctan(sqrt(-x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{3\,x^3} - \frac{1}{3}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x^4/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(-x^3 - 1)/x^3 - 1/3*arctan(sqrt(-x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 194, normalized size of antiderivative = 5.54

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{3\,x^3} + \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^4*(- x^3 - 1)^(1/2)),x)
```

output
```
(- x^3 - 1)^(1/2)/(3*x^3) + (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*
(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^4\sqrt{-1-x^3}}\,dx = \frac{i\left(2\sqrt{x^3+1}+\log\left(\sqrt{x^3+1}-1\right)x^3-\log\left(\sqrt{x^3+1}+1\right)x^3\right)}{6x^3}$$

input
```
int(1/x^4/(-x^3-1)^(1/2),x)
```

output
```
(i*(2*sqrt(x**3 + 1) + log(sqrt(x**3 + 1) - 1)*x**3 - log(sqrt(x**3 + 1) +
 1)*x**3))/(6*x**3)
```

3.292 $\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{6x^6} - \frac{\sqrt{-1-x^3}}{4x^3} + \frac{1}{4}\arctan\left(\sqrt{-1-x^3}\right)$$

output
```
1/6*(-x^3-1)^(1/2)/x^6-1/4*(-x^3-1)^(1/2)/x^3+1/4*arctan((-x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = \frac{(2-3x^3)\sqrt{-1-x^3}}{12x^6} + \frac{1}{4}\arctan\left(\sqrt{-1-x^3}\right)$$

input
```
Integrate[1/(x^7*Sqrt[-1 - x^3]),x]
```

output
```
((2 - 3*x^3)*Sqrt[-1 - x^3])/(12*x^6) + ArcTan[Sqrt[-1 - x^3]]/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{-x^3-1}}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3}\int \frac{1}{x^9\sqrt{-x^3-1}}dx^3$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\int \frac{1}{x^6\sqrt{-x^3-1}}dx^3\right)$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\left(\frac{\sqrt{-x^3-1}}{x^3}-\frac{1}{2}\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3\right)\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\left(\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}+\frac{\sqrt{-x^3-1}}{x^3}\right)\right)$$
$$\Big\downarrow \text{217}$$
$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\left(\frac{\sqrt{-x^3-1}}{x^3}-\arctan\left(\sqrt{-x^3-1}\right)\right)\right)$$

input
```
Int[1/(x^7*Sqrt[-1 - x^3]),x]
```

output
```
(Sqrt[-1 - x^3]/(2*x^6) - (3*(Sqrt[-1 - x^3]/x^3 - ArcTan[Sqrt[-1 - x^3]])
)/4)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{3x^6+x^3-2}{12x^6\sqrt{-x^3-1}}+\frac{\arctan\left(\sqrt{-x^3-1}\right)}{4}$	38
default	$\frac{\sqrt{-x^3-1}}{6x^6}-\frac{\sqrt{-x^3-1}}{4x^3}+\frac{\arctan\left(\sqrt{-x^3-1}\right)}{4}$	42
elliptic	$\frac{\sqrt{-x^3-1}}{6x^6}-\frac{\sqrt{-x^3-1}}{4x^3}+\frac{\arctan\left(\sqrt{-x^3-1}\right)}{4}$	42
pseudoelliptic	$\frac{3\arctan\left(\sqrt{-x^3-1}\right)x^6-3x^3\sqrt{-x^3-1}+2\sqrt{-x^3-1}}{12x^6}$	47
trager	$-\frac{(3x^3-2)\sqrt{-x^3-1}}{12x^6}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{-x^3-1}+2\operatorname{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{8}$	67
meijerg	$-\frac{i\left(-\frac{\sqrt{\pi}}{2x^6}+\frac{\sqrt{\pi}}{2x^3}+\frac{3\left(\frac{7}{6}-2\ln(2)+3\ln(x)\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^6-8x^3+8\right)}{16x^6}-\frac{\sqrt{\pi}\left(-12x^3+8\right)\sqrt{x^3+1}}{16x^6}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)}{4}\right)}{3\sqrt{\pi}}$	98

input
```
int(1/x^7/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*x^6+x^3-2)/x^6/(-x^3-1)^(1/2)+1/4*arctan((-x^3-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx=\frac{3\,x^6\arctan\left(\sqrt{-x^3-1}\right)-(3\,x^3-2)\sqrt{-x^3-1}}{12\,x^6}$$

input
```
integrate(1/x^7/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(3*x^6*arctan(sqrt(-x^3 - 1)) - (3*x^3 - 2)*sqrt(-x^3 - 1))/x^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = \frac{i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{4} - \frac{i}{4x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}} - \frac{i}{12x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}} + \frac{i}{6x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}}$$

input
```
integrate(1/x**7/(-x**3-1)**(1/2),x)
```

output
```
I*asinh(x**(-3/2))/4 - I/(4*x**(3/2)*sqrt(1 + x**(-3))) - I/(12*x**(9/2)*s
qrt(1 + x**(-3))) + I/(6*x**(15/2)*sqrt(1 + x**(-3)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = -\frac{3\left(-x^3-1\right)^{\frac{3}{2}} + 5\sqrt{-x^3-1}}{12\left(2x^3-\left(x^3+1\right)^2+1\right)} + \frac{1}{4}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x^7/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(3*(-x^3 - 1)^(3/2) + 5*sqrt(-x^3 - 1))/(2*x^3 - (x^3 + 1)^2 + 1) +
1/4*arctan(sqrt(-x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = \frac{3\left(-x^3-1\right)^{\frac{3}{2}} + 5\sqrt{-x^3-1}}{12x^6} + \frac{1}{4}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x^7/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
1/12*(3*(-x^3 - 1)^(3/2) + 5*sqrt(-x^3 - 1))/x^6 + 1/4*arctan(sqrt(-x^3 -
1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.94

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{6\,x^6} - \frac{\sqrt{-x^3-1}}{4\,x^3} - \frac{3\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^7*(- x^3 - 1)^(1/2)),x)
```

output
```
(- x^3 - 1)^(1/2)/(6*x^6) - (- x^3 - 1)^(1/2)/(4*x^3) - (3*((3^(1/2)*1i)/2
 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x +
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))/(4*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^7\sqrt{-1-x^3}}\,dx = \frac{i\left(-6\sqrt{x^3+1}\,x^3+4\sqrt{x^3+1}-3\log\left(\sqrt{x^3+1}-1\right)x^6+3\log\left(\sqrt{x^3+1}+1\right)x^6\right)}{24x^6}$$

input
```
int(1/x^7/(-x^3-1)^(1/2),x)
```

output
```
(i*( - 6*sqrt(x**3 + 1)*x**3 + 4*sqrt(x**3 + 1) - 3*log(sqrt(x**3 + 1) - 1
)*x**6 + 3*log(sqrt(x**3 + 1) + 1)*x**6))/(24*x**6)
```

3.293 $\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{9x^9} - \frac{5\sqrt{-1-x^3}}{36x^6} + \frac{5\sqrt{-1-x^3}}{24x^3} - \frac{5}{24}\arctan\left(\sqrt{-1-x^3}\right)$$

output
```
1/9*(-x^3-1)^(1/2)/x^9-5/36*(-x^3-1)^(1/2)/x^6+5/24*(-x^3-1)^(1/2)/x^3-5/2
4*arctan((-x^3-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}(8-10x^3+15x^6)}{72x^9} - \frac{5}{24}\arctan\left(\sqrt{-1-x^3}\right)$$

input
```
Integrate[1/(x^10*Sqrt[-1 - x^3]),x]
```

output
```
(Sqrt[-1 - x^3]*(8 - 10*x^3 + 15*x^6))/(72*x^9) - (5*ArcTan[Sqrt[-1 - x^3]
])/24
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 52, 52, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^{12}\sqrt{-x^3-1}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{3x^9}-\frac{5}{6}\int \frac{1}{x^9\sqrt{-x^3-1}}dx^3\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{3x^9}-\frac{5}{6}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\int \frac{1}{x^6\sqrt{-x^3-1}}dx^3\right)\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{3x^9}-\frac{5}{6}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\left(\frac{\sqrt{-x^3-1}}{x^3}-\frac{1}{2}\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3\right)\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{3x^9}-\frac{5}{6}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\left(\int \frac{1}{-x^6-1}d\sqrt{-x^3-1}+\frac{\sqrt{-x^3-1}}{x^3}\right)\right)\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\sqrt{-x^3-1}}{3x^9}-\frac{5}{6}\left(\frac{\sqrt{-x^3-1}}{2x^6}-\frac{3}{4}\left(\frac{\sqrt{-x^3-1}}{x^3}-\arctan\left(\sqrt{-x^3-1}\right)\right)\right)\right)$$

input
```
Int[1/(x^10*Sqrt[-1 - x^3]),x]
```

output
```
(Sqrt[-1 - x^3]/(3*x^9) - (5*(Sqrt[-1 - x^3]/(2*x^6) - (3*(Sqrt[-1 - x^3]/
x^3 - ArcTan[Sqrt[-1 - x^3]]))/4))/6)/3
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{(15x^6-10x^3+8)\sqrt{-x^3-1}-15\arctan\left(\sqrt{-x^3-1}\right)x^9}{72x^9}$
risch	$-\frac{15x^9+5x^6-2x^3+8}{72x^9\sqrt{-x^3-1}}-\frac{5\arctan\left(\sqrt{-x^3-1}\right)}{24}$
default	$\frac{\sqrt{-x^3-1}}{9x^9}-\frac{5\sqrt{-x^3-1}}{36x^6}+\frac{5\sqrt{-x^3-1}}{24x^3}-\frac{5\arctan\left(\sqrt{-x^3-1}\right)}{24}$
elliptic	$\frac{\sqrt{-x^3-1}}{9x^9}-\frac{5\sqrt{-x^3-1}}{36x^6}+\frac{5\sqrt{-x^3-1}}{24x^3}-\frac{5\arctan\left(\sqrt{-x^3-1}\right)}{24}$
trager	$\frac{(15x^6-10x^3+8)\sqrt{-x^3-1}}{72x^9}-\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2\sqrt{-x^3-1}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x^3}\right)}{48}$
meijerg	$-\frac{i\left(-\frac{\sqrt{\pi}}{3x^9}+\frac{\sqrt{\pi}}{4x^6}-\frac{3\sqrt{\pi}}{8x^3}-\frac{5\left(\frac{37}{30}-2\ln(2)+3\ln(x)\right)\sqrt{\pi}}{16}+\frac{\sqrt{\pi}\left(148x^9+144x^6-96x^3+128\right)}{384x^9}-\frac{\sqrt{\pi}\left(240x^6-160x^3+128\right)\sqrt{x^3+1}}{384x^9}+\frac{5\sqrt{\pi}\ln}{}\right.}{3\sqrt{\pi}}$

input
```
int(1/x^10/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/72*((15*x^6-10*x^3+8)*(-x^3-1)^(1/2)-15*arctan((-x^3-1)^(1/2))*x^9)/x^9
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = -\frac{15\,x^9\arctan\left(\sqrt{-x^3-1}\right)-(15\,x^6-10\,x^3+8)\sqrt{-x^3-1}}{72\,x^9}$$

input
```
integrate(1/x^10/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/72*(15*x^9*arctan(sqrt(-x^3 - 1)) - (15*x^6 - 10*x^3 + 8)*sqrt(-x^3 - 1
))/x^9
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.68 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = -\frac{5i\,\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{24} + \frac{5i}{24x^{\frac{3}{2}}\sqrt{1+\frac{1}{x^3}}} + \frac{5i}{72x^{\frac{9}{2}}\sqrt{1+\frac{1}{x^3}}} - \frac{i}{36x^{\frac{15}{2}}\sqrt{1+\frac{1}{x^3}}} + \frac{i}{9x^{\frac{21}{2}}\sqrt{1+\frac{1}{x^3}}}$$

input
```
integrate(1/x**10/(-x**3-1)**(1/2),x)
```

output
```
-5*I*asinh(x**(-3/2))/24 + 5*I/(24*x**(3/2)*sqrt(1 + x**(-3))) + 5*I/(72*x
**(9/2)*sqrt(1 + x**(-3))) - I/(36*x**(15/2)*sqrt(1 + x**(-3))) + I/(9*x**
(21/2)*sqrt(1 + x**(-3)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = \frac{15\,(-x^3-1)^{\frac{5}{2}} + 40\,(-x^3-1)^{\frac{3}{2}} + 33\,\sqrt{-x^3-1}}{72\,\left((x^3+1)^3 + 3\,x^3 - 3\,(x^3+1)^2 + 2\right)} - \frac{5}{24}\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x^10/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
1/72*(15*(-x^3 - 1)^(5/2) + 40*(-x^3 - 1)^(3/2) + 33*sqrt(-x^3 - 1))/((x^3
 + 1)^3 + 3*x^3 - 3*(x^3 + 1)^2 + 2) - 5/24*arctan(sqrt(-x^3 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = \frac{15\,(x^3+1)^2\sqrt{-x^3-1}+40\,(-x^3-1)^{\frac{3}{2}}+33\,\sqrt{-x^3-1}}{72\,x^9} - \frac{5}{24}\,\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
integrate(1/x^10/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
1/72*(15*(x^3 + 1)^2*sqrt(-x^3 - 1) + 40*(-x^3 - 1)^(3/2) + 33*sqrt(-x^3 -
 1))/x^9 - 5/24*arctan(sqrt(-x^3 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.14

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = \frac{5\,\sqrt{-x^3-1}}{24\,x^3} - \frac{5\,\sqrt{-x^3-1}}{36\,x^6} + \frac{\sqrt{-x^3-1}}{9\,x^9} + \frac{5\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{8\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^10*(- x^3 - 1)^(1/2)),x)
```

output
```
(5*(- x^3 - 1)^(1/2))/(24*x^3) - (5*(- x^3 - 1)^(1/2))/(36*x^6) + (- x^3 -
 1)^(1/2)/(9*x^9) + (5*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(8*(- x^3 - 1)^(1/2)*(x^3
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{10}\sqrt{-1-x^3}}\,dx = \frac{i\left(30\sqrt{x^3+1}\,x^6-20\sqrt{x^3+1}\,x^3+16\sqrt{x^3+1}+15\log\left(\sqrt{x^3+1}-1\right)x^9-15\log\left(\sqrt{x^3+1}+1\right)x^9\right)}{144x^9}$$

input
```
int(1/x^10/(-x^3-1)^(1/2),x)
```

output
```
(i*(30*sqrt(x**3 + 1)*x**6 - 20*sqrt(x**3 + 1)*x**3 + 16*sqrt(x**3 + 1) +
15*log(sqrt(x**3 + 1) - 1)*x**9 - 15*log(sqrt(x**3 + 1) + 1)*x**9))/(144*x
**9)
```

3.294 $\int \frac{x^6}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = \frac{16}{55}x\sqrt{-1-x^3}-\frac{2}{11}x^4\sqrt{-1-x^3} + \frac{32\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{55\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
16/55*x*(-x^3-1)^(1/2)-2/11*x^4*(-x^3-1)^(1/2)+32/165*(1/2*6^(1/2)-1/2*2^(
1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x
-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = \frac{2x\left(-8-3x^3+5x^6+8\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)\right)}{55\sqrt{-1-x^3}}$$

input
```
Integrate[x^6/Sqrt[-1 - x^3],x]
```

output
```
(2*x*(-8 - 3*x^3 + 5*x^6 + 8*Sqrt[1 + x^3]*Hypergeometric2F1[1/3, 1/2, 4/3
, -x^3]))/(55*Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {843, 843, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{843}$$

$$-\frac{8}{11}\int \frac{x^3}{\sqrt{-x^3-1}}dx-\frac{2}{11}\sqrt{-x^3-1}x^4$$

$$\Big\downarrow \text{843}$$

$$-\frac{8}{11}\left(-\frac{2}{5}\int \frac{1}{\sqrt{-x^3-1}}dx-\frac{2}{5}\sqrt{-x^3-1}x\right)-\frac{2}{11}\sqrt{-x^3-1}x^4$$

$$\Big\downarrow \text{760}$$

$$-\frac{8}{11}\left(-\frac{4\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{5}\sqrt{-x^3-1}x\right)-\frac{2}{11}\sqrt{-x^3-1}x^4$$

input
```
Int[x^6/Sqrt[-1 - x^3],x]
```

output
```
(-2*x^4*Sqrt[-1 - x^3])/11 - (8*((-2*x*Sqrt[-1 - x^3])/5 - (4*Sqrt[2 - Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1
+ Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[-((1 +
 x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])))/11
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.12

method	result
meijerg	$-\frac{i x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{7}{3}\right],\left[\frac{10}{3}\right],-x^{3}\right)}{7}$
risch	$\frac{2 x\left(5 x^{3}-8\right)\left(x^{3}+1\right)}{55 \sqrt{-x^{3}-1}}-\frac{32 i \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{165 \sqrt{-x^{3}-1}}$
default	$-\frac{2 x^{4} \sqrt{-x^{3}-1}}{11}+\frac{16 x \sqrt{-x^{3}-1}}{55}-\frac{32 i \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{}}}{3}\right.}{165 \sqrt{-x^{3}-1}}$
elliptic	$-\frac{2 x^{4} \sqrt{-x^{3}-1}}{11}+\frac{16 x \sqrt{-x^{3}-1}}{55}-\frac{32 i \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \operatorname{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{}}}{3}\right.}{165 \sqrt{-x^{3}-1}}$

input
```
int(x^6/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*I*x^7*hypergeom([1/2,7/3],[10/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = -\frac{2}{55}\left(5\,x^4-8\,x\right)\sqrt{-x^3-1}-\frac{32}{55}i\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^6/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(5*x^4 - 8*x)*sqrt(-x^3 - 1) - 32/55*I*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.21

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = -\frac{ix^7\Gamma\left(\frac{7}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(-x**3-1)**(1/2),x)
```

output
```
-I*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), x**3*exp_polar(I*pi))/(3*gam
ma(10/3))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = \int \frac{x^6}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^6/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = \int \frac{x^6}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^6/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = \frac{16\,x\,\sqrt{-x^3-1}}{55} - \frac{2\,x^4\,\sqrt{-x^3-1}}{11} + \frac{32\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{55\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x^6/(- x^3 - 1)^(1/2),x)
```

output
```
(16*x*(- x^3 - 1)^(1/2))/55 - (2*x^4*(- x^3 - 1)^(1/2))/11 + (32*((3^(1/2)
*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2
- 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(55*(-
 x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{-1-x^3}}\,dx = \frac{2i\left(-5\sqrt{x^3+1}\,x^4+8\sqrt{x^3+1}\,x-8\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)\right)}{55}$$

input
```
int(x^6/(-x^3-1)^(1/2),x)
```

output
```
(2*i*( - 5*sqrt(x**3 + 1)*x**4 + 8*sqrt(x**3 + 1)*x - 8*int(sqrt(x**3 + 1)
/(x**3 + 1),x)))/55
```

3.295 $\int \frac{x^3}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = -\frac{2}{5}x\sqrt{-1-x^3} - \frac{4\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2/5*x*(-x^3-1)^(1/2)-4/15*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x
-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^
(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.36

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = \frac{2\left(x+x^4-x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)\right)}{5\sqrt{-1-x^3}}$$

input
```
Integrate[x^3/Sqrt[-1 - x^3],x]
```

output
```
(2*(x + x^4 - x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3]))/(5*
Sqrt[-1 - x^3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {843, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 843}$$

$$-\frac{2}{5}\int \frac{1}{\sqrt{-x^3-1}}dx-\frac{2}{5}\sqrt{-x^3-1}x$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{4\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{5}\sqrt{-x^3-1}x$$

input
```
Int[x^3/Sqrt[-1 - x^3],x]
```

output
```
(-2*x*Sqrt[-1 - x^3])/5 - (4*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/
(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)]
, -7 + 4*Sqrt[3]])/(5*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
 - x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.14

method	result	si
meijerg	$-\frac{i x^4 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{4}{3}\right],\left[\frac{7}{3}\right],-x^3\right)}{4}$	18
default	$-\frac{2x\sqrt{-x^3-1}}{5}+\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{15\sqrt{-x^3-1}}$	12
elliptic	$-\frac{2x\sqrt{-x^3-1}}{5}+\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{15\sqrt{-x^3-1}}$	12
risch	$\frac{2x\left(x^3+1\right)}{5\sqrt{-x^3-1}}+\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{15\sqrt{-x^3-1}}$	12

input
```
int(x^3/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*x^4*hypergeom([1/2,4/3],[7/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.15

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = -\frac{2}{5}\sqrt{-x^3-1}x+\frac{4}{5}i\operatorname{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x^3/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(-x^3 - 1)*x + 4/5*I*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.24

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = -\frac{ix^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right| x^3e^{i\pi}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(-x**3-1)**(1/2),x)
```

output
```
-I*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), x**3*exp_polar(I*pi))/(3*gamm
a(7/3))
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = \int \frac{x^3}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^3/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = \int \frac{x^3}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^3/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.41

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = -\frac{2\,x\,\sqrt{-x^3-1}}{5} - \frac{4\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{5\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x^3/(- x^3 - 1)^(1/2),x)
```

output
```
- (2*x*(- x^3 - 1)^(1/2))/5 - (4*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((
x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(5*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{-1-x^3}}\,dx = \frac{2i\left(-\sqrt{x^3+1}\,x+\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)}{5}$$

input
```
int(x^3/(-x^3-1)^(1/2),x)
```

output
```
(2*i*( - sqrt(x**3 + 1)*x + int(sqrt(x**3 + 1)/(x**3 + 1),x)))/5
```

3.296 $\int \frac{1}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 112

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*Elli
pticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1
/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = \frac{x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[1/Sqrt[-1 - x^3],x]
```

output
```
(x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{760\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{760}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[1/Sqrt[-1 - x^3],x]
```

output
```
(2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4
)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.14

method	result	size
meijerg	$-ix \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}\right], \left[\frac{4}{3}\right], -x^3\right)$	16
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}$	107
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}$	107

input
```
int(1/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*x*hypergeom([1/3,1/2],[4/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = -2i\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(0, -4, x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(-x**3-1)**(1/2),x)
```

output
```
-I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4
/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(1/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(1/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(1/(- x^3 - 1)^(1/2),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1-x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^3+1}dx\right)i$$

input
```
int(1/(-x^3-1)^(1/2),x)
```

output
```
- int(sqrt(x**3 + 1)/(x**3 + 1),x)*i
```

3.297 $\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 133

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{2x^2} - \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
1/2*(-x^3-1)^(1/2)/x^2-1/6*(1/2*6^(1/2)-1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x
-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^
(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},-x^3\right)}{2x^2\sqrt{-1-x^3}}$$

input
```
Integrate[1/(x^3*Sqrt[-1 - x^3]),x]
```

output
```
-1/2*(Sqrt[1 + x^3]*Hypergeometric2F1[-2/3, 1/2, 1/3, -x^3])/(x^2*Sqrt[-1
- x^3])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {847, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{-x^3-1}}{2x^2}-\frac{1}{4}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{-x^3-1}}{2x^2}-\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[1/(x^3*Sqrt[-1 - x^3]),x]
```

output
```
Sqrt[-1 - x^3]/(2*x^2) - (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
- Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 -
x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.14

method	result	si
meijerg	$\frac{i\,\text{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{2}\right],\left[\frac{1}{3}\right],-x^3\right)}{2x^2}$	18
default	$\frac{\sqrt{-x^3-1}}{2x^2}+\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^3-1}}$	12
elliptic	$\frac{\sqrt{-x^3-1}}{2x^2}+\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^3-1}}$	12
risch	$-\frac{x^3+1}{2x^2\sqrt{-x^3-1}}+\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{6\sqrt{-x^3-1}}$	12

input
```
int(1/x^3/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/x^2*hypergeom([-2/3,1/2],[1/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = \frac{i\,x^2 \text{weierstrassPInverse}(0,-4,x)+\sqrt{-x^3-1}}{2\,x^2}$$

input
```
integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(I*x^2*weierstrassPInverse(0, -4, x) + sqrt(-x^3 - 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = -\frac{i\Gamma\left(-\frac{2}{3}\right)\,{}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{1}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(-x**3-1)**(1/2),x)
```

output
```
-I*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), x**3*exp_polar(I*pi))/(3*x**2*ga
mma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^3}\,dx$$

input
```
integrate(1/x^3/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{2\,x^2} - \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^3*(- x^3 - 1)^(1/2)),x)
```

output
```
(- x^3 - 1)^(1/2)/(2*x^2) - (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt{-1-x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^6+x^3}dx\right)i$$

input
```
int(1/x^3/(-x^3-1)^(1/2),x)
```

output
```
 - int(sqrt(x**3 + 1)/(x**6 + x**3),x)*i
```

3.298 $\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 151

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-1-x^3}}{5x^5}-\frac{7\sqrt{-1-x^3}}{20x^2} + \frac{7\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{20\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
1/5*(-x^3-1)^(1/2)/x^5-7/20*(-x^3-1)^(1/2)/x^2+7/60*(1/2*6^(1/2)-1/2*2^(1/
2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3
^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},-x^3\right)}{5x^5\sqrt{-1-x^3}}$$

input
```
Integrate[1/(x^6*Sqrt[-1 - x^3]),x]
```

output
```
-1/5*(Sqrt[1 + x^3]*Hypergeometric2F1[-5/3, 1/2, -2/3, -x^3])/(x^5*Sqrt[-1
 - x^3])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 847, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{-x^3-1}}{5x^5}-\frac{7}{10}\int \frac{1}{x^3\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{\sqrt{-x^3-1}}{5x^5}-\frac{7}{10}\left(\frac{\sqrt{-x^3-1}}{2x^2}-\frac{1}{4}\int \frac{1}{\sqrt{-x^3-1}}dx\right)$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{-x^3-1}}{5x^5} - \frac{7}{10}\left(\frac{\sqrt{-x^3-1}}{2x^2} - \frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}\right)$$

input
```
Int[1/(x^6*Sqrt[-1 - x^3]),x]
```

output
```
Sqrt[-1 - x^3]/(5*x^5) - (7*(Sqrt[-1 - x^3]/(2*x^2) - (Sqrt[2 - Sqrt[3]]*(
1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[
3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(2*3^(1/4)*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])))/10
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.12

method	result
meijerg	$\frac{i\,\text{hypergeom}\left(\left[-\frac{5}{3},\frac{1}{2}\right],\left[-\frac{2}{3}\right],-x^{3}\right)}{5x^{5}}$
risch	$\frac{7x^{6}+3x^{3}-4}{20x^{5}\sqrt{-x^{3}-1}}-\frac{7i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{60\sqrt{-x^{3}-1}}$
default	$\frac{\sqrt{-x^{3}-1}}{5x^{5}}-\frac{7\sqrt{-x^{3}-1}}{20x^{2}}-\frac{7i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\cdots}}\right.}{60\sqrt{-x^{3}-1}}$
elliptic	$\frac{\sqrt{-x^{3}-1}}{5x^{5}}-\frac{7\sqrt{-x^{3}-1}}{20x^{2}}-\frac{7i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\cdots}}\right.}{60\sqrt{-x^{3}-1}}$

input
```
int(1/x^6/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*I/x^5*hypergeom([-5/3,1/2],[-2/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^{6}\sqrt{-1-x^{3}}}\,dx=\frac{-7i\,x^{5}\text{weierstrassPInverse}(0,-4,x)-\left(7\,x^{3}-4\right)\sqrt{-x^{3}-1}}{20\,x^{5}}$$

input
```
integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/20*(-7*I*x^5*weierstrassPInverse(0, -4, x) - (7*x^3 - 4)*sqrt(-x^3 - 1))
/x^5
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = -\frac{i\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{5}{3},\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| x^3e^{i\pi}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(-x**3-1)**(1/2),x)
```

output
```
-I*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), x**3*exp_polar(I*pi))/(3*x**5*g
amma(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{5\,x^5} - \frac{7\,\sqrt{-x^3-1}}{20\,x^2} + \frac{7\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{20\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/(x^6*(- x^3 - 1)^(1/2)),x)
```

output
```
(- x^3 - 1)^(1/2)/(5*x^5) - (7*(- x^3 - 1)^(1/2))/(20*x^2) + (7*((3^(1/2)*
1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(20*(-
x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{-1-x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^9+x^6}dx\right) i$$

input
```
int(1/x^6/(-x^3-1)^(1/2),x)
```

output
```
 - int(sqrt(x**3 + 1)/(x**9 + x**6),x)*i
```

3.299 $\int \frac{x^7}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 282

$$\begin{aligned}\int \frac{x^7}{\sqrt{-1-x^3}}\,dx &= \frac{20}{91}x^2\sqrt{-1-x^3}-\frac{2}{13}x^5\sqrt{-1-x^3}-\frac{80\sqrt{-1-x^3}}{91\left(1-\sqrt{3}+x\right)}\\ &+\frac{40\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{91\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\\ &-\frac{80\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{91\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

output
```
20/91*x^2*(-x^3-1)^(1/2)-2/13*x^5*(-x^3-1)^(1/2)-80*(-x^3-1)^(1/2)/(91-91*
3^(1/2)+91*x)+40/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+
x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))/(
-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)-80/273*2^(1/2)*(1+x)*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1
/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.20

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = \frac{2x^2\left(-10-3x^3+7x^6+10\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{91\sqrt{-1-x^3}}$$

input
```
Integrate[x^7/Sqrt[-1 - x^3],x]
```

output
```
(2*x^2*(-10 - 3*x^3 + 7*x^6 + 10*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 2/3,
 5/3, -x^3]))/(91*Sqrt[-1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {843, 843, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \; 843$$

$$-\frac{10}{13}\int \frac{x^4}{\sqrt{-x^3-1}}dx-\frac{2}{13}\sqrt{-x^3-1}x^5$$

$$\Big\downarrow \; 843$$

$$-\frac{10}{13}\left(-\frac{4}{7}\int \frac{x}{\sqrt{-x^3-1}}dx-\frac{2}{7}\sqrt{-x^3-1}x^2\right)-\frac{2}{13}\sqrt{-x^3-1}x^5$$

$$\Big\downarrow \; 833$$

$$-\frac{10}{13}\left(-\frac{4}{7}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx\right)-\frac{2}{7}\sqrt{-x^3-1}x^2\right)-\frac{2}{13}\sqrt{-x^3-1}x^5$$

$\Big\downarrow$ 760

$$-\frac{10}{13}\left(-\frac{4}{7}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{\right.}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}\right.\right.$$

$$\frac{2}{13}\sqrt{-x^3-1}x^5$$

$\Big\downarrow$ 2418

$$-\frac{10}{13}\left(-\frac{4}{7}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}}{\ldots}\right.\right.$$

$$\frac{2}{13}\sqrt{-x^3-1}x^5$$

input
```
Int[x^7/Sqrt[-1 - x^3],x]
```

output
```
(-2*x^5*Sqrt[-1 - x^3])/13 - (10*((-2*x^2*Sqrt[-1 - x^3])/7 - (4*((-2*Sqrt
[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - S
qrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[
-1 - x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + x)*Sqrt[(1 - x + x^2)
/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1
- x^3])))/7))/13
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.06

method	result
meijerg	$-\frac{i x^{8} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{8}{3}\right],\left[\frac{11}{3}\right],-x^{3}\right)}{8}$
risch	$\frac{2 x^{2} \left(7 x^{3}-10\right) \left(x^{3}+1\right)}{91 \sqrt{-x^{3}-1}}-\frac{80 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{}}}{3}\right.\right.}{273 \sqrt{-x^{3}-1}}$
default	$-\frac{2 x^{5} \sqrt{-x^{3}-1}}{13}+\frac{20 x^{2} \sqrt{-x^{3}-1}}{91}-\frac{80 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\frac{\sqrt{3}\, \sqrt{i (}}{}\right.\right.}{273 \sqrt{-x^{3}-1}}$
elliptic	$-\frac{2 x^{5} \sqrt{-x^{3}-1}}{13}+\frac{20 x^{2} \sqrt{-x^{3}-1}}{91}-\frac{80 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i \sqrt{3}}{2}\right) \operatorname{EllipticE}\left(\frac{\sqrt{3}\, \sqrt{i (}}{}\right.\right.}{273 \sqrt{-x^{3}-1}}$

input
```
int(x^7/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*x^8*hypergeom([1/2,8/3],[11/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = -\frac{2}{91}\left(7\,x^5-10\,x^2\right)\sqrt{-x^3-1} + \frac{80}{91} i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x^7/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/91*(7*x^5 - 10*x^2)*sqrt(-x^3 - 1) + 80/91*I*weierstrassZeta(0, -4, wei
erstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = -\frac{i x^8 \Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(-x**3-1)**(1/2),x)
```

output
```
-I*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), x**3*exp_polar(I*pi))/(3*gam
ma(11/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = \int \frac{x^7}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^7/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^7/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = \int \frac{x^7}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^7/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^7/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = \frac{20\,x^2\,\sqrt{-x^3-1}}{91} - \frac{2\,x^5\,\sqrt{-x^3-1}}{13} - \frac{80\left(\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\right)}{91\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}\right)}}$$

input
```
int(x^7/(- x^3 - 1)^(1/2),x)
```

output
```
(20*x^2*(- x^3 - 1)^(1/2))/91 - (2*x^5*(- x^3 - 1)^(1/2))/13 - (80*(((3^(1
/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -
((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*el
lipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
)/(91*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt{-1-x^3}}\,dx = \frac{2i\left(-7\sqrt{x^3+1}\,x^5+10\sqrt{x^3+1}\,x^2-20\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)\right)}{91}$$

input
```
int(x^7/(-x^3-1)^(1/2),x)
```

output
```
(2*i*( - 7*sqrt(x**3 + 1)*x**5 + 10*sqrt(x**3 + 1)*x**2 - 20*int((sqrt(x**
3 + 1)*x)/(x**3 + 1),x)))/91
```

3.300 $\int \frac{x^4}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 264

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = -\frac{2}{7}x^2\sqrt{-1-x^3}+\frac{8\sqrt{-1-x^3}}{7\left(1-\sqrt{3}+x\right)} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{7\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{8\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{7\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2/7*x^2*(-x^3-1)^(1/2)+8*(-x^3-1)^(1/2)/(7-7*3^(1/2)+7*x)-4/7*3^(1/4)*(1/
2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((
1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(
-x^3-1)^(1/2)+8/21*2^(1/2)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*Ellipti
cF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2)
)^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.19

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = \frac{2x^2\left(1+x^3-\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)\right)}{7\sqrt{-1-x^3}}$$

input
```
Integrate[x^4/Sqrt[-1 - x^3],x]
```

output
```
(2*x^2*(1 + x^3 - Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3]))/(
7*Sqrt[-1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {843, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{843}$$

$$-\frac{4}{7}\int \frac{x}{\sqrt{-x^3-1}}dx-\frac{2}{7}\sqrt{-x^3-1}x^2$$

$$\Big\downarrow \text{833}$$

$$-\frac{4}{7}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx\right)-\frac{2}{7}\sqrt{-x^3-1}x^2$$

$$\Big\downarrow \text{760}$$

$$-\frac{4}{7}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}\right)-$$
$$\frac{2}{7}\sqrt{-x^3-1}x^2$$

$\downarrow$ 2418

$$-\frac{4}{7}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1}{}\right.$$
$$\frac{2}{7}\sqrt{-x^3-1}x^2$$

input
```
Int[x^4/Sqrt[-1 - x^3],x]
```

output
```
(-2*x^2*Sqrt[-1 - x^3])/7 - (4*((-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sq
rt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (2*Sqrt[2 - Sqrt[3]]*
(1 + Sqrt[3])*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[Ar
cSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[
-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]))/7
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

method	result
meijerg	$-\frac{i x^5 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{3}\right],\left[\frac{8}{3}\right],-x^3\right)}{5}$
default	$-\frac{2x^2\sqrt{-x^3-1}}{7}+\frac{8i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{3}{2}\cdots}\right)\cdots\right)}{21\sqrt{-x^3-1}}$
elliptic	$-\frac{2x^2\sqrt{-x^3-1}}{7}+\frac{8i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{3}{2}\cdots}\right)\cdots\right)}{21\sqrt{-x^3-1}}$
risch	$\frac{2x^2\left(x^3+1\right)}{7\sqrt{-x^3-1}}+\frac{8i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{\frac{i\sqrt{3}}{\cdots}}{\frac{3}{2}+\frac{i\sqrt{\cdots}}{2}}}\right)\cdots\right)}{21\sqrt{-x^3-1}}$

input
```
int(x^4/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*I*x^5*hypergeom([1/2,5/3],[8/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = -\frac{2}{7}\sqrt{-x^3-1}x^2 - \frac{8}{7}i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x^4/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/7*sqrt(-x^3 - 1)*x^2 - 8/7*I*weierstrassZeta(0, -4, weierstrassPInverse
(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = -\frac{i x^5 \Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| x^3 e^{i\pi}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(-x**3-1)**(1/2),x)
```

output
```
-I*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), x**3*exp_polar(I*pi))/(3*gamm
a(8/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = \int \frac{x^4}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^4/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = \int \frac{x^4}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x^4/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = -\frac{2\,x^2\,\sqrt{-x^3-1}}{7} + \frac{8\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\cdots\right.}{7\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}\cdots\right.}}$$

input
```
int(x^4/(- x^3 - 1)^(1/2),x)
```

output
```
(8*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2
 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1
/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2))/(7*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
) - (2*x^2*(- x^3 - 1)^(1/2))/7
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{-1-x^3}}\,dx = \frac{2i\left(-\sqrt{x^3+1}\,x^2+2\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)\right)}{7}$$

input
```
int(x^4/(-x^3-1)^(1/2),x)
```

output
```
(2*i*( - sqrt(x**3 + 1)*x**2 + 2*int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)))/7
```

3.301 $\int \frac{x}{\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 239

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = -\frac{2\sqrt{-1-x^3}}{1-\sqrt{3}+x} + \frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{2\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2*(-x^3-1)^(1/2)/(1+x-3^(1/2))+3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((
x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-
I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)-2/3*2^(1/2)*(1+x)
*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2
*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = \frac{x^2\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)}{2\sqrt{-1-x^3}}$$

input
```
Integrate[x/Sqrt[-1 - x^3],x]
```

output
```
(x^2*Sqrt[1 + x^3]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/(2*Sqrt[-1 - x^
3])
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{833}$$

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2418}$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\sqrt{-x^3-1}}{x-\sqrt{3}+1}$$

input
```
Int[x/Sqrt[-1 - x^3],x]
```

output
```
(-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x
)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[-1 - x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + x)*Sqrt[(1 -
x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt
[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.08

method	result
meijerg	$-\frac{i x^{2} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^{3}\right)}{2}$
default	$-\frac{2 i \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)-\text{Ellipticl}\right.}{3\sqrt{-x^{3}-1}}$
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, \sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\, \left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\operatorname{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)-\text{Ellipticl}\right.}{3\sqrt{-x^{3}-1}}$

input
```
int(x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*x^2*hypergeom([1/2,2/3],[5/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.04

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = 2i\,\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))$$

input
```
integrate(x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2*I*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.13

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = -\frac{ix^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(-x**3-1)**(1/2),x)
```

output
```
-I*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), x**3*exp_polar(I*pi))/(3*gamm
a(5/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(-x^3 - 1), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-x^3 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.96

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = -\frac{2\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{F}\bigg(\operatorname{asin}\bigg(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\bigg)\bigg|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\bigg)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\mathrm{E}\bigg(\operatorname{asin}\bigg(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\bigg)\bigg|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\bigg)\cdots}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}\cdots\right.}}$$

input
```
int(x/(- x^3 - 1)^(1/2),x)
```

output
```
-(2*(((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/
2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2
)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(
1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 +
3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{x}{\sqrt{-1-x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}\,x}{x^3+1}dx\right)i$$

input
```
int(x/(-x^3-1)^(1/2),x)
```

output
```
- int((sqrt(x**3 + 1)*x)/(x**3 + 1),x)*i
```

3.302 $\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 257

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx &= \frac{\sqrt{-1-x^3}}{x}-\frac{\sqrt{-1-x^3}}{1-\sqrt{3}+x} \\
&\quad+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{2\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&\quad-\frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

output
```
(-x^3-1)^(1/2)/x-(-x^3-1)^(1/2)/(1+x-3^(1/2))+1/2*3^(1/4)*(1/2*6^(1/2)+1/2
*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticE((1+x+3^(1/2))/
(1+x-3^(1/2)),2*I-I*3^(1/2))/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
-1/3*2^(1/2)*(1+x)*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*EllipticF((1+x+3^(1/2
))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x
^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-x^3\right)}{x\sqrt{-1-x^3}}$$

input
```
Integrate[1/(x^2*Sqrt[-1 - x^3]),x]
```

output
```
-((Sqrt[1 + x^3]*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])/(x*Sqrt[-1 - x^3
]))
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {847, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{2}\int \frac{x}{\sqrt{-x^3-1}}dx+\frac{\sqrt{-x^3-1}}{x}$$

$$\Big\downarrow \text{833}$$

$$\frac{1}{2}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx\right)+\frac{\sqrt{-x^3-1}}{x}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{2}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}\right)+\frac{\sqrt{-x^3-1}}{x}$$

$$\Big\downarrow \text{2418}$$

$$\frac{1}{2}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)}{}\right.$$
$$\frac{\sqrt{-x^3-1}}{x}$$

input
```
Int[1/(x^2*Sqrt[-1 - x^3]),x]
```

output
```
Sqrt[-1 - x^3]/x + ((-2*Sqrt[-1 - x^3])/(1 - Sqrt[3] + x) + (3^(1/4)*Sqrt[
2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticE[Arc
Sin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(Sqrt[-((1 + x)
/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3]
)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sq
rt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1
 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]))/2
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.07

method	result
meijerg	$\frac{i\,\text{hypergeom}\left(\left[-\frac{1}{3},\frac{1}{2}\right],\left[\frac{2}{3}\right],-x^{3}\right)}{x}$
default	$\frac{\sqrt{-x^{3}-1}}{x}-\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)\right.}{3\sqrt{-x^{3}-1}}$
elliptic	$\frac{\sqrt{-x^{3}-1}}{x}-\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)\right.}{3\sqrt{-x^{3}-1}}$
risch	$-\frac{x^{3}+1}{x\sqrt{-x^{3}-1}}-\frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)\right.}{3\sqrt{-x^{3}-1}}$

input
```
int(1/(-x^3-1)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
I/x*hypergeom([-1/3,1/2],[2/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = \frac{i\,x\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))+\sqrt{-x^3-1}}{x}$$

input
```
integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
(I*x*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) + sqrt(-x^3 - 1
))/x
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = -\frac{i\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| x^3e^{i\pi}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(-x**3-1)**(1/2),x)
```

output
```
-I*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), x**3*exp_polar(I*pi))/(3*x*gamma
(2/3))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{x}$$

$$-\frac{\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\right.}}$$

input
```
int(1/(x^2*(- x^3 - 1)^(1/2)),x)
```

output
```
(- x^3 - 1)^(1/2)/x - ((((3^(1/2)*1i)/2 - 1/2)*ellipticF(asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)) - ((3^(1/2)*1i)/2 - 3/2)*ellipticE(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2
 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2)
)^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{-1-x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^5+x^2}dx\right)i$$

input
```
int(1/x^2/(-x^3-1)^(1/2),x)
```

output
```
- int(sqrt(x**3 + 1)/(x**5 + x**2),x)*i
```

3.303 $\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 282

$$\begin{aligned}
&\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx \\
&= \frac{\sqrt{-1-x^3}}{4x^4}-\frac{5\sqrt{-1-x^3}}{8x}+\frac{5\sqrt{-1-x^3}}{8\left(1-\sqrt{3}+x\right)} \\
&-\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{16\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\
&+\frac{5(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}
\end{aligned}$$

output
```
1/4*(-x^3-1)^(1/2)/x^4-5/8*(-x^3-1)^(1/2)/x+5*(-x^3-1)^(1/2)/(8-8*3^(1/2)+
8*x)-5/16*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(1+x)*((x^2-x+1)/(1+x-3^(1/2))
^2)^(1/2)*EllipticE((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))/(-(1+x)/(1+
x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)+5/24*2^(1/2)*(1+x)*((x^2-x+1)/(1+x-3^(1
/2))^2)^(1/2)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*3^(3/4)
/(-(1+x)/(1+x-3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},-x^3\right)}{4x^4\sqrt{-1-x^3}}$$

input
```
Integrate[1/(x^5*Sqrt[-1 - x^3]),x]
```

output
```
-1/4*(Sqrt[1 + x^3]*Hypergeometric2F1[-4/3, 1/2, -1/3, -x^3])/(x^4*Sqrt[-1
 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {847, 847, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 847}$$

$$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5}{8}\int \frac{1}{x^2\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 847}$$

$$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5}{8}\left(\frac{1}{2}\int \frac{x}{\sqrt{-x^3-1}}dx+\frac{\sqrt{-x^3-1}}{x}\right)$$

$$\Big\downarrow \text{ 833}$$

$$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5}{8}\left(\frac{1}{2}\left(\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx\right)+\frac{\sqrt{-x^3-1}}{x}\right)$$

$$\Big\downarrow \; 760$$

$$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5}{8}\left(\frac{1}{2}\left(\int\frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}\right)\right)$$

$$\Big\downarrow \; 2418$$

$$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5}{8}\left(\frac{1}{2}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+\right.\right.$$

input
```
Int[1/(x^5*Sqrt[-1 - x^3]),x]
```

output
```
Sqrt[-1 - x^3]/(4*x^4) - (5*(Sqrt[-1 - x^3]/x + ((-2*Sqrt[-1 - x^3])/(1 -
Sqrt[3] + x) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7
+ 4*Sqrt[3]])/(Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (2*S
qrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x
)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]
])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]))/2))/8
```

Defintions of rubi rules used

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.06

method	result
meijerg	$\frac{i\,\mathrm{hypergeom}\left(\left[-\frac{4}{3},\frac{1}{2}\right],\left[-\frac{1}{3}\right],-x^3\right)}{4x^4}$
risch	$\frac{5x^6+3x^3-2}{8x^4\sqrt{-x^3-1}}+\frac{5i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{}}{\frac{3}{2}+i}}\right.\right.}{24\sqrt{-x^3-1}}$
default	$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5\sqrt{-x^3-1}}{8x}+\frac{5i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{3}\right.\right.}{24\sqrt{-x^3-1}}$
elliptic	$\frac{\sqrt{-x^3-1}}{4x^4}-\frac{5\sqrt{-x^3-1}}{8x}+\frac{5i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\mathrm{EllipticE}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}}{3}\right.\right.}{24\sqrt{-x^3-1}}$

input
```
int(1/x^5/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/x^4*hypergeom([-4/3,1/2],[-1/3],-x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = \frac{-5i\,x^4\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))-(5\,x^3-2)\sqrt{-x^3-1}}{8\,x^4}$$

input
```
integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(-5*I*x^4*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) - (5*x
^3 - 2)*sqrt(-x^3 - 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = -\frac{i\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3},\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| x^3e^{i\pi}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(-x**3-1)**(1/2),x)
```

output
```
-I*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), x**3*exp_polar(I*pi))/(3*x**4*g
amma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}x^5}\,dx$$

input
```
integrate(1/x^5/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = \frac{\sqrt{-x^3-1}}{4\,x^4} - \frac{5\,\sqrt{-x^3-1}}{8\,x} + \frac{5\left(\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\left(-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right.}{8\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}\right.}}$$

input
```
int(1/(x^5*(- x^3 - 1)^(1/2)),x)
```

output
```
(- x^3 - 1)^(1/2)/(4*x^4) - (5*(- x^3 - 1)^(1/2))/(8*x) + (5*(((3^(1/2)*1i
)/2 - 1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ((3^(1/2)*1i)/2 - 3/2)*elliptic
E(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(8*(
- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt{-1-x^3}}\,dx = -\left(\int \frac{\sqrt{x^3+1}}{x^8+x^5}dx\right)i$$

input
```
int(1/x^5/(-x^3-1)^(1/2),x)
```

output
```
- int(sqrt(x**3 + 1)/(x**8 + x**5),x)*i
```

3.304 $\int (cx)^{7/2}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 102

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx = \frac{ac^2(cx)^{3/2}\sqrt{a+bx^3}}{12b} + \frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c} - \frac{a^2c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}$$

output
```
1/12*a*c^2*(c*x)^(3/2)*(b*x^3+a)^(1/2)/b+1/6*(c*x)^(9/2)*(b*x^3+a)^(1/2)/c
-1/12*a^2*c^(7/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a)^(1/2))/b^(
3/2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.83

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx = \frac{(cx)^{7/2}\sqrt{a+bx^3}(a+2bx^3)}{12bx^2} - \frac{a^2(cx)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{12b^{3/2}x^{7/2}}$$

input
```
Integrate[(c*x)^(7/2)*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(7/2)*Sqrt[a + b*x^3]*(a + 2*b*x^3))/(12*b*x^2) - (a^2*(c*x)^(7/2)*
Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(12*b^(3/2)*x^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {811, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{4}a\int \frac{(cx)^{7/2}}{\sqrt{bx^3+a}}dx+\frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{4}a\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{2b}\right)+\frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c}$$

$$\Big\downarrow \text{851}$$

$$\frac{1}{4}a\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{b}\right)+\frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}a\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3b}\right)+\frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}a\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3b}\right)+\frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}a\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3b^{3/2}}\right)+\frac{(cx)^{9/2}\sqrt{a+bx^3}}{6c}$$

input
```
Int[(c*x)^(7/2)*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(9/2)*Sqrt[a + b*x^3])/(6*c) + (a*((c^2*(c*x)^(3/2)*Sqrt[a + b*x^3]
)/(3*b) - (a*c^(7/2)*ArcTanh[(Sqrt[b]*(c*x)^(3/2))/(c^(3/2)*Sqrt[a + (b*x)
/c^2])])/(3*b^(3/2))))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{x^{2} \left(2 b \,x^{3}+a \right) \sqrt{b \,x^{3}+a}\, c^{4}}{12 b \sqrt{c x}}-\frac{a^{2} \operatorname{arctanh}\left(\frac{\sqrt{c x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right) c^{4} \sqrt{c x \left(b \,x^{3}+a \right)}}{12 b \sqrt{b c}\, \sqrt{c x}\, \sqrt{b \,x^{3}+a}}$	99
default	$-\frac{c^{3} \sqrt{c x}\, \sqrt{b \,x^{3}+a}\, \left(-2 \sqrt{c x \left(b \,x^{3}+a \right)}\, \sqrt{b c}\, b \,x^{4}+\operatorname{arctanh}\left(\frac{\sqrt{c x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right) c \,a^{2}-a x \sqrt{c x \left(b \,x^{3}+a \right)}\, \sqrt{b c}\right)}{12 \sqrt{c x \left(b \,x^{3}+a \right)}\, \sqrt{b c}\, b}$	112
elliptic	Expression too large to display	1063

input
```
int((c*x)^(7/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(2*b*x^3+a)*(b*x^3+a)^(1/2)/b*c^4/(c*x)^(1/2)-1/12/b*a^2/(b*c)^(1
/2)*arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b*c)^(1/2))*c^4*(c*x*(b*x^3+a))^(1/
2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.03

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx = \left[\frac{a^2c^3\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c+4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{cx}\sqrt{\frac{c}{b}}\right)+4\,(\right.$$

$$48\,b$$

input
```
integrate((c*x)^(7/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(a^2*c^3*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x^3 - a^2*c + 4*(2*b^2
*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)) + 4*(2*b*c^3*x^4 + a*c^
3*x)*sqrt(b*x^3 + a)*sqrt(c*x))/b, 1/24*(a^2*c^3*sqrt(-c/b)*arctan(2*sqrt(
b*x^3 + a)*sqrt(c*x)*b*x*sqrt(-c/b)/(2*b*c*x^3 + a*c)) + 2*(2*b*c^3*x^4 +
a*c^3*x)*sqrt(b*x^3 + a)*sqrt(c*x))/b]
```

Sympy [A] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx = \frac{a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{3}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}}+\frac{\sqrt{a}c^{\frac{7}{2}}x^{\frac{9}{2}}}{4\sqrt{1+\frac{bx^3}{a}}}-\frac{a^2c^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{12b^{\frac{3}{2}}}+\frac{bc^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((c*x)**(7/2)*(b*x**3+a)**(1/2),x)
```

output
```
a**(3/2)*c**(7/2)*x**(3/2)/(12*b*sqrt(1 + b*x**3/a)) + sqrt(a)*c**(7/2)*x*
*(9/2)/(4*sqrt(1 + b*x**3/a)) - a**2*c**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(
a))/(12*b**(3/2)) + b*c**(7/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx=\frac{a^2c^4\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{12\sqrt{bc}b}+\frac{\sqrt{bc^4x^3+ac^4}\left(2c^3x^3+\frac{ac^3}{b}\right)\sqrt{cx}x|c|^2}{12c^4}$$

input
```
integrate((c*x)^(7/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*a^2*c^4*log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))/
(sqrt(b*c)*b) + 1/12*sqrt(b*c^4*x^3 + a*c^4)*(2*c^3*x^3 + a*c^3/b)*sqrt(c*
x)*x*abs(c)^2/c^4
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx=\int (c\,x)^{7/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((c*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int (cx)^{7/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{c}\,c^3\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,abx+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^4+\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)\right.}{24b^2}$$

input
```
int((c*x)^(7/2)*(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**3*(2*sqrt(x)*sqrt(a + b*x**3)*a*b*x + 4*sqrt(x)*sqrt(a + b*x**
3)*b**2*x**4 + sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**2 - sq
rt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**2))/(24*b**2)
```

3.305 $\int \sqrt{cx}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx=\frac{(cx)^{3/2}\sqrt{a+bx^3}}{3c}+\frac{a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}}$$

output
```
1/3*(c*x)^(3/2)*(b*x^3+a)^(1/2)/c+1/3*a*c^(1/2)*arctanh(b^(1/2)*(c*x)^(3/2
)/c^(3/2)/(b*x^3+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx=\frac{\sqrt{cx}\left(x^{3/2}\sqrt{a+bx^3}+\frac{a\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{\sqrt{b}}\right)}{3\sqrt{x}}$$

input
```
Integrate[Sqrt[c*x]*Sqrt[a + b*x^3],x]
```

output
```
(Sqrt[c*x]*(x^(3/2)*Sqrt[a + b*x^3] + (a*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*
x^3]])/Sqrt[b]))/(3*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {811, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{2}a\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx+\frac{(cx)^{3/2}\sqrt{a+bx^3}}{3c}$$

$$\Big\downarrow \text{851}$$

$$\frac{a\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{c}+\frac{(cx)^{3/2}\sqrt{a+bx^3}}{3c}$$

$$\Big\downarrow \text{807}$$

$$\frac{a\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3c}+\frac{(cx)^{3/2}\sqrt{a+bx^3}}{3c}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3c}+\frac{(cx)^{3/2}\sqrt{a+bx^3}}{3c}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3\sqrt{b}}+\frac{(cx)^{3/2}\sqrt{a+bx^3}}{3c}$$

input
```
Int[Sqrt[c*x]*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(3/2)*Sqrt[a + b*x^3])/(3*c) + (a*Sqrt[c]*ArcTanh[(Sqrt[b]*(c*x)^(3
/2))/(c^(3/2)*Sqrt[a + (b*x)/c^2])])/(3*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\sqrt{cx}\,\sqrt{b\,x^{3}+a}\,\left(\sqrt{cx\left(b\,x^{3}+a\right)}\,x\sqrt{bc}+\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)ac\right)}{3\sqrt{cx\left(b\,x^{3}+a\right)}\,\sqrt{bc}}$	79
risch	$\frac{x^{2}\sqrt{b\,x^{3}+a}\,c}{3\sqrt{cx}}+\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)c\sqrt{cx\left(b\,x^{3}+a\right)}}{3\sqrt{bc}\,\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	79
elliptic	Expression too large to display	1031

input
```
int((c*x)^(1/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(c*x)^(1/2)*(b*x^3+a)^(1/2)/(c*x*(b*x^3+a))^(1/2)*((c*x*(b*x^3+a))^(1/
2)*x*(b*c)^(1/2)+arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b*c)^(1/2))*a*c)/(b*c)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.24

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx = \left[\frac{1}{12}\,a\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c-4\,\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{cx}\sqrt{\frac{c}{b}}\right)+\frac{1}{3}\,\sqrt{bx^3+a}\sqrt{cx}x,\ -\frac{1}{6}\,a\sqrt{-\frac{c}{b}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{cx}bx\sqrt{-\frac{c}{b}}}{2\,bcx^3+ac}\right)+\frac{1}{3}\,\sqrt{bx^3+a}\sqrt{cx}x\right]$$

input
```
integrate((c*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*a*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x^3 - a^2*c - 4*(2*b^2*x^4 +
a*b*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)) + 1/3*sqrt(b*x^3 + a)*sqrt(c*x
)*x, -1/6*a*sqrt(-c/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(c*x)*b*x*sqrt(-c/b)/(
2*b*c*x^3 + a*c)) + 1/3*sqrt(b*x^3 + a)*sqrt(c*x)*x]
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3} + \frac{a\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}}$$

input
```
integrate((c*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*sqrt(c)*x**(3/2)*sqrt(1 + b*x**3/a)/3 + a*sqrt(c)*asinh(sqrt(b)*x*
*(3/2)/sqrt(a))/(3*sqrt(b))
```

Maxima [F]

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*sqrt(c*x), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx = -\frac{\left(\frac{ac^4 \log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{\sqrt{bc}} - \sqrt{bc^4x^3+ac^4}\sqrt{cx}cx\right)|c|^2}{3\,c^5}$$

input
```
integrate((c*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-1/3*(a*c^4*log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))/s
qrt(b*c) - sqrt(b*c^4*x^3 + a*c^4)*sqrt(c*x)*c*x)*abs(c)^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx = \int \sqrt{c\,x}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((c*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \sqrt{cx}\sqrt{a+bx^3}\,dx = \frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx-\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a+\sqrt{b}\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)a\right)}{6b}$$

input
```
int((c*x)^(1/2)*(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a + b*x**3)*b*x - sqrt(b)*log(sqrt(a + b*x**3) -
sqrt(x)*sqrt(b)*x)*a + sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a
))/(6*b)
```

3.306 $\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{a+bx^3}}{3c(cx)^{3/2}} + \frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{3c^{5/2}}$$

output
```
-2/3*(b*x^3+a)^(1/2)/c/(c*x)^(3/2)+2/3*b^(1/2)*arctanh(b^(1/2)*(c*x)^(3/2)
/c^(3/2)/(b*x^3+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = \frac{x\left(-2\sqrt{a+bx^3} + 2\sqrt{b}x^{3/2}\log\left(\sqrt{b}x^{3/2} + \sqrt{a+bx^3}\right)\right)}{3(cx)^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(5/2),x]
```

output
```
(x*(-2*Sqrt[a + b*x^3] + 2*Sqrt[b]*x^(3/2)*Log[Sqrt[b]*x^(3/2) + Sqrt[a +
b*x^3]]))/(3*(c*x)^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {809, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{c^3}-\frac{2\sqrt{a+bx^3}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2b\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{c^4}-\frac{2\sqrt{a+bx^3}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2b\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3c^4}-\frac{2\sqrt{a+bx^3}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2b\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3c^4}-\frac{2\sqrt{a+bx^3}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3c^{5/2}}-\frac{2\sqrt{a+bx^3}}{3c(cx)^{3/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(5/2),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(3*c*(c*x)^(3/2)) + (2*Sqrt[b]*ArcTanh[(Sqrt[b]*(c*x)
^(3/2))/(c^(3/2)*Sqrt[a + (b*x)/c^2])])/(3*c^(5/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{2\sqrt{b\,x^{3}+a}}{3x\,c^{2}\sqrt{cx}}+\frac{2b\,\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)\sqrt{cx(b\,x^{3}+a)}}{3\sqrt{bc}\,c^{2}\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	83
default	$\frac{2\sqrt{b\,x^{3}+a}\left(bc\,\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)x^{2}-\sqrt{cx(b\,x^{3}+a)}\,\sqrt{bc}\right)}{3x\,c^{2}\sqrt{cx}\,\sqrt{cx(b\,x^{3}+a)}\,\sqrt{bc}}$	88
elliptic	Expression too large to display	1034

input
```
int((b*x^3+a)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x^3+a)^(1/2)/x/c^2/(c*x)^(1/2)+2/3*b/(b*c)^(1/2)*arctanh((c*x*(b*x
^3+a))^(1/2)/x^2/(b*c)^(1/2))/c^2*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3
+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx=\left[\frac{cx^2\sqrt{\frac{b}{c}}\log\left(-8\,b^2x^6-8\,abx^3-4\,(2\,bx^4+ax)\sqrt{bx^3+a}\sqrt{cx}\sqrt{\frac{b}{c}}-a^2\right)-4\,\sqrt{bx^3+a}\sqrt{}}{6\,c^3x^2}\right.$$
$$\left.-\frac{cx^2\sqrt{-\frac{b}{c}}\arctan\left(\frac{2\,\sqrt{bx^3+a}\sqrt{cx}x\sqrt{-\frac{b}{c}}}{2\,bx^3+a}\right)+2\,\sqrt{bx^3+a}\sqrt{cx}}{3\,c^3x^2}\right]$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(c*x^2*sqrt(b/c)*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqrt(
b*x^3 + a)*sqrt(c*x)*sqrt(b/c) - a^2) - 4*sqrt(b*x^3 + a)*sqrt(c*x))/(c^3*
x^2), -1/3*(c*x^2*sqrt(-b/c)*arctan(2*sqrt(b*x^3 + a)*sqrt(c*x)*x*sqrt(-b/
c)/(2*b*x^3 + a)) + 2*sqrt(b*x^3 + a)*sqrt(c*x))/(c^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{a}}{3c^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}} + \frac{2\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3c^{\frac{5}{2}}} - \frac{2bx^{\frac{3}{2}}}{3\sqrt{a}c^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(5/2),x)
```

output
```
-2*sqrt(a)/(3*c**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*sqrt(b)*asinh(sqrt
(b)*x**(3/2)/sqrt(a))/(3*c**(5/2)) - 2*b*x**(3/2)/(3*sqrt(a)*c**(5/2)*sqrt
(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = -\frac{2\left(\left(\frac{b\arctan\left(\frac{\sqrt{bc+\frac{ac}{x^3}}}{\sqrt{-bc}}\right)}{\sqrt{-bc}}+\frac{\sqrt{bc+\frac{ac}{x^3}}}{c}\right)c-\frac{bc\arctan\left(\frac{\sqrt{bc}}{\sqrt{-bc}}\right)+\sqrt{bc}\sqrt{-bc}}{\sqrt{-bc}}\right)|c|^2}{3\,c^5}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3*((b*arctan(sqrt(b*c + a*c/x^3)/sqrt(-b*c))/sqrt(-b*c) + sqrt(b*c + a*
c/x^3)/c)*c - (b*c*arctan(sqrt(b*c)/sqrt(-b*c)) + sqrt(b*c)*sqrt(-b*c))/sq
rt(-b*c))*abs(c)^2/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{{(c\,x)}^{5/2}}\,dx$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{5/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)x+\sqrt{x}\,\sqrt{b}\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\right.}{3\sqrt{x}\,c^3x}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*log(sqrt(a + b*x**3) - s
qrt(x)*sqrt(b)*x)*x + sqrt(x)*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(
b)*x)*x))/(3*sqrt(x)*c**3*x)
```

3.307 $\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}} \, dx = -\frac{2(a+bx^3)^{3/2}}{9ac(cx)^{9/2}}$$

output
```
-2/9*(b*x^3+a)^(3/2)/a/c/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}} \, dx = -\frac{2x(a+bx^3)^{3/2}}{9a(cx)^{11/2}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(11/2),x]
```

output
```
(-2*x*(a + b*x^3)^(3/2))/(9*a*(c*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{9/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(11/2),x]
```

output
```
(-2*(a + b*x^3)^(3/2))/(9*a*c*(c*x)^(9/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{2x\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9a\left(cx\right)^{\frac{11}{2}}}$	21
orering	$-\frac{2x\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9a\left(cx\right)^{\frac{11}{2}}}$	21
default	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9x^{4}a\,c^{5}\sqrt{cx}}$	26
risch	$-\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}}{9x^{4}a\,c^{5}\sqrt{cx}}$	26
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{b c\,x^{4}+a c x}}{9c^{6}x^{5}}-\frac{2b\sqrt{b c\,x^{4}+a c x}}{9a\,c^{6}x^{2}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	75

input
```
int((b*x^3+a)^(1/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x*(b*x^3+a)^(3/2)/a/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx=-\frac{2\,(bx^3+a)^{\frac{3}{2}}\sqrt{cx}}{9\,ac^6x^5}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
-2/9*(b*x^3 + a)^(3/2)*sqrt(c*x)/(a*c^6*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 49.96 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9c^{\frac{11}{2}}x^3} - \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9ac^{\frac{11}{2}}}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(11/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*c**(11/2)*x**3) - 2*b**(3/2)*sqrt(a/(b*
x**3) + 1)/(9*a*c**(11/2))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2\left(b\sqrt{c}x^4 + a\sqrt{c}x\right)\sqrt{bx^3+a}}{9\,ac^6x^{\frac{11}{2}}}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
-2/9*(b*sqrt(c)*x^4 + a*sqrt(c)*x)*sqrt(b*x^3 + a)/(a*c^6*x^(11/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx = \frac{2\left(\frac{\sqrt{bc}bc^2}{a} - \frac{\left(bc+\frac{ac}{x^3}\right)^{\frac{3}{2}}c}{a}\right)|c|^2}{9\,c^{10}}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
2/9*(sqrt(b*c)*b*c^2/a - (b*c + a*c/x^3)^(3/2)*c/a)*abs(c)^2/c^10
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2\,{\left(b\,x^3+a\right)}^{3/2}}{9\,a\,c^5\,x^4\,\sqrt{c\,x}}$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(11/2),x)
```

output
```
-(2*(a + b*x^3)^(3/2))/(9*a*c^5*x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{11/2}}\,dx = -\frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(b\,x^3+a)}{9\sqrt{x}\,a\,c^6x^4}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(11/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a + b*x**3)*(a + b*x**3))/(9*sqrt(x)*a*c**6*x**4)
```

3.308 $\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = -\frac{2(a+bx^3)^{3/2}}{15ac(cx)^{15/2}} + \frac{4b(a+bx^3)^{3/2}}{45a^2c^4(cx)^{9/2}}$$

output
```
-2/15*(b*x^3+a)^(3/2)/a/c/(c*x)^(15/2)+4/45*b*(b*x^3+a)^(3/2)/a^2/c^4/(c*x
)^(9/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = -\frac{2x\sqrt{a+bx^3}(3a^2+abx^3-2b^2x^6)}{45a^2(cx)^{17/2}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(17/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*(3*a^2 + a*b*x^3 - 2*b^2*x^6))/(45*a^2*(c*x)^(17/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx$$
$$\Big\downarrow \text{805}$$
$$-\frac{2\int \frac{\left(bx^3+a\right)^{3/2}}{(cx)^{17/2}}dx}{3a}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{15/2}}$$
$$\Big\downarrow \text{796}$$
$$\frac{4\left(a+bx^3\right)^{5/2}}{45a^2c(cx)^{15/2}}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{15/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(17/2),x]
```

output
```
(-2*(a + b*x^3)^(3/2))/(9*a*c*(c*x)^(15/2)) + (4*(a + b*x^3)^(5/2))/(45*a^
2*c*(c*x)^(15/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2x\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-2b\,x^{3}+3a\right)}{45a^{2}\left(cx\right)^{\frac{17}{2}}}$	31
orering	$-\frac{2x\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(-2b\,x^{3}+3a\right)}{45a^{2}\left(cx\right)^{\frac{17}{2}}}$	31
default	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-2b^{2}x^{6}+ab\,x^{3}+3a^{2}\right)}{45x^{7}c^{8}\sqrt{cx}\,a^{2}}$	46
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-2b^{2}x^{6}+ab\,x^{3}+3a^{2}\right)}{45x^{7}c^{8}\sqrt{cx}\,a^{2}}$	46
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{15c^{9}x^{8}}-\frac{2b\sqrt{bc\,x^{4}+acx}}{45a\,c^{9}x^{5}}+\frac{4b^{2}\sqrt{bc\,x^{4}+acx}}{45a^{2}c^{9}x^{2}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	102

input
```
int((b*x^3+a)^(1/2)/(c*x)^(17/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(b*x^3+a)^(3/2)*(-2*b*x^3+3*a)/a^2/(c*x)^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = \frac{2\,(2\,b^2x^6-abx^3-3\,a^2)\sqrt{bx^3+a}\sqrt{cx}}{45\,a^2c^9x^8}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(17/2),x, algorithm="fricas")
```

output
```
2/45*(2*b^2*x^6 - a*b*x^3 - 3*a^2)*sqrt(b*x^3 + a)*sqrt(c*x)/(a^2*c^9*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{17}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(17/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(17/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = -\frac{2\left(\frac{2\sqrt{bc}b^2}{a^2c^6} - \frac{5\left(bc+\frac{ac}{x^3}\right)^{\frac{3}{2}}bc-3\left(bc+\frac{ac}{x^3}\right)^{\frac{5}{2}}}{a^2c^8}\right)|c|^2}{45\,c^5}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(17/2),x, algorithm="giac")
```

output
```
-2/45*(2*sqrt(b*c)*b^2/(a^2*c^6) - (5*(b*c + a*c/x^3)^(3/2)*b*c - 3*(b*c +
 a*c/x^3)^(5/2))/(a^2*c^8))*abs(c)^2/c^5
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2}{15\,c^8}+\frac{2\,b\,x^3}{45\,a\,c^8}-\frac{4\,b^2\,x^6}{45\,a^2\,c^8}\right)}{x^7\,\sqrt{c\,x}}$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(17/2),x)
```

output
```
-((a + b*x^3)^(1/2)*(2/(15*c^8) + (2*b*x^3)/(45*a*c^8) - (4*b^2*x^6)/(45*a
^2*c^8)))/(x^7*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{17/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(2b^2x^6-ab\,x^3-3a^2)}{45\sqrt{x}\,a^2c^9x^7}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(17/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - 3*a**2 - a*b*x**3 + 2*b**2*x**6))/(45*sqrt
(x)*a**2*c**9*x**7)
```

3.309 $\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = -\frac{2(a+bx^3)^{3/2}}{21ac(cx)^{21/2}} + \frac{8b(a+bx^3)^{3/2}}{105a^2c^4(cx)^{15/2}} - \frac{16b^2(a+bx^3)^{3/2}}{315a^3c^7(cx)^{9/2}}$$

output
```
-2/21*(b*x^3+a)^(3/2)/a/c/(c*x)^(21/2)+8/105*b*(b*x^3+a)^(3/2)/a^2/c^4/(c*
x)^(15/2)-16/315*b^2*(b*x^3+a)^(3/2)/a^3/c^7/(c*x)^(9/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = -\frac{2x\sqrt{a+bx^3}(15a^3+3a^2bx^3-4ab^2x^6+8b^3x^9)}{315a^3(cx)^{23/2}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(23/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*(15*a^3 + 3*a^2*b*x^3 - 4*a*b^2*x^6 + 8*b^3*x^9))/(3
15*a^3*(c*x)^(23/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{4\int \frac{\left(bx^3+a\right)^{3/2}}{(cx)^{23/2}}dx}{3a}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{21/2}}$$

$$\Big\downarrow \text{805}$$

$$-\frac{4\left(-\frac{2\int \frac{\left(bx^3+a\right)^{5/2}}{(cx)^{23/2}}dx}{5a}-\frac{2\left(a+bx^3\right)^{5/2}}{15ac(cx)^{21/2}}\right)}{3a}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{21/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{4\left(\frac{4\left(a+bx^3\right)^{7/2}}{105a^2c(cx)^{21/2}}-\frac{2\left(a+bx^3\right)^{5/2}}{15ac(cx)^{21/2}}\right)}{3a}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{21/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(23/2),x]
```

output
```
(-2*(a + b*x^3)^(3/2))/(9*a*c*(c*x)^(21/2)) - (4*((-2*(a + b*x^3)^(5/2))/(
15*a*c*(c*x)^(21/2)) + (4*(a + b*x^3)^(7/2))/(105*a^2*c*(c*x)^(21/2))))/(3
*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(8b^{2}x^{6}-12ab\,x^{3}+15a^{2}\right)}{315a^{3}\left(cx\right)^{\frac{23}{2}}}$	42
orering	$-\frac{2x\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(8b^{2}x^{6}-12ab\,x^{3}+15a^{2}\right)}{315a^{3}\left(cx\right)^{\frac{23}{2}}}$	42
default	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(8b^{3}x^{9}-4a\,b^{2}x^{6}+3a^{2}b\,x^{3}+15a^{3}\right)}{315x^{10}c^{11}\sqrt{cx}\,a^{3}}$	58
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(8b^{3}x^{9}-4a\,b^{2}x^{6}+3a^{2}b\,x^{3}+15a^{3}\right)}{315x^{10}c^{11}\sqrt{cx}\,a^{3}}$	58
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{21c^{12}x^{11}}-\frac{2b\sqrt{bc\,x^{4}+acx}}{105a\,c^{12}x^{8}}+\frac{8b^{2}\sqrt{bc\,x^{4}+acx}}{315a^{2}c^{12}x^{5}}-\frac{16b^{3}\sqrt{bc\,x^{4}+acx}}{315a^{3}c^{12}x^{2}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	129

input
```
int((b*x^3+a)^(1/2)/(c*x)^(23/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*x*(b*x^3+a)^(3/2)*(8*b^2*x^6-12*a*b*x^3+15*a^2)/a^3/(c*x)^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = -\frac{2\,(8\,b^3x^9-4\,ab^2x^6+3\,a^2bx^3+15\,a^3)\sqrt{bx^3+a}\sqrt{cx}}{315\,a^3c^{12}x^{11}}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(23/2),x, algorithm="fricas")
```

output
```
-2/315*(8*b^3*x^9 - 4*a*b^2*x^6 + 3*a^2*b*x^3 + 15*a^3)*sqrt(b*x^3 + a)*sq
rt(c*x)/(a^3*c^12*x^11)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(23/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = -\frac{2\left(8\,b^3\sqrt{c}x^{10}-4\,ab^2\sqrt{c}x^7+3\,a^2b\sqrt{c}x^4+15\,a^3\sqrt{c}x\right)\sqrt{bx^3+a}}{315\,a^3c^{12}x^{\frac{23}{2}}}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(23/2),x, algorithm="maxima")
```

output
```
-2/315*(8*b^3*sqrt(c)*x^10 - 4*a*b^2*sqrt(c)*x^7 + 3*a^2*b*sqrt(c)*x^4 + 1
5*a^3*sqrt(c)*x)*sqrt(b*x^3 + a)/(a^3*c^12*x^(23/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = \frac{2\left(\frac{8\sqrt{bc}b^3c^2}{a^3} - \frac{35\left(bc+\frac{ac}{x^3}\right)^{\frac{3}{2}}b^2c^2 - 42\left(bc+\frac{ac}{x^3}\right)^{\frac{5}{2}}bc + 15\left(bc+\frac{ac}{x^3}\right)^{\frac{7}{2}}}{a^3c}\right)|c|^2}{315\,c^{16}}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(23/2),x, algorithm="giac")
```

output
```
2/315*(8*sqrt(b*c)*b^3*c^2/a^3 - (35*(b*c + a*c/x^3)^(3/2)*b^2*c^2 - 42*(b
*c + a*c/x^3)^(5/2)*b*c + 15*(b*c + a*c/x^3)^(7/2))/(a^3*c))*abs(c)^2/c^16
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = -\frac{\sqrt{b\,x^3+a}\left(\frac{2}{21\,c^{11}} + \frac{2\,b\,x^3}{105\,a\,c^{11}} - \frac{8\,b^2\,x^6}{315\,a^2\,c^{11}} + \frac{16\,b^3\,x^9}{315\,a^3\,c^{11}}\right)}{x^{10}\,\sqrt{c\,x}}$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(23/2),x)
```

output
```
-((a + b*x^3)^(1/2)*(2/(21*c^11) + (2*b*x^3)/(105*a*c^11) - (8*b^2*x^6)/(3
15*a^2*c^11) + (16*b^3*x^9)/(315*a^3*c^11)))/(x^10*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{23/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(-8b^3x^9+4a\,b^2x^6-3a^2b\,x^3-15a^3)}{315\sqrt{x}\,a^3c^{12}x^{10}}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(23/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - 15*a**3 - 3*a**2*b*x**3 + 4*a*b**2*x**6 -
8*b**3*x**9))/(315*sqrt(x)*a**3*c**12*x**10)
```

3.310 $\int (cx)^{11/2}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 296

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = -\frac{21a^2c^5\sqrt{cx}\sqrt{a+bx^3}}{320b^2} + \frac{3ac^2(cx)^{7/2}\sqrt{a+bx^3}}{80b} + \frac{(cx)^{13/2}\sqrt{a+bx^3}}{8c} + \frac{7\,3^{3/4}a^{8/3}c^5\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-21/320*a^2*c^5*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^2+3/80*a*c^2*(c*x)^(7/2)*(b*
x^3+a)^(1/2)/b+1/8*(c*x)^(13/2)*(b*x^3+a)^(1/2)/c+7/640*3^(3/4)*a^(8/3)*c^
5*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+
(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^
(1/2))/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^
2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.35

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = \frac{c^5\sqrt{cx}\sqrt{a+bx^3}\left(\sqrt{1+\frac{bx^3}{a}}(-7a^2+3abx^3+10b^2x^6)+7a^2\,\text{Hypergeometric2F1}\left(\right.\right.}{80b^2\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c*x)^(11/2)*Sqrt[a + b*x^3],x]
```

output
```
(c^5*Sqrt[c*x]*Sqrt[a + b*x^3]*(Sqrt[1 + (b*x^3)/a]*(-7*a^2 + 3*a*b*x^3 +
10*b^2*x^6) + 7*a^2*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(80*
b^2*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {811, 843, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{16}a\int \frac{(cx)^{11/2}}{\sqrt{bx^3+a}}dx+\frac{(cx)^{13/2}\sqrt{a+bx^3}}{8c}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{16}a\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\int \frac{(cx)^{5/2}}{\sqrt{bx^3+a}}dx}{10b}\right)+\frac{(cx)^{13/2}\sqrt{a+bx^3}}{8c}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{16}a\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^3\int\frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{4b}\right)}{10b}\right)+\frac{(cx)^{13/2}\sqrt{a+bx^3}}{8c}$$

$\Big\downarrow$ 851

$$\frac{3}{16}a\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b}\right)}{10b}\right)+\frac{(cx)^{13/2}\sqrt{a+bx^3}}{8c}$$

$\Big\downarrow$ 766

$$\frac{3}{16}a\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{10b}\right.$$

$$\frac{(cx)^{13/2}\sqrt{a+bx^3}}{8c}$$

input
```
Int[(c*x)^(11/2)*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(13/2)*Sqrt[a + b*x^3])/(8*c) + (3*a*((c^2*(c*x)^(7/2)*Sqrt[a + b*x
^3])/(5*b) - (7*a*c^3*((c^2*Sqrt[c*x]*Sqrt[a + b*x^3])/(2*b) - (a^(2/3)*c*
Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^
2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Elliptic
F[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3]
)*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*c*x*(a^(1/3)
*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x
^3]))/(10*b)))/16
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.54

method	result
risch	$-\frac{(-40b^2x^6-12ab\,x^3+21a^2)x\sqrt{b\,x^3+a}\,c^6}{320b^2\sqrt{cx}}+\frac{21a^3\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a\,b^2)^{\frac{1}{3}}}{b}\right)}}}{}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^3+a)}\left(\frac{c^5x^6\sqrt{bc\,x^4+acx}}{8}+\frac{3a\,c^5x^3\sqrt{bc\,x^4+acx}}{80b}-\frac{21c^5a^2\sqrt{bc\,x^4+acx}}{320b^2}+\frac{21c^6a^3\left(\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a\,b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-}{}\right.}{\left(-\frac{(-a\,b^2)^{\frac{1}{3}}}{2b}\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((c*x)^(11/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/320*(-40*b^2*x^6-12*a*b*x^3+21*a^2)*x*(b*x^3+a)^(1/2)/b^2*c^6/(c*x)^(1/
2)+21/320*a^3/b*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3
/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-
a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*c^6*(c*x*(b*x^3+a))^(1/2)/(
c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}(cx)^{\frac{11}{2}}\,dx$$

input
```
integrate((c*x)^(11/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^5*x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 134.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.16

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}c^{\frac{11}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((c*x)**(11/2)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c**(11/2)*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**
3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

Maxima [F]

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{11}{2}}\,dx$$

input
```
integrate((c*x)^(11/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(11/2), x)
```

Giac [F]

