Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.3-General-binomial/1.1.3.2/44-1.1.3.2-b

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**727**]. This is test number [44].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (727)	0.00 (0)
Mathematica	100.00 (727)	0.00 (0)
Sympy	97.52 (709)	2.48 (18)
Maple	81.43 (592)	18.57 (135)
Fricas	78.54 (571)	21.46 (156)
Mupad	60.66 (441)	39.34 (286)
Maxima	59.42 (432)	40.58 (295)
Giac	53.37 (388)	46.63 (339)
Reduce	48.42 (352)	51.58 (375)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

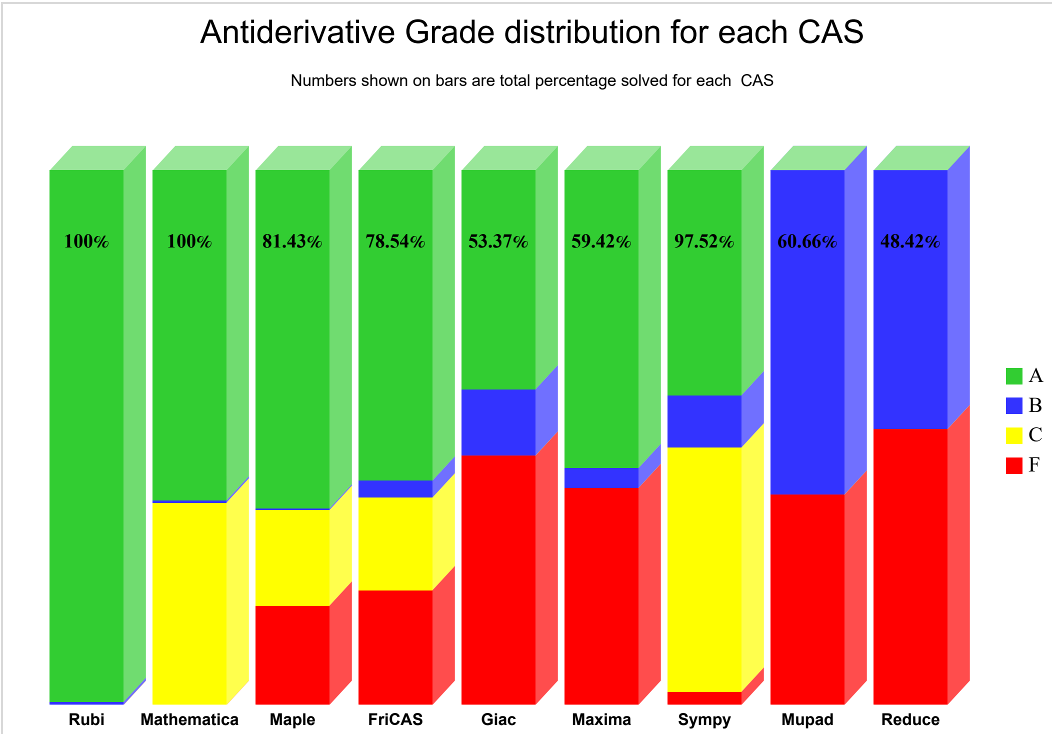
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

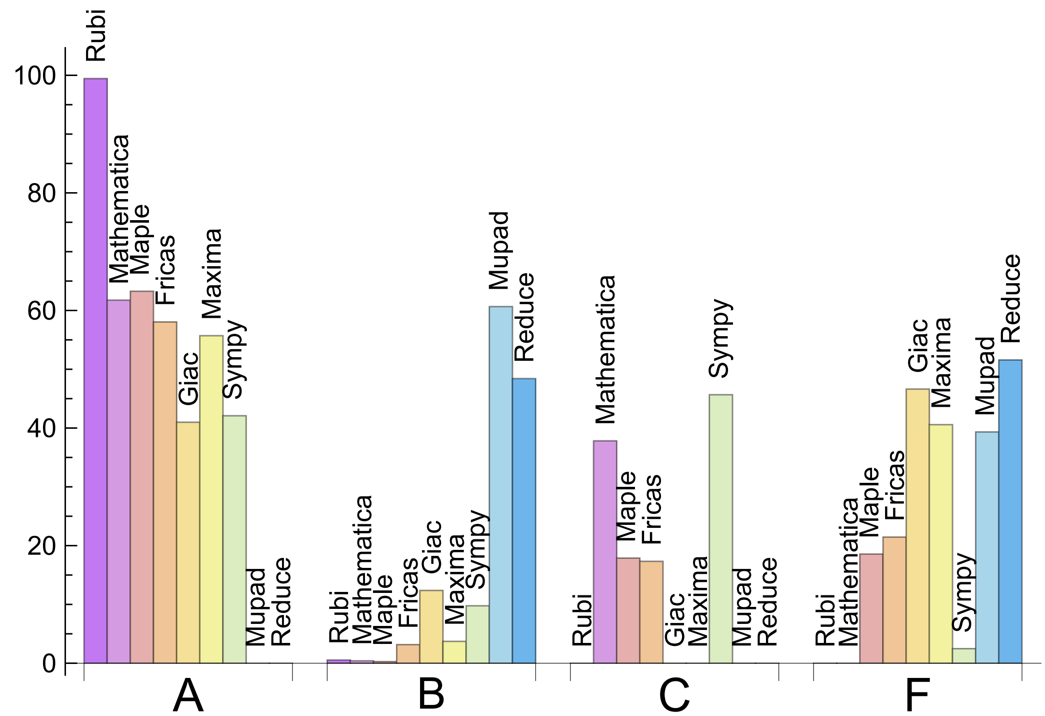
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.450	0.550	0.000	0.000
Maple	63.274	0.275	17.882	18.569
Mathematica	61.761	0.413	37.827	0.000
Fricas	58.047	3.164	17.331	21.458
Maxima	55.708	3.714	0.000	40.578
Sympy	42.091	9.766	45.667	2.476
Giac	40.990	12.380	0.000	46.630
Mupad	0.000	60.660	0.000	39.340
Reduce	0.000	48.418	0.000	51.582

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	18	0.00	100.00	0.00
Maple	135	100.00	0.00	0.00
Fricas	156	96.79	3.21	0.00
Mupad	286	0.00	100.00	0.00
Maxima	295	100.00	0.00	0.00
Giac	339	100.00	0.00	0.00
Reduce	375	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Fricas	0.08
Giac	0.13
Reduce	0.22
Mupad	0.32
Rubi	0.35
Maple	0.61
Sympy	0.99
Mathematica	3.07

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	44.77	0.75	37.00	0.74
Maple	47.83	0.71	36.00	0.75
Mathematica	60.48	0.79	51.00	0.81
Maxima	61.81	1.00	43.00	0.85
Fricas	89.54	1.15	50.00	0.88
Sympy	91.18	1.44	39.00	0.75
Rubi	95.26	1.09	77.00	1.05
Giac	97.23	1.27	45.00	0.97
Reduce	181.02	2.23	57.00	1.41

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

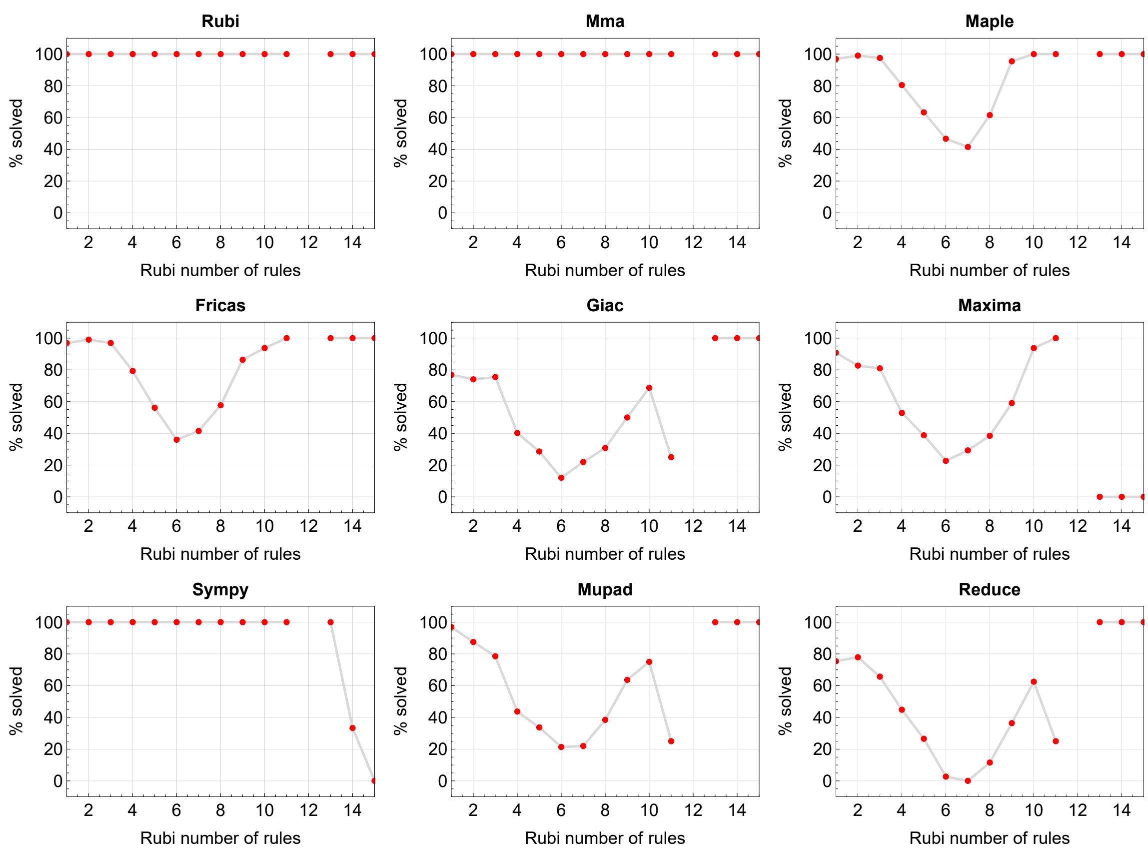


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

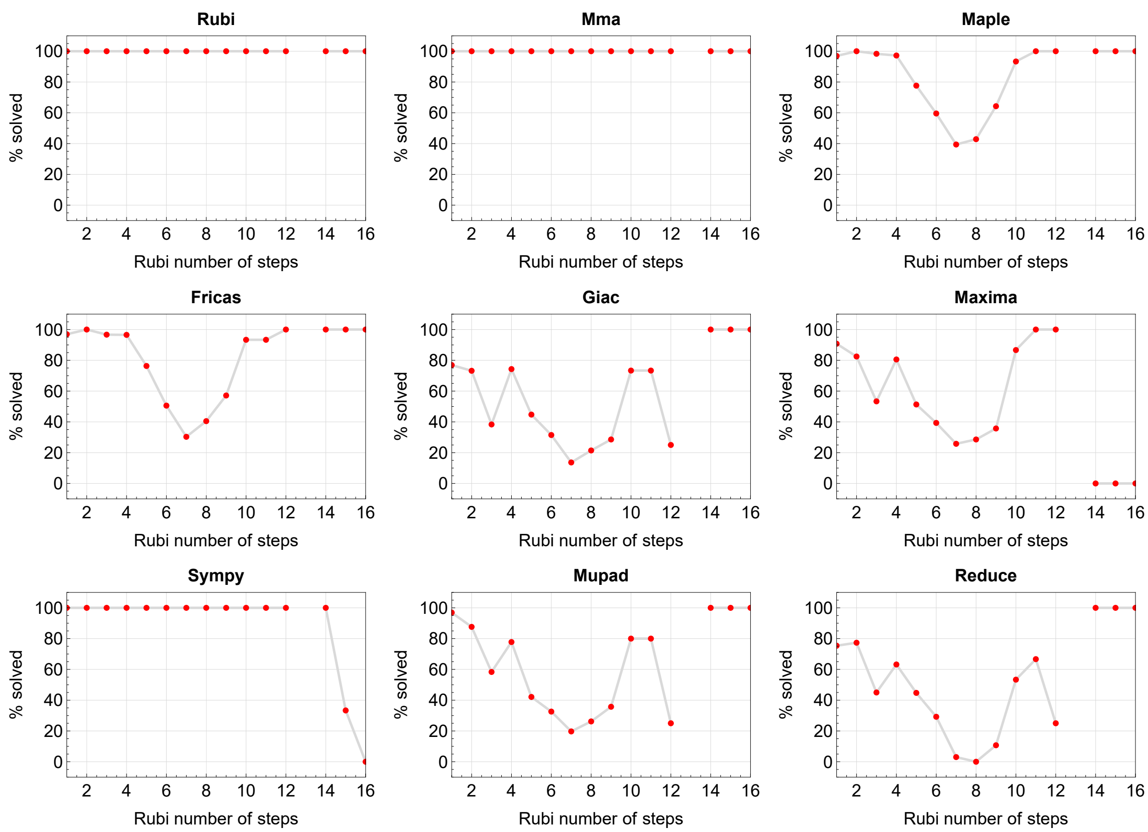


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

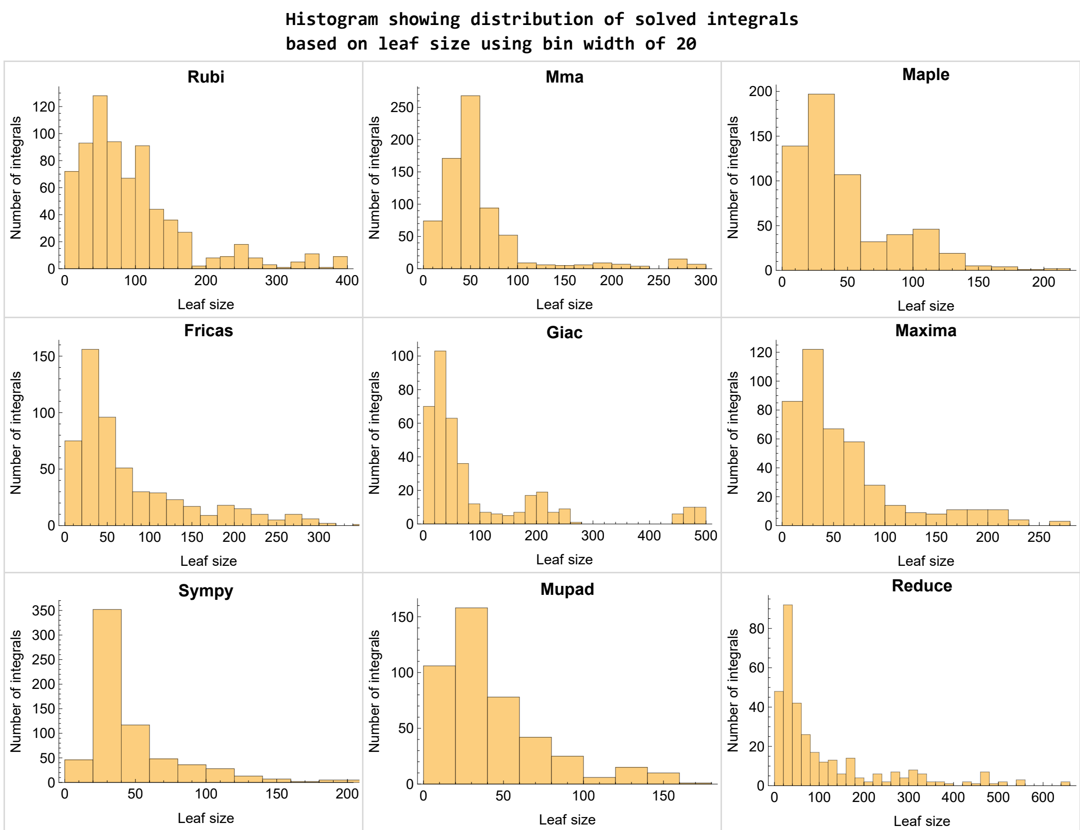


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

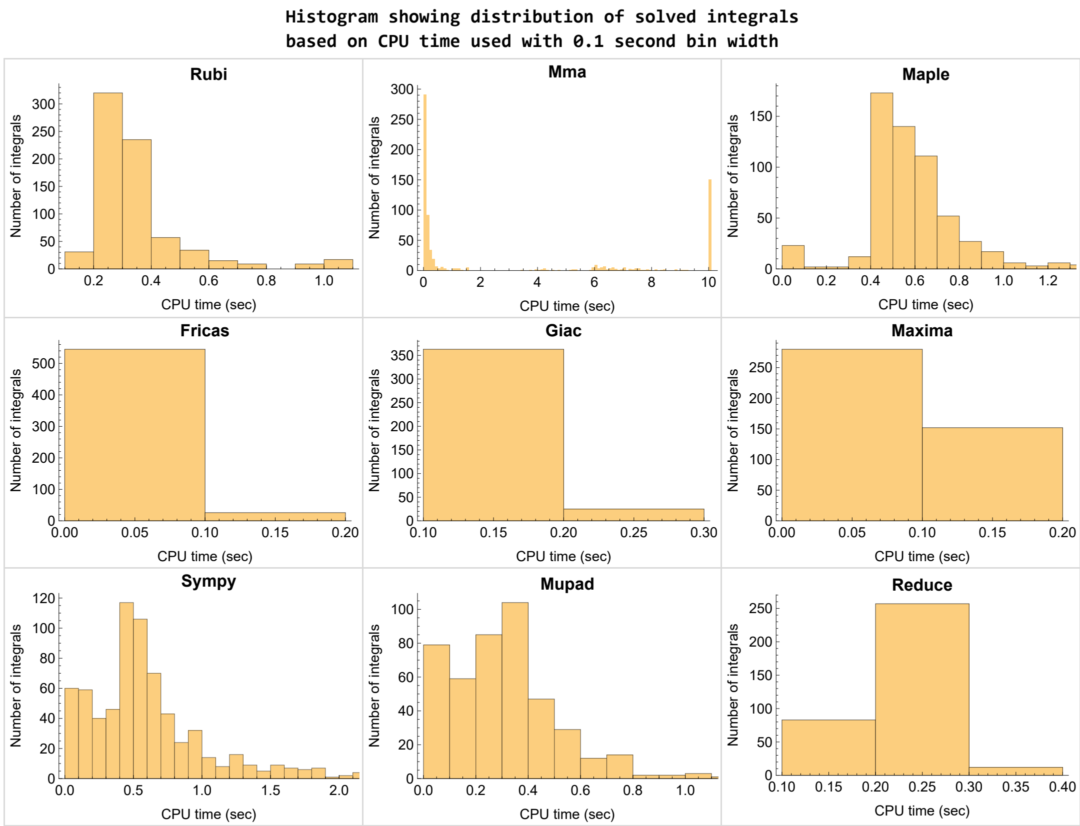


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

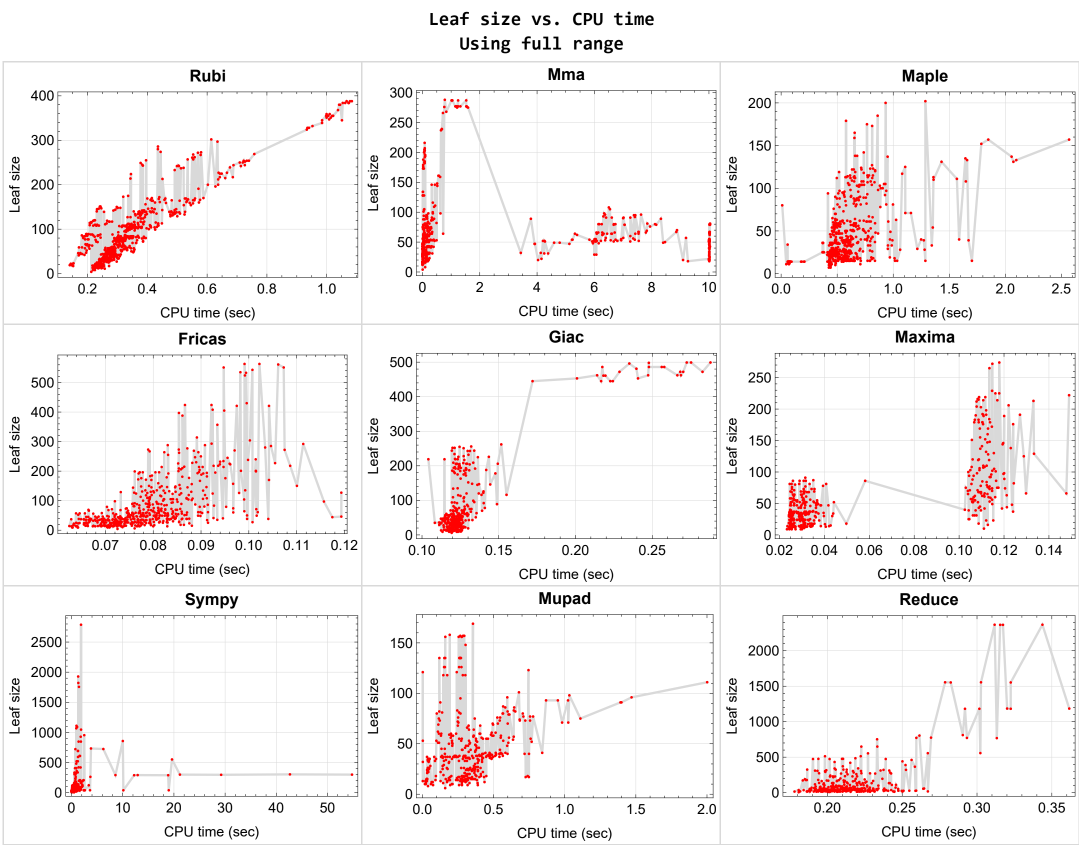


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{300, 471, 495\}$

Mathematica {}

Maple $\{14, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 625, 631\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

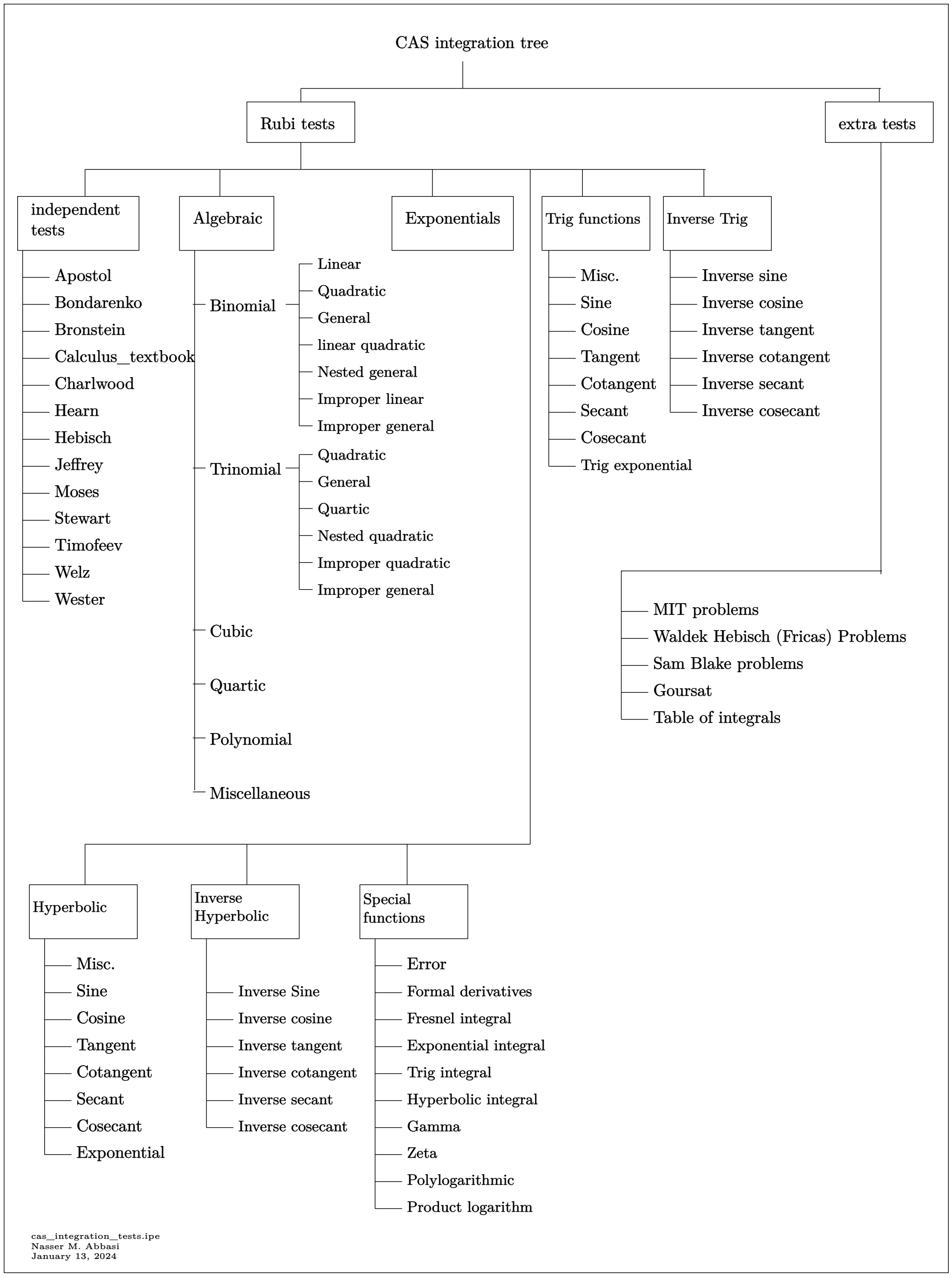
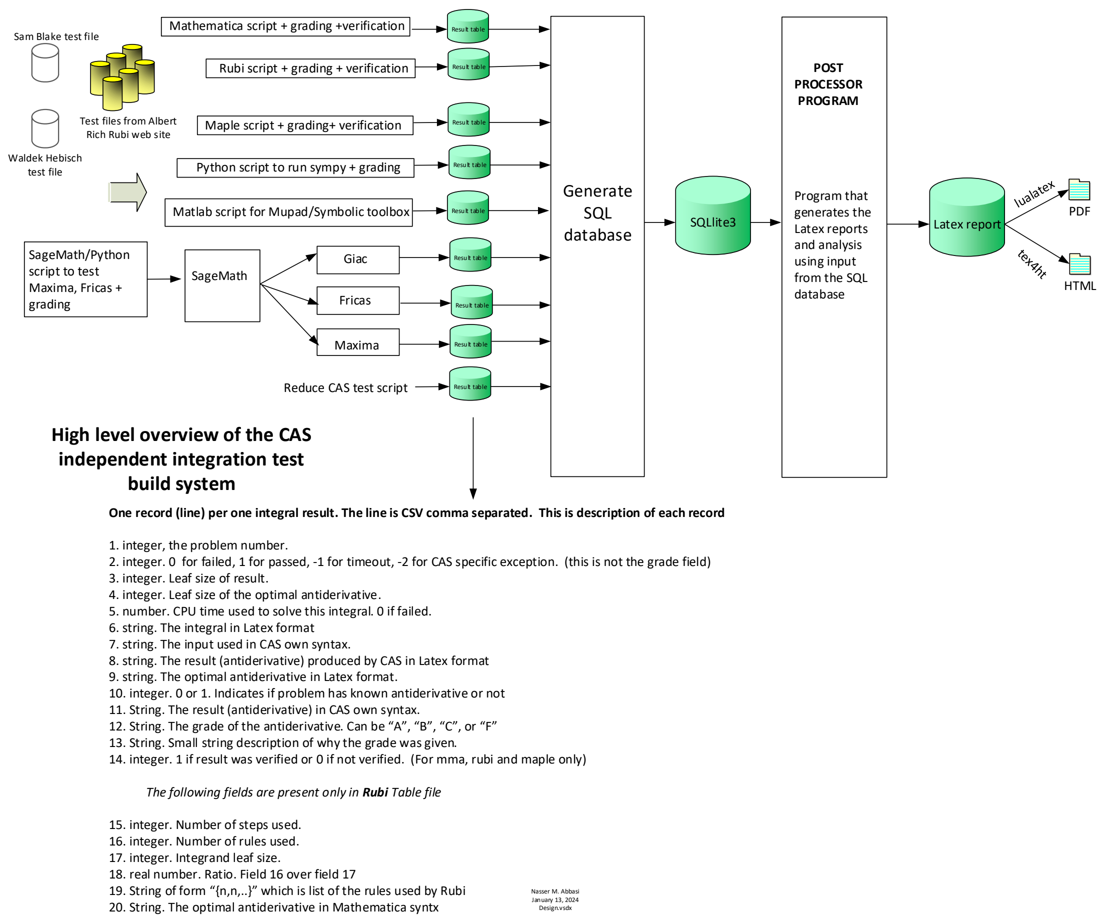


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 303, 304, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727 }

B grade { 302, 306, 307, 308 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 183, 184, 185, 186, 187, 188, 193, 194, 195, 203, 204, 205, 206, 207, 208, 211, 212, 213, 224, 225, 226, 227, 228, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 302, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 396, 397, 398, 399, 400, 401, 403, 404, 405, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 457, 458, 459, 460, 461, 462, 463, 472, 473, 474, 475, 476, 477, 478, 479, 486, 487, 488, 489, 490, 491, 492, 493, 494, 502, 503, 504, 505, 506, 507, 508, 509, 517, 518, 519, 520, 521, 522, 523, 524, 525, 534, 535, 536, 537, 538, 539, 540, 541, 549, 550, 551, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 614, 615, 636, 637, 638, 639, 640, 641, 642, 643, 650, 651, 652, 653, 654, 655, 656, 665, 666, 667, 668, 669, 670, 671, 672, 680, 681, 682, 683, 684, 685, 686, 687, 695, 696, 697, 698, 699, 700, 701, 702, 710, 711, 712, 713, 714, 715, 716, 725, 726, 727 }

B grade { 34, 39, 402 }

C grade { 170, 171, 172, 176, 177, 178, 179, 180, 181, 182, 189, 190, 191, 192, 196, 197, 198, 199, 200, 201, 202, 209, 210, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 229, 230, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 256, 257, 259, 260, 261, 262, 263, 264, 265, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 300, 301, 303, 304, 305, 306, 307, 308, 309, 310, 311, 324, 325, 326, 327, 328, 329, 330, 344, 345, 346, 347, 348, 349, 350, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 470, 471, 480, 481, 482, 483, 484, 485, 495, 496, 497, 498, 499, 500, 501, 510, 511, 512, 513, 514, 515, 516, 526, 527, 528, 529, 530, 531, 532, 533, 542, 543, 544, 545, 546, 547, 548, 557, 558, 559, 560, 561, 562, 563, 572, 573, 574, 575, 576, 577, 578, 586, 587, 588, 589, 590, 591, 592, 601, 602, 603, 604, 605, 606, 607, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 644, 645, 646, 647, 648, 649, 657, 658, 659, 660, 661, 662, 663, 664, 673, 674, 675, 676, 677, 678, 679, 688, 689, 690, 691, 692, 693, 694, 703, 704, 705, 706, 707, 708, 709, 717, 718, 719, 720, 721, 722, 723, 724 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 105, 106, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 457, 458, 459, 460, 461, 462, 463, 472, 473, 474, 475, 476, 477, 478, 479, 486, 487, 488, 489, 490, 491, 492, 493, 494, 502, 503, 504, 505, 506, 507, 508, 509, 517, 518, 519, 520, 521, 522, 523, 524, 525, 534, 535, 536, 537, 538, 539, 540, 541, 549, 550, 551, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 614, 615, 636, 637, 638, 639, 640, 641, 642, 643, 650, 651, 652, 653, 654, 655, 656, 665, 666, 667, 668, 669, 670, 671, 672, 680, 681, 682, 683, 684, 685, 686, 687, 695, 696, 697, 698, 699, 700, 701, 702, 710, 711, 712, 713, 714, 715, 716, 725, 726 }

B grade { 39, 159 }

C grade { 61, 62, 63, 64, 65, 66, 78, 79, 80, 81, 82, 83, 84, 85, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 111, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 324, 325, 326, 327, 328, 329, 330, 344, 345, 346, 347, 348, 349, 350, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 624, 625, 627, 628, 630, 631, 633, 634 }

F normal fail { 162, 163, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 470, 471, 480, 481, 482, 483, 484, 485, 495, 496, 497, 498, 499, 500, 501, 510, 511, 512, 513, 514, 515, 516, 526, 527, 528, 529, 530, 531, 532, 533, 542, 543, 544, 545, 546, 547, 548, 557, 558, 559, 560, 561, 562, 563, 572, 573, 574, 575, 576, 577, 578, 586, 587, 588, 589, 590, 591, 592, 601, 602, 603, 604, 605, 606, 607, 616, 617, 618, 619, 620, 621, 622, 623, 626, 629, 632, 635, 644, 645, 646, 647, 648, 649, 657, 658, 659, 660, 661, 662, 663, 664, 673, 674, 675, 676, 677, 678, 679, 688, 689, 690, 691, 692, 693, 694, 703, 704, 705, 706, 707, 708, 709, 717, 718, 719, 720, 721, 722, 723, 724, 727 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 86, 87, 89, 90, 91, 92, 93, 94, 95, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 271, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 332, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 347, 348, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 403, 404, 405, 416, 417, 418, 421, 424, 425, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 460, 461, 462, 463, 472, 473, 474, 475, 476, 491, 492, 493, 494, 502, 503, 504, 505, 523, 524, 525, 534, 535, 536, 537, 538, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 582, 583, 584, 585, 593, 594, 595, 596, 597, 611, 612, 613, 614, 615, 636, 637, 638, 639, 640, 653, 654, 655, 656, 665, 666, 667, 668, 669, 683, 684, 685, 686, 687, 695, 696, 697, 698, 699, 713, 714, 715, 716, 725, 726 }

B grade { 34, 39, 88, 185, 227, 248, 252, 263, 269, 270, 272, 284, 311, 333, 341, 399, 402, 419, 420, 422, 423, 506, 522 }

C grade { 61, 62, 63, 64, 65, 66, 78, 79, 80, 81, 82, 83, 84, 85, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 448, 449, 450, 457, 458, 477, 478, 479, 486, 487, 488, 489, 507, 508, 509, 517, 518, 519, 539, 540, 541, 549, 550, 551, 569, 570, 571, 579, 580, 581, 598, 599, 600, 608, 609, 610, 641, 642, 643, 650, 651, 670, 671, 672, 680, 681, 682, 700, 701, 702, 710, 711, 712 }

F normal fail { 162, 163, 181, 198, 201, 202, 329, 346, 349, 350, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 470, 471, 480, 481, 482, 483, 484, 485, 495, 496, 497, 498, 499, 500, 501, 510, 511, 512, 513, 514, 515, 516, 526, 527, 528, 529, 530, 531, 532, 533, 542, 543, 544, 545, 546, 547, 548, 557, 558, 559, 560, 561, 562, 563, 572, 573, 574, 575, 576, 577, 578, 586, 587, 588, 589, 590, 591, 592, 601, 602, 603, 604, 605, 606, 607, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 644, 645, 646, 647, 648, 649, 657, 658, 659, 660, 661, 662, 663, 664, 673, 674, 675, 676, 677, 678, 679, 688, 689, 690, 691, 692, 693, 694, 703, 704, 705, 706, 707, 708, 709, 717, 718, 719, 720, 721, 722, 723, 724, 727 }

F(-1) timedout fail { 459, 490, 520, 521, 652 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 245, 246, 247, 249, 253, 254, 255, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 288, 289, 290, 291, 292, 293, 297, 298, 299, 312, 313, 314, 315, 316, 317, 320, 321, 322, 323, 331, 332, 333, 334, 335, 336, 339, 340, 341, 342, 343, 351, 352, 354, 355, 356, 357, 358, 359, 361, 362, 363, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 396, 397, 398, 400, 403, 404, 405, 416, 417, 418, 419, 420, 423, 424, 425, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 457, 458, 459, 460, 461, 462, 463, 472, 473, 474, 475, 476, 477, 478, 479, 486, 487, 488, 489, 490, 491, 492, 493, 494, 502, 503, 504, 505, 506, 507, 508, 509, 517, 518, 519, 520, 521, 522, 523, 524, 525, 534, 535, 536, 537, 538, 539, 540, 541, 549, 550, 551, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 614, 615, 636, 637, 638, 639, 640, 641, 642, 643, 650, 651, 652, 653, 654, 655, 656, 665, 666, 667, 668, 669, 670, 671, 672, 680, 681, 682, 683, 684, 685, 686, 687, 695, 696, 697, 698, 699, 700, 701, 702, 710, 711, 712, 713, 714, 715, 716, 725, 726 }

B grade { 39, 88, 103, 104, 107, 108, 203, 248, 250, 251, 252, 294, 295, 296, 318, 319, 337, 338, 353, 360, 399, 401, 402, 421, 422, 438, 439 }

C grade { }

F normal fail { 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 162, 163, 176, 177, 178, 179, 180, 181, 182, 196, 197, 198, 199, 200, 201, 202, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 324, 325, 326, 327, 328, 329, 330, 344, 345, 346, 347, 348, 349, 350, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 451, 452, 453, 454, 455, 456, 464, 465, 466, 467, 468, 469, 470, 471, 480, 481, 482, 483, 484, 485, 495, 496, 497, 498, 499, 500, 501, 510, 511, 512, 513, 514, 515, 516, 526, 527, 528, 529, 530, 531, 532, 533, 542, 543, 544, 545, 546, 547, 548, 557, 558, 559, 560, 561, 562, 563, 572, 573, 574, 575, 576, 577, 578, 586, 587, 588, 589, 590, 591, 592, 601, 602, 603, 604, 605, 606, 607, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 644, 645, 646, 647, 648, 649, 657, 658, 659, 660, 661, 662, 663, 664, 673, 674, 675, 676, 677, 678, 679, 688, 689, 690, 691, 692, 693, 694, 703, 704, 705, 706, 707, 708, 709, 717, 718, 719, 720, 721, 722, 723, 724, 727 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 183, 184, 185, 186, 187, 188, 189, 190, 191, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 224, 225, 226, 227, 228, 229, 230, 231, 232, 245, 246, 247, 249, 250, 251, 252, 266, 267, 268, 269, 270, 271, 272, 273, 274, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 312, 313, 314, 315, 316, 317, 318, 319, 320, 331, 332, 333, 334, 335, 336, 337, 338, 339, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 374, 375, 376, 377, 378, 379, 380, 381, 382, 396, 397, 398, 400, 401, 403, 404, 405, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 476, 502, 503, 504, 505, 506, 534, 535, 536, 537, 538, 564, 565, 566, 567, 568, 593, 594, 595, 596, 597, 636, 637, 638, 639, 640, 665, 666, 667, 668, 669, 695, 696, 697, 698, 699, 725, 726 }

B grade { 39, 63, 64, 103, 104, 107, 108, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 173, 174, 175, 192, 193, 194, 195, 233, 234, 248, 253, 254, 255, 275, 321, 322, 323, 340, 341, 342, 343, 383, 399, 402, 448, 449, 450, 472, 473, 474, 475, 477, 478, 479, 507, 508, 509, 539, 540, 541, 569, 570, 571, 598, 599, 600, 641, 642, 643, 670, 671, 672, 700, 701, 702 }

C grade { }

F normal fail { 162, 163, 176, 177, 178, 179, 180, 181, 182, 196, 197, 198, 199, 200, 201, 202, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 324, 325, 326, 327, 328, 329, 330, 344, 345, 346, 347, 348, 349, 350, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 727 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 181, 183, 184, 185, 186, 187, 188, 193, 194, 195, 197, 201, 204, 205, 206, 207, 208, 211, 212, 213, 216, 222, 224, 225, 226, 227, 228, 231, 232, 233, 234, 237, 243, 245, 246, 247, 248, 249, 252, 253, 254, 255, 258, 264, 266, 267, 268, 269, 270, 273, 274, 275, 279, 286, 288, 289, 290, 291, 292, 293, 296, 297, 298, 299, 302, 308, 312, 313, 314, 315, 316, 317, 321, 322, 323, 325, 329, 331, 332, 333, 334, 335, 336, 341, 342, 343, 345, 349, 351, 352, 354, 355, 356, 357, 358, 361, 362, 363, 366, 372, 374, 375, 376, 377, 378, 381, 382, 383, 387, 394, 396, 397, 398, 399, 400, 402, 403, 404, 405, 408, 414, 416, 417, 418, 419, 420, 423, 424, 425, 429, 436, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 459, 460, 461, 462, 463, 467, 472, 473, 474, 475, 476, 477, 478, 479, 489, 491, 492, 493, 494, 498, 502, 503, 504, 505, 506, 507, 508, 509, 520, 522, 523, 524, 525, 529, 534, 535, 536, 537, 538, 539, 540, 541, 551, 552, 553, 554, 555, 556, 560, 564, 565, 566, 567, 568, 569, 570, 571, 582, 583, 584, 585, 589, 593, 594, 595, 596, 597, 598, 599, 600, 611, 612, 613, 614, 615, 620, 624, 625, 626, 627, 628, 629, 636, 637, 638, 639, 640, 641, 642, 643, 652, 653, 654, 655, 656, 660, 665, 666, 667, 668, 669, 670, 671, 672, 682, 683, 684, 685, 686, 687, 691, 695, 696, 697, 698, 699, 700, 701, 702, 713, 714, 715, 716, 720, 725, 726 }

C grade { }

F normal fail { }

F(-1) timedout fail { 162, 163, 170, 171, 172, 176, 178, 179, 180, 182, 189, 190, 191, 192, 196, 198, 199, 200, 202, 203, 209, 210, 214, 215, 217, 218, 219, 220, 221, 223, 229, 230, 235, 236, 238, 239, 240, 241, 242, 244, 250, 251, 256, 257, 259, 260, 261, 262, 263, 265, 271, 272, 276, 277, 278, 280, 281, 282, 283, 284, 285, 287, 294, 295, 300, 301, 303, 304, 305, 306, 307, 309, 310, 311, 318, 319, 320, 324, 326, 327, 328, 330, 337, 338, 339, 340, 344, 346, 347, 348, 350, 353, 359, 360, 364, 365, 367, 368, 369, 370, 371, 373, 379, 380, 384, 385, 386, 388, 389, 390, 391, 392, 393, 395, 401, 406, 407, 409, 410, 411, 412, 413, 415, 421, 422, 426, 427, 428, 430, 431, 432, 433, 434, 435, 437, 451, 452, 453, 454, 455, 456, 457, 458, 464, 465, 466, 468, 469, 470, 471, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 495, 496, 497, 499, 500, 501, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 521, 526, 527, 528, 530, 531, 532, 533, 542, 543, 544, 545, 546, 547, 548, 549, 550, 557, 558, 559, 561, 562, 563, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 586, 587, 588, 590, 591, 592, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 616, 617, 618, 619, 621, 622, 623, 630, 631, 632, 633, 634, 635, 644, 645, 646, 647, 648, 649, 650, 651, 657, 658, 659, 661, 662, 663, 664, 673, 674, 675, 676, 677, 678, 679, 680, 681, 688, 689, 690, 692, 693, 694, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 717, 718, 719, 721, 722, 723, 724, 727 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 164, 165, 176, 177, 178, 179, 180, 181, 182, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 256, 259, 260, 261, 262, 265, 266, 267, 268, 276, 277, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 322, 334, 335, 336, 337, 338, 339, 340, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 374, 375, 376, 378, 379, 380, 381, 382, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 416, 417, 418, 420, 421, 422, 423, 424, 425, 439, 440, 441, 442, 443, 444, 445, 472, 473, 474, 502, 503, 534, 535, 536, 537, 538, 552, 553, 564, 565, 566, 567, 568, 583, 593, 594, 595, 596, 597, 612, 636, 637, 638, 665, 666, 667, 668, 669, 695, 696, 697, 698, 699 }

B grade { 20, 34, 39, 56, 58, 60, 74, 75, 88, 93, 105, 106, 159, 160, 161, 166, 183, 184, 185, 257, 258, 263, 264, 278, 314, 321, 323, 331, 332, 333, 341, 342, 343, 352, 363, 377, 383, 419, 446, 447, 460, 461, 462, 463, 475, 476, 491, 492, 493, 494, 504, 505, 506, 522, 523, 524, 525, 554, 555, 556, 582, 584, 585, 611, 613, 614, 615, 639, 640, 725, 726 }

C grade { 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 203, 207, 208, 209, 210, 211, 212, 213, 227, 228, 229, 230, 231, 232, 233, 234, 248, 249, 250, 251, 252, 253, 254, 255, 269, 270, 271, 272, 273, 274, 275, 291, 292, 293, 294, 295, 296, 297, 298, 299, 324, 325, 326, 327, 328, 329, 330, 344, 345, 346, 347, 348, 349, 350, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 464, 465, 466, 467, 468, 469, 470, 471, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 495, 496, 497, 498, 499, 500, 501, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 557, 558, 559, 560, 561, 562, 563, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 586, 587, 588, 589, 590, 591, 592, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 727 }

F normal fail { }

F(-1) timedout fail { 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 438, 439, 440, 443, 444, 445, 446, 447, 460, 461, 462, 463, 472, 473, 474, 475, 476, 491, 492, 493, 494, 502, 503, 504, 505, 506, 522, 523, 524, 525, 568, 636, 637, 638, 639, 640, 653, 654, 655, 656, 699, 725, 726 }

C grade { }

F normal fail { 162, 163, 176, 177, 178, 179, 180, 181, 182, 196, 197, 198, 199, 200, 201, 202, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 324, 325, 326, 327, 328, 329, 330, 344, 345, 346, 347, 348, 349, 350, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 464, 465, 466, 467, 468, 469, 470, 471, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 495, 496, 497, 498, 499, 500, 501, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 727 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.233	0.002	0.184	0.032	0.073	0.017	0.126	0.209	0.026

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	17	17	14	14	13	12	13	14	13
N.S.	1	1.06	1.06	0.88	0.88	0.81	0.75	0.81	0.88	0.81
time (sec)	N/A	0.241	0.002	0.178	0.025	0.083	0.017	0.118	0.202	0.023

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85	0.85
time (sec)	N/A	0.238	0.004	0.075	0.029	0.070	0.036	0.124	0.216	0.024

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	17	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	1.31	0.85
time (sec)	N/A	0.247	0.003	0.059	0.034	0.063	0.056	0.119	0.197	0.146

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.88	0.76
time (sec)	N/A	0.251	0.002	0.064	0.026	0.063	0.072	0.123	0.188	0.031

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.245	0.002	0.075	0.043	0.066	0.094	0.114	0.178	0.032

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.249	0.002	0.208	0.027	0.064	0.021	0.120	0.223	0.022

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.253	0.002	0.204	0.027	0.072	0.021	0.120	0.224	0.024

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.250	0.002	0.078	0.027	0.064	0.024	0.124	0.207	0.022

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	14	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.82	0.76
time (sec)	N/A	0.247	0.002	0.070	0.030	0.069	0.021	0.116	0.210	0.022

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.235	0.000	0.046	0.025	0.071	0.027	0.125	0.229	0.018

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	14	8	13	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.93	0.53	0.87	0.93	0.87
time (sec)	N/A	0.245	0.002	0.075	0.041	0.072	0.038	0.116	0.196	0.027

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	14	12	13	14	13
N.S.	1	1.00	1.00	0.82	0.76	0.82	0.71	0.76	0.82	0.76
time (sec)	N/A	0.249	0.001	0.070	0.025	0.074	0.040	0.120	0.222	0.025

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	15	8	10	15	10
N.S.	1	1.00	1.00	0.92	0.83	1.25	0.67	0.83	1.25	0.83
time (sec)	N/A	0.247	0.001	0.070	0.026	0.070	0.046	0.125	0.208	0.024

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	1.00	0.87
time (sec)	N/A	0.247	0.002	0.059	0.025	0.072	0.065	0.122	0.250	0.027

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.88	0.76
time (sec)	N/A	0.255	0.003	0.059	0.026	0.067	0.065	0.123	0.205	0.030

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.272	0.002	0.062	0.030	0.066	0.087	0.121	0.191	0.030

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.253	0.003	0.062	0.034	0.068	0.098	0.126	0.194	0.029

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.277	0.001	0.484	0.030	0.071	0.019	0.126	0.212	0.040

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	15	14	24	24	14	25	24
N.S.	1	1.00	1.88	0.94	0.88	1.50	1.50	0.88	1.56	1.50
time (sec)	N/A	0.231	0.001	0.473	0.026	0.073	0.020	0.123	0.238	0.036

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	30	26	23	25	22	22	25	22	22
N.S.	1	1.15	1.00	0.88	0.96	0.85	0.85	0.96	0.85	0.85
time (sec)	N/A	0.270	0.001	0.517	0.029	0.075	0.039	0.118	0.200	0.128

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	25	27	24	33	27	23
N.S.	1	1.04	1.00	0.89	0.93	1.00	0.89	1.22	1.00	0.85
time (sec)	N/A	0.276	0.001	0.460	0.030	0.070	0.070	0.119	0.231	0.035

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	30	26	23	26	28	24	34	28	25
N.S.	1	1.15	1.00	0.88	1.00	1.08	0.92	1.31	1.08	0.96
time (sec)	N/A	0.283	0.001	0.435	0.026	0.073	0.109	0.124	0.211	0.046

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	1.37
time (sec)	N/A	0.234	0.001	0.428	0.032	0.071	0.127	0.123	0.204	0.038

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.279	0.001	0.438	0.025	0.066	0.146	0.122	0.205	0.042

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.268	0.001	0.498	0.028	0.066	0.018	0.116	0.259	0.033

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.270	0.001	0.501	0.031	0.071	0.020	0.120	0.227	0.033

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.275	0.002	0.507	0.033	0.065	0.020	0.116	0.251	0.034

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.271	0.001	0.423	0.025	0.066	0.021	0.122	0.200	0.033

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	22	24	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.79	0.86	0.93	0.89
time (sec)	N/A	0.274	0.001	0.448	0.028	0.068	0.061	0.121	0.202	0.037

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	25	22	23	25	25
N.S.	1	1.00	1.00	0.89	0.85	0.93	0.81	0.85	0.93	0.93
time (sec)	N/A	0.271	0.001	0.441	0.031	0.072	0.056	0.128	0.195	0.037

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	26	22	22	26	22
N.S.	1	1.00	1.00	0.88	0.85	1.00	0.85	0.85	1.00	0.85
time (sec)	N/A	0.270	0.001	0.612	0.029	0.072	0.053	0.123	0.227	0.038

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	43	36	35	35	37	35	37	35
N.S.	1	1.12	1.26	1.06	1.03	1.03	1.09	1.03	1.09	1.03
time (sec)	N/A	0.288	0.004	0.487	0.033	0.072	0.022	0.121	0.196	0.045

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	43	15	14	35	37	14	36	35
N.S.	1	1.00	2.69	0.94	0.88	2.19	2.31	0.88	2.25	2.19
time (sec)	N/A	0.233	0.002	0.473	0.034	0.066	0.026	0.117	0.215	0.044

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	36	33	37	36	33	33
N.S.	1	1.10	1.00	0.87	0.92	0.85	0.95	0.92	0.85	0.85
time (sec)	N/A	0.290	0.004	0.453	0.036	0.072	0.069	0.111	0.202	0.039

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	35	36	38	37	46	38	34
N.S.	1	1.05	1.00	0.88	0.90	0.95	0.92	1.15	0.95	0.85
time (sec)	N/A	0.291	0.005	0.508	0.025	0.064	0.083	0.116	0.205	0.037

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	40	35	37	39	37	46	39	37
N.S.	1	1.02	1.00	0.88	0.92	0.98	0.92	1.15	0.98	0.92
time (sec)	N/A	0.300	0.007	0.450	0.033	0.067	0.127	0.121	0.195	0.132

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	39	39	37	47	39	36
N.S.	1	1.10	1.00	0.87	1.00	1.00	0.95	1.21	1.00	0.92
time (sec)	N/A	0.297	0.004	0.438	0.025	0.070	0.159	0.121	0.205	0.143

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37	37
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95	1.95
time (sec)	N/A	0.228	0.004	0.437	0.031	0.067	0.183	0.123	0.215	0.032

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	43	36	37	37	39	37	37	37
N.S.	1	1.10	1.08	0.90	0.92	0.92	0.98	0.92	0.92	0.92
time (sec)	N/A	0.250	0.006	0.441	0.035	0.063	0.218	0.122	0.237	0.134

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.277	0.002	0.493	0.032	0.066	0.025	0.117	0.194	0.045

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.275	0.002	0.493	0.026	0.071	0.024	0.123	0.201	0.045

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.279	0.002	0.490	0.025	0.072	0.023	0.118	0.196	0.044

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	33	32	32	34	32	35	32
N.S.	1	1.00	1.00	0.87	0.84	0.84	0.89	0.84	0.92	0.84
time (sec)	N/A	0.275	0.001	0.428	0.025	0.067	0.020	0.122	0.203	0.044

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	34	37	32	34	37	34
N.S.	1	1.00	1.00	0.92	0.89	0.97	0.84	0.89	0.97	0.89
time (sec)	N/A	0.282	0.005	0.460	0.026	0.069	0.044	0.123	0.199	0.044

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	36	37	35	36	35
N.S.	1	1.00	1.00	0.84	0.81	0.84	0.86	0.81	0.84	0.81
time (sec)	N/A	0.292	0.005	0.454	0.033	0.068	0.049	0.113	0.222	0.045

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	33	37	36	33	37	33
N.S.	1	1.00	1.00	0.87	0.85	0.95	0.92	0.85	0.95	0.85
time (sec)	N/A	0.289	0.003	0.454	0.025	0.072	0.083	0.112	0.220	0.045

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	35	37	36	35	37	37
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.95	0.92	0.97	0.97
time (sec)	N/A	0.289	0.005	0.458	0.031	0.065	0.118	0.121	0.194	0.135

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	36	36	39	36	36	38
N.S.	1	1.00	1.00	0.84	0.84	0.84	0.91	0.84	0.84	0.88
time (sec)	N/A	0.285	0.004	0.454	0.026	0.066	0.084	0.118	0.181	0.137

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	37	36	34	37	35
N.S.	1	1.00	1.00	0.92	0.92	1.00	0.97	0.92	1.00	0.95
time (sec)	N/A	0.287	0.004	0.443	0.028	0.066	0.101	0.117	0.200	0.040

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	34	34	33	32	35	71	33
N.S.	1	0.98	1.00	0.85	0.85	0.82	0.80	0.88	1.78	0.82
time (sec)	N/A	0.293	0.007	0.642	0.032	0.068	0.106	0.123	0.228	0.151

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	58	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	2.15	0.81
time (sec)	N/A	0.265	0.006	0.572	0.035	0.063	0.123	0.123	0.234	0.042

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	47	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	3.13	0.87
time (sec)	N/A	0.231	0.004	0.555	0.025	0.077	0.088	0.122	0.195	0.128

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	15	24	55	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.68	1.09	2.50	0.82
time (sec)	N/A	0.245	0.005	0.562	0.038	0.070	0.139	0.123	0.196	0.095

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	35	32	33	33	31	43	71	31
N.S.	1	1.03	1.00	0.91	0.94	0.94	0.89	1.23	2.03	0.89
time (sec)	N/A	0.299	0.006	0.589	0.033	0.071	0.235	0.117	0.205	0.185

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	50	48	43	42	107	87	45	88	37
N.S.	1	0.98	0.94	0.84	0.82	2.10	1.71	0.88	1.73	0.73
time (sec)	N/A	0.314	0.022	0.596	0.119	0.079	0.111	0.124	0.202	0.083

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	32	31	89	63	31	75	28
N.S.	1	0.98	1.00	0.80	0.78	2.22	1.58	0.78	1.88	0.70
time (sec)	N/A	0.263	0.010	0.585	0.107	0.076	0.104	0.124	0.195	0.142

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	18	72	56	18	67	19
N.S.	1	1.00	1.00	0.66	0.62	2.48	1.93	0.62	2.31	0.66
time (sec)	N/A	0.248	0.005	0.572	0.109	0.084	0.081	0.123	0.216	0.148

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	79	32	31	90	71	31	82	28
N.S.	1	1.00	1.98	0.80	0.78	2.25	1.78	0.78	2.05	0.70
time (sec)	N/A	0.265	0.027	0.628	0.106	0.078	0.149	0.125	0.218	0.056

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	57	88	43	43	110	90	43	95	40
N.S.	1	1.12	1.73	0.84	0.84	2.16	1.76	0.84	1.86	0.78
time (sec)	N/A	0.285	0.028	0.630	0.109	0.079	0.168	0.125	0.208	0.158

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	217	176	37	183	150	34	175	147	51
N.S.	1	1.51	1.22	0.26	1.27	1.04	0.24	1.22	1.02	0.35
time (sec)	N/A	0.686	0.025	0.687	0.107	0.079	0.099	0.127	0.211	0.209

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	212	173	34	179	107	22	172	144	48
N.S.	1	1.53	1.24	0.24	1.29	0.77	0.16	1.24	1.04	0.35
time (sec)	N/A	0.637	0.020	0.622	0.110	0.087	0.086	0.129	0.213	0.210

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	200	134	27	169	124	26	179	112	35
N.S.	1	1.49	1.00	0.20	1.26	0.93	0.19	1.34	0.84	0.26
time (sec)	N/A	0.637	0.016	0.615	0.109	0.076	0.084	0.128	0.222	0.096

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	200	134	27	169	112	20	179	112	33
N.S.	1	1.49	1.00	0.20	1.26	0.84	0.15	1.34	0.84	0.25
time (sec)	N/A	0.603	0.013	0.569	0.113	0.079	0.085	0.121	0.211	0.088

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	215	179	50	183	128	29	187	150	51
N.S.	1	1.51	1.26	0.35	1.29	0.90	0.20	1.32	1.06	0.36
time (sec)	N/A	0.661	0.023	0.650	0.109	0.080	0.104	0.131	0.221	0.117

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	217	188	56	184	153	32	175	159	51
N.S.	1	1.51	1.31	0.39	1.28	1.06	0.22	1.22	1.10	0.35
time (sec)	N/A	0.642	0.027	0.648	0.113	0.085	0.117	0.123	0.217	0.109

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	38	41	43	56	41	49	135	45
N.S.	1	0.93	0.83	0.89	0.93	1.22	0.89	1.07	2.93	0.98
time (sec)	N/A	0.311	0.020	0.631	0.029	0.067	0.187	0.116	0.203	0.157

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	48	117	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	1.45	3.55	0.88
time (sec)	N/A	0.289	0.011	0.487	0.032	0.066	0.156	0.114	0.197	0.151

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.228	0.004	0.470	0.038	0.064	0.129	0.123	0.201	0.026

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	37	47	34	47	134	34
N.S.	1	1.03	0.87	0.92	0.97	1.24	0.89	1.24	3.53	0.89
time (sec)	N/A	0.308	0.010	0.452	0.034	0.085	0.259	0.120	0.241	0.165

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	54	53	73	53	52	162	53
N.S.	1	1.00	0.79	1.04	1.02	1.40	1.02	1.00	3.12	1.02
time (sec)	N/A	0.315	0.030	0.453	0.035	0.074	0.299	0.120	0.206	0.098

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	64	60	65	172	112	67	189	62
N.S.	1	1.08	0.90	0.85	0.92	2.42	1.58	0.94	2.66	0.87
time (sec)	N/A	0.327	0.034	0.464	0.104	0.077	0.201	0.132	0.185	0.109

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	66	60	49	52	144	92	49	172	50
N.S.	1	1.10	1.00	0.82	0.87	2.40	1.53	0.82	2.87	0.83
time (sec)	N/A	0.289	0.033	0.460	0.106	0.076	0.218	0.125	0.184	0.211

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	53	49	40	40	127	83	39	167	37
N.S.	1	1.08	1.00	0.82	0.82	2.59	1.69	0.80	3.41	0.76
time (sec)	N/A	0.269	0.021	0.490	0.103	0.076	0.167	0.120	0.218	0.176

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	53	49	40	39	129	83	39	166	37
N.S.	1	1.08	1.00	0.82	0.80	2.63	1.69	0.80	3.39	0.76
time (sec)	N/A	0.266	0.021	0.484	0.110	0.073	0.178	0.117	0.212	0.144

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	67	94	49	53	148	97	51	178	50
N.S.	1	1.12	1.57	0.82	0.88	2.47	1.62	0.85	2.97	0.83
time (sec)	N/A	0.293	0.044	0.469	0.106	0.077	0.221	0.124	0.186	0.186

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	84	104	59	66	176	117	63	195	60
N.S.	1	1.20	1.49	0.84	0.94	2.51	1.67	0.90	2.79	0.86
time (sec)	N/A	0.311	0.054	0.480	0.130	0.078	0.302	0.127	0.237	0.201

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	244	196	55	204	226	60	193	312	76
N.S.	1	1.49	1.20	0.34	1.24	1.38	0.37	1.18	1.90	0.46
time (sec)	N/A	0.688	0.086	0.472	0.108	0.085	0.184	0.129	0.236	0.251

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	239	192	50	198	196	48	188	310	71
N.S.	1	1.52	1.22	0.32	1.26	1.25	0.31	1.20	1.97	0.45
time (sec)	N/A	0.676	0.079	0.471	0.112	0.080	0.171	0.129	0.192	0.253

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	227	185	45	192	192	44	196	307	60
N.S.	1	1.48	1.21	0.29	1.25	1.25	0.29	1.28	2.01	0.39
time (sec)	N/A	0.638	0.077	0.470	0.110	0.077	0.196	0.121	0.193	0.237

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	225	182	43	190	181	39	194	303	58
N.S.	1	1.49	1.21	0.28	1.26	1.20	0.26	1.28	2.01	0.38
time (sec)	N/A	0.643	0.076	0.488	0.107	0.086	0.150	0.125	0.198	0.130

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	227	184	48	191	196	46	196	305	60
N.S.	1	1.48	1.20	0.31	1.25	1.28	0.30	1.28	1.99	0.39
time (sec)	N/A	0.666	0.067	0.490	0.127	0.081	0.137	0.124	0.217	0.132

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	225	183	46	189	183	39	194	305	58
N.S.	1	1.49	1.21	0.30	1.25	1.21	0.26	1.28	2.02	0.38
time (sec)	N/A	0.649	0.068	0.437	0.111	0.079	0.154	0.121	0.222	0.204

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	242	196	70	203	204	56	205	316	69
N.S.	1	1.49	1.21	0.43	1.25	1.26	0.35	1.27	1.95	0.43
time (sec)	N/A	0.713	0.090	0.481	0.110	0.090	0.189	0.122	0.199	0.112

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	244	194	76	206	230	58	191	324	73
N.S.	1	1.51	1.20	0.47	1.27	1.42	0.36	1.18	2.00	0.45
time (sec)	N/A	0.726	0.098	0.469	0.122	0.089	0.227	0.124	0.213	0.122

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	62	48	54	66	91	68	62	217	68
N.S.	1	0.95	0.74	0.83	1.02	1.40	1.05	0.95	3.34	1.05
time (sec)	N/A	0.341	0.043	0.461	0.031	0.076	0.333	0.124	0.204	0.249

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	39	43	55	69	53	42	200	53
N.S.	1	0.98	0.75	0.83	1.06	1.33	1.02	0.81	3.85	1.02
time (sec)	N/A	0.315	0.014	0.443	0.031	0.066	0.261	0.124	0.209	0.208

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.233	0.010	0.441	0.036	0.067	0.237	0.128	0.208	0.161

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	25	25
N.S.	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.56	1.56
time (sec)	N/A	0.235	0.005	0.438	0.030	0.076	0.180	0.122	0.250	0.142

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	43	46	60	90	56	59	228	56
N.S.	1	1.02	0.80	0.85	1.11	1.67	1.04	1.09	4.22	1.04
time (sec)	N/A	0.321	0.025	0.458	0.030	0.076	0.303	0.124	0.205	0.199

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	59	65	77	119	80	82	258	75
N.S.	1	1.00	0.86	0.94	1.12	1.72	1.16	1.19	3.74	1.09
time (sec)	N/A	0.351	0.039	0.458	0.032	0.080	0.442	0.128	0.213	0.115

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	80	58	52	63	196	116	49	273	60
N.S.	1	1.18	0.85	0.76	0.93	2.88	1.71	0.72	4.01	0.88
time (sec)	N/A	0.318	0.037	0.476	0.111	0.078	0.263	0.126	0.194	0.116

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	62	54	66	199	116	54	272	58
N.S.	1	1.13	0.87	0.76	0.93	2.80	1.63	0.76	3.83	0.82
time (sec)	N/A	0.302	0.024	0.483	0.111	0.076	0.250	0.126	0.229	0.215

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	80	58	63	62	196	110	49	273	59
N.S.	1	1.18	0.85	0.93	0.91	2.88	1.62	0.72	4.01	0.87
time (sec)	N/A	0.290	0.034	0.497	0.107	0.077	0.239	0.117	0.223	0.188

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	94	105	58	75	214	121	61	284	72
N.S.	1	1.18	1.31	0.72	0.94	2.68	1.51	0.76	3.55	0.90
time (sec)	N/A	0.329	0.062	0.501	0.118	0.082	0.330	0.128	0.218	0.248

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	254	205	56	215	264	70	206	476	83
N.S.	1	1.48	1.19	0.33	1.25	1.53	0.41	1.20	2.77	0.48
time (sec)	N/A	0.740	0.070	0.487	0.109	0.082	0.267	0.129	0.210	0.245

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	252	201	54	213	262	68	204	474	81
N.S.	1	1.48	1.18	0.32	1.25	1.54	0.40	1.20	2.79	0.48
time (sec)	N/A	0.726	0.064	0.500	0.133	0.085	0.275	0.131	0.201	0.139

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	254	207	58	219	277	71	209	476	81
N.S.	1	1.45	1.18	0.33	1.25	1.58	0.41	1.19	2.72	0.46
time (sec)	N/A	0.733	0.079	0.507	0.109	0.087	0.253	0.133	0.194	0.258

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	250	203	56	216	267	66	206	474	80
N.S.	1	1.46	1.19	0.33	1.26	1.56	0.39	1.20	2.77	0.47
time (sec)	N/A	0.719	0.079	0.484	0.108	0.079	0.233	0.123	0.190	0.268

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	254	204	59	214	269	71	206	476	82
N.S.	1	1.48	1.19	0.34	1.24	1.56	0.41	1.20	2.77	0.48
time (sec)	N/A	0.722	0.069	0.513	0.111	0.089	0.224	0.129	0.210	0.119

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	250	200	57	212	255	63	204	474	80
N.S.	1	1.49	1.19	0.34	1.26	1.52	0.38	1.21	2.82	0.48
time (sec)	N/A	0.709	0.054	0.468	0.108	0.083	0.287	0.124	0.239	0.101

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	269	216	81	225	270	80	217	485	91
N.S.	1	1.48	1.19	0.45	1.24	1.48	0.44	1.19	2.66	0.50
time (sec)	N/A	0.758	0.075	0.503	0.116	0.097	0.296	0.128	0.219	0.126

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	27	179	273	42	219	136	121
N.S.	1	1.00	1.62	0.34	2.27	3.46	0.53	2.77	1.72	1.53
time (sec)	N/A	0.340	0.056	0.494	0.107	0.079	0.139	0.123	0.194	0.241

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	27	179	273	42	219	136	121
N.S.	1	1.00	1.62	0.34	2.27	3.46	0.53	2.77	1.72	1.53
time (sec)	N/A	0.301	0.010	0.438	0.109	0.085	0.159	0.104	0.221	0.003

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	85	110	24	78	32
N.S.	1	1.00	1.00	0.79	0.76	2.58	3.33	0.73	2.36	0.97
time (sec)	N/A	0.261	0.009	0.452	0.104	0.079	0.142	0.125	0.203	0.188

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	85	110	24	78	32
N.S.	1	1.00	1.00	0.79	0.76	2.58	3.33	0.73	2.36	0.97
time (sec)	N/A	0.252	0.005	0.451	0.105	0.083	0.122	0.122	0.204	0.003

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	27	179	145	29	219	134	53
N.S.	1	1.00	1.62	0.34	2.27	1.84	0.37	2.77	1.70	0.67
time (sec)	N/A	0.308	0.018	0.464	0.114	0.080	0.108	0.122	0.242	0.273

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	128	27	179	145	29	219	134	53
N.S.	1	1.00	1.62	0.34	2.27	1.84	0.37	2.77	1.70	0.67
time (sec)	N/A	0.296	0.009	0.443	0.110	0.087	0.124	0.115	0.224	0.003

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	14	13	13	12	14	54	13
N.S.	1	1.00	1.07	1.00	0.93	0.93	0.86	1.00	3.86	0.93
time (sec)	N/A	0.223	0.005	0.435	0.029	0.068	0.085	0.122	0.192	0.175

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	15	14	13	13	12	14	54	13
N.S.	1	1.00	1.07	1.00	0.93	0.93	0.86	1.00	3.86	0.93
time (sec)	N/A	0.232	0.003	0.432	0.030	0.072	0.087	0.122	0.198	0.003

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	138	101	22	107	75	124	95	83	45
N.S.	1	1.41	1.03	0.22	1.09	0.77	1.27	0.97	0.85	0.46
time (sec)	N/A	0.507	0.036	0.412	0.111	0.077	0.199	0.134	0.206	0.134

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.251	0.013	0.083	0.025	0.086	0.408	0.116	0.267	0.164

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	18	19	13	17	15
N.S.	1	1.00	0.90	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.247	0.012	0.071	0.025	0.073	0.280	0.119	0.232	0.033

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	16	19	13	15	15
N.S.	1	1.00	0.90	0.67	0.62	0.76	0.90	0.62	0.71	0.71
time (sec)	N/A	0.254	0.012	0.069	0.032	0.073	0.392	0.112	0.186	0.031

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	14	13	14	17	13	13	14
N.S.	1	1.00	1.05	0.74	0.68	0.74	0.89	0.68	0.68	0.74
time (sec)	N/A	0.243	0.012	0.083	0.025	0.069	0.146	0.116	0.197	0.031

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	14	17	13	15	15
N.S.	1	1.00	1.00	0.74	0.68	0.74	0.89	0.68	0.79	0.79
time (sec)	N/A	0.252	0.017	0.093	0.026	0.072	0.218	0.125	0.221	0.033

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	15	19	13	19	15
N.S.	1	1.00	0.90	0.67	0.62	0.71	0.90	0.62	0.90	0.71
time (sec)	N/A	0.246	0.016	0.086	0.034	0.073	0.251	0.124	0.199	0.033

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	15	19	13	19	15
N.S.	1	1.00	0.90	0.67	0.62	0.71	0.90	0.62	0.90	0.71
time (sec)	N/A	0.246	0.016	0.079	0.031	0.074	0.313	0.125	0.201	0.034

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.262	0.018	0.414	0.029	0.069	0.911	0.116	0.211	0.177

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.72
time (sec)	N/A	0.269	0.017	0.374	0.025	0.072	0.697	0.123	0.259	0.048

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	27	34	24	26	26
N.S.	1	1.00	0.83	0.69	0.67	0.75	0.94	0.67	0.72	0.72
time (sec)	N/A	0.269	0.016	0.375	0.026	0.067	0.652	0.127	0.211	0.172

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	25	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.74	0.76
time (sec)	N/A	0.266	0.016	0.374	0.024	0.070	0.380	0.123	0.194	0.040

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	27	25
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.79	0.74
time (sec)	N/A	0.263	0.020	0.369	0.026	0.081	0.470	0.121	0.223	0.042

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	26	34	24	30	26
N.S.	1	1.00	0.83	0.69	0.67	0.72	0.94	0.67	0.83	0.72
time (sec)	N/A	0.260	0.020	0.375	0.028	0.074	0.530	0.124	0.211	0.039

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	26	34	24	30	26
N.S.	1	1.00	0.83	0.69	0.67	0.72	0.94	0.67	0.83	0.72
time (sec)	N/A	0.265	0.021	0.382	0.025	0.070	0.674	0.117	0.195	0.039

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	40	49	35	39	35
N.S.	1	1.00	0.92	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.292	0.021	0.509	0.026	0.073	1.831	0.118	0.208	0.051

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	35	40	49	35	39	35
N.S.	1	1.00	0.92	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.284	0.022	0.371	0.030	0.085	1.352	0.117	0.228	0.046

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	38	49	35	37	35
N.S.	1	1.00	0.80	0.71	0.69	0.75	0.96	0.69	0.73	0.69
time (sec)	N/A	0.271	0.020	0.373	0.024	0.067	1.209	0.112	0.216	0.048

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	36	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.73	0.71
time (sec)	N/A	0.275	0.018	0.371	0.024	0.071	1.003	0.108	0.233	0.045

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	38	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.78	0.71
time (sec)	N/A	0.278	0.022	0.373	0.031	0.069	1.047	0.115	0.211	0.049

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	37	49	35	41	35
N.S.	1	1.00	0.80	0.71	0.69	0.73	0.96	0.69	0.80	0.69
time (sec)	N/A	0.276	0.023	0.374	0.025	0.083	1.189	0.126	0.221	0.048

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	41	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.84	0.71
time (sec)	N/A	0.275	0.024	0.370	0.040	0.072	1.715	0.117	0.212	0.048

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	345	268	39	0	314	299	453	752	126
N.S.	1	1.44	1.12	0.16	0.00	1.31	1.25	1.89	3.13	0.52
time (sec)	N/A	1.051	0.831	0.486	0.000	0.089	54.788	0.241	0.233	0.158

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	347	266	39	0	250	296	453	371	126
N.S.	1	1.46	1.12	0.16	0.00	1.05	1.24	1.90	1.56	0.53
time (sec)	N/A	1.008	0.715	0.490	0.000	0.090	29.225	0.201	0.210	0.274

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	324	237	29	0	288	289	445	321	118
N.S.	1	1.42	1.04	0.13	0.00	1.26	1.27	1.95	1.41	0.52
time (sec)	N/A	0.933	0.645	0.459	0.000	0.083	18.877	0.223	0.253	0.174

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	328	239	29	0	304	291	445	649	118
N.S.	1	1.44	1.05	0.13	0.00	1.33	1.28	1.95	2.85	0.52
time (sec)	N/A	0.941	0.674	0.452	0.000	0.100	12.917	0.217	0.223	0.302

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	328	239	29	0	288	291	445	649	118
N.S.	1	1.44	1.05	0.13	0.00	1.26	1.28	1.95	2.85	0.52
time (sec)	N/A	0.935	0.675	0.454	0.000	0.091	8.578	0.224	0.233	0.147

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	332	239	29	0	264	289	445	321	118
N.S.	1	1.46	1.05	0.13	0.00	1.16	1.27	1.95	1.41	0.52
time (sec)	N/A	0.952	0.675	0.452	0.000	0.089	12.229	0.172	0.238	0.284

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	339	276	38	0	293	299	461	389	126
N.S.	1	1.42	1.16	0.16	0.00	1.23	1.26	1.94	1.63	0.53
time (sec)	N/A	0.984	0.762	0.483	0.000	0.092	21.169	0.220	0.252	0.258

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	345	288	38	0	357	303	461	804	126
N.S.	1	1.44	1.20	0.16	0.00	1.49	1.26	1.92	3.35	0.52
time (sec)	N/A	0.985	0.779	0.479	0.000	0.093	42.662	0.218	0.262	0.152

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	351	277	47	0	408	0	486	775	135
N.S.	1	1.41	1.11	0.19	0.00	1.64	0.00	1.95	3.11	0.54
time (sec)	N/A	1.014	1.196	0.522	0.000	0.092	0.000	0.256	0.269	0.260

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	355	277	47	0	424	0	486	1555	135
N.S.	1	1.43	1.11	0.19	0.00	1.70	0.00	1.95	6.24	0.54
time (sec)	N/A	0.998	1.245	0.513	0.000	0.087	0.000	0.258	0.303	0.260

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	355	277	47	0	424	0	486	1553	135
N.S.	1	1.43	1.11	0.19	0.00	1.70	0.00	1.95	6.24	0.54
time (sec)	N/A	1.021	1.323	0.513	0.000	0.092	0.000	0.248	0.282	0.119

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	359	275	47	0	388	0	486	768	135
N.S.	1	1.44	1.10	0.19	0.00	1.56	0.00	1.95	3.08	0.54
time (sec)	N/A	1.010	1.208	0.511	0.000	0.086	0.000	0.217	0.313	0.154

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	351	277	50	0	421	0	462	771	135
N.S.	1	1.41	1.11	0.20	0.00	1.69	0.00	1.86	3.10	0.54
time (sec)	N/A	1.000	1.147	0.515	0.000	0.097	0.000	0.269	0.293	0.267

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	355	277	50	0	421	0	462	1555	135
N.S.	1	1.43	1.11	0.20	0.00	1.69	0.00	1.86	6.24	0.54
time (sec)	N/A	1.018	1.160	0.454	0.000	0.104	0.000	0.266	0.279	0.267

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	355	277	50	0	405	0	462	1553	135
N.S.	1	1.43	1.11	0.20	0.00	1.63	0.00	1.86	6.24	0.54
time (sec)	N/A	1.005	1.170	0.458	0.000	0.095	0.000	0.247	0.323	0.271

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	359	277	50	0	397	0	462	772	135
N.S.	1	1.44	1.11	0.20	0.00	1.59	0.00	1.86	3.10	0.54
time (sec)	N/A	1.000	1.202	0.463	0.000	0.085	0.000	0.214	0.259	0.160

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	368	287	56	0	430	0	481	811	148
N.S.	1	1.41	1.10	0.21	0.00	1.65	0.00	1.84	3.11	0.57
time (sec)	N/A	1.038	1.244	0.513	0.000	0.100	0.000	0.240	0.290	0.303

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	388	287	59	0	535	0	496	1182	158
N.S.	1	1.44	1.06	0.22	0.00	1.98	0.00	1.84	4.38	0.59
time (sec)	N/A	1.078	1.528	0.838	0.000	0.098	0.000	0.235	0.323	0.192

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	380	275	61	0	563	0	499	1185	156
N.S.	1	1.39	1.01	0.22	0.00	2.06	0.00	1.83	4.34	0.57
time (sec)	N/A	1.045	1.591	0.800	0.000	0.099	0.000	0.275	0.320	0.275

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	384	277	61	0	563	0	499	2367	156
N.S.	1	1.41	1.01	0.22	0.00	2.06	0.00	1.83	8.67	0.57
time (sec)	N/A	1.074	1.528	0.786	0.000	0.102	0.000	0.288	0.344	0.277

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	384	277	61	0	561	0	499	2365	156
N.S.	1	1.41	1.01	0.22	0.00	2.05	0.00	1.83	8.66	0.57
time (sec)	N/A	1.062	1.544	0.761	0.000	0.106	0.000	0.272	0.315	0.255

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	388	276	61	0	543	0	498	1182	156
N.S.	1	1.42	1.01	0.22	0.00	1.99	0.00	1.82	4.33	0.57
time (sec)	N/A	1.084	1.554	0.766	0.000	0.101	0.000	0.248	0.292	0.161

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	380	287	62	0	551	0	472	1185	157
N.S.	1	1.41	1.06	0.23	0.00	2.04	0.00	1.75	4.39	0.58
time (sec)	N/A	1.042	1.043	0.737	0.000	0.107	0.000	0.283	0.362	0.286

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	384	287	62	0	551	0	472	2367	157
N.S.	1	1.42	1.06	0.23	0.00	2.04	0.00	1.75	8.77	0.58
time (sec)	N/A	1.054	1.015	0.484	0.000	0.095	0.000	0.268	0.312	0.294

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	384	287	62	0	533	0	472	2365	157
N.S.	1	1.42	1.06	0.23	0.00	1.97	0.00	1.75	8.76	0.58
time (sec)	N/A	1.073	1.035	0.546	0.000	0.099	0.000	0.270	0.317	0.266

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	C	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	388	287	62	0	524	0	472	1182	157
N.S.	1	1.44	1.06	0.23	0.00	1.94	0.00	1.75	4.38	0.58
time (sec)	N/A	1.067	1.023	0.550	0.000	0.099	0.000	0.229	0.302	0.290

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	57	179	78	159	722	256	179	169
N.S.	1	1.00	0.70	2.21	0.96	1.96	8.91	3.16	2.21	2.09
time (sec)	N/A	0.372	0.071	0.579	0.039	0.081	0.749	0.132	0.194	0.355

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	94	56	87	333	135	94	95
N.S.	1	1.00	0.71	1.62	0.97	1.50	5.74	2.33	1.62	1.64
time (sec)	N/A	0.332	0.053	0.418	0.034	0.079	0.569	0.123	0.208	0.267

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	34	34	35	105	51	36	35
N.S.	1	1.00	0.71	0.97	0.97	1.00	3.00	1.46	1.03	1.00
time (sec)	N/A	0.288	0.027	0.058	0.037	0.083	0.287	0.135	0.243	0.207

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	95	0	19	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.16	0.00	0.43	0.00
time (sec)	N/A	0.268	0.029	0.000	0.000	0.000	1.624	0.000	0.196	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	551	0	30	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	12.52	0.00	0.68	0.00
time (sec)	N/A	0.256	0.035	0.000	0.000	0.000	19.626	0.000	0.183	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	51	37	50	47	87	68	46	46
N.S.	1	1.06	0.82	0.60	0.81	0.76	1.40	1.10	0.74	0.74
time (sec)	N/A	0.308	0.031	0.617	0.031	0.073	0.434	0.127	0.215	0.397

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	40	26	32	36	61	42	35	35
N.S.	1	1.10	1.00	0.65	0.80	0.90	1.52	1.05	0.88	0.88
time (sec)	N/A	0.290	0.031	0.600	0.025	0.073	0.298	0.121	0.196	0.332

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	24	39	15	23	15
N.S.	1	1.00	1.00	0.84	0.79	1.26	2.05	0.79	1.21	0.79
time (sec)	N/A	0.231	0.014	0.580	0.025	0.071	0.122	0.118	0.202	0.313

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	52	96	146	38	35	33
N.S.	1	1.00	1.00	0.76	1.16	2.13	3.24	0.84	0.78	0.73
time (sec)	N/A	0.288	0.031	0.573	0.106	0.085	1.008	0.121	0.202	0.394

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	48	49	42	56	120	126	47	44	37
N.S.	1	0.98	1.00	0.86	1.14	2.45	2.57	0.96	0.90	0.76
time (sec)	N/A	0.272	0.068	0.628	0.115	0.085	1.264	0.112	0.217	0.497

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	63	57	108	145	206	75	77	57
N.S.	1	1.04	0.85	0.77	1.46	1.96	2.78	1.01	1.04	0.77
time (sec)	N/A	0.309	0.096	0.622	0.106	0.085	2.368	0.128	0.201	0.642

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	84	73	54	105	134	206	64	59	0
N.S.	1	1.09	0.95	0.70	1.36	1.74	2.68	0.83	0.77	0.00
time (sec)	N/A	0.324	0.168	1.353	0.116	0.090	2.314	0.127	0.196	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	56	58	41	59	107	126	47	35	0
N.S.	1	1.08	1.12	0.79	1.13	2.06	2.42	0.90	0.67	0.00
time (sec)	N/A	0.271	0.145	0.803	0.114	0.088	1.045	0.123	0.184	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	57	40	39	102	146	70	35	0
N.S.	1	1.00	1.12	0.78	0.76	2.00	2.86	1.37	0.69	0.00
time (sec)	N/A	0.279	0.149	0.711	0.107	0.088	0.971	0.139	0.239	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	27	94	75	26	18
N.S.	1	1.00	1.00	0.86	0.82	1.23	4.27	3.41	1.18	0.82
time (sec)	N/A	0.237	0.125	0.805	0.026	0.081	0.668	0.131	0.220	0.449

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	29	37	38	260	152	37	38
N.S.	1	1.00	0.91	0.63	0.80	0.83	5.65	3.30	0.80	0.83
time (sec)	N/A	0.272	0.153	0.952	0.031	0.085	0.813	0.130	0.216	0.565

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	54	40	55	50	733	188	49	77
N.S.	1	1.08	0.76	0.56	0.77	0.70	10.32	2.65	0.69	1.08
time (sec)	N/A	0.317	0.167	1.249	0.028	0.081	3.797	0.142	0.197	0.757

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	102	64	98	0	57	41	0	61	0
N.S.	1	1.05	0.66	1.01	0.00	0.59	0.42	0.00	0.63	0.00
time (sec)	N/A	0.369	5.322	0.619	0.000	0.081	0.549	0.000	0.221	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	88	80	0	42	39	0	38	38
N.S.	1	1.00	1.19	1.08	0.00	0.57	0.53	0.00	0.51	0.51
time (sec)	N/A	0.299	0.075	0.579	0.000	0.073	0.542	0.000	0.217	0.225

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	82	0	43	36	0	48	0
N.S.	1	1.00	0.68	1.08	0.00	0.57	0.47	0.00	0.63	0.00
time (sec)	N/A	0.308	10.011	0.648	0.000	0.084	0.697	0.000	0.213	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	52	100	0	59	48	0	49	0
N.S.	1	1.05	0.53	1.01	0.00	0.60	0.48	0.00	0.49	0.00
time (sec)	N/A	0.356	10.012	0.889	0.000	0.076	0.702	0.000	0.246	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	133	52	104	0	90	41	0	43	0
N.S.	1	1.01	0.39	0.79	0.00	0.68	0.31	0.00	0.33	0.00
time (sec)	N/A	0.524	4.258	0.605	0.000	0.081	0.574	0.000	0.210	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	129	50	104	0	0	36	0	44	40
N.S.	1	1.02	0.40	0.83	0.00	0.00	0.29	0.00	0.35	0.32
time (sec)	N/A	0.500	8.467	0.609	0.000	0.000	0.587	0.000	0.216	0.519

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	160	52	115	0	86	37	0	49	0
N.S.	1	1.03	0.34	0.74	0.00	0.55	0.24	0.00	0.32	0.00
time (sec)	N/A	0.573	10.010	0.784	0.000	0.091	0.748	0.000	0.205	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	40	37	50	58	109	79	57	55
N.S.	1	1.06	0.65	0.60	0.81	0.94	1.76	1.27	0.92	0.89
time (sec)	N/A	0.318	0.042	0.577	0.036	0.069	0.648	0.121	0.220	0.377

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	29	26	32	46	83	53	46	44
N.S.	1	1.10	0.72	0.65	0.80	1.15	2.08	1.32	1.15	1.10
time (sec)	N/A	0.293	0.036	0.567	0.029	0.069	0.436	0.120	0.210	0.339

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	33	60	26	35	15
N.S.	1	1.00	1.00	0.84	0.79	1.74	3.16	1.37	1.84	0.79
time (sec)	N/A	0.231	0.015	0.578	0.026	0.071	0.189	0.120	0.228	0.343

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	64	55	44	65	115	196	53	53	46
N.S.	1	1.03	0.89	0.71	1.05	1.85	3.16	0.85	0.85	0.74
time (sec)	N/A	0.304	0.044	0.582	0.113	0.082	1.439	0.117	0.219	0.382

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	68	59	51	70	130	206	65	64	51
N.S.	1	1.01	0.88	0.76	1.04	1.94	3.07	0.97	0.96	0.76
time (sec)	N/A	0.307	0.082	0.639	0.115	0.085	1.686	0.123	0.210	0.604

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	73	61	59	106	150	189	73	64	55
N.S.	1	1.01	0.85	0.82	1.47	2.08	2.62	1.01	0.89	0.76
time (sec)	N/A	0.296	0.095	0.651	0.109	0.110	2.140	0.128	0.215	0.760

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	109	84	67	147	157	262	147	82	0
N.S.	1	1.10	0.85	0.68	1.48	1.59	2.65	1.48	0.83	0.00
time (sec)	N/A	0.358	0.193	0.783	0.113	0.097	3.682	0.137	0.204	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	81	70	53	103	135	189	116	57	0
N.S.	1	1.09	0.95	0.72	1.39	1.82	2.55	1.57	0.77	0.00
time (sec)	N/A	0.300	0.174	0.697	0.109	0.099	1.737	0.139	0.238	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	77	68	50	76	124	206	89	52	0
N.S.	1	1.05	0.93	0.68	1.04	1.70	2.82	1.22	0.71	0.00
time (sec)	N/A	0.313	0.178	0.741	0.113	0.092	1.578	0.149	0.225	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	67	48	55	125	201	146	52	0
N.S.	1	1.03	0.93	0.67	0.76	1.74	2.79	2.03	0.72	0.00
time (sec)	N/A	0.323	0.214	0.842	0.107	0.086	1.596	0.144	0.200	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	36	143	116	38	18
N.S.	1	1.00	1.00	0.86	0.82	1.64	6.50	5.27	1.73	0.82
time (sec)	N/A	0.233	0.171	0.970	0.036	0.075	0.875	0.155	0.194	0.740

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	29	37	49	335	226	49	75
N.S.	1	1.00	0.70	0.63	0.80	1.07	7.28	4.91	1.07	1.63
time (sec)	N/A	0.271	0.213	1.215	0.028	0.082	1.203	0.144	0.218	1.110

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	43	40	55	61	857	262	60	96
N.S.	1	1.08	0.61	0.56	0.77	0.86	12.07	3.69	0.85	1.35
time (sec)	N/A	0.319	0.216	1.588	0.027	0.102	10.017	0.152	0.213	1.470

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	127	70	109	0	71	41	0	81	0
N.S.	1	1.07	0.59	0.92	0.00	0.60	0.34	0.00	0.68	0.00
time (sec)	N/A	0.392	8.885	0.632	0.000	0.083	0.580	0.000	0.219	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	97	48	92	0	52	39	0	56	38
N.S.	1	1.05	0.52	1.00	0.00	0.57	0.42	0.00	0.61	0.41
time (sec)	N/A	0.327	0.003	0.562	0.000	0.077	0.587	0.000	0.256	0.232

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	53	90	0	0	39	0	67	0
N.S.	1	1.02	0.56	0.95	0.00	0.00	0.41	0.00	0.71	0.00
time (sec)	N/A	0.335	10.011	0.622	0.000	0.000	0.585	0.000	0.250	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	101	53	92	0	55	37	0	66	0
N.S.	1	1.04	0.55	0.95	0.00	0.57	0.38	0.00	0.68	0.00
time (sec)	N/A	0.348	10.011	0.927	0.000	0.084	0.671	0.000	0.221	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	53	114	0	105	41	0	61	0
N.S.	1	1.04	0.35	0.75	0.00	0.69	0.27	0.00	0.40	0.00
time (sec)	N/A	0.553	6.938	0.593	0.000	0.082	0.561	0.000	0.225	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	154	51	113	0	0	42	0	65	41
N.S.	1	1.02	0.34	0.75	0.00	0.00	0.28	0.00	0.43	0.27
time (sec)	N/A	0.522	10.010	0.581	0.000	0.000	0.620	0.000	0.265	0.842

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	154	53	112	0	0	37	0	66	0
N.S.	1	1.01	0.35	0.73	0.00	0.00	0.24	0.00	0.43	0.00
time (sec)	N/A	0.546	10.011	0.813	0.000	0.000	0.685	0.000	0.221	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	35	28	58	29	88	29	117	0
N.S.	1	1.06	1.00	0.80	1.66	0.83	2.51	0.83	3.34	0.00
time (sec)	N/A	0.255	0.086	0.459	0.025	0.081	0.968	0.122	0.192	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	64	40	37	50	36	70	61	35	38
N.S.	1	1.03	0.65	0.60	0.81	0.58	1.13	0.98	0.56	0.61
time (sec)	N/A	0.318	0.029	0.585	0.030	0.078	0.380	0.116	0.204	0.400

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	29	25	32	24	44	32	24	24
N.S.	1	1.05	0.72	0.62	0.80	0.60	1.10	0.80	0.60	0.60
time (sec)	N/A	0.284	0.023	0.567	0.026	0.073	0.271	0.122	0.221	0.390

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	24	15	14	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.26	0.79	0.74	0.79
time (sec)	N/A	0.231	0.015	0.583	0.035	0.070	0.102	0.115	0.200	0.370

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	39	73	53	24	22	20
N.S.	1	1.00	1.00	0.75	1.39	2.61	1.89	0.86	0.79	0.71
time (sec)	N/A	0.262	0.023	0.574	0.104	0.082	0.578	0.116	0.205	0.411

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	44	71	119	129	54	46	40
N.S.	1	1.00	1.00	0.85	1.37	2.29	2.48	1.04	0.88	0.77
time (sec)	N/A	0.281	0.070	0.620	0.105	0.087	1.298	0.124	0.228	0.544

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	61	44	62	105	128	53	39	0
N.S.	1	1.07	1.11	0.80	1.13	1.91	2.33	0.96	0.71	0.00
time (sec)	N/A	0.279	0.168	0.775	0.110	0.082	1.322	0.125	0.196	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	37	24	23	65	53	30	17	0
N.S.	1	1.00	1.19	0.77	0.74	2.10	1.71	0.97	0.55	0.00
time (sec)	N/A	0.244	0.125	0.665	0.115	0.082	0.570	0.126	0.199	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	51	36	17	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	2.32	1.64	0.77	0.82
time (sec)	N/A	0.227	0.148	0.677	0.034	0.072	0.423	0.127	0.182	0.312

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	27	36	26	189	68	27	26
N.S.	1	1.00	0.70	0.59	0.78	0.57	4.11	1.48	0.59	0.57
time (sec)	N/A	0.272	0.172	0.793	0.028	0.078	0.690	0.127	0.247	0.409

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	43	40	55	39	609	97	38	39
N.S.	1	1.08	0.61	0.56	0.77	0.55	8.58	1.37	0.54	0.55
time (sec)	N/A	0.310	0.202	0.971	0.037	0.088	1.568	0.127	0.217	0.518

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	80	98	0	58	39	0	61	0
N.S.	1	1.08	0.80	0.98	0.00	0.58	0.39	0.00	0.61	0.00
time (sec)	N/A	0.351	10.020	0.865	0.000	0.081	0.551	0.000	0.213	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	86	0	46	39	0	42	0
N.S.	1	1.00	0.83	1.12	0.00	0.60	0.51	0.00	0.55	0.00
time (sec)	N/A	0.304	10.027	0.661	0.000	0.081	0.594	0.000	0.216	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	72	64	0	26	37	0	22	38
N.S.	1	1.00	1.36	1.21	0.00	0.49	0.70	0.00	0.42	0.72
time (sec)	N/A	0.258	0.020	0.483	0.000	0.077	0.452	0.000	0.227	0.260

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	88	0	47	42	0	26	0
N.S.	1	1.00	0.66	1.11	0.00	0.59	0.53	0.00	0.33	0.00
time (sec)	N/A	0.299	10.010	0.648	0.000	0.083	0.538	0.000	0.191	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	110	52	100	0	59	46	0	26	0
N.S.	1	1.08	0.51	0.98	0.00	0.58	0.45	0.00	0.25	0.00
time (sec)	N/A	0.358	10.010	0.906	0.000	0.076	0.699	0.000	0.220	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	170	81	117	0	105	39	0	66	0
N.S.	1	1.08	0.51	0.74	0.00	0.66	0.25	0.00	0.42	0.00
time (sec)	N/A	0.588	10.039	1.080	0.000	0.084	0.804	0.000	0.224	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	139	66	107	0	89	39	0	48	0
N.S.	1	1.03	0.49	0.79	0.00	0.66	0.29	0.00	0.36	0.00
time (sec)	N/A	0.526	10.020	0.771	0.000	0.081	0.519	0.000	0.220	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	52	88	0	76	39	0	25	0
N.S.	1	1.00	0.48	0.81	0.00	0.70	0.36	0.00	0.23	0.00
time (sec)	N/A	0.437	10.017	0.498	0.000	0.081	0.460	0.000	0.205	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	135	50	106	0	68	41	0	26	41
N.S.	1	1.05	0.39	0.83	0.00	0.53	0.32	0.00	0.20	0.32
time (sec)	N/A	0.509	10.013	0.597	0.000	0.079	0.497	0.000	0.202	0.472

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	166	52	115	0	84	39	0	26	0
N.S.	1	1.05	0.33	0.73	0.00	0.53	0.25	0.00	0.16	0.00
time (sec)	N/A	0.576	10.010	0.819	0.000	0.082	0.654	0.000	0.247	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	62	39	37	49	48	68	55	37	44
N.S.	1	1.05	0.66	0.63	0.83	0.81	1.15	0.93	0.63	0.75
time (sec)	N/A	0.319	0.039	0.601	0.037	0.070	0.409	0.121	0.202	0.545

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	32	37	41	35	26	22
N.S.	1	1.00	0.72	0.65	0.80	0.92	1.02	0.88	0.65	0.55
time (sec)	N/A	0.289	0.032	0.621	0.032	0.074	0.267	0.124	0.224	0.425

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	26	24	15	16	15
N.S.	1	1.00	1.00	0.84	0.79	1.37	1.26	0.79	0.84	0.79
time (sec)	N/A	0.231	0.020	0.592	0.031	0.083	0.106	0.121	0.227	0.352

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	55	149	520	43	58	36
N.S.	1	1.00	1.00	0.77	1.15	3.10	10.83	0.90	1.21	0.75
time (sec)	N/A	0.281	0.056	0.582	0.107	0.082	1.085	0.122	0.223	0.473

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	62	57	92	193	165	77	81	56
N.S.	1	1.07	0.86	0.79	1.28	2.68	2.29	1.07	1.12	0.78
time (sec)	N/A	0.320	0.093	0.641	0.114	0.089	1.801	0.122	0.206	0.608

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	84	70	63	84	177	163	75	60	0
N.S.	1	1.09	0.91	0.82	1.09	2.30	2.12	0.97	0.78	0.00
time (sec)	N/A	0.332	0.259	1.026	0.112	0.087	1.847	0.132	0.203	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	60	43	42	148	100	63	49	0
N.S.	1	0.98	1.11	0.80	0.78	2.74	1.85	1.17	0.91	0.00
time (sec)	N/A	0.284	0.223	0.779	0.111	0.085	0.916	0.136	0.210	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	29	53	29	19	18
N.S.	1	1.00	1.00	0.86	0.82	1.32	2.41	1.32	0.86	0.82
time (sec)	N/A	0.230	0.144	0.648	0.034	0.074	0.386	0.124	0.231	0.203

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	30	27	38	41	133	70	29	26
N.S.	1	1.00	0.70	0.63	0.88	0.95	3.09	1.63	0.67	0.60
time (sec)	N/A	0.270	0.190	0.699	0.035	0.093	0.518	0.122	0.206	0.335

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	41	38	58	52	481	145	40	73
N.S.	1	1.09	0.60	0.56	0.85	0.76	7.07	2.13	0.59	1.07
time (sec)	N/A	0.311	0.294	0.795	0.025	0.087	0.919	0.136	0.201	0.543

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	105	52	49	76	63	724	225	51	80
N.S.	1	1.13	0.56	0.53	0.82	0.68	7.78	2.42	0.55	0.86
time (sec)	N/A	0.360	0.423	0.975	0.026	0.086	6.212	0.136	0.231	0.705

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	107	67	108	0	79	39	0	122	0
N.S.	1	1.08	0.68	1.09	0.00	0.80	0.39	0.00	1.23	0.00
time (sec)	N/A	0.346	10.018	1.648	0.000	0.095	0.541	0.000	0.195	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	57	90	0	64	39	0	101	0
N.S.	1	1.00	0.74	1.17	0.00	0.83	0.51	0.00	1.31	0.00
time (sec)	N/A	0.302	6.400	0.566	0.000	0.099	0.501	0.000	0.208	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	90	0	66	37	0	32	38
N.S.	1	1.00	0.73	1.17	0.00	0.86	0.48	0.00	0.42	0.49
time (sec)	N/A	0.291	0.007	0.483	0.000	0.075	0.496	0.000	0.217	0.302

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	109	55	110	0	80	42	0	36	0
N.S.	1	1.08	0.54	1.09	0.00	0.79	0.42	0.00	0.36	0.00
time (sec)	N/A	0.341	10.011	1.360	0.000	0.091	0.575	0.000	0.223	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	140	55	133	0	94	46	0	36	0
N.S.	1	1.13	0.44	1.07	0.00	0.76	0.37	0.00	0.29	0.00
time (sec)	N/A	0.385	10.019	1.659	0.000	0.084	0.692	0.000	0.209	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	169	67	131	0	134	39	0	130	0
N.S.	1	1.08	0.43	0.83	0.00	0.85	0.25	0.00	0.83	0.00
time (sec)	N/A	0.560	10.029	2.072	0.000	0.082	0.600	0.000	0.213	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	55	112	0	117	39	0	111	0
N.S.	1	1.02	0.41	0.83	0.00	0.87	0.29	0.00	0.82	0.00
time (sec)	N/A	0.498	10.019	0.660	0.000	0.081	0.503	0.000	0.222	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	55	112	0	100	39	0	35	0
N.S.	1	1.02	0.41	0.83	0.00	0.74	0.29	0.00	0.26	0.00
time (sec)	N/A	0.491	6.048	0.497	0.000	0.078	0.464	0.000	0.203	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	165	53	131	0	109	41	0	36	41
N.S.	1	1.05	0.34	0.83	0.00	0.69	0.26	0.00	0.23	0.26
time (sec)	N/A	0.558	10.011	1.431	0.000	0.087	0.536	0.000	0.226	0.583

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	196	55	152	0	132	46	0	36	0
N.S.	1	1.09	0.31	0.84	0.00	0.73	0.26	0.00	0.20	0.00
time (sec)	N/A	0.628	10.011	1.786	0.000	0.080	0.688	0.000	0.208	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	48	27	23	34	23	41	41	22	23
N.S.	1	1.04	0.59	0.50	0.74	0.50	0.89	0.89	0.48	0.50
time (sec)	N/A	0.274	0.020	0.476	0.031	0.069	0.192	0.121	0.225	0.390

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	22	17	23	16	24	23	17	16
N.S.	1	1.06	0.71	0.55	0.74	0.52	0.77	0.74	0.55	0.52
time (sec)	N/A	0.264	0.016	0.435	0.031	0.070	0.117	0.122	0.186	0.323

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	10	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.67	0.73
time (sec)	N/A	0.225	0.013	0.435	0.031	0.071	0.066	0.120	0.202	0.388

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	29	29	24	31	10	12
N.S.	1	1.00	1.00	0.81	1.81	1.81	1.50	1.94	0.62	0.75
time (sec)	N/A	0.237	0.018	0.447	0.030	0.072	0.481	0.122	0.200	0.436

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	43	50	73	45	28	27
N.S.	1	1.00	1.00	0.80	1.23	1.43	2.09	1.29	0.80	0.77
time (sec)	N/A	0.254	0.042	0.500	0.030	0.072	1.019	0.119	0.230	0.390

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	54	34	30	74	41	110	28	33	0
N.S.	1	1.20	0.76	0.67	1.64	0.91	2.44	0.62	0.73	0.00
time (sec)	N/A	0.272	0.136	0.461	0.120	0.074	2.112	0.125	0.215	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	31	27	22	44	33	61	21	20	0
N.S.	1	1.15	1.00	0.81	1.63	1.22	2.26	0.78	0.74	0.00
time (sec)	N/A	0.239	0.123	0.454	0.121	0.071	0.984	0.122	0.203	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	16	18	19	6	6	16
N.S.	1	1.00	1.00	0.88	2.00	2.25	2.38	0.75	0.75	2.00
time (sec)	N/A	0.218	0.094	0.426	0.113	0.072	0.465	0.119	0.181	0.185

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	34	35	13	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.89	1.94	0.72	0.78
time (sec)	N/A	0.221	0.116	0.431	0.026	0.072	0.353	0.125	0.220	0.284

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	29	21	63	73	20	21
N.S.	1	1.00	0.68	0.59	0.78	0.57	1.70	1.97	0.54	0.57
time (sec)	N/A	0.253	0.133	0.440	0.024	0.072	0.474	0.128	0.196	0.296

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	30	27	43	26	104	109	25	26
N.S.	1	1.09	0.55	0.49	0.78	0.47	1.89	1.98	0.45	0.47
time (sec)	N/A	0.314	0.134	0.447	0.036	0.072	0.690	0.127	0.194	0.336

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	42	15	0	29	31	0	45	0
N.S.	1	1.12	0.98	0.35	0.00	0.67	0.72	0.00	1.05	0.00
time (sec)	N/A	0.273	10.032	0.771	0.000	0.085	0.453	0.000	0.244	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	15	0	21	31	0	32	0
N.S.	1	1.00	1.28	0.60	0.00	0.84	1.24	0.00	1.28	0.00
time (sec)	N/A	0.240	10.014	0.569	0.000	0.075	0.377	0.000	0.226	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	12	0	4	29	0	20	10
N.S.	1	1.00	1.00	3.00	0.00	1.00	7.25	0.00	5.00	2.50
time (sec)	N/A	0.213	0.006	0.430	0.000	0.073	0.357	0.000	0.220	0.207

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	15	0	25	34	0	24	0
N.S.	1	1.00	0.74	0.56	0.00	0.93	1.26	0.00	0.89	0.00
time (sec)	N/A	0.243	10.007	0.582	0.000	0.077	0.435	0.000	0.226	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	20	15	0	33	37	0	24	0
N.S.	1	1.11	0.44	0.33	0.00	0.73	0.82	0.00	0.53	0.00
time (sec)	N/A	0.278	10.006	0.837	0.000	0.079	0.521	0.000	0.211	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	43	15	0	47	31	0	50	0
N.S.	1	1.08	0.81	0.28	0.00	0.89	0.58	0.00	0.94	0.00
time (sec)	N/A	0.345	10.032	1.035	0.000	0.079	0.487	0.000	0.203	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	34	15	0	40	31	0	37	0
N.S.	1	0.97	0.97	0.43	0.00	1.14	0.89	0.00	1.06	0.00
time (sec)	N/A	0.302	10.012	0.648	0.000	0.095	0.409	0.000	0.212	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	15	0	34	31	0	23	0
N.S.	1	1.00	1.82	1.36	0.00	3.09	2.82	0.00	2.09	0.00
time (sec)	N/A	0.258	10.022	0.488	0.000	0.077	0.371	0.000	0.205	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	0	28	32	0	24	13
N.S.	1	1.00	0.67	0.56	0.00	1.04	1.19	0.00	0.89	0.48
time (sec)	N/A	0.298	10.012	0.563	0.000	0.080	0.429	0.000	0.279	0.266

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	20	15	0	41	37	0	24	0
N.S.	1	0.94	0.38	0.28	0.00	0.77	0.70	0.00	0.45	0.00
time (sec)	N/A	0.347	10.007	0.649	0.000	0.081	0.480	0.000	0.219	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	46	25	22	32	28	54	32	24	25
N.S.	1	1.10	0.60	0.52	0.76	0.67	1.29	0.76	0.57	0.60
time (sec)	N/A	0.285	0.025	0.440	0.025	0.072	0.286	0.123	0.238	0.371

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	20	17	23	23	34	23	19	16
N.S.	1	1.00	0.65	0.55	0.74	0.74	1.10	0.74	0.61	0.52
time (sec)	N/A	0.260	0.021	0.434	0.026	0.071	0.201	0.125	0.228	0.333

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	18	10	11	12	11
N.S.	1	1.00	1.00	0.80	0.73	1.20	0.67	0.73	0.80	0.73
time (sec)	N/A	0.222	0.012	0.431	0.030	0.069	0.157	0.120	0.245	0.331

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	25	40	58	228	42	41	24
N.S.	1	1.00	0.94	0.78	1.25	1.81	7.12	1.31	1.28	0.75
time (sec)	N/A	0.250	0.034	0.480	0.025	0.077	0.767	0.113	0.218	0.325

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	41	35	61	78	95	63	56	38
N.S.	1	1.06	0.82	0.70	1.22	1.56	1.90	1.26	1.12	0.76
time (sec)	N/A	0.268	0.055	0.501	0.032	0.079	1.455	0.126	0.214	0.384

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	49	32	27	60	52	82	33	37	0
N.S.	1	1.09	0.71	0.60	1.33	1.16	1.82	0.73	0.82	0.00
time (sec)	N/A	0.266	0.183	0.484	0.107	0.079	1.514	0.124	0.195	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	26	22	31	46	46	28	30	0
N.S.	1	0.96	0.96	0.81	1.15	1.70	1.70	1.04	1.11	0.00
time (sec)	N/A	0.238	0.141	0.457	0.108	0.077	0.752	0.122	0.200	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	21	32	21	15	14
N.S.	1	1.00	1.00	0.83	0.78	1.17	1.78	1.17	0.83	0.78
time (sec)	N/A	0.216	0.109	0.428	0.042	0.076	0.340	0.127	0.220	0.194

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	25	22	29	29	90	56	22	18
N.S.	1	0.97	0.71	0.63	0.83	0.83	2.57	1.60	0.63	0.51
time (sec)	N/A	0.248	0.141	0.435	0.029	0.073	0.426	0.121	0.198	0.301

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	30	27	43	34	151	94	27	30
N.S.	1	1.04	0.55	0.49	0.78	0.62	2.75	1.71	0.49	0.55
time (sec)	N/A	0.283	0.163	0.467	0.031	0.079	0.609	0.119	0.211	0.370

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	71	54	15	0	50	31	0	100	0
N.S.	1	1.16	0.89	0.25	0.00	0.82	0.51	0.00	1.64	0.00
time (sec)	N/A	0.306	5.220	1.287	0.000	0.081	0.608	0.000	0.201	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	48	49	15	0	45	31	0	87	0
N.S.	1	1.12	1.14	0.35	0.00	1.05	0.72	0.00	2.02	0.00
time (sec)	N/A	0.271	4.595	0.793	0.000	0.078	0.449	0.000	0.197	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	15	0	31	31	0	71	0
N.S.	1	1.00	1.28	0.60	0.00	1.24	1.24	0.00	2.84	0.00
time (sec)	N/A	0.242	4.129	0.597	0.000	0.072	0.413	0.000	0.212	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	30	12	0	32	29	0	23	10
N.S.	1	1.00	1.20	0.48	0.00	1.28	1.16	0.00	0.92	0.40
time (sec)	N/A	0.232	0.004	0.487	0.000	0.074	0.414	0.000	0.225	0.212

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	20	15	0	47	34	0	25	0
N.S.	1	1.11	0.44	0.33	0.00	1.04	0.76	0.00	0.56	0.00
time (sec)	N/A	0.285	10.005	0.566	0.000	0.078	0.497	0.000	0.251	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	73	20	15	0	52	37	0	25	0
N.S.	1	1.16	0.32	0.24	0.00	0.83	0.59	0.00	0.40	0.00
time (sec)	N/A	0.319	10.008	0.792	0.000	0.078	0.655	0.000	0.237	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	56	15	0	74	31	0	109	0
N.S.	1	1.13	0.79	0.21	0.00	1.04	0.44	0.00	1.54	0.00
time (sec)	N/A	0.395	7.356	1.702	0.000	0.081	0.642	0.000	0.203	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	49	15	0	69	31	0	95	0
N.S.	1	1.08	0.92	0.28	0.00	1.30	0.58	0.00	1.79	0.00
time (sec)	N/A	0.328	4.789	1.016	0.000	0.082	0.586	0.000	0.231	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	31	15	0	64	31	0	80	0
N.S.	1	0.97	0.89	0.43	0.00	1.83	0.89	0.00	2.29	0.00
time (sec)	N/A	0.304	4.386	0.656	0.000	0.080	0.414	0.000	0.208	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	34	20	15	0	44	31	0	26	0
N.S.	1	0.97	0.57	0.43	0.00	1.26	0.89	0.00	0.74	0.00
time (sec)	N/A	0.301	4.034	0.638	0.000	0.075	0.377	0.000	0.202	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	50	18	15	0	55	32	0	25	13
N.S.	1	0.94	0.34	0.28	0.00	1.04	0.60	0.00	0.47	0.25
time (sec)	N/A	0.355	9.266	0.622	0.000	0.076	0.437	0.000	0.240	0.309

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	20	15	0	66	37	0	25	0
N.S.	1	1.03	0.28	0.21	0.00	0.93	0.52	0.00	0.35	0.00
time (sec)	N/A	0.386	10.005	0.711	0.000	0.081	0.547	0.000	0.226	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	25	21	28	21	39	28	131	20
N.S.	1	1.05	0.62	0.52	0.70	0.52	0.98	0.70	3.28	0.50
time (sec)	N/A	0.275	0.017	0.478	0.025	0.074	0.181	0.114	0.214	0.326

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	18	15	19	14	22	19	83	14
N.S.	1	1.07	0.67	0.56	0.70	0.52	0.81	0.70	3.07	0.52
time (sec)	N/A	0.258	0.015	0.487	0.024	0.068	0.111	0.117	0.195	0.311

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	32	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	2.46	0.69
time (sec)	N/A	0.217	0.011	0.484	0.030	0.071	0.067	0.113	0.204	0.389

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	24	10	11	10
N.S.	1	1.00	1.00	0.79	0.71	0.71	1.71	0.71	0.79	0.71
time (sec)	N/A	0.238	0.018	0.707	0.111	0.074	0.478	0.123	0.207	0.277

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	28	28	24	23	25	71	23	109	23
N.S.	1	0.90	0.90	0.77	0.74	0.81	2.29	0.74	3.52	0.74
time (sec)	N/A	0.244	0.026	0.628	0.112	0.073	1.055	0.124	0.243	0.367

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	49	38	36	48	34	129	35	195	35
N.S.	1	1.04	0.81	0.77	1.02	0.72	2.74	0.74	4.15	0.74
time (sec)	N/A	0.262	0.039	0.555	0.124	0.077	2.111	0.120	0.219	0.388

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	60	42	35	86	37	110	36	186	0
N.S.	1	1.18	0.82	0.69	1.69	0.73	2.16	0.71	3.65	0.00
time (sec)	N/A	0.283	0.167	0.521	0.026	0.074	2.156	0.124	0.225	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	32	28	58	29	61	29	105	0
N.S.	1	1.11	0.91	0.80	1.66	0.83	1.74	0.83	3.00	0.00
time (sec)	N/A	0.255	0.114	0.475	0.026	0.073	0.986	0.125	0.204	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	33	16	19	16	13	14
N.S.	1	1.00	1.00	0.83	1.83	0.89	1.06	0.89	0.72	0.78
time (sec)	N/A	0.245	0.168	0.438	0.032	0.073	0.471	0.118	0.193	0.092

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	16	29	19	35	12
N.S.	1	1.00	1.00	0.81	0.75	1.00	1.81	1.19	2.19	0.75
time (sec)	N/A	0.233	0.104	0.441	0.025	0.076	0.357	0.123	0.196	0.303

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	25	26	60	40	55	25
N.S.	1	1.00	0.70	0.61	0.76	0.79	1.82	1.21	1.67	0.76
time (sec)	N/A	0.240	0.118	0.446	0.029	0.075	0.479	0.121	0.264	0.314

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	28	25	37	31	100	57	85	24
N.S.	1	1.10	0.57	0.51	0.76	0.63	2.04	1.16	1.73	0.49
time (sec)	N/A	0.272	0.125	0.461	0.033	0.079	0.706	0.123	0.227	0.356

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	93	52	33	0	27	27	0	39	0
N.S.	1	1.58	0.88	0.56	0.00	0.46	0.46	0.00	0.66	0.00
time (sec)	N/A	0.303	10.022	0.953	0.000	0.076	0.440	0.000	0.199	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	72	44	33	0	19	27	0	28	0
N.S.	1	1.67	1.02	0.77	0.00	0.44	0.63	0.00	0.65	0.00
time (sec)	N/A	0.285	10.014	0.598	0.000	0.073	0.371	0.000	0.199	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	25	54	25	30	0	6	26	0	16	26
N.S.	1	2.16	1.00	1.20	0.00	0.24	1.04	0.00	0.64	1.04
time (sec)	N/A	0.258	10.026	0.451	0.000	0.076	0.343	0.000	0.223	0.272

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	74	40	33	0	22	31	0	20	0
N.S.	1	1.64	0.89	0.73	0.00	0.49	0.69	0.00	0.44	0.00
time (sec)	N/A	0.288	10.008	0.622	0.000	0.078	0.447	0.000	0.200	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	61	95	40	33	0	30	34	0	20	0
N.S.	1	1.56	0.66	0.54	0.00	0.49	0.56	0.00	0.33	0.00
time (sec)	N/A	0.323	10.006	0.987	0.000	0.084	0.510	0.000	0.198	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	168	54	33	0	44	27	0	44	0
N.S.	1	1.89	0.61	0.37	0.00	0.49	0.30	0.00	0.49	0.00
time (sec)	N/A	0.438	10.019	1.344	0.000	0.080	0.490	0.000	0.226	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	73	147	46	33	0	37	27	0	33	0
N.S.	1	2.01	0.63	0.45	0.00	0.51	0.37	0.00	0.45	0.00
time (sec)	N/A	0.415	10.017	0.795	0.000	0.077	0.407	0.000	0.216	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	52	126	40	33	0	29	27	0	19	0
N.S.	1	2.42	0.77	0.63	0.00	0.56	0.52	0.00	0.37	0.00
time (sec)	N/A	0.370	10.018	0.567	0.000	0.076	0.361	0.000	0.199	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	A	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	140	38	33	0	26	29	0	20	18
N.S.	1	2.15	0.58	0.51	0.00	0.40	0.45	0.00	0.31	0.28
time (sec)	N/A	0.409	10.017	0.608	0.000	0.076	0.426	0.000	0.192	0.379

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	89	161	40	33	0	39	34	0	20	0
N.S.	1	1.81	0.45	0.37	0.00	0.44	0.38	0.00	0.22	0.00
time (sec)	N/A	0.442	10.008	0.776	0.000	0.083	0.474	0.000	0.215	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	29	20	0	66	39	0	25	0
N.S.	1	1.00	0.60	0.42	0.00	1.38	0.81	0.00	0.52	0.00
time (sec)	N/A	0.353	10.011	0.678	0.000	0.077	0.390	0.000	0.214	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	30	21	0	96	39	0	26	0
N.S.	1	1.00	0.56	0.39	0.00	1.78	0.72	0.00	0.48	0.00
time (sec)	N/A	0.357	10.012	0.588	0.000	0.081	0.421	0.000	0.237	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	50	36	47	46	87	43	325	44
N.S.	1	1.07	0.85	0.61	0.80	0.78	1.47	0.73	5.51	0.75
time (sec)	N/A	0.323	0.030	0.678	0.028	0.119	0.372	0.122	0.187	0.376

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	25	30	34	61	29	229	33
N.S.	1	1.11	1.00	0.66	0.79	0.89	1.61	0.76	6.03	0.87
time (sec)	N/A	0.305	0.028	0.608	0.024	0.091	0.268	0.124	0.220	0.313

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	131	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	7.28	0.78
time (sec)	N/A	0.248	0.014	0.606	0.035	0.070	0.096	0.119	0.220	0.300

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	32	49	81	66	36	178	31
N.S.	1	1.00	1.00	0.74	1.14	1.88	1.53	0.84	4.14	0.72
time (sec)	N/A	0.295	0.029	0.623	0.115	0.076	0.767	0.123	0.186	0.388

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	41	53	105	46	43	289	35
N.S.	1	1.00	1.00	0.87	1.13	2.23	0.98	0.91	6.15	0.74
time (sec)	N/A	0.281	0.064	0.642	0.114	0.084	0.996	0.119	0.213	0.486

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	62	56	100	130	95	72	556	54
N.S.	1	1.03	0.87	0.79	1.41	1.83	1.34	1.01	7.83	0.76
time (sec)	N/A	0.314	0.103	0.689	0.104	0.079	2.032	0.123	0.267	0.637

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	81	64	53	120	130	95	54	342	0
N.S.	1	1.09	0.86	0.72	1.62	1.76	1.28	0.73	4.62	0.00
time (sec)	N/A	0.330	0.329	0.799	0.106	0.088	2.005	0.126	0.221	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	51	40	78	105	44	41	168	0
N.S.	1	1.08	1.02	0.80	1.56	2.10	0.88	0.82	3.36	0.00
time (sec)	N/A	0.291	0.126	0.747	0.110	0.084	0.922	0.115	0.207	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	48	50	39	60	99	66	60	108	0
N.S.	1	0.98	1.02	0.80	1.22	2.02	1.35	1.22	2.20	0.00
time (sec)	N/A	0.292	0.147	0.766	0.113	0.079	0.807	0.129	0.215	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	42	63	136	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	2.00	3.00	6.48	0.81
time (sec)	N/A	0.239	0.142	0.822	0.031	0.071	0.453	0.129	0.220	0.433

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	28	35	38	66	120	165	37
N.S.	1	1.00	0.95	0.64	0.80	0.86	1.50	2.73	3.75	0.84
time (sec)	N/A	0.276	0.152	1.062	0.026	0.087	0.619	0.129	0.231	0.541

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	53	39	52	49	359	148	229	73
N.S.	1	1.09	0.78	0.57	0.76	0.72	5.28	2.18	3.37	1.07
time (sec)	N/A	0.316	0.166	1.273	0.025	0.079	1.010	0.133	0.217	0.757

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	132	62	103	0	57	39	0	57	0
N.S.	1	1.04	0.49	0.81	0.00	0.45	0.31	0.00	0.45	0.00
time (sec)	N/A	0.378	5.396	0.677	0.000	0.079	0.484	0.000	0.245	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	41	37	0	35	37
N.S.	1	1.00	0.85	0.81	0.00	0.39	0.35	0.00	0.33	0.35
time (sec)	N/A	0.315	3.784	0.574	0.000	0.084	0.449	0.000	0.202	0.239

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	51	87	0	44	42	0	45	0
N.S.	1	1.00	0.48	0.81	0.00	0.41	0.39	0.00	0.42	0.00
time (sec)	N/A	0.306	10.011	0.672	0.000	0.117	0.509	0.000	0.216	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	134	51	105	0	60	46	0	46	0
N.S.	1	1.04	0.40	0.81	0.00	0.47	0.36	0.00	0.36	0.00
time (sec)	N/A	0.359	10.010	0.918	0.000	0.082	0.607	0.000	0.235	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	237	51	112	0	88	39	0	40	0
N.S.	1	1.01	0.22	0.48	0.00	0.38	0.17	0.00	0.17	0.00
time (sec)	N/A	0.513	4.275	0.655	0.000	0.077	0.466	0.000	0.231	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	233	49	112	0	0	41	0	41	39
N.S.	1	1.04	0.22	0.50	0.00	0.00	0.18	0.00	0.18	0.17
time (sec)	N/A	0.491	8.402	0.619	0.000	0.000	0.538	0.000	0.210	0.489

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	262	51	123	0	87	46	0	46	0
N.S.	1	1.02	0.20	0.48	0.00	0.34	0.18	0.00	0.18	0.00
time (sec)	N/A	0.558	10.009	0.821	0.000	0.080	0.542	0.000	0.212	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	57	109	43	421	53
N.S.	1	1.07	0.66	0.61	0.80	0.97	1.85	0.73	7.14	0.90
time (sec)	N/A	0.301	0.035	0.601	0.036	0.064	0.546	0.120	0.215	0.377

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	45	83	29	325	42
N.S.	1	1.11	0.74	0.66	0.79	1.18	2.18	0.76	8.55	1.11
time (sec)	N/A	0.288	0.031	0.600	0.025	0.066	0.397	0.119	0.206	0.371

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	60	14	227	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.33	0.78	12.61	0.78
time (sec)	N/A	0.228	0.014	0.585	0.035	0.070	0.180	0.119	0.202	0.382

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	52	41	61	100	80	50	432	43
N.S.	1	1.03	0.88	0.69	1.03	1.69	1.36	0.85	7.32	0.73
time (sec)	N/A	0.290	0.040	0.627	0.111	0.082	1.207	0.121	0.205	0.372

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	57	52	66	118	95	57	441	48
N.S.	1	1.02	0.89	0.81	1.03	1.84	1.48	0.89	6.89	0.75
time (sec)	N/A	0.299	0.102	0.682	0.148	0.078	1.364	0.124	0.250	0.596

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	59	57	98	135	75	70	554	52
N.S.	1	1.03	0.86	0.83	1.42	1.96	1.09	1.01	8.03	0.75
time (sec)	N/A	0.297	0.118	0.664	0.120	0.078	1.775	0.126	0.227	0.768

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	105	77	66	160	156	122	126	513	0
N.S.	1	1.11	0.81	0.69	1.68	1.64	1.28	1.33	5.40	0.00
time (sec)	N/A	0.350	0.178	0.816	0.113	0.088	3.366	0.140	0.198	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	78	63	52	119	135	73	99	340	0
N.S.	1	1.10	0.89	0.73	1.68	1.90	1.03	1.39	4.79	0.00
time (sec)	N/A	0.296	0.170	0.732	0.113	0.081	1.475	0.133	0.205	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	74	60	50	94	123	95	78	270	0
N.S.	1	1.06	0.86	0.71	1.34	1.76	1.36	1.11	3.86	0.00
time (sec)	N/A	0.309	0.215	0.780	0.104	0.084	1.352	0.141	0.243	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	47	75	119	80	122	269	0
N.S.	1	1.00	0.87	0.68	1.09	1.72	1.16	1.77	3.90	0.00
time (sec)	N/A	0.319	0.173	0.865	0.117	0.091	1.228	0.140	0.209	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	66	92	232	17
N.S.	1	1.00	1.00	0.86	0.81	1.67	3.14	4.38	11.05	0.81
time (sec)	N/A	0.235	0.153	1.012	0.031	0.074	0.617	0.140	0.212	0.748

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	49	92	178	261	71
N.S.	1	1.00	0.70	0.64	0.80	1.11	2.09	4.05	5.93	1.61
time (sec)	N/A	0.292	0.194	1.280	0.028	0.075	0.892	0.148	0.219	1.025

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	60	420	206	325	91
N.S.	1	1.09	0.62	0.57	0.76	0.88	6.18	3.03	4.78	1.34
time (sec)	N/A	0.320	0.229	1.672	0.032	0.087	1.382	0.149	0.211	1.399

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	156	67	114	0	70	39	0	76	0
N.S.	1	1.05	0.45	0.77	0.00	0.47	0.26	0.00	0.51	0.00
time (sec)	N/A	0.394	8.861	0.654	0.000	0.078	0.526	0.000	0.226	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	47	96	0	51	37	0	52	37
N.S.	1	1.04	0.39	0.79	0.00	0.42	0.30	0.00	0.43	0.30
time (sec)	N/A	0.334	5.128	0.565	0.000	0.083	0.465	0.000	0.213	0.217

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	127	52	96	0	0	42	0	62	0
N.S.	1	1.02	0.42	0.77	0.00	0.00	0.34	0.00	0.50	0.00
time (sec)	N/A	0.344	10.011	0.688	0.000	0.000	0.512	0.000	0.239	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	131	52	97	0	54	46	0	62	0
N.S.	1	1.03	0.41	0.76	0.00	0.43	0.36	0.00	0.49	0.00
time (sec)	N/A	0.361	10.012	0.938	0.000	0.085	0.612	0.000	0.264	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	261	52	122	0	104	39	0	57	0
N.S.	1	1.02	0.20	0.48	0.00	0.41	0.15	0.00	0.22	0.00
time (sec)	N/A	0.550	6.803	0.648	0.000	0.082	0.489	0.000	0.222	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	257	50	122	0	0	41	0	60	40
N.S.	1	1.02	0.20	0.48	0.00	0.00	0.16	0.00	0.24	0.16
time (sec)	N/A	0.535	10.009	0.633	0.000	0.000	0.505	0.000	0.242	0.718

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	257	52	120	0	0	46	0	61	0
N.S.	1	1.02	0.21	0.47	0.00	0.00	0.18	0.00	0.24	0.00
time (sec)	N/A	0.554	10.010	0.839	0.000	0.000	0.563	0.000	0.254	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	19	23	23	42	23	163	18
N.S.	1	1.13	0.71	0.61	0.74	0.74	1.35	0.74	5.26	0.58
time (sec)	N/A	0.275	0.017	0.450	0.033	0.064	0.123	0.125	0.196	0.313

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	24	9	83	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	1.85	0.69	6.38	0.69
time (sec)	N/A	0.227	0.014	0.441	0.027	0.064	0.070	0.123	0.205	0.278

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	50	30	75	45	51	39	154	0
N.S.	1	1.10	1.25	0.75	1.88	1.12	1.28	0.98	3.85	0.00
time (sec)	N/A	0.257	0.145	0.499	0.103	0.075	0.796	0.122	0.226	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	39	36	47	35	68	47	229	36
N.S.	1	1.03	0.66	0.61	0.80	0.59	1.15	0.80	3.88	0.61
time (sec)	N/A	0.318	0.026	0.684	0.042	0.069	0.396	0.113	0.204	0.358

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	27	24	30	23	42	30	133	24
N.S.	1	1.05	0.71	0.63	0.79	0.61	1.11	0.79	3.50	0.63
time (sec)	N/A	0.292	0.021	0.590	0.030	0.072	0.231	0.119	0.230	0.320

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	43	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	2.39	0.78
time (sec)	N/A	0.244	0.012	0.655	0.025	0.062	0.087	0.116	0.196	0.310

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	37	60	22	23	58	19
N.S.	1	1.00	1.00	0.74	1.37	2.22	0.81	0.85	2.15	0.70
time (sec)	N/A	0.258	0.021	0.602	0.124	0.074	0.509	0.114	0.231	0.373

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	50	43	68	104	46	48	292	38
N.S.	1	0.98	1.00	0.86	1.36	2.08	0.92	0.96	5.84	0.76
time (sec)	N/A	0.284	0.061	0.669	0.110	0.079	1.128	0.125	0.199	0.474

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	54	43	81	104	46	44	172	0
N.S.	1	1.08	1.02	0.81	1.53	1.96	0.87	0.83	3.25	0.00
time (sec)	N/A	0.294	0.147	0.816	0.106	0.087	1.110	0.123	0.211	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	31	23	45	66	20	25	27	0
N.S.	1	1.00	1.03	0.77	1.50	2.20	0.67	0.83	0.90	0.00
time (sec)	N/A	0.256	0.219	0.673	0.109	0.080	0.495	0.122	0.188	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	20	31	50	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.95	1.48	2.38	0.81
time (sec)	N/A	0.240	0.137	0.705	0.025	0.073	0.361	0.122	0.221	0.282

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	26	35	27	44	59	73	25
N.S.	1	1.00	0.70	0.59	0.80	0.61	1.00	1.34	1.66	0.57
time (sec)	N/A	0.275	0.167	0.813	0.026	0.072	0.538	0.130	0.198	0.359

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	38	298	83	133	38
N.S.	1	1.09	0.62	0.57	0.76	0.56	4.38	1.22	1.96	0.56
time (sec)	N/A	0.326	0.181	0.993	0.044	0.073	0.765	0.130	0.203	0.467

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	138	79	103	0	56	37	0	57	0
N.S.	1	1.06	0.61	0.79	0.00	0.43	0.28	0.00	0.44	0.00
time (sec)	N/A	0.376	10.018	0.881	0.000	0.086	0.514	0.000	0.214	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	91	0	45	37	0	39	0
N.S.	1	1.00	0.57	0.84	0.00	0.42	0.34	0.00	0.36	0.00
time (sec)	N/A	0.322	10.024	0.625	0.000	0.076	0.430	0.000	0.227	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	29	36	0	20	37
N.S.	1	1.00	0.84	0.80	0.00	0.33	0.41	0.00	0.23	0.42
time (sec)	N/A	0.284	0.020	0.487	0.000	0.078	0.392	0.000	0.224	0.236

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	51	93	0	48	41	0	24	0
N.S.	1	1.00	0.46	0.85	0.00	0.44	0.37	0.00	0.22	0.00
time (sec)	N/A	0.332	10.009	0.769	0.000	0.090	0.507	0.000	0.214	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	140	51	105	0	60	44	0	24	0
N.S.	1	1.06	0.39	0.80	0.00	0.45	0.33	0.00	0.18	0.00
time (sec)	N/A	0.355	10.010	0.930	0.000	0.080	0.605	0.000	0.239	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	273	80	125	0	104	37	0	62	0
N.S.	1	1.05	0.31	0.48	0.00	0.40	0.14	0.00	0.24	0.00
time (sec)	N/A	0.580	10.033	1.107	0.000	0.085	0.560	0.000	0.206	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	243	64	115	0	89	37	0	44	0
N.S.	1	1.03	0.27	0.49	0.00	0.38	0.16	0.00	0.19	0.00
time (sec)	N/A	0.504	10.016	0.770	0.000	0.086	0.524	0.000	0.206	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	213	51	97	0	74	37	0	23	0
N.S.	1	1.01	0.24	0.46	0.00	0.35	0.18	0.00	0.11	0.00
time (sec)	N/A	0.451	10.019	0.521	0.000	0.074	0.405	0.000	0.215	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	238	49	115	0	71	39	0	24	40
N.S.	1	1.03	0.21	0.50	0.00	0.31	0.17	0.00	0.10	0.17
time (sec)	N/A	0.521	10.010	0.615	0.000	0.071	0.453	0.000	0.267	0.420

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	268	51	123	0	89	44	0	24	0
N.S.	1	1.03	0.20	0.47	0.00	0.34	0.17	0.00	0.09	0.00
time (sec)	N/A	0.553	10.010	0.795	0.000	0.079	0.598	0.000	0.198	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	38	35	47	46	68	57	100	41
N.S.	1	1.04	0.67	0.61	0.82	0.81	1.19	1.00	1.75	0.72
time (sec)	N/A	0.303	0.033	0.601	0.038	0.069	0.395	0.115	0.206	0.443

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	30	35	41	33	68	21
N.S.	1	1.00	0.71	0.63	0.79	0.92	1.08	0.87	1.79	0.55
time (sec)	N/A	0.282	0.028	0.602	0.036	0.067	0.254	0.123	0.211	0.365

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	46	14
N.S.	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	2.56	0.78
time (sec)	N/A	0.236	0.016	0.612	0.027	0.066	0.102	0.121	0.227	0.306

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	52	126	184	41	241	34
N.S.	1	1.00	1.00	0.76	1.13	2.74	4.00	0.89	5.24	0.74
time (sec)	N/A	0.280	0.052	0.606	0.108	0.079	0.854	0.124	0.202	0.419

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	60	55	86	170	76	72	513	53
N.S.	1	1.07	0.87	0.80	1.25	2.46	1.10	1.04	7.43	0.77
time (sec)	N/A	0.306	0.088	0.668	0.116	0.104	1.736	0.123	0.222	0.568

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	82	63	61	103	170	75	55	312	0
N.S.	1	1.11	0.85	0.82	1.39	2.30	1.01	0.74	4.22	0.00
time (sec)	N/A	0.328	0.257	0.956	0.105	0.080	1.806	0.130	0.241	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	51	53	41	63	141	44	43	141	0
N.S.	1	0.98	1.02	0.79	1.21	2.71	0.85	0.83	2.71	0.00
time (sec)	N/A	0.292	0.176	0.768	0.121	0.086	0.853	0.131	0.225	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	26	20	17	55	17
N.S.	1	1.00	1.00	0.86	0.81	1.24	0.95	0.81	2.62	0.81
time (sec)	N/A	0.231	0.144	0.672	0.040	0.075	0.383	0.120	0.195	0.194

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	42	31	26	36	37	46	53	104	25
N.S.	1	1.02	0.76	0.63	0.88	0.90	1.12	1.29	2.54	0.61
time (sec)	N/A	0.279	0.344	0.730	0.026	0.081	0.474	0.122	0.198	0.356

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	72	42	37	56	50	233	115	105	70
N.S.	1	1.11	0.65	0.57	0.86	0.77	3.58	1.77	1.62	1.08
time (sec)	N/A	0.316	0.265	0.837	0.026	0.078	0.739	0.124	0.224	0.530

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	167	79	133	0	88	37	0	135	0
N.S.	1	1.11	0.52	0.88	0.00	0.58	0.25	0.00	0.89	0.00
time (sec)	N/A	0.413	10.038	2.098	0.000	0.082	0.669	0.000	0.252	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	137	66	111	0	75	37	0	116	0
N.S.	1	1.06	0.51	0.86	0.00	0.58	0.29	0.00	0.90	0.00
time (sec)	N/A	0.376	10.015	1.568	0.000	0.080	0.547	0.000	0.231	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	62	37	0	97	0
N.S.	1	1.00	0.51	0.87	0.00	0.57	0.34	0.00	0.90	0.00
time (sec)	N/A	0.329	6.009	0.557	0.000	0.081	0.455	0.000	0.295	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	64	36	0	31	37
N.S.	1	1.00	0.51	0.87	0.00	0.59	0.33	0.00	0.29	0.34
time (sec)	N/A	0.317	0.010	0.510	0.000	0.084	0.472	0.000	0.235	0.288

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	139	54	113	0	79	41	0	35	0
N.S.	1	1.06	0.41	0.86	0.00	0.60	0.31	0.00	0.27	0.00
time (sec)	N/A	0.371	10.012	1.359	0.000	0.090	0.568	0.000	0.202	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	169	54	135	0	93	44	0	35	0
N.S.	1	1.10	0.35	0.88	0.00	0.61	0.29	0.00	0.23	0.00
time (sec)	N/A	0.400	10.019	1.644	0.000	0.084	0.696	0.000	0.219	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	302	80	157	0	145	37	0	144	0
N.S.	1	1.07	0.28	0.56	0.00	0.51	0.13	0.00	0.51	0.00
time (sec)	N/A	0.614	10.036	2.569	0.000	0.081	0.797	0.000	0.275	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	272	66	137	0	131	37	0	124	0
N.S.	1	1.05	0.26	0.53	0.00	0.51	0.14	0.00	0.48	0.00
time (sec)	N/A	0.569	10.017	2.056	0.000	0.078	0.584	0.000	0.229	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	242	57	119	0	113	37	0	106	0
N.S.	1	1.03	0.24	0.50	0.00	0.48	0.16	0.00	0.45	0.00
time (sec)	N/A	0.499	9.199	0.661	0.000	0.077	0.474	0.000	0.243	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	242	54	119	0	98	37	0	34	0
N.S.	1	1.01	0.23	0.50	0.00	0.41	0.15	0.00	0.14	0.00
time (sec)	N/A	0.506	5.828	0.517	0.000	0.072	0.483	0.000	0.199	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	267	52	137	0	109	39	0	35	40
N.S.	1	1.03	0.20	0.53	0.00	0.42	0.15	0.00	0.13	0.15
time (sec)	N/A	0.578	10.011	0.950	0.000	0.079	0.521	0.000	0.201	0.541

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	297	54	157	0	132	44	0	35	0
N.S.	1	1.05	0.19	0.56	0.00	0.47	0.16	0.00	0.12	0.00
time (sec)	N/A	0.635	10.012	1.848	0.000	0.083	0.656	0.000	0.221	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	25	21	28	21	39	28	131	20
N.S.	1	1.05	0.62	0.52	0.70	0.52	0.98	0.70	3.28	0.50
time (sec)	N/A	0.264	0.019	0.433	0.030	0.072	0.178	0.121	0.198	0.315

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	29	18	15	19	14	22	19	83	14
N.S.	1	1.07	0.67	0.56	0.70	0.52	0.81	0.70	3.07	0.52
time (sec)	N/A	0.261	0.016	0.434	0.027	0.071	0.113	0.125	0.209	0.303

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	8	9	32	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.62	0.69	2.46	0.69
time (sec)	N/A	0.223	0.010	0.433	0.025	0.070	0.067	0.118	0.239	0.195

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	25	25	8	25	29	10
N.S.	1	1.00	1.00	0.79	1.79	1.79	0.57	1.79	2.07	0.71
time (sec)	N/A	0.238	0.016	0.452	0.034	0.064	0.452	0.122	0.188	0.060

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	29	31	24	37	44	22	37	168	23
N.S.	1	0.94	1.00	0.77	1.19	1.42	0.71	1.19	5.42	0.74
time (sec)	N/A	0.258	0.039	0.494	0.026	0.075	0.967	0.119	0.201	0.390

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	29	35	20	58	29	19	29	104	0
N.S.	1	1.16	1.40	0.80	2.32	1.16	0.76	1.16	4.16	0.00
time (sec)	N/A	0.252	0.105	0.486	0.031	0.073	0.954	0.120	0.220	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	18	7	33	16	5	16	13	6
N.S.	1	1.00	2.25	0.88	4.12	2.00	0.62	2.00	1.62	0.75
time (sec)	N/A	0.225	0.144	0.441	0.030	0.071	0.482	0.125	0.220	0.160

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	16	12	19	35	12
N.S.	1	1.00	1.00	0.81	0.75	1.00	0.75	1.19	2.19	0.75
time (sec)	N/A	0.233	0.090	0.434	0.027	0.071	0.347	0.130	0.216	0.278

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	25	26	26	40	53	19
N.S.	1	1.00	0.70	0.61	0.76	0.79	0.79	1.21	1.61	0.58
time (sec)	N/A	0.272	0.113	0.427	0.031	0.072	0.439	0.114	0.188	0.310

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	28	25	37	31	44	57	85	24
N.S.	1	1.10	0.57	0.51	0.76	0.63	0.90	1.16	1.73	0.49
time (sec)	N/A	0.284	0.127	0.447	0.025	0.067	0.635	0.120	0.217	0.345

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	40	17	0	34	29	0	39	0
N.S.	1	1.07	0.54	0.23	0.00	0.46	0.39	0.00	0.53	0.00
time (sec)	N/A	0.307	10.019	0.599	0.000	0.078	0.424	0.000	0.192	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	32	17	0	26	29	0	28	0
N.S.	1	1.00	0.55	0.29	0.00	0.45	0.50	0.00	0.48	0.00
time (sec)	N/A	0.276	10.010	0.523	0.000	0.077	0.362	0.000	0.221	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	21	14	0	13	27	0	16	12
N.S.	1	1.00	0.49	0.33	0.00	0.30	0.63	0.00	0.37	0.28
time (sec)	N/A	0.245	0.006	0.413	0.000	0.080	0.324	0.000	0.211	0.064

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	22	17	0	31	32	0	18	0
N.S.	1	1.00	0.37	0.28	0.00	0.52	0.53	0.00	0.30	0.00
time (sec)	N/A	0.282	10.005	0.525	0.000	0.082	0.440	0.000	0.232	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	22	17	0	37	36	0	18	0
N.S.	1	1.07	0.29	0.22	0.00	0.49	0.47	0.00	0.24	0.00
time (sec)	N/A	0.310	10.006	0.584	0.000	0.083	0.498	0.000	0.230	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	42	17	0	58	29	0	44	0
N.S.	1	1.04	0.30	0.12	0.00	0.41	0.21	0.00	0.31	0.00
time (sec)	N/A	0.424	10.023	0.648	0.000	0.075	0.533	0.000	0.195	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	34	17	0	51	29	0	33	0
N.S.	1	1.00	0.27	0.14	0.00	0.41	0.23	0.00	0.27	0.00
time (sec)	N/A	0.378	10.012	0.575	0.000	0.075	0.389	0.000	0.198	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	22	17	0	44	29	0	19	0
N.S.	1	1.00	0.21	0.17	0.00	0.43	0.28	0.00	0.18	0.00
time (sec)	N/A	0.345	10.019	0.462	0.000	0.073	0.360	0.000	0.236	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	20	17	0	42	31	0	18	15
N.S.	1	1.00	0.17	0.15	0.00	0.36	0.26	0.00	0.15	0.13
time (sec)	N/A	0.378	10.007	0.532	0.000	0.073	0.377	0.000	0.196	0.275

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	138	22	17	0	53	36	0	18	0
N.S.	1	0.99	0.16	0.12	0.00	0.38	0.26	0.00	0.13	0.00
time (sec)	N/A	0.421	10.006	0.572	0.000	0.075	0.483	0.000	0.204	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	23	20	28	19	39	28	64	25
N.S.	1	1.05	0.61	0.53	0.74	0.50	1.03	0.74	1.68	0.66
time (sec)	N/A	0.282	0.032	0.433	0.036	0.066	0.320	0.118	0.191	0.362

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	19	14	22	19	48	14
N.S.	1	1.00	0.67	0.56	0.70	0.52	0.81	0.70	1.78	0.52
time (sec)	N/A	0.271	0.024	0.427	0.031	0.066	0.211	0.118	0.233	0.319

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	35	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	2.69	0.69
time (sec)	N/A	0.226	0.013	0.434	0.023	0.063	0.159	0.119	0.197	0.317

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	34	52	87	34	135	20
N.S.	1	1.00	1.00	0.75	1.21	1.86	3.11	1.21	4.82	0.71
time (sec)	N/A	0.256	0.028	0.500	0.044	0.064	0.736	0.124	0.204	0.332

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	47	38	31	53	66	42	53	281	32
N.S.	1	1.07	0.86	0.70	1.20	1.50	0.95	1.20	6.39	0.73
time (sec)	N/A	0.273	0.055	0.508	0.026	0.067	1.482	0.115	0.242	0.419

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	41	25	73	54	36	34	176	0
N.S.	1	1.12	1.00	0.61	1.78	1.32	0.88	0.83	4.29	0.00
time (sec)	N/A	0.278	0.148	0.464	0.026	0.072	1.586	0.123	0.208	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	23	35	20	45	45	19	29	86	0
N.S.	1	0.92	1.40	0.80	1.80	1.80	0.76	1.16	3.44	0.00
time (sec)	N/A	0.252	0.119	0.460	0.030	0.072	0.731	0.124	0.192	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	25	12	12	39	12
N.S.	1	1.00	1.00	0.81	0.75	1.56	0.75	0.75	2.44	0.75
time (sec)	N/A	0.228	0.095	0.429	0.024	0.066	0.329	0.121	0.219	0.173

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	20	25	37	42	32	69	17
N.S.	1	1.00	0.74	0.65	0.81	1.19	1.35	1.03	2.23	0.55
time (sec)	N/A	0.254	0.244	0.440	0.030	0.072	0.435	0.119	0.215	0.292

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	28	25	36	42	70	70	69	28
N.S.	1	1.06	0.57	0.51	0.73	0.86	1.43	1.43	1.41	0.57
time (sec)	N/A	0.282	0.142	0.459	0.025	0.068	0.588	0.127	0.189	0.362

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	100	52	17	0	55	29	0	90	0
N.S.	1	1.11	0.58	0.19	0.00	0.61	0.32	0.00	1.00	0.00
time (sec)	N/A	0.332	4.206	0.683	0.000	0.085	0.540	0.000	0.216	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	47	17	0	50	29	0	79	0
N.S.	1	1.07	0.64	0.23	0.00	0.68	0.39	0.00	1.07	0.00
time (sec)	N/A	0.308	3.912	0.593	0.000	0.083	0.438	0.000	0.211	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	32	17	0	39	29	0	64	0
N.S.	1	1.00	0.55	0.29	0.00	0.67	0.50	0.00	1.10	0.00
time (sec)	N/A	0.278	3.432	0.591	0.000	0.086	0.363	0.000	0.232	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	30	14	0	38	27	0	21	12
N.S.	1	1.00	0.52	0.24	0.00	0.66	0.47	0.00	0.36	0.21
time (sec)	N/A	0.260	0.003	0.541	0.000	0.080	0.388	0.000	0.218	0.072

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	22	17	0	51	32	0	23	0
N.S.	1	1.07	0.29	0.22	0.00	0.67	0.42	0.00	0.30	0.00
time (sec)	N/A	0.305	10.007	0.542	0.000	0.087	0.463	0.000	0.210	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	102	22	17	0	57	36	0	23	0
N.S.	1	1.11	0.24	0.18	0.00	0.62	0.39	0.00	0.25	0.00
time (sec)	N/A	0.339	10.005	0.599	0.000	0.083	0.595	0.000	0.200	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	166	54	17	0	84	29	0	99	0
N.S.	1	1.06	0.35	0.11	0.00	0.54	0.19	0.00	0.63	0.00
time (sec)	N/A	0.457	5.237	0.809	0.000	0.077	0.616	0.000	0.227	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	47	17	0	79	29	0	87	0
N.S.	1	1.04	0.34	0.12	0.00	0.56	0.21	0.00	0.62	0.00
time (sec)	N/A	0.424	3.926	0.623	0.000	0.079	0.494	0.000	0.197	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	31	17	0	74	29	0	74	0
N.S.	1	1.00	0.25	0.14	0.00	0.60	0.23	0.00	0.60	0.00
time (sec)	N/A	0.380	4.251	0.556	0.000	0.097	0.388	0.000	0.202	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	22	17	0	59	29	0	24	0
N.S.	1	1.00	0.18	0.14	0.00	0.48	0.23	0.00	0.19	0.00
time (sec)	N/A	0.386	4.153	0.540	0.000	0.077	0.345	0.000	0.212	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	138	20	17	0	69	31	0	23	15
N.S.	1	0.99	0.14	0.12	0.00	0.49	0.22	0.00	0.16	0.11
time (sec)	N/A	0.412	9.096	0.536	0.000	0.083	0.423	0.000	0.237	0.279

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	159	22	17	0	81	36	0	23	0
N.S.	1	1.02	0.14	0.11	0.00	0.52	0.23	0.00	0.15	0.00
time (sec)	N/A	0.462	10.006	0.577	0.000	0.076	0.523	0.000	0.203	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	33	16	22	16	17	14
N.S.	1	1.00	1.00	0.83	1.83	0.89	1.22	0.89	0.94	0.78
time (sec)	N/A	0.244	0.182	0.468	0.025	0.073	0.474	0.121	0.187	0.248

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	9	33	16	7	16	17	8
N.S.	1	1.00	1.50	0.75	2.75	1.33	0.58	1.33	1.42	0.67
time (sec)	N/A	0.236	0.181	0.442	0.030	0.069	0.493	0.126	0.244	0.039

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	31	20	17	19	19	41	19	41	16
N.S.	1	1.15	0.74	0.63	0.70	0.70	1.52	0.70	1.52	0.59
time (sec)	N/A	0.274	0.020	0.431	0.024	0.069	0.135	0.120	0.218	0.243

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	12	9	27	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.92	0.69	2.08	0.69
time (sec)	N/A	0.235	0.017	0.427	0.024	0.064	0.160	0.119	0.210	0.258

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	9	10	9	13	9
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	1.00	0.69
time (sec)	N/A	0.249	0.013	0.429	0.029	0.063	0.064	0.123	0.211	0.075

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	61	58	81	68	134	71	99	66
N.S.	1	1.04	0.60	0.57	0.80	0.67	1.33	0.70	0.98	0.65
time (sec)	N/A	0.382	0.046	0.512	0.033	0.080	0.915	0.120	0.227	0.315

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	61	47	64	57	110	57	88	55
N.S.	1	1.05	0.76	0.59	0.80	0.71	1.38	0.71	1.10	0.69
time (sec)	N/A	0.353	0.035	0.497	0.030	0.072	0.633	0.125	0.213	0.258

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	46	87	43	77	44
N.S.	1	1.07	0.66	0.61	0.80	0.78	1.47	0.73	1.31	0.75
time (sec)	N/A	0.321	0.034	0.498	0.025	0.071	0.418	0.126	0.212	0.278

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	25	30	34	63	29	65	33
N.S.	1	1.11	1.00	0.66	0.79	0.89	1.66	0.76	1.71	0.87
time (sec)	N/A	0.294	0.033	0.502	0.044	0.070	0.274	0.124	0.210	0.257

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	52	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	2.89	0.78
time (sec)	N/A	0.242	0.014	0.492	0.025	0.071	0.109	0.122	0.247	0.236

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	81	66	71	66	92	42	183	35	48
N.S.	1	1.23	1.00	1.08	1.00	1.39	0.64	2.77	0.53	0.73
time (sec)	N/A	0.328	0.067	1.158	0.110	0.079	0.605	0.121	0.242	0.306

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	77	75	78	73	171	41	205	45	55
N.S.	1	1.03	1.00	1.04	0.97	2.28	0.55	2.73	0.60	0.73
time (sec)	N/A	0.313	0.126	0.885	0.114	0.078	0.682	0.124	0.237	0.449

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	110	92	104	129	209	41	243	67	79
N.S.	1	1.09	0.91	1.03	1.28	2.07	0.41	2.41	0.66	0.78
time (sec)	N/A	0.349	0.155	0.715	0.133	0.085	1.251	0.131	0.267	0.605

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	140	95	0	0	0	29	0	73	0
N.S.	1	1.12	0.76	0.00	0.00	0.00	0.23	0.00	0.58	0.00
time (sec)	N/A	0.390	7.382	0.000	0.000	0.000	0.617	0.000	0.239	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	110	64	0	0	0	29	0	53	0
N.S.	1	1.09	0.63	0.00	0.00	0.00	0.29	0.00	0.52	0.00
time (sec)	N/A	0.345	6.446	0.000	0.000	0.000	0.514	0.000	0.231	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	83	51	0	0	0	29	0	31	0
N.S.	1	1.05	0.65	0.00	0.00	0.00	0.37	0.00	0.39	0.00
time (sec)	N/A	0.308	6.049	0.000	0.000	0.000	0.450	0.000	0.253	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	51	0	0	0	32	0	47	0
N.S.	1	0.99	0.65	0.00	0.00	0.00	0.41	0.00	0.59	0.00
time (sec)	N/A	0.308	10.012	0.000	0.000	0.000	0.492	0.000	0.249	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	51	0	0	0	34	0	48	0
N.S.	1	1.05	0.50	0.00	0.00	0.00	0.34	0.00	0.48	0.00
time (sec)	N/A	0.353	10.010	0.000	0.000	0.000	0.566	0.000	0.247	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	51	0	0	0	34	0	48	0
N.S.	1	1.09	0.41	0.00	0.00	0.00	0.27	0.00	0.38	0.00
time (sec)	N/A	0.383	10.011	0.000	0.000	0.000	0.760	0.000	0.264	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	111	87	104	152	218	39	0	55	0
N.S.	1	1.08	0.84	1.01	1.48	2.12	0.38	0.00	0.53	0.00
time (sec)	N/A	0.387	0.317	0.809	0.112	0.093	1.074	0.000	0.219	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	81	74	81	101	179	39	0	33	0
N.S.	1	1.05	0.96	1.05	1.31	2.32	0.51	0.00	0.43	0.00
time (sec)	N/A	0.338	0.237	0.938	0.116	0.083	0.627	0.000	0.209	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	76	73	81	82	0	41	0	15	40
N.S.	1	1.04	1.00	1.11	1.12	0.00	0.56	0.00	0.21	0.55
time (sec)	N/A	0.317	0.184	0.998	0.124	0.000	0.608	0.000	0.226	0.427

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81	0.81
time (sec)	N/A	0.233	0.125	0.537	0.035	0.080	0.440	0.000	0.212	0.349

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	28	35	38	109	0	38	37
N.S.	1	1.00	0.95	0.64	0.80	0.86	2.48	0.00	0.86	0.84
time (sec)	N/A	0.270	0.142	0.552	0.028	0.087	0.627	0.000	0.207	0.448