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{11}{2}}\,dx$$

input
```
integrate((c*x)^(11/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = \int (c\,x)^{11/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((c*x)^(11/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(11/2)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{11/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{c}\,c^5\Big(-42\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2+24\sqrt{x}\,\sqrt{b\,x^3+a}\,ab\,x^3+80\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^6+21}{640b^2}$$

input
```
int((c*x)^(11/2)*(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**5*( - 42*sqrt(x)*sqrt(a + b*x**3)*a**2 + 24*sqrt(x)*sqrt(a + b
*x**3)*a*b*x**3 + 80*sqrt(x)*sqrt(a + b*x**3)*b**2*x**6 + 21*int((sqrt(x)*
sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**3))/(640*b**2)
```

3.311 $\int (cx)^{5/2}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 265

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx=\frac{3ac^2\sqrt{cx}\sqrt{a+bx^3}}{20b}+\frac{(cx)^{7/2}\sqrt{a+bx^3}}{5c}-\frac{3^{3/4}a^{5/3}c^2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
3/20*a*c^2*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b+1/5*(c*x)^(7/2)*(b*x^3+a)^(1/2)/c
-1/40*3^(3/4)*a^(5/3)*c^2*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJ
acobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.33

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx=\frac{c^2\sqrt{cx}\sqrt{a+bx^3}\left((a+bx^3)\sqrt{1+\frac{bx^3}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{5b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c*x)^(5/2)*Sqrt[a + b*x^3],x]
```

output
```
(c^2*Sqrt[c*x]*Sqrt[a + b*x^3]*((a + b*x^3)*Sqrt[1 + (b*x^3)/a] - a*Hyperg
eometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {811, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{10}a\int \frac{(cx)^{5/2}}{\sqrt{bx^3+a}}dx+\frac{(cx)^{7/2}\sqrt{a+bx^3}}{5c}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{10}a\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^3\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{4b}\right)+\frac{(cx)^{7/2}\sqrt{a+bx^3}}{5c}$$

$$\Big\downarrow \text{851}$$

$$\frac{3}{10}a\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b}\right)+\frac{(cx)^{7/2}\sqrt{a+bx^3}}{5c}$$

$$\Big\downarrow \text{766}$$

$$\frac{3}{10}a\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.$$

$$\frac{(cx)^{7/2}\sqrt{a+bx^3}}{5c}$$

input
```
Int[(c*x)^(5/2)*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(7/2)*Sqrt[a + b*x^3])/(5*c) + (3*a*((c^2*Sqrt[c*x]*Sqrt[a + b*x^3]
)/(2*b) - (a^(2/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2
 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(
1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[
(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)
*c*x)^2]*Sqrt[a + b*x^3])))/10
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 737, normalized size of antiderivative = 2.78

method	result
risch	$\frac{(4b\,x^3+3a)x\sqrt{b\,x^3+a}\,c^3}{20b\sqrt{cx}} - \frac{3a^2\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2}{20}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^3+a)}\left(\frac{c^2x^3\sqrt{bc\,x^4+acx}}{5}+\frac{3a\,c^2\sqrt{bc\,x^4+acx}}{20b}-\frac{3a^2c^3\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((c*x)^(5/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(4*b*x^3+3*a)*x*(b*x^3+a)^(1/2)/b*c^3/(c*x)^(1/2)-3/20*a^2*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b
^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a
*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b
^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3)))^(1/2))*c^3*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^2*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(5/2)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(13/6))
```

Maxima [F]

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(5/2), x)
```

Giac [F]

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx=\int (c\,x)^{5/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((c*x)^(5/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(5/2)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{5/2}\sqrt{a+bx^3}\,dx=\frac{\sqrt{c}\,c^2\left(6\sqrt{x}\,\sqrt{b\,x^3+a}\,a+8\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^3-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a^2\right)}{40b}$$

input
```
int((c*x)^(5/2)*(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**2*(6*sqrt(x)*sqrt(a + b*x**3)*a + 8*sqrt(x)*sqrt(a + b*x**3)*b
*x**3 - 3*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**2))/(40*b)
```

3.312 $\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 233

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx = \frac{\sqrt{cx}\sqrt{a+bx^3}}{2c} + \frac{3^{3/4}a^{2/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4c\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/2*(c*x)^(1/2)*(b*x^3+a)^(1/2)/c+1/4*3^(3/4)*a^(2/3)*(c*x)^(1/2)*(a^(1/3)
+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)
/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/c/(b^(1/3)*x*(a
^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx=\frac{2x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{cx}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/Sqrt[c*x],x]
```

output
```
(2*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)])/(Sqr
t[c*x]*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {811, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{4}a\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx+\frac{\sqrt{cx}\sqrt{a+bx^3}}{2c}$$

$$\Big\downarrow \text{851}$$

$$\frac{3a\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2c}+\frac{\sqrt{cx}\sqrt{a+bx^3}}{2c}$$

$$\Big\downarrow \text{766}$$

$$\frac{3^{3/4}a^{2/3}\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4c^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}+\frac{\sqrt{cx}\sqrt{a+bx^3}}{2c}$$

input
```
Int[Sqrt[a + b*x^3]/Sqrt[c*x],x]
```

output
```
(Sqrt[c*x]*Sqrt[a + b*x^3])/(2*c) + (3^(3/4)*a^(2/3)*Sqrt[c*x]*(a^(1/3)*c
+ b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2
)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c +
 (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 +
 Sqrt[3])/4])/(4*c^2*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)
*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.08

method	result
risch	$\frac{x\sqrt{bx^3+a}}{2\sqrt{cx}}+\frac{3a\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{(-ab^2)^{\frac{1}{3}}\cdots}{b\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}\cdots\right.}}\cdots}{2\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\cdots\right.}$
elliptic	$\frac{\sqrt{cx(bx^3+a)}\left(\frac{\sqrt{bc\,x^4+acx}}{2c}+\frac{3a\left(\frac{(-ab^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-ab^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-ab^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-ab^2)^{\frac{1}{3}}}{b}\right)^2\cdots}{2\left(\cdots\right.}\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^3+a)^(1/2)/(c*x)^(1/2)+3/2*a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)
^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(
x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*c*x*(x-
1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF
(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))*(
c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^3+a}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx = \frac{\sqrt{a}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{c}\,\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(1/2),x)
```

output
```
sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*p
i)/a)/(3*sqrt(c)*gamma(7/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^3+a}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^3+a}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{\sqrt{c\,x}}\,dx$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{\sqrt{cx}}\,dx=\frac{\sqrt{c}\left(2\sqrt{x}\,\sqrt{b\,x^3+a}+3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)a\right)}{4c}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*sqrt(a + b*x**3) + 3*int((sqrt(x)*sqrt(a + b*x**3))/(a
*x + b*x**4),x)*a))/(4*c)
```

3.313 $\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{a+bx^3}}{5c(cx)^{5/2}} + \frac{3^{3/4}b\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[3]{a}c^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*(b*x^3+a)^(1/2)/c/(c*x)^(5/2)+1/5*3^(3/4)*b*(c*x)^(1/2)*(a^(1/3)+b^(1
/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/a^(1/3)/c^4/(b^(1/3)
*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx = -\frac{2x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5(cx)^{7/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(7/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/(5
*(c*x)^(7/2)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {809, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}dx$$

$$\Big\downarrow \text{ 809}$$

$$\frac{3b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{5c^3}-\frac{2\sqrt{a+bx^3}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{6b\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{5c^4}-\frac{2\sqrt{a+bx^3}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{3^{3/4}b\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[3]{a}c^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2\sqrt{a+bx^3}}{5c(cx)^{5/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(7/2),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(5*c*(c*x)^(5/2)) + (3^(3/4)*b*Sqrt[c*x]*(a^(1/3)*c +
 b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)
/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c +
(1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 +
Sqrt[3])/4])/(5*a^(1/3)*c^5*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(
a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.10

method	result
risch	$-\frac{2\sqrt{b\,x^{3}+a}}{5x^{2}c^{3}\sqrt{cx}}+\frac{6b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b \cdots\right.}{b\left(-\frac{\left(-a \cdots\right.}{2 \cdots}\right.}}\cdots}{5\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots\right.}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{5c^{4}x^{3}}+\frac{6b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\cdots}{5\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(b*x^3+a)^(1/2)/x^2/c^3/(c*x)^(1/2)+6/5*b^2*(1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2
)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*
c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*El
lipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(
1/2))/c^3*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx = -\frac{2\left(3\sqrt{ac}bx^3\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)+\sqrt{bx^3+a}\sqrt{cx}a\right)}{5\,ac^4x^3}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(3*sqrt(a*c)*b*x^3*weierstrassPInverse(0, -4*b/a, 1/x) + sqrt(b*x^3 +
 a)*sqrt(c*x)*a)/(a*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(7/2),x)
```

output
```
sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/
(3*c**(7/2)*x**(5/2)*gamma(1/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{{\left(c\,x\right)}^{7/2}}\,dx$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(7/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{7/2}}\,dx=\frac{\sqrt{c}\left(-2\sqrt{b\,x^3+a}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^7+a\,x^4}dx\right)a\,x^2\right)}{2\sqrt{x}\,c^4x^2}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(7/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a + b*x**3) - 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))
/(a*x**4 + b*x**7),x)*a*x**2))/(2*sqrt(x)*c**4*x**2)
```

3.314 $\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 265

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt{a+bx^3}}{11c(cx)^{11/2}}-\frac{6b\sqrt{a+bx^3}}{55ac^4(cx)^{5/2}} -\frac{2\,3^{3/4}b^2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}c^7\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/11*(b*x^3+a)^(1/2)/c/(c*x)^(11/2)-6/55*b*(b*x^3+a)^(1/2)/a/c^4/(c*x)^(5
/2)-2/55*3^(3/4)*b^2*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobi
AM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x))
,1/4*6^(1/2)+1/4*2^(1/2))/a^(4/3)/c^7/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx = -\frac{2x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{1}{2},-\frac{5}{6},-\frac{bx^3}{a}\right)}{11(cx)^{13/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(13/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-11/6, -1/2, -5/6, -((b*x^3)/a)])/
(11*(c*x)^(13/2)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {809, 847, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3b\int \frac{1}{(cx)^{7/2}\sqrt{bx^3+a}}dx}{11c^3}-\frac{2\sqrt{a+bx^3}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{5ac^3}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11c^3}-\frac{2\sqrt{a+bx^3}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{3b\left(-\frac{4b\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{5ac^4}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11c^3}-\frac{2\sqrt{a+bx^3}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{766}$$

$$\frac{3b\left(-\frac{2b\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}c^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11c^3}-\frac{2\sqrt{a+bx^3}}{11c(cx)^{11/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(13/2),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(11*c*(c*x)^(11/2)) + (3*b*((-2*Sqrt[a + b*x^3])/(5*a
*c*(c*x)^(5/2)) - (2*b*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c
^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b
^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a
^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/
3)*c^5*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt
[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(11*c^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.80

method	result
risch	$-\frac{2\sqrt{b\,x^3+a}\,(3b\,x^3+5a)}{55x^5 a\,c^6\sqrt{cx}}-\frac{12b^3\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}{55a\left(\right.}$
elliptic	$\frac{\sqrt{cx(b\,x^3+a)}\left(-\frac{2\sqrt{bc\,x^4+acx}}{11c^7x^6}-\frac{6b\sqrt{bc\,x^4+acx}}{55a\,c^7x^3}-\frac{12b^3\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x^3+a)^(1/2)/(c*x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*(b*x^3+a)^(1/2)*(3*b*x^3+5*a)/x^5/a/c^6/(c*x)^(1/2)-12/55/a*b^3*(1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b
*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))/c^6*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx = \frac{2\left(6\sqrt{ac}b^2x^6\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\left(3abx^3+5a^2\right)\sqrt{bx^3+a}\sqrt{cx}\right)}{55a^2c^7x^6}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(13/2),x, algorithm="fricas")
```

output
```
2/55*(6*sqrt(a*c)*b^2*x^6*weierstrassPInverse(0, -4*b/a, 1/x) - (3*a*b*x^3
 + 5*a^2)*sqrt(b*x^3 + a)*sqrt(c*x))/(a^2*c^7*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 158.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{11}{6},-\frac{1}{2}\\ -\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3c^{\frac{13}{2}}x^{\frac{11}{2}}\Gamma\left(-\frac{5}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(13/2),x)
```

output
```
sqrt(a)*gamma(-11/6)*hyper((-11/6, -1/2), (-5/6,), b*x**3*exp_polar(I*pi)/
a)/(3*c**(13/2)*x**(11/2)*gamma(-5/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(13/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{{\left(c\,x\right)}^{13/2}} \,d x$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(13/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{13/2}}\,dx=\frac{\sqrt{c}\,\left(-2\sqrt{b\,x^{3}+a}-3\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{3}+a}}{b\,x^{10}+a\,x^{7}}d x \right) a\,x^{5}\right)}{8\sqrt{x}\,c^{7} x^{5}}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(13/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a + b*x**3) - 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))
/(a*x**7 + b*x**10),x)*a*x**5))/(8*sqrt(x)*c**7*x**5)
```

3.315 $\int (cx)^{9/2}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 550

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx=\frac{3ac^2(cx)^{5/2}\sqrt{a+bx^3}}{56b}$$
$$+\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}-\frac{15\left(1+\sqrt{3}\right)a^2c^4\sqrt{cx}\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{15\sqrt[4]{3}a^{7/3}c^4\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{5\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}c^4\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
3/56*a*c^2*(c*x)^(5/2)*(b*x^3+a)^(1/2)/b+1/7*(c*x)^(11/2)*(b*x^3+a)^(1/2)/
c-15/112*(1+3^(1/2))*a^2*c^4*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)+15/112*3^(1/4)*a^(7/3)*c^4*(c*x)^(1/2)*(a^(1/3)+b^(
1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1
/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3)/(b^(1/3)*x*
(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/
2)+5/224*3^(3/4)*(1-3^(1/2))*a^(7/3)*c^4*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)
^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+
3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^(5/3)/(b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.16

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx=\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}\left((a+bx^3)\sqrt{1+\frac{bx^3}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{7b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c*x)^(9/2)*Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(5/2)*Sqrt[a + b*x^3]*((a + b*x^3)*Sqrt[1 + (b*x^3)/a] - a*Hype
rgeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(7*b*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {811, 843, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{3}{14}a\int \frac{(cx)^{9/2}}{\sqrt{bx^3+a}}dx+\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{14}a\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}\right)+\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{3}{14}a\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{4b}\right)+\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{3}{14}a\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)+\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3}{14}a\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)+\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{3}{14}a\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b\cdots}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots\right.}}\cdots}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\cdots}}\cdots\right.}{4b}\cdots\right.$$

$$\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

$\downarrow$ 2420

$$\frac{3}{14}a\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\cdots\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\cdots}{2b^{2/3}}\cdots\right.}{\cdots}\cdots\right.$$

$$\frac{(cx)^{11/2}\sqrt{a+bx^3}}{7c}$$

input
```
Int[(c*x)^(9/2)*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(11/2)*Sqrt[a + b*x^3])/(7*c) + (3*a*((c^2*(c*x)^(5/2)*Sqrt[a + b*x
^3])/(4*b) - (5*a*c^2*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3
)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2
*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)
*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)],
(2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c
 + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqr
t[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^
(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*
c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*
c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt
[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3
)*c*x)^2]*Sqrt[a + b*x^3])))/(4*b)))/14
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.05

method	result	size
risch	Expression too large to display	1127
elliptic	Expression too large to display	1145
default	Expression too large to display	2825

input
```
int((c*x)^(9/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/56*x^3*(8*b*x^3+3*a)*(b*x^3+a)^(1/2)/b*c^5/(c*x)^(1/2)-15/112/b*a^2*(x*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b
^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^
2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(...
```

Fricas [F]

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{9}{2}}\,dx$$

input
```
integrate((c*x)^(9/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^4*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 40.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{a}c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((c*x)**(9/2)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c**(9/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3
*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

Maxima [F]

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{9}{2}}\,dx$$

input
```
integrate((c*x)^(9/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(9/2), x)
```

Giac [F]

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{9}{2}}\,dx$$

input
```
integrate((c*x)^(9/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx = \int (c\,x)^{9/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((c*x)^(9/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(9/2)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{9/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{c}\,c^4\left(6\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,x^2+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^5-15\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2\right)}{112b}$$

input
```
int((c*x)^(9/2)*(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**4*(6*sqrt(x)*sqrt(a + b*x**3)*a*x**2 + 16*sqrt(x)*sqrt(a + b*x
**3)*b*x**5 - 15*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**2))/(
112*b)
```

3.316 $\int (cx)^{3/2}\sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 513

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx=\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}+\frac{3\left(1+\sqrt{3}\right)ac\sqrt{cx}\sqrt{a+bx^3}}{8b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$-\frac{3\sqrt[4]{3}a^{4/3}c\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}c\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2-}{16b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/4*(c*x)^(5/2)*(b*x^3+a)^(1/2)/c+3/8*(1+3^(1/2))*a*c*(c*x)^(1/2)*(b*x^3+a
)^(1/2)/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-3/8*3^(1/4)*a^(4/3)*c*(c*x
)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1
/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)
^(1/2)/(b*x^3+a)^(1/2)-1/16*3^(3/4)*(1-3^(1/2))*a^(4/3)*c*(c*x)^(1/2)*(a^(
1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3
)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/(b^
(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^
3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.11

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx=\frac{2x(cx)^{3/2}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c*x)^(3/2)*Sqrt[a + b*x^3],x]
```

output
```
(2*x*(c*x)^(3/2)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x
^3)/a)])/(5*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.316$, Rules used = {811, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx$$

$$\Big\downarrow \; 811$$

$$\frac{3}{8}a\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx+\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}$$

$$\Big\downarrow \; 851$$

$$\frac{3a\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{4c}+\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}$$

$$\Big\downarrow \; 837$$

$$\frac{3a\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4c}+\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}$$

$$\Big\downarrow \; 25$$

$$\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4c}+\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}$$

$$\Big\downarrow \; 766$$

$$\frac{3a\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{4c}$$

$$\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}$$

$$\Big\downarrow \; 2420$$

$$3a\left(\frac{\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}} - \cdots\right.$$

$$4c$$

$$\frac{(cx)^{5/2}\sqrt{a+bx^3}}{4c}$$

input
```
Int[(c*x)^(3/2)*Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(5/2)*Sqrt[a + b*x^3])/(4*c) + (3*a*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*
Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3
)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3
)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Elli
pticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqr
t[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(
1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2
*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*S
qrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (
1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b
^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4
*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c +
 (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 1109, normalized size of antiderivative = 2.16

method	result	size
risch	Expression too large to display	1109
elliptic	Expression too large to display	1113
default	Expression too large to display	2599

input
```
int((c*x)^(3/2)*(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^3*(b*x^3+a)^(1/2)*c^2/(c*x)^(1/2)+3/8*a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b
^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1
/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1...
```

Fricas [F]

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx=\frac{\sqrt{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(3/2)*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*ex
p_polar(I*pi)/a)/(3*gamma(11/6))
```

Maxima [F]

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx=\int \sqrt{bx^3+a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(3/2), x)
```

Giac [F]

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx = \int (c\,x)^{3/2}\,\sqrt{b\,x^3+a}\,dx$$

input
```
int((c*x)^(3/2)*(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(3/2)*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{3/2}\sqrt{a+bx^3}\,dx = \frac{\sqrt{c}\,c\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,x^2+3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a\right)}{8}$$

input
```
int((c*x)^(3/2)*(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c*(2*sqrt(x)*sqrt(a + b*x**3)*x**2 + 3*int((sqrt(x)*sqrt(a + b*x*
*3)*x)/(a + b*x**3),x)*a))/8
```

3.317 $\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 512

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}+\frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{cx}\sqrt{a+bx^3}}{c^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{c^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[3]{b}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{2c^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2*(b*x^3+a)^(1/2)/c/(c*x)^(1/2)+3*(1+3^(1/2))*b^(1/3)*(c*x)^(1/2)*(b*x^3+
a)^(1/2)/c^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-3*3^(1/4)*a^(1/3)*b^(1/3)*(c*
x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(
1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)/c^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1
/2)/(b*x^3+a)^(1/2)-1/2*3^(3/4)*(1-3^(1/2))*a^(1/3)*b^(1/3)*(c*x)^(1/2)*(a
^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(
1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1
/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))/c^2/(b^(1
/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx=-\frac{2x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{5}{6},-\frac{bx^3}{a}\right)}{(cx)^{3/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(3/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/((
c*x)^(3/2)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {809, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx$$

$\big\downarrow$ 809

$$\frac{3b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{c^3}-\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}$$

$\big\downarrow$ 851

$$\frac{6b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{c^4}-\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}$$

$\big\downarrow$ 837

$$\frac{6b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{c^4}-\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}$$

$\big\downarrow$ 25

$$\frac{6b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{c^4}-\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}$$

$\big\downarrow$ 766

$$\frac{6b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\vphantom{b}}}{\left(1+\sqrt{3}\right)\sqrt[3]{\vphantom{b}}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{c^4}$$

$$\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}$$

$\big\downarrow$ 2420

$$\frac{6b\left(\frac{\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}} - \cdots\right)}{c^4}$$

$$\frac{2\sqrt{a+bx^3}}{c\sqrt{cx}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(3/2),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(c*Sqrt[c*x]) + (6*b*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*S
qrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)
*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)
*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Ellip
ticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt
[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1
/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*
b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sq
rt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1
 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^
(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*
3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c +
(1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/c^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 1106, normalized size of antiderivative = 2.16

method	result	size
risch	Expression too large to display	1106
elliptic	Expression too large to display	1117
default	Expression too large to display	2878

input
```
int((b*x^3+a)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^3+a)^(1/2)/c/(c*x)^(1/2)+3*b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3)
)^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^
2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I...
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)/(c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(3/2),x)
```

output
```
sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a)/
(3*c**(3/2)*sqrt(x)*gamma(5/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a}}{(c\,x)^{3/2}}\,dx$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{b\,x^3+a}+3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)a\right)}{2\sqrt{x}\,c^2}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a + b*x**3) + 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))/(a
*x**2 + b*x**5),x)*a))/(2*sqrt(x)*c**2)
```

3.318 $\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 550

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx = -\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}} - \frac{6b\sqrt{a+bx^3}}{7ac^4\sqrt{cx}} + \frac{6\left(1+\sqrt{3}\right)b^{4/3}\sqrt{cx}\sqrt{a+bx^3}}{7ac^5\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{6\sqrt[4]{3}\,b^{4/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{2/3}c^5\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)b^{4/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\Big(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+}{7a^{2/3}c^5\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/7*(b*x^3+a)^(1/2)/c/(c*x)^(7/2)-6/7*b*(b*x^3+a)^(1/2)/a/c^4/(c*x)^(1/2)
+6/7*(1+3^(1/2))*b^(4/3)*(c*x)^(1/2)*(b*x^3+a)^(1/2)/a/c^5/(a^(1/3)+(1+3^(
1/2))*b^(1/3)*x)-6/7*3^(1/4)*b^(4/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/
2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(2/3)/c^5/(b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/7*3^
(3/4)*(1-3^(1/2))*b^(4/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJ
acobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)),1/4*6^(1/2)+1/4*2^(1/2))/a^(2/3)/c^5/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx = -\frac{2x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},-\frac{1}{6},-\frac{bx^3}{a}\right)}{7(cx)^{9/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(9/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-7/6, -1/2, -1/6, -((b*x^3)/a)])/(
7*(c*x)^(9/2)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {809, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+a}}dx}{7c^3}-\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3b\left(\frac{2b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{ac^3}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7c^3}-\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{3b\left(\frac{4b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7c^3}-\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{3b\left(\frac{4b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7c^3}-\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{3b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7c^3}-\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{766}$$

$$3b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{ac^4}\right.$$

$$\overline{\qquad 7c^3 \qquad}$$

$$\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

$\downarrow$ 2420

$$3b\left(\frac{4b\left(\frac{\frac{(1+\sqrt{3})c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}{(1+\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}} - \cdots\right)}{ac^4}\cdots\right)}{7c^3}$$

$$\frac{2\sqrt{a+bx^3}}{7c(cx)^{7/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(9/2),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(7*c*(c*x)^(7/2)) + (3*b*((-2*Sqrt[a + b*x^3])/(a*c*S
qrt[c*x]) + (4*b*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*
c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c +
 b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)
/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c +
(1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 +
Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1
 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])
*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)
*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^
2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (
1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(
1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x
)^2]*Sqrt[a + b*x^3]))))/(a*c^4)))/(7*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.05

method	result	size
risch	Expression too large to display	1125
elliptic	Expression too large to display	1172
default	Expression too large to display	3074

input
```
int((b*x^3+a)^(1/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^3+a)^(1/2)*(3*b*x^3+a)/x^3/a/c^4/(c*x)^(1/2)+6/7/a*b^2*(x*(x+1/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(
2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1
/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Elliptic
E(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx = \frac{2\left(3\sqrt{ac}bx^4\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)-\sqrt{bx^3+a}\sqrt{cx}a\right)}{7\,ac^5x^4}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(9/2),x, algorithm="fricas")
```

output
```
2/7*(3*sqrt(a*c)*b*x^4*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -
4*b/a, 1/x)) - sqrt(b*x^3 + a)*sqrt(c*x)*a)/(a*c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.61 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{7}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{6}, -\frac{1}{2} \\ -\frac{1}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(9/2),x)
```

output
```
sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/a)
/(3*c**(9/2)*x**(7/2)*gamma(-1/6))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{{\left(c\,x\right)}^{9/2}} \,d x$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(9/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{9/2}}\,dx=\frac{\sqrt{c}\,\left(-2\sqrt{b\,x^{3}+a}-3\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{3}+a}}{b\,x^{8}+a\,x^{5}}d x \right) a\,x^{3}\right)}{4\sqrt{x}\,c^{5} x^{3}}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(9/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a + b*x**3) - 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))
/(a*x**5 + b*x**8),x)*a*x**3))/(4*sqrt(x)*c**5*x**3)
```

3.319 $\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 581

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx = -\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}} - \frac{6b\sqrt{a+bx^3}}{91ac^4(cx)^{7/2}} + \frac{24b^2\sqrt{a+bx^3}}{91a^2c^7\sqrt{cx}} - \frac{24\left(1+\sqrt{3}\right)b^{7/3}\sqrt{cx}\sqrt{a+bx^3}}{91a^2c^8\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{24\sqrt[4]{3}b^{7/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{91a^{5/3}c^8\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{4\,3^{3/4}\left(1-\sqrt{3}\right)b^{7/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2 \right.\right.}{91a^{5/3}c^8\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/13*(b*x^3+a)^(1/2)/c/(c*x)^(13/2)-6/91*b*(b*x^3+a)^(1/2)/a/c^4/(c*x)^(7
/2)+24/91*b^2*(b*x^3+a)^(1/2)/a^2/c^7/(c*x)^(1/2)-24/91*(1+3^(1/2))*b^(7/3
)*(c*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/c^8/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)+24/9
1*3^(1/4)*b^(7/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^
(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))/a^(5/3)/c^8/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)
+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+4/91*3^(3/4)*(1-3^(1/2))*
b^(7/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^
(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+
1/4*2^(1/2))/a^(5/3)/c^8/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx=-\frac{2x\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{13}{6},-\frac{1}{2},-\frac{7}{6},-\frac{bx^3}{a}\right)}{13(cx)^{15/2}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^3]/(c*x)^(15/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*Hypergeometric2F1[-13/6, -1/2, -7/6, -((b*x^3)/a)])/
(13*(c*x)^(15/2)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {809, 847, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3b\int \frac{1}{(cx)^{9/2}\sqrt{bx^3+a}}dx}{13c^3}-\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3b\left(-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+a}}dx}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{13c^3}-\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3b\left(-\frac{4b\left(\frac{2b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{ac^3}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{13c^3}-\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{3b\left(-\frac{4b\left(\frac{4b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{13c^3}-\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{3b\left(-\frac{4b\left(\frac{4b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{13c^3}-\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{3b\left(-\frac{4b\left(\frac{4b\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{13c^3}-\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \ 766$$

$$3b\left(-\frac{4b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{ac^4}\right.}{7ac^3}\right.}{13c^3}$$

$$\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

$$\Big\downarrow \; 2420$$

$$3b\left(-\frac{4b\left(\frac{4b\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{\vphantom{3}}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{2b^{2/3}}\right)}{ac^4}\right)$$

$$\frac{2\sqrt{a+bx^3}}{13c(cx)^{13/2}}$$

input
```
Int[Sqrt[a + b*x^3]/(c*x)^(15/2),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(13*c*(c*x)^(13/2)) + (3*b*((-2*Sqrt[a + b*x^3])/(7*a
*c*(c*x)^(7/2)) - (4*b*((-2*Sqrt[a + b*x^3])/(a*c*Sqrt[c*x]) + (4*b*((((1
+ Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/
3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(
2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt
[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c
*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(
1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x
)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a
^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3
)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^
(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*
x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b
^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))
/(a*c^4)))/(7*a*c^3)))/(13*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 1138, normalized size of antiderivative = 1.96

method	result	size
risch	Expression too large to display	1138
elliptic	Expression too large to display	1201
default	Expression too large to display	3275

input
```
int((b*x^3+a)^(1/2)/(c*x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/91*(b*x^3+a)^(1/2)*(-12*b^2*x^6+3*a*b*x^3+7*a^2)/x^6/a^2/c^7/(c*x)^(1/2
)-24/91/a^2*b^3*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/
3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b
*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^
2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx = -\frac{2\left(12\sqrt{ac}b^2x^7\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\left(3abx^3+7a^2\right)\sqrt{bx^3+a}\sqrt{cx}\right)}{91a^2c^8x^7}$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(15/2),x, algorithm="fricas")
```

output
```
-2/91*(12*sqrt(a*c)*b^2*x^7*weierstrassZeta(0, -4*b/a, weierstrassPInverse
(0, -4*b/a, 1/x)) + (3*a*b*x^3 + 7*a^2)*sqrt(b*x^3 + a)*sqrt(c*x))/(a^2*c^
8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**3+a)**(1/2)/(c*x)**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{15}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(15/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(15/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx = \int \frac{\sqrt{bx^3+a}}{(cx)^{\frac{15}{2}}}\,dx$$

input
```
integrate((b*x^3+a)^(1/2)/(c*x)^(15/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)/(c*x)^(15/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx=\int \frac{\sqrt{b\,x^3+a}}{{\left(c\,x\right)}^{15/2}} \,d x$$

input
```
int((a + b*x^3)^(1/2)/(c*x)^(15/2),x)
```

output
```
int((a + b*x^3)^(1/2)/(c*x)^(15/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^3}}{(cx)^{15/2}}\,dx=\frac{\sqrt{c}\,\left(-2\sqrt{b\,x^{3}+a}-3\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{3}+a}}{b\,x^{11}+a\,x^{8}}d x \right) a\,x^{6}\right)}{10\sqrt{x}\,c^{8} x^{6}}$$

input
```
int((b*x^3+a)^(1/2)/(c*x)^(15/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a + b*x**3) - 3*sqrt(x)*int((sqrt(x)*sqrt(a + b*x**3))
/(a*x**8 + b*x**11),x)*a*x**6))/(10*sqrt(x)*c**8*x**6)
```

3.320 $\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx=-\frac{ac^5(cx)^{3/2}\sqrt{a+bx^3}}{4b^2}+\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}+\frac{a^2c^{13/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{4b^{5/2}}$$

output
```
-1/4*a*c^5*(c*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/6*c^2*(c*x)^(9/2)*(b*x^3+a)^(
1/2)/b+1/4*a^2*c^(13/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a)^(1/2
))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx=\frac{c^6\sqrt{cx}\left(\sqrt{b}x^{3/2}\sqrt{a+bx^3}(-3a+2bx^3)+3a^2\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{12b^{5/2}\sqrt{x}}$$

input
```
Integrate[(c*x)^(13/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^6*Sqrt[c*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(-3*a + 2*b*x^3) + 3*a^2*L
og[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(12*b^(5/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {843, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\int \frac{(cx)^{7/2}}{\sqrt{bx^3+a}}dx}{4b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{b}\right)}{4b}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3b}\right)}{4b}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3b}\right)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3b^{3/2}}\right)}{4b}$$

input
```
Int[(c*x)^(13/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(9/2)*Sqrt[a + b*x^3])/(6*b) - (3*a*c^3*((c^2*(c*x)^(3/2)*Sqrt[
a + b*x^3])/(3*b) - (a*c^(7/2)*ArcTanh[(Sqrt[b]*(c*x)^(3/2))/(c^(3/2)*Sqrt
[a + (b*x)/c^2])])/(3*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{x^{2}\left(-2 b\,x^{3}+3 a \right) \sqrt{b\,x^{3}+a}\, c^{7}}{12 b^{2} \sqrt{c x}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{\sqrt{c x \left(b\,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right) c^{7} \sqrt{c x \left(b\,x^{3}+a \right)}}{4 b^{2} \sqrt{b c}\, \sqrt{c x}\, \sqrt{b\,x^{3}+a}}$	101
default	$\frac{c^{6} \sqrt{c x}\, \sqrt{b\,x^{3}+a}\, \left(2 \sqrt{c x \left(b\,x^{3}+a \right)}\, \sqrt{b c}\, b\,x^{4}+3 \operatorname{arctanh}\left(\frac{\sqrt{c x \left(b\,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right) c\,a^{2}-3 a x \sqrt{c x \left(b\,x^{3}+a \right)}\, \sqrt{b c}\right)}{12 \sqrt{c x \left(b\,x^{3}+a \right)}\, b^{2} \sqrt{b c}}$	113
elliptic	Expression too large to display	1069

input
```
int((c*x)^(13/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*x^2*(-2*b*x^3+3*a)*(b*x^3+a)^(1/2)/b^2*c^7/(c*x)^(1/2)+1/4*a^2/b^2/(
b*c)^(1/2)*arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b*c)^(1/2))*c^7*(c*x*(b*x^3+
a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.01

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx = \left[\frac{3\,a^2c^6\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c-4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{cx}\sqrt{\frac{c}{b}}\right)+4\,(2\,bc^{6}\right.$$

$$\left.-\frac{3\,a^2c^6\sqrt{-\frac{c}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{cx}bx\sqrt{-\frac{c}{b}}}{2\,bcx^3+ac}\right)-2\,(2\,bc^6x^4-3\,ac^6x)\sqrt{bx^3+a}\sqrt{cx}}{24\,b^2}\right]$$

input
```
integrate((c*x)^(13/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*a^2*c^6*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x^3 - a^2*c - 4*(2*b
^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)) + 4*(2*b*c^6*x^4 - 3*
a*c^6*x)*sqrt(b*x^3 + a)*sqrt(c*x))/b^2, -1/24*(3*a^2*c^6*sqrt(-c/b)*arcta
n(2*sqrt(b*x^3 + a)*sqrt(c*x)*b*x*sqrt(-c/b)/(2*b*c*x^3 + a*c)) - 2*(2*b*c
^6*x^4 - 3*a*c^6*x)*sqrt(b*x^3 + a)*sqrt(c*x))/b^2]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(13/2)/(b*x**3+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{13}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(13/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(13/2)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{bc^4x^3+ac^4}\sqrt{cx}c^{11}x\left(\frac{2x^3}{bc^5}-\frac{3a}{b^2c^5}\right)}{12\,|c|^2} - \frac{a^2c^{11}\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{4\sqrt{bc}b^2|c|^4}$$

input
```
integrate((c*x)^(13/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/12*sqrt(b*c^4*x^3 + a*c^4)*sqrt(c*x)*c^11*x*(2*x^3/(b*c^5) - 3*a/(b^2*c^
5))/abs(c)^2 - 1/4*a^2*c^11*log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*
x^3 + a*c^4)))/(sqrt(b*c)*b^2*abs(c)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(c\,x)^{13/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c*x)^(13/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(13/2)/(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(cx)^{13/2}}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\,c^6\left(-6\sqrt{x}\,\sqrt{b\,x^3+a}\,abx+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^4-3\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a^2}{24b^3}$$

input
```
int((c*x)^(13/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**6*( - 6*sqrt(x)*sqrt(a + b*x**3)*a*b*x + 4*sqrt(x)*sqrt(a + b*
x**3)*b**2*x**4 - 3*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**2
 + 3*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**2))/(24*b**3)
```

3.321 $\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx=\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

output
```
1/3*c^2*(c*x)^(3/2)*(b*x^3+a)^(1/2)/b-1/3*a*c^(7/2)*arctanh(b^(1/2)*(c*x)^
(3/2)/c^(3/2)/(b*x^3+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx=\frac{(cx)^{7/2}\sqrt{a+bx^3}}{3bx^2}-\frac{a(cx)^{7/2}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3b^{3/2}x^{7/2}}$$

input
```
Integrate[(c*x)^(7/2)/Sqrt[a + b*x^3],x]
```

output
```
((c*x)^(7/2)*Sqrt[a + b*x^3])/(3*b*x^2) - (a*(c*x)^(7/2)*Log[Sqrt[b]*x^(3/
2) + Sqrt[a + b*x^3]])/(3*b^(3/2)*x^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{2b}$$

$$\Big\downarrow \text{851}$$

$$\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{b}$$

$$\Big\downarrow \text{807}$$

$$\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3b^{3/2}}$$

input
```
Int[(c*x)^(7/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(3/2)*Sqrt[a + b*x^3])/(3*b) - (a*c^(7/2)*ArcTanh[(Sqrt[b]*(c*x
)^(3/2))/(c^(3/2)*Sqrt[a + (b*x)/c^2])])/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{c^{3}\sqrt{cx}\,\sqrt{b\,x^{3}+a}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)ac-\sqrt{cx\left(b\,x^{3}+a\right)}\,x\sqrt{bc}\right)}{3\sqrt{cx\left(b\,x^{3}+a\right)}\,b\sqrt{bc}}$	86
risch	$\frac{x^{2}\sqrt{b\,x^{3}+a}\,c^{4}}{3b\sqrt{cx}}-\frac{a\,\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)c^{4}\sqrt{cx\left(b\,x^{3}+a\right)}}{3b\sqrt{bc}\,\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	89
elliptic	Expression too large to display	1039

input
```
int((c*x)^(7/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^3*(c*x)^(1/2)*(b*x^3+a)^(1/2)*(arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b
*c)^(1/2))*a*c-(c*x*(b*x^3+a))^(1/2)*x*(b*c)^(1/2))/(c*x*(b*x^3+a))^(1/2)/
b/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.42

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx = \left[\frac{ac^3\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c+4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{cx}\sqrt{\frac{c}{b}}\right)+4\sqrt{bx^3+}}{12\,b}\right.$$

input
```
integrate((c*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(a*c^3*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x^3 - a^2*c + 4*(2*b^2*x
^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)) + 4*sqrt(b*x^3 + a)*sqrt(
c*x)*c^3*x)/b, 1/6*(a*c^3*sqrt(-c/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(c*x)*b*
x*sqrt(-c/b)/(2*b*c*x^3 + a*c)) + 2*sqrt(b*x^3 + a)*sqrt(c*x)*c^3*x)/b]
```

Sympy [A] (verification not implemented)

Time = 13.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{a}c^{\frac{7}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3b} - \frac{ac^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}$$

input
```
integrate((c*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*c**(7/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*b) - a*c**(7/2)*asinh(sqrt
(b)*x**(3/2)/sqrt(a))/(3*b**(3/2))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx = \frac{ac^8\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{3\sqrt{bc}b|c|^4} + \frac{\sqrt{bc^4x^3+ac^4}\sqrt{cx}c^3x}{3\,b|c|^2}$$

input
```
integrate((c*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*a*c^8*log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))/(sq
rt(b*c)*b*abs(c)^4) + 1/3*sqrt(b*c^4*x^3 + a*c^4)*sqrt(c*x)*c^3*x/(b*abs(c
)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{{\left(c\,x\right)}^{7/2}}{\sqrt{b\,x^3+a}} \,d x$$

input
```
int((c*x)^(7/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(7/2)/(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\,c^{3} \left(2 \sqrt{x}\, \sqrt{b\,x^{3}+a}\, b x +\sqrt{b}\, \log\left(\sqrt{b\,x^{3}+a}-\sqrt{x}\, \sqrt{b}\, x \right) a -\sqrt{b}\, \log\left(\sqrt{b\,x^{3}+a}+\sqrt{x}\right.}{6 b^{2}}$$

input
```
int((c*x)^(7/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**3*(2*sqrt(x)*sqrt(a + b*x**3)*b*x + sqrt(b)*log(sqrt(a + b*x**
3) - sqrt(x)*sqrt(b)*x)*a - sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)
*x)*a))/(6*b**2)
```

3.322 $\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}}$$

output
```
2/3*c^(1/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{cx}\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)}{3\sqrt{b}\sqrt{x}}$$

input
```
Integrate[Sqrt[c*x]/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[c*x]*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]])/(3*Sqrt[b]*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 851}$$

$$\frac{2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3c}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3\sqrt{b}}$$

input
```
Int[Sqrt[c*x]/Sqrt[a + b*x^3],x]
```

output
```
(2*Sqrt[c]*ArcTanh[(Sqrt[b]*(c*x)^(3/2))/(c^(3/2)*Sqrt[a + (b*x)/c^2])])/(
3*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{2\sqrt{cx}\,\sqrt{b\,x^{3}+a}\,c\,\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)}{3\sqrt{cx(b\,x^{3}+a)}\,\sqrt{bc}}$	57
elliptic	Expression too large to display	1009

input
```
int((c*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x)^(1/2)*(b*x^3+a)^(1/2)/(c*x*(b*x^3+a))^(1/2)*c/(b*c)^(1/2)*arctan
h((c*x*(b*x^3+a))^(1/2)/x^2/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \left[\frac{1}{6}\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6 - 8\,abcx^3 - a^2c - 4\left(2\,b^2x^4 + abx\right)\sqrt{bx^3+a}\sqrt{cx}\sqrt{\frac{c}{b}}\right),\ -\frac{1}{3}\sqrt{-\frac{c}{b}}\arctan\left(\frac{2\sqrt{bx^3+a}\sqrt{cx}bx\sqrt{-\frac{c}{b}}}{2\,bcx^3+ac}\right)\right]$$

input
```
integrate((c*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x^3 - a^2*c - 4*(2*b^2*x^4 + a*b
*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)), -1/3*sqrt(-c/b)*arctan(2*sqrt(b*
x^3 + a)*sqrt(c*x)*b*x*sqrt(-c/b)/(2*b*c*x^3 + a*c))]
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}}$$

input
```
integrate((c*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
2*sqrt(c)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/sqrt(b*x^3 + a), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = -\frac{2\,c^3\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{3\,\sqrt{bc}|c|^2}$$

input
```
integrate((c*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*c^3*log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))/(sqr
t(b*c)*abs(c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \int \frac{\sqrt{c\,x}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\,\sqrt{b}\left(-\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)+\log\left(\sqrt{b\,x^3+a}+\sqrt{x}\,\sqrt{b}\,x\right)\right)}{3b}$$

input
```
int((c*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(b)*( - log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x) + log(sqrt(
a + b*x**3) + sqrt(x)*sqrt(b)*x)))/(3*b)
```

3.323 $\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{3ac(cx)^{3/2}}$$

output
```
-2/3*(b*x^3+a)^(1/2)/a/c/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\sqrt{a+bx^3}}{3a(cx)^{5/2}}$$

input
```
Integrate[1/((c*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*Sqrt[a + b*x^3])/(3*a*(c*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(3*a*c*(c*x)^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{2x\sqrt{b\,x^{3}+a}}{3a\left(cx\right)^{\frac{5}{2}}}$	21
orering	$-\frac{2x\sqrt{b\,x^{3}+a}}{3a\left(cx\right)^{\frac{5}{2}}}$	21
default	$-\frac{2\sqrt{b\,x^{3}+a}}{3xa\,c^{2}\sqrt{cx}}$	26
risch	$-\frac{2\sqrt{b\,x^{3}+a}}{3xa\,c^{2}\sqrt{cx}}$	26
elliptic	$-\frac{2\sqrt{cx\left(b\,x^{3}+a\right)}\,\sqrt{bc\,x^{4}+acx}}{3\sqrt{cx}\,\sqrt{b\,x^{3}+a}\,a\,c^{3}x^{2}}$	51

input
```
int(1/(c*x)^(5/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(b*x^3+a)^(1/2)/a/(c*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,\sqrt{bx^3+a}\sqrt{cx}}{3\,ac^3x^2}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^3 + a)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ac^{\frac{5}{2}}}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*c**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\left(b\sqrt{c}x^4+a\sqrt{c}x\right)}{3\sqrt{bx^3+a}ac^3x^{\frac{5}{2}}}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(b*sqrt(c)*x^4 + a*sqrt(c)*x)/(sqrt(b*x^3 + a)*a*c^3*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,c^3\left(\frac{\sqrt{bc+\frac{ac}{x^3}}}{ac^4}-\frac{\sqrt{bc}}{ac^4}\right)}{3\,|c|^2}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*c^3*(sqrt(b*c + a*c/x^3)/(a*c^4) - sqrt(b*c)/(a*c^4))/abs(c)^2
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{b\,x^3+a}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^3)^(1/2)),x)
```

output
```
-(2*(a + b*x^3)^(1/2))/(3*a*c^2*x*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{c}\,\sqrt{b\,x^3+a}}{3\sqrt{x}\,a\,c^3x}$$

input
```
int(1/(c*x)^(5/2)/(b*x^3+a)^(1/2),x)
```

output
```
( - 2*sqrt(c)*sqrt(a + b*x**3))/(3*sqrt(x)*a*c**3*x)
```

3.324 $\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{9ac(cx)^{9/2}}+\frac{4b\sqrt{a+bx^3}}{9a^2c^4(cx)^{3/2}}$$

output
```
-2/9*(b*x^3+a)^(1/2)/a/c/(c*x)^(9/2)+4/9*b*(b*x^3+a)^(1/2)/a^2/c^4/(c*x)^(
3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = -\frac{2x(a-2bx^3)\sqrt{a+bx^3}}{9a^2(cx)^{11/2}}$$

input
```
Integrate[1/((c*x)^(11/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*(a - 2*b*x^3)*Sqrt[a + b*x^3])/(9*a^2*(c*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{2\int \frac{\sqrt{bx^3+a}}{(cx)^{11/2}}dx}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{9/2}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4\left(a+bx^3\right)^{3/2}}{9a^2c(cx)^{9/2}}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{9/2}}$$

input
```
Int[1/((c*x)^(11/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(3*a*c*(c*x)^(9/2)) + (4*(a + b*x^3)^(3/2))/(9*a^2*c*
(c*x)^(9/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2x\sqrt{b\,x^{3}+a}\,\left(-2b\,x^{3}+a\right)}{9a^{2}\left(cx\right)^{\frac{11}{2}}}$	29
orering	$-\frac{2x\sqrt{b\,x^{3}+a}\,\left(-2b\,x^{3}+a\right)}{9a^{2}\left(cx\right)^{\frac{11}{2}}}$	29
default	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-2b\,x^{3}+a\right)}{9x^{4}a^{2}c^{5}\sqrt{cx}}$	34
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-2b\,x^{3}+a\right)}{9x^{4}a^{2}c^{5}\sqrt{cx}}$	34
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{9a\,c^{6}x^{5}}+\frac{4b\sqrt{bc\,x^{4}+acx}}{9a^{2}c^{6}x^{2}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	78

input
```
int(1/(c*x)^(11/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x*(b*x^3+a)^(1/2)*(-2*b*x^3+a)/a^2/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = \frac{2\,(2\,bx^3-a)\sqrt{bx^3+a}\sqrt{cx}}{9\,a^2c^6x^5}$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(2*b*x^3 - a)*sqrt(b*x^3 + a)*sqrt(c*x)/(a^2*c^6*x^5)
```

Sympy [A] (verification not implemented)

Time = 66.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9ac^{\frac{11}{2}}x^3} + \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a^2c^{\frac{11}{2}}}$$

input
```
integrate(1/(c*x)**(11/2)/(b*x**3+a)**(1/2),x)
```

output
```
-2*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*a*c**(11/2)*x**3) + 4*b**(3/2)*sqrt(a/(
b*x**3) + 1)/(9*a**2*c**(11/2))
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(11/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = \frac{2\left(\frac{3\sqrt{bc+\frac{ac}{x^3}}\,b}{a^2c^2} - \frac{2\sqrt{bc}\,b}{a^2c^2} - \frac{\left(bc+\frac{ac}{x^3}\right)^{\frac{3}{2}}}{a^2c^3}\right)}{9\,c^2|c|^2}$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
2/9*(3*sqrt(b*c + a*c/x^3)*b/(a^2*c^2) - 2*sqrt(b*c)*b/(a^2*c^2) - (b*c +
a*c/x^3)^(3/2)/(a^2*c^3))/(c^2*abs(c)^2)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2}{9\,a\,c^5}-\frac{4\,b\,x^3}{9\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(11/2)*(a + b*x^3)^(1/2)),x)
```

output
```
-((a + b*x^3)^(1/2)*(2/(9*a*c^5) - (4*b*x^3)/(9*a^2*c^5)))/(x^4*(c*x)^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{11/2}\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(2b\,x^3-a)}{9\sqrt{x}\,a^2c^6x^4}$$

input
```
int(1/(c*x)^(11/2)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - a + 2*b*x**3))/(9*sqrt(x)*a**2*c**6*x**4)
```

3.325 $\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{15ac(cx)^{15/2}}+\frac{8b\sqrt{a+bx^3}}{45a^2c^4(cx)^{9/2}}-\frac{16b^2\sqrt{a+bx^3}}{45a^3c^7(cx)^{3/2}}$$

output
```
-2/15*(b*x^3+a)^(1/2)/a/c/(c*x)^(15/2)+8/45*b*(b*x^3+a)^(1/2)/a^2/c^4/(c*x
)^(9/2)-16/45*b^2*(b*x^3+a)^(1/2)/a^3/c^7/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\sqrt{a+bx^3}(3a^2-4abx^3+8b^2x^6)}{45a^3(cx)^{17/2}}$$

input
```
Integrate[1/((c*x)^(17/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*Sqrt[a + b*x^3]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*(c*x)^(17/2
))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{805}$$

$$-\frac{4\int \frac{\sqrt{bx^3+a}}{(cx)^{17/2}}dx}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{15/2}}$$

$$\Big\downarrow \text{805}$$

$$-\frac{4\left(-\frac{2\int \frac{\left(bx^3+a\right)^{3/2}}{(cx)^{17/2}}dx}{3a}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{15/2}}\right)}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{15/2}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{4\left(\frac{4\left(a+bx^3\right)^{5/2}}{45a^2c(cx)^{15/2}}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{15/2}}\right)}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{15/2}}$$

input
```
Int[1/((c*x)^(17/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(3*a*c*(c*x)^(15/2)) - (4*((-2*(a + b*x^3)^(3/2))/(9*
a*c*(c*x)^(15/2)) + (4*(a + b*x^3)^(5/2))/(45*a^2*c*(c*x)^(15/2))))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\sqrt{b\,x^{3}+a}\,\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45a^{3}\left(cx\right)^{\frac{17}{2}}}$	42
orering	$-\frac{2x\sqrt{b\,x^{3}+a}\,\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45a^{3}\left(cx\right)^{\frac{17}{2}}}$	42
default	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45x^{7}a^{3}c^{8}\sqrt{cx}}$	47
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(8b^{2}x^{6}-4ab\,x^{3}+3a^{2}\right)}{45x^{7}a^{3}c^{8}\sqrt{cx}}$	47
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{15a\,c^{9}x^{8}}+\frac{8b\sqrt{bc\,x^{4}+acx}}{45a^{2}c^{9}x^{5}}-\frac{16b^{2}\sqrt{bc\,x^{4}+acx}}{45a^{3}c^{9}x^{2}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	105

input
```
int(1/(c*x)^(17/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(b*x^3+a)^(1/2)*(8*b^2*x^6-4*a*b*x^3+3*a^2)/a^3/(c*x)^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,(8\,b^2x^6-4\,abx^3+3\,a^2)\sqrt{bx^3+a}\sqrt{cx}}{45\,a^3c^9x^8}$$

input
```
integrate(1/(c*x)^(17/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^3 + a)*sqrt(c*x)/(a^3*c^9*x
^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(17/2)/(b*x**3+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,\left(8\,b^3\sqrt{c}x^{10}+4\,ab^2\sqrt{c}x^7-a^2b\sqrt{c}x^4+3\,a^3\sqrt{c}x\right)}{45\,\sqrt{bx^3+a}a^3c^9x^{\frac{17}{2}}}$$

input
```
integrate(1/(c*x)^(17/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/45*(8*b^3*sqrt(c)*x^10 + 4*a*b^2*sqrt(c)*x^7 - a^2*b*sqrt(c)*x^4 + 3*a^
3*sqrt(c)*x)/(sqrt(b*x^3 + a)*a^3*c^9*x^(17/2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = -\frac{2\,c^3\left(\frac{15\,\sqrt{bc+\frac{ac}{x^3}}\,b^2}{a^3c^{10}} - \frac{8\,\sqrt{bc}b^2}{a^3c^{10}} - \frac{10\,\left(bc+\frac{ac}{x^3}\right)^{\frac{3}{2}}bc-3\,\left(bc+\frac{ac}{x^3}\right)^{\frac{5}{2}}}{a^3c^{12}}\right)}{45\,|c|^2}$$

input
```
integrate(1/(c*x)^(17/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
-2/45*c^3*(15*sqrt(b*c + a*c/x^3)*b^2/(a^3*c^10) - 8*sqrt(b*c)*b^2/(a^3*c^
10) - (10*(b*c + a*c/x^3)^(3/2)*b*c - 3*(b*c + a*c/x^3)^(5/2))/(a^3*c^12))
/abs(c)^2
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2}{15\,a\,c^8} - \frac{8\,b\,x^3}{45\,a^2\,c^8} + \frac{16\,b^2\,x^6}{45\,a^3\,c^8}\right)}{x^7\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(17/2)*(a + b*x^3)^(1/2)),x)
```

output
```
-((a + b*x^3)^(1/2)*(2/(15*a*c^8) - (8*b*x^3)/(45*a^2*c^8) + (16*b^2*x^6)/
(45*a^3*c^8)))/(x^7*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{17/2}\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(-8b^2x^6+4ab\,x^3-3a^2)}{45\sqrt{x}\,a^3c^9x^7}$$

input
```
int(1/(c*x)^(17/2)/(b*x^3+a)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - 3*a**2 + 4*a*b*x**3 - 8*b**2*x**6))/(45*sq
rt(x)*a**3*c**9*x**7)
```

3.326 $\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx = -\frac{7ac^5\sqrt{cx}\sqrt{a+bx^3}}{20b^2}+\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b} + \frac{7a^{5/3}c^5\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-7/20*a*c^5*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^2+1/5*c^2*(c*x)^(7/2)*(b*x^3+a)^
(1/2)/b+7/120*a^(5/3)*c^5*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJ
acobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/
(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.32

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^5\sqrt{cx}\left(-7a^2-3abx^3+4b^2x^6+7a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{20b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(11/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^5*Sqrt[c*x]*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {843, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\int \frac{(cx)^{5/2}}{\sqrt{bx^3+a}}dx}{10b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^3\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{4b}\right)}{10b}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b}\right)}{10b}$$

$\downarrow$ 766

$$\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{10b}$$

input
```
Int[(c*x)^(11/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(7/2)*Sqrt[a + b*x^3])/(5*b) - (7*a*c^3*((c^2*Sqrt[c*x]*Sqrt[a
+ b*x^3])/(2*b) - (a^(2/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(
2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt
[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c
*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)
*b*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])
*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(10*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.76

method	result
risch	$-\frac{(-4b x^3+7a) x\sqrt{b x^3+a}\, c^6}{20 b^2\sqrt{cx}} + \frac{7a^2\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{(-a b^2)^{\frac{1}{3}}}{b}\right.}{}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b x^3+a)}\left(\frac{c^5 x^3\sqrt{bc x^4+acx}}{5b}-\frac{7a c^5\sqrt{bc x^4+acx}}{20b^2}+\frac{7a^2c^6\left(\frac{(-a b^2)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)}{\left(-\frac{(-a b^2)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,(-a b^2)^{\frac{1}{3}}}{2b}\right)\left(x-\right.}}}{}\right.}{}$
default	Expression too large to display

input
```
int((c*x)^(11/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-4*b*x^3+7*a)*x*(b*x^3+a)^(1/2)/b^2*c^6/(c*x)^(1/2)+7/20*a^2/b*(1/2
/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b
*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)
))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3)))^(1/2))*c^6*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)
^(1/2)
```

Fricas [F]

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx=\int \frac{(cx)^{\frac{11}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c^5*x^5/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 81.77 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^{\frac{11}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((c*x)**(11/2)/(b*x**3+a)**(1/2),x)
```

output
```
c**(11/2)*x**(13/2)*gamma(13/6)*hyper((1/2, 13/6), (19/6,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(11/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(11/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx=\int \frac{{\left(c\,x\right)}^{11/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c*x)^(11/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(11/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{11/2}}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{c}\,c^{5}\left(-14\sqrt{x}\,\sqrt{b\,x^{3}+a}\,a+8\sqrt{x}\,\sqrt{b\,x^{3}+a}\,b\,x^{3}+7\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{3}+a}}{b\,x^{4}+a x}d x\right)a^{2}\right)}{40b^{2}}$$

input
```
int((c*x)^(11/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**5*( - 14*sqrt(x)*sqrt(a + b*x**3)*a + 8*sqrt(x)*sqrt(a + b*x**
3)*b*x**3 + 7*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x)*a**2))/(40*
b**2)
```

3.327 $\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b} - \frac{a^{2/3}c^2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/2*c^2*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b-1/12*a^(2/3)*c^2*(c*x)^(1/2)*(a^(1/3
)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))
*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x
)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.30

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^2\sqrt{cx}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{2b\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(5/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*Sqrt[c*x]*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1
/2, 7/6, -((b*x^3)/a)]))/(2*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^3\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{4b}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b} - \frac{a^{2/3}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}$$

input
```
Int[(c*x)^(5/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*Sqrt[c*x]*Sqrt[a + b*x^3])/(2*b) - (a^(2/3)*c*Sqrt[c*x]*(a^(1/3)*c +
b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/
(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (
1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + S
qrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 725, normalized size of antiderivative = 3.03

method	result
risch	$\frac{x\sqrt{b\,x^{3}+a}\,c^{3}}{2b\sqrt{cx}}-\frac{a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)\cdots}{b\left(-\frac{\left(-a\,b^{2}\right)\cdots}{2b}\cdots\right.}}\cdots}{2\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\cdots\right.}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^{3}+a\right)}\left(\frac{c^{2}\sqrt{b c\,x^{4}+a c x}}{2b}-\frac{c^{3}a\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)\cdots}{b}\right.\cdots}{\cdots}\right.\cdots}{\cdots}$
default	Expression too large to display

input
```
int((c*x)^(5/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*x*(b*x^3+a)^(1/2)*c^3/(c*x)^(1/2)-1/2*a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(
1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*c*x
*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),
((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))*c^3*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx=\int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c^2*x^2/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx=\frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(a)*gamma(13/6))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx=\int \frac{{\left(c\,x\right)}^{5/2}}{\sqrt{b\,x^3+a}} \,d x$$

input
```
int((c*x)^(5/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{c}\,c^{2} \left(2 \sqrt{x}\, \sqrt{b \,x^{3}+a}-\left(\int \frac{\sqrt{x}\, \sqrt{b \,x^{3}+a}}{b \,x^{4}+a x}d x \right) a \right)}{4 b}$$

input
```
int((c*x)^(5/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**2*(2*sqrt(x)*sqrt(a + b*x**3) - int((sqrt(x)*sqrt(a + b*x**3))
/(a*x + b*x**4),x)*a))/(4*b)
```

3.328 $\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 204

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
1/3*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3
)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*
2^(1/2))*3^(3/4)/a^(1/3)/c/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1
/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)}{\sqrt{cx}\sqrt{a+bx^3}}$$

input
```
Integrate[1/(Sqrt[c*x]*Sqrt[a + b*x^3]),x]
```

output
```
(2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/(
Sqrt[c*x]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 851$$

$$\frac{2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \; 766$$

$$\frac{\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}c^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}$$

input
```
Int[1/(Sqrt[c*x]*Sqrt[a + b*x^3]),x]
```

output
```
(Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c
^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Ellipti
cF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3
])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(1/3)*c^2*Sqrt[(b^(1/3)*c*x
*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqr
t[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.12

method	result
default	$-\frac{4\sqrt{b x^{3}+a}\, x c \sqrt{-\frac{\left(i \sqrt{3}-3\right) x b}{\left(i \sqrt{3}-1\right) \left(-b x+\left(-a b^{2}\right)^{\frac{1}{3}}\right)}}\, \sqrt{\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}+2 b x+\left(-a b^{2}\right)^{\frac{1}{3}}}{\left(1+i \sqrt{3}\right) \left(-b x+\left(-a b^{2}\right)^{\frac{1}{3}}\right)}}\, \sqrt{\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}-2 b x-\left(-a b^{2}\right)^{\frac{1}{3}}}{\left(i \sqrt{3}-1\right) \left(-b x+\left(-a b^{2}\right)^{\frac{1}{3}}\right)}}\, \mathrm{EllipticF}\left(\sqrt{-\cdots}\right.}{b \left(-a b^{2}\right)^{\frac{1}{3}} \sqrt{c x}\, \sqrt{c x \left(b x^{3}+a\right)}\, \left(i \sqrt{3}-3\right) \sqrt{\frac{c x \left(-b x+\left(-a b^{2}\right)^{\frac{1}{3}}\right) \left(\cdots\right.}{\cdots}}}$
elliptic	$\frac{2 \sqrt{c x \left(b x^{3}+a\right)}\, \left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{\frac{\left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \left(x -\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\, \left(x -\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2} \sqrt{\frac{\left(-a b^{2}\right)^{\frac{1}{3}} \left(\cdots\right.}{b \left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2 b} \cdots\right.}}}{\sqrt{c x}\, \sqrt{b x^{3}+a}\, \left(-\frac{3 \left(-a b^{2}\right)^{\frac{1}{3}}}{2 b}+\cdots\right.}$

input
```
int(1/(c*x)^(1/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(b*x^3+a)^(1/2)*x/b/(-a*b^2)^(1/3)*c*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*3^(1/2)*b^2*x^2-
2*I*3^(1/2)*(-a*b^2)^(1/3)*b*x+I*3^(1/2)*(-a*b^2)^(2/3)-b^2*x^2+2*(-a*b^2)
^(1/3)*b*x-(-a*b^2)^(2/3))/(c*x)^(1/2)/(c*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)
/(1/b^2*c*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = -\frac{2\,\sqrt{ac}\mathrm{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)}{ac}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*c)*weierstrassPInverse(0, -4*b/a, 1/x)/(a*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{1}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\sqrt{c}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

output
```
sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*
sqrt(a)*sqrt(c)*gamma(7/6))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^4+ax}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x + b*x**4),x))/c
```

3.329 $\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 237

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}} - \frac{2b\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}c^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/5*(b*x^3+a)^(1/2)/a/c/(c*x)^(5/2)-2/15*b*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)/c^4/(b^(1
/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.24

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5(cx)^{7/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])
/(5*(c*x)^(7/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {847, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{5ac^3}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{4b\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{5ac^4}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{2b\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}c^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}$$

input
```
Int[1/((c*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(5*a*c*(c*x)^(5/2)) - (2*b*Sqrt[c*x]*(a^(1/3)*c + b^(
1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^
(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 -
 Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt
[3])/4])/(5*3^(1/4)*a^(4/3)*c^5*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x
))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.09

method	result
risch	$-\frac{2\sqrt{b x^3+a}}{5 a x^2 c^3 \sqrt{cx}}-\frac{4b^2\left(\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)^2\sqrt{\frac{\left(-a\cdots\right.}{b\left(-\frac{\left(-\cdots\right.}{}\right)}}\cdots}{5a\left(-\frac{3\left(-a b^2\right)\cdots}{2b}\cdots\right.}$
elliptic	$\frac{\sqrt{cx\left(b x^3+a\right)}\left(-\frac{2\sqrt{b c x^4+a c x}}{5 a c^4 x^3}-\frac{4b^2\left(\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)^2\cdots}{5a\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(c*x)^(7/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/a*(b*x^3+a)^(1/2)/x^2/c^3/(c*x)^(1/2)-4/5*b^2/a*(1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a
*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(
1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)
/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))
^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b
^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
))^(1/2))/c^3*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx = \frac{2\left(2\sqrt{ac}bx^3\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^3+a}\sqrt{cx}a\right)}{5\,a^2c^4x^3}$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(2*sqrt(a*c)*b*x^3*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b*x^3 +
a)*sqrt(c*x)*a)/(a^2*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, \frac{1}{2} \\ \frac{1}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*c**(7/2)*x**(5/2)*gamma(1/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{7/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^3}}\,dx=\frac{\sqrt{c}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^7+a\,x^4}dx\right)}{c^4}$$

input
```
int(1/(c*x)^(7/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**4 + b*x**7),x))/c**4
```

3.330 $\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}+\frac{16b\sqrt{a+bx^3}}{55a^2c^4(cx)^{5/2}} + \frac{16b^2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}c^7\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/11*(b*x^3+a)^(1/2)/a/c/(c*x)^(11/2)+16/55*b*(b*x^3+a)^(1/2)/a^2/c^4/(c*
x)^(5/2)+16/165*b^2*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiA
M(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),
1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/c^7/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{1}{2},-\frac{5}{6},-\frac{bx^3}{a}\right)}{11(cx)^{13/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(13/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)
])/(11*(c*x)^(13/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {847, 847, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{8b\int \frac{1}{(cx)^{7/2}\sqrt{bx^3+a}}dx}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{8b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{5ac^3}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{8b\left(-\frac{4b\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{5ac^4}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}$$

$\Big\downarrow$ 766

$$-\frac{8b\left(-\frac{2b\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}c^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}$$

input
```
Int[1/((c*x)^(13/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(11*a*c*(c*x)^(11/2)) - (8*b*((-2*Sqrt[a + b*x^3])/(5
*a*c*(c*x)^(5/2)) - (2*b*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)
*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])
*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/
(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(
4/3)*c^5*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sq
rt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(11*a*c^3)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.24 (sec) , antiderivative size = 742, normalized size of antiderivative = 2.77

method	result
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-8b\,x^{3}+5a\right)}{55a^{2}x^{5}c^{6}\sqrt{cx}}+\frac{32b^{3}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right.}{55a}$
elliptic	$\frac{\sqrt{cx(b\,x^{3}+a)}\left(-\frac{2\sqrt{b c\,x^{4}+a c x}}{11a\,c^{7}x^{6}}+\frac{16b\sqrt{b c\,x^{4}+a c x}}{55a^{2}c^{7}x^{3}}+\frac{32b^{3}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(c*x)^(13/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/55*(b*x^3+a)^(1/2)*(-8*b*x^3+5*a)/a^2/x^5/c^6/(c*x)^(1/2)+32/55*b^3/a^2
*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2
*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-
1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(
x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))/c^6*(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x
^3+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = -\frac{2\left(16\,\sqrt{ac}b^2x^6\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)-\left(8\,abx^3-5\,a^2\right)\sqrt{bx^3+a}\sqrt{cx}\right)}{55\,a^3c^7x^6}$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/55*(16*sqrt(a*c)*b^2*x^6*weierstrassPInverse(0, -4*b/a, 1/x) - (8*a*b*x
^3 - 5*a^2)*sqrt(b*x^3 + a)*sqrt(c*x))/(a^3*c^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(13/2)/(b*x**3+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(13/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{(c\,x)^{13/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/((c*x)^(13/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(13/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^{10}+a\,x^7}dx\right)}{c^7}$$

input
```
int(1/(c*x)^(13/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**7 + b*x**10),x))/c**7
```

3.331 $\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 553

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx = -\frac{11ac^5(cx)^{5/2}\sqrt{a+bx^3}}{56b^2} + \frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b} + \frac{55\left(1+\sqrt{3}\right)a^2c^7\sqrt{cx}\sqrt{a+bx^3}}{112b^{8/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{55\sqrt[4]{3}a^{7/3}c^7\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112b^{8/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{55\left(1-\sqrt{3}\right)a^{7/3}c^7\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\right.}{224\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-11/56*a*c^5*(c*x)^(5/2)*(b*x^3+a)^(1/2)/b^2+1/7*c^2*(c*x)^(11/2)*(b*x^3+a
)^(1/2)/b+55/112*(1+3^(1/2))*a^2*c^7*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^(8/3)/(
a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-55/112*3^(1/4)*a^(7/3)*c^7*(c*x)^(1/2)*(a^(
1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^
(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(8/3)/(b^
(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^
3+a)^(1/2)-55/672*(1-3^(1/2))*a^(7/3)*c^7*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2
)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1
+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(8/3)/(b^(1/3)*x*
(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^5(cx)^{5/2}\left(-11a^2-3abx^3+8b^2x^6+11a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{56b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(15/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^5*(c*x)^(5/2)*(-11*a^2 - 3*a*b*x^3 + 8*b^2*x^6 + 11*a^2*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(56*b^2*Sqrt[a + b*
x^3])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {843, 843, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \ 843$$

$$\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\int \frac{(cx)^{9/2}}{\sqrt{bx^3+a}}dx}{14b}$$

$$\Big\downarrow \ 843$$

$$\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}\right)}{14b}$$

$$\Big\downarrow \ 851$$

$$\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{4b}\right)}{14b}$$

$$\Big\downarrow \ 837$$

$$\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)}{14b}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)}{14b}}{}$$

$\downarrow$ 766

$$\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{Elli}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}\right.}{\left(\sqrt[3]{a}c+\left(1+\sqrt{\right.}\right.}}}\right)}{4b}\right)}{14b}$$

$\downarrow$ 2420

$$\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{2b^{2/3}}\right)}{}$$

input
```
Int[(c*x)^(15/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(11/2)*Sqrt[a + b*x^3])/(7*b) - (11*a*c^3*((c^2*(c*x)^(5/2)*Sqr
t[a + b*x^3])/(4*b) - (5*a*c^2*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x
^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x
]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^
(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos
[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3
)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(
a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) -
((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)
*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])
*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/
(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(
2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3
])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(4*b)))/(14*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 1127, normalized size of antiderivative = 2.04

method	result	size
risch	Expression too large to display	1127
elliptic	Expression too large to display	1148
default	Expression too large to display	2825

input
```
int((c*x)^(15/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/56*x^3*(-8*b*x^3+11*a)*(b*x^3+a)^(1/2)/b^2*c^8/(c*x)^(1/2)+55/112*a^2/b
^2*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*
b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/
3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^
2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b
/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-
1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1...
```

Fricas [F]

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{15}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(15/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c^7*x^7/sqrt(b*x^3 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(15/2)/(b*x**3+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{15}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(15/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(15/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{15}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(15/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(15/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx=\int \frac{(c\,x)^{15/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c*x)^(15/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(15/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{15/2}}{\sqrt{a+bx^3}}\,dx=\frac{\sqrt{c}\,c^7\left(-22\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,x^2+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^5+55\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a^2\right)}{112b^2}$$

input
```
int((c*x)^(15/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**7*( - 22*sqrt(x)*sqrt(a + b*x**3)*a*x**2 + 16*sqrt(x)*sqrt(a +
 b*x**3)*b*x**5 + 55*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)*a**2
))/(112*b**2)
```

3.332 $\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 522

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b} - \frac{5\left(1+\sqrt{3}\right)ac^4\sqrt{cx}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{5\sqrt[4]{3}a^{4/3}c^4\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{5\left(1-\sqrt{3}\right)a^{4/3}c^4\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\cdots}{16\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
1/4*c^2*(c*x)^(5/2)*(b*x^3+a)^(1/2)/b-5/8*(1+3^(1/2))*a*c^4*(c*x)^(1/2)*(b
*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)+5/8*3^(1/4)*a^(4/3)*
c^4*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/
2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/
3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+5/48*(1-3^(1/2))*a^(4/3)*c^4*(c*x)^(1/2)*(a
^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(
1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1
/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b
^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(
1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.14

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^2(cx)^{5/2}\left(a+bx^3-a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{4b\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(9/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(5/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2,
 5/6, 11/6, -((b*x^3)/a)]))/(4*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {843, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx$$

$\Big\downarrow$ 843

$$\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}$$

$\Big\downarrow$ 851

$$\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{4b}$$

$\Big\downarrow$ 837

$$\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}$$

$\Big\downarrow$ 25

$$\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}$$

$\Big\downarrow$ 766

$$\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-$$

$$\frac{5ac^2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{4b}$$

$\Big\downarrow$ 2420

$$\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b} - \frac{5ac^2\left(\frac{\frac{(1+\sqrt{3})c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx} - \frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}(\sqrt[3]{a}c+\sqrt[3]{b}cx)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}{(1+\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx(\sqrt[3]{a}c+\sqrt[3]{b}cx)}{(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx)^2}}}}{2b^{2/3}} - \right)}{4b}$$

input
```
Int[(c*x)^(9/2)/Sqrt[a + b*x^3],x]
```

output
```
(c^2*(c*x)^(5/2)*Sqrt[a + b*x^3])/(4*b) - (5*a*c^2*((((1 + Sqrt[3])*c^3*Sq
rt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4
)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3
)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)
^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c +
(1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)
*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x
^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3
)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/
3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sq
rt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3]
)/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.14

method	result	size
risch	Expression too large to display	1115
elliptic	Expression too large to display	1121
default	Expression too large to display	2601

input
```
int((c*x)^(9/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^3/b*(b*x^3+a)^(1/2)*c^5/(c*x)^(1/2)-5/8*a/b*(x*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(
-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF((
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2...
```

Fricas [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c^4*x^4/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 31.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**3+a)**(1/2),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/6)*hyper((1/2, 11/6), (17/6,), b*x**3*exp_pola
r(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(c\,x)^{9/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\,c^4\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)a\right)}{8b}$$

input
```
int((c*x)^(9/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*c**4*(2*sqrt(x)*sqrt(a + b*x**3)*x**2 - 5*int((sqrt(x)*sqrt(a + b
*x**3)*x)/(a + b*x**3),x)*a))/(8*b)
```

3.333 $\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 482

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \frac{\left(1+\sqrt{3}\right)c\sqrt{cx}\sqrt{a+bx^3}}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{2\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
(1+3^(1/2))*c*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(
1/3)*x)-3^(1/4)*a^(1/3)*c*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Elliptic
E((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-1/6*(1-3^(1/2))*a^(1/
3)*c*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/
3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4
*2^(1/2))*3^(3/4)/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.12

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \frac{2x(cx)^{3/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\frac{bx^3}{a}\right)}{5\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(3/2)/Sqrt[a + b*x^3],x]
```

output
```
(2*x*(c*x)^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((
b*x^3)/a)])/(5*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 851$$

$$\frac{2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \; 837$$

$$\frac{2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{c}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{c}$$

$$\Big\downarrow \; 766$$

$$\frac{2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\ldots}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\ldots\right.}{c}$$

$$\Big\downarrow \; 2420$$

$$\frac{2\left(\frac{\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}}-\ldots\right.}{c}$$

input
```
Int[(c*x)^(3/2)/Sqrt[a + b*x^3],x]
```

output
```
(2*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[
3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*
Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c +
(1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*
b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(
Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^
(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqr
t[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x
 + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[A
rcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b
^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1
/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a +
b*x^3])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 1085, normalized size of antiderivative = 2.25

method	result	size
elliptic	Expression too large to display	1085
default	Expression too large to display	2370

input
```
int((c*x)^(3/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(x*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1
/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2
/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*Ellipti
cF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a...
```

Fricas [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c*x/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(a)*gamma(11/6))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \int \frac{(c\,x)^{3/2}}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^3)^(1/2),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^3}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b\,x^3+a}dx\right)c$$

input
```
int((c*x)^(3/2)/(b*x^3+a)^(1/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a + b*x**3),x)*c
```

3.334 $\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 516

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}+\frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\sqrt{cx}\sqrt{a+bx^3}}{ac^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$-\frac{2\sqrt[4]{3}\sqrt[3]{b}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{a^{2/3}c^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}a^{2/3}c^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2*(b*x^3+a)^(1/2)/a/c/(c*x)^(1/2)+2*(1+3^(1/2))*b^(1/3)*(c*x)^(1/2)*(b*x^
3+a)^(1/2)/a/c^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-2*3^(1/4)*b^(1/3)*(c*x)^(
1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)
+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)
*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^
(2/3)/c^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2
)^(1/2)/(b*x^3+a)^(1/2)-1/3*(1-3^(1/2))*b^(1/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/
3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/c^2/(
b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*
x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.10

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{(cx)^{3/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])
/((c*x)^(3/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 847$$

$$\frac{2b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{ac^3}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}$$

$$\Big\downarrow \; 851$$

$$\frac{4b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}$$

$$\Big\downarrow \; 837$$

$$\frac{4b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}$$

$$\Big\downarrow \; 25$$

$$\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}$$

$$\Big\downarrow \; 766$$

$$\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{ac^4}$$

$$\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}$$

$$\Big\downarrow \; 2420$$

$$\frac{4b\left(\frac{\frac{(1+\sqrt{3})c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}{(1+\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}}-\cdots\right)}{ac^4}$$

$$\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}$$

input

```
Int[1/((c*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

output

```
(-2*Sqrt[a + b*x^3])/(a*c*Sqrt[c*x]) + (4*b*((((1 + Sqrt[3])*c^3*Sqrt[c*x]
*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/
3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/
3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Ell
ipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sq
rt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^
(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(
2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*
Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c +
(1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*
b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(
4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c
+ (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))))/(a*c^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 1112, normalized size of antiderivative = 2.16

method	result	size
risch	Expression too large to display	1112
elliptic	Expression too large to display	1123
default	Expression too large to display	2881

input
```
int(1/(c*x)^(3/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(b*x^3+a)^(1/2)/a/c/(c*x)^(1/2)+2*b/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(
1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1...
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{2\,\sqrt{ac}\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)}{ac^2}$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*c)*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x)
)/(a*c^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{1}{2}\\ \frac{5}{6}\end{matrix}\middle|\,\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a
)*c**(3/2)*sqrt(x)*gamma(5/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^3}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b\,x^5+a\,x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**2 + b*x**5),x))/c**2
```

3.335 $\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 553

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx = -\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}} + \frac{8b\sqrt{a+bx^3}}{7a^2c^4\sqrt{cx}} - \frac{8\left(1+\sqrt{3}\right)b^{4/3}\sqrt{cx}\sqrt{a+bx^3}}{7a^2c^5\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{8\sqrt[4]{3}b^{4/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7a^{5/3}c^5\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{4\left(1-\sqrt{3}\right)b^{4/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right.}{7\sqrt[4]{3}a^{5/3}c^5\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/7*(b*x^3+a)^(1/2)/a/c/(c*x)^(7/2)+8/7*b*(b*x^3+a)^(1/2)/a^2/c^4/(c*x)^(
1/2)-8/7*(1+3^(1/2))*b^(4/3)*(c*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/c^5/(a^(1/3)+
(1+3^(1/2))*b^(1/3)*x)+8/7*3^(1/4)*b^(4/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^
2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2
))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(5/3)/c^5/(b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+
4/21*(1-3^(1/2))*b^(4/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJa
cobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)/c^5/(b^(1/3)*x*(a^(1/3)+b^(1
/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.10

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx = -\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},-\frac{bx^3}{a}\right)}{7(cx)^{9/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(9/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)]
)/(7*(c*x)^(9/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {847, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \ 847$$

$$-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+a}}dx}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 847$$

$$-\frac{4b\left(\frac{2b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{ac^3}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 851$$

$$-\frac{4b\left(\frac{4b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 837$$

$$-\frac{4b\left(\frac{4b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$-\frac{4b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 766$$

$$-\frac{4b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x c}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{ac^4}\right.}{7ac^3}$$

$$\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

$\downarrow$ 2420

$$-\frac{4b\left(\frac{4b\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\cdots}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\cdots}}{2b^{2/3}\cdots}\cdots}{ac^4\cdots}\cdots}{7a\cdots}$$

$$\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}$$

input

```
Int[1/((c*x)^(9/2)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*Sqrt[a + b*x^3])/(7*a*c*(c*x)^(7/2)) - (4*b*((-2*Sqrt[a + b*x^3])/(a*c
*Sqrt[c*x]) + (4*b*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3
)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c
 + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^
2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c
+ (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2
+ Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c +
(1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3
])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/
3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x
)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c +
 (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b
^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c
*x)^2]*Sqrt[a + b*x^3])))/(a*c^4)))/(7*a*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.29 (sec) , antiderivative size = 1125, normalized size of antiderivative = 2.03

method	result	size
risch	Expression too large to display	1125
elliptic	Expression too large to display	1175
default	Expression too large to display	3074

input
```
int(1/(c*x)^(9/2)/(b*x^3+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(b*x^3+a)^(1/2)*(-4*b*x^3+a)/a^2/x^3/c^4/(c*x)^(1/2)-8/7*b^2/a^2*(x*(
x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x
/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)
^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b
^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^
2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^
(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.10

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx = -\frac{2\left(4\sqrt{ac}bx^4\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\sqrt{bx^3+a}\sqrt{cx}a\right)}{7a^2c^5x^4}$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/7*(4*sqrt(a*c)*b*x^4*weierstrassZeta(0, -4*b/a, weierstrassPInverse(0,
-4*b/a, 1/x)) + sqrt(b*x^3 + a)*sqrt(c*x)*a)/(a^2*c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.54 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{7}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{6}, \frac{1}{2} \\ -\frac{1}{6} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)}$$

input
```
integrate(1/(c*x)**(9/2)/(b*x**3+a)**(1/2),x)
```

output
```
gamma(-7/6)*hyper((-7/6, 1/2), (-1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*c**(9/2)*x**(7/2)*gamma(-1/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx = \int \frac{1}{\sqrt{bx^3+a}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(c*x)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{9/2}\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(1/((c*x)^(9/2)*(a + b*x^3)^(1/2)),x)
```

output
```
int(1/((c*x)^(9/2)*(a + b*x^3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^3}}\,dx=\frac{\sqrt{c}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{3}+a}}{b\,x^{8}+a\,x^{5}}d x \right)}{c^{5}}$$

input
```
int(1/(c*x)^(9/2)/(b*x^3+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a*x**5 + b*x**8),x))/c**5
```

3.336 $\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}} - \frac{5ac^8(cx)^{3/2}\sqrt{a+bx^3}}{4b^3} + \frac{5c^5(cx)^{9/2}\sqrt{a+bx^3}}{6b^2} + \frac{5a^2c^{19/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{4b^{7/2}}$$

output
```
-2/3*c^2*(c*x)^(15/2)/b/(b*x^3+a)^(1/2)-5/4*a*c^8*(c*x)^(3/2)*(b*x^3+a)^(1
/2)/b^3+5/6*c^5*(c*x)^(9/2)*(b*x^3+a)^(1/2)/b^2+5/4*a^2*c^(19/2)*arctanh(b
^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^9\sqrt{cx}\left(\frac{\sqrt{b}x^{3/2}(-15a^2-5abx^3+2b^2x^6)}{\sqrt{a+bx^3}} + 15a^2\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{12b^{7/2}\sqrt{x}}$$

input
```
Integrate[(c*x)^(19/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^9*Sqrt[c*x]*((Sqrt[b]*x^(3/2)*(-15*a^2 - 5*a*b*x^3 + 2*b^2*x^6))/Sqrt[a
 + b*x^3] + 15*a^2*Log[Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(12*b^(7/2)*Sq
rt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {817, 843, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5c^3\int \frac{(cx)^{13/2}}{\sqrt{bx^3+a}}dx}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{5c^3\left(\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\int \frac{(cx)^{7/2}}{\sqrt{bx^3+a}}dx}{4b}\right)}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{5c^3\left(\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{2b}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{5c^3\left(\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{b}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow 807$$

$$\frac{5c^3\left(\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3b}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow 224$$

$$\frac{5c^3\left(\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3b}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow 219$$

$$\frac{5c^3\left(\frac{c^2(cx)^{9/2}\sqrt{a+bx^3}}{6b}-\frac{3ac^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3b^{3/2}}\right)}{4b}\right)}{b}-\frac{2c^2(cx)^{15/2}}{3b\sqrt{a+bx^3}}$$

input

```
Int[(c*x)^(19/2)/(a + b*x^3)^(3/2),x]
```

output

```
(-2*c^2*(c*x)^(15/2))/(3*b*Sqrt[a + b*x^3]) + (5*c^3*((c^2*(c*x)^(9/2)*Sqr
t[a + b*x^3])/(6*b) - (3*a*c^3*((c^2*(c*x)^(3/2)*Sqrt[a + b*x^3])/(3*b) -
(a*c^(7/2)*ArcTanh[(Sqrt[b]*(c*x)^(3/2))/(c^(3/2)*Sqrt[a + (b*x)/c^2])])/(
3*b^(3/2))))/(4*b)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{c^{9} \sqrt{c x}\, \left(2 \sqrt{b c}\, b^{2} x^{8}-5 \sqrt{b c}\, a b \,x^{5}-15 a^{2} x^{2} \sqrt{b c}+15 \,\operatorname{arctanh}\left(\frac{\sqrt{c x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right) a^{2} \sqrt{c x \left(b \,x^{3}+a \right)}\right)}{12 x \sqrt{b \,x^{3}+a}\, b^{3} \sqrt{b c}}$	109
risch	$-\frac{x^{2} \left(-2 b \,x^{3}+7 a \right) \sqrt{b \,x^{3}+a}\, c^{10}}{12 b^{3} \sqrt{c x}}+\frac{a^{2} \left(\frac{10 \,\operatorname{arctanh}\left(\frac{\sqrt{c x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right)}{\sqrt{b c}}-\frac{16 x^{2}}{3 \sqrt{\left(x^{3}+\frac{a}{b}\right) b c x}}\right) c^{10} \sqrt{c x \left(b \,x^{3}+a \right)}}{8 b^{3} \sqrt{c x}\, \sqrt{b \,x^{3}+a}}$	124
elliptic	Expression too large to display	1098

input
```
int((c*x)^(19/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*c^9/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(2*(b*c)^(1/2)*b^2*x^8-5*(b*c)^(1/2
)*a*b*x^5-15*a^2*x^2*(b*c)^(1/2)+15*arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b*c
)^(1/2))*a^2*(c*x*(b*x^3+a))^(1/2))/b^3/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.17

$$\int \frac{(cx)^{19/2}}{\left(a+bx^3\right)^{3/2}}\,dx=\left[\frac{15\left(a^2bc^9x^3+a^3c^9\right)\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{c}\right.}{48\left(b^4x^3+ab^3\right)}\right.$$

input
```
integrate((c*x)^(19/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(15*(a^2*b*c^9*x^3 + a^3*c^9)*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x
^3 - a^2*c - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)) +
4*(2*b^2*c^9*x^7 - 5*a*b*c^9*x^4 - 15*a^2*c^9*x)*sqrt(b*x^3 + a)*sqrt(c*x)
)/(b^4*x^3 + a*b^3), -1/24*(15*(a^2*b*c^9*x^3 + a^3*c^9)*sqrt(-c/b)*arctan
(2*sqrt(b*x^3 + a)*sqrt(c*x)*b*x*sqrt(-c/b)/(2*b*c*x^3 + a*c)) - 2*(2*b^2*
c^9*x^7 - 5*a*b*c^9*x^4 - 15*a^2*c^9*x)*sqrt(b*x^3 + a)*sqrt(c*x))/(b^4*x^
3 + a*b^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(19/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{19}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(19/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(19/2)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{5\,a^2c^{12}\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{4\,\sqrt{bc}b^3|c|^2} - \frac{\left(\frac{15\,a^2c^{11}}{b^3}-\left(\frac{2\,c^8x^3}{b}-\frac{5\,ac^8}{b^2}\right)c^3x^3\right)\sqrt{cx}x}{12\,\sqrt{bc^4x^3+ac^4}}$$

input
```
integrate((c*x)^(19/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-5/4*a^2*c^12*log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))
/(sqrt(b*c)*b^3*abs(c)^2) - 1/12*(15*a^2*c^11/b^3 - (2*c^8*x^3/b - 5*a*c^8
/b^2)*c^3*x^3)*sqrt(c*x)*x/sqrt(b*c^4*x^3 + a*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{19/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(19/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(19/2)/(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int \frac{(cx)^{19/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^9\left(-30\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2bx-10\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,b^2x^4+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b^3x^7-15\sqrt{b}\right.}{}$$

input
```
int((c*x)^(19/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**9*( - 30*sqrt(x)*sqrt(a + b*x**3)*a**2*b*x - 10*sqrt(x)*sqrt(a
 + b*x**3)*a*b**2*x**4 + 4*sqrt(x)*sqrt(a + b*x**3)*b**3*x**7 - 15*sqrt(b)
*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**3 - 15*sqrt(b)*log(sqrt(a +
b*x**3) - sqrt(x)*sqrt(b)*x)*a**2*b*x**3 + 15*sqrt(b)*log(sqrt(a + b*x**3)
 + sqrt(x)*sqrt(b)*x)*a**3 + 15*sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqr
t(b)*x)*a**2*b*x**3))/(24*b**4*(a + b*x**3))
```

3.337 $\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx=-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}+\frac{c^5(cx)^{3/2}\sqrt{a+bx^3}}{b^2}-\frac{ac^{13/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{b^{5/2}}$$

output
```
-2/3*c^2*(c*x)^(9/2)/b/(b*x^3+a)^(1/2)+c^5*(c*x)^(3/2)*(b*x^3+a)^(1/2)/b^2
-a*c^(13/2)*arctanh(b^(1/2)*(c*x)^(3/2)/c^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx=\frac{c^6\sqrt{cx}\left(\frac{\sqrt{b}x^{3/2}(3a+bx^3)}{\sqrt{a+bx^3}}-3a\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{5/2}\sqrt{x}}$$

input
```
Integrate[(c*x)^(13/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^6*Sqrt[c*x]*((Sqrt[b]*x^(3/2)*(3*a + b*x^3))/Sqrt[a + b*x^3] - 3*a*Log[
Sqrt[b]*x^(3/2) + Sqrt[a + b*x^3]]))/(3*b^(5/2)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {817, 843, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{3c^3\int \frac{(cx)^{7/2}}{\sqrt{bx^3+a}}dx}{b}-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{3c^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{2b}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{3c^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{b}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{807}$$

$$\frac{3c^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3b}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{224}$$

$$\frac{3c^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3b}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3c^3\left(\frac{c^2(cx)^{3/2}\sqrt{a+bx^3}}{3b}-\frac{ac^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3b^{3/2}}\right)}{b}-\frac{2c^2(cx)^{9/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(13/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(9/2))/(3*b*Sqrt[a + b*x^3]) + (3*c^3*((c^2*(c*x)^(3/2)*Sqrt
[a + b*x^3])/(3*b) - (a*c^(7/2)*ArcTanh[(Sqrt[b]*(c*x)^(3/2))/(c^(3/2)*Sqr
t[a + (b*x)/c^2])])/(3*b^(3/2))))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{c^{6} \sqrt{c x}\, \left(-\sqrt{b c}\, b \,x^{5}-3 a \,x^{2} \sqrt{b c}+3 \,\operatorname{arctanh}\left(\frac{\sqrt{c x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right) a \sqrt{c x \left(b \,x^{3}+a \right)}\right)}{3 x \sqrt{b \,x^{3}+a}\, b^{2} \sqrt{b c}}$	91
risch	$\frac{x^{2} \sqrt{b \,x^{3}+a}\, c^{7}}{3 b^{2} \sqrt{c x}}-\frac{a \left(\frac{2 \,\operatorname{arctanh}\left(\frac{\sqrt{c x \left(b \,x^{3}+a \right)}}{x^{2} \sqrt{b c}}\right)}{\sqrt{b c}}-\frac{4 x^{2}}{3 \sqrt{\left(x^{3}+\frac{a}{b}\right) b c x}}\right) c^{7} \sqrt{c x \left(b \,x^{3}+a \right)}}{2 b^{2} \sqrt{c x}\, \sqrt{b \,x^{3}+a}}$	112
elliptic	Expression too large to display	1069

input
```
int((c*x)^(13/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c^6/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(-(b*c)^(1/2)*b*x^5-3*a*x^2*(b*c)^(
1/2)+3*arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b*c)^(1/2))*a*(c*x*(b*x^3+a))^(1
/2))/b^2/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.63

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx = \left[\frac{3\,(abc^6x^3+a^2c^6)\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c+4\,(2\,b^2x^4+abx)\sqrt{bx^3+a}\sqrt{cx}\right.}{12\,(b^3x^3+ab^2)}\right.$$

input
```
integrate((c*x)^(13/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*b*c^6*x^3 + a^2*c^6)*sqrt(c/b)*log(-8*b^2*c*x^6 - 8*a*b*c*x^3
- a^2*c + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(c*x)*sqrt(c/b)) + 4*(
b*c^6*x^4 + 3*a*c^6*x)*sqrt(b*x^3 + a)*sqrt(c*x))/(b^3*x^3 + a*b^2), 1/6*(
3*(a*b*c^6*x^3 + a^2*c^6)*sqrt(-c/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(c*x)*b*
x*sqrt(-c/b)/(2*b*c*x^3 + a*c)) + 2*(b*c^6*x^4 + 3*a*c^6*x)*sqrt(b*x^3 + a
)*sqrt(c*x))/(b^3*x^3 + a*b^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(13/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{13}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(13/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(13/2)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\left(\frac{cx^3}{b}+\frac{3\,ac}{b^2}\right)\sqrt{cx}c^7x}{3\sqrt{bc^4x^3+ac^4}} + \frac{ac^9\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{\sqrt{bc}b^2|c|^2}$$

input
```
integrate((c*x)^(13/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
1/3*(c*x^3/b + 3*a*c/b^2)*sqrt(c*x)*c^7*x/sqrt(b*c^4*x^3 + a*c^4) + a*c^9*
log(abs(-sqrt(b*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))/(sqrt(b*c)*b^
2*abs(c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{13/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(13/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(13/2)/(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.56

$$\int \frac{(cx)^{13/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^6\left(6\sqrt{x}\,\sqrt{b\,x^3+a}\,abx+2\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^4+3\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a^2}{}$$

input
```
int((c*x)^(13/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**6*(6*sqrt(x)*sqrt(a + b*x**3)*a*b*x + 2*sqrt(x)*sqrt(a + b*x**
3)*b**2*x**4 + 3*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a**2 +
3*sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt(b)*x)*a*b*x**3 - 3*sqrt(b)*l
og(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a**2 - 3*sqrt(b)*log(sqrt(a + b*x
**3) + sqrt(x)*sqrt(b)*x)*a*b*x**3))/(6*b**3*(a + b*x**3))
```

3.338 $\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{3/2}}{3b\sqrt{a+bx^3}}+\frac{2c^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

output
```
-2/3*c^2*(c*x)^(3/2)/b/(b*x^3+a)^(1/2)+2/3*c^(7/2)*arctanh(b^(1/2)*(c*x)^(
3/2)/c^(3/2)/(b*x^3+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx = \frac{2(cx)^{7/2}\left(-\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}+\log\left(\sqrt{b}x^{3/2}+\sqrt{a+bx^3}\right)\right)}{3b^{3/2}x^{7/2}}$$

input
```
Integrate[(c*x)^(7/2)/(a + b*x^3)^(3/2),x]
```

output
```
(2*(c*x)^(7/2)*(-((Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]) + Log[Sqrt[b]*x^(3/2)
 + Sqrt[a + b*x^3]]))/(3*b^(3/2)*x^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {817, 851, 807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{bx^3+a}}dx}{b}-\frac{2c^2(cx)^{3/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2c^2\int \frac{cx}{\sqrt{bx^3+a}}d\sqrt{cx}}{b}-\frac{2c^2(cx)^{3/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2c^2\int \frac{1}{\sqrt{a+\frac{bx}{c^2}}}d(cx)^{3/2}}{3b}-\frac{2c^2(cx)^{3/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{224}$$

$$\frac{2c^2\int \frac{1}{1-\frac{bx}{c^2}}d\frac{(cx)^{3/2}}{\sqrt{a+\frac{bx}{c^2}}}}{3b}-\frac{2c^2(cx)^{3/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2c^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(cx)^{3/2}}{c^{3/2}\sqrt{a+\frac{bx}{c^2}}}\right)}{3b^{3/2}}-\frac{2c^2(cx)^{3/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(7/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(3/2))/(3*b*Sqrt[a + b*x^3]) + (2*c^(7/2)*ArcTanh[(Sqrt[b]*(
c*x)^(3/2))/(c^(3/2)*Sqrt[a + (b*x)/c^2])])/(3*b^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{2c^{3}\sqrt{cx}\,\left(-x^{2}\sqrt{bc}+\operatorname{arctanh}\left(\frac{\sqrt{cx\left(b\,x^{3}+a\right)}}{x^{2}\sqrt{bc}}\right)\sqrt{cx\left(b\,x^{3}+a\right)}\right)}{3x\sqrt{b\,x^{3}+a}\,b\sqrt{bc}}$	77
elliptic	Expression too large to display	1042

input
```
int((c*x)^(7/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*c^3*(c*x)^(1/2)*(-x^2*(b*c)^(1/2)+arctanh((c*x*(b*x^3+a))^(1/2)/x^2/(b
*c)^(1/2))*(c*x*(b*x^3+a))^(1/2))/x/(b*x^3+a)^(1/2)/b/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.01

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx = \left[-\frac{4\sqrt{bx^3+a}\sqrt{cx}c^3x-(bc^3x^3+ac^3)\sqrt{\frac{c}{b}}\log\left(-8\,b^2cx^6-8\,abcx^3-a^2c-4\,(2\,b^2x^4+\right.}{6\,(b^2x^3+ab)}\right.$$

input
```
integrate((c*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/6*(4*sqrt(b*x^3 + a)*sqrt(c*x)*c^3*x - (b*c^3*x^3 + a*c^3)*sqrt(c/b)*l
og(-8*b^2*c*x^6 - 8*a*b*c*x^3 - a^2*c - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 +
 a)*sqrt(c*x)*sqrt(c/b)))/(b^2*x^3 + a*b), -1/3*(2*sqrt(b*x^3 + a)*sqrt(c*
x)*c^3*x + (b*c^3*x^3 + a*c^3)*sqrt(-c/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(c*
x)*b*x*sqrt(-c/b)/(2*b*c*x^3 + a*c)))/(b^2*x^3 + a*b)]
```

Sympy [A] (verification not implemented)

Time = 12.94 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx = \frac{2c^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}} - \frac{2c^{\frac{7}{2}}x^{\frac{3}{2}}}{3\sqrt{a}b\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((c*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

output
```
2*c**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*c**(7/2)*x**(3
/2)/(3*sqrt(a)*b*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{cx}c^5x}{3\sqrt{bc^4x^3+ac^4}b} - \frac{2c^6\log\left(\left|-\sqrt{bc}\sqrt{cx}cx+\sqrt{bc^4x^3+ac^4}\right|\right)}{3\sqrt{bc}b|c|^2}$$

input
```
integrate((c*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
-2/3*sqrt(c*x)*c^5*x/(sqrt(b*c^4*x^3 + a*c^4)*b) - 2/3*c^6*log(abs(-sqrt(b
*c)*sqrt(c*x)*c*x + sqrt(b*c^4*x^3 + a*c^4)))/(sqrt(b*c)*b*abs(c)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(c\,x)^{7/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(7/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(a + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{(cx)^{7/2}}{(a+bx^3)^{3/2}}\,dx=\frac{\sqrt{c}\,c^3\left(-2\sqrt{x}\,\sqrt{b\,x^3+a}\,bx-\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\sqrt{x}\,\sqrt{b}\,x\right)a-\sqrt{b}\log\left(\sqrt{b\,x^3+a}-\right.\right.}{3b^2\,(b}$$

input
```
int((c*x)^(7/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**3*( - 2*sqrt(x)*sqrt(a + b*x**3)*b*x - sqrt(b)*log(sqrt(a + b*
x**3) - sqrt(x)*sqrt(b)*x)*a - sqrt(b)*log(sqrt(a + b*x**3) - sqrt(x)*sqrt
(b)*x)*b*x**3 + sqrt(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*a + sqrt
(b)*log(sqrt(a + b*x**3) + sqrt(x)*sqrt(b)*x)*b*x**3))/(3*b**2*(a + b*x**3
))
```

3.339 $\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2(cx)^{3/2}}{3ac\sqrt{a+bx^3}}$$

output
```
2/3*(c*x)^(3/2)/a/c/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2x\sqrt{cx}}{3a\sqrt{a+bx^3}}$$

input
```
Integrate[Sqrt[c*x]/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*Sqrt[c*x])/(3*a*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 796}$$
$$\frac{2(cx)^{3/2}}{3ac\sqrt{a+bx^3}}$$

input
```
Int[Sqrt[c*x]/(a + b*x^3)^(3/2),x]
```

output
```
(2*(c*x)^(3/2))/(3*a*c*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{2x\sqrt{cx}}{3\sqrt{b\,x^{3}+a}\,a}$	21
default	$\frac{2x\sqrt{cx}}{3\sqrt{b\,x^{3}+a}\,a}$	21
orering	$\frac{2x\sqrt{cx}}{3\sqrt{b\,x^{3}+a}\,a}$	21
elliptic	$\frac{2x\sqrt{cx}\,\sqrt{cx(b\,x^{3}+a)}}{3\sqrt{b\,x^{3}+a}\,a\sqrt{\left(x^{3}+\frac{a}{b}\right)bcx}}$	48

input
```
int((c*x)^(1/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x/(b*x^3+a)^(1/2)/a*(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{bx^3+a}\sqrt{cx}x}{3\,(abx^3+a^2)}$$

input
```
integrate((c*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a)*sqrt(c*x)*x/(a*b*x^3 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{c}x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}$$

input
```
integrate((c*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

output
```
2*sqrt(c)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(b*x^3 + a)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{cx}c^2x}{3\sqrt{bc^4x^3+ac^4}a}$$

input
```
integrate((c*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
2/3*sqrt(c*x)*c^2*x/(sqrt(b*c^4*x^3 + a*c^4)*a)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2\,x\,\sqrt{c\,x}}{3\,a\,\sqrt{b\,x^3+a}}$$

input
```
int((c*x)^(1/2)/(a + b*x^3)^(3/2),x)
```

output
```
(2*x*(c*x)^(1/2))/(3*a*(a + b*x^3)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{cx}}{(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{c}\,\sqrt{b\,x^3+a}\,x}{3a\,(b\,x^3+a)}$$

input
```
int((c*x)^(1/2)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(c)*sqrt(a + b*x**3)*x)/(3*a*(a + b*x**3))
```

3.340 $\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{2}{3ac(cx)^{3/2}\sqrt{a+bx^3}}-\frac{4\sqrt{a+bx^3}}{3a^2c(cx)^{3/2}}$$

output
```
2/3/a/c/(c*x)^(3/2)/(b*x^3+a)^(1/2)-4/3*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2x(a+2bx^3)}{3a^2(cx)^{5/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*(a + 2*b*x^3))/(3*a^2*(c*x)^(5/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}dx$$
$$\Big\downarrow \text{805}$$
$$\frac{2\int \frac{1}{(cx)^{5/2}\sqrt{bx^3+a}}dx}{a}+\frac{2}{3ac(cx)^{3/2}\sqrt{a+bx^3}}$$
$$\Big\downarrow \text{796}$$
$$\frac{2}{3ac(cx)^{3/2}\sqrt{a+bx^3}}-\frac{4\sqrt{a+bx^3}}{3a^2c(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*(c*x)^(3/2)*Sqrt[a + b*x^3]) - (4*Sqrt[a + b*x^3])/(3*a^2*c*(c*x)
^(3/2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{2x(2b\,x^3+a)}{3\sqrt{b\,x^3+a}\,a^2(cx)^{\frac{5}{2}}}$	29
orering	$-\frac{2x(2b\,x^3+a)}{3\sqrt{b\,x^3+a}\,a^2(cx)^{\frac{5}{2}}}$	29
default	$-\frac{2(2b\,x^3+a)}{3x\sqrt{b\,x^3+a}\,a^2c^2\sqrt{cx}}$	34
risch	$-\frac{2\sqrt{b\,x^3+a}}{3a^2x\,c^2\sqrt{cx}}-\frac{2b\,x^2}{3a^2c^2\sqrt{cx}\,\sqrt{b\,x^3+a}}$	53
elliptic	$\frac{\sqrt{cx(b\,x^3+a)}\left(-\frac{2\sqrt{bc\,x^4+acx}}{3a^2c^3x^2}-\frac{2b\,x^2}{3c^2a^2\sqrt{\left(x^3+\frac{a}{b}\right)bcx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^3+a}}$	80

input
```
int(1/(c*x)^(5/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(2*b*x^3+a)/(b*x^3+a)^(1/2)/a^2/(c*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^3)^{3/2}}\,dx=-\frac{2\,(2\,bx^3+a)\sqrt{bx^3+a}\sqrt{cx}}{3\,(a^2bc^3x^5+a^3c^3x^2)}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(2*b*x^3 + a)*sqrt(b*x^3 + a)*sqrt(c*x)/(a^2*b*c^3*x^5 + a^3*c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2}{3a\sqrt{b}c^{\frac{5}{2}}x^3\sqrt{\frac{a}{bx^3}+1}} - \frac{4\sqrt{b}}{3a^2c^{\frac{5}{2}}\sqrt{\frac{a}{bx^3}+1}}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

output
```
-2/(3*a*sqrt(b)*c**(5/2)*x**3*sqrt(a/(b*x**3) + 1)) - 4*sqrt(b)/(3*a**2*c*
*(5/2)*sqrt(a/(b*x**3) + 1))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(5/2)), x)
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx = Recursiveaassumption \geq -\frac{2\,c^3\left(\frac{\sqrt{bc+\frac{ac}{x^3}}}{ac^4}-\frac{\sqrt{bc}}{ac^4}\right)}{3\,a|c|^2} - \frac{2\sqrt{cx}bx}{3\sqrt{bc^4x^3+ac^4}a^2c} - \frac{bignored}{c^3t_{nostep}^6}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
Recursive*a*assumption >= -2/3*c^3*(sqrt(b*c + a*c/x^3)/(a*c^4) - sqrt(b*c
)/(a*c^4))/(a*abs(c)^2) - 2/3*sqrt(c*x)*b*x/(sqrt(b*c^4*x^3 + a*c^4)*a^2*c
) - b*ignored/(c^3*t_nostep^6)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^3)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2}{3\,a\,b\,c^2}+\frac{4\,x^3}{3\,a^2\,c^2}\right)}{x^4\,\sqrt{c\,x}+\frac{a\,x\,\sqrt{c\,x}}{b}}$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

output
```
-((a + b*x^3)^(1/2)*(2/(3*a*b*c^2) + (4*x^3)/(3*a^2*c^2)))/(x^4*(c*x)^(1/2
) + (a*x*(c*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(-2b\,x^3-a)}{3\sqrt{x}\,a^2c^3x\,(b\,x^3+a)}$$

input
```
int(1/(c*x)^(5/2)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - a - 2*b*x**3))/(3*sqrt(x)*a**2*c**3*x*(a +
 b*x**3))
```

3.341 $\int \frac{1}{(cx)^{11/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{1}{(cx)^{11/2}\,(a+bx^3)^{3/2}}\,dx = \frac{2}{3ac(cx)^{9/2}\sqrt{a+bx^3}} - \frac{8\sqrt{a+bx^3}}{9a^2c(cx)^{9/2}} + \frac{16b\sqrt{a+bx^3}}{9a^3c^4(cx)^{3/2}}$$

output
```
2/3/a/c/(c*x)^(9/2)/(b*x^3+a)^(1/2)-8/9*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(9/2)+
16/9*b*(b*x^3+a)^(1/2)/a^3/c^4/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{11/2}\,(a+bx^3)^{3/2}}\,dx = -\frac{2x(a^2-4abx^3-8b^2x^6)}{9a^3(cx)^{11/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*(a^2 - 4*a*b*x^3 - 8*b^2*x^6))/(9*a^3*(c*x)^(11/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2} (a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{805}$$

$$\frac{4\int \frac{1}{(cx)^{11/2}\sqrt{bx^3+a}}dx}{a}+\frac{2}{3ac(cx)^{9/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{805}$$

$$\frac{4\left(-\frac{2\int \frac{\sqrt{bx^3+a}}{(cx)^{11/2}}dx}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{9/2}}\right)}{a}+\frac{2}{3ac(cx)^{9/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4\left(\frac{4(a+bx^3)^{3/2}}{9a^2c(cx)^{9/2}}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{9/2}}\right)}{a}+\frac{2}{3ac(cx)^{9/2}\sqrt{a+bx^3}}$$

input
```
Int[1/((c*x)^(11/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*(c*x)^(9/2)*Sqrt[a + b*x^3]) + (4*((-2*Sqrt[a + b*x^3])/(3*a*c*(c
*x)^(9/2)) + (4*(a + b*x^3)^(3/2))/(9*a^2*c*(c*x)^(9/2))))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(-8b^{2}x^{6}-4ab\,x^{3}+a^{2}\right)}{9\sqrt{b\,x^{3}+a}\,a^{3}\left(cx\right)^{\frac{11}{2}}}$	40
orering	$-\frac{2x\left(-8b^{2}x^{6}-4ab\,x^{3}+a^{2}\right)}{9\sqrt{b\,x^{3}+a}\,a^{3}\left(cx\right)^{\frac{11}{2}}}$	40
default	$-\frac{2\left(-8b^{2}x^{6}-4ab\,x^{3}+a^{2}\right)}{9x^{4}\sqrt{b\,x^{3}+a}\,a^{3}c^{5}\sqrt{cx}}$	45
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(-5b\,x^{3}+a\right)}{9a^{3}x^{4}c^{5}\sqrt{cx}}+\frac{2b^{2}x^{2}}{3a^{3}c^{5}\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	63
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{9a^{2}c^{6}x^{5}}+\frac{10b\sqrt{bc\,x^{4}+acx}}{9a^{3}c^{6}x^{2}}+\frac{2b^{2}x^{2}}{3c^{5}a^{3}\sqrt{\left(x^{3}+\frac{a}{b}\right)bcx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	107

input
```
int(1/(c*x)^(11/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*x*(-8*b^2*x^6-4*a*b*x^3+a^2)/(b*x^3+a)^(1/2)/a^3/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int \frac{1}{(cx)^{11/2}\,(a+bx^3)^{3/2}}\,dx = \frac{2\,(8\,b^2x^6+4\,abx^3-a^2)\sqrt{bx^3+a}\sqrt{cx}}{9\,(a^3bc^6x^8+a^4c^6x^5)}$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(8*b^2*x^6 + 4*a*b*x^3 - a^2)*sqrt(b*x^3 + a)*sqrt(c*x)/(a^3*b*c^6*x^8
 + a^4*c^6*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(76) = 152.

Time = 121.67 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.44

$$\begin{aligned}\int \frac{1}{(cx)^{11/2}\,(a+bx^3)^{3/2}}\,dx = &-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a^5b^4c^{\frac{11}{2}}x^3+18a^4b^5c^{\frac{11}{2}}x^6+9a^3b^6c^{\frac{11}{2}}x^9}\\ &+\frac{6a^2b^{\frac{11}{2}}x^3\sqrt{\frac{a}{bx^3}+1}}{9a^5b^4c^{\frac{11}{2}}x^3+18a^4b^5c^{\frac{11}{2}}x^6+9a^3b^6c^{\frac{11}{2}}x^9}+\frac{24ab^{\frac{13}{2}}x^6\sqrt{\frac{a}{bx^3}+1}}{9a^5b^4c^{\frac{11}{2}}x^3+18a^4b^5c^{\frac{11}{2}}x^6+9a^3b^6c^{\frac{11}{2}}x^9}\\ &+\frac{16b^{\frac{15}{2}}x^9\sqrt{\frac{a}{bx^3}+1}}{9a^5b^4c^{\frac{11}{2}}x^3+18a^4b^5c^{\frac{11}{2}}x^6+9a^3b^6c^{\frac{11}{2}}x^9}\end{aligned}$$

input
```
integrate(1/(c*x)**(11/2)/(b*x**3+a)**(3/2),x)
```

output
```
-2*a**3*b**(9/2)*sqrt(a/(b*x**3) + 1)/(9*a**5*b**4*c**(11/2)*x**3 + 18*a**
4*b**5*c**(11/2)*x**6 + 9*a**3*b**6*c**(11/2)*x**9) + 6*a**2*b**(11/2)*x**
3*sqrt(a/(b*x**3) + 1)/(9*a**5*b**4*c**(11/2)*x**3 + 18*a**4*b**5*c**(11/2
)*x**6 + 9*a**3*b**6*c**(11/2)*x**9) + 24*a*b**(13/2)*x**6*sqrt(a/(b*x**3)
 + 1)/(9*a**5*b**4*c**(11/2)*x**3 + 18*a**4*b**5*c**(11/2)*x**6 + 9*a**3*b
**6*c**(11/2)*x**9) + 16*b**(15/2)*x**9*sqrt(a/(b*x**3) + 1)/(9*a**5*b**4*
c**(11/2)*x**3 + 18*a**4*b**5*c**(11/2)*x**6 + 9*a**3*b**6*c**(11/2)*x**9)
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2} (a+bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}} (cx)^{\frac{11}{2}}} \, dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(11/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(cx)^{11/2} (a+bx^3)^{3/2}} \, dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: Recu
rsive assumption sageVARa>=(-sageVARb*sageVARc/(sageVARc^4*t_nostep^6)) ig
nored324*sageVARb^2/486/sageVARc^5/sageVARa^3*sqrt(sageVARc*sageVARx)*sqrt
(sageVARc*sageVARx)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{1}{(cx)^{11/2} (a+bx^3)^{3/2}} \, dx = \frac{\sqrt{b\,x^3+a}\,\left(\frac{8\,x^3}{9\,a^2\,c^5}-\frac{2}{9\,a\,b\,c^5}+\frac{16\,b\,x^6}{9\,a^3\,c^5}\right)}{x^7\,\sqrt{c\,x}+\frac{a\,x^4\,\sqrt{c\,x}}{b}}$$

input
```
int(1/((c*x)^(11/2)*(a + b*x^3)^(3/2)),x)
```

output
```
((a + b*x^3)^(1/2)*((8*x^3)/(9*a^2*c^5) - 2/(9*a*b*c^5) + (16*b*x^6)/(9*a^
3*c^5)))/(x^7*(c*x)^(1/2) + (a*x^4*(c*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.64

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,(8b^2x^6+4ab\,x^3-a^2)}{9\sqrt{x}\,a^3c^6x^4\,(b\,x^3+a)}$$

input
```
int(1/(c*x)^(11/2)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - a**2 + 4*a*b*x**3 + 8*b**2*x**6))/(9*sqrt(
x)*a**3*c**6*x**4*(a + b*x**3))
```

3.342 $\int \frac{1}{(cx)^{17/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{1}{(cx)^{17/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2}{3ac(cx)^{15/2}\sqrt{a+bx^3}} - \frac{4\sqrt{a+bx^3}}{5a^2c(cx)^{15/2}} + \frac{16b\sqrt{a+bx^3}}{15a^3c^4(cx)^{9/2}} - \frac{32b^2\sqrt{a+bx^3}}{15a^4c^7(cx)^{3/2}}$$

output
```
2/3/a/c/(c*x)^(15/2)/(b*x^3+a)^(1/2)-4/5*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(15/2
)+16/15*b*(b*x^3+a)^(1/2)/a^3/c^4/(c*x)^(9/2)-32/15*b^2*(b*x^3+a)^(1/2)/a^
4/c^7/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48

$$\int \frac{1}{(cx)^{17/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2x(a^3-2a^2bx^3+8ab^2x^6+16b^3x^9)}{15a^4(cx)^{17/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(17/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*(a^3 - 2*a^2*b*x^3 + 8*a*b^2*x^6 + 16*b^3*x^9))/(15*a^4*(c*x)^(17/2)
*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {805, 805, 805, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{17/2} (a+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \ 805$$

$$\frac{6 \int \frac{1}{(cx)^{17/2}\sqrt{bx^3+a}}dx}{a}+\frac{2}{3ac(cx)^{15/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 805$$

$$\frac{6\left(-\frac{4\int \frac{\sqrt{bx^3+a}}{(cx)^{17/2}}dx}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{15/2}}\right)}{a}+\frac{2}{3ac(cx)^{15/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 805$$

$$\frac{6\left(-\frac{4\left(-\frac{2\int \frac{\left(bx^3+a\right)^{3/2}}{(cx)^{17/2}}dx}{3a}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{15/2}}\right)}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{15/2}}\right)}{a}+\frac{2}{3ac(cx)^{15/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 796$$

$$\frac{6\left(-\frac{4\left(\frac{4\left(a+bx^3\right)^{5/2}}{45a^2c(cx)^{15/2}}-\frac{2\left(a+bx^3\right)^{3/2}}{9ac(cx)^{15/2}}\right)}{a}-\frac{2\sqrt{a+bx^3}}{3ac(cx)^{15/2}}\right)}{a}+\frac{2}{3ac(cx)^{15/2}\sqrt{a+bx^3}}$$

input
```
Int[1/((c*x)^(17/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*(c*x)^(15/2)*Sqrt[a + b*x^3]) + (6*((-2*Sqrt[a + b*x^3])/(3*a*c*(
c*x)^(15/2)) - (4*((-2*(a + b*x^3)^(3/2))/(9*a*c*(c*x)^(15/2)) + (4*(a + b
*x^3)^(5/2))/(45*a^2*c*(c*x)^(15/2))))/a))/a
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 805
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{2x\left(16b^{3}x^{9}+8a\,b^{2}x^{6}-2a^{2}b\,x^{3}+a^{3}\right)}{15\sqrt{b\,x^{3}+a}\,a^{4}\left(cx\right)^{\frac{17}{2}}}$	51
orering	$-\frac{2x\left(16b^{3}x^{9}+8a\,b^{2}x^{6}-2a^{2}b\,x^{3}+a^{3}\right)}{15\sqrt{b\,x^{3}+a}\,a^{4}\left(cx\right)^{\frac{17}{2}}}$	51
default	$-\frac{2\left(16b^{3}x^{9}+8a\,b^{2}x^{6}-2a^{2}b\,x^{3}+a^{3}\right)}{15x^{7}\sqrt{b\,x^{3}+a}\,a^{4}c^{8}\sqrt{cx}}$	56
risch	$-\frac{2\sqrt{b\,x^{3}+a}\,\left(11b^{2}x^{6}-3ab\,x^{3}+a^{2}\right)}{15a^{4}x^{7}c^{8}\sqrt{cx}}-\frac{2b^{3}x^{2}}{3a^{4}c^{8}\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	74
elliptic	$\frac{\sqrt{cx\left(b\,x^{3}+a\right)}\,\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{15a^{2}c^{9}x^{8}}+\frac{2b\sqrt{bc\,x^{4}+acx}}{5c^{9}a^{3}x^{5}}-\frac{22b^{2}\sqrt{bc\,x^{4}+acx}}{15a^{4}c^{9}x^{2}}-\frac{2b^{3}x^{2}}{3c^{8}a^{4}\sqrt{\left(x^{3}+\frac{a}{b}\right)bcx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{3}+a}}$	134

input
```
int(1/(c*x)^(17/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(16*b^3*x^9+8*a*b^2*x^6-2*a^2*b*x^3+a^3)/(b*x^3+a)^(1/2)/a^4/(c*x)
^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{17/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2\left(16\,b^3x^9+8\,ab^2x^6-2\,a^2bx^3+a^3\right)\sqrt{bx^3+a}\sqrt{cx}}{15\left(a^4bc^9x^{11}+a^5c^9x^8\right)}$$

input
```
integrate(1/(c*x)^(17/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(16*b^3*x^9 + 8*a*b^2*x^6 - 2*a^2*b*x^3 + a^3)*sqrt(b*x^3 + a)*sqrt(
c*x)/(a^4*b*c^9*x^11 + a^5*c^9*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{17/2}(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(17/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{17/2}(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}(cx)^{\frac{17}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(17/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(17/2)), x)
```

Giac [B] (verification not implemented)

Error detected during grading. Assigning place holder grade for now.

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int \frac{1}{(cx)^{17/2}(a+bx^3)^{3/2}}\,dx = Recursiveaassumption \geq -\frac{2\left(\frac{15\sqrt{bc+\frac{ac}{x^3}}b^2}{ac^7}-\frac{11\sqrt{bc}b^2}{ac^7}-\frac{5\left(bc+\frac{ac}{x^3}\right)^{\frac{3}{2}}bc-\left(bc+\frac{ac}{x^3}\right)^{\frac{5}{2}}}{ac^9}\right)}{15\,a^3|c|^2} - \frac{bignored}{c^3t_{nostep}^6} - \frac{2\sqrt{cx}b^3x}{3\sqrt{bc^4x^3+ac^4}a^4c^7}$$

input
```
integrate(1/(c*x)^(17/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
Recursive*a*assumption >= -2/15*(15*sqrt(b*c + a*c/x^3)*b^2/(a*c^7) - 11*s
qrt(b*c)*b^2/(a*c^7) - (5*(b*c + a*c/x^3)^(3/2)*b*c - (b*c + a*c/x^3)^(5/2
))/(a*c^9))/(a^3*abs(c)^2) - b*ignored/(c^3*t_nostep^6) - 2/3*sqrt(c*x)*b^
3*x/(sqrt(b*c^4*x^3 + a*c^4)*a^4*c^7)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.73

$$\int \frac{1}{(cx)^{17/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a}\,\left(\frac{2}{15\,a\,b\,c^8}-\frac{4\,x^3}{15\,a^2\,c^8}+\frac{16\,b\,x^6}{15\,a^3\,c^8}+\frac{32\,b^2\,x^9}{15\,a^4\,c^8}\right)}{x^{10}\,\sqrt{c\,x}+\frac{a\,x^7\,\sqrt{c\,x}}{b}}$$

input
```
int(1/((c*x)^(17/2)*(a + b*x^3)^(3/2)),x)
```

output
```
-((a + b*x^3)^(1/2)*(2/(15*a*b*c^8) - (4*x^3)/(15*a^2*c^8) + (16*b*x^6)/(1
5*a^3*c^8) + (32*b^2*x^9)/(15*a^4*c^8)))/(x^10*(c*x)^(1/2) + (a*x^7*(c*x)^
(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx)^{17/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{b\,x^3+a}\,\left(-16b^3x^9-8a\,b^2x^6+2a^2b\,x^3-a^3\right)}{15\sqrt{x}\,a^4c^9x^7\left(b\,x^3+a\right)}$$

input
```
int(1/(c*x)^(17/2)/(b*x^3+a)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + b*x**3)*( - a**3 + 2*a**2*b*x**3 - 8*a*b**2*x**6 - 16*
b**3*x**9))/(15*sqrt(x)*a**4*c**9*x**7*(a + b*x**3))
```

3.343 $\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 296

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{13/2}}{3b\sqrt{a+bx^3}} - \frac{91ac^8\sqrt{cx}\sqrt{a+bx^3}}{60b^3} + \frac{13c^5(cx)^{7/2}\sqrt{a+bx^3}}{15b^2} + \frac{91a^{5/3}c^8\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{120\sqrt[4]{3}b^3\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output

```
-2/3*c^2*(c*x)^(13/2)/b/(b*x^3+a)^(1/2)-91/60*a*c^8*(c*x)^(1/2)*(b*x^3+a)^
(1/2)/b^3+13/15*c^5*(c*x)^(7/2)*(b*x^3+a)^(1/2)/b^2+91/360*a^(5/3)*c^8*(c*
x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^
(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2)
)*3^(3/4)/b^3/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.29

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^8\sqrt{cx}\left(-91a^2-39abx^3+12b^2x^6+91a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{60b^3\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(17/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^8*Sqrt[c*x]*(-91*a^2 - 39*a*b*x^3 + 12*b^2*x^6 + 91*a^2*Sqrt[1 + (b*x^3
)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(60*b^3*Sqrt[a + b*x
^3])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {817, 843, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{13c^3\int \frac{(cx)^{11/2}}{\sqrt{bx^3+a}}dx}{3b}-\frac{2c^2(cx)^{13/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{13c^3\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\int \frac{(cx)^{5/2}}{\sqrt{bx^3+a}}dx}{10b}\right)}{3b}-\frac{2c^2(cx)^{13/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 843$$

$$\frac{13c^3\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^3\int\frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{4b}\right)}{10b}\right)}{3b}-\frac{2c^2(cx)^{13/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 851$$

$$\frac{13c^3\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b}\right)}{10b}\right)}{3b}-\frac{2c^2(cx)^{13/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 766$$

$$\frac{13c^3\left(\frac{c^2(cx)^{7/2}\sqrt{a+bx^3}}{5b}-\frac{7ac^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc-}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc-}\right.\right.}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{10b}\right.}{3b}$$

$$\frac{2c^2(cx)^{13/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(17/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(13/2))/(3*b*Sqrt[a + b*x^3]) + (13*c^3*((c^2*(c*x)^(7/2)*Sq
rt[a + b*x^3])/(5*b) - (7*a*c^3*((c^2*Sqrt[c*x]*Sqrt[a + b*x^3])/(2*b) - (
a^(2/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*
b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2
]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1
 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*c*
x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sq
rt[a + b*x^3])))/(10*b)))/(3*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.37 (sec) , antiderivative size = 787, normalized size of antiderivative = 2.66

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^3+a)}\left(-\frac{2c^9 x\,a^2}{3b^3\sqrt{\left(x^3+\frac{a}{b}\right)bcx}}+\frac{c^8x^3\sqrt{bc\,x^4+acx}}{5b^2}-\frac{17a\,c^8\sqrt{bc\,x^4+acx}}{20b^3}+\frac{91a^2c^9\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-\right.}{}\right.}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\right.}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x)^(17/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(-2/3/b^3*c^9*x*a^
2/((x^3+a/b)*b*c*x)^(1/2)+1/5/b^2*c^8*x^3*(b*c*x^4+a*c*x)^(1/2)-17/20*a/b^
3*c^8*(b*c*x^4+a*c*x)^(1/2)+91/60*a^2*c^9/b^2*(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(
1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-
3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*c*x
*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),
((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
)))
```

Fricas [F]

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(cx)^{\frac{17}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(17/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^8*x^8/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(17/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{17}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(17/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(17/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{17}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(17/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(17/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{17/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(17/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(17/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{17/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^8\Big(-182\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2-52\sqrt{x}\,\sqrt{b\,x^3+a}\,ab\,x^3+16\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^6+91\Big(\int}{80b^3\,(b\,x^3+a)}$$

input
```
int((c*x)^(17/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**8*( - 182*sqrt(x)*sqrt(a + b*x**3)*a**2 - 52*sqrt(x)*sqrt(a +
b*x**3)*a*b*x**3 + 16*sqrt(x)*sqrt(a + b*x**3)*b**2*x**6 + 91*int((sqrt(x)
*sqrt(a + b*x**3))/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)*a**4 + 91*int((sqr
t(x)*sqrt(a + b*x**3))/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)*a**3*b*x**3))/
(80*b**3*(a + b*x**3))
```

3.344 $\int \frac{(cx)^{11/2}}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 267

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{7/2}}{3b\sqrt{a+bx^3}}+\frac{7c^5\sqrt{cx}\sqrt{a+bx^3}}{6b^2} - \frac{7a^{2/3}c^5\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c^2*(c*x)^(7/2)/b/(b*x^3+a)^(1/2)+7/6*c^5*(c*x)^(1/2)*(b*x^3+a)^(1/2)
/b^2-7/36*a^(2/3)*c^5*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacob
iAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.28

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^5\sqrt{cx}\left(7a+3bx^3-7a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{6b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(11/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^5*Sqrt[c*x]*(7*a + 3*b*x^3 - 7*a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[
1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {817, 843, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7c^3\int \frac{(cx)^{5/2}}{\sqrt{bx^3+a}}dx}{3b}-\frac{2c^2(cx)^{7/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{7c^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^3\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{4b}\right)}{3b}-\frac{2c^2(cx)^{7/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{7c^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{ac^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b}\right)}{3b}-\frac{2c^2(cx)^{7/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 766

$$\frac{7c^3\left(\frac{c^2\sqrt{cx}\sqrt{a+bx^3}}{2b}-\frac{a^{2/3}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right)}{3b}$$
$$\frac{2c^2(cx)^{7/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(11/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(7/2))/(3*b*Sqrt[a + b*x^3]) + (7*c^3*((c^2*Sqrt[c*x]*Sqrt[a
 + b*x^3])/(2*b) - (a^(2/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^
(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqr
t[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*
c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4
)*b*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3]
)*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(3*b)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.88 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.84

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^3+a)}\left(\frac{2c^6xa}{3b^2\sqrt{\left(x^3+\frac{a}{b}\right)bcx}}+\frac{c^5\sqrt{bc\,x^4+acx}}{2b^2}-\frac{7a\,c^6\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\right.}{}\right)}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x)^(11/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(2/3/b^2*c^6*x*a/(
(x^3+a/b)*b*c*x)^(1/2)+1/2*c^5/b^2*(b*c*x^4+a*c*x)^(1/2)-7/6*a*c^6/b*(1/2/
b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*
(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3))
)^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^5*x^5/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 98.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^{\frac{11}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((c*x)**(11/2)/(b*x**3+a)**(3/2),x)
```

output
```
c**(11/2)*x**(13/2)*gamma(13/6)*hyper((3/2, 13/6), (19/6,), b*x**3*exp_pol
ar(I*pi)/a)/(3*a**(3/2)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(11/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(11/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{11/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(11/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(11/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^5\left(14\sqrt{x}\,\sqrt{b\,x^3+a}\,a+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^3-7\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)a^3-7\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)a^2b\,x^3\right)}{8b^2\left(b\,x^3+a\right)}$$

input
```
int((c*x)^(11/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**5*(14*sqrt(x)*sqrt(a + b*x**3)*a + 4*sqrt(x)*sqrt(a + b*x**3)*
b*x**3 - 7*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x + 2*a*b*x**4 + b**2*x**7
),x)*a**3 - 7*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x + 2*a*b*x**4 + b**2*x
**7),x)*a**2*b*x**3))/(8*b**2*(a + b*x**3))
```

3.345 $\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 239

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2\sqrt{cx}}{3b\sqrt{a+bx^3}} + \frac{c^2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}\sqrt[3]{a}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c^2*(c*x)^(1/2)/b/(b*x^3+a)^(1/2)+1/9*c^2*(c*x)^(1/2)*(a^(1/3)+b^(1/3
)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)
*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/
3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(1/3)/b/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.27

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \frac{2c^2\sqrt{cx}\left(-1+\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{3b\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(5/2)/(a + b*x^3)^(3/2),x]
```

output
```
(2*c^2*Sqrt[c*x]*(-1 + Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6
, -((b*x^3)/a)]))/(3*b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {817, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{c^3\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{3b}-\frac{2c^2\sqrt{cx}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{2c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{3b}-\frac{2c^2\sqrt{cx}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}(2+\sqrt{3})\right)}{3\sqrt[4]{3}\sqrt[3]{a}b\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2c^2\sqrt{cx}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(5/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*Sqrt[c*x])/(3*b*Sqrt[a + b*x^3]) + (c*Sqrt[c*x]*(a^(1/3)*c + b^(1/
3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1
/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - S
qrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3
])/4])/(3*3^(1/4)*a^(1/3)*b*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(
a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 732, normalized size of antiderivative = 3.06

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^3+a)}\left(-\frac{2c^3x}{3b\sqrt{\left(x^3+\frac{a}{b}\right)bcx}}+\frac{2c^3\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\right.}{}\right.}{}\right.}{}$
default	Expression too large to display

input

```
int((c*x)^(5/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(-2/3/b*c^3*x/((x^
3+a/b)*b*c*x)^(1/2)+2/3*c^3*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*
b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(
1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/
2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.29

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{bx^3+a}\sqrt{cx}ac^2+(bc^2x^3+ac^2)\sqrt{ac}\,\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)}{3\,(ab^2x^3+a^2b)}$$

input
```
integrate((c*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*sqrt(c*x)*a*c^2 + (b*c^2*x^3 + a*c^2)*sqrt(a*c)*weie
rstrassPInverse(0, -4*b/a, 1/x))/(a*b^2*x^3 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.18

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (13/6,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(3/2)*gamma(13/6))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{{\left(c\,x\right)}^{5/2}}{{\left(b\,x^3+a\right)}^{3/2}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^3)^{3/2}}\,dx=\frac{\sqrt{c}\,c^2\left(-2\sqrt{x}\,\sqrt{b\,x^3+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)a^2+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)ab\,x^3\right)}{2b\left(b\,x^3+a\right)}$$

input
```
int((c*x)^(5/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**2*( - 2*sqrt(x)*sqrt(a + b*x**3) + int((sqrt(x)*sqrt(a + b*x**
3))/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)*a**2 + int((sqrt(x)*sqrt(a + b*x*
*3))/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)*a*b*x**3))/(2*b*(a + b*x**3))
```

3.346 $\int \frac{1}{\sqrt{cx}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 236

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \frac{2\sqrt{cx}}{3ac\sqrt{a+bx^3}} + \frac{2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}c\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(c*x)^(1/2)/a/c/(b*x^3+a)^(1/2)+2/9*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^
(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3
^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)/c/(b^(1/3)*x*
(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.27

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \frac{2\left(x+2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{bx^3}{a}\right)\right)}{3a\sqrt{cx}\sqrt{a+bx^3}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a + b*x^3)^(3/2)),x]
```

output
```
(2*(x + 2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)
/a)]))/(3*a*Sqrt[c*x]*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {819, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{2\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{3a}+\frac{2\sqrt{cx}}{3ac\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{4\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{3ac}+\frac{2\sqrt{cx}}{3ac\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{2\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}(2+\sqrt{3})\right)}{3\sqrt[4]{3}a^{4/3}c^2\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}+\frac{2\sqrt{cx}}{3ac\sqrt{a+bx^3}}$$

input
```
Int[1/(Sqrt[c*x]*(a + b*x^3)^(3/2)),x]
```

output
```
(2*Sqrt[c*x])/(3*a*c*Sqrt[a + b*x^3]) + (2*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*
c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)
*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt
[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/
4]])/(3*3^(1/4)*a^(4/3)*c^2*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a
^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.07

method	result
elliptic	$\frac{\sqrt{cx(b\,x^3+a)}\left(\frac{2x}{3a\sqrt{\left(x^3+\frac{a}{b}\right)bcx}}+\frac{4\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)^2 \cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int(1/(c*x)^(1/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)*(2/3*x/a/((x^3+a/b)*b*c*
x)^(1/2)+4/3/a*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a
*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)
-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = -\frac{2\left(2\,(bx^3+a)\sqrt{ac}\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\sqrt{bx^3+a}\sqrt{cx}a\right)}{3\,(a^2bcx^3+a^3c)}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(2*(b*x^3 + a)*sqrt(a*c)*weierstrassPInverse(0, -4*b/a, 1/x) - sqrt(b
*x^3 + a)*sqrt(c*x)*a)/(a^2*b*c*x^3 + a^3*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{c}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

output
```
sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*
a**(3/2)*sqrt(c)*gamma(7/6))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^7+2ab\,x^4+a^2x}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x + 2*a*b*x**4 + b**2*x**7),
x))/c
```

3.347 $\int \frac{1}{(cx)^{7/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 265

$$\int \frac{1}{(cx)^{7/2}(a+bx^3)^{3/2}}\,dx = \frac{2}{3ac(cx)^{5/2}\sqrt{a+bx^3}} - \frac{16\sqrt{a+bx^3}}{15a^2c(cx)^{5/2}} - \frac{16b\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}c^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/c/(c*x)^(5/2)/(b*x^3+a)^(1/2)-16/15*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(5/2
)-16/45*b*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((
a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2
)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/c^4/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)
+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.22

$$\int \frac{1}{(cx)^{7/2}(a+bx^3)^{3/2}}\,dx=-\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{3}{2},\frac{1}{6},-\frac{bx^3}{a}\right)}{5a(cx)^{7/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 3/2, 1/6, -((b*x^3)/a)])
/(5*a*(c*x)^(7/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {819, 847, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{8\int \frac{1}{(cx)^{7/2}\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ac(cx)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$\frac{8\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{5ac^3}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{3a}+\frac{2}{3ac(cx)^{5/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{8\left(-\frac{4b\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{5ac^4}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{3a}+\frac{2}{3ac(cx)^{5/2}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 766

$$\frac{8\left(-\frac{2b\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}c^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{3a}+\frac{2}{3ac(cx)^{5/2}\sqrt{a+bx^3}}$$

input
```
Int[1/((c*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*(c*x)^(5/2)*Sqrt[a + b*x^3]) + (8*((-2*Sqrt[a + b*x^3])/(5*a*c*(c
*x)^(5/2)) - (2*b*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 -
a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3
)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3
)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*c^
5*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*
b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(3*a)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.01 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.86

method	result
elliptic	$\frac{\sqrt{cx(b\,x^{3}+a)}\left(-\frac{2\sqrt{bc\,x^{4}+acx}}{5a^{2}c^{4}x^{3}}-\frac{2bx}{3c^{3}a^{2}\sqrt{\left(x^{3}+\frac{a}{b}\right)bcx}}-\frac{32b^{2}\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)}{b}\right.}}}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/(c*x)^(7/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/5/a^2/c^4*(b*c*x^4+a
*c*x)^(1/2)/x^3-2/3*b/c^3*x/a^2/((x^3+a/b)*b*c*x)^(1/2)-32/15*b^2/a^2/c^3*
(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*
(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(
1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1
/b*(-a*b^2)^(1/3)))^(1/2)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/(-a*b^2)^(1/3)/(b*c*x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3)))^(1/2)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x
-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.33

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2\left(16\left(b^2x^6+abx^3\right)\sqrt{ac}\,\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\left(8\,abx^3+3\,a^2\right)\sqrt{bx^3}}{15\left(a^3bc^4x^6+a^4c^4x^3\right)}$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(16*(b^2*x^6 + a*b*x^3)*sqrt(a*c)*weierstrassPInverse(0, -4*b/a, 1/x)
 - (8*a*b*x^3 + 3*a^2)*sqrt(b*x^3 + a)*sqrt(c*x))/(a^3*b*c^4*x^6 + a^4*c^4
*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{\Gamma\left(-\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{6},\frac{3}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-5/6)*hyper((-5/6, 3/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*c**(7/2)*x**(5/2)*gamma(1/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{1}{(c\,x)^{7/2}\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^{10}+2ab\,x^7+a^2x^4}dx\right)}{c^4}$$

input
```
int(1/(c*x)^(7/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x**4 + 2*a*b*x**7 + b**2*x**
10),x))/c**4
```

3.348 $\int \frac{1}{(cx)^{13/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 296

$$\int \frac{1}{(cx)^{13/2}(a+bx^3)^{3/2}}\,dx = \frac{2}{3ac(cx)^{11/2}\sqrt{a+bx^3}} - \frac{28\sqrt{a+bx^3}}{33a^2c(cx)^{11/2}} + \frac{224b\sqrt{a+bx^3}}{165a^3c^4(cx)^{5/2}} + \frac{224b^2\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{165\sqrt[4]{3}a^{10/3}c^7\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/c/(c*x)^(11/2)/(b*x^3+a)^(1/2)-28/33*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(11
/2)+224/165*b*(b*x^3+a)^(1/2)/a^3/c^4/(c*x)^(5/2)+224/495*b^2*(c*x)^(1/2)*
(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3
^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^
(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)
/a^(10/3)/c^7/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*
x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.20

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{3}{2},-\frac{5}{6},-\frac{bx^3}{a}\right)}{11a(cx)^{13/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(13/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 3/2, -5/6, -((b*x^3)/a)
])/(11*a*(c*x)^(13/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {819, 847, 847, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{14\int \frac{1}{(cx)^{13/2}\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ac(cx)^{11/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$\frac{14\left(-\frac{8b\int \frac{1}{(cx)^{7/2}\sqrt{bx^3+a}}dx}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}\right)}{3a}+\frac{2}{3ac(cx)^{11/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{847}$$

$$\frac{14\left(-\frac{8b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^3+a}}dx}{5ac^3}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}\right)}{3a}+\frac{2}{3ac(cx)^{11/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 851$$

$$\frac{14\left(-\frac{8b\left(-\frac{4b\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{5ac^4}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11ac^3}-\frac{2\sqrt{a+bx^3}}{11ac(cx)^{11/2}}\right)}{3a}+\frac{2}{3ac(cx)^{11/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 766$$

$$\frac{14\left(-\frac{8b\left(-\frac{2b\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}c^5\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}-\frac{2\sqrt{a+bx^3}}{5ac(cx)^{5/2}}\right)}{11ac^3}-\frac{2\cdots}{11a\cdots}\right)}{3a}$$

$$\frac{2}{3ac(cx)^{11/2}\sqrt{a+bx^3}}$$

input

```
Int[1/((c*x)^(13/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*(c*x)^(11/2)*Sqrt[a + b*x^3]) + (14*((-2*Sqrt[a + b*x^3])/(11*a*c
*(c*x)^(11/2)) - (8*b*((-2*Sqrt[a + b*x^3])/(5*a*c*(c*x)^(5/2)) - (2*b*Sqr
t[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x
 + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[A
rcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b
^(1/3)*c*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*c^5*Sqrt[(b^(1/3)*c*x*(
a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[
a + b*x^3])))/(11*a*c^3)))/(3*a)
```

Defintions of rubi rules used

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 784, normalized size of antiderivative = 2.65

method	result
elliptic	$\frac{\sqrt{cx(b\,x^3+a)}\left(-\frac{2\sqrt{bc\,x^4+acx}}{11a^2c^7x^6}+\frac{38b\sqrt{bc\,x^4+acx}}{55a^3c^7x^3}+\frac{2b^2x}{3c^6a^3\sqrt{\left(x^3+\frac{a}{b}\right)bcx}}+\frac{448b^3\left(\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i \cdots}{\cdots}\right)\cdots}{\left(-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b \cdots\right.}{2b}\cdots\right.}}\cdots}{\cdots}\right.\cdots}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(1/(c*x)^(13/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/11/a^2/c^7*(b*c*x^4+
a*c*x)^(1/2)/x^6+38/55*b/a^3/c^7*(b*c*x^4+a*c*x)^(1/2)/x^3+2/3*b^2/c^6*x/a
^3/((x^3+a/b)*b*c*x)^(1/2)+448/165*b^3/a^3/c^6*(1/2/b*(-a*b^2)^(1/3)-1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^
(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(b*c*
x*(x-1/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*Elli
pticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-
a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)
,((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.34

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2\left(224\left(b^3x^9+ab^2x^6\right)\sqrt{ac}\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)-\left(112\,ab^2x^6+42\,a^2bx^3-15\,a^3\right)\sqrt{bx^3+a}\sqrt{cx}\right)}{165\left(a^4bc^7x^9+a^5c^7x^6\right)}$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/165*(224*(b^3*x^9 + a*b^2*x^6)*sqrt(a*c)*weierstrassPInverse(0, -4*b/a,
 1/x) - (112*a*b^2*x^6 + 42*a^2*b*x^3 - 15*a^3)*sqrt(b*x^3 + a)*sqrt(c*x))
/(a^4*b*c^7*x^9 + a^5*c^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2}\,(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(13/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\,(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\,(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(13/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^3\right)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{13/2}\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(13/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(13/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^3\right)^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^{13}+2ab\,x^{10}+a^2x^7}dx\right)}{c^7}$$

input
```
int(1/(c*x)^(13/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x**7 + 2*a*b*x**10 + b**2*x*
*13),x))/c**7
```

3.349 $\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 581

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}} - \frac{187ac^8(cx)^{5/2}\sqrt{a+bx^3}}{168b^3} + \frac{17c^5(cx)^{11/2}\sqrt{a+bx^3}}{21b^2} + \frac{935\left(1+\sqrt{3}\right)a^2c^{10}\sqrt{cx}\sqrt{a+bx^3}}{336b^{11/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)} - \frac{935a^{7/3}c^{10}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{112\ 3^{3/4}b^{11/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{935\left(1-\sqrt{3}\right)a^{7/3}c^{10}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2}{672\sqrt[4]{3}b^{11/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c^2*(c*x)^(17/2)/b/(b*x^3+a)^(1/2)-187/168*a*c^8*(c*x)^(5/2)*(b*x^3+a
)^(1/2)/b^3+17/21*c^5*(c*x)^(11/2)*(b*x^3+a)^(1/2)/b^2+935/336*(1+3^(1/2))
*a^2*c^10*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^(11/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)-935/336*a^(7/3)*c^10*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Elliptic
E((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/b^(11/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3
)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-935/2016*(1-
3^(1/2))*a^(7/3)*c^10*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacob
iAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(11/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)
/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.15

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^8(cx)^{5/2}\left(187a^2-34abx^3+16b^2x^6-187a^2\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},\right.\right.}{112b^3\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(21/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^8*(c*x)^(5/2)*(187*a^2 - 34*a*b*x^3 + 16*b^2*x^6 - 187*a^2*Sqrt[1 + (b*
x^3)/a]*Hypergeometric2F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(112*b^3*Sqrt[a
+ b*x^3])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {817, 843, 843, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{17c^3\int \frac{(cx)^{15/2}}{\sqrt{bx^3+a}}dx}{3b}-\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\int \frac{(cx)^{9/2}}{\sqrt{bx^3+a}}dx}{14b}\right)}{3b}-\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{843}$$

$$\frac{17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}\right)}{14b}\right)}{3b}-\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{851}$$

$$\frac{17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{4b}\right)}{14b}\right)}{3b}-\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{837}$$

$$\frac{17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int-\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)}{14b}\right)}{3b}-$$

$$\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 25$$

$$\frac{17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int\frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)}{14b}\right)}{3b}-$$

$$\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 766$$

$$17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-}{\left(\sqrt[3]{a}\right.}}}{4\sqrt[4]{3}b^2}\right)}{4b}\right)}{14b}\right)$$

$$3b$$

$$\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 2420

$$\frac{17c^3\left(\frac{c^2(cx)^{11/2}\sqrt{a+bx^3}}{7b}-\frac{11ac^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b\cdots}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}\cdots\right.}}}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\cdots\right.}{\left(\sqrt[3]{a}c+\cdots\right.}}}\cdots\right.}{2b^{2/3}\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{2c^2(cx)^{17/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(21/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(17/2))/(3*b*Sqrt[a + b*x^3]) + (17*c^3*((c^2*(c*x)^(11/2)*S
qrt[a + b*x^3])/(7*b) - (11*a*c^3*((c^2*(c*x)^(5/2)*Sqrt[a + b*x^3])/(4*b)
 - (5*a*c^2*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (
1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1
/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 -
Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[
3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sq
rt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1
/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1
/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*El
lipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + S
qrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*
c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*
Sqrt[a + b*x^3])))/(4*b)))/(14*b)))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.32 (sec) , antiderivative size = 1177, normalized size of antiderivative = 2.03

method	result	size
elliptic	Expression too large to display	1177
risch	Expression too large to display	2215
default	Expression too large to display	3113

input
```
int((c*x)^(21/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(-2/3/b^3*c^11*x^3
*a^2/((x^3+a/b)*b*c*x)^(1/2)+1/7/b^2*c^10*x^5*(b*c*x^4+a*c*x)^(1/2)-25/56*
a/b^3*c^10*x^2*(b*c*x^4+a*c*x)^(1/2)+935/336*a^2*c^11/b^3*(x*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*
(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2
)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*Ell
ipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2
),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(
1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1
/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((...
```

Fricas [F]

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(cx)^{\frac{21}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(21/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^10*x^10/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(21/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{21}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(21/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(21/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{21}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(21/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(21/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{21/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(21/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(21/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{21/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^{10}\Big(374\sqrt{x}\,\sqrt{b\,x^3+a}\,a^2x^2 - 68\sqrt{x}\,\sqrt{b\,x^3+a}\,ab\,x^5 + 32\sqrt{x}\,\sqrt{b\,x^3+a}\,b^2x^8 - 935\Big(}{224b^3\,(b\,x^3+a)}$$

input
```
int((c*x)^(21/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**10*(374*sqrt(x)*sqrt(a + b*x**3)*a**2*x**2 - 68*sqrt(x)*sqrt(a
 + b*x**3)*a*b*x**5 + 32*sqrt(x)*sqrt(a + b*x**3)*b**2*x**8 - 935*int((sqr
t(x)*sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**4 - 935*int
((sqrt(x)*sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**3*b*x*
*3))/(224*b**3*(a + b*x**3))
```

3.350 $\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 550

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}} + \frac{11c^5(cx)^{5/2}\sqrt{a+bx^3}}{12b^2} - \frac{55\left(1+\sqrt{3}\right)ac^7\sqrt{cx}\sqrt{a+bx^3}}{24b^{8/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+\frac{55a^{4/3}c^7\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{8\,3^{3/4}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+\frac{55\left(1-\sqrt{3}\right)a^{4/3}c^7\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}(2+\right.}{48\sqrt[4]{3}b^{8/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c^2*(c*x)^(11/2)/b/(b*x^3+a)^(1/2)+11/12*c^5*(c*x)^(5/2)*(b*x^3+a)^(1
/2)/b^2-55/24*(1+3^(1/2))*a*c^7*(c*x)^(1/2)*(b*x^3+a)^(1/2)/b^(8/3)/(a^(1/
3)+(1+3^(1/2))*b^(1/3)*x)+55/24*a^(4/3)*c^7*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/b^(8/3)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(
1/2)+55/144*(1-3^(1/2))*a^(4/3)*c^7*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2
)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/
2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^(8/3)/(b^(1/3)*x*(a^(1/
3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.13

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx=\frac{c^5(cx)^{5/2}\left(-11a+2bx^3+11a\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{8b^2\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(15/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^5*(c*x)^(5/2)*(-11*a + 2*b*x^3 + 11*a*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(8*b^2*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}=0.368$, Rules used = {817, 843, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx$$

$\downarrow$ 817

$$\frac{11c^3\int \frac{(cx)^{9/2}}{\sqrt{bx^3+a}}dx}{3b}-\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 843

$$\frac{11c^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{8b}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 851

$$\frac{11c^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{4b}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 837

$$\frac{11c^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 25

$$\frac{11c^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{4b}\right)}{3b}-\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 766

$$\frac{11c^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\int\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{Ellipt}}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c-\right.}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right.\right.}}}\right)}{4b}\right)}{3b}$$

$$\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

$\downarrow$ 2420

$$\frac{11c^3\left(\frac{c^2(cx)^{5/2}\sqrt{a+bx^3}}{4b}-\frac{5ac^2\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{2b^{2/3}}\right.}{\cdots}$$

$$\frac{2c^2(cx)^{11/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(15/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(11/2))/(3*b*Sqrt[a + b*x^3]) + (11*c^3*((c^2*(c*x)^(5/2)*Sq
rt[a + b*x^3])/(4*b) - (5*a*c^2*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*
x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*
x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b
^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCo
s[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/
3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/
(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) -
 ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3
)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3]
)*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)
/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^
(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[
3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))))/(4*b)))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.52 (sec) , antiderivative size = 1148, normalized size of antiderivative = 2.09

method	result	size
elliptic	Expression too large to display	1148
risch	Expression too large to display	2203
default	Expression too large to display	2909

input
```
int((c*x)^(15/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(2/3/b^2*c^8*x^3*a
/((x^3+a/b)*b*c*x)^(1/2)+1/4/b^2*c^7*x^2*(b*c*x^4+a*c*x)^(1/2)-55/24*a*c^8
/b^2*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-
a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(
1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b
^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*((
(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/
b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*
(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))...
```

Fricas [F]

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(cx)^{\frac{15}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(15/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^7*x^7/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(15/2)/(b*x**3+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{15}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(15/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(15/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{15}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(15/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(15/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{15/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(15/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(15/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{15/2}}{(a+bx^3)^{3/2}}\,dx = \frac{\sqrt{c}\,c^7\left(-22\sqrt{x}\,\sqrt{b\,x^3+a}\,a\,x^2+4\sqrt{x}\,\sqrt{b\,x^3+a}\,b\,x^5+55\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^3+55(}{16b^2\,(b\,x^3+a)}$$

input
```
int((c*x)^(15/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**7*( - 22*sqrt(x)*sqrt(a + b*x**3)*a*x**2 + 4*sqrt(x)*sqrt(a +
b*x**3)*b*x**5 + 55*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 +
b**2*x**6),x)*a**3 + 55*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**
3 + b**2*x**6),x)*a**2*b*x**3))/(16*b**2*(a + b*x**3))
```

3.351 $\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 519

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}+\frac{5\left(1+\sqrt{3}\right)c^4\sqrt{cx}\sqrt{a+bx^3}}{3b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$-\frac{5\sqrt[3]{a}c^4\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{5\left(1-\sqrt{3}\right)\sqrt[3]{a}c^4\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/3*c^2*(c*x)^(5/2)/b/(b*x^3+a)^(1/2)+5/3*(1+3^(1/2))*c^4*(c*x)^(1/2)*(b*
x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)-5/3*a^(1/3)*c^4*(c*x)
^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/
3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
3^(1/4)/b^(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3
)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-5/18*(1-3^(1/2))*a^(1/3)*c^4*(c*x)^(1/2)*(a^
(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1
/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/
3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^
(5/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1
/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.12

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^2(cx)^{5/2}\left(1-\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)\right)}{b\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(9/2)/(a + b*x^3)^(3/2),x]
```

output
```
(c^2*(c*x)^(5/2)*(1 - Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/6, 3/2, 11/6
, -((b*x^3)/a)]))/(b*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {817, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{\left(a+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{5c^3\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{3b}-\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{10c^2\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{3b}-\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 837}$$

$$\frac{10c^2\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{3b}-\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{10c^2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{3b}-\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 766}$$

$$\frac{10c^2\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{3b}$$

$$\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{ 2420}$$

$$\frac{10c^2\left(\frac{(1+\sqrt{3})c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx}-\frac{\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}{(1+\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}}-\right.}{3b}$$

$$\frac{2c^2(cx)^{5/2}}{3b\sqrt{a+bx^3}}$$

input
```
Int[(c*x)^(9/2)/(a + b*x^3)^(3/2),x]
```

output
```
(-2*c^2*(c*x)^(5/2))/(3*b*Sqrt[a + b*x^3]) + (10*c^2*((((1 + Sqrt[3])*c^3*
Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1
/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1
/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*
x)^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c
+ (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/
3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b
*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1
/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 -
Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[
3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a
^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 1122, normalized size of antiderivative = 2.16

method	result	size
elliptic	Expression too large to display	1122
default	Expression too large to display	2715

input
```
int((c*x)^(9/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(-2/3/b*c^5*x^3/((
x^3+a/b)*b*c*x)^(1/2)+5/3*c^5/b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(
1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
/(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1
/3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/
b*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2
)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1
/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)
+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b...
```

Fricas [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*sqrt(c*x)*c^4*x^4/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 39.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**3+a)**(3/2),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/6)*hyper((3/2, 11/6), (17/6,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(3/2)*gamma(17/6))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(cx)^{\frac{9}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx=\int \frac{(c\,x)^{9/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^3)^{3/2}}\,dx=\frac{\sqrt{c}\,c^4\left(2\sqrt{x}\,\sqrt{b\,x^3+a}\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)ab\,x^3\right)}{2b\,(b\,x^3+a)}$$

input
```
int((c*x)^(9/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*c**4*(2*sqrt(x)*sqrt(a + b*x**3)*x**2 - 5*int((sqrt(x)*sqrt(a + b
*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**2 - 5*int((sqrt(x)*sqrt(a
+ b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*x**3))/(2*b*(a + b*x**
3))
```

3.352 $\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 516

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx=\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}}-\frac{2\left(1+\sqrt{3}\right)c\sqrt{cx}\sqrt{a+bx^3}}{3ab^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$
$$+\frac{2c\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3^{3/4}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$+\frac{\left(1-\sqrt{3}\right)c\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3*(c*x)^(5/2)/a/c/(b*x^3+a)^(1/2)-2/3*(1+3^(1/2))*c*(c*x)^(1/2)*(b*x^3+a
)^(1/2)/a/b^(2/3)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)+2/3*c*(c*x)^(1/2)*(a^(1/
3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2)
)*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1
/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(2/
3)/b^(2/3)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^
2)^(1/2)/(b*x^3+a)^(1/2)+1/9*(1-3^(1/2))*c*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^
2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(
1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/b^(2/3)/(b
^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x
^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.11

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \frac{2x(cx)^{3/2}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{6},\frac{3}{2},\frac{11}{6},-\frac{bx^3}{a}\right)}{5a\sqrt{a+bx^3}}$$

input
```
Integrate[(c*x)^(3/2)/(a + b*x^3)^(3/2),x]
```

output
```
(2*x*(c*x)^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/6, 3/2, 11/6, -((
b*x^3)/a)])/(5*a*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {819, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx$$

$\Big\downarrow$ 819

$$\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}}-\frac{2\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{3a}$$

$\Big\downarrow$ 851

$$\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}}-\frac{4\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{3ac}$$

$\Big\downarrow$ 837

$$\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}}-\frac{4\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{3ac}$$

$\Big\downarrow$ 25

$$\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}}-\frac{4\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{3ac}$$

$\Big\downarrow$ 766

$$\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}}-$$
$$\frac{4\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\ldots}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{3ac}$$

$\Big\downarrow$ 2420

$$\frac{\frac{2(cx)^{5/2}}{3ac\sqrt{a+bx^3}} - 4\left(\frac{\frac{(1+\sqrt{3})c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx} - \frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{(1-\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}{(1+\sqrt{3})\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+(1+\sqrt{3})\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}} - \right.}{3ac}$$

input
```
Int[(c*x)^(3/2)/(a + b*x^3)^(3/2),x]
```

output
```
(2*(c*x)^(5/2))/(3*a*c*Sqrt[a + b*x^3]) - (4*((((1 + Sqrt[3])*c^3*Sqrt[c*x
]*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1
/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1
/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*El
lipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + S
qrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b
^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/
(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)
*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c +
 (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])
*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/
(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c
 + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(3*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 1122, normalized size of antiderivative = 2.17

method	result	size
elliptic	Expression too large to display	1122
default	Expression too large to display	2716

input
```
int((c*x)^(3/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^3+a)^(1/2)*(c*x*(b*x^3+a))^(1/2)*(2/3*c^2*x^3/a/((x
^3+a/b)*b*c*x)^(1/2)-2/3*c^2/a*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*
(-a*b^2)^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1
/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1
/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/
(-1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/
3)))^(1/2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a
*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b
*(-a*b^2)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))/b*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I
*3^(1/2)/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1
/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^
2)^(1/3))/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.14

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = -\frac{2\left(\sqrt{bx^3+a}\sqrt{cx}ac+(bcx^4+acx)\sqrt{ac}\,\text{weierstrassZeta}\left(0,-\frac{4\,b}{a},\text{weierstrassPInverse}\left(0,-\frac{4\,b}{a},\frac{1}{x}\right)\right)\right)}{3\,(ab^2x^4+a^2bx)}$$

input
```
integrate((c*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*x^3 + a)*sqrt(c*x)*a*c + (b*c*x^4 + a*c*x)*sqrt(a*c)*weierstr
assZeta(0, -4*b/a, weierstrassPInverse(0, -4*b/a, 1/x)))/(a*b^2*x^4 + a^2*
b*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{6},\frac{3}{2}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2), (11/6,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(3/2)*gamma(11/6))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \int \frac{(c\,x)^{3/2}}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^3)^(3/2),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^3)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}\,x}{b^2x^6+2ab\,x^3+a^2}dx\right)c$$

input
```
int((c*x)^(3/2)/(b*x^3+a)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x
)*c
```

3.353 $\int \frac{1}{(cx)^{3/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 550

$$\int \frac{1}{(cx)^{3/2}(a+bx^3)^{3/2}}\,dx = \frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}} - \frac{8\sqrt{a+bx^3}}{3a^2c\sqrt{cx}} + \frac{8(1+\sqrt{3})\sqrt[3]{b}\sqrt{cx}\sqrt{a+bx^3}}{3a^2c^2\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)}$$
$$-\frac{8\sqrt[3]{b}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right)|\frac{1}{4}(2+\sqrt{3})\right)}{3^{3/4}a^{5/3}c^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$-\frac{4(1-\sqrt{3})\sqrt[3]{b}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+(1-\sqrt{3})\sqrt[3]{b}x}{\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x}\right),\frac{1}{4}(2+\sqrt{3}}{3\sqrt[4]{3}a^{5/3}c^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/c/(c*x)^(1/2)/(b*x^3+a)^(1/2)-8/3*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(1/2)+
8/3*(1+3^(1/2))*b^(1/3)*(c*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/c^2/(a^(1/3)+(1+3^
(1/2))*b^(1/3)*x)-8/3*b^(1/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*Elli
pticE((1-(a^(1/3)+(1-3^(1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)
^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(5/3)/c^2/(b^(1/3)*x*(a^(1/3)
+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)-4/9*(
1-3^(1/2))*b^(1/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM
(arccos((a^(1/3)+(1-3^(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1
/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)/c^2/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)
/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.10

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{3}{2},\frac{5}{6},-\frac{bx^3}{a}\right)}{a(cx)^{3/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 3/2, 5/6, -((b*x^3)/a)])
/(a*(c*x)^(3/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {819, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2} \left(a+bx^3\right)^{3/2}}dx$$