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	53	39	52	49	520	0	49	73
N.S.	1	1.09	0.78	0.57	0.76	0.72	7.65	0.00	0.72	1.07
time (sec)	N/A	0.316	0.157	0.587	0.031	0.082	0.981	0.000	0.219	0.684

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	60	847	0	60	93
N.S.	1	1.13	0.58	0.54	0.75	0.65	9.21	0.00	0.65	1.01
time (sec)	N/A	0.364	0.192	0.635	0.030	0.079	1.382	0.000	0.210	0.952

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	167	105	0	0	0	39	0	90	0
N.S.	1	1.11	0.70	0.00	0.00	0.00	0.26	0.00	0.60	0.00
time (sec)	N/A	0.536	6.545	0.000	0.000	0.000	0.725	0.000	0.241	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	93	0	0	0	39	0	70	0
N.S.	1	1.09	0.74	0.00	0.00	0.00	0.31	0.00	0.56	0.00
time (sec)	N/A	0.485	6.315	0.000	0.000	0.000	0.577	0.000	0.209	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	107	62	0	0	0	39	0	49	0
N.S.	1	1.05	0.61	0.00	0.00	0.00	0.38	0.00	0.48	0.00
time (sec)	N/A	0.411	6.278	0.000	0.000	0.000	0.497	0.000	0.231	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	0	0	0	37	0	27	37
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.46	0.00	0.34	0.46
time (sec)	N/A	0.347	0.003	0.000	0.000	0.000	0.451	0.000	0.246	0.192

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	51	0	0	0	31	0	48	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.38	0.00	0.59	0.00
time (sec)	N/A	0.362	10.011	0.000	0.000	0.000	0.533	0.000	0.239	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	109	51	0	0	0	31	0	48	0
N.S.	1	1.05	0.49	0.00	0.00	0.00	0.30	0.00	0.46	0.00
time (sec)	N/A	0.409	10.011	0.000	0.000	0.000	0.644	0.000	0.221	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	139	51	0	0	0	46	0	48	0
N.S.	1	1.09	0.40	0.00	0.00	0.00	0.36	0.00	0.38	0.00
time (sec)	N/A	0.480	10.011	0.000	0.000	0.000	0.739	0.000	0.205	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	169	51	0	0	0	46	0	48	0
N.S.	1	1.11	0.34	0.00	0.00	0.00	0.30	0.00	0.32	0.00
time (sec)	N/A	0.529	10.010	0.000	0.000	0.000	0.983	0.000	0.276	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	61	58	81	68	136	163	557	66
N.S.	1	1.04	0.60	0.57	0.80	0.67	1.35	1.61	5.51	0.65
time (sec)	N/A	0.358	0.044	0.500	0.041	0.071	1.293	0.125	0.302	0.314

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	57	110	134	461	55
N.S.	1	1.05	0.62	0.59	0.80	0.71	1.38	1.68	5.76	0.69
time (sec)	N/A	0.342	0.034	0.543	0.026	0.066	0.903	0.120	0.256	0.277

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	46	87	106	365	44
N.S.	1	1.07	0.66	0.61	0.80	0.78	1.47	1.80	6.19	0.75
time (sec)	N/A	0.309	0.028	0.556	0.026	0.068	0.598	0.127	0.258	0.282

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	39	25	30	35	65	78	269	33
N.S.	1	1.11	1.03	0.66	0.79	0.92	1.71	2.05	7.08	0.87
time (sec)	N/A	0.291	0.026	0.565	0.025	0.074	0.444	0.124	0.232	0.298

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	171	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	9.50	0.78
time (sec)	N/A	0.222	0.014	0.497	0.034	0.071	0.191	0.120	0.200	0.285

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	78	70	73	71	126	44	185	37	50
N.S.	1	1.11	1.00	1.04	1.01	1.80	0.63	2.64	0.53	0.71
time (sec)	N/A	0.324	0.056	0.556	0.108	0.082	0.645	0.125	0.208	0.331

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	75	82	74	185	39	206	46	55
N.S.	1	1.05	1.00	1.09	0.99	2.47	0.52	2.75	0.61	0.73
time (sec)	N/A	0.320	0.112	0.762	0.115	0.080	0.711	0.122	0.294	0.460

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	108	92	108	125	210	41	243	67	79
N.S.	1	1.07	0.91	1.07	1.24	2.08	0.41	2.41	0.66	0.78
time (sec)	N/A	0.350	0.158	0.734	0.129	0.086	1.306	0.130	0.268	0.611

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	166	95	0	0	0	29	0	74	0
N.S.	1	1.11	0.64	0.00	0.00	0.00	0.19	0.00	0.50	0.00
time (sec)	N/A	0.411	7.430	0.000	0.000	0.000	0.753	0.000	0.238	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	64	0	0	0	29	0	54	0
N.S.	1	1.09	0.51	0.00	0.00	0.00	0.23	0.00	0.43	0.00
time (sec)	N/A	0.374	7.542	0.000	0.000	0.000	0.599	0.000	0.260	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	106	51	0	0	0	29	0	31	0
N.S.	1	1.08	0.52	0.00	0.00	0.00	0.30	0.00	0.32	0.00
time (sec)	N/A	0.329	6.674	0.000	0.000	0.000	0.524	0.000	0.226	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	104	51	0	0	0	32	0	45	0
N.S.	1	1.06	0.52	0.00	0.00	0.00	0.33	0.00	0.46	0.00
time (sec)	N/A	0.333	10.013	0.000	0.000	0.000	0.570	0.000	0.260	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	134	51	0	0	0	34	0	48	0
N.S.	1	1.07	0.41	0.00	0.00	0.00	0.27	0.00	0.38	0.00
time (sec)	N/A	0.369	10.013	0.000	0.000	0.000	0.626	0.000	0.286	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	164	51	0	0	0	34	0	48	0
N.S.	1	1.10	0.34	0.00	0.00	0.00	0.23	0.00	0.32	0.00
time (sec)	N/A	0.415	10.012	0.000	0.000	0.000	0.821	0.000	0.255	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	169	109	142	225	245	39	0	90	0
N.S.	1	1.13	0.73	0.95	1.51	1.64	0.26	0.00	0.60	0.00
time (sec)	N/A	0.460	0.405	0.737	0.118	0.096	18.987	0.000	0.198	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	139	98	122	189	234	39	0	70	0
N.S.	1	1.11	0.78	0.98	1.51	1.87	0.31	0.00	0.56	0.00
time (sec)	N/A	0.396	0.361	0.717	0.120	0.095	3.612	0.000	0.198	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	109	87	102	148	215	39	0	50	0
N.S.	1	1.08	0.86	1.01	1.47	2.13	0.39	0.00	0.50	0.00
time (sec)	N/A	0.351	0.321	0.691	0.114	0.086	1.189	0.000	0.212	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	79	75	80	102	188	37	0	27	37
N.S.	1	1.05	1.00	1.07	1.36	2.51	0.49	0.00	0.36	0.49
time (sec)	N/A	0.297	0.008	0.010	0.115	0.090	0.648	0.000	0.229	0.225

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	78	75	86	85	0	42	0	15	0
N.S.	1	1.04	1.00	1.15	1.13	0.00	0.56	0.00	0.20	0.00
time (sec)	N/A	0.322	0.214	0.693	0.110	0.000	0.689	0.000	0.207	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	68	0	17	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	3.24	0.00	0.81	0.81
time (sec)	N/A	0.231	0.149	0.536	0.025	0.077	0.563	0.000	0.231	0.400

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	38	110	0	38	56
N.S.	1	1.00	0.70	0.64	0.80	0.86	2.50	0.00	0.86	1.27
time (sec)	N/A	0.263	0.165	0.556	0.026	0.104	0.847	0.000	0.212	0.550

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	49	520	0	49	73
N.S.	1	1.09	0.62	0.57	0.76	0.72	7.65	0.00	0.72	1.07
time (sec)	N/A	0.302	0.185	0.602	0.035	0.102	1.234	0.000	0.192	0.723

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	60	847	0	60	93
N.S.	1	1.13	0.58	0.54	0.75	0.65	9.21	0.00	0.65	1.01
time (sec)	N/A	0.345	0.196	0.646	0.025	0.077	1.832	0.000	0.192	1.023

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	164	96	0	0	0	39	0	76	0
N.S.	1	1.09	0.64	0.00	0.00	0.00	0.26	0.00	0.51	0.00
time (sec)	N/A	0.508	7.630	0.000	0.000	0.000	0.787	0.000	0.252	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	134	64	0	0	0	39	0	55	0
N.S.	1	1.06	0.51	0.00	0.00	0.00	0.31	0.00	0.44	0.00
time (sec)	N/A	0.489	7.429	0.000	0.000	0.000	0.644	0.000	0.206	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	51	0	0	0	39	0	33	0
N.S.	1	1.05	0.52	0.00	0.00	0.00	0.39	0.00	0.33	0.00
time (sec)	N/A	0.424	7.254	0.000	0.000	0.000	0.568	0.000	0.202	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	49	0	0	0	41	0	44	40
N.S.	1	1.03	0.51	0.00	0.00	0.00	0.42	0.00	0.45	0.41
time (sec)	N/A	0.405	10.009	0.000	0.000	0.000	0.531	0.000	0.221	0.560

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	51	0	0	0	31	0	48	0
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.31	0.00	0.48	0.00
time (sec)	N/A	0.426	10.012	0.000	0.000	0.000	0.634	0.000	0.230	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	129	51	0	0	0	31	0	48	0
N.S.	1	1.02	0.40	0.00	0.00	0.00	0.25	0.00	0.38	0.00
time (sec)	N/A	0.477	10.011	0.000	0.000	0.000	0.771	0.000	0.207	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	159	51	0	0	0	46	0	48	0
N.S.	1	1.06	0.34	0.00	0.00	0.00	0.31	0.00	0.32	0.00
time (sec)	N/A	0.542	10.012	0.000	0.000	0.000	0.985	0.000	0.231	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	61	58	81	79	156	71	110	75
N.S.	1	1.04	0.60	0.57	0.80	0.78	1.54	0.70	1.09	0.74
time (sec)	N/A	0.380	0.047	0.500	0.029	0.070	1.901	0.126	0.246	0.368

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	68	134	57	99	64
N.S.	1	1.05	0.62	0.59	0.80	0.85	1.68	0.71	1.24	0.80
time (sec)	N/A	0.351	0.039	0.498	0.027	0.073	1.325	0.118	0.234	0.327

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	57	110	43	88	53
N.S.	1	1.07	0.66	0.61	0.80	0.97	1.86	0.73	1.49	0.90
time (sec)	N/A	0.322	0.030	0.494	0.025	0.065	0.894	0.124	0.210	0.328

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	45	85	29	76	42
N.S.	1	1.11	0.74	0.66	0.79	1.18	2.24	0.76	2.00	1.11
time (sec)	N/A	0.296	0.028	0.481	0.040	0.066	0.597	0.121	0.217	0.325

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	61	14	63	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.39	0.78	3.50	0.78
time (sec)	N/A	0.234	0.014	0.473	0.025	0.068	0.327	0.116	0.225	0.295

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	99	79	78	79	127	48	200	55	64
N.S.	1	1.19	0.95	0.94	0.95	1.53	0.58	2.41	0.66	0.77
time (sec)	N/A	0.341	0.074	0.521	0.112	0.119	0.970	0.125	0.256	0.361

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	103	81	88	89	167	42	206	63	70
N.S.	1	1.16	0.91	0.99	1.00	1.88	0.47	2.31	0.71	0.79
time (sec)	N/A	0.356	0.170	0.674	0.120	0.090	0.963	0.129	0.237	0.599

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	101	89	104	120	195	41	232	64	77
N.S.	1	1.02	0.90	1.05	1.21	1.97	0.41	2.34	0.65	0.78
time (sec)	N/A	0.338	0.163	0.698	0.109	0.093	1.186	0.130	0.304	0.778

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	164	81	0	0	0	29	0	94	0
N.S.	1	1.12	0.55	0.00	0.00	0.00	0.20	0.00	0.64	0.00
time (sec)	N/A	0.418	7.848	0.000	0.000	0.000	0.997	0.000	0.266	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	134	69	0	0	0	29	0	74	0
N.S.	1	1.10	0.57	0.00	0.00	0.00	0.24	0.00	0.61	0.00
time (sec)	N/A	0.374	7.830	0.000	0.000	0.000	0.733	0.000	0.251	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	107	52	0	0	0	29	0	49	0
N.S.	1	1.09	0.53	0.00	0.00	0.00	0.30	0.00	0.50	0.00
time (sec)	N/A	0.323	7.549	0.000	0.000	0.000	0.603	0.000	0.277	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	105	52	0	0	0	32	0	65	0
N.S.	1	1.07	0.53	0.00	0.00	0.00	0.33	0.00	0.66	0.00
time (sec)	N/A	0.339	10.013	0.000	0.000	0.000	0.679	0.000	0.250	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	52	0	0	0	34	0	65	0
N.S.	1	1.04	0.53	0.00	0.00	0.00	0.35	0.00	0.66	0.00
time (sec)	N/A	0.334	10.012	0.000	0.000	0.000	0.707	0.000	0.261	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	130	52	0	0	0	34	0	66	0
N.S.	1	1.07	0.43	0.00	0.00	0.00	0.28	0.00	0.54	0.00
time (sec)	N/A	0.376	10.013	0.000	0.000	0.000	0.945	0.000	0.284	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	160	52	0	0	0	34	0	66	0
N.S.	1	1.10	0.36	0.00	0.00	0.00	0.23	0.00	0.45	0.00
time (sec)	N/A	0.426	10.014	0.000	0.000	0.000	1.205	0.000	0.300	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	165	111	118	229	247	39	0	96	0
N.S.	1	1.11	0.75	0.80	1.55	1.67	0.26	0.00	0.65	0.00
time (sec)	N/A	0.482	0.444	0.737	0.115	0.089	10.122	0.000	0.204	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	135	100	124	191	228	39	0	76	0
N.S.	1	1.09	0.81	1.00	1.54	1.84	0.31	0.00	0.61	0.00
time (sec)	N/A	0.431	0.358	0.714	0.112	0.088	2.373	0.000	0.249	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	105	91	104	144	200	39	0	51	0
N.S.	1	1.05	0.91	1.04	1.44	2.00	0.39	0.00	0.51	0.00
time (sec)	N/A	0.375	0.326	0.740	0.119	0.085	0.963	0.000	0.233	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	101	86	90	119	0	41	0	52	40
N.S.	1	1.07	0.91	0.96	1.27	0.00	0.44	0.00	0.55	0.43
time (sec)	N/A	0.359	0.307	0.704	0.108	0.000	0.966	0.000	0.217	0.749

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	97	81	89	97	0	46	0	54	0
N.S.	1	1.05	0.88	0.97	1.05	0.00	0.50	0.00	0.59	0.00
time (sec)	N/A	0.358	0.241	0.736	0.120	0.000	1.026	0.000	0.207	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	105	0	38	17
N.S.	1	1.00	1.00	0.86	0.81	1.67	5.00	0.00	1.81	0.81
time (sec)	N/A	0.226	0.173	0.546	0.035	0.093	0.759	0.000	0.218	0.726

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	49	148	0	49	71
N.S.	1	1.00	0.70	0.64	0.80	1.11	3.36	0.00	1.11	1.61
time (sec)	N/A	0.271	0.205	0.576	0.031	0.089	1.069	0.000	0.199	0.982

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	60	609	0	60	91
N.S.	1	1.09	0.62	0.57	0.76	0.88	8.96	0.00	0.88	1.34
time (sec)	N/A	0.311	0.258	0.609	0.029	0.097	1.653	0.000	0.223	1.392

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	71	954	0	71	111
N.S.	1	1.13	0.58	0.54	0.75	0.77	10.37	0.00	0.77	1.21
time (sec)	N/A	0.363	0.289	0.648	0.035	0.102	2.487	0.000	0.246	2.001

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	191	89	0	0	0	39	0	110	0
N.S.	1	1.12	0.52	0.00	0.00	0.00	0.23	0.00	0.64	0.00
time (sec)	N/A	0.571	8.323	0.000	0.000	0.000	1.129	0.000	0.235	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	161	76	0	0	0	39	0	90	0
N.S.	1	1.10	0.52	0.00	0.00	0.00	0.27	0.00	0.61	0.00
time (sec)	N/A	0.531	7.945	0.000	0.000	0.000	0.892	0.000	0.223	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	67	0	0	0	39	0	70	0
N.S.	1	1.07	0.54	0.00	0.00	0.00	0.32	0.00	0.57	0.00
time (sec)	N/A	0.484	7.537	0.000	0.000	0.000	0.737	0.000	0.215	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	102	47	0	0	0	37	0	45	37
N.S.	1	1.05	0.48	0.00	0.00	0.00	0.38	0.00	0.46	0.38
time (sec)	N/A	0.397	0.003	0.000	0.000	0.000	0.560	0.000	0.238	0.223

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	52	0	0	0	42	0	66	0
N.S.	1	1.05	0.53	0.00	0.00	0.00	0.42	0.00	0.67	0.00
time (sec)	N/A	0.413	10.009	0.000	0.000	0.000	0.612	0.000	0.262	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	52	0	0	0	31	0	66	0
N.S.	1	1.05	0.51	0.00	0.00	0.00	0.31	0.00	0.65	0.00
time (sec)	N/A	0.410	10.011	0.000	0.000	0.000	0.749	0.000	0.274	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	52	0	0	0	31	0	65	0
N.S.	1	1.06	0.42	0.00	0.00	0.00	0.25	0.00	0.52	0.00
time (sec)	N/A	0.477	10.010	0.000	0.000	0.000	1.029	0.000	0.282	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	163	52	0	0	0	46	0	66	0
N.S.	1	1.09	0.35	0.00	0.00	0.00	0.31	0.00	0.44	0.00
time (sec)	N/A	0.532	10.011	0.000	0.000	0.000	1.276	0.000	0.249	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	61	58	81	57	116	71	15	58
N.S.	1	1.04	0.60	0.57	0.80	0.56	1.15	0.70	0.15	0.57
time (sec)	N/A	0.371	0.039	0.490	0.025	0.083	0.881	0.119	0.216	0.361

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	46	92	57	15	48
N.S.	1	1.05	0.62	0.59	0.80	0.58	1.15	0.71	0.19	0.60
time (sec)	N/A	0.347	0.032	0.490	0.030	0.077	0.558	0.118	0.221	0.378

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	35	68	43	15	36
N.S.	1	1.07	0.66	0.61	0.80	0.59	1.15	0.73	0.25	0.61
time (sec)	N/A	0.319	0.029	0.486	0.032	0.078	0.365	0.133	0.246	0.317

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	24	44	29	15	26
N.S.	1	1.11	0.74	0.66	0.79	0.63	1.16	0.76	0.39	0.68
time (sec)	N/A	0.297	0.028	0.470	0.026	0.073	0.247	0.111	0.226	0.292

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	22	14	15	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.22	0.78	0.83	0.78
time (sec)	N/A	0.241	0.019	0.474	0.024	0.077	0.095	0.120	0.216	0.292

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	57	57	83	37	186	15	36
N.S.	1	1.00	0.87	1.04	1.04	1.51	0.67	3.38	0.27	0.65
time (sec)	N/A	0.323	0.041	0.497	0.105	0.093	0.504	0.127	0.212	0.355

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	78	85	92	194	39	208	15	58
N.S.	1	1.08	1.00	1.09	1.18	2.49	0.50	2.67	0.19	0.74
time (sec)	N/A	0.335	0.107	0.655	0.114	0.091	0.765	0.127	0.216	0.491

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	114	92	108	128	218	39	244	15	82
N.S.	1	1.10	0.88	1.04	1.23	2.10	0.38	2.35	0.14	0.79
time (sec)	N/A	0.368	0.130	0.652	0.113	0.094	1.566	0.134	0.225	0.599

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	172	91	0	0	0	27	0	15	0
N.S.	1	1.13	0.60	0.00	0.00	0.00	0.18	0.00	0.10	0.00
time (sec)	N/A	0.440	6.409	0.000	0.000	0.000	0.698	0.000	0.224	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	80	0	0	0	27	0	15	0
N.S.	1	1.11	0.62	0.00	0.00	0.00	0.21	0.00	0.12	0.00
time (sec)	N/A	0.387	6.141	0.000	0.000	0.000	0.566	0.000	0.247	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	64	0	0	0	27	0	15	0
N.S.	1	1.08	0.62	0.00	0.00	0.00	0.26	0.00	0.14	0.00
time (sec)	N/A	0.338	6.084	0.000	0.000	0.000	0.518	0.000	0.207	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	82	51	0	0	0	27	0	13	0
N.S.	1	1.11	0.69	0.00	0.00	0.00	0.36	0.00	0.18	0.00
time (sec)	N/A	0.303	5.924	0.000	0.000	0.000	0.414	0.000	0.203	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	51	0	0	0	31	0	15	0
N.S.	1	1.06	0.49	0.00	0.00	0.00	0.30	0.00	0.14	0.00
time (sec)	N/A	0.343	10.012	0.000	0.000	0.000	0.467	0.000	0.200	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	140	51	0	0	0	32	0	15	0
N.S.	1	1.09	0.40	0.00	0.00	0.00	0.25	0.00	0.12	0.00
time (sec)	N/A	0.373	10.012	0.000	0.000	0.000	0.583	0.000	0.264	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	170	51	0	0	0	32	0	15	0
N.S.	1	1.12	0.34	0.00	0.00	0.00	0.21	0.00	0.10	0.00
time (sec)	N/A	0.415	10.014	0.000	0.000	0.000	0.709	0.000	0.225	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	115	87	102	151	225	37	0	15	0
N.S.	1	1.11	0.84	0.98	1.45	2.16	0.36	0.00	0.14	0.00
time (sec)	N/A	0.385	0.358	0.697	0.106	0.096	1.526	0.000	0.228	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	85	73	79	108	201	37	0	15	0
N.S.	1	1.09	0.94	1.01	1.38	2.58	0.47	0.00	0.19	0.00
time (sec)	N/A	0.325	0.305	0.665	0.111	0.098	0.687	0.000	0.217	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	61	68	106	36	0	11	37
N.S.	1	1.00	1.33	1.07	1.19	1.86	0.63	0.00	0.19	0.65
time (sec)	N/A	0.274	0.017	0.545	0.109	0.090	0.493	0.000	0.215	0.249

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	31	0	15	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	1.48	0.00	0.71	0.81
time (sec)	N/A	0.232	0.214	0.527	0.031	0.080	0.388	0.000	0.184	0.260

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	35	27	70	0	15	36
N.S.	1	1.00	0.70	0.64	0.80	0.61	1.59	0.00	0.34	0.82
time (sec)	N/A	0.271	0.210	0.522	0.032	0.085	0.557	0.000	0.194	0.323

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	38	406	0	15	56
N.S.	1	1.09	0.62	0.57	0.76	0.56	5.97	0.00	0.22	0.82
time (sec)	N/A	0.311	0.220	0.556	0.037	0.076	0.907	0.000	0.218	0.416

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	49	692	0	15	76
N.S.	1	1.13	0.58	0.54	0.75	0.53	7.52	0.00	0.16	0.83
time (sec)	N/A	0.347	0.235	0.587	0.031	0.080	1.261	0.000	0.202	0.526

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	134	64	61	86	60	1046	0	15	96
N.S.	1	1.16	0.55	0.53	0.74	0.52	9.02	0.00	0.13	0.83
time (sec)	N/A	0.399	0.279	0.618	0.058	0.084	1.859	0.000	0.213	0.597

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	140	80	0	0	0	37	0	15	0
N.S.	1	1.09	0.62	0.00	0.00	0.00	0.29	0.00	0.12	0.00
time (sec)	N/A	0.480	6.679	0.000	0.000	0.000	0.563	0.000	0.225	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	110	64	0	0	0	37	0	15	0
N.S.	1	1.05	0.61	0.00	0.00	0.00	0.35	0.00	0.14	0.00
time (sec)	N/A	0.436	6.613	0.000	0.000	0.000	0.489	0.000	0.226	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	51	0	0	0	37	0	15	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.46	0.00	0.19	0.00
time (sec)	N/A	0.388	6.052	0.000	0.000	0.000	0.414	0.000	0.199	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	49	0	0	0	39	0	15	40
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.52	0.00	0.20	0.53
time (sec)	N/A	0.366	10.010	0.000	0.000	0.000	0.471	0.000	0.208	0.476

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	51	0	0	0	29	0	15	0
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.28	0.00	0.14	0.00
time (sec)	N/A	0.432	10.011	0.000	0.000	0.000	0.576	0.000	0.203	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	135	51	0	0	0	44	0	15	0
N.S.	1	1.05	0.40	0.00	0.00	0.00	0.34	0.00	0.12	0.00
time (sec)	N/A	0.493	10.010	0.000	0.000	0.000	0.654	0.000	0.211	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	165	51	0	0	0	44	0	15	0
N.S.	1	1.08	0.33	0.00	0.00	0.00	0.29	0.00	0.10	0.00
time (sec)	N/A	0.553	10.012	0.000	0.000	0.000	0.876	0.000	0.232	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	61	58	80	57	116	74	15	58
N.S.	1	1.05	0.62	0.59	0.82	0.58	1.18	0.76	0.15	0.59
time (sec)	N/A	0.381	0.041	0.492	0.027	0.075	0.953	0.123	0.198	0.360

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	50	47	64	46	92	61	15	48
N.S.	1	1.05	0.64	0.60	0.82	0.59	1.18	0.78	0.19	0.62
time (sec)	N/A	0.352	0.039	0.490	0.026	0.081	0.625	0.119	0.218	0.321

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	39	36	46	35	68	46	15	36
N.S.	1	1.09	0.70	0.64	0.82	0.62	1.21	0.82	0.27	0.64
time (sec)	N/A	0.322	0.030	0.477	0.034	0.081	0.400	0.122	0.232	0.305

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	27	24	30	23	44	30	15	24
N.S.	1	1.11	0.75	0.67	0.83	0.64	1.22	0.83	0.42	0.67
time (sec)	N/A	0.293	0.027	0.471	0.025	0.079	0.253	0.120	0.219	0.306

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.87	0.87
time (sec)	N/A	0.249	0.015	0.486	0.029	0.077	0.093	0.121	0.201	0.294

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	61	46	55	57	104	39	186	15	34
N.S.	1	1.11	0.84	1.00	1.04	1.89	0.71	3.38	0.27	0.62
time (sec)	N/A	0.315	0.050	0.526	0.106	0.099	0.524	0.120	0.223	0.395

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	86	78	86	94	192	39	209	15	58
N.S.	1	1.10	1.00	1.10	1.21	2.46	0.50	2.68	0.19	0.74
time (sec)	N/A	0.327	0.098	0.670	0.106	0.083	0.776	0.120	0.200	0.552

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	116	92	108	132	218	39	244	15	82
N.S.	1	1.12	0.88	1.04	1.27	2.10	0.38	2.35	0.14	0.79
time (sec)	N/A	0.357	0.127	0.718	0.112	0.109	1.626	0.119	0.200	0.653

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	146	91	0	0	0	27	0	15	0
N.S.	1	1.14	0.71	0.00	0.00	0.00	0.21	0.00	0.12	0.00
time (sec)	N/A	0.403	7.256	0.000	0.000	0.000	0.689	0.000	0.190	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	116	79	0	0	0	27	0	15	0
N.S.	1	1.12	0.76	0.00	0.00	0.00	0.26	0.00	0.14	0.00
time (sec)	N/A	0.355	7.053	0.000	0.000	0.000	0.559	0.000	0.221	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	64	0	0	0	27	0	15	0
N.S.	1	1.05	0.78	0.00	0.00	0.00	0.33	0.00	0.18	0.00
time (sec)	N/A	0.303	6.593	0.000	0.000	0.000	0.473	0.000	0.234	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	0	0	0	27	0	13	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.47	0.00	0.23	0.00
time (sec)	N/A	0.268	5.993	0.000	0.000	0.000	0.426	0.000	0.198	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	51	0	0	0	31	0	15	0
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.38	0.00	0.18	0.00
time (sec)	N/A	0.300	10.013	0.000	0.000	0.000	0.522	0.000	0.191	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	51	0	0	0	32	0	15	0
N.S.	1	1.08	0.49	0.00	0.00	0.00	0.31	0.00	0.14	0.00
time (sec)	N/A	0.335	10.017	0.000	0.000	0.000	0.694	0.000	0.229	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	51	0	0	0	32	0	15	0
N.S.	1	1.11	0.40	0.00	0.00	0.00	0.25	0.00	0.12	0.00
time (sec)	N/A	0.389	10.011	0.000	0.000	0.000	0.819	0.000	0.197	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	117	89	104	155	227	37	0	15	0
N.S.	1	1.10	0.84	0.98	1.46	2.14	0.35	0.00	0.14	0.00
time (sec)	N/A	0.390	0.405	0.722	0.109	0.105	1.590	0.000	0.197	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	87	75	81	110	201	37	0	15	0
N.S.	1	1.09	0.94	1.01	1.38	2.51	0.46	0.00	0.19	0.00
time (sec)	N/A	0.325	0.300	0.710	0.112	0.101	0.700	0.000	0.205	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	61	68	126	37	0	15	0
N.S.	1	1.00	0.88	1.07	1.19	2.21	0.65	0.00	0.26	0.00
time (sec)	N/A	0.278	0.222	0.528	0.109	0.098	0.511	0.000	0.230	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	31	0	15	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.63	0.00	0.79	0.89
time (sec)	N/A	0.231	0.200	0.506	0.033	0.078	0.385	0.000	0.229	0.252

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	26	35	27	68	0	15	25
N.S.	1	1.00	0.70	0.59	0.80	0.61	1.55	0.00	0.34	0.57
time (sec)	N/A	0.259	0.240	0.520	0.026	0.083	0.583	0.000	0.196	0.332

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	52	38	406	0	15	38
N.S.	1	1.09	0.62	0.57	0.76	0.56	5.97	0.00	0.22	0.56
time (sec)	N/A	0.313	0.297	0.541	0.032	0.084	0.852	0.000	0.210	0.415

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	69	49	692	0	15	76
N.S.	1	1.13	0.58	0.54	0.75	0.53	7.52	0.00	0.16	0.83
time (sec)	N/A	0.356	0.348	0.572	0.031	0.081	1.239	0.000	0.202	0.546

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	143	90	0	0	0	37	0	15	0
N.S.	1	1.11	0.70	0.00	0.00	0.00	0.29	0.00	0.12	0.00
time (sec)	N/A	0.450	7.004	0.000	0.000	0.000	0.676	0.000	0.222	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	79	0	0	0	37	0	15	0
N.S.	1	1.08	0.75	0.00	0.00	0.00	0.35	0.00	0.14	0.00
time (sec)	N/A	0.437	6.674	0.000	0.000	0.000	0.510	0.000	0.207	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	62	0	0	0	37	0	15	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.45	0.00	0.18	0.00
time (sec)	N/A	0.388	6.286	0.000	0.000	0.000	0.436	0.000	0.228	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	46	0	0	0	36	0	11	37
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.59	0.00	0.18	0.61
time (sec)	N/A	0.326	0.003	0.000	0.000	0.000	0.415	0.000	0.198	0.241

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	51	0	0	0	41	0	15	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.48	0.00	0.18	0.00
time (sec)	N/A	0.368	10.012	0.000	0.000	0.000	0.508	0.000	0.197	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	51	0	0	0	44	0	15	0
N.S.	1	1.07	0.48	0.00	0.00	0.00	0.41	0.00	0.14	0.00
time (sec)	N/A	0.428	10.010	0.000	0.000	0.000	0.680	0.000	0.192	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	145	51	0	0	0	44	0	15	0
N.S.	1	1.11	0.39	0.00	0.00	0.00	0.34	0.00	0.11	0.00
time (sec)	N/A	0.497	10.010	0.000	0.000	0.000	0.837	0.000	0.238	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	61	58	81	69	116	92	34	69
N.S.	1	1.04	0.62	0.59	0.82	0.70	1.17	0.93	0.34	0.70
time (sec)	N/A	0.379	0.050	0.530	0.025	0.085	0.941	0.123	0.208	0.497

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	80	50	47	62	58	92	69	34	55
N.S.	1	1.08	0.68	0.64	0.84	0.78	1.24	0.93	0.46	0.74
time (sec)	N/A	0.351	0.040	0.527	0.026	0.078	0.660	0.126	0.236	0.425

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	39	36	47	47	68	58	34	39
N.S.	1	1.07	0.68	0.63	0.82	0.82	1.19	1.02	0.60	0.68
time (sec)	N/A	0.315	0.030	0.506	0.030	0.076	0.387	0.120	0.188	0.400

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	40	27	24	29	35	44	34	34	23
N.S.	1	1.14	0.77	0.69	0.83	1.00	1.26	0.97	0.97	0.66
time (sec)	N/A	0.288	0.026	0.497	0.030	0.077	0.256	0.118	0.221	0.348

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	34	14
N.S.	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	2.12	0.88
time (sec)	N/A	0.239	0.015	0.495	0.029	0.079	0.121	0.122	0.212	0.269

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	81	66	88	75	182	39	199	31	52
N.S.	1	1.16	0.94	1.26	1.07	2.60	0.56	2.84	0.44	0.74
time (sec)	N/A	0.343	0.071	0.522	0.113	0.091	0.568	0.124	0.226	0.385

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	109	81	108	110	272	39	234	33	87
N.S.	1	1.12	0.84	1.11	1.13	2.80	0.40	2.41	0.34	0.90
time (sec)	N/A	0.358	0.153	0.799	0.111	0.107	0.918	0.121	0.234	0.567

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	139	98	127	146	285	39	255	33	123
N.S.	1	1.11	0.78	1.02	1.17	2.28	0.31	2.04	0.26	0.98
time (sec)	N/A	0.392	0.205	0.811	0.120	0.105	2.562	0.124	0.269	0.745

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	79	0	0	0	27	0	34	0
N.S.	1	1.11	0.62	0.00	0.00	0.00	0.21	0.00	0.27	0.00
time (sec)	N/A	0.389	8.159	0.000	0.000	0.000	0.750	0.000	0.208	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	66	0	0	0	27	0	34	0
N.S.	1	1.08	0.63	0.00	0.00	0.00	0.26	0.00	0.33	0.00
time (sec)	N/A	0.347	7.798	0.000	0.000	0.000	0.551	0.000	0.221	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	54	0	0	0	27	0	34	0
N.S.	1	1.06	0.70	0.00	0.00	0.00	0.35	0.00	0.44	0.00
time (sec)	N/A	0.301	7.411	0.000	0.000	0.000	0.464	0.000	0.233	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	27	0	32	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.47	0.00	0.56	0.00
time (sec)	N/A	0.269	7.010	0.000	0.000	0.000	0.451	0.000	0.222	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	0	0	0	31	0	33	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.38	0.00	0.40	0.00
time (sec)	N/A	0.306	10.013	0.000	0.000	0.000	0.575	0.000	0.224	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	54	0	0	0	32	0	33	0
N.S.	1	1.08	0.52	0.00	0.00	0.00	0.31	0.00	0.32	0.00
time (sec)	N/A	0.338	10.012	0.000	0.000	0.000	0.743	0.000	0.248	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	142	54	0	0	0	32	0	33	0
N.S.	1	1.11	0.42	0.00	0.00	0.00	0.25	0.00	0.26	0.00
time (sec)	N/A	0.382	10.013	0.000	0.000	0.000	0.914	0.000	0.238	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	140	98	121	172	292	37	0	34	0
N.S.	1	1.14	0.80	0.98	1.40	2.37	0.30	0.00	0.28	0.00
time (sec)	N/A	0.417	0.628	0.839	0.115	0.111	2.438	0.000	0.201	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	110	82	107	130	280	37	0	34	0
N.S.	1	1.13	0.85	1.10	1.34	2.89	0.38	0.00	0.35	0.00
time (sec)	N/A	0.370	0.490	0.833	0.115	0.103	0.857	0.000	0.199	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	67	95	88	206	37	0	34	0
N.S.	1	1.07	0.91	1.28	1.19	2.78	0.50	0.00	0.46	0.00
time (sec)	N/A	0.310	0.340	0.547	0.106	0.086	0.552	0.000	0.261	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	29	0	30	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.81	0.00	1.88	0.88
time (sec)	N/A	0.221	0.001	0.509	0.032	0.088	0.355	0.000	0.229	0.185

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	31	26	34	37	68	0	33	25
N.S.	1	1.05	0.78	0.65	0.85	0.92	1.70	0.00	0.82	0.62
time (sec)	N/A	0.254	0.238	0.539	0.029	0.086	0.524	0.000	0.205	0.320

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	42	39	53	50	323	0	33	70
N.S.	1	1.16	0.68	0.63	0.85	0.81	5.21	0.00	0.53	1.13
time (sec)	N/A	0.298	0.388	0.563	0.028	0.080	0.849	0.000	0.205	0.494

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	102	53	50	71	61	592	0	33	74
N.S.	1	1.19	0.62	0.58	0.83	0.71	6.88	0.00	0.38	0.86
time (sec)	N/A	0.342	0.492	0.580	0.027	0.089	1.265	0.000	0.220	0.683

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	132	64	61	87	72	928	0	33	93
N.S.	1	1.20	0.58	0.55	0.79	0.65	8.44	0.00	0.30	0.85
time (sec)	N/A	0.394	0.721	0.616	0.035	0.086	1.789	0.000	0.192	0.869

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	143	80	0	0	0	37	0	34	0
N.S.	1	1.11	0.62	0.00	0.00	0.00	0.29	0.00	0.26	0.00
time (sec)	N/A	0.491	8.118	0.000	0.000	0.000	0.782	0.000	0.200	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	113	67	0	0	0	37	0	34	0
N.S.	1	1.08	0.64	0.00	0.00	0.00	0.35	0.00	0.32	0.00
time (sec)	N/A	0.449	7.549	0.000	0.000	0.000	0.574	0.000	0.200	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	60	0	0	0	37	0	34	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.45	0.00	0.41	0.00
time (sec)	N/A	0.370	7.256	0.000	0.000	0.000	0.493	0.000	0.250	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	0	0	0	37	0	34	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.63	0.00	0.58	0.00
time (sec)	N/A	0.340	6.740	0.000	0.000	0.000	0.465	0.000	0.202	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	0	0	0	39	0	33	40
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.49	0.00	0.42	0.51
time (sec)	N/A	0.382	10.008	0.000	0.000	0.000	0.517	0.000	0.193	0.598

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	54	0	0	0	44	0	33	0
N.S.	1	1.04	0.51	0.00	0.00	0.00	0.42	0.00	0.31	0.00
time (sec)	N/A	0.437	10.009	0.000	0.000	0.000	0.639	0.000	0.190	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	139	54	0	0	0	44	0	33	0
N.S.	1	1.08	0.42	0.00	0.00	0.00	0.34	0.00	0.26	0.00
time (sec)	N/A	0.474	10.010	0.000	0.000	0.000	0.883	0.000	0.246	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	169	54	0	0	0	44	0	33	0
N.S.	1	1.10	0.35	0.00	0.00	0.00	0.29	0.00	0.22	0.00
time (sec)	N/A	0.526	10.010	0.000	0.000	0.000	1.121	0.000	0.208	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	27	20	0	0	29	0	15	36
N.S.	1	1.00	0.39	0.29	0.00	0.00	0.42	0.00	0.22	0.52
time (sec)	N/A	0.363	10.012	0.678	0.000	0.000	0.428	0.000	0.220	0.477

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	53	46	42	0	0	31	0	15	36
N.S.	1	1.39	1.21	1.11	0.00	0.00	0.82	0.00	0.39	0.95
time (sec)	N/A	0.333	10.012	0.543	0.000	0.000	0.453	0.000	0.207	0.488

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	0	0	0	39	0	15	37
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.54	0.00	0.21	0.51
time (sec)	N/A	0.386	10.010	0.000	0.000	0.000	0.471	0.000	0.200	0.496

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	27	20	0	0	31	0	15	36
N.S.	1	1.00	0.51	0.38	0.00	0.00	0.58	0.00	0.28	0.68
time (sec)	N/A	0.332	10.008	0.523	0.000	0.000	0.443	0.000	0.197	0.421

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	69	46	23	0	0	36	0	15	36
N.S.	1	1.23	0.82	0.41	0.00	0.00	0.64	0.00	0.27	0.64
time (sec)	N/A	0.362	10.009	0.488	0.000	0.000	0.430	0.000	0.202	0.426

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	0	0	0	31	0	15	38
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.54	0.00	0.26	0.67
time (sec)	N/A	0.355	10.014	0.000	0.000	0.000	0.469	0.000	0.218	0.452

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	29	20	0	0	37	0	34	0
N.S.	1	1.00	0.55	0.38	0.00	0.00	0.70	0.00	0.64	0.00
time (sec)	N/A	0.311	5.998	0.512	0.000	0.000	0.411	0.000	0.207	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	56	71	48	42	0	0	44	0	34	0
N.S.	1	1.27	0.86	0.75	0.00	0.00	0.79	0.00	0.61	0.00
time (sec)	N/A	0.234	5.951	0.528	0.000	0.000	0.425	0.000	0.230	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	37	0	34	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.66	0.00	0.61	0.00
time (sec)	N/A	0.206	6.378	0.000	0.000	0.000	0.438	0.000	0.197	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	29	20	0	0	39	0	36	0
N.S.	1	1.00	0.40	0.28	0.00	0.00	0.54	0.00	0.50	0.00
time (sec)	N/A	0.229	6.069	0.510	0.000	0.000	0.422	0.000	0.212	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	52	48	23	0	0	46	0	36	0
N.S.	1	1.33	1.23	0.59	0.00	0.00	1.18	0.00	0.92	0.00
time (sec)	N/A	0.197	6.152	0.498	0.000	0.000	0.418	0.000	0.223	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	53	0	0	0	39	0	34	0
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.51	0.00	0.45	0.00
time (sec)	N/A	0.227	6.570	0.000	0.000	0.000	0.513	0.000	0.211	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	110	73	59	86	69	134	120	69	68
N.S.	1	1.04	0.69	0.56	0.81	0.65	1.26	1.13	0.65	0.64
time (sec)	N/A	0.225	0.046	0.495	0.031	0.085	0.916	0.137	0.204	0.406

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	88	62	48	68	58	110	94	58	57
N.S.	1	1.05	0.74	0.57	0.81	0.69	1.31	1.12	0.69	0.68
time (sec)	N/A	0.211	0.048	0.494	0.032	0.076	0.633	0.121	0.265	0.358

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	51	37	50	47	87	68	47	46
N.S.	1	1.06	0.82	0.60	0.81	0.76	1.40	1.10	0.76	0.74
time (sec)	N/A	0.193	0.032	0.488	0.025	0.078	0.417	0.120	0.198	0.300

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	40	26	32	36	63	42	36	35
N.S.	1	1.10	1.00	0.65	0.80	0.90	1.58	1.05	0.90	0.88
time (sec)	N/A	0.175	0.026	0.479	0.024	0.075	0.284	0.128	0.189	0.284

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	24	39	15	24	15
N.S.	1	1.00	1.00	0.84	0.79	1.26	2.05	0.79	1.26	0.79
time (sec)	N/A	0.141	0.015	0.484	0.030	0.079	0.113	0.120	0.214	0.270

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	84	69	71	70	97	44	190	38	51
N.S.	1	1.22	1.00	1.03	1.01	1.41	0.64	2.75	0.55	0.74
time (sec)	N/A	0.191	0.056	1.108	0.104	0.116	0.684	0.129	0.238	0.335

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	80	74	81	77	176	42	220	49	58
N.S.	1	1.03	0.95	1.04	0.99	2.26	0.54	2.82	0.63	0.74
time (sec)	N/A	0.186	0.112	0.672	0.114	0.095	0.748	0.121	0.242	0.499

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	114	94	113	138	214	42	251	70	83
N.S.	1	1.09	0.90	1.08	1.31	2.04	0.40	2.39	0.67	0.79
time (sec)	N/A	0.207	0.152	0.574	0.123	0.095	1.238	0.127	0.284	0.642

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	145	98	0	0	0	31	0	78	0
N.S.	1	1.12	0.75	0.00	0.00	0.00	0.24	0.00	0.60	0.00
time (sec)	N/A	0.232	6.342	0.000	0.000	0.000	0.715	0.000	0.273	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	114	66	0	0	0	31	0	57	0
N.S.	1	1.09	0.63	0.00	0.00	0.00	0.30	0.00	0.54	0.00
time (sec)	N/A	0.211	6.137	0.000	0.000	0.000	0.551	0.000	0.226	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	52	0	0	0	31	0	33	0
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.38	0.00	0.40	0.00
time (sec)	N/A	0.181	6.025	0.000	0.000	0.000	0.498	0.000	0.229	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	52	0	0	0	34	0	50	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.41	0.00	0.61	0.00
time (sec)	N/A	0.185	10.011	0.000	0.000	0.000	0.530	0.000	0.230	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	52	0	0	0	36	0	51	0
N.S.	1	1.04	0.50	0.00	0.00	0.00	0.34	0.00	0.49	0.00
time (sec)	N/A	0.207	10.014	0.000	0.000	0.000	0.607	0.000	0.257	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	140	52	0	0	0	36	0	51	0
N.S.	1	1.08	0.40	0.00	0.00	0.00	0.28	0.00	0.39	0.00
time (sec)	N/A	0.231	10.013	0.000	0.000	0.000	0.795	0.000	0.277	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	280	161	185	272	232	41	0	59	0
N.S.	1	1.41	0.81	0.93	1.37	1.17	0.21	0.00	0.30	0.00
time (sec)	N/A	0.435	0.556	0.861	0.115	0.094	1.117	0.000	0.197	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	249	146	159	214	192	41	0	35	0
N.S.	1	1.46	0.86	0.94	1.26	1.13	0.24	0.00	0.21	0.00
time (sec)	N/A	0.378	0.432	0.654	0.117	0.090	0.647	0.000	0.239	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	245	146	165	191	0	42	0	16	41
N.S.	1	1.48	0.88	1.00	1.16	0.00	0.25	0.00	0.10	0.25
time (sec)	N/A	0.383	0.321	0.656	0.111	0.000	0.691	0.000	0.209	0.515

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	27	158	0	27	18
N.S.	1	1.00	1.00	0.86	0.82	1.23	7.18	0.00	1.23	0.82
time (sec)	N/A	0.144	0.133	0.550	0.025	0.101	0.500	0.000	0.255	0.411

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	29	37	38	406	0	38	38
N.S.	1	1.00	0.91	0.63	0.80	0.83	8.83	0.00	0.83	0.83
time (sec)	N/A	0.169	0.147	0.559	0.032	0.100	0.676	0.000	0.192	0.512

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	43	40	55	50	1090	0	50	77
N.S.	1	1.08	0.61	0.56	0.77	0.70	15.35	0.00	0.70	1.08
time (sec)	N/A	0.192	0.167	0.582	0.030	0.100	1.094	0.000	0.220	0.710

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	108	54	51	73	61	1756	0	61	98
N.S.	1	1.12	0.56	0.53	0.76	0.64	18.29	0.00	0.64	1.02
time (sec)	N/A	0.219	0.178	0.620	0.034	0.090	1.448	0.000	0.207	1.033

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	156	173	108	0	0	0	41	0	95	0
N.S.	1	1.11	0.69	0.00	0.00	0.00	0.26	0.00	0.61	0.00
time (sec)	N/A	0.320	6.506	0.000	0.000	0.000	0.730	0.000	0.209	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	142	96	0	0	0	41	0	74	0
N.S.	1	1.08	0.73	0.00	0.00	0.00	0.31	0.00	0.56	0.00
time (sec)	N/A	0.287	6.376	0.000	0.000	0.000	0.648	0.000	0.231	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	111	64	0	0	0	41	0	52	0
N.S.	1	1.05	0.60	0.00	0.00	0.00	0.39	0.00	0.49	0.00
time (sec)	N/A	0.259	6.306	0.000	0.000	0.000	0.505	0.000	0.236	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	47	0	0	0	39	0	29	38
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.47	0.00	0.35	0.46
time (sec)	N/A	0.224	0.004	0.000	0.000	0.000	0.471	0.000	0.225	0.234

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	0	0	0	34	0	51	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.40	0.00	0.60	0.00
time (sec)	N/A	0.234	10.011	0.000	0.000	0.000	0.542	0.000	0.196	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	52	0	0	0	34	0	51	0
N.S.	1	1.05	0.48	0.00	0.00	0.00	0.31	0.00	0.47	0.00
time (sec)	N/A	0.264	10.013	0.000	0.000	0.000	0.650	0.000	0.239	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	144	52	0	0	0	34	0	51	0
N.S.	1	1.08	0.39	0.00	0.00	0.00	0.26	0.00	0.38	0.00
time (sec)	N/A	0.293	10.012	0.000	0.000	0.000	0.840	0.000	0.212	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	175	52	0	0	0	34	0	51	0
N.S.	1	1.11	0.33	0.00	0.00	0.00	0.22	0.00	0.32	0.00
time (sec)	N/A	0.336	10.012	0.000	0.000	0.000	1.030	0.000	0.207	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	110	62	59	86	58	117	109	16	60
N.S.	1	1.04	0.58	0.56	0.81	0.55	1.10	1.03	0.15	0.57
time (sec)	N/A	0.232	0.043	0.562	0.032	0.078	0.861	0.120	0.210	0.401

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	88	51	48	68	47	94	83	16	49
N.S.	1	1.05	0.61	0.57	0.81	0.56	1.12	0.99	0.19	0.58
time (sec)	N/A	0.216	0.033	0.577	0.033	0.084	0.562	0.121	0.226	0.372

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	66	40	37	50	36	70	57	16	38
N.S.	1	1.06	0.65	0.60	0.81	0.58	1.13	0.92	0.26	0.61
time (sec)	N/A	0.199	0.032	0.572	0.027	0.079	0.422	0.116	0.198	0.378

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	29	26	32	25	46	31	16	27
N.S.	1	1.10	0.72	0.65	0.80	0.62	1.15	0.78	0.40	0.68
time (sec)	N/A	0.182	0.029	0.585	0.026	0.082	0.261	0.123	0.218	0.376

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	24	15	16	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.26	0.79	0.84	0.79
time (sec)	N/A	0.147	0.015	0.581	0.030	0.076	0.100	0.120	0.193	0.362

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	64	60	87	39	192	16	38
N.S.	1	1.00	0.88	1.12	1.05	1.53	0.68	3.37	0.28	0.67
time (sec)	N/A	0.191	0.043	0.659	0.116	0.089	0.528	0.135	0.229	0.414

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	88	81	89	96	199	41	226	16	61
N.S.	1	1.09	1.00	1.10	1.19	2.46	0.51	2.79	0.20	0.75
time (sec)	N/A	0.201	0.108	0.756	0.114	0.101	0.786	0.124	0.209	0.514

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	119	95	113	137	223	41	252	16	86
N.S.	1	1.10	0.88	1.05	1.27	2.06	0.38	2.33	0.15	0.80
time (sec)	N/A	0.219	0.139	0.773	0.109	0.096	1.624	0.120	0.193	0.645

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	151	91	0	0	0	29	0	16	0
N.S.	1	1.14	0.68	0.00	0.00	0.00	0.22	0.00	0.12	0.00
time (sec)	N/A	0.243	6.608	0.000	0.000	0.000	0.740	0.000	0.214	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	80	0	0	0	29	0	16	0
N.S.	1	1.11	0.74	0.00	0.00	0.00	0.27	0.00	0.15	0.00
time (sec)	N/A	0.223	6.257	0.000	0.000	0.000	0.580	0.000	0.233	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	66	0	0	0	29	0	16	0
N.S.	1	1.05	0.78	0.00	0.00	0.00	0.34	0.00	0.19	0.00
time (sec)	N/A	0.197	6.072	0.000	0.000	0.000	0.500	0.000	0.202	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	0	0	0	29	0	14	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.49	0.00	0.24	0.00
time (sec)	N/A	0.182	5.975	0.000	0.000	0.000	0.433	0.000	0.208	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	0	0	0	32	0	16	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.38	0.00	0.19	0.00
time (sec)	N/A	0.199	10.011	0.000	0.000	0.000	0.481	0.000	0.243	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	116	52	0	0	0	34	0	16	0
N.S.	1	1.07	0.48	0.00	0.00	0.00	0.31	0.00	0.15	0.00
time (sec)	N/A	0.212	10.012	0.000	0.000	0.000	0.615	0.000	0.204	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	147	52	0	0	0	34	0	16	0
N.S.	1	1.11	0.39	0.00	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.246	10.013	0.000	0.000	0.000	0.791	0.000	0.209	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	274	159	200	265	238	39	0	16	0
N.S.	1	1.37	0.80	1.00	1.32	1.19	0.20	0.00	0.08	0.00
time (sec)	N/A	0.444	0.617	0.932	0.114	0.100	1.512	0.000	0.227	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	243	144	173	219	214	39	0	16	0
N.S.	1	1.42	0.84	1.01	1.28	1.25	0.23	0.00	0.09	0.00
time (sec)	N/A	0.385	0.519	0.812	0.112	0.090	0.673	0.000	0.218	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	214	173	138	176	146	37	0	12	38
N.S.	1	1.45	1.17	0.93	1.19	0.99	0.25	0.00	0.08	0.26
time (sec)	N/A	0.344	0.048	0.660	0.111	0.083	0.506	0.000	0.201	0.301

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	78	0	16	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	3.55	0.00	0.73	0.82
time (sec)	N/A	0.150	0.196	0.681	0.025	0.077	0.474	0.000	0.210	0.286

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	29	37	28	303	0	16	38
N.S.	1	1.00	0.70	0.63	0.80	0.61	6.59	0.00	0.35	0.83
time (sec)	N/A	0.176	0.230	0.706	0.025	0.077	0.595	0.000	0.227	0.414

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	43	40	55	39	1068	0	16	59
N.S.	1	1.08	0.61	0.56	0.77	0.55	15.04	0.00	0.23	0.83
time (sec)	N/A	0.203	0.249	0.628	0.030	0.076	0.928	0.000	0.208	0.482

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	108	54	51	73	50	1821	0	16	80
N.S.	1	1.12	0.56	0.53	0.76	0.52	18.97	0.00	0.17	0.83
time (sec)	N/A	0.230	0.284	0.731	0.036	0.081	1.329	0.000	0.191	0.566

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	139	65	62	91	61	2788	0	16	101
N.S.	1	1.15	0.54	0.51	0.75	0.50	23.04	0.00	0.13	0.83
time (sec)	N/A	0.255	0.317	0.701	0.032	0.091	1.851	0.000	0.204	0.678

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	148	80	0	0	0	39	0	16	0
N.S.	1	1.10	0.60	0.00	0.00	0.00	0.29	0.00	0.12	0.00
time (sec)	N/A	0.301	6.170	0.000	0.000	0.000	0.589	0.000	0.219	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	117	66	0	0	0	39	0	16	0
N.S.	1	1.07	0.61	0.00	0.00	0.00	0.36	0.00	0.15	0.00
time (sec)	N/A	0.282	6.275	0.000	0.000	0.000	0.485	0.000	0.210	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	39	0	16	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.45	0.00	0.19	0.00
time (sec)	N/A	0.237	6.084	0.000	0.000	0.000	0.437	0.000	0.186	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	0	0	0	31	0	16	41
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.51	0.00	0.26	0.67
time (sec)	N/A	0.204	10.015	0.000	0.000	0.000	0.544	0.000	0.193	0.550

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	52	0	0	0	34	0	16	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.40	0.00	0.19	0.00
time (sec)	N/A	0.237	10.011	0.000	0.000	0.000	0.579	0.000	0.204	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	117	52	0	0	0	31	0	16	0
N.S.	1	1.07	0.48	0.00	0.00	0.00	0.28	0.00	0.15	0.00
time (sec)	N/A	0.272	10.010	0.000	0.000	0.000	0.676	0.000	0.216	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	148	52	0	0	0	34	0	16	0
N.S.	1	1.10	0.39	0.00	0.00	0.00	0.25	0.00	0.12	0.00
time (sec)	N/A	0.310	10.010	0.000	0.000	0.000	0.898	0.000	0.210	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	62	59	86	58	117	113	16	60
N.S.	1	1.04	0.60	0.57	0.83	0.56	1.12	1.09	0.15	0.58
time (sec)	N/A	0.238	0.039	0.667	0.029	0.077	0.968	0.125	0.198	0.406

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	86	51	48	68	47	94	87	16	49
N.S.	1	1.05	0.62	0.59	0.83	0.57	1.15	1.06	0.20	0.60
time (sec)	N/A	0.219	0.035	0.754	0.025	0.076	0.606	0.129	0.280	0.372

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	40	37	50	36	70	61	16	38
N.S.	1	1.07	0.67	0.62	0.83	0.60	1.17	1.02	0.27	0.63
time (sec)	N/A	0.207	0.032	0.550	0.036	0.083	0.390	0.121	0.228	0.349

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	29	25	32	24	46	32	16	24
N.S.	1	1.11	0.76	0.66	0.84	0.63	1.21	0.84	0.42	0.63
time (sec)	N/A	0.190	0.026	0.559	0.029	0.070	0.252	0.116	0.224	0.323

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	22	15	15	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.29	0.88	0.88	0.88
time (sec)	N/A	0.152	0.018	0.546	0.026	0.088	0.095	0.118	0.245	0.303

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	64	48	62	60	108	42	192	16	36
N.S.	1	1.12	0.84	1.09	1.05	1.89	0.74	3.37	0.28	0.63
time (sec)	N/A	0.196	0.052	0.590	0.106	0.091	0.528	0.122	0.257	0.328

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	89	81	89	98	197	39	228	16	61
N.S.	1	1.10	1.00	1.10	1.21	2.43	0.48	2.81	0.20	0.75
time (sec)	N/A	0.204	0.081	0.692	0.108	0.090	0.803	0.127	0.222	0.487

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	95	113	141	223	42	252	16	86
N.S.	1	1.11	0.88	1.05	1.31	2.06	0.39	2.33	0.15	0.80
time (sec)	N/A	0.223	0.144	0.648	0.110	0.094	1.662	0.121	0.220	0.582

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	151	92	0	0	0	29	0	16	0
N.S.	1	1.14	0.69	0.00	0.00	0.00	0.22	0.00	0.12	0.00
time (sec)	N/A	0.241	7.416	0.000	0.000	0.000	0.768	0.000	0.227	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	79	0	0	0	29	0	16	0
N.S.	1	1.11	0.73	0.00	0.00	0.00	0.27	0.00	0.15	0.00
time (sec)	N/A	0.225	7.057	0.000	0.000	0.000	0.574	0.000	0.223	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	66	0	0	0	29	0	16	0
N.S.	1	1.05	0.78	0.00	0.00	0.00	0.34	0.00	0.19	0.00
time (sec)	N/A	0.195	6.710	0.000	0.000	0.000	0.485	0.000	0.200	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	52	0	0	0	29	0	14	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.49	0.00	0.24	0.00
time (sec)	N/A	0.171	6.228	0.000	0.000	0.000	0.446	0.000	0.215	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	84	52	0	0	0	32	0	16	0
N.S.	1	0.99	0.61	0.00	0.00	0.00	0.38	0.00	0.19	0.00
time (sec)	N/A	0.189	10.012	0.000	0.000	0.000	0.545	0.000	0.255	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	115	52	0	0	0	34	0	16	0
N.S.	1	1.06	0.48	0.00	0.00	0.00	0.31	0.00	0.15	0.00
time (sec)	N/A	0.213	10.013	0.000	0.000	0.000	0.704	0.000	0.269	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	146	52	0	0	0	34	0	16	0
N.S.	1	1.10	0.39	0.00	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.237	10.014	0.000	0.000	0.000	0.845	0.000	0.229	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	286	161	202	274	240	39	0	16	0
N.S.	1	1.42	0.80	1.00	1.36	1.19	0.19	0.00	0.08	0.00
time (sec)	N/A	0.436	0.571	1.287	0.118	0.101	1.641	0.000	0.203	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	255	148	175	222	214	39	0	16	0
N.S.	1	1.47	0.86	1.01	1.28	1.24	0.23	0.00	0.09	0.00
time (sec)	N/A	0.396	0.514	0.766	0.149	0.090	0.718	0.000	0.227	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	224	116	138	176	146	39	0	16	0
N.S.	1	1.51	0.78	0.93	1.19	0.99	0.26	0.00	0.11	0.00
time (sec)	N/A	0.345	0.361	0.678	0.124	0.091	0.521	0.000	0.237	0.000

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	80	0	16	18
N.S.	1	1.00	1.00	0.95	0.90	0.90	4.00	0.00	0.80	0.90
time (sec)	N/A	0.148	0.215	0.585	0.050	0.086	0.414	0.000	0.232	0.277

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	32	27	36	26	311	0	16	26
N.S.	1	1.00	0.70	0.59	0.78	0.57	6.76	0.00	0.35	0.57
time (sec)	N/A	0.176	0.249	0.604	0.032	0.079	0.587	0.000	0.264	0.361

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	77	43	40	55	39	1110	0	16	39
N.S.	1	1.08	0.61	0.56	0.77	0.55	15.63	0.00	0.23	0.55
time (sec)	N/A	0.201	0.322	0.623	0.031	0.081	0.963	0.000	0.245	0.463

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	108	54	51	73	50	1928	0	16	80
N.S.	1	1.12	0.56	0.53	0.76	0.52	20.08	0.00	0.17	0.83
time (sec)	N/A	0.229	0.391	0.642	0.031	0.075	1.298	0.000	0.209	0.564

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	148	91	0	0	0	39	0	16	0
N.S.	1	1.10	0.68	0.00	0.00	0.00	0.29	0.00	0.12	0.00
time (sec)	N/A	0.300	7.019	0.000	0.000	0.000	0.733	0.000	0.203	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	117	80	0	0	0	39	0	16	0
N.S.	1	1.07	0.73	0.00	0.00	0.00	0.36	0.00	0.15	0.00
time (sec)	N/A	0.274	6.735	0.000	0.000	0.000	0.530	0.000	0.220	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	64	0	0	0	39	0	16	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.45	0.00	0.19	0.00
time (sec)	N/A	0.234	6.323	0.000	0.000	0.000	0.460	0.000	0.239	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	47	0	0	0	37	0	12	38
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.59	0.00	0.19	0.60
time (sec)	N/A	0.216	0.004	0.000	0.000	0.000	0.441	0.000	0.275	0.291

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	0	0	0	34	0	16	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.39	0.00	0.18	0.00
time (sec)	N/A	0.243	10.013	0.000	0.000	0.000	0.572	0.000	0.202	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	119	52	0	0	0	31	0	16	0
N.S.	1	1.07	0.47	0.00	0.00	0.00	0.28	0.00	0.14	0.00
time (sec)	N/A	0.266	10.011	0.000	0.000	0.000	0.684	0.000	0.208	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	136	150	52	0	0	0	34	0	16	0
N.S.	1	1.10	0.38	0.00	0.00	0.00	0.25	0.00	0.12	0.00
time (sec)	N/A	0.298	10.013	0.000	0.000	0.000	0.955	0.000	0.212	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	0	39	0	37	0
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.48	0.00	0.46	0.00
time (sec)	N/A	0.244	6.937	0.000	0.000	0.000	0.472	0.000	0.196	0.000

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	47	40	42	47	58	418	51	52	68
N.S.	1	0.98	0.83	0.88	0.98	1.21	8.71	1.06	1.08	1.42
time (sec)	N/A	0.195	0.051	0.595	0.039	0.090	1.766	0.126	0.209	0.324

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	21	25	104	21	26	21
N.S.	1	1.00	1.00	0.96	0.91	1.09	4.52	0.91	1.13	0.91
time (sec)	N/A	0.153	0.029	0.540	0.031	0.091	0.570	0.120	0.241	0.298

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	0	39	0	42	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	1.02	0.00
time (sec)	N/A	0.171	0.043	0.000	0.000	0.000	2.604	0.000	0.235	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [101] had the largest ratio of [1.1111100000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.06	11	0.182
3	A	2	2	1.00	11	0.182
4	A	2	2	1.00	11	0.182
5	A	2	2	1.00	11	0.182
6	A	2	2	1.00	11	0.182
7	A	2	2	1.00	11	0.182
8	A	2	2	1.00	11	0.182
9	A	2	2	1.00	11	0.182
10	A	2	2	1.00	9	0.222
11	A	1	1	1.00	7	0.143
12	A	2	2	1.00	11	0.182
13	A	2	2	1.00	11	0.182
14	A	2	2	1.00	11	0.182
15	A	2	2	1.00	11	0.182
16	A	2	2	1.00	11	0.182
17	A	2	2	1.00	11	0.182
18	A	2	2	1.00	11	0.182
19	A	4	3	1.13	13	0.231
20	A	1	1	1.00	13	0.077
21	A	4	3	1.15	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	1.04	13	0.231
23	A	4	3	1.15	13	0.231
24	A	1	1	1.00	13	0.077
25	A	4	3	1.13	13	0.231
26	A	2	2	1.00	13	0.154
27	A	2	2	1.00	13	0.154
28	A	2	2	1.00	11	0.182
29	A	2	2	1.00	9	0.222
30	A	2	2	1.00	13	0.154
31	A	2	2	1.00	13	0.154
32	A	2	2	1.00	13	0.154
33	A	4	3	1.12	13	0.231
34	A	1	1	1.00	13	0.077
35	A	4	3	1.10	13	0.231
36	A	4	3	1.05	13	0.231
37	A	4	3	1.02	13	0.231
38	A	4	3	1.10	13	0.231
39	A	1	1	1.00	13	0.077
40	A	4	3	1.10	13	0.231
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	13	0.154
43	A	2	2	1.00	11	0.182
44	A	2	2	1.00	9	0.222
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	2	2	1.00	13	0.154
48	A	2	2	1.00	13	0.154
49	A	2	2	1.00	13	0.154
50	A	2	2	1.00	13	0.154
51	A	4	3	0.98	13	0.231
52	A	4	3	0.96	13	0.231
53	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	5	4	1.18	13	0.308
55	A	4	3	1.03	13	0.231
56	A	4	3	0.98	13	0.231
57	A	4	3	0.98	13	0.231
58	A	3	2	1.00	11	0.182
59	A	4	3	1.00	13	0.231
60	A	5	4	1.12	13	0.308
61	A	10	9	1.51	13	0.692
62	A	10	9	1.53	13	0.692
63	A	9	8	1.49	13	0.615
64	A	9	8	1.49	9	0.889
65	A	10	9	1.51	13	0.692
66	A	10	9	1.51	13	0.692
67	A	4	3	0.93	13	0.231
68	A	4	3	0.94	13	0.231
69	A	1	1	1.00	13	0.077
70	A	4	3	1.03	13	0.231
71	A	4	3	1.00	13	0.231
72	A	5	4	1.08	13	0.308
73	A	5	4	1.10	13	0.308
74	A	4	3	1.08	13	0.231
75	A	4	3	1.08	11	0.273
76	A	5	4	1.12	13	0.308
77	A	6	5	1.20	13	0.385
78	A	11	10	1.49	13	0.769
79	A	11	10	1.52	13	0.769
80	A	10	9	1.48	13	0.692
81	A	10	9	1.49	13	0.692
82	A	10	9	1.48	13	0.692
83	A	10	9	1.49	9	1.000
84	A	11	10	1.49	13	0.769
85	A	11	10	1.51	13	0.769