$\Big\downarrow$ 819

$$\frac{4\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 847

$$\frac{4\left(\frac{2b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{ac^3}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{3a}+\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 851

$$\frac{4\left(\frac{4b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{3a}+\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 837

$$\frac{4\left(\frac{4b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{3a}+\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 25

$$\frac{4\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{3a}+\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 766

$$\frac{4\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{\cdots}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{ac^4}\right.}{3a}$$

$$\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2420}$$

$$4\left(\frac{4b\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}}{2b^{2/3}}-\cdots\right)}{ac^4}\cdots}{3a}$$

$$\frac{2}{3ac\sqrt{cx}\sqrt{a+bx^3}}$$

input

```
Int[1/((c*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*Sqrt[c*x]*Sqrt[a + b*x^3]) + (4*((-2*Sqrt[a + b*x^3])/(a*c*Sqrt[c
*x]) + (4*b*((((1 + Sqrt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (
1 + Sqrt[3])*b^(1/3)*c*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1
/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(
1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 -
Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[
3])/4])/(Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sq
rt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1
/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1
/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*El
lipticF[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + S
qrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*
c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*
Sqrt[a + b*x^3])))/(a*c^4)))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 1150, normalized size of antiderivative = 2.09

method	result	size
elliptic	Expression too large to display	1150
risch	Expression too large to display	2199
default	Expression too large to display	2881

input
```
int(1/(c*x)^(3/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)*(-2*(b*c*x^3+a*c)/a^2/c^
2/(x*(b*c*x^3+a*c))^(1/2)-2/3*b/c*x^3/a^2/((x^3+a/b)*b*c*x)^(1/2)+8/3/a^2/
c*b*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*(x+1/2/b*(-
a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a*b^2)^(1/3)-1/2*I*3
^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a
*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*
(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(1/b*(-a*b^2)^(1
/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(((
-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b*(-a*b^2)^(1/3)+1/b
^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(
-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(1/2/b*(-a*b^2)^(
1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.14

$$\int \frac{1}{(cx)^{3/2}\,(a+bx^3)^{3/2}}\,dx = \frac{2\left(4\left(bx^4+ax\right)\sqrt{ac}\text{weierstrassZeta}\left(0,-\frac{4b}{a},\text{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\cdots\right)}{3\left(a^2bc^2x^4+a^3c^2x\right)}$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(4*(b*x^4 + a*x)*sqrt(a*c)*weierstrassZeta(0, -4*b/a, weierstrassPInve
rse(0, -4*b/a, 1/x)) + sqrt(b*x^3 + a)*sqrt(c*x)*a)/(a^2*b*c^2*x^4 + a^3*c
^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{3/2}\,(a+bx^3)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{1}{6}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{3}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/
2)*c**(3/2)*sqrt(x)*gamma(5/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\,(a+bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2} (a+bx^3)^{3/2}} \, dx = \int \frac{1}{(c\,x)^{3/2} \, (b\,x^3+a)^{3/2}} \, dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2} (a+bx^3)^{3/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^8+2ab\,x^5+a^2x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x**2 + 2*a*b*x**5 + b**2*x**
8),x))/c**2
```

3.354 $\int \frac{1}{(cx)^{9/2}(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 581

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}} - \frac{20\sqrt{a+bx^3}}{21a^2c(cx)^{7/2}}$$

$$+ \frac{80b\sqrt{a+bx^3}}{21a^3c^4\sqrt{cx}} - \frac{80\left(1+\sqrt{3}\right)b^{4/3}\sqrt{cx}\sqrt{a+bx^3}}{21a^3c^5\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

$$+ \frac{80b^{4/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\;3^{3/4}a^{8/3}c^5\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$+ \frac{40\left(1-\sqrt{3}\right)b^{4/3}\sqrt{cx}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right),\frac{1}{4}\left(2+\right)\right)}{21\sqrt[4]{3}a^{8/3}c^5\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/3/a/c/(c*x)^(7/2)/(b*x^3+a)^(1/2)-20/21*(b*x^3+a)^(1/2)/a^2/c/(c*x)^(7/2
)+80/21*b*(b*x^3+a)^(1/2)/a^3/c^4/(c*x)^(1/2)-80/21*(1+3^(1/2))*b^(4/3)*(c
*x)^(1/2)*(b*x^3+a)^(1/2)/a^3/c^5/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)+80/21*b^
(4/3)*(c*x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*EllipticE((1-(a^(1/3)+(1-3^(
1/2))*b^(1/3)*x)^2/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*3^(1/4)/a^(8/3)/c^5/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+
3^(1/2))*b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)+40/63*(1-3^(1/2))*b^(4/3)*(c*
x)^(1/2)*(a^(1/3)+b^(1/3)*x)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+(1+3^(1/2))*b^(1/3)*x)^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)+(1-3^
(1/2))*b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*b^(1/3)*x)),1/4*6^(1/2)+1/4*2^(1/2)
)*3^(3/4)/a^(8/3)/c^5/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+(1+3^(1/2))*
b^(1/3)*x)^2)^(1/2)/(b*x^3+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.10

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^3\right)^{3/2}}\,dx=-\frac{2x\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{3}{2},-\frac{1}{6},-\frac{bx^3}{a}\right)}{7a(cx)^{9/2}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((c*x)^(9/2)*(a + b*x^3)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 3/2, -1/6, -((b*x^3)/a)]
)/(7*a*(c*x)^(9/2)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {819, 847, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2} \left(a+bx^3\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 819$$

$$\frac{10 \int \frac{1}{(cx)^{9/2}\sqrt{bx^3+a}}dx}{3a}+\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 847$$

$$\frac{10 \left(-\frac{4b\int \frac{1}{(cx)^{3/2}\sqrt{bx^3+a}}dx}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{3a}+\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 847$$

$$\frac{10 \left(-\frac{4b\left(\frac{2b\int \frac{(cx)^{3/2}}{\sqrt{bx^3+a}}dx}{ac^3}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{3a}+\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 851$$

$$\frac{10 \left(-\frac{4b\left(\frac{4b\int \frac{c^2x^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{3a}+\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 837$$

$$\frac{10 \left(-\frac{4b\left(\frac{4b\left(-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\int -\frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{3a}+\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow 25$$

$$\frac{10\left(-\frac{4b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)a^{2/3}c^2\int \frac{1}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}\right)}{ac^4}-\frac{2\sqrt{a+bx^3}}{ac\sqrt{cx}}\right)}{7ac^3}-\frac{2\sqrt{a+bx^3}}{7ac(cx)^{7/2}}\right)}{3a}+\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\Big\downarrow 766$$

$$\frac{10\left(-\frac{4b\left(\frac{4b\left(\frac{\int \frac{2b^{2/3}x^2c^2+\left(1-\sqrt{3}\right)a^{2/3}c^2}{\sqrt{bx^3+a}}d\sqrt{cx}}{2b^{2/3}}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{4\sqrt[4]{3}b^{2/3}\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}}\right.}{ac^4}\right.}{7ac^3}\right.}{3a}$$

$$\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

$$\downarrow \text{2420}$$

$$10\left(-\frac{4b\left(\frac{4b\left(\frac{\left(1+\sqrt{3}\right)c^3\sqrt{cx}\sqrt{a+bx^3}}{\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx}-\frac{\sqrt[4]{3}\sqrt[3]{a}c\sqrt{cx}\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)\sqrt{\frac{a^{2/3}c^2-\sqrt[3]{a}\sqrt[3]{b}c^2x+b^{2/3}c^2x^2}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}{\left(1+\sqrt{3}\right)\sqrt[3]{b}xc+\sqrt[3]{a}c}\right)|\frac{1}{4}\left(2+\sqrt{\cdots}\right.\right.}{\sqrt{a+bx^3}\sqrt{\frac{\sqrt[3]{b}cx\left(\sqrt[3]{a}c+\sqrt[3]{b}cx\right)}{\left(\sqrt[3]{a}c+\left(1+\sqrt{3}\right)\sqrt[3]{b}cx\right)^2}}\cdots}}{2b^{2/3}\cdots}\cdots\right)}{ac^4\cdots}\cdots\right)}{\cdots}\cdots\right)$$

$$\frac{2}{3ac(cx)^{7/2}\sqrt{a+bx^3}}$$

input
```
Int[1/((c*x)^(9/2)*(a + b*x^3)^(3/2)),x]
```

output
```
2/(3*a*c*(c*x)^(7/2)*Sqrt[a + b*x^3]) + (10*((-2*Sqrt[a + b*x^3])/(7*a*c*(
c*x)^(7/2)) - (4*b*((-2*Sqrt[a + b*x^3])/(a*c*Sqrt[c*x]) + (4*b*((((1 + Sq
rt[3])*c^3*Sqrt[c*x]*Sqrt[a + b*x^3])/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c
*x) - (3^(1/4)*a^(1/3)*c*Sqrt[c*x]*(a^(1/3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)
*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])
*b^(1/3)*c*x)^2]*EllipticE[ArcCos[(a^(1/3)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/
(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)], (2 + Sqrt[3])/4])/(Sqrt[(b^(1/3)
*c*x*(a^(1/3)*c + b^(1/3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]
*Sqrt[a + b*x^3]))/(2*b^(2/3)) - ((1 - Sqrt[3])*a^(1/3)*c*Sqrt[c*x]*(a^(1/
3)*c + b^(1/3)*c*x)*Sqrt[(a^(2/3)*c^2 - a^(1/3)*b^(1/3)*c^2*x + b^(2/3)*c^
2*x^2)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*EllipticF[ArcCos[(a^(1/3
)*c + (1 - Sqrt[3])*b^(1/3)*c*x)/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)],
 (2 + Sqrt[3])/4])/(4*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*c*x*(a^(1/3)*c + b^(1/
3)*c*x))/(a^(1/3)*c + (1 + Sqrt[3])*b^(1/3)*c*x)^2]*Sqrt[a + b*x^3])))/(a*
c^4)))/(7*a*c^3)))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.56 (sec) , antiderivative size = 1204, normalized size of antiderivative = 2.07

method	result	size
elliptic	Expression too large to display	1204
risch	Expression too large to display	2213
default	Expression too large to display	3074

input
```
int(1/(c*x)^(9/2)/(b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^3+a))^(1/2)/(c*x)^(1/2)/(b*x^3+a)^(1/2)*(-2/7/a^2/c^5*(b*c*x^4+a
*c*x)^(1/2)/x^4+8/21*b/a^3/c^5/x*(b*c*x^4+a*c*x)^(1/2)+58/21*(b*c*x^3+a*c)
*b/a^3/c^5/(x*(b*c*x^3+a*c))^(1/2)+2/3*b^2/c^4*x^3/a^3/((x^3+a/b)*b*c*x)^(
1/2)-80/21*b^2/a^3/c^4*(x*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)
^(1/3))*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))+(1/2/b*(-a
*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2)*(x-1/b*(-a*b^2)^(1/3))^2*(1/b*(-a*b
^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(-1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/
2)*(1/b*(-a*b^2)^(1/3)*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(x-1/b*(-a*b^2
)^(1/3)))^(1/2)*(((-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/b
*(-a*b^2)^(1/3)+1/b^2*(-a*b^2)^(2/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)
/b*(-a*b^2)^(1/3))*b/(-a*b^2)^(1/3)*EllipticF(((-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*x/(-1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))/(x-1/b*(-a*b^2)^(1/3)))^(1/2),((3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1
/2)/b*(-a*b^2)^(1/3))*(1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)
)/(1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))/(3/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+(1/2/b*(-a*b^2)^(1/3)+1/2*I...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.16

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2\left(40\left(b^2x^7+abx^4\right)\sqrt{ac}\,\mathrm{weierstrassZeta}\left(0,-\frac{4b}{a},\mathrm{weierstrassPInverse}\left(0,-\frac{4b}{a},\frac{1}{x}\right)\right)+\left(10\,abx^3+3\,a^2\right)\sqrt{bx^3}}{21\left(a^3bc^5x^7+a^4c^5x^4\right)}$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/21*(40*(b^2*x^7 + a*b*x^4)*sqrt(a*c)*weierstrassZeta(0, -4*b/a, weierst
rassPInverse(0, -4*b/a, 1/x)) + (10*a*b*x^3 + 3*a^2)*sqrt(b*x^3 + a)*sqrt(
c*x))/(a^3*b*c^5*x^7 + a^4*c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 47.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^3\right)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{7}{6},\frac{3}{2}\\-\frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)}$$

input
```
integrate(1/(c*x)**(9/2)/(b*x**3+a)**(3/2),x)
```

output
```
gamma(-7/6)*hyper((-7/6, 3/2), (-1/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*c**(9/2)*x**(7/2)*gamma(-1/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^3\right)^{3/2}}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{3}{2}}(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2} (a+bx^3)^{3/2}} \, dx = \int \frac{1}{(bx^3+a)^{\frac{3}{2}} (cx)^{\frac{9}{2}}} \, dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(3/2)*(c*x)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{9/2} (a+bx^3)^{3/2}} \, dx = \int \frac{1}{(c\,x)^{9/2}\,(b\,x^3+a)^{3/2}} \, dx$$

input
```
int(1/((c*x)^(9/2)*(a + b*x^3)^(3/2)),x)
```

output
```
int(1/((c*x)^(9/2)*(a + b*x^3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{9/2} (a+bx^3)^{3/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^3+a}}{b^2x^{11}+2ab\,x^8+a^2x^5}dx\right)}{c^5}$$

input
```
int(1/(c*x)^(9/2)/(b*x^3+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**3))/(a**2*x**5 + 2*a*b*x**8 + b**2*x**
11),x))/c**5
```

3.355 $\int x^{11}\sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx=-\frac{a^3(a+bx^3)^{4/3}}{4b^4}+\frac{3a^2(a+bx^3)^{7/3}}{7b^4}-\frac{3a(a+bx^3)^{10/3}}{10b^4}+\frac{(a+bx^3)^{13/3}}{13b^4}$$

output
```
-1/4*a^3*(b*x^3+a)^(4/3)/b^4+3/7*a^2*(b*x^3+a)^(7/3)/b^4-3/10*a*(b*x^3+a)^
(10/3)/b^4+1/13*(b*x^3+a)^(13/3)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx=\frac{(a+bx^3)^{4/3}(-81a^3+108a^2bx^3-126ab^2x^6+140b^3x^9)}{1820b^4}$$

input
```
Integrate[x^11*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(4/3)*(-81*a^3 + 108*a^2*b*x^3 - 126*a*b^2*x^6 + 140*b^3*x^9)
)/(1820*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \sqrt[3]{a+bx^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^9 \sqrt[3]{bx^3+a} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{10/3}}{b^3} - \frac{3a\left(bx^3+a\right)^{7/3}}{b^3} + \frac{3a^2\left(bx^3+a\right)^{4/3}}{b^3} - \frac{a^3 \sqrt[3]{bx^3+a}}{b^3}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3a^3\left(a+bx^3\right)^{4/3}}{4b^4} + \frac{9a^2\left(a+bx^3\right)^{7/3}}{7b^4} + \frac{3\left(a+bx^3\right)^{13/3}}{13b^4} - \frac{9a\left(a+bx^3\right)^{10/3}}{10b^4}\right)$$

input
```
Int[x^11*(a + b*x^3)^(1/3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(4/3))/(4*b^4) + (9*a^2*(a + b*x^3)^(7/3))/(7*b^4) -
(9*a*(a + b*x^3)^(10/3))/(10*b^4) + (3*(a + b*x^3)^(13/3))/(13*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-140 b^{3} x^{9}+126 a\,b^{2} x^{6}-108 a^{2} b\,x^{3}+81 a^{3}\right)}{1820 b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-140 b^{3} x^{9}+126 a\,b^{2} x^{6}-108 a^{2} b\,x^{3}+81 a^{3}\right)}{1820 b^{4}}$	47
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-140 b^{3} x^{9}+126 a\,b^{2} x^{6}-108 a^{2} b\,x^{3}+81 a^{3}\right)}{1820 b^{4}}$	47
trager	$-\frac{\left(-140 b^{4} x^{12}-14 a\,b^{3} x^{9}+18 a^{2} b^{2} x^{6}-27 a^{3} b\,x^{3}+81 a^{4}\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{1820 b^{4}}$	58
risch	$-\frac{\left(-140 b^{4} x^{12}-14 a\,b^{3} x^{9}+18 a^{2} b^{2} x^{6}-27 a^{3} b\,x^{3}+81 a^{4}\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{1820 b^{4}}$	58

input
```
int(x^11*(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/1820*(b*x^3+a)^(4/3)*(-140*b^3*x^9+126*a*b^2*x^6-108*a^2*b*x^3+81*a^3)/
b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx=\frac{(140\,b^4x^{12}+14\,ab^3x^9-18\,a^2b^2x^6+27\,a^3bx^3-81\,a^4)(bx^3+a)^{\frac{1}{3}}}{1820\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/1820*(140*b^4*x^12 + 14*a*b^3*x^9 - 18*a^2*b^2*x^6 + 27*a^3*b*x^3 - 81*a
^4)*(b*x^3 + a)^(1/3)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx$$
$$=\begin{cases} -\frac{81a^4\sqrt[3]{a+bx^3}}{1820b^4}+\frac{27a^3x^3\sqrt[3]{a+bx^3}}{1820b^3}-\frac{9a^2x^6\sqrt[3]{a+bx^3}}{910b^2}+\frac{ax^9\sqrt[3]{a+bx^3}}{130b}+\frac{x^{12}\sqrt[3]{a+bx^3}}{13} & \text{for } b\neq 0\\ \frac{\sqrt[3]{a}x^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((-81*a**4*(a + b*x**3)**(1/3)/(1820*b**4) + 27*a**3*x**3*(a + b*
x**3)**(1/3)/(1820*b**3) - 9*a**2*x**6*(a + b*x**3)**(1/3)/(910*b**2) + a*
x**9*(a + b*x**3)**(1/3)/(130*b) + x**12*(a + b*x**3)**(1/3)/13, Ne(b, 0))
, (a**(1/3)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx=\frac{(bx^3+a)^{\frac{13}{3}}}{13\,b^4}-\frac{3\,(bx^3+a)^{\frac{10}{3}}a}{10\,b^4}+\frac{3\,(bx^3+a)^{\frac{7}{3}}a^2}{7\,b^4}-\frac{(bx^3+a)^{\frac{4}{3}}a^3}{4\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/13*(b*x^3 + a)^(13/3)/b^4 - 3/10*(b*x^3 + a)^(10/3)*a/b^4 + 3/7*(b*x^3 +
 a)^(7/3)*a^2/b^4 - 1/4*(b*x^3 + a)^(4/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx = \frac{140\,(bx^3+a)^{\frac{13}{3}} - 546\,(bx^3+a)^{\frac{10}{3}}a + 780\,(bx^3+a)^{\frac{7}{3}}a^2 - 455\,(bx^3+a)^{\frac{4}{3}}a^3}{1820\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/1820*(140*(b*x^3 + a)^(13/3) - 546*(b*x^3 + a)^(10/3)*a + 780*(b*x^3 + a
)^(7/3)*a^2 - 455*(b*x^3 + a)^(4/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx = \left(b\,x^3+a\right)^{1/3}\left(\frac{x^{12}}{13} - \frac{81\,a^4}{1820\,b^4} + \frac{a\,x^9}{130\,b} + \frac{27\,a^3\,x^3}{1820\,b^3} - \frac{9\,a^2\,x^6}{910\,b^2}\right)$$

input
```
int(x^11*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(1/3)*(x^12/13 - (81*a^4)/(1820*b^4) + (a*x^9)/(130*b) + (27*a
^3*x^3)/(1820*b^3) - (9*a^2*x^6)/(910*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\sqrt[3]{a+bx^3}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(140b^4x^{12} + 14a\,b^3x^9 - 18a^2b^2x^6 + 27a^3b\,x^3 - 81a^4)}{1820b^4}$$

input
```
int(x^11*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*( - 81*a**4 + 27*a**3*b*x**3 - 18*a**2*b**2*x**6 + 14
*a*b**3*x**9 + 140*b**4*x**12))/(1820*b**4)
```

3.356 $\int x^8 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^8 \sqrt[3]{a+bx^3}\,dx = \frac{a^2(a+bx^3)^{4/3}}{4b^3} - \frac{2a(a+bx^3)^{7/3}}{7b^3} + \frac{(a+bx^3)^{10/3}}{10b^3}$$

output
```
1/4*a^2*(b*x^3+a)^(4/3)/b^3-2/7*a*(b*x^3+a)^(7/3)/b^3+1/10*(b*x^3+a)^(10/3
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^8 \sqrt[3]{a+bx^3}\,dx = \frac{(a+bx^3)^{4/3}(9a^2 - 12abx^3 + 14b^2x^6)}{140b^3}$$

input
```
Integrate[x^8*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(4/3)*(9*a^2 - 12*a*b*x^3 + 14*b^2*x^6))/(140*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt[3]{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\int x^6 \sqrt[3]{bx^3+a}dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{7/3}}{b^2}-\frac{2a\left(bx^3+a\right)^{4/3}}{b^2}+\frac{a^2\sqrt[3]{bx^3+a}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{3a^2\left(a+bx^3\right)^{4/3}}{4b^3}+\frac{3\left(a+bx^3\right)^{10/3}}{10b^3}-\frac{6a\left(a+bx^3\right)^{7/3}}{7b^3}\right)$$

input
```
Int[x^8*(a + b*x^3)^(1/3),x]
```

output
```
((3*a^2*(a + b*x^3)^(4/3))/(4*b^3) - (6*a*(a + b*x^3)^(7/3))/(7*b^3) + (3*
(a + b*x^3)^(10/3))/(10*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(b x^3+a)^{\frac{4}{3}} (14b^2x^6-12ab x^3+9a^2)}{140b^3}$	36
pseudoelliptic	$\frac{(b x^3+a)^{\frac{4}{3}} (14b^2x^6-12ab x^3+9a^2)}{140b^3}$	36
orering	$\frac{(b x^3+a)^{\frac{4}{3}} (14b^2x^6-12ab x^3+9a^2)}{140b^3}$	36
trager	$\frac{(14b^3x^9+2a b^2x^6-3a^2b x^3+9a^3) (b x^3+a)^{\frac{1}{3}}}{140b^3}$	47
risch	$\frac{(14b^3x^9+2a b^2x^6-3a^2b x^3+9a^3) (b x^3+a)^{\frac{1}{3}}}{140b^3}$	47

input
```
int(x^8*(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/140*(b*x^3+a)^(4/3)*(14*b^2*x^6-12*a*b*x^3+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^8 \sqrt[3]{a+bx^3}\,dx = \frac{(14\,b^3x^9+2\,ab^2x^6-3\,a^2bx^3+9\,a^3)(bx^3+a)^{\frac{1}{3}}}{140\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/140*(14*b^3*x^9 + 2*a*b^2*x^6 - 3*a^2*b*x^3 + 9*a^3)*(b*x^3 + a)^(1/3)/b
^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x^8\sqrt[3]{a+bx^3}\,dx = \begin{cases} \frac{9a^3\sqrt[3]{a+bx^3}}{140b^3} - \frac{3a^2x^3\sqrt[3]{a+bx^3}}{140b^2} + \frac{ax^6\sqrt[3]{a+bx^3}}{70b} + \frac{x^9\sqrt[3]{a+bx^3}}{10} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{a}x^9}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((9*a**3*(a + b*x**3)**(1/3)/(140*b**3) - 3*a**2*x**3*(a + b*x**3
)**(1/3)/(140*b**2) + a*x**6*(a + b*x**3)**(1/3)/(70*b) + x**9*(a + b*x**3
)**(1/3)/10, Ne(b, 0)), (a**(1/3)*x**9/9, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^8\sqrt[3]{a+bx^3}\,dx = \frac{(bx^3+a)^{\frac{10}{3}}}{10\,b^3} - \frac{2\,(bx^3+a)^{\frac{7}{3}}a}{7\,b^3} + \frac{(bx^3+a)^{\frac{4}{3}}a^2}{4\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/10*(b*x^3 + a)^(10/3)/b^3 - 2/7*(b*x^3 + a)^(7/3)*a/b^3 + 1/4*(b*x^3 + a
)^(4/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^8 \sqrt[3]{a+bx^3}\, dx = \frac{14\,(bx^3+a)^{\frac{10}{3}} - 40\,(bx^3+a)^{\frac{7}{3}}a + 35\,(bx^3+a)^{\frac{4}{3}}a^2}{140\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/140*(14*(b*x^3 + a)^(10/3) - 40*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/
3)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^8 \sqrt[3]{a+bx^3}\, dx = \left(b\,x^3+a\right)^{1/3}\left(\frac{x^9}{10}+\frac{9\,a^3}{140\,b^3}+\frac{a\,x^6}{70\,b}-\frac{3\,a^2\,x^3}{140\,b^2}\right)$$

input
```
int(x^8*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(1/3)*(x^9/10 + (9*a^3)/(140*b^3) + (a*x^6)/(70*b) - (3*a^2*x^
3)/(140*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^8 \sqrt[3]{a+bx^3}\, dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(14b^3x^9+2a\,b^2x^6-3a^2b\,x^3+9a^3)}{140b^3}$$

input
```
int(x^8*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*(9*a**3 - 3*a**2*b*x**3 + 2*a*b**2*x**6 + 14*b**3*x**
9))/(140*b**3)
```

3.357 $\int x^5 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^5 \sqrt[3]{a+bx^3}\,dx = -\frac{a(a+bx^3)^{4/3}}{4b^2} + \frac{(a+bx^3)^{7/3}}{7b^2}$$

output
```
-1/4*a*(b*x^3+a)^(4/3)/b^2+1/7*(b*x^3+a)^(7/3)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^5 \sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a+bx^3}(-3a^2+abx^3+4b^2x^6)}{28b^2}$$

input
```
Integrate[x^5*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(-3*a^2 + a*b*x^3 + 4*b^2*x^6))/(28*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^3 \sqrt[3]{bx^3+a}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{(bx^3+a)^{4/3}}{b}-\frac{a\sqrt[3]{bx^3+a}}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{7/3}}{7b^2}-\frac{3a(a+bx^3)^{4/3}}{4b^2}\right)$$

input
```
Int[x^5*(a + b*x^3)^(1/3),x]
```

output
```
((-3*a*(a + b*x^3)^(4/3))/(4*b^2) + (3*(a + b*x^3)^(7/3))/(7*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(-4b\,x^{3}+3a\right)}{28b^{2}}$	25
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(-4b\,x^{3}+3a\right)}{28b^{2}}$	25
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(-4b\,x^{3}+3a\right)}{28b^{2}}$	25
trager	$-\frac{\left(-4b^{2}x^{6}-ab\,x^{3}+3a^{2}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{28b^{2}}$	36
risch	$-\frac{\left(-4b^{2}x^{6}-ab\,x^{3}+3a^{2}\right)\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{28b^{2}}$	36

input
```
int(x^5*(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/28*(b*x^3+a)^(4/3)*(-4*b*x^3+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^5\sqrt[3]{a+bx^3}\,dx=\frac{(4\,b^2x^6+abx^3-3\,a^2)(bx^3+a)^{\frac{1}{3}}}{28\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/28*(4*b^2*x^6 + a*b*x^3 - 3*a^2)*(b*x^3 + a)^(1/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int x^5 \sqrt[3]{a+bx^3}\,dx = \begin{cases} -\frac{3a^2\sqrt[3]{a+bx^3}}{28b^2} + \frac{ax^3\sqrt[3]{a+bx^3}}{28b} + \frac{x^6\sqrt[3]{a+bx^3}}{7} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{a}x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((-3*a**2*(a + b*x**3)**(1/3)/(28*b**2) + a*x**3*(a + b*x**3)**(1
/3)/(28*b) + x**6*(a + b*x**3)**(1/3)/7, Ne(b, 0)), (a**(1/3)*x**6/6, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^5 \sqrt[3]{a+bx^3}\,dx = \frac{(bx^3+a)^{\frac{7}{3}}}{7\,b^2} - \frac{(bx^3+a)^{\frac{4}{3}}a}{4\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/7*(b*x^3 + a)^(7/3)/b^2 - 1/4*(b*x^3 + a)^(4/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^5 \sqrt[3]{a+bx^3}\, dx = \frac{4\,(bx^3+a)^{\frac{7}{3}} - 7\,(bx^3+a)^{\frac{4}{3}}a}{28\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/28*(4*(b*x^3 + a)^(7/3) - 7*(b*x^3 + a)^(4/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^5 \sqrt[3]{a+bx^3}\, dx = \left(b\,x^3+a\right)^{1/3}\left(\frac{x^6}{7} - \frac{3\,a^2}{28\,b^2} + \frac{a\,x^3}{28\,b}\right)$$

input
```
int(x^5*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(1/3)*(x^6/7 - (3*a^2)/(28*b^2) + (a*x^3)/(28*b))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^5 \sqrt[3]{a+bx^3}\, dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(4b^2x^6 + ab\,x^3 - 3a^2)}{28b^2}$$

input
```
int(x^5*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*( - 3*a**2 + a*b*x**3 + 4*b**2*x**6))/(28*b**2)
```

3.358 $\int x^2 \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^2 \sqrt[3]{a+bx^3}\,dx = \frac{(a+bx^3)^{4/3}}{4b}$$

output
```
1/4*(b*x^3+a)^(4/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2 \sqrt[3]{a+bx^3}\,dx = \frac{(a+bx^3)^{4/3}}{4b}$$

input
```
Integrate[x^2*(a + b*x^3)^(1/3),x]
```

output
```
(a + b*x^3)^(4/3)/(4*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^3\right)^{4/3}}{4b}$$

input
```
Int[x^2*(a + b*x^3)^(1/3),x]
```

output
```
(a + b*x^3)^(4/3)/(4*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15
derivativedivides	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15
default	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15
trager	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15
risch	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{4b}$	15

input
```
int(x^2*(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^3+a)^(4/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt[3]{a+bx^3}\,dx=\frac{(bx^3+a)^{\frac{4}{3}}}{4\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/4*(b*x^3 + a)^(4/3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int x^2\sqrt[3]{a+bx^3}\,dx = \begin{cases} \frac{a\sqrt[3]{a+bx^3}}{4b} + \frac{x^3\sqrt[3]{a+bx^3}}{4} & \text{for } b\neq 0 \\ \frac{\sqrt[3]{a}x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((a*(a + b*x**3)**(1/3)/(4*b) + x**3*(a + b*x**3)**(1/3)/4, Ne(b,
 0)), (a**(1/3)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt[3]{a+bx^3}\,dx = \frac{(bx^3+a)^{\frac{4}{3}}}{4\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/4*(b*x^3 + a)^(4/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2\sqrt[3]{a+bx^3}\,dx = \frac{(bx^3+a)^{\frac{4}{3}}}{4\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/4*(b*x^3 + a)^(4/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2 \sqrt[3]{a + bx^3} \, dx = \frac{(b\,x^3 + a)^{4/3}}{4\,b}$$

input
```
int(x^2*(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(4/3)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2 \sqrt[3]{a + bx^3} \, dx = \frac{(b\,x^3 + a)^{\frac{4}{3}}}{4b}$$

input
```
int(x^2*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*(a + b*x**3))/(4*b)
```

3.359 $\int \frac{\sqrt[3]{a+bx^3}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{\sqrt[3]{a+bx^3}}{x} \, dx = \sqrt[3]{a+bx^3} - \frac{\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{1}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx^3}\right)$$

output
```
(b*x^3+a)^(1/3)-1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)
/a^(1/3))*3^(1/2)-1/2*a^(1/3)*ln(x)+1/2*a^(1/3)*ln(a^(1/3)-(b*x^3+a)^(1/3)
)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = \sqrt[3]{a+bx^3} - \frac{\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\sqrt[3]{a}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - \frac{1}{6}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)$$

input
```
Integrate[(a + b*x^3)^(1/3)/x,x]
```

output
```
(a + b*x^3)^(1/3) - (a^(1/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]])/Sqrt[3] + (a^(1/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)])/3 - (a^(1/3)
*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^3}dx^3$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3+3\sqrt[3]{a+bx^3}\right)$$

$\Big\downarrow$ 69

$$\frac{1}{3}\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)$$

$\Big\downarrow$ 16

$$\frac{1}{3}\left(a\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)$$

$\Big\downarrow$ 1082

$$\frac{1}{3}\left(a\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{3}\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^3}\right)$$

input
```
Int[(a + b*x^3)^(1/3)/x,x]
```

output
```
(3*(a + b*x^3)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*a^(2/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\left(b\,x^{3}+a\right)^{\frac{1}{3}}+\frac{a^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3}-\frac{a^{\frac{1}{3}}\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6}-\frac{a^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{3}+a\right.}{3a^{\frac{1}{3}}}\right.}{3}$

input
```
int((b*x^3+a)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/3)+1/3*a^(1/3)*ln((b*x^3+a)^(1/3)-a^(1/3))-1/6*a^(1/3)*ln((b*
x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))-1/3*a^(1/3)*3^(1/2)*arctan(2
/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = -\frac{1}{3}\,\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right) - \frac{1}{6}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{3}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\left(bx^3+a\right)^{\frac{1}{3}}$$

input
```
integrate((b*x^3+a)^(1/3)/x,x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqr
t(3)*a)/a) - 1/6*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 1/3*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (b*x^3 + a)^(
1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = -\frac{\sqrt[3]{b}x\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3},-\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x,x)
```

output
```
-b**(1/3)*x*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*pi)/(b*x
**3))/(3*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = &-\frac{1}{3}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)\\ &-\frac{1}{6}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)\\ &+\frac{1}{3}\,a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\left(bx^3+a\right)^{\frac{1}{3}}\end{aligned}$$

input
```
integrate((b*x^3+a)^(1/3)/x,x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3)) - 1/6*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3)) + 1/3*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (b*x^3 + a)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = -\frac{1}{3}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{6}\,a^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{3}\,a^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+(bx^3+a)^{\frac{1}{3}}$$

input
```
integrate((b*x^3+a)^(1/3)/x,x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^
(1/3)) - 1/6*a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3)) + 1/3*a^(1/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3))) + (b*x^3 + a)^
(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = \frac{a^{1/3}\ln\left(a\,(b\,x^3+a)^{1/3}-a^{4/3}\right)}{3}+(b\,x^3+a)^{1/3} - \frac{a^{1/3}\ln\left(3\,a^{4/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+3\,a\,(b\,x^3+a)^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3} + a^{1/3}\ln\left(3\,a\,(b\,x^3+a)^{1/3}-9\,a^{4/3}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int((a + b*x^3)^(1/3)/x,x)
```

output
```
(a^(1/3)*log(a*(a + b*x^3)^(1/3) - a^(4/3)))/3 + (a + b*x^3)^(1/3) - (a^(1
/3)*log(3*a^(4/3)*((3^(1/2)*1i)/2 + 1/2) + 3*a*(a + b*x^3)^(1/3))*((3^(1/2
)*1i)/2 + 1/2))/3 + a^(1/3)*log(3*a*(a + b*x^3)^(1/3) - 9*a^(4/3)*((3^(1/2
)*1i)/6 - 1/6))*((3^(1/2)*1i)/6 - 1/6)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x}\,dx = \left(b\,x^{3}+a\right)^{\frac{1}{3}}+\left(\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{b\,x^{4}+ax}dx\right)a$$

input
```
int((b*x^3+a)^(1/3)/x,x)
```

output
```
(a + b*x**3)**(1/3) + int((a + b*x**3)**(1/3)/(a*x + b*x**4),x)*a
```

3.360 $\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = -\frac{\sqrt[3]{a+bx^3}}{3x^3} - \frac{b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}}$$

output
```
-1/3*(b*x^3+a)^(1/3)/x^3-1/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1
/2)/a^(1/3))*3^(1/2)/a^(2/3)-1/6*b*ln(x)/a^(2/3)+1/6*b*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = -\frac{6a^{2/3}\sqrt[3]{a+bx^3}+2\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx^3\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)+bx^3\log\left(a^{2/3}+\cdots\right.}{18a^{2/3}x^3}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^4,x]
```

output
```
-1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b
*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1/3)]
 + b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(a^
(2/3)*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{bx^3+a}}{x^6}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{1}{3}b\int \frac{1}{x^3\left(bx^3+a\right)^{2/3}}dx^3-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{1}{3}b\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{1}{3}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x^3}\right)$$

input
```
Int[(a + b*x^3)^(1/3)/x^4,x]
```

output
```
(-((a + b*x^3)^(1/3)/x^3) + (b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(2*a^(2/3))))/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x^{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b\,x^{3}-\frac{\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)b\,x^{3}}{2}-3\left(b\,x^{3}+a\right.}{9a^{\frac{2}{3}}x^{3}}$

input
```
int((b*x^3+a)^(1/3)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*(-arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*b*x^
3+ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3-1/2*ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+
a)^(1/3)+a^(2/3))*b*x^3-3*(b*x^3+a)^(1/3)*a^(2/3))/a^(2/3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}abx^3\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^3+a)^{\frac{1}{3}}\,(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+(a^2)^{\frac{2}{3}}bx^3\log\left((bx^3+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx\right.}{18\,a^2x^3}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4,x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(1/3)*(a^2)^(1/6)*a*b*x^3*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)
^(1/3)*a + 2*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^3*log((
b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(a
^2)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/
3)*a^2)/(a^2*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = -\frac{\sqrt[3]{b}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x**4,x)
```

output
```
-b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**3)
)/(3*x**2*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{2}{3}}} - \frac{b\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{2}{3}}} + \frac{b\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}} - \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{3x^3}$$

input
```
integrate((b*x^3+a)^(1/3)/x^4,x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(2/3) - 1/18*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(2/3) + 1/9*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/3*(b*x^3
 + a)^(1/3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = -\frac{1}{18}\,b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2\log\left(\left|(bx^3+a\right.\right.}{a^{\frac{2}{3}}}\right.$$

input
```
integrate((b*x^3+a)^(1/3)/x^4,x, algorithm="giac")
```

output
```
-1/18*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(2/3) + log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x^3 + a
)^(1/3)/(b*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = \frac{b\,\ln\left(b\,(b\,x^3+a)^{1/3}-a^{1/3}\,b\right)}{9\,a^{2/3}}-\frac{(b\,x^3+a)^{1/3}}{3\,x^3} - \frac{\ln\left(\frac{a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+b\,(b\,x^3+a)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{18\,a^{2/3}} - \frac{\ln\left(\frac{a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2}+b\,(b\,x^3+a)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{18\,a^{2/3}}$$

input
```
int((a + b*x^3)^(1/3)/x^4,x)
```

output
```
(b*log(b*(a + b*x^3)^(1/3) - a^(1/3)*b))/(9*a^(2/3)) - (a + b*x^3)^(1/3)/(
3*x^3) - (log((a^(1/3)*(b - 3^(1/2)*b*1i))/2 + b*(a + b*x^3)^(1/3))*(b - 3
^(1/2)*b*1i))/(18*a^(2/3)) - (log((a^(1/3)*(b + 3^(1/2)*b*1i))/2 + b*(a +
b*x^3)^(1/3))*(b + 3^(1/2)*b*1i))/(18*a^(2/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4}\,dx = \frac{-(b\,x^3+a)^{\frac{1}{3}}+\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^4+ax}dx\right)b\,x^3}{3x^3}$$

input
```
int((b*x^3+a)^(1/3)/x^4,x)
```

output
```
( - (a + b*x**3)**(1/3) + int((a + b*x**3)**(1/3)/(a*x + b*x**4),x)*b*x**3
)/(3*x**3)
```

3.361 $\int x^4 \sqrt[3]{a+bx^3}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int x^4 \sqrt[3]{a+bx^3}\, dx = \frac{ax^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{6}x^5\sqrt[3]{a+bx^3}+\frac{a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}+\frac{a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}$$

output
```
1/18*a*x^2*(b*x^3+a)^(1/3)/b+1/6*x^5*(b*x^3+a)^(1/3)+1/27*a^2*arctan(1/3*(
1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)+1/18*a^2*ln(b^(1/3
)*x-(b*x^3+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.40

$$\int x^4 \sqrt[3]{a+bx^3}\,dx = \frac{x^2\sqrt[3]{a+bx^3}(a+3bx^3)}{18b} + \frac{a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}b^{5/3}} + \frac{a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27b^{5/3}} - \frac{a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54b^{5/3}}$$

input
```
Integrate[x^4*(a + b*x^3)^(1/3),x]
```

output
```
(x^2*(a + b*x^3)^(1/3)*(a + 3*b*x^3))/(18*b) + (a^2*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]])/(9*Sqrt[3]*b^(5/3)) + (a^2*Log[-(
b^(1/3)*x) + (a + b*x^3)^(1/3)]])/(27*b^(5/3)) - (a^2*Log[b^(2/3)*x^2 + b^(
1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]])/(54*b^(5/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {811, 843, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{6}a\int \frac{x^4}{(bx^3+a)^{2/3}}dx + \frac{1}{6}x^5\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{6}a\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\int\frac{x}{(bx^3+a)^{2/3}}dx}{3b}\right)+\frac{1}{6}x^5\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{853}$$

$$\frac{1}{6}a\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b}\right)+\frac{1}{6}x^5\sqrt[3]{a+bx^3}$$

input
```
Int[x^4*(a + b*x^3)^(1/3),x]
```

output
```
(x^5*(a + b*x^3)^(1/3))/6 + (a*((x^2*(a + b*x^3)^(1/3))/(3*b) - (2*a*(-(Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - L
og[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/(3*b)))/6
```

Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

method	result
pseudoelliptic	$\frac{9x^5\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{5}{3}}+3ax^2b^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a^2+2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)a^2-\ln\left(\frac{b^{\frac{2}{3}}\cdots}{\cdots}\right.}{54b^{\frac{5}{3}}}$

input
```
int(x^4*(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*x^5*(b*x^3+a)^(1/3)*b^(5/3)+3*a*x^2*b^(2/3)*(b*x^3+a)^(1/3)-2*3^(1
/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a^2+2*ln((
-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)
*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.48

$$\int x^4\sqrt[3]{a+bx^3}\,dx =$$

$$-\frac{6\sqrt{\frac{1}{3}}a^2(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-2\,a^2(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}b}{x}\right)+a^2(b^2)\cdots}{54\,b^3}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/54*(6*sqrt(1/3)*a^2*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2
*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*a^2*(b^2)^(2/3)*l
og(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + a^2*(b^2)^(2/3)*log(((b^2)^
(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2)
- 3*(3*b^3*x^5 + a*b^2*x^2)*(b*x^3 + a)^(1/3))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.32

$$\int x^4\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.51

$$\int x^4 \sqrt[3]{a+bx^3}\,dx = -\frac{\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{5}{3}}} - \frac{a^2 \log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{54\,b^{\frac{5}{3}}} + \frac{a^2 \log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{5}{3}}} + \frac{\frac{2\,(bx^3+a)^{\frac{1}{3}}a^2b}{x}+\frac{(bx^3+a)^{\frac{4}{3}}a^2}{x^4}}{18\left(b^3-\frac{2\,(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}\right)}$$

input
```
integrate(x^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(5/3) - 1/54*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^
3 + a)^(2/3)/x^2)/b^(5/3) + 1/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b
^(5/3) + 1/18*(2*(b*x^3 + a)^(1/3)*a^2*b/x + (b*x^3 + a)^(4/3)*a^2/x^4)/(b
^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6)
```

Giac [F]

$$\int x^4 \sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}} x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt[3]{a+bx^3}\,dx=\int x^4\,{\left(b\,x^3+a\right)}^{1/3}\,dx$$

input
```
int(x^4*(a + b*x^3)^(1/3),x)
```

output
```
int(x^4*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int x^4\sqrt[3]{a+bx^3}\,dx=\frac{(b\,x^3+a)^{\frac{1}{3}}\,a\,x^2+3(b\,x^3+a)^{\frac{1}{3}}\,b\,x^5-2\left(\int\frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2}{18b}$$

input
```
int(x^4*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*a*x**2 + 3*(a + b*x**3)**(1/3)*b*x**5 - 2*int(((a + b
*x**3)**(1/3)*x)/(a + b*x**3),x)*a**2)/(18*b)
```

3.362 $\int x\sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int x\sqrt[3]{a+bx^3}\,dx=\frac{1}{3}x^2\sqrt[3]{a+bx^3}-\frac{a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

output
```
1/3*x^2*(b*x^3+a)^(1/3)-1/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/b^(2/3)-1/6*a*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.50

$$\int x\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{6b^{2/3}x^2\sqrt[3]{a+bx^3}-2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)-2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)+a\log\left(b^{2/3}x^2+\sqrt[3]{\ldots}\right.}{18b^{2/3}}$$

input
```
Integrate[x*(a + b*x^3)^(1/3),x]
```

output
```
(6*b^(2/3)*x^2*(a + b*x^3)^(1/3) - 2*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/
(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] - 2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1
/3)] + a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/(18*b^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {811, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{3}a\int \frac{x}{(bx^3+a)^{2/3}}dx+\frac{1}{3}x^2\sqrt[3]{a+bx^3}$$

$$\Big\downarrow \text{853}$$

$$\frac{1}{3}a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)+\frac{1}{3}x^2\sqrt[3]{a+bx^3}$$

input
```
Int[x*(a + b*x^3)^(1/3),x]
```

output
```
(x^2*(a + b*x^3)^(1/3))/3 + (a*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b
^(2/3))))/3
```

Defintions of rubi rules used

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

method	result
pseudoelliptic	$\frac{6\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{2} b^{\frac{2}{3}}+2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a-2\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a+\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}\right.}{x^{2}}\right.}{18 b^{\frac{2}{3}}}$

input
```
int(x*(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(6*(b*x^3+a)^(1/3)*x^2*b^(2/3)+2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*
x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a+l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.02

$$\int x\sqrt[3]{a+bx^3}\,dx = \frac{6\left(bx^3+a\right)^{\frac{1}{3}}b^2x^2 + 6\sqrt{\frac{1}{3}}ab\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-b^2\right)^{\frac{1}{3}}bx-2\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{b^2x}\right) - 2\left(-b^2\right)^{\frac{2}{3}}a}{18\,b^2}$$

input
```
integrate(x*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/18*(6*(b*x^3 + a)^(1/3)*b^2*x^2 + 6*sqrt(1/3)*a*b*sqrt(-(-b^2)^(1/3))*ar
ctan(-sqrt(1/3)*((-b^2)^(1/3)*b*x - 2*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt
(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*a*log(-((-b^2)^(2/3)*x - (b*x^3
+ a)^(1/3)*b)/x) + (-b^2)^(2/3)*a*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(
1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int x\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.43

$$\int x\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{2}{3}}} + \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{18\,b^{\frac{2}{3}}} - \frac{a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9\,b^{\frac{2}{3}}} - \frac{\left(bx^3+a\right)^{\frac{1}{3}}a}{3\left(b-\frac{bx^3+a}{x^3}\right)x}$$

input
```
integrate(x*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3)
)/b^(2/3) + 1/18*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(2/3) - 1/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3) -
1/3*(b*x^3 + a)^(1/3)*a/((b - (b*x^3 + a)/x^3)*x)
```

Giac [F]

$$\int x\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[3]{a+bx^3}\,dx=\int x\,\left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int(x*(a + b*x^3)^(1/3),x)
```

output
```
int(x*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int x\sqrt[3]{a+bx^3}\,dx=\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^2}{3}+\frac{\left(\int\frac{x}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)a}{3}$$

input
```
int(x*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*x**2 + int(((a + b*x**3)**(1/3)*x)/(a + b*x**3),x)*a)
/3
```

3.363 $\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx=-\frac{\sqrt[3]{a+bx^3}}{x}-\frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)$$

output
```
-(b*x^3+a)^(1/3)/x-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*
3^(1/2))*3^(1/2)-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx=&-\frac{\sqrt[3]{a+bx^3}}{x}-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}}\\&-\frac{1}{3}\sqrt[3]{b}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)\\&+\frac{1}{6}\sqrt[3]{b}\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)\end{aligned}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^2,x]
```

output
```
-((a + b*x^3)^(1/3)/x) - (b^(1/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x +
2*(a + b*x^3)^(1/3))])/Sqrt[3] - (b^(1/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(
1/3)])/3 + (b^(1/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {809, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx$$

$$\Big\downarrow \text{809}$$

$$b\int \frac{x}{(bx^3+a)^{2/3}}dx-\frac{\sqrt[3]{a+bx^3}}{x}$$

$$\Big\downarrow \text{853}$$

$$b\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)-\frac{\sqrt[3]{a+bx^3}}{x}$$

input
```
Int[(a + b*x^3)^(1/3)/x^2,x]
```

output
```
-((a + b*x^3)^(1/3)/x) + b*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))
/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/
3)))
```

Defintions of rubi rules used

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}-\frac{b^{\frac{1}{3}}\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)}{3}+\frac{b^{\frac{1}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6}+\frac{b^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\sqrt{3}\left(\frac{2(b}{}\right.\right.}{3}$

input
```
int((b*x^3+a)^(1/3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x^3+a)^(1/3)/x-1/3*b^(1/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/6*b^(1
/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)+1/3*b^
(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(69) = 138$.

Time = 73.73 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx =$$
$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\,x\arctan\left(\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2+2\sqrt{3}(bx^3+a)^{\frac{2}{3}}(-b)^{\frac{1}{3}}x+\sqrt{3}a}{3\,(2\,bx^3+a)}\right)+(-b)^{\frac{1}{3}}\,x\log\left(-(-b)^{\frac{1}{3}}\,bx^2+(bx^3+\right.}{6\,x}$$

input
```
integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="fricas")
```

output
```
-1/6*(2*sqrt(3)*(-b)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^
(2/3)*x^2 + 2*sqrt(3)*(b*x^3 + a)^(2/3)*(-b)^(1/3)*x + sqrt(3)*a)/(2*b*x^3
 + a)) + (-b)^(1/3)*x*log(-(-b)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*b*x + (b*x
^3 + a)^(2/3)*(-b)^(2/3)) - 2*(-b)^(1/3)*x*log((-b)^(1/3)*x + (b*x^3 + a)^
(1/3)) + 6*(b*x^3 + a)^(1/3))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx = \frac{\sqrt[3]{a}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x**2,x)
```

output
```
a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx = \frac{1}{3}\sqrt{3}b^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right) + \frac{1}{6}b^{\frac{1}{3}}\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3}b^{\frac{1}{3}}\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right) - \frac{(bx^3+a)^{\frac{1}{3}}}{x}$$

input
```
integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b^(1/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3)) + 1/6*b^(1/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 +
 a)^(2/3)/x^2) - 1/3*b^(1/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) - (b*x^3
+ a)^(1/3)/x
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx = -\frac{(b\,x^3+a)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ -\frac{b\,x^3}{a}\right)}{x\,\left(\frac{b\,x^3}{a}+1\right)^{1/3}}$$

input
```
int((a + b*x^3)^(1/3)/x^2,x)
```

output
```
-((a + b*x^3)^(1/3)*hypergeom([-1/3, -1/3], 2/3, -(b*x^3)/a))/(x*((b*x^3)/
a + 1)^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2}\,dx = \int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^2}dx$$

input
```
int((b*x^3+a)^(1/3)/x^2,x)
```

output
```
int((a + b*x**3)**(1/3)/x**2,x)
```

3.364 $\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = -\frac{(a+bx^3)^{4/3}}{4ax^4}$$

output
```
-1/4*(b*x^3+a)^(4/3)/a/x^4
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = -\frac{(a+bx^3)^{4/3}}{4ax^4}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^5,x]
```

output
```
-1/4*(a + b*x^3)^(4/3)/(a*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5} \, dx$$
$$\Big\downarrow \; 796$$
$$-\frac{\left(a+bx^3\right)^{4/3}}{4ax^4}$$

input
```
Int[(a + b*x^3)^(1/3)/x^5,x]
```

output
```
-1/4*(a + b*x^3)^(4/3)/(a*x^4)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{4}{3}}}{4a\,x^4}$	18
trager	$-\frac{(b\,x^3+a)^{\frac{4}{3}}}{4a\,x^4}$	18
risch	$-\frac{(b\,x^3+a)^{\frac{4}{3}}}{4a\,x^4}$	18
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{4}{3}}}{4a\,x^4}$	18
orering	$-\frac{(b\,x^3+a)^{\frac{4}{3}}}{4a\,x^4}$	18

input
```
int((b*x^3+a)^(1/3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^3+a)^(4/3)/a/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = -\frac{(bx^3+a)^{\frac{4}{3}}}{4\,ax^4}$$

input
```
integrate((b*x^3+a)^(1/3)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(b*x^3 + a)^(4/3)/(a*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(17) = 34.

Time = 0.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = \frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3x^3\Gamma\left(-\frac{1}{3}\right)} + \frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3a\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x**5,x)
```

output
```
b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*x**3*gamma(-1/3)) + b**(4/
3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a*gamma(-1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = -\frac{(bx^3+a)^{\frac{4}{3}}}{4\,ax^4}$$

input
```
integrate((b*x^3+a)^(1/3)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(b*x^3 + a)^(4/3)/(a*x^4)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5} \, dx = -\frac{(b\,x^3+a)^{4/3}}{4\,a\,x^4}$$

input
```
int((a + b*x^3)^(1/3)/x^5,x)
```

output
```
-(a + b*x^3)^(4/3)/(4*a*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5} \, dx = -\frac{(b\,x^3+a)^{\frac{4}{3}}}{4a\,x^4}$$

input
```
int((b*x^3+a)^(1/3)/x^5,x)
```

output
```
( - (a + b*x**3)**(1/3)*(a + b*x**3))/(4*a*x**4)
```

3.365 $\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = -\frac{(a+bx^3)^{4/3}}{7ax^7}+\frac{3b(a+bx^3)^{4/3}}{28a^2x^4}$$

output
```
-1/7*(b*x^3+a)^(4/3)/a/x^7+3/28*b*(b*x^3+a)^(4/3)/a^2/x^4
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = \frac{\sqrt[3]{a+bx^3}(-4a^2-abx^3+3b^2x^6)}{28a^2x^7}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^8,x]
```

output
```
((a + b*x^3)^(1/3)*(-4*a^2 - a*b*x^3 + 3*b^2*x^6))/(28*a^2*x^7)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b\int \frac{\sqrt[3]{bx^3+a}}{x^5}dx}{7a}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}$$

input
```
Int[(a + b*x^3)^(1/3)/x^8,x]
```

output
```
-1/7*(a + b*x^3)^(4/3)/(a*x^7) + (3*b*(a + b*x^3)^(4/3))/(28*a^2*x^4)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{4}{3}}\,(-3b\,x^3+4a)}{28a^2x^7}$	28
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{4}{3}}\,(-3b\,x^3+4a)}{28a^2x^7}$	28
orering	$-\frac{(b\,x^3+a)^{\frac{4}{3}}\,(-3b\,x^3+4a)}{28a^2x^7}$	28
trager	$-\frac{(-3b^2x^6+ab\,x^3+4a^2)\,(b\,x^3+a)^{\frac{1}{3}}}{28a^2x^7}$	38
risch	$-\frac{(-3b^2x^6+ab\,x^3+4a^2)\,(b\,x^3+a)^{\frac{1}{3}}}{28a^2x^7}$	38

input
```
int((b*x^3+a)^(1/3)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/28*(b*x^3+a)^(4/3)*(-3*b*x^3+4*a)/a^2/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = \frac{(3\,b^2x^6 - abx^3 - 4\,a^2)(bx^3+a)^{\frac{1}{3}}}{28\,a^2x^7}$$

input
```
integrate((b*x^3+a)^(1/3)/x^8,x, algorithm="fricas")
```

output
```
1/28*(3*b^2*x^6 - a*b*x^3 - 4*a^2)*(b*x^3 + a)^(1/3)/(a^2*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(37) = 74$.

Time = 0.65 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = -\frac{4\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{9x^6\Gamma\left(-\frac{1}{3}\right)} - \frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{9ax^3\Gamma\left(-\frac{1}{3}\right)} + \frac{b^{\frac{7}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{3a^2\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x**8,x)
```

output
```
-4*b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*x**6*gamma(-1/3)) - b**
(4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(9*a*x**3*gamma(-1/3)) + b**(7/3
)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(3*a**2*gamma(-1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = \frac{\frac{7\,(bx^3+a)^{\frac{4}{3}}b}{x^4} - \frac{4\,(bx^3+a)^{\frac{7}{3}}}{x^7}}{28\,a^2}$$

input
```
integrate((b*x^3+a)^(1/3)/x^8,x, algorithm="maxima")
```

output
```
1/28*(7*(b*x^3 + a)^(4/3)*b/x^4 - 4*(b*x^3 + a)^(7/3)/x^7)/a^2
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^8}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = -\frac{7\,a\,{\left(b\,x^3+a\right)}^{4/3}-3\,{\left(b\,x^3+a\right)}^{7/3}}{28\,a^2\,x^7}$$

input
```
int((a + b*x^3)^(1/3)/x^8,x)
```

output
```
-(7*a*(a + b*x^3)^(4/3) - 3*(a + b*x^3)^(7/3))/(28*a^2*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(3b^2x^6-ab\,x^3-4a^2)}{28a^2x^7}$$

input
```
int((b*x^3+a)^(1/3)/x^8,x)
```

output
```
((a + b*x**3)**(1/3)*( - 4*a**2 - a*b*x**3 + 3*b**2*x**6))/(28*a**2*x**7)
```

3.366 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx=-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}+\frac{3b\left(a+bx^3\right)^{4/3}}{35a^2x^7}-\frac{9b^2\left(a+bx^3\right)^{4/3}}{140a^3x^4}$$

output
```
-1/10*(b*x^3+a)^(4/3)/a/x^10+3/35*b*(b*x^3+a)^(4/3)/a^2/x^7-9/140*b^2*(b*x
^3+a)^(4/3)/a^3/x^4
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx=\frac{\sqrt[3]{a+bx^3}\left(-14a^3-2a^2bx^3+3ab^2x^6-9b^3x^9\right)}{140a^3x^{10}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^11,x]
```

output
```
((a + b*x^3)^(1/3)*(-14*a^3 - 2*a^2*b*x^3 + 3*a*b^2*x^6 - 9*b^3*x^9))/(140
*a^3*x^10)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b \int \frac{\sqrt[3]{bx^3+a}}{x^8}dx}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b \left(-\frac{3b \int \frac{\sqrt[3]{bx^3+a}}{x^5}dx}{7a}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}\right)}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{3b \left(\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}\right)}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}$$

input
```
Int[(a + b*x^3)^(1/3)/x^11,x]
```

output
```
-1/10*(a + b*x^3)^(4/3)/(a*x^10) - (3*b*(-1/7*(a + b*x^3)^(4/3)/(a*x^7) +
(3*b*(a + b*x^3)^(4/3))/(28*a^2*x^4)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{(b\,x^{3}+a)^{\frac{4}{3}} (9b^{2}x^{6}-12ab\,x^{3}+14a^{2})}{140a^{3}x^{10}}$	39
pseudoelliptic	$-\frac{(b\,x^{3}+a)^{\frac{4}{3}} (9b^{2}x^{6}-12ab\,x^{3}+14a^{2})}{140a^{3}x^{10}}$	39
orering	$-\frac{(b\,x^{3}+a)^{\frac{4}{3}} (9b^{2}x^{6}-12ab\,x^{3}+14a^{2})}{140a^{3}x^{10}}$	39
trager	$-\frac{(9b^{3}x^{9}-3a\,b^{2}x^{6}+2a^{2}b\,x^{3}+14a^{3}) (b\,x^{3}+a)^{\frac{1}{3}}}{140a^{3}x^{10}}$	50
risch	$-\frac{(9b^{3}x^{9}-3a\,b^{2}x^{6}+2a^{2}b\,x^{3}+14a^{3}) (b\,x^{3}+a)^{\frac{1}{3}}}{140a^{3}x^{10}}$	50

input
```
int((b*x^3+a)^(1/3)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/140*(b*x^3+a)^(4/3)*(9*b^2*x^6-12*a*b*x^3+14*a^2)/a^3/x^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx = -\frac{(9\,b^3x^9 - 3\,ab^2x^6 + 2\,a^2bx^3 + 14\,a^3)(bx^3+a)^{\frac{1}{3}}}{140\,a^3x^{10}}$$

input
```
integrate((b*x^3+a)^(1/3)/x^11,x, algorithm="fricas")
```

output
```
-1/140*(9*b^3*x^9 - 3*a*b^2*x^6 + 2*a^2*b*x^3 + 14*a^3)*(b*x^3 + a)^(1/3)/
(a^3*x^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(61) = 122.

Time = 0.88 (sec) , antiderivative size = 520, normalized size of antiderivative = 7.65

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx &= \frac{28a^5b^{\frac{13}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)} \\ &+ \frac{60a^4b^{\frac{16}{3}}x^3\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)} \\ &+ \frac{30a^3b^{\frac{19}{3}}x^6\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)} \\ &+ \frac{10a^2b^{\frac{22}{3}}x^9\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)} \\ &+ \frac{30ab^{\frac{25}{3}}x^{12}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)} \\ &+ \frac{18b^{\frac{28}{3}}x^{15}\sqrt[3]{\frac{a}{bx^3}+1}\,\Gamma\left(-\frac{10}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{1}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{1}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{1}{3}\right)}\end{aligned}$$

input
```
integrate((b*x**3+a)**(1/3)/x**11,x)
```

output
```
28*a**5*b**(13/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*
gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1
/3)) + 60*a**4*b**(16/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**
5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x*
*15*gamma(-1/3)) + 30*a**3*b**(19/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-1
0/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma(-1/3) + 27*
a**3*b**6*x**15*gamma(-1/3)) + 10*a**2*b**(22/3)*x**9*(a/(b*x**3) + 1)**(1
/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x**12*gamma
(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3)) + 30*a*b**(25/3)*x**12*(a/(b*x**3
) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**4*b**5*x
**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3)) + 18*b**(28/3)*x**15*(a
/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(27*a**5*b**4*x**9*gamma(-1/3) + 54*a**
4*b**5*x**12*gamma(-1/3) + 27*a**3*b**6*x**15*gamma(-1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx = -\frac{\frac{35\,(bx^3+a)^{\frac{4}{3}}b^2}{x^4} - \frac{40\,(bx^3+a)^{\frac{7}{3}}b}{x^7} + \frac{14\,(bx^3+a)^{\frac{10}{3}}}{x^{10}}}{140\,a^3}$$

input
```
integrate((b*x^3+a)^(1/3)/x^11,x, algorithm="maxima")
```

output
```
-1/140*(35*(b*x^3 + a)^(4/3)*b^2/x^4 - 40*(b*x^3 + a)^(7/3)*b/x^7 + 14*(b*
x^3 + a)^(10/3)/x^10)/a^3
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^{11}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^11,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^11, x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx = \frac{3\,b^2\,(b\,x^3+a)^{1/3}}{140\,a^2\,x^4} - \frac{b\,(b\,x^3+a)^{1/3}}{70\,a\,x^7} - \frac{9\,b^3\,(b\,x^3+a)^{1/3}}{140\,a^3\,x} - \frac{(b\,x^3+a)^{1/3}}{10\,x^{10}}$$

input
```
int((a + b*x^3)^(1/3)/x^11,x)
```

output
```
(3*b^2*(a + b*x^3)^(1/3))/(140*a^2*x^4) - (b*(a + b*x^3)^(1/3))/(70*a*x^7)
 - (9*b^3*(a + b*x^3)^(1/3))/(140*a^3*x) - (a + b*x^3)^(1/3)/(10*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(-9b^3x^9+3a\,b^2x^6-2a^2b\,x^3-14a^3)}{140a^3x^{10}}$$

input
```
int((b*x^3+a)^(1/3)/x^11,x)
```

output
```
((a + b*x**3)**(1/3)*( - 14*a**3 - 2*a**2*b*x**3 + 3*a*b**2*x**6 - 9*b**3*
x**9))/(140*a**3*x**10)
```

3.367 $\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = -\frac{(a+bx^3)^{4/3}}{13ax^{13}} + \frac{9b(a+bx^3)^{4/3}}{130a^2x^{10}} - \frac{27b^2(a+bx^3)^{4/3}}{455a^3x^7} + \frac{81b^3(a+bx^3)^{4/3}}{1820a^4x^4}$$

output
```
-1/13*(b*x^3+a)^(4/3)/a/x^13+9/130*b*(b*x^3+a)^(4/3)/a^2/x^10-27/455*b^2*(
b*x^3+a)^(4/3)/a^3/x^7+81/1820*b^3*(b*x^3+a)^(4/3)/a^4/x^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \frac{(a+bx^3)^{4/3}\,(-140a^3+126a^2bx^3-108ab^2x^6+81b^3x^9)}{1820a^4x^{13}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^14,x]
```

output
```
((a + b*x^3)^(4/3)*(-140*a^3 + 126*a^2*b*x^3 - 108*a*b^2*x^6 + 81*b^3*x^9)
)/(1820*a^4*x^13)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{9b\int \frac{\sqrt[3]{bx^3+a}}{x^{11}}dx}{13a}-\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{9b\left(-\frac{3b\int \frac{\sqrt[3]{bx^3+a}}{x^8}dx}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}\right)}{13a}-\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{9b\left(-\frac{3b\left(-\frac{3b\int \frac{\sqrt[3]{bx^3+a}}{x^5}dx}{7a}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}\right)}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}\right)}{13a}-\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{9b\left(-\frac{3b\left(\frac{3b\left(a+bx^3\right)^{4/3}}{28a^2x^4}-\frac{\left(a+bx^3\right)^{4/3}}{7ax^7}\right)}{5a}-\frac{\left(a+bx^3\right)^{4/3}}{10ax^{10}}\right)}{13a}-\frac{\left(a+bx^3\right)^{4/3}}{13ax^{13}}$$

input
```
Int[(a + b*x^3)^(1/3)/x^14,x]
```

output
```
-1/13*(a + b*x^3)^(4/3)/(a*x^13) - (9*b*(-1/10*(a + b*x^3)^(4/3)/(a*x^10)
- (3*b*(-1/7*(a + b*x^3)^(4/3)/(a*x^7) + (3*b*(a + b*x^3)^(4/3))/(28*a^2*x
^4)))/(5*a)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-81 b^{3} x^{9}+108 a\,b^{2} x^{6}-126 a^{2} b\,x^{3}+140 a^{3}\right)}{1820 x^{13} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-81 b^{3} x^{9}+108 a\,b^{2} x^{6}-126 a^{2} b\,x^{3}+140 a^{3}\right)}{1820 x^{13} a^{4}}$	50
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(-81 b^{3} x^{9}+108 a\,b^{2} x^{6}-126 a^{2} b\,x^{3}+140 a^{3}\right)}{1820 x^{13} a^{4}}$	50
trager	$-\frac{\left(-81 b^{4} x^{12}+27 a\,b^{3} x^{9}-18 a^{2} b^{2} x^{6}+14 a^{3} b\,x^{3}+140 a^{4}\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{1820 x^{13} a^{4}}$	61
risch	$-\frac{\left(-81 b^{4} x^{12}+27 a\,b^{3} x^{9}-18 a^{2} b^{2} x^{6}+14 a^{3} b\,x^{3}+140 a^{4}\right) \left(b\,x^{3}+a\right)^{\frac{1}{3}}}{1820 x^{13} a^{4}}$	61

input
```
int((b*x^3+a)^(1/3)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/1820*(b*x^3+a)^(4/3)*(-81*b^3*x^9+108*a*b^2*x^6-126*a^2*b*x^3+140*a^3)/
x^13/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \frac{(81\,b^4x^{12} - 27\,ab^3x^9 + 18\,a^2b^2x^6 - 14\,a^3bx^3 - 140\,a^4)(bx^3+a)^{\frac{1}{3}}}{1820\,a^4x^{13}}$$

input
```
integrate((b*x^3+a)^(1/3)/x^14,x, algorithm="fricas")
```

output
```
1/1820*(81*b^4*x^12 - 27*a*b^3*x^9 + 18*a^2*b^2*x^6 - 14*a^3*b*x^3 - 140*a
^4)*(b*x^3 + a)^(1/3)/(a^4*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(85) = 170.

Time = 1.28 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.21

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(1/3)/x**14,x)
```

output
```
-280*a**7*b**(28/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**
12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*g
amma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) - 868*a**6*b**(31/3)*x**3*(a
/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a
**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b
**12*x**21*gamma(-1/3)) - 888*a**5*b**(34/3)*x**6*(a/(b*x**3) + 1)**(1/3)*
gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(
-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)
) - 310*a**4*b**(37/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*
b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11
*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 80*a**3*b**(40/3)*
x**12*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3)
 + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 8
1*a**4*b**12*x**21*gamma(-1/3)) + 360*a**2*b**(43/3)*x**15*(a/(b*x**3) + 1
)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**
15*gamma(-1/3) + 243*a**5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*ga
mma(-1/3)) + 432*a*b**(46/3)*x**18*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(8
1*a**7*b**9*x**12*gamma(-1/3) + 243*a**6*b**10*x**15*gamma(-1/3) + 243*a**
5*b**11*x**18*gamma(-1/3) + 81*a**4*b**12*x**21*gamma(-1/3)) + 162*b**(49/
3)*x**21*(a/(b*x**3) + 1)**(1/3)*gamma(-13/3)/(81*a**7*b**9*x**12*gamma...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \frac{\frac{455\,\left(bx^3+a\right)^{\frac{4}{3}}b^3}{x^4} - \frac{780\,\left(bx^3+a\right)^{\frac{7}{3}}b^2}{x^7} + \frac{546\,\left(bx^3+a\right)^{\frac{10}{3}}b}{x^{10}} - \frac{140\,\left(bx^3+a\right)^{\frac{13}{3}}}{x^{13}}}{1820\,a^4}$$

input
```
integrate((b*x^3+a)^(1/3)/x^14,x, algorithm="maxima")
```

output
```
1/1820*(455*(b*x^3 + a)^(4/3)*b^3/x^4 - 780*(b*x^3 + a)^(7/3)*b^2/x^7 + 54
6*(b*x^3 + a)^(10/3)*b/x^10 - 140*(b*x^3 + a)^(13/3)/x^13)/a^4
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^{14}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^14,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \frac{81\,b^4\,(b\,x^3+a)^{1/3}}{1820\,a^4\,x} - \frac{b\,(b\,x^3+a)^{1/3}}{130\,a\,x^{10}} - \frac{(b\,x^3+a)^{1/3}}{13\,x^{13}} - \frac{27\,b^3\,(b\,x^3+a)^{1/3}}{1820\,a^3\,x^4} + \frac{9\,b^2\,(b\,x^3+a)^{1/3}}{910\,a^2\,x^7}$$

input
```
int((a + b*x^3)^(1/3)/x^14,x)
```

output
```
(81*b^4*(a + b*x^3)^(1/3))/(1820*a^4*x) - (b*(a + b*x^3)^(1/3))/(130*a*x^1
0) - (a + b*x^3)^(1/3)/(13*x^13) - (27*b^3*(a + b*x^3)^(1/3))/(1820*a^3*x^
4) + (9*b^2*(a + b*x^3)^(1/3))/(910*a^2*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{14}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,(81b^4x^{12} - 27a\,b^3x^9 + 18a^2b^2x^6 - 14a^3b\,x^3 - 140a^4)}{1820a^4x^{13}}$$

input
```
int((b*x^3+a)^(1/3)/x^14,x)
```

output
```
((a + b*x**3)**(1/3)*( - 140*a**4 - 14*a**3*b*x**3 + 18*a**2*b**2*x**6 - 2
7*a*b**3*x**9 + 81*b**4*x**12))/(1820*a**4*x**13)
```

3.368 $\int x^3 \sqrt[3]{a+bx^3}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^3 \sqrt[3]{a+bx^3}\, dx = \frac{x^4 \sqrt[3]{a+bx^3}\, \text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/4*x^4*(b*x^3+a)^(1/3)*hypergeom([-1/3, 4/3],[7/3],-b*x^3/a)/(1+b*x^3/a)^
(1/3)
```

Mathematica [A] (verified)

Time = 4.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^3 \sqrt[3]{a+bx^3}\, dx = \frac{x^4 \sqrt[3]{a+bx^3}\, \text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^3*(a + b*x^3)^(1/3),x]
```

output
```
(x^4*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])/(4
*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{a+bx^3} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{a+bx^3} \int x^3 \sqrt[3]{\frac{bx^3}{a}+1}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^4 \sqrt[3]{a+bx^3} \text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[x^3*(a + b*x^3)^(1/3),x]
```

output
```
(x^4*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])/(4
*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{3} \left(b \,x^{3}+a \right)^{\frac{1}{3}}d x$$

input
```
int(x^3*(b*x^3+a)^(1/3),x)
```

output
```
int(x^3*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int x^3 \sqrt[3]{a+bx^3}\,dx = \int \left(bx^{3} + a\right)^{\frac{1}{3}} x^{3}\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^3\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int x^3\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^3, x)
```

Giac [F]

$$\int x^3\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt[3]{a+bx^3}\,dx = \int x^3 \left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int(x^3*(a + b*x^3)^(1/3),x)
```

output
```
int(x^3*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int x^3 \sqrt[3]{a+bx^3}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}\,ax+2(b\,x^3+a)^{\frac{1}{3}}\,b\,x^4-\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2}{10b}$$

input
```
int(x^3*(b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*a*x + 2*(a + b*x**3)**(1/3)*b*x**4 - int((a + b*x**3)
**(1/3)/(a + b*x**3),x)*a**2)/(10*b)
```

3.369 $\int \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt[3]{a+bx^3}\,dx=\frac{x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
x*(b*x^3+a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.41

$$\int \sqrt[3]{a+bx^3}\,dx =\frac{3\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},-\frac{1}{3},\frac{7}{3},-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}},\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{b}x}{\sqrt[3]{a}}}{3i+\sqrt{3}}\right)}{4\sqrt[3]{2}\sqrt[3]{b}\sqrt[3]{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt[3]{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}}$$

input
```
Integrate[(a + b*x^3)^(1/3),x]
```

output
```
(3*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3,
-1/3, 7/3, -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (-1)^(1/3
))*a^(1/3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/
(4*2^(1/3)*b^(1/3)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3)))^(1/3)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}\,dx$$

$$\Big\downarrow \text{779}$$

$$\frac{\sqrt[3]{a+bx^3}\int \sqrt[3]{\frac{bx^3}{a}+1}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{778}$$

$$\frac{x\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(1 +
 (b*x^3)/a)^(1/3)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{3}+a \right)^{\frac{1}{3}}d x$$

input
```
int((b*x^3+a)^(1/3),x)
```

output
```
int((b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^3}\,dx=\int \left(bx^{3}+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3))
```