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	0.95	13	0.231
87	A	4	3	0.98	13	0.231
88	A	1	1	1.00	13	0.077
89	A	1	1	1.00	13	0.077
90	A	4	3	1.02	13	0.231
91	A	4	3	1.00	13	0.231
92	A	5	4	1.18	13	0.308
93	A	5	4	1.13	13	0.308
94	A	5	4	1.18	11	0.364
95	A	6	5	1.18	13	0.385
96	A	11	10	1.48	13	0.769
97	A	11	10	1.48	13	0.769
98	A	11	10	1.45	13	0.769
99	A	11	10	1.46	13	0.769
100	A	11	10	1.48	13	0.769
101	A	11	10	1.49	9	1.111
102	A	12	11	1.48	13	0.846
103	A	3	3	1.00	12	0.250
104	A	3	3	1.00	11	0.273
105	A	3	2	1.00	14	0.143
106	A	3	2	1.00	13	0.154
107	A	3	3	1.00	16	0.188
108	A	3	3	1.00	15	0.200
109	A	1	1	1.00	16	0.062
110	A	1	1	1.00	15	0.067
111	A	9	8	1.41	11	0.727
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	13	0.154
114	A	2	2	1.00	13	0.154
115	A	2	2	1.00	13	0.154
116	A	2	2	1.00	13	0.154
117	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	13	0.154
119	A	2	2	1.00	15	0.133
120	A	2	2	1.00	15	0.133
121	A	2	2	1.00	15	0.133
122	A	2	2	1.00	15	0.133
123	A	2	2	1.00	15	0.133
124	A	2	2	1.00	15	0.133
125	A	2	2	1.00	15	0.133
126	A	2	2	1.00	15	0.133
127	A	2	2	1.00	15	0.133
128	A	2	2	1.00	15	0.133
129	A	2	2	1.00	15	0.133
130	A	2	2	1.00	15	0.133
131	A	2	2	1.00	15	0.133
132	A	2	2	1.00	15	0.133
133	A	15	14	1.44	15	0.933
134	A	15	14	1.46	15	0.933
135	A	14	13	1.42	15	0.867
136	A	14	13	1.44	15	0.867
137	A	14	13	1.44	15	0.867
138	A	14	13	1.46	15	0.867
139	A	15	14	1.42	15	0.933
140	A	15	14	1.44	15	0.933
141	A	15	14	1.41	15	0.933
142	A	15	14	1.43	15	0.933
143	A	15	14	1.43	15	0.933
144	A	15	14	1.44	15	0.933
145	A	15	14	1.41	15	0.933
146	A	15	14	1.43	15	0.933
147	A	15	14	1.43	15	0.933
148	A	15	14	1.44	15	0.933
149	A	16	15	1.41	15	1.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	16	15	1.44	15	1.000
151	A	16	15	1.39	15	1.000
152	A	16	15	1.41	15	1.000
153	A	16	15	1.41	15	1.000
154	A	16	15	1.42	15	1.000
155	A	16	15	1.41	15	1.000
156	A	16	15	1.42	15	1.000
157	A	16	15	1.42	15	1.000
158	A	16	15	1.44	15	1.000
159	A	2	2	1.00	15	0.133
160	A	2	2	1.00	15	0.133
161	A	2	2	1.00	13	0.154
162	A	1	1	1.00	15	0.067
163	A	1	1	1.00	15	0.067
164	A	4	3	1.06	16	0.188
165	A	4	3	1.10	16	0.188
166	A	1	1	1.00	16	0.062
167	A	5	4	1.00	16	0.250
168	A	5	4	0.98	16	0.250
169	A	6	5	1.04	16	0.312
170	A	6	5	1.09	16	0.312
171	A	5	4	1.08	14	0.286
172	A	5	4	1.00	16	0.250
173	A	1	1	1.00	16	0.062
174	A	2	2	1.00	16	0.125
175	A	3	3	1.08	16	0.188
176	A	4	4	1.05	16	0.250
177	A	3	3	1.00	12	0.250
178	A	3	3	1.00	16	0.188
179	A	4	4	1.05	16	0.250
180	A	8	8	1.01	16	0.500
181	A	8	8	1.02	16	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	A	9	9	1.03	16	0.562
183	A	4	3	1.06	16	0.188
184	A	4	3	1.10	16	0.188
185	A	1	1	1.00	16	0.062
186	A	6	5	1.03	16	0.312
187	A	6	5	1.01	16	0.312
188	A	6	5	1.01	16	0.312
189	A	7	6	1.10	16	0.375
190	A	6	5	1.09	14	0.357
191	A	6	5	1.05	16	0.312
192	A	6	5	1.03	16	0.312
193	A	1	1	1.00	16	0.062
194	A	2	2	1.00	16	0.125
195	A	3	3	1.08	16	0.188
196	A	5	5	1.07	16	0.312
197	A	4	4	1.05	12	0.333
198	A	4	4	1.02	16	0.250
199	A	4	4	1.04	16	0.250
200	A	9	9	1.04	16	0.562
201	A	9	9	1.02	16	0.562
202	A	9	9	1.01	16	0.562
203	A	5	4	1.06	11	0.364
204	A	4	3	1.03	16	0.188
205	A	4	3	1.05	16	0.188
206	A	1	1	1.00	16	0.062
207	A	4	3	1.00	16	0.188
208	A	5	4	1.00	16	0.250
209	A	5	4	1.07	16	0.250
210	A	4	3	1.00	14	0.214
211	A	1	1	1.00	16	0.062
212	A	2	2	1.00	16	0.125
213	A	3	3	1.08	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	4	4	1.08	16	0.250
215	A	3	3	1.00	16	0.188
216	A	2	2	1.00	12	0.167
217	A	3	3	1.00	16	0.188
218	A	4	4	1.08	16	0.250
219	A	9	9	1.08	16	0.562
220	A	8	8	1.03	16	0.500
221	A	7	7	1.00	16	0.438
222	A	8	8	1.05	16	0.500
223	A	9	9	1.05	16	0.562
224	A	4	3	1.05	16	0.188
225	A	4	3	1.00	16	0.188
226	A	1	1	1.00	16	0.062
227	A	5	4	1.00	16	0.250
228	A	6	5	1.07	16	0.312
229	A	6	5	1.09	16	0.312
230	A	5	4	0.98	16	0.250
231	A	1	1	1.00	14	0.071
232	A	2	2	1.00	16	0.125
233	A	3	3	1.09	16	0.188
234	A	4	4	1.13	16	0.250
235	A	4	4	1.08	16	0.250
236	A	3	3	1.00	16	0.188
237	A	3	3	1.00	12	0.250
238	A	4	4	1.08	16	0.250
239	A	5	5	1.13	16	0.312
240	A	9	9	1.08	16	0.562
241	A	8	8	1.02	16	0.500
242	A	8	8	1.02	16	0.500
243	A	9	9	1.05	16	0.562
244	A	10	10	1.09	16	0.625
245	A	4	3	1.04	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	4	3	1.06	15	0.200
247	A	1	1	1.00	15	0.067
248	A	4	3	1.00	15	0.200
249	A	5	4	1.00	15	0.267
250	A	5	4	1.20	15	0.267
251	A	4	3	1.15	15	0.200
252	A	3	2	1.00	13	0.154
253	A	1	1	1.00	15	0.067
254	A	2	2	1.00	15	0.133
255	A	3	3	1.09	15	0.200
256	A	3	3	1.12	15	0.200
257	A	2	2	1.00	15	0.133
258	A	1	1	1.00	11	0.091
259	A	2	2	1.00	15	0.133
260	A	3	3	1.11	15	0.200
261	A	6	6	1.08	15	0.400
262	A	5	5	0.97	15	0.333
263	A	4	4	1.00	15	0.267
264	A	5	5	1.00	15	0.333
265	A	6	6	0.94	15	0.400
266	A	4	3	1.10	15	0.200
267	A	4	3	1.00	15	0.200
268	A	1	1	1.00	15	0.067
269	A	5	4	1.00	15	0.267
270	A	6	5	1.06	15	0.333
271	A	5	4	1.09	15	0.267
272	A	4	3	0.96	15	0.200
273	A	1	1	1.00	13	0.077
274	A	2	2	0.97	15	0.133
275	A	3	3	1.04	15	0.200
276	A	4	4	1.16	15	0.267
277	A	3	3	1.12	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	2	2	1.00	15	0.133
279	A	2	2	1.00	11	0.182
280	A	3	3	1.11	15	0.200
281	A	4	4	1.16	15	0.267
282	A	7	7	1.13	15	0.467
283	A	6	6	1.08	15	0.400
284	A	5	5	0.97	15	0.333
285	A	5	5	0.97	15	0.333
286	A	6	6	0.94	15	0.400
287	A	7	7	1.03	15	0.467
288	A	4	3	1.05	13	0.231
289	A	4	3	1.07	13	0.231
290	A	1	1	1.00	13	0.077
291	A	4	3	1.00	13	0.231
292	A	5	4	0.90	13	0.308
293	A	6	5	1.04	13	0.385
294	A	6	5	1.18	13	0.385
295	A	5	4	1.11	13	0.308
296	A	4	3	1.00	11	0.273
297	A	1	1	1.00	13	0.077
298	A	2	2	1.00	13	0.154
299	A	3	3	1.10	13	0.231
300	A	3	3	1.58	13	0.231
301	A	2	2	1.67	13	0.154
302	B	1	1	2.16	9	0.111
303	A	2	2	1.64	13	0.154
304	A	3	3	1.56	13	0.231
305	A	5	5	1.89	13	0.385
306	B	4	4	2.01	13	0.308
307	B	3	3	2.42	13	0.231
308	B	4	4	2.15	13	0.308
309	A	5	5	1.81	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	5	5	1.00	15	0.333
311	A	5	5	1.00	16	0.312
312	A	4	3	1.07	15	0.200
313	A	4	3	1.11	15	0.200
314	A	1	1	1.00	15	0.067
315	A	5	4	1.00	15	0.267
316	A	5	4	1.00	15	0.267
317	A	6	5	1.03	15	0.333
318	A	6	5	1.09	15	0.333
319	A	5	4	1.08	13	0.308
320	A	5	4	0.98	15	0.267
321	A	1	1	1.00	15	0.067
322	A	2	2	1.00	15	0.133
323	A	3	3	1.09	15	0.200
324	A	3	3	1.04	15	0.200
325	A	2	2	1.00	11	0.182
326	A	2	2	1.00	15	0.133
327	A	3	3	1.04	15	0.200
328	A	5	5	1.01	15	0.333
329	A	5	5	1.04	15	0.333
330	A	6	6	1.02	15	0.400
331	A	4	3	1.07	15	0.200
332	A	4	3	1.11	15	0.200
333	A	1	1	1.00	15	0.067
334	A	6	5	1.03	15	0.333
335	A	6	5	1.02	15	0.333
336	A	6	5	1.03	15	0.333
337	A	7	6	1.11	15	0.400
338	A	6	5	1.10	13	0.385
339	A	6	5	1.06	15	0.333
340	A	6	5	1.00	15	0.333
341	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	2	2	1.00	15	0.133
343	A	3	3	1.09	15	0.200
344	A	4	4	1.05	15	0.267
345	A	3	3	1.04	11	0.273
346	A	3	3	1.02	15	0.200
347	A	3	3	1.03	15	0.200
348	A	6	6	1.02	15	0.400
349	A	6	6	1.02	15	0.400
350	A	6	6	1.02	15	0.400
351	A	4	3	1.13	15	0.200
352	A	1	1	1.00	13	0.077
353	A	4	3	1.10	13	0.231
354	A	4	3	1.03	15	0.200
355	A	4	3	1.05	15	0.200
356	A	1	1	1.00	15	0.067
357	A	4	3	1.00	15	0.200
358	A	5	4	0.98	15	0.267
359	A	5	4	1.08	15	0.267
360	A	4	3	1.00	13	0.231
361	A	1	1	1.00	15	0.067
362	A	2	2	1.00	15	0.133
363	A	3	3	1.09	15	0.200
364	A	3	3	1.06	15	0.200
365	A	2	2	1.00	15	0.133
366	A	1	1	1.00	11	0.091
367	A	2	2	1.00	15	0.133
368	A	3	3	1.06	15	0.200
369	A	6	6	1.05	15	0.400
370	A	5	5	1.03	15	0.333
371	A	4	4	1.01	15	0.267
372	A	5	5	1.03	15	0.333
373	A	6	6	1.03	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	4	3	1.04	15	0.200
375	A	4	3	1.00	15	0.200
376	A	1	1	1.00	15	0.067
377	A	5	4	1.00	15	0.267
378	A	6	5	1.07	15	0.333
379	A	6	5	1.11	15	0.333
380	A	5	4	0.98	15	0.267
381	A	1	1	1.00	13	0.077
382	A	2	2	1.02	15	0.133
383	A	3	3	1.11	15	0.200
384	A	4	4	1.11	15	0.267
385	A	3	3	1.06	15	0.200
386	A	2	2	1.00	15	0.133
387	A	2	2	1.00	11	0.182
388	A	3	3	1.06	15	0.200
389	A	4	4	1.10	15	0.267
390	A	7	7	1.07	15	0.467
391	A	6	6	1.05	15	0.400
392	A	5	5	1.03	15	0.333
393	A	5	5	1.01	15	0.333
394	A	6	6	1.03	15	0.400
395	A	7	7	1.05	15	0.467
396	A	4	3	1.05	13	0.231
397	A	4	3	1.07	13	0.231
398	A	1	1	1.00	13	0.077
399	A	4	3	1.00	13	0.231
400	A	5	4	0.94	13	0.308
401	A	4	3	1.16	13	0.231
402	A	3	2	1.00	11	0.182
403	A	1	1	1.00	13	0.077
404	A	2	2	1.00	13	0.154
405	A	3	3	1.10	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
406	A	3	3	1.07	13	0.231
407	A	2	2	1.00	13	0.154
408	A	1	1	1.00	9	0.111
409	A	2	2	1.00	13	0.154
410	A	3	3	1.07	13	0.231
411	A	5	5	1.04	13	0.385
412	A	4	4	1.00	13	0.308
413	A	3	3	1.00	13	0.231
414	A	4	4	1.00	13	0.308
415	A	5	5	0.99	13	0.385
416	A	4	3	1.05	13	0.231
417	A	4	3	1.00	13	0.231
418	A	1	1	1.00	13	0.077
419	A	5	4	1.00	13	0.308
420	A	6	5	1.07	13	0.385
421	A	5	4	1.12	13	0.308
422	A	4	3	0.92	13	0.231
423	A	1	1	1.00	11	0.091
424	A	2	2	1.00	13	0.154
425	A	3	3	1.06	13	0.231
426	A	4	4	1.11	13	0.308
427	A	3	3	1.07	13	0.231
428	A	2	2	1.00	13	0.154
429	A	2	2	1.00	9	0.222
430	A	3	3	1.07	13	0.231
431	A	4	4	1.11	13	0.308
432	A	6	6	1.06	13	0.462
433	A	5	5	1.04	13	0.385
434	A	4	4	1.00	13	0.308
435	A	4	4	1.00	13	0.308
436	A	5	5	0.99	13	0.385
437	A	6	6	1.02	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	4	3	1.00	11	0.273
439	A	3	2	1.00	11	0.182
440	A	4	3	1.15	13	0.231
441	A	1	1	1.00	13	0.077
442	A	1	1	1.00	13	0.077
443	A	4	3	1.04	15	0.200
444	A	4	3	1.05	15	0.200
445	A	4	3	1.07	15	0.200
446	A	4	3	1.11	15	0.200
447	A	1	1	1.00	15	0.067
448	A	7	6	1.23	15	0.400
449	A	7	6	1.03	15	0.400
450	A	8	7	1.09	15	0.467
451	A	7	6	1.12	15	0.400
452	A	6	5	1.09	15	0.333
453	A	5	4	1.05	13	0.308
454	A	5	4	0.99	15	0.267
455	A	6	5	1.05	15	0.333
456	A	7	6	1.09	15	0.400
457	A	7	6	1.08	15	0.400
458	A	6	5	1.05	15	0.333
459	A	6	5	1.04	15	0.333
460	A	1	1	1.00	15	0.067
461	A	2	2	1.00	15	0.133
462	A	3	3	1.09	15	0.200
463	A	4	4	1.13	15	0.267
464	A	9	8	1.11	15	0.533
465	A	8	7	1.09	15	0.467
466	A	7	6	1.05	15	0.400
467	A	6	5	1.00	11	0.455
468	A	6	5	1.00	15	0.333
469	A	7	6	1.05	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	8	7	1.09	15	0.467
471	A	9	8	1.11	15	0.533
472	A	4	3	1.04	15	0.200
473	A	4	3	1.05	15	0.200
474	A	4	3	1.07	15	0.200
475	A	4	3	1.11	15	0.200
476	A	1	1	1.00	15	0.067
477	A	9	8	1.11	15	0.533
478	A	9	8	1.05	15	0.533
479	A	10	9	1.07	15	0.600
480	A	8	7	1.11	15	0.467
481	A	7	6	1.09	15	0.400
482	A	6	5	1.08	13	0.385
483	A	6	5	1.06	15	0.333
484	A	7	6	1.07	15	0.400
485	A	8	7	1.10	15	0.467
486	A	9	8	1.13	15	0.533
487	A	8	7	1.11	15	0.467
488	A	7	6	1.08	15	0.400
489	A	6	5	1.05	11	0.455
490	A	6	5	1.04	15	0.333
491	A	1	1	1.00	15	0.067
492	A	2	2	1.00	15	0.133
493	A	3	3	1.09	15	0.200
494	A	4	4	1.13	15	0.267
495	A	9	8	1.09	15	0.533
496	A	8	7	1.06	15	0.467
497	A	7	6	1.05	15	0.400
498	A	7	6	1.03	15	0.400
499	A	7	6	1.00	15	0.400
500	A	8	7	1.02	15	0.467
501	A	9	8	1.06	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	4	3	1.04	15	0.200
503	A	4	3	1.05	15	0.200
504	A	4	3	1.07	15	0.200
505	A	4	3	1.11	15	0.200
506	A	1	1	1.00	15	0.067
507	A	8	7	1.19	15	0.467
508	A	8	7	1.16	15	0.467
509	A	8	7	1.02	15	0.467
510	A	8	7	1.12	15	0.467
511	A	7	6	1.10	15	0.400
512	A	6	5	1.09	13	0.385
513	A	6	5	1.07	15	0.333
514	A	6	5	1.04	15	0.333
515	A	7	6	1.07	15	0.400
516	A	8	7	1.10	15	0.467
517	A	9	8	1.11	15	0.533
518	A	8	7	1.09	15	0.467
519	A	7	6	1.05	15	0.400
520	A	7	6	1.07	15	0.400
521	A	7	6	1.05	15	0.400
522	A	1	1	1.00	15	0.067
523	A	2	2	1.00	15	0.133
524	A	3	3	1.09	15	0.200
525	A	4	4	1.13	15	0.267
526	A	10	9	1.12	15	0.600
527	A	9	8	1.10	15	0.533
528	A	8	7	1.07	15	0.467
529	A	7	6	1.05	11	0.545
530	A	7	6	1.05	15	0.400
531	A	7	6	1.05	15	0.400
532	A	8	7	1.06	15	0.467
533	A	9	8	1.09	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
534	A	4	3	1.04	15	0.200
535	A	4	3	1.05	15	0.200
536	A	4	3	1.07	15	0.200
537	A	4	3	1.11	15	0.200
538	A	1	1	1.00	15	0.067
539	A	8	7	1.00	15	0.467
540	A	9	8	1.08	15	0.533
541	A	10	9	1.10	15	0.600
542	A	8	7	1.13	15	0.467
543	A	7	6	1.11	15	0.400
544	A	6	5	1.08	15	0.333
545	A	5	4	1.11	13	0.308
546	A	6	5	1.06	15	0.333
547	A	7	6	1.09	15	0.400
548	A	8	7	1.12	15	0.467
549	A	7	6	1.11	15	0.400
550	A	6	5	1.09	15	0.333
551	A	5	4	1.00	11	0.364
552	A	1	1	1.00	15	0.067
553	A	2	2	1.00	15	0.133
554	A	3	3	1.09	15	0.200
555	A	4	4	1.13	15	0.267
556	A	5	5	1.16	15	0.333
557	A	8	7	1.09	15	0.467
558	A	7	6	1.05	15	0.400
559	A	6	5	1.00	15	0.333
560	A	6	5	1.00	15	0.333
561	A	7	6	1.00	15	0.400
562	A	8	7	1.05	15	0.467
563	A	9	8	1.08	15	0.533
564	A	4	3	1.05	15	0.200
565	A	4	3	1.05	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	4	3	1.09	15	0.200
567	A	4	3	1.11	15	0.200
568	A	1	1	1.00	15	0.067
569	A	6	5	1.11	15	0.333
570	A	7	6	1.10	15	0.400
571	A	8	7	1.12	15	0.467
572	A	7	6	1.14	15	0.400
573	A	6	5	1.12	15	0.333
574	A	5	4	1.05	15	0.267
575	A	4	3	1.00	13	0.231
576	A	5	4	1.00	15	0.267
577	A	6	5	1.08	15	0.333
578	A	7	6	1.11	15	0.400
579	A	7	6	1.10	15	0.400
580	A	6	5	1.09	15	0.333
581	A	5	4	1.00	15	0.267
582	A	1	1	1.00	15	0.067
583	A	2	2	1.00	15	0.133
584	A	3	3	1.09	15	0.200
585	A	4	4	1.13	15	0.267
586	A	8	7	1.11	15	0.467
587	A	7	6	1.08	15	0.400
588	A	6	5	1.00	15	0.333
589	A	5	4	1.00	11	0.364
590	A	6	5	1.00	15	0.333
591	A	7	6	1.07	15	0.400
592	A	8	7	1.11	15	0.467
593	A	4	3	1.04	15	0.200
594	A	4	3	1.08	15	0.200
595	A	4	3	1.07	15	0.200
596	A	4	3	1.14	15	0.200
597	A	1	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	9	8	1.16	15	0.533
599	A	10	9	1.12	15	0.600
600	A	11	10	1.11	15	0.667
601	A	7	6	1.11	15	0.400
602	A	6	5	1.08	15	0.333
603	A	5	4	1.06	15	0.267
604	A	4	3	1.00	13	0.231
605	A	5	4	1.00	15	0.267
606	A	6	5	1.08	15	0.333
607	A	7	6	1.11	15	0.400
608	A	8	7	1.14	15	0.467
609	A	7	6	1.13	15	0.400
610	A	6	5	1.07	15	0.333
611	A	1	1	1.00	11	0.091
612	A	2	2	1.05	15	0.133
613	A	3	3	1.16	15	0.200
614	A	4	4	1.19	15	0.267
615	A	5	5	1.20	15	0.333
616	A	8	7	1.11	15	0.467
617	A	7	6	1.08	15	0.400
618	A	6	5	1.00	15	0.333
619	A	5	4	1.00	15	0.267
620	A	6	5	1.00	15	0.333
621	A	7	6	1.04	15	0.400
622	A	8	7	1.08	15	0.467
623	A	9	8	1.10	15	0.533
624	A	7	6	1.00	15	0.400
625	A	6	5	1.39	15	0.333
626	A	7	6	1.00	15	0.400
627	A	6	5	1.00	15	0.333
628	A	7	6	1.23	15	0.400
629	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	6	5	1.00	15	0.333
631	A	7	6	1.27	15	0.400
632	A	6	5	1.00	15	0.333
633	A	7	6	1.00	15	0.400
634	A	6	5	1.33	15	0.333
635	A	7	6	1.00	15	0.400
636	A	4	3	1.04	16	0.188
637	A	4	3	1.05	16	0.188
638	A	4	3	1.06	16	0.188
639	A	4	3	1.10	16	0.188
640	A	1	1	1.00	16	0.062
641	A	7	6	1.22	16	0.375
642	A	7	6	1.03	16	0.375
643	A	8	7	1.09	16	0.438
644	A	7	6	1.12	16	0.375
645	A	6	5	1.09	16	0.312
646	A	5	4	1.05	14	0.286
647	A	5	4	1.00	16	0.250
648	A	6	5	1.04	16	0.312
649	A	7	6	1.08	16	0.375
650	A	12	11	1.41	16	0.688
651	A	11	10	1.46	16	0.625
652	A	11	10	1.48	16	0.625
653	A	1	1	1.00	16	0.062
654	A	2	2	1.00	16	0.125
655	A	3	3	1.08	16	0.188
656	A	4	4	1.12	16	0.250
657	A	9	8	1.11	16	0.500
658	A	8	7	1.08	16	0.438
659	A	7	6	1.05	16	0.375
660	A	6	5	1.00	12	0.417
661	A	6	5	1.00	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	7	6	1.05	16	0.375
663	A	8	7	1.08	16	0.438
664	A	9	8	1.11	16	0.500
665	A	4	3	1.04	16	0.188
666	A	4	3	1.05	16	0.188
667	A	4	3	1.06	16	0.188
668	A	4	3	1.10	16	0.188
669	A	1	1	1.00	16	0.062
670	A	7	6	1.00	16	0.375
671	A	8	7	1.09	16	0.438
672	A	9	8	1.10	16	0.500
673	A	7	6	1.14	16	0.375
674	A	6	5	1.11	16	0.312
675	A	5	4	1.05	16	0.250
676	A	4	3	1.00	14	0.214
677	A	5	4	1.00	16	0.250
678	A	6	5	1.07	16	0.312
679	A	7	6	1.11	16	0.375
680	A	12	11	1.37	16	0.688
681	A	11	10	1.42	16	0.625
682	A	10	9	1.45	12	0.750
683	A	1	1	1.00	16	0.062
684	A	2	2	1.00	16	0.125
685	A	3	3	1.08	16	0.188
686	A	4	4	1.12	16	0.250
687	A	5	5	1.15	16	0.312
688	A	8	7	1.10	16	0.438
689	A	7	6	1.07	16	0.375
690	A	6	5	1.00	16	0.312
691	A	5	4	1.00	16	0.250
692	A	6	5	1.00	16	0.312
693	A	7	6	1.07	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
694	A	8	7	1.10	16	0.438
695	A	4	3	1.04	16	0.188
696	A	4	3	1.05	16	0.188
697	A	4	3	1.07	16	0.188
698	A	4	3	1.11	16	0.188
699	A	1	1	1.00	16	0.062
700	A	6	5	1.12	16	0.312
701	A	7	6	1.10	16	0.375
702	A	8	7	1.11	16	0.438
703	A	7	6	1.14	16	0.375
704	A	6	5	1.11	16	0.312
705	A	5	4	1.05	16	0.250
706	A	4	3	1.00	14	0.214
707	A	5	4	0.99	16	0.250
708	A	6	5	1.06	16	0.312
709	A	7	6	1.10	16	0.375
710	A	12	11	1.42	16	0.688
711	A	11	10	1.47	16	0.625
712	A	10	9	1.51	16	0.562
713	A	1	1	1.00	16	0.062
714	A	2	2	1.00	16	0.125
715	A	3	3	1.08	16	0.188
716	A	4	4	1.12	16	0.250
717	A	8	7	1.10	16	0.438
718	A	7	6	1.07	16	0.375
719	A	6	5	1.00	16	0.312
720	A	5	4	1.00	12	0.333
721	A	6	5	1.00	16	0.312
722	A	7	6	1.07	16	0.375
723	A	8	7	1.10	16	0.438
724	A	6	5	1.00	16	0.312
725	A	4	3	0.98	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	1	1	1.00	13	0.077
727	A	3	2	1.00	13	0.154

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^7(a+bx^4)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^7\left(a+bx^4\right)\,dx=\frac{ax^8}{8}+\frac{bx^{12}}{12}$$

output
```
1/8*a*x^8+1/12*b*x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^7\left(a+bx^4\right)\,dx=\frac{ax^8}{8}+\frac{bx^{12}}{12}$$

input
```
Integrate[x^7*(a + b*x^4),x]
```

output
```
(a*x^8)/8 + (b*x^12)/12
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^4\right) dx$$

$$\Big\downarrow \ 802$$

$$\int \left(ax^7+bx^{11}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^8}{8}+\frac{bx^{12}}{12}$$

input
```
Int[x^7*(a + b*x^4),x]
```

output
```
(a*x^8)/8 + (b*x^12)/12
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{8}a\,x^{8}+\frac{1}{12}b\,x^{12}$	14
default	$\frac{1}{8}a\,x^{8}+\frac{1}{12}b\,x^{12}$	14
norman	$\frac{1}{8}a\,x^{8}+\frac{1}{12}b\,x^{12}$	14
risch	$\frac{1}{8}a\,x^{8}+\frac{1}{12}b\,x^{12}$	14
parallelrisch	$\frac{1}{8}a\,x^{8}+\frac{1}{12}b\,x^{12}$	14
orering	$\frac{x^{8}\left(2b\,x^{4}+3a\right)}{24}$	16

input
```
int(x^7*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*x^8+1/12*b*x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^7\left(a+bx^4\right)\,dx=\frac{1}{12}\,bx^{12}+\frac{1}{8}\,ax^8$$

input
```
integrate(x^7*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/12*b*x^12 + 1/8*a*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^7(a+bx^4)\,dx=\frac{ax^8}{8}+\frac{bx^{12}}{12}$$

input
```
integrate(x**7*(b*x**4+a),x)
```

output
```
a*x**8/8 + b*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^7(a+bx^4)\,dx=\frac{1}{12}\,bx^{12}+\frac{1}{8}\,ax^8$$

input
```
integrate(x^7*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/12*b*x^12 + 1/8*a*x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^7(a+bx^4)\,dx=\frac{1}{12}\,bx^{12}+\frac{1}{8}\,ax^8$$

input
```
integrate(x^7*(b*x^4+a),x, algorithm="giac")
```

output
```
1/12*b*x^12 + 1/8*a*x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^7 \left(a + bx^4\right) \, dx = \frac{b\,x^{12}}{12} + \frac{a\,x^8}{8}$$

input
```
int(x^7*(a + b*x^4),x)
```

output
```
(a*x^8)/8 + (b*x^12)/12
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^7 \left(a + bx^4\right) \, dx = \frac{x^8 (2b\,x^4 + 3a)}{24}$$

input
```
int(x^7*(b*x^4+a),x)
```

output
```
(x**8*(3*a + 2*b*x**4))/24
```

3.2 $\int x^3(a+bx^4)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x^3(a+bx^4)\,dx=\frac{(a+bx^4)^2}{8b}$$

output
```
1/8*(b*x^4+a)^2/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int x^3(a+bx^4)\,dx=\frac{ax^4}{4}+\frac{bx^8}{8}$$

input
```
Integrate[x^3*(a + b*x^4),x]
```

output
```
(a*x^4)/4 + (b*x^8)/8
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^4)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int (ax^3+bx^7)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4}+\frac{bx^8}{8}$$

input
```
Int[x^3*(a + b*x^4),x]
```

output
```
(a*x^4)/4 + (b*x^8)/8
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{1}{8}b\,x^8+\frac{1}{4}a\,x^4$	14
norman	$\frac{1}{8}b\,x^8+\frac{1}{4}a\,x^4$	14
parallelrisch	$\frac{1}{8}b\,x^8+\frac{1}{4}a\,x^4$	14
default	$\frac{(b\,x^4+a)^2}{8b}$	15
orering	$\frac{x^4(b\,x^4+2a)}{8}$	15
risch	$\frac{b\,x^8}{8}+\frac{a\,x^4}{4}+\frac{a^2}{8b}$	22

input
```
int(x^3*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*b*x^8+1/4*a*x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^3\left(a+bx^4\right)\,dx=\frac{1}{8}\,bx^8+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*b*x^8 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int x^3\left(a+bx^4\right)\,dx=\frac{ax^4}{4}+\frac{bx^8}{8}$$

input
```
integrate(x**3*(b*x**4+a),x)
```

output
```
a*x**4/4 + b*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3\left(a+bx^4\right)\,dx=\frac{\left(bx^4+a\right)^2}{8\,b}$$

input
```
integrate(x^3*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(b*x^4 + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^3\left(a+bx^4\right)\,dx=\frac{1}{8}\,bx^8+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*b*x^8 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int x^3 (a + bx^4) \, dx = \frac{b\,x^8}{8} + \frac{a\,x^4}{4}$$

input
```
int(x^3*(a + b*x^4),x)
```

output
```
(a*x^4)/4 + (b*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3 (a + bx^4) \, dx = \frac{x^4(b\,x^4 + 2a)}{8}$$

input
```
int(x^3*(b*x^4+a),x)
```

output
```
(x**4*(2*a + b*x**4))/8
```

3.3 $\int \frac{a+bx^4}{x} dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^4}{x} dx = \frac{bx^4}{4} + a\log(x)$$

output
```
1/4*b*x^4+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x} dx = \frac{bx^4}{4} + a\log(x)$$

input
```
Integrate[(a + b*x^4)/x,x]
```

output
```
(b*x^4)/4 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x}+bx^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a\log(x)+\frac{bx^4}{4}$$

input
```
Int[(a + b*x^4)/x,x]
```

output
```
(b*x^4)/4 + a*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b\,x^{4}}{4}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^{4}}{4}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^{4}}{4}+a\ln\left(x\right)$	12
parallelrisch	$\frac{b\,x^{4}}{4}+a\ln\left(x\right)$	12

input
```
int((b*x^4+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*x^4+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^4}{x}\,dx=\frac{1}{4}\,bx^4+a\log\left(x\right)$$

input
```
integrate((b*x^4+a)/x,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^4}{x}\,dx=a\log\left(x\right)+\frac{bx^4}{4}$$

input
```
integrate((b*x**4+a)/x,x)
```

output
```
a*log(x) + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^4}{x}\,dx = \frac{1}{4}\,bx^4 + \frac{1}{4}\,a\log\left(x^4\right)$$

input
```
integrate((b*x^4+a)/x,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/4*a*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^4}{x}\,dx = \frac{1}{4}\,bx^4 + \frac{1}{4}\,a\log\left(x^4\right)$$

input
```
integrate((b*x^4+a)/x,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/4*a*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^4}{x}\,dx = \frac{b\,x^4}{4} + a\,\ln\left(x\right)$$

input
```
int((a + b*x^4)/x,x)
```

output
```
(b*x^4)/4 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^4}{x}\,dx=\log(x)\,a+\frac{b\,x^4}{4}$$

input
```
int((b*x^4+a)/x,x)
```

output
```
(4*log(x)*a + b*x**4)/4
```

3.4 $\int \frac{a+bx^4}{x^5} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^4}{x^5} \, dx = -\frac{a}{4x^4} + b\log(x)$$

output
```
-1/4*a/x^4+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^5} \, dx = -\frac{a}{4x^4} + b\log(x)$$

input
```
Integrate[(a + b*x^4)/x^5,x]
```

output
```
-1/4*a/x^4 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^5}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\log(x)-\frac{a}{4x^4}$$

input
```
Int[(a + b*x^4)/x^5,x]
```

output
```
-1/4*a/x^4 + b*Log[x]
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a}{4x^4}+b\ln\left(x\right)$	12
norman	$-\frac{a}{4x^4}+b\ln\left(x\right)$	12
risch	$-\frac{a}{4x^4}+b\ln\left(x\right)$	12
parallelrisch	$\frac{4b\ln(x)x^4-a}{4x^4}$	18

input
```
int((b*x^4+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^4}{x^5}\,dx=\frac{4\,bx^4\log\left(x\right)-a}{4\,x^4}$$

input
```
integrate((b*x^4+a)/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*b*x^4*log(x) - a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^4}{x^5}\,dx=-\frac{a}{4x^4}+b\log\left(x\right)$$

input
```
integrate((b*x**4+a)/x**5,x)
```

output
```
-a/(4*x**4) + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^4}{x^5}\,dx = \frac{1}{4}\,b\log\left(x^4\right) - \frac{a}{4\,x^4}$$

input
```
integrate((b*x^4+a)/x^5,x, algorithm="maxima")
```

output
```
1/4*b*log(x^4) - 1/4*a/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{a+bx^4}{x^5}\,dx = \frac{1}{4}\,b\log\left(x^4\right) - \frac{bx^4+a}{4\,x^4}$$

input
```
integrate((b*x^4+a)/x^5,x, algorithm="giac")
```

output
```
1/4*b*log(x^4) - 1/4*(b*x^4 + a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^4}{x^5}\,dx = b\,\ln\left(x\right) - \frac{a}{4\,x^4}$$

input
```
int((a + b*x^4)/x^5,x)
```

output
```
b*log(x) - a/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^4}{x^5}\,dx = \frac{4\log(x)\,b\,x^4-a}{4x^4}$$

input
```
int((b*x^4+a)/x^5,x)
```

output
```
(4*log(x)*b*x**4 - a)/(4*x**4)
```

3.5 $\int \frac{a+bx^4}{x^9}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^4}{x^9}\,dx = -\frac{a}{8x^8}-\frac{b}{4x^4}$$

output
```
-1/8*a/x^8-1/4*b/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^9}\,dx = -\frac{a}{8x^8}-\frac{b}{4x^4}$$

input
```
Integrate[(a + b*x^4)/x^9,x]
```

output
```
-1/8*a/x^8 - b/(4*x^4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^9} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^9}+\frac{b}{x^5}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{8x^8}-\frac{b}{4x^4}$$

input
```
Int[(a + b*x^4)/x^9,x]
```

output
```
-1/8*a/x^8 - b/(4*x^4)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{2b\,x^4+a}{8x^8}$	14
default	$-\frac{a}{8x^8}-\frac{b}{4x^4}$	14
orering	$-\frac{2b\,x^4+a}{8x^8}$	14
norman	$\frac{-\frac{b\,x^4}{4}-\frac{a}{8}}{x^8}$	15
risch	$\frac{-\frac{b\,x^4}{4}-\frac{a}{8}}{x^8}$	15
parallelrisch	$\frac{-2b\,x^4-a}{8x^8}$	16

input
```
int((b*x^4+a)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*b*x^4+a)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^9}\,dx=-\frac{2\,bx^4+a}{8\,x^8}$$

input
```
integrate((b*x^4+a)/x^9,x, algorithm="fricas")
```

output
```
-1/8*(2*b*x^4 + a)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^4}{x^9}\,dx = \frac{-a-2bx^4}{8x^8}$$

input
```
integrate((b*x**4+a)/x**9,x)
```

output
```
(-a - 2*b*x**4)/(8*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^9}\,dx = -\frac{2\,bx^4+a}{8\,x^8}$$

input
```
integrate((b*x^4+a)/x^9,x, algorithm="maxima")
```

output
```
-1/8*(2*b*x^4 + a)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^9}\,dx = -\frac{2\,bx^4+a}{8\,x^8}$$

input
```
integrate((b*x^4+a)/x^9,x, algorithm="giac")
```

output
```
-1/8*(2*b*x^4 + a)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^9}\,dx = -\frac{2\,b\,x^4+a}{8\,x^8}$$

input
```
int((a + b*x^4)/x^9,x)
```

output
```
-(a + 2*b*x^4)/(8*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^9}\,dx = \frac{-2b\,x^4-a}{8x^8}$$

input
```
int((b*x^4+a)/x^9,x)
```

output
```
( - a - 2*b*x**4)/(8*x**8)
```

3.6 $\int \frac{a+bx^4}{x^{13}} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^4}{x^{13}} dx = -\frac{a}{12x^{12}} - \frac{b}{8x^8}$$

output
```
-1/12*a/x^12-1/8*b/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^{13}} dx = -\frac{a}{12x^{12}} - \frac{b}{8x^8}$$

input
```
Integrate[(a + b*x^4)/x^13,x]
```

output
```
-1/12*a/x^12 - b/(8*x^8)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^{13}}\,dx$$

$$\Big\downarrow \ 802$$

$$\int \left(\frac{a}{x^{13}}+\frac{b}{x^9}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a}{12x^{12}}-\frac{b}{8x^8}$$

input
```
Int[(a + b*x^4)/x^13,x]
```

output
```
-1/12*a/x^12 - b/(8*x^8)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{12x^{12}}-\frac{b}{8x^{8}}$	14
norman	$\frac{-\frac{b\,x^{4}}{8}-\frac{a}{12}}{x^{12}}$	15
risch	$\frac{-\frac{b\,x^{4}}{8}-\frac{a}{12}}{x^{12}}$	15
gosper	$-\frac{3b\,x^{4}+2a}{24x^{12}}$	16
parallelrisch	$\frac{-3b\,x^{4}-2a}{24x^{12}}$	16
orering	$-\frac{3b\,x^{4}+2a}{24x^{12}}$	16

input
```
int((b*x^4+a)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a/x^12-1/8*b/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{13}}\,dx=-\frac{3\,bx^4+2\,a}{24\,x^{12}}$$

input
```
integrate((b*x^4+a)/x^13,x, algorithm="fricas")
```

output
```
-1/24*(3*b*x^4 + 2*a)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{13}}\,dx = \frac{-2a-3bx^4}{24x^{12}}$$

input
```
integrate((b*x**4+a)/x**13,x)
```

output
```
(-2*a - 3*b*x**4)/(24*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{13}}\,dx = -\frac{3\,bx^4+2\,a}{24\,x^{12}}$$

input
```
integrate((b*x^4+a)/x^13,x, algorithm="maxima")
```

output
```
-1/24*(3*b*x^4 + 2*a)/x^12
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{13}}\,dx = -\frac{3\,bx^4+2\,a}{24\,x^{12}}$$

input
```
integrate((b*x^4+a)/x^13,x, algorithm="giac")
```

output
```
-1/24*(3*b*x^4 + 2*a)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{13}}\,dx=-\frac{3\,b\,x^4+2\,a}{24\,x^{12}}$$

input
```
int((a + b*x^4)/x^13,x)
```

output
```
-(2*a + 3*b*x^4)/(24*x^12)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{13}}\,dx=\frac{-3b\,x^4-2a}{24x^{12}}$$

input
```
int((b*x^4+a)/x^13,x)
```

output
```
( - 2*a - 3*b*x**4)/(24*x**12)
```

3.7 $\int x^5(a+bx^4)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^5\left(a+bx^4\right)\,dx=\frac{ax^6}{6}+\frac{bx^{10}}{10}$$

output
```
1/6*a*x^6+1/10*b*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^5\left(a+bx^4\right)\,dx=\frac{ax^6}{6}+\frac{bx^{10}}{10}$$

input
```
Integrate[x^5*(a + b*x^4),x]
```

output
```
(a*x^6)/6 + (b*x^10)/10
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^4\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^5+bx^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^6}{6}+\frac{bx^{10}}{10}$$

input
```
Int[x^5*(a + b*x^4),x]
```

output
```
(a*x^6)/6 + (b*x^10)/10
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{6}a\,x^{6}+\frac{1}{10}b\,x^{10}$	14
default	$\frac{1}{6}a\,x^{6}+\frac{1}{10}b\,x^{10}$	14
norman	$\frac{1}{6}a\,x^{6}+\frac{1}{10}b\,x^{10}$	14
risch	$\frac{1}{6}a\,x^{6}+\frac{1}{10}b\,x^{10}$	14
parallelrisch	$\frac{1}{6}a\,x^{6}+\frac{1}{10}b\,x^{10}$	14
orering	$\frac{x^{6}\left(3b\,x^{4}+5a\right)}{30}$	16

input
```
int(x^5*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+1/10*b*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^5\left(a+bx^4\right)\,dx=\frac{1}{10}\,bx^{10}+\frac{1}{6}\,ax^6$$

input
```
integrate(x^5*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/10*b*x^10 + 1/6*a*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^5 (a + bx^4) \, dx = \frac{ax^6}{6} + \frac{bx^{10}}{10}$$

input
```
integrate(x**5*(b*x**4+a),x)
```

output
```
a*x**6/6 + b*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^5 (a + bx^4) \, dx = \frac{1}{10} bx^{10} + \frac{1}{6} ax^6$$

input
```
integrate(x^5*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/10*b*x^10 + 1/6*a*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^5 (a + bx^4) \, dx = \frac{1}{10} bx^{10} + \frac{1}{6} ax^6$$

input
```
integrate(x^5*(b*x^4+a),x, algorithm="giac")
```

output
```
1/10*b*x^10 + 1/6*a*x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^5(a+bx^4)\,dx=\frac{b\,x^{10}}{10}+\frac{a\,x^6}{6}$$

input
```
int(x^5*(a + b*x^4),x)
```

output
```
(a*x^6)/6 + (b*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^5(a+bx^4)\,dx=\frac{x^6(3b\,x^4+5a)}{30}$$

input
```
int(x^5*(b*x^4+a),x)
```

output
```
(x**6*(5*a + 3*b*x**4))/30
```

3.8 $\int x^4(a+bx^4)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^4(a+bx^4)\,dx=\frac{ax^5}{5}+\frac{bx^9}{9}$$

output
```
1/5*a*x^5+1/9*b*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^4)\,dx=\frac{ax^5}{5}+\frac{bx^9}{9}$$

input
```
Integrate[x^4*(a + b*x^4),x]
```

output
```
(a*x^5)/5 + (b*x^9)/9
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^4\right) dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^4+bx^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^5}{5}+\frac{bx^9}{9}$$

input
```
Int[x^4*(a + b*x^4),x]
```

output
```
(a*x^5)/5 + (b*x^9)/9
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{5}a\,x^5+\frac{1}{9}b\,x^9$	14
default	$\frac{1}{5}a\,x^5+\frac{1}{9}b\,x^9$	14
norman	$\frac{1}{5}a\,x^5+\frac{1}{9}b\,x^9$	14
risch	$\frac{1}{5}a\,x^5+\frac{1}{9}b\,x^9$	14
parallelrisch	$\frac{1}{5}a\,x^5+\frac{1}{9}b\,x^9$	14
orering	$\frac{x^5(5b\,x^4+9a)}{45}$	16

input
```
int(x^4*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/9*b*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^4)\,dx=\frac{1}{9}\,bx^9+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/9*b*x^9 + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^4(a+bx^4)\,dx=\frac{ax^5}{5}+\frac{bx^9}{9}$$

input
```
integrate(x**4*(b*x**4+a),x)
```

output
```
a*x**5/5 + b*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^4)\,dx=\frac{1}{9}\,bx^9+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/9*b*x^9 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^4)\,dx=\frac{1}{9}\,bx^9+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*(b*x^4+a),x, algorithm="giac")
```

output
```
1/9*b*x^9 + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4 \left(a+bx^4\right) \, dx = \frac{b\,x^9}{9}+\frac{a\,x^5}{5}$$

input
```
int(x^4*(a + b*x^4),x)
```

output
```
(a*x^5)/5 + (b*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^4 \left(a+bx^4\right) \, dx = \frac{x^5 (5b\,x^4+9a)}{45}$$

input
```
int(x^4*(b*x^4+a),x)
```

output
```
(x**5*(9*a + 5*b*x**4))/45
```

3.9 $\int x^2(a+bx^4)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^2\left(a+bx^4\right)\,dx=\frac{ax^3}{3}+\frac{bx^7}{7}$$

output
```
1/3*a*x^3+1/7*b*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^4\right)\,dx=\frac{ax^3}{3}+\frac{bx^7}{7}$$

input
```
Integrate[x^2*(a + b*x^4),x]
```

output
```
(a*x^3)/3 + (b*x^7)/7
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^4\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^2+bx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^7}{7}$$

input
```
Int[x^2*(a + b*x^4),x]
```

output
```
(a*x^3)/3 + (b*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{3}a\,x^3+\frac{1}{7}b\,x^7$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{7}b\,x^7$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{7}b\,x^7$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{7}b\,x^7$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{7}b\,x^7$	14
orering	$\frac{x^3(3b\,x^4+7a)}{21}$	16

input
```
int(x^2*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/7*b*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx^4)\,dx=\frac{1}{7}\,bx^7+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/7*b*x^7 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2(a+bx^4)\ dx = \frac{ax^3}{3} + \frac{bx^7}{7}$$

input
```
integrate(x**2*(b*x**4+a),x)
```

output
```
a*x**3/3 + b*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx^4)\ dx = \frac{1}{7}bx^7 + \frac{1}{3}ax^3$$

input
```
integrate(x^2*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/7*b*x^7 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx^4)\ dx = \frac{1}{7}bx^7 + \frac{1}{3}ax^3$$

input
```
integrate(x^2*(b*x^4+a),x, algorithm="giac")
```

output
```
1/7*b*x^7 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(a+bx^4\right)\,dx=\frac{b\,x^7}{7}+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a + b*x^4),x)
```

output
```
(a*x^3)/3 + (b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2\left(a+bx^4\right)\,dx=\frac{x^3(3b\,x^4+7a)}{21}$$

input
```
int(x^2*(b*x^4+a),x)
```

output
```
(x**3*(7*a + 3*b*x**4))/21
```

3.10 $\int x(a+bx^4)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int x(a+bx^4)\,dx=\frac{ax^2}{2}+\frac{bx^6}{6}$$

output
```
1/2*a*x^2+1/6*b*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(a+bx^4)\,dx=\frac{ax^2}{2}+\frac{bx^6}{6}$$

input
```
Integrate[x*(a + b*x^4),x]
```

output
```
(a*x^2)/2 + (b*x^6)/6
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^4\right)\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(ax+bx^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^2}{2}+\frac{bx^6}{6}$$

input
```
Int[x*(a + b*x^4),x]
```

output
```
(a*x^2)/2 + (b*x^6)/6
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{2}a\,x^{2}+\frac{1}{6}b\,x^{6}$	14
default	$\frac{1}{2}a\,x^{2}+\frac{1}{6}b\,x^{6}$	14
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{6}b\,x^{6}$	14
risch	$\frac{1}{2}a\,x^{2}+\frac{1}{6}b\,x^{6}$	14
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{6}b\,x^{6}$	14
orering	$\frac{x^{2}\left(b\,x^{4}+3a\right)}{6}$	15

input
```
int(x*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/6*b*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(a+bx^{4}\right)\,dx=\frac{1}{6}\,bx^{6}+\frac{1}{2}\,ax^{2}$$

input
```
integrate(x*(b*x^4+a),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(a+bx^4)\,dx=\frac{ax^2}{2}+\frac{bx^6}{6}$$

input
```
integrate(x*(b*x**4+a),x)
```

output
```
a*x**2/2 + b*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^4)\,dx=\frac{1}{6}\,bx^6+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^4+a),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/2*a*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^4)\,dx=\frac{1}{6}\,bx^6+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^4+a),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^4)\,dx=\frac{b\,x^6}{6}+\frac{a\,x^2}{2}$$

input
```
int(x*(a + b*x^4),x)
```

output
```
(a*x^2)/2 + (b*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int x(a+bx^4)\,dx=\frac{x^2(b\,x^4+3a)}{6}$$

input
```
int(x*(b*x^4+a),x)
```

output
```
(x**2*(3*a + b*x**4))/6
```

3.11 $\int (a+bx^4) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (a+bx^4) \, dx = ax + \frac{bx^5}{5}$$

output
```
a*x+1/5*b*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+bx^4) \, dx = ax + \frac{bx^5}{5}$$

input
```
Integrate[a + b*x^4,x]
```

output
```
a*x + (b*x^5)/5
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{bx^5}{5}$$

input
```
Int[a + b*x^4,x]
```

output
```
a*x + (b*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$ax + \frac{1}{5}b\,x^5$	11
default	$ax + \frac{1}{5}b\,x^5$	11
norman	$ax + \frac{1}{5}b\,x^5$	11
risch	$ax + \frac{1}{5}b\,x^5$	11
parallelrisch	$ax + \frac{1}{5}b\,x^5$	11
parts	$ax + \frac{1}{5}b\,x^5$	11
orering	$\frac{x(b\,x^4+5a)}{5}$	13

input
```
int(b*x^4+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/5*b*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + bx^4)\;dx = \frac{1}{5}\,bx^5 + ax$$

input
```
integrate(b*x^4+a,x, algorithm="fricas")
```

output
```
1/5*b*x^5 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \left(a + bx^4\right) \, dx = ax + \frac{bx^5}{5}$$

input
```
integrate(b*x**4+a,x)
```

output
```
a*x + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a + bx^4\right) \, dx = \frac{1}{5} \, bx^5 + ax$$

input
```
integrate(b*x^4+a,x, algorithm="maxima")
```

output
```
1/5*b*x^5 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a + bx^4\right) \, dx = \frac{1}{5} \, bx^5 + ax$$

input
```
integrate(b*x^4+a,x, algorithm="giac")
```

output
```
1/5*b*x^5 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + bx^4) \, dx = \frac{b\,x^5}{5} + a\,x$$

input
```
int(a + b*x^4,x)
```

output
```
a*x + (b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + bx^4) \, dx = \frac{x(b\,x^4 + 5a)}{5}$$

input
```
int(b*x^4+a,x)
```

output
```
(x*(5*a + b*x**4))/5
```

3.12 $\int \frac{a+bx^4}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{a+bx^4}{x^2} dx = -\frac{a}{x} + \frac{bx^3}{3}$$

output
```
-a/x+1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^2} dx = -\frac{a}{x} + \frac{bx^3}{3}$$

input
```
Integrate[(a + b*x^4)/x^2,x]
```

output
```
-(a/x) + (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^2} \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a}{x^2}+bx^2\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^3}{3}-\frac{a}{x}$$

input
```
Int[(a + b*x^4)/x^2,x]
```

output
```
-(a/x) + (b*x^3)/3
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a}{x}+\frac{b\,x^{3}}{3}$	14
risch	$-\frac{a}{x}+\frac{b\,x^{3}}{3}$	14
norman	$\frac{\frac{b\,x^{4}}{3}-a}{x}$	15
parallelrisch	$\frac{b\,x^{4}-3a}{3x}$	15
gosper	$-\frac{-b\,x^{4}+3a}{3x}$	16
orering	$-\frac{-b\,x^{4}+3a}{3x}$	16

input
```
int((b*x^4+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+1/3*b*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^4}{x^2}\,dx=\frac{bx^4-3\,a}{3\,x}$$

input
```
integrate((b*x^4+a)/x^2,x, algorithm="fricas")
```

output
```
1/3*(b*x^4 - 3*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.53

$$\int \frac{a+bx^4}{x^2}\,dx=-\frac{a}{x}+\frac{bx^3}{3}$$

input
```
integrate((b*x**4+a)/x**2,x)
```

output
```
-a/x + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^2}\,dx=\frac{1}{3}\,bx^3-\frac{a}{x}$$

input
```
integrate((b*x^4+a)/x^2,x, algorithm="maxima")
```

output
```
1/3*b*x^3 - a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^2}\,dx=\frac{1}{3}\,bx^3-\frac{a}{x}$$

input
```
integrate((b*x^4+a)/x^2,x, algorithm="giac")
```

output
```
1/3*b*x^3 - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^2}\,dx=\frac{b\,x^3}{3}-\frac{a}{x}$$

input
```
int((a + b*x^4)/x^2,x)
```

output
```
(b*x^3)/3 - a/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^4}{x^2}\,dx=\frac{b\,x^4-3a}{3x}$$

input
```
int((b*x^4+a)/x^2,x)
```

output
```
( - 3*a + b*x**4)/(3*x)
```

3.13 $\int \frac{a+bx^4}{x^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^4}{x^3} dx = -\frac{a}{2x^2} + \frac{bx^2}{2}$$

output
```
-1/2*a/x^2+1/2*b*x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^3} dx = -\frac{a}{2x^2} + \frac{bx^2}{2}$$

input
```
Integrate[(a + b*x^4)/x^3,x]
```

output
```
-1/2*a/x^2 + (b*x^2)/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^3}+bx\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bx^2}{2}-\frac{a}{2x^2}$$

input
```
Int[(a + b*x^4)/x^3,x]
```

output
```
-1/2*a/x^2 + (b*x^2)/2
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{-b\,x^{4}+a}{2x^{2}}$	14
default	$-\frac{a}{2x^{2}}+\frac{b\,x^{2}}{2}$	14
risch	$-\frac{a}{2x^{2}}+\frac{b\,x^{2}}{2}$	14
orering	$-\frac{-b\,x^{4}+a}{2x^{2}}$	14
norman	$\frac{\frac{b\,x^{4}}{2}-\frac{a}{2}}{x^{2}}$	15
parallelrisch	$\frac{b\,x^{4}-a}{2x^{2}}$	15

input
```
int((b*x^4+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^4+a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^4}{x^3}\,dx=\frac{bx^4-a}{2\,x^2}$$

input
```
integrate((b*x^4+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*x^4 - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{a+bx^4}{x^3}\,dx = -\frac{a}{2x^2} + \frac{bx^2}{2}$$

input
```
integrate((b*x**4+a)/x**3,x)
```

output
```
-a/(2*x**2) + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^3}\,dx = \frac{1}{2}\,bx^2 - \frac{a}{2\,x^2}$$

input
```
integrate((b*x^4+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*b*x^2 - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^3}\,dx = \frac{1}{2}\,bx^2 - \frac{a}{2\,x^2}$$

input
```
integrate((b*x^4+a)/x^3,x, algorithm="giac")
```

output
```
1/2*b*x^2 - 1/2*a/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^3}\,dx = -\frac{a-b\,x^4}{2\,x^2}$$

input
```
int((a + b*x^4)/x^3,x)
```

output
```
-(a - b*x^4)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^4}{x^3}\,dx = \frac{b\,x^4-a}{2x^2}$$

input
```
int((b*x^4+a)/x^3,x)
```

output
```
( - a + b*x**4)/(2*x**2)
```

3.14 $\int \frac{a+bx^4}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{a+bx^4}{x^4} dx = -\frac{a}{3x^3} + bx$$

output
```
-1/3*a/x^3+b*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^4} dx = -\frac{a}{3x^3} + bx$$

input
```
Integrate[(a + b*x^4)/x^4,x]
```

output
```
-1/3*a/x^3 + b*x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^4} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^4}+b\right) dx$$
$$\Big\downarrow \text{2009}$$
$$bx-\frac{a}{3x^3}$$

input
```
Int[(a + b*x^4)/x^4,x]
```

output
```
-1/3*a/x^3 + b*x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a}{3x^3}+bx$	11
risch	$-\frac{a}{3x^3}+bx$	11
gosper	$-\frac{-3b\,x^4+a}{3x^3}$	14
norman	$\frac{b\,x^4-\frac{a}{3}}{x^3}$	14
orering	$-\frac{-3b\,x^4+a}{3x^3}$	14
parallelrisch	$\frac{3b\,x^4-a}{3x^3}$	16

input
```
int((b*x^4+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{a+bx^4}{x^4}\,dx=\frac{3\,bx^4-a}{3\,x^3}$$

input
```
integrate((b*x^4+a)/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b*x^4 - a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{a+bx^4}{x^4}\,dx = -\frac{a}{3x^3} + bx$$

input
```
integrate((b*x**4+a)/x**4,x)
```

output
```
-a/(3*x**3) + b*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^4}{x^4}\,dx = bx - \frac{a}{3\,x^3}$$

input
```
integrate((b*x^4+a)/x^4,x, algorithm="maxima")
```

output
```
b*x - 1/3*a/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^4}{x^4}\,dx = bx - \frac{a}{3\,x^3}$$

input
```
integrate((b*x^4+a)/x^4,x, algorithm="giac")
```

output
```
b*x - 1/3*a/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^4}{x^4}\,dx=b\,x-\frac{a}{3\,x^3}$$

input
```
int((a + b*x^4)/x^4,x)
```

output
```
b*x - a/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{a+bx^4}{x^4}\,dx=\frac{3b\,x^4-a}{3x^3}$$

input
```
int((b*x^4+a)/x^4,x)
```

output
```
( - a + 3*b*x**4)/(3*x**3)
```

3.15 $\int \frac{a+bx^4}{x^6} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{a+bx^4}{x^6} \, dx = -\frac{a}{5x^5} - \frac{b}{x}$$

output
```
-1/5*a/x^5-b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^6} \, dx = -\frac{a}{5x^5} - \frac{b}{x}$$

input
```
Integrate[(a + b*x^4)/x^6,x]
```

output
```
-1/5*a/x^5 - b/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^6} \, dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^6}+\frac{b}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{5x^5}-\frac{b}{x}$$

input
```
Int[(a + b*x^4)/x^6,x]
```

output
```
-1/5*a/x^5 - b/x
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{5b\,x^{4}+a}{5x^{5}}$	14
default	$-\frac{a}{5x^{5}}-\frac{b}{x}$	14
orering	$-\frac{5b\,x^{4}+a}{5x^{5}}$	14
norman	$\frac{-b\,x^{4}-\frac{a}{5}}{x^{5}}$	15
risch	$\frac{-b\,x^{4}-\frac{a}{5}}{x^{5}}$	15
parallelrisch	$\frac{-5b\,x^{4}-a}{5x^{5}}$	16

input
```
int((b*x^4+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(5*b*x^4+a)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^6}\,dx=-\frac{5\,bx^4+a}{5\,x^5}$$

input
```
integrate((b*x^4+a)/x^6,x, algorithm="fricas")
```

output
```
-1/5*(5*b*x^4 + a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a+bx^4}{x^6}\,dx = \frac{-a-5bx^4}{5x^5}$$

input
```
integrate((b*x**4+a)/x**6,x)
```

output
```
(-a - 5*b*x**4)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^6}\,dx = -\frac{5\,bx^4+a}{5\,x^5}$$

input
```
integrate((b*x^4+a)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(5*b*x^4 + a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^6}\,dx = -\frac{5\,bx^4+a}{5\,x^5}$$

input
```
integrate((b*x^4+a)/x^6,x, algorithm="giac")
```

output
```
-1/5*(5*b*x^4 + a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^4}{x^6}\,dx = -\frac{5\,b\,x^4+a}{5\,x^5}$$

input
```
int((a + b*x^4)/x^6,x)
```

output
```
-(a + 5*b*x^4)/(5*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^6}\,dx = \frac{-5b\,x^4-a}{5x^5}$$

input
```
int((b*x^4+a)/x^6,x)
```

output
```
( - a - 5*b*x**4)/(5*x**5)
```

3.16 $\int \frac{a+bx^4}{x^7}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^4}{x^7}\,dx=-\frac{a}{6x^6}-\frac{b}{2x^2}$$

output
```
-1/6*a/x^6-1/2*b/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^7}\,dx=-\frac{a}{6x^6}-\frac{b}{2x^2}$$

input
```
Integrate[(a + b*x^4)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(2*x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^7} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^7}+\frac{b}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{6x^6}-\frac{b}{2x^2}$$

input
```
Int[(a + b*x^4)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(2*x^2)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{3b\,x^{4}+a}{6x^{6}}$	14
default	$-\frac{a}{6x^{6}}-\frac{b}{2x^{2}}$	14
orering	$-\frac{3b\,x^{4}+a}{6x^{6}}$	14
norman	$\frac{-\frac{b\,x^{4}}{2}-\frac{a}{6}}{x^{6}}$	15
risch	$\frac{-\frac{b\,x^{4}}{2}-\frac{a}{6}}{x^{6}}$	15
parallelrisch	$\frac{-3b\,x^{4}-a}{6x^{6}}$	16

input
```
int((b*x^4+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*b*x^4+a)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^7}\,dx=-\frac{3\,bx^{4}+a}{6\,x^{6}}$$

input
```
integrate((b*x^4+a)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(3*b*x^4 + a)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^4}{x^7}\,dx = \frac{-a-3bx^4}{6x^6}$$

input
```
integrate((b*x**4+a)/x**7,x)
```

output
```
(-a - 3*b*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^7}\,dx = -\frac{3\,bx^4+a}{6\,x^6}$$

input
```
integrate((b*x^4+a)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(3*b*x^4 + a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^7}\,dx = -\frac{3\,bx^4+a}{6\,x^6}$$

input
```
integrate((b*x^4+a)/x^7,x, algorithm="giac")
```

output
```
-1/6*(3*b*x^4 + a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^4}{x^7}\,dx = -\frac{3\,b\,x^4+a}{6\,x^6}$$

input
```
int((a + b*x^4)/x^7,x)
```

output
```
-(a + 3*b*x^4)/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^7}\,dx = \frac{-3b\,x^4-a}{6x^6}$$

input
```
int((b*x^4+a)/x^7,x)
```

output
```
( - a - 3*b*x**4)/(6*x**6)
```

3.17 $\int \frac{a+bx^4}{x^8} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^4}{x^8} \, dx = -\frac{a}{7x^7} - \frac{b}{3x^3}$$

output
```
-1/7*a/x^7-1/3*b/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^8} \, dx = -\frac{a}{7x^7} - \frac{b}{3x^3}$$

input
```
Integrate[(a + b*x^4)/x^8,x]
```

output
```
-1/7*a/x^7 - b/(3*x^3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^8}\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a}{x^8}+\frac{b}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{7x^7}-\frac{b}{3x^3}$$

input
```
Int[(a + b*x^4)/x^8,x]
```

output
```
-1/7*a/x^7 - b/(3*x^3)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{7x^7}-\frac{b}{3x^3}$	14
norman	$\frac{-\frac{b\,x^4}{3}-\frac{a}{7}}{x^7}$	15
risch	$\frac{-\frac{b\,x^4}{3}-\frac{a}{7}}{x^7}$	15
gosper	$-\frac{7b\,x^4+3a}{21x^7}$	16
parallelrisch	$\frac{-7b\,x^4-3a}{21x^7}$	16
orering	$-\frac{7b\,x^4+3a}{21x^7}$	16

input
```
int((b*x^4+a)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a/x^7-1/3*b/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^8}\,dx=-\frac{7\,bx^4+3\,a}{21\,x^7}$$

input
```
integrate((b*x^4+a)/x^8,x, algorithm="fricas")
```

output
```
-1/21*(7*b*x^4 + 3*a)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^8}\,dx = \frac{-3a-7bx^4}{21x^7}$$

input
```
integrate((b*x**4+a)/x**8,x)
```

output
```
(-3*a - 7*b*x**4)/(21*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^8}\,dx = -\frac{7\,bx^4+3\,a}{21\,x^7}$$

input
```
integrate((b*x^4+a)/x^8,x, algorithm="maxima")
```

output
```
-1/21*(7*b*x^4 + 3*a)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^8}\,dx = -\frac{7\,bx^4+3\,a}{21\,x^7}$$

input
```
integrate((b*x^4+a)/x^8,x, algorithm="giac")
```

output
```
-1/21*(7*b*x^4 + 3*a)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^8}\,dx = -\frac{7\,b\,x^4+3\,a}{21\,x^7}$$

input
```
int((a + b*x^4)/x^8,x)
```

output
```
-(3*a + 7*b*x^4)/(21*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^8}\,dx = \frac{-7b\,x^4-3a}{21x^7}$$

input
```
int((b*x^4+a)/x^8,x)
```

output
```
( - 3*a - 7*b*x**4)/(21*x**7)
```

3.18 $\int \frac{a+bx^4}{x^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^4}{x^{10}}\,dx=-\frac{a}{9x^9}-\frac{b}{5x^5}$$

output
```
-1/9*a/x^9-1/5*b/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^4}{x^{10}}\,dx=-\frac{a}{9x^9}-\frac{b}{5x^5}$$

input
```
Integrate[(a + b*x^4)/x^10,x]
```

output
```
-1/9*a/x^9 - b/(5*x^5)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^4}{x^{10}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^{10}}+\frac{b}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{9x^9}-\frac{b}{5x^5}$$

input
```
Int[(a + b*x^4)/x^10,x]
```

output
```
-1/9*a/x^9 - b/(5*x^5)
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{9x^{9}}-\frac{b}{5x^{5}}$	14
norman	$\frac{-\frac{b\,x^{4}}{5}-\frac{a}{9}}{x^{9}}$	15
risch	$\frac{-\frac{b\,x^{4}}{5}-\frac{a}{9}}{x^{9}}$	15
gosper	$-\frac{9b\,x^{4}+5a}{45x^{9}}$	16
parallelrisch	$\frac{-9b\,x^{4}-5a}{45x^{9}}$	16
orering	$-\frac{9b\,x^{4}+5a}{45x^{9}}$	16

input
```
int((b*x^4+a)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a/x^9-1/5*b/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{10}}\,dx=-\frac{9\,bx^4+5\,a}{45\,x^9}$$

input
```
integrate((b*x^4+a)/x^10,x, algorithm="fricas")
```

output
```
-1/45*(9*b*x^4 + 5*a)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{10}}\,dx = \frac{-5a-9bx^4}{45x^9}$$

input
```
integrate((b*x**4+a)/x**10,x)
```

output
```
(-5*a - 9*b*x**4)/(45*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{10}}\,dx = -\frac{9\,bx^4+5\,a}{45\,x^9}$$

input
```
integrate((b*x^4+a)/x^10,x, algorithm="maxima")
```

output
```
-1/45*(9*b*x^4 + 5*a)/x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{10}}\,dx = -\frac{9\,bx^4+5\,a}{45\,x^9}$$

input
```
integrate((b*x^4+a)/x^10,x, algorithm="giac")
```

output
```
-1/45*(9*b*x^4 + 5*a)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{10}}\,dx = -\frac{9\,b\,x^4+5\,a}{45\,x^9}$$

input
```
int((a + b*x^4)/x^10,x)
```

output
```
-(5*a + 9*b*x^4)/(45*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^4}{x^{10}}\,dx = \frac{-9b\,x^4-5a}{45x^9}$$

input
```
int((b*x^4+a)/x^10,x)
```

output
```
( - 5*a - 9*b*x**4)/(45*x**9)
```

3.19 $\int x^7(a+bx^4)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^7\left(a+bx^4\right)^2\,dx=\frac{a^2x^8}{8}+\frac{1}{6}abx^{12}+\frac{b^2x^{16}}{16}$$

output
```
1/8*a^2*x^8+1/6*a*b*x^12+1/16*b^2*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^7\left(a+bx^4\right)^2\,dx=\frac{a^2x^8}{8}+\frac{1}{6}abx^{12}+\frac{b^2x^{16}}{16}$$

input
```
Integrate[x^7*(a + b*x^4)^2,x]
```

output
```
(a^2*x^8)/8 + (a*b*x^12)/6 + (b^2*x^16)/16
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^4\right)^2 \, dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{1}{4}\int x^4 \left(bx^4+a\right)^2 dx^4$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{4}\int \left(b^2x^{12}+2abx^8+a^2x^4\right) dx^4$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{4}\left(\frac{a^2x^8}{2}+\frac{2}{3}abx^{12}+\frac{b^2x^{16}}{4}\right)$$

input
```
Int[x^7*(a + b*x^4)^2,x]
```

output
```
((a^2*x^8)/2 + (2*a*b*x^12)/3 + (b^2*x^16)/4)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{8}a^2x^8 + \frac{1}{6}ab\,x^{12} + \frac{1}{16}b^2x^{16}$	25
default	$\frac{1}{8}a^2x^8 + \frac{1}{6}ab\,x^{12} + \frac{1}{16}b^2x^{16}$	25
norman	$\frac{1}{8}a^2x^8 + \frac{1}{6}ab\,x^{12} + \frac{1}{16}b^2x^{16}$	25
risch	$\frac{1}{8}a^2x^8 + \frac{1}{6}ab\,x^{12} + \frac{1}{16}b^2x^{16}$	25
parallelrisch	$\frac{1}{8}a^2x^8 + \frac{1}{6}ab\,x^{12} + \frac{1}{16}b^2x^{16}$	25
orering	$\frac{x^8(3b^2x^8+8ab\,x^4+6a^2)}{48}$	27

input
```
int(x^7*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^2*x^8+1/6*a*b*x^12+1/16*b^2*x^16
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^7 (a + bx^4)^2 \, dx = \frac{1}{16}\,b^2x^{16} + \frac{1}{6}\,abx^{12} + \frac{1}{8}\,a^2x^8$$

input
```
integrate(x^7*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*b^2*x^16 + 1/6*a*b*x^12 + 1/8*a^2*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^7 (a + bx^4)^2 \, dx = \frac{a^2x^8}{8} + \frac{abx^{12}}{6} + \frac{b^2x^{16}}{16}$$

input
```
integrate(x**7*(b*x**4+a)**2,x)
```

output
```
a**2*x**8/8 + a*b*x**12/6 + b**2*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^7 (a + bx^4)^2 \, dx = \frac{1}{16} b^2x^{16} + \frac{1}{6} abx^{12} + \frac{1}{8} a^2x^8$$

input
```
integrate(x^7*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/16*b^2*x^16 + 1/6*a*b*x^12 + 1/8*a^2*x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^7 (a + bx^4)^2 \, dx = \frac{1}{16} b^2x^{16} + \frac{1}{6} abx^{12} + \frac{1}{8} a^2x^8$$

input
```
integrate(x^7*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*b^2*x^16 + 1/6*a*b*x^12 + 1/8*a^2*x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^7 (a+bx^4)^2 \, dx = \frac{a^2 x^8}{8} + \frac{a\, b\, x^{12}}{6} + \frac{b^2 x^{16}}{16}$$

input
```
int(x^7*(a + b*x^4)^2,x)
```

output
```
(a^2*x^8)/8 + (b^2*x^16)/16 + (a*b*x^12)/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^7 (a+bx^4)^2 \, dx = \frac{x^8(3b^2x^8 + 8ab\, x^4 + 6a^2)}{48}$$

input
```
int(x^7*(b*x^4+a)^2,x)
```

output
```
(x**8*(6*a**2 + 8*a*b*x**4 + 3*b**2*x**8))/48
```

3.20 $\int x^3(a+bx^4)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^3\left(a+bx^4\right)^2\,dx=\frac{\left(a+bx^4\right)^3}{12b}$$

output
```
1/12*(b*x^4+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int x^3\left(a+bx^4\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{4}abx^8+\frac{b^2x^{12}}{12}$$

input
```
Integrate[x^3*(a + b*x^4)^2,x]
```

output
```
(a^2*x^4)/4 + (a*b*x^8)/4 + (b^2*x^12)/12
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^4\right)^2 \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{\left(a+bx^4\right)^3}{12b}$$

input
```
Int[x^3*(a + b*x^4)^2,x]
```

output
```
(a + b*x^4)^3/(12*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^4+a)^3}{12b}$	15
gosper	$\frac{1}{12}b^2x^{12}+\frac{1}{4}ab\,x^8+\frac{1}{4}a^2x^4$	25
norman	$\frac{1}{12}b^2x^{12}+\frac{1}{4}ab\,x^8+\frac{1}{4}a^2x^4$	25
parallelrisch	$\frac{1}{12}b^2x^{12}+\frac{1}{4}ab\,x^8+\frac{1}{4}a^2x^4$	25
orering	$\frac{x^4(b^2x^8+3ab\,x^4+3a^2)}{12}$	26
risch	$\frac{b^2x^{12}}{12}+\frac{ab\,x^8}{4}+\frac{a^2x^4}{4}+\frac{a^3}{12b}$	33

input
```
int(x^3*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(b*x^4+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^3\left(a+bx^4\right)^2\,dx=\frac{1}{12}\,b^2x^{12}+\frac{1}{4}\,abx^8+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/12*b^2*x^12 + 1/4*a*b*x^8 + 1/4*a^2*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^3(a+bx^4)^2\,dx = \frac{a^2x^4}{4} + \frac{abx^8}{4} + \frac{b^2x^{12}}{12}$$

input
```
integrate(x**3*(b*x**4+a)**2,x)
```

output
```
a**2*x**4/4 + a*b*x**8/4 + b**2*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^4)^2\,dx = \frac{(bx^4+a)^3}{12\,b}$$

input
```
integrate(x^3*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*(b*x^4 + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^4)^2\,dx = \frac{(bx^4+a)^3}{12\,b}$$

input
```
integrate(x^3*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/12*(b*x^4 + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x^3(a+bx^4)^2\,dx=\frac{a^2\,x^4}{4}+\frac{a\,b\,x^8}{4}+\frac{b^2\,x^{12}}{12}$$

input
```
int(x^3*(a + b*x^4)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^12)/12 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int x^3(a+bx^4)^2\,dx=\frac{x^4(b^2x^8+3ab\,x^4+3a^2)}{12}$$

input
```
int(x^3*(b*x^4+a)^2,x)
```

output
```
(x**4*(3*a**2 + 3*a*b*x**4 + b**2*x**8))/12
```

3.21 $\int \frac{(a+bx^4)^2}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(a+bx^4)^2}{x} dx = \frac{1}{2}abx^4 + \frac{b^2x^8}{8} + a^2\log(x)$$

output
```
1/2*a*b*x^4+1/8*b^2*x^8+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x} dx = \frac{1}{2}abx^4 + \frac{b^2x^8}{8} + a^2\log(x)$$

input
```
Integrate[(a + b*x^4)^2/x,x]
```

output
```
(a*b*x^4)/2 + (b^2*x^8)/8 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^2}{x^4}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(b^2x^4+2ab+\frac{a^2}{x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(a^2\log\left(x^4\right)+2abx^4+\frac{b^2x^8}{2}\right)$$

input
```
Int[(a + b*x^4)^2/x,x]
```

output
```
(2*a*b*x^4 + (b^2*x^8)/2 + a^2*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{ab\,x^4}{2}+\frac{b^2x^8}{8}+a^2\ln\left(x\right)$	23
norman	$\frac{ab\,x^4}{2}+\frac{b^2x^8}{8}+a^2\ln\left(x\right)$	23
parallelrisch	$\frac{ab\,x^4}{2}+\frac{b^2x^8}{8}+a^2\ln\left(x\right)$	23
risch	$\frac{b^2x^8}{8}+\frac{ab\,x^4}{2}+\frac{a^2}{2}+a^2\ln\left(x\right)$	28

input
```
int((b*x^4+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*b*x^4+1/8*b^2*x^8+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x}\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{2}\,abx^4+a^2\log\left(x\right)$$

input
```
integrate((b*x^4+a)^2/x,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 1/2*a*b*x^4 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x}\,dx = a^2\log(x) + \frac{abx^4}{2} + \frac{b^2x^8}{8}$$

input
```
integrate((b*x**4+a)**2/x,x)
```

output
```
a**2*log(x) + a*b*x**4/2 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2}{x}\,dx = \frac{1}{8}\,b^2x^8 + \frac{1}{2}\,abx^4 + \frac{1}{4}\,a^2\log\left(x^4\right)$$

input
```
integrate((b*x^4+a)^2/x,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 1/2*a*b*x^4 + 1/4*a^2*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2}{x}\,dx = \frac{1}{8}\,b^2x^8 + \frac{1}{2}\,abx^4 + \frac{1}{4}\,a^2\log\left(x^4\right)$$

input
```
integrate((b*x^4+a)^2/x,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 1/2*a*b*x^4 + 1/4*a^2*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x}\,dx = a^2\,\ln(x) + \frac{b^2\,x^8}{8} + \frac{a\,b\,x^4}{2}$$

input
```
int((a + b*x^4)^2/x,x)
```

output
```
a^2*log(x) + (b^2*x^8)/8 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x}\,dx = \log(x)\,a^2 + \frac{ab\,x^4}{2} + \frac{b^2x^8}{8}$$

input
```
int((b*x^4+a)^2/x,x)
```

output
```
(8*log(x)*a**2 + 4*a*b*x**4 + b**2*x**8)/8
```

3.22 $\int \frac{(a+bx^4)^2}{x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{(a+bx^4)^2}{x^5} \, dx = -\frac{a^2}{4x^4} + \frac{b^2x^4}{4} + 2ab\log(x)$$

output
```
-1/4*a^2/x^4+1/4*b^2*x^4+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^5} \, dx = -\frac{a^2}{4x^4} + \frac{b^2x^4}{4} + 2ab\log(x)$$

input
```
Integrate[(a + b*x^4)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 + (b^2*x^4)/4 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^5}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^2}{x^8}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{a^2}{x^8}+\frac{2ba}{x^4}+b^2\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^2}{x^4}+2ab\log\left(x^4\right)+b^2x^4\right)$$

input
```
Int[(a + b*x^4)^2/x^5,x]
```

output
```
(-(a^2/x^4) + b^2*x^4 + 2*a*b*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{4x^4}+\frac{b^2x^4}{4}+2ab\ln(x)$	24
risch	$-\frac{a^2}{4x^4}+\frac{b^2x^4}{4}+2ab\ln(x)$	24
norman	$\frac{-\frac{a^2}{4}+\frac{b^2x^8}{4}}{x^4}+2ab\ln(x)$	26
parallelrisch	$\frac{b^2x^8+8ab\ln(x)x^4-a^2}{4x^4}$	28

input
```
int((b*x^4+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4+1/4*b^2*x^4+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^5}\,dx=\frac{b^2x^8+8\,abx^4\log(x)-a^2}{4\,x^4}$$

input
```
integrate((b*x^4+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(b^2*x^8 + 8*a*b*x^4*log(x) - a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^4)^2}{x^5}\,dx = -\frac{a^2}{4x^4} + 2ab\log(x) + \frac{b^2x^4}{4}$$

input
```
integrate((b*x**4+a)**2/x**5,x)
```

output
```
-a**2/(4*x**4) + 2*a*b*log(x) + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^2}{x^5}\,dx = \frac{1}{4}\,b^2x^4 + \frac{1}{2}\,ab\log\left(x^4\right) - \frac{a^2}{4\,x^4}$$

input
```
integrate((b*x^4+a)^2/x^5,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + 1/2*a*b*log(x^4) - 1/4*a^2/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^4)^2}{x^5}\,dx = \frac{1}{4}\,b^2x^4 + \frac{1}{2}\,ab\log\left(x^4\right) - \frac{2\,abx^4 + a^2}{4\,x^4}$$

input
```
integrate((b*x^4+a)^2/x^5,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + 1/2*a*b*log(x^4) - 1/4*(2*a*b*x^4 + a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^5}\,dx = \frac{b^2\,x^4}{4} - \frac{a^2}{4\,x^4} + 2\,a\,b\,\ln{(x)}$$

input
```
int((a + b*x^4)^2/x^5,x)
```

output
```
(b^2*x^4)/4 - a^2/(4*x^4) + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^5}\,dx = \frac{8\log(x)\,ab\,x^4 - a^2 + b^2x^8}{4x^4}$$

input
```
int((b*x^4+a)^2/x^5,x)
```

output
```
(8*log(x)*a*b*x**4 - a**2 + b**2*x**8)/(4*x**4)
```

3.23 $\int \frac{(a+bx^4)^2}{x^9}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = -\frac{a^2}{8x^8} - \frac{ab}{2x^4} + b^2\log(x)$$

output
```
-1/8*a^2/x^8-1/2*a*b/x^4+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = -\frac{a^2}{8x^8} - \frac{ab}{2x^4} + b^2\log(x)$$

input
```
Integrate[(a + b*x^4)^2/x^9,x]
```

output
```
-1/8*a^2/x^8 - (a*b)/(2*x^4) + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^9} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^2}{x^{12}}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{a^2}{x^{12}}+\frac{2ba}{x^8}+\frac{b^2}{x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^2}{2x^8}-\frac{2ab}{x^4}+b^2\log\left(x^4\right)\right)$$

input
```
Int[(a + b*x^4)^2/x^9,x]
```

output
```
(-1/2*a^2/x^8 - (2*a*b)/x^4 + b^2*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^2}{8x^8}-\frac{ab}{2x^4}+b^2\ln\left(x\right)$	23
norman	$\frac{-\frac{1}{8}a^2-\frac{1}{2}ab\,x^4}{x^8}+b^2\ln\left(x\right)$	25
risch	$\frac{-\frac{1}{8}a^2-\frac{1}{2}ab\,x^4}{x^8}+b^2\ln\left(x\right)$	25
parallelrisch	$\frac{8b^2\ln(x)x^8-4ab\,x^4-a^2}{8x^8}$	29

input
```
int((b*x^4+a)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2/x^8-1/2*a*b/x^4+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int\frac{(a+bx^4)^2}{x^9}\,dx=\frac{8\,b^2x^8\log\left(x\right)-4\,abx^4-a^2}{8\,x^8}$$

input
```
integrate((b*x^4+a)^2/x^9,x, algorithm="fricas")
```

output
```
1/8*(8*b^2*x^8*log(x) - 4*a*b*x^4 - a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = b^2\log(x) + \frac{-a^2-4abx^4}{8x^8}$$

input
```
integrate((b*x**4+a)**2/x**9,x)
```

output
```
b**2*log(x) + (-a**2 - 4*a*b*x**4)/(8*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = \frac{1}{4}\,b^2\log\left(x^4\right) - \frac{4\,abx^4+a^2}{8\,x^8}$$

input
```
integrate((b*x^4+a)^2/x^9,x, algorithm="maxima")
```

output
```
1/4*b^2*log(x^4) - 1/8*(4*a*b*x^4 + a^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = \frac{1}{4}\,b^2\log\left(x^4\right) - \frac{3\,b^2x^8+4\,abx^4+a^2}{8\,x^8}$$

input
```
integrate((b*x^4+a)^2/x^9,x, algorithm="giac")
```

output
```
1/4*b^2*log(x^4) - 1/8*(3*b^2*x^8 + 4*a*b*x^4 + a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = b^2 \ln(x) - \frac{\frac{a^2}{8} + \frac{b\,a\,x^4}{2}}{x^8}$$

input
```
int((a + b*x^4)^2/x^9,x)
```

output
```
b^2*log(x) - (a^2/8 + (a*b*x^4)/2)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^4)^2}{x^9}\,dx = \frac{8\log(x)\,b^2x^8 - a^2 - 4ab\,x^4}{8x^8}$$

input
```
int((b*x^4+a)^2/x^9,x)
```

output
```
(8*log(x)*b**2*x**8 - a**2 - 4*a*b*x**4)/(8*x**8)
```

3.24 $\int \frac{(a+bx^4)^2}{x^{13}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^4)^2}{x^{13}} \, dx = -\frac{(a+bx^4)^3}{12ax^{12}}$$

output
```
-1/12*(b*x^4+a)^3/a/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^4)^2}{x^{13}} \, dx = -\frac{a^2}{12x^{12}} - \frac{ab}{4x^8} - \frac{b^2}{4x^4}$$

input
```
Integrate[(a + b*x^4)^2/x^13,x]
```

output
```
-1/12*a^2/x^12 - (a*b)/(4*x^8) - b^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^{13}} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+bx^4)^3}{12ax^{12}}$$

input
```
Int[(a + b*x^4)^2/x^13,x]
```

output
```
-1/12*(a + b*x^4)^3/(a*x^12)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
gosper	$-\frac{3b^2x^8+3ab\,x^4+a^2}{12x^{12}}$	25
default	$-\frac{b^2}{4x^4}-\frac{ab}{4x^8}-\frac{a^2}{12x^{12}}$	25
orering	$-\frac{3b^2x^8+3ab\,x^4+a^2}{12x^{12}}$	25
norman	$\frac{-\frac{1}{4}b^2x^8-\frac{1}{4}ab\,x^4-\frac{1}{12}a^2}{x^{12}}$	26
risch	$\frac{-\frac{1}{4}b^2x^8-\frac{1}{4}ab\,x^4-\frac{1}{12}a^2}{x^{12}}$	26
parallelrisch	$\frac{-3b^2x^8-3ab\,x^4-a^2}{12x^{12}}$	27

input
```
int((b*x^4+a)^2/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*b^2*x^8+3*a*b*x^4+a^2)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^4)^2}{x^{13}}\,dx = -\frac{3\,b^2x^8+3\,abx^4+a^2}{12\,x^{12}}$$

input
```
integrate((b*x^4+a)^2/x^13,x, algorithm="fricas")
```

output
```
-1/12*(3*b^2*x^8 + 3*a*b*x^4 + a^2)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^4)^2}{x^{13}}\,dx = \frac{-a^2-3abx^4-3b^2x^8}{12x^{12}}$$

input
```
integrate((b*x**4+a)**2/x**13,x)
```

output
```
(-a**2 - 3*a*b*x**4 - 3*b**2*x**8)/(12*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^4)^2}{x^{13}}\,dx = -\frac{3\,b^2x^8 + 3\,abx^4 + a^2}{12\,x^{12}}$$

input
```
integrate((b*x^4+a)^2/x^13,x, algorithm="maxima")
```

output
```
-1/12*(3*b^2*x^8 + 3*a*b*x^4 + a^2)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^4)^2}{x^{13}}\,dx = -\frac{3\,b^2x^8 + 3\,abx^4 + a^2}{12\,x^{12}}$$

input
```
integrate((b*x^4+a)^2/x^13,x, algorithm="giac")
```

output
```
-1/12*(3*b^2*x^8 + 3*a*b*x^4 + a^2)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^4)^2}{x^{13}}\,dx = -\frac{\frac{a^2}{12} + \frac{a\,b\,x^4}{4} + \frac{b^2\,x^8}{4}}{x^{12}}$$

input
```
int((a + b*x^4)^2/x^13,x)
```

output
```
-(a^2/12 + (b^2*x^8)/4 + (a*b*x^4)/4)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^4)^2}{x^{13}}\,dx = \frac{-3b^2x^8-3ab\,x^4-a^2}{12x^{12}}$$

input
```
int((b*x^4+a)^2/x^13,x)
```

output
```
( - a**2 - 3*a*b*x**4 - 3*b**2*x**8)/(12*x**12)
```

3.25 $\int \frac{(a+bx^4)^2}{x^{17}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^4)^2}{x^{17}} \, dx = -\frac{a^2}{16x^{16}} - \frac{ab}{6x^{12}} - \frac{b^2}{8x^8}$$

output
```
-1/16*a^2/x^16-1/6*a*b/x^12-1/8*b^2/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^{17}} \, dx = -\frac{a^2}{16x^{16}} - \frac{ab}{6x^{12}} - \frac{b^2}{8x^8}$$

input
```
Integrate[(a + b*x^4)^2/x^17,x]
```

output
```
-1/16*a^2/x^16 - (a*b)/(6*x^12) - b^2/(8*x^8)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^{17}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^2}{x^{20}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{x^{20}}+\frac{2ba}{x^{16}}+\frac{b^2}{x^{12}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^2}{4x^{16}}-\frac{2ab}{3x^{12}}-\frac{b^2}{2x^8}\right)$$

input
```
Int[(a + b*x^4)^2/x^17,x]
```

output
```
(-1/4*a^2/x^16 - (2*a*b)/(3*x^12) - b^2/(2*x^8))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{16x^{16}}-\frac{ab}{6x^{12}}-\frac{b^2}{8x^8}$	25
norman	$\frac{-\frac{1}{16}a^2-\frac{1}{6}ab\,x^4-\frac{1}{8}b^2x^8}{x^{16}}$	26
risch	$\frac{-\frac{1}{16}a^2-\frac{1}{6}ab\,x^4-\frac{1}{8}b^2x^8}{x^{16}}$	26
gosper	$-\frac{6b^2x^8+8ab\,x^4+3a^2}{48x^{16}}$	27
parallelrisch	$\frac{-6b^2x^8-8ab\,x^4-3a^2}{48x^{16}}$	27
orering	$-\frac{6b^2x^8+8ab\,x^4+3a^2}{48x^{16}}$	27

input
```
int((b*x^4+a)^2/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^2/x^16-1/6*a*b/x^12-1/8*b^2/x^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^4)^2}{x^{17}}\,dx = -\frac{6\,b^2x^8+8\,abx^4+3\,a^2}{48\,x^{16}}$$

input
```
integrate((b*x^4+a)^2/x^17,x, algorithm="fricas")
```

output
```
-1/48*(6*b^2*x^8 + 8*a*b*x^4 + 3*a^2)/x^16
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^4)^2}{x^{17}}\,dx = \frac{-3a^2-8abx^4-6b^2x^8}{48x^{16}}$$

input
```
integrate((b*x**4+a)**2/x**17,x)
```

output
```
(-3*a**2 - 8*a*b*x**4 - 6*b**2*x**8)/(48*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^4)^2}{x^{17}}\,dx = -\frac{6\,b^2x^8+8\,abx^4+3\,a^2}{48\,x^{16}}$$

input
```
integrate((b*x^4+a)^2/x^17,x, algorithm="maxima")
```

output
```
-1/48*(6*b^2*x^8 + 8*a*b*x^4 + 3*a^2)/x^16
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^4)^2}{x^{17}}\,dx = -\frac{6\,b^2x^8+8\,abx^4+3\,a^2}{48\,x^{16}}$$

input
```
integrate((b*x^4+a)^2/x^17,x, algorithm="giac")
```

output
```
-1/48*(6*b^2*x^8 + 8*a*b*x^4 + 3*a^2)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^4)^2}{x^{17}}\,dx = -\frac{\frac{a^2}{16}+\frac{a\,b\,x^4}{6}+\frac{b^2\,x^8}{8}}{x^{16}}$$

input
```
int((a + b*x^4)^2/x^17,x)
```

output
```
-(a^2/16 + (b^2*x^8)/8 + (a*b*x^4)/6)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^4)^2}{x^{17}}\,dx = \frac{-6b^2x^8-8ab\,x^4-3a^2}{48x^{16}}$$

input
```
int((b*x^4+a)^2/x^17,x)
```

output
```
( - 3*a**2 - 8*a*b*x**4 - 6*b**2*x**8)/(48*x**16)
```

3.26 $\int x^4(a+bx^4)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^4(a+bx^4)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}$$

output
```
1/5*a^2*x^5+2/9*a*b*x^9+1/13*b^2*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^4)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}$$

input
```
Integrate[x^4*(a + b*x^4)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^9)/9 + (b^2*x^13)/13
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^4\right)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^4+2abx^8+b^2x^{12}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^5}{5}+\frac{2}{9}abx^9+\frac{b^2x^{13}}{13}$$

input
```
Int[x^4*(a + b*x^4)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^9)/9 + (b^2*x^13)/13
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{5}a^2x^5+\frac{2}{9}ab\,x^9+\frac{1}{13}b^2x^{13}$	25
default	$\frac{1}{5}a^2x^5+\frac{2}{9}ab\,x^9+\frac{1}{13}b^2x^{13}$	25
norman	$\frac{1}{5}a^2x^5+\frac{2}{9}ab\,x^9+\frac{1}{13}b^2x^{13}$	25
risch	$\frac{1}{5}a^2x^5+\frac{2}{9}ab\,x^9+\frac{1}{13}b^2x^{13}$	25
parallelrisch	$\frac{1}{5}a^2x^5+\frac{2}{9}ab\,x^9+\frac{1}{13}b^2x^{13}$	25
orering	$\frac{x^5\left(45b^2x^8+130ab\,x^4+117a^2\right)}{585}$	27

input
```
int(x^4*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+2/9*a*b*x^9+1/13*b^2*x^13
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4\left(a+bx^4\right)^2\,dx=\frac{1}{13}\,b^2x^{13}+\frac{2}{9}\,abx^9+\frac{1}{5}\,a^2x^5$$

input
```
integrate(x^4*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/13*b^2*x^13 + 2/9*a*b*x^9 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4 \left(a + b x^4\right)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2 a b x^9}{9} + \frac{b^2 x^{13}}{13}$$

input
```
integrate(x**4*(b*x**4+a)**2,x)
```

output
```
a**2*x**5/5 + 2*a*b*x**9/9 + b**2*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 \left(a + b x^4\right)^2 \, dx = \frac{1}{13} \, b^2 x^{13} + \frac{2}{9} \, a b x^9 + \frac{1}{5} \, a^2 x^5$$

input
```
integrate(x^4*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/13*b^2*x^13 + 2/9*a*b*x^9 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 \left(a + b x^4\right)^2 \, dx = \frac{1}{13} \, b^2 x^{13} + \frac{2}{9} \, a b x^9 + \frac{1}{5} \, a^2 x^5$$

input
```
integrate(x^4*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/13*b^2*x^13 + 2/9*a*b*x^9 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^4)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2 a b x^9}{9} + \frac{b^2 x^{13}}{13}$$

input
```
int(x^4*(a + b*x^4)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^13)/13 + (2*a*b*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4 (a + bx^4)^2 \, dx = \frac{x^5(45b^2x^8 + 130ab\,x^4 + 117a^2)}{585}$$

input
```
int(x^4*(b*x^4+a)^2,x)
```

output
```
(x**5*(117*a**2 + 130*a*b*x**4 + 45*b**2*x**8))/585
```

3.27 $\int x^2(a+bx^4)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^2(a+bx^4)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}$$

output
```
1/3*a^2*x^3+2/7*a*b*x^7+1/11*b^2*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^4)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}$$

input
```
Integrate[x^2*(a + b*x^4)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^7)/7 + (b^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^4\right)^2 \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(a^2x^2+2abx^6+b^2x^{10}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2x^3}{3}+\frac{2}{7}abx^7+\frac{b^2x^{11}}{11}$$

input
```
Int[x^2*(a + b*x^4)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^7)/7 + (b^2*x^11)/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{3}a^2x^3+\frac{2}{7}ab\,x^7+\frac{1}{11}b^2x^{11}$	25
default	$\frac{1}{3}a^2x^3+\frac{2}{7}ab\,x^7+\frac{1}{11}b^2x^{11}$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{7}ab\,x^7+\frac{1}{11}b^2x^{11}$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{7}ab\,x^7+\frac{1}{11}b^2x^{11}$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{7}ab\,x^7+\frac{1}{11}b^2x^{11}$	25
orering	$\frac{x^3(21b^2x^8+66ab\,x^4+77a^2)}{231}$	27

input
```
int(x^2*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/7*a*b*x^7+1/11*b^2*x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx^4)^2\,dx=\frac{1}{11}\,b^2x^{11}+\frac{2}{7}\,abx^7+\frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/11*b^2*x^11 + 2/7*a*b*x^7 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2 (a + bx^4)^2 \, dx = \frac{a^2 x^3}{3} + \frac{2abx^7}{7} + \frac{b^2 x^{11}}{11}$$

input
```
integrate(x**2*(b*x**4+a)**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x**7/7 + b**2*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (a + bx^4)^2 \, dx = \frac{1}{11} b^2 x^{11} + \frac{2}{7} abx^7 + \frac{1}{3} a^2 x^3$$

input
```
integrate(x^2*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/11*b^2*x^11 + 2/7*a*b*x^7 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (a + bx^4)^2 \, dx = \frac{1}{11} b^2 x^{11} + \frac{2}{7} abx^7 + \frac{1}{3} a^2 x^3$$

input
```
integrate(x^2*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/11*b^2*x^11 + 2/7*a*b*x^7 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 \left(a + bx^4\right)^2 \, dx = \frac{a^2 x^3}{3} + \frac{2 a b x^7}{7} + \frac{b^2 x^{11}}{11}$$

input
```
int(x^2*(a + b*x^4)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^11)/11 + (2*a*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2 \left(a + bx^4\right)^2 \, dx = \frac{x^3(21b^2x^8 + 66ab\,x^4 + 77a^2)}{231}$$

input
```
int(x^2*(b*x^4+a)^2,x)
```

output
```
(x**3*(77*a**2 + 66*a*b*x**4 + 21*b**2*x**8))/231
```

3.28 $\int x(a+bx^4)^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int x\left(a+bx^4\right)^2 \, dx = \frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}$$

output
```
1/2*a^2*x^2+1/3*a*b*x^6+1/10*b^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^4\right)^2 \, dx = \frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}$$

input
```
Integrate[x*(a + b*x^4)^2,x]
```

output
```
(a^2*x^2)/2 + (a*b*x^6)/3 + (b^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^4)^2 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int (a^2x+2abx^5+b^2x^9) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^2}{2}+\frac{1}{3}abx^6+\frac{b^2x^{10}}{10}$$

input
```
Int[x*(a + b*x^4)^2,x]
```

output
```
(a^2*x^2)/2 + (a*b*x^6)/3 + (b^2*x^10)/10
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{2}a^2x^2+\frac{1}{3}ab\,x^6+\frac{1}{10}b^2x^{10}$	25
default	$\frac{1}{2}a^2x^2+\frac{1}{3}ab\,x^6+\frac{1}{10}b^2x^{10}$	25
norman	$\frac{1}{2}a^2x^2+\frac{1}{3}ab\,x^6+\frac{1}{10}b^2x^{10}$	25
risch	$\frac{1}{2}a^2x^2+\frac{1}{3}ab\,x^6+\frac{1}{10}b^2x^{10}$	25
parallelrisch	$\frac{1}{2}a^2x^2+\frac{1}{3}ab\,x^6+\frac{1}{10}b^2x^{10}$	25
orering	$\frac{x^2(3b^2x^8+10ab\,x^4+15a^2)}{30}$	27

input
```
int(x*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+1/3*a*b*x^6+1/10*b^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^4)^2\,dx=\frac{1}{10}\,b^2x^{10}+\frac{1}{3}\,abx^6+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*x^10 + 1/3*a*b*x^6 + 1/2*a^2*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^4)^2\,dx=\frac{a^2x^2}{2}+\frac{abx^6}{3}+\frac{b^2x^{10}}{10}$$

input
```
integrate(x*(b*x**4+a)**2,x)
```

output
```
a**2*x**2/2 + a*b*x**6/3 + b**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^4)^2\,dx=\frac{1}{10}\,b^2x^{10}+\frac{1}{3}\,abx^6+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*x^10 + 1/3*a*b*x^6 + 1/2*a^2*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^4)^2\,dx=\frac{1}{10}\,b^2x^{10}+\frac{1}{3}\,abx^6+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/10*b^2*x^10 + 1/3*a*b*x^6 + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x(a+bx^4)^2\,dx=\frac{a^2\,x^2}{2}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^{10}}{10}$$

input
```
int(x*(a + b*x^4)^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^10)/10 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x(a+bx^4)^2\,dx=\frac{x^2(3b^2x^8+10ab\,x^4+15a^2)}{30}$$

input
```
int(x*(b*x^4+a)^2,x)
```

output
```
(x**2*(15*a**2 + 10*a*b*x**4 + 3*b**2*x**8))/30
```

3.29 $\int (a+bx^4)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+bx^4)^2 \, dx = a^2x + \frac{2}{5}abx^5 + \frac{b^2x^9}{9}$$

output
```
a^2*x+2/5*a*b*x^5+1/9*b^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^2 \, dx = a^2x + \frac{2}{5}abx^5 + \frac{b^2x^9}{9}$$

input
```
Integrate[(a + b*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^5)/5 + (b^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 dx$$

$$\Big\downarrow \text{ 747}$$

$$\int \left(a^2+2abx^4+b^2x^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x+\frac{2}{5}abx^5+\frac{b^2x^9}{9}$$

input
```
Int[(a + b*x^4)^2,x]
```

output
```
a^2*x + (2*a*b*x^5)/5 + (b^2*x^9)/9
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x + \frac{2}{5}ab\,x^5 + \frac{1}{9}b^2x^9$	22
default	$a^2x + \frac{2}{5}ab\,x^5 + \frac{1}{9}b^2x^9$	22
norman	$a^2x + \frac{2}{5}ab\,x^5 + \frac{1}{9}b^2x^9$	22
risch	$a^2x + \frac{2}{5}ab\,x^5 + \frac{1}{9}b^2x^9$	22
parallelrisch	$a^2x + \frac{2}{5}ab\,x^5 + \frac{1}{9}b^2x^9$	22
orering	$\frac{x(5b^2x^8+18ab\,x^4+45a^2)}{45}$	25

input
```
int((b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/5*a*b*x^5+1/9*b^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^2\,dx = \frac{1}{9}\,b^2x^9 + \frac{2}{5}\,abx^5 + a^2x$$

input
```
integrate((b*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 2/5*a*b*x^5 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+bx^4\right)^2 \, dx = a^2x + \frac{2abx^5}{5} + \frac{b^2x^9}{9}$$

input
```
integrate((b*x**4+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**5/5 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^2 \, dx = \frac{1}{9}\,b^2x^9 + \frac{2}{5}\,abx^5 + a^2x$$

input
```
integrate((b*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 2/5*a*b*x^5 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^2 \, dx = \frac{1}{9}\,b^2x^9 + \frac{2}{5}\,abx^5 + a^2x$$

input
```
integrate((b*x^4+a)^2,x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 2/5*a*b*x^5 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^2\,dx=a^2\,x+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^9}{9}$$

input
```
int((a + b*x^4)^2,x)
```

output
```
a^2*x + (b^2*x^9)/9 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a+bx^4\right)^2\,dx=\frac{x(5b^2x^8+18ab\,x^4+45a^2)}{45}$$

input
```
int((b*x^4+a)^2,x)
```

output
```
(x*(45*a**2 + 18*a*b*x***4 + 5*b**2*x**8))/45
```

3.30 $\int \frac{(a+bx^4)^2}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(a+bx^4)^2}{x^2} dx = -\frac{a^2}{x} + \frac{2}{3}abx^3 + \frac{b^2x^7}{7}$$

output
```
-a^2/x+2/3*a*b*x^3+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^2} dx = -\frac{a^2}{x} + \frac{2}{3}abx^3 + \frac{b^2x^7}{7}$$

input
```
Integrate[(a + b*x^4)^2/x^2,x]
```

output
```
-(a^2/x) + (2*a*b*x^3)/3 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^2} \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^2}{x^2}+2abx^2+b^2x^6\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{x}+\frac{2}{3}abx^3+\frac{b^2x^7}{7}$$

input
```
Int[(a + b*x^4)^2/x^2,x]
```

output
```
-(a^2/x) + (2*a*b*x^3)/3 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{x}+\frac{2ab\,x^3}{3}+\frac{b^2x^7}{7}$	25
risch	$-\frac{a^2}{x}+\frac{2ab\,x^3}{3}+\frac{b^2x^7}{7}$	25
norman	$\frac{\frac{1}{7}b^2x^8+\frac{2}{3}ab\,x^4-a^2}{x}$	26
gosper	$-\frac{-3b^2x^8-14ab\,x^4+21a^2}{21x}$	27
parallelrisch	$\frac{3b^2x^8+14ab\,x^4-21a^2}{21x}$	27
orering	$-\frac{-3b^2x^8-14ab\,x^4+21a^2}{21x}$	27

input
```
int((b*x^4+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2/3*a*b*x^3+1/7*b^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^2}{x^2}\,dx = \frac{3\,b^2x^8+14\,abx^4-21\,a^2}{21\,x}$$

input
```
integrate((b*x^4+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/21*(3*b^2*x^8 + 14*a*b*x^4 - 21*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^4)^2}{x^2}\,dx = -\frac{a^2}{x} + \frac{2abx^3}{3} + \frac{b^2x^7}{7}$$

input
```
integrate((b*x**4+a)**2/x**2,x)
```

output
```
-a**2/x + 2*a*b*x**3/3 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)^2}{x^2}\,dx = \frac{1}{7}\,b^2x^7 + \frac{2}{3}\,abx^3 - \frac{a^2}{x}$$

input
```
integrate((b*x^4+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 2/3*a*b*x^3 - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)^2}{x^2}\,dx = \frac{1}{7}\,b^2x^7 + \frac{2}{3}\,abx^3 - \frac{a^2}{x}$$

input
```
integrate((b*x^4+a)^2/x^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 2/3*a*b*x^3 - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^4)^2}{x^2}\,dx = \frac{-a^2 + \frac{2\,a\,b\,x^4}{3} + \frac{b^2\,x^8}{7}}{x}$$

input
```
int((a + b*x^4)^2/x^2,x)
```

output
```
((b^2*x^8)/7 - a^2 + (2*a*b*x^4)/3)/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^2}{x^2}\,dx = \frac{3b^2x^8 + 14ab\,x^4 - 21a^2}{21x}$$

input
```
int((b*x^4+a)^2/x^2,x)
```

output
```
( - 21*a**2 + 14*a*b*x**4 + 3*b**2*x**8)/(21*x)
```

3.31 $\int \frac{(a+bx^4)^2}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{(a+bx^4)^2}{x^3} \, dx = -\frac{a^2}{2x^2} + abx^2 + \frac{b^2x^6}{6}$$

output
```
-1/2*a^2/x^2+a*b*x^2+1/6*b^2*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^3} \, dx = -\frac{a^2}{2x^2} + abx^2 + \frac{b^2x^6}{6}$$

input
```
Integrate[(a + b*x^4)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + a*b*x^2 + (b^2*x^6)/6
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{x^3}+2abx+b^2x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2x^2}+abx^2+\frac{b^2x^6}{6}$$

input
```
Int[(a + b*x^4)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + a*b*x^2 + (b^2*x^6)/6
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{2x^2}+ab\,x^2+\frac{b^2x^6}{6}$	24
risch	$-\frac{a^2}{2x^2}+ab\,x^2+\frac{b^2x^6}{6}$	24
norman	$\frac{\frac{1}{6}b^2x^8+ab\,x^4-\frac{1}{2}a^2}{x^2}$	25
parallelrisch	$\frac{b^2x^8+6ab\,x^4-3a^2}{6x^2}$	26
gosper	$-\frac{-b^2x^8-6ab\,x^4+3a^2}{6x^2}$	27
orering	$-\frac{-b^2x^8-6ab\,x^4+3a^2}{6x^2}$	27

input
```
int((b*x^4+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+a*b*x^2+1/6*b^2*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^2}{x^3}\,dx=\frac{b^2x^8+6\,abx^4-3\,a^2}{6\,x^2}$$

input
```
integrate((b*x^4+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/6*(b^2*x^8 + 6*a*b*x^4 - 3*a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^2}{x^3}\,dx = -\frac{a^2}{2x^2} + abx^2 + \frac{b^2x^6}{6}$$

input
```
integrate((b*x**4+a)**2/x**3,x)
```

output
```
-a**2/(2*x**2) + a*b*x**2 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^3}\,dx = \frac{1}{6}\,b^2x^6 + abx^2 - \frac{a^2}{2\,x^2}$$

input
```
integrate((b*x^4+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6 + a*b*x^2 - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^3}\,dx = \frac{1}{6}\,b^2x^6 + abx^2 - \frac{a^2}{2\,x^2}$$

input
```
integrate((b*x^4+a)^2/x^3,x, algorithm="giac")
```

output
```
1/6*b^2*x^6 + a*b*x^2 - 1/2*a^2/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^2}{x^3}\,dx = \frac{-3\,a^2+6\,a\,b\,x^4+b^2\,x^8}{6\,x^2}$$

input
```
int((a + b*x^4)^2/x^3,x)
```

output
```
(b^2*x^8 - 3*a^2 + 6*a*b*x^4)/(6*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^4)^2}{x^3}\,dx = \frac{b^2x^8+6ab\,x^4-3a^2}{6x^2}$$

input
```
int((b*x^4+a)^2/x^3,x)
```

output
```
( - 3*a**2 + 6*a*b*x**4 + b**2*x**8)/(6*x**2)
```

3.32 $\int \frac{(a+bx^4)^2}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{(a+bx^4)^2}{x^4} dx = -\frac{a^2}{3x^3} + 2abx + \frac{b^2x^5}{5}$$

output
```
-1/3*a^2/x^3+2*a*b*x+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^4} dx = -\frac{a^2}{3x^3} + 2abx + \frac{b^2x^5}{5}$$

input
```
Integrate[(a + b*x^4)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 + 2*a*b*x + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^2}{x^4} \, dx$$
$$\Big\downarrow \text{ 802}$$
$$\int \left(\frac{a^2}{x^4}+2ab+b^2x^4\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^2}{3x^3}+2abx+\frac{b^2x^5}{5}$$

input
```
Int[(a + b*x^4)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 + 2*a*b*x + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{2}}{3x^{3}}+2abx+\frac{b^{2}x^{5}}{5}$	23
risch	$-\frac{a^{2}}{3x^{3}}+2abx+\frac{b^{2}x^{5}}{5}$	23
norman	$\frac{\frac{1}{5}b^{2}x^{8}+2ab\,x^{4}-\frac{1}{3}a^{2}}{x^{3}}$	26
gosper	$-\frac{-3b^{2}x^{8}-30ab\,x^{4}+5a^{2}}{15x^{3}}$	27
parallelrisch	$\frac{3b^{2}x^{8}+30ab\,x^{4}-5a^{2}}{15x^{3}}$	27
orering	$-\frac{-3b^{2}x^{8}-30ab\,x^{4}+5a^{2}}{15x^{3}}$	27

input
```
int((b*x^4+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+2*a*b*x+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^4}\,dx=\frac{3\,b^2x^8+30\,abx^4-5\,a^2}{15\,x^3}$$

input
```
integrate((b*x^4+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/15*(3*b^2*x^8 + 30*a*b*x^4 - 5*a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^4}\,dx = -\frac{a^2}{3x^3} + 2abx + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**4+a)**2/x**4,x)
```

output
```
-a**2/(3*x**3) + 2*a*b*x + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^4}\,dx = \frac{1}{5}\,b^2x^5 + 2\,abx - \frac{a^2}{3\,x^3}$$

input
```
integrate((b*x^4+a)^2/x^4,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2*a*b*x - 1/3*a^2/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^4}\,dx = \frac{1}{5}\,b^2x^5 + 2\,abx - \frac{a^2}{3\,x^3}$$

input
```
integrate((b*x^4+a)^2/x^4,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2*a*b*x - 1/3*a^2/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^2}{x^4}\,dx = \frac{b^2\,x^5}{5} - \frac{a^2}{3\,x^3} + 2\,a\,b\,x$$

input
```
int((a + b*x^4)^2/x^4,x)
```

output
```
(b^2*x^5)/5 - a^2/(3*x^3) + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^2}{x^4}\,dx = \frac{3b^2x^8 + 30ab\,x^4 - 5a^2}{15x^3}$$

input
```
int((b*x^4+a)^2/x^4,x)
```

output
```
( - 5*a**2 + 30*a*b*x**4 + 3*b**2*x**8)/(15*x**3)
```

3.33 $\int x^7(a+bx^4)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^7\left(a+bx^4\right)^3\,dx=-\frac{a(a+bx^4)^4}{16b^2}+\frac{(a+bx^4)^5}{20b^2}$$

output
```
-1/16*a*(b*x^4+a)^4/b^2+1/20*(b*x^4+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int x^7\left(a+bx^4\right)^3\,dx=\frac{a^3x^8}{8}+\frac{1}{4}a^2bx^{12}+\frac{3}{16}ab^2x^{16}+\frac{b^3x^{20}}{20}$$

input
```
Integrate[x^7*(a + b*x^4)^3,x]
```

output
```
(a^3*x^8)/8 + (a^2*b*x^12)/4 + (3*a*b^2*x^16)/16 + (b^3*x^20)/20
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^4\right)^3 \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int x^4 \left(bx^4+a\right)^3 dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4} \int \left(\frac{\left(bx^4+a\right)^4}{b}-\frac{a\left(bx^4+a\right)^3}{b}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{\left(a+bx^4\right)^5}{5b^2}-\frac{a\left(a+bx^4\right)^4}{4b^2}\right)$$

input
```
Int[x^7*(a + b*x^4)^3,x]
```

output
```
(-1/4*(a*(a + b*x^4)^4)/b^2 + (a + b*x^4)^5/(5*b^2))/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{1}{8}a^3x^8+\frac{1}{4}a^2b\,x^{12}+\frac{3}{16}a\,b^2x^{16}+\frac{1}{20}b^3x^{20}$	36
default	$\frac{1}{8}a^3x^8+\frac{1}{4}a^2b\,x^{12}+\frac{3}{16}a\,b^2x^{16}+\frac{1}{20}b^3x^{20}$	36
norman	$\frac{1}{8}a^3x^8+\frac{1}{4}a^2b\,x^{12}+\frac{3}{16}a\,b^2x^{16}+\frac{1}{20}b^3x^{20}$	36
risch	$\frac{1}{8}a^3x^8+\frac{1}{4}a^2b\,x^{12}+\frac{3}{16}a\,b^2x^{16}+\frac{1}{20}b^3x^{20}$	36
parallelrisch	$\frac{1}{8}a^3x^8+\frac{1}{4}a^2b\,x^{12}+\frac{3}{16}a\,b^2x^{16}+\frac{1}{20}b^3x^{20}$	36
orering	$\frac{x^8\left(4b^3x^{12}+15a\,b^2x^8+20a^2b\,x^4+10a^3\right)}{80}$	38

input
```
int(x^7*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^3*x^8+1/4*a^2*b*x^12+3/16*a*b^2*x^16+1/20*b^3*x^20
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^7\left(a+bx^4\right)^3\,dx=\frac{1}{20}\,b^3x^{20}+\frac{3}{16}\,ab^2x^{16}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/20*b^3*x^20 + 3/16*a*b^2*x^16 + 1/4*a^2*b*x^12 + 1/8*a^3*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int x^7 \left(a+bx^4\right)^3 \, dx = \frac{a^3x^8}{8} + \frac{a^2bx^{12}}{4} + \frac{3ab^2x^{16}}{16} + \frac{b^3x^{20}}{20}$$

input
```
integrate(x**7*(b*x**4+a)**3,x)
```

output
```
a**3*x**8/8 + a**2*b*x**12/4 + 3*a*b**2*x**16/16 + b**3*x**20/20
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^7 \left(a+bx^4\right)^3 \, dx = \frac{1}{20}\,b^3x^{20} + \frac{3}{16}\,ab^2x^{16} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/20*b^3*x^20 + 3/16*a*b^2*x^16 + 1/4*a^2*b*x^12 + 1/8*a^3*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^7 \left(a+bx^4\right)^3 \, dx = \frac{1}{20}\,b^3x^{20} + \frac{3}{16}\,ab^2x^{16} + \frac{1}{4}\,a^2bx^{12} + \frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/20*b^3*x^20 + 3/16*a*b^2*x^16 + 1/4*a^2*b*x^12 + 1/8*a^3*x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^7 (a + bx^4)^3 \, dx = \frac{a^3 x^8}{8} + \frac{a^2 b x^{12}}{4} + \frac{3 a b^2 x^{16}}{16} + \frac{b^3 x^{20}}{20}$$

input
```
int(x^7*(a + b*x^4)^3,x)
```

output
```
(a^3*x^8)/8 + (b^3*x^20)/20 + (a^2*b*x^12)/4 + (3*a*b^2*x^16)/16
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int x^7 (a + bx^4)^3 \, dx = \frac{x^8 (4b^3 x^{12} + 15a b^2 x^8 + 20a^2 b x^4 + 10a^3)}{80}$$

input
```
int(x^7*(b*x^4+a)^3,x)
```

output
```
(x**8*(10*a**3 + 20*a**2*b*x**4 + 15*a*b**2*x**8 + 4*b**3*x**12))/80
```

3.34 $\int x^3(a+bx^4)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int x^3\left(a+bx^4\right)^3\,dx=\frac{\left(a+bx^4\right)^4}{16b}$$

output
```
1/16*(b*x^4+a)^4/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(16) = 32.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.69

$$\int x^3\left(a+bx^4\right)^3\,dx=\frac{a^3x^4}{4}+\frac{3}{8}a^2bx^8+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{16}}{16}$$

input
```
Integrate[x^3*(a + b*x^4)^3,x]
```

output
```
(a^3*x^4)/4 + (3*a^2*b*x^8)/8 + (a*b^2*x^12)/4 + (b^3*x^16)/16
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx^4)^3 \, dx$$

$$\Big\downarrow \, 793$$

$$\frac{(a + bx^4)^4}{16b}$$

input
```
Int[x^3*(a + b*x^4)^3,x]
```

output
```
(a + b*x^4)^4/(16*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\left(b\,x^{4}+a\right)^{4}}{16 b}$	15
gosper	$\frac{1}{4} a^{3} x^{4}+\frac{3}{8} a^{2} b\,x^{8}+\frac{1}{4} a\,b^{2} x^{12}+\frac{1}{16} b^{3} x^{16}$	36
norman	$\frac{1}{4} a^{3} x^{4}+\frac{3}{8} a^{2} b\,x^{8}+\frac{1}{4} a\,b^{2} x^{12}+\frac{1}{16} b^{3} x^{16}$	36
parallelrisch	$\frac{1}{4} a^{3} x^{4}+\frac{3}{8} a^{2} b\,x^{8}+\frac{1}{4} a\,b^{2} x^{12}+\frac{1}{16} b^{3} x^{16}$	36
orering	$\frac{x^{4} \left(b^{3} x^{12}+4 a\,b^{2} x^{8}+6 a^{2} b\,x^{4}+4 a^{3}\right)}{16}$	37
risch	$\frac{b^{3} x^{16}}{16}+\frac{a\,b^{2} x^{12}}{4}+\frac{3 a^{2} b\,x^{8}}{8}+\frac{a^{3} x^{4}}{4}+\frac{a^{4}}{16 b}$	44

input
```
int(x^3*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(b*x^4+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int x^3\left(a+bx^4\right)^3\,dx=\frac{1}{16}\,b^3x^{16}+\frac{1}{4}\,ab^2x^{12}+\frac{3}{8}\,a^2bx^8+\frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/16*b^3*x^16 + 1/4*a*b^2*x^12 + 3/8*a^2*b*x^8 + 1/4*a^3*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int x^3 (a + bx^4)^3 \, dx = \frac{a^3x^4}{4} + \frac{3a^2bx^8}{8} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{16}}{16}$$

input
```
integrate(x**3*(b*x**4+a)**3,x)
```

output
```
a**3*x**4/4 + 3*a**2*b*x**8/8 + a*b**2*x**12/4 + b**3*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3 (a + bx^4)^3 \, dx = \frac{(bx^4 + a)^4}{16 \, b}$$

input
```
integrate(x^3*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*(b*x^4 + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x^3 (a + bx^4)^3 \, dx = \frac{(bx^4 + a)^4}{16 \, b}$$

input
```
integrate(x^3*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/16*(b*x^4 + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int x^3 (a + bx^4)^3 \, dx = \frac{a^3 x^4}{4} + \frac{3 a^2 b x^8}{8} + \frac{a b^2 x^{12}}{4} + \frac{b^3 x^{16}}{16}$$

input
```
int(x^3*(a + b*x^4)^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^16)/16 + (3*a^2*b*x^8)/8 + (a*b^2*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int x^3 (a + bx^4)^3 \, dx = \frac{x^4 (b^3 x^{12} + 4a b^2 x^8 + 6a^2 b x^4 + 4a^3)}{16}$$

input
```
int(x^3*(b*x^4+a)^3,x)
```

output
```
(x**4*(4*a**3 + 6*a**2*b*x**4 + 4*a*b**2*x**8 + b**3*x**12))/16
```

3.35 $\int \frac{(a+bx^4)^3}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^4)^3}{x} dx = \frac{3}{4}a^2bx^4 + \frac{3}{8}ab^2x^8 + \frac{b^3x^{12}}{12} + a^3\log(x)$$

output
```
3/4*a^2*b*x^4+3/8*a*b^2*x^8+1/12*b^3*x^12+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x} dx = \frac{3}{4}a^2bx^4 + \frac{3}{8}ab^2x^8 + \frac{b^3x^{12}}{12} + a^3\log(x)$$

input
```
Integrate[(a + b*x^4)^3/x,x]
```

output
```
(3*a^2*b*x^4)/4 + (3*a*b^2*x^8)/8 + (b^3*x^12)/12 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^3}{x^4}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(b^3x^8+3ab^2x^4+3a^2b+\frac{a^3}{x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(a^3\log\left(x^4\right)+3a^2bx^4+\frac{3}{2}ab^2x^8+\frac{b^3x^{12}}{3}\right)$$

input
```
Int[(a + b*x^4)^3/x,x]
```

output
```
(3*a^2*b*x^4 + (3*a*b^2*x^8)/2 + (b^3*x^12)/3 + a^3*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{3a^2b\,x^4}{4}+\frac{3a\,b^2x^8}{8}+\frac{b^3x^{12}}{12}+a^3\ln\left(x\right)$	34
norman	$\frac{3a^2b\,x^4}{4}+\frac{3a\,b^2x^8}{8}+\frac{b^3x^{12}}{12}+a^3\ln\left(x\right)$	34
risch	$\frac{3a^2b\,x^4}{4}+\frac{3a\,b^2x^8}{8}+\frac{b^3x^{12}}{12}+a^3\ln\left(x\right)$	34
parallelrisch	$\frac{3a^2b\,x^4}{4}+\frac{3a\,b^2x^8}{8}+\frac{b^3x^{12}}{12}+a^3\ln\left(x\right)$	34

input
```
int((b*x^4+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
3/4*a^2*b*x^4+3/8*a*b^2*x^8+1/12*b^3*x^12+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int\frac{(a+bx^4)^3}{x}\,dx=\frac{1}{12}\,b^3x^{12}+\frac{3}{8}\,ab^2x^8+\frac{3}{4}\,a^2bx^4+a^3\log\left(x\right)$$

input
```
integrate((b*x^4+a)^3/x,x, algorithm="fricas")
```

output
```
1/12*b^3*x^12 + 3/8*a*b^2*x^8 + 3/4*a^2*b*x^4 + a^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x}\,dx = a^3\log(x) + \frac{3a^2bx^4}{4} + \frac{3ab^2x^8}{8} + \frac{b^3x^{12}}{12}$$

input
```
integrate((b*x**4+a)**3/x,x)
```

output
```
a**3*log(x) + 3*a**2*b*x**4/4 + 3*a*b**2*x**8/8 + b**3*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x}\,dx = \frac{1}{12}b^3x^{12} + \frac{3}{8}ab^2x^8 + \frac{3}{4}a^2bx^4 + \frac{1}{4}a^3\log(x^4)$$

input
```
integrate((b*x^4+a)^3/x,x, algorithm="maxima")
```

output
```
1/12*b^3*x^12 + 3/8*a*b^2*x^8 + 3/4*a^2*b*x^4 + 1/4*a^3*log(x^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x}\,dx = \frac{1}{12}b^3x^{12} + \frac{3}{8}ab^2x^8 + \frac{3}{4}a^2bx^4 + \frac{1}{4}a^3\log(x^4)$$

input
```
integrate((b*x^4+a)^3/x,x, algorithm="giac")
```

output
```
1/12*b^3*x^12 + 3/8*a*b^2*x^8 + 3/4*a^2*b*x^4 + 1/4*a^3*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^3}{x}\,dx = a^3\,\ln(x) + \frac{b^3\,x^{12}}{12} + \frac{3\,a^2\,b\,x^4}{4} + \frac{3\,a\,b^2\,x^8}{8}$$

input
```
int((a + b*x^4)^3/x,x)
```

output
```
a^3*log(x) + (b^3*x^12)/12 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^3}{x}\,dx = \log(x)\,a^3 + \frac{3a^2b\,x^4}{4} + \frac{3a\,b^2x^8}{8} + \frac{b^3x^{12}}{12}$$

input
```
int((b*x^4+a)^3/x,x)
```

output
```
(24*log(x)*a**3 + 18*a**2*b*x**4 + 9*a*b**2*x**8 + 2*b**3*x**12)/24
```

3.36 $\int \frac{(a+bx^4)^3}{x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = -\frac{a^3}{4x^4} + \frac{3}{4}ab^2x^4 + \frac{b^3x^8}{8} + 3a^2b\log(x)$$

output
```
-1/4*a^3/x^4+3/4*a*b^2*x^4+1/8*b^3*x^8+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = -\frac{a^3}{4x^4} + \frac{3}{4}ab^2x^4 + \frac{b^3x^8}{8} + 3a^2b\log(x)$$

input
```
Integrate[(a + b*x^4)^3/x^5,x]
```

output
```
-1/4*a^3/x^4 + (3*a*b^2*x^4)/4 + (b^3*x^8)/8 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^5}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^3}{x^8}dx^4$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{4}\int \left(b^3x^4+3ab^2+\frac{3a^2b}{x^4}+\frac{a^3}{x^8}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{a^3}{x^4}+3a^2b\log\left(x^4\right)+3ab^2x^4+\frac{b^3x^8}{2}\right)$$

input
```
Int[(a + b*x^4)^3/x^5,x]
```

output
```
(-(a^3/x^4) + 3*a*b^2*x^4 + (b^3*x^8)/2 + 3*a^2*b*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{4x^{4}}+\frac{3a\,b^{2}x^{4}}{4}+\frac{b^{3}x^{8}}{8}+3a^{2}b\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{4}a^{3}+\frac{1}{8}b^{3}x^{12}+\frac{3}{4}a\,b^{2}x^{8}}{x^{4}}+3a^{2}b\ln\left(x\right)$	37
parallelrisch	$\frac{b^{3}x^{12}+6a\,b^{2}x^{8}+24a^{2}b\ln(x)x^{4}-2a^{3}}{8x^{4}}$	39
risch	$\frac{b^{3}x^{8}}{8}+\frac{3a\,b^{2}x^{4}}{4}+\frac{9a^{2}b}{8}-\frac{a^{3}}{4x^{4}}+3a^{2}b\ln\left(x\right)$	41

input
```
int((b*x^4+a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4+3/4*a*b^2*x^4+1/8*b^3*x^8+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^5}\,dx=\frac{b^3x^{12}+6\,ab^2x^8+24\,a^2bx^4\log\left(x\right)-2\,a^3}{8\,x^4}$$

input
```
integrate((b*x^4+a)^3/x^5,x, algorithm="fricas")
```

output
```
1/8*(b^3*x^12 + 6*a*b^2*x^8 + 24*a^2*b*x^4*log(x) - 2*a^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = -\frac{a^3}{4x^4} + 3a^2b\log{(x)} + \frac{3ab^2x^4}{4} + \frac{b^3x^8}{8}$$

input
```
integrate((b*x**4+a)**3/x**5,x)
```

output
```
-a**3/(4*x**4) + 3*a**2*b*log(x) + 3*a*b**2*x**4/4 + b**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = \frac{1}{8}\,b^3x^8 + \frac{3}{4}\,ab^2x^4 + \frac{3}{4}\,a^2b\log\left(x^4\right) - \frac{a^3}{4\,x^4}$$

input
```
integrate((b*x^4+a)^3/x^5,x, algorithm="maxima")
```

output
```
1/8*b^3*x^8 + 3/4*a*b^2*x^4 + 3/4*a^2*b*log(x^4) - 1/4*a^3/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = \frac{1}{8}\,b^3x^8 + \frac{3}{4}\,ab^2x^4 + \frac{3}{4}\,a^2b\log\left(x^4\right) - \frac{3\,a^2bx^4 + a^3}{4\,x^4}$$

input
```
integrate((b*x^4+a)^3/x^5,x, algorithm="giac")
```

output
```
1/8*b^3*x^8 + 3/4*a*b^2*x^4 + 3/4*a^2*b*log(x^4) - 1/4*(3*a^2*b*x^4 + a^3)
/x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = \frac{b^3\,x^8}{8} - \frac{a^3}{4\,x^4} + \frac{3\,a\,b^2\,x^4}{4} + 3\,a^2\,b\,\ln(x)$$

input
```
int((a + b*x^4)^3/x^5,x)
```

output
```
(b^3*x^8)/8 - a^3/(4*x^4) + (3*a*b^2*x^4)/4 + 3*a^2*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^5}\,dx = \frac{24\log(x)\,a^2b\,x^4 - 2a^3 + 6a\,b^2x^8 + b^3x^{12}}{8x^4}$$

input
```
int((b*x^4+a)^3/x^5,x)
```

output
```
(24*log(x)*a**2*b*x**4 - 2*a**3 + 6*a*b**2*x**8 + b**3*x**12)/(8*x**4)
```

3.37 $\int \frac{(a+bx^4)^3}{x^9}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = -\frac{a^3}{8x^8} - \frac{3a^2b}{4x^4} + \frac{b^3x^4}{4} + 3ab^2\log(x)$$

output
```
-1/8*a^3/x^8-3/4*a^2*b/x^4+1/4*b^3*x^4+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = -\frac{a^3}{8x^8} - \frac{3a^2b}{4x^4} + \frac{b^3x^4}{4} + 3ab^2\log(x)$$

input
```
Integrate[(a + b*x^4)^3/x^9,x]
```

output
```
-1/8*a^3/x^8 - (3*a^2*b)/(4*x^4) + (b^3*x^4)/4 + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^9} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^3}{x^{12}}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{a^3}{x^{12}}+\frac{3ba^2}{x^8}+\frac{3b^2a}{x^4}+b^3\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^3}{2x^8}-\frac{3a^2b}{x^4}+3ab^2\log\left(x^4\right)+b^3x^4\right)$$

input
```
Int[(a + b*x^4)^3/x^9,x]
```

output
```
(-1/2*a^3/x^8 - (3*a^2*b)/x^4 + b^3*x^4 + 3*a*b^2*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^3}{8x^8}-\frac{3a^2b}{4x^4}+\frac{b^3x^4}{4}+3a\,b^2\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{8}a^3+\frac{1}{4}b^3x^{12}-\frac{3}{4}a^2b\,x^4}{x^8}+3a\,b^2\ln\left(x\right)$	37
risch	$\frac{b^3x^4}{4}+\frac{-\frac{3}{4}a^2b\,x^4-\frac{1}{8}a^3}{x^8}+3a\,b^2\ln\left(x\right)$	37
parallelrisch	$\frac{2b^3x^{12}+24a\,b^2\ln(x)x^8-6a^2b\,x^4-a^3}{8x^8}$	40

input
```
int((b*x^4+a)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^3/x^8-3/4*a^2*b/x^4+1/4*b^3*x^4+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int\frac{(a+bx^4)^3}{x^9}\,dx=\frac{2\,b^3x^{12}+24\,ab^2x^8\log\left(x\right)-6\,a^2bx^4-a^3}{8\,x^8}$$

input
```
integrate((b*x^4+a)^3/x^9,x, algorithm="fricas")
```

output
```
1/8*(2*b^3*x^12 + 24*a*b^2*x^8*log(x) - 6*a^2*b*x^4 - a^3)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = 3ab^2\log(x) + \frac{b^3x^4}{4} + \frac{-a^3-6a^2bx^4}{8x^8}$$

input
```
integrate((b*x**4+a)**3/x**9,x)
```

output
```
3*a*b**2*log(x) + b**3*x**4/4 + (-a**3 - 6*a**2*b*x**4)/(8*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = \frac{1}{4}\,b^3x^4 + \frac{3}{4}\,ab^2\log\left(x^4\right) - \frac{6\,a^2bx^4+a^3}{8\,x^8}$$

input
```
integrate((b*x^4+a)^3/x^9,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 3/4*a*b^2*log(x^4) - 1/8*(6*a^2*b*x^4 + a^3)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = \frac{1}{4}\,b^3x^4 + \frac{3}{4}\,ab^2\log\left(x^4\right) - \frac{9\,ab^2x^8+6\,a^2bx^4+a^3}{8\,x^8}$$

input
```
integrate((b*x^4+a)^3/x^9,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + 3/4*a*b^2*log(x^4) - 1/8*(9*a*b^2*x^8 + 6*a^2*b*x^4 + a^3)/x
^8
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = \frac{b^3\,x^4}{4} - \frac{\frac{a^3}{8} + \frac{3\,b\,a^2\,x^4}{4}}{x^8} + 3\,a\,b^2\,\ln(x)$$

input
```
int((a + b*x^4)^3/x^9,x)
```

output
```
(b^3*x^4)/4 - (a^3/8 + (3*a^2*b*x^4)/4)/x^8 + 3*a*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^4)^3}{x^9}\,dx = \frac{24\log(x)\,a\,b^2x^8 - a^3 - 6a^2b\,x^4 + 2b^3x^{12}}{8x^8}$$

input
```
int((b*x^4+a)^3/x^9,x)
```

output
```
(24*log(x)*a*b**2*x**8 - a**3 - 6*a**2*b*x**4 + 2*b**3*x**12)/(8*x**8)
```

3.38 $\int \frac{(a+bx^4)^3}{x^{13}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^4)^3}{x^{13}} \, dx = -\frac{a^3}{12x^{12}} - \frac{3a^2b}{8x^8} - \frac{3ab^2}{4x^4} + b^3 \log(x)$$

output
```
-1/12*a^3/x^12-3/8*a^2*b/x^8-3/4*a*b^2/x^4+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^{13}} \, dx = -\frac{a^3}{12x^{12}} - \frac{3a^2b}{8x^8} - \frac{3ab^2}{4x^4} + b^3 \log(x)$$

input
```
Integrate[(a + b*x^4)^3/x^13,x]
```

output
```
-1/12*a^3/x^12 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(4*x^4) + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{(bx^4+a)^3}{x^{16}}dx^4$$

$$\Big\downarrow \; 49$$

$$\frac{1}{4}\int \left(\frac{a^3}{x^{16}}+\frac{3ba^2}{x^{12}}+\frac{3b^2a}{x^8}+\frac{b^3}{x^4}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(-\frac{a^3}{3x^{12}}-\frac{3a^2b}{2x^8}-\frac{3ab^2}{x^4}+b^3\log\left(x^4\right)\right)$$

input
```
Int[(a + b*x^4)^3/x^13,x]
```

output
```
(-1/3*a^3/x^12 - (3*a^2*b)/(2*x^8) - (3*a*b^2)/x^4 + b^3*Log[x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{3}}{12x^{12}}-\frac{3a^{2}b}{8x^{8}}-\frac{3a\,b^{2}}{4x^{4}}+b^{3}\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{12}a^{3}-\frac{3}{4}a\,b^{2}x^{8}-\frac{3}{8}a^{2}b\,x^{4}}{x^{12}}+b^{3}\ln\left(x\right)$	36
risch	$\frac{-\frac{1}{12}a^{3}-\frac{3}{4}a\,b^{2}x^{8}-\frac{3}{8}a^{2}b\,x^{4}}{x^{12}}+b^{3}\ln\left(x\right)$	36
parallelrisch	$\frac{24b^{3}\ln\left(x\right)x^{12}-18a\,b^{2}x^{8}-9a^{2}b\,x^{4}-2a^{3}}{24x^{12}}$	40

input
```
int((b*x^4+a)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^3/x^12-3/8*a^2*b/x^8-3/4*a*b^2/x^4+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx = \frac{24\,b^3x^{12}\log\left(x\right)-18\,ab^2x^8-9\,a^2bx^4-2\,a^3}{24\,x^{12}}$$

input
```
integrate((b*x^4+a)^3/x^13,x, algorithm="fricas")
```

output
```
1/24*(24*b^3*x^12*log(x) - 18*a*b^2*x^8 - 9*a^2*b*x^4 - 2*a^3)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx = b^3\log{(x)} + \frac{-2a^3 - 9a^2bx^4 - 18ab^2x^8}{24x^{12}}$$

input
```
integrate((b*x**4+a)**3/x**13,x)
```

output
```
b**3*log(x) + (-2*a**3 - 9*a**2*b*x**4 - 18*a*b**2*x**8)/(24*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx = \frac{1}{4}\,b^3\log\left(x^4\right) - \frac{18\,ab^2x^8 + 9\,a^2bx^4 + 2\,a^3}{24\,x^{12}}$$

input
```
integrate((b*x^4+a)^3/x^13,x, algorithm="maxima")
```

output
```
1/4*b^3*log(x^4) - 1/24*(18*a*b^2*x^8 + 9*a^2*b*x^4 + 2*a^3)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx = \frac{1}{4}\,b^3\log\left(x^4\right) - \frac{11\,b^3x^{12} + 18\,ab^2x^8 + 9\,a^2bx^4 + 2\,a^3}{24\,x^{12}}$$

input
```
integrate((b*x^4+a)^3/x^13,x, algorithm="giac")
```

output
```
1/4*b^3*log(x^4) - 1/24*(11*b^3*x^12 + 18*a*b^2*x^8 + 9*a^2*b*x^4 + 2*a^3)
/x^12
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx = b^3\,\ln(x) - \frac{\frac{a^3}{12} + \frac{3\,a^2\,b\,x^4}{8} + \frac{3\,a\,b^2\,x^8}{4}}{x^{12}}$$

input
```
int((a + b*x^4)^3/x^13,x)
```

output
```
b^3*log(x) - (a^3/12 + (3*a^2*b*x^4)/8 + (3*a*b^2*x^8)/4)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^{13}}\,dx = \frac{24\log(x)\,b^3x^{12} - 2a^3 - 9a^2b\,x^4 - 18a\,b^2x^8}{24x^{12}}$$

input
```
int((b*x^4+a)^3/x^13,x)
```

output
```
(24*log(x)*b**3*x**12 - 2*a**3 - 9*a**2*b*x**4 - 18*a*b**2*x**8)/(24*x**12
)
```

3.39 $\int \frac{(a+bx^4)^3}{x^{17}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = -\frac{(a+bx^4)^4}{16ax^{16}}$$

output
```
-1/16*(b*x^4+a)^4/a/x^16
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = -\frac{a^3}{16x^{16}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{8x^8} - \frac{b^3}{4x^4}$$

input
```
Integrate[(a + b*x^4)^3/x^17,x]
```

output
```
-1/16*a^3/x^16 - (a^2*b)/(4*x^12) - (3*a*b^2)/(8*x^8) - b^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+bx^4)^4}{16ax^{16}}$$

input
```
Int[(a + b*x^4)^3/x^17,x]
```

output
```
-1/16*(a + b*x^4)^4/(a*x^16)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result	size
gosper	$-\frac{4b^3x^{12}+6a\,b^2x^8+4a^2b\,x^4+a^3}{16x^{16}}$	36
default	$-\frac{b^3}{4x^4}-\frac{3a\,b^2}{8x^8}-\frac{a^3}{16x^{16}}-\frac{a^2b}{4x^{12}}$	36
orering	$-\frac{4b^3x^{12}+6a\,b^2x^8+4a^2b\,x^4+a^3}{16x^{16}}$	36
norman	$\frac{-\frac{1}{16}a^3-\frac{1}{4}a^2b\,x^4-\frac{3}{8}a\,b^2x^8-\frac{1}{4}b^3x^{12}}{x^{16}}$	37
risch	$\frac{-\frac{1}{16}a^3-\frac{1}{4}a^2b\,x^4-\frac{3}{8}a\,b^2x^8-\frac{1}{4}b^3x^{12}}{x^{16}}$	37
parallelrisch	$\frac{-4b^3x^{12}-6a\,b^2x^8-4a^2b\,x^4-a^3}{16x^{16}}$	38

input
```
int((b*x^4+a)^3/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(4*b^3*x^12+6*a*b^2*x^8+4*a^2*b*x^4+a^3)/x^16
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = -\frac{4\,b^3x^{12}+6\,ab^2x^8+4\,a^2bx^4+a^3}{16\,x^{16}}$$

input
```
integrate((b*x^4+a)^3/x^17,x, algorithm="fricas")
```

output
```
-1/16*(4*b^3*x^12 + 6*a*b^2*x^8 + 4*a^2*b*x^4 + a^3)/x^16
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = \frac{-a^3 - 4a^2bx^4 - 6ab^2x^8 - 4b^3x^{12}}{16x^{16}}$$

input
```
integrate((b*x**4+a)**3/x**17,x)
```

output
```
(-a**3 - 4*a**2*b*x**4 - 6*a*b**2*x**8 - 4*b**3*x**12)/(16*x**16)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = -\frac{4\,b^3x^{12} + 6\,ab^2x^8 + 4\,a^2bx^4 + a^3}{16\,x^{16}}$$

input
```
integrate((b*x^4+a)^3/x^17,x, algorithm="maxima")
```

output
```
-1/16*(4*b^3*x^12 + 6*a*b^2*x^8 + 4*a^2*b*x^4 + a^3)/x^16
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = -\frac{4\,b^3x^{12} + 6\,ab^2x^8 + 4\,a^2bx^4 + a^3}{16\,x^{16}}$$

input
```
integrate((b*x^4+a)^3/x^17,x, algorithm="giac")
```

output
```
-1/16*(4*b^3*x^12 + 6*a*b^2*x^8 + 4*a^2*b*x^4 + a^3)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = -\frac{\frac{a^3}{16}+\frac{a^2\,b\,x^4}{4}+\frac{3\,a\,b^2\,x^8}{8}+\frac{b^3\,x^{12}}{4}}{x^{16}}$$

input
```
int((a + b*x^4)^3/x^17,x)
```

output
```
-(a^3/16 + (b^3*x^12)/4 + (a^2*b*x^4)/4 + (3*a*b^2*x^8)/8)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^4)^3}{x^{17}}\,dx = \frac{-4b^3x^{12}-6a\,b^2x^8-4a^2b\,x^4-a^3}{16x^{16}}$$

input
```
int((b*x^4+a)^3/x^17,x)
```

output
```
( - a**3 - 4*a**2*b*x**4 - 6*a*b**2*x**8 - 4*b**3*x**12)/(16*x**16)
```

3.40 $\int \frac{(a+bx^4)^3}{x^{21}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = -\frac{(a+bx^4)^4}{20ax^{20}} + \frac{b(a+bx^4)^4}{80a^2x^{16}}$$

output
```
-1/20*(b*x^4+a)^4/a/x^20+1/80*b*(b*x^4+a)^4/a^2/x^16
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = -\frac{a^3}{20x^{20}} - \frac{3a^2b}{16x^{16}} - \frac{ab^2}{4x^{12}} - \frac{b^3}{8x^8}$$

input
```
Integrate[(a + b*x^4)^3/x^21,x]
```

output
```
-1/20*a^3/x^20 - (3*a^2*b)/(16*x^16) - (a*b^2)/(4*x^12) - b^3/(8*x^8)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^{21}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^3}{x^{24}}dx^4$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{4}\left(-\frac{b\int \frac{(bx^4+a)^3}{x^{20}}dx^4}{5a}-\frac{(a+bx^4)^4}{5ax^{20}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{4}\left(\frac{b(a+bx^4)^4}{20a^2x^{16}}-\frac{(a+bx^4)^4}{5ax^{20}}\right)$$

input
```
Int[(a + b*x^4)^3/x^21,x]
```

output
```
(-1/5*(a + b*x^4)^4/(a*x^20) + (b*(a + b*x^4)^4)/(20*a^2*x^16))/4
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{b^{3}}{8 x^{8}}-\frac{a^{3}}{20 x^{20}}-\frac{3 a^{2} b}{16 x^{16}}-\frac{a \,b^{2}}{4 x^{12}}$	36
norman	$\frac{-\frac{1}{20} a^{3}-\frac{3}{16} a^{2} b \,x^{4}-\frac{1}{4} a \,b^{2} x^{8}-\frac{1}{8} b^{3} x^{12}}{x^{20}}$	37
risch	$\frac{-\frac{1}{20} a^{3}-\frac{3}{16} a^{2} b \,x^{4}-\frac{1}{4} a \,b^{2} x^{8}-\frac{1}{8} b^{3} x^{12}}{x^{20}}$	37
gosper	$-\frac{10 b^{3} x^{12}+20 a \,b^{2} x^{8}+15 a^{2} b \,x^{4}+4 a^{3}}{80 x^{20}}$	38
parallelrisch	$\frac{-10 b^{3} x^{12}-20 a \,b^{2} x^{8}-15 a^{2} b \,x^{4}-4 a^{3}}{80 x^{20}}$	38
orering	$-\frac{10 b^{3} x^{12}+20 a \,b^{2} x^{8}+15 a^{2} b \,x^{4}+4 a^{3}}{80 x^{20}}$	38

input
```
int((b*x^4+a)^3/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b^3/x^8-1/20*a^3/x^20-3/16*a^2*b/x^16-1/4*a*b^2/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = -\frac{10\,b^3x^{12}+20\,ab^2x^8+15\,a^2bx^4+4\,a^3}{80\,x^{20}}$$

input
```
integrate((b*x^4+a)^3/x^21,x, algorithm="fricas")
```

output
```
-1/80*(10*b^3*x^12 + 20*a*b^2*x^8 + 15*a^2*b*x^4 + 4*a^3)/x^20
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = \frac{-4a^3-15a^2bx^4-20ab^2x^8-10b^3x^{12}}{80x^{20}}$$

input
```
integrate((b*x**4+a)**3/x**21,x)
```

output
```
(-4*a**3 - 15*a**2*b*x**4 - 20*a*b**2*x**8 - 10*b**3*x**12)/(80*x**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = -\frac{10\,b^3x^{12}+20\,ab^2x^8+15\,a^2bx^4+4\,a^3}{80\,x^{20}}$$

input
```
integrate((b*x^4+a)^3/x^21,x, algorithm="maxima")
```

output
```
-1/80*(10*b^3*x^12 + 20*a*b^2*x^8 + 15*a^2*b*x^4 + 4*a^3)/x^20
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = -\frac{10\,b^3x^{12}+20\,ab^2x^8+15\,a^2bx^4+4\,a^3}{80\,x^{20}}$$

input
```
integrate((b*x^4+a)^3/x^21,x, algorithm="giac")
```

output
```
-1/80*(10*b^3*x^12 + 20*a*b^2*x^8 + 15*a^2*b*x^4 + 4*a^3)/x^20
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = -\frac{\frac{a^3}{20}+\frac{3\,a^2\,b\,x^4}{16}+\frac{a\,b^2\,x^8}{4}+\frac{b^3\,x^{12}}{8}}{x^{20}}$$

input
```
int((a + b*x^4)^3/x^21,x)
```

output
```
-(a^3/20 + (b^3*x^12)/8 + (3*a^2*b*x^4)/16 + (a*b^2*x^8)/4)/x^20
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^{21}}\,dx = \frac{-10b^3x^{12}-20a\,b^2x^8-15a^2b\,x^4-4a^3}{80x^{20}}$$

input
```
int((b*x^4+a)^3/x^21,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x**4 - 20*a*b**2*x**8 - 10*b**3*x**12)/(80*x**20)
```

3.41 $\int x^4(a+bx^4)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^4\left(a+bx^4\right)^3\,dx=\frac{a^3x^5}{5}+\frac{1}{3}a^2bx^9+\frac{3}{13}ab^2x^{13}+\frac{b^3x^{17}}{17}$$

output
```
1/5*a^3*x^5+1/3*a^2*b*x^9+3/13*a*b^2*x^13+1/17*b^3*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^4\right)^3\,dx=\frac{a^3x^5}{5}+\frac{1}{3}a^2bx^9+\frac{3}{13}ab^2x^{13}+\frac{b^3x^{17}}{17}$$

input
```
Integrate[x^4*(a + b*x^4)^3,x]
```

output
```
(a^3*x^5)/5 + (a^2*b*x^9)/3 + (3*a*b^2*x^13)/13 + (b^3*x^17)/17
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + bx^4)^3 \, dx$$

$$\Big\downarrow \; 802$$

$$\int (a^3x^4 + 3a^2bx^8 + 3ab^2x^{12} + b^3x^{16}) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3x^5}{5} + \frac{1}{3}a^2bx^9 + \frac{3}{13}ab^2x^{13} + \frac{b^3x^{17}}{17}$$

input
```
Int[x^4*(a + b*x^4)^3,x]
```

output
```
(a^3*x^5)/5 + (a^2*b*x^9)/3 + (3*a*b^2*x^13)/13 + (b^3*x^17)/17
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5+\frac{1}{3}a^2b\,x^9+\frac{3}{13}a\,b^2x^{13}+\frac{1}{17}b^3x^{17}$	36
default	$\frac{1}{5}a^3x^5+\frac{1}{3}a^2b\,x^9+\frac{3}{13}a\,b^2x^{13}+\frac{1}{17}b^3x^{17}$	36
norman	$\frac{1}{5}a^3x^5+\frac{1}{3}a^2b\,x^9+\frac{3}{13}a\,b^2x^{13}+\frac{1}{17}b^3x^{17}$	36
risch	$\frac{1}{5}a^3x^5+\frac{1}{3}a^2b\,x^9+\frac{3}{13}a\,b^2x^{13}+\frac{1}{17}b^3x^{17}$	36
parallelrisch	$\frac{1}{5}a^3x^5+\frac{1}{3}a^2b\,x^9+\frac{3}{13}a\,b^2x^{13}+\frac{1}{17}b^3x^{17}$	36
orering	$\frac{x^5(195b^3x^{12}+765a\,b^2x^8+1105a^2b\,x^4+663a^3)}{3315}$	38

input
```
int(x^4*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+1/3*a^2*b*x^9+3/13*a*b^2*x^13+1/17*b^3*x^17
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx^4)^3\,dx=\frac{1}{17}\,b^3x^{17}+\frac{3}{13}\,ab^2x^{13}+\frac{1}{3}\,a^2bx^9+\frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/17*b^3*x^17 + 3/13*a*b^2*x^13 + 1/3*a^2*b*x^9 + 1/5*a^3*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^4 (a+bx^4)^3 \, dx = \frac{a^3x^5}{5} + \frac{a^2bx^9}{3} + \frac{3ab^2x^{13}}{13} + \frac{b^3x^{17}}{17}$$

input
```
integrate(x**4*(b*x**4+a)**3,x)
```

output
```
a**3*x**5/5 + a**2*b*x**9/3 + 3*a*b**2*x**13/13 + b**3*x**17/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 (a+bx^4)^3 \, dx = \frac{1}{17} b^3x^{17} + \frac{3}{13} ab^2x^{13} + \frac{1}{3} a^2bx^9 + \frac{1}{5} a^3x^5$$

input
```
integrate(x^4*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/17*b^3*x^17 + 3/13*a*b^2*x^13 + 1/3*a^2*b*x^9 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 (a+bx^4)^3 \, dx = \frac{1}{17} b^3x^{17} + \frac{3}{13} ab^2x^{13} + \frac{1}{3} a^2bx^9 + \frac{1}{5} a^3x^5$$

input
```
integrate(x^4*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/17*b^3*x^17 + 3/13*a*b^2*x^13 + 1/3*a^2*b*x^9 + 1/5*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx^4)^3\,dx=\frac{a^3\,x^5}{5}+\frac{a^2\,b\,x^9}{3}+\frac{3\,a\,b^2\,x^{13}}{13}+\frac{b^3\,x^{17}}{17}$$

input
```
int(x^4*(a + b*x^4)^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^17)/17 + (a^2*b*x^9)/3 + (3*a*b^2*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^4(a+bx^4)^3\,dx=\frac{x^5(195b^3x^{12}+765a\,b^2x^8+1105a^2b\,x^4+663a^3)}{3315}$$

input
```
int(x^4*(b*x^4+a)^3,x)
```

output
```
(x**5*(663*a**3 + 1105*a**2*b*x**4 + 765*a*b**2*x**8 + 195*b**3*x**12))/33
15
```

3.42 $\int x^2(a+bx^4)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^2\left(a+bx^4\right)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{7}a^2bx^7+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{15}}{15}$$

output
```
1/3*a^3*x^3+3/7*a^2*b*x^7+3/11*a*b^2*x^11+1/15*b^3*x^15
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^4\right)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{7}a^2bx^7+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{15}}{15}$$

input
```
Integrate[x^2*(a + b*x^4)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^11)/11 + (b^3*x^15)/15
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + bx^4)^3 \, dx$$

$$\Big\downarrow \text{ 802}$$

$$\int (a^3x^2 + 3a^2bx^6 + 3ab^2x^{10} + b^3x^{14}) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3x^3}{3} + \frac{3}{7}a^2bx^7 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{15}}{15}$$

input
```
Int[x^2*(a + b*x^4)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^11)/11 + (b^3*x^15)/15
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{3}a^3x^3+\frac{3}{7}a^2b\,x^7+\frac{3}{11}a\,b^2x^{11}+\frac{1}{15}b^3x^{15}$	36
default	$\frac{1}{3}a^3x^3+\frac{3}{7}a^2b\,x^7+\frac{3}{11}a\,b^2x^{11}+\frac{1}{15}b^3x^{15}$	36
norman	$\frac{1}{3}a^3x^3+\frac{3}{7}a^2b\,x^7+\frac{3}{11}a\,b^2x^{11}+\frac{1}{15}b^3x^{15}$	36
risch	$\frac{1}{3}a^3x^3+\frac{3}{7}a^2b\,x^7+\frac{3}{11}a\,b^2x^{11}+\frac{1}{15}b^3x^{15}$	36
parallelrisch	$\frac{1}{3}a^3x^3+\frac{3}{7}a^2b\,x^7+\frac{3}{11}a\,b^2x^{11}+\frac{1}{15}b^3x^{15}$	36
orering	$\frac{x^3(77b^3x^{12}+315a\,b^2x^8+495a^2b\,x^4+385a^3)}{1155}$	38

input
```
int(x^2*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+3/7*a^2*b*x^7+3/11*a*b^2*x^11+1/15*b^3*x^15
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2(a+bx^4)^3\,dx=\frac{1}{15}\,b^3x^{15}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{7}\,a^2bx^7+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/15*b^3*x^15 + 3/11*a*b^2*x^11 + 3/7*a^2*b*x^7 + 1/3*a^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^2 (a + bx^4)^3 \, dx = \frac{a^3x^3}{3} + \frac{3a^2bx^7}{7} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{15}}{15}$$

input
```
integrate(x**2*(b*x**4+a)**3,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x**7/7 + 3*a*b**2*x**11/11 + b**3*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2 (a + bx^4)^3 \, dx = \frac{1}{15} b^3x^{15} + \frac{3}{11} ab^2x^{11} + \frac{3}{7} a^2bx^7 + \frac{1}{3} a^3x^3$$

input
```
integrate(x^2*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/15*b^3*x^15 + 3/11*a*b^2*x^11 + 3/7*a^2*b*x^7 + 1/3*a^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2 (a + bx^4)^3 \, dx = \frac{1}{15} b^3x^{15} + \frac{3}{11} ab^2x^{11} + \frac{3}{7} a^2bx^7 + \frac{1}{3} a^3x^3$$

input
```
integrate(x^2*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/15*b^3*x^15 + 3/11*a*b^2*x^11 + 3/7*a^2*b*x^7 + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2 (a + bx^4)^3 \, dx = \frac{a^3 x^3}{3} + \frac{3 a^2 b x^7}{7} + \frac{3 a b^2 x^{11}}{11} + \frac{b^3 x^{15}}{15}$$

input
```
int(x^2*(a + b*x^4)^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^15)/15 + (3*a^2*b*x^7)/7 + (3*a*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^2 (a + bx^4)^3 \, dx = \frac{x^3(77b^3x^{12} + 315a b^2x^8 + 495a^2b x^4 + 385a^3)}{1155}$$

input
```
int(x^2*(b*x^4+a)^3,x)
```

output
```
(x**3*(385*a**3 + 495*a**2*b*x**4 + 315*a*b**2*x**8 + 77*b**3*x**12))/1155
```

3.43 $\int x(a+bx^4)^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int x\left(a+bx^4\right)^3\,dx=\frac{a^3x^2}{2}+\frac{1}{2}a^2bx^6+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}$$

output
```
1/2*a^3*x^2+1/2*a^2*b*x^6+3/10*a*b^2*x^10+1/14*b^3*x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^4\right)^3\,dx=\frac{a^3x^2}{2}+\frac{1}{2}a^2bx^6+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}$$

input
```
Integrate[x*(a + b*x^4)^3,x]
```

output
```
(a^3*x^2)/2 + (a^2*b*x^6)/2 + (3*a*b^2*x^10)/10 + (b^3*x^14)/14
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^4)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int (a^3x+3a^2bx^5+3ab^2x^9+b^3x^{13})\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^2}{2}+\frac{1}{2}a^2bx^6+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{14}}{14}$$

input
```
Int[x*(a + b*x^4)^3,x]
```

output
```
(a^3*x^2)/2 + (a^2*b*x^6)/2 + (3*a*b^2*x^10)/10 + (b^3*x^14)/14
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{2}a^3x^2+\frac{1}{2}a^2b\,x^6+\frac{3}{10}a\,b^2x^{10}+\frac{1}{14}b^3x^{14}$	36
default	$\frac{1}{2}a^3x^2+\frac{1}{2}a^2b\,x^6+\frac{3}{10}a\,b^2x^{10}+\frac{1}{14}b^3x^{14}$	36
norman	$\frac{1}{2}a^3x^2+\frac{1}{2}a^2b\,x^6+\frac{3}{10}a\,b^2x^{10}+\frac{1}{14}b^3x^{14}$	36
risch	$\frac{1}{2}a^3x^2+\frac{1}{2}a^2b\,x^6+\frac{3}{10}a\,b^2x^{10}+\frac{1}{14}b^3x^{14}$	36
parallelrisch	$\frac{1}{2}a^3x^2+\frac{1}{2}a^2b\,x^6+\frac{3}{10}a\,b^2x^{10}+\frac{1}{14}b^3x^{14}$	36
orering	$\frac{x^2(5b^3x^{12}+21a\,b^2x^8+35a^2b\,x^4+35a^3)}{70}$	38

input
```
int(x*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+1/2*a^2*b*x^6+3/10*a*b^2*x^10+1/14*b^3*x^14
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^4)^3\,dx=\frac{1}{14}\,b^3x^{14}+\frac{3}{10}\,ab^2x^{10}+\frac{1}{2}\,a^2bx^6+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/14*b^3*x^14 + 3/10*a*b^2*x^10 + 1/2*a^2*b*x^6 + 1/2*a^3*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x(a+bx^4)^3\,dx=\frac{a^3x^2}{2}+\frac{a^2bx^6}{2}+\frac{3ab^2x^{10}}{10}+\frac{b^3x^{14}}{14}$$

input
```
integrate(x*(b*x**4+a)**3,x)
```

output
```
a**3*x**2/2 + a**2*b*x**6/2 + 3*a*b**2*x**10/10 + b**3*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^4)^3\,dx=\frac{1}{14}\,b^3x^{14}+\frac{3}{10}\,ab^2x^{10}+\frac{1}{2}\,a^2bx^6+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/14*b^3*x^14 + 3/10*a*b^2*x^10 + 1/2*a^2*b*x^6 + 1/2*a^3*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^4)^3\,dx=\frac{1}{14}\,b^3x^{14}+\frac{3}{10}\,ab^2x^{10}+\frac{1}{2}\,a^2bx^6+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/14*b^3*x^14 + 3/10*a*b^2*x^10 + 1/2*a^2*b*x^6 + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x(a+bx^4)^3\,dx=\frac{a^3\,x^2}{2}+\frac{a^2\,b\,x^6}{2}+\frac{3\,a\,b^2\,x^{10}}{10}+\frac{b^3\,x^{14}}{14}$$

input
```
int(x*(a + b*x^4)^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^14)/14 + (a^2*b*x^6)/2 + (3*a*b^2*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x(a+bx^4)^3\,dx=\frac{x^2(5b^3x^{12}+21a\,b^2x^8+35a^2b\,x^4+35a^3)}{70}$$

input
```
int(x*(b*x^4+a)^3,x)
```

output
```
(x**2*(35*a**3 + 35*a**2*b*x**4 + 21*a*b**2*x**8 + 5*b**3*x**12))/70
```

3.44 $\int (a+bx^4)^3\,dx$

Optimal result

Integrand size = 9, antiderivative size = 38

$$\int (a+bx^4)^3\,dx = a^3x+\frac{3}{5}a^2bx^5+\frac{1}{3}ab^2x^9+\frac{b^3x^{13}}{13}$$

output
```
a^3*x+3/5*a^2*b*x^5+1/3*a*b^2*x^9+1/13*b^3*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int (a+bx^4)^3\,dx = a^3x+\frac{3}{5}a^2bx^5+\frac{1}{3}ab^2x^9+\frac{b^3x^{13}}{13}$$

input
```
Integrate[(a + b*x^4)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^5)/5 + (a*b^2*x^9)/3 + (b^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^3\,dx$$

$$\Big\downarrow \text{747}$$

$$\int (a^3+3a^2bx^4+3ab^2x^8+b^3x^{12})\,dx$$

$$\Big\downarrow \text{2009}$$

$$a^3x+\frac{3}{5}a^2bx^5+\frac{1}{3}ab^2x^9+\frac{b^3x^{13}}{13}$$

input
```
Int[(a + b*x^4)^3,x]
```

output
```
a^3*x + (3*a^2*b*x^5)/5 + (a*b^2*x^9)/3 + (b^3*x^13)/13
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
gosper	$a^3x+\frac{3}{5}a^2b\,x^5+\frac{1}{3}a\,b^2x^9+\frac{1}{13}b^3x^{13}$	33
default	$a^3x+\frac{3}{5}a^2b\,x^5+\frac{1}{3}a\,b^2x^9+\frac{1}{13}b^3x^{13}$	33
norman	$a^3x+\frac{3}{5}a^2b\,x^5+\frac{1}{3}a\,b^2x^9+\frac{1}{13}b^3x^{13}$	33
risch	$a^3x+\frac{3}{5}a^2b\,x^5+\frac{1}{3}a\,b^2x^9+\frac{1}{13}b^3x^{13}$	33
parallelrisch	$a^3x+\frac{3}{5}a^2b\,x^5+\frac{1}{3}a\,b^2x^9+\frac{1}{13}b^3x^{13}$	33
orering	$\frac{x(15b^3x^{12}+65a\,b^2x^8+117a^2b\,x^4+195a^3)}{195}$	36

input
```
int((b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+3/5*a^2*b*x^5+1/3*a*b^2*x^9+1/13*b^3*x^13
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^3\,dx=\frac{1}{13}\,b^3x^{13}+\frac{1}{3}\,ab^2x^9+\frac{3}{5}\,a^2bx^5+a^3x$$

input
```
integrate((b*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/13*b^3*x^13 + 1/3*a*b^2*x^9 + 3/5*a^2*b*x^5 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \left(a+bx^4\right)^3\,dx=a^3x+\frac{3a^2bx^5}{5}+\frac{ab^2x^9}{3}+\frac{b^3x^{13}}{13}$$

input
```
integrate((b*x**4+a)**3,x)
```

output
```
a**3*x + 3*a**2*b*x**5/5 + a*b**2*x**9/3 + b**3*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^3\,dx=\frac{1}{13}\,b^3x^{13}+\frac{1}{3}\,ab^2x^9+\frac{3}{5}\,a^2bx^5+a^3x$$

input
```
integrate((b*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/13*b^3*x^13 + 1/3*a*b^2*x^9 + 3/5*a^2*b*x^5 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^3\,dx=\frac{1}{13}\,b^3x^{13}+\frac{1}{3}\,ab^2x^9+\frac{3}{5}\,a^2bx^5+a^3x$$

input
```
integrate((b*x^4+a)^3,x, algorithm="giac")
```

output
```
1/13*b^3*x^13 + 1/3*a*b^2*x^9 + 3/5*a^2*b*x^5 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \left(a+bx^4\right)^3 \, dx = a^3\,x + \frac{3\,a^2\,b\,x^5}{5} + \frac{a\,b^2\,x^9}{3} + \frac{b^3\,x^{13}}{13}$$

input
```
int((a + b*x^4)^3,x)
```

output
```
a^3*x + (b^3*x^13)/13 + (3*a^2*b*x^5)/5 + (a*b^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \left(a+bx^4\right)^3 \, dx = \frac{x(15b^3x^{12} + 65a\,b^2x^8 + 117a^2b\,x^4 + 195a^3)}{195}$$

input
```
int((b*x^4+a)^3,x)
```

output
```
(x*(195*a**3 + 117*a**2*b*x**4 + 65*a*b**2*x**8 + 15*b**3*x**12))/195
```