Maxima [F]

$$\int \sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{a+bx^3}\,dx=\frac{x\,\left(b\,x^3+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^{1/3}}$$

input
```
int((a + b*x^3)^(1/3),x)
```

output
```
(x*(a + b*x^3)^(1/3)*hypergeom([-1/3, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
 1)^(1/3)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^3}\,dx=\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x}{2}+\frac{\left(\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}}dx\right)a}{2}$$

input
```
int((b*x^3+a)^(1/3),x)
```

output
```
((a + b*x**3)**(1/3)*x + int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a)/2
```

3.370 $\int \frac{\sqrt[3]{a+bx^3}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3} dx = -\frac{\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*(b*x^3+a)^(1/3)*hypergeom([-2/3, -1/3],[1/3],-b*x^3/a)/x^2/(1+b*x^3/a
)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3} dx = -\frac{\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^3,x]
```

output
```
-1/2*((a + b*x^3)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, -((b*x^3)/a)])/
(x^2*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3} dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt[3]{a+bx^3} \text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/x^3,x]
```

output
```
-1/2*((a + b*x^3)^(1/3)*Hypergeometric2F1[-2/3, -1/3, 1/3, -((b*x^3)/a)])/
(x^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{x^3}dx$$

input
```
int((b*x^3+a)^(1/3)/x^3,x)
```

output
```
int((b*x^3+a)^(1/3)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx = \frac{\sqrt[3]{a}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x**3,x)
```

output
```
a**(1/3)*gamma(-2/3)*hyper((-2/3, -1/3), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx = \int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^3}\,dx$$

input
```
int((a + b*x^3)^(1/3)/x^3,x)
```

output
```
int((a + b*x^3)^(1/3)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3}\,dx = \frac{-\left(b\,x^3+a\right)^{\frac{1}{3}}-\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{b\,x^6+a\,x^3}dx\right)a\,x^2}{x^2}$$

input
```
int((b*x^3+a)^(1/3)/x^3,x)
```

output
```
( - ((a + b*x**3)**(1/3) + int((a + b*x**3)**(1/3)/(a*x**3 + b*x**6),x)*a*
x**2))/x**2
```

3.371 $\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx=-\frac{\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*(b*x^3+a)^(1/3)*hypergeom([-5/3, -1/3],[-2/3],-b*x^3/a)/x^5/(1+b*x^3/
a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx=-\frac{\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(1/3)/x^6,x]
```

output
```
-1/5*((a + b*x^3)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, -((b*x^3)/a)])
/(x^5*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{\frac{bx^3}{a}+1}}{x^6}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt[3]{a+bx^3}\, \text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{1}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(1/3)/x^6,x]
```

output
```
-1/5*((a + b*x^3)^(1/3)*Hypergeometric2F1[-5/3, -1/3, -2/3, -((b*x^3)/a)])
/(x^5*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{x^{6}}dx$$

input
```
int((b*x^3+a)^(1/3)/x^6,x)
```

output
```
int((b*x^3+a)^(1/3)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx = \frac{\sqrt[3]{b}\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(1/3)/x**6,x)
```

output
```
b**(1/3)*gamma(-4/3)*hyper((-1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**3)
)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx = \int \frac{{\left(b\,x^3+a\right)}^{1/3}}{x^6}\,dx$$

input
```
int((a + b*x^3)^(1/3)/x^6,x)
```

output
```
int((a + b*x^3)^(1/3)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6}\,dx = \frac{-\left(b\,x^3+a\right)^{\frac{1}{3}}-\left(\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{b\,x^9+a\,x^6}dx\right)a\,x^5}{4x^5}$$

input
```
int((b*x^3+a)^(1/3)/x^6,x)
```

output
```
( - ((a + b*x**3)**(1/3) + int((a + b*x**3)**(1/3)/(a*x**6 + b*x**9),x)*a*
x**5))/(4*x**5)
```

3.372 $\int x^{11}(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{11}\left(a+bx^3\right)^{2/3}\,dx=-\frac{a^3(a+bx^3)^{5/3}}{5b^4}+\frac{3a^2(a+bx^3)^{8/3}}{8b^4}-\frac{3a(a+bx^3)^{11/3}}{11b^4}+\frac{(a+bx^3)^{14/3}}{14b^4}$$

output
```
-1/5*a^3*(b*x^3+a)^(5/3)/b^4+3/8*a^2*(b*x^3+a)^(8/3)/b^4-3/11*a*(b*x^3+a)^
(11/3)/b^4+1/14*(b*x^3+a)^(14/3)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^{11}\left(a+bx^3\right)^{2/3}\,dx=\frac{(a+bx^3)^{5/3}\left(-81a^3+135a^2bx^3-180ab^2x^6+220b^3x^9\right)}{3080b^4}$$

input
```
Integrate[x^11*(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(5/3)*(-81*a^3 + 135*a^2*b*x^3 - 180*a*b^2*x^6 + 220*b^3*x^9)
)/(3080*b^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^3\right)^{2/3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^9\left(bx^3+a\right)^{2/3}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int\left(\frac{\left(bx^3+a\right)^{11/3}}{b^3}-\frac{3a\left(bx^3+a\right)^{8/3}}{b^3}+\frac{3a^2\left(bx^3+a\right)^{5/3}}{b^3}-\frac{a^3\left(bx^3+a\right)^{2/3}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3a^3\left(a+bx^3\right)^{5/3}}{5b^4}+\frac{9a^2\left(a+bx^3\right)^{8/3}}{8b^4}+\frac{3\left(a+bx^3\right)^{14/3}}{14b^4}-\frac{9a\left(a+bx^3\right)^{11/3}}{11b^4}\right)$$

input
```
Int[x^11*(a + b*x^3)^(2/3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(5/3))/(5*b^4) + (9*a^2*(a + b*x^3)^(8/3))/(8*b^4) -
(9*a*(a + b*x^3)^(11/3))/(11*b^4) + (3*(a + b*x^3)^(14/3))/(14*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(-220 b^{3} x^{9}+180 a\,b^{2} x^{6}-135 a^{2} b\,x^{3}+81 a^{3}\right)}{3080 b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(-220 b^{3} x^{9}+180 a\,b^{2} x^{6}-135 a^{2} b\,x^{3}+81 a^{3}\right)}{3080 b^{4}}$	47
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(-220 b^{3} x^{9}+180 a\,b^{2} x^{6}-135 a^{2} b\,x^{3}+81 a^{3}\right)}{3080 b^{4}}$	47
trager	$-\frac{\left(-220 b^{4} x^{12}-40 a\,b^{3} x^{9}+45 a^{2} b^{2} x^{6}-54 a^{3} b\,x^{3}+81 a^{4}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{3080 b^{4}}$	58
risch	$-\frac{\left(-220 b^{4} x^{12}-40 a\,b^{3} x^{9}+45 a^{2} b^{2} x^{6}-54 a^{3} b\,x^{3}+81 a^{4}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{3080 b^{4}}$	58

input
```
int(x^11*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3080*(b*x^3+a)^(5/3)*(-220*b^3*x^9+180*a*b^2*x^6-135*a^2*b*x^3+81*a^3)/
b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}\left(a+bx^3\right)^{2/3}\,dx=\frac{(220\,b^4x^{12}+40\,ab^3x^9-45\,a^2b^2x^6+54\,a^3bx^3-81\,a^4)(bx^3+a)^{\frac{2}{3}}}{3080\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/3080*(220*b^4*x^12 + 40*a*b^3*x^9 - 45*a^2*b^2*x^6 + 54*a^3*b*x^3 - 81*a
^4)*(b*x^3 + a)^(2/3)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int x^{11}(a+bx^3)^{2/3}\,dx=\begin{cases}-\frac{81a^4(a+bx^3)^{\frac{2}{3}}}{3080b^4}+\frac{27a^3x^3(a+bx^3)^{\frac{2}{3}}}{1540b^3}-\frac{9a^2x^6(a+bx^3)^{\frac{2}{3}}}{616b^2}+\frac{ax^9(a+bx^3)^{\frac{2}{3}}}{77b}+\frac{x^{12}(a+bx^3)^{\frac{2}{3}}}{14} & \text{for } b\neq 0\\ \frac{a^{\frac{2}{3}}x^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((-81*a**4*(a + b*x**3)**(2/3)/(3080*b**4) + 27*a**3*x**3*(a + b*
x**3)**(2/3)/(1540*b**3) - 9*a**2*x**6*(a + b*x**3)**(2/3)/(616*b**2) + a*
x**9*(a + b*x**3)**(2/3)/(77*b) + x**12*(a + b*x**3)**(2/3)/14, Ne(b, 0)),
 (a**(2/3)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{11}(a+bx^3)^{2/3}\,dx=\frac{(bx^3+a)^{\frac{14}{3}}}{14\,b^4}-\frac{3\,(bx^3+a)^{\frac{11}{3}}a}{11\,b^4}+\frac{3\,(bx^3+a)^{\frac{8}{3}}a^2}{8\,b^4}-\frac{(bx^3+a)^{\frac{5}{3}}a^3}{5\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/14*(b*x^3 + a)^(14/3)/b^4 - 3/11*(b*x^3 + a)^(11/3)*a/b^4 + 3/8*(b*x^3 +
 a)^(8/3)*a^2/b^4 - 1/5*(b*x^3 + a)^(5/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}(a+bx^3)^{2/3}\,dx = \frac{220\,(bx^3+a)^{\frac{14}{3}} - 840\,(bx^3+a)^{\frac{11}{3}}a + 1155\,(bx^3+a)^{\frac{8}{3}}a^2 - 616\,(bx^3+a)^{\frac{5}{3}}a^3}{3080\,b^4}$$

input
```
integrate(x^11*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/3080*(220*(b*x^3 + a)^(14/3) - 840*(b*x^3 + a)^(11/3)*a + 1155*(b*x^3 +
a)^(8/3)*a^2 - 616*(b*x^3 + a)^(5/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^{11}(a+bx^3)^{2/3}\,dx = (b\,x^3+a)^{2/3}\left(\frac{x^{12}}{14} - \frac{81\,a^4}{3080\,b^4} + \frac{a\,x^9}{77\,b} + \frac{27\,a^3\,x^3}{1540\,b^3} - \frac{9\,a^2\,x^6}{616\,b^2}\right)$$

input
```
int(x^11*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(2/3)*(x^12/14 - (81*a^4)/(3080*b^4) + (a*x^9)/(77*b) + (27*a^
3*x^3)/(1540*b^3) - (9*a^2*x^6)/(616*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{11}(a+bx^3)^{2/3}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,(220b^4x^{12}+40a\,b^3x^9-45a^2b^2x^6+54a^3b\,x^3-81a^4)}{3080b^4}$$

input
```
int(x^11*(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*( - 81*a**4 + 54*a**3*b*x**3 - 45*a**2*b**2*x**6 + 40
*a*b**3*x**9 + 220*b**4*x**12))/(3080*b**4)
```

3.373 $\int x^8(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^8\left(a+bx^3\right)^{2/3}\,dx=\frac{a^2(a+bx^3)^{5/3}}{5b^3}-\frac{a(a+bx^3)^{8/3}}{4b^3}+\frac{(a+bx^3)^{11/3}}{11b^3}$$

output
```
1/5*a^2*(b*x^3+a)^(5/3)/b^3-1/4*a*(b*x^3+a)^(8/3)/b^3+1/11*(b*x^3+a)^(11/3
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^8\left(a+bx^3\right)^{2/3}\,dx=\frac{(a+bx^3)^{5/3}\left(9a^2-15abx^3+20b^2x^6\right)}{220b^3}$$

input
```
Integrate[x^8*(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(5/3)*(9*a^2 - 15*a*b*x^3 + 20*b^2*x^6))/(220*b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^{2/3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^{2/3} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{8/3}}{b^2}-\frac{2a\left(bx^3+a\right)^{5/3}}{b^2}+\frac{a^2\left(bx^3+a\right)^{2/3}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{3a^2\left(a+bx^3\right)^{5/3}}{5b^3}+\frac{3\left(a+bx^3\right)^{11/3}}{11b^3}-\frac{3a\left(a+bx^3\right)^{8/3}}{4b^3}\right)$$

input
```
Int[x^8*(a + b*x^3)^(2/3),x]
```

output
```
((3*a^2*(a + b*x^3)^(5/3))/(5*b^3) - (3*a*(a + b*x^3)^(8/3))/(4*b^3) + (3*
(a + b*x^3)^(11/3))/(11*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(20 b^{2} x^{6}-15 a b\,x^{3}+9 a^{2}\right)}{220 b^{3}}$	36
pseudoelliptic	$\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(20 b^{2} x^{6}-15 a b\,x^{3}+9 a^{2}\right)}{220 b^{3}}$	36
orering	$\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(20 b^{2} x^{6}-15 a b\,x^{3}+9 a^{2}\right)}{220 b^{3}}$	36
trager	$\frac{\left(20 b^{3} x^{9}+5 a\,b^{2} x^{6}-6 a^{2} b\,x^{3}+9 a^{3}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{220 b^{3}}$	47
risch	$\frac{\left(20 b^{3} x^{9}+5 a\,b^{2} x^{6}-6 a^{2} b\,x^{3}+9 a^{3}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{220 b^{3}}$	47

input
```
int(x^8*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/220*(b*x^3+a)^(5/3)*(20*b^2*x^6-15*a*b*x^3+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^8 \left(a+bx^3\right)^{2/3} \, dx = \frac{(20 \, b^3 x^9 + 5 \, ab^2 x^6 - 6 \, a^2 b x^3 + 9 \, a^3)(bx^3 + a)^{\frac{2}{3}}}{220 \, b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/220*(20*b^3*x^9 + 5*a*b^2*x^6 - 6*a^2*b*x^3 + 9*a^3)*(b*x^3 + a)^(2/3)/b
^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x^8 \left(a+bx^3\right)^{2/3}\, dx = \begin{cases} \frac{9a^3\left(a+bx^3\right)^{\frac{2}{3}}}{220b^3} - \frac{3a^2x^3\left(a+bx^3\right)^{\frac{2}{3}}}{110b^2} + \frac{ax^6\left(a+bx^3\right)^{\frac{2}{3}}}{44b} + \frac{x^9\left(a+bx^3\right)^{\frac{2}{3}}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^9}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8*(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((9*a**3*(a + b*x**3)**(2/3)/(220*b**3) - 3*a**2*x**3*(a + b*x**3
)**(2/3)/(110*b**2) + a*x**6*(a + b*x**3)**(2/3)/(44*b) + x**9*(a + b*x**3
)**(2/3)/11, Ne(b, 0)), (a**(2/3)*x**9/9, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^8 \left(a+bx^3\right)^{2/3}\, dx = \frac{\left(bx^3+a\right)^{\frac{11}{3}}}{11\, b^3} - \frac{\left(bx^3+a\right)^{\frac{8}{3}}a}{4\, b^3} + \frac{\left(bx^3+a\right)^{\frac{5}{3}}a^2}{5\, b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/11*(b*x^3 + a)^(11/3)/b^3 - 1/4*(b*x^3 + a)^(8/3)*a/b^3 + 1/5*(b*x^3 + a
)^(5/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^8(a+bx^3)^{2/3}\,dx=\frac{20\,(bx^3+a)^{\frac{11}{3}}-55\,(bx^3+a)^{\frac{8}{3}}a+44\,(bx^3+a)^{\frac{5}{3}}a^2}{220\,b^3}$$

input
```
integrate(x^8*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/220*(20*(b*x^3 + a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 44*(b*x^3 + a)^(5/
3)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^8(a+bx^3)^{2/3}\,dx=(b\,x^3+a)^{2/3}\left(\frac{x^9}{11}+\frac{9\,a^3}{220\,b^3}+\frac{a\,x^6}{44\,b}-\frac{3\,a^2\,x^3}{110\,b^2}\right)$$

input
```
int(x^8*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(2/3)*(x^9/11 + (9*a^3)/(220*b^3) + (a*x^6)/(44*b) - (3*a^2*x^
3)/(110*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^8(a+bx^3)^{2/3}\,dx=\frac{(b\,x^3+a)^{\frac{2}{3}}\,(20b^3x^9+5a\,b^2x^6-6a^2b\,x^3+9a^3)}{220b^3}$$

input
```
int(x^8*(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*(9*a**3 - 6*a**2*b*x**3 + 5*a*b**2*x**6 + 20*b**3*x**
9))/(220*b**3)
```

3.374 $\int x^5(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^5\left(a+bx^3\right)^{2/3}\,dx=-\frac{a(a+bx^3)^{5/3}}{5b^2}+\frac{(a+bx^3)^{8/3}}{8b^2}$$

output
```
-1/5*a*(b*x^3+a)^(5/3)/b^2+1/8*(b*x^3+a)^(8/3)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int x^5\left(a+bx^3\right)^{2/3}\,dx=\frac{(a+bx^3)^{2/3}\left(-3a^2+2abx^3+5b^2x^6\right)}{40b^2}$$

input
```
Integrate[x^5*(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-3*a^2 + 2*a*b*x^3 + 5*b^2*x^6))/(40*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^{2/3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^3 \left(bx^3+a\right)^{2/3} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\frac{\left(bx^3+a\right)^{5/3}}{b} - \frac{a\left(bx^3+a\right)^{2/3}}{b}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{3\left(a+bx^3\right)^{8/3}}{8b^2} - \frac{3a\left(a+bx^3\right)^{5/3}}{5b^2}\right)$$

input
```
Int[x^5*(a + b*x^3)^(2/3),x]
```

output
```
((-3*a*(a + b*x^3)^(5/3))/(5*b^2) + (3*(a + b*x^3)^(8/3))/(8*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-5b\,x^{3}+3a\right)}{40b^{2}}$	25
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-5b\,x^{3}+3a\right)}{40b^{2}}$	25
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-5b\,x^{3}+3a\right)}{40b^{2}}$	25
trager	$-\frac{\left(-5b^{2}x^{6}-2ab\,x^{3}+3a^{2}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{40b^{2}}$	36
risch	$-\frac{\left(-5b^{2}x^{6}-2ab\,x^{3}+3a^{2}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{40b^{2}}$	36

input
```
int(x^5*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(b*x^3+a)^(5/3)*(-5*b*x^3+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x^5\left(a+bx^3\right)^{2/3}\,dx=\frac{(5\,b^2x^6+2\,abx^3-3\,a^2)(bx^3+a)^{\frac{2}{3}}}{40\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/40*(5*b^2*x^6 + 2*a*b*x^3 - 3*a^2)*(b*x^3 + a)^(2/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int x^5 \left(a + bx^3\right)^{2/3} \, dx = \begin{cases} -\frac{3a^2(a+bx^3)^{\frac{2}{3}}}{40b^2} + \frac{ax^3(a+bx^3)^{\frac{2}{3}}}{20b} + \frac{x^6(a+bx^3)^{\frac{2}{3}}}{8} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((-3*a**2*(a + b*x**3)**(2/3)/(40*b**2) + a*x**3*(a + b*x**3)**(2
/3)/(20*b) + x**6*(a + b*x**3)**(2/3)/8, Ne(b, 0)), (a**(2/3)*x**6/6, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^5 \left(a + bx^3\right)^{2/3} \, dx = \frac{(bx^3 + a)^{\frac{8}{3}}}{8\,b^2} - \frac{(bx^3 + a)^{\frac{5}{3}}a}{5\,b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/8*(b*x^3 + a)^(8/3)/b^2 - 1/5*(b*x^3 + a)^(5/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^5 (a + bx^3)^{2/3} \, dx = \frac{5 (bx^3 + a)^{\frac{8}{3}} - 8 (bx^3 + a)^{\frac{5}{3}} a}{40 b^2}$$

input
```
integrate(x^5*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/40*(5*(b*x^3 + a)^(8/3) - 8*(b*x^3 + a)^(5/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^5 (a + bx^3)^{2/3} \, dx = (b\, x^3 + a)^{2/3} \left(\frac{x^6}{8} - \frac{3\, a^2}{40\, b^2} + \frac{a\, x^3}{20\, b} \right)$$

input
```
int(x^5*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(2/3)*(x^6/8 - (3*a^2)/(40*b^2) + (a*x^3)/(20*b))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x^5 (a + bx^3)^{2/3} \, dx = \frac{(b\, x^3 + a)^{\frac{2}{3}} (5b^2 x^6 + 2ab\, x^3 - 3a^2)}{40b^2}$$

input
```
int(x^5*(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*( - 3*a**2 + 2*a*b*x**3 + 5*b**2*x**6))/(40*b**2)
```

3.375 $\int x^2(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^2\left(a+bx^3\right)^{2/3}\,dx=\frac{\left(a+bx^3\right)^{5/3}}{5b}$$

output
```
1/5*(b*x^3+a)^(5/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^3\right)^{2/3}\,dx=\frac{\left(a+bx^3\right)^{5/3}}{5b}$$

input
```
Integrate[x^2*(a + b*x^3)^(2/3),x]
```

output
```
(a + b*x^3)^(5/3)/(5*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^{2/3} dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^3\right)^{5/3}}{5b}$$

input
```
Int[x^2*(a + b*x^3)^(2/3),x]
```

output
```
(a + b*x^3)^(5/3)/(5*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15
derivativedivides	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15
default	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15
trager	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15
risch	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15
orering	$\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$	15

input
```
int(x^2*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/5*(b*x^3+a)^(5/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2(a+bx^3)^{2/3}\,dx=\frac{(bx^3+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int x^2 \left(a+bx^3\right)^{2/3} dx = \begin{cases} \frac{a(a+bx^3)^{\frac{2}{3}}}{5b} + \frac{x^3(a+bx^3)^{\frac{2}{3}}}{5} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((a*(a + b*x**3)**(2/3)/(5*b) + x**3*(a + b*x**3)**(2/3)/5, Ne(b,
 0)), (a**(2/3)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2 \left(a+bx^3\right)^{2/3} dx = \frac{(bx^3+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2 \left(a+bx^3\right)^{2/3} dx = \frac{(bx^3+a)^{\frac{5}{3}}}{5\,b}$$

input
```
integrate(x^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2(a+bx^3)^{2/3}\,dx=\frac{(b\,x^3+a)^{5/3}}{5\,b}$$

input
```
int(x^2*(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(5/3)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^2(a+bx^3)^{2/3}\,dx=\frac{(b\,x^3+a)^{\frac{5}{3}}}{5b}$$

input
```
int(x^2*(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*(a + b*x**3))/(5*b)
```

3.376 $\int \frac{(a+bx^3)^{2/3}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{(a+bx^3)^{2/3}}{x} dx = \frac{1}{2}(a+bx^3)^{2/3} + \frac{a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{1}{2}a^{2/3}\log(x) + \frac{1}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)$$

output
```
1/2*(b*x^3+a)^(2/3)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(
1/2)/a^(1/3))*3^(1/2)-1/2*a^(2/3)*ln(x)+1/2*a^(2/3)*ln(a^(1/3)-(b*x^3+a)^(
1/3))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = \frac{1}{6}\left(3(a+bx^3)^{2/3}\right.$$
$$\left. +2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx\right.\right.$$

input
```
Integrate[(a + b*x^3)^(2/3)/x,x]
```

output
```
(3*(a + b*x^3)^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^3)^(1/3))
/a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - a^(2/3)
*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{3}\int \frac{(bx^3+a)^{2/3}}{x^3}dx^3$$
$$\Big\downarrow \text{60}$$

$$\frac{1}{3}\left(a\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)$$

$\big\downarrow$ 67

$$\frac{1}{3}\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/}$$

$\big\downarrow$ 16

$$\frac{1}{3}\left(a\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)$$

$\big\downarrow$ 1082

$$\frac{1}{3}\left(a\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)$$

$\big\downarrow$ 217

$$\frac{1}{3}\left(a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^3\right)^{2/3}\right)$$

input
```
Int[(a + b*x^3)^(2/3)/x,x]
```

output
```
((3*(a + b*x^3)^(2/3))/2 + a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a
^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b
*x^3)^(1/3)])/(2*a^(1/3))))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2}+\frac{a^{\frac{2}{3}}\ln\left((b\,x^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3}-\frac{a^{\frac{2}{3}}\ln\left((b\,x^3+a)^{\frac{2}{3}}+a^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6}+\frac{a^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(b\,x^3+a)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}\right.}{3}$

input
```
int((b*x^3+a)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^3+a)^(2/3)+1/3*a^(2/3)*ln((b*x^3+a)^(1/3)-a^(1/3))-1/6*a^(2/3)*ln
((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2/3))+1/3*a^(2/3)*3^(1/2)*arct
an(2/3*3^(1/2)/a^(1/3)*(b*x^3+a)^(1/3)+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = \frac{1}{3}\,\sqrt{3}(a^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\,\sqrt{3}(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}}}{3\,a}\right) - \frac{1}{6}\,(a^2)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^3+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right) + \frac{1}{3}\,(a^2)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+\frac{1}{2}\,(bx^3+a)^{\frac{2}{3}}$$

input
```
integrate((b*x^3+a)^(2/3)/x,x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3
)*(a^2)^(1/3))/a) - 1/6*(a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*
a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) + 1/3*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)
*a - (a^2)^(2/3)) + 1/2*(b*x^3 + a)^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = -\frac{b^{\frac{2}{3}}x^2\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x,x)
```

output
```
-b**(2/3)*x**2*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*pi)/(
b*x**3))/(3*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{6}\,a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{3}\,a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + \frac{1}{2}\,(bx^3+a)^{\frac{2}{3}}$$

input
```
integrate((b*x^3+a)^(2/3)/x,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 1/3*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + 1/2*(b*x^3 + a)^(2
/3)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = \frac{1}{3}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{6}\,a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{3}\,a^{\frac{2}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{1}{2}\,(bx^3+a)^{\frac{2}{3}}$$

input
```
integrate((b*x^3+a)^(2/3)/x,x, algorithm="giac")
```

output
```
1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3))) + 1/2*(b*x^3 +
a)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = \frac{(b\,x^3+a)^{2/3}}{2} + \frac{a^{2/3}\,\ln\left(a^2\,(b\,x^3+a)^{1/3}-a^{7/3}\right)}{3} - \frac{a^{2/3}\,\ln\left(a^2\,(b\,x^3+a)^{1/3}-a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{3} + a^{2/3}\,\ln\left(a^2\,(b\,x^3+a)^{1/3}-9\,a^{7/3}\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int((a + b*x^3)^(2/3)/x,x)
```

output
```
(a + b*x^3)^(2/3)/2 + (a^(2/3)*log(a^2*(a + b*x^3)^(1/3) - a^(7/3)))/3 - (
a^(2/3)*log(a^2*(a + b*x^3)^(1/3) - a^(7/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^
(1/2)*1i)/2 + 1/2))/3 + a^(2/3)*log(a^2*(a + b*x^3)^(1/3) - 9*a^(7/3)*((3^
(1/2)*1i)/6 - 1/6)^2)*((3^(1/2)*1i)/6 - 1/6)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}}{2} + \left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^4+ax}dx\right)a$$

input
```
int((b*x^3+a)^(2/3)/x,x)
```

output
```
((a + b*x**3)**(2/3) + 2*int((a + b*x**3)**(2/3)/(a*x + b*x**4),x)*a)/2
```

3.377 $\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx=-\frac{(a+bx^3)^{2/3}}{3x^3}+\frac{2b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{a}}$$

output
```
-1/3*(b*x^3+a)^(2/3)/x^3+2/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1
/2)/a^(1/3))*3^(1/2)/a^(1/3)-1/3*b*ln(x)/a^(1/3)+1/3*b*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx=\frac{-3\sqrt[3]{a}(a+bx^3)^{2/3}+2\sqrt{3}bx^3\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2bx^3\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)}{9\sqrt[3]{a}x^3}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^4,x]
```

output
```
(-3*a^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]] + 2*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1/3)] - b*
x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*a^(1/
3)*x^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 51, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{\left(bx^3+a\right)^{2/3}}{x^6}dx^3$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{3}\left(\frac{2}{3}b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^3\right)^{2/}}{x^3}\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{2}{3}b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{2}{3}b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^3\right)^{2/3}}{x^3}\right)$$

input
```
Int[(a + b*x^3)^(2/3)/x^4,x]
```

output
```
(-((a + b*x^3)^(2/3)/x^3) + (2*b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a
 + b*x^3)^(1/3)])/(2*a^(1/3))))/3)/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x^{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b\,x^{3}-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)b\,x^{3}-3\left(b\,x^{3}\right.}{9x^{3}a^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*b*x
^3+2*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a
)^(1/3)+a^(2/3))*b*x^3-3*(b*x^3+a)^(2/3)*a^(1/3))/x^3/a^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.71

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx^3\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)}{}\right.$$

input
```
integrate((b*x^3+a)^(2/3)/x^4,x, algorithm="fricas")
```

output
```
[1/9*(3*sqrt(1/3)*a*b*x^3*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(
b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3))
 - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - a^(2/3)*b*x^3*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^3*log((b*x^3
 + a)^(1/3) - a^(1/3)) - 3*(b*x^3 + a)^(2/3)*a)/(a*x^3), 1/9*(6*sqrt(1/3)*
a^(2/3)*b*x^3*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) -
a^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))
 + 2*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) - 3*(b*x^3 + a)^(2/3)*
a)/(a*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx = -\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x**4,x)
```

output
```
-b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**3)
)/(3*x*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx = \frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{1}{3}}} - \frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{1}{3}}} + \frac{2\,b\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{1}{3}}} - \frac{(bx^3+a)^{\frac{2}{3}}}{3\,x^3}$$

input
```
integrate((b*x^3+a)^(2/3)/x^4,x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(1/3) - 1/9*b*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(1/3) + 2/9*b*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(1/3) - 1/3*(b*x^3 +
 a)^(2/3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx = \frac{1}{9}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}} + \frac{2}{}\right.$$

input
```
integrate((b*x^3+a)^(2/3)/x^4,x, algorithm="giac")
```

output
```
1/9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(1/3) - log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^3 + a)^(2
/3)/(b*x^3))*b
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx = \frac{2\,b\,\ln\left(\frac{4\,a^{1/3}\,b^2}{9}-\frac{4\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)}{9\,a^{1/3}}-\frac{(b\,x^3+a)^{2/3}}{3\,x^3} - \frac{\ln\left(\frac{a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{1/3}} - \frac{\ln\left(\frac{a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{9}-\frac{4\,b^2\,(b\,x^3+a)^{1/3}}{9}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{1/3}}$$

input
```
int((a + b*x^3)^(2/3)/x^4,x)
```

output
```
(2*b*log((4*a^(1/3)*b^2)/9 - (4*b^2*(a + b*x^3)^(1/3))/9))/(9*a^(1/3)) - (
a + b*x^3)^(2/3)/(3*x^3) - (log((a^(1/3)*(b - 3^(1/2)*b*1i)^2)/9 - (4*b^2*
(a + b*x^3)^(1/3))/9)*(b - 3^(1/2)*b*1i))/(9*a^(1/3)) - (log((a^(1/3)*(b +
 3^(1/2)*b*1i)^2)/9 - (4*b^2*(a + b*x^3)^(1/3))/9)*(b + 3^(1/2)*b*1i))/(9*
a^(1/3))
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^4}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}}+2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^4+a x}dx\right)b\,x^3}{3x^3}$$

input
```
int((b*x^3+a)^(2/3)/x^4,x)
```

output
```
( - (a + b*x**3)**(2/3) + 2*int((a + b*x**3)**(2/3)/(a*x + b*x**4),x)*b*x*
*3)/(3*x**3)
```

3.378 $\int x^3(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int x^3(a+bx^3)^{2/3}\,dx = \frac{ax(a+bx^3)^{2/3}}{9b} + \frac{1}{6}x^4(a+bx^3)^{2/3} - \frac{a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} + \frac{a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{18b^{4/3}}$$

output
```
1/9*a*x*(b*x^3+a)^(2/3)/b+1/6*x^4*(b*x^3+a)^(2/3)-1/27*a^2*arctan(1/3*(1+2
*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)+1/18*a^2*ln(-b^(1/3)*
x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.44

$$\int x^3(a+bx^3)^{2/3}\,dx = \frac{(a+bx^3)^{2/3}(2ax+3bx^4)}{18b} - \frac{a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}b^{4/3}} + \frac{a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27b^{4/3}} - \frac{a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54b^{4/3}}$$

input
```
Integrate[x^3*(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(2/3)*(2*a*x + 3*b*x^4))/(18*b) - (a^2*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(4/3)) + (a^2*Log[-(
b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(4/3)) - (a^2*Log[b^(2/3)*x^2 + b^(
1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(4/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {811, 843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^{2/3} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{3}a\int \frac{x^3}{\sqrt[3]{bx^3+a}}dx+\frac{1}{6}x^4\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{3}a\left(\frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}\right)+\frac{1}{6}x^4\left(a+bx^3\right)^{2/3}$$

$$\Big\downarrow \text{769}$$

$$\frac{1}{3}a\left(\frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}\right)+$$

$$\frac{1}{6}x^4\left(a+bx^3\right)^{2/3}$$

input
```
Int[x^3*(a + b*x^3)^(2/3),x]
```

output
```
(x^4*(a + b*x^3)^(2/3))/6 + (a*((x*(a + b*x^3)^(2/3))/(3*b) - (a*(ArcTan[(
1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/3
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$\frac{9\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}} x^{4}+6 a x \left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{1}{3}}+2 \sqrt{3}\, \arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2 \left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}+2 \ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a^{2}-\ln\left(\frac{b^{\frac{2}{3}} x^{2}}{\cdots}\right.}{54 b^{\frac{4}{3}}}$

input
```
int(x^3*(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*(b*x^3+a)^(2/3)*b^(4/3)*x^4+6*a*x*(b*x^3+a)^(2/3)*b^(1/3)+2*3^(1/2
)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a^2+2*ln((-b
^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2-ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x
+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.95

$$\int x^3(a+bx^3)^{2/3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+(bx^3+a)^{\frac{1}{3}}bx^2-2\,(bx^3+a\cdots\right.\right.}{\cdots}\right.$$

input
```
integrate(x^3*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
[1/54*(3*sqrt(1/3)*a^2*b*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3
)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*
x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*a^2*b^(2/3)*log(-(b^
(1/3)*x - (b*x^3 + a)^(1/3))/x) - a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 +
a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 2*a*b*x)*(b*
x^3 + a)^(2/3))/b^2, 1/54*(6*sqrt(1/3)*a^2*b^(2/3)*arctan(sqrt(1/3)*(b^(1/
3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) + 2*a^2*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) - a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*
b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 2*a*b*x)*(b*x^3 + a)^
(2/3))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int x^3\left(a+bx^3\right)^{2/3}\,dx=\frac{a^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(2/3),x)
```

output
```
a**(2/3)*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(90) = 180.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.55

$$\int x^3(a+bx^3)^{2/3}\,dx=\frac{\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{54\,b^{\frac{4}{3}}}+\frac{a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{4}{3}}}+\frac{\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}+\frac{2\,(bx^3+a)^{\frac{5}{3}}a^2}{x^5}}{18\left(b^3-\frac{2\,(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}\right)}$$

input
```
integrate(x^3*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) - 1/54*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(4/3) + 1/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^
(4/3) + 1/18*((b*x^3 + a)^(2/3)*a^2*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(
b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 + a)^2*b/x^6)
```

Giac [F]

$$\int x^3(a+bx^3)^{2/3}\,dx=\int (bx^3+a)^{\frac{2}{3}}x^3\,dx$$

input
```
integrate(x^3*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^3\right)^{2/3} \, dx=\int x^3 \, {\left (b\,x^3+a\right)}^{2/3} \,d x$$

input
```
int(x^3*(a + b*x^3)^(2/3),x)
```

output
```
int(x^3*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int x^3 \left(a+bx^3\right)^{2/3} \, dx=\frac{2(b\,x^{3}+a)^{\frac{2}{3}} a x +3(b\,x^{3}+a)^{\frac{2}{3}} b\,x^{4}-2\left(\int \frac{1}{(b\,x^{3}+a)^{\frac{1}{3}}}d x \right) a^{2}}{18b}$$

input
```
int(x^3*(b*x^3+a)^(2/3),x)
```

output
```
(2*(a + b*x**3)**(2/3)*a*x + 3*(a + b*x**3)**(2/3)*b*x**4 - 2*int((a + b*x
**3)**(2/3)/(a + b*x**3),x)*a**2)/(18*b)
```

3.379 $\int (a+bx^3)^{2/3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 91

$$\int (a+bx^3)^{2/3} \, dx = \frac{1}{3}x(a+bx^3)^{2/3} + \frac{2a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} - \frac{a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

output
```
1/3*x*(b*x^3+a)^(2/3)+2/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))*3^(1/2)/b^(1/3)-1/3*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.23

$$\int (a+bx^3)^{2/3}\,dx=\frac{3\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)(a+bx^3)^{2/3}\,\text{AppellF1}\left(\frac{5}{3},-\frac{2}{3},-\frac{2}{3},\frac{8}{3},-\frac{(-1)^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}},\frac{i+\sqrt{3}\cdots}{3\cdots}\right)}{5\ 2^{2/3}\sqrt[3]{b}\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{2/3}\left(\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(2/3),x]
```

output
```
(3*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3,
-2/3, 8/3, -(((-1)^(2/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/((1 + (-1)^(1/3
))*a^(1/3))), (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/
(5*2^(2/3)*b^(1/3)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(
1/3)))^(2/3)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {748, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^{2/3}\,dx$$

$$\Big\downarrow \ 748$$

$$\frac{2}{3}a\int \frac{1}{\sqrt[3]{bx^3+a}}dx+\frac{1}{3}x(a+bx^3)^{2/3}$$

$$\Big\downarrow \ 769$$

$$\frac{2}{3}a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)+\frac{1}{3}x\left(a+bx^3\right)^{2/3}$$

input
```
Int[(a + b*x^3)^(2/3),x]
```

output
```
(x*(a + b*x^3)^(2/3))/3 + (2*a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b
^(1/3))))/3
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

method	result
pseudoelliptic	$-\frac{2\left(\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a-\frac{3\left(b\,x^{3}+a\right)^{\frac{2}{3}}x\,b^{\frac{1}{3}}}{2}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\right.}{x^{2}}\right.}{2}\right.}{9b^{\frac{1}{3}}}$

input
```
int((b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*
a-3/2*(b*x^3+a)^(2/3)*x*b^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-1/2*l
n((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(68) = 136$.

Time = 0.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.98

$$\int (a+bx^3)^{2/3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\right.\right.}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-3\,(bx^3+a)^{\frac{2}{3}}bx+2\,a(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(b}{x}\right.}{9\,b}$$

input
```
integrate((b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
[1/9*(3*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)
*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2
+ 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 3*(b*x^3 +
 a)^(2/3)*b*x - 2*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) +
 a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^
3 + a)^(2/3))/x^2))/b, -1/9*(6*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-s
qrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) - 3*(
b*x^3 + a)^(2/3)*b*x + 2*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3
))/x) - a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.41

$$\int \left(a+bx^3\right)^{2/3} dx = \frac{a^{\frac{2}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3),x)
```

output
```
a**(2/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.47

$$\int \left(a+bx^3\right)^{2/3}\,dx = -\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{1}{3}}} + \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9\,b^{\frac{1}{3}}} - \frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9\,b^{\frac{1}{3}}} - \frac{\left(bx^3+a\right)^{\frac{2}{3}}a}{3\left(b-\frac{bx^3+a}{x^3}\right)x^2}$$

input
```
integrate((b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) + 1/9*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(1/3) - 2/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) -
1/3*(b*x^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2)
```

Giac [F]

$$\int \left(a+bx^3\right)^{2/3}\,dx = \int \left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.41

$$\int \left(a+bx^3\right)^{2/3}\,dx=\frac{x\,\left(b\,x^3+a\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^{2/3}}$$

input
```
int((a + b*x^3)^(2/3),x)
```

output
```
(x*(a + b*x^3)^(2/3)*hypergeom([-2/3, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
 1)^(2/3)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{2/3}\,dx=\frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x}{3}+\frac{2\left(\int\frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx\right)a}{3}$$

input
```
int((b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*x + 2*int((a + b*x**3)**(2/3)/(a + b*x**3),x)*a)/3
```

3.380 $\int \frac{(a+bx^3)^{2/3}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{(a+bx^3)^{2/3}}{x^3} dx = -\frac{(a+bx^3)^{2/3}}{2x^2} + \frac{b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{2}b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)$$

output
```
-1/2*(b*x^3+a)^(2/3)/x^2+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(
1/3))*3^(1/2))*3^(1/2)-1/2*b^(2/3)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx^3)^{2/3}}{x^3} dx = -\frac{(a+bx^3)^{2/3}}{2x^2} + \frac{b^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}} - \frac{1}{3}b^{2/3}\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) + \frac{1}{6}b^{2/3}\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^3,x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/x^2 + (b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*
x + 2*(a + b*x^3)^(1/3))])/Sqrt[3] - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)])/3 + (b^(2/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a
 + b*x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {809, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx$$

$$\Big\downarrow \; 809$$

$$b\int \frac{1}{\sqrt[3]{bx^3+a}}dx-\frac{(a+bx^3)^{2/3}}{2x^2}$$

$$\Big\downarrow \; 769$$

$$b\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)-\frac{(a+bx^3)^{2/3}}{2x^2}$$

input
```
Int[(a + b*x^3)^(2/3)/x^3,x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/x^2 + b*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b
^(1/3)))
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

method	result
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{2}{3}}}{2x^2}-\frac{b^{\frac{2}{3}}\ln\left(\frac{-b^{\frac{1}{3}}x+(b\,x^3+a)^{\frac{1}{3}}}{x}\right)}{3}+\frac{b^{\frac{2}{3}}\ln\left(\frac{b^{\frac{2}{3}}x^2+b^{\frac{1}{3}}(b\,x^3+a)^{\frac{1}{3}}x+(b\,x^3+a)^{\frac{2}{3}}}{x^2}\right)}{6}-\frac{b^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\sqrt{3}\left(2(b \ldots\right.\right.}{3}$

input
```
int((b*x^3+a)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^3+a)^(2/3)/x^2-1/3*b^(2/3)*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+1/
6*b^(2/3)*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)-
1/3*b^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x^3+a)^(1/3)/b^(1/3)+x)/x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(2/3)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx = \frac{a^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x**3,x)
```

output
```
a**(2/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx = -\frac{1}{3}\sqrt{3}b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right) + \frac{1}{6}b^{\frac{2}{3}}\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right) - \frac{1}{3}b^{\frac{2}{3}}\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right) - \frac{(bx^3+a)^{\frac{2}{3}}}{2\,x^2}$$

input
```
integrate((b*x^3+a)^(2/3)/x^3,x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*b^(2/3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3)) + 1/6*b^(2/3)*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
+ a)^(2/3)/x^2) - 1/3*b^(2/3)*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x) - 1/2*(b
*x^3 + a)^(2/3)/x^2
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx=\int \frac{(b\,x^3+a)^{2/3}}{x^3}\,dx$$

input
```
int((a + b*x^3)^(2/3)/x^3,x)
```

output
```
int((a + b*x^3)^(2/3)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^3}\,dx=\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{x^3}dx$$

input
```
int((b*x^3+a)^(2/3)/x^3,x)
```

output
```
int((a + b*x**3)**(2/3)/x**3,x)
```

3.381 $\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = -\frac{(a+bx^3)^{5/3}}{5ax^5}$$

output
```
-1/5*(b*x^3+a)^(5/3)/a/x^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = -\frac{(a+bx^3)^{5/3}}{5ax^5}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^6,x]
```

output
```
-1/5*(a + b*x^3)^(5/3)/(a*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^6} \, dx$$

$$\Big\downarrow \; 796$$

$$-\frac{(a+bx^3)^{5/3}}{5ax^5}$$

input
```
Int[(a + b*x^3)^(2/3)/x^6,x]
```

output
```
-1/5*(a + b*x^3)^(5/3)/(a*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{5}{3}}}{5a\,x^5}$	18
trager	$-\frac{(b\,x^3+a)^{\frac{5}{3}}}{5a\,x^5}$	18
risch	$-\frac{(b\,x^3+a)^{\frac{5}{3}}}{5a\,x^5}$	18
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{5}{3}}}{5a\,x^5}$	18
orering	$-\frac{(b\,x^3+a)^{\frac{5}{3}}}{5a\,x^5}$	18

input
```
int((b*x^3+a)^(2/3)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^3+a)^(5/3)/a/x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = -\frac{(bx^3+a)^{\frac{5}{3}}}{5\,ax^5}$$

input
```
integrate((b*x^3+a)^(2/3)/x^6,x, algorithm="fricas")
```

output
```
-1/5*(b*x^3 + a)^(5/3)/(a*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(17) = 34.

Time = 0.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = \frac{b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3x^3\Gamma\left(-\frac{2}{3}\right)} + \frac{b^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x**6,x)
```

output
```
b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*x**3*gamma(-2/3)) + b**(5/
3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a*gamma(-2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = -\frac{(bx^3+a)^{\frac{5}{3}}}{5\,ax^5}$$

input
```
integrate((b*x^3+a)^(2/3)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(b*x^3 + a)^(5/3)/(a*x^5)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = -\frac{(b\,x^3+a)^{5/3}}{5\,a\,x^5}$$

input
```
int((a + b*x^3)^(2/3)/x^6,x)
```

output
```
-(a + b*x^3)^(5/3)/(5*a*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^{2/3}}{x^6}\,dx = -\frac{(b\,x^3+a)^{\frac{5}{3}}}{5a\,x^5}$$

input
```
int((b*x^3+a)^(2/3)/x^6,x)
```

output
```
( - (a + b*x**3)**(2/3)*(a + b*x**3))/(5*a*x**5)
```

3.382 $\int \frac{(a+bx^3)^{2/3}}{x^9} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx^3)^{2/3}}{x^9} dx = -\frac{(a+bx^3)^{5/3}}{8ax^8} + \frac{3b(a+bx^3)^{5/3}}{40a^2x^5}$$

output
```
-1/8*(b*x^3+a)^(5/3)/a/x^8+3/40*b*(b*x^3+a)^(5/3)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^3)^{2/3}}{x^9} dx = \frac{(a+bx^3)^{2/3} (-5a^2 - 2abx^3 + 3b^2x^6)}{40a^2x^8}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^9,x]
```

output
```
((a + b*x^3)^(2/3)*(-5*a^2 - 2*a*b*x^3 + 3*b^2*x^6))/(40*a^2*x^8)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^9} dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b\int \frac{(bx^3+a)^{2/3}}{x^6}dx}{8a}-\frac{(a+bx^3)^{5/3}}{8ax^8}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{3b(a+bx^3)^{5/3}}{40a^2x^5}-\frac{(a+bx^3)^{5/3}}{8ax^8}$$

input
```
Int[(a + b*x^3)^(2/3)/x^9,x]
```

output
```
-1/8*(a + b*x^3)^(5/3)/(a*x^8) + (3*b*(a + b*x^3)^(5/3))/(40*a^2*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{5}{3}}(-3b\,x^3+5a)}{40x^8a^2}$	28
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{5}{3}}(-3b\,x^3+5a)}{40x^8a^2}$	28
orering	$-\frac{(b\,x^3+a)^{\frac{5}{3}}(-3b\,x^3+5a)}{40x^8a^2}$	28
trager	$-\frac{(-3b^2x^6+2ab\,x^3+5a^2)(b\,x^3+a)^{\frac{2}{3}}}{40x^8a^2}$	39
risch	$-\frac{(-3b^2x^6+2ab\,x^3+5a^2)(b\,x^3+a)^{\frac{2}{3}}}{40x^8a^2}$	39

input
```
int((b*x^3+a)^(2/3)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(b*x^3+a)^(5/3)*(-3*b*x^3+5*a)/x^8/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^{2/3}}{x^9}\,dx = \frac{(3\,b^2x^6-2\,abx^3-5\,a^2)(bx^3+a)^{\frac{2}{3}}}{40\,a^2x^8}$$

input
```
integrate((b*x^3+a)^(2/3)/x^9,x, algorithm="fricas")
```

output
```
1/40*(3*b^2*x^6 - 2*a*b*x^3 - 5*a^2)*(b*x^3 + a)^(2/3)/(a^2*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(37) = 74$.

Time = 0.80 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx^3)^{2/3}}{x^9}\,dx = -\frac{5b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9x^6\Gamma\left(-\frac{2}{3}\right)} - \frac{2b^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{9ax^3\Gamma\left(-\frac{2}{3}\right)} + \frac{b^{\frac{8}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{3a^2\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x**9,x)
```

output
```
-5*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*x**6*gamma(-2/3)) - 2*b
**(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(9*a*x**3*gamma(-2/3)) + b**(8
/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(3*a**2*gamma(-2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)^{2/3}}{x^9}\,dx = \frac{\frac{8\,(bx^3+a)^{\frac{5}{3}}b}{x^5} - \frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}}{40\,a^2}$$

input
```
integrate((b*x^3+a)^(2/3)/x^9,x, algorithm="maxima")
```

output
```
1/40*(8*(b*x^3 + a)^(5/3)*b/x^5 - 5*(b*x^3 + a)^(8/3)/x^8)/a^2
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^9}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^9}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^9,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^9, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^{2/3}}{x^9}\,dx = -\frac{(b\,x^3+a)^{2/3}\,(5\,a^2+2\,a\,b\,x^3-3\,b^2\,x^6)}{40\,a^2\,x^8}$$

input
```
int((a + b*x^3)^(2/3)/x^9,x)
```

output
```
-((a + b*x^3)^(2/3)*(5*a^2 - 3*b^2*x^6 + 2*a*b*x^3))/(40*a^2*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^3)^{2/3}}{x^9}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,(3b^2x^6-2ab\,x^3-5a^2)}{40a^2x^8}$$

input
```
int((b*x^3+a)^(2/3)/x^9,x)
```

output
```
((a + b*x**3)**(2/3)*( - 5*a**2 - 2*a*b*x**3 + 3*b**2*x**6))/(40*a**2*x**8
)
```

3.383 $\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = -\frac{(a+bx^3)^{5/3}}{11ax^{11}} + \frac{3b(a+bx^3)^{5/3}}{44a^2x^8} - \frac{9b^2(a+bx^3)^{5/3}}{220a^3x^5}$$

output
```
-1/11*(b*x^3+a)^(5/3)/a/x^11+3/44*b*(b*x^3+a)^(5/3)/a^2/x^8-9/220*b^2*(b*x
^3+a)^(5/3)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = \frac{(a+bx^3)^{2/3}\,(-20a^3 - 5a^2bx^3 + 6ab^2x^6 - 9b^3x^9)}{220a^3x^{11}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^12,x]
```

output
```
((a + b*x^3)^(2/3)*(-20*a^3 - 5*a^2*b*x^3 + 6*a*b^2*x^6 - 9*b^3*x^9))/(220
*a^3*x^11)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\int \frac{\left(bx^3+a\right)^{2/3}}{x^9}dx}{11a}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\left(-\frac{3b\int \frac{\left(bx^3+a\right)^{2/3}}{x^6}dx}{8a}-\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}\right)}{11a}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{6b\left(\frac{3b\left(a+bx^3\right)^{5/3}}{40a^2x^5}-\frac{\left(a+bx^3\right)^{5/3}}{8ax^8}\right)}{11a}-\frac{\left(a+bx^3\right)^{5/3}}{11ax^{11}}$$

input
```
Int[(a + b*x^3)^(2/3)/x^12,x]
```

output
```
-1/11*(a + b*x^3)^(5/3)/(a*x^11) - (6*b*(-1/8*(a + b*x^3)^(5/3)/(a*x^8) +
(3*b*(a + b*x^3)^(5/3))/(40*a^2*x^5)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(9 b^{2} x^{6}-15 a b\,x^{3}+20 a^{2}\right)}{220 x^{11} a^{3}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(9 b^{2} x^{6}-15 a b\,x^{3}+20 a^{2}\right)}{220 x^{11} a^{3}}$	39
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}} \left(9 b^{2} x^{6}-15 a b\,x^{3}+20 a^{2}\right)}{220 x^{11} a^{3}}$	39
trager	$-\frac{\left(9 b^{3} x^{9}-6 a\,b^{2} x^{6}+5 a^{2} b\,x^{3}+20 a^{3}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{220 x^{11} a^{3}}$	50
risch	$-\frac{\left(9 b^{3} x^{9}-6 a\,b^{2} x^{6}+5 a^{2} b\,x^{3}+20 a^{3}\right) \left(b\,x^{3}+a\right)^{\frac{2}{3}}}{220 x^{11} a^{3}}$	50

input
```
int((b*x^3+a)^(2/3)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/220*(b*x^3+a)^(5/3)*(9*b^2*x^6-15*a*b*x^3+20*a^2)/x^11/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = -\frac{(9\,b^3x^9 - 6\,ab^2x^6 + 5\,a^2bx^3 + 20\,a^3)(bx^3+a)^{\frac{2}{3}}}{220\,a^3x^{11}}$$

input
```
integrate((b*x^3+a)^(2/3)/x^12,x, algorithm="fricas")
```

output
```
-1/220*(9*b^3*x^9 - 6*a*b^2*x^6 + 5*a^2*b*x^3 + 20*a^3)*(b*x^3 + a)^(2/3)/
(a^3*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(61) = 122.

Time = 1.42 (sec) , antiderivative size = 520, normalized size of antiderivative = 7.65

$$\begin{aligned}\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx &= \frac{40a^5b^{\frac{14}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)} \\ &+ \frac{90a^4b^{\frac{17}{3}}x^3\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)} \\ &+ \frac{48a^3b^{\frac{20}{3}}x^6\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)} \\ &+ \frac{4a^2b^{\frac{23}{3}}x^9\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)} \\ &+ \frac{24ab^{\frac{26}{3}}x^{12}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)} \\ &+ \frac{18b^{\frac{29}{3}}x^{15}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{11}{3}\right)}{27a^5b^4x^9\Gamma\left(-\frac{2}{3}\right)+54a^4b^5x^{12}\Gamma\left(-\frac{2}{3}\right)+27a^3b^6x^{15}\Gamma\left(-\frac{2}{3}\right)}\end{aligned}$$

input
```
integrate((b*x**3+a)**(2/3)/x**12,x)
```

output
```
40*a**5*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*
gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2
/3)) + 90*a**4*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**
5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x*
*15*gamma(-2/3)) + 48*a**3*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-1
1/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(-2/3) + 27*
a**3*b**6*x**15*gamma(-2/3)) + 4*a**2*b**(23/3)*x**9*(a/(b*x**3) + 1)**(2/
3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x**12*gamma(
-2/3) + 27*a**3*b**6*x**15*gamma(-2/3)) + 24*a*b**(26/3)*x**12*(a/(b*x**3)
 + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4*b**5*x*
*12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3)) + 18*b**(29/3)*x**15*(a/
(b*x**3) + 1)**(2/3)*gamma(-11/3)/(27*a**5*b**4*x**9*gamma(-2/3) + 54*a**4
*b**5*x**12*gamma(-2/3) + 27*a**3*b**6*x**15*gamma(-2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = -\frac{\frac{44\,(bx^3+a)^{\frac{5}{3}}b^2}{x^5} - \frac{55\,(bx^3+a)^{\frac{8}{3}}b}{x^8} + \frac{20\,(bx^3+a)^{\frac{11}{3}}}{x^{11}}}{220\,a^3}$$

input
```
integrate((b*x^3+a)^(2/3)/x^12,x, algorithm="maxima")
```

output
```
-1/220*(44*(b*x^3 + a)^(5/3)*b^2/x^5 - 55*(b*x^3 + a)^(8/3)*b/x^8 + 20*(b*
x^3 + a)^(11/3)/x^11)/a^3
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^{12}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^12,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^12, x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = \frac{3\,b^2\,(b\,x^3+a)^{2/3}}{110\,a^2\,x^5} - \frac{b\,(b\,x^3+a)^{2/3}}{44\,a\,x^8} - \frac{9\,b^3\,(b\,x^3+a)^{2/3}}{220\,a^3\,x^2} - \frac{(b\,x^3+a)^{2/3}}{11\,x^{11}}$$

input
```
int((a + b*x^3)^(2/3)/x^12,x)
```

output
```
(3*b^2*(a + b*x^3)^(2/3))/(110*a^2*x^5) - (b*(a + b*x^3)^(2/3))/(44*a*x^8)
 - (9*b^3*(a + b*x^3)^(2/3))/(220*a^3*x^2) - (a + b*x^3)^(2/3)/(11*x^11)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}\,(-9b^3x^9+6a\,b^2x^6-5a^2b\,x^3-20a^3)}{220a^3x^{11}}$$

input
```
int((b*x^3+a)^(2/3)/x^12,x)
```

output
```
((a + b*x**3)**(2/3)*( - 20*a**3 - 5*a**2*b*x**3 + 6*a*b**2*x**6 - 9*b**3*
x**9))/(220*a**3*x**11)
```

3.384 $\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx = -\frac{(a+bx^3)^{5/3}}{14ax^{14}} + \frac{9b(a+bx^3)^{5/3}}{154a^2x^{11}} - \frac{27b^2(a+bx^3)^{5/3}}{616a^3x^8} + \frac{81b^3(a+bx^3)^{5/3}}{3080a^4x^5}$$

output
```
-1/14*(b*x^3+a)^(5/3)/a/x^14+9/154*b*(b*x^3+a)^(5/3)/a^2/x^11-27/616*b^2*(
b*x^3+a)^(5/3)/a^3/x^8+81/3080*b^3*(b*x^3+a)^(5/3)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx = \frac{(a+bx^3)^{5/3}\,(-220a^3 + 180a^2bx^3 - 135ab^2x^6 + 81b^3x^9)}{3080a^4x^{14}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^15,x]
```

output
```
((a + b*x^3)^(5/3)*(-220*a^3 + 180*a^2*b*x^3 - 135*a*b^2*x^6 + 81*b^3*x^9)
)/(3080*a^4*x^14)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{9b\int \frac{(bx^3+a)^{2/3}}{x^{12}}dx}{14a}-\frac{(a+bx^3)^{5/3}}{14ax^{14}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{9b\left(-\frac{6b\int \frac{(bx^3+a)^{2/3}}{x^9}dx}{11a}-\frac{(a+bx^3)^{5/3}}{11ax^{11}}\right)}{14a}-\frac{(a+bx^3)^{5/3}}{14ax^{14}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{9b\left(-\frac{6b\left(-\frac{3b\int \frac{(bx^3+a)^{2/3}}{x^6}dx}{8a}-\frac{(a+bx^3)^{5/3}}{8ax^8}\right)}{11a}-\frac{(a+bx^3)^{5/3}}{11ax^{11}}\right)}{14a}-\frac{(a+bx^3)^{5/3}}{14ax^{14}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{9b\left(-\frac{6b\left(\frac{3b(a+bx^3)^{5/3}}{40a^2x^5}-\frac{(a+bx^3)^{5/3}}{8ax^8}\right)}{11a}-\frac{(a+bx^3)^{5/3}}{11ax^{11}}\right)}{14a}-\frac{(a+bx^3)^{5/3}}{14ax^{14}}$$

input
```
Int[(a + b*x^3)^(2/3)/x^15,x]
```

output
```
-1/14*(a + b*x^3)^(5/3)/(a*x^14) - (9*b*(-1/11*(a + b*x^3)^(5/3)/(a*x^11)
- (6*b*(-1/8*(a + b*x^3)^(5/3)/(a*x^8) + (3*b*(a + b*x^3)^(5/3))/(40*a^2*x
^5)))/(11*a)))/(14*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-81 b^{3} x^{9}+135 a\,b^{2} x^{6}-180 a^{2} b\,x^{3}+220 a^{3}\right)}{3080 x^{14} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-81 b^{3} x^{9}+135 a\,b^{2} x^{6}-180 a^{2} b\,x^{3}+220 a^{3}\right)}{3080 x^{14} a^{4}}$	50
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}\left(-81 b^{3} x^{9}+135 a\,b^{2} x^{6}-180 a^{2} b\,x^{3}+220 a^{3}\right)}{3080 x^{14} a^{4}}$	50
trager	$-\frac{\left(-81 b^{4} x^{12}+54 a\,b^{3} x^{9}-45 a^{2} b^{2} x^{6}+40 a^{3} b\,x^{3}+220 a^{4}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{3080 x^{14} a^{4}}$	61
risch	$-\frac{\left(-81 b^{4} x^{12}+54 a\,b^{3} x^{9}-45 a^{2} b^{2} x^{6}+40 a^{3} b\,x^{3}+220 a^{4}\right)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{3080 x^{14} a^{4}}$	61

input
```
int((b*x^3+a)^(2/3)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/3080*(b*x^3+a)^(5/3)*(-81*b^3*x^9+135*a*b^2*x^6-180*a^2*b*x^3+220*a^3)/
x^14/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx = \frac{(81\,b^4x^{12}-54\,ab^3x^9+45\,a^2b^2x^6-40\,a^3bx^3-220\,a^4)(bx^3+a)^{\frac{2}{3}}}{3080\,a^4x^{14}}$$

input
```
integrate((b*x^3+a)^(2/3)/x^15,x, algorithm="fricas")
```

output
```
1/3080*(81*b^4*x^12 - 54*a*b^3*x^9 + 45*a^2*b^2*x^6 - 40*a^3*b*x^3 - 220*a
^4)*(b*x^3 + a)^(2/3)/(a^4*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(85) = 170.

Time = 1.49 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.21

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**3+a)**(2/3)/x**15,x)
```

output
```
-440*a**7*b**(29/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**
12*gamma(-2/3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18*g
amma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3)) - 1400*a**6*b**(32/3)*x**3*(
a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) + 243*
a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a**4*
b**12*x**21*gamma(-2/3)) - 1470*a**5*b**(35/3)*x**6*(a/(b*x**3) + 1)**(2/3
)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x**15*gamm
a(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/
3)) - 518*a**4*b**(38/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**
7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**
11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3)) + 28*a**3*b**(41/3
)*x**12*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/
3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) +
 81*a**4*b**12*x**21*gamma(-2/3)) + 252*a**2*b**(44/3)*x**15*(a/(b*x**3) +
 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x
**15*gamma(-2/3) + 243*a**5*b**11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*
gamma(-2/3)) + 378*a*b**(47/3)*x**18*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/
(81*a**7*b**9*x**12*gamma(-2/3) + 243*a**6*b**10*x**15*gamma(-2/3) + 243*a
**5*b**11*x**18*gamma(-2/3) + 81*a**4*b**12*x**21*gamma(-2/3)) + 162*b**(5
0/3)*x**21*(a/(b*x**3) + 1)**(2/3)*gamma(-14/3)/(81*a**7*b**9*x**12*gam...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx = \frac{\frac{616\,(bx^3+a)^{\frac{5}{3}}b^3}{x^5} - \frac{1155\,(bx^3+a)^{\frac{8}{3}}b^2}{x^8} + \frac{840\,(bx^3+a)^{\frac{11}{3}}b}{x^{11}} - \frac{220\,(bx^3+a)^{\frac{14}{3}}}{x^{14}}}{3080\,a^4}$$

input
```
integrate((b*x^3+a)^(2/3)/x^15,x, algorithm="maxima")
```

output
```
1/3080*(616*(b*x^3 + a)^(5/3)*b^3/x^5 - 1155*(b*x^3 + a)^(8/3)*b^2/x^8 + 8
40*(b*x^3 + a)^(11/3)*b/x^11 - 220*(b*x^3 + a)^(14/3)/x^14)/a^4
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{x^{15}}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^15,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^15, x)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx=\frac{81\,b^4\,(b\,x^3+a)^{2/3}}{3080\,a^4\,x^2}-\frac{b\,(b\,x^3+a)^{2/3}}{77\,a\,x^{11}}-\frac{(b\,x^3+a)^{2/3}}{14\,x^{14}}-\frac{27\,b^3\,(b\,x^3+a)^{2/3}}{1540\,a^3\,x^5}+\frac{9\,b^2\,(b\,x^3+a)^{2/3}}{616\,a^2\,x^8}$$

input
```
int((a + b*x^3)^(2/3)/x^15,x)
```

output
```
(81*b^4*(a + b*x^3)^(2/3))/(3080*a^4*x^2) - (b*(a + b*x^3)^(2/3))/(77*a*x^
11) - (a + b*x^3)^(2/3)/(14*x^14) - (27*b^3*(a + b*x^3)^(2/3))/(1540*a^3*x
^5) + (9*b^2*(a + b*x^3)^(2/3))/(616*a^2*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^3)^{2/3}}{x^{15}}\,dx=\frac{(b\,x^3+a)^{\frac{2}{3}}\,(81b^4x^{12}-54a\,b^3x^9+45a^2b^2x^6-40a^3b\,x^3-220a^4)}{3080a^4x^{14}}$$

input
```
int((b*x^3+a)^(2/3)/x^15,x)
```

output
```
((a + b*x**3)**(2/3)*( - 220*a**4 - 40*a**3*b*x**3 + 45*a**2*b**2*x**6 - 5
4*a*b**3*x**9 + 81*b**4*x**12))/(3080*a**4*x**14)
```

3.385 $\int x^4 (a+bx^3)^{2/3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^4 (a+bx^3)^{2/3} \, dx = \frac{x^5 (a+bx^3)^{2/3} \, \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/5*x^5*(b*x^3+a)^(2/3)*hypergeom([-2/3, 5/3],[8/3],-b*x^3/a)/(1+b*x^3/a)^
(2/3)
```

Mathematica [A] (verified)

Time = 5.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^4 (a+bx^3)^{2/3} \, dx = \frac{x^5 (a+bx^3)^{2/3} \, \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[x^4*(a + b*x^3)^(2/3),x]
```

output
```
(x^5*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)])/(5
*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^3\right)^{2/3} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+bx^3\right)^{2/3} \int x^4 \left(\frac{bx^3}{a}+1\right)^{2/3}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^5 \left(a+bx^3\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5 \left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[x^4*(a + b*x^3)^(2/3),x]
```

output
```
(x^5*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x^3)/a)])/(5
*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^4 \left(b\,x^3+a\right)^{\frac{2}{3}}dx$$

input
```
int(x^4*(b*x^3+a)^(2/3),x)
```

output
```
int(x^4*(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^3\right)^{2/3}\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^4 \left(a+bx^3\right)^{2/3}\,dx=\frac{a^{\frac{2}{3}}x^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4*(b*x**3+a)**(2/3),x)
```

output
```
a**(2/3)*x**5*gamma(5/3)*hyper((-2/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3))
```

Maxima [F]

$$\int x^4(a+bx^3)^{2/3}\,dx=\int (bx^3+a)^{\frac{2}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^4, x)
```

Giac [F]

$$\int x^4(a+bx^3)^{2/3}\,dx=\int (bx^3+a)^{\frac{2}{3}}x^4\,dx$$

input
```
integrate(x^4*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+bx^3\right)^{2/3}\,dx=\int x^4\,{\left(b\,x^3+a\right)}^{2/3}\,dx$$

input
```
int(x^4*(a + b*x^3)^(2/3),x)
```

output
```
int(x^4*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int x^4\left(a+bx^3\right)^{2/3}\,dx=\frac{(b\,x^3+a)^{\frac{2}{3}}\,a\,x^2+2(b\,x^3+a)^{\frac{2}{3}}\,b\,x^5-2\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a^2}{14b}$$

input
```
int(x^4*(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*a*x**2 + 2*(a + b*x**3)**(2/3)*b*x**5 - 2*int(((a + b
*x**3)**(2/3)*x)/(a + b*x**3),x)*a**2)/(14*b)
```

3.386 $\int x(a+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int x\left(a+bx^3\right)^{2/3}\,dx=\frac{x^2(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
1/2*x^2*(b*x^3+a)^(2/3)*hypergeom([-2/3, 2/3],[5/3],-b*x^3/a)/(1+b*x^3/a)^
(2/3)
```

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^3\right)^{2/3}\,dx=\frac{x^2(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[x*(a + b*x^3)^(2/3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(2
*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^{2/3}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(a+bx^3\right)^{2/3}\int x\left(\frac{bx^3}{a}+1\right)^{2/3}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\left(a+bx^3\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[x*(a + b*x^3)^(2/3),x]
```

output
```
(x^2*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 5/3, -((b*x^3)/a)])/(2
*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x\left(b\,x^{3}+a\right)^{\frac{2}{3}}dx$$

input
```
int(x*(b*x^3+a)^(2/3),x)
```

output
```
int(x*(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int x\left(a+bx^3\right)^{2/3}\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x\left(a+bx^3\right)^{2/3}\,dx=\frac{a^{\frac{2}{3}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x*(b*x**3+a)**(2/3),x)
```

output
```
a**(2/3)*x**2*gamma(2/3)*hyper((-2/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3))
```

Maxima [F]

$$\int x\left(a+bx^3\right)^{2/3}\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*x, x)
```

Giac [F]

$$\int x\left(a+bx^3\right)^{2/3}\,dx=\int \left(bx^3+a\right)^{\frac{2}{3}}x\,dx$$

input
```
integrate(x*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^3)^{2/3}\,dx=\int x\,{\left(b\,x^3+a\right)}^{2/3}\,dx$$

input
```
int(x*(a + b*x^3)^(2/3),x)
```

output
```
int(x*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int x(a+bx^3)^{2/3}\,dx=\frac{(b\,x^3+a)^{\frac{2}{3}}\,x^2}{4}+\frac{\left(\int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx\right)a}{2}$$

input
```
int(x*(b*x^3+a)^(2/3),x)
```

output
```
((a + b*x**3)**(2/3)*x**2 + 2*int(((a + b*x**3)**(2/3)*x)/(a + b*x**3),x)*
a)/4
```

3.387 $\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx = -\frac{(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-(b*x^3+a)^(2/3)*hypergeom([-2/3, -1/3],[2/3],-b*x^3/a)/x/(1+b*x^3/a)^(2/3
)
```

Mathematica [A] (verified)

Time = 8.98 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx = -\frac{(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^2,x]
```

output
```
-(((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, -((b*x^3)/a)])/(x*
(1 + (b*x^3)/a)^(2/3)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^2} \, dx$$

$$\Big\downarrow \ 889$$

$$\frac{(a+bx^3)^{2/3} \int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^2}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \ 888$$

$$-\frac{(a+bx^3)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(a + b*x^3)^(2/3)/x^2,x]
```

output
```
-(((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, -1/3, 2/3, -((b*x^3)/a)])/(x*
(1 + (b*x^3)/a)^(2/3)))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{2}}dx$$

input
```
int((b*x^3+a)^(2/3)/x^2,x)
```

output
```
int((b*x^3+a)^(2/3)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx=\frac{a^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x**2,x)
```

output
```
a**(2/3)*gamma(-1/3)*hyper((-2/3, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx = \frac{(b\,x^3+a)^{2/3}\,{}_2\mathrm{F}_1\!\left(-\frac{2}{3},-\frac{1}{3};\ \frac{2}{3};\ -\frac{a}{b\,x^3}\right)}{x\,\left(\frac{a}{b\,x^3}+1\right)^{2/3}}$$

input
```
int((a + b*x^3)^(2/3)/x^2,x)
```

output
```
((a + b*x^3)^(2/3)*hypergeom([-2/3, -1/3], 2/3, -a/(b*x^3)))/(x*(a/(b*x^3)
 + 1)^(2/3))
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^2}\,dx = \frac{(b\,x^3+a)^{\frac{2}{3}}+2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^5+a\,x^2}dx\right)a x}{x}$$

input
```
int((b*x^3+a)^(2/3)/x^2,x)
```

output
```
((a + b*x**3)**(2/3) + 2*int((a + b*x**3)**(2/3)/(a*x**2 + b*x**5),x)*a*x)
/x
```

3.388 $\int \frac{(a+bx^3)^{2/3}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^3)^{2/3}}{x^5} dx = -\frac{(a+bx^3)^{2/3} \text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4 \left(1+\frac{bx^3}{a}\right)^{2/3}}$$

output
```
-1/4*(b*x^3+a)^(2/3)*hypergeom([-4/3, -2/3],[-1/3],-b*x^3/a)/x^4/(1+b*x^3/
a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{2/3}}{x^5} dx = -\frac{(a+bx^3)^{2/3} \text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4 \left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(2/3)/x^5,x]
```

output
```
-1/4*((a + b*x^3)^(2/3)*Hypergeometric2F1[-4/3, -2/3, -1/3, -((b*x^3)/a)])
/(x^4*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{2/3}}{x^5}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{(a+bx^3)^{2/3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}}{x^5}dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{(a+bx^3)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(a + b*x^3)^(2/3)/x^5,x]
```

output
```
-1/4*((a + b*x^3)^(2/3)*Hypergeometric2F1[-4/3, -2/3, -1/3, -((b*x^3)/a)])
/(x^4*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{2}{3}}}{x^{5}}dx$$

input
```
int((b*x^3+a)^(2/3)/x^5,x)
```

output
```
int((b*x^3+a)^(2/3)/x^5,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5}\,dx=\int \frac{(bx^3+a)^{\frac{2}{3}}}{x^5}\,dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)/x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^{2/3}}{x^5}\,dx=\frac{a^{\frac{2}{3}}\Gamma\left(-\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{4}{3},-\frac{2}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(2/3)/x**5,x)
```

output
```
a**(2/3)*gamma(-4/3)*hyper((-4/3, -2/3), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5} \, dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^5} \, dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^5, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5} \, dx = \int \frac{(bx^3+a)^{\frac{2}{3}}}{x^5} \, dx$$

input
```
integrate((b*x^3+a)^(2/3)/x^5,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{2/3}}{x^5}\,dx = \int \frac{(b\,x^3+a)^{2/3}}{x^5}\,dx$$

input
```
int((a + b*x^3)^(2/3)/x^5,x)
```

output
```
int((a + b*x^3)^(2/3)/x^5, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{2/3}}{x^5}\,dx = \frac{-(b\,x^3+a)^{\frac{2}{3}} - 2\left(\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{b\,x^8+a\,x^5}dx\right) a\,x^4}{2x^4}$$

input
```
int((b*x^3+a)^(2/3)/x^5,x)
```

output
```
( - (a + b*x**3)**(2/3) - 2*int((a + b*x**3)**(2/3)/(a*x**5 + b*x**8),x)*a
*x**4)/(2*x**4)
```

3.389 $\int x^3(a+bx^3)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int x^3\left(a+bx^3\right)^{4/3}\,dx=\frac{ax^4\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
1/4*a*x^4*(b*x^3+a)^(1/3)*hypergeom([-4/3, 4/3],[7/3],-b*x^3/a)/(1+b*x^3/a
)^(1/3)
```

Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^3\right)^{4/3}\,dx=\frac{ax^4\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^3*(a + b*x^3)^(4/3),x]
```

output
```
(a*x^4*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 4/3, 7/3, -((b*x^3)/a)])/
(4*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^3\right)^{4/3} dx$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt[3]{a+bx^3}\int x^3\left(\frac{bx^3}{a}+1\right)^{4/3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{ax^4\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[x^3*(a + b*x^3)^(4/3),x]
```

output

```
(a*x^4*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 4/3, 7/3, -((b*x^3)/a)])/
(4*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{3} \left(b\,x^{3}+a \right)^{\frac{4}{3}}dx$$

input
```
int(x^3*(b*x^3+a)^(4/3),x)
```

output
```
int(x^3*(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int x^3 \left(a+bx^3\right)^{4/3} \, dx = \int \left(bx^{3} + a\right)^{\frac{4}{3}} x^{3} \, dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^6 + a*x^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int x^3 (a+bx^3)^{4/3} \, dx = \frac{a^{\frac{4}{3}} x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3*(b*x**3+a)**(4/3),x)
```

output
```
a**(4/3)*x**4*gamma(4/3)*hyper((-4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3))
```

Maxima [F]

$$\int x^3 (a+bx^3)^{4/3} \, dx = \int (bx^3+a)^{\frac{4}{3}} x^3 \, dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^3, x)
```

Giac [F]

$$\int x^3 (a+bx^3)^{4/3} \, dx = \int (bx^3+a)^{\frac{4}{3}} x^3 \, dx$$

input
```
integrate(x^3*(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+bx^3\right)^{4/3} dx = \int x^3 \left(b x^3+a\right)^{4/3} dx$$

input
```
int(x^3*(a + b*x^3)^(4/3),x)
```

output
```
int(x^3*(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int x^3 (a+bx^3)^{4/3} dx = \frac{2(b x^3+a)^{\frac{1}{3}} a^2 x+9(b x^3+a)^{\frac{1}{3}} ab x^4+5(b x^3+a)^{\frac{1}{3}} b^2 x^7-2\left(\int \frac{1}{(b x^3+a)^{\frac{2}{3}}}dx\right) a^3}{40b}$$

input
```
int(x^3*(b*x^3+a)^(4/3),x)
```

output
```
(2*(a + b*x**3)**(1/3)*a**2*x + 9*(a + b*x**3)**(1/3)*a*b*x**4 + 5*(a + b*
x**3)**(1/3)*b**2*x**7 - 2*int((a + b*x**3)**(1/3)/(a + b*x**3),x)*a**3)/(
40*b)
```

3.390 $\int (a+bx^3)^{4/3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int (a+bx^3)^{4/3}\,dx=\frac{ax\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*x*(b*x^3+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^3/a)/(1+b*x^3/a)^(1/3
)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.17 (sec) , antiderivative size = 272, normalized size of antiderivative = 5.79

$$\int (a+bx^3)^{4/3}\,dx = \frac{3\left(\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}+x\right)(a+bx^3)^{4/3}\operatorname{AppellF1}\left(\frac{7}{3},-\frac{4}{3},-\frac{4}{3},\frac{10}{3},-\frac{\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}+x}{-\frac{\sqrt[3]{-1}\sqrt[3]{a}}{\sqrt[3]{b}}-\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}},-\frac{\frac{(-1)^{2/}}{}}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}-}\right.}{7\left(1+\frac{\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}+x}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}-\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}}\right)^{4/3}\left(1+\frac{\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}+x}{-\frac{\sqrt[3]{-1}\sqrt[3]{a}}{\sqrt[3]{b}}-\frac{(-1)^{2/3}\sqrt[3]{a}}{\sqrt[3]{b}}}\right)^{4/3}}$$

input
```
Integrate[(a + b*x^3)^(4/3),x]
```

output
```
(3*(((-1)^(2/3)*a^(1/3))/b^(1/3) + x)*(a + b*x^3)^(4/3)*AppellF1[7/3, -4/3
, -4/3, 10/3, -((((-1)^(2/3)*a^(1/3))/b^(1/3) + x)/(-(((-1)^(1/3)*a^(1/3))
/b^(1/3)) - ((-1)^(2/3)*a^(1/3))/b^(1/3))), -((((-1)^(2/3)*a^(1/3))/b^(1/3
) + x)/(a^(1/3)/b^(1/3) - ((-1)^(2/3)*a^(1/3))/b^(1/3)))])/(7*(1 + (((-1)^
(2/3)*a^(1/3))/b^(1/3) + x)/(a^(1/3)/b^(1/3) - ((-1)^(2/3)*a^(1/3))/b^(1/3
)))^(4/3)*(1 + (((-1)^(2/3)*a^(1/3))/b^(1/3) + x)/(-(((-1)^(1/3)*a^(1/3))/
b^(1/3)) - ((-1)^(2/3)*a^(1/3))/b^(1/3)))^(4/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^3)^{4/3}\,dx$$

$$\Big\downarrow \text{779}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{778}$$

$$\frac{ax\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3),x]
```

output
```
(a*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1
 + (b*x^3)/a)^(1/3)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{3}+a\right)^{\frac{4}{3}}d x$$

input
```
int((b*x^3+a)^(4/3),x)
```

output
```
int((b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \left(a+bx^3\right)^{4/3}\,dx=\int \left(bx^{3}+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(4/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \left(a+bx^3\right)^{4/3}\,dx=\frac{a^{\frac{4}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{4}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3),x)
```

output
```
a**(4/3)*x*gamma(1/3)*hyper((-4/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3))
```

Maxima [F]

$$\int \left(a+bx^3\right)^{4/3}\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \left(a+bx^3\right)^{4/3}\,dx=\int \left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \left(a+bx^3\right)^{4/3}\,dx=\frac{x\,\left(b\,x^3+a\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(\frac{b\,x^3}{a}+1\right)^{4/3}}$$

input
```
int((a + b*x^3)^(4/3),x)
```

output
```
(x*(a + b*x^3)^(4/3)*hypergeom([-4/3, 1/3], 4/3, -(b*x^3)/a))/((b*x^3)/a +
 1)^(4/3)
```

Reduce [F]

$$\int \left(a+bx^3\right)^{4/3}\,dx=\frac{3(b\,x^3+a)^{\frac{1}{3}}\,ax}{5}+\frac{(b\,x^3+a)^{\frac{1}{3}}\,b\,x^4}{5}+\frac{2\left(\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx\right)a^2}{5}$$

input
```
int((b*x^3+a)^(4/3),x)
```

output
```
(3*(a + b*x**3)**(1/3)*a*x + (a + b*x**3)**(1/3)*b*x**4 + 2*int((a + b*x**
3)**(1/3)/(a + b*x**3),x)*a**2)/5
```

3.391 $\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/2*a*(b*x^3+a)^(1/3)*hypergeom([-4/3, -2/3],[1/3],-b*x^3/a)/x^2/(1+b*x^3
/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx = -\frac{a\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(4/3)/x^3,x]
```

output
```
-1/2*(a*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, -2/3, 1/3, -((b*x^3)/a)]
)/(x^2*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^3}dx$$
$$\Big\downarrow \text{ 889}$$
$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{x^3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$
$$\Big\downarrow \text{ 888}$$
$$-\frac{a\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/x^3,x]
```

output
```
-1/2*(a*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, -2/3, 1/3, -((b*x^3)/a)]
)/(x^2*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^3}dx$$

input
```
int((b*x^3+a)^(4/3)/x^3,x)
```

output
```
int((b*x^3+a)^(4/3)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(4/3)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx = \frac{a^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{2}{3} \\ \frac{1}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3)/x**3,x)
```

output
```
a**(4/3)*gamma(-2/3)*hyper((-4/3, -2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{x^3}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^3}\,dx$$

input
```
int((a + b*x^3)^(4/3)/x^3,x)
```

output
```
int((a + b*x^3)^(4/3)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^3}\,dx=\frac{-3(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3-4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^6+a\,x^3}dx\right)a^2x^2}{2x^2}$$

input
```
int((b*x^3+a)^(4/3)/x^3,x)
```

output
```
( - 3*(a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3 - 4*int((a + b*x*
*3)**(1/3)/(a*x**3 + b*x**6),x)*a**2*x**2)/(2*x**2)
```

3.392 $\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx=-\frac{a\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{4}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
-1/5*a*(b*x^3+a)^(1/3)*hypergeom([-5/3, -4/3],[-2/3],-b*x^3/a)/x^5/(1+b*x^
3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx=-\frac{a\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{4}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(a + b*x^3)^(4/3)/x^6,x]
```

output
```
-1/5*(a*(a + b*x^3)^(1/3)*Hypergeometric2F1[-5/3, -4/3, -2/3, -((b*x^3)/a)
])/(x^5*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{a\sqrt[3]{a+bx^3}\int \frac{\left(\frac{bx^3}{a}+1\right)^{4/3}}{x^6}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},-\frac{4}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(a + b*x^3)^(4/3)/x^6,x]
```

output
```
-1/5*(a*(a + b*x^3)^(1/3)*Hypergeometric2F1[-5/3, -4/3, -2/3, -((b*x^3)/a)
])/(x^5*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{4}{3}}}{x^6}dx$$

input
```
int((b*x^3+a)^(4/3)/x^6,x)
```

output
```
int((b*x^3+a)^(4/3)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(4/3)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx = \frac{a^{\frac{4}{3}}\Gamma\left(-\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{4}{3}\\ -\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate((b*x**3+a)**(4/3)/x**6,x)
```

output
```
a**(4/3)*gamma(-5/3)*hyper((-5/3, -4/3), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx = \int \frac{(bx^3+a)^{\frac{4}{3}}}{x^6}\,dx$$

input
```
integrate((b*x^3+a)^(4/3)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx=\int \frac{(b\,x^3+a)^{4/3}}{x^6}\,dx$$

input
```
int((a + b*x^3)^(4/3)/x^6,x)
```

output
```
int((a + b*x^3)^(4/3)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^{4/3}}{x^6}\,dx=\frac{-(b\,x^3+a)^{\frac{1}{3}}\,a-5(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3-4\left(\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{b\,x^6+a\,x^3}dx\right)ab\,x^5}{5x^5}$$

input
```
int((b*x^3+a)^(4/3)/x^6,x)
```

output
```
( - (a + b*x**3)**(1/3)*a - 5*(a + b*x**3)**(1/3)*b*x**3 - 4*int((a + b*x*
*3)**(1/3)/(a*x**3 + b*x**6),x)*a*b*x**5)/(5*x**5)
```

3.393 $\int x^8(1-x^3)^{6/5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int x^8\left(1-x^3\right)^{6/5}\,dx=-\frac{5}{33}\left(1-x^3\right)^{11/5}+\frac{5}{24}\left(1-x^3\right)^{16/5}-\frac{5}{63}\left(1-x^3\right)^{21/5}$$

output
```
-5/33*(-x^3+1)^(11/5)+5/24*(-x^3+1)^(16/5)-5/63*(-x^3+1)^(21/5)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^8\left(1-x^3\right)^{6/5}\,dx=-\frac{5(1-x^3)^{11/5}\left(25+55x^3+88x^6\right)}{5544}$$

input
```
Integrate[x^8*(1 - x^3)^(6/5),x]
```

output
```
(-5*(1 - x^3)^(11/5)*(25 + 55*x^3 + 88*x^6))/5544
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(1-x^3\right)^{6/5} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int x^6 \left(1-x^3\right)^{6/5} dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3} \int \left(\left(1-x^3\right)^{16/5}-2 \left(1-x^3\right)^{11/5}+\left(1-x^3\right)^{6/5}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{5}{21} \left(1-x^3\right)^{21/5}+\frac{5}{8} \left(1-x^3\right)^{16/5}-\frac{5}{11} \left(1-x^3\right)^{11/5}\right)$$

input
```
Int[x^8*(1 - x^3)^(6/5),x]
```

output
```
((-5*(1 - x^3)^(11/5))/11 + (5*(1 - x^3)^(16/5))/8 - (5*(1 - x^3)^(21/5))/
21)/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

method	result	size
meijerg	$\frac{x^{9} \operatorname{hypergeom}\left(\left[-\frac{6}{5},3\right],\left[4\right],x^{3}\right)}{9}$	15
pseudoelliptic	$-\frac{5\left(-x^{3}+1\right)^{\frac{11}{5}} \left(88 x^{6}+55 x^{3}+25\right)}{5544}$	24
gosper	$\frac{5\left(-1+x \right) \left(x^{2}+x +1\right) \left(88 x^{6}+55 x^{3}+25\right) \left(-x^{3}+1\right)^{\frac{6}{5}}}{5544}$	33
trager	$\left(-\frac{5}{63} x^{12}+\frac{55}{504} x^{9}-\frac{5}{1848} x^{6}-\frac{25}{5544} x^{3}-\frac{125}{5544}\right) \left(-x^{3}+1\right)^{\frac{1}{5}}$	33
orering	$\frac{5\left(-1+x \right) \left(x^{2}+x +1\right) \left(88 x^{6}+55 x^{3}+25\right) \left(-x^{3}+1\right)^{\frac{6}{5}}}{5544}$	33
risch	$\frac{5 \left(88 x^{12}-121 x^{9}+3 x^{6}+5 x^{3}+25\right) \left(x^{3}-1\right)}{5544 \left(-x^{3}+1\right)^{\frac{4}{5}}}$	39

input
```
int(x^8*(-x^3+1)^(6/5),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*hypergeom([-6/5,3],[4],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int x^8 (1-x^3)^{6/5} \, dx = -\frac{5}{5544} \left(88 x^{12} - 121 x^9 + 3 x^6 + 5 x^3 + 25\right) \left(-x^3 + 1\right)^{\frac{1}{5}}$$

input
```
integrate(x^8*(-x^3+1)^(6/5),x, algorithm="fricas")
```

output
```
-5/5544*(88*x^12 - 121*x^9 + 3*x^6 + 5*x^3 + 25)*(-x^3 + 1)^(1/5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(34) = 68.