3.45 $\int \frac{(a+bx^4)^3}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = -\frac{a^3}{x} + a^2bx^3 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{11}}{11}$$

output
```
-a^3/x+a^2*b*x^3+3/7*a*b^2*x^7+1/11*b^3*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = -\frac{a^3}{x} + a^2bx^3 + \frac{3}{7}ab^2x^7 + \frac{b^3x^{11}}{11}$$

input
```
Integrate[(a + b*x^4)^3/x^2,x]
```

output
```
-(a^3/x) + a^2*b*x^3 + (3*a*b^2*x^7)/7 + (b^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^3}{x^2} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^2}+3a^2bx^2+3ab^2x^6+b^3x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{x}+a^2bx^3+\frac{3}{7}ab^2x^7+\frac{b^3x^{11}}{11}$$

input
```
Int[(a + b*x^4)^3/x^2,x]
```

output
```
-(a^3/x) + a^2*b*x^3 + (3*a*b^2*x^7)/7 + (b^3*x^11)/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{3}}{x}+a^{2}b\,x^{3}+\frac{3a\,b^{2}x^{7}}{7}+\frac{b^{3}x^{11}}{11}$	35
risch	$-\frac{a^{3}}{x}+a^{2}b\,x^{3}+\frac{3a\,b^{2}x^{7}}{7}+\frac{b^{3}x^{11}}{11}$	35
norman	$\frac{\frac{1}{11}b^{3}x^{12}+\frac{3}{7}a\,b^{2}x^{8}+a^{2}b\,x^{4}-a^{3}}{x}$	36
gosper	$-\frac{-7b^{3}x^{12}-33a\,b^{2}x^{8}-77a^{2}b\,x^{4}+77a^{3}}{77x}$	38
parallelrisch	$\frac{7b^{3}x^{12}+33a\,b^{2}x^{8}+77a^{2}b\,x^{4}-77a^{3}}{77x}$	38
orering	$-\frac{-7b^{3}x^{12}-33a\,b^{2}x^{8}-77a^{2}b\,x^{4}+77a^{3}}{77x}$	38

input
```
int((b*x^4+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+a^2*b*x^3+3/7*a*b^2*x^7+1/11*b^3*x^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^3}{x^2}\,dx=\frac{7\,b^3x^{12}+33\,ab^2x^8+77\,a^2bx^4-77\,a^3}{77\,x}$$

input
```
integrate((b*x^4+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/77*(7*b^3*x^12 + 33*a*b^2*x^8 + 77*a^2*b*x^4 - 77*a^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = -\frac{a^3}{x} + a^2bx^3 + \frac{3ab^2x^7}{7} + \frac{b^3x^{11}}{11}$$

input
```
integrate((b*x**4+a)**3/x**2,x)
```

output
```
-a**3/x + a**2*b*x**3 + 3*a*b**2*x**7/7 + b**3*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = \frac{1}{11}\,b^3x^{11} + \frac{3}{7}\,ab^2x^7 + a^2bx^3 - \frac{a^3}{x}$$

input
```
integrate((b*x^4+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/11*b^3*x^11 + 3/7*a*b^2*x^7 + a^2*b*x^3 - a^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = \frac{1}{11}\,b^3x^{11} + \frac{3}{7}\,ab^2x^7 + a^2bx^3 - \frac{a^3}{x}$$

input
```
integrate((b*x^4+a)^3/x^2,x, algorithm="giac")
```

output
```
1/11*b^3*x^11 + 3/7*a*b^2*x^7 + a^2*b*x^3 - a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = \frac{b^3\,x^{11}}{11} - \frac{a^3}{x} + a^2\,b\,x^3 + \frac{3\,a\,b^2\,x^7}{7}$$

input
```
int((a + b*x^4)^3/x^2,x)
```

output
```
(b^3*x^11)/11 - a^3/x + a^2*b*x^3 + (3*a*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^3}{x^2}\,dx = \frac{7b^3x^{12} + 33a\,b^2x^8 + 77a^2b\,x^4 - 77a^3}{77x}$$

input
```
int((b*x^4+a)^3/x^2,x)
```

output
```
( - 77*a**3 + 77*a**2*b*x**4 + 33*a*b**2*x**8 + 7*b**3*x**12)/(77*x)
```

3.46 $\int \frac{(a+bx^4)^3}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = -\frac{a^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}$$

output
```
-1/2*a^3/x^2+3/2*a^2*b*x^2+1/2*a*b^2*x^6+1/10*b^3*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = -\frac{a^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}$$

input
```
Integrate[(a + b*x^4)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 + (3*a^2*b*x^2)/2 + (a*b^2*x^6)/2 + (b^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^3} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^3}+3a^2bx+3ab^2x^5+b^3x^9\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{2x^2}+\frac{3}{2}a^2bx^2+\frac{1}{2}ab^2x^6+\frac{b^3x^{10}}{10}$$

input
```
Int[(a + b*x^4)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 + (3*a^2*b*x^2)/2 + (a*b^2*x^6)/2 + (b^3*x^10)/10
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^3}{2x^2}+\frac{3a^2b\,x^2}{2}+\frac{a\,b^2x^6}{2}+\frac{b^3x^{10}}{10}$	36
risch	$-\frac{a^3}{2x^2}+\frac{3a^2b\,x^2}{2}+\frac{a\,b^2x^6}{2}+\frac{b^3x^{10}}{10}$	36
norman	$\frac{\frac{1}{10}b^3x^{12}+\frac{1}{2}a\,b^2x^8+\frac{3}{2}a^2b\,x^4-\frac{1}{2}a^3}{x^2}$	37
parallelrisch	$\frac{b^3x^{12}+5a\,b^2x^8+15a^2b\,x^4-5a^3}{10x^2}$	37
gosper	$-\frac{-b^3x^{12}-5a\,b^2x^8-15a^2b\,x^4+5a^3}{10x^2}$	38
orering	$-\frac{-b^3x^{12}-5a\,b^2x^8-15a^2b\,x^4+5a^3}{10x^2}$	38

input
```
int((b*x^4+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2+3/2*a^2*b*x^2+1/2*a*b^2*x^6+1/10*b^3*x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)^3}{x^3}\,dx=\frac{b^3x^{12}+5\,ab^2x^8+15\,a^2bx^4-5\,a^3}{10\,x^2}$$

input
```
integrate((b*x^4+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/10*(b^3*x^12 + 5*a*b^2*x^8 + 15*a^2*b*x^4 - 5*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + \frac{3a^2bx^2}{2} + \frac{ab^2x^6}{2} + \frac{b^3x^{10}}{10}$$

input
```
integrate((b*x**4+a)**3/x**3,x)
```

output
```
-a**3/(2*x**2) + 3*a**2*b*x**2/2 + a*b**2*x**6/2 + b**3*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = \frac{1}{10}\,b^3x^{10} + \frac{1}{2}\,ab^2x^6 + \frac{3}{2}\,a^2bx^2 - \frac{a^3}{2\,x^2}$$

input
```
integrate((b*x^4+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 1/2*a*b^2*x^6 + 3/2*a^2*b*x^2 - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = \frac{1}{10}\,b^3x^{10} + \frac{1}{2}\,ab^2x^6 + \frac{3}{2}\,a^2bx^2 - \frac{a^3}{2\,x^2}$$

input
```
integrate((b*x^4+a)^3/x^3,x, algorithm="giac")
```

output
```
1/10*b^3*x^10 + 1/2*a*b^2*x^6 + 3/2*a^2*b*x^2 - 1/2*a^3/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = \frac{b^3\,x^{10}}{10} - \frac{a^3}{2\,x^2} + \frac{3\,a^2\,b\,x^2}{2} + \frac{a\,b^2\,x^6}{2}$$

input
```
int((a + b*x^4)^3/x^3,x)
```

output
```
(b^3*x^10)/10 - a^3/(2*x^2) + (3*a^2*b*x^2)/2 + (a*b^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)^3}{x^3}\,dx = \frac{b^3x^{12} + 5a\,b^2x^8 + 15a^2b\,x^4 - 5a^3}{10x^2}$$

input
```
int((b*x^4+a)^3/x^3,x)
```

output
```
( - 5*a**3 + 15*a**2*b*x**4 + 5*a*b**2*x**8 + b**3*x**12)/(10*x**2)
```

3.47 $\int \frac{(a+bx^4)^3}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^4)^3}{x^4} dx = -\frac{a^3}{3x^3} + 3a^2bx + \frac{3}{5}ab^2x^5 + \frac{b^3x^9}{9}$$

output
```
-1/3*a^3/x^3+3*a^2*b*x+3/5*a*b^2*x^5+1/9*b^3*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^4} dx = -\frac{a^3}{3x^3} + 3a^2bx + \frac{3}{5}ab^2x^5 + \frac{b^3x^9}{9}$$

input
```
Integrate[(a + b*x^4)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 + 3*a^2*b*x + (3*a*b^2*x^5)/5 + (b^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^4}\,dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a^3}{x^4}+3a^2b+3ab^2x^4+b^3x^8\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3}{3x^3}+3a^2bx+\frac{3}{5}ab^2x^5+\frac{b^3x^9}{9}$$

input
```
Int[(a + b*x^4)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 + 3*a^2*b*x + (3*a*b^2*x^5)/5 + (b^3*x^9)/9
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{3}}{3x^{3}}+3a^{2}bx+\frac{3a\,b^{2}x^{5}}{5}+\frac{b^{3}x^{9}}{9}$	34
risch	$-\frac{a^{3}}{3x^{3}}+3a^{2}bx+\frac{3a\,b^{2}x^{5}}{5}+\frac{b^{3}x^{9}}{9}$	34
norman	$\frac{\frac{1}{9}b^{3}x^{12}+\frac{3}{5}a\,b^{2}x^{8}+3a^{2}b\,x^{4}-\frac{1}{3}a^{3}}{x^{3}}$	37
gosper	$-\frac{-5b^{3}x^{12}-27a\,b^{2}x^{8}-135a^{2}b\,x^{4}+15a^{3}}{45x^{3}}$	38
parallelrisch	$\frac{5b^{3}x^{12}+27a\,b^{2}x^{8}+135a^{2}b\,x^{4}-15a^{3}}{45x^{3}}$	38
orering	$-\frac{-5b^{3}x^{12}-27a\,b^{2}x^{8}-135a^{2}b\,x^{4}+15a^{3}}{45x^{3}}$	38

input
```
int((b*x^4+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3/x^3+3*a^2*b*x+3/5*a*b^2*x^5+1/9*b^3*x^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^4}\,dx=\frac{5\,b^3x^{12}+27\,ab^2x^8+135\,a^2bx^4-15\,a^3}{45\,x^3}$$

input
```
integrate((b*x^4+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/45*(5*b^3*x^12 + 27*a*b^2*x^8 + 135*a^2*b*x^4 - 15*a^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^4}\,dx = -\frac{a^3}{3x^3} + 3a^2bx + \frac{3ab^2x^5}{5} + \frac{b^3x^9}{9}$$

input
```
integrate((b*x**4+a)**3/x**4,x)
```

output
```
-a**3/(3*x**3) + 3*a**2*b*x + 3*a*b**2*x**5/5 + b**3*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^3}{x^4}\,dx = \frac{1}{9}\,b^3x^9 + \frac{3}{5}\,ab^2x^5 + 3\,a^2bx - \frac{a^3}{3\,x^3}$$

input
```
integrate((b*x^4+a)^3/x^4,x, algorithm="maxima")
```

output
```
1/9*b^3*x^9 + 3/5*a*b^2*x^5 + 3*a^2*b*x - 1/3*a^3/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^3}{x^4}\,dx = \frac{1}{9}\,b^3x^9 + \frac{3}{5}\,ab^2x^5 + 3\,a^2bx - \frac{a^3}{3\,x^3}$$

input
```
integrate((b*x^4+a)^3/x^4,x, algorithm="giac")
```

output
```
1/9*b^3*x^9 + 3/5*a*b^2*x^5 + 3*a^2*b*x - 1/3*a^3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^4)^3}{x^4}\,dx = \frac{b^3\,x^9}{9} - \frac{a^3}{3\,x^3} + \frac{3\,a\,b^2\,x^5}{5} + 3\,a^2\,b\,x$$

input
```
int((a + b*x^4)^3/x^4,x)
```

output
```
(b^3*x^9)/9 - a^3/(3*x^3) + (3*a*b^2*x^5)/5 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^4}\,dx = \frac{5b^3x^{12} + 27a\,b^2x^8 + 135a^2b\,x^4 - 15a^3}{45x^3}$$

input
```
int((b*x^4+a)^3/x^4,x)
```

output
```
( - 15*a**3 + 135*a**2*b*x**4 + 27*a*b**2*x**8 + 5*b**3*x**12)/(45*x**3)
```

3.48 $\int \frac{(a+bx^4)^3}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{(a+bx^4)^3}{x^6} dx = -\frac{a^3}{5x^5} - \frac{3a^2b}{x} + ab^2x^3 + \frac{b^3x^7}{7}$$

output
```
-1/5*a^3/x^5-3*a^2*b/x+a*b^2*x^3+1/7*b^3*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^6} dx = -\frac{a^3}{5x^5} - \frac{3a^2b}{x} + ab^2x^3 + \frac{b^3x^7}{7}$$

input
```
Integrate[(a + b*x^4)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/x + a*b^2*x^3 + (b^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^6} \, dx$$

$$\Big\downarrow \; 802$$

$$\int \left(\frac{a^3}{x^6}+\frac{3a^2b}{x^2}+3ab^2x^2+b^3x^6\right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{x}+ab^2x^3+\frac{b^3x^7}{7}$$

input
```
Int[(a + b*x^4)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (3*a^2*b)/x + a*b^2*x^3 + (b^3*x^7)/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{3}}{5x^{5}}-\frac{3a^{2}b}{x}+a\,b^{2}x^{3}+\frac{b^{3}x^{7}}{7}$	35
norman	$\frac{\frac{1}{7}b^{3}x^{12}+a\,b^{2}x^{8}-3a^{2}b\,x^{4}-\frac{1}{5}a^{3}}{x^{5}}$	36
risch	$\frac{b^{3}x^{7}}{7}+a\,b^{2}x^{3}+\frac{-3a^{2}b\,x^{4}-\frac{1}{5}a^{3}}{x^{5}}$	37
gosper	$-\frac{-5b^{3}x^{12}-35a\,b^{2}x^{8}+105a^{2}b\,x^{4}+7a^{3}}{35x^{5}}$	38
parallelrisch	$\frac{5b^{3}x^{12}+35a\,b^{2}x^{8}-105a^{2}b\,x^{4}-7a^{3}}{35x^{5}}$	38
orering	$-\frac{-5b^{3}x^{12}-35a\,b^{2}x^{8}+105a^{2}b\,x^{4}+7a^{3}}{35x^{5}}$	38

input
```
int((b*x^4+a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^3/x^5-3*a^2*b/x+a*b^2*x^3+1/7*b^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^3}{x^6}\,dx=\frac{5\,b^3x^{12}+35\,ab^2x^8-105\,a^2bx^4-7\,a^3}{35\,x^5}$$

input
```
integrate((b*x^4+a)^3/x^6,x, algorithm="fricas")
```

output
```
1/35*(5*b^3*x^12 + 35*a*b^2*x^8 - 105*a^2*b*x^4 - 7*a^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^6}\,dx = ab^2x^3 + \frac{b^3x^7}{7} + \frac{-a^3-15a^2bx^4}{5x^5}$$

input
```
integrate((b*x**4+a)**3/x**6,x)
```

output
```
a*b**2*x**3 + b**3*x**7/7 + (-a**3 - 15*a**2*b*x**4)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^6}\,dx = \frac{1}{7}\,b^3x^7 + ab^2x^3 - \frac{15\,a^2bx^4+a^3}{5\,x^5}$$

input
```
integrate((b*x^4+a)^3/x^6,x, algorithm="maxima")
```

output
```
1/7*b^3*x^7 + a*b^2*x^3 - 1/5*(15*a^2*b*x^4 + a^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^6}\,dx = \frac{1}{7}\,b^3x^7 + ab^2x^3 - \frac{15\,a^2bx^4+a^3}{5\,x^5}$$

input
```
integrate((b*x^4+a)^3/x^6,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 + a*b^2*x^3 - 1/5*(15*a^2*b*x^4 + a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^3}{x^6}\,dx = \frac{b^3\,x^7}{7} - \frac{\frac{a^3}{5} + 3\,b\,a^2\,x^4}{x^5} + a\,b^2\,x^3$$

input
```
int((a + b*x^4)^3/x^6,x)
```

output
```
(b^3*x^7)/7 - (a^3/5 + 3*a^2*b*x^4)/x^5 + a*b^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^3}{x^6}\,dx = \frac{5b^3x^{12} + 35a\,b^2x^8 - 105a^2b\,x^4 - 7a^3}{35x^5}$$

input
```
int((b*x^4+a)^3/x^6,x)
```

output
```
( - 7*a**3 - 105*a**2*b*x**4 + 35*a*b**2*x**8 + 5*b**3*x**12)/(35*x**5)
```

3.49 $\int \frac{(a+bx^4)^3}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^4)^3}{x^7} dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^6}{6}$$

output
```
-1/6*a^3/x^6-3/2*a^2*b/x^2+3/2*a*b^2*x^2+1/6*b^3*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^7} dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^6}{6}$$

input
```
Integrate[(a + b*x^4)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (3*a^2*b)/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^6)/6
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^7} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^3}+3ab^2x+b^3x^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^6}{6}$$

input
```
Int[(a + b*x^4)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (3*a^2*b)/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^6)/6
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{-b^3x^{12}-9a\,b^2x^8+9a^2b\,x^4+a^3}{6x^6}$	36
default	$-\frac{a^3}{6x^6}-\frac{3a^2b}{2x^2}+\frac{3a\,b^2x^2}{2}+\frac{b^3x^6}{6}$	36
orering	$-\frac{-b^3x^{12}-9a\,b^2x^8+9a^2b\,x^4+a^3}{6x^6}$	36
norman	$\frac{\frac{1}{6}b^3x^{12}+\frac{3}{2}a\,b^2x^8-\frac{3}{2}a^2b\,x^4-\frac{1}{6}a^3}{x^6}$	37
parallelrisch	$\frac{b^3x^{12}+9a\,b^2x^8-9a^2b\,x^4-a^3}{6x^6}$	37
risch	$\frac{b^3x^6}{6}+\frac{3a\,b^2x^2}{2}+\frac{-\frac{3}{2}a^2b\,x^4-\frac{1}{6}a^3}{x^6}$	38

input
```
int((b*x^4+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b^3*x^12-9*a*b^2*x^8+9*a^2*b*x^4+a^3)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)^3}{x^7}\,dx=\frac{b^3x^{12}+9\,ab^2x^8-9\,a^2bx^4-a^3}{6\,x^6}$$

input
```
integrate((b*x^4+a)^3/x^7,x, algorithm="fricas")
```

output
```
1/6*(b^3*x^12 + 9*a*b^2*x^8 - 9*a^2*b*x^4 - a^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^3}{x^7}\,dx = \frac{3ab^2x^2}{2} + \frac{b^3x^6}{6} + \frac{-a^3-9a^2bx^4}{6x^6}$$

input
```
integrate((b*x**4+a)**3/x**7,x)
```

output
```
3*a*b**2*x**2/2 + b**3*x**6/6 + (-a**3 - 9*a**2*b*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)^3}{x^7}\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{2}\,ab^2x^2 - \frac{9\,a^2bx^4+a^3}{6\,x^6}$$

input
```
integrate((b*x^4+a)^3/x^7,x, algorithm="maxima")
```

output
```
1/6*b^3*x^6 + 3/2*a*b^2*x^2 - 1/6*(9*a^2*b*x^4 + a^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^4)^3}{x^7}\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{2}\,ab^2x^2 - \frac{9\,a^2bx^4+a^3}{6\,x^6}$$

input
```
integrate((b*x^4+a)^3/x^7,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 + 3/2*a*b^2*x^2 - 1/6*(9*a^2*b*x^4 + a^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+bx^4\right)^3}{x^7}\,dx = \frac{b^3\,x^6}{6} - \frac{\frac{a^3}{6}+\frac{3\,b\,a^2\,x^4}{2}}{x^6} + \frac{3\,a\,b^2\,x^2}{2}$$

input
```
int((a + b*x^4)^3/x^7,x)
```

output
```
(b^3*x^6)/6 - (a^3/6 + (3*a^2*b*x^4)/2)/x^6 + (3*a*b^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+bx^4\right)^3}{x^7}\,dx = \frac{b^3x^{12}+9a\,b^2x^8-9a^2b\,x^4-a^3}{6x^6}$$

input
```
int((b*x^4+a)^3/x^7,x)
```

output
```
( - a**3 - 9*a**2*b*x**4 + 9*a*b**2*x**8 + b**3*x**12)/(6*x**6)
```

3.50 $\int \frac{(a+bx^4)^3}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{(a+bx^4)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{a^2b}{x^3} + 3ab^2x + \frac{b^3x^5}{5}$$

output
```
-1/7*a^3/x^7-a^2*b/x^3+3*a*b^2*x+1/5*b^3*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{a^2b}{x^3} + 3ab^2x + \frac{b^3x^5}{5}$$

input
```
Integrate[(a + b*x^4)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (a^2*b)/x^3 + 3*a*b^2*x + (b^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^3}{x^8} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^4}+3ab^2+b^3x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{7x^7}-\frac{a^2b}{x^3}+3ab^2x+\frac{b^3x^5}{5}$$

input
```
Int[(a + b*x^4)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (a^2*b)/x^3 + 3*a*b^2*x + (b^3*x^5)/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^3}{7x^7}-\frac{a^2b}{x^3}+3a\,b^2x+\frac{b^3x^5}{5}$	34
risch	$\frac{b^3x^5}{5}+3a\,b^2x+\frac{-a^2b\,x^4-\frac{1}{7}a^3}{x^7}$	36
norman	$\frac{\frac{1}{5}b^3x^{12}+3a\,b^2x^8-a^2b\,x^4-\frac{1}{7}a^3}{x^7}$	37
gosper	$-\frac{-7b^3x^{12}-105a\,b^2x^8+35a^2b\,x^4+5a^3}{35x^7}$	38
parallelrisch	$\frac{7b^3x^{12}+105a\,b^2x^8-35a^2b\,x^4-5a^3}{35x^7}$	38
orering	$-\frac{-7b^3x^{12}-105a\,b^2x^8+35a^2b\,x^4+5a^3}{35x^7}$	38

input
```
int((b*x^4+a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^3/x^7-a^2*b/x^3+3*a*b^2*x+1/5*b^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^8}\,dx=\frac{7\,b^3x^{12}+105\,ab^2x^8-35\,a^2bx^4-5\,a^3}{35\,x^7}$$

input
```
integrate((b*x^4+a)^3/x^8,x, algorithm="fricas")
```

output
```
1/35*(7*b^3*x^12 + 105*a*b^2*x^8 - 35*a^2*b*x^4 - 5*a^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^4)^3}{x^8}\,dx = 3ab^2x + \frac{b^3x^5}{5} + \frac{-a^3 - 7a^2bx^4}{7x^7}$$

input
```
integrate((b*x**4+a)**3/x**8,x)
```

output
```
3*a*b**2*x + b**3*x**5/5 + (-a**3 - 7*a**2*b*x**4)/(7*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^8}\,dx = \frac{1}{5}\,b^3x^5 + 3\,ab^2x - \frac{7\,a^2bx^4 + a^3}{7\,x^7}$$

input
```
integrate((b*x^4+a)^3/x^8,x, algorithm="maxima")
```

output
```
1/5*b^3*x^5 + 3*a*b^2*x - 1/7*(7*a^2*b*x^4 + a^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^4)^3}{x^8}\,dx = \frac{1}{5}\,b^3x^5 + 3\,ab^2x - \frac{7\,a^2bx^4 + a^3}{7\,x^7}$$

input
```
integrate((b*x^4+a)^3/x^8,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 + 3*a*b^2*x - 1/7*(7*a^2*b*x^4 + a^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^3}{x^8}\,dx = \frac{b^3\,x^5}{5} - \frac{\frac{a^3}{7} + b\,a^2\,x^4}{x^7} + 3\,a\,b^2\,x$$

input
```
int((a + b*x^4)^3/x^8,x)
```

output
```
(b^3*x^5)/5 - (a^3/7 + a^2*b*x^4)/x^7 + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^3}{x^8}\,dx = \frac{7b^3x^{12} + 105a\,b^2x^8 - 35a^2b\,x^4 - 5a^3}{35x^7}$$

input
```
int((b*x^4+a)^3/x^8,x)
```

output
```
( - 5*a**3 - 35*a**2*b*x**4 + 105*a*b**2*x**8 + 7*b**3*x**12)/(35*x**7)
```

3.51 $\int \frac{x^{11}}{a+cx^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^{11}}{a+cx^4} dx = -\frac{ax^4}{4c^2} + \frac{x^8}{8c} + \frac{a^2 \log (a+cx^4)}{4c^3}$$

output
```
-1/4*a*x^4/c^2+1/8*x^8/c+1/4*a^2*ln(c*x^4+a)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{a+cx^4} dx = -\frac{ax^4}{4c^2} + \frac{x^8}{8c} + \frac{a^2 \log (a+cx^4)}{4c^3}$$

input
```
Integrate[x^11/(a + c*x^4),x]
```

output
```
-1/4*(a*x^4)/c^2 + x^8/(8*c) + (a^2*Log[a + c*x^4])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{a+cx^4} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4} \int \frac{x^8}{cx^4+a}dx^4$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{4} \int \left(\frac{x^4}{c}+\frac{a^2}{c^2 (cx^4+a)}-\frac{a}{c^2}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} \left(\frac{a^2 \log (a+cx^4)}{c^3}-\frac{ax^4}{c^2}+\frac{x^8}{2c}\right)$$

input
```
Int[x^11/(a + c*x^4),x]
```

output
```
(-((a*x^4)/c^2) + x^8/(2*c) + (a^2*Log[a + c*x^4])/c^3)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{c^{2} x^{8}-2 a \,x^{4} c +2 a^{2} \ln \left(c \,x^{4}+a \right)}{8 c^{3}}$	34
default	$-\frac{-\frac{1}{2} c \,x^{8}+a \,x^{4}}{4 c^{2}}+\frac{a^{2} \ln \left(c \,x^{4}+a \right)}{4 c^{3}}$	35
norman	$-\frac{a \,x^{4}}{4 c^{2}}+\frac{x^{8}}{8 c}+\frac{a^{2} \ln \left(c \,x^{4}+a \right)}{4 c^{3}}$	35
risch	$\frac{x^{8}}{8 c}-\frac{a \,x^{4}}{4 c^{2}}+\frac{a^{2}}{8 c^{3}}+\frac{a^{2} \ln \left(c \,x^{4}+a \right)}{4 c^{3}}$	43

input
```
int(x^11/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/8*(c^2*x^8-2*a*x^4*c+2*a^2*ln(c*x^4+a))/c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{a+cx^4}\,dx=\frac{c^{2} x^{8} - 2 \, a c x^{4} + 2 \, a^{2} \log \left(c x^{4} + a\right)}{8 \, c^{3}}$$

input
```
integrate(x^11/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(c^2*x^8 - 2*a*c*x^4 + 2*a^2*log(c*x^4 + a))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{a+cx^4}\,dx = \frac{a^2\log\left(a+cx^4\right)}{4c^3} - \frac{ax^4}{4c^2} + \frac{x^8}{8c}$$

input
```
integrate(x**11/(c*x**4+a),x)
```

output
```
a**2*log(a + c*x**4)/(4*c**3) - a*x**4/(4*c**2) + x**8/(8*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{a+cx^4}\,dx = \frac{a^2\log\left(cx^4+a\right)}{4\,c^3} + \frac{cx^8-2\,ax^4}{8\,c^2}$$

input
```
integrate(x^11/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*a^2*log(c*x^4 + a)/c^3 + 1/8*(c*x^8 - 2*a*x^4)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}}{a+cx^4}\,dx = \frac{a^2\log\left(|cx^4+a|\right)}{4\,c^3} + \frac{cx^8-2\,ax^4}{8\,c^2}$$

input
```
integrate(x^11/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*a^2*log(abs(c*x^4 + a))/c^3 + 1/8*(c*x^8 - 2*a*x^4)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{a+cx^4}\,dx = \frac{2\,a^2\,\ln\left(c\,x^4+a\right)+c^2\,x^8-2\,a\,c\,x^4}{8\,c^3}$$

input
```
int(x^11/(a + c*x^4),x)
```

output
```
(2*a^2*log(a + c*x^4) + c^2*x^8 - 2*a*c*x^4)/(8*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \frac{x^{11}}{a+cx^4}\,dx = \frac{2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2+2\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2-2ac\,x^4+c^2x^8}{8c^3}$$

input
```
int(x^11/(c*x^4+a),x)
```

output
```
(2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 2*l
og(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 2*a*c*x**4
 + c**2*x**8)/(8*c**3)
```

3.52 $\int \frac{x^7}{a+cx^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^7}{a+cx^4} dx = \frac{x^4}{4c} - \frac{a \log(a+cx^4)}{4c^2}$$

output
```
1/4*x^4/c-1/4*a*ln(c*x^4+a)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{a+cx^4} dx = \frac{x^4}{4c} - \frac{a \log(a+cx^4)}{4c^2}$$

input
```
Integrate[x^7/(a + c*x^4),x]
```

output
```
x^4/(4*c) - (a*Log[a + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+cx^4}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{x^4}{cx^4+a}dx^4$$

$$\Big\downarrow \ 49$$

$$\frac{1}{4}\int \left(\frac{1}{c}-\frac{a}{c\,(cx^4+a)}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{x^4}{c}-\frac{a\log\left(a+cx^4\right)}{c^2}\right)$$

input
```
Int[x^7/(a + c*x^4),x]
```

output
```
(x^4/c - (a*Log[a + c*x^4])/c^2)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-c\,x^{4}+a\ln\left(c\,x^{4}+a\right)}{4c^{2}}$	23
default	$\frac{x^{4}}{4c}-\frac{a\ln\left(c\,x^{4}+a\right)}{4c^{2}}$	24
norman	$\frac{x^{4}}{4c}-\frac{a\ln\left(c\,x^{4}+a\right)}{4c^{2}}$	24
risch	$\frac{x^{4}}{4c}-\frac{a\ln\left(c\,x^{4}+a\right)}{4c^{2}}$	24

input
```
int(x^7/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-c*x^4+a*ln(c*x^4+a))/c^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{a+cx^4}\,dx = \frac{cx^4 - a\log\left(cx^4+a\right)}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(c*x^4 - a*log(c*x^4 + a))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{a+cx^4}\,dx = -\frac{a\log\left(a+cx^4\right)}{4c^2} + \frac{x^4}{4c}$$

input
```
integrate(x**7/(c*x**4+a),x)
```

output
```
-a*log(a + c*x**4)/(4*c**2) + x**4/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{a+cx^4}\,dx = \frac{x^4}{4\,c} - \frac{a\log\left(cx^4+a\right)}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*x^4/c - 1/4*a*log(c*x^4 + a)/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^7}{a+cx^4}\,dx = \frac{x^4}{4\,c} - \frac{a\log\left(|cx^4+a|\right)}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*x^4/c - 1/4*a*log(abs(c*x^4 + a))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{a+cx^4}\,dx = -\frac{a\,\ln\left(c\,x^4+a\right)-c\,x^4}{4\,c^2}$$

input
```
int(x^7/(a + c*x^4),x)
```

output
```
-(a*log(a + c*x^4) - c*x^4)/(4*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{x^7}{a+cx^4}\,dx = \frac{-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a-\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a+c\,x^4}{4c^2}$$

input
```
int(x^7/(c*x^4+a),x)
```

output
```
( - log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - log(c
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + c*x**4)/(4*c**2)
```

3.53 $\int \frac{x^3}{a+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x^3}{a+cx^4}\,dx=\frac{\log\left(a+cx^4\right)}{4c}$$

output
```
1/4*ln(c*x^4+a)/c
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+cx^4}\,dx=\frac{\log\left(a+cx^4\right)}{4c}$$

input
```
Integrate[x^3/(a + c*x^4),x]
```

output
```
Log[a + c*x^4]/(4*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+cx^4} \, dx$$

$$\Big\downarrow \; 792$$

$$\frac{\log \left(a+cx^4\right)}{4c}$$

input
```
Int[x^3/(a + c*x^4),x]
```

output
```
Log[a + c*x^4]/(4*c)
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln(c\,x^{4}+a)}{4c}$	14
default	$\frac{\ln(c\,x^{4}+a)}{4c}$	14
norman	$\frac{\ln(c\,x^{4}+a)}{4c}$	14
risch	$\frac{\ln(c\,x^{4}+a)}{4c}$	14
parallelrisch	$\frac{\ln(c\,x^{4}+a)}{4c}$	14

input
```
int(x^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(c*x^4+a)/c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a+cx^4}\,dx=\frac{\log\left(cx^4+a\right)}{4\,c}$$

input
```
integrate(x^3/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*log(c*x^4 + a)/c
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{a+cx^4}\,dx = \frac{\log\left(a+cx^4\right)}{4c}$$

input
```
integrate(x**3/(c*x**4+a),x)
```

output
```
log(a + c*x**4)/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a+cx^4}\,dx = \frac{\log\left(cx^4+a\right)}{4\,c}$$

input
```
integrate(x^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*log(c*x^4 + a)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+cx^4}\,dx = \frac{\log\left(|cx^4+a|\right)}{4\,c}$$

input
```
integrate(x^3/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*log(abs(c*x^4 + a))/c
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{a+cx^4}\,dx=\frac{\ln\left(c\,x^4+a\right)}{4\,c}$$

input
```
int(x^3/(a + c*x^4),x)
```

output
```
log(a + c*x^4)/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{x^3}{a+cx^4}\,dx=\frac{\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4c}$$

input
```
int(x^3/(c*x^4+a),x)
```

output
```
(log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2) + log(c**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2))/(4*c)
```

3.54 $\int \frac{1}{x(a+cx^4)} dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(a+cx^4)} dx = \frac{\log(x)}{a} - \frac{\log(a+cx^4)}{4a}$$

output
```
ln(x)/a-1/4*ln(c*x^4+a)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+cx^4)} dx = \frac{\log(x)}{a} - \frac{\log(a+cx^4)}{4a}$$

input
```
Integrate[1/(x*(a + c*x^4)),x]
```

output
```
Log[x]/a - Log[a + c*x^4]/(4*a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+cx^4\right)}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(cx^4+a\right)}dx^4$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^4}dx^4}{a}-\frac{c\int \frac{1}{cx^4+a}dx^4}{a}\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{c\int \frac{1}{cx^4+a}dx^4}{a}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}\left(\frac{\log\left(x^4\right)}{a}-\frac{\log\left(a+cx^4\right)}{a}\right)$$

input
```
Int[1/(x*(a + c*x^4)),x]
```

output
```
(Log[x^4]/a - Log[a + c*x^4]/a)/4
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(c\,x^{4}+a)}{4a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(c\,x^{4}+a)}{4a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(c\,x^{4}+a)}{4a}$	21
parallelrisch	$\frac{4\ln(x)-\ln(c\,x^{4}+a)}{4a}$	21

input
```
int(1/x/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/4*ln(c*x^4+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\left(a+cx^4\right)}\,dx = -\frac{\log\left(cx^4+a\right)-4\,\log\left(x\right)}{4\,a}$$

input
```
integrate(1/x/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(log(c*x^4 + a) - 4*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\left(a+cx^4\right)}\,dx = \frac{\log\left(x\right)}{a} - \frac{\log\left(\frac{a}{c}+x^4\right)}{4a}$$

input
```
integrate(1/x/(c*x**4+a),x)
```

output
```
log(x)/a - log(a/c + x**4)/(4*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+cx^4\right)}\,dx = -\frac{\log\left(cx^4+a\right)}{4\,a} + \frac{\log\left(x^4\right)}{4\,a}$$

input
```
integrate(1/x/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/4*log(c*x^4 + a)/a + 1/4*log(x^4)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(a+cx^4)}\,dx = \frac{\log\left(x^4\right)}{4\,a} - \frac{\log\left(|cx^4+a|\right)}{4\,a}$$

input
```
integrate(1/x/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*log(x^4)/a - 1/4*log(abs(c*x^4 + a))/a
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(a+cx^4)}\,dx = -\frac{\ln\left(c\,x^4+a\right)-4\,\ln\left(x\right)}{4\,a}$$

input
```
int(1/(x*(a + c*x^4)),x)
```

output
```
-(log(a + c*x^4) - 4*log(x))/(4*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\,(a+cx^4)}\,dx = \frac{-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+4\log(x)}{4a}$$

input
```
int(1/x/(c*x^4+a),x)
```

output
```
( - log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2) - log(c**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2) + 4*log(x))/(4*a)
```

3.55 $\int \frac{1}{x^5(a+cx^4)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{x^5\left(a+cx^4\right)} \, dx = -\frac{1}{4ax^4} - \frac{c\log(x)}{a^2} + \frac{c\log\left(a+cx^4\right)}{4a^2}$$

output
```
-1/4/a/x^4-c*ln(x)/a^2+1/4*c*ln(c*x^4+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\left(a+cx^4\right)} \, dx = -\frac{1}{4ax^4} - \frac{c\log(x)}{a^2} + \frac{c\log\left(a+cx^4\right)}{4a^2}$$

input
```
Integrate[1/(x^5*(a + c*x^4)),x]
```

output
```
-1/4*1/(a*x^4) - (c*Log[x])/a^2 + (c*Log[a + c*x^4])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^4)} \, dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{1}{x^8 (cx^4+a)}dx^4$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{4}\int \left(\frac{c^2}{a^2 (cx^4+a)}-\frac{c}{a^2x^4}+\frac{1}{ax^8}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(-\frac{c\log (x^4)}{a^2}+\frac{c\log (a+cx^4)}{a^2}-\frac{1}{ax^4}\right)$$

input
```
Int[1/(x^5*(a + c*x^4)),x]
```

output
```
(-(1/(a*x^4)) - (c*Log[x^4])/a^2 + (c*Log[a + c*x^4])/a^2)/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{4a\,x^{4}}-\frac{c\ln\left(x\right)}{a^{2}}+\frac{c\ln\left(c\,x^{4}+a\right)}{4a^{2}}$	32
norman	$-\frac{1}{4a\,x^{4}}-\frac{c\ln\left(x\right)}{a^{2}}+\frac{c\ln\left(c\,x^{4}+a\right)}{4a^{2}}$	32
parallelrisch	$-\frac{4c\ln\left(x\right)x^{4}-c\ln\left(c\,x^{4}+a\right)x^{4}+a}{4a^{2}x^{4}}$	33
risch	$-\frac{1}{4a\,x^{4}}-\frac{c\ln\left(x\right)}{a^{2}}+\frac{c\ln\left(-c\,x^{4}-a\right)}{4a^{2}}$	35

input
```
int(1/x^5/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4-c*ln(x)/a^2+1/4*c*ln(c*x^4+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5\left(a+cx^4\right)}\,dx=\frac{cx^4\log\left(cx^4+a\right)-4\,cx^4\log\left(x\right)-a}{4\,a^2x^4}$$

input
```
integrate(1/x^5/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(c*x^4*log(c*x^4 + a) - 4*c*x^4*log(x) - a)/(a^2*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (a+cx^4)} \, dx = -\frac{1}{4ax^4} - \frac{c\log(x)}{a^2} + \frac{c\log\left(\frac{a}{c}+x^4\right)}{4a^2}$$

input
```
integrate(1/x**5/(c*x**4+a),x)
```

output
```
-1/(4*a*x**4) - c*log(x)/a**2 + c*log(a/c + x**4)/(4*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (a+cx^4)} \, dx = \frac{c\log(cx^4+a)}{4\,a^2} - \frac{c\log(x^4)}{4\,a^2} - \frac{1}{4\,ax^4}$$

input
```
integrate(1/x^5/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*c*log(c*x^4 + a)/a^2 - 1/4*c*log(x^4)/a^2 - 1/4/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5 (a+cx^4)} \, dx = -\frac{c\log(x^4)}{4\,a^2} + \frac{c\log(|cx^4+a|)}{4\,a^2} + \frac{cx^4-a}{4\,a^2x^4}$$

input
```
integrate(1/x^5/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/4*c*log(x^4)/a^2 + 1/4*c*log(abs(c*x^4 + a))/a^2 + 1/4*(c*x^4 - a)/(a^2
*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^5 (a+cx^4)} \, dx = \frac{c \ln (c x^4 + a)}{4 a^2} - \frac{1}{4 a x^4} - \frac{c \ln (x)}{a^2}$$

input
```
int(1/(x^5*(a + c*x^4)),x)
```

output
```
(c*log(a + c*x^4))/(4*a^2) - 1/(4*a*x^4) - (c*log(x))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5 (a+cx^4)} \, dx = \frac{\log\left(-c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{a} + \sqrt{c}\, x^2\right) c x^4 + \log\left(c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{a} + \sqrt{c}\, x^2\right) c x^4 - 4 \log(x) c x^4 - a}{4a^2 x^4}$$

input
```
int(1/x^5/(c*x^4+a),x)
```

output
```
(log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + log
(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 - 4*log(x)*c
*x**4 - a)/(4*a**2*x**4)
```

3.56 $\int \frac{x^9}{a+cx^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^9}{a+cx^4} dx = -\frac{ax^2}{2c^2} + \frac{x^6}{6c} + \frac{a^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{5/2}}$$

output
```
-1/2*a*x^2/c^2+1/6*x^6/c+1/2*a^(3/2)*arctan(c^(1/2)*x^2/a^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{a+cx^4} dx = \frac{1}{6}\left(\frac{-3ax^2+cx^6}{c^2} + \frac{3a^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{5/2}}\right)$$

input
```
Integrate[x^9/(a + c*x^4),x]
```

output
```
((-3*a*x^2 + c*x^6)/c^2 + (3*a^(3/2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(5/2
))/6
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{a+cx^4}\,dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{x^8}{cx^4+a}dx^2$$
$$\Big\downarrow \text{254}$$
$$\frac{1}{2}\int \left(\frac{x^4}{c}+\frac{a^2}{c^2\left(cx^4+a\right)}-\frac{a}{c^2}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{5/2}}-\frac{ax^2}{c^2}+\frac{x^6}{3c}\right)$$

input
```
Int[x^9/(a + c*x^4),x]
```

output
```
(-((a*x^2)/c^2) + x^6/(3*c) + (a^(3/2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(5
/2))/2
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{-\frac{1}{3}c\,x^{6}+a\,x^{2}}{2c^{2}}+\frac{a^{2}\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{2c^{2}\sqrt{ac}}$	43
risch	$\frac{x^{6}}{6c}-\frac{a\,x^{2}}{2c^{2}}+\frac{\sqrt{-ac}\,a\ln\left(c\,x^{2}+\sqrt{-ac}\right)}{4c^{3}}-\frac{\sqrt{-ac}\,a\ln\left(c\,x^{2}-\sqrt{-ac}\right)}{4c^{3}}$	71

input
```
int(x^9/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/c^2*(-1/3*c*x^6+a*x^2)+1/2/c^2*a^2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.10

$$\int\frac{x^9}{a+cx^4}\,dx$$
$$=\left[\frac{2\,cx^{6}-6\,ax^{2}+3\,a\sqrt{-\frac{a}{c}}\log\left(\frac{cx^{4}+2\,cx^{2}\sqrt{-\frac{a}{c}}-a}{cx^{4}+a}\right)}{12\,c^{2}},\ \frac{cx^{6}-3\,ax^{2}+3\,a\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^{2}\sqrt{\frac{a}{c}}}{a}\right)}{6\,c^{2}}\right]$$

input
```
integrate(x^9/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/12*(2*c*x^6 - 6*a*x^2 + 3*a*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c)
- a)/(c*x^4 + a)))/c^2, 1/6*(c*x^6 - 3*a*x^2 + 3*a*sqrt(a/c)*arctan(c*x^2*
sqrt(a/c)/a))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{x^9}{a+cx^4}\,dx = -\frac{ax^2}{2c^2} - \frac{\sqrt{-\frac{a^3}{c^5}}\log\left(x^2 - \frac{c^2\sqrt{-\frac{a^3}{c^5}}}{a}\right)}{4} + \frac{\sqrt{-\frac{a^3}{c^5}}\log\left(x^2 + \frac{c^2\sqrt{-\frac{a^3}{c^5}}}{a}\right)}{4} + \frac{x^6}{6c}$$

input
```
integrate(x**9/(c*x**4+a),x)
```

output
```
-a*x**2/(2*c**2) - sqrt(-a**3/c**5)*log(x**2 - c**2*sqrt(-a**3/c**5)/a)/4
+ sqrt(-a**3/c**5)*log(x**2 + c**2*sqrt(-a**3/c**5)/a)/4 + x**6/(6*c)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{a+cx^4}\,dx = \frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}c^2} + \frac{cx^6 - 3\,ax^2}{6\,c^2}$$

input
```
integrate(x^9/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*a^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/6*(c*x^6 - 3*a*x^2)/c^
2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x^9}{a+cx^4}\,dx = \frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}c^2} + \frac{c^2x^6 - 3\,acx^2}{6\,c^3}$$

input
```
integrate(x^9/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*a^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2) + 1/6*(c^2*x^6 - 3*a*c*x^2
)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{a+cx^4}\,dx = \frac{x^6}{6\,c} - \frac{a\,x^2}{2\,c^2} + \frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\,c^{5/2}}$$

input
```
int(x^9/(a + c*x^4),x)
```

output
```
x^6/(6*c) - (a*x^2)/(2*c^2) + (a^(3/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*c^(
5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{x^9}{a+cx^4}\,dx = \frac{-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a - 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a - 3ac\,x^2 + c^2x^6}{6c^3}$$

input
```
int(x^9/(c*x^4+a),x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*a - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 3*a*c*x**2 + c**2*x**6
)/(6*c**3)
```

3.57 $\int \frac{x^5}{a+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{x^2}{2c} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}}$$

output
```
1/2*x^2/c-1/2*a^(1/2)*arctan(c^(1/2)*x^2/a^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{x^2}{2c} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}}$$

input
```
Integrate[x^5/(a + c*x^4),x]
```

output
```
x^2/(2*c) - (Sqrt[a]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {807, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+cx^4} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{cx^4+a}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{x^2}{c}-\frac{a \int \frac{1}{cx^4+a}dx^2}{c}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2} \left(\frac{x^2}{c}-\frac{\sqrt{a} \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}}\right)$$

input
```
Int[x^5/(a + c*x^4),x]
```

output
```
(x^2/c - (Sqrt[a]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2}{2c} - \frac{a \arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2c\sqrt{ac}}$	32
risch	$\frac{x^2}{2c} + \frac{\sqrt{-ac}\,\ln\left(c\,x^2 - \sqrt{-ac}\right)}{4c^2} - \frac{\sqrt{-ac}\,\ln\left(c\,x^2 + \sqrt{-ac}\right)}{4c^2}$	60

input
```
int(x^5/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c-1/2/c*a/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \frac{x^5}{a + cx^4}\,dx = \left[\frac{2\,x^2 + \sqrt{-\frac{a}{c}}\log\left(\frac{cx^4 - 2\,cx^2\sqrt{-\frac{a}{c}} - a}{cx^4 + a}\right)}{4\,c}, \frac{x^2 - \sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)}{2\,c}\right]$$

input
```
integrate(x^5/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(2*x^2 + sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a))
)/c, 1/2*(x^2 - sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a))/c]
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{\sqrt{-\frac{a}{c^3}}\log\left(-c\sqrt{-\frac{a}{c^3}}+x^2\right)}{4} - \frac{\sqrt{-\frac{a}{c^3}}\log\left(c\sqrt{-\frac{a}{c^3}}+x^2\right)}{4} + \frac{x^2}{2c}$$

input
```
integrate(x**5/(c*x**4+a),x)
```

output
```
sqrt(-a/c**3)*log(-c*sqrt(-a/c**3) + x**2)/4 - sqrt(-a/c**3)*log(c*sqrt(-a
/c**3) + x**2)/4 + x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,\sqrt{ac}c}$$

input
```
integrate(x^5/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*x^2/c - 1/2*a*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\,\sqrt{ac}c}$$

input
```
integrate(x^5/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*x^2/c - 1/2*a*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{x^2}{2\,c} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\,c^{3/2}}$$

input
```
int(x^5/(a + c*x^4),x)
```

output
```
x^2/(2*c) - (a^(1/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int \frac{x^5}{a+cx^4}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+c\,x^2}{2c^2}$$

input
```
int(x^5/(c*x^4+a),x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2))) + sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + c*x**2)/(2*c**2)
```

3.58 $\int \frac{x}{a+cx^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{x}{a+cx^4} dx = \frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

output
```
1/2*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+cx^4} dx = \frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

input
```
Integrate[x/(a + c*x^4),x]
```

output
```
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {807, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+cx^4} \, dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{cx^4+a}dx^2$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{c}}$$

input
```
Int[x/(a + c*x^4),x]
```

output
```
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*Sqrt[c])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
default	$\frac{\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}$	19
risch	$-\frac{\ln(x^2\sqrt{-ac}-a)}{4\sqrt{-ac}}+\frac{\ln(x^2\sqrt{-ac}+a)}{4\sqrt{-ac}}$	46

input
```
int(x/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int \frac{x}{a+cx^4}\,dx = \left[-\frac{\sqrt{-ac}\log\left(\frac{cx^4-2\sqrt{-ac}x^2-a}{cx^4+a}\right)}{4\,ac}, -\frac{\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{2\,ac}\right]$$

input
```
integrate(x/(c*x^4+a),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a))/(a*c), -1
/2*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2))/(a*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{x}{a+cx^4}\,dx = -\frac{\sqrt{-\frac{1}{ac}}\log\left(-a\sqrt{-\frac{1}{ac}}+x^2\right)}{4} + \frac{\sqrt{-\frac{1}{ac}}\log\left(a\sqrt{-\frac{1}{ac}}+x^2\right)}{4}$$

input
```
integrate(x/(c*x**4+a),x)
```

output
```
-sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x**2)/4 + sqrt(-1/(a*c))*log(a*sqr
t(-1/(a*c)) + x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{x}{a+cx^4}\,dx=\frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}$$

input
```
integrate(x/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*arctan(c*x^2/sqrt(a*c))/sqrt(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.62

$$\int \frac{x}{a+cx^4}\,dx=\frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}$$

input
```
integrate(x/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*arctan(c*x^2/sqrt(a*c))/sqrt(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{x}{a+cx^4}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\sqrt{c}}$$

input
```
int(x/(a + c*x^4),x)
```

output
```
atan((c^(1/2)*x^2)/a^(1/2))/(2*a^(1/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.31

$$\int \frac{x}{a+cx^4}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)\right)}{2ac}$$

input
```
int(x/(c*x^4+a),x)
```

output
```
( - sqrt(c)*sqrt(a)*(atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1
/4)*a**(1/4)*sqrt(2))) + atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c
**(1/4)*a**(1/4)*sqrt(2)))))/(2*a*c)
```

3.59 $\int \frac{1}{x^3(a+cx^4)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx=-\frac{1}{2ax^2}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/2/a/x^2-1/2*c^(1/2)*arctan(c^(1/2)*x^2/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx=\frac{-\sqrt{a}+\sqrt{c}x^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{c}x^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2a^{3/2}x^2}$$

input
```
Integrate[1/(x^3*(a + c*x^4)),x]
```

output
```
(-Sqrt[a] + Sqrt[c]*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[c]*
x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*a^(3/2)*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {807, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + cx^4)} dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^4 (cx^4 + a)} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{c \int \frac{1}{cx^4+a} dx^2}{a} - \frac{1}{ax^2} \right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2} \left(-\frac{\sqrt{c} \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax^2} \right)$$

input
```
Int[1/(x^3*(a + c*x^4)),x]
```

output
```
(-(1/(a*x^2)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{2a\,x^{2}}-\frac{c\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{2a\sqrt{ac}}$	32
risch	$-\frac{1}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{3}\_Z^{2}+c\right)}\_R\ln\left(\left(-5a^{3}\_R^{2}-4c\right)x^{2}-a^{2}\_R\right)\right)}{4}$	51

input
```
int(1/x^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-1/2/a*c/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx=\left[\frac{x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-2}{4\,ax^2},-\frac{x^2\sqrt{\frac{c}{a}}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)+1}{2\,ax^2}\right]$$

input
```
integrate(1/x^3/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/4*(x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 2
)/(a*x^2), -1/2*(x^2*sqrt(c/a)*arctan(x^2*sqrt(c/a)) + 1)/(a*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx = \frac{\sqrt{-\frac{c}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{c}{a^3}}}{c}+x^2\right)}{4} - \frac{\sqrt{-\frac{c}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{c}{a^3}}}{c}+x^2\right)}{4} - \frac{1}{2ax^2}$$

input
```
integrate(1/x**3/(c*x**4+a),x)
```

output
```
sqrt(-c/a**3)*log(-a**2*sqrt(-c/a**3)/c + x**2)/4 - sqrt(-c/a**3)*log(a**2
*sqrt(-c/a**3)/c + x**2)/4 - 1/(2*a*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx = -\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}a} - \frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/2*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a) - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx=-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}a}-\frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a) - 1/2/(a*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx=-\frac{1}{2\,a\,x^2}-\frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int(1/(x^3*(a + c*x^4)),x)
```

output
```
- 1/(2*a*x^2) - (c^(1/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^3\left(a+cx^4\right)}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2-a}{2a^2x^2}$$

input
```
int(1/x^3/(c*x^4+a),x)
```

output
```
(sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*x**2 + sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*x**2 - a)/(2*a**2*x**2)
```

3.60 $\int \frac{1}{x^7(a+cx^4)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{1}{x^7\left(a+cx^4\right)}\,dx=-\frac{1}{6ax^6}+\frac{c}{2a^2x^2}+\frac{c^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/6/a/x^6+1/2*c/a^2/x^2+1/2*c^(3/2)*arctan(c^(1/2)*x^2/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.73

$$\int \frac{1}{x^7\left(a+cx^4\right)}\,dx = -\frac{\sqrt{a}(a-3cx^4)+3c^{3/2}x^6\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+3c^{3/2}x^6\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{6a^{5/2}x^6}$$

input
```
Integrate[1/(x^7*(a + c*x^4)),x]
```

output
```
-1/6*(Sqrt[a]*(a - 3*c*x^4) + 3*c^(3/2)*x^6*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)] + 3*c^(3/2)*x^6*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2
)*x^6)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+cx^4)}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^8 (cx^4+a)}dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{c\int \frac{1}{x^4(cx^4+a)}dx^2}{a}-\frac{1}{3ax^6}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{c\left(-\frac{c\int \frac{1}{cx^4+a}dx^2}{a}-\frac{1}{ax^2}\right)}{a}-\frac{1}{3ax^6}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax^2}\right)}{a}-\frac{1}{3ax^6}\right)$$

input
```
Int[1/(x^7*(a + c*x^4)),x]
```

output
```
(-1/3*1/(a*x^6) - (c*(-(1/(a*x^2)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/a^(3/2)))/a)/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{1}{6a\,x^{6}}+\frac{c}{2a^{2}x^{2}}+\frac{c^{2}\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{2a^{2}\sqrt{ac}}$	43
risch	$\frac{\frac{c\,x^{4}}{2a^{2}}-\frac{1}{6a}}{x^{6}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(5a^{5}\_R^{2}+4c^{3}\right)x^{2}-a^{3}c\_R\right)\right)}{4}$	67

input
```
int(1/x^7/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/x^6+1/2*c/a^2/x^2+1/2*c^2/a^2/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^7\left(a+cx^4\right)}\,dx = \left[\frac{3\,cx^6\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+6\,cx^4-2\,a}{12\,a^2x^6}, \frac{3\,cx^6\sqrt{\frac{c}{a}}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)+3\,cx^4-a}{6\,a^2x^6}\right]$$

input
```
integrate(1/x^7/(c*x^4+a),x, algorithm="fricas")
```

output
```
[1/12*(3*c*x^6*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)
) + 6*c*x^4 - 2*a)/(a^2*x^6), 1/6*(3*c*x^6*sqrt(c/a)*arctan(x^2*sqrt(c/a))
 + 3*c*x^4 - a)/(a^2*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(44) = 88.

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^7\left(a+cx^4\right)}\,dx = -\frac{\sqrt{-\frac{c^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{c^3}{a^5}}}{c^2}+x^2\right)}{4} + \frac{\sqrt{-\frac{c^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{c^3}{a^5}}}{c^2}+x^2\right)}{4} + \frac{-a+3cx^4}{6a^2x^6}$$

input
```
integrate(1/x**7/(c*x**4+a),x)
```

output
```
-sqrt(-c**3/a**5)*log(-a**3*sqrt(-c**3/a**5)/c**2 + x**2)/4 + sqrt(-c**3/a
**5)*log(a**3*sqrt(-c**3/a**5)/c**2 + x**2)/4 + (-a + 3*c*x**4)/(6*a**2*x*
*6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7 (a + cx^4)} \, dx = \frac{c^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}a^2} + \frac{3\, cx^4 - a}{6\, a^2 x^6}$$

input
```
integrate(1/x^7/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/2*c^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/6*(3*c*x^4 - a)/(a^2*x
^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^7 (a + cx^4)} \, dx = \frac{c^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\sqrt{ac}a^2} + \frac{3\, cx^4 - a}{6\, a^2 x^6}$$

input
```
integrate(1/x^7/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*c^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/6*(3*c*x^4 - a)/(a^2*x
^6)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (a + cx^4)} \, dx = \frac{c^{3/2} \operatorname{atan}\left(\frac{\sqrt{c}\, x^2}{\sqrt{a}}\right)}{2\, a^{5/2}} - \frac{\frac{1}{6\, a} - \frac{c\, x^4}{2\, a^2}}{x^6}$$

input
```
int(1/(x^7*(a + c*x^4)),x)
```

output
```
(c^(3/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(2*a^(5/2)) - (1/(6*a) - (c*x^4)/(2*
a^2))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

$$\int \frac{1}{x^7 (a+cx^4)}\,dx$$
$$= \frac{-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c\,x^6-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c\,x^6-a^2+3ac\,x^4}{6a^3x^6}$$

input
```
int(1/x^7/(c*x^4+a),x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*c*x**6 - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)
*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**6 - a**2 + 3*a*c
*x**4)/(6*a**3*x**6)
```

3.61 $\int \frac{x^6}{a+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int \frac{x^6}{a+cx^4}\,dx = \frac{x^3}{3c} + \frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}} - \frac{a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}} + \frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}c^{7/4}}$$

output
```
1/3*x^3/c-1/4*a^(3/4)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/c^(7/4)
-1/4*a^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/c^(7/4)+1/4*a^(3/
4)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/c^(7/4
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{a+cx^4}\,dx = \frac{8c^{3/4}x^3 + 6\sqrt{2}a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - 6\sqrt{2}a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - 3\sqrt{2}a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\right.}{24c^{7/4}}$$

input
```
Integrate[x^6/(a + c*x^4),x]
```

output
```
(8*c^(3/4)*x^3 + 6*Sqrt[2]*a^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
 - 6*Sqrt[2]*a^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a
^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*
a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*c^(7/4
))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+cx^4}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^3}{3c}-\frac{a\int \frac{x^2}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{x^3}{3c}-\frac{a\left(\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 217

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 1479

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 25

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

$\downarrow$ 27

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{c}$$

$\big\downarrow$ 1103

$$\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}$$

input
```
Int[x^6/(a + c*x^4),x]
```

output
```
x^3/(3*c) - (a*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4
)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(
1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

method	result	size
risch	$\frac{x^3}{3c}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4c^2}$	37
default	$\frac{x^3}{3c}-\frac{a\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	112

input
```
int(x^6/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c-1/4/c^2*a*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04

$$\int\frac{x^6}{a+cx^4}\,dx=\frac{4\,x^3-3\,c\left(-\frac{a^3}{c^7}\right)^{\frac{1}{4}}\log\left(c^5\left(-\frac{a^3}{c^7}\right)^{\frac{3}{4}}+a^2x\right)+3i\,c\left(-\frac{a^3}{c^7}\right)^{\frac{1}{4}}\log\left(i\,c^5\left(-\frac{a^3}{c^7}\right)^{\frac{3}{4}}+a^2x\right)-3i\,c\left(-\frac{a^3}{c^7}\right)^{\frac{1}{4}}\log\left(-\right.}{12\,c}$$

input
```
integrate(x^6/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/12*(4*x^3 - 3*c*(-a^3/c^7)^(1/4)*log(c^5*(-a^3/c^7)^(3/4) + a^2*x) + 3*I
*c*(-a^3/c^7)^(1/4)*log(I*c^5*(-a^3/c^7)^(3/4) + a^2*x) - 3*I*c*(-a^3/c^7)
^(1/4)*log(-I*c^5*(-a^3/c^7)^(3/4) + a^2*x) + 3*c*(-a^3/c^7)^(1/4)*log(-c^
5*(-a^3/c^7)^(3/4) + a^2*x))/c
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{a+cx^4}\,dx = \text{RootSum}\left(256t^4c^7+a^3,\left(t\mapsto t\log\left(-\frac{64t^3c^5}{a^2}+x\right)\right)\right)+\frac{x^3}{3c}$$

input
```
integrate(x**6/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*c**7 + a**3, Lambda(_t, _t*log(-64*_t**3*c**5/a**2 + x))
) + x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.27

$$\int \frac{x^6}{a+cx^4}\,dx = \frac{x^3}{3\,c} - \frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\cdots\right)}{a\cdots}\right)}{8\,c}$$

input
```
integrate(x^6/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/3*x^3/c - 1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*s
qrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqr
t(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 +
 sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt
(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

$$\int \frac{x^6}{a+cx^4}\,dx = \frac{x^3}{3\,c} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,c^4} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,c^4} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,c^4} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,c^4}$$

input
```
integrate(x^6/(c*x^4+a),x, algorithm="giac")
```

output
```
1/3*x^3/c - 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/c^4 - 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/c^4 + 1/8*sqrt(2)*(a*c^3)^(3/4)*
log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^4 - 1/8*sqrt(2)*(a*c^3)^(3/
4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{a+cx^4}\,dx = \frac{x^3}{3\,c} + \frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,c^{7/4}} - \frac{(-a)^{3/4}\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,c^{7/4}}$$

input
```
int(x^6/(a + c*x^4),x)
```

output
```
x^3/(3*c) + ((-a)^(3/4)*atan((c^(1/4)*x)/(-a)^(1/4)))/(2*c^(7/4)) - ((-a)^
(3/4)*atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*c^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.02

$$\int \frac{x^6}{a+cx^4}\,dx$$
$$= \frac{6c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-3c^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{}\right.}{24c^2}$$

input
```
int(x^6/(c*x^4+a),x)
```

output
```
(6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2))) - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) - 3*c**(1/4
)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*
x**2) + 3*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt
(a) + sqrt(c)*x**2) + 8*c*x**3)/(24*c**2)
```

3.62 $\int \frac{x^4}{a+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 139

$$\int \frac{x^4}{a+cx^4}\,dx=\frac{x}{c}+\frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}c^{5/4}}$$

output
```
x/c-1/4*a^(1/4)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/c^(5/4)-1/4*a
^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/c^(5/4)-1/4*a^(1/4)*arc
tanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{a+cx^4}\,dx=\frac{8\sqrt[4]{c}x+2\sqrt{2}\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x\right.}{8c^{5/4}}$$

input
```
Integrate[x^4/(a + c*x^4),x]
```

output
```
(8*c^(1/4)*x + 2*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 2*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*a^(1/
4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*a^(1/4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*c^(5/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.53, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+cx^4}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x}{c}-\frac{a\int \frac{1}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{x}{c}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{c}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{x}{c}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{c}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{x}{c}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}$$

$\downarrow$ 217

$$\frac{x}{c}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}$$

$\downarrow$ 1479

$$\frac{x}{c}-\frac{a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}$$

$\downarrow$ 25

$$\frac{x}{c}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}$$

$\downarrow$ 27

$$\frac{x}{c}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}$$

$$\Big\downarrow \text{1103}$$

$$\frac{x}{c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}$$

input
```
Int[x^4/(a + c*x^4),x]
```

output
```
x/c - (a*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1
/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))
/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.24

method	result	size
risch	$\frac{x}{c}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4c^{2}}$	34
default	$\frac{x}{c}-\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c}$	108

input
```
int(x^4/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
x/c-1/4/c^2*a*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.77

$$\int \frac{x^4}{a+cx^4}\,dx = -\frac{c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}+x\right)+i\,c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}\log\left(i\,c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}+x\right)-i\,c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}\log\left(-i\,c\left(-\frac{a}{c^5}\right)^{\frac{1}{4}}+x\right)-c(}{4\,c}$$

input
```
integrate(x^4/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(c*(-a/c^5)^(1/4)*log(c*(-a/c^5)^(1/4) + x) + I*c*(-a/c^5)^(1/4)*log(
I*c*(-a/c^5)^(1/4) + x) - I*c*(-a/c^5)^(1/4)*log(-I*c*(-a/c^5)^(1/4) + x)
- c*(-a/c^5)^(1/4)*log(-c*(-a/c^5)^(1/4) + x) - 4*x)/c
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

$$\int \frac{x^4}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4c^5+a,\left(t\mapsto t\log\left(-4tc+x\right)\right)\right)+\frac{x}{c}$$

input
```
integrate(x**4/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*c**5 + a, Lambda(_t, _t*log(-4*_t*c + x))) + x/c
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \frac{x^4}{a+cx^4}\,dx = -\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{c^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log \cdots}{\cdots}}{8\,c}+\frac{x}{c}$$

input
```
integrate(x^4/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*
c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(sqrt(a)*sqrt(c)) + 2*sqrt(2)*sqrt(a)*
arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sq
rt(c)))/sqrt(sqrt(a)*sqrt(c)) + sqrt(2)*a^(1/4)*log(sqrt(c)*x^2 + sqrt(2)*
a^(1/4)*c^(1/4)*x + sqrt(a))/c^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(c)*x^2 - s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/c^(1/4))/c + x/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{a+cx^4}\,dx = \frac{x}{c} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4c^2} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4c^2} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8c^2} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8c^2}$$

input
```
integrate(x^4/(c*x^4+a),x, algorithm="giac")
```

output
```
x/c - 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/c^2 - 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
- sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/c^2 - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^
2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^2 + 1/8*sqrt(2)*(a*c^3)^(1/4)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{x^4}{a+cx^4}\,dx = \frac{x}{c} - \frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2c^{5/4}} - \frac{(-a)^{1/4}\,\mathrm{atanh}\left(\frac{c^{1/4}x}{(-a)^{1/4}}\right)}{2c^{5/4}}$$

input
```
int(x^4/(a + c*x^4),x)
```

output
```
x/c - ((-a)^(1/4)*atan((c^(1/4)*x)/(-a)^(1/4)))/(2*c^(5/4)) - ((-a)^(1/4)*
atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*c^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{a+cx^4}\,dx = \frac{2c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}}{8c^2}$$

input
```
int(x^4/(c*x^4+a),x)
```

output
```
(2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2))) - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + c**(3/4)*
a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x*
*2) - c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2) + 8*c*x)/(8*c**2)
```

3.63 $\int \frac{x^2}{a+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{x^2}{a+cx^4}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{3/4}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}c^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}\sqrt[4]{a}c^{3/4}}$$

output
```
1/4*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/c^(3/4)+1/4*arcta
n(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/c^(3/4)-1/4*arctanh(2^(1/2)
*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(1/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+cx^4}\,dx$$
$$=\frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{4\sqrt{2}\sqrt[4]{a}c^{3/4}}$$

input
```
Integrate[x^2/(a + c*x^4),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] + Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*
c^(3/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+cx^4}dx$$

$$\Big\downarrow \text{826}$$

$$\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}$$

input
```
Int[x^2/(a + c*x^4),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c
]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]
*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}}{4c}$	27
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8c\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	102

input
```
int(x^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{a+cx^4}\,dx = \frac{1}{4}\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}}\log\left(ac^2\left(-\frac{1}{ac^3}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}}\log\left(i\,ac^2\left(-\frac{1}{ac^3}\right)^{\frac{3}{4}}+x\right) + \frac{1}{4}i\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}}\log\left(-i\,ac^2\left(-\frac{1}{ac^3}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{ac^3}\right)^{\frac{1}{4}}\log\left(-ac^2\left(-\frac{1}{ac^3}\right)^{\frac{3}{4}}+x\right)$$

input
```
integrate(x^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(-1/(a*c^3))^(1/4)*log(a*c^2*(-1/(a*c^3))^(3/4) + x) - 1/4*I*(-1/(a*c^
3))^(1/4)*log(I*a*c^2*(-1/(a*c^3))^(3/4) + x) + 1/4*I*(-1/(a*c^3))^(1/4)*l
og(-I*a*c^2*(-1/(a*c^3))^(3/4) + x) - 1/4*(-1/(a*c^3))^(1/4)*log(-a*c^2*(-
1/(a*c^3))^(3/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{x^2}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4ac^3+1, \left(t \mapsto t\log\left(64t^3ac^2+x\right)\right)\right)$$

input
```
integrate(x**2/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a*c**3 + 1, Lambda(_t, _t*log(64*_t**3*a*c**2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{1}{4}}c^{\frac{3}{4}}}$$

input
```
integrate(x^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + 1/8*sqrt(2)*log(sqrt(c)*x^2
 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^3} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^3}$$

input
```
integrate(x^2/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) - 1/8*sqrt(2)*(a*c^3)^(3/4)*log(
x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) + 1/8*sqrt(2)*(a*c^3)^(3/
4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{a+cx^4}\,dx = \frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)-\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{1/4}\,c^{3/4}}$$

input
```
int(x^2/(a + c*x^4),x)
```

output
```
(atan((c^(1/4)*x)/(-a)^(1/4)) - atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(1/
4)*c^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+cx^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)-\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{\cdots}\right.\right.}{8c^{\frac{3}{4}}a^{\frac{1}{4}}}$$

input
```
int(x^2/(c*x^4+a),x)
```

output
```
(c**(1/4)*a**(3/4)*sqrt(2)*( - 2*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + log( - c**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(c)*x**2) - log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2)))/(8*a*c)
```

3.64 $\int \frac{1}{a+cx^4}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 134

$$\int \frac{1}{a+cx^4}\,dx=-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

output
```
1/4*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*arcta
n(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(1/4)+1/4*arctanh(2^(1/2)
*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+cx^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{\right.}}{4\sqrt{2}a^{3/4}\sqrt[4]{c}}$$

input
```
Integrate[(a + c*x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
c^(1/4))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.49, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+cx^4} \, dx$$

$$\Big\downarrow \text{755}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow \text{1479}$$

$$\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

$$\Big\downarrow 1103$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}$$

input

```
Int[(a + c*x^4)^(-1),x]
```

output

```
(-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a
]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]
*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 c+a\right)} \frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

input
```
int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+cx^4}\,dx = \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) + \frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}i\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right) - \frac{1}{4}\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3c}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(c*x^4+a),x, algorithm="fricas")
```

output
```
1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/(a^3*c))^(1/4) + x) + 1/4*I*(-1/(a^3*c))^
(1/4)*log(I*a*(-1/(a^3*c))^(1/4) + x) - 1/4*I*(-1/(a^3*c))^(1/4)*log(-I*a*
(-1/(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4)
 + x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

$$\int \frac{1}{a+cx^4}\,dx = \operatorname{RootSum}\left(256t^4a^3c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/
(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2
 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(87) = 174$.

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

input
```
integrate(1/(c*x^4+a),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sq
rt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
+ sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int \frac{1}{a+cx^4}\,dx = -\frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{2\,(-a)^{3/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4),x)
```

output
```
-(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3
/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.84

$$\int \frac{1}{a+cx^4}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)+\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{\right.\right.}{8c^{\frac{1}{4}}a^{\frac{3}{4}}}$$

input
```
int(1/(c*x^4+a),x)
```

output
```
(c**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2))) - log( - c**(1/4)*a**(1/4)*sqrt(2
)*x + sqrt(a) + sqrt(c)*x**2) + log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2)))/(8*a*c)
```

3.65 $\int \frac{1}{x^2(a+cx^4)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 142

$$\int \frac{1}{x^2(a+cx^4)}\,dx = -\frac{1}{ax}+\frac{\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} - \frac{\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}} + \frac{\sqrt[4]{c}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{5/4}}$$

output

```
-1/a/x-1/4*c^(1/4)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)-1/
4*c^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)+1/4*c^(1/4)*
arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2(a+cx^4)}\,dx = \frac{-8\sqrt[4]{a}+2\sqrt{2}\sqrt[4]{c}x\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-2\sqrt{2}\sqrt[4]{c}x\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-\sqrt{2}\sqrt[4]{c}x\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{8a^{5/4}x}$$

input
```
Integrate[1/(x^2*(a + c*x^4)),x]
```

output
```
(-8*a^(1/4) + 2*Sqrt[2]*c^(1/4)*x*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 2*Sqrt[2]*c^(1/4)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*c^
(1/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*c
^(1/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(5/4
)*x)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^4)} dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{c\int \frac{x^2}{cx^4+a}dx}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{826}$$

$$-\frac{c\left(\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{c\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 217

$$-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 1479

$$-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 25

$$-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$\Big\downarrow$ 27

$$-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

$\downarrow$ 1103

$$-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + c*x^4)),x]
```

output
```
-(1/(a*x)) - (c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/
4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^
(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.35

method	result	size
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^{5}\_Z^{4}+c\right)}\_R\ln\left(\left(5\_R^{4}a^{5}+4c\right)x+a^{4}\_R^{3}\right)\right)}{4}$	50
default	$-\frac{1}{ax}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	111

input
```
int(1/x^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/a/x+1/4*sum(_R*ln((5*_R^4*a^5+4*c)*x+a^4*_R^3),_R=RootOf(_Z^4*a^5+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\left(a+cx^4\right)}\,dx = -\frac{ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{c}{a^5}\right)^{\frac{3}{4}}+cx\right)-i\,ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(-\frac{c}{a^5}\right)^{\frac{3}{4}}+cx\right)+i\,ax\left(-\frac{c}{a^5}\right)^{\frac{1}{4}}\log\left(-i\,a^4\left(-\frac{c}{a^5}\right)^{\frac{3}{4}}\right.}{4\,ax}$$

input
```
integrate(1/x^2/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/4*(a*x*(-c/a^5)^(1/4)*log(a^4*(-c/a^5)^(3/4) + c*x) - I*a*x*(-c/a^5)^(1
/4)*log(I*a^4*(-c/a^5)^(3/4) + c*x) + I*a*x*(-c/a^5)^(1/4)*log(-I*a^4*(-c/
a^5)^(3/4) + c*x) - a*x*(-c/a^5)^(1/4)*log(-a^4*(-c/a^5)^(3/4) + c*x) + 4)
/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2 (a + c x^4)}\, dx = \operatorname{RootSum}\left(256t^4a^5 + c, \left(t \mapsto t \log\left(-\frac{64t^3a^4}{c} + x\right)\right)\right) - \frac{1}{ax}$$

input
```
integrate(1/x**2/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**5 + c, Lambda(_t, _t*log(-64*_t**3*a**4/c + x))) - 1/
(a*x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^2 (a + c x^4)}\, dx = -\frac{c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}}\right)}{8\,a} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c
)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2\,(a+cx^4)}\,dx = -\frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2c^2} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,a^2c^2} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,a^2c^2} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,a^2c^2} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^2*c^2) - 1/4*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 1/8*sqrt(2)*(a*c^3)^(3/4)
*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) - 1/8*sqrt(2)*(a*c
^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2\,(a+cx^4)}\,dx = \frac{(-c)^{1/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{(-c)^{1/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{1}{a\,x}$$

input
```
int(1/(x^2*(a + c*x^4)),x)
```

output
```
((-c)^(1/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(2*a^(5/4)) - ((-c)^(1/4)*atan(
((-c)^(1/4)*x)/a^(1/4)))/(2*a^(5/4)) - 1/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 (a + cx^4)} \, dx$$
$$= \frac{2c^{\frac{1}{4}} a^{\frac{3}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} - 2\sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x - 2c^{\frac{1}{4}} a^{\frac{3}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} + 2\sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x - c^{\frac{1}{4}} a^{\frac{3}{4}} \sqrt{2} \log\left(-c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{a} + \right.}{8a^2 x}$$

input
```
int(1/x^2/(c*x^4+a),x)
```

output
```
(2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*x - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*x - c**(1
/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c
)*x**2)*x + c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sq
rt(a) + sqrt(c)*x**2)*x - 8*a)/(8*a**2*x)
```