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int x^8 (1-x^3)^{6/5} \, dx = -\frac{5x^{12} \sqrt[5]{1-x^3}}{63} + \frac{55x^9 \sqrt[5]{1-x^3}}{504} - \frac{5x^6 \sqrt[5]{1-x^3}}{1848} - \frac{25x^3 \sqrt[5]{1-x^3}}{5544} - \frac{125 \sqrt[5]{1-x^3}}{5544}$$

input
```
integrate(x**8*(-x**3+1)**(6/5),x)
```

output
```
-5*x**12*(1 - x**3)**(1/5)/63 + 55*x**9*(1 - x**3)**(1/5)/504 - 5*x**6*(1
- x**3)**(1/5)/1848 - 25*x**3*(1 - x**3)**(1/5)/5544 - 125*(1 - x**3)**(1/
5)/5544
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^8 (1-x^3)^{6/5}\, dx = -\frac{5}{63} (-x^3+1)^{\frac{21}{5}} + \frac{5}{24} (-x^3+1)^{\frac{16}{5}} - \frac{5}{33} (-x^3+1)^{\frac{11}{5}}$$

input
```
integrate(x^8*(-x^3+1)^(6/5),x, algorithm="maxima")
```

output
```
-5/63*(-x^3 + 1)^(21/5) + 5/24*(-x^3 + 1)^(16/5) - 5/33*(-x^3 + 1)^(11/5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20

$$\int x^8 (1-x^3)^{6/5}\, dx = -\frac{5}{63} (x^3-1)^4 (-x^3+1)^{\frac{1}{5}} - \frac{5}{24} (x^3-1)^3 (-x^3+1)^{\frac{1}{5}} - \frac{5}{33} (x^3-1)^2 (-x^3+1)^{\frac{1}{5}}$$

input
```
integrate(x^8*(-x^3+1)^(6/5),x, algorithm="giac")
```

output
```
-5/63*(x^3 - 1)^4*(-x^3 + 1)^(1/5) - 5/24*(x^3 - 1)^3*(-x^3 + 1)^(1/5) - 5
/33*(x^3 - 1)^2*(-x^3 + 1)^(1/5)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int x^8 (1-x^3)^{6/5}\, dx = -(1-x^3)^{1/5} \left(\frac{5\,x^{12}}{63} - \frac{55\,x^9}{504} + \frac{5\,x^6}{1848} + \frac{25\,x^3}{5544} + \frac{125}{5544}\right)$$

input
```
int(x^8*(1 - x^3)^(6/5),x)
```

output
```
-(1 - x^3)^(1/5)*((25*x^3)/5544 + (5*x^6)/1848 - (55*x^9)/504 + (5*x^12)/6
3 + 125/5544)
```

Reduce [F]

$$\int x^8 (1-x^3)^{6/5} \, dx = -\left(\int (-x^3+1)^{\frac{1}{5}} x^{11} dx\right) + \int (-x^3+1)^{\frac{1}{5}} x^8 dx$$

input
```
int(x^8*(-x^3+1)^(6/5),x)
```

output
```
- int(( - x**3 + 1)**(1/5)*x**11,x) + int(( - x**3 + 1)**(1/5)*x**8,x)
```

3.394 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = -\frac{a^3(a+bx^3)^{2/3}}{2b^4}+\frac{3a^2(a+bx^3)^{5/3}}{5b^4}-\frac{3a(a+bx^3)^{8/3}}{8b^4}+\frac{(a+bx^3)^{11/3}}{11b^4}$$

output
```
-1/2*a^3*(b*x^3+a)^(2/3)/b^4+3/5*a^2*(b*x^3+a)^(5/3)/b^4-3/8*a*(b*x^3+a)^(
8/3)/b^4+1/11*(b*x^3+a)^(11/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = \frac{(a+bx^3)^{2/3}(-81a^3+54a^2bx^3-45ab^2x^6+40b^3x^9)}{440b^4}$$

input
```
Integrate[x^11/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-81*a^3 + 54*a^2*b*x^3 - 45*a*b^2*x^6 + 40*b^3*x^9))/(
440*b^4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^9}{\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^3\sqrt[3]{bx^3+a}}+\frac{3\left(bx^3+a\right)^{2/3}a^2}{b^3}-\frac{3\left(bx^3+a\right)^{5/3}a}{b^3}+\frac{\left(bx^3+a\right)^{8/3}}{b^3}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{3a^3\left(a+bx^3\right)^{2/3}}{2b^4}+\frac{9a^2\left(a+bx^3\right)^{5/3}}{5b^4}+\frac{3\left(a+bx^3\right)^{11/3}}{11b^4}-\frac{9a\left(a+bx^3\right)^{8/3}}{8b^4}\right)$$

input
```
Int[x^11/(a + b*x^3)^(1/3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(2/3))/(2*b^4) + (9*a^2*(a + b*x^3)^(5/3))/(5*b^4) -
(9*a*(a + b*x^3)^(8/3))/(8*b^4) + (3*(a + b*x^3)^(11/3))/(11*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(b\,x^{3}+a)^{\frac{2}{3}}\left(-40b^{3}x^{9}+45a\,b^{2}x^{6}-54a^{2}b\,x^{3}+81a^{3}\right)}{440b^{4}}$	47
trager	$-\frac{(b\,x^{3}+a)^{\frac{2}{3}}\left(-40b^{3}x^{9}+45a\,b^{2}x^{6}-54a^{2}b\,x^{3}+81a^{3}\right)}{440b^{4}}$	47
risch	$-\frac{(b\,x^{3}+a)^{\frac{2}{3}}\left(-40b^{3}x^{9}+45a\,b^{2}x^{6}-54a^{2}b\,x^{3}+81a^{3}\right)}{440b^{4}}$	47
pseudoelliptic	$-\frac{(b\,x^{3}+a)^{\frac{2}{3}}\left(-40b^{3}x^{9}+45a\,b^{2}x^{6}-54a^{2}b\,x^{3}+81a^{3}\right)}{440b^{4}}$	47
orering	$-\frac{(b\,x^{3}+a)^{\frac{2}{3}}\left(-40b^{3}x^{9}+45a\,b^{2}x^{6}-54a^{2}b\,x^{3}+81a^{3}\right)}{440b^{4}}$	47

input
```
int(x^11/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/440*(b*x^3+a)^(2/3)*(-40*b^3*x^9+45*a*b^2*x^6-54*a^2*b*x^3+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = \frac{(40\,b^3x^9 - 45\,ab^2x^6 + 54\,a^2bx^3 - 81\,a^3)(bx^3+a)^{\frac{2}{3}}}{440\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/440*(40*b^3*x^9 - 45*a*b^2*x^6 + 54*a^2*b*x^3 - 81*a^3)*(b*x^3 + a)^(2/3
)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = \begin{cases} -\frac{81a^3(a+bx^3)^{\frac{2}{3}}}{440b^4} + \frac{27a^2x^3(a+bx^3)^{\frac{2}{3}}}{220b^3} - \frac{9ax^6(a+bx^3)^{\frac{2}{3}}}{88b^2} + \frac{x^9(a+bx^3)^{\frac{2}{3}}}{11b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((-81*a**3*(a + b*x**3)**(2/3)/(440*b**4) + 27*a**2*x**3*(a + b*x
**3)**(2/3)/(220*b**3) - 9*a*x**6*(a + b*x**3)**(2/3)/(88*b**2) + x**9*(a
+ b*x**3)**(2/3)/(11*b), Ne(b, 0)), (x**12/(12*a**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{11}{3}}}{11\,b^4} - \frac{3\,(bx^3+a)^{\frac{8}{3}}a}{8\,b^4} + \frac{3\,(bx^3+a)^{\frac{5}{3}}a^2}{5\,b^4} - \frac{(bx^3+a)^{\frac{2}{3}}a^3}{2\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/11*(b*x^3 + a)^(11/3)/b^4 - 3/8*(b*x^3 + a)^(8/3)*a/b^4 + 3/5*(b*x^3 + a
)^(5/3)*a^2/b^4 - 1/2*(b*x^3 + a)^(2/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = -\frac{(bx^3+a)^{\frac{2}{3}}a^3}{2\,b^4} + \frac{40\,(bx^3+a)^{\frac{11}{3}} - 165\,(bx^3+a)^{\frac{8}{3}}a + 264\,(bx^3+a)^{\frac{5}{3}}a^2}{440\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
-1/2*(b*x^3 + a)^(2/3)*a^3/b^4 + 1/440*(40*(b*x^3 + a)^(11/3) - 165*(b*x^3
 + a)^(8/3)*a + 264*(b*x^3 + a)^(5/3)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = -\left(b\,x^3+a\right)^{2/3}\left(\frac{81\,a^3}{440\,b^4} - \frac{x^9}{11\,b} + \frac{9\,a\,x^6}{88\,b^2} - \frac{27\,a^2\,x^3}{220\,b^3}\right)$$

input
```
int(x^11/(a + b*x^3)^(1/3),x)
```

output
```
-(a + b*x^3)^(2/3)*((81*a^3)/(440*b^4) - x^9/(11*b) + (9*a*x^6)/(88*b^2) -
 (27*a^2*x^3)/(220*b^3))
```

Reduce [F]

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^{11}}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^11/(b*x^3+a)^(1/3),x)
```

output
```
int(x**11/(a + b*x**3)**(1/3),x)
```

3.395 $\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx=\frac{a^2(a+bx^3)^{2/3}}{2b^3}-\frac{2a(a+bx^3)^{5/3}}{5b^3}+\frac{(a+bx^3)^{8/3}}{8b^3}$$

output
```
1/2*a^2*(b*x^3+a)^(2/3)/b^3-2/5*a*(b*x^3+a)^(5/3)/b^3+1/8*(b*x^3+a)^(8/3)/
b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx=\frac{(a+bx^3)^{2/3}(9a^2-6abx^3+5b^2x^6)}{40b^3}$$

input
```
Integrate[x^8/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(9*a^2 - 6*a*b*x^3 + 5*b^2*x^6))/(40*b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^6}{\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{a^2}{b^2\sqrt[3]{bx^3+a}}-\frac{2\left(bx^3+a\right)^{2/3}a}{b^2}+\frac{\left(bx^3+a\right)^{5/3}}{b^2}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3a^2\left(a+bx^3\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^3\right)^{8/3}}{8b^3}-\frac{6a\left(a+bx^3\right)^{5/3}}{5b^3}\right)$$

input
```
Int[x^8/(a + b*x^3)^(1/3),x]
```

output
```
((3*a^2*(a + b*x^3)^(2/3))/(2*b^3) - (6*a*(a + b*x^3)^(5/3))/(5*b^3) + (3*
(a + b*x^3)^(8/3))/(8*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{2}{3}}(5b^2x^6-6ab\,x^3+9a^2)}{40b^3}$	36
trager	$\frac{(b\,x^3+a)^{\frac{2}{3}}(5b^2x^6-6ab\,x^3+9a^2)}{40b^3}$	36
risch	$\frac{(b\,x^3+a)^{\frac{2}{3}}(5b^2x^6-6ab\,x^3+9a^2)}{40b^3}$	36
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{2}{3}}(5b^2x^6-6ab\,x^3+9a^2)}{40b^3}$	36
orering	$\frac{(b\,x^3+a)^{\frac{2}{3}}(5b^2x^6-6ab\,x^3+9a^2)}{40b^3}$	36

input
```
int(x^8/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/40*(b*x^3+a)^(2/3)*(5*b^2*x^6-6*a*b*x^3+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx = \frac{(5\,b^2x^6 - 6\,abx^3 + 9\,a^2)(bx^3+a)^{\frac{2}{3}}}{40\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/40*(5*b^2*x^6 - 6*a*b*x^3 + 9*a^2)*(b*x^3 + a)^(2/3)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx = \begin{cases} \frac{9a^2(a+bx^3)^{\frac{2}{3}}}{40b^3} - \frac{3ax^3(a+bx^3)^{\frac{2}{3}}}{20b^2} + \frac{x^6(a+bx^3)^{\frac{2}{3}}}{8b} & \text{for } b \neq 0 \\ \frac{x^9}{9\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((9*a**2*(a + b*x**3)**(2/3)/(40*b**3) - 3*a*x**3*(a + b*x**3)**(
2/3)/(20*b**2) + x**6*(a + b*x**3)**(2/3)/(8*b), Ne(b, 0)), (x**9/(9*a**(1
/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{8}{3}}}{8\,b^3} - \frac{2\,(bx^3+a)^{\frac{5}{3}}a}{5\,b^3} + \frac{(bx^3+a)^{\frac{2}{3}}a^2}{2\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/8*(b*x^3 + a)^(8/3)/b^3 - 2/5*(b*x^3 + a)^(5/3)*a/b^3 + 1/2*(b*x^3 + a)^
(2/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{2}{3}}a^2}{2\,b^3} + \frac{5\,(bx^3+a)^{\frac{8}{3}} - 16\,(bx^3+a)^{\frac{5}{3}}a}{40\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/2*(b*x^3 + a)^(2/3)*a^2/b^3 + 1/40*(5*(b*x^3 + a)^(8/3) - 16*(b*x^3 + a)
^(5/3)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx = \left(b\,x^3+a\right)^{2/3}\left(\frac{9\,a^2}{40\,b^3} + \frac{x^6}{8\,b} - \frac{3\,a\,x^3}{20\,b^2}\right)$$

input
```
int(x^8/(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(2/3)*((9*a^2)/(40*b^3) + x^6/(8*b) - (3*a*x^3)/(20*b^2))
```

Reduce [F]

$$\int \frac{x^8}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx$$

input
```
int(x^8/(b*x^3+a)^(1/3),x)
```

output
```
int(x**8/(a + b*x**3)**(1/3),x)
```

3.396 $\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx=-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2}$$

output
```
-1/2*a*(b*x^3+a)^(2/3)/b^2+1/5*(b*x^3+a)^(5/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx=\frac{\left(a+bx^3\right)^{2/3}\left(-3a+2bx^3\right)}{10b^2}$$

input
```
Integrate[x^5/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-3*a + 2*b*x^3))/(10*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{x^3}{\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \ 53$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{2/3}}{b}-\frac{a}{b\sqrt[3]{bx^3+a}}\right)dx^3$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}\left(\frac{3\left(a+bx^3\right)^{5/3}}{5b^2}-\frac{3a\left(a+bx^3\right)^{2/3}}{2b^2}\right)$$

input
```
Int[x^5/(a + b*x^3)^(1/3),x]
```

output
```
((-3*a*(a + b*x^3)^(2/3))/(2*b^2) + (3*(a + b*x^3)^(5/3))/(5*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-2b\,x^3+3a)}{10b^2}$	25
trager	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-2b\,x^3+3a)}{10b^2}$	25
risch	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-2b\,x^3+3a)}{10b^2}$	25
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-2b\,x^3+3a)}{10b^2}$	25
orering	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-2b\,x^3+3a)}{10b^2}$	25

input
```
int(x^5/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(2/3)*(-2*b*x^3+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx = \frac{(2\,bx^3-3\,a)(bx^3+a)^{\frac{2}{3}}}{10\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/10*(2*b*x^3 - 3*a)*(b*x^3 + a)^(2/3)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx = \begin{cases} -\frac{3a(a+bx^3)^{\frac{2}{3}}}{10b^2} + \frac{x^3(a+bx^3)^{\frac{2}{3}}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise((-3*a*(a + b*x**3)**(2/3)/(10*b**2) + x**3*(a + b*x**3)**(2/3)/(
5*b), Ne(b, 0)), (x**6/(6*a**(1/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{5}{3}}}{5\,b^2} - \frac{(bx^3+a)^{\frac{2}{3}}a}{2\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b^2 - 1/2*(b*x^3 + a)^(2/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{5}{3}}}{5\,b^2} - \frac{(bx^3+a)^{\frac{2}{3}}a}{2\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/5*(b*x^3 + a)^(5/3)/b^2 - 1/2*(b*x^3 + a)^(2/3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx = -\frac{(b\,x^3+a)^{2/3}\,(3\,a-2\,b\,x^3)}{10\,b^2}$$

input
```
int(x^5/(a + b*x^3)^(1/3),x)
```

output
```
-((a + b*x^3)^(2/3)*(3*a - 2*b*x^3))/(10*b^2)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^5}{(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

input
```
int(x^5/(b*x^3+a)^(1/3),x)
```

output
```
int(x**5/(a + b*x**3)**(1/3),x)
```

3.397 $\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx=\frac{\left(a+bx^3\right)^{2/3}}{2b}$$

output
```
1/2*(b*x^3+a)^(2/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx=\frac{\left(a+bx^3\right)^{2/3}}{2b}$$

input
```
Integrate[x^2/(a + b*x^3)^(1/3),x]
```

output
```
(a + b*x^3)^(2/3)/(2*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx$$
$$\Big\downarrow \; 793$$
$$\frac{\left(a+bx^3\right)^{2/3}}{2b}$$

input
```
Int[x^2/(a + b*x^3)^(1/3),x]
```

output
```
(a + b*x^3)^(2/3)/(2*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15
derivativedivides	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15
default	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15
trager	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15
risch	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15
orering	$\frac{(b\,x^3+a)^{\frac{2}{3}}}{2b}$	15

input
```
int(x^2/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^3+a)^(2/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/2*(b*x^3 + a)^(2/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \begin{cases} \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{for } b \neq 0 \\ \frac{x^3}{3\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)**(1/3),x)
```

output
```
Piecewise(((a + b*x**3)**(2/3)/(2*b), Ne(b, 0)), (x**3/(3*a**(1/3)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/2*(b*x^3 + a)^(2/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{(bx^3+a)^{\frac{2}{3}}}{2\,b}$$

input
```
integrate(x^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/2*(b*x^3 + a)^(2/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{(b\,x^3+a)^{2/3}}{2\,b}$$

input
```
int(x^2/(a + b*x^3)^(1/3),x)
```

output
```
(a + b*x^3)^(2/3)/(2*b)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^2}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^2/(b*x^3+a)^(1/3),x)
```

output
```
int(x**2/(a + b*x**3)**(1/3),x)
```

3.398 $\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx=\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}$$

output
```
1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(1/3
)-1/2*ln(x)/a^(1/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{6\sqrt[3]{a}}$$

input
```
Integrate[1/(x*(a + b*x^3)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[-a^
(1/3) + (a + b*x^3)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a
+ b*x^3)^(2/3)])/(6*a^(1/3))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^3\right)}{2\sqrt[3]{a}}\right)$$

input
```
Int[1/(x*(a + b*x^3)^(1/3)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - L
og[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(1/3)))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}$	83

input
```
int(1/x/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+2*l
n((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(2
/3)))/a^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.84

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\,\sqrt{\frac{1}{3}}\left(2\,(bx^3+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^3+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^3+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)-a^{\frac{2}{3}}\log\left((bx^3+a)^{\frac{2}{3}}\right.}{6\,a}\right.$$

input
```
integrate(1/x/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3
+ a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(
b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*
x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1
/3)))/a, 1/6*(6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) +
a^(1/3))/a^(1/3)) - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1
/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.45

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx=-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(b*x**3+a)**(1/3),x)
```

output
```
-gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(1
/3)*x*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}}-\frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{1}{3}}}+\frac{\log\left(\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(1/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(1/3) + 1/3*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}} + \frac{\log\left(\left|(bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(1/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(1/3) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx = \frac{\ln\left((b\,x^3+a)^{1/3}-a^{1/3}\right)}{3\,a^{1/3}} + \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{a^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}} - \frac{\ln\left((b\,x^3+a)^{1/3}-\frac{a^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{6\,a^{1/3}}$$

input
```
int(1/(x*(a + b*x^3)^(1/3)),x)
```

output
```
log((a + b*x^3)^(1/3) - a^(1/3))/(3*a^(1/3)) + (log((a + b*x^3)^(1/3) - (a
^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(6*a^(1/3)) - (log((a + b*
x^3)^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(6*a^(1/3))
```

Reduce [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x}dx$$

input
```
int(1/x/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x),x)
```

3.399 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}} \, dx = -\frac{(a+bx^3)^{2/3}}{3ax^3} - \frac{b \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}}$$

output
```
-1/3*(b*x^3+a)^(2/3)/a/x^3-1/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^
(1/2)/a^(1/3))*3^(1/2)/a^(4/3)+1/6*b*ln(x)/a^(4/3)-1/6*b*ln(a^(1/3)-(b*x^3
+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}} \, dx = -\frac{6\sqrt[3]{a}(a+bx^3)^{2/3} + 2\sqrt{3}bx^3 \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2bx^3\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) - bx^3\log\left(a^{2/3} +\right.}{18a^{4/3}x^3}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(1/3)),x]
```

output
```
-1/18*(6*a^(1/3)*(a + b*x^3)^(2/3) + 2*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b
*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 2*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1/3)]
 - b*x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(a^
(4/3)*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\int \frac{1}{x^6 \sqrt[3]{bx^3+a}}dx^3$$

$$\Big\downarrow \ 52$$

$$\frac{1}{3}\left(-\frac{b\int \frac{1}{x^3 \sqrt[3]{bx^3+a}}dx^3}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

$$\Big\downarrow \ 67$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

$$\Big\downarrow \ 16$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{b\left(-\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^3\right)^{2/3}}{ax^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^(1/3)),x]
```

output
```
(-((a + b*x^3)^(2/3)/(a*x^3)) - (b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1
/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^3]/(2*a^(1/3)) + (3*Log[a^(1/3) -
(a + b*x^3)^(1/3)])/(2*a^(1/3))))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x^{3}-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b\,x^{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)b\,x^{3}-6(b\,x}{18a^{\frac{4}{3}}x^{3}}$

input
```
int(1/x^4/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*b
*x^3-2*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3
+a)^(1/3)+a^(2/3))*b*x^3-6*(b*x^3+a)^(2/3)*a^(1/3))/a^(4/3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.13

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx^3\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^3-3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^3}\right)+(-a)^{\frac{2}{3}}}{1}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}abx^3\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}\,bx^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-(\right.}{18\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*a*b*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*sqrt(1/3)*(
2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt(
(-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (-a)^(2/3)*b*
x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2
*(-a)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3
)*a)/(a^2*x^3), -1/18*(6*sqrt(1/3)*a*b*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x^3 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)
*b*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
+ 2*(-a)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^3 + a)^(
2/3)*a)/(a^2*x^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3\sqrt[3]{b}x^4\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(1/x**4/(b*x**3+a)**(1/3),x)
```

output
```
-gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(1
/3)*x**4*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{4}{3}}} - \frac{(bx^3+a)^{\frac{2}{3}}b}{3\,((bx^3+a)a-a^2)} + \frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{4}{3}}} - \frac{b\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{4}{3}}}$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(4/3) - 1/3*(b*x^3 + a)^(2/3)*b/((b*x^3 + a)*a - a^2) + 1/18*b*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/9*b*log((
b*x^3 + a)^(1/3) - a^(1/3))/a^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{18}\,b\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \frac{2\,\log\left(\left|(bx^3+a\right.\right.}{a^{\frac{4}{3}}}\right.$$

input
```
integrate(1/x^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
-1/18*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(4/3) - log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(4/3) + 2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x^3 + a
)^(2/3)/(a*b*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx = -\frac{b\ln\left(\left(b\,x^3+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{4/3}} - \frac{\left(b\,x^3+a\right)^{2/3}}{3\,a\,x^3} + \frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a^2}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{18\,a^{4/3}} + \frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)^2}{36\,a^{5/3}}-\frac{b^2\,\left(b\,x^3+a\right)^{1/3}}{9\,a^2}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{18\,a^{4/3}}$$

input
```
int(1/(x^4*(a + b*x^3)^(1/3)),x)
```

output
```
(log((b - 3^(1/2)*b*1i)^2/(36*a^(5/3)) - (b^2*(a + b*x^3)^(1/3))/(9*a^2))*
(b - 3^(1/2)*b*1i))/(18*a^(4/3)) - (a + b*x^3)^(2/3)/(3*a*x^3) - (b*log((a
 + b*x^3)^(1/3) - a^(1/3)))/(9*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(36*a^
(5/3)) - (b^2*(a + b*x^3)^(1/3))/(9*a^2))*(b + 3^(1/2)*b*1i))/(18*a^(4/3))
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}x^4}dx$$

input
```
int(1/x^4/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**4),x)
```

3.400 $\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx = -\frac{2ax(a+bx^3)^{2/3}}{9b^2}+\frac{x^4(a+bx^3)^{2/3}}{6b}+\frac{2a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}}-\frac{a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{9b^{7/3}}$$

output

```
-2/9*a*x*(b*x^3+a)^(2/3)/b^2+1/6*x^4*(b*x^3+a)^(2/3)/b+2/27*a^2*arctan(1/3
*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(7/3)-1/9*a^2*ln(-b^(1
/3)*x+(b*x^3+a)^(1/3))/b^(7/3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.40

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx = \frac{(a+bx^3)^{2/3}(-4ax+3bx^4)}{18b^2} + \frac{2a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}b^{7/3}} - \frac{2a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27b^{7/3}} + \frac{a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{27b^{7/3}}$$

input
```
Integrate[x^6/(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(2/3)*(-4*a*x + 3*b*x^4))/(18*b^2) + (2*a^2*ArcTan[(Sqrt[3]*b
^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(7/3)) - (2*a^2
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(7/3)) + (a^2*Log[b^(2/3)*x^
2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(27*b^(7/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {843, 843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^4(a+bx^3)^{2/3}}{6b} - \frac{2a\int \frac{x^3}{\sqrt[3]{bx^3+a}}dx}{3b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6b}-\frac{2a\left(\frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\int\frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}\right)}{3b}$$

$\big\downarrow$ 769

$$\frac{x^4\left(a+bx^3\right)^{2/3}}{6b}-\frac{2a\left(\frac{x\left(a+bx^3\right)^{2/3}}{3b}-\frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}\right)}{3b}$$

input
```
Int[x^6/(a + b*x^3)^(1/3),x]
```

output
```
(x^4*(a + b*x^3)^(2/3))/(6*b) - (2*a*((x*(a + b*x^3)^(2/3))/(3*b) - (a*(Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))))/(3*b)))/(3*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$\frac{9\left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{4}{3}} x^{4}-12 a x \left(b\,x^{3}+a\right)^{\frac{2}{3}} b^{\frac{1}{3}}-4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a^{2}-4\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a^{2}+2\ln\left(b^{\frac{2}{3}}\ldots\right.}{54 b^{\frac{7}{3}}}$

input
```
int(x^6/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/54*(9*(b*x^3+a)^(2/3)*b^(4/3)*x^4-12*a*x*(b*x^3+a)^(2/3)*b^(1/3)-4*3^(1/
2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a^2-4*ln((-
b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3
)*x+(b*x^3+a)^(2/3))/x^2)*a^2)/b^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.32

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{6\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}\right.\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+4\,a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-2}{54\,b^3}$$

input
```
integrate(x^6/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/54*(6*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x
^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*a^2*(
-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*a^2*(-b)^(2/3)*log
(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b^3, -1/54*(12*sqrt(1/3)*a^
2*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1
/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3
+ a)^(1/3))/x) - 2*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*
(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 +
a)^(2/3))/b^3]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.31

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(b*x**3+a)**(1/3),x)
```

output
```
x**7*gamma(7/3)*hyper((1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(10/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.53

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx = -\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{7}{3}}} + \frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{27\,b^{\frac{7}{3}}} - \frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{7}{3}}} + \frac{\frac{7\,(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}-\frac{4\,(bx^3+a)^{\frac{5}{3}}a^2}{x^5}}{18\left(b^4-\frac{2\,(bx^3+a)b^3}{x^3}+\frac{(bx^3+a)^2b^2}{x^6}\right)}$$

input
```
integrate(x^6/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-2/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(
1/3))/b^(7/3) + 1/27*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^
3 + a)^(2/3)/x^2)/b^(7/3) - 2/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b
^(7/3) + 1/18*(7*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5
)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3 + a)^2*b^2/x^6)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^6}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int(x^6/(a + b*x^3)^(1/3),x)
```

output
```
int(x^6/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{x^6}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx$$

input
```
int(x^6/(b*x^3+a)^(1/3),x)
```

output
```
int(x**6/(a + b*x**3)**(1/3),x)
```

3.401 $\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{x\left(a+bx^3\right)^{2/3}}{3b} - \frac{a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
1/3*x*(b*x^3+a)^(2/3)/b-1/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3
^(1/2))*3^(1/2)/b^(4/3)+1/6*a*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.49

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{6\sqrt[3]{b}x\left(a+bx^3\right)^{2/3} - 2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) + 2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right) - a\log\left(b^{2/3}x^2 + \cdots\right.}{18b^{4/3}}$$

input
```
Integrate[x^3/(a + b*x^3)^(1/3),x]
```

output
```
(6*b^(1/3)*x*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b
^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3
)] - a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])
/(18*b^(4/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {843, 769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x(a+bx^3)^{2/3}}{3b} - \frac{a\int \frac{1}{\sqrt[3]{bx^3+a}}dx}{3b}$$

$$\Big\downarrow \text{769}$$

$$\frac{x(a+bx^3)^{2/3}}{3b} - \frac{a\left(\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}\right)}{3b}$$

input
```
Int[x^3/(a + b*x^3)^(1/3),x]
```

output
```
(x*(a + b*x^3)^(2/3))/(3*b) - (a*(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2
*b^(1/3))))/(3*b)
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a+3\left(b\,x^{3}+a\right)^{\frac{2}{3}}x\,b^{\frac{1}{3}}+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)a-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right.}{x^{2}}\right.}{2}}{9b^{\frac{4}{3}}}$

input
```
int(x^3/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a
+3*(b*x^3+a)^(2/3)*x*b^(1/3)+ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-1/2*ln((
b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.29

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3 - 3\,(bx^3+a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3+a)^{\frac{1}{3}}bx^2 - 2\,(bx^3+a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\right.}{18\,b^2}\right.$$

input
```
integrate(x^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/18*(3*sqrt(1/3)*a*b*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^
3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 6*(b*x^3 + a)^(2/3)*b*x
+ 2*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - a*b^(2/3)*log((b^(
2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2, 1/1
8*(6*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3)
)/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x + 2*a*b^(2/3)*log(-(b^(1/3)*x - (
b*x^3 + a)^(1/3))/x) - a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1
/3)*x + (b*x^3 + a)^(2/3))/x^2))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.39

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(b*x**3+a)**(1/3),x)
```

output
```
x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(1/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.46

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{18\,b^{\frac{4}{3}}} + \frac{a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{9\,b^{\frac{4}{3}}} - \frac{\left(bx^3+a\right)^{\frac{2}{3}}a}{3\left(b^2-\frac{\left(bx^3+a\right)b}{x^3}\right)x^2}$$

input
```
integrate(x^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3)
)/b^(4/3) - 1/18*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(4/3) + 1/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) -
1/3*(b*x^3 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2)
```

Giac [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^3}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^3/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^3}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

input
```
int(x^3/(a + b*x^3)^(1/3),x)
```

output
```
int(x^3/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx$$

input
```
int(x^3/(b*x^3+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**3)**(1/3),x)
```

3.402 $\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 70

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx=\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(1/3)-1/
2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx=\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}}$$

input
```
Integrate[(a + b*x^3)^(-1/3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1
 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3
) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {769}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{769}$$

$$\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}$$

input
```
Int[(a + b*x^3)^(-1/3),x]
```

output
```
ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) -
Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.41

method	result	size
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right)+\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}}{3 b^{\frac{1}{3}}}$	99

input
```
int(1/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)+
ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(
1/3)*x+(b*x^3+a)^(2/3))/x^2))/b^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(54) = 108$.

Time = 0.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.71

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\,(bx^3+a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-(bx^3+a)^{\frac{1}{3}}bx^2+2\,(bx^3+a)^{\frac{2}{3}}(-\right.\right.}{6}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+2\,(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+a)^{\frac{1}{3}}}{x}\right)-(-b)^{\frac{2}{3}}}{6\,b}$$

input
```
integrate(1/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(
-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(-b)^(2/3)
*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^
2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b, -1/6*(6*s
qrt(1/3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3
+ a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x
^3 + a)^(1/3))/x) - (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b
)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(b*x**3+a)**(1/3),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{1}{3}}} + \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6\,b^{\frac{1}{3}}} - \frac{\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}$$

input
```
integrate(1/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))
/b^(1/3) + 1/6*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/
3)/x^2)/b^(1/3) - 1/3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx = \frac{x\left(\frac{b\,x^3}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{\left(b\,x^3+a\right)^{1/3}}$$

input
```
int(1/(a + b*x^3)^(1/3),x)
```

output
```
(x*((b*x^3)/a + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -(b*x^3)/a))/(a + b*x^
3)^(1/3)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(a + b*x**3)**(1/3),x)
```

3.403 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}} \, dx = -\frac{(a+bx^3)^{2/3}}{2ax^2}$$

output
```
-1/2*(b*x^3+a)^(2/3)/a/x^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}} \, dx = -\frac{(a+bx^3)^{2/3}}{2ax^2}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(1/3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(a*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{(a+bx^3)^{2/3}}{2ax^2}$$

input
```
Int[1/(x^3*(a + b*x^3)^(1/3)),x]
```

output
```
-1/2*(a + b*x^3)^(2/3)/(a*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2 a\,x^{2}}$	18
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2 a\,x^{2}}$	18
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2 a\,x^{2}}$	18
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2 a\,x^{2}}$	18
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{2 a\,x^{2}}$	18

input
```
int(1/x^3/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^3+a)^(2/3)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}}\,dx = -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(b*x^3 + a)^(2/3)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}}\,dx = \frac{b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3}\right)}{3a\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**3+a)**(1/3),x)
```

output
```
b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-2/3)/(3*a*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}}\,dx = -\frac{(bx^3+a)^{\frac{2}{3}}}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/2*(b*x^3 + a)^(2/3)/(a*x^2)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}}\,dx = -\frac{\left(b\,x^3+a\right)^{2/3}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + b*x^3)^(1/3)),x)
```

output
```
-(a + b*x^3)^(2/3)/(2*a*x^2)
```

Reduce [F]

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^3}dx$$

input
```
int(1/x^3/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**3),x)
```

3.404 $\int \frac{1}{x^6 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}} \, dx = -\frac{(a+bx^3)^{2/3}}{5ax^5} + \frac{3b(a+bx^3)^{2/3}}{10a^2x^2}$$

output
```
-1/5*(b*x^3+a)^(2/3)/a/x^5+3/10*b*(b*x^3+a)^(2/3)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}} \, dx = \frac{(a+bx^3)^{2/3} (-2a+3bx^3)}{10a^2x^5}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(1/3)),x]
```

output
```
((a + b*x^3)^(2/3)*(-2*a + 3*b*x^3))/(10*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{3b \int \frac{1}{x^3 \sqrt[3]{bx^3+a}}dx}{5a} - \frac{\left(a+bx^3\right)^{2/3}}{5ax^5}$$

$$\Big\downarrow \text{796}$$

$$\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2} - \frac{\left(a+bx^3\right)^{2/3}}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^3)^(1/3)),x]
```

output
```
-1/5*(a + b*x^3)^(2/3)/(a*x^5) + (3*b*(a + b*x^3)^(2/3))/(10*a^2*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-3b\,x^3+2a)}{10a^2x^5}$	28
trager	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-3b\,x^3+2a)}{10a^2x^5}$	28
risch	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-3b\,x^3+2a)}{10a^2x^5}$	28
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-3b\,x^3+2a)}{10a^2x^5}$	28
orering	$-\frac{(b\,x^3+a)^{\frac{2}{3}}(-3b\,x^3+2a)}{10a^2x^5}$	28

input
```
int(1/x^6/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(b*x^3+a)^(2/3)*(-3*b*x^3+2*a)/a^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx = \frac{(3\,bx^3-2\,a)(bx^3+a)^{\frac{2}{3}}}{10\,a^2x^5}$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/10*(3*b*x^3 - 2*a)*(b*x^3 + a)^(2/3)/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx = -\frac{2b^{\frac{2}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{9ax^3\Gamma\left(\frac{1}{3}\right)} + \frac{b^{\frac{5}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{5}{3}\right)}{3a^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**6/(b*x**3+a)**(1/3),x)
```

output
```
-2*b**(2/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(9*a*x**3*gamma(1/3)) + b*
*(5/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-5/3)/(3*a**2*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx = \frac{\frac{5\,(bx^3+a)^{\frac{2}{3}}b}{x^2} - \frac{2\,(bx^3+a)^{\frac{5}{3}}}{x^5}}{10\,a^2}$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/10*(5*(b*x^3 + a)^(2/3)*b/x^2 - 2*(b*x^3 + a)^(5/3)/x^5)/a^2
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx = -\frac{\left(b\,x^3+a\right)^{2/3}\,\left(2\,a-3\,b\,x^3\right)}{10\,a^2\,x^5}$$

input
```
int(1/(x^6*(a + b*x^3)^(1/3)),x)
```

output
```
-((a + b*x^3)^(2/3)*(2*a - 3*b*x^3))/(10*a^2*x^5)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^6}dx$$

input
```
int(1/x^6/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**6),x)
```

3.405 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}} dx = -\frac{(a+bx^3)^{2/3}}{8ax^8} + \frac{3b(a+bx^3)^{2/3}}{20a^2x^5} - \frac{9b^2(a+bx^3)^{2/3}}{40a^3x^2}$$

output
```
-1/8*(b*x^3+a)^(2/3)/a/x^8+3/20*b*(b*x^3+a)^(2/3)/a^2/x^5-9/40*b^2*(b*x^3+
a)^(2/3)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}} dx = \frac{(a+bx^3)^{2/3}(-5a^2+6abx^3-9b^2x^6)}{40a^3x^8}$$

input
```
Integrate[1/(x^9*(a + b*x^3)^(1/3)),x]
```

output
```
((a + b*x^3)^(2/3)*(-5*a^2 + 6*a*b*x^3 - 9*b^2*x^6))/(40*a^3*x^8)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b\int \frac{1}{x^6\sqrt[3]{bx^3+a}}dx}{4a}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b\left(-\frac{3b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx}{5a}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{3b\left(\frac{3b\left(a+bx^3\right)^{2/3}}{10a^2x^2}-\frac{\left(a+bx^3\right)^{2/3}}{5ax^5}\right)}{4a}-\frac{\left(a+bx^3\right)^{2/3}}{8ax^8}$$

input
```
Int[1/(x^9*(a + b*x^3)^(1/3)),x]
```

output
```
-1/8*(a + b*x^3)^(2/3)/(a*x^8) - (3*b*(-1/5*(a + b*x^3)^(2/3)/(a*x^5) + (3
*b*(a + b*x^3)^(2/3))/(10*a^2*x^2)))/(4*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 b^{2} x^{6}-6 a b\,x^{3}+5 a^{2}\right)}{40 a^{3} x^{8}}$	39
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 b^{2} x^{6}-6 a b\,x^{3}+5 a^{2}\right)}{40 a^{3} x^{8}}$	39
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 b^{2} x^{6}-6 a b\,x^{3}+5 a^{2}\right)}{40 a^{3} x^{8}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 b^{2} x^{6}-6 a b\,x^{3}+5 a^{2}\right)}{40 a^{3} x^{8}}$	39
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(9 b^{2} x^{6}-6 a b\,x^{3}+5 a^{2}\right)}{40 a^{3} x^{8}}$	39

input
```
int(1/x^9/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(b*x^3+a)^(2/3)*(9*b^2*x^6-6*a*b*x^3+5*a^2)/a^3/x^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}}\,dx = -\frac{(9\,b^2x^6 - 6\,abx^3 + 5\,a^2)(bx^3+a)^{\frac{2}{3}}}{40\,a^3x^8}$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
-1/40*(9*b^2*x^6 - 6*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3)/(a^3*x^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(61) = 122.

Time = 0.77 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.97

$$\begin{aligned}\int \frac{1}{x^9 \sqrt[3]{a+bx^3}}\,dx &= \frac{10a^4b^{\frac{14}{3}}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{8a^3b^{\frac{17}{3}}x^3\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{4a^2b^{\frac{20}{3}}x^6\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{24ab^{\frac{23}{3}}x^9\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)} \\ &+ \frac{18b^{\frac{26}{3}}x^{12}\left(\frac{a}{bx^3}+1\right)^{\frac{2}{3}}\Gamma\left(-\frac{8}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{1}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{1}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{1}{3}\right)}\end{aligned}$$

input
```
integrate(1/x**9/(b*x**3+a)**(1/3),x)
```

output
```
10*a**4*b**(14/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b**4*x**6*g
amma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamma(1/3))
+ 8*a**3*b**(17/3)*x**3*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5*b**4*
x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*gamma(
1/3)) + 4*a**2*b**(20/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*a**5
*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x**12*
gamma(1/3)) + 24*a*b**(23/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(27*
a**5*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6*x*
*12*gamma(1/3)) + 18*b**(26/3)*x**12*(a/(b*x**3) + 1)**(2/3)*gamma(-8/3)/(
27*a**5*b**4*x**6*gamma(1/3) + 54*a**4*b**5*x**9*gamma(1/3) + 27*a**3*b**6
*x**12*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}}\,dx = -\frac{\frac{20\,(bx^3+a)^{\frac{2}{3}}b^2}{x^2} - \frac{16\,(bx^3+a)^{\frac{5}{3}}b}{x^5} + \frac{5\,(bx^3+a)^{\frac{8}{3}}}{x^8}}{40\,a^3}$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/40*(20*(b*x^3 + a)^(2/3)*b^2/x^2 - 16*(b*x^3 + a)^(5/3)*b/x^5 + 5*(b*x^
3 + a)^(8/3)/x^8)/a^3
```

Giac [F]

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^9}\,dx$$

input
```
integrate(1/x^9/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^9), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}} \, dx = -\frac{(b\,x^3+a)^{2/3}\,(5\,a^2-6\,a\,b\,x^3+9\,b^2\,x^6)}{40\,a^3\,x^8}$$

input
```
int(1/(x^9*(a + b*x^3)^(1/3)),x)
```

output
```
-((a + b*x^3)^(2/3)*(5*a^2 + 9*b^2*x^6 - 6*a*b*x^3))/(40*a^3*x^8)
```

Reduce [F]

$$\int \frac{1}{x^9 \sqrt[3]{a+bx^3}} \, dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^9} dx$$

input
```
int(1/x^9/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**9),x)
```

3.406 $\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = -\frac{(a+bx^3)^{2/3}}{11ax^{11}}+\frac{9b(a+bx^3)^{2/3}}{88a^2x^8}-\frac{27b^2(a+bx^3)^{2/3}}{220a^3x^5}+\frac{81b^3(a+bx^3)^{2/3}}{440a^4x^2}$$

output
```
-1/11*(b*x^3+a)^(2/3)/a/x^11+9/88*b*(b*x^3+a)^(2/3)/a^2/x^8-27/220*b^2*(b*
x^3+a)^(2/3)/a^3/x^5+81/440*b^3*(b*x^3+a)^(2/3)/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \frac{(a+bx^3)^{2/3}(-40a^3+45a^2bx^3-54ab^2x^6+81b^3x^9)}{440a^4x^{11}}$$

input
```
Integrate[1/(x^12*(a + b*x^3)^(1/3)),x]
```

output
```
((a + b*x^3)^(2/3)*(-40*a^3 + 45*a^2*b*x^3 - 54*a*b^2*x^6 + 81*b^3*x^9))/(
440*a^4*x^11)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{9b\int \frac{1}{x^9\sqrt[3]{bx^3+a}}dx}{11a}-\frac{(a+bx^3)^{2/3}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{9b\left(-\frac{3b\int \frac{1}{x^6\sqrt[3]{bx^3+a}}dx}{4a}-\frac{(a+bx^3)^{2/3}}{8ax^8}\right)}{11a}-\frac{(a+bx^3)^{2/3}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{9b\left(-\frac{3b\left(-\frac{3b\int \frac{1}{x^3\sqrt[3]{bx^3+a}}dx}{5a}-\frac{(a+bx^3)^{2/3}}{5ax^5}\right)}{4a}-\frac{(a+bx^3)^{2/3}}{8ax^8}\right)}{11a}-\frac{(a+bx^3)^{2/3}}{11ax^{11}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{9b\left(-\frac{3b\left(\frac{3b(a+bx^3)^{2/3}}{10a^2x^2}-\frac{(a+bx^3)^{2/3}}{5ax^5}\right)}{4a}-\frac{(a+bx^3)^{2/3}}{8ax^8}\right)}{11a}-\frac{(a+bx^3)^{2/3}}{11ax^{11}}$$

input
```
Int[1/(x^12*(a + b*x^3)^(1/3)),x]
```

output
```
-1/11*(a + b*x^3)^(2/3)/(a*x^11) - (9*b*(-1/8*(a + b*x^3)^(2/3)/(a*x^8) -
(3*b*(-1/5*(a + b*x^3)^(2/3)/(a*x^5) + (3*b*(a + b*x^3)^(2/3))/(10*a^2*x^2
)))/(4*a)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 b^{3} x^{9}+54 a\,b^{2} x^{6}-45 a^{2} b\,x^{3}+40 a^{3}\right)}{440 x^{11} a^{4}}$	50
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 b^{3} x^{9}+54 a\,b^{2} x^{6}-45 a^{2} b\,x^{3}+40 a^{3}\right)}{440 x^{11} a^{4}}$	50
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 b^{3} x^{9}+54 a\,b^{2} x^{6}-45 a^{2} b\,x^{3}+40 a^{3}\right)}{440 x^{11} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 b^{3} x^{9}+54 a\,b^{2} x^{6}-45 a^{2} b\,x^{3}+40 a^{3}\right)}{440 x^{11} a^{4}}$	50
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(-81 b^{3} x^{9}+54 a\,b^{2} x^{6}-45 a^{2} b\,x^{3}+40 a^{3}\right)}{440 x^{11} a^{4}}$	50

input
```
int(1/x^12/(b*x^3+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/440*(b*x^3+a)^(2/3)*(-81*b^3*x^9+54*a*b^2*x^6-45*a^2*b*x^3+40*a^3)/x^11
/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \frac{(81\,b^3x^9-54\,ab^2x^6+45\,a^2bx^3-40\,a^3)(bx^3+a)^{\frac{2}{3}}}{440\,a^4x^{11}}$$

input
```
integrate(1/x^12/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
1/440*(81*b^3*x^9 - 54*a*b^2*x^6 + 45*a^2*b*x^3 - 40*a^3)*(b*x^3 + a)^(2/3
)/(a^4*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(85) = 170.

Time = 1.15 (sec) , antiderivative size = 692, normalized size of antiderivative = 7.52

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**12/(b*x**3+a)**(1/3),x)
```

output
```
-80*a**6*b**(29/3)*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9
*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma
(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) - 150*a**5*b**(32/3)*x**3*(a/(b*x*
*3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**1
0*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18
*gamma(1/3)) - 78*a**4*b**(35/3)*x**6*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)
/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**
5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 28*a**3*b**(3
8/3)*x**9*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/
3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 8
1*a**4*b**12*x**18*gamma(1/3)) + 252*a**2*b**(41/3)*x**12*(a/(b*x**3) + 1)
**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*
gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1
/3)) + 378*a*b**(44/3)*x**15*(a/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7
*b**9*x**9*gamma(1/3) + 243*a**6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x
**15*gamma(1/3) + 81*a**4*b**12*x**18*gamma(1/3)) + 162*b**(47/3)*x**18*(a
/(b*x**3) + 1)**(2/3)*gamma(-11/3)/(81*a**7*b**9*x**9*gamma(1/3) + 243*a**
6*b**10*x**12*gamma(1/3) + 243*a**5*b**11*x**15*gamma(1/3) + 81*a**4*b**12
*x**18*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx=\frac{\frac{220\,(bx^3+a)^{\frac{2}{3}}b^3}{x^2}-\frac{264\,(bx^3+a)^{\frac{5}{3}}b^2}{x^5}+\frac{165\,(bx^3+a)^{\frac{8}{3}}b}{x^8}-\frac{40\,(bx^3+a)^{\frac{11}{3}}}{x^{11}}}{440\,a^4}$$

input
```
integrate(1/x^12/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
1/440*(220*(b*x^3 + a)^(2/3)*b^3/x^2 - 264*(b*x^3 + a)^(5/3)*b^2/x^5 + 165
*(b*x^3 + a)^(8/3)*b/x^8 - 40*(b*x^3 + a)^(11/3)/x^11)/a^4
```

Giac [F]

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^{12}}\,dx$$

input
```
integrate(1/x^12/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \frac{9\,b\,(b\,x^3+a)^{2/3}}{88\,a^2\,x^8} - \frac{(b\,x^3+a)^{2/3}}{11\,a\,x^{11}} + \frac{81\,b^3\,(b\,x^3+a)^{2/3}}{440\,a^4\,x^2} - \frac{27\,b^2\,(b\,x^3+a)^{2/3}}{220\,a^3\,x^5}$$

input
```
int(1/(x^12*(a + b*x^3)^(1/3)),x)
```

output
```
(9*b*(a + b*x^3)^(2/3))/(88*a^2*x^8) - (a + b*x^3)^(2/3)/(11*a*x^11) + (81
*b^3*(a + b*x^3)^(2/3))/(440*a^4*x^2) - (27*b^2*(a + b*x^3)^(2/3))/(220*a^
3*x^5)
```

Reduce [F]

$$\int \frac{1}{x^{12}\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,x^{12}}dx$$

input
```
int(1/x^12/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**12),x)
```

3.407 $\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8\sqrt[3]{a+bx^3}}$$

output
```
1/8*x^8*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 8/3],[11/3],-b*x^3/a)/(b*x^3+a)^
(1/3)
```

Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^7/(a + b*x^3)^(1/3),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^3)/a)]
)/(8*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^7}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8\sqrt[3]{a+bx^3}}$$

input
```
Int[x^7/(a + b*x^3)^(1/3),x]
```

output

```
(x^8*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^3)/a)]
)/(8*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^7}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^7/(b*x^3+a)^(1/3),x)
```

output
```
int(x^7/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(x^7/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(b*x**3+a)**(1/3),x)
```

output
```
x**8*gamma(8/3)*hyper((1/3, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^7/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^7/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int(x^7/(a + b*x^3)^(1/3),x)
```

output
```
int(x^7/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^7/(b*x^3+a)^(1/3),x)
```

output
```
int(x**7/(a + b*x**3)**(1/3),x)
```

3.408 $\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

output
```
1/5*x^5*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 5/3],[8/3],-b*x^3/a)/(b*x^3+a)^(
1/3)
```

Mathematica [A] (verified)

Time = 4.97 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^4/(a + b*x^3)^(1/3),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])
/(5*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^4}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^5 \sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

input
```
Int[x^4/(a + b*x^3)^(1/3),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])
/(5*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x^4/(b*x^3+a)^(1/3),x)
```

output
```
int(x^4/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(b*x**3+a)**(1/3),x)
```

output
```
x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(1/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{x^4}{{\left(b\,x^3+a\right)}^{1/3}} \,dx$$

input
```
int(x^4/(a + b*x^3)^(1/3),x)
```

output
```
int(x^4/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{x^4}{\left(b\,x^{3}+a \right)^{\frac{1}{3}}}d x$$

input
```
int(x^4/(b*x^3+a)^(1/3),x)
```

output
```
int(x**4/(a + b*x**3)**(1/3),x)
```

3.409 $\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

output
```
1/2*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(
1/3)
```

Mathematica [A] (verified)

Time = 4.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x/(a + b*x^3)^(1/3),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])
/(2*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt[3]{\frac{bx^3}{a}+1}}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

input
```
Int[x/(a + b*x^3)^(1/3),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])
/(2*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}d x$$

input
```
int(x/(b*x^3+a)^(1/3),x)
```

output
```
int(x/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx=\int \frac{x}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(x/(b*x^3 + a)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx=\frac{x^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(b*x**3+a)**(1/3),x)
```

output
```
x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(1/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x/(b*x^3 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int(x/(a + b*x^3)^(1/3),x)
```

output
```
int(x/(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(x/(b*x^3+a)^(1/3),x)
```

output
```
int(x/(a + b*x**3)**(1/3),x)
```

3.410 $\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt[3]{a+bx^3}}$$

output
```
-(1+b*x^3/a)^(1/3)*hypergeom([-1/3, 1/3],[2/3],-b*x^3/a)/x/(b*x^3+a)^(1/3)
```

Mathematica [A] (verified)

Time = 8.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(1/3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, -((b*x^3)/a)])/
(x*(a + b*x^3)^(1/3)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^2 \sqrt[3]{\frac{bx^3}{a}+1}}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{x\sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^(1/3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, -((b*x^3)/a)])/
(x*(a + b*x^3)^(1/3)))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a \right)^{\frac{1}{3}}}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/3),x)
```

output
```
int(1/x^2/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)/(b*x^5 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\,dx=\frac{\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{3},\frac{1}{3} \\ \frac{2}{3} \end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(b*x**3+a)**(1/3),x)
```

output
```
gamma(-1/3)*hyper((-1/3, 1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}} \, dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}} \, dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\, dx = -\frac{\left(\frac{a}{b\,x^3}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{a}{b\,x^3}\right)}{2\,x\,\left(b\,x^3+a\right)^{1/3}}$$

input
```
int(1/(x^2*(a + b*x^3)^(1/3)),x)
```

output
```
-((a/(b*x^3) + 1)^(1/3)*hypergeom([1/3, 2/3], 5/3, -a/(b*x^3)))/(2*x*(a +
b*x^3)^(1/3))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}}\, dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\,x^2}dx$$

input
```
int(1/x^2/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**2),x)
```

3.411 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}} \, dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}} \, \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4 \sqrt[3]{a+bx^3}}$$

output
```
-1/4*(1+b*x^3/a)^(1/3)*hypergeom([-4/3, 1/3],[-1/3],-b*x^3/a)/x^4/(b*x^3+a
)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}} \, dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}} \, \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4 \sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(1/3)),x]
```

output
```
-1/4*((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, -((b*x^3)/a
)])/(x^4*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^5 \sqrt[3]{\frac{bx^3}{a}+1}}dx}{\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4x^4 \sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^5*(a + b*x^3)^(1/3)),x]
```

output
```
-1/4*((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3, 1/3, -1/3, -((b*x^3)/a
)])/(x^4*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/x^5/(b*x^3+a)^(1/3),x)
```

output
```
int(1/x^5/(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)/(b*x^8 + a*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}}\,dx = \frac{\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{1}{3} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x^4\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**3+a)**(1/3),x)
```

output
```
gamma(-4/3)*hyper((-4/3, 1/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{1}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{x^5\,{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

input
```
int(1/(x^5*(a + b*x^3)^(1/3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}} x^5}dx$$

input
```
int(1/x^5/(b*x^3+a)^(1/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*x**5),x)
```

3.412 $\int \frac{x^{11}}{(a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}} \, dx = -\frac{a^3 \sqrt[3]{a+bx^3}}{b^4} + \frac{3a^2(a+bx^3)^{4/3}}{4b^4} - \frac{3a(a+bx^3)^{7/3}}{7b^4} + \frac{(a+bx^3)^{10/3}}{10b^4}$$

output
```
-a^3*(b*x^3+a)^(1/3)/b^4+3/4*a^2*(b*x^3+a)^(4/3)/b^4-3/7*a*(b*x^3+a)^(7/3)
/b^4+1/10*(b*x^3+a)^(10/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}} \, dx = \frac{\sqrt[3]{a+bx^3}(-81a^3 + 27a^2bx^3 - 18ab^2x^6 + 14b^3x^9)}{140b^4}$$

input
```
Integrate[x^11/(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(1/3)*(-81*a^3 + 27*a^2*b*x^3 - 18*a*b^2*x^6 + 14*b^3*x^9))/(
140*b^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{3}\int \frac{x^9}{(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \; 53$$

$$\frac{1}{3}\int \left(-\frac{a^3}{b^3(bx^3+a)^{2/3}}+\frac{3\sqrt[3]{bx^3+a}a^2}{b^3}-\frac{3(bx^3+a)^{4/3}a}{b^3}+\frac{(bx^3+a)^{7/3}}{b^3}\right)dx^3$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{3a^3\sqrt[3]{a+bx^3}}{b^4}+\frac{9a^2(a+bx^3)^{4/3}}{4b^4}+\frac{3(a+bx^3)^{10/3}}{10b^4}-\frac{9a(a+bx^3)^{7/3}}{7b^4}\right)$$

input
```
Int[x^11/(a + b*x^3)^(2/3),x]
```

output
```
((-3*a^3*(a + b*x^3)^(1/3))/b^4 + (9*a^2*(a + b*x^3)^(4/3))/(4*b^4) - (9*a
*(a + b*x^3)^(7/3))/(7*b^4) + (3*(a + b*x^3)^(10/3))/(10*b^4))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{(b x^3+a)^{\frac{1}{3}}(-14b^3x^9+18a b^2x^6-27a^2b x^3+81a^3)}{140b^4}$	47
trager	$-\frac{(b x^3+a)^{\frac{1}{3}}(-14b^3x^9+18a b^2x^6-27a^2b x^3+81a^3)}{140b^4}$	47
risch	$-\frac{(b x^3+a)^{\frac{1}{3}}(-14b^3x^9+18a b^2x^6-27a^2b x^3+81a^3)}{140b^4}$	47
pseudoelliptic	$-\frac{(b x^3+a)^{\frac{1}{3}}(-14b^3x^9+18a b^2x^6-27a^2b x^3+81a^3)}{140b^4}$	47
orering	$-\frac{(b x^3+a)^{\frac{1}{3}}(-14b^3x^9+18a b^2x^6-27a^2b x^3+81a^3)}{140b^4}$	47

input
```
int(x^11/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/140*(b*x^3+a)^(1/3)*(-14*b^3*x^9+18*a*b^2*x^6-27*a^2*b*x^3+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx = \frac{(14\,b^3x^9 - 18\,ab^2x^6 + 27\,a^2bx^3 - 81\,a^3)(bx^3+a)^{\frac{1}{3}}}{140\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/140*(14*b^3*x^9 - 18*a*b^2*x^6 + 27*a^2*b*x^3 - 81*a^3)*(b*x^3 + a)^(1/3
)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx = \begin{cases} -\frac{81a^3\sqrt[3]{a+bx^3}}{140b^4} + \frac{27a^2x^3\sqrt[3]{a+bx^3}}{140b^3} - \frac{9ax^6\sqrt[3]{a+bx^3}}{70b^2} + \frac{x^9\sqrt[3]{a+bx^3}}{10b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((-81*a**3*(a + b*x**3)**(1/3)/(140*b**4) + 27*a**2*x**3*(a + b*x
**3)**(1/3)/(140*b**3) - 9*a*x**6*(a + b*x**3)**(1/3)/(70*b**2) + x**9*(a
+ b*x**3)**(1/3)/(10*b), Ne(b, 0)), (x**12/(12*a**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{10}{3}}}{10\,b^4} - \frac{3\,(bx^3+a)^{\frac{7}{3}}a}{7\,b^4} + \frac{3\,(bx^3+a)^{\frac{4}{3}}a^2}{4\,b^4} - \frac{(bx^3+a)^{\frac{1}{3}}a^3}{b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/10*(b*x^3 + a)^(10/3)/b^4 - 3/7*(b*x^3 + a)^(7/3)*a/b^4 + 3/4*(b*x^3 + a
)^(4/3)*a^2/b^4 - (b*x^3 + a)^(1/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx = -\frac{(bx^3+a)^{\frac{1}{3}}a^3}{b^4} + \frac{14\,(bx^3+a)^{\frac{10}{3}} - 60\,(bx^3+a)^{\frac{7}{3}}a + 105\,(bx^3+a)^{\frac{4}{3}}a^2}{140\,b^4}$$

input
```
integrate(x^11/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
-(b*x^3 + a)^(1/3)*a^3/b^4 + 1/140*(14*(b*x^3 + a)^(10/3) - 60*(b*x^3 + a)
^(7/3)*a + 105*(b*x^3 + a)^(4/3)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx = -\left(b\,x^3+a\right)^{1/3}\left(\frac{81\,a^3}{140\,b^4} - \frac{x^9}{10\,b} + \frac{9\,a\,x^6}{70\,b^2} - \frac{27\,a^2\,x^3}{140\,b^3}\right)$$

input
```
int(x^11/(a + b*x^3)^(2/3),x)
```

output
```
-(a + b*x^3)^(1/3)*((81*a^3)/(140*b^4) - x^9/(10*b) + (9*a*x^6)/(70*b^2) -
 (27*a^2*x^3)/(140*b^3))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^{11}}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^11/(b*x^3+a)^(2/3),x)
```

output
```
int(x**11/(a + b*x**3)**(2/3),x)
```

3.413 $\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx=\frac{a^2\sqrt[3]{a+bx^3}}{b^3}-\frac{a(a+bx^3)^{4/3}}{2b^3}+\frac{(a+bx^3)^{7/3}}{7b^3}$$

output
```
a^2*(b*x^3+a)^(1/3)/b^3-1/2*a*(b*x^3+a)^(4/3)/b^3+1/7*(b*x^3+a)^(7/3)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^3}(9a^2-3abx^3+2b^2x^6)}{14b^3}$$

input
```
Integrate[x^8/(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(1/3)*(9*a^2 - 3*a*b*x^3 + 2*b^2*x^6))/(14*b^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3} \int \frac{x^6}{(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3} \int \left(\frac{a^2}{b^2 (bx^3+a)^{2/3}} - \frac{2\sqrt[3]{bx^3+a}a}{b^2} + \frac{(bx^3+a)^{4/3}}{b^2}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{3a^2\sqrt[3]{a+bx^3}}{b^3} + \frac{3(a+bx^3)^{7/3}}{7b^3} - \frac{3a(a+bx^3)^{4/3}}{2b^3}\right)$$

input
```
Int[x^8/(a + b*x^3)^(2/3),x]
```

output
```
((3*a^2*(a + b*x^3)^(1/3))/b^3 - (3*a*(a + b*x^3)^(4/3))/(2*b^3) + (3*(a +
 b*x^3)^(7/3))/(7*b^3))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(2b^2x^6-3ab\,x^3+9a^2)}{14b^3}$	36
trager	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(2b^2x^6-3ab\,x^3+9a^2)}{14b^3}$	36
risch	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(2b^2x^6-3ab\,x^3+9a^2)}{14b^3}$	36
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(2b^2x^6-3ab\,x^3+9a^2)}{14b^3}$	36
orering	$\frac{(b\,x^3+a)^{\frac{1}{3}}\,(2b^2x^6-3ab\,x^3+9a^2)}{14b^3}$	36

input
```
int(x^8/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/14*(b*x^3+a)^(1/3)*(2*b^2*x^6-3*a*b*x^3+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx = \frac{(2\,b^2x^6-3\,abx^3+9\,a^2)(bx^3+a)^{\frac{1}{3}}}{14\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/14*(2*b^2*x^6 - 3*a*b*x^3 + 9*a^2)*(b*x^3 + a)^(1/3)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx = \begin{cases} \frac{9a^2\sqrt[3]{a+bx^3}}{14b^3} - \frac{3ax^3\sqrt[3]{a+bx^3}}{14b^2} + \frac{x^6\sqrt[3]{a+bx^3}}{7b} & \text{for } b \neq 0 \\ \frac{x^9}{9a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**8/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((9*a**2*(a + b*x**3)**(1/3)/(14*b**3) - 3*a*x**3*(a + b*x**3)**(
1/3)/(14*b**2) + x**6*(a + b*x**3)**(1/3)/(7*b), Ne(b, 0)), (x**9/(9*a**(2
/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{7}{3}}}{7\,b^3} - \frac{(bx^3+a)^{\frac{4}{3}}a}{2\,b^3} + \frac{(bx^3+a)^{\frac{1}{3}}a^2}{b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/7*(b*x^3 + a)^(7/3)/b^3 - 1/2*(b*x^3 + a)^(4/3)*a/b^3 + (b*x^3 + a)^(1/3
)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{1}{3}}a^2}{b^3} + \frac{2\,(bx^3+a)^{\frac{7}{3}} - 7\,(bx^3+a)^{\frac{4}{3}}a}{14\,b^3}$$

input
```
integrate(x^8/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
(b*x^3 + a)^(1/3)*a^2/b^3 + 1/14*(2*(b*x^3 + a)^(7/3) - 7*(b*x^3 + a)^(4/3
)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx = \left(b\,x^3+a\right)^{1/3}\left(\frac{9\,a^2}{14\,b^3} + \frac{x^6}{7\,b} - \frac{3\,a\,x^3}{14\,b^2}\right)$$

input
```
int(x^8/(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(1/3)*((9*a^2)/(14*b^3) + x^6/(7*b) - (3*a*x^3)/(14*b^2))
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^8}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^8/(b*x^3+a)^(2/3),x)
```

output
```
int(x**8/(a + b*x**3)**(2/3),x)
```

3.414 $\int \frac{x^5}{(a+bx^3)^{2/3}} dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x^5}{(a+bx^3)^{2/3}} dx = -\frac{a\sqrt[3]{a+bx^3}}{b^2} + \frac{(a+bx^3)^{4/3}}{4b^2}$$

output
```
-a*(b*x^3+a)^(1/3)/b^2+1/4*(b*x^3+a)^(4/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^3)^{2/3}} dx = \frac{(-3a+bx^3)\sqrt[3]{a+bx^3}}{4b^2}$$

input
```
Integrate[x^5/(a + b*x^3)^(2/3),x]
```

output
```
((-3*a + b*x^3)*(a + b*x^3)^(1/3))/(4*b^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{x^3}{(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{\sqrt[3]{bx^3+a}}{b}-\frac{a}{b(bx^3+a)^{2/3}}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{3(a+bx^3)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx^3}}{b^2}\right)$$

input
```
Int[x^5/(a + b*x^3)^(2/3),x]
```

output
```
((-3*a*(a + b*x^3)^(1/3))/b^2 + (3*(a + b*x^3)^(4/3))/(4*b^2))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{(b\,x^3-3a)\,(b\,x^3+a)^{\frac{1}{3}}}{4b^2}$	24
gosper	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\,(-b\,x^3+3a)}{4b^2}$	25
trager	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\,(-b\,x^3+3a)}{4b^2}$	25
risch	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\,(-b\,x^3+3a)}{4b^2}$	25
orering	$-\frac{(b\,x^3+a)^{\frac{1}{3}}\,(-b\,x^3+3a)}{4b^2}$	25

input
```
int(x^5/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^3-3*a)*(b*x^3+a)^(1/3)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{1}{3}}(bx^3-3\,a)}{4\,b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(b*x^3 + a)^(1/3)*(b*x^3 - 3*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{(a+bx^3)^{2/3}}\,dx = \begin{cases} -\frac{3a\sqrt[3]{a+bx^3}}{4b^2} + \frac{x^3\sqrt[3]{a+bx^3}}{4b} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise((-3*a*(a + b*x**3)**(1/3)/(4*b**2) + x**3*(a + b*x**3)**(1/3)/(4
*b), Ne(b, 0)), (x**6/(6*a**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{4}{3}}}{4\,b^2} - \frac{(bx^3+a)^{\frac{1}{3}}a}{b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/4*(b*x^3 + a)^(4/3)/b^2 - (b*x^3 + a)^(1/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{4}{3}}}{4\,b^2} - \frac{(bx^3+a)^{\frac{1}{3}}a}{b^2}$$

input
```
integrate(x^5/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/4*(b*x^3 + a)^(4/3)/b^2 - (b*x^3 + a)^(1/3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{x^5}{(a+bx^3)^{2/3}}\,dx = -\frac{(b\,x^3+a)^{1/3}\,(3\,a-b\,x^3)}{4\,b^2}$$

input
```
int(x^5/(a + b*x^3)^(2/3),x)
```

output
```
-((a + b*x^3)^(1/3)*(3*a - b*x^3))/(4*b^2)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^5}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^5/(b*x^3+a)^(2/3),x)
```

output
```
int(x**5/(a + b*x**3)**(2/3),x)
```

3.415 $\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^3}}{b}$$

output
```
(b*x^3+a)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx=\frac{\sqrt[3]{a+bx^3}}{b}$$

input
```
Integrate[x^2/(a + b*x^3)^(2/3),x]
```

output
```
(a + b*x^3)^(1/3)/b
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{\sqrt[3]{a+bx^3}}{b}$$

input
```
Int[x^2/(a + b*x^3)^(2/3),x]
```

output
```
(a + b*x^3)^(1/3)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14
derivativedivides	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14
default	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14
trager	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14
risch	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14
pseudoelliptic	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14
orering	$\frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$	14

input
```
int(x^2/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{1}{3}}}{b}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
(b*x^3 + a)^(1/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx = \begin{cases} \frac{\sqrt[3]{a+bx^3}}{b} & \text{for } b \neq 0 \\ \frac{x^3}{3a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(b*x**3+a)**(2/3),x)
```

output
```
Piecewise(((a + b*x**3)**(1/3)/b, Ne(b, 0)), (x**3/(3*a**(2/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{1}{3}}}{b}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
(b*x^3 + a)^(1/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx = \frac{(bx^3+a)^{\frac{1}{3}}}{b}$$

input
```
integrate(x^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
(b*x^3 + a)^(1/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx = \frac{(b\,x^3+a)^{1/3}}{b}$$

input
```
int(x^2/(a + b*x^3)^(2/3),x)
```

output
```
(a + b*x^3)^(1/3)/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx = \frac{(b\,x^3+a)^{\frac{1}{3}}}{b}$$

input
```
int(x^2/(b*x^3+a)^(2/3),x)
```

output
```
(a + b*x**3)**(1/3)/b
```

3.416 $\int \frac{1}{x(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{1}{x(a+bx^3)^{2/3}}\,dx = -\frac{\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}$$

output
```
-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/
3)-1/2*ln(x)/a^(2/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(a+bx^3)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right) + \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{6a^{2/3}}$$

input
```
Integrate[1/(x*(a + b*x^3)^(2/3)),x]
```

output
```
-1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Lo
g[-a^(1/3) + (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3)
+ (a + b*x^3)^(2/3)])/a^(2/3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int \frac{1}{x^3(bx^3+a)^{2/3}}dx^3$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}-\frac{\log(x^3)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log\left(x^3\right)}{2a^{2/3}}\right)$$

input
```
Int[1/(x*(a + b*x^3)^(2/3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
- Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*a^(2/3)))
/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}+2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}$	83

input
```
int(1/x/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)+2*
ln((b*x^3+a)^(1/3)-a^(1/3))-ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a)^(1/3)+a^(
2/3)))/a^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx =$$
$$-\frac{6\sqrt{\frac{1}{3}}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(bx^3+a\right)^{\frac{1}{3}}\left(a^2\right)^{\frac{2}{3}}\right)}{a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(bx^3+a\right)^{\frac{1}{3}}(a\right.}{6\,a^2}$$

input
```
integrate(1/x/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/6*(6*sqrt(1/3)*(a^2)^(1/6)*a*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)^(1/3)*
a + 2*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^3 + a)^(2
/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log
((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.46

$$\int \frac{1}{x\,(a+bx^3)^{2/3}}\,dx = -\frac{\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(b*x**3+a)**(2/3),x)
```

output
```
-gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(2
/3)*x**2*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\,(a+bx^3)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}} + \frac{\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(2/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 1/3*log((b*x^3 + a)^(1/3) - a^(1/3))/a^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{2}{3}}} - \frac{\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{2}{3}}} + \frac{\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(2/3) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx = \frac{\ln\left(3\left(b\,x^3+a\right)^{1/3}-3\,a^{1/3}\right)}{3\,a^{2/3}} + \frac{\ln\left(\frac{3\,a^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2}-3\left(b\,x^3+a\right)^{1/3}\right)\left(-1+\sqrt{3}\,1i\right)}{6\,a^{2/3}} - \frac{\ln\left(\frac{3\,a^{1/3}\left(1+\sqrt{3}\,1i\right)}{2}+3\left(b\,x^3+a\right)^{1/3}\right)\left(1+\sqrt{3}\,1i\right)}{6\,a^{2/3}}$$

input
```
int(1/(x*(a + b*x^3)^(2/3)),x)
```

output
```
log(3*(a + b*x^3)^(1/3) - 3*a^(1/3))/(3*a^(2/3)) + (log((3*a^(1/3)*(3^(1/2
)*1i - 1))/2 - 3*(a + b*x^3)^(1/3))*(3^(1/2)*1i - 1))/(6*a^(2/3)) - (log((
3*a^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(a + b*x^3)^(1/3))*(3^(1/2)*1i + 1))/(6*
a^(2/3))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}x}dx$$

input
```
int(1/x/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x),x)
```

3.417 $\int \frac{1}{x^4 (a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{x^4 (a+bx^3)^{2/3}} \, dx = -\frac{\sqrt[3]{a+bx^3}}{3ax^3} + \frac{2b \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}} + \frac{b \log(x)}{3a^{5/3}} - \frac{b \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}}$$

output
```
-1/3*(b*x^3+a)^(1/3)/a/x^3+2/9*b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^
(1/2)/a^(1/3))*3^(1/2)/a^(5/3)+1/3*b*ln(x)/a^(5/3)-1/3*b*ln(a^(1/3)-(b*x^3
+a)^(1/3))/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 (a+bx^3)^{2/3}} \, dx = \frac{-3a^{2/3}\sqrt[3]{a+bx^3} + 2\sqrt{3}bx^3 \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - 2bx^3 \log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^3}\right.}{9a^{5/3}x^3}$$

input
```
Integrate[1/(x^4*(a + b*x^3)^(2/3)),x]
```

output
```
(-3*a^(2/3)*(a + b*x^3)^(1/3) + 2*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)
^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x^3*Log[-a^(1/3) + (a + b*x^3)^(1/3)] + b*
x^3*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*a^(5/
3)*x^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b x^3\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3} \int \frac{1}{x^6 \left(b x^3+a\right)^{2/3}}dx^3$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3} \left(-\frac{2 b \int \frac{1}{x^3 \left(b x^3+a\right)^{2/3}}dx^3}{3 a}-\frac{\sqrt[3]{a+b x^3}}{a x^3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{3} \left(-\frac{2 b \left(-\frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{b x^3+a}}d\sqrt[3]{b x^3+a}}{2 a^{2/3}}-\frac{3 \int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a} \sqrt[3]{b x^3+a}}d\sqrt[3]{b x^3+a}}{2 \sqrt[3]{a}}-\frac{\log \left(x^3\right)}{2 a^{2/3}}\right)}{3 a}-\frac{\sqrt[3]{a+b x^3}}{a x^3}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\left(-\frac{2b\left(-\frac{3\int \frac{1}{x^6+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^3+a}}d\sqrt[3]{bx^3+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{3}\left(-\frac{2b\left(\frac{3\int \frac{1}{-x^6-3}d\left(\frac{2\sqrt[3]{bx^3+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}}-\frac{\log(x^3)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^3}}{ax^3}\right)$$

input
```
Int[1/(x^4*(a + b*x^3)^(2/3)),x]
```

output
```
(-((a + b*x^3)^(1/3)/(a*x^3)) - (2*b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3
)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^3]/(2*a^(2/3)) + (3*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(2*a^(2/3))))/(3*a))/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,b\,x^{3}-2\ln\left(\left(b\,x^{3}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)b\,x^{3}+\ln\left(\left(b\,x^{3}+a\right)^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}+a^{\frac{2}{3}}\right)b\,x^{3}-3(b\,x^{3}}{9a^{\frac{5}{3}}x^{3}}$

input
```
int(1/x^4/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*(2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*b*x
^3-2*ln((b*x^3+a)^(1/3)-a^(1/3))*b*x^3+ln((b*x^3+a)^(2/3)+a^(1/3)*(b*x^3+a
)^(1/3)+a^(2/3))*b*x^3-3*(b*x^3+a)^(1/3)*a^(2/3))/a^(5/3)/x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(81) = 162.

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}}\,dx = \frac{6\sqrt{\frac{1}{3}}abx^3\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-a^2\right)^{\frac{1}{3}}a-2\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}}{}$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/9*(6*sqrt(1/3)*a*b*x^3*sqrt(-(-a^2)^(1/3))*arctan(-sqrt(1/3)*((-a^2)^(1/
3)*a - 2*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^2)
^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3
)) - 3*(b*x^3 + a)^(1/3)*a^2)/(a^3*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4 (a+bx^3)^{2/3}}\,dx = -\frac{\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^3}\right)}{3b^{\frac{2}{3}}x^5\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(1/x**4/(b*x**3+a)**(2/3),x)
```

output
```
-gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**3))/(3*b**(2
/3)*x**5*gamma(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^4 (a+bx^3)^{2/3}}\,dx = \frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}} - \frac{(bx^3+a)^{\frac{1}{3}}b}{3\left((bx^3+a)a-a^2\right)} + \frac{b\log\left((bx^3+a)^{\frac{2}{3}}+(bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{5}{3}}} - \frac{2\,b\log\left((bx^3+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}}$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(5/3) - 1/3*(b*x^3 + a)^(1/3)*b/((b*x^3 + a)*a - a^2) + 1/9*b*log((b*x^3
 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/9*b*log((b*
x^3 + a)^(1/3) - a^(1/3))/a^(5/3)
```

Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^4 (a+bx^3)^{2/3}}\,dx = \frac{1}{9}\,b\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^3+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} + \frac{\log\left((bx^3+a)^{\frac{2}{3}} + (bx^3+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}} -\right.$$

input
```
integrate(1/x^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
1/9*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 2*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^3 + a)^
(1/3)/(a*b*x^3))
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^4 (a+bx^3)^{2/3}}\,dx = \frac{\ln\left(\frac{b-\sqrt{3}\,b\,1\mathrm{i}}{a^{2/3}} + \frac{2\,b\,(b\,x^3+a)^{1/3}}{a}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{5/3}} + \frac{\ln\left(\frac{b+\sqrt{3}\,b\,1\mathrm{i}}{a^{2/3}} + \frac{2\,b\,(b\,x^3+a)^{1/3}}{a}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{9\,a^{5/3}} - \frac{2\,b\,\ln\left((b\,x^3+a)^{1/3}-a^{1/3}\right)}{9\,a^{5/3}} - \frac{(b\,x^3+a)^{1/3}}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a + b*x^3)^(2/3)),x)
```

output
```
(log((b - 3^(1/2)*b*1i)/a^(2/3) + (2*b*(a + b*x^3)^(1/3))/a)*(b - 3^(1/2)*
b*1i))/(9*a^(5/3)) + (log((b + 3^(1/2)*b*1i)/a^(2/3) + (2*b*(a + b*x^3)^(1
/3))/a)*(b + 3^(1/2)*b*1i))/(9*a^(5/3)) - (2*b*log((a + b*x^3)^(1/3) - a^(
1/3)))/(9*a^(5/3)) - (a + b*x^3)^(1/3)/(3*a*x^3)
```

Reduce [F]

$$\int \frac{1}{x^4 (a + bx^3)^{2/3}} \, dx = \int \frac{1}{(b\, x^3 + a)^{\frac{2}{3}} \, x^4} dx$$

input
```
int(1/x^4/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**4),x)
```

3.418 $\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = -\frac{5ax^2\sqrt[3]{a+bx^3}}{18b^2}+\frac{x^5\sqrt[3]{a+bx^3}}{6b} - \frac{5a^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{8/3}} - \frac{5a^2\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}}$$

output
```
-5/18*a*x^2*(b*x^3+a)^(1/3)/b^2+1/6*x^5*(b*x^3+a)^(1/3)/b-5/27*a^2*arctan(
1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)-5/18*a^2*ln(b
^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.38

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}(-5ax^2+3bx^5)}{18b^2} - \frac{5a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}b^{8/3}} - \frac{5a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{27b^{8/3}} + \frac{5a^2\log\left(b^{2/3}x^2+\sqrt[3]{b}x\sqrt[3]{a+bx^3}+(a+bx^3)^{2/3}\right)}{54b^{8/3}}$$

input
```
Integrate[x^7/(a + b*x^3)^(2/3),x]
```

output
```
((a + b*x^3)^(1/3)*(-5*a*x^2 + 3*b*x^5))/(18*b^2) - (5*a^2*ArcTan[(Sqrt[3]
*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(8/3)) - (5*a
^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(8/3)) + (5*a^2*Log[b^(2/3
)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(8/3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {843, 843, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6b} - \frac{5a\int \frac{x^4}{(bx^3+a)^{2/3}}dx}{6b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^5\sqrt[3]{a+bx^3}}{6b}-\frac{5a\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\int\frac{x}{(bx^3+a)^{2/3}}dx}{3b}\right)}{6b}$$

$\Big\downarrow$ 853

$$\frac{x^5\sqrt[3]{a+bx^3}}{6b}-\frac{5a\left(\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b}\right)}{6b}$$

input
```
Int[x^7/(a + b*x^3)^(2/3),x]
```

output
```
(x^5*(a + b*x^3)^(1/3))/(6*b) - (5*a*((x^2*(a + b*x^3)^(1/3))/(3*b) - (2*a
*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)
)) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))))/(3*b)))/(6*b)
```

Defintions of rubi rules used

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{9x^5\left(b\,x^3+a\right)^{\frac{1}{3}}b^{\frac{5}{3}}-15a\,x^2b^{\frac{2}{3}}\left(b\,x^3+a\right)^{\frac{1}{3}}+10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^3+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)a^2-10\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^3+a\right)^{\frac{1}{3}}}{x}\right)a^2+5\ln}{54b^{\frac{8}{3}}}$

input
```
int(x^7/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/54/b^(8/3)*(9*x^5*(b*x^3+a)^(1/3)*b^(5/3)-15*a*x^2*b^(2/3)*(b*x^3+a)^(1/
3)+10*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*
a^2-10*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a^2+5*ln((b^(2/3)*x^2+b^(1/3)*(b
*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.46

$$\int\frac{x^7}{\left(a+bx^3\right)^{2/3}}\,dx=\frac{30\sqrt{\frac{1}{3}}a^2\left(b^2\right)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(\left(b^2\right)^{\frac{1}{3}}bx+2\left(bx^3+a\right)^{\frac{1}{3}}\left(b^2\right)^{\frac{2}{3}}\right)\left(b^2\right)^{\frac{1}{6}}}{b^2x}\right)-10\,a^2\left(b^2\right)^{\frac{2}{3}}\log\left(-\frac{\left(b^2\right)^{\frac{2}{3}}x-}{}\right.}{}$$

input
```
integrate(x^7/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/54*(30*sqrt(1/3)*a^2*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2
*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 10*a^2*(b^2)^(2/3)*
log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 5*a^2*(b^2)^(2/3)*log(((b^
2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^
2) + 3*(3*b^3*x^5 - 5*a*b^2*x^2)*(b*x^3 + a)^(1/3))/b^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(b*x**3+a)**(2/3),x)
```

output
```
x**8*gamma(8/3)*hyper((2/3, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(2/3)*gamma(11/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = \frac{5\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{27\,b^{\frac{8}{3}}} + \frac{5\,a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{54\,b^{\frac{8}{3}}} - \frac{5\,a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{27\,b^{\frac{8}{3}}} + \frac{\frac{8\,(bx^3+a)^{\frac{1}{3}}a^2b}{x}-\frac{5\,(bx^3+a)^{\frac{4}{3}}a^2}{x^4}}{18\left(b^4-\frac{2\,(bx^3+a)b^3}{x^3}+\frac{(bx^3+a)^2b^2}{x^6}\right)}$$

input
```
integrate(x^7/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
5/27*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(8/3) + 5/54*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(8/3) - 5/27*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^
(8/3) + 1/18*(8*(b*x^3 + a)^(1/3)*a^2*b/x - 5*(b*x^3 + a)^(4/3)*a^2/x^4)/(
b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3 + a)^2*b^2/x^6)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^7/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int(x^7/(a + b*x^3)^(2/3),x)
```

output
```
int(x^7/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^7/(b*x^3+a)^(2/3),x)
```

output
```
int(x**7/(a + b*x**3)**(2/3),x)
```

3.419 $\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx=\frac{x^2\sqrt[3]{a+bx^3}}{3b}+\frac{2a\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}+\frac{a\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}$$

output
```
1/3*x^2*(b*x^3+a)^(1/3)/b+2/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))*3^(1/2)/b^(5/3)+1/3*a*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx=\frac{3b^{2/3}x^2\sqrt[3]{a+bx^3}+2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right)+2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)-}{9b^{5/3}}$$

input
```
Integrate[x^4/(a + b*x^3)^(2/3),x]
```

output
```
(3*b^(2/3)*x^2*(a + b*x^3)^(1/3) + 2*Sqrt[3]*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/
(b^(1/3)*x + 2*(a + b*x^3)^(1/3))] + 2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1
/3)] - a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/(9*b^(5/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {843, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+a)^{2/3}}dx}{3b}$$

$$\Big\downarrow \text{853}$$

$$\frac{x^2\sqrt[3]{a+bx^3}}{3b}-\frac{2a\left(-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}\right)}{3b}$$

input
```
Int[x^4/(a + b*x^3)^(2/3),x]
```

output
```
(x^2*(a + b*x^3)^(1/3))/(3*b) - (2*a*(-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3))) - Log[b^(1/3)*x - (a + b*x^3)^(1/3)
]/(2*b^(2/3))))/(3*b)
```

Defintions of rubi rules used

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

method	result
pseudoelliptic	$\frac{3\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{2} b^{\frac{2}{3}}-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(b^{\frac{1}{3}} x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3 b^{\frac{1}{3}} x}\right) a+2\ln\left(\frac{-b^{\frac{1}{3}} x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right) a-\ln\left(\frac{b^{\frac{2}{3}} x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}} x+\left(b\,x^{3}\right.}{x^{2}}\right.}{9 b^{\frac{5}{3}}}$

input
```
int(x^4/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*(3*(b*x^3+a)^(1/3)*x^2*b^(2/3)-2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x
+2*(b*x^3+a)^(1/3))/b^(1/3)/x)*a+2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)*a-ln
((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1/3)*x+(b*x^3+a)^(2/3))/x^2)*a)/b^(5/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(74) = 148.

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.66

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx = \frac{3\,(bx^3+a)^{\frac{1}{3}}b^2x^2 - 6\,\sqrt{\frac{1}{3}}a(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+a)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right) + 2\,a(b^2)^{\frac{2}{3}}}{9\,b^3}$$

input
```
integrate(x^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/9*(3*(b*x^3 + a)^(1/3)*b^2*x^2 - 6*sqrt(1/3)*a*(b^2)^(1/6)*b*arctan(sqrt
(1/3)*((b^2)^(1/3)*b*x + 2*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2
*x)) + 2*a*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - a*(
b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x
^3 + a)^(2/3)*b)/x^2))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(b*x**3+a)**(2/3),x)
```

output
```
x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(2/3)*gamma(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx = -\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{9\,b^{\frac{5}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{9\,b^{\frac{5}{3}}} + \frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{9\,b^{\frac{5}{3}}} - \frac{(bx^3+a)^{\frac{1}{3}}a}{3\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x}$$

input
```
integrate(x^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-2/9*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(5/3) - 1/9*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(5/3) + 2/9*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(5/3) -
1/3*(b*x^3 + a)^(1/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^4}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int(x^4/(a + b*x^3)^(2/3),x)
```

output
```
int(x^4/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^4}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^4/(b*x^3+a)^(2/3),x)
```

output
```
int(x**4/(a + b*x**3)**(2/3),x)
```

3.420 $\int \frac{x}{\left(a+bx^3\right)^{2/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}} \, dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{\log\left(\sqrt[3]{b}x - \sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)-1
/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(a+bx^3\right)^{2/3}} \, dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x}{\sqrt[3]{b}x+2\sqrt[3]{a+bx^3}}\right) - 2\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right) + \log\left(b^{2/3}x^2 + \sqrt[3]{b}x\cdots\right.}{6b^{2/3}}$$

input
```
Integrate[x/(a + b*x^3)^(2/3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))]
- 2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)] + Log[b^(2/3)*x^2 + b^(1/3)*x*(a
 + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{853\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \; 853$$

$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

input
```
Int[x/(a + b*x^3)^(2/3),x]
```

output
```
-(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)))
 - Log[b^(1/3)*x - (a + b*x^3)^(1/3)]/(2*b^(2/3))
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(b\,x^{3}+a\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)-2\ln\left(\frac{-b^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{x}\right)+\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(b\,x^{3}+a\right)^{\frac{1}{3}}x+\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{x^{2}}\right)}{6b^{\frac{2}{3}}}$	100

input
```
int(x/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(b*x^3+a)^(1/3))/b^(1/3)/x)
-2*ln((-b^(1/3)*x+(b*x^3+a)^(1/3))/x)+ln((b^(2/3)*x^2+b^(1/3)*(b*x^3+a)^(1
/3)*x+(b*x^3+a)^(2/3))/x^2))/b^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(55) = 110.