3.66 $\int \frac{1}{x^4(a+cx^4)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int \frac{1}{x^4(a+cx^4)}\,dx = -\frac{1}{3ax^3}+\frac{c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}} - \frac{c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}} - \frac{c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{2\sqrt{2}a^{7/4}}$$

output
```
-1/3/a/x^3-1/4*c^(3/4)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4
)-1/4*c^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)-1/4*c^(3
/4)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(7/
4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^4(a+cx^4)}\,dx = \frac{-8a^{3/4}+6\sqrt{2}c^{3/4}x^3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-6\sqrt{2}c^{3/4}x^3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+3\sqrt{2}c^{3/4}x^3\log\left(\sqrt{a}-\sqrt{\right.}}{24a^{7/4}x^3}$$

input
```
Integrate[1/(x^4*(a + c*x^4)),x]
```

output
```
(-8*a^(3/4) + 6*Sqrt[2]*c^(3/4)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
] - 6*Sqrt[2]*c^(3/4)*x^3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt
[2]*c^(3/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3
*Sqrt[2]*c^(3/4)*x^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
])/(24*a^(7/4)*x^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4(a+cx^4)}\,dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{c\int \frac{1}{cx^4+a}dx}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{755}$$

$$-\frac{c\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{1082}$$

$$-\frac{c\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{c\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{1479}$$

$$-\frac{c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

$$\Big\downarrow \text{1103}$$

$$-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}$$

input
```
Int[1/(x^4*(a + c*x^4)),x]
```

output
```
-1/3*1/(a*x^3) - (c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4
)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.39

method	result	size
risch	$-\frac{1}{3a\,x^{3}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}\_Z^{4}+c^{3}\right)}\_R\ln\left(\left(-5\_R^{4}a^{7}-4c^{3}\right)x-a^{2}c^{2}\_R\right)\right)}{4}$	56
default	$-\frac{1}{3a\,x^{3}}-\frac{c\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a^{2}}$	112

input
```
int(1/x^4/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a/x^3+1/4*sum(_R*ln((-5*_R^4*a^7-4*c^3)*x-a^2*c^2*_R),_R=RootOf(_Z^4*
a^7+c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (a + cx^4)} \, dx = -\frac{3\,ax^3\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}} \log\left(a^2\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}} + cx\right) + 3i\,ax^3\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}} \log\left(i\,a^2\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}} + cx\right) - 3i\,ax^3\left(-\frac{c^3}{a^7}\right)^{\frac{1}{4}} \log\left(\right.}{12\,ax^3}$$

input
```
integrate(1/x^4/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/12*(3*a*x^3*(-c^3/a^7)^(1/4)*log(a^2*(-c^3/a^7)^(1/4) + c*x) + 3*I*a*x^
3*(-c^3/a^7)^(1/4)*log(I*a^2*(-c^3/a^7)^(1/4) + c*x) - 3*I*a*x^3*(-c^3/a^7
)^(1/4)*log(-I*a^2*(-c^3/a^7)^(1/4) + c*x) - 3*a*x^3*(-c^3/a^7)^(1/4)*log(
-a^2*(-c^3/a^7)^(1/4) + c*x) + 4)/(a*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^4 (a + cx^4)} \, dx = \operatorname{RootSum}\left(256t^4a^7 + c^3, \left(t \mapsto t \log\left(-\frac{4ta^2}{c} + x\right)\right)\right) - \frac{1}{3ax^3}$$

input
```
integrate(1/x**4/(c*x**4+a),x)
```

output
```
RootSum(256*_t**4*a**7 + c**3, Lambda(_t, _t*log(-4*_t*a**2/c + x))) - 1/(
3*a*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^4 (a + cx^4)} \, dx = -\frac{\frac{2\sqrt{2}c \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}c \arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}c^{\frac{3}{4}} \log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}c^{\frac{3}{4}} \log\left(\sqrt{c}\right.}{}}{8a} - \frac{1}{3ax^3}$$

input
```
integrate(1/x^4/(c*x^4+a),x, algorithm="maxima")
```

output
```
-1/8*(2*sqrt(2)*c*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*c*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt
(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(c)*x^2 +
sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*c^(3/4)*log(sqrt(c)
*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/a^(3/4))/a - 1/3/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^4 (a + cx^4)} \, dx = -\frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4a^2} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4a^2} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8a^2} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8a^2} - \frac{1}{3ax^3}$$

input
```
integrate(1/x^4/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/a^2 - 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqr
t(2)*(a/c)^(1/4))/(a/c)^(1/4))/a^2 - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^2 + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
- sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^2 - 1/3/(a*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4\left(a+cx^4\right)}\,dx = \frac{(-c)^{3/4}\,\mathrm{atan}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{7/4}} - \frac{1}{3\,a\,x^3} + \frac{(-c)^{3/4}\,\mathrm{atanh}\left(\frac{(-c)^{1/4}\,x}{a^{1/4}}\right)}{2\,a^{7/4}}$$

input
```
int(1/(x^4*(a + c*x^4)),x)
```

output
```
((-c)^(3/4)*atan(((-c)^(1/4)*x)/a^(1/4)))/(2*a^(7/4)) - 1/(3*a*x^3) + ((-c
)^(3/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(2*a^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^4\left(a+cx^4\right)}\,dx = \frac{6c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^3 - 6c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^3 + 3c^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{c}\right.}{24a^2x^3}$$

input
```
int(1/x^4/(c*x^4+a),x)
```

output
```
(6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x
)/(c**(1/4)*a**(1/4)*sqrt(2)))*x**3 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*x**3 +
 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
+ sqrt(c)*x**2)*x**3 - 3*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*s
qrt(2)*x + sqrt(a) + sqrt(c)*x**2)*x**3 - 8*a)/(24*a**2*x**3)
```

3.67 $\int \frac{x^{11}}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{x^{11}}{(a+cx^4)^2}\,dx = \frac{x^4}{4c^2} - \frac{a^2}{4c^3(a+cx^4)} - \frac{a\log(a+cx^4)}{2c^3}$$

output
```
1/4*x^4/c^2-1/4*a^2/c^3/(c*x^4+a)-1/2*a*ln(c*x^4+a)/c^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{(a+cx^4)^2}\,dx = \frac{cx^4 - \frac{a^2}{a+cx^4} - 2a\log(a+cx^4)}{4c^3}$$

input
```
Integrate[x^11/(a + c*x^4)^2,x]
```

output
```
(c*x^4 - a^2/(a + c*x^4) - 2*a*Log[a + c*x^4])/(4*c^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+cx^4)^2} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(cx^4+a)^2}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{a^2}{c^2(cx^4+a)^2}-\frac{2a}{c^2(cx^4+a)}+\frac{1}{c^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^2}{c^3(a+cx^4)}-\frac{2a\log(a+cx^4)}{c^3}+\frac{x^4}{c^2}\right)$$

input
```
Int[x^11/(a + c*x^4)^2,x]
```

output
```
(x^4/c^2 - a^2/(c^3*(a + c*x^4)) - (2*a*Log[a + c*x^4])/c^3)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x^{4}}{4c^{2}}-\frac{a^{2}}{4c^{3}\left(c\,x^{4}+a\right)}-\frac{a\ln\left(c\,x^{4}+a\right)}{2c^{3}}$	41
norman	$\frac{\frac{x^{8}}{4c}-\frac{a^{2}}{2c^{3}}}{c\,x^{4}+a}-\frac{a\ln\left(c\,x^{4}+a\right)}{2c^{3}}$	43
default	$\frac{x^{4}}{4c^{2}}-\frac{a\left(\frac{\ln\left(c\,x^{4}+a\right)}{c}+\frac{a}{2c\left(c\,x^{4}+a\right)}\right)}{2c^{2}}$	44
parallelrisch	$-\frac{-c^{2}x^{8}+2\ln\left(c\,x^{4}+a\right)x^{4}ac+2a^{2}\ln\left(c\,x^{4}+a\right)+2a^{2}}{4c^{3}\left(c\,x^{4}+a\right)}$	57

input
```
int(x^11/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4/c^2-1/4*a^2/c^3/(c*x^4+a)-1/2*a*ln(c*x^4+a)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{x^{11}}{\left(a+cx^{4}\right)^{2}}\,dx=\frac{c^{2}x^{8}+acx^{4}-a^{2}-2\left(acx^{4}+a^{2}\right)\log\left(cx^{4}+a\right)}{4\left(c^{4}x^{4}+ac^{3}\right)}$$

input
```
integrate(x^11/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/4*(c^2*x^8 + a*c*x^4 - a^2 - 2*(a*c*x^4 + a^2)*log(c*x^4 + a))/(c^4*x^4
+ a*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}}{\left(a+cx^4\right)^2}\,dx = -\frac{a^2}{4ac^3+4c^4x^4} - \frac{a\log\left(a+cx^4\right)}{2c^3} + \frac{x^4}{4c^2}$$

input
```
integrate(x**11/(c*x**4+a)**2,x)
```

output
```
-a**2/(4*a*c**3 + 4*c**4*x**4) - a*log(a + c*x**4)/(2*c**3) + x**4/(4*c**2
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{\left(a+cx^4\right)^2}\,dx = \frac{x^4}{4\,c^2} - \frac{a^2}{4\,\left(c^4x^4+ac^3\right)} - \frac{a\log\left(cx^4+a\right)}{2\,c^3}$$

input
```
integrate(x^11/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^4/c^2 - 1/4*a^2/(c^4*x^4 + a*c^3) - 1/2*a*log(c*x^4 + a)/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^{11}}{(a+cx^4)^2}\,dx = \frac{x^4}{4\,c^2} - \frac{a\log\left(|cx^4+a|\right)}{2\,c^3} + \frac{2\,acx^4+a^2}{4\,(cx^4+a)c^3}$$

input
```
integrate(x^11/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x^4/c^2 - 1/2*a*log(abs(c*x^4 + a))/c^3 + 1/4*(2*a*c*x^4 + a^2)/((c*x^
4 + a)*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{(a+cx^4)^2}\,dx = \frac{x^4}{4\,c^2} - \frac{a^2}{4\,(c^4\,x^4+a\,c^3)} - \frac{a\,\ln\left(c\,x^4+a\right)}{2\,c^3}$$

input
```
int(x^11/(a + c*x^4)^2,x)
```

output
```
x^4/(4*c^2) - a^2/(4*(a*c^3 + c^4*x^4)) - (a*log(a + c*x^4))/(2*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.93

$$\int \frac{x^{11}}{(a+cx^4)^2}\,dx = \frac{-2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2-2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)ac\,x^4-2\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\right.}{4c^3\left(c\,x^4+a\right)}$$

input
```
int(x^11/(c*x^4+a)^2,x)
```

output
```
( - 2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 -
2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 -
2*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 2*log(c
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 2*a*c*x**4
 + c**2*x**8)/(4*c**3*(a + c*x**4))
```

3.68 $\int \frac{x^7}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{x^7}{(a+cx^4)^2} dx = \frac{a}{4c^2(a+cx^4)} + \frac{\log(a+cx^4)}{4c^2}$$

output
```
1/4*a/c^2/(c*x^4+a)+1/4*ln(c*x^4+a)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^7}{(a+cx^4)^2} dx = \frac{\frac{a}{a+cx^4} + \log(a+cx^4)}{4c^2}$$

input
```
Integrate[x^7/(a + c*x^4)^2,x]
```

output
```
(a/(a + c*x^4) + Log[a + c*x^4])/(4*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+cx^4)^2} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(cx^4+a)^2}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{1}{c(cx^4+a)}-\frac{a}{c(cx^4+a)^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{a}{c^2(a+cx^4)}+\frac{\log(a+cx^4)}{c^2}\right)$$

input
```
Int[x^7/(a + c*x^4)^2,x]
```

output
```
(a/(c^2*(a + c*x^4)) + Log[a + c*x^4]/c^2)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{a}{4c^2(c\,x^4+a)}+\frac{\ln(c\,x^4+a)}{4c^2}$	30
norman	$\frac{a}{4c^2(c\,x^4+a)}+\frac{\ln(c\,x^4+a)}{4c^2}$	30
risch	$\frac{a}{4c^2(c\,x^4+a)}+\frac{\ln(c\,x^4+a)}{4c^2}$	30
parallelrisch	$\frac{c\ln(c\,x^4+a)x^4+a\ln(c\,x^4+a)+a}{4c^2(c\,x^4+a)}$	40

input
```
int(x^7/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a/c^2/(c*x^4+a)+1/4*ln(c*x^4+a)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{x^7}{(a+cx^4)^2}\,dx=\frac{(cx^4+a)\log(cx^4+a)+a}{4\,(c^3x^4+ac^2)}$$

input
```
integrate(x^7/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/4*((c*x^4 + a)*log(c*x^4 + a) + a)/(c^3*x^4 + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(a+cx^4)^2}\,dx = \frac{a}{4ac^2+4c^3x^4} + \frac{\log(a+cx^4)}{4c^2}$$

input
```
integrate(x**7/(c*x**4+a)**2,x)
```

output
```
a/(4*a*c**2 + 4*c**3*x**4) + log(a + c*x**4)/(4*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(a+cx^4)^2}\,dx = \frac{a}{4\,(c^3x^4+ac^2)} + \frac{\log(cx^4+a)}{4\,c^2}$$

input
```
integrate(x^7/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*a/(c^3*x^4 + a*c^2) + 1/4*log(c*x^4 + a)/c^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{x^7}{(a+cx^4)^2}\,dx = -\frac{\frac{\log\left(\frac{|cx^4+a|}{(cx^4+a)^2|c|}\right)}{c} - \frac{a}{(cx^4+a)c}}{4\,c}$$

input
```
integrate(x^7/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(log(abs(c*x^4 + a)/((c*x^4 + a)^2*abs(c)))/c - a/((c*x^4 + a)*c))/c
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(a+cx^4)^2}\,dx = \frac{\ln\left(c\,x^4+a\right)}{4\,c^2} + \frac{a}{4\,c^2\,\left(c\,x^4+a\right)}$$

input
```
int(x^7/(a + c*x^4)^2,x)
```

output
```
log(a + c*x^4)/(4*c^2) + a/(4*c^2*(a + c*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.55

$$\int \frac{x^7}{(a+cx^4)^2}\,dx = \frac{\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a+\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)c\,x^4+\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\right.}{4c^2\left(c\,x^4+a\right)}$$

input
```
int(x^7/(c*x^4+a)^2,x)
```

output
```
(log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + log( - c
**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + log(c**(1/4)
*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + log(c**(1/4)*a**(1/4)*sq
rt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 - c*x**4)/(4*c**2*(a + c*x**4))
```

3.69 $\int \frac{x^3}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^3}{(a+cx^4)^2} dx = -\frac{1}{4c(a+cx^4)}$$

output
```
-1/4/c/(c*x^4+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+cx^4)^2} dx = -\frac{1}{4c(a+cx^4)}$$

input
```
Integrate[x^3/(a + c*x^4)^2,x]
```

output
```
-1/4*1/(c*(a + c*x^4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+cx^4)^2} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{4c\,(a+cx^4)}$$

input
```
Int[x^3/(a + c*x^4)^2,x]
```

output
```
-1/4*1/(c*(a + c*x^4))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{4c(c\,x^{4}+a)}$	15
derivativedivides	$-\frac{1}{4c(c\,x^{4}+a)}$	15
default	$-\frac{1}{4c(c\,x^{4}+a)}$	15
norman	$-\frac{1}{4c(c\,x^{4}+a)}$	15
risch	$-\frac{1}{4c(c\,x^{4}+a)}$	15
parallelrisch	$-\frac{1}{4c(c\,x^{4}+a)}$	15
orering	$-\frac{1}{4c(c\,x^{4}+a)}$	15

input
```
int(x^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/c/(c*x^4+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+cx^4)^2}\,dx = -\frac{1}{4\,(c^2x^4+ac)}$$

input
```
integrate(x^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/4/(c^2*x^4 + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+cx^4)^2}\,dx = -\frac{1}{4ac+4c^2x^4}$$

input
```
integrate(x**3/(c*x**4+a)**2,x)
```

output
```
-1/(4*a*c + 4*c**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+cx^4)^2}\,dx = -\frac{1}{4\,(cx^4+a)c}$$

input
```
integrate(x^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4/((c*x^4 + a)*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+cx^4)^2}\,dx = -\frac{1}{4\,(cx^4+a)c}$$

input
```
integrate(x^3/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4/((c*x^4 + a)*c)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+cx^4)^2}\,dx = -\frac{1}{4\,c\,(c\,x^4+a)}$$

input
```
int(x^3/(a + c*x^4)^2,x)
```

output
```
-1/(4*c*(a + c*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(a+cx^4)^2}\,dx = \frac{x^4}{4a\,(c\,x^4+a)}$$

input
```
int(x^3/(c*x^4+a)^2,x)
```

output
```
x**4/(4*a*(a + c*x**4))
```

3.70 $\int \frac{1}{x(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{x(a+cx^4)^2}\,dx = \frac{1}{4a(a+cx^4)} + \frac{\log(x)}{a^2} - \frac{\log(a+cx^4)}{4a^2}$$

output
```
1/4/a/(c*x^4+a)+ln(x)/a^2-1/4*ln(c*x^4+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(a+cx^4)^2}\,dx = \frac{\frac{a}{a+cx^4} + 4\log(x) - \log(a+cx^4)}{4a^2}$$

input
```
Integrate[1/(x*(a + c*x^4)^2),x]
```

output
```
(a/(a + c*x^4) + 4*Log[x] - Log[a + c*x^4])/(4*a^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+cx^4\right)^2}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(cx^4+a\right)^2}dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\int \left(-\frac{c}{a^2\left(cx^4+a\right)}-\frac{c}{a\left(cx^4+a\right)^2}+\frac{1}{a^2x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{\log\left(a+cx^4\right)}{a^2}+\frac{\log\left(x^4\right)}{a^2}+\frac{1}{a\left(a+cx^4\right)}\right)$$

input
```
Int[1/(x*(a + c*x^4)^2),x]
```

output
```
(1/(a*(a + c*x^4)) + Log[x^4]/a^2 - Log[a + c*x^4]/a^2)/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{4a(c\,x^4+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(c\,x^4+a)}{4a^2}$	35
norman	$-\frac{c\,x^4}{4a^2(c\,x^4+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(c\,x^4+a)}{4a^2}$	39
default	$\frac{\ln(x)}{a^2}-\frac{c\left(\frac{\ln(c\,x^4+a)}{2c}-\frac{a}{2c(c\,x^4+a)}\right)}{2a^2}$	43
parallelrisch	$\frac{4c\ln(x)x^4-c\ln(c\,x^4+a)x^4-c\,x^4+4a\ln(x)-a\ln(c\,x^4+a)}{4a^2(c\,x^4+a)}$	60

input
```
int(1/x/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/a/(c*x^4+a)+ln(x)/a^2-1/4*ln(c*x^4+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+cx^4\right)^2}\,dx=-\frac{(cx^4+a)\log\left(cx^4+a\right)-4\left(cx^4+a\right)\log\left(x\right)-a}{4\left(a^2cx^4+a^3\right)}$$

input
```
integrate(1/x/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/4*((c*x^4 + a)*log(c*x^4 + a) - 4*(c*x^4 + a)*log(x) - a)/(a^2*c*x^4 +
a^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+cx^4\right)^2}\,dx = \frac{1}{4a^2+4acx^4} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{c}+x^4\right)}{4a^2}$$

input
```
integrate(1/x/(c*x**4+a)**2,x)
```

output
```
1/(4*a**2 + 4*a*c*x**4) + log(x)/a**2 - log(a/c + x**4)/(4*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+cx^4\right)^2}\,dx = \frac{1}{4\left(acx^4+a^2\right)} - \frac{\log\left(cx^4+a\right)}{4\,a^2} + \frac{\log\left(x^4\right)}{4\,a^2}$$

input
```
integrate(1/x/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4/(a*c*x^4 + a^2) - 1/4*log(c*x^4 + a)/a^2 + 1/4*log(x^4)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+cx^4\right)^2}\,dx = \frac{\log\left(x^4\right)}{4\,a^2} - \frac{\log\left(\left|cx^4+a\right|\right)}{4\,a^2} + \frac{cx^4+2\,a}{4\left(cx^4+a\right)a^2}$$

input
```
integrate(1/x/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*log(x^4)/a^2 - 1/4*log(abs(c*x^4 + a))/a^2 + 1/4*(c*x^4 + 2*a)/((c*x^4
 + a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\,(a+cx^4)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{1}{4\,a\,(c\,x^4+a)} - \frac{\ln(c\,x^4+a)}{4\,a^2}$$

input
```
int(1/(x*(a + c*x^4)^2),x)
```

output
```
log(x)/a^2 + 1/(4*a*(a + c*x^4)) - log(a + c*x^4)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.53

$$\int \frac{1}{x\,(a+cx^4)^2}\,dx$$
$$= \frac{-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a-\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)c\,x^4-\log\left(c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{}\right.}{4a^2\,(c\,x^4+a)}$$

input
```
int(1/x/(c*x^4+a)^2,x)
```

output
```
( - log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - log(
- c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 - log(c**(1
/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - log(c**(1/4)*a**(1/4)
*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + 4*log(x)*a + 4*log(x)*c*x**4
 - c*x**4)/(4*a**2*(a + c*x**4))
```

3.71 $\int \frac{1}{x^5(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{1}{x^5\left(a+cx^4\right)^2}\,dx = -\frac{1}{4a^2x^4}-\frac{c}{4a^2\left(a+cx^4\right)}-\frac{2c\log(x)}{a^3}+\frac{c\log\left(a+cx^4\right)}{2a^3}$$

output
```
-1/4/a^2/x^4-1/4*c/a^2/(c*x^4+a)-2*c*ln(x)/a^3+1/2*c*ln(c*x^4+a)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\left(a+cx^4\right)^2}\,dx = -\frac{a\left(\frac{1}{x^4}+\frac{c}{a+cx^4}\right)+8c\log(x)-2c\log\left(a+cx^4\right)}{4a^3}$$

input
```
Integrate[1/(x^5*(a + c*x^4)^2),x]
```

output
```
-1/4*(a*(x^(-4) + c/(a + c*x^4)) + 8*c*Log[x] - 2*c*Log[a + c*x^4])/a^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+c x^4\right)^2} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4} \int \frac{1}{x^8 \left(c x^4+a\right)^2}dx^4$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{4} \int \left(\frac{2 c^2}{a^3 \left(c x^4+a\right)}+\frac{c^2}{a^2 \left(c x^4+a\right)^2}-\frac{2 c}{a^3 x^4}+\frac{1}{a^2 x^8}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} \left(-\frac{2 c \log \left(x^4\right)}{a^3}+\frac{2 c \log \left(a+c x^4\right)}{a^3}-\frac{c}{a^2 \left(a+c x^4\right)}-\frac{1}{a^2 x^4}\right)$$

input
```
Int[1/(x^5*(a + c*x^4)^2),x]
```

output
```
(-(1/(a^2*x^4)) - c/(a^2*(a + c*x^4)) - (2*c*Log[x^4])/a^3 + (2*c*Log[a + 
c*x^4])/a^3)/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{1}{4a^2x^4}-\frac{2c\ln(x)}{a^3}+\frac{c^2\left(\frac{\ln(cx^4+a)}{c}-\frac{a}{2c(cx^4+a)}\right)}{2a^3}$	54
norman	$\frac{-\frac{1}{4a}+\frac{c^2x^8}{2a^3}}{x^4(cx^4+a)}-\frac{2c\ln(x)}{a^3}+\frac{c\ln(cx^4+a)}{2a^3}$	54
risch	$\frac{-\frac{cx^4}{2a^2}-\frac{1}{4a}}{x^4(cx^4+a)}-\frac{2c\ln(x)}{a^3}+\frac{c\ln(-cx^4-a)}{2a^3}$	55
parallelrisch	$-\frac{8\ln(x)x^8c^2-2\ln(cx^4+a)x^8c^2-2c^2x^8+8\ln(x)x^4ac-2\ln(cx^4+a)x^4ac+a^2}{4a^3x^4(cx^4+a)}$	80

input
```
int(1/x^5/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/x^4-2*c*ln(x)/a^3+1/2*c^2/a^3*(ln(c*x^4+a)/c-1/2/c*a/(c*x^4+a))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{1}{x^5(a+cx^4)^2}\,dx = -\frac{2\,acx^4+a^2-2\,(c^2x^8+acx^4)\log(cx^4+a)+8\,(c^2x^8+acx^4)\log(x)}{4\,(a^3cx^8+a^4x^4)}$$

input
```
integrate(1/x^5/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a*c*x^4 + a^2 - 2*(c^2*x^8 + a*c*x^4)*log(c*x^4 + a) + 8*(c^2*x^8
+ a*c*x^4)*log(x))/(a^3*c*x^8 + a^4*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5 (a+cx^4)^2} \, dx = \frac{-a-2cx^4}{4a^3x^4+4a^2cx^8} - \frac{2c\log(x)}{a^3} + \frac{c\log\left(\frac{a}{c}+x^4\right)}{2a^3}$$

input
```
integrate(1/x**5/(c*x**4+a)**2,x)
```

output
```
(-a - 2*c*x**4)/(4*a**3*x**4 + 4*a**2*c*x**8) - 2*c*log(x)/a**3 + c*log(a/
c + x**4)/(2*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5 (a+cx^4)^2} \, dx = -\frac{2\,cx^4+a}{4\,(a^2cx^8+a^3x^4)} + \frac{c\log(cx^4+a)}{2\,a^3} - \frac{c\log(x^4)}{2\,a^3}$$

input
```
integrate(1/x^5/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*c*x^4 + a)/(a^2*c*x^8 + a^3*x^4) + 1/2*c*log(c*x^4 + a)/a^3 - 1/2*
c*log(x^4)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a+cx^4)^2} \, dx = -\frac{c \log(x^4)}{2 a^3} + \frac{c \log(|cx^4+a|)}{2 a^3} - \frac{2 cx^4 + a}{4 (cx^8 + ax^4) a^2}$$

input
```
integrate(1/x^5/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/2*c*log(x^4)/a^3 + 1/2*c*log(abs(c*x^4 + a))/a^3 - 1/4*(2*c*x^4 + a)/((
c*x^8 + a*x^4)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5 (a+cx^4)^2} \, dx = \frac{c \ln(c x^4 + a)}{2 a^3} - \frac{\frac{1}{4 a} + \frac{c x^4}{2 a^2}}{c x^8 + a x^4} - \frac{2 c \ln(x)}{a^3}$$

input
```
int(1/(x^5*(a + c*x^4)^2),x)
```

output
```
(c*log(a + c*x^4))/(2*a^3) - (1/(4*a) + (c*x^4)/(2*a^2))/(a*x^4 + c*x^8) -
 (2*c*log(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.12

$$\int \frac{1}{x^5 (a+cx^4)^2} \, dx = \frac{2 \log\left(-c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{a} + \sqrt{c}\, x^2\right) ac x^4 + 2 \log\left(-c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{a} + \sqrt{c}\, x^2\right) c^2 x^8 + 2 \log\left(c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}\, x + \sqrt{}\right.}{4 a^3 x^4 (c}$$

input
```
int(1/x^5/(c*x^4+a)^2,x)
```

output
```
(2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 +
 2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*x**8
+ 2*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 2
*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*x**8 - 8*l
og(x)*a*c*x**4 - 8*log(x)*c**2*x**8 - a**2 + 2*c**2*x**8)/(4*a**3*x**4*(a
+ c*x**4))
```

3.72 $\int \frac{x^{13}}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^{13}}{(a+cx^4)^2} dx = -\frac{ax^2}{c^3} + \frac{x^6}{6c^2} - \frac{a^2x^2}{4c^3(a+cx^4)} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{7/2}}$$

output
```
-a*x^2/c^3+1/6*x^6/c^2-1/4*a^2*x^2/c^3/(c*x^4+a)+5/4*a^(3/2)*arctan(c^(1/2
)*x^2/a^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{x^{13}}{(a+cx^4)^2} dx = \frac{x^2\left(-12a + 2cx^4 - \frac{3a^2}{a+cx^4}\right)}{12c^3} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{7/2}}$$

input
```
Integrate[x^13/(a + c*x^4)^2,x]
```

output
```
(x^2*(-12*a + 2*c*x^4 - (3*a^2)/(a + c*x^4)))/(12*c^3) + (5*a^(3/2)*ArcTan
[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+cx^4)^2}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{ 252}$$

$$\frac{1}{2}\left(\frac{5\int \frac{x^8}{cx^4+a}dx^2}{2c}-\frac{x^{10}}{2c\,(a+cx^4)}\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{2}\left(\frac{5\int \left(\frac{x^4}{c}+\frac{a^2}{c^2(cx^4+a)}-\frac{a}{c^2}\right)dx^2}{2c}-\frac{x^{10}}{2c\,(a+cx^4)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{5/2}}-\frac{ax^2}{c^2}+\frac{x^6}{3c}\right)}{2c}-\frac{x^{10}}{2c\,(a+cx^4)}\right)$$

input
```
Int[x^13/(a + c*x^4)^2,x]
```

output
```
(-1/2*x^10/(c*(a + c*x^4)) + (5*(-((a*x^2)/c^2) + x^6/(3*c) + (a^(3/2)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(5/2)))/(2*c))/2
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{-\frac{1}{6}c\,x^{6}+a\,x^{2}}{c^{3}}+\frac{a^{2}\left(-\frac{x^{2}}{2\left(c\,x^{4}+a\right)}+\frac{5\arctan\left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{2\sqrt{a c}}\right)}{2c^{3}}$	60
risch	$\frac{x^{6}}{6c^{2}}-\frac{a\,x^{2}}{c^{3}}-\frac{a^{2}x^{2}}{4c^{3}\left(c\,x^{4}+a\right)}+\frac{5\sqrt{-a c}\,a\ln\left(c\,x^{2}+\sqrt{-a c}\right)}{8c^{4}}-\frac{5\sqrt{-a c}\,a\ln\left(c\,x^{2}-\sqrt{-a c}\right)}{8c^{4}}$	91

input
```
int(x^13/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/6*c*x^6+a*x^2)+1/2/c^3*a^2*(-1/2*x^2/(c*x^4+a)+5/2/(a*c)^(1/2)*
arctan(c*x^2/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.42

$$\int \frac{x^{13}}{(a+cx^4)^2}\,dx = \left[\frac{4\,c^2x^{10}-20\,acx^6-30\,a^2x^2+15\,(acx^4+a^2)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)}{24\,(c^4x^4+ac^3)}, \frac{2\,c^2x^{10}-10\,acx^6-15\,a^2x^2 \ldots}{12\,(\ldots}\right.$$

input
```
integrate(x^13/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/24*(4*c^2*x^10 - 20*a*c*x^6 - 30*a^2*x^2 + 15*(a*c*x^4 + a^2)*sqrt(-a/c
)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)))/(c^4*x^4 + a*c^3), 1/
12*(2*c^2*x^10 - 10*a*c*x^6 - 15*a^2*x^2 + 15*(a*c*x^4 + a^2)*sqrt(a/c)*ar
ctan(c*x^2*sqrt(a/c)/a))/(c^4*x^4 + a*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \frac{x^{13}}{(a+cx^4)^2}\,dx = -\frac{a^2x^2}{4ac^3+4c^4x^4}-\frac{ax^2}{c^3}-\frac{5\sqrt{-\frac{a^3}{c^7}}\log\left(x^2-\frac{c^3\sqrt{-\frac{a^3}{c^7}}}{a}\right)}{8} + \frac{5\sqrt{-\frac{a^3}{c^7}}\log\left(x^2+\frac{c^3\sqrt{-\frac{a^3}{c^7}}}{a}\right)}{8}+\frac{x^6}{6c^2}$$

input
```
integrate(x**13/(c*x**4+a)**2,x)
```

output
```
-a**2*x**2/(4*a*c**3 + 4*c**4*x**4) - a*x**2/c**3 - 5*sqrt(-a**3/c**7)*log
(x**2 - c**3*sqrt(-a**3/c**7)/a)/8 + 5*sqrt(-a**3/c**7)*log(x**2 + c**3*sq
rt(-a**3/c**7)/a)/8 + x**6/(6*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{x^{13}}{(a+cx^4)^2}\,dx = -\frac{a^2x^2}{4\,(c^4x^4+ac^3)} + \frac{5\,a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}c^3} + \frac{cx^6-6\,ax^2}{6\,c^3}$$

input
```
integrate(x^13/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*x^2/(c^4*x^4 + a*c^3) + 5/4*a^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c
)*c^3) + 1/6*(c*x^6 - 6*a*x^2)/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{x^{13}}{(a+cx^4)^2}\,dx = -\frac{a^2x^2}{4\,(cx^4+a)c^3} + \frac{5\,a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}c^3} + \frac{c^4x^6-6\,ac^3x^2}{6\,c^6}$$

input
```
integrate(x^13/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*a^2*x^2/((c*x^4 + a)*c^3) + 5/4*a^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c
)*c^3) + 1/6*(c^4*x^6 - 6*a*c^3*x^2)/c^6
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{x^{13}}{(a+cx^4)^2}\,dx = \frac{x^6}{6\,c^2} - \frac{a^2\,x^2}{4\,(c^4\,x^4+a\,c^3)} - \frac{a\,x^2}{c^3} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,c^{7/2}}$$

input
```
int(x^13/(a + c*x^4)^2,x)
```

output
```
x^6/(6*c^2) - (a^2*x^2)/(4*(a*c^3 + c^4*x^4)) - (a*x^2)/c^3 + (5*a^(3/2)*a
tan((c^(1/2)*x^2)/a^(1/2)))/(4*c^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.66

$$\int \frac{x^{13}}{(a+cx^4)^2}\,dx$$

$$= \frac{-15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2-15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)ac\,x^4-15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right.}{12c^4\,(c\,x^4+a)}$$

input
```
int(x^13/(c*x^4+a)^2,x)
```

output
```
( - 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**
(1/4)*a**(1/4)*sqrt(2)))*a**2 - 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - 15*sqrt(c)
*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)
*sqrt(2)))*a**2 - 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*s
qrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - 15*a**2*c*x**2 - 10*a*c*
*2*x**6 + 2*c**3*x**10)/(12*c**4*(a + c*x**4))
```

3.73 $\int \frac{x^9}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{x^9}{(a+cx^4)^2}\,dx=\frac{x^2}{2c^2}+\frac{ax^2}{4c^2(a+cx^4)}-\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{5/2}}$$

output
```
1/2*x^2/c^2+1/4*a*x^2/c^2/(c*x^4+a)-3/4*a^(1/2)*arctan(c^(1/2)*x^2/a^(1/2)
)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{(a+cx^4)^2}\,dx=\frac{x^2}{2c^2}+\frac{ax^2}{4c^2(a+cx^4)}-\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{5/2}}$$

input
```
Integrate[x^9/(a + c*x^4)^2,x]
```

output
```
x^2/(2*c^2) + (a*x^2)/(4*c^2*(a + c*x^4)) - (3*Sqrt[a]*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+cx^4)^2} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{3\int \frac{x^4}{cx^4+a}dx^2}{2c}-\frac{x^6}{2c(a+cx^4)}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{x^2}{c}-\frac{a\int \frac{1}{cx^4+a}dx^2}{c}\right)}{2c}-\frac{x^6}{2c(a+cx^4)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{x^2}{c}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}}\right)}{2c}-\frac{x^6}{2c(a+cx^4)}\right)$$

input
```
Int[x^9/(a + c*x^4)^2,x]
```

output
```
(-1/2*x^6/(c*(a + c*x^4)) + (3*(x^2/c - (Sqrt[a]*ArcTan[(Sqrt[c]*x^2)/Sqrt
[a]])/c^(3/2)))/(2*c))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^{2}}{2 c^{2}}-\frac{a \left(-\frac{x^{2}}{2 \left(c \,x^{4}+a \right)}+\frac{3 \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{2 \sqrt{a c}}\right)}{2 c^{2}}$	49
risch	$\frac{x^{2}}{2 c^{2}}+\frac{a \,x^{2}}{4 c^{2} \left(c \,x^{4}+a \right)}+\frac{3 \sqrt{-a c}\, \ln \left(c \,x^{2}-\sqrt{-a c}\right)}{8 c^{3}}-\frac{3 \sqrt{-a c}\, \ln \left(c \,x^{2}+\sqrt{-a c}\right)}{8 c^{3}}$	78

input
```
int(x^9/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/c^2-1/2/c^2*a*(-1/2*x^2/(c*x^4+a)+3/2/(a*c)^(1/2)*arctan(c*x^2/(a*
c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.40

$$\int \frac{x^9}{(a+cx^4)^2}\,dx = \left[\frac{4\,cx^6+6\,ax^2+3\,(cx^4+a)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)}{8\,(c^3x^4+ac^2)}, \frac{2\,cx^6+3\,ax^2-3\,(cx^4+a)\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right.}{4\,(c^3x^4+ac^2)}\right.$$

input
```
integrate(x^9/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(4*c*x^6 + 6*a*x^2 + 3*(c*x^4 + a)*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sq
rt(-a/c) - a)/(c*x^4 + a)))/(c^3*x^4 + a*c^2), 1/4*(2*c*x^6 + 3*a*x^2 - 3*
(c*x^4 + a)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a))/(c^3*x^4 + a*c^2)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{x^9}{(a+cx^4)^2}\,dx = \frac{ax^2}{4ac^2+4c^3x^4} + \frac{3\sqrt{-\frac{a}{c^5}}\log\left(-c^2\sqrt{-\frac{a}{c^5}}+x^2\right)}{8} - \frac{3\sqrt{-\frac{a}{c^5}}\log\left(c^2\sqrt{-\frac{a}{c^5}}+x^2\right)}{8} + \frac{x^2}{2c^2}$$

input
```
integrate(x**9/(c*x**4+a)**2,x)
```

output
```
a*x**2/(4*a*c**2 + 4*c**3*x**4) + 3*sqrt(-a/c**5)*log(-c**2*sqrt(-a/c**5)
+ x**2)/8 - 3*sqrt(-a/c**5)*log(c**2*sqrt(-a/c**5) + x**2)/8 + x**2/(2*c**
2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \frac{x^9}{(a+cx^4)^2}\,dx = \frac{ax^2}{4\,(c^3x^4+ac^2)} + \frac{x^2}{2\,c^2} - \frac{3\,a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}c^2}$$

input
```
integrate(x^9/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*a*x^2/(c^3*x^4 + a*c^2) + 1/2*x^2/c^2 - 3/4*a*arctan(c*x^2/sqrt(a*c))/
(sqrt(a*c)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{(a+cx^4)^2}\,dx = \frac{ax^2}{4\,(cx^4+a)c^2} + \frac{x^2}{2\,c^2} - \frac{3\,a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}c^2}$$

input
```
integrate(x^9/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*a*x^2/((c*x^4 + a)*c^2) + 1/2*x^2/c^2 - 3/4*a*arctan(c*x^2/sqrt(a*c))/
(sqrt(a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{(a+cx^4)^2}\,dx = \frac{x^2}{2\,c^2} + \frac{a\,x^2}{4\,\left(c^3\,x^4+a\,c^2\right)} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,c^{5/2}}$$

input
```
int(x^9/(a + c*x^4)^2,x)
```

output
```
x^2/(2*c^2) + (a*x^2)/(4*(a*c^2 + c^3*x^4)) - (3*a^(1/2)*atan((c^(1/2)*x^2
)/a^(1/2)))/(4*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.87

$$\int \frac{x^9}{(a+cx^4)^2}\,dx$$
$$= \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c\,x^4+3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a+3}{4c^3\,(c\,x^4+a)}$$

input
```
int(x^9/(c*x^4+a)^2,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4
)*a**(1/4)*sqrt(2)))*a + 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4 + 3*sqrt(c)*sqrt(a)*at
an((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*
a + 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*c*x**4 + 3*a*c*x**2 + 2*c**2*x**6)/(4*c**3*(a + c*
x**4))
```

3.74 $\int \frac{x^5}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{x^5}{(a+cx^4)^2} dx = -\frac{x^2}{4c(a+cx^4)} + \frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}c^{3/2}}$$

output
```
-1/4*x^2/c/(c*x^4+a)+1/4*arctan(c^(1/2)*x^2/a^(1/2))/a^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+cx^4)^2} dx = -\frac{x^2}{4c(a+cx^4)} + \frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}c^{3/2}}$$

input
```
Integrate[x^5/(a + c*x^4)^2,x]
```

output
```
-1/4*x^2/(c*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*Sqrt[a]*c^(3/2
))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {807, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+cx^4)^2} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{cx^4+a}dx^2}{2c}-\frac{x^2}{2c(a+cx^4)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}c^{3/2}}-\frac{x^2}{2c(a+cx^4)}\right)$$

input
```
Int[x^5/(a + c*x^4)^2,x]
```

output
```
(-1/2*x^2/(c*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*c^(3/
2)))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^{2}}{4c\left(c\,x^{4}+a\right)}+\frac{\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{4c\sqrt{ac}}$	40
risch	$-\frac{x^{2}}{4c\left(c\,x^{4}+a\right)}-\frac{\ln\left(x^{2}\sqrt{-ac}-a\right)}{8\sqrt{-ac}\,c}+\frac{\ln\left(x^{2}\sqrt{-ac}+a\right)}{8\sqrt{-ac}\,c}$	69

input
```
int(x^5/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2/c/(c*x^4+a)+1/4/c/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.59

$$\int \frac{x^5}{(a+cx^4)^2}\,dx = \left[-\frac{2\,acx^2 + (cx^4+a)\sqrt{-ac}\log\left(\frac{cx^4-2\,\sqrt{-ac}x^2-a}{cx^4+a}\right)}{8\,(ac^3x^4+a^2c^2)},\right.$$
$$\left. -\frac{acx^2 + (cx^4+a)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{4\,(ac^3x^4+a^2c^2)}\right]$$

input
```
integrate(x^5/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(2*a*c*x^2 + (c*x^4 + a)*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 -
a)/(c*x^4 + a)))/(a*c^3*x^4 + a^2*c^2), -1/4*(a*c*x^2 + (c*x^4 + a)*sqrt(a
*c)*arctan(sqrt(a*c)/(c*x^2)))/(a*c^3*x^4 + a^2*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(39) = 78.

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{x^5}{(a+cx^4)^2}\,dx = -\frac{x^2}{4ac+4c^2x^4} - \frac{\sqrt{-\frac{1}{ac^3}}\log\left(-ac\sqrt{-\frac{1}{ac^3}}+x^2\right)}{8}$$
$$+\frac{\sqrt{-\frac{1}{ac^3}}\log\left(ac\sqrt{-\frac{1}{ac^3}}+x^2\right)}{8}$$

input
```
integrate(x**5/(c*x**4+a)**2,x)
```

output
```
-x**2/(4*a*c + 4*c**2*x**4) - sqrt(-1/(a*c**3))*log(-a*c*sqrt(-1/(a*c**3))
 + x**2)/8 + sqrt(-1/(a*c**3))*log(a*c*sqrt(-1/(a*c**3)) + x**2)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+cx^4)^2}\,dx = -\frac{x^2}{4\,(c^2x^4+ac)} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}c}$$

input
```
integrate(x^5/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^2/(c^2*x^4 + a*c) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(a+cx^4)^2}\,dx = -\frac{x^2}{4\,(cx^4+a)c} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}c}$$

input
```
integrate(x^5/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*x^2/((c*x^4 + a)*c) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x^5}{(a+cx^4)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,\sqrt{a}\,c^{3/2}} - \frac{x^2}{4\,c\,\left(c\,x^4+a\right)}$$

input
```
int(x^5/(a + c*x^4)^2,x)
```

output
```
atan((c^(1/2)*x^2)/a^(1/2))/(4*a^(1/2)*c^(3/2)) - x^2/(4*c*(a + c*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.41

$$\int \frac{x^5}{(a+cx^4)^2}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c\,x^4-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a-\sqrt{}}{4a\,c^2\left(c\,x^4+a\right)}$$

input
```
int(x^5/(c*x^4+a)^2,x)
```

output
```
( - (sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1
/4)*a**(1/4)*sqrt(2)))*a + sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4 + sqrt(c)*sqrt(a)*atan
((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
+ sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)
*a**(1/4)*sqrt(2)))*c*x**4 + a*c*x**2))/(4*a*c**2*(a + c*x**4))
```

3.75 $\int \frac{x}{(a+cx^4)^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{x}{(a+cx^4)^2} \, dx = \frac{x^2}{4a(a+cx^4)} + \frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}$$

output
```
1/4*x^2/a/(c*x^4+a)+1/4*arctan(c^(1/2)*x^2/a^(1/2))/a^(3/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+cx^4)^2} \, dx = \frac{x^2}{4a(a+cx^4)} + \frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{c}}$$

input
```
Integrate[x/(a + c*x^4)^2,x]
```

output
```
x^2/(4*a*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(4*a^(3/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {807, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{(cx^4+a)^2}dx^2$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{cx^4+a}dx^2}{2a}+\frac{x^2}{2a(a+cx^4)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x^2}{2a(a+cx^4)}\right)$$

input
```
Int[x/(a + c*x^4)^2,x]
```

output
```
(x^2/(2*a*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])
)/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^{2}}{4a(c\,x^{4}+a)}+\frac{\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{4a\sqrt{ac}}$	40
risch	$\frac{x^{2}}{4a(c\,x^{4}+a)}-\frac{\ln\left(x^{2}\sqrt{-ac}-a\right)}{8\sqrt{-ac}\,a}+\frac{\ln\left(x^{2}\sqrt{-ac}+a\right)}{8\sqrt{-ac}\,a}$	69

input
```
int(x/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2/a/(c*x^4+a)+1/4/a/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.63

$$\int \frac{x}{\left(a+cx^4\right)^2}\,dx = \left[\frac{2\,acx^2-\left(cx^4+a\right)\sqrt{-ac}\log\left(\frac{cx^4-2\,\sqrt{-ac}x^2-a}{cx^4+a}\right)}{8\,\left(a^2c^2x^4+a^3c\right)}, \frac{acx^2-\left(cx^4+a\right)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{4\,\left(a^2c^2x^4+a^3c\right)}\right]$$

input
```
integrate(x/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/8*(2*a*c*x^2 - (c*x^4 + a)*sqrt(-a*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a
)/(c*x^4 + a)))/(a^2*c^2*x^4 + a^3*c), 1/4*(a*c*x^2 - (c*x^4 + a)*sqrt(a*c
)*arctan(sqrt(a*c)/(c*x^2)))/(a^2*c^2*x^4 + a^3*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(39) = 78.

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \frac{x}{\left(a+cx^4\right)^2}\,dx = \frac{x^2}{4a^2+4acx^4} - \frac{\sqrt{-\frac{1}{a^3c}}\log\left(-a^2\sqrt{-\frac{1}{a^3c}}+x^2\right)}{8} + \frac{\sqrt{-\frac{1}{a^3c}}\log\left(a^2\sqrt{-\frac{1}{a^3c}}+x^2\right)}{8}$$

input
```
integrate(x/(c*x**4+a)**2,x)
```

output
```
x**2/(4*a**2 + 4*a*c*x**4) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a**3*c))
 + x**2)/8 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x**2)/8
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+cx^4)^2}\,dx = \frac{x^2}{4\,(acx^4+a^2)} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}a}$$

input
```
integrate(x/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^2/(a*c*x^4 + a^2) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+cx^4)^2}\,dx = \frac{x^2}{4\,(cx^4+a)a} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}a}$$

input
```
integrate(x/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x^2/((c*x^4 + a)*a) + 1/4*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x}{(a+cx^4)^2}\,dx = \frac{x^2}{4\,a\,\left(c\,x^4+a\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,a^{3/2}\,\sqrt{c}}$$

input
```
int(x/(a + c*x^4)^2,x)
```

output
```
x^2/(4*a*(a + c*x^4)) + atan((c^(1/2)*x^2)/a^(1/2))/(4*a^(3/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.39

$$\int \frac{x}{(a+cx^4)^2}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c\,x^4-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a-\sqrt{}}{4a^2c\,(c\,x^4+a)}$$

input
```
int(x/(c*x^4+a)^2,x)
```

output
```
( - sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/
4)*a**(1/4)*sqrt(2)))*a - sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2)
- 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4 - sqrt(c)*sqrt(a)*atan(
(c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a -
 sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*c*x**4 + a*c*x**2)/(4*a**2*c*(a + c*x**4))
```

3.76 $\int \frac{1}{x^3(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{1}{x^3\left(a+cx^4\right)^2}\,dx=-\frac{1}{2a^2x^2}-\frac{cx^2}{4a^2\left(a+cx^4\right)}-\frac{3\sqrt{c}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-1/2/a^2/x^2-1/4*c*x^2/a^2/(c*x^4+a)-3/4*c^(1/2)*arctan(c^(1/2)*x^2/a^(1/2
))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^3\left(a+cx^4\right)^2}\,dx = \frac{-\frac{\sqrt{a}\left(2a+3cx^4\right)}{x^2\left(a+cx^4\right)}+3\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+3\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[1/(x^3*(a + c*x^4)^2),x]
```

output
```
(-((Sqrt[a]*(2*a + 3*c*x^4))/(x^2*(a + c*x^4))) + 3*Sqrt[c]*ArcTan[1 - (Sq
rt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[c]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {807, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+cx^4)^2} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2} \int \frac{1}{x^4 (cx^4+a)^2} dx^2$$

$$\Big\downarrow \text{ 253}$$

$$\frac{1}{2} \left(\frac{3 \int \frac{1}{x^4 (cx^4+a)} dx^2}{2a} + \frac{1}{2ax^2 (a+cx^4)} \right)$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2} \left(\frac{3 \left(-\frac{c \int \frac{1}{cx^4+a} dx^2}{a} - \frac{1}{ax^2} \right)}{2a} + \frac{1}{2ax^2 (a+cx^4)} \right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2} \left(\frac{3 \left(-\frac{\sqrt{c} \arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax^2} \right)}{2a} + \frac{1}{2ax^2 (a+cx^4)} \right)$$

input
```
Int[1/(x^3*(a + c*x^4)^2),x]
```

output
```
(1/(2*a*x^2*(a + c*x^4)) + (3*(-(1/(a*x^2)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]])/a^(3/2)))/(2*a))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{2a^2x^2}-\frac{c\left(\frac{x^2}{2c\,x^4+2a}+\frac{3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2a^2}$	49
risch	$\frac{-\frac{3c\,x^4}{4a^2}-\frac{1}{2a}}{x^2(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\text{RootOf}\left(a^5\_Z^2+c\right)}\_R\ln\left(\left(-5a^5\_R^2-4c\right)x^2-a^3\_R\right)\right)}{8}$	71

input
```
int(1/x^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/x^2-1/2/a^2*c*(1/2*x^2/(c*x^4+a)+3/2/(a*c)^(1/2)*arctan(c*x^2/(a*
c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^3\left(a+cx^4\right)^2}\,dx = \left[-\frac{6\,cx^4-3\left(cx^6+ax^2\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+4\,a}{8\left(a^2cx^6+a^3x^2\right)},\ -\frac{3\,cx^4+3\left(cx^6+ax^2\right)\sqrt{\frac{c}{a}}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)+2\,a}{4\left(a^2cx^6+a^3x^2\right)}\right]$$

input
```
integrate(1/x^3/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[-1/8*(6*c*x^4 - 3*(c*x^6 + a*x^2)*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c
/a) - a)/(c*x^4 + a)) + 4*a)/(a^2*c*x^6 + a^3*x^2), -1/4*(3*c*x^4 + 3*(c*x
^6 + a*x^2)*sqrt(c/a)*arctan(x^2*sqrt(c/a)) + 2*a)/(a^2*c*x^6 + a^3*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^3(a+cx^4)^2}\,dx = \frac{3\sqrt{-\frac{c}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{c}{a^5}}}{c}+x^2\right)}{8} - \frac{3\sqrt{-\frac{c}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{c}{a^5}}}{c}+x^2\right)}{8} + \frac{-2a-3cx^4}{4a^3x^2+4a^2cx^6}$$

input
```
integrate(1/x**3/(c*x**4+a)**2,x)
```

output
```
3*sqrt(-c/a**5)*log(-a**3*sqrt(-c/a**5)/c + x**2)/8 - 3*sqrt(-c/a**5)*log(
a**3*sqrt(-c/a**5)/c + x**2)/8 + (-2*a - 3*c*x**4)/(4*a**3*x**2 + 4*a**2*c
*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3(a+cx^4)^2}\,dx = -\frac{3\,cx^4+2\,a}{4\,(a^2cx^6+a^3x^2)} - \frac{3\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}a^2}$$

input
```
integrate(1/x^3/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(3*c*x^4 + 2*a)/(a^2*c*x^6 + a^3*x^2) - 3/4*c*arctan(c*x^2/sqrt(a*c))
/(sqrt(a*c)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (a+cx^4)^2} \, dx = -\frac{3 \, c \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4 \sqrt{ac} a^2} - \frac{3 \, cx^4 + 2 \, a}{4 \, (cx^6 + ax^2) a^2}$$

input
```
integrate(1/x^3/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-3/4*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) - 1/4*(3*c*x^4 + 2*a)/((c*x
^6 + a*x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a+cx^4)^2} \, dx = -\frac{\frac{1}{2 \, a} + \frac{3 \, c \, x^4}{4 \, a^2}}{c \, x^6 + a \, x^2} - \frac{3 \, \sqrt{c} \, \mathrm{atan}\left(\frac{\sqrt{c} \, x^2}{\sqrt{a}}\right)}{4 \, a^{5/2}}$$

input
```
int(1/(x^3*(a + c*x^4)^2),x)
```

output
```
- (1/(2*a) + (3*c*x^4)/(4*a^2))/(a*x^2 + c*x^6) - (3*c^(1/2)*atan((c^(1/2)
*x^2)/a^(1/2)))/(4*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.97

$$\int \frac{1}{x^3 (a+cx^4)^2} \, dx = \frac{3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} - 2\sqrt{c}\,x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) a\,x^2 + 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} - 2\sqrt{c}\,x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) c\,x^6 + 3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2} + 2\sqrt{c}\,x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) a}{4a^3 x^2 \left(c\,x^4 + a\right)}$$

input
```
int(1/x^3/(c*x^4+a)^2,x)
```

output
```
(3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4
)*a**(1/4)*sqrt(2)))*a*x**2 + 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sq
rt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**6 + 3*sqrt(c)*sqrt(
a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(
2)))*a*x**2 + 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**6 - 2*a**2 - 3*a*c*x**4)/(4*a**3*x*
*2*(a + c*x**4))
```

3.77 $\int \frac{1}{x^7(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{1}{x^7\left(a+cx^4\right)^2}\,dx=-\frac{1}{6a^2x^6}+\frac{c}{a^3x^2}+\frac{c^2x^2}{4a^3\left(a+cx^4\right)}+\frac{5c^{3/2}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{7/2}}$$

output
```
-1/6/a^2/x^6+c/a^3/x^2+1/4*c^2*x^2/a^3/(c*x^4+a)+5/4*c^(3/2)*arctan(c^(1/2
)*x^2/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^7\left(a+cx^4\right)^2}\,dx=\frac{\frac{\sqrt{a}\left(-2a^2+10acx^4+15c^2x^8\right)}{x^6\left(a+cx^4\right)}-15c^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-15c^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{12a^{7/2}}$$

input
```
Integrate[1/(x^7*(a + c*x^4)^2),x]
```

output
```
((Sqrt[a]*(-2*a^2 + 10*a*c*x^4 + 15*c^2*x^8))/(x^6*(a + c*x^4)) - 15*c^(3/
2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 15*c^(3/2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(12*a^(7/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {807, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+cx^4)^2} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^8 (cx^4+a)^2} dx^2$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{2} \left(\frac{5 \int \frac{1}{x^8 (cx^4+a)} dx^2}{2a} + \frac{1}{2ax^6 (a+cx^4)} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{5 \left(-\frac{c \int \frac{1}{x^4 (cx^4+a)} dx^2}{a} - \frac{1}{3ax^6} \right)}{2a} + \frac{1}{2ax^6 (a+cx^4)} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{5 \left(-\frac{c \left(-\frac{c \int \frac{1}{cx^4+a} dx^2}{a} - \frac{1}{ax^2} \right)}{a} - \frac{1}{3ax^6} \right)}{2a} + \frac{1}{2ax^6 (a+cx^4)} \right)$$

$$\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{5\left(-\frac{c\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax^2}\right)}{a}-\frac{1}{3ax^6}\right)}{2a}+\frac{1}{2ax^6\left(a+cx^4\right)}\right)$$

input
```
Int[1/(x^7*(a + c*x^4)^2),x]
```

output
```
(1/(2*a*x^6*(a + c*x^4)) + (5*(-1/3*1/(a*x^6) - (c*(-(1/(a*x^2)) - (Sqrt[c
]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2)))/a))/(2*a))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{1}{6a^2x^6}+\frac{c}{a^3x^2}+\frac{c^2\left(\frac{x^2}{2c\,x^4+2a}+\frac{5\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2a^3}$	59
risch	$\frac{\frac{5c^2x^8}{4a^3}+\frac{5c\,x^4}{6a^2}-\frac{1}{6a}}{x^6(c\,x^4+a)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(a^7\_Z^2+c^3\right)}\_R\ln\left(\left(5a^7\_R^2+4c^3\right)x^2-a^4c\_R\right)\right)}{8}$	87

input
```
int(1/x^7/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6/a^2/x^6+c/a^3/x^2+1/2*c^2/a^3*(1/2*x^2/(c*x^4+a)+5/2/(a*c)^(1/2)*arct
an(c*x^2/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.51

$$\int \frac{1}{x^7(a+cx^4)^2}\,dx$$
$$=\left[\frac{30\,c^2x^8+20\,acx^4+15\,(c^2x^{10}+acx^6)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-4\,a^2}{24\,(a^3cx^{10}+a^4x^6)},\frac{15\,c^2x^8+10\,acx^4+15\,(c^2x^{10}}{12\,(a^3}\right.$$

input
```
integrate(1/x^7/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
[1/24*(30*c^2*x^8 + 20*a*c*x^4 + 15*(c^2*x^10 + a*c*x^6)*sqrt(-c/a)*log((c
*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - 4*a^2)/(a^3*c*x^10 + a^4*x^6
), 1/12*(15*c^2*x^8 + 10*a*c*x^4 + 15*(c^2*x^10 + a*c*x^6)*sqrt(c/a)*arcta
n(x^2*sqrt(c/a)) - 2*a^2)/(a^3*c*x^10 + a^4*x^6)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^7 (a+cx^4)^2}\,dx = -\frac{5\sqrt{-\frac{c^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{c^3}{a^7}}}{c^2}+x^2\right)}{8} + \frac{5\sqrt{-\frac{c^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{c^3}{a^7}}}{c^2}+x^2\right)}{8} + \frac{-2a^2+10acx^4+15c^2x^8}{12a^4x^6+12a^3cx^{10}}$$

input
```
integrate(1/x**7/(c*x**4+a)**2,x)
```

output
```
-5*sqrt(-c**3/a**7)*log(-a**4*sqrt(-c**3/a**7)/c**2 + x**2)/8 + 5*sqrt(-c*
*3/a**7)*log(a**4*sqrt(-c**3/a**7)/c**2 + x**2)/8 + (-2*a**2 + 10*a*c*x**4
 + 15*c**2*x**8)/(12*a**4*x**6 + 12*a**3*c*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^7 (a+cx^4)^2}\,dx = \frac{15\,c^2x^8+10\,acx^4-2\,a^2}{12\,(a^3cx^{10}+a^4x^6)} + \frac{5\,c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}a^3}$$

input
```
integrate(1/x^7/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/12*(15*c^2*x^8 + 10*a*c*x^4 - 2*a^2)/(a^3*c*x^10 + a^4*x^6) + 5/4*c^2*ar
ctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^7 (a+cx^4)^2} \, dx = \frac{c^2x^2}{4\,(cx^4+a)a^3} + \frac{5\,c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,\sqrt{ac}a^3} + \frac{6\,cx^4-a}{6\,a^3x^6}$$

input
```
integrate(1/x^7/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*c^2*x^2/((c*x^4 + a)*a^3) + 5/4*c^2*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)
*a^3) + 1/6*(6*c*x^4 - a)/(a^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7 (a+cx^4)^2} \, dx = \frac{\frac{5\,c\,x^4}{6\,a^2} - \frac{1}{6\,a} + \frac{5\,c^2\,x^8}{4\,a^3}}{c\,x^{10}+a\,x^6} + \frac{5\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

input
```
int(1/(x^7*(a + c*x^4)^2),x)
```

output
```
((5*c*x^4)/(6*a^2) - 1/(6*a) + (5*c^2*x^8)/(4*a^3))/(a*x^6 + c*x^10) + (5*
c^(3/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(4*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.79

$$\int \frac{1}{x^7 (a+cx^4)^2} \, dx$$
$$= \frac{-15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)ac\,x^6 - 15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^2x^{10} - 15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{}}\right.}{12a^4x^6\,(c\,x^4+a)}$$

input
```
int(1/x^7/(c*x^4+a)^2,x)
```

output
```
( - 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**
(1/4)*a**(1/4)*sqrt(2)))*a*c*x**6 - 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(
1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**10 - 15*s
qrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a*
*(1/4)*sqrt(2)))*a*c*x**6 - 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqr
t(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**10 - 2*a**3 + 10*
a**2*c*x**4 + 15*a*c**2*x**8)/(12*a**4*x**6*(a + c*x**4))
```

3.78 $\int \frac{x^{10}}{\left(a+cx^4\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 164

$$\int \frac{x^{10}}{\left(a+cx^4\right)^2}\,dx = \frac{x^3}{3c^2}+\frac{ax^3}{4c^2\left(a+cx^4\right)}+\frac{7a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{11/4}} - \frac{7a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{11/4}}+\frac{7a^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}c^{11/4}}$$

output
```
1/3*x^3/c^2+1/4*a*x^3/c^2/(c*x^4+a)-7/16*a^(3/4)*arctan(-1+2^(1/2)*c^(1/4)
*x/a^(1/4))*2^(1/2)/c^(11/4)-7/16*a^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/
4))*2^(1/2)/c^(11/4)+7/16*a^(3/4)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/
2)+c^(1/2)*x^2))*2^(1/2)/c^(11/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx = \frac{32c^{3/4}x^3+\frac{24ac^{3/4}x^3}{a+cx^4}+42\sqrt{2}a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-42\sqrt{2}a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-21\sqrt{2}a^{3/4}\log}{96c^{11/4}}$$

input
```
Integrate[x^10/(a + c*x^4)^2,x]
```

output
```
(32*c^(3/4)*x^3 + (24*a*c^(3/4)*x^3)/(a + c*x^4) + 42*Sqrt[2]*a^(3/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 42*Sqrt[2]*a^(3/4)*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)] - 21*Sqrt[2]*a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2] + 21*Sqrt[2]*a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(96*c^(11/4))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 843, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7\int \frac{x^6}{cx^4+a}dx}{4c}-\frac{x^7}{4c(a+cx^4)}$$

$$\Big\downarrow \text{843}$$

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\int\frac{x^2}{cx^4+a}dx}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 826

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\int\frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1082

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{7\left(\frac{x^3}{3c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{c}\right)}{4c}-\frac{x^7}{4c\left(a+cx^4\right)}$$

input
```
Int[x^10/(a + c*x^4)^2,x]
```

output
```
-1/4*x^7/(c*(a + c*x^4)) + (7*(x^3/(3*c) - (a*((-(ArcTan[1 - (Sqrt[2]*c^(1
/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)
))/(2*Sqrt[c])))/c))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{x^3}{3c^2}+\frac{a\,x^3}{4c^2(c\,x^4+a)}-\frac{7a\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16c^3}$	55
default	$\frac{x^3}{3c^2}-\frac{a\left(-\frac{x^3}{4\left(c\,x^4+a\right)}+\frac{7\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{c^2}$	132

input
```
int(x^10/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/c^2+1/4*a*x^3/c^2/(c*x^4+a)-7/16/c^3*a*sum(1/_R*ln(x-_R),_R=RootOf
(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.38

$$\int\frac{x^{10}}{\left(a+cx^4\right)^2}\,dx$$
$$=\frac{16\,cx^7+28\,ax^3-21\left(c^3x^4+ac^2\right)\left(-\frac{a^3}{c^{11}}\right)^{\frac{1}{4}}\log\left(343\,c^8\left(-\frac{a^3}{c^{11}}\right)^{\frac{3}{4}}+343\,a^2x\right)-21\left(-i\,c^3x^4-i\,ac^2\right)\left(-\frac{a^3}{c^{11}}\right)\cdots}{}$$

input
```
integrate(x^10/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/48*(16*c*x^7 + 28*a*x^3 - 21*(c^3*x^4 + a*c^2)*(-a^3/c^11)^(1/4)*log(343
*c^8*(-a^3/c^11)^(3/4) + 343*a^2*x) - 21*(-I*c^3*x^4 - I*a*c^2)*(-a^3/c^11
)^(1/4)*log(343*I*c^8*(-a^3/c^11)^(3/4) + 343*a^2*x) - 21*(I*c^3*x^4 + I*a
*c^2)*(-a^3/c^11)^(1/4)*log(-343*I*c^8*(-a^3/c^11)^(3/4) + 343*a^2*x) + 21
*(c^3*x^4 + a*c^2)*(-a^3/c^11)^(1/4)*log(-343*c^8*(-a^3/c^11)^(3/4) + 343*
a^2*x))/(c^3*x^4 + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.37

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx = \frac{ax^3}{4ac^2+4c^3x^4} + \operatorname{RootSum}\left(65536t^4c^{11}+2401a^3, \left(t \mapsto t\log\left(-\frac{4096t^3c^8}{343a^2}+x\right)\right)\right) + \frac{x^3}{3c^2}$$

input
```
integrate(x**10/(c*x**4+a)**2,x)
```

output
```
a*x**3/(4*a*c**2 + 4*c**3*x**4) + RootSum(65536*_t**4*c**11 + 2401*a**3, L
ambda(_t, _t*log(-4096*_t**3*c**8/(343*a**2) + x))) + x**3/(3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx = \frac{ax^3}{4\,(c^3x^4+ac^2)} + \frac{x^3}{3\,c^2} - \frac{7\,a\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x\cdots\right.}{\cdots}\right.}{32\,c^2}$$

input
```
integrate(x^10/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*a*x^3/(c^3*x^4 + a*c^2) + 1/3*x^3/c^2 - 7/32*a*(2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqr
t(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqr
t(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^
(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqr
t(a))/(a^(1/4)*c^(3/4)))/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.18

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx = \frac{ax^3}{4\,(cx^4+a)c^2} + \frac{x^3}{3\,c^2} - \frac{7\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,c^5} - \frac{7\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,c^5} + \frac{7\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,c^5} - \frac{7\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,c^5}$$

input
```
integrate(x^10/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*a*x^3/((c*x^4 + a)*c^2) + 1/3*x^3/c^2 - 7/16*sqrt(2)*(a*c^3)^(3/4)*arc
tan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/c^5 - 7/16*sqrt(2
)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/c^5 + 7/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(
a/c))/c^5 - 7/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/c^5
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.46

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx = \frac{x^3}{3\,c^2} + \frac{7\,(-a)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,c^{11/4}} + \frac{a\,x^3}{4\,(c^3\,x^4+a\,c^2)} + \frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,x\,1\mathrm{i}}{(-a)^{1/4}}\right)\,7\mathrm{i}}{8\,c^{11/4}}$$

input
```
int(x^10/(a + c*x^4)^2,x)
```

output
```
x^3/(3*c^2) + (7*(-a)^(3/4)*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*c^(11/4)) + (
(-a)^(3/4)*atan((c^(1/4)*x*1i)/(-a)^(1/4))*7i)/(8*c^(11/4)) + (a*x^3)/(4*(
a*c^2 + c^3*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.90

$$\int \frac{x^{10}}{(a+cx^4)^2}\,dx = \frac{42c^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+42c^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4-42c^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input
```
int(x^10/(c*x^4+a)^2,x)
```

output
```
(42*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 42*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
- 42*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 42*c**(1/4)*a**(3/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 - 21*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(
a) + sqrt(c)*x**2)*a - 21*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/
4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + 21*c**(1/4)*a**(3/4)*sqrt(
2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + 21*c**(1/
4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x*
*2)*c*x**4 + 56*a*c*x**3 + 32*c**2*x**7)/(96*c**3*(a + c*x**4))
```

3.79 $\int \frac{x^8}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 157

$$\int \frac{x^8}{(a+cx^4)^2}\,dx = \frac{x}{c^2} + \frac{ax}{4c^2(a+cx^4)} + \frac{5\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{9/4}} - \frac{5\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}c^{9/4}} - \frac{5\sqrt[4]{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}c^{9/4}}$$

output
```
x/c^2+1/4*a*x/c^2/(c*x^4+a)-5/16*a^(1/4)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/
4))*2^(1/2)/c^(9/4)-5/16*a^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/
2)/c^(9/4)-5/16*a^(1/4)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)
*x^2))*2^(1/2)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.22

$$\int \frac{x^8}{(a+cx^4)^2}\,dx$$
$$= \frac{32\sqrt[4]{c}x + \frac{8a\sqrt[4]{c}x}{a+cx^4} + 10\sqrt{2}\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - 10\sqrt{2}\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{a}\right.}{32c^{9/4}}$$

input
```
Integrate[x^8/(a + c*x^4)^2,x]
```

output
```
(32*c^(1/4)*x + (8*a*c^(1/4)*x)/(a + c*x^4) + 10*Sqrt[2]*a^(1/4)*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 10*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)] + 5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4
)*x + Sqrt[c]*x^2] - 5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/
4)*x + Sqrt[c]*x^2])/(32*c^(9/4))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {817, 843, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{817}$$
$$\frac{5\int \frac{x^4}{cx^4+a}dx}{4c} - \frac{x^5}{4c(a+cx^4)}$$
$$\Big\downarrow \text{843}$$
$$\frac{5\left(\frac{x}{c} - \frac{a\int \frac{1}{cx^4+a}dx}{c}\right)}{4c} - \frac{x^5}{4c(a+cx^4)}$$

$$\Big\downarrow \text{755}$$

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$\downarrow$ 1479

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$\downarrow$ 25

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{x}{c}-\frac{a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{c}\right)}{4c}-\frac{x^5}{4c\left(a+cx^4\right)}$$

input
```
Int[x^8/(a + c*x^4)^2,x]
```

output
```
-1/4*x^5/(c*(a + c*x^4)) + (5*(x/c - (a*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1
/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*
Sqrt[a])))/c))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{x}{c^2}+\frac{ax}{4c^2(c\,x^4+a)}-\frac{5a\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16c^3}$	50
default	$\frac{x}{c^2}-\frac{a\left(-\frac{x}{4\left(c\,x^4+a\right)}+\frac{5\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{c^2}$	127

input
```
int(x^8/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/c^2+1/4*a*x/c^2/(c*x^4+a)-5/16/c^3*a*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*
c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.25

$$\int\frac{x^8}{(a+cx^4)^2}\,dx$$
$$=\frac{16\,cx^5-5\,(c^3x^4+ac^2)\left(-\frac{a}{c^9}\right)^{\frac{1}{4}}\log\left(5\,c^2\left(-\frac{a}{c^9}\right)^{\frac{1}{4}}+5\,x\right)-5\,(i\,c^3x^4+i\,ac^2)\left(-\frac{a}{c^9}\right)^{\frac{1}{4}}\log\left(5i\,c^2\left(-\frac{a}{c^9}\right)^{\frac{1}{4}}+5\,x\right.}{16}$$

input
```
integrate(x^8/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(16*c*x^5 - 5*(c^3*x^4 + a*c^2)*(-a/c^9)^(1/4)*log(5*c^2*(-a/c^9)^(1/
4) + 5*x) - 5*(I*c^3*x^4 + I*a*c^2)*(-a/c^9)^(1/4)*log(5*I*c^2*(-a/c^9)^(1
/4) + 5*x) - 5*(-I*c^3*x^4 - I*a*c^2)*(-a/c^9)^(1/4)*log(-5*I*c^2*(-a/c^9)
^(1/4) + 5*x) + 5*(c^3*x^4 + a*c^2)*(-a/c^9)^(1/4)*log(-5*c^2*(-a/c^9)^(1/
4) + 5*x) + 20*a*x)/(c^3*x^4 + a*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{x^8}{(a+cx^4)^2}\,dx = \frac{ax}{4ac^2+4c^3x^4} + \operatorname{RootSum}\left(65536t^4c^9+625a, \left(t\mapsto t\log\left(-\frac{16tc^2}{5}+x\right)\right)\right) + \frac{x}{c^2}$$

input
```
integrate(x**8/(c*x**4+a)**2,x)
```

output
```
a*x/(4*a*c**2 + 4*c**3*x**4) + RootSum(65536*_t**4*c**9 + 625*a, Lambda(_t
, _t*log(-16*_t*c**2/5 + x))) + x/c**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.26

$$\int \frac{x^8}{(a+cx^4)^2}\,dx = \frac{ax}{4\,(c^3x^4+ac^2)} - \frac{5\left(\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{c^{\frac{1}{4}}} - \sqrt{2}a\cdots\right)}{32\,c^2} + \frac{x}{c^2}$$

input
```
integrate(x^8/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*a*x/(c^3*x^4 + a*c^2) - 5/32*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*
sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(sqrt(a)*s
qrt(c)) + 2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1
/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/sqrt(sqrt(a)*sqrt(c)) + sqrt(2)*a^(1/4
)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/c^(1/4) - sqrt(2)
*a^(1/4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/c^(1/4))/c
^2 + x/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{(a+cx^4)^2}\,dx = \frac{ax}{4\,(cx^4+a)c^2} + \frac{x}{c^2} - \frac{5\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,c^3} - \frac{5\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,c^3} - \frac{5\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,c^3} + \frac{5\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,c^3}$$

input
```
integrate(x^8/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*a*x/((c*x^4 + a)*c^2) + x/c^2 - 5/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*
sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/c^3 - 5/16*sqrt(2)*(a*c^3
)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/c^3 -
5/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/c^
3 + 5/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.45

$$\int \frac{x^8}{(a+cx^4)^2}\,dx = \frac{x}{c^2} - \frac{5\,(-a)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,c^{9/4}} + \frac{a\,x}{4\,(c^3\,x^4+a\,c^2)} + \frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{c^{1/4}\,x\,1\mathrm{i}}{(-a)^{1/4}}\right)\,5\mathrm{i}}{8\,c^{9/4}}$$

input
```
int(x^8/(a + c*x^4)^2,x)
```

output
```
x/c^2 - (5*(-a)^(1/4)*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*c^(9/4)) + ((-a)^(1
/4)*atan((c^(1/4)*x*1i)/(-a)^(1/4))*5i)/(8*c^(9/4)) + (a*x)/(4*(a*c^2 + c^
3*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.97

$$\int \frac{x^8}{(a+cx^4)^2}\,dx = \frac{10c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10c^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4-10c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input
```
int(x^8/(c*x^4+a)^2,x)
```

output
```
(10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
- 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a
) + sqrt(c)*x**2)*a + 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)
*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 - 5*c**(3/4)*a**(1/4)*sqrt(2)*
log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - 5*c**(3/4)*a
**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*
c*x**4 + 40*a*c*x + 32*c**2*x**5)/(32*c**3*(a + c*x**4))
```

3.80 $\int \frac{x^6}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = -\frac{x^3}{4c(a+cx^4)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{7/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{7/4}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}\sqrt[4]{a}c^{7/4}}$$

output

```
-1/4*x^3/c/(c*x^4+a)+3/16*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(
1/4)/c^(7/4)+3/16*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/c^(7
/4)-3/16*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/
a^(1/4)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.21

$$\int \frac{x^6}{(a+cx^4)^2}\,dx$$
$$= \frac{-\frac{8c^{3/4}x^3}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}} + \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{a}} - \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{c}\right.}{\sqrt[4]{a}}}{32c^{7/4}}$$

input
```
Integrate[x^6/(a + c*x^4)^2,x]
```

output
```
((-8*c^(3/4)*x^3)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/a^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^
(1/4) + (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/a^(1/4) - (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/a^(1/4))/(32*c^(7/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {817, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{ 817}$$
$$\frac{3\int \frac{x^2}{cx^4+a}dx}{4c} - \frac{x^3}{4c\,(a+cx^4)}$$
$$\Big\downarrow \text{ 826}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1479