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.94

$$\int \frac{x}{(a+bx^3)^{2/3}}\,dx = \frac{6\sqrt{\frac{1}{3}}\,(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+a)^{\frac{1}{3}}\,(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-2\,(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+a)^{\frac{1}{3}}}{x}\right.}{6\,b^2}$$

input
```
integrate(x/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/6*(6*sqrt(1/3)*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2*(b*x^
3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)
^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 +
(b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{x}{(a+bx^3)^{2/3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(b*x**3+a)**(2/3),x)
```

output
```
x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(2/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.39

$$\int \frac{x}{(a+bx^3)^{2/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{3\,b^{\frac{2}{3}}} + \frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{6\,b^{\frac{2}{3}}} - \frac{\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{2}{3}}}$$

input
```
integrate(x/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/
b^(2/3) + 1/6*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(2/3) - 1/3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(2/3)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^{2/3}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{2/3}}\,dx = \int \frac{x}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int(x/(a + b*x^3)^(2/3),x)
```

output
```
int(x/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^{2/3}}\,dx = \int \frac{x}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x/(b*x^3+a)^(2/3),x)
```

output
```
int(x/(a + b*x**3)**(2/3),x)
```

3.421 $\int \frac{1}{x^2(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{x^2(a+bx^3)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^3}}{ax}$$

output
```
-(b*x^3+a)^(1/3)/a/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^3)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^3}}{ax}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(2/3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(a*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{\sqrt[3]{a+bx^3}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^3)^(2/3)),x]
```

output
```
-((a + b*x^3)^(1/3)/(a*x))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{1}{3}}}{ax}$	18
trager	$-\frac{(b\,x^3+a)^{\frac{1}{3}}}{ax}$	18
risch	$-\frac{(b\,x^3+a)^{\frac{1}{3}}}{ax}$	18
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{1}{3}}}{ax}$	18
orering	$-\frac{(b\,x^3+a)^{\frac{1}{3}}}{ax}$	18

input
```
int(1/x^2/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^3+a)^(1/3)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}}\,dx=-\frac{(bx^3+a)^{\frac{1}{3}}}{ax}$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-(b*x^3 + a)^(1/3)/(a*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^2 (a + bx^3)^{2/3}}\,dx = \frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3} + 1}\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(b*x**3+a)**(2/3),x)
```

output
```
b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-1/3)/(3*a*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a + bx^3)^{2/3}}\,dx = -\frac{(bx^3 + a)^{\frac{1}{3}}}{ax}$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-(b*x^3 + a)^(1/3)/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3 + a)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a+bx^3)^{2/3}}\,dx = -\frac{(b\,x^3+a)^{1/3}}{a\,x}$$

input
```
int(1/(x^2*(a + b*x^3)^(2/3)),x)
```

output
```
-(a + b*x^3)^(1/3)/(a*x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^2}dx$$

input
```
int(1/x^2/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**2),x)
```

3.422 $\int \frac{1}{x^5(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^5(a+bx^3)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^3}}{4ax^4}+\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}$$

output
```
-1/4*(b*x^3+a)^(1/3)/a/x^4+3/4*b*(b*x^3+a)^(1/3)/a^2/x
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^5(a+bx^3)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}(-a+3bx^3)}{4a^2x^4}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(2/3)),x]
```

output
```
((a + b*x^3)^(1/3)*(-a + 3*b*x^3))/(4*a^2*x^4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{3b \int \frac{1}{x^2 (bx^3+a)^{2/3}}dx}{4a} - \frac{\sqrt[3]{a+bx^3}}{4ax^4}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{3b\sqrt[3]{a+bx^3}}{4a^2x} - \frac{\sqrt[3]{a+bx^3}}{4ax^4}$$

input
```
Int[1/(x^5*(a + b*x^3)^(2/3)),x]
```

output
```
-1/4*(a + b*x^3)^(1/3)/(a*x^4) + (3*b*(a + b*x^3)^(1/3))/(4*a^2*x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(b\,x^3+a)^{\frac{1}{3}}(-3b\,x^3+a)}{4a^2x^4}$	26
trager	$-\frac{(b\,x^3+a)^{\frac{1}{3}}(-3b\,x^3+a)}{4a^2x^4}$	26
risch	$-\frac{(b\,x^3+a)^{\frac{1}{3}}(-3b\,x^3+a)}{4a^2x^4}$	26
pseudoelliptic	$-\frac{(b\,x^3+a)^{\frac{1}{3}}(-3b\,x^3+a)}{4a^2x^4}$	26
orering	$-\frac{(b\,x^3+a)^{\frac{1}{3}}(-3b\,x^3+a)}{4a^2x^4}$	26

input
```
int(1/x^5/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x^3+a)^(1/3)*(-3*b*x^3+a)/a^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx = \frac{(3\,bx^3-a)(bx^3+a)^{\frac{1}{3}}}{4\,a^2x^4}$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/4*(3*b*x^3 - a)*(b*x^3 + a)^(1/3)/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx = -\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{9ax^3\Gamma\left(\frac{2}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{4}{3}\right)}{3a^2\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**3+a)**(2/3),x)
```

output
```
-b**(1/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(9*a*x**3*gamma(2/3)) + b**(
4/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-4/3)/(3*a**2*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx = \frac{\frac{4\left(bx^3+a\right)^{\frac{1}{3}}b}{x}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{x^4}}{4\,a^2}$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/4*(4*(b*x^3 + a)^(1/3)*b/x - (b*x^3 + a)^(4/3)/x^4)/a^2
```

Giac [F]

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^5), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^5 (a+bx^3)^{2/3}}\,dx = -\frac{(b\,x^3+a)^{1/3}\,(a-3\,b\,x^3)}{4\,a^2\,x^4}$$

input
```
int(1/(x^5*(a + b*x^3)^(2/3)),x)
```

output
```
-((a + b*x^3)^(1/3)*(a - 3*b*x^3))/(4*a^2*x^4)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^5}dx$$

input
```
int(1/x^5/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**5),x)
```

3.423 $\int \frac{1}{x^8 (a+bx^3)^{2/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}} \, dx = -\frac{\sqrt[3]{a+bx^3}}{7ax^7} + \frac{3b\sqrt[3]{a+bx^3}}{14a^2x^4} - \frac{9b^2\sqrt[3]{a+bx^3}}{14a^3x}$$

output
```
-1/7*(b*x^3+a)^(1/3)/a/x^7+3/14*b*(b*x^3+a)^(1/3)/a^2/x^4-9/14*b^2*(b*x^3+
a)^(1/3)/a^3/x
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}} \, dx = \frac{\sqrt[3]{a+bx^3}(-2a^2+3abx^3-9b^2x^6)}{14a^3x^7}$$

input
```
Integrate[1/(x^8*(a + b*x^3)^(2/3)),x]
```

output
```
((a + b*x^3)^(1/3)*(-2*a^2 + 3*a*b*x^3 - 9*b^2*x^6))/(14*a^3*x^7)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\int \frac{1}{x^5(bx^3+a)^{2/3}}dx}{7a}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{803}$$

$$-\frac{6b\left(-\frac{3b\int \frac{1}{x^2\left(bx^3+a\right)^{2/3}}dx}{4a}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}$$

$$\Big\downarrow \text{796}$$

$$-\frac{6b\left(\frac{3b\sqrt[3]{a+bx^3}}{4a^2x}-\frac{\sqrt[3]{a+bx^3}}{4ax^4}\right)}{7a}-\frac{\sqrt[3]{a+bx^3}}{7ax^7}$$

input
```
Int[1/(x^8*(a + b*x^3)^(2/3)),x]
```

output
```
-1/7*(a + b*x^3)^(1/3)/(a*x^7) - (6*b*(-1/4*(a + b*x^3)^(1/3)/(a*x^4) + (3
*b*(a + b*x^3)^(1/3))/(4*a^2*x)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{6}-3ab\,x^{3}+2a^{2}\right)}{14a^{3}x^{7}}$	39
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{6}-3ab\,x^{3}+2a^{2}\right)}{14a^{3}x^{7}}$	39
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{6}-3ab\,x^{3}+2a^{2}\right)}{14a^{3}x^{7}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{6}-3ab\,x^{3}+2a^{2}\right)}{14a^{3}x^{7}}$	39
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{6}-3ab\,x^{3}+2a^{2}\right)}{14a^{3}x^{7}}$	39

input
```
int(1/x^8/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/14*(b*x^3+a)^(1/3)*(9*b^2*x^6-3*a*b*x^3+2*a^2)/a^3/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}}\,dx = -\frac{(9\,b^2x^6-3\,abx^3+2\,a^2)(bx^3+a)^{\frac{1}{3}}}{14\,a^3x^7}$$

input
```
integrate(1/x^8/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/14*(9*b^2*x^6 - 3*a*b*x^3 + 2*a^2)*(b*x^3 + a)^(1/3)/(a^3*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(60) = 120.

Time = 0.79 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.97

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}}\,dx = \frac{4a^4b^{\frac{13}{3}}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{2a^3b^{\frac{16}{3}}x^3\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{10a^2b^{\frac{19}{3}}x^6\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{30ab^{\frac{22}{3}}x^9\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)} + \frac{18b^{\frac{25}{3}}x^{12}\sqrt[3]{\frac{a}{bx^3}+1}\Gamma\left(-\frac{7}{3}\right)}{27a^5b^4x^6\Gamma\left(\frac{2}{3}\right)+54a^4b^5x^9\Gamma\left(\frac{2}{3}\right)+27a^3b^6x^{12}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**8/(b*x**3+a)**(2/3),x)
```

output
```
4*a**4*b**(13/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b**4*x**6*ga
mma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamma(2/3)) +
 2*a**3*b**(16/3)*x**3*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5*b**4*x
**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*gamma(2
/3)) + 10*a**2*b**(19/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*a**5
*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x**12*
gamma(2/3)) + 30*a*b**(22/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(27*
a**5*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6*x*
*12*gamma(2/3)) + 18*b**(25/3)*x**12*(a/(b*x**3) + 1)**(1/3)*gamma(-7/3)/(
27*a**5*b**4*x**6*gamma(2/3) + 54*a**4*b**5*x**9*gamma(2/3) + 27*a**3*b**6
*x**12*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}}\,dx = -\frac{\frac{14\,(bx^3+a)^{\frac{1}{3}}b^2}{x} - \frac{7\,(bx^3+a)^{\frac{4}{3}}b}{x^4} + \frac{2\,(bx^3+a)^{\frac{7}{3}}}{x^7}}{14\,a^3}$$

input
```
integrate(1/x^8/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/14*(14*(b*x^3 + a)^(1/3)*b^2/x - 7*(b*x^3 + a)^(4/3)*b/x^4 + 2*(b*x^3 +
 a)^(7/3)/x^7)/a^3
```

Giac [F]

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}}\,dx = -\frac{\left(b\,x^3+a\right)^{1/3}\,\left(2\,a^2-3\,a\,b\,x^3+9\,b^2\,x^6\right)}{14\,a^3\,x^7}$$

input
```
int(1/(x^8*(a + b*x^3)^(2/3)),x)
```

output
```
-((a + b*x^3)^(1/3)*(2*a^2 + 9*b^2*x^6 - 3*a*b*x^3))/(14*a^3*x^7)
```

Reduce [F]

$$\int \frac{1}{x^8 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\,x^8}dx$$

input
```
int(1/x^8/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**8),x)
```

3.424 $\int \frac{1}{x^{11}(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{11}(a+bx^3)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^3}}{10ax^{10}} + \frac{9b\sqrt[3]{a+bx^3}}{70a^2x^7} - \frac{27b^2\sqrt[3]{a+bx^3}}{140a^3x^4} + \frac{81b^3\sqrt[3]{a+bx^3}}{140a^4x}$$

output
```
-1/10*(b*x^3+a)^(1/3)/a/x^10+9/70*b*(b*x^3+a)^(1/3)/a^2/x^7-27/140*b^2*(b*
x^3+a)^(1/3)/a^3/x^4+81/140*b^3*(b*x^3+a)^(1/3)/a^4/x
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{11}(a+bx^3)^{2/3}}\,dx = \frac{\sqrt[3]{a+bx^3}(-14a^3+18a^2bx^3-27ab^2x^6+81b^3x^9)}{140a^4x^{10}}$$

input
```
Integrate[1/(x^11*(a + b*x^3)^(2/3)),x]
```

output
```
((a + b*x^3)^(1/3)*(-14*a^3 + 18*a^2*b*x^3 - 27*a*b^2*x^6 + 81*b^3*x^9))/(
140*a^4*x^10)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11} \left(a+b x^3\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{9 b \int \frac{1}{x^8 \left(b x^3+a\right)^{2/3}}dx}{10 a}-\frac{\sqrt[3]{a+b x^3}}{10 a x^{10}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{9 b \left(-\frac{6 b \int \frac{1}{x^5 \left(b x^3+a\right)^{2/3}}dx}{7 a}-\frac{\sqrt[3]{a+b x^3}}{7 a x^7}\right)}{10 a}-\frac{\sqrt[3]{a+b x^3}}{10 a x^{10}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{9 b \left(-\frac{6 b \left(-\frac{3 b \int \frac{1}{x^2 \left(b x^3+a\right)^{2/3}}dx}{4 a}-\frac{\sqrt[3]{a+b x^3}}{4 a x^4}\right)}{7 a}-\frac{\sqrt[3]{a+b x^3}}{7 a x^7}\right)}{10 a}-\frac{\sqrt[3]{a+b x^3}}{10 a x^{10}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{9 b \left(-\frac{6 b \left(\frac{3 b \sqrt[3]{a+b x^3}}{4 a^2 x}-\frac{\sqrt[3]{a+b x^3}}{4 a x^4}\right)}{7 a}-\frac{\sqrt[3]{a+b x^3}}{7 a x^7}\right)}{10 a}-\frac{\sqrt[3]{a+b x^3}}{10 a x^{10}}$$

input
```
Int[1/(x^11*(a + b*x^3)^(2/3)),x]
```

output
```
-1/10*(a + b*x^3)^(1/3)/(a*x^10) - (9*b*(-1/7*(a + b*x^3)^(1/3)/(a*x^7) -
(6*b*(-1/4*(a + b*x^3)^(1/3)/(a*x^4) + (3*b*(a + b*x^3)^(1/3))/(4*a^2*x)))
/(7*a)))/(10*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81 b^{3} x^{9}+27 a\,b^{2} x^{6}-18 a^{2} b\,x^{3}+14 a^{3}\right)}{140 x^{10} a^{4}}$	50
trager	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81 b^{3} x^{9}+27 a\,b^{2} x^{6}-18 a^{2} b\,x^{3}+14 a^{3}\right)}{140 x^{10} a^{4}}$	50
risch	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81 b^{3} x^{9}+27 a\,b^{2} x^{6}-18 a^{2} b\,x^{3}+14 a^{3}\right)}{140 x^{10} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81 b^{3} x^{9}+27 a\,b^{2} x^{6}-18 a^{2} b\,x^{3}+14 a^{3}\right)}{140 x^{10} a^{4}}$	50
orering	$-\frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(-81 b^{3} x^{9}+27 a\,b^{2} x^{6}-18 a^{2} b\,x^{3}+14 a^{3}\right)}{140 x^{10} a^{4}}$	50

input
```
int(1/x^11/(b*x^3+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/140*(b*x^3+a)^(1/3)*(-81*b^3*x^9+27*a*b^2*x^6-18*a^2*b*x^3+14*a^3)/x^10
/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{11} (a + bx^3)^{2/3}} \, dx = \frac{(81 \, b^3 x^9 - 27 \, a b^2 x^6 + 18 \, a^2 b x^3 - 14 \, a^3)(bx^3 + a)^{\frac{1}{3}}}{140 \, a^4 x^{10}}$$

input
```
integrate(1/x^11/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
1/140*(81*b^3*x^9 - 27*a*b^2*x^6 + 18*a^2*b*x^3 - 14*a^3)*(b*x^3 + a)^(1/3
)/(a^4*x^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(83) = 166.

Time = 1.15 (sec) , antiderivative size = 692, normalized size of antiderivative = 7.52

$$\int \frac{1}{x^{11} (a + bx^3)^{2/3}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**11/(b*x**3+a)**(2/3),x)
```

output
```
-28*a**6*b**(28/3)*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9
*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma
(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) - 48*a**5*b**(31/3)*x**3*(a/(b*x**
3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10
*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*
gamma(2/3)) - 30*a**4*b**(34/3)*x**6*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/
(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5
*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 80*a**3*b**(37
/3)*x**9*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3
) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81
*a**4*b**12*x**18*gamma(2/3)) + 360*a**2*b**(40/3)*x**12*(a/(b*x**3) + 1)*
*(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*g
amma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/
3)) + 432*a*b**(43/3)*x**15*(a/(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*
b**9*x**9*gamma(2/3) + 243*a**6*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x*
*15*gamma(2/3) + 81*a**4*b**12*x**18*gamma(2/3)) + 162*b**(46/3)*x**18*(a/
(b*x**3) + 1)**(1/3)*gamma(-10/3)/(81*a**7*b**9*x**9*gamma(2/3) + 243*a**6
*b**10*x**12*gamma(2/3) + 243*a**5*b**11*x**15*gamma(2/3) + 81*a**4*b**12*
x**18*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{11} (a + bx^3)^{2/3}} \, dx = \frac{\frac{140 \, {\left(b x^{3} + a\right)}^{\frac{1}{3}} b^{3}}{x} - \frac{105 \, {\left(b x^{3} + a\right)}^{\frac{4}{3}} b^{2}}{x^{4}} + \frac{60 \, {\left(b x^{3} + a\right)}^{\frac{7}{3}} b}{x^{7}} - \frac{14 \, {\left(b x^{3} + a\right)}^{\frac{10}{3}}}{x^{10}}}{140 \, a^{4}}$$

input
```
integrate(1/x^11/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
1/140*(140*(b*x^3 + a)^(1/3)*b^3/x - 105*(b*x^3 + a)^(4/3)*b^2/x^4 + 60*(b
*x^3 + a)^(7/3)*b/x^7 - 14*(b*x^3 + a)^(10/3)/x^10)/a^4
```

Giac [F]

$$\int \frac{1}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^11), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx=\frac{9\,b\,\left(b\,x^3+a\right)^{1/3}}{70\,a^2\,x^7}-\frac{\left(b\,x^3+a\right)^{1/3}}{10\,a\,x^{10}}+\frac{81\,b^3\,\left(b\,x^3+a\right)^{1/3}}{140\,a^4\,x}-\frac{27\,b^2\,\left(b\,x^3+a\right)^{1/3}}{140\,a^3\,x^4}$$

input
```
int(1/(x^11*(a + b*x^3)^(2/3)),x)
```

output
```
(9*b*(a + b*x^3)^(1/3))/(70*a^2*x^7) - (a + b*x^3)^(1/3)/(10*a*x^10) + (81
*b^3*(a + b*x^3)^(1/3))/(140*a^4*x) - (27*b^2*(a + b*x^3)^(1/3))/(140*a^3*
x^4)
```

Reduce [F]

$$\int \frac{1}{x^{11}\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}x^{11}}dx$$

input
```
int(1/x^11/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**11),x)
```

3.425 $\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx=\frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)}{7\,(a+bx^3)^{2/3}}$$

output
```
1/7*x^7*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 7/3],[10/3],-b*x^3/a)/(b*x^3+a)^
(2/3)
```

Mathematica [A] (verified)

Time = 5.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx=\frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)}{7\,(a+bx^3)^{2/3}}$$

input
```
Integrate[x^6/(a + b*x^3)^(2/3),x]
```

output
```
(x^7*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^3)/a)]
)/(7*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \int \frac{x^6}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{x^7 \left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{2}{3},\frac{7}{3},\frac{10}{3},-\frac{bx^3}{a}\right)}{7 (a+bx^3)^{2/3}}$$

input
```
Int[x^6/(a + b*x^3)^(2/3),x]
```

output
```
(x^7*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^3)/a)]
)/(7*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^6/(b*x^3+a)^(2/3),x)
```

output
```
int(x^6/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx = \frac{x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(x**6/(b*x**3+a)**(2/3),x)
```

output
```
x**7*gamma(7/3)*hyper((2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(2/3)*gamma(10/3))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^6}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^6/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^6}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int(x^6/(a + b*x^3)^(2/3),x)
```

output
```
int(x^6/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^6}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^6/(b*x^3+a)^(2/3),x)
```

output
```
int(x**6/(a + b*x**3)**(2/3),x)
```

3.426 $\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx=\frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}$$

output
```
1/4*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a)/(b*x^3+a)^(
2/3)
```

Mathematica [A] (verified)

Time = 5.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx=\frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\,(a+bx^3)^{2/3}}$$

input
```
Integrate[x^3/(a + b*x^3)^(2/3),x]
```

output
```
(x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])
/(4*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}dx$$

$$\Big\downarrow \ 889$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\int \frac{x^3}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{\left(a+bx^3\right)^{2/3}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}$$

input
```
Int[x^3/(a + b*x^3)^(2/3),x]
```

output
```
(x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])
/(4*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^3/(b*x^3+a)^(2/3),x)
```

output
```
int(x^3/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(x^3/(b*x^3 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx = \frac{x^4\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(x**3/(b*x**3+a)**(2/3),x)
```

output
```
x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(2/3)*gamma(7/3))
```

Maxima [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*x^3 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^3/(b*x^3 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int(x^3/(a + b*x^3)^(2/3),x)
```

output
```
int(x^3/(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{x^3}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(x^3/(b*x^3+a)^(2/3),x)
```

output
```
int(x**3/(a + b*x**3)**(2/3),x)
```

3.427 $\int \frac{1}{(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \frac{3\sqrt[3]{2}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{2/3}\sqrt[3]{\frac{i\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}}\,\text{Hypergeometric2F1}}{\sqrt[3]{b}\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(a + b*x^3)^(-2/3),x]
```

output
```
(3*2^(1/3)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a^(1/3) + (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3)))^(2/3)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I
+ Sqrt[3]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 4/3, ((2*I)*Sqrt[3]*a^(1/3)
+ (3 - I*Sqrt[3])*b^(1/3)*x)/(2*(1 + (-1)^(1/3))*(a^(1/3) + b^(1/3)*x))])/
(b^(1/3)*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 779}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \int \frac{1}{\left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 778}$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}$$

input
```
Int[(a + b*x^3)^(-2/3),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
a + b*x^3)^(2/3)
```

Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Si
mplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(-2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(b*x**3+a)**(2/3),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/
3)*gamma(4/3))
```

Maxima [F]

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(\frac{b\,x^3}{a}+1\right)^{2/3}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{4}{3};\ -\frac{b\,x^3}{a}\right)}{(b\,x^3+a)^{2/3}}$$

input
```
int(1/(a + b*x^3)^(2/3),x)
```

output
```
(x*((b*x^3)/a + 1)^(2/3)*hypergeom([1/3, 2/3], 4/3, -(b*x^3)/a))/(a + b*x^
3)^(2/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(a + b*x**3)**(2/3),x)
```

3.428 $\int \frac{1}{x^3(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^3(a+bx^3)^{2/3}}\,dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2(a+bx^3)^{2/3}}$$

output
```
-1/2*(1+b*x^3/a)^(2/3)*hypergeom([-2/3, 2/3],[1/3],-b*x^3/a)/x^2/(b*x^3+a)
^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx^3)^{2/3}}\,dx = -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2(a+bx^3)^{2/3}}$$

input
```
Integrate[1/(x^3*(a + b*x^3)^(2/3)),x]
```

output
```
-1/2*((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^3)/a)
])/(x^2*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \int \frac{1}{x^3 \left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{2}{3},\frac{1}{3},-\frac{bx^3}{a}\right)}{2x^2 (a+bx^3)^{2/3}}$$

input
```
Int[1/(x^3*(a + b*x^3)^(2/3)),x]
```

output
```
-1/2*((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, 2/3, 1/3, -((b*x^3)/a)
])/(x^2*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}dx$$

input
```
int(1/x^3/(b*x^3+a)^(2/3),x)
```

output
```
int(1/x^3/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}}\,dx=\int \frac{1}{(bx^3+a)^{\frac{2}{3}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)/(b*x^6 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}}\,dx=\frac{\Gamma\left(-\frac{2}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{2}{3}, \frac{2}{3} \\ \frac{1}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**3+a)**(2/3),x)
```

output
```
gamma(-2/3)*hyper((-2/3, 2/3), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/
3)*x**2*gamma(1/3))
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}} \, dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} x^3} \, dx$$

input
```
integrate(1/x^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}}\,dx=\int \frac{1}{x^3\,{\left(b\,x^3+a\right)}^{2/3}} \,d x$$

input
```
int(1/(x^3*(a + b*x^3)^(2/3)),x)
```

output
```
int(1/(x^3*(a + b*x^3)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^3)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{3}}d x$$

input
```
int(1/x^3/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**3),x)
```

3.429 $\int \frac{1}{x^6(a+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^6\left(a+bx^3\right)^{2/3}}\,dx=-\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{2}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\left(a+bx^3\right)^{2/3}}$$

output
```
-1/5*(1+b*x^3/a)^(2/3)*hypergeom([-5/3, 2/3],[-2/3],-b*x^3/a)/x^5/(b*x^3+a
)^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6\left(a+bx^3\right)^{2/3}}\,dx=-\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{2}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[1/(x^6*(a + b*x^3)^(2/3)),x]
```

output
```
-1/5*((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, -((b*x^3)/a
)])/(x^5*(a + b*x^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \int \frac{1}{x^6 \left(\frac{bx^3}{a}+1\right)^{2/3}}dx}{(a+bx^3)^{2/3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{2}{3},-\frac{2}{3},-\frac{bx^3}{a}\right)}{5x^5 (a+bx^3)^{2/3}}$$

input
```
Int[1/(x^6*(a + b*x^3)^(2/3)),x]
```

output
```
-1/5*((1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[-5/3, 2/3, -2/3, -((b*x^3)/a
)])/(x^5*(a + b*x^3)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{3}+a \right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^6/(b*x^3+a)^(2/3),x)
```

output
```
int(1/x^6/(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)/(b*x^9 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}}\,dx = \frac{\Gamma\left(-\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{3}, \frac{2}{3} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^5\Gamma\left(-\frac{2}{3}\right)}$$

input
```
integrate(1/x**6/(b*x**3+a)**(2/3),x)
```

output
```
gamma(-5/3)*hyper((-5/3, 2/3), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*x**5*gamma(-2/3))
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{x^6 (b\,x^3+a)^{2/3}}\,dx$$

input
```
int(1/(x^6*(a + b*x^3)^(2/3)),x)
```

output
```
int(1/(x^6*(a + b*x^3)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{\frac{2}{3}}\,x^6}dx$$

input
```
int(1/x^6/(b*x^3+a)^(2/3),x)
```

output
```
int(1/((a + b*x**3)**(2/3)*x**6),x)
```

3.430 $\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8a\sqrt[3]{a+bx^3}}$$

output
```
1/8*x^8*(1+b*x^3/a)^(1/3)*hypergeom([4/3, 8/3],[11/3],-b*x^3/a)/a/(b*x^3+a
)^(1/3)
```

Mathematica [A] (verified)

Time = 6.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^7/(a + b*x^3)^(4/3),x]
```

output
```
(x^8*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 8/3, 11/3, -((b*x^3)/a)]
)/(8*a*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^7}{\left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{8}{3},\frac{11}{3},-\frac{bx^3}{a}\right)}{8a\sqrt[3]{a+bx^3}}$$

input
```
Int[x^7/(a + b*x^3)^(4/3),x]
```

output

```
(x^8*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 8/3, 11/3, -((b*x^3)/a)]
)/(8*a*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^7}{(b\,x^3+a)^{\frac{4}{3}}}dx$$

input
```
int(x^7/(b*x^3+a)^(4/3),x)
```

output
```
int(x^7/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)*x^7/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \frac{x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(x**7/(b*x**3+a)**(4/3),x)
```

output
```
x**8*gamma(8/3)*hyper((4/3, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(4/3)*gamma(11/3))
```

Maxima [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(x^7/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^7/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(x^7/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int(x^7/(a + b*x^3)^(4/3),x)
```

output
```
int(x^7/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^7}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx$$

input
```
int(x^7/(b*x^3+a)^(4/3),x)
```

output
```
int(x**7/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)
```

3.431 $\int \frac{x^4}{(a+bx^3)^{4/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^4}{(a+bx^3)^{4/3}} \, dx = \frac{x^5 \sqrt[3]{1+\frac{bx^3}{a}} \, \text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5a\sqrt[3]{a+bx^3}}$$

output
```
1/5*x^5*(1+b*x^3/a)^(1/3)*hypergeom([4/3, 5/3],[8/3],-b*x^3/a)/a/(b*x^3+a)
^(1/3)
```

Mathematica [A] (verified)

Time = 5.93 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^3)^{4/3}} \, dx = \frac{x^5 \sqrt[3]{1+\frac{bx^3}{a}} \, \text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x^4/(a + b*x^3)^(4/3),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 5/3, 8/3, -((b*x^3)/a)])
/(5*a*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x^4}{\left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{4}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5a\sqrt[3]{a+bx^3}}$$

input
```
Int[x^4/(a + b*x^3)^(4/3),x]
```

output
```
(x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 5/3, 8/3, -((b*x^3)/a)])
/(5*a*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{4}{3}}}dx$$

input
```
int(x^4/(b*x^3+a)^(4/3),x)
```

output
```
int(x^4/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)*x^4/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{(a+bx^3)^{4/3}}\,dx = \frac{x^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(x**4/(b*x**3+a)**(4/3),x)
```

output
```
x**5*gamma(5/3)*hyper((4/3, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(4/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int(x^4/(a + b*x^3)^(4/3),x)
```

output
```
int(x^4/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^3)^{4/3}}\,dx = \int \frac{x^4}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx$$

input
```
int(x^4/(b*x^3+a)^(4/3),x)
```

output
```
int(x**4/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)
```

3.432 $\int \frac{x}{(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}$$

output
```
1/2*x^2*(1+b*x^3/a)^(1/3)*hypergeom([2/3, 4/3],[5/3],-b*x^3/a)/a/(b*x^3+a)
^(1/3)
```

Mathematica [A] (verified)

Time = 5.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx=\frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}$$

input
```
Integrate[x/(a + b*x^3)^(4/3),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3, 4/3, 5/3, -((b*x^3)/a)])
/(2*a*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^3)^{4/3}}dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1}\int \frac{x}{\left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2a\sqrt[3]{a+bx^3}}$$

input
```
Int[x/(a + b*x^3)^(4/3),x]
```

output
```
(x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[2/3, 4/3, 5/3, -((b*x^3)/a)])
/(2*a*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x}{(b\,x^3+a)^{\frac{4}{3}}}dx$$

input
```
int(x/(b*x^3+a)^(4/3),x)
```

output
```
int(x/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)*x/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(b*x**3+a)**(4/3),x)
```

output
```
x**2*gamma(2/3)*hyper((2/3, 4/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**
(4/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^3 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx = \int \frac{x}{(bx^3+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(x/(b*x^3 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx = \int \frac{x}{(b\,x^3+a)^{4/3}}\,dx$$

input
```
int(x/(a + b*x^3)^(4/3),x)
```

output
```
int(x/(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^3)^{4/3}}\,dx = \int \frac{x}{(b\,x^3+a)^{\frac{1}{3}}\,a+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^3}dx$$

input
```
int(x/(b*x^3+a)^(4/3),x)
```

output
```
int(x/((a + b*x**3)**(1/3)*a + (a + b*x**3)**(1/3)*b*x**3),x)
```

3.433 $\int \frac{1}{x^2(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{1}{x^2(a+bx^3)^{4/3}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{ax\sqrt[3]{a+bx^3}}$$

output
```
-(1+b*x^3/a)^(1/3)*hypergeom([-1/3, 4/3],[2/3],-b*x^3/a)/a/x/(b*x^3+a)^(1/
3)
```

Mathematica [A] (verified)

Time = 8.91 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^3)^{4/3}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{ax\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^2*(a + b*x^3)^(4/3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, -((b*x^3)/a)])/
(a*x*(a + b*x^3)^(1/3)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^2\left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \ 888$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \, \text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx^3}{a}\right)}{ax\sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^2*(a + b*x^3)^(4/3)),x]
```

output
```
-(((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, -((b*x^3)/a)])/
(a*x*(a + b*x^3)^(1/3)))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{3}+a \right)^{\frac{4}{3}}}d x$$

input
```
int(1/x^2/(b*x^3+a)^(4/3),x)
```

output
```
int(1/x^2/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^3\right)^{4/3}}\,dx=\int \frac{1}{{\left(bx^{3}+a\right)}^{\frac{4}{3}} x^{2}}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2 (a+bx^3)^{4/3}}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}} x \Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/x**2/(b*x**3+a)**(4/3),x)
```

output
```
gamma(-1/3)*hyper((-1/3, 4/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/
3)*x*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (a+bx^3)^{4/3}} \, dx = -\frac{\left(\frac{a}{b\,x^3}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{5}{3};\ \frac{8}{3};\ -\frac{a}{b\,x^3}\right)}{5\,x\,(b\,x^3+a)^{4/3}}$$

input
```
int(1/(x^2*(a + b*x^3)^(4/3)),x)
```

output
```
-((a/(b*x^3) + 1)^(4/3)*hypergeom([4/3, 5/3], 8/3, -a/(b*x^3)))/(5*x*(a +
b*x^3)^(4/3))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^3)^{4/3}} \, dx = \int \frac{1}{(b\,x^3+a)^{\frac{1}{3}}\,a\,x^2+(b\,x^3+a)^{\frac{1}{3}}\,b\,x^5} dx$$

input
```
int(1/x^2/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*x**2 + (a + b*x**3)**(1/3)*b*x**5),x)
```

3.434 $\int \frac{1}{x^5(a+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{x^5(a+bx^3)^{4/3}}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4ax^4\sqrt[3]{a+bx^3}}$$

output
```
-1/4*(1+b*x^3/a)^(1/3)*hypergeom([-4/3, 4/3],[-1/3],-b*x^3/a)/a/x^4/(b*x^3
+a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(a+bx^3)^{4/3}}\,dx=-\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4ax^4\sqrt[3]{a+bx^3}}$$

input
```
Integrate[1/(x^5*(a + b*x^3)^(4/3)),x]
```

output
```
-1/4*((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3, 4/3, -1/3, -((b*x^3)/a
)])/(a*x^4*(a + b*x^3)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{\frac{bx^3}{a}+1} \int \frac{1}{x^5 \left(\frac{bx^3}{a}+1\right)^{4/3}}dx}{a\sqrt[3]{a+bx^3}}$$

$$\Big\downarrow \text{888}$$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1} \,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{4}{3},-\frac{1}{3},-\frac{bx^3}{a}\right)}{4ax^4\sqrt[3]{a+bx^3}}$$

input
```
Int[1/(x^5*(a + b*x^3)^(4/3)),x]
```

output

```
-1/4*((1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[-4/3, 4/3, -1/3, -((b*x^3)/a
)])/(a*x^4*(a + b*x^3)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{5} \left(b\,x^{3}+a \right)^{\frac{4}{3}}}d x$$

input
```
int(1/x^5/(b*x^3+a)^(4/3),x)
```

output
```
int(1/x^5/(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{x^5 \left(a+bx^3\right)^{4/3}}\, dx = \int \frac{1}{{\left(bx^{3} + a\right)}^{\frac{4}{3}} x^{5}}\, dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)/(b^2*x^11 + 2*a*b*x^8 + a^2*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5 (a+bx^3)^{4/3}}\,dx = \frac{\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{4}{3} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}} x^4 \Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**3+a)**(4/3),x)
```

output
```
gamma(-4/3)*hyper((-4/3, 4/3), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4
/3)*x**4*gamma(-1/3))
```

Maxima [F]

$$\int \frac{1}{x^5 (a+bx^3)^{4/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*x^5), x)
```

Giac [F]

$$\int \frac{1}{x^5 (a+bx^3)^{4/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{4}{3}} x^5}\,dx$$

input
```
integrate(1/x^5/(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(4/3)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 (a+bx^3)^{4/3}} \, dx = \int \frac{1}{x^5 \, (b \, x^3 + a)^{4/3}} \, dx$$

input
```
int(1/(x^5*(a + b*x^3)^(4/3)),x)
```

output
```
int(1/(x^5*(a + b*x^3)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^3)^{4/3}} \, dx = \int \frac{1}{(b \, x^3 + a)^{\frac{1}{3}} \, a \, x^5 + (b \, x^3 + a)^{\frac{1}{3}} \, b \, x^8} dx$$

input
```
int(1/x^5/(b*x^3+a)^(4/3),x)
```

output
```
int(1/((a + b*x**3)**(1/3)*a*x**5 + (a + b*x**3)**(1/3)*b*x**8),x)
```

3.435 $\int \frac{x}{(1-x^3)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx=-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-x-\sqrt[3]{1-x^3}\right)$$

output
```
-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)-1/2*ln(-x-(-x^3+1)
^(1/3))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.57

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx=\frac{1}{6}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}x}{x-2\sqrt[3]{1-x^3}}\right)-2\log\left(x+\sqrt[3]{1-x^3}\right)+\log\left(x^2-x\sqrt[3]{1-x^3}+\left(1-x^3\right)^{2/3}\right)\right)$$

input
```
Integrate[x/(1 - x^3)^(2/3),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(Sqrt[3]*x)/(x - 2*(1 - x^3)^(1/3))] - 2*Log[x + (1 - x
^3)^(1/3)] + Log[x^2 - x*(1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{853\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^3)^{2/3}} \, dx$$

$$\Big\downarrow \text{853}$$

$$-\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)$$

input
```
Int[x/(1 - x^3)^(2/3),x]
```

output
```
-(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) - Log[-x - (1 - x^3
)^(1/3)]/2
```

Defintions of rubi rules used

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 4.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.28

method	result
meijerg	$\frac{x^{2} \operatorname{hypergeom}\left(\left[\frac{2}{3}, \frac{2}{3}\right], \left[\frac{5}{3}\right], x^{3}\right)}{2}$
pseudoelliptic	$\frac{\ln\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+x^{2}}{x^{2}}\right)}{6}+\frac{\sqrt{3}\, \arctan\left(\frac{\left(-2\left(-x^{3}+1\right)^{\frac{1}{3}}+x\right) \sqrt{3}}{3 x}\right)}{3}-\frac{\ln\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)}{3}$
trager	$-\frac{\ln\left(-2 \operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)^{2} x^{3}+3 \operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}} x-\operatorname{RootOf}\left(\_Z^{2}-\_Z+1\right) x^{3}+3\left(-x^{3}+1\right.\right.}{3}$

input
```
int(x/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*hypergeom([2/3,2/3],[5/3],x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.55

$$\int \frac{x}{\left(1-x^{3}\right)^{2/3}}\, dx = -\frac{1}{3}\, \sqrt{3} \arctan\left(-\frac{\sqrt{3} x-2\, \sqrt{3}\left(-x^{3}+1\right)^{\frac{1}{3}}}{3\, x}\right) - \frac{1}{3}\, \log\left(\frac{x+\left(-x^{3}+1\right)^{\frac{1}{3}}}{x}\right)+\frac{1}{6}\, \log\left(\frac{x^{2}-\left(-x^{3}+1\right)^{\frac{1}{3}} x+\left(-x^{3}+1\right)^{\frac{2}{3}}}{x^{2}}\right)$$

input
```
integrate(x/(-x^3+1)^(2/3),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 1/3
*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3
 + 1)^(2/3))/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx = \frac{x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(x/(-x**3+1)**(2/3),x)
```

output
```
x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), x**3*exp_polar(2*I*pi))/(3*gamma
(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2(-x^3+1)^{\frac{1}{3}}}{x}-1\right)\right) - \frac{1}{3}\log\left(\frac{(-x^3+1)^{\frac{1}{3}}}{x}+1\right) + \frac{1}{6}\log\left(-\frac{(-x^3+1)^{\frac{1}{3}}}{x}+\frac{(-x^3+1)^{\frac{2}{3}}}{x^2}+1\right)$$

input
```
integrate(x/(-x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3)/x - 1)) - 1/3*log((-x^
3 + 1)^(1/3)/x + 1) + 1/6*log(-(-x^3 + 1)^(1/3)/x + (-x^3 + 1)^(2/3)/x^2 +
 1)
```

Giac [F]

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx = \int \frac{x}{(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x/(-x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate(x/(-x^3 + 1)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx = \int \frac{x}{(1-x^3)^{2/3}}\,dx$$

input
```
int(x/(1 - x^3)^(2/3),x)
```

output
```
int(x/(1 - x^3)^(2/3), x)
```

Reduce [F]

$$\int \frac{x}{(1-x^3)^{2/3}}\,dx = \int \frac{x}{(-x^3+1)^{\frac{2}{3}}}\,dx$$

input
```
int(x/(-x^3+1)^(2/3),x)
```

output
```
int(x/( - x**3 + 1)**(2/3),x)
```

3.436 $\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4}{9}\left(2+x^3\right)^{3/4}$$

output
```
4/9*(x^3+2)^(3/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4}{9}\left(2+x^3\right)^{3/4}$$

input
```
Integrate[x^2/(2 + x^3)^(1/4),x]
```

output
```
(4*(2 + x^3)^(3/4))/9
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{x^3+2}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{4}{9}\left(x^3+2\right)^{3/4}$$

input
```
Int[x^2/(2 + x^3)^(1/4),x]
```

output
```
(4*(2 + x^3)^(3/4))/9
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
derivativedivides	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
default	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
trager	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
risch	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
pseudoelliptic	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
orering	$\frac{4(x^3+2)^{\frac{3}{4}}}{9}$	10
meijerg	$\frac{2^{\frac{3}{4}} x^3 \operatorname{hypergeom}\left([\frac{1}{4},1],[2],-\frac{x^3}{2}\right)}{6}$	20

input
```
int(x^2/(x^3+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/9*(x^3+2)^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4}{9}\left(x^3+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^2/(x^3+2)^(1/4),x, algorithm="fricas")
```

output
```
4/9*(x^3 + 2)^(3/4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4(x^3+2)^{\frac{3}{4}}}{9}$$

input
```
integrate(x**2/(x**3+2)**(1/4),x)
```

output
```
4*(x**3 + 2)**(3/4)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4}{9}\left(x^3+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^2/(x^3+2)^(1/4),x, algorithm="maxima")
```

output
```
4/9*(x^3 + 2)^(3/4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4}{9}\left(x^3+2\right)^{\frac{3}{4}}$$

input
```
integrate(x^2/(x^3+2)^(1/4),x, algorithm="giac")
```

output
```
4/9*(x^3 + 2)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \frac{4\,(x^3+2)^{3/4}}{9}$$

input
```
int(x^2/(x^3 + 2)^(1/4),x)
```

output
```
(4*(x^3 + 2)^(3/4))/9
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{2+x^3}}\,dx = \int \frac{x^2}{(x^3+2)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(x^3+2)^(1/4),x)
```

output
```
int(x**2/(x**3 + 2)**(1/4),x)
```

3.437 $\int x^m(a+bx^3)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 151

$$\int x^m\left(a+bx^3\right)^8\,dx=\frac{a^8x^{1+m}}{1+m}+\frac{8a^7bx^{4+m}}{4+m}+\frac{28a^6b^2x^{7+m}}{7+m}+\frac{56a^5b^3x^{10+m}}{10+m}+\frac{70a^4b^4x^{13+m}}{13+m}+\frac{56a^3b^5x^{16+m}}{16+m}+\frac{28a^2b^6x^{19+m}}{19+m}+\frac{8ab^7x^{22+m}}{22+m}+\frac{b^8x^{25+m}}{25+m}$$

output
```
a^8*x^(1+m)/(1+m)+8*a^7*b*x^(4+m)/(4+m)+28*a^6*b^2*x^(7+m)/(7+m)+56*a^5*b^
3*x^(10+m)/(10+m)+70*a^4*b^4*x^(13+m)/(13+m)+56*a^3*b^5*x^(16+m)/(16+m)+28
*a^2*b^6*x^(19+m)/(19+m)+8*a*b^7*x^(22+m)/(22+m)+b^8*x^(25+m)/(25+m)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

$$\int x^m\left(a+bx^3\right)^8\,dx=x^{1+m}\left(\frac{a^8}{1+m}+\frac{8a^7bx^3}{4+m}+\frac{28a^6b^2x^6}{7+m}+\frac{56a^5b^3x^9}{10+m}+\frac{70a^4b^4x^{12}}{13+m}+\frac{56a^3b^5x^{15}}{16+m}+\frac{28a^2b^6x^{18}}{19+m}+\frac{8ab^7x^{21}}{22+m}+\frac{b^8x^{24}}{25+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^8,x]
```

output
```
x^(1 + m)*(a^8/(1 + m) + (8*a^7*b*x^3)/(4 + m) + (28*a^6*b^2*x^6)/(7 + m)
+ (56*a^5*b^3*x^9)/(10 + m) + (70*a^4*b^4*x^12)/(13 + m) + (56*a^3*b^5*x^1
5)/(16 + m) + (28*a^2*b^6*x^18)/(19 + m) + (8*a*b^7*x^21)/(22 + m) + (b^8*
x^24)/(25 + m))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^8 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^8x^m+8a^7bx^{m+3}+28a^6b^2x^{m+6}+56a^5b^3x^{m+9}+70a^4b^4x^{m+12}+56a^3b^5x^{m+15}+28a^2b^6x^{m+18}+8ab^7x^{m+2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^{m+1}}{m+1}+\frac{8a^7bx^{m+4}}{m+4}+\frac{28a^6b^2x^{m+7}}{m+7}+\frac{56a^5b^3x^{m+10}}{m+10}+\frac{70a^4b^4x^{m+13}}{m+13}+\frac{56a^3b^5x^{m+16}}{m+16}+\frac{28a^2b^6x^{m+19}}{m+19}+\frac{8ab^7x^{m+22}}{m+22}+\frac{b^8x^{m+25}}{m+25}$$

input
```
Int[x^m*(a + b*x^3)^8,x]
```

output
```
(a^8*x^(1 + m))/(1 + m) + (8*a^7*b*x^(4 + m))/(4 + m) + (28*a^6*b^2*x^(7 +
 m))/(7 + m) + (56*a^5*b^3*x^(10 + m))/(10 + m) + (70*a^4*b^4*x^(13 + m))/
(13 + m) + (56*a^3*b^5*x^(16 + m))/(16 + m) + (28*a^2*b^6*x^(19 + m))/(19
+ m) + (8*a*b^7*x^(22 + m))/(22 + m) + (b^8*x^(25 + m))/(25 + m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(151) = 302$.

Time = 2.65 (sec) , antiderivative size = 1022, normalized size of antiderivative = 6.77

method	result	size
risch	Expression too large to display	1022
orering	Expression too large to display	1022
gosper	Expression too large to display	1023
parallelrisch	Expression too large to display	1270

input
```
int(x^m*(b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

output
```
x*(b^8*m^8*x^24+92*b^8*m^7*x^24+3514*b^8*m^6*x^24+8*a*b^7*m^8*x^21+72128*b
^8*m^5*x^24+760*a*b^7*m^7*x^21+859369*b^8*m^4*x^24+29792*a*b^7*m^6*x^21+59
74388*b^8*m^3*x^24+28*a^2*b^6*m^8*x^18+624400*a*b^7*m^5*x^21+22963996*b^8*
m^2*x^24+2744*a^2*b^6*m^7*x^18+7563752*a*b^7*m^4*x^21+42124592*b^8*m*x^24+
110656*a^2*b^6*m^6*x^18+53266360*a*b^7*m^3*x^21+24344320*b^8*x^24+56*a^3*b
^5*m^8*x^15+2376920*a^2*b^6*m^5*x^18+206729648*a*b^7*m^2*x^21+5656*a^3*b^5
*m^7*x^15+29390452*a^2*b^6*m^4*x^18+381743680*a*b^7*m*x^21+235088*a^3*b^5*
m^6*x^15+210422576*a^2*b^6*m^3*x^18+221312000*a*b^7*x^21+70*a^4*b^4*m^8*x^
12+5197360*a^3*b^5*m^5*x^15+827034544*a^2*b^6*m^2*x^18+7280*a^4*b^4*m^7*x^
12+65946104*a^3*b^5*m^4*x^15+1540629440*a^2*b^6*m*x^18+312340*a^4*b^4*m^6*
x^12+482544664*a^3*b^5*m^3*x^15+896896000*a^2*b^6*x^18+56*a^5*b^3*m^8*x^9+
7138040*a^4*b^4*m^5*x^12+1929412352*a^3*b^5*m^2*x^15+5992*a^5*b^3*m^7*x^9+
93585310*a^4*b^4*m^4*x^12+3637973920*a^3*b^5*m*x^15+265664*a^5*b^3*m^6*x^9
+705493880*a^4*b^4*m^3*x^12+2130128000*a^3*b^5*x^15+28*a^6*b^2*m^8*x^6+630
2128*a^5*b^3*m^5*x^9+2891238280*a^4*b^4*m^2*x^12+3080*a^6*b^2*m^7*x^6+8608
2584*a^5*b^3*m^4*x^9+5549616800*a^4*b^4*m*x^12+141232*a^6*b^2*m^6*x^6+6768
56488*a^5*b^3*m^3*x^9+3277120000*a^4*b^4*x^12+8*a^7*b*m^8*x^3+3490760*a^6*
b^2*m^5*x^6+2881562096*a^5*b^3*m^2*x^9+904*a^7*b*m^7*x^3+50116612*a^6*b^2*
m^4*x^6+5692950592*a^5*b^3*m*x^9+42896*a^7*b*m^6*x^3+418024880*a^6*b^2*m^3
*x^6+3408204800*a^5*b^3*x^9+a^8*m^8+1108240*a^7*b*m^5*x^3+1898889328*a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(151) = 302$.

Time = 0.14 (sec) , antiderivative size = 847, normalized size of antiderivative = 5.61

$$\int x^m \left(a+bx^3\right)^8 \, dx = \text{Too large to display}$$

input
```
integrate(x^m*(b*x^3+a)^8,x, algorithm="fricas")
```

output
```
((b^8*m^8 + 92*b^8*m^7 + 3514*b^8*m^6 + 72128*b^8*m^5 + 859369*b^8*m^4 + 5
974388*b^8*m^3 + 22963996*b^8*m^2 + 42124592*b^8*m + 24344320*b^8)*x^25 +
8*(a*b^7*m^8 + 95*a*b^7*m^7 + 3724*a*b^7*m^6 + 78050*a*b^7*m^5 + 945469*a*
b^7*m^4 + 6658295*a*b^7*m^3 + 25841206*a*b^7*m^2 + 47717960*a*b^7*m + 2766
4000*a*b^7)*x^22 + 28*(a^2*b^6*m^8 + 98*a^2*b^6*m^7 + 3952*a^2*b^6*m^6 + 8
4890*a^2*b^6*m^5 + 1049659*a^2*b^6*m^4 + 7515092*a^2*b^6*m^3 + 29536948*a^
2*b^6*m^2 + 55022480*a^2*b^6*m + 32032000*a^2*b^6)*x^19 + 56*(a^3*b^5*m^8
+ 101*a^3*b^5*m^7 + 4198*a^3*b^5*m^6 + 92810*a^3*b^5*m^5 + 1177609*a^3*b^5
*m^4 + 8616869*a^3*b^5*m^3 + 34453792*a^3*b^5*m^2 + 64963820*a^3*b^5*m + 3
8038000*a^3*b^5)*x^16 + 70*(a^4*b^4*m^8 + 104*a^4*b^4*m^7 + 4462*a^4*b^4*m
^6 + 101972*a^4*b^4*m^5 + 1336933*a^4*b^4*m^4 + 10078484*a^4*b^4*m^3 + 413
03404*a^4*b^4*m^2 + 79280240*a^4*b^4*m + 46816000*a^4*b^4)*x^13 + 56*(a^5*
b^3*m^8 + 107*a^5*b^3*m^7 + 4744*a^5*b^3*m^6 + 112538*a^5*b^3*m^5 + 153718
9*a^5*b^3*m^4 + 12086723*a^5*b^3*m^3 + 51456466*a^5*b^3*m^2 + 101659832*a^
5*b^3*m + 60860800*a^5*b^3)*x^10 + 28*(a^6*b^2*m^8 + 110*a^6*b^2*m^7 + 504
4*a^6*b^2*m^6 + 124670*a^6*b^2*m^5 + 1789879*a^6*b^2*m^4 + 14929460*a^6*b^
2*m^3 + 67817476*a^6*b^2*m^2 + 141502160*a^6*b^2*m + 86944000*a^6*b^2)*x^7
 + 8*(a^7*b*m^8 + 113*a^7*b*m^7 + 5362*a^7*b*m^6 + 138530*a^7*b*m^5 + 2108
449*a^7*b*m^4 + 19024817*a^7*b*m^3 + 96224428*a^7*b*m^2 + 231326780*a^7*b*
m + 152152000*a^7*b)*x^4 + (a^8*m^8 + 116*a^8*m^7 + 5698*a^8*m^6 + 1542...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5902 vs. $2(138) = 276$.

Time = 2.52 (sec) , antiderivative size = 5902, normalized size of antiderivative = 39.09

$$\int x^m (a + bx^3)^8 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**3+a)**8,x)
```

output
```
Piecewise((-a**8/(24*x**24) - 8*a**7*b/(21*x**21) - 14*a**6*b**2/(9*x**18)
 - 56*a**5*b**3/(15*x**15) - 35*a**4*b**4/(6*x**12) - 56*a**3*b**5/(9*x**9
) - 14*a**2*b**6/(3*x**6) - 8*a*b**7/(3*x**3) + b**8*log(x), Eq(m, -25)),
(-a**8/(21*x**21) - 4*a**7*b/(9*x**18) - 28*a**6*b**2/(15*x**15) - 14*a**5
*b**3/(3*x**12) - 70*a**4*b**4/(9*x**9) - 28*a**3*b**5/(3*x**6) - 28*a**2*
b**6/(3*x**3) + 8*a*b**7*log(x) + b**8*x**3/3, Eq(m, -22)), (-a**8/(18*x**
18) - 8*a**7*b/(15*x**15) - 7*a**6*b**2/(3*x**12) - 56*a**5*b**3/(9*x**9)
- 35*a**4*b**4/(3*x**6) - 56*a**3*b**5/(3*x**3) + 28*a**2*b**6*log(x) + 8*
a*b**7*x**3/3 + b**8*x**6/6, Eq(m, -19)), (-a**8/(15*x**15) - 2*a**7*b/(3*
x**12) - 28*a**6*b**2/(9*x**9) - 28*a**5*b**3/(3*x**6) - 70*a**4*b**4/(3*x
**3) + 56*a**3*b**5*log(x) + 28*a**2*b**6*x**3/3 + 4*a*b**7*x**6/3 + b**8*
x**9/9, Eq(m, -16)), (-a**8/(12*x**12) - 8*a**7*b/(9*x**9) - 14*a**6*b**2/
(3*x**6) - 56*a**5*b**3/(3*x**3) + 70*a**4*b**4*log(x) + 56*a**3*b**5*x**3
/3 + 14*a**2*b**6*x**6/3 + 8*a*b**7*x**9/9 + b**8*x**12/12, Eq(m, -13)), (
-a**8/(9*x**9) - 4*a**7*b/(3*x**6) - 28*a**6*b**2/(3*x**3) + 56*a**5*b**3*
log(x) + 70*a**4*b**4*x**3/3 + 28*a**3*b**5*x**6/3 + 28*a**2*b**6*x**9/9 +
 2*a*b**7*x**12/3 + b**8*x**15/15, Eq(m, -10)), (-a**8/(6*x**6) - 8*a**7*b
/(3*x**3) + 28*a**6*b**2*log(x) + 56*a**5*b**3*x**3/3 + 35*a**4*b**4*x**6/
3 + 56*a**3*b**5*x**9/9 + 7*a**2*b**6*x**12/3 + 8*a*b**7*x**15/15 + b**8*x
**18/18, Eq(m, -7)), (-a**8/(3*x**3) + 8*a**7*b*log(x) + 28*a**6*b**2*x...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int x^m \left(a+bx^3\right)^8 \,dx = \frac{b^8 x^{m+25}}{m+25} + \frac{8\,ab^7 x^{m+22}}{m+22} + \frac{28\,a^2 b^6 x^{m+19}}{m+19} + \frac{56\,a^3 b^5 x^{m+16}}{m+16} + \frac{70\,a^4 b^4 x^{m+13}}{m+13} + \frac{56\,a^5 b^3 x^{m+10}}{m+10} + \frac{28\,a^6 b^2 x^{m+7}}{m+7} + \frac{8\,a^7 b x^{m+4}}{m+4} + \frac{a^8 x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^8,x, algorithm="maxima")
```

output
```
b^8*x^(m + 25)/(m + 25) + 8*a*b^7*x^(m + 22)/(m + 22) + 28*a^2*b^6*x^(m +
19)/(m + 19) + 56*a^3*b^5*x^(m + 16)/(m + 16) + 70*a^4*b^4*x^(m + 13)/(m +
 13) + 56*a^5*b^3*x^(m + 10)/(m + 10) + 28*a^6*b^2*x^(m + 7)/(m + 7) + 8*a
^7*b*x^(m + 4)/(m + 4) + a^8*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1269 vs. $2(151) = 302$.

Time = 0.19 (sec) , antiderivative size = 1269, normalized size of antiderivative = 8.40

$$\int x^m (a + bx^3)^8 \, dx = \text{Too large to display}$$

input
```
integrate(x^m*(b*x^3+a)^8,x, algorithm="giac")
```

output
```
(b^8*m^8*x^25*x^m + 92*b^8*m^7*x^25*x^m + 3514*b^8*m^6*x^25*x^m + 8*a*b^7*
m^8*x^22*x^m + 72128*b^8*m^5*x^25*x^m + 760*a*b^7*m^7*x^22*x^m + 859369*b^
8*m^4*x^25*x^m + 29792*a*b^7*m^6*x^22*x^m + 5974388*b^8*m^3*x^25*x^m + 28*
a^2*b^6*m^8*x^19*x^m + 624400*a*b^7*m^5*x^22*x^m + 22963996*b^8*m^2*x^25*x
^m + 2744*a^2*b^6*m^7*x^19*x^m + 7563752*a*b^7*m^4*x^22*x^m + 42124592*b^8
*m*x^25*x^m + 110656*a^2*b^6*m^6*x^19*x^m + 53266360*a*b^7*m^3*x^22*x^m +
24344320*b^8*x^25*x^m + 56*a^3*b^5*m^8*x^16*x^m + 2376920*a^2*b^6*m^5*x^19
*x^m + 206729648*a*b^7*m^2*x^22*x^m + 5656*a^3*b^5*m^7*x^16*x^m + 29390452
*a^2*b^6*m^4*x^19*x^m + 381743680*a*b^7*m*x^22*x^m + 235088*a^3*b^5*m^6*x^
16*x^m + 210422576*a^2*b^6*m^3*x^19*x^m + 221312000*a*b^7*x^22*x^m + 70*a^
4*b^4*m^8*x^13*x^m + 5197360*a^3*b^5*m^5*x^16*x^m + 827034544*a^2*b^6*m^2*
x^19*x^m + 7280*a^4*b^4*m^7*x^13*x^m + 65946104*a^3*b^5*m^4*x^16*x^m + 154
0629440*a^2*b^6*m*x^19*x^m + 312340*a^4*b^4*m^6*x^13*x^m + 482544664*a^3*b
^5*m^3*x^16*x^m + 896896000*a^2*b^6*x^19*x^m + 56*a^5*b^3*m^8*x^10*x^m + 7
138040*a^4*b^4*m^5*x^13*x^m + 1929412352*a^3*b^5*m^2*x^16*x^m + 5992*a^5*b
^3*m^7*x^10*x^m + 93585310*a^4*b^4*m^4*x^13*x^m + 3637973920*a^3*b^5*m*x^1
6*x^m + 265664*a^5*b^3*m^6*x^10*x^m + 705493880*a^4*b^4*m^3*x^13*x^m + 213
0128000*a^3*b^5*x^16*x^m + 28*a^6*b^2*m^8*x^7*x^m + 6302128*a^5*b^3*m^5*x^
10*x^m + 2891238280*a^4*b^4*m^2*x^13*x^m + 3080*a^6*b^2*m^7*x^7*x^m + 8608
2584*a^5*b^3*m^4*x^10*x^m + 5549616800*a^4*b^4*m*x^13*x^m + 141232*a^6*...
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 860, normalized size of antiderivative = 5.70

$$\int x^m \left(a+bx^3\right)^8 \, dx = \text{Too large to display}$$

input
```
int(x^m*(a + b*x^3)^8,x)
```

output
```
(a^8*x*x^m*(468851120*m + 147373372*m^2 + 24950324*m^3 + 2508289*m^4 + 154
280*m^5 + 5698*m^6 + 116*m^7 + m^8 + 608608000))/(1077459120*m + 616224492
*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7
+ 117*m^8 + m^9 + 608608000) + (b^8*x^m*x^25*(42124592*m + 22963996*m^2 +
5974388*m^3 + 859369*m^4 + 72128*m^5 + 3514*m^6 + 92*m^7 + m^8 + 24344320)
)/(1077459120*m + 616224492*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m
^5 + 159978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 608608000) + (28*a^2*b^6*x^m*
x^19*(55022480*m + 29536948*m^2 + 7515092*m^3 + 1049659*m^4 + 84890*m^5 +
3952*m^6 + 98*m^7 + m^8 + 32032000))/(1077459120*m + 616224492*m^2 + 17232
3696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 117*m^8 +
m^9 + 608608000) + (56*a^3*b^5*x^m*x^16*(64963820*m + 34453792*m^2 + 86168
69*m^3 + 1177609*m^4 + 92810*m^5 + 4198*m^6 + 101*m^7 + m^8 + 38038000))/(
1077459120*m + 616224492*m^2 + 172323696*m^3 + 27458613*m^4 + 2662569*m^5
+ 159978*m^6 + 5814*m^7 + 117*m^8 + m^9 + 608608000) + (70*a^4*b^4*x^m*x^1
3*(79280240*m + 41303404*m^2 + 10078484*m^3 + 1336933*m^4 + 101972*m^5 + 4
462*m^6 + 104*m^7 + m^8 + 46816000))/(1077459120*m + 616224492*m^2 + 17232
3696*m^3 + 27458613*m^4 + 2662569*m^5 + 159978*m^6 + 5814*m^7 + 117*m^8 +
m^9 + 608608000) + (56*a^5*b^3*x^m*x^10*(101659832*m + 51456466*m^2 + 1208
6723*m^3 + 1537189*m^4 + 112538*m^5 + 4744*m^6 + 107*m^7 + m^8 + 60860800)
)/(1077459120*m + 616224492*m^2 + 172323696*m^3 + 27458613*m^4 + 266256...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1021, normalized size of antiderivative = 6.76

$$\int x^m \left(a+bx^3\right)^8 \, dx = \text{Too large to display}$$

input
```
int(x^m*(b*x^3+a)^8,x)
```

output

```
(x**m*x*(a**8*m**8 + 116*a**8*m**7 + 5698*a**8*m**6 + 154280*a**8*m**5 + 2
508289*a**8*m**4 + 24950324*a**8*m**3 + 147373372*a**8*m**2 + 468851120*a*
*8*m + 608608000*a**8 + 8*a**7*b*m**8*x**3 + 904*a**7*b*m**7*x**3 + 42896*
a**7*b*m**6*x**3 + 1108240*a**7*b*m**5*x**3 + 16867592*a**7*b*m**4*x**3 +
152198536*a**7*b*m**3*x**3 + 769795424*a**7*b*m**2*x**3 + 1850614240*a**7*
b*m*x**3 + 1217216000*a**7*b*x**3 + 28*a**6*b**2*m**8*x**6 + 3080*a**6*b**
2*m**7*x**6 + 141232*a**6*b**2*m**6*x**6 + 3490760*a**6*b**2*m**5*x**6 + 5
0116612*a**6*b**2*m**4*x**6 + 418024880*a**6*b**2*m**3*x**6 + 1898889328*a
**6*b**2*m**2*x**6 + 3962060480*a**6*b**2*m*x**6 + 2434432000*a**6*b**2*x*
*6 + 56*a**5*b**3*m**8*x**9 + 5992*a**5*b**3*m**7*x**9 + 265664*a**5*b**3*
m**6*x**9 + 6302128*a**5*b**3*m**5*x**9 + 86082584*a**5*b**3*m**4*x**9 + 6
76856488*a**5*b**3*m**3*x**9 + 2881562096*a**5*b**3*m**2*x**9 + 5692950592
*a**5*b**3*m*x**9 + 3408204800*a**5*b**3*x**9 + 70*a**4*b**4*m**8*x**12 +
7280*a**4*b**4*m**7*x**12 + 312340*a**4*b**4*m**6*x**12 + 7138040*a**4*b**
4*m**5*x**12 + 93585310*a**4*b**4*m**4*x**12 + 705493880*a**4*b**4*m**3*x*
*12 + 2891238280*a**4*b**4*m**2*x**12 + 5549616800*a**4*b**4*m*x**12 + 327
7120000*a**4*b**4*x**12 + 56*a**3*b**5*m**8*x**15 + 5656*a**3*b**5*m**7*x*
*15 + 235088*a**3*b**5*m**6*x**15 + 5197360*a**3*b**5*m**5*x**15 + 6594610
4*a**3*b**5*m**4*x**15 + 482544664*a**3*b**5*m**3*x**15 + 1929412352*a**3*
b**5*m**2*x**15 + 3637973920*a**3*b**5*m*x**15 + 2130128000*a**3*b**5*x...
```

3.438 $\int x^m(a+bx^3)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int x^m\left(a+bx^3\right)^5\,dx=\frac{a^5x^{1+m}}{1+m}+\frac{5a^4bx^{4+m}}{4+m}+\frac{10a^3b^2x^{7+m}}{7+m}+\frac{10a^2b^3x^{10+m}}{10+m}+\frac{5ab^4x^{13+m}}{13+m}+\frac{b^5x^{16+m}}{16+m}$$

output
```
a^5*x^(1+m)/(1+m)+5*a^4*b*x^(4+m)/(4+m)+10*a^3*b^2*x^(7+m)/(7+m)+10*a^2*b^
3*x^(10+m)/(10+m)+5*a*b^4*x^(13+m)/(13+m)+b^5*x^(16+m)/(16+m)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int x^m\left(a+bx^3\right)^5\,dx=x^{1+m}\left(\frac{a^5}{1+m}+\frac{5a^4bx^3}{4+m}+\frac{10a^3b^2x^6}{7+m}+\frac{10a^2b^3x^9}{10+m}+\frac{5ab^4x^{12}}{13+m}+\frac{b^5x^{15}}{16+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^5,x]
```

output
```
x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10*a^3*b^2*x^6)/(7 + m)
+ (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m) + (b^5*x^15)/(16 + m
))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^5 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^5x^m+5a^4bx^{m+3}+10a^3b^2x^{m+6}+10a^2b^3x^{m+9}+5ab^4x^{m+12}+b^5x^{m+15}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5x^{m+1}}{m+1}+\frac{5a^4bx^{m+4}}{m+4}+\frac{10a^3b^2x^{m+7}}{m+7}+\frac{10a^2b^3x^{m+10}}{m+10}+\frac{5ab^4x^{m+13}}{m+13}+\frac{b^5x^{m+16}}{m+16}$$

input
```
Int[x^m*(a + b*x^3)^5,x]
```

output
```
(a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(4 + m))/(4 + m) + (10*a^3*b^2*x^(7 +
 m))/(7 + m) + (10*a^2*b^3*x^(10 + m))/(10 + m) + (5*a*b^4*x^(13 + m))/(13
 + m) + (b^5*x^(16 + m))/(16 + m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(97) = 194$.

Time = 0.91 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.44

method	result
risch	$x(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5$
orering	$x(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5$
gosper	$x^{1+m}(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+36$
parallelrisch	$2555x^{13}x^m a\,b^4m^3+14810x^{13}x^m a\,b^4m^2+10x^{10}x^m a^2b^3m^5+34840x^{13}x^m a\,b^4m+410x^{10}x^m a^2b^3m^4+5950x^{10}x^m a^2b^3m^3+3$

input
```
int(x^m*(b*x^3+a)^5,x,method=_RETURNVERBOSE)
```

output
```
x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5
*m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*
x^15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b
^4*m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a
^2*b^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^
6+58240*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^
3+123920*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*
a^4*b*m^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+86
50*a^5*m^2+36824*a^5*m+58240*a^5)*x^m/(1+m)/(4+m)/(7+m)/(10+m)/(13+m)/(16+
m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(97) = 194$.

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.78

$$\int x^m (a + bx^3)^5 \, dx = \frac{((b^5m^5 + 35\, b^5m^4 + 445\, b^5m^3 + 2485\, b^5m^2 + 5714\, b^5m + 3640\, b^5)x^{16} + 5\,(ab^4m^5 + 38\, ab^4m^4 + 511\, ab^4m}{}$$

input
```
integrate(x^m*(b*x^3+a)^5,x, algorithm="fricas")
```

output
```
((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^
5)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 6
968*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^
2*b^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a
^3*b^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a
^3*b^2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3
 + 6337*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m
^4 + 955*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*x^m/(m^6 + 5
1*m^5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(87) = 174$.

Time = 1.07 (sec) , antiderivative size = 2006, normalized size of antiderivative = 20.68

$$\int x^m (a + bx^3)^5 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**3+a)**5,x)
```

output
```
Piecewise((-a**5/(15*x**15) - 5*a**4*b/(12*x**12) - 10*a**3*b**2/(9*x**9)
- 5*a**2*b**3/(3*x**6) - 5*a*b**4/(3*x**3) + b**5*log(x), Eq(m, -16)), (-a
**5/(12*x**12) - 5*a**4*b/(9*x**9) - 5*a**3*b**2/(3*x**6) - 10*a**2*b**3/(
3*x**3) + 5*a*b**4*log(x) + b**5*x**3/3, Eq(m, -13)), (-a**5/(9*x**9) - 5*
a**4*b/(6*x**6) - 10*a**3*b**2/(3*x**3) + 10*a**2*b**3*log(x) + 5*a*b**4*x
**3/3 + b**5*x**6/6, Eq(m, -10)), (-a**5/(6*x**6) - 5*a**4*b/(3*x**3) + 10
*a**3*b**2*log(x) + 10*a**2*b**3*x**3/3 + 5*a*b**4*x**6/6 + b**5*x**9/9, E
q(m, -7)), (-a**5/(3*x**3) + 5*a**4*b*log(x) + 10*a**3*b**2*x**3/3 + 5*a**
2*b**3*x**6/3 + 5*a*b**4*x**9/9 + b**5*x**12/12, Eq(m, -4)), (a**5*log(x)
+ 5*a**4*b*x**3/3 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**9/9 + 5*a*b**4*x*
*12/12 + b**5*x**15/15, Eq(m, -1)), (a**5*m**5*x*x**m/(m**6 + 51*m**5 + 10
05*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 50*a**5*m**4*x*x**m/
(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) +
955*a**5*m**3*x*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2
+ 95064*m + 58240) + 8650*a**5*m**2*x*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9
605*m**3 + 45474*m**2 + 95064*m + 58240) + 36824*a**5*m*x*x**m/(m**6 + 51*
m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240) + 58240*a**5*
x*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58
240) + 5*a**4*b*m**5*x**4*x**m/(m**6 + 51*m**5 + 1005*m**4 + 9605*m**3 + 4
5474*m**2 + 95064*m + 58240) + 235*a**4*b*m**4*x**4*x**m/(m**6 + 51*m**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int x^m (a+bx^3)^5 \, dx = \frac{b^5 x^{m+16}}{m+16} + \frac{5\, ab^4 x^{m+13}}{m+13} + \frac{10\, a^2 b^3 x^{m+10}}{m+10} + \frac{10\, a^3 b^2 x^{m+7}}{m+7} + \frac{5\, a^4 b x^{m+4}}{m+4} + \frac{a^5 x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^5,x, algorithm="maxima")
```

output
```
b^5*x^(m + 16)/(m + 16) + 5*a*b^4*x^(m + 13)/(m + 13) + 10*a^2*b^3*x^(m +
10)/(m + 10) + 10*a^3*b^2*x^(m + 7)/(m + 7) + 5*a^4*b*x^(m + 4)/(m + 4) +
a^5*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(97) = 194$.

Time = 0.15 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.57

$$\int x^m (a + bx^3)^5 \, dx$$
$$= \frac{b^5m^5x^{16}x^m + 35\,b^5m^4x^{16}x^m + 445\,b^5m^3x^{16}x^m + 5\,ab^4m^5x^{13}x^m + 2485\,b^5m^2x^{16}x^m + 190\,ab^4m^4x^{13}x^m +}{}$$

input
```
integrate(x^m*(b*x^3+a)^5,x, algorithm="giac")
```

output
```
(b^5*m^5*x^16*x^m + 35*b^5*m^4*x^16*x^m + 445*b^5*m^3*x^16*x^m + 5*a*b^4*m
^5*x^13*x^m + 2485*b^5*m^2*x^16*x^m + 190*a*b^4*m^4*x^13*x^m + 5714*b^5*m*
x^16*x^m + 2555*a*b^4*m^3*x^13*x^m + 3640*b^5*x^16*x^m + 10*a^2*b^3*m^5*x^
10*x^m + 14810*a*b^4*m^2*x^13*x^m + 410*a^2*b^3*m^4*x^10*x^m + 34840*a*b^4
*m*x^13*x^m + 5950*a^2*b^3*m^3*x^10*x^m + 22400*a*b^4*x^13*x^m + 10*a^3*b^
2*m^5*x^7*x^m + 36550*a^2*b^3*m^2*x^10*x^m + 440*a^3*b^2*m^4*x^7*x^m + 892
40*a^2*b^3*m*x^10*x^m + 6970*a^3*b^2*m^3*x^7*x^m + 58240*a^2*b^3*x^10*x^m
 + 5*a^4*b*m^5*x^4*x^m + 47260*a^3*b^2*m^2*x^7*x^m + 235*a^4*b*m^4*x^4*x^m
 + 123920*a^3*b^2*m*x^7*x^m + 4085*a^4*b*m^3*x^4*x^m + 83200*a^3*b^2*x^7*x^
m + a^5*m^5*x*x^m + 31685*a^4*b*m^2*x^4*x^m + 50*a^5*m^4*x*x^m + 100630*a^
4*b*m*x^4*x^m + 955*a^5*m^3*x*x^m + 72800*a^4*b*x^4*x^m + 8650*a^5*m^2*x*x
^m + 36824*a^5*m*x*x^m + 58240*a^5*x*x^m)/(m^6 + 51*m^5 + 1005*m^4 + 9605*
m^3 + 45474*m^2 + 95064*m + 58240)
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.01

$$\int x^m (a+bx^3)^5 \, dx = \frac{a^5 \, x \, x^m \, (m^5+50\,m^4+955\,m^3+8650\,m^2+36824\,m+58240)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240} + \frac{b^5 \, x^m \, x^{16} \, (m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240} + \frac{5\,a\,b^4 \, x^m \, x^{13} \, (m^5+38\,m^4+511\,m^3+2962\,m^2+6968\,m+4480)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240} + \frac{5\,a^4\,b\,x^m \, x^4 \, (m^5+47\,m^4+817\,m^3+6337\,m^2+20126\,m+14560)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240} + \frac{10\,a^2\,b^3 \, x^m \, x^{10} \, (m^5+41\,m^4+595\,m^3+3655\,m^2+8924\,m+5824)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240} + \frac{10\,a^3\,b^2 \, x^m \, x^7 \, (m^5+44\,m^4+697\,m^3+4726\,m^2+12392\,m+8320)}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240}$$

input

```
int(x^m*(a + b*x^3)^5,x)
```

output

```
(a^5*x*x^m*(36824*m + 8650*m^2 + 955*m^3 + 50*m^4 + m^5 + 58240))/(95064*m
 + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 + 58240) + (b^5*x^m*x^16
*(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640))/(95064*m + 45474*m^2
 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 + 58240) + (5*a*b^4*x^m*x^13*(6968*m
 + 2962*m^2 + 511*m^3 + 38*m^4 + m^5 + 4480))/(95064*m + 45474*m^2 + 9605*
m^3 + 1005*m^4 + 51*m^5 + m^6 + 58240) + (5*a^4*b*x^m*x^4*(20126*m + 6337*
m^2 + 817*m^3 + 47*m^4 + m^5 + 14560))/(95064*m + 45474*m^2 + 9605*m^3 + 1
005*m^4 + 51*m^5 + m^6 + 58240) + (10*a^2*b^3*x^m*x^10*(8924*m + 3655*m^2
+ 595*m^3 + 41*m^4 + m^5 + 5824))/(95064*m + 45474*m^2 + 9605*m^3 + 1005*m
^4 + 51*m^5 + m^6 + 58240) + (10*a^3*b^2*x^m*x^7*(12392*m + 4726*m^2 + 697
*m^3 + 44*m^4 + m^5 + 8320))/(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 +
51*m^5 + m^6 + 58240)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.43

$$\int x^m (a+bx^3)^5 \, dx$$
$$= \frac{x^m x(b^5m^5x^{15} + 35b^5m^4x^{15} + 445b^5m^3x^{15} + 5a\,b^4m^5x^{12} + 2485b^5m^2x^{15} + 190a\,b^4m^4x^{12} + 5714b^5m\,x^{15} -}{}$$

input
```
int(x^m*(b*x^3+a)^5,x)
```

output
```
(x**m*x*(a**5*m**5 + 50*a**5*m**4 + 955*a**5*m**3 + 8650*a**5*m**2 + 36824
*a**5*m + 58240*a**5 + 5*a**4*b*m**5*x**3 + 235*a**4*b*m**4*x**3 + 4085*a*
*4*b*m**3*x**3 + 31685*a**4*b*m**2*x**3 + 100630*a**4*b*m*x**3 + 72800*a**
4*b*x**3 + 10*a**3*b**2*m**5*x**6 + 440*a**3*b**2*m**4*x**6 + 6970*a**3*b*
*2*m**3*x**6 + 47260*a**3*b**2*m**2*x**6 + 123920*a**3*b**2*m*x**6 + 83200
*a**3*b**2*x**6 + 10*a**2*b**3*m**5*x**9 + 410*a**2*b**3*m**4*x**9 + 5950*
a**2*b**3*m**3*x**9 + 36550*a**2*b**3*m**2*x**9 + 89240*a**2*b**3*m*x**9 +
 58240*a**2*b**3*x**9 + 5*a*b**4*m**5*x**12 + 190*a*b**4*m**4*x**12 + 2555
*a*b**4*m**3*x**12 + 14810*a*b**4*m**2*x**12 + 34840*a*b**4*m*x**12 + 2240
0*a*b**4*x**12 + b**5*m**5*x**15 + 35*b**5*m**4*x**15 + 445*b**5*m**3*x**1
5 + 2485*b**5*m**2*x**15 + 5714*b**5*m*x**15 + 3640*b**5*x**15))/(m**6 + 5
1*m**5 + 1005*m**4 + 9605*m**3 + 45474*m**2 + 95064*m + 58240)
```

3.439 $\int x^m(a+bx^3)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^m\left(a+bx^3\right)^3\,dx=\frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{4+m}}{4+m}+\frac{3ab^2x^{7+m}}{7+m}+\frac{b^3x^{10+m}}{10+m}$$

output
```
a^3*x^(1+m)/(1+m)+3*a^2*b*x^(4+m)/(4+m)+3*a*b^2*x^(7+m)/(7+m)+b^3*x^(10+m)
/(10+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int x^m\left(a+bx^3\right)^3\,dx=x^{1+m}\left(\frac{a^3}{1+m}+\frac{3a^2bx^3}{4+m}+\frac{3ab^2x^6}{7+m}+\frac{b^3x^9}{10+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^3,x]
```

output
```
x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^3)/(4 + m) + (3*a*b^2*x^6)/(7 + m) + (
b^3*x^9)/(10 + m))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^3 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^m+3a^2bx^{m+3}+3ab^2x^{m+6}+b^3x^{m+9}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+4}}{m+4}+\frac{3ab^2x^{m+7}}{m+7}+\frac{b^3x^{m+10}}{m+10}$$

input
```
Int[x^m*(a + b*x^3)^3,x]
```

output
```
(a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(4 + m))/(4 + m) + (3*a*b^2*x^(7 + m)
)/(7 + m) + (b^3*x^(10 + m))/(10 + m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(61) = 122.

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.90

method	result
risch	$\frac{x\left(b^{3} m^{3} x^{9}+12 b^{3} m^{2} x^{9}+39 m\,x^{9} b^{3}+3 a\,b^{2} m^{3} x^{6}+28 b^{3} x^{9}+45 a\,b^{2} m^{2} x^{6}+162 m\,x^{6} a\,b^{2}+3 a^{2} b\,m^{3} x^{3}+120 a\,b^{2} x^{6}+54 a^{2} b\,m^{2} x^{3}+2}{(10+m)(7+m)(4+m)(1+m)}$
orering	$\frac{x\left(b^{3} m^{3} x^{9}+12 b^{3} m^{2} x^{9}+39 m\,x^{9} b^{3}+3 a\,b^{2} m^{3} x^{6}+28 b^{3} x^{9}+45 a\,b^{2} m^{2} x^{6}+162 m\,x^{6} a\,b^{2}+3 a^{2} b\,m^{3} x^{3}+120 a\,b^{2} x^{6}+54 a^{2} b\,m^{2} x^{3}+2}{(10+m)(7+m)(4+m)(1+m)}$
gosper	$\frac{x^{1+m}\left(b^{3} m^{3} x^{9}+12 b^{3} m^{2} x^{9}+39 m\,x^{9} b^{3}+3 a\,b^{2} m^{3} x^{6}+28 b^{3} x^{9}+45 a\,b^{2} m^{2} x^{6}+162 m\,x^{6} a\,b^{2}+3 a^{2} b\,m^{3} x^{3}+120 a\,b^{2} x^{6}+54 a^{2} b\,m^{2} x}{(1+m)(4+m)(7+m)(10+m)}$
parallelrisch	$\frac{x^{10} x^{m} b^{3} m^{3}+12 x^{10} x^{m} b^{3} m^{2}+39 x^{10} x^{m} b^{3} m+28 x^{10} x^{m} b^{3}+3 x^{7} x^{m} a\,b^{2} m^{3}+45 x^{7} x^{m} a\,b^{2} m^{2}+162 x^{7} x^{m} a\,b^{2} m+120 x^{7} x^{m} a\,b^{2}+}{(10+m)(7+m)(4}$

input
```
int(x^m*(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a
*b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^
3+261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*x^m/
(10+m)/(7+m)/(4+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.57

$$\int x^m\left(a+bx^3\right)^3\,dx$$
$$=\frac{((b^3m^3+12\,b^3m^2+39\,b^3m+28\,b^3)x^{10}+3\,(ab^2m^3+15\,ab^2m^2+54\,ab^2m+40\,ab^2)x^7+3\,(a^2bm^3+18\,a}{m^4+22\,m^3+159\,m^2+418\,m+}$$

input
```
integrate(x^m*(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2
*m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b
*m + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*x^m/(
m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(53) = 106$.

Time = 0.57 (sec) , antiderivative size = 666, normalized size of antiderivative = 10.92

$$\int x^m (a+bx^3)^3 \, dx$$
$$= \begin{cases} -\frac{a^3}{9x^9} - \frac{a^2b}{2x^6} - \frac{ab^2}{x^3} + b^3\log(x) \\ -\frac{a^3}{6x^6} - \frac{a^2b}{x^3} + 3ab^2\log(x) + \frac{b^3x^3}{3} \\ -\frac{a^3}{3x^3} + 3a^2b\log(x) + ab^2x^3 + \frac{b^3x^6}{6} \\ a^3\log(x) + a^2bx^3 + \frac{ab^2x^6}{2} + \frac{b^3x^9}{9} \\ \frac{a^3m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21a^3m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138a^3mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280a^3xx^m}{m^4+22m^3+159m^2+418m+2} \end{cases}$$

input
```
integrate(x**m*(b*x**3+a)**3,x)
```

output
```
Piecewise((-a**3/(9*x**9) - a**2*b/(2*x**6) - a*b**2/x**3 + b**3*log(x), E
q(m, -10)), (-a**3/(6*x**6) - a**2*b/x**3 + 3*a*b**2*log(x) + b**3*x**3/3,
 Eq(m, -7)), (-a**3/(3*x**3) + 3*a**2*b*log(x) + a*b**2*x**3 + b**3*x**6/6
, Eq(m, -4)), (a**3*log(x) + a**2*b*x**3 + a*b**2*x**6/2 + b**3*x**9/9, Eq
(m, -1)), (a**3*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 21
*a**3*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 138*a**3*m*x
*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*a**3*x*x**m/(m**4 +
22*m**3 + 159*m**2 + 418*m + 280) + 3*a**2*b*m**3*x**4*x**m/(m**4 + 22*m**
3 + 159*m**2 + 418*m + 280) + 54*a**2*b*m**2*x**4*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 261*a**2*b*m*x**4*x**m/(m**4 + 22*m**3 + 159*m**2
 + 418*m + 280) + 210*a**2*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m
+ 280) + 3*a*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280)
 + 45*a*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 16
2*a*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 120*a*b**
2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + b**3*m**3*x**10*x*
*m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*b**3*m**2*x**10*x**m/(m*
*4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*b**3*m*x**10*x**m/(m**4 + 22*m
**3 + 159*m**2 + 418*m + 280) + 28*b**3*x**10*x**m/(m**4 + 22*m**3 + 159*m
**2 + 418*m + 280), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x^m \left(a+bx^3\right)^3 \, dx = \frac{b^3x^{m+10}}{m+10} + \frac{3\,ab^2x^{m+7}}{m+7} + \frac{3\,a^2bx^{m+4}}{m+4} + \frac{a^3x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
b^3*x^(m + 10)/(m + 10) + 3*a*b^2*x^(m + 7)/(m + 7) + 3*a^2*b*x^(m + 4)/(m
 + 4) + a^3*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(61) = 122$.

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.67

$$\int x^m (a + bx^3)^3 \, dx$$
$$= \frac{b^3m^3x^{10}x^m + 12\,b^3m^2x^{10}x^m + 39\,b^3mx^{10}x^m + 3\,ab^2m^3x^7x^m + 28\,b^3x^{10}x^m + 45\,ab^2m^2x^7x^m + 162\,ab^2mx}{}$$

input
```
integrate(x^m*(b*x^3+a)^3,x, algorithm="giac")
```

output
```
(b^3*m^3*x^10*x^m + 12*b^3*m^2*x^10*x^m + 39*b^3*m*x^10*x^m + 3*a*b^2*m^3*
x^7*x^m + 28*b^3*x^10*x^m + 45*a*b^2*m^2*x^7*x^m + 162*a*b^2*m*x^7*x^m + 3
*a^2*b*m^3*x^4*x^m + 120*a*b^2*x^7*x^m + 54*a^2*b*m^2*x^4*x^m + 261*a^2*b*
m*x^4*x^m + a^3*m^3*x*x^m + 210*a^2*b*x^4*x^m + 21*a^3*m^2*x*x^m + 138*a^3
*m*x*x^m + 280*a^3*x*x^m)/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.74

$$\int x^m (a + bx^3)^3 \, dx = x^m \left(\frac{a^3\,x\,(m^3 + 21\,m^2 + 138\,m + 280)}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280} + \frac{b^3\,x^{10}\,(m^3 + 12\,m^2 + 39\,m + 28)}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280} + \frac{3\,a\,b^2\,x^7\,(m^3 + 15\,m^2 + 54\,m + 40)}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280} + \frac{3\,a^2\,b\,x^4\,(m^3 + 18\,m^2 + 87\,m + 70)}{m^4 + 22\,m^3 + 159\,m^2 + 418\,m + 280} \right)$$

input
```
int(x^m*(a + b*x^3)^3,x)
```

output
```
x^m*((a^3*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m^3 + m^4
+ 280) + (b^3*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 +
 m^4 + 280) + (3*a*b^2*x^7*(54*m + 15*m^2 + m^3 + 40))/(418*m + 159*m^2 +
22*m^3 + m^4 + 280) + (3*a^2*b*x^4*(87*m + 18*m^2 + m^3 + 70))/(418*m + 15
9*m^2 + 22*m^3 + m^4 + 280))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.89

$$\int x^m \left(a+bx^3\right)^3 \, dx$$
$$= \frac{x^m x(b^3m^3x^9+12b^3m^2x^9+39b^3m\,x^9+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162a\,b^2m\,x^6+3a^2b\,m^3x^3+}{m^4+22m^3+159m^2+4}$$

input
```
int(x^m*(b*x^3+a)^3,x)
```

output
```
(x**m*x*(a**3*m**3 + 21*a**3*m**2 + 138*a**3*m + 280*a**3 + 3*a**2*b*m**3*
x**3 + 54*a**2*b*m**2*x**3 + 261*a**2*b*m*x**3 + 210*a**2*b*x**3 + 3*a*b**
2*m**3*x**6 + 45*a*b**2*m**2*x**6 + 162*a*b**2*m*x**6 + 120*a*b**2*x**6 +
b**3*m**3*x**9 + 12*b**3*m**2*x**9 + 39*b**3*m*x**9 + 28*b**3*x**9))/(m**4
 + 22*m**3 + 159*m**2 + 418*m + 280)
```

3.440 $\int x^m (a+bx^3)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^m \left(a+bx^3\right)^2 \, dx = \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{4+m}}{4+m} + \frac{b^2 x^{7+m}}{7+m}$$

output
```
a^2*x^(1+m)/(1+m)+2*a*b*x^(4+m)/(4+m)+b^2*x^(7+m)/(7+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^m \left(a+bx^3\right)^2 \, dx = x^{1+m}\left(\frac{a^2}{1+m} + \frac{2abx^3}{4+m} + \frac{b^2x^6}{7+m}\right)$$

input
```
Integrate[x^m*(a + b*x^3)^2,x]
```

output
```
x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^3)/(4 + m) + (b^2*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^2 dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^m+2abx^{m+3}+b^2x^{m+6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+4}}{m+4}+\frac{b^2x^{m+7}}{m+7}$$

input
```
Int[x^m*(a + b*x^3)^2,x]
```

output
```
(a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(4 + m))/(4 + m) + (b^2*x^(7 + m))/(7 +
 m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
norman	$\frac{a^{2} x\,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b^{2} x^{7} {\mathrm e}^{m \ln \left(x \right)}}{7+m}+\frac{2 a b\,x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}$	51
risch	$\frac{x \left(b^{2} m^{2} x^{6}+5 m\,x^{6} b^{2}+4 b^{2} x^{6}+2 a b\,m^{2} x^{3}+16 m\,x^{3} a b+14 a b\,x^{3}+a^{2} m^{2}+11 m\,a^{2}+28 a^{2}\right) x^{m}}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$	92
orering	$\frac{x \left(b^{2} m^{2} x^{6}+5 m\,x^{6} b^{2}+4 b^{2} x^{6}+2 a b\,m^{2} x^{3}+16 m\,x^{3} a b+14 a b\,x^{3}+a^{2} m^{2}+11 m\,a^{2}+28 a^{2}\right) x^{m}}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$	92
gosper	$\frac{x^{1+m} \left(b^{2} m^{2} x^{6}+5 m\,x^{6} b^{2}+4 b^{2} x^{6}+2 a b\,m^{2} x^{3}+16 m\,x^{3} a b+14 a b\,x^{3}+a^{2} m^{2}+11 m\,a^{2}+28 a^{2}\right)}{\left(1+m \right) \left(4+m \right) \left(7+m \right)}$	93
parallelrisch	$\frac{x^{7} x^{m} b^{2} m^{2}+5 x^{7} x^{m} b^{2} m+4 x^{7} x^{m} b^{2}+2 x^{4} x^{m} a b\,m^{2}+16 x^{4} x^{m} a b m+14 x^{4} x^{m} a b+x\,x^{m} a^{2} m^{2}+11 x\,x^{m} a^{2} m+28 x\,x^{m} a^{2}}{\left(7+m \right) \left(4+m \right) \left(1+m \right)}$	118

input
```
int(x^m*(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(x))+b^2/(7+m)*x^7*exp(m*ln(x))+2*a*b/(4+m)*x^4*exp(m*
ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int x^m \left(a+bx^3\right)^2 \,dx = \frac{((b^2m^2 + 5\,b^2m + 4\,b^2)x^7 + 2\,(abm^2 + 8\,abm + 7\,ab)x^4 + (a^2m^2 + 11\,a^2m + 28\,a^2)x)x^m}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate(x^m*(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 5*b^2*m + 4*b^2)*x^7 + 2*(a*b*m^2 + 8*a*b*m + 7*a*b)*x^4 + (a^
2*m^2 + 11*a^2*m + 28*a^2)*x)*x^m/(m^3 + 12*m^2 + 39*m + 28)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(36) = 72$.

Time = 0.41 (sec) , antiderivative size = 313, normalized size of antiderivative = 7.28

$$\int x^m (a+bx^3)^2 \, dx = \begin{cases} -\frac{a^2}{6x^6} - \frac{2ab}{3x^3} + b^2 \log(x) \\ -\frac{a^2}{3x^3} + 2ab\log(x) + \frac{b^2x^3}{3} \\ a^2\log(x) + \frac{2abx^3}{3} + \frac{b^2x^6}{6} \\ \frac{a^2m^2xx^m}{m^3+12m^2+39m+28} + \frac{11a^2mxx^m}{m^3+12m^2+39m+28} + \frac{28a^2xx^m}{m^3+12m^2+39m+28} + \frac{2abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{16abmx^4x^m}{m^3+12m^2+39m+28} + \frac{14abx}{m^3+12m^2} \end{cases}$$

input
```
integrate(x**m*(b*x**3+a)**2,x)
```

output
```
Piecewise((-a**2/(6*x**6) - 2*a*b/(3*x**3) + b**2*log(x), Eq(m, -7)), (-a*
*2/(3*x**3) + 2*a*b*log(x) + b**2*x**3/3, Eq(m, -4)), (a**2*log(x) + 2*a*b
*x**3/3 + b**2*x**6/6, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 12*m**2 + 39*
m + 28) + 11*a**2*m*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a**2*x*x**m/(
m**3 + 12*m**2 + 39*m + 28) + 2*a*b*m**2*x**4*x**m/(m**3 + 12*m**2 + 39*m
+ 28) + 16*a*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 14*a*b*x**4*x**m
/(m**3 + 12*m**2 + 39*m + 28) + b**2*m**2*x**7*x**m/(m**3 + 12*m**2 + 39*m
 + 28) + 5*b**2*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*b**2*x**7*x**
m/(m**3 + 12*m**2 + 39*m + 28), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^m (a+bx^3)^2 \, dx = \frac{b^2x^{m+7}}{m+7} + \frac{2\,abx^{m+4}}{m+4} + \frac{a^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
b^2*x^(m + 7)/(m + 7) + 2*a*b*x^(m + 4)/(m + 4) + a^2*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(43) = 86.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.72

$$\int x^m (a+bx^3)^2 \, dx$$
$$= \frac{b^2m^2x^7x^m + 5\,b^2mx^7x^m + 4\,b^2x^7x^m + 2\,abm^2x^4x^m + 16\,abmx^4x^m + 14\,abx^4x^m + a^2m^2xx^m + 11\,a^2mx}{m^3 + 12\,m^2 + 39\,m + 28}$$

input
```
integrate(x^m*(b*x^3+a)^2,x, algorithm="giac")
```

output
```
(b^2*m^2*x^7*x^m + 5*b^2*m*x^7*x^m + 4*b^2*x^7*x^m + 2*a*b*m^2*x^4*x^m + 1
6*a*b*m*x^4*x^m + 14*a*b*x^4*x^m + a^2*m^2*x*x^m + 11*a^2*m*x*x^m + 28*a^2
*x*x^m)/(m^3 + 12*m^2 + 39*m + 28)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int x^m (a+bx^3)^2 \, dx = x^m \left(\frac{a^2\,x\,(m^2+11\,m+28)}{m^3+12\,m^2+39\,m+28} + \frac{b^2\,x^7\,(m^2+5\,m+4)}{m^3+12\,m^2+39\,m+28} + \frac{2\,a\,b\,x^4\,(m^2+8\,m+7)}{m^3+12\,m^2+39\,m+28} \right)$$

input
```
int(x^m*(a + b*x^3)^2,x)
```

output
```
x^m*((a^2*x*(11*m + m^2 + 28))/(39*m + 12*m^2 + m^3 + 28) + (b^2*x^7*(5*m
+ m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (2*a*b*x^4*(8*m + m^2 + 7))/(39*m
 + 12*m^2 + m^3 + 28))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.12

$$\int x^m (a + bx^3)^2 \, dx = \frac{x^m x(b^2m^2x^6 + 5b^2m\,x^6 + 4b^2x^6 + 2ab\,m^2x^3 + 16abm\,x^3 + 14ab\,x^3 + a^2m^2 + 11a^2m + 28a^2)}{m^3 + 12m^2 + 39m + 28}$$

input
```
int(x^m*(b*x^3+a)^2,x)
```

output
```
(x**m*x*(a**2*m**2 + 11*a**2*m + 28*a**2 + 2*a*b*m**2*x**3 + 16*a*b*m*x**3
 + 14*a*b*x**3 + b**2*m**2*x**6 + 5*b**2*m*x**6 + 4*b**2*x**6))/(m**3 + 12
*m**2 + 39*m + 28)
```

3.441 $\int x^m(a+bx^3)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int x^m\left(a+bx^3\right)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{4+m}}{4+m}$$

output
```
a*x^(1+m)/(1+m)+b*x^(4+m)/(4+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^m\left(a+bx^3\right)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{4+m}}{4+m}$$

input
```
Integrate[x^m*(a + b*x^3),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(4 + m))/(4 + m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right) \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(ax^m+bx^{m+3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+4}}{m+4}$$

input
```
Int[x^m*(a + b*x^3),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(4 + m))/(4 + m)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{a x \,{\mathrm e}^{m \ln \left(x \right)}}{1+m}+\frac{b \,x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}$	30
risch	$\frac{x \left(b m \,x^{3}+b \,x^{3}+a m +4 a \right) x^{m}}{\left(4+m \right) \left(1+m \right)}$	34
orering	$\frac{x \left(b m \,x^{3}+b \,x^{3}+a m +4 a \right) x^{m}}{\left(4+m \right) \left(1+m \right)}$	34
gosper	$\frac{x^{1+m} \left(b m \,x^{3}+b \,x^{3}+a m +4 a \right)}{\left(1+m \right) \left(4+m \right)}$	35
parallelrisch	$\frac{x^{4} x^{m} b m +x^{4} x^{m} b +x \,x^{m} a m +4 x \,x^{m} a}{\left(4+m \right) \left(1+m \right)}$	44

input
```
int(x^m*(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*exp(m*ln(x))+b/(4+m)*x^4*exp(m*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^m \left(a + bx^3\right) \, dx = \frac{((bm + b)x^4 + (am + 4\,a)x)x^m}{m^2 + 5\,m + 4}$$

input
```
integrate(x^m*(b*x^3+a),x, algorithm="fricas")
```

output
```
((b*m + b)*x^4 + (a*m + 4*a)*x)*x^m/(m^2 + 5*m + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.76

$$\int x^m (a + bx^3) \, dx = \begin{cases} -\frac{a}{3x^3} + b \log(x) & \text{for } m = -4 \\ a \log(x) + \frac{bx^3}{3} & \text{for } m = -1 \\ \frac{amxx^m}{m^2+5m+4} + \frac{4axx^m}{m^2+5m+4} + \frac{bmx^4x^m}{m^2+5m+4} + \frac{bx^4x^m}{m^2+5m+4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(b*x**3+a),x)
```

output
```
Piecewise((-a/(3*x**3) + b*log(x), Eq(m, -4)), (a*log(x) + b*x**3/3, Eq(m,
 -1)), (a*m*x*x**m/(m**2 + 5*m + 4) + 4*a*x*x**m/(m**2 + 5*m + 4) + b*m*x*
*4*x**m/(m**2 + 5*m + 4) + b*x**4*x**m/(m**2 + 5*m + 4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^m (a + bx^3) \, dx = \frac{bx^{m+4}}{m+4} + \frac{ax^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^3+a),x, algorithm="maxima")
```

output
```
b*x^(m + 4)/(m + 4) + a*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int x^m (a + bx^3)\ dx = \frac{bmx^4x^m + bx^4x^m + amxx^m + 4\,axx^m}{m^2 + 5\,m + 4}$$

input
```
integrate(x^m*(b*x^3+a),x, algorithm="giac")
```

output
```
(b*m*x^4*x^m + b*x^4*x^m + a*m*x*x^m + 4*a*x*x^m)/(m^2 + 5*m + 4)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int x^m (a + bx^3)\ dx = \frac{x^{m+1}\,(4\,a + a\,m + b\,x^3 + b\,m\,x^3)}{m^2 + 5\,m + 4}$$

input
```
int(x^m*(a + b*x^3),x)
```

output
```
(x^(m + 1)*(4*a + a*m + b*x^3 + b*m*x^3))/(5*m + m^2 + 4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^m (a + bx^3)\ dx = \frac{x^m x(bm\,x^3 + b\,x^3 + am + 4a)}{m^2 + 5m + 4}$$

input
```
int(x^m*(b*x^3+a),x)
```

output
```
(x**m*x*(a*m + 4*a + b*m*x**3 + b*x**3))/(m**2 + 5*m + 4)
```

3.442 $\int \frac{x^m}{a+bx^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^m}{a+bx^3} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{3}, \frac{4+m}{3}, -\frac{bx^3}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{a+bx^3} \, dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{3}, 1+\frac{1+m}{3}, -\frac{bx^3}{a}\right)}{a(1+m)}$$

input
```
Integrate[x^m/(a + b*x^3),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b*x^3)/a)])/(
a*(1 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^3}\,dx$$

$$\Big\downarrow \ 888$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a(m+1)}$$

input
```
Int[x^m/(a + b*x^3),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*(1
 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{b\,x^3+a}dx$$

input
```
int(x^m/(b*x^3+a),x)
```

output
```
int(x^m/(b*x^3+a),x)
```

Fricas [F]

$$\int \frac{x^m}{a+bx^3}\,dx = \int \frac{x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral(x^m/(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{x^m}{a+bx^3}\,dx = \frac{mx^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{x^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate(x**m/(b*x**3+a),x)
```

output
```
m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 +
1/3)/(9*a*gamma(m/3 + 4/3)) + x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a
, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int \frac{x^m}{a+bx^3}\,dx = \int \frac{x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^m}{a+bx^3}\,dx = \int \frac{x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^3}\,dx = \int \frac{x^m}{b\,x^3+a}\,dx$$

input
```
int(x^m/(a + b*x^3),x)
```

output
```
int(x^m/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^m}{a+bx^3}\,dx = \int \frac{x^m}{b\,x^3+a}dx$$

input
```
int(x^m/(b*x^3+a),x)
```

output
```
int(x**m/(a + b*x**3),x)
```

3.443 $\int \frac{x^m}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^m}{(a+bx^3)^2} dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^2(1+m)}$$

output
```
x^(1+m)*hypergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{(a+bx^3)^2} dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[x^m/(a + b*x^3)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, 1 + (1 + m)/3, -((b*x^3)/a)])/(
a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \; 888$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(2,\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a^2(m+1)}$$

input
```
Int[x^m/(a + b*x^3)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a^2*
(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(b\,x^3+a)^2}dx$$

input
```
int(x^m/(b*x^3+a)^2,x)
```

output
```
int(x^m/(b*x^3+a)^2,x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^3)^2}\,dx = \int \frac{x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.65 (sec) , antiderivative size = 520, normalized size of antiderivative = 13.33

$$\int \frac{x^m}{(a+bx^3)^2}\,dx = -\frac{am^2x^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{amx^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{3amx^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{2ax^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{3ax^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} - \frac{bm^2x^3x^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{bmx^3x^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{2bx^3x^{m+1}\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{27a^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)+27a^2bx^3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate(x**m/(b*x**3+a)**2,x)
```

output
```
-a*m**2*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(
m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) +
a*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3
+ 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 3*a*
m*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*g
amma(m/3 + 4/3)) + 2*a*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/
3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) + 27*a**2*b*x**3*gamma
(m/3 + 4/3)) + 3*a*x**(m + 1)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/3) +
 27*a**2*b*x**3*gamma(m/3 + 4/3)) - b*m**2*x**3*x**(m + 1)*lerchphi(b*x**3
*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4/
3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + b*m*x**3*x**(m + 1)*lerchphi(b*x**
3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 + 4
/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3)) + 2*b*x**3*x**(m + 1)*lerchphi(b*x*
*3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(27*a**3*gamma(m/3 +
4/3) + 27*a**2*b*x**3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^3)^2}\,dx = \int \frac{x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^3)^2}\,dx = \int \frac{x^m}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^3)^2}\,dx = \int \frac{x^m}{(b\,x^3+a)^2}\,dx$$

input
```
int(x^m/(a + b*x^3)^2,x)
```

output
```
int(x^m/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^3)^2}\,dx = \int \frac{x^m}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x^m/(b*x^3+a)^2,x)
```

output
```
int(x**m/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.444 $\int \frac{x^m}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^m}{(a+bx^3)^3}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a^3(1+m)}$$

output
```
x^(1+m)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{(a+bx^3)^3}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(3,\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{a^3(1+m)}$$

input
```
Integrate[x^m/(a + b*x^3)^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, (1 + m)/3, 1 + (1 + m)/3, -((b*x^3)/a)])/(
a^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^3)^3} \, dx$$
$$\Big\downarrow \text{ 888}$$
$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(3, \frac{m+1}{3}, \frac{m+4}{3}, -\frac{bx^3}{a}\right)}{a^3(m+1)}$$

input
```
Int[x^m/(a + b*x^3)^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(a^3*
(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\left(b\,x^3+a\right)^3}dx$$

input
```
int(x^m/(b*x^3+a)^3,x)
```

output
```
int(x^m/(b*x^3+a)^3,x)
```

Fricas [F]

$$\int \frac{x^m}{\left(a+bx^3\right)^3}\,dx = \int \frac{x^m}{\left(bx^3+a\right)^3}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 121.23 (sec) , antiderivative size = 1578, normalized size of antiderivative = 40.46

$$\int \frac{x^m}{\left(a+bx^3\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**m/(b*x**3+a)**3,x)
```

output
```
a**2*m**3*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamm
a(m/3 + 1/3)/(162*a**5*gamma(m/3 + 4/3) + 324*a**4*b*x**3*gamma(m/3 + 4/3)
 + 162*a**3*b**2*x**6*gamma(m/3 + 4/3)) - 6*a**2*m**2*x**(m + 1)*lerchphi(
b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(162*a**5*gamma(m
/3 + 4/3) + 324*a**4*b*x**3*gamma(m/3 + 4/3) + 162*a**3*b**2*x**6*gamma(m/
3 + 4/3)) - 3*a**2*m**2*x**(m + 1)*gamma(m/3 + 1/3)/(162*a**5*gamma(m/3 +
4/3) + 324*a**4*b*x**3*gamma(m/3 + 4/3) + 162*a**3*b**2*x**6*gamma(m/3 + 4
/3)) + 3*a**2*m*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3
)*gamma(m/3 + 1/3)/(162*a**5*gamma(m/3 + 4/3) + 324*a**4*b*x**3*gamma(m/3
+ 4/3) + 162*a**3*b**2*x**6*gamma(m/3 + 4/3)) + 21*a**2*m*x**(m + 1)*gamma
(m/3 + 1/3)/(162*a**5*gamma(m/3 + 4/3) + 324*a**4*b*x**3*gamma(m/3 + 4/3)
+ 162*a**3*b**2*x**6*gamma(m/3 + 4/3)) + 10*a**2*x**(m + 1)*lerchphi(b*x**
3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(162*a**5*gamma(m/3 +
4/3) + 324*a**4*b*x**3*gamma(m/3 + 4/3) + 162*a**3*b**2*x**6*gamma(m/3 + 4
/3)) + 24*a**2*x**(m + 1)*gamma(m/3 + 1/3)/(162*a**5*gamma(m/3 + 4/3) + 32
4*a**4*b*x**3*gamma(m/3 + 4/3) + 162*a**3*b**2*x**6*gamma(m/3 + 4/3)) + 2*
a*b*m**3*x**3*x**(m + 1)*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*
gamma(m/3 + 1/3)/(162*a**5*gamma(m/3 + 4/3) + 324*a**4*b*x**3*gamma(m/3 +
4/3) + 162*a**3*b**2*x**6*gamma(m/3 + 4/3)) - 12*a*b*m**2*x**3*x**(m + 1)*
lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3)/(162*...
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^3)^3}\,dx=\int \frac{x^m}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^3 + a)^3, x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^3)^3} \, dx = \int \frac{x^m}{(bx^3+a)^3} \, dx$$

input
```
integrate(x^m/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^3)^3} \, dx = \int \frac{x^m}{(b\,x^3+a)^3} \, dx$$

input
```
int(x^m/(a + b*x^3)^3,x)
```

output
```
int(x^m/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^3)^3} \, dx = \int \frac{x^m}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx$$

input
```
int(x^m/(b*x^3+a)^3,x)
```

output
```
int(x**m/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)
```

3.445 $\int x^m(a+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int x^m\left(a+bx^3\right)^{3/2}\,dx=\frac{ax^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt{1+\frac{bx^3}{a}}}$$

output
```
a*x^(1+m)*(b*x^3+a)^(1/2)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a
)/(1+m)/(1+b*x^3/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int x^m\left(a+bx^3\right)^{3/2}\,dx=\frac{ax^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^m*(a + b*x^3)^(3/2),x]
```

output
```
(a*x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1 + m)/3, 1 + (1 + m
)/3, -((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^3\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a\sqrt{a+bx^3} \int x^m \left(\frac{bx^3}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{ax^{m+1}\sqrt{a+bx^3}\, \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[x^m*(a + b*x^3)^(3/2),x]
```

output
```
(a*x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1 + m)/3, (4 + m)/3,
 -((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0
] && !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{m} \left(b \,x^{3}+a \right)^{\frac{3}{2}}d x$$

input
```
int(x^m*(b*x^3+a)^(3/2),x)
```

output
```
int(x^m*(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int x^m \left(a+bx^3\right)^{3/2} \, dx=\int { {\left(b x^{3} + a\right)}^{\frac{3}{2}} x^{m} \,d x }$$

input
```
integrate(x^m*(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(3/2)*x^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int x^m \left(a+bx^3\right)^{3/2} \, dx=\frac{a^{\frac{3}{2}} x^{m+1} \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate(x**m*(b*x**3+a)**(3/2),x)
```

output
```
a**(3/2)*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-3/2, m/3 + 1/3), (m/3 + 4/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int x^m \left(a+bx^3\right)^{3/2} dx = \int \left(bx^3+a\right)^{\frac{3}{2}} x^m \, dx$$

input
```
integrate(x^m*(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^m, x)
```

Giac [F]

$$\int x^m \left(a+bx^3\right)^{3/2} dx = \int \left(bx^3+a\right)^{\frac{3}{2}} x^m \, dx$$

input
```
integrate(x^m*(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(3/2)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \left(a+bx^3\right)^{3/2} dx = \int x^m \left(b x^3+a\right)^{3/2} dx$$

input
```
int(x^m*(a + b*x^3)^(3/2),x)
```

output
```
int(x^m*(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int x^m \left(a+bx^3\right)^{3/2} dx = \frac{4x^m \sqrt{b x^3+a}\, amx + 28x^m \sqrt{b x^3+a}\, ax + 4x^m \sqrt{b x^3+a}\, bm x^4 + 10x^m \sqrt{b x^3+a}\, b x^4 + 108}{}$$

input
```
int(x^m*(b*x^3+a)^(3/2),x)
```

output
```
(4*x**m*sqrt(a + b*x**3)*a*m*x + 28*x**m*sqrt(a + b*x**3)*a*x + 4*x**m*sqr
t(a + b*x**3)*b*m*x**4 + 10*x**m*sqrt(a + b*x**3)*b*x**4 + 108*int((x**m*s
qrt(a + b*x**3))/(4*a*m**2 + 32*a*m + 55*a + 4*b*m**2*x**3 + 32*b*m*x**3 +
 55*b*x**3),x)*a**2*m**2 + 864*int((x**m*sqrt(a + b*x**3))/(4*a*m**2 + 32*
a*m + 55*a + 4*b*m**2*x**3 + 32*b*m*x**3 + 55*b*x**3),x)*a**2*m + 1485*int
((x**m*sqrt(a + b*x**3))/(4*a*m**2 + 32*a*m + 55*a + 4*b*m**2*x**3 + 32*b*
m*x**3 + 55*b*x**3),x)*a**2)/(4*m**2 + 32*m + 55)
```

3.446 $\int x^m \sqrt{a+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int x^m\sqrt{a+bx^3}\,dx=\frac{x^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt{1+\frac{bx^3}{a}}}$$

output
```
x^(1+m)*(b*x^3+a)^(1/2)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/
(1+m)/(1+b*x^3/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int x^m\sqrt{a+bx^3}\,dx=\frac{x^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt{1+\frac{bx^3}{a}}}$$

input
```
Integrate[x^m*Sqrt[a + b*x^3],x]
```

output
```
(x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 + m)/3, 1 + (1 + m)/
3, -((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{a+bx^3} \, dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt{a+bx^3} \int x^m \sqrt{\frac{bx^3}{a}+1} dx}{\sqrt{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{x^{m+1}\sqrt{a+bx^3} \text{ Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{(m+1)\sqrt{\frac{bx^3}{a}+1}}$$

input
```
Int[x^m*Sqrt[a + b*x^3],x]
```

output
```
(x^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 + m)/3, (4 + m)/3, -
((b*x^3)/a)])/((1 + m)*Sqrt[1 + (b*x^3)/a])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^m \sqrt{b\,x^3+a}dx$$

input
```
int(x^m*(b*x^3+a)^(1/2),x)
```

output
```
int(x^m*(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int x^m \sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x^m\,dx$$

input
```
integrate(x^m*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*x^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int x^m \sqrt{a+bx^3}\,dx = \frac{\sqrt{a}x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate(x**m*(b*x**3+a)**(1/2),x)
```

output
```
sqrt(a)*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int x^m\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x^m\,dx$$

input
```
integrate(x^m*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a)*x^m, x)
```

Giac [F]

$$\int x^m\sqrt{a+bx^3}\,dx = \int \sqrt{bx^3+a}x^m\,dx$$

input
```
integrate(x^m*(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^3 + a)*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m\sqrt{a+bx^3}\,dx=\int x^m\,\sqrt{b\,x^3+a}\,dx$$

input
```
int(x^m*(a + b*x^3)^(1/2),x)
```

output
```
int(x^m*(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int x^m\sqrt{a+bx^3}\,dx$$
$$=\frac{2x^m\sqrt{b\,x^3+a}\,x+6\left(\int\frac{x^m\sqrt{b\,x^3+a}}{2bm\,x^3+5b\,x^3+2am+5a}dx\right)am+15\left(\int\frac{x^m\sqrt{b\,x^3+a}}{2bm\,x^3+5b\,x^3+2am+5a}dx\right)a}{2m+5}$$

input
```
int(x^m*(b*x^3+a)^(1/2),x)
```

output
```
(2*x**m*sqrt(a + b*x**3)*x + 6*int((x**m*sqrt(a + b*x**3))/(2*a*m + 5*a +
2*b*m*x**3 + 5*b*x**3),x)*a*m + 15*int((x**m*sqrt(a + b*x**3))/(2*a*m + 5*
a + 2*b*m*x**3 + 5*b*x**3),x)*a)/(2*m + 5)
```

3.447 $\int \frac{x^m}{\sqrt{a+bx^3}} dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{x^m}{\sqrt{a+bx^3}} dx = \frac{x^{1+m}\sqrt{a+bx^3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(5+2m),\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*(b*x^3+a)^(1/2)*hypergeom([1, 5/6+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(
1+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.25

$$\int \frac{x^m}{\sqrt{a+bx^3}} dx = \frac{x^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt{a+bx^3}}$$

input
```
Integrate[x^m/Sqrt[a + b*x^3],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m)/3, 1 + (1 +
m)/3, -((b*x^3)/a)])/((1 + m)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx$$
$$\Big\downarrow \text{ 889}$$
$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^3}{a}+1}}dx}{\sqrt{a+bx^3}}$$
$$\Big\downarrow \text{ 888}$$
$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{(m+1)\sqrt{a+bx^3}}$$

input
```
Int[x^m/Sqrt[a + b*x^3],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m)/3, (4 + m)/3
, -((b*x^3)/a)])/((1 + m)*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{b\,x^3+a}}dx$$

input
```
int(x^m/(b*x^3+a)^(1/2),x)
```

output
```
int(x^m/(b*x^3+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx = \int \frac{x^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/sqrt(b*x^3 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx = \frac{x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate(x**m/(b*x**3+a)**(1/2),x)
```

output
```
x**(m + 1)*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx=\int \frac{x^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x^3 + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx=\int \frac{x^m}{\sqrt{bx^3+a}}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx = \int \frac{x^m}{\sqrt{b\,x^3+a}}\,dx$$

input
```
int(x^m/(a + b*x^3)^(1/2),x)
```

output
```
int(x^m/(a + b*x^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^3}}\,dx = \int \frac{x^m\sqrt{b\,x^3+a}}{b\,x^3+a}dx$$

input
```
int(x^m/(b*x^3+a)^(1/2),x)
```

output
```
int((x**m*sqrt(a + b*x**3))/(a + b*x**3),x)
```

3.448 $\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{a+bx^3}}$$

output
```
x^(1+m)*(1+b*x^3/a)^(1/2)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)
/a/(1+m)/(b*x^3+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx = \frac{x^{1+m}\sqrt{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{a+bx^3}}$$

input
```
Integrate[x^m/(a + b*x^3)^(3/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, (1 + m)/3, 1 + (1 +
m)/3, -((b*x^3)/a)])/(a*(1 + m)*Sqrt[a + b*x^3])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 889$$

$$\frac{\sqrt{\frac{bx^3}{a}+1}\int \frac{x^m}{\left(\frac{bx^3}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 888$$

$$\frac{x^{m+1}\sqrt{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{a(m+1)\sqrt{a+bx^3}}$$

input
```
Int[x^m/(a + b*x^3)^(3/2),x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2, (1 + m)/3, (4 + m)/3
, -((b*x^3)/a)])/(a*(1 + m)*Sqrt[a + b*x^3])
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(b\,x^3+a)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(b*x^3+a)^(3/2),x)
```

output
```
int(x^m/(b*x^3+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^3 + a)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx = \frac{x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{3}+\frac{1}{3}\\\frac{m}{3}+\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate(x**m/(b*x**3+a)**(3/2),x)
```

output
```
x**(m + 1)*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx=\int \frac{x^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^3 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx=\int \frac{x^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(b*x^3+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^3 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^m}{(b\,x^3+a)^{3/2}}\,dx$$

input
```
int(x^m/(a + b*x^3)^(3/2),x)
```

output
```
int(x^m/(a + b*x^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^3)^{3/2}}\,dx = \int \frac{x^m\sqrt{b\,x^3+a}}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x^m/(b*x^3+a)^(3/2),x)
```

output
```
int((x**m*sqrt(a + b*x**3))/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.449 $\int (cx)^m \left(a+bx^3\right)^{4/3} dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (cx)^m \left(a+bx^3\right)^{4/3} dx = \frac{a(cx)^{1+m}\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1+m}{3},\frac{4+m}{3},-\frac{bx^3}{a}\right)}{c(1+m)\sqrt[3]{1+\frac{bx^3}{a}}}$$

output
```
a*(c*x)^(1+m)*(b*x^3+a)^(1/3)*hypergeom([-4/3, 1/3+1/3*m],[4/3+1/3*m],-b*x
^3/a)/c/(1+m)/(1+b*x^3/a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (cx)^m (a + bx^3)^{4/3} dx = \frac{ax(cx)^m\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c*x)^m*(a + b*x^3)^(4/3),x]
```

output
```
(a*x*(c*x)^m*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, (1 + m)/3, 1 + (1 +
 m)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{4/3} (cx)^m \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{a\sqrt[3]{a+bx^3}\int (cx)^m \left(\frac{bx^3}{a}+1\right)^{4/3}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{a\sqrt[3]{a+bx^3}(cx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{c(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(c*x)^m*(a + b*x^3)^(4/3),x]
```

output
```
(a*(c*x)^(1 + m)*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, (1 + m)/3, (4 +
 m)/3, -((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \left(b\,x^3+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x)^m*(b*x^3+a)^(4/3),x)
```

output
```
int((c*x)^m*(b*x^3+a)^(4/3),x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^3\right)^{4/3}\,dx = \int \left(bx^3+a\right)^{\frac{4}{3}}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(4/3)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (cx)^m \left(a+bx^3\right)^{4/3} dx = \frac{a^{\frac{4}{3}} c^m x^{m+1} \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate((c*x)**m*(b*x**3+a)**(4/3),x)
```

output
```
a**(4/3)*c**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-4/3, m/3 + 1/3), (m/3 +
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int (cx)^m \left(a+bx^3\right)^{4/3} dx = \int \left(bx^3+a\right)^{\frac{4}{3}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(4/3)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^3\right)^{4/3} dx = \int \left(bx^3+a\right)^{\frac{4}{3}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(4/3)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^3\right)^{4/3} dx = \int (c\,x)^m \left(b\,x^3+a\right)^{4/3} dx$$

input
```
int((c*x)^m*(a + b*x^3)^(4/3),x)
```

output
```
int((c*x)^m*(a + b*x^3)^(4/3), x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^3\right)^{4/3} dx = \frac{c^m \left(x^m (b\,x^3+a)^{\frac{1}{3}}\,amx + 6x^m (b\,x^3+a)^{\frac{1}{3}}\,ax + x^m (b\,x^3+a)^{\frac{1}{3}}\,bm\,x^4 + 2x^m (b\,x^3+a)^{\frac{1}{3}}\,b\,x^4 + \right.}{}$$

input
```
int((c*x)^m*(b*x^3+a)^(4/3),x)
```

output
```
(c**m*(x**m*(a + b*x**3)**(1/3)*a*m*x + 6*x**m*(a + b*x**3)**(1/3)*a*x + x
**m*(a + b*x**3)**(1/3)*b*m*x**4 + 2*x**m*(a + b*x**3)**(1/3)*b*x**4 + 4*i
nt((x**m*(a + b*x**3)**(1/3))/(a*m**2 + 7*a*m + 10*a + b*m**2*x**3 + 7*b*m
*x**3 + 10*b*x**3),x)*a**2*m**2 + 28*int((x**m*(a + b*x**3)**(1/3))/(a*m**
2 + 7*a*m + 10*a + b*m**2*x**3 + 7*b*m*x**3 + 10*b*x**3),x)*a**2*m + 40*in
t((x**m*(a + b*x**3)**(1/3))/(a*m**2 + 7*a*m + 10*a + b*m**2*x**3 + 7*b*m*
x**3 + 10*b*x**3),x)*a**2))/(m**2 + 7*m + 10)
```

3.450 $\int (cx)^m \left(a+bx^3\right)^{2/3} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int (cx)^m \left(a+bx^3\right)^{2/3} \, dx=\frac{(cx)^{1+m} \left(a+bc^3x^3\right)^{5/3} \text{Hypergeometric2F1}\left(1,\frac{6+m}{3},\frac{4+m}{3},-\frac{bc^3x^3}{a}\right)}{ac(1+m)}$$

output
```
(c*x)^(1+m)*(b*c^3*x^3+a)^(5/3)*hypergeom([1, 2+1/3*m],[4/3+1/3*m],-b*c^3*
x^3/a)/a/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int (cx)^m \left(a+bx^3\right)^{2/3} \, dx=\frac{x(cx)^m \left(a+bx^3\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{(1+m) \left(1+\frac{bx^3}{a}\right)^{2/3}}$$

input
```
Integrate[(c*x)^m*(a + b*x^3)^(2/3),x]
```

output
```
(x*(c*x)^m*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, (1 + m)/3, 1 + (1 + m
)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2/3))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^{2/3} (cx)^m \, dx$$

$$\Big\downarrow \text{ 889}$$

$$\frac{\left(a+bx^3\right)^{2/3} \int (cx)^m \left(\frac{bx^3}{a}+1\right)^{2/3} dx}{\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{\left(a+bx^3\right)^{2/3} (cx)^{m+1} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{c(m+1) \left(\frac{bx^3}{a}+1\right)^{2/3}}$$

input
```
Int[(c*x)^m*(a + b*x^3)^(2/3),x]
```

output
```
((c*x)^(1 + m)*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^(2/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \left(b\,x^3+a\right)^{\frac{2}{3}}dx$$

input
```
int((c*x)^m*(b*x^3+a)^(2/3),x)
```

output
```
int((c*x)^m*(b*x^3+a)^(2/3),x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^3\right)^{2/3} \, dx = \int \left(bx^3+a\right)^{\frac{2}{3}}(cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(2/3)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int (cx)^m \left(a+bx^3\right)^{2/3} dx = \frac{a^{\frac{2}{3}} c^m x^{m+1} \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate((c*x)**m*(b*x**3+a)**(2/3),x)
```

output
```
a**(2/3)*c**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-2/3, m/3 + 1/3), (m/3 +
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int (cx)^m \left(a+bx^3\right)^{2/3} dx = \int \left(bx^3+a\right)^{\frac{2}{3}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(2/3)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^3\right)^{2/3} dx = \int \left(bx^3+a\right)^{\frac{2}{3}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(2/3)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^3\right)^{2/3} \, dx = \int (c\,x)^m\,\left(b\,x^3+a\right)^{2/3} \,dx$$

input
```
int((c*x)^m*(a + b*x^3)^(2/3),x)
```

output
```
int((c*x)^m*(a + b*x^3)^(2/3), x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^3\right)^{2/3} \, dx = \frac{c^m \left(x^m (b\,x^3+a)^{\frac{2}{3}} x+2 \left(\int \frac{x^m (b\,x^3+a)^{\frac{2}{3}}}{bm\,x^3+3b\,x^3+am+3a}dx\right) am+6 \left(\int \frac{x^m (b\,x^3+a)^{\frac{2}{3}}}{bm\,x^3+3b\,x^3+am+3a}dx\right) a\right)}{m+3}$$

input
```
int((c*x)^m*(b*x^3+a)^(2/3),x)
```

output
```
(c**m*(x**m*(a + b*x**3)**(2/3)*x + 2*int((x**m*(a + b*x**3)**(2/3))/(a*m
+ 3*a + b*m*x**3 + 3*b*x**3),x)*a*m + 6*int((x**m*(a + b*x**3)**(2/3))/(a*
m + 3*a + b*m*x**3 + 3*b*x**3),x)*a))/(m + 3)
```

3.451 $\int (cx)^m \sqrt[3]{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 61

$$\int (cx)^m \sqrt[3]{a+bx^3}\,dx=\frac{(cx)^{1+m}\left(a+bc^3x^3\right)^{4/3}\text{Hypergeometric2F1}\left(1,\frac{5+m}{3},\frac{4+m}{3},-\frac{bc^3x^3}{a}\right)}{ac(1+m)}$$

output
```
(c*x)^(1+m)*(b*c^3*x^3+a)^(4/3)*hypergeom([1, 5/3+1/3*m],[4/3+1/3*m],-b*c^
3*x^3/a)/a/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int (cx)^m \sqrt[3]{a+bx^3}\,dx=\frac{x(cx)^m\sqrt[3]{a+bx^3}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1+m}{3},1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{(1+m)\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c*x)^m*(a + b*x^3)^(1/3),x]
```

output
```
(x*(c*x)^m*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, (1 + m)/3, 1 + (1 + m
)/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(cx)^m\,dx$$

$$\Big\downarrow \text{889}$$

$$\frac{\sqrt[3]{a+bx^3}\int (cx)^m\sqrt[3]{\frac{bx^3}{a}+1}dx}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

$$\Big\downarrow \text{888}$$

$$\frac{\sqrt[3]{a+bx^3}(cx)^{m+1}\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{m+1}{3},\frac{m+4}{3},-\frac{bx^3}{a}\right)}{c(m+1)\sqrt[3]{\frac{bx^3}{a}+1}}$$

input
```
Int[(c*x)^m*(a + b*x^3)^(1/3),x]
```

output
```
((c*x)^(1 + m)*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, (1 + m)/3, (4 + m
)/3, -((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^(1/3))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \left(b\,x^3+a\right)^{\frac{1}{3}}dx$$

input
```
int((c*x)^m*(b*x^3+a)^(1/3),x)
```

output
```
int((c*x)^m*(b*x^3+a)^(1/3),x)
```

Fricas [F]

$$\int (cx)^m \sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int (cx)^m \sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{m}{3}+\frac{1}{3}\\ \frac{m}{3}+\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate((c*x)**m*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-1/3, m/3 + 1/3), (m/3 +
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int (cx)^m \sqrt[3]{a+bx^3}\, dx = \int \left(bx^3+a\right)^{\frac{1}{3}} (cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \sqrt[3]{a+bx^3}\, dx = \int \left(bx^3+a\right)^{\frac{1}{3}} (cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \sqrt[3]{a+bx^3}\,dx=\int (c\,x)^m\,\left(b\,x^3+a\right)^{1/3}\,dx$$

input
```
int((c*x)^m*(a + b*x^3)^(1/3),x)
```

output
```
int((c*x)^m*(a + b*x^3)^(1/3), x)
```

Reduce [F]

$$\int (cx)^m \sqrt[3]{a+bx^3}\,dx = \frac{c^m \left(x^m (b\,x^3+a)^{\frac{1}{3}}\,x+\left(\int \frac{x^m (b\,x^3+a)^{\frac{1}{3}}}{bm\,x^3+2b\,x^3+am+2a}dx\right) am+2\left(\int \frac{x^m (b\,x^3+a)^{\frac{1}{3}}}{bm\,x^3+2b\,x^3+am+2a}dx\right) a\right)}{m+2}$$

input
```
int((c*x)^m*(b*x^3+a)^(1/3),x)
```

output
```
(c**m*(x**m*(a + b*x**3)**(1/3)*x + int((x**m*(a + b*x**3)**(1/3))/(a*m +
2*a + b*m*x**3 + 2*b*x**3),x)*a*m + 2*int((x**m*(a + b*x**3)**(1/3))/(a*m
+ 2*a + b*m*x**3 + 2*b*x**3),x)*a))/(m + 2)
```

3.452 $\int x^2(a+bx^3)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int x^2\left(a+bx^3\right)^p\,dx=\frac{\left(a+bx^3\right)^{1+p}}{3b(1+p)}$$

output
```
1/3*(b*x^3+a)^(p+1)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^3\right)^p\,dx=\frac{\left(a+bx^3\right)^{1+p}}{3b(1+p)}$$

input
```
Integrate[x^2*(a + b*x^3)^p,x]
```

output
```
(a + b*x^3)^(1 + p)/(3*b*(1 + p))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^3\right)^{p+1}}{3b(p+1)}$$

input
```
Int[x^2*(a + b*x^3)^p,x]
```

output
```
(a + b*x^3)^(1 + p)/(3*b*(1 + p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(b\,x^3+a)^{p+1}}{3b(p+1)}$	22
derivativedivides	$\frac{(b\,x^3+a)^{p+1}}{3b(p+1)}$	22
default	$\frac{(b\,x^3+a)^{p+1}}{3b(p+1)}$	22
risch	$\frac{(b\,x^3+a)\,(b\,x^3+a)^{p}}{3b(p+1)}$	27
orering	$\frac{(b\,x^3+a)\,(b\,x^3+a)^{p}}{3b(p+1)}$	27
parallelrisch	$\frac{x^3\,(b\,x^3+a)^{p}ab+a^2\,(b\,x^3+a)^{p}}{3ab(p+1)}$	43
norman	$\frac{x^3\mathrm{e}^{p\ln\left(b\,x^3+a\right)}}{3p+3}+\frac{a\,\mathrm{e}^{p\ln\left(b\,x^3+a\right)}}{3b(p+1)}$	45

input
```
int(x^2*(b*x^3+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^3+a)^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(a+bx^3)^p\,dx=\frac{(bx^3+a)(bx^3+a)^p}{3\,(bp+b)}$$

input
```
integrate(x^2*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
1/3*(b*x^3 + a)*(b*x^3 + a)^p/(b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.52

$$\int x^2(a+bx^3)^p\,dx = \begin{cases} \frac{x^3}{3a} & \text{for } b=0 \wedge p=-1 \\ \frac{a^px^3}{3} & \text{for } b=0 \\ \frac{\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3b} + \frac{\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b} & \text{for } p=-1 \\ \frac{a(a+bx^3)^p}{3bp+3b} + \frac{bx^3(a+bx^3)^p}{3bp+3b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)**p,x)
```

output
```
Piecewise((x**3/(3*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**3/3, Eq(b, 0)), (lo
g(x - (-a/b)**(1/3))/(3*b) + log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2
/3))/(3*b), Eq(p, -1)), (a*(a + b*x**3)**p/(3*b*p + 3*b) + b*x**3*(a + b*x
**3)**p/(3*b*p + 3*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^3)^p\,dx = \frac{(bx^3+a)^{p+1}}{3\,b(p+1)}$$

input
```
integrate(x^2*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
1/3*(b*x^3 + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^3)^p\,dx=\frac{(bx^3+a)^{p+1}}{3\,b(p+1)}$$

input
```
integrate(x^2*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
1/3*(b*x^3 + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+bx^3)^p\,dx=\frac{(b\,x^3+a)^{p+1}}{3\,b\,(p+1)}$$

input
```
int(x^2*(a + b*x^3)^p,x)
```

output
```
(a + b*x^3)^(p + 1)/(3*b*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^2(a+bx^3)^p\,dx=\frac{(b\,x^3+a)^p\,(b\,x^3+a)}{3b\,(p+1)}$$

input
```
int(x^2*(b*x^3+a)^p,x)
```

output
```
((a + b*x**3)**p*(a + b*x**3))/(3*b*(p + 1))
```

3.453 $\int x^5(a+bx^3)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x^5\left(a+bx^3\right)^p\,dx=-\frac{a\left(a+bx^3\right)^{1+p}}{3b^2(1+p)}+\frac{\left(a+bx^3\right)^{2+p}}{3b^2(2+p)}$$

output
```
-1/3*a*(b*x^3+a)^(p+1)/b^2/(p+1)+1/3*(b*x^3+a)^(2+p)/b^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int x^5\left(a+bx^3\right)^p\,dx=\frac{\left(a+bx^3\right)^{1+p}\left(-a+b(1+p)x^3\right)}{3b^2(1+p)(2+p)}$$

input
```
Integrate[x^5*(a + b*x^3)^p,x]
```

output
```
((a + b*x^3)^(1 + p)*(-a + b*(1 + p)*x^3))/(3*b^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^3\right)^p dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^3 \left(bx^3+a\right)^p dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(\frac{\left(bx^3+a\right)^{p+1}}{b}-\frac{a\left(bx^3+a\right)^p}{b}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\left(a+bx^3\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+bx^3\right)^{p+1}}{b^2(p+1)}\right)$$

input
```
Int[x^5*(a + b*x^3)^p,x]
```

output
```
(-((a*(a + b*x^3)^(1 + p))/(b^2*(1 + p))) + (a + b*x^3)^(2 + p)/(b^2*(2 +
p)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{\left(b\,x^{3}+a\right)^{p+1}\left(-x^{3} p b-b\,x^{3}+a\right)}{3 b^{2}\left(p^{2}+3 p+2\right)}$	42
orering	$-\frac{\left(b\,x^{3}+a\right)^{p}\left(-x^{3} p b-b\,x^{3}+a\right)\left(b\,x^{3}+a\right)}{3 b^{2}\left(p^{2}+3 p+2\right)}$	47
risch	$-\frac{\left(-b^{2} x^{6} p-b^{2} x^{6}-a p\,x^{3} b+a^{2}\right)\left(b\,x^{3}+a\right)^{p}}{3 b^{2}\left(2+p\right)\left(p+1\right)}$	54
parallelrisch	$\frac{x^{6}\left(b\,x^{3}+a\right)^{p} b^{2} p+x^{6}\left(b\,x^{3}+a\right)^{p} b^{2}+x^{3}\left(b\,x^{3}+a\right)^{p} a b p-a^{2}\left(b\,x^{3}+a\right)^{p}}{3 b^{2}\left(p^{2}+3 p+2\right)}$	80
norman	$\frac{x^{6} {\mathrm e}^{p \ln\left(b\,x^{3}+a\right)}}{3 p+6}-\frac{a^{2} {\mathrm e}^{p \ln\left(b\,x^{3}+a\right)}}{3 b^{2}\left(p^{2}+3 p+2\right)}+\frac{p a\,x^{3} {\mathrm e}^{p \ln\left(b\,x^{3}+a\right)}}{3 b\left(p^{2}+3 p+2\right)}$	83

input
```
int(x^5*(b*x^3+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^2*(b*x^3+a)^(p+1)/(p^2+3*p+2)*(-b*p*x^3-b*x^3+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x^5 \left(a+b x^3\right)^p \, dx=\frac{{\left({\left(b^{2} p + b^{2}\right)} x^{6} + a b p x^{3} - a^{2}\right)} {\left(b x^{3} + a\right)}^{p}}{3 \, {\left(b^{2} p^{2} + 3 \, b^{2} p + 2 \, b^{2}\right)}}$$

input
```
integrate(x^5*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
1/3*((b^2*p + b^2)*x^6 + a*b*p*x^3 - a^2)*(b*x^3 + a)^p/(b^2*p^2 + 3*b^2*p
 + 2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(37) = 74$.

Time = 1.94 (sec) , antiderivative size = 432, normalized size of antiderivative = 9.00

$$\int x^5 (a + bx^3)^p \, dx$$
$$= \begin{cases} \frac{a^p x^6}{6} \\ \frac{a \log\left(x - \sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} + \frac{a \log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3} - \frac{2a\log(2)}{3ab^2+3b^3x^3} + \frac{a}{3ab^2+3b^3x^3} + \frac{bx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3} + \frac{bx^3 \log\left(4x^2+\ldots\right.}{3a\ldots} \\ -\frac{a \log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3b^2} - \frac{a\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^2} + \frac{x^3}{3b} \\ -\frac{a^2(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2} + \frac{abpx^3(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2} + \frac{b^2px^6(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2} + \frac{b^2x^6(a+bx^3)^p}{3b^2p^2+9b^2p+6b^2} \end{cases}$$

input
```
integrate(x**5*(b*x**3+a)**p,x)
```

output
```
Piecewise((a**p*x**6/6, Eq(b, 0)), (a*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3
*b**3*x**3) + a*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**
2 + 3*b**3*x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3*b
**3*x**3) + b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x**3) + b*x**
3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2 + 3*b**3*x**
3) - 2*b*x**3*log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -2)), (-a*log(x - (-a
/b)**(1/3))/(3*b**2) - a*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))
/(3*b**2) + x**3/(3*b), Eq(p, -1)), (-a**2*(a + b*x**3)**p/(3*b**2*p**2 +
9*b**2*p + 6*b**2) + a*b*p*x**3*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p +
6*b**2) + b**2*p*x**6*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2) +
b**2*x**6*(a + b*x**3)**p/(3*b**2*p**2 + 9*b**2*p + 6*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x^5 \left(a+bx^3\right)^p \, dx = \frac{(b^2(p+1)x^6 + abpx^3 - a^2)(bx^3+a)^p}{3\,(p^2+3\,p+2)b^2}$$

input
```
integrate(x^5*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
1/3*(b^2*(p + 1)*x^6 + a*b*p*x^3 - a^2)*(b*x^3 + a)^p/((p^2 + 3*p + 2)*b^2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^5 \left(a+bx^3\right)^p \, dx = \frac{\left(bx^3+a\right)^2 \left(bx^3+a\right)^p}{3\,b^2(p+2)} - \frac{\left(bx^3+a\right)^{p+1} a}{3\,b^2(p+1)}$$

input
```
integrate(x^5*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
1/3*(b*x^3 + a)^2*(b*x^3 + a)^p/(b^2*(p + 2)) - 1/3*(b*x^3 + a)^(p + 1)*a/
(b^2*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int x^5 \left(a+bx^3\right)^p \, dx = \left(b\,x^3+a\right)^p \left(\frac{x^6\,(p+1)}{3\,\left(p^2+3\,p+2\right)} - \frac{a^2}{3\,b^2\,\left(p^2+3\,p+2\right)} + \frac{a\,p\,x^3}{3\,b\,\left(p^2+3\,p+2\right)}\right)$$

input
```
int(x^5*(a + b*x^3)^p,x)
```

output
```
(a + b*x^3)^p*((x^6*(p + 1))/(3*(3*p + p^2 + 2)) - a^2/(3*b^2*(3*p + p^2 +
 2)) + (a*p*x^3)/(3*b*(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int x^5\left(a+bx^3\right)^p\,dx=\frac{\left(b\,x^3+a\right)^p\left(b^2p\,x^6+b^2x^6+abp\,x^3-a^2\right)}{3b^2\left(p^2+3p+2\right)}$$

input
```
int(x^5*(b*x^3+a)^p,x)
```

output
```
((a + b*x**3)**p*( - a**2 + a*b*p*x**3 + b**2*p*x**6 + b**2*x**6))/(3*b**2
*(p**2 + 3*p + 2))
```

3.454 $\int x^8(a+bx^3)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int x^8(a+bx^3)^p\,dx=\frac{a^2(a+bx^3)^{1+p}}{3b^3(1+p)}-\frac{2a(a+bx^3)^{2+p}}{3b^3(2+p)}+\frac{(a+bx^3)^{3+p}}{3b^3(3+p)}$$

output
```
1/3*a^2*(b*x^3+a)^(p+1)/b^3/(p+1)-2/3*a*(b*x^3+a)^(2+p)/b^3/(2+p)+1/3*(b*x
^3+a)^(3+p)/b^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^8(a+bx^3)^p\,dx=\frac{(a+bx^3)^{1+p}\left(2a^2-2ab(1+p)x^3+b^2(2+3p+p^2)\,x^6\right)}{3b^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^8*(a + b*x^3)^p,x]
```

output
```
((a + b*x^3)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^3 + b^2*(2 + 3*p + p^2)*x^6)
)/(3*b^3*(1 + p)*(2 + p)*(3 + p))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3} \int x^6 \left(bx^3+a\right)^p dx^3$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3} \int \left(\frac{a^2 \left(bx^3+a\right)^p}{b^2} - \frac{2a\left(bx^3+a\right)^{p+1}}{b^2} + \frac{\left(bx^3+a\right)^{p+2}}{b^2}\right) dx^3$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3} \left(\frac{a^2 \left(a+bx^3\right)^{p+1}}{b^3(p+1)} - \frac{2a\left(a+bx^3\right)^{p+2}}{b^3(p+2)} + \frac{\left(a+bx^3\right)^{p+3}}{b^3(p+3)}\right)$$

input
```
Int[x^8*(a + b*x^3)^p,x]
```

output
```
((a^2*(a + b*x^3)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x^3)^(2 + p))/(b^3*
(2 + p)) + (a + b*x^3)^(3 + p)/(b^3*(3 + p)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

method	result
gosper	$\frac{(b x^3+a)^{p+1}(b^2p^2x^6+3b^2x^6p+2b^2x^6-2apx^3b-2abx^3+2a^2)}{3b^3(p^3+6p^2+11p+6)}$
orering	$\frac{(b x^3+a)(b^2p^2x^6+3b^2x^6p+2b^2x^6-2apx^3b-2abx^3+2a^2)(b x^3+a)^p}{3b^3(p^3+6p^2+11p+6)}$
risch	$\frac{(b^3p^2x^9+3b^3p x^9+2b^3x^9+a b^2p^2x^6+apx^6b^2-2a^2px^3b+2a^3)(b x^3+a)^p}{3(2+p)(3+p)(p+1)b^3}$
parallelrisch	$\frac{x^9(b x^3+a)^p a b^3p^2+3x^9(b x^3+a)^p a b^3p+2x^9(b x^3+a)^p a b^3+x^6(b x^3+a)^p a^2b^2p^2+x^6(b x^3+a)^p a^2b^2p-2x^3(b x^3+a)^p a^3bp+}{3(3+p)(2+p)a(p+1)b^3}$

input
```
int(x^8*(b*x^3+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/3/b^3*(b*x^3+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^6+3*b^2*p*x^6+2*b^2*
x^6-2*a*b*p*x^3-2*a*b*x^3+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int x^8(a+bx^3)^p\,dx = \frac{((b^3p^2+3\,b^3p+2\,b^3)x^9-2\,a^2bpx^3+(ab^2p^2+ab^2p)x^6+2\,a^3)(bx^3+a)^p}{3\,(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3)}$$

input
```
integrate(x^8*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
1/3*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^9 - 2*a^2*b*p*x^3 + (a*b^2*p^2 + a*b^2*
p)*x^6 + 2*a^3)*(b*x^3 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1187 vs. 2(61) = 122.

Time = 8.41 (sec) , antiderivative size = 1187, normalized size of antiderivative = 16.04

$$\int x^8 (a+bx^3)^p \, dx = \text{Too large to display}$$

input
```
integrate(x**8*(b*x**3+a)**p,x)
```

output
```
Piecewise((a**p*x**9/9, Eq(b, 0)), (2*a**2*log(x - (-a/b)**(1/3))/(6*a**2*
b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 2*a**2*log(4*x**2 + 4*x*(-a/b)**(1/
3) + 4*(-a/b)**(2/3))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) - 4*a**
2*log(2)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 3*a**2/(6*a**2*b**
3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 4*a*b*x**3*log(x - (-a/b)**(1/3))/(6*a
**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 4*a*b*x**3*log(4*x**2 + 4*x*(-a
/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
 - 8*a*b*x**3*log(2)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 4*a*b*
x**3/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 2*b**2*x**6*log(x - (-
a/b)**(1/3))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6) + 2*b**2*x**6*lo
g(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a**2*b**3 + 12*a*b**4*x
**3 + 6*b**5*x**6) - 4*b**2*x**6*log(2)/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*
b**5*x**6), Eq(p, -3)), (-2*a**2*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4
*x**3) - 2*a**2*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**
3 + 3*b**4*x**3) - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b*
*3 + 3*b**4*x**3) - 2*a*b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*x
**3) - 2*a*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b
**3 + 3*b**4*x**3) + 4*a*b*x**3*log(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x**
6/(3*a*b**3 + 3*b**4*x**3), Eq(p, -2)), (a**2*log(x - (-a/b)**(1/3))/(3*b*
*3) + a**2*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*b**3) -...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int x^8 (a + bx^3)^p \, dx = \frac{((p^2 + 3p + 2)b^3x^9 + (p^2 + p)ab^2x^6 - 2a^2bpx^3 + 2a^3)(bx^3 + a)^p}{3(p^3 + 6p^2 + 11p + 6)b^3}$$

input
```
integrate(x^8*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
1/3*((p^2 + 3*p + 2)*b^3*x^9 + (p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + 2*a^3
)*(b*x^3 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

$$\int x^8 (a + bx^3)^p \, dx$$
$$= \frac{(bx^3 + a)^3 (bx^3 + a)^p p - 2(bx^3 + a)^2 (bx^3 + a)^p ap + 2(bx^3 + a)^3 (bx^3 + a)^p - 6(bx^3 + a)^2 (bx^3 + a)^p a}{3(b^3p^2 + 5b^3p + 6b^3)} + \frac{(bx^3 + a)^{p+1} a^2}{3b^3(p + 1)}$$

input
```
integrate(x^8*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
1/3*((b*x^3 + a)^3*(b*x^3 + a)^p*p - 2*(b*x^3 + a)^2*(b*x^3 + a)^p*a*p + 2
*(b*x^3 + a)^3*(b*x^3 + a)^p - 6*(b*x^3 + a)^2*(b*x^3 + a)^p*a)/(b^3*p^2 +
 5*b^3*p + 6*b^3) + 1/3*(b*x^3 + a)^(p + 1)*a^2/(b^3*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.59

$$\int x^8 (a + bx^3)^p \, dx = (b\,x^3 + a)^p \left(\frac{2\,a^3}{3\,b^3\,(p^3 + 6\,p^2 + 11\,p + 6)} + \frac{x^9\,(p^2 + 3\,p + 2)}{3\,(p^3 + 6\,p^2 + 11\,p + 6)} - \frac{2\,a^2\,p\,x^3}{3\,b^2\,(p^3 + 6\,p^2 + 11\,p + 6)} + \frac{a\,p\,x^6\,(p + 1)}{3\,b\,(p^3 + 6\,p^2 + 11\,p + 6)} \right)$$

input
```
int(x^8*(a + b*x^3)^p,x)
```

output
```
(a + b*x^3)^p*((2*a^3)/(3*b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^9*(3*p + p^2
+ 2))/(3*(11*p + 6*p^2 + p^3 + 6)) - (2*a^2*p*x^3)/(3*b^2*(11*p + 6*p^2 +
p^3 + 6)) + (a*p*x^6*(p + 1))/(3*b*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int x^8 (a + bx^3)^p \, dx = \frac{(b\,x^3 + a)^p\,(b^3p^2x^9 + 3b^3p\,x^9 + 2b^3x^9 + a\,b^2p^2x^6 + a\,b^2p\,x^6 - 2a^2bp\,x^3 + 2a^3)}{3b^3\,(p^3 + 6p^2 + 11p + 6)}$$

input
```
int(x^8*(b*x^3+a)^p,x)
```

output
```
((a + b*x**3)**p*(2*a**3 - 2*a**2*b*p*x**3 + a*b**2*p**2*x**6 + a*b**2*p*x
**6 + b**3*p**2*x**9 + 3*b**3*p*x**9 + 2*b**3*x**9))/(3*b**3*(p**3 + 6*p**
2 + 11*p + 6))
```

3.455 $\int x^{11}(a+bx^3)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int x^{11}\left(a+bx^3\right)^p \, dx = -\frac{a^3(a+bx^3)^{1+p}}{3b^4(1+p)}+\frac{a^2(a+bx^3)^{2+p}}{b^4(2+p)}-\frac{a(a+bx^3)^{3+p}}{b^4(3+p)}+\frac{(a+bx^3)^{4+p}}{3b^4(4+p)}$$

output
```
-1/3*a^3*(b*x^3+a)^(p+1)/b^4/(p+1)+a^2*(b*x^3+a)^(2+p)/b^4/(2+p)-a*(b*x^3+
a)^(3+p)/b^4/(3+p)+1/3*(b*x^3+a)^(4+p)/b^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int x^{11}\left(a+bx^3\right)^p \, dx = -\frac{a^3(a+bx^3)^{1+p}}{3b^4(1+p)}+\frac{a^2(a+bx^3)^{2+p}}{b^4(2+p)}-\frac{a(a+bx^3)^{3+p}}{b^4(3+p)}+\frac{(a+bx^3)^{4+p}}{3b^4(4+p)}$$

input
```
Integrate[x^11*(a + b*x^3)^p,x]
```

output
```
-1/3*(a^3*(a + b*x^3)^(1 + p))/(b^4*(1 + p)) + (a^2*(a + b*x^3)^(2 + p))/(
b^4*(2 + p)) - (a*(a + b*x^3)^(3 + p))/(b^4*(3 + p)) + (a + b*x^3)^(4 + p)
/(3*b^4*(4 + p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} (a+bx^3)^p \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\int x^9 (bx^3+a)^p \, dx^3$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{3}\int \left(-\frac{a^3 (bx^3+a)^p}{b^3}+\frac{3a^2 (bx^3+a)^{p+1}}{b^3}-\frac{3a (bx^3+a)^{p+2}}{b^3}+\frac{(bx^3+a)^{p+3}}{b^3}\right) dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-\frac{a^3 (a+bx^3)^{p+1}}{b^4 (p+1)}+\frac{3a^2 (a+bx^3)^{p+2}}{b^4 (p+2)}-\frac{3a (a+bx^3)^{p+3}}{b^4 (p+3)}+\frac{(a+bx^3)^{p+4}}{b^4 (p+4)}\right)$$

input
```
Int[x^11*(a + b*x^3)^p,x]
```

output
```
(-((a^3*(a + b*x^3)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^3)^(2 + p))/
(b^4*(2 + p)) - (3*a*(a + b*x^3)^(3 + p))/(b^4*(3 + p)) + (a + b*x^3)^(4 +
 p)/(b^4*(4 + p)))/3
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

method	result
gosper	$-\frac{(b\,x^{3}+a)^{p+1}(-b^{3}p^{3}x^{9}-6b^{3}p^{2}x^{9}-11b^{3}p\,x^{9}-6b^{3}x^{9}+3a\,b^{2}p^{2}x^{6}+9ap\,x^{6}b^{2}+6a\,b^{2}x^{6}-6a^{2}p\,x^{3}b-6a^{2}b\,x^{3}+6a^{3})}{3b^{4}(p^{4}+10p^{3}+35p^{2}+50p+24)}$
orering	$-\frac{(b\,x^{3}+a)^{p}(-b^{3}p^{3}x^{9}-6b^{3}p^{2}x^{9}-11b^{3}p\,x^{9}-6b^{3}x^{9}+3a\,b^{2}p^{2}x^{6}+9ap\,x^{6}b^{2}+6a\,b^{2}x^{6}-6a^{2}p\,x^{3}b-6a^{2}b\,x^{3}+6a^{3})(b\,x^{3}+a)}{3b^{4}(p^{4}+10p^{3}+35p^{2}+50p+24)}$
risch	$-\frac{(-b^{4}p^{3}x^{12}-6b^{4}p^{2}x^{12}-11b^{4}p\,x^{12}-a\,b^{3}p^{3}x^{9}-6b^{4}x^{12}-3a\,b^{3}p^{2}x^{9}-2ap\,x^{9}b^{3}+3a^{2}b^{2}p^{2}x^{6}+3a^{2}p\,x^{6}b^{2}-6a^{3}p\,x^{3}b+6a^{4})(b\,x^{3}}{3(3+p)(4+p)(2+p)(p+1)b^{4}}$
parallelrisch	$\frac{x^{12}(b\,x^{3}+a)^{p}b^{4}p^{3}+6x^{12}(b\,x^{3}+a)^{p}b^{4}p^{2}+11x^{12}(b\,x^{3}+a)^{p}b^{4}p+6x^{12}(b\,x^{3}+a)^{p}b^{4}+x^{9}(b\,x^{3}+a)^{p}a\,b^{3}p^{3}+3x^{9}(b\,x^{3}+a)^{p}a\,b^{3}p^{2}}{3b^{4}(p^{4}+10p^{3}+35p^{2}+50p+}$

input
```
int(x^11*(b*x^3+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/3/b^4*(b*x^3+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p^3*x^9-6*b^3*p
^2*x^9-11*b^3*p*x^9-6*b^3*x^9+3*a*b^2*p^2*x^6+9*a*b^2*p*x^6+6*a*b^2*x^6-6*
a^2*b*p*x^3-6*a^2*b*x^3+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.56

$$\int x^{11} \left(a+bx^3\right)^p \, dx = \frac{((b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4)x^{12}+(ab^3p^3+3\,ab^3p^2+2\,ab^3p)x^9+6\,a^3bpx^3-3\,(a^2b^2p^2+a^2b^2p)x^6-6}{3\,(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4)}$$

input
```
integrate(x^11*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
1/3*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^12 + (a*b^3*p^3 + 3*a*b^3*
p^2 + 2*a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^6 - 6
*a^4)*(b*x^3 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2467 vs. 2(78) = 156.

Time = 22.49 (sec) , antiderivative size = 2467, normalized size of antiderivative = 25.97

$$\int x^{11} \left(a+bx^3\right)^p \, dx = \text{Too large to display}$$

input
```
integrate(x**11*(b*x**3+a)**p,x)
```

output
```
Piecewise((a**p*x**12/12, Eq(b, 0)), (6*a**3*log(x - (-a/b)**(1/3))/(18*a*
*3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*a**3*log(
4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b**5
*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4 + 54
*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b**4 +
 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*log(x
 - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*
b**7*x**9) + 18*a**2*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/
3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 3
6*a**2*b*x**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 +
18*b**7*x**9) + 27*a**2*b*x**3/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b*
*6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(x - (-a/b)**(1/3))/(18*a**3*b
**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*
log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*
b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b**2*x**6*log(2)/(18*a**
3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x*
*6/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*
b**3*x**9*log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*
b**6*x**6 + 18*b**7*x**9) + 6*b**3*x**9*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4
*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12

$$\int x^{11}(a+bx^3)^p\,dx = \frac{((p^3+6\,p^2+11\,p+6)b^4x^{12}+(p^3+3\,p^2+2\,p)ab^3x^9-3\,(p^2+p)a^2b^2x^6+6\,a^3bpx^3-6\,a^4)(bx^3+a)^p}{3\,(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}$$

input
```
integrate(x^11*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
1/3*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^12 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^9 - 3
*(p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 6*a^4)*(b*x^3 + a)^p/((p^4 + 10*p
^3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.74

$$\int x^{11}(a+bx^3)^p\,dx = \frac{(bx^3+a)^4(bx^3+a)^p p^2 - 3(bx^3+a)^3(bx^3+a)^p ap^2 + 3(bx^3+a)^2(bx^3+a)^p a^2p^2 + 5(bx^3+a)^4(bx^3+a}{}$$

$$-\frac{(bx^3+a)^{p+1}a^3}{3\,b^4(p+1)}$$

input
```
integrate(x^11*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
1/3*((b*x^3 + a)^4*(b*x^3 + a)^p*p^2 - 3*(b*x^3 + a)^3*(b*x^3 + a)^p*a*p^2
 + 3*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2*p^2 + 5*(b*x^3 + a)^4*(b*x^3 + a)^p*p
 - 18*(b*x^3 + a)^3*(b*x^3 + a)^p*a*p + 21*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2
*p + 6*(b*x^3 + a)^4*(b*x^3 + a)^p - 24*(b*x^3 + a)^3*(b*x^3 + a)^p*a + 36
*(b*x^3 + a)^2*(b*x^3 + a)^p*a^2)/(b^4*p^3 + 9*b^4*p^2 + 26*b^4*p + 24*b^4
) - 1/3*(b*x^3 + a)^(p + 1)*a^3/(b^4*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.93

$$\int x^{11}(a+bx^3)^p\,dx = (b\,x^3+a)^p\left(\frac{x^{12}\,(p^3+6\,p^2+11\,p+6)}{3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{2\,a^4}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{2\,a^3\,p\,x^3}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{a\,p\,x^9\,(p^2+3\,p+2)}{3\,b\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{a^2\,p\,x^6\,(p+1)}{b^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}\right)$$

input
```
int(x^11*(a + b*x^3)^p,x)
```

output
```
(a + b*x^3)^p*((x^12*(11*p + 6*p^2 + p^3 + 6))/(3*(50*p + 35*p^2 + 10*p^3
+ p^4 + 24)) - (2*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (2*a^3*
p*x^3)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^9*(3*p + p^2 + 2
))/(3*b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (a^2*p*x^6*(p + 1))/(b^2*(5
0*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\int x^{11}\left(a+bx^3\right)^p\,dx$$
$$= \frac{(b\,x^3+a)^p\,(b^4p^3x^{12}+6b^4p^2x^{12}+11b^4p\,x^{12}+a\,b^3p^3x^9+6b^4x^{12}+3a\,b^3p^2x^9+2a\,b^3p\,x^9-3a^2b^2p^2x^6-3a}{3b^4\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^11*(b*x^3+a)^p,x)
```

output
```
((a + b*x**3)**p*( - 6*a**4 + 6*a**3*b*p*x**3 - 3*a**2*b**2*p**2*x**6 - 3*
a**2*b**2*p*x**6 + a*b**3*p**3*x**9 + 3*a*b**3*p**2*x**9 + 2*a*b**3*p*x**9
 + b**4*p**3*x**12 + 6*b**4*p**2*x**12 + 11*b**4*p*x**12 + 6*b**4*x**12))/
(3*b**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.456 $\int (cx)^m (a+bx^3)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int (cx)^m (a+bx^3)^p \, dx = \frac{(cx)^{1+m} (a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{3},-p,\frac{4+m}{3},-\frac{bx^3}{a}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*(b*x^3+a)^p*hypergeom([-p, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/c/
(1+m)/((1+b*x^3/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (cx)^m (a+bx^3)^p \, dx = \frac{x(cx)^m (a+bx^3)^p \left(1+\frac{bx^3}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{3},-p,1+\frac{1+m}{3},-\frac{bx^3}{a}\right)}{1+m}$$

input
```
Integrate[(c*x)^m*(a + b*x^3)^p,x]
```

output
```
(x*(c*x)^m*(a + b*x^3)^p*Hypergeometric2F1[(1 + m)/3, -p, 1 + (1 + m)/3, -
((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^p)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+bx^3\right)^p \, dx$$

$$\Big\downarrow \text{889}$$

$$\left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \int (cx)^m \left(\frac{bx^3}{a}+1\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1} \left(a+bx^3\right)^p \left(\frac{bx^3}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{3},-p,\frac{m+4}{3},-\frac{bx^3}{a}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m*(a + b*x^3)^p,x]
```

output
```
((c*x)^(1 + m)*(a + b*x^3)^p*Hypergeometric2F1[(1 + m)/3, -p, (4 + m)/3, -
((b*x^3)/a)])/(c*(1 + m)*(1 + (b*x^3)/a)^p)
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \left(b\,x^3+a\right)^p dx$$

input
```
int((c*x)^m*(b*x^3+a)^p,x)
```

output
```
int((c*x)^m*(b*x^3+a)^p,x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^3\right)^p\,dx=\int \left(bx^3+a\right)^p(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^p*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 123.82 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (cx)^m \left(a+bx^3\right)^p \, dx = \frac{a^p c^m x^{m+1}\Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}$$

input
```
integrate((c*x)**m*(b*x**3+a)**p,x)
```

output
```
a**p*c**m*x**(m + 1)*gamma(m/3 + 1/3)*hyper((-p, m/3 + 1/3), (m/3 + 4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3))
```

Maxima [F]

$$\int (cx)^m \left(a+bx^3\right)^p \, dx = \int \left(bx^3+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^3\right)^p \, dx = \int \left(bx^3+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^3+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^3\right)^p \, dx = \int (c\,x)^m \,\left(b\,x^3+a\right)^p \, dx$$

input
```
int((c*x)^m*(a + b*x^3)^p,x)
```

output
```
int((c*x)^m*(a + b*x^3)^p, x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^3\right)^p \, dx = \frac{c^m \left(x^m (b\,x^3+a)^p\, x + 3\left(\int \frac{x^m (b\,x^3+a)^p}{bm\,x^3+3bp\,x^3+b\,x^3+am+3ap+a} dx\right) amp + 9\left(\int \frac{x^m (b\,x^3+a)^p}{bm\,x^3+3bp\,x^3+b\,x^3+am+3ap+a} dx\right) a\,p^2 +}{m+3p+1}$$

input
```
int((c*x)^m*(b*x^3+a)^p,x)
```

output
```
(c**m*(x**m*(a + b*x**3)**p*x + 3*int((x**m*(a + b*x**3)**p)/(a*m + 3*a*p
+ a + b*m*x**3 + 3*b*p*x**3 + b*x**3),x)*a*m*p + 9*int((x**m*(a + b*x**3)*
*p)/(a*m + 3*a*p + a + b*m*x**3 + 3*b*p*x**3 + b*x**3),x)*a*p**2 + 3*int((
x**m*(a + b*x**3)**p)/(a*m + 3*a*p + a + b*m*x**3 + 3*b*p*x**3 + b*x**3),x
)*a*p))/(m + 3*p + 1)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file