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}$$

input
```
Int[x^6/(a + c*x^4)^2,x]
```

output
```
-1/4*x^3/(c*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/
(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^
(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c])))
/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.29

method	result	size
risch	$-\frac{x^3}{4c(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}(\_Z^4c+a)}\frac{\ln(x-\_R)}{\_R}\right)}{16c^2}$	45
default	$-\frac{x^3}{4c(c\,x^4+a)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c^2\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	120

input
```
int(x^6/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^3/c/(c*x^4+a)+3/16/c^2*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = -\frac{4\,x^3-3\,(c^2x^4+ac)\left(-\frac{1}{ac^7}\right)^{\frac{1}{4}}\log\left(ac^5\left(-\frac{1}{ac^7}\right)^{\frac{3}{4}}+x\right)+3\,(i\,c^2x^4+i\,ac)\left(-\frac{1}{ac^7}\right)^{\frac{1}{4}}\log\left(i\,ac^5\left(-\frac{1}{ac^7}\right)^{\frac{3}{4}}+x\right)}{16\,(c^2x}$$

input
```
integrate(x^6/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(4*x^3 - 3*(c^2*x^4 + a*c)*(-1/(a*c^7))^(1/4)*log(a*c^5*(-1/(a*c^7))
^(3/4) + x) + 3*(I*c^2*x^4 + I*a*c)*(-1/(a*c^7))^(1/4)*log(I*a*c^5*(-1/(a*
c^7))^(3/4) + x) + 3*(-I*c^2*x^4 - I*a*c)*(-1/(a*c^7))^(1/4)*log(-I*a*c^5*
(-1/(a*c^7))^(3/4) + x) + 3*(c^2*x^4 + a*c)*(-1/(a*c^7))^(1/4)*log(-a*c^5*
(-1/(a*c^7))^(3/4) + x))/(c^2*x^4 + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = -\frac{x^3}{4ac+4c^2x^4} + \operatorname{RootSum}\left(65536t^4ac^7+81, \left(t \mapsto t\log\left(\frac{4096t^3ac^5}{27}+x\right)\right)\right)$$

input
```
integrate(x**6/(c*x**4+a)**2,x)
```

output
```
-x**3/(4*a*c + 4*c**2*x**4) + RootSum(65536*_t**4*a*c**7 + 81, Lambda(_t,
_t*log(4096*_t**3*a*c**5/27 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = -\frac{x^3}{4\,(c^2x^4+ac)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2 \cdots\right.}{c \cdots}\right.}{32\,c}$$

input
```
integrate(x^6/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^3/(c^2*x^4 + a*c) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(
1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log
(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sq
rt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3
/4)))/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.28

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = -\frac{x^3}{4\,(cx^4+a)c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^4} + \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^4} - \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,ac^4} + \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,ac^4}$$

input
```
integrate(x^6/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*x^3/((c*x^4 + a)*c) + 3/16*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 3/16*sqrt(2)*(a*c^3)^(3/
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) - 3
/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*
c^4) + 3/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a
/c))/(a*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{1/4}\,c^{7/4}} - \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{1/4}\,c^{7/4}} - \frac{x^3}{4\,c\,(c\,x^4+a)}$$

input
```
int(x^6/(a + c*x^4)^2,x)
```

output
```
(3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(1/4)*c^(7/4)) - (3*atanh((c^(1/4
)*x)/(-a)^(1/4)))/(8*(-a)^(1/4)*c^(7/4)) - x^3/(4*c*(a + c*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.01

$$\int \frac{x^6}{(a+cx^4)^2}\,dx = \frac{-6c^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6c^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4+6c^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+}{}$$

input
```
int(x^6/(c*x^4+a)^2,x)
```

output
```
( - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
+ 3*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
 + sqrt(c)*x**2)*a + 3*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*
sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 - 3*c**(1/4)*a**(3/4)*sqrt(2)*l
og(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - 3*c**(1/4)*a*
*(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c
*x**4 - 8*a*c*x**3)/(32*a*c**2*(a + c*x**4))
```

3.81 $\int \frac{x^4}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 151

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = -\frac{x}{4c\,(a+cx^4)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{5/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{3/4}c^{5/4}}$$

output
```
-1/4*x/c/(c*x^4+a)+1/16*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/
4)/c^(5/4)+1/16*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(5/4
)+1/16*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^
(3/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = \frac{-\frac{8\sqrt[4]{c}x}{a+cx^4} - \frac{2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{2\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{3/4}} + \frac{\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{3/4}}}{32c^{5/4}}$$

input
```
Integrate[x^4/(a + c*x^4)^2,x]
```

output
```
((-8*c^(1/4)*x)/(a + c*x^4) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/a^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3
/4) - (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(
3/4) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^
(3/4))/(32*c^(5/4))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{\int \frac{1}{cx^4+a}dx}{4c} - \frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c\left(a+cx^4\right)}$$

input
```
Int[x^4/(a + c*x^4)^2,x]
```

output
```
-1/4*x/(c*(a + c*x^4)) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.28

method	result	size
risch	$-\frac{x}{4c(c\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{16c^2}$	43
default	$-\frac{x}{4c(c\,x^4+a)}+\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32ca}$	121

input
```
int(x^4/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x/c/(c*x^4+a)+1/16/c^2*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{(a+cx^4)^2}\,dx$$
$$=\frac{(c^2x^4+ac)\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}}\log\left(ac\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}}+x\right)-(-i\,c^2x^4-i\,ac)\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}}\log\left(i\,ac\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{4}}+x\right)-(i\,c^2x^4}{16\,(c^2x^4+}$$

input
```
integrate(x^4/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*((c^2*x^4 + a*c)*(-1/(a^3*c^5))^(1/4)*log(a*c*(-1/(a^3*c^5))^(1/4) +
x) - (-I*c^2*x^4 - I*a*c)*(-1/(a^3*c^5))^(1/4)*log(I*a*c*(-1/(a^3*c^5))^(1
/4) + x) - (I*c^2*x^4 + I*a*c)*(-1/(a^3*c^5))^(1/4)*log(-I*a*c*(-1/(a^3*c^
5))^(1/4) + x) - (c^2*x^4 + a*c)*(-1/(a^3*c^5))^(1/4)*log(-a*c*(-1/(a^3*c^
5))^(1/4) + x) - 4*x)/(c^2*x^4 + a*c)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = -\frac{x}{4ac+4c^2x^4} + \mathrm{RootSum}\left(65536t^4a^3c^5+1, \left(t \mapsto t\log\left(16tac+x\right)\right)\right)$$

input
```
integrate(x**4/(c*x**4+a)**2,x)
```

output
```
-x/(4*a*c + 4*c**2*x**4) + RootSum(65536*_t**4*a**3*c**5 + 1, Lambda(_t, _
t*log(16*_t*a*c + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = -\frac{x}{4(c^2x^4+ac)} + \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}\right.}{a^{\frac{3}{4}}c^{\frac{1}{4}}}}{32\,c}$$

input
```
integrate(x^4/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x/(c^2*x^4 + a*c) + 1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
 sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqr
t(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/
4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(s
qrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt
(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4
)))/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = -\frac{x}{4\,(cx^4+a)c} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^2} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,ac^2} + \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,ac^2} - \frac{\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,ac^2}$$

input

```
integrate(x^4/(c*x^4+a)^2,x, algorithm="giac")
```

output

```
-1/4*x/((c*x^4 + a)*c) + 1/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) + 1/16*sqrt(2)*(a*c^3)^(1/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^2) + 1/3
2*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^
2) - 1/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c
))/(a*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = -\frac{x}{4\,c\,\left(c\,x^4+a\right)} - \frac{\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{3/4}\,c^{5/4}} - \frac{\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{3/4}\,c^{5/4}}$$

input

```
int(x^4/(a + c*x^4)^2,x)
```

output
```
- x/(4*c*(a + c*x^4)) - atan((c^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(3/4)*c^(5/4)
) - atanh((c^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(3/4)*c^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.01

$$\int \frac{x^4}{(a+cx^4)^2}\,dx = \frac{-2c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2c^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4+2c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+}{}$$

input
```
int(x^4/(c*x^4+a)^2,x)
```

output
```
( - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 2*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
- c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(c)*x**2)*a - c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt
(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + c**(3/4)*a**(1/4)*sqrt(2)*log(c**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + c**(3/4)*a**(1/4)*s
qrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 -
8*a*c*x)/(32*a*c**2*(a + c*x**4))
```

3.82 $\int \frac{x^2}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 153

$$\int \frac{x^2}{(a+cx^4)^2}\,dx = \frac{x^3}{4a(a+cx^4)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}c^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}c^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{5/4}c^{3/4}}$$

output
```
1/4*x^3/a/(c*x^4+a)+1/16*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(5
/4)/c^(3/4)+1/16*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(5/4)/c^(3/
4)-1/16*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a
^(5/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(a+cx^4)^2}\,dx$$
$$=\frac{\frac{8\sqrt[4]{a}x^3}{a+cx^4}-\frac{2\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{2\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{c^{3/4}}-\frac{\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{c^{3/4}}}{32a^{5/4}}$$

input
```
Integrate[x^2/(a + c*x^4)^2,x]
```

output
```
((8*a^(1/4)*x^3)/(a + c*x^4) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/c^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(
3/4) + (Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(3/4) - (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c
^(3/4))/(32*a^(5/4))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {819, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \ 819$$
$$\frac{\int \frac{x^2}{cx^4+a}dx}{4a}+\frac{x^3}{4a(a+cx^4)}$$
$$\Big\downarrow \ 826$$

$$\frac{\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a\left(a+cx^4\right)}$$

input

```
Int[x^2/(a + c*x^4)^2,x]
```

output

```
x^3/(4*a*(a + c*x^4)) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[
2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^
(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]))/(4*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{x^3}{4a(c\,x^4+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(\_Z^4c+a)}\frac{\ln(x-\_R)}{\_R}}{16ac}$	48
default	$\frac{x^3}{4a(c\,x^4+a)}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32ac\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	123

input
```
int(x^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^3/a/(c*x^4+a)+1/16/a/c*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(a+cx^4)^2}\,dx = \frac{4\,x^3+(acx^4+a^2)\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{4}}\log\left(a^4c^2\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{4}}+x\right)-(i\,acx^4+i\,a^2)\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{4}}\log\left(i\,a^4c^2\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{4}}+x\right)}{16\,(ac}$$

input
```
integrate(x^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(4*x^3 + (a*c*x^4 + a^2)*(-1/(a^5*c^3))^(1/4)*log(a^4*c^2*(-1/(a^5*c^
3))^(3/4) + x) - (I*a*c*x^4 + I*a^2)*(-1/(a^5*c^3))^(1/4)*log(I*a^4*c^2*(-
1/(a^5*c^3))^(3/4) + x) - (-I*a*c*x^4 - I*a^2)*(-1/(a^5*c^3))^(1/4)*log(-I
*a^4*c^2*(-1/(a^5*c^3))^(3/4) + x) - (a*c*x^4 + a^2)*(-1/(a^5*c^3))^(1/4)*
log(-a^4*c^2*(-1/(a^5*c^3))^(3/4) + x))/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{(a+cx^4)^2}\,dx = \frac{x^3}{4a^2+4acx^4} + \operatorname{RootSum}\left(65536t^4a^5c^3+1,\left(t\mapsto t\log\left(4096t^3a^4c^2+x\right)\right)\right)$$

input
```
integrate(x**2/(c*x**4+a)**2,x)
```

output
```
x**3/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**5*c**3 + 1, Lambda(_t,
 _t*log(4096*_t**3*a**4*c**2 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(a+cx^4)^2}\,dx = \frac{x^3}{4(acx^4+a^2)} + \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2-\sqrt{2}\right.}{a^{\frac{1}{4}}c^{\frac{3}{4}}}}{32\,a}$$

input
```
integrate(x^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^3/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
+ sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1
/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(
sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqr
t(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/
4)))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(a+cx^4)^2}\,dx = \frac{x^3}{4\,(cx^4+a)a} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^3} - \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3} + \frac{\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

input
```
integrate(x^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x^3/((c*x^4 + a)*a) + 1/16*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2
*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(2)*(a*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3)
- 1/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/
(a^2*c^3) + 1/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(a^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.39

$$\int \frac{x^2}{(a+cx^4)^2}\,dx = \frac{\operatorname{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{5/4}\,c^{3/4}} - \frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{5/4}\,c^{3/4}} + \frac{x^3}{4\,a\,\left(c\,x^4+a\right)}$$

input
```
int(x^2/(a + c*x^4)^2,x)
```

output
```
atanh((c^(1/4)*x)/(-a)^(1/4))/(8*(-a)^(5/4)*c^(3/4)) - atan((c^(1/4)*x)/(-
a)^(1/4))/(8*(-a)^(5/4)*c^(3/4)) + x^3/(4*a*(a + c*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.99

$$\int \frac{x^2}{(a+cx^4)^2}\,dx$$

$$= \frac{-2c^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2c^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4+2c^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+}{}$$

input
```
int(x^2/(c*x^4+a)^2,x)
```

output
```
( - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 2*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
+ c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(c)*x**2)*a + c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt
(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 - c**(1/4)*a**(3/4)*sqrt(2)*log(c**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a - c**(1/4)*a**(3/4)*s
qrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 +
8*a*c*x**3)/(32*a**2*c*(a + c*x**4))
```

3.83 $\int \frac{1}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 151

$$\int \frac{1}{(a+cx^4)^2} dx = \frac{x}{4a(a+cx^4)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

output
```
1/4*x/a/(c*x^4+a)+3/16*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4
)/c^(1/4)+3/16*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(7/4)/c^(1/4)
+3/16*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(
7/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$= \frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{\sqrt[4]{c}}}{32a^{7/4}}$$

input
```
Integrate[(a + c*x^4)^(-2),x]
```

output
```
((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/
4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^
(1/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/c^(1/4))/(32*a^(7/4))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.49, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$\Big\downarrow \text{749}$$
$$\frac{3\int \frac{1}{cx^4+a}dx}{4a} + \frac{x}{4a(a+cx^4)}$$
$$\Big\downarrow \text{755}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1479

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a\left(a+cx^4\right)}$$

input
```
Int[(a + c*x^4)^(-2),x]
```

output
```
x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt
[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a
^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a])))/(4*a
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46
default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

input
```
int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(c*x^4+a)+3/16/a/c*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+cx^4)^2}\,dx$$
$$=\frac{3\left(acx^4+a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(-i\,acx^4-i\,a^2\right)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\left(i\,ac\right.}{16\left(acx^4+\right.}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/16*(3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x)
 - 3*(-I*a*c*x^4 - I*a^2)*(-1/(a^7*c))^(1/4)*log(I*a^2*(-1/(a^7*c))^(1/4)
+ x) - 3*(I*a*c*x^4 + I*a^2)*(-1/(a^7*c))^(1/4)*log(-I*a^2*(-1/(a^7*c))^(1
/4) + x) - 3*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4
) + x) + 4*x)/(a*c*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a^2+4acx^4} + \operatorname{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**2,x)
```

output
```
x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*l
og(16*_t*a**2/3 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4\,(acx^4+a^2)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2 \cdots\right.}{\cdots}\right.}{32\,a}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt
(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sq
rt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(
2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)
))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4\,(cx^4+a)a} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c} + \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c} - \frac{3\,\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

input
```
integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32
*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
) - 3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c)
)/(a^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.38

$$\int \frac{1}{\left(a+cx^4\right)^2}\,dx = \frac{x}{4\,a\,\left(c\,x^4+a\right)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{8\,(-a)^{7/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^2,x)
```

output
```
x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/
4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{-6c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6c^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^4+6c^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+}{}$$

input
```
int(1/(c*x^4+a)^2,x)
```

output
```
( - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c*
*(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
 + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)
*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a + 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**4
- 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a)
 + sqrt(c)*x**2)*a - 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*
sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**4 + 3*c**(3/4)*a**(1/4)*sqrt(2)*l
og(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a + 3*c**(3/4)*a*
*(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c
*x**4 + 8*a*c*x)/(32*a**2*c*(a + c*x**4))
```

3.84 $\int \frac{1}{x^2(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 162

$$\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx = -\frac{1}{a^2x}-\frac{cx^3}{4a^2\left(a+cx^4\right)}+\frac{5\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}}-\frac{5\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{c}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{9/4}}$$

output
```
-1/a^2/x-1/4*c*x^3/a^2/(c*x^4+a)-5/16*c^(1/4)*arctan(-1+2^(1/2)*c^(1/4)*x/
a^(1/4))*2^(1/2)/a^(9/4)-5/16*c^(1/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*
2^(1/2)/a^(9/4)+5/16*c^(1/4)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^
(1/2)*x^2))*2^(1/2)/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2 (a+cx^4)^2} dx$$
$$= \frac{-\frac{32\sqrt[4]{a}}{x} - \frac{8\sqrt[4]{a}cx^3}{a+cx^4} + 10\sqrt{2}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - 10\sqrt{2}\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) - 5\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{a}\right.}{32a^{9/4}}$$

input
```
Integrate[1/(x^2*(a + c*x^4)^2),x]
```

output
```
((-32*a^(1/4))/x - (8*a^(1/4)*c*x^3)/(a + c*x^4) + 10*Sqrt[2]*c^(1/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 10*Sqrt[2]*c^(1/4)*ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c
^(1/4)*x + Sqrt[c]*x^2] + 5*Sqrt[2]*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(9/4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {819, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^4)^2} dx$$
$$\Big\downarrow \text{ 819}$$
$$\frac{5\int \frac{1}{x^2(cx^4+a)}dx}{4a} + \frac{1}{4ax(a+cx^4)}$$
$$\Big\downarrow \text{ 847}$$

$$\frac{5\left(-\frac{c\int\frac{x^2}{cx^4+a}dx}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$\Big\downarrow$ 826

$$\frac{5\left(-\frac{c\left(\frac{\int\frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{5\left(-\frac{c\left(\frac{\frac{\int\frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int\frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$\Big\downarrow$ 1082

$$\frac{5\left(-\frac{c\left(\frac{\frac{\int\frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int\frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax\left(a+cx^4\right)}$$

input
```
Int[1/(x^2*(a + c*x^4)^2),x]
```

output
```
1/(4*a*x*(a + c*x^4)) + (5*(-(1/(a*x)) - (c*((-(ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))
/(2*Sqrt[c])))/a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{-\frac{5 c x^4}{4 a^2}-\frac{1}{a}}{x(c x^4+a)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(a^9 \_Z^4+c\right)} \_R\ln\left(\left(5\_R^4 a^9+4c\right)x+a^7\_R^3\right)\right)}{16}$	70
default	$-\frac{1}{a^2 x}-\frac{c\left(\frac{x^3}{4c x^4+4a}+\frac{5\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{a^2}$	132

input
```
int(1/x^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-5/4/a^2*c*x^4-1/a)/x/(c*x^4+a)+5/16*sum(_R*ln((5*_R^4*a^9+4*c)*x+a^7*_R^
3),_R=RootOf(_Z^4*a^9+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx =$$
$$-\frac{20\,cx^4+5\left(a^2cx^5+a^3x\right)\left(-\frac{c}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(-\frac{c}{a^9}\right)^{\frac{3}{4}}+125\,cx\right)+5\left(-i\,a^2cx^5-i\,a^3x\right)\left(-\frac{c}{a^9}\right)^{\frac{1}{4}}\log\left(125a\right.}{}$$

input
```
integrate(1/x^2/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/16*(20*c*x^4 + 5*(a^2*c*x^5 + a^3*x)*(-c/a^9)^(1/4)*log(125*a^7*(-c/a^9
)^(3/4) + 125*c*x) + 5*(-I*a^2*c*x^5 - I*a^3*x)*(-c/a^9)^(1/4)*log(125*I*a
^7*(-c/a^9)^(3/4) + 125*c*x) + 5*(I*a^2*c*x^5 + I*a^3*x)*(-c/a^9)^(1/4)*lo
g(-125*I*a^7*(-c/a^9)^(3/4) + 125*c*x) - 5*(a^2*c*x^5 + a^3*x)*(-c/a^9)^(1
/4)*log(-125*a^7*(-c/a^9)^(3/4) + 125*c*x) + 16*a)/(a^2*c*x^5 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx = \frac{-4a-5cx^4}{4a^3x+4a^2cx^5} + \operatorname{RootSum}\left(65536t^4a^9+625c,\left(t\mapsto t\log\left(-\frac{4096t^3a^7}{125c}+x\right)\right)\right)$$

input
```
integrate(1/x**2/(c*x**4+a)**2,x)
```

output
```
(-4*a - 5*c*x**4)/(4*a**3*x + 4*a**2*c*x**5) + RootSum(65536*_t**4*a**9 +
625*c, Lambda(_t, _t*log(-4096*_t**3*a**7/(125*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx = -\frac{5\,cx^4+4\,a}{4\,(a^2cx^5+a^3x)} - \frac{5\,c\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x\right.}{}\right.}{32\,a^2}$$

input
```
integrate(1/x^2/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(5*c*x^4 + 4*a)/(a^2*c*x^5 + a^3*x) - 5/32*c*(2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt
(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt
(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(
1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt
(a))/(a^(1/4)*c^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^2\left(a+cx^4\right)^2}\,dx = -\frac{5\,cx^4+4\,a}{4\,(cx^5+ax)a^2} - \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3c^2} - \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^3c^2} + \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^3c^2} - \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^3c^2}$$

input
```
integrate(1/x^2/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*(5*c*x^4 + 4*a)/((c*x^5 + a*x)*a^2) - 5/16*sqrt(2)*(a*c^3)^(3/4)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^2) - 5/16*s
qrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^
(1/4))/(a^3*c^2) + 5/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1
/4) + sqrt(a/c))/(a^3*c^2) - 5/32*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*
x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2 (a+cx^4)^2} \, dx = \frac{5 (-c)^{1/4} \operatorname{atanh}\left(\frac{(-c)^{1/4} x}{a^{1/4}}\right)}{8 a^{9/4}} - \frac{5 (-c)^{1/4} \operatorname{atan}\left(\frac{(-c)^{1/4} x}{a^{1/4}}\right)}{8 a^{9/4}} - \frac{\frac{1}{a} + \frac{5 c x^4}{4 a^2}}{c x^5 + a x}$$

input
```
int(1/(x^2*(a + c*x^4)^2),x)
```

output
```
(5*(-c)^(1/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(8*a^(9/4)) - (5*(-c)^(1/4)*a
tan(((-c)^(1/4)*x)/a^(1/4)))/(8*a^(9/4)) - (1/a + (5*c*x^4)/(4*a^2))/(a*x
+ c*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.95

$$\int \frac{1}{x^2 (a+cx^4)^2} \, dx$$
$$= \frac{10c^{\frac{1}{4}} a^{\frac{7}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}-2\sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x + 10c^{\frac{5}{4}} a^{\frac{3}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}-2\sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x^5 - 10c^{\frac{1}{4}} a^{\frac{7}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}+2\sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right)}{}$$

input
```
int(1/x^2/(c*x^4+a)^2,x)
```

output
```
(10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*x + 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c
**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*x**
5 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*x - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*
x**5 - 5*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sq
rt(a) + sqrt(c)*x**2)*a*x - 5*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a*
*(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**5 + 5*c**(1/4)*a**(3/4)*sq
rt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*x + 5*c*
*(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c
)*x**2)*c*x**5 - 32*a**2 - 40*a*c*x**4)/(32*a**3*x*(a + c*x**4))
```

3.85 $\int \frac{1}{x^4(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 162

$$\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx = -\frac{1}{3a^2x^3}-\frac{cx}{4a^2\left(a+cx^4\right)}+\frac{7c^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}} - \frac{7c^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{11/4}}-\frac{7c^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{8\sqrt{2}a^{11/4}}$$

output
```
-1/3/a^2/x^3-1/4*c*x/a^2/(c*x^4+a)-7/16*c^(3/4)*arctan(-1+2^(1/2)*c^(1/4)*
x/a^(1/4))*2^(1/2)/a^(11/4)-7/16*c^(3/4)*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4
))*2^(1/2)/a^(11/4)-7/16*c^(3/4)*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2
)+c^(1/2)*x^2))*2^(1/2)/a^(11/4)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^4 \left(a+c x^4\right)^2} \, dx$$
$$= \frac{-\frac{32 a^{3/4}}{x^3}-\frac{24 a^{3/4} c x}{a+c x^4}+42 \sqrt{2} c^{3/4} \arctan \left(1-\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)-42 \sqrt{2} c^{3/4} \arctan \left(1+\frac{\sqrt{2} \sqrt[4]{c} x}{\sqrt[4]{a}}\right)+21 \sqrt{2} c^{3/4} \log (}{96 a^{11/4}}$$

input
```
Integrate[1/(x^4*(a + c*x^4)^2),x]
```

output
```
((-32*a^(3/4))/x^3 - (24*a^(3/4)*c*x)/(a + c*x^4) + 42*Sqrt[2]*c^(3/4)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 42*Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 21*Sqrt[2]*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2] - 21*Sqrt[2]*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(96*a^(11/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {819, 847, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+c x^4\right)^2} \, dx$$
$$\Big\downarrow \; 819$$
$$\frac{7 \int \frac{1}{x^4 \left(c x^4+a\right)}dx}{4 a}+\frac{1}{4 a x^3 \left(a+c x^4\right)}$$
$$\Big\downarrow \; 847$$
$$\frac{7 \left(-\frac{c \int \frac{1}{c x^4+a}dx}{a}-\frac{1}{3 a x^3}\right)}{4 a}+\frac{1}{4 a x^3 \left(a+c x^4\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{7\left(-\frac{c\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7\left(-\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7\left(-\frac{c\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{7\left(-\frac{c\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3(a+cx^4)}$$

$\Big\downarrow$ 1479

$$\frac{7\left(-\frac{c\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3(a+cx^4)}$$

$\Big\downarrow$ 25

$$\frac{7\left(-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{7\left(-\frac{c\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{7\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{a}-\frac{1}{3ax^3}\right)}{4a}+\frac{1}{4ax^3\left(a+cx^4\right)}$$

input
```
Int[1/(x^4*(a + c*x^4)^2),x]
```

output
```
1/(4*a*x^3*(a + c*x^4)) + (7*(-1/3*1/(a*x^3) - (c*((-(ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a]
- Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*c^(
1/4)))/(2*Sqrt[a])))/a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{-\frac{7c\,x^4}{12a^2}-\frac{1}{3a}}{x^3(c\,x^4+a)}+\frac{7\left(\sum_{\_R=\text{RootOf}\left(a^{11}\_Z^4+c^3\right)}\_R\ln\left(\left(-5\_R^4a^{11}-4c^3\right)x-a^3c^2\_R\right)\right)}{16}$	76
default	$-\frac{1}{3a^2x^3}-\frac{c\left(\frac{x}{4c\,x^4+4a}+\frac{7\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{a^2}$	130

input
```
int(1/x^4/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-7/12/a^2*c*x^4-1/3/a)/x^3/(c*x^4+a)+7/16*sum(_R*ln((-5*_R^4*a^11-4*c^3)*
x-a^3*c^2*_R),_R=RootOf(_Z^4*a^11+c^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx =$$

$$-\frac{28\,cx^4+21\left(a^2cx^7+a^3x^3\right)\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7\,a^3\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{4}}+7\,cx\right)+21\left(i\,a^2cx^7+i\,a^3x^3\right)\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7\right.}{}$$

input
```
integrate(1/x^4/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/48*(28*c*x^4 + 21*(a^2*c*x^7 + a^3*x^3)*(-c^3/a^11)^(1/4)*log(7*a^3*(-c
^3/a^11)^(1/4) + 7*c*x) + 21*(I*a^2*c*x^7 + I*a^3*x^3)*(-c^3/a^11)^(1/4)*l
og(7*I*a^3*(-c^3/a^11)^(1/4) + 7*c*x) + 21*(-I*a^2*c*x^7 - I*a^3*x^3)*(-c^
3/a^11)^(1/4)*log(-7*I*a^3*(-c^3/a^11)^(1/4) + 7*c*x) - 21*(a^2*c*x^7 + a^
3*x^3)*(-c^3/a^11)^(1/4)*log(-7*a^3*(-c^3/a^11)^(1/4) + 7*c*x) + 16*a)/(a^
2*c*x^7 + a^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx = \frac{-4a-7cx^4}{12a^3x^3+12a^2cx^7} + \operatorname{RootSum}\left(65536t^4a^{11}+2401c^3, \left(t\mapsto t\log\left(-\frac{16ta^3}{7c}+x\right)\right)\right)$$

input
```
integrate(1/x**4/(c*x**4+a)**2,x)
```

output
```
(-4*a - 7*c*x**4)/(12*a**3*x**3 + 12*a**2*c*x**7) + RootSum(65536*_t**4*a*
*11 + 2401*c**3, Lambda(_t, _t*log(-16*_t*a**3/(7*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^4\left(a+cx^4\right)^2}\,dx = -\frac{7\,cx^4+4\,a}{12\,\left(a^2cx^7+a^3x^3\right)} - \frac{7\left(\frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}c^{\frac{3}{4}}\log\cdots}{\cdots}\right.}{32\,a^2}$$

input
```
integrate(1/x^4/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/12*(7*c*x^4 + 4*a)/(a^2*c*x^7 + a^3*x^3) - 7/32*(2*sqrt(2)*c*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*c*arctan(1/2*sqrt(2)*(2*sqrt(c)*
x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(c))) + sqrt(2)*c^(3/4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x +
sqrt(a))/a^(3/4) - sqrt(2)*c^(3/4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/a^(3/4))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^4 (a+cx^4)^2} dx = -\frac{cx}{4 (cx^4+a)a^2} - \frac{7\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^3} - \frac{7\sqrt{2}(ac^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16a^3} - \frac{7\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^3} + \frac{7\sqrt{2}(ac^3)^{\frac{1}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32a^3} - \frac{1}{3a^2x^3}$$

input
```
integrate(1/x^4/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*c*x/((c*x^4 + a)*a^2) - 7/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)
*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/a^3 - 7/16*sqrt(2)*(a*c^3)^(1/4)
*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/a^3 - 7/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^3 + 7/3
2*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/a^3 -
 1/3/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^4 \left(a+cx^4\right)^2} \, dx = \frac{7 \left(-c\right)^{3/4} \operatorname{atan}\left(\frac{\left(-c\right)^{1/4} x}{a^{1/4}}\right)}{8 a^{11/4}} - \frac{\frac{1}{3 a} + \frac{7 c x^4}{12 a^2}}{c x^7 + a x^3} + \frac{7 \left(-c\right)^{3/4} \operatorname{atanh}\left(\frac{\left(-c\right)^{1/4} x}{a^{1/4}}\right)}{8 a^{11/4}}$$

input
```
int(1/(x^4*(a + c*x^4)^2),x)
```

output
```
(7*(-c)^(3/4)*atan(((-c)^(1/4)*x)/a^(1/4)))/(8*a^(11/4)) - (1/(3*a) + (7*c
*x^4)/(12*a^2))/(a*x^3 + c*x^7) + (7*(-c)^(3/4)*atanh(((-c)^(1/4)*x)/a^(1/
4)))/(8*a^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^4 \left(a+cx^4\right)^2} \, dx = \frac{42 c^{\frac{3}{4}} a^{\frac{5}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}-2 \sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x^3 + 42 c^{\frac{7}{4}} a^{\frac{1}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}-2 \sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right) x^7 - 42 c^{\frac{3}{4}} a^{\frac{5}{4}} \sqrt{2}\, atan\left(\frac{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}+2 \sqrt{c}\, x}{c^{\frac{1}{4}} a^{\frac{1}{4}} \sqrt{2}}\right.}{}$$

input
```
int(1/x^4/(c*x^4+a)^2,x)
```

output
```
(42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*x**3 + 42*c**(3/4)*a**(1/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c*
x**7 - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*x**3 - 42*c**(3/4)*a**(1/4)*sqrt(2
)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2
)))*c*x**7 + 21*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)
*x + sqrt(a) + sqrt(c)*x**2)*a*x**3 + 21*c**(3/4)*a**(1/4)*sqrt(2)*log( -
c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c*x**7 - 21*c**(3/4)
*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2
)*a*x**3 - 21*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x +
sqrt(a) + sqrt(c)*x**2)*c*x**7 - 32*a**2 - 56*a*c*x**4)/(96*a**3*x**3*(a +
 c*x**4))
```

3.86 $\int \frac{x^{15}}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^{15}}{(a+cx^4)^3} dx = \frac{x^4}{4c^3} + \frac{a^3}{8c^4(a+cx^4)^2} - \frac{3a^2}{4c^4(a+cx^4)} - \frac{3a\log(a+cx^4)}{4c^4}$$

output
```
1/4*x^4/c^3+1/8*a^3/c^4/(c*x^4+a)^2-3/4*a^2/c^4/(c*x^4+a)-3/4*a*ln(c*x^4+a
)/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x^{15}}{(a+cx^4)^3} dx = -\frac{-2cx^4 + \frac{a^2(5a+6cx^4)}{(a+cx^4)^2} + 6a\log(a+cx^4)}{8c^4}$$

input
```
Integrate[x^15/(a + c*x^4)^3,x]
```

output
```
-1/8*(-2*c*x^4 + (a^2*(5*a + 6*c*x^4))/(a + c*x^4)^2 + 6*a*Log[a + c*x^4])
/c^4
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{x^{12}}{(cx^4+a)^3}dx^4$$

$$\Big\downarrow \ 49$$

$$\frac{1}{4}\int \left(-\frac{a^3}{c^3\,(cx^4+a)^3}+\frac{3a^2}{c^3\,(cx^4+a)^2}-\frac{3a}{c^3\,(cx^4+a)}+\frac{1}{c^3}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{a^3}{2c^4\,(a+cx^4)^2}-\frac{3a^2}{c^4\,(a+cx^4)}-\frac{3a\log\left(a+cx^4\right)}{c^4}+\frac{x^4}{c^3}\right)$$

input
```
Int[x^15/(a + c*x^4)^3,x]
```

output
```
(x^4/c^3 + a^3/(2*c^4*(a + c*x^4)^2) - (3*a^2)/(c^4*(a + c*x^4)) - (3*a*Lo
g[a + c*x^4])/c^4)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{\frac{x^{12}}{4c}-\frac{9a^3}{8c^4}-\frac{3a^2x^4}{2c^3}}{\left(c\,x^4+a\right)^2}-\frac{3a\ln\left(c\,x^4+a\right)}{4c^4}$	54
risch	$\frac{x^4}{4c^3}+\frac{-\frac{3a^2x^4}{4}-\frac{5a^3}{8c}}{c^3\left(c\,x^4+a\right)^2}-\frac{3a\ln\left(c\,x^4+a\right)}{4c^4}$	54
default	$\frac{x^4}{4c^3}-\frac{a\left(\frac{3\ln\left(c\,x^4+a\right)}{2c}+\frac{3a}{2c\left(c\,x^4+a\right)}-\frac{a^2}{4c\left(c\,x^4+a\right)^2}\right)}{2c^3}$	62
parallelrisch	$-\frac{-2c^3x^{12}+6\ln\left(c\,x^4+a\right)x^8a\,c^2+12\ln\left(c\,x^4+a\right)x^4a^2c+12a^2c\,x^4+6\ln\left(c\,x^4+a\right)a^3+9a^3}{8c^4\left(c\,x^4+a\right)^2}$	85

input
```
int(x^15/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4/c*x^12-9/8*a^3/c^4-3/2*a^2/c^3*x^4)/(c*x^4+a)^2-3/4*a*ln(c*x^4+a)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int \frac{x^{15}}{\left(a+cx^4\right)^3}\,dx = \frac{2\,c^3x^{12}+4\,ac^2x^8-4\,a^2cx^4-5\,a^3-6\,\left(ac^2x^8+2\,a^2cx^4+a^3\right)\log\left(cx^4+a\right)}{8\,\left(c^6x^8+2\,ac^5x^4+a^2c^4\right)}$$

input
```
integrate(x^15/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/8*(2*c^3*x^12 + 4*a*c^2*x^8 - 4*a^2*c*x^4 - 5*a^3 - 6*(a*c^2*x^8 + 2*a^2
*c*x^4 + a^3)*log(c*x^4 + a))/(c^6*x^8 + 2*a*c^5*x^4 + a^2*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^{15}}{(a+cx^4)^3}\,dx = -\frac{3a\log(a+cx^4)}{4c^4} + \frac{-5a^3-6a^2cx^4}{8a^2c^4+16ac^5x^4+8c^6x^8} + \frac{x^4}{4c^3}$$

input
```
integrate(x**15/(c*x**4+a)**3,x)
```

output
```
-3*a*log(a + c*x**4)/(4*c**4) + (-5*a**3 - 6*a**2*c*x**4)/(8*a**2*c**4 + 1
6*a*c**5*x**4 + 8*c**6*x**8) + x**4/(4*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x^{15}}{(a+cx^4)^3}\,dx = \frac{x^4}{4\,c^3} - \frac{6\,a^2cx^4+5\,a^3}{8\,(c^6x^8+2\,ac^5x^4+a^2c^4)} - \frac{3\,a\log(cx^4+a)}{4\,c^4}$$

input
```
integrate(x^15/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/4*x^4/c^3 - 1/8*(6*a^2*c*x^4 + 5*a^3)/(c^6*x^8 + 2*a*c^5*x^4 + a^2*c^4)
- 3/4*a*log(c*x^4 + a)/c^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^{15}}{(a+cx^4)^3}\,dx = \frac{x^4}{4\,c^3} - \frac{3\,a\log\left(|cx^4+a|\right)}{4\,c^4} + \frac{9\,ac^2x^8+12\,a^2cx^4+4\,a^3}{8\,(cx^4+a)^2c^4}$$

input
```
integrate(x^15/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/4*x^4/c^3 - 3/4*a*log(abs(c*x^4 + a))/c^4 + 1/8*(9*a*c^2*x^8 + 12*a^2*c*
x^4 + 4*a^3)/((c*x^4 + a)^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^{15}}{(a+cx^4)^3}\,dx = \frac{x^4}{4\,c^3} - \frac{\frac{5\,a^3}{8\,c}+\frac{3\,a^2\,x^4}{4}}{a^2\,c^3+2\,a\,c^4\,x^4+c^5\,x^8} - \frac{3\,a\,\ln\left(c\,x^4+a\right)}{4\,c^4}$$

input
```
int(x^15/(a + c*x^4)^3,x)
```

output
```
x^4/(4*c^3) - ((5*a^3)/(8*c) + (3*a^2*x^4)/4)/(a^2*c^3 + c^5*x^8 + 2*a*c^4
*x^4) - (3*a*log(a + c*x^4))/(4*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.34

$$\int \frac{x^{15}}{(a+cx^4)^3}\,dx$$
$$= \frac{-6\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^3-12\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2c\,x^4-6\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x\right.}{}$$

input
```
int(x^15/(c*x^4+a)^3,x)
```

output
```
( - 6*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**3 -
12*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c*x**
4 - 6*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2*
x**8 - 6*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**3 -
12*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c*x**4 -
 6*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2*x**8 -
 3*a**3 + 6*a*c**2*x**8 + 2*c**3*x**12)/(8*c**4*(a**2 + 2*a*c*x**4 + c**2*
x**8))
```

3.87 $\int \frac{x^{11}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = -\frac{a^2}{8c^3(a+cx^4)^2}+\frac{a}{2c^3(a+cx^4)}+\frac{\log(a+cx^4)}{4c^3}$$

output
```
-1/8*a^2/c^3/(c*x^4+a)^2+1/2*a/c^3/(c*x^4+a)+1/4*ln(c*x^4+a)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{\frac{a(3a+4cx^4)}{(a+cx^4)^2}+2\log(a+cx^4)}{8c^3}$$

input
```
Integrate[x^11/(a + c*x^4)^3,x]
```

output
```
((a*(3*a + 4*c*x^4))/(a + c*x^4)^2 + 2*Log[a + c*x^4])/(8*c^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(cx^4+a)^3}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}\int \left(\frac{a^2}{c^2(cx^4+a)^3}-\frac{2a}{c^2(cx^4+a)^2}+\frac{1}{c^2(cx^4+a)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^2}{2c^3(a+cx^4)^2}+\frac{2a}{c^3(a+cx^4)}+\frac{\log(a+cx^4)}{c^3}\right)$$

input
```
Int[x^11/(a + c*x^4)^3,x]
```

output
```
(-1/2*a^2/(c^3*(a + c*x^4)^2) + (2*a)/(c^3*(a + c*x^4)) + Log[a + c*x^4]/c
^3)/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{\frac{3a^2}{8c^3}+\frac{a\,x^4}{2c^2}}{(c\,x^4+a)^2}+\frac{\ln(c\,x^4+a)}{4c^3}$	43
risch	$\frac{\frac{3a^2}{8c^3}+\frac{a\,x^4}{2c^2}}{(c\,x^4+a)^2}+\frac{\ln(c\,x^4+a)}{4c^3}$	43
default	$-\frac{a^2}{8c^3(c\,x^4+a)^2}+\frac{a}{2c^3(c\,x^4+a)}+\frac{\ln(c\,x^4+a)}{4c^3}$	47
parallelrisch	$\frac{2\ln(c\,x^4+a)x^8c^2+4\ln(c\,x^4+a)x^4ac+4a\,x^4c+2a^2\ln(c\,x^4+a)+3a^2}{8c^3(c\,x^4+a)^2}$	72

input
```
int(x^11/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/8/c^3*a^2+1/2*a*x^4/c^2)/(c*x^4+a)^2+1/4*ln(c*x^4+a)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.33

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{4\,acx^4+3\,a^2+2\,(c^2x^8+2\,acx^4+a^2)\log(cx^4+a)}{8\,(c^5x^8+2\,ac^4x^4+a^2c^3)}$$

input
```
integrate(x^11/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/8*(4*a*c*x^4 + 3*a^2 + 2*(c^2*x^8 + 2*a*c*x^4 + a^2)*log(c*x^4 + a))/(c^
5*x^8 + 2*a*c^4*x^4 + a^2*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{3a^2+4acx^4}{8a^2c^3+16ac^4x^4+8c^5x^8} + \frac{\log(a+cx^4)}{4c^3}$$

input
```
integrate(x**11/(c*x**4+a)**3,x)
```

output
```
(3*a**2 + 4*a*c*x**4)/(8*a**2*c**3 + 16*a*c**4*x**4 + 8*c**5*x**8) + log(a
 + c*x**4)/(4*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{4\,acx^4+3\,a^2}{8\,(c^5x^8+2\,ac^4x^4+a^2c^3)} + \frac{\log(cx^4+a)}{4\,c^3}$$

input
```
integrate(x^11/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*a*c*x^4 + 3*a^2)/(c^5*x^8 + 2*a*c^4*x^4 + a^2*c^3) + 1/4*log(c*x^4
+ a)/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{\log(|cx^4+a|)}{4\,c^3} - \frac{3\,cx^8+2\,ax^4}{8\,(cx^4+a)^2c^2}$$

input
```
integrate(x^11/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/4*log(abs(c*x^4 + a))/c^3 - 1/8*(3*c*x^8 + 2*a*x^4)/((c*x^4 + a)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{\frac{3a^2}{8c^3}+\frac{a\,x^4}{2\,c^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{\ln\left(c\,x^4+a\right)}{4\,c^3}$$

input
```
int(x^11/(a + c*x^4)^3,x)
```

output
```
((3*a^2)/(8*c^3) + (a*x^4)/(2*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + log(a +
c*x^4)/(4*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.85

$$\int \frac{x^{11}}{(a+cx^4)^3}\,dx = \frac{2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2+4\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)ac\,x^4+2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\right.}{}$$

input
```
int(x^11/(c*x^4+a)^3,x)
```

output
```
(2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 4*l
og( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 2*l
og( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*x**8 + 2*
log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 4*log(c**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 2*log(c**(1/
4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*x**8 + a**2 - 2*c**2*
x**8)/(8*c**3*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.88 $\int \frac{x^7}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^7}{(a+cx^4)^3} dx = \frac{x^8}{8a(a+cx^4)^2}$$

output
```
1/8*x^8/a/(c*x^4+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^7}{(a+cx^4)^3} dx = -\frac{a+2cx^4}{8c^2(a+cx^4)^2}$$

input
```
Integrate[x^7/(a + c*x^4)^3,x]
```

output
```
-1/8*(a + 2*c*x^4)/(c^2*(a + c*x^4)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+cx^4)^3} \, dx$$

$$\Big\downarrow \text{796}$$

$$\frac{x^8}{8a\,(a+cx^4)^2}$$

input
```
Int[x^7/(a + c*x^4)^3,x]
```

output
```
x^8/(8*a*(a + c*x^4)^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2c\,x^4+a}{8(c\,x^4+a)^2 c^2}$	23
orering	$-\frac{2c\,x^4+a}{8(c\,x^4+a)^2 c^2}$	23
parallelrisch	$\frac{-2c\,x^4-a}{8c^2(c\,x^4+a)^2}$	25
norman	$\frac{-\frac{x^4}{4c}-\frac{a}{8c^2}}{(c\,x^4+a)^2}$	26
risch	$\frac{-\frac{x^4}{4c}-\frac{a}{8c^2}}{(c\,x^4+a)^2}$	26
default	$-\frac{1}{4c^2(c\,x^4+a)}+\frac{a}{8c^2(c\,x^4+a)^2}$	31

input
```
int(x^7/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*c*x^4+a)/(c*x^4+a)^2/c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^7}{(a+cx^4)^3}\,dx = -\frac{2\,cx^4+a}{8\,(c^4x^8+2\,ac^3x^4+a^2c^2)}$$

input
```
integrate(x^7/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*c*x^4 + a)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^7}{(a+cx^4)^3}\,dx = \frac{-a-2cx^4}{8a^2c^2+16ac^3x^4+8c^4x^8}$$

input
```
integrate(x**7/(c*x**4+a)**3,x)
```

output
```
(-a - 2*c*x**4)/(8*a**2*c**2 + 16*a*c**3*x**4 + 8*c**4*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^7}{(a+cx^4)^3}\,dx = -\frac{2\,cx^4+a}{8\,(c^4x^8+2\,ac^3x^4+a^2c^2)}$$

input
```
integrate(x^7/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(2*c*x^4 + a)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^7}{(a+cx^4)^3}\,dx = -\frac{2\,cx^4+a}{8\,(cx^4+a)^2c^2}$$

input
```
integrate(x^7/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/8*(2*c*x^4 + a)/((c*x^4 + a)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^7}{(a+cx^4)^3}\,dx = -\frac{\frac{a}{8\,c^2}+\frac{x^4}{4\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$

input
```
int(x^7/(a + c*x^4)^3,x)
```

output
```
-(a/(8*c^2) + x^4/(4*c))/(a^2 + c^2*x^8 + 2*a*c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{x^7}{(a+cx^4)^3}\,dx = \frac{x^8}{8a\,(c^2x^8+2ac\,x^4+a^2)}$$

input
```
int(x^7/(c*x^4+a)^3,x)
```

output
```
x**8/(8*a*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.89 $\int \frac{x^3}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x^3}{(a+cx^4)^3} dx = -\frac{1}{8c(a+cx^4)^2}$$

output
```
-1/8/c/(c*x^4+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+cx^4)^3} dx = -\frac{1}{8c(a+cx^4)^2}$$

input
```
Integrate[x^3/(a + c*x^4)^3,x]
```

output
```
-1/8*1/(c*(a + c*x^4)^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^3/(a + c*x^4)^3,x]
```

output
```
-1/8*1/(c*(a + c*x^4)^2)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{8c(c\,x^4+a)^2}$	15
derivativedivides	$-\frac{1}{8c(c\,x^4+a)^2}$	15
default	$-\frac{1}{8c(c\,x^4+a)^2}$	15
norman	$-\frac{1}{8c(c\,x^4+a)^2}$	15
risch	$-\frac{1}{8c(c\,x^4+a)^2}$	15
parallelrisch	$-\frac{1}{8c(c\,x^4+a)^2}$	15
orering	$-\frac{1}{8c(c\,x^4+a)^2}$	15

input
```
int(x^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/c/(c*x^4+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{(a+cx^4)^3}\,dx = -\frac{1}{8\,(c^3x^8+2\,ac^2x^4+a^2c)}$$

input
```
integrate(x^3/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/8/(c^3*x^8 + 2*a*c^2*x^4 + a^2*c)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{x^3}{(a+cx^4)^3}\,dx = -\frac{1}{8a^2c+16ac^2x^4+8c^3x^8}$$

input
```
integrate(x**3/(c*x**4+a)**3,x)
```

output
```
-1/(8*a**2*c + 16*a*c**2*x**4 + 8*c**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+cx^4)^3}\,dx = -\frac{1}{8\,(cx^4+a)^2c}$$

input
```
integrate(x^3/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/8/((c*x^4 + a)^2*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+cx^4)^3}\,dx = -\frac{1}{8\,(cx^4+a)^2c}$$

input
```
integrate(x^3/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/8/((c*x^4 + a)^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(a+cx^4)^3}\,dx = -\frac{1}{8\,c\,\left(a^2+2\,a\,c\,x^4+c^2\,x^8\right)}$$

input
```
int(x^3/(a + c*x^4)^3,x)
```

output
```
-1/(8*c*(a^2 + c^2*x^8 + 2*a*c*x^4))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x^3}{(a+cx^4)^3}\,dx = -\frac{1}{8c\left(c^2x^8+2ac\,x^4+a^2\right)}$$

input
```
int(x^3/(c*x^4+a)^3,x)
```

output
```
( - 1)/(8*c*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.90 $\int \frac{1}{x(a+cx^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a+cx^4)^3} dx = \frac{1}{8a(a+cx^4)^2} + \frac{1}{4a^2(a+cx^4)} + \frac{\log(x)}{a^3} - \frac{\log(a+cx^4)}{4a^3}$$

output
```
1/8/a/(c*x^4+a)^2+1/4/a^2/(c*x^4+a)+ln(x)/a^3-1/4*ln(c*x^4+a)/a^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(a+cx^4)^3} dx = \frac{\frac{a(3a+2cx^4)}{(a+cx^4)^2} + 8\log(x) - 2\log(a+cx^4)}{8a^3}$$

input
```
Integrate[1/(x*(a + c*x^4)^3),x]
```

output
```
((a*(3*a + 2*c*x^4))/(a + c*x^4)^2 + 8*Log[x] - 2*Log[a + c*x^4])/(8*a^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+cx^4\right)^3}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(cx^4+a\right)^3}dx^4$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{4}\int \left(-\frac{c}{a^3\left(cx^4+a\right)}-\frac{c}{a^2\left(cx^4+a\right)^2}-\frac{c}{a\left(cx^4+a\right)^3}+\frac{1}{a^3x^4}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{\log\left(a+cx^4\right)}{a^3}+\frac{\log\left(x^4\right)}{a^3}+\frac{1}{a^2\left(a+cx^4\right)}+\frac{1}{2a\left(a+cx^4\right)^2}\right)$$

input
```
Int[1/(x*(a + c*x^4)^3),x]
```

output
```
(1/(2*a*(a + c*x^4)^2) + 1/(a^2*(a + c*x^4)) + Log[x^4]/a^3 - Log[a + c*x^
4]/a^3)/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\frac{c\,x^{4}}{4a^{2}}+\frac{3}{8a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(c\,x^{4}+a\right)}{4a^{3}}$	46
norman	$\frac{-\frac{c\,x^{4}}{2a^{2}}-\frac{3c^{2}x^{8}}{8a^{3}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(c\,x^{4}+a\right)}{4a^{3}}$	52
default	$\frac{\ln\left(x\right)}{a^{3}}-\frac{c\left(\frac{\ln\left(c\,x^{4}+a\right)}{2c}-\frac{a}{2c\left(c\,x^{4}+a\right)}-\frac{a^{2}}{4c\left(c\,x^{4}+a\right)^{2}}\right)}{2a^{3}}$	60
parallelrisch	$\frac{8\ln\left(x\right)x^{8}c^{2}-2\ln\left(c\,x^{4}+a\right)x^{8}c^{2}-3c^{2}x^{8}+16\ln\left(x\right)x^{4}ac-4\ln\left(c\,x^{4}+a\right)x^{4}ac-4a\,x^{4}c+8a^{2}\ln\left(x\right)-2a^{2}\ln\left(c\,x^{4}+a\right)}{8a^{3}\left(c\,x^{4}+a\right)^{2}}$	101

input
```
int(1/x/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4/a^2*c*x^4+3/8/a)/(c*x^4+a)^2+ln(x)/a^3-1/4*ln(c*x^4+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(a+cx^4\right)^3}\,dx = \frac{2\,acx^{4}+3\,a^{2}-2\left(c^{2}x^{8}+2\,acx^{4}+a^{2}\right)\log\left(cx^{4}+a\right)+8\left(c^{2}x^{8}+2\,acx^{4}+a^{2}\right)\log\left(x\right)}{8\left(a^{3}c^{2}x^{8}+2\,a^{4}cx^{4}+a^{5}\right)}$$

input
```
integrate(1/x/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/8*(2*a*c*x^4 + 3*a^2 - 2*(c^2*x^8 + 2*a*c*x^4 + a^2)*log(c*x^4 + a) + 8*
(c^2*x^8 + 2*a*c*x^4 + a^2)*log(x))/(a^3*c^2*x^8 + 2*a^4*c*x^4 + a^5)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+cx^4\right)^3}\,dx = \frac{3a+2cx^4}{8a^4+16a^3cx^4+8a^2c^2x^8}+\frac{\log(x)}{a^3}-\frac{\log\left(\frac{a}{c}+x^4\right)}{4a^3}$$

input
```
integrate(1/x/(c*x**4+a)**3,x)
```

output
```
(3*a + 2*c*x**4)/(8*a**4 + 16*a**3*c*x**4 + 8*a**2*c**2*x**8) + log(x)/a**
3 - log(a/c + x**4)/(4*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+cx^4\right)^3}\,dx = \frac{2\,cx^4+3\,a}{8\left(a^2c^2x^8+2\,a^3cx^4+a^4\right)}-\frac{\log\left(cx^4+a\right)}{4\,a^3}+\frac{\log\left(x^4\right)}{4\,a^3}$$

input
```
integrate(1/x/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/8*(2*c*x^4 + 3*a)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) - 1/4*log(c*x^4 + a)
/a^3 + 1/4*log(x^4)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(a+cx^4)^3}\,dx = \frac{\log(x^4)}{4\,a^3} - \frac{\log(|cx^4+a|)}{4\,a^3} + \frac{3\,c^2x^8+8\,acx^4+6\,a^2}{8\,(cx^4+a)^2a^3}$$

input
```
integrate(1/x/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/4*log(x^4)/a^3 - 1/4*log(abs(c*x^4 + a))/a^3 + 1/8*(3*c^2*x^8 + 8*a*c*x^
4 + 6*a^2)/((c*x^4 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\,(a+cx^4)^3}\,dx = \frac{\ln(x)}{a^3} + \frac{\frac{3}{8\,a}+\frac{c\,x^4}{4\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{\ln(c\,x^4+a)}{4\,a^3}$$

input
```
int(1/(x*(a + c*x^4)^3),x)
```

output
```
log(x)/a^3 + (3/(8*a) + (c*x^4)/(4*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - log
(a + c*x^4)/(4*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.22

$$\int \frac{1}{x\,(a+cx^4)^3}\,dx$$
$$= \frac{-2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2-4\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)ac\,x^4-2\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\right.}{}$$

input
```
int(1/x/(c*x^4+a)^3,x)
```

output
```
( - 2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 -
4*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 -
2*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*x**8 -
 2*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 4*log(
c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 - 2*log(c**
(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**2*x**8 + 8*log(x)*a*
*2 + 16*log(x)*a*c*x**4 + 8*log(x)*c**2*x**8 + 2*a**2 - c**2*x**8)/(8*a**3
*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.91 $\int \frac{1}{x^5(a+cx^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{x^5 (a+cx^4)^3} dx = -\frac{1}{4a^3x^4} - \frac{c}{8a^2 (a+cx^4)^2} - \frac{c}{2a^3 (a+cx^4)} - \frac{3c\log(x)}{a^4} + \frac{3c\log(a+cx^4)}{4a^4}$$

output
```
-1/4/a^3/x^4-1/8*c/a^2/(c*x^4+a)^2-1/2*c/a^3/(c*x^4+a)-3*c*ln(x)/a^4+3/4*c
*ln(c*x^4+a)/a^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 (a+cx^4)^3} dx = -\frac{\frac{a(2a^2+9acx^4+6c^2x^8)}{x^4(a+cx^4)^2} + 24c\log(x) - 6c\log(a+cx^4)}{8a^4}$$

input
```
Integrate[1/(x^5*(a + c*x^4)^3),x]
```

output
```
-1/8*((a*(2*a^2 + 9*a*c*x^4 + 6*c^2*x^8))/(x^4*(a + c*x^4)^2) + 24*c*Log[x
] - 6*c*Log[a + c*x^4])/a^4
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{798, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+cx^4)^3} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8 (cx^4+a)^3}dx^4$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{4}\int \left(\frac{3c^2}{a^4 (cx^4+a)}+\frac{2c^2}{a^3 (cx^4+a)^2}+\frac{c^2}{a^2 (cx^4+a)^3}-\frac{3c}{a^4x^4}+\frac{1}{a^3x^8}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{3c\log(x^4)}{a^4}+\frac{3c\log(a+cx^4)}{a^4}-\frac{2c}{a^3 (a+cx^4)}-\frac{1}{a^3x^4}-\frac{c}{2a^2 (a+cx^4)^2}\right)$$

input
```
Int[1/(x^5*(a + c*x^4)^3),x]
```

output
```
(-(1/(a^3*x^4)) - c/(2*a^2*(a + c*x^4)^2) - (2*c)/(a^3*(a + c*x^4)) - (3*c
*Log[x^4])/a^4 + (3*c*Log[a + c*x^4])/a^4)/4
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result
norman	$\frac{-\frac{1}{4a}+\frac{3c^2x^8}{2a^3}+\frac{9c^3x^{12}}{8a^4}}{x^4\left(c\,x^4+a\right)^2}-\frac{3c\ln\left(x\right)}{a^4}+\frac{3c\ln\left(c\,x^4+a\right)}{4a^4}$
risch	$\frac{-\frac{3c^2x^8}{4a^3}-\frac{9c\,x^4}{8a^2}-\frac{1}{4a}}{x^4\left(c\,x^4+a\right)^2}-\frac{3c\ln\left(x\right)}{a^4}+\frac{3c\ln\left(-c\,x^4-a\right)}{4a^4}$
default	$-\frac{1}{4a^3x^4}-\frac{3c\ln\left(x\right)}{a^4}+\frac{c^2\left(\frac{3\ln\left(c\,x^4+a\right)}{2c}-\frac{a}{c\left(c\,x^4+a\right)}-\frac{a^2}{4c\left(c\,x^4+a\right)^2}\right)}{2a^4}$
parallelrisch	$-\frac{24\ln\left(x\right)x^{12}c^3-6\ln\left(c\,x^4+a\right)x^{12}c^3-9c^3x^{12}+48\ln\left(x\right)x^8a\,c^2-12\ln\left(c\,x^4+a\right)x^8a\,c^2-12a\,c^2x^8+24\ln\left(x\right)x^4a^2c-6\ln\left(c\,x^4+a\right)}{8a^4x^4\left(c\,x^4+a\right)^2}$

input
```
int(1/x^5/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a+3/2*c^2/a^3*x^8+9/8*c^3/a^4*x^12)/x^4/(c*x^4+a)^2-3*c*ln(x)/a^4+3/
4*c*ln(c*x^4+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^5 (a + cx^4)^3} \, dx = -\frac{6\, ac^2x^8 + 9\, a^2cx^4 + 2\, a^3 - 6\, (c^3x^{12} + 2\, ac^2x^8 + a^2cx^4) \log (cx^4 + a) + 24\, (c^3x^{12} + 2\, ac^2x^8 + a^2cx^4) \log}{8\, (a^4c^2x^{12} + 2\, a^5cx^8 + a^6x^4)}$$

input
```
integrate(1/x^5/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/8*(6*a*c^2*x^8 + 9*a^2*c*x^4 + 2*a^3 - 6*(c^3*x^12 + 2*a*c^2*x^8 + a^2*
c*x^4)*log(c*x^4 + a) + 24*(c^3*x^12 + 2*a*c^2*x^8 + a^2*c*x^4)*log(x))/(a
^4*c^2*x^12 + 2*a^5*c*x^8 + a^6*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^5 (a + cx^4)^3} \, dx = \frac{-2a^2 - 9acx^4 - 6c^2x^8}{8a^5x^4 + 16a^4cx^8 + 8a^3c^2x^{12}} - \frac{3c \log (x)}{a^4} + \frac{3c \log \left(\frac{a}{c} + x^4\right)}{4a^4}$$

input
```
integrate(1/x**5/(c*x**4+a)**3,x)
```

output
```
(-2*a**2 - 9*a*c*x**4 - 6*c**2*x**8)/(8*a**5*x**4 + 16*a**4*c*x**8 + 8*a**
3*c**2*x**12) - 3*c*log(x)/a**4 + 3*c*log(a/c + x**4)/(4*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^5 (a + cx^4)^3} \, dx = -\frac{6\, c^2x^8 + 9\, acx^4 + 2\, a^2}{8\, (a^3c^2x^{12} + 2\, a^4cx^8 + a^5x^4)} + \frac{3\, c \log (cx^4 + a)}{4\, a^4} - \frac{3\, c \log (x^4)}{4\, a^4}$$

input
```
integrate(1/x^5/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(6*c^2*x^8 + 9*a*c*x^4 + 2*a^2)/(a^3*c^2*x^12 + 2*a^4*c*x^8 + a^5*x^4
) + 3/4*c*log(c*x^4 + a)/a^4 - 3/4*c*log(x^4)/a^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^5 (a + cx^4)^3} \, dx = -\frac{3 \, c \log \left(x^4\right)}{4 \, a^4} + \frac{3 \, c \log \left(|cx^4 + a|\right)}{4 \, a^4} - \frac{9 \, c^3 x^8 + 22 \, a c^2 x^4 + 14 \, a^2 c}{8 \, (cx^4 + a)^2 a^4} + \frac{3 \, cx^4 - a}{4 \, a^4 x^4}$$

input
```
integrate(1/x^5/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-3/4*c*log(x^4)/a^4 + 3/4*c*log(abs(c*x^4 + a))/a^4 - 1/8*(9*c^3*x^8 + 22*
a*c^2*x^4 + 14*a^2*c)/((c*x^4 + a)^2*a^4) + 1/4*(3*c*x^4 - a)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^5 (a + cx^4)^3} \, dx = \frac{3 \, c \, \ln \left(c \, x^4 + a\right)}{4 \, a^4} - \frac{\frac{1}{4 \, a} + \frac{9 \, c \, x^4}{8 \, a^2} + \frac{3 \, c^2 \, x^8}{4 \, a^3}}{a^2 \, x^4 + 2 \, a \, c \, x^8 + c^2 \, x^{12}} - \frac{3 \, c \, \ln \left(x\right)}{a^4}$$

input
```
int(1/(x^5*(a + c*x^4)^3),x)
```

output
```
(3*c*log(a + c*x^4))/(4*a^4) - (1/(4*a) + (9*c*x^4)/(8*a^2) + (3*c^2*x^8)/
(4*a^3))/(a^2*x^4 + c^2*x^12 + 2*a*c*x^8) - (3*c*log(x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.74

$$\int \frac{1}{x^5 (a+cx^4)^3}\,dx$$
$$= \frac{6\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a^2c\,x^4+12\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)a\,c^2x^8+6\log\left(-c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}\right.}{}$$

input
```
int(1/x^5/(c*x^4+a)^3,x)
```

output
```
(6*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2*c*x**
4 + 12*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c**2
*x**8 + 6*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**
3*x**12 + 6*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2
*c*x**4 + 12*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c
**2*x**8 + 6*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*c**
3*x**12 - 24*log(x)*a**2*c*x**4 - 48*log(x)*a*c**2*x**8 - 24*log(x)*c**3*x
**12 - 2*a**3 - 6*a**2*c*x**4 + 3*c**3*x**12)/(8*a**4*x**4*(a**2 + 2*a*c*x
**4 + c**2*x**8))
```

3.92 $\int \frac{x^9}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = -\frac{x^6}{8c\,(a+cx^4)^2} - \frac{3x^2}{16c^2\,(a+cx^4)} + \frac{3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16\sqrt{a}c^{5/2}}$$

output
```
-1/8*x^6/c/(c*x^4+a)^2-3/16*x^2/c^2/(c*x^4+a)+3/16*arctan(c^(1/2)*x^2/a^(1
/2))/a^(1/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = \frac{1}{16}\left(\frac{-3ax^2-5cx^6}{c^2\,(a+cx^4)^2} + \frac{3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}c^{5/2}}\right)$$

input
```
Integrate[x^9/(a + c*x^4)^3,x]
```

output
```
((-3*a*x^2 - 5*c*x^6)/(c^2*(a + c*x^4)^2) + (3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(Sqrt[a]*c^(5/2)))/16
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(cx^4+a)^3}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{3\int \frac{x^4}{(cx^4+a)^2}dx^2}{4c}-\frac{x^6}{4c(a+cx^4)^2}\right)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{\int \frac{1}{cx^4+a}dx^2}{2c}-\frac{x^2}{2c(a+cx^4)}\right)}{4c}-\frac{x^6}{4c(a+cx^4)^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}c^{3/2}}-\frac{x^2}{2c(a+cx^4)}\right)}{4c}-\frac{x^6}{4c(a+cx^4)^2}\right)$$

input
```
Int[x^9/(a + c*x^4)^3,x]
```

output
```
(-1/4*x^6/(c*(a + c*x^4)^2) + (3*(-1/2*x^2/(c*(a + c*x^4)) + ArcTan[(Sqrt[
c]*x^2)/Sqrt[a]]/(2*Sqrt[a]*c^(3/2))))/(4*c))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{-\frac{5x^6}{8c}-\frac{3a\,x^2}{8c^2}}{2(c\,x^4+a)^2}+\frac{3\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{16c^2\sqrt{ac}}$	52
risch	$\frac{-\frac{5x^6}{16c}-\frac{3a\,x^2}{16c^2}}{(c\,x^4+a)^2}-\frac{3\ln\left(x^2\sqrt{-ac}-a\right)}{32\sqrt{-ac}\,c^2}+\frac{3\ln\left(x^2\sqrt{-ac}+a\right)}{32\sqrt{-ac}\,c^2}$	80

input
```
int(x^9/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-5/8*x^6/c-3/8*a*x^2/c^2)/(c*x^4+a)^2+3/16/c^2/(a*c)^(1/2)*arctan(c*x
^2/(a*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.88

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = \left[-\frac{10\,ac^2x^6+6\,a^2cx^2+3\,(c^2x^8+2\,acx^4+a^2)\sqrt{-ac}\log\left(\frac{cx^4-2\,\sqrt{-ac}x^2-a}{cx^4+a}\right)}{32\,(ac^5x^8+2\,a^2c^4x^4+a^3c^3)}, -\frac{5\,ac^2x^6+3\,a^2cx^2+3\,(c^2x^8+2\,acx^4+a^2)\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{16\,(ac^5x^8+2\,a^2c^4x^4+a^3c^3)}\right]$$

input
```
integrate(x^9/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
[-1/32*(10*a*c^2*x^6 + 6*a^2*c*x^2 + 3*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(-a
*c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a*c^5*x^8 + 2*a^2*c^
4*x^4 + a^3*c^3), -1/16*(5*a*c^2*x^6 + 3*a^2*c*x^2 + 3*(c^2*x^8 + 2*a*c*x^
4 + a^2)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)))/(a*c^5*x^8 + 2*a^2*c^4*x^4 +
 a^3*c^3)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = -\frac{3\sqrt{-\frac{1}{ac^5}}\log\left(-ac^2\sqrt{-\frac{1}{ac^5}}+x^2\right)}{32} + \frac{3\sqrt{-\frac{1}{ac^5}}\log\left(ac^2\sqrt{-\frac{1}{ac^5}}+x^2\right)}{32} + \frac{-3ax^2-5cx^6}{16a^2c^2+32ac^3x^4+16c^4x^8}$$

input
```
integrate(x**9/(c*x**4+a)**3,x)
```

output
```
-3*sqrt(-1/(a*c**5))*log(-a*c**2*sqrt(-1/(a*c**5)) + x**2)/32 + 3*sqrt(-1/
(a*c**5))*log(a*c**2*sqrt(-1/(a*c**5)) + x**2)/32 + (-3*a*x**2 - 5*c*x**6)
/(16*a**2*c**2 + 32*a*c**3*x**4 + 16*c**4*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = -\frac{5\,cx^6+3\,ax^2}{16\,(c^4x^8+2\,ac^3x^4+a^2c^2)} + \frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}c^2}$$

input
```
integrate(x^9/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(5*c*x^6 + 3*a*x^2)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2) + 3/16*arctan(
c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = \frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}c^2} - \frac{5\,cx^6+3\,ax^2}{16\,(cx^4+a)^2c^2}$$

input
```
integrate(x^9/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
3/16*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*c^2) - 1/16*(5*c*x^6 + 3*a*x^2)/((
c*x^4 + a)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^9}{(a+cx^4)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,\sqrt{a}\,c^{5/2}} - \frac{\frac{5\,x^6}{16\,c}+\frac{3\,a\,x^2}{16\,c^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}$$

input
```
int(x^9/(a + c*x^4)^3,x)
```

output
```
(3*atan((c^(1/2)*x^2)/a^(1/2)))/(16*a^(1/2)*c^(5/2)) - ((5*x^6)/(16*c) + (
3*a*x^2)/(16*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.01

$$\int \frac{x^9}{(a+cx^4)^3}\,dx$$
$$= \frac{-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2-6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)ac\,x^4-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c}{}$$

input
```
int(x^9/(c*x^4+a)^3,x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*a**2 - 6*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - 3*sqrt(c)*sq
rt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sq
rt(2)))*c**2*x**8 - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 6*sqrt(c)*sqrt(a)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**
4 - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 - 3*a**2*c*x**2 - 5*a*c**2*x**6)/(16*a*c
**3*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.93 $\int \frac{x^5}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^5}{(a+cx^4)^3}\,dx=-\frac{x^2}{8c(a+cx^4)^2}+\frac{x^2}{16ac(a+cx^4)}+\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{3/2}c^{3/2}}$$

output
```
-1/8*x^2/c/(c*x^4+a)^2+1/16*x^2/a/c/(c*x^4+a)+1/16*arctan(c^(1/2)*x^2/a^(1
/2))/a^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{(a+cx^4)^3}\,dx=\frac{\frac{\sqrt{a}\sqrt{c}x^2(-a+cx^4)}{(a+cx^4)^2}+\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{3/2}c^{3/2}}$$

input
```
Integrate[x^5/(a + c*x^4)^3,x]
```

output
```
((Sqrt[a]*Sqrt[c]*x^2*(-a + c*x^4))/(a + c*x^4)^2 + ArcTan[(Sqrt[c]*x^2)/S
qrt[a]])/(16*a^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+cx^4)^3} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^4}{(cx^4+a)^3}dx^2$$

$$\Big\downarrow \text{ 252}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(cx^4+a)^2}dx^2}{4c}-\frac{x^2}{4c\,(a+cx^4)^2}\right)$$

$$\Big\downarrow \text{ 215}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{cx^4+a}dx^2}{2a}+\frac{x^2}{2a(a+cx^4)}}{4c}-\frac{x^2}{4c\,(a+cx^4)^2}\right)$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{2}\left(\frac{\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x^2}{2a(a+cx^4)}}{4c}-\frac{x^2}{4c\,(a+cx^4)^2}\right)$$

input
```
Int[x^5/(a + c*x^4)^3,x]
```

output
```
(-1/4*x^2/(c*(a + c*x^4)^2) + (x^2/(2*a*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c]))/(4*c))/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\frac{x^{6}}{8a}-\frac{x^{2}}{8c}}{2\left(c\,x^{4}+a\right)^{2}}+\frac{\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{16ac\sqrt{ac}}$	54
risch	$\frac{\frac{x^{6}}{16a}-\frac{x^{2}}{16c}}{\left(c\,x^{4}+a\right)^{2}}-\frac{\ln\left(x^{2}\sqrt{-ac}-a\right)}{32\sqrt{-ac}\,ca}+\frac{\ln\left(x^{2}\sqrt{-ac}+a\right)}{32\sqrt{-ac}\,ca}$	85

input
```
int(x^5/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(1/8/a*x^6-1/8*x^2/c)/(c*x^4+a)^2+1/16/a/c/(a*c)^(1/2)*arctan(c*x^2/(a
*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.80

$$\int \frac{x^5}{(a+cx^4)^3}\,dx = \left[\frac{2\,ac^2x^6 - 2\,a^2cx^2 - (c^2x^8 + 2\,acx^4 + a^2)\sqrt{-ac}\log\left(\frac{cx^4 - 2\,\sqrt{-ac}x^2 - a}{cx^4 + a}\right)}{32\,(a^2c^4x^8 + 2\,a^3c^3x^4 + a^4c^2)}, \frac{ac^2x^6 - a^2cx^2 - (c^2x^8 + 2\,acx^4 +}{16\,(a^2c^4x^8 + 2\,a^3c^3x}\right.$$

input
```
integrate(x^5/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
[1/32*(2*a*c^2*x^6 - 2*a^2*c*x^2 - (c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(-a*c)*
log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a^2*c^4*x^8 + 2*a^3*c^3*
x^4 + a^4*c^2), 1/16*(a*c^2*x^6 - a^2*c*x^2 - (c^2*x^8 + 2*a*c*x^4 + a^2)*
sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)))/(a^2*c^4*x^8 + 2*a^3*c^3*x^4 + a^4*c^
2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(56) = 112.

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.63

$$\int \frac{x^5}{(a+cx^4)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^3c^3}}\log\left(-a^2c\sqrt{-\frac{1}{a^3c^3}} + x^2\right)}{32} + \frac{\sqrt{-\frac{1}{a^3c^3}}\log\left(a^2c\sqrt{-\frac{1}{a^3c^3}} + x^2\right)}{32} + \frac{-ax^2 + cx^6}{16a^3c + 32a^2c^2x^4 + 16ac^3x^8}$$

input
```
integrate(x**5/(c*x**4+a)**3,x)
```

output
```
-sqrt(-1/(a**3*c**3))*log(-a**2*c*sqrt(-1/(a**3*c**3)) + x**2)/32 + sqrt(-
1/(a**3*c**3))*log(a**2*c*sqrt(-1/(a**3*c**3)) + x**2)/32 + (-a*x**2 + c*x
**6)/(16*a**3*c + 32*a**2*c**2*x**4 + 16*a*c**3*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{(a+cx^4)^3}\,dx = \frac{cx^6-ax^2}{16\,(ac^3x^8+2\,a^2c^2x^4+a^3c)} + \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}ac}$$

input
```
integrate(x^5/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*(c*x^6 - a*x^2)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 1/16*arctan(c*x
^2/sqrt(a*c))/(sqrt(a*c)*a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{x^5}{(a+cx^4)^3}\,dx = \frac{\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}ac} + \frac{cx^6-ax^2}{16\,(cx^4+a)^2ac}$$

input
```
integrate(x^5/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/16*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a*c) + 1/16*(c*x^6 - a*x^2)/((c*x^
4 + a)^2*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{(a+cx^4)^3}\,dx = \frac{\frac{x^6}{16\,a}-\frac{x^2}{16\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,a^{3/2}\,c^{3/2}}$$

input
```
int(x^5/(a + c*x^4)^3,x)
```

output
```
(x^6/(16*a) - x^2/(16*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) + atan((c^(1/2)*x^2)
/a^(1/2))/(16*a^(3/2)*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.83

$$\int \frac{x^5}{(a+cx^4)^3}\,dx$$
$$= \frac{-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2-2\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)ac\,x^4-\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)c^2x}{16}$$

input
```
int(x^5/(c*x^4+a)^3,x)
```

output
```
( - sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/
4)*a**(1/4)*sqrt(2)))*a**2 - 2*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqr
t(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - sqrt(c)*sqrt(a
)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2
)))*c**2*x**8 - sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 2*sqrt(c)*sqrt(a)*atan((c**(1/4)*
a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - sq
rt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*c**2*x**8 - a**2*c*x**2 + a*c**2*x**6)/(16*a**2*c**2*(a**2
 + 2*a*c*x**4 + c**2*x**8))
```

3.94 $\int \frac{x}{(a+cx^4)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 68

$$\int \frac{x}{(a+cx^4)^3} \, dx = \frac{x^2}{8a(a+cx^4)^2} + \frac{3x^2}{16a^2(a+cx^4)} + \frac{3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{c}}$$

output
```
1/8*x^2/a/(c*x^4+a)^2+3/16*x^2/a^2/(c*x^4+a)+3/16*arctan(c^(1/2)*x^2/a^(1/
2))/a^(5/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{x}{(a+cx^4)^3} \, dx = \frac{1}{16}\left(\frac{5ax^2+3cx^6}{a^2(a+cx^4)^2} + \frac{3\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{5/2}\sqrt{c}}\right)$$

input
```
Integrate[x/(a + c*x^4)^3,x]
```

output
```
((5*a*x^2 + 3*c*x^6)/(a^2*(a + c*x^4)^2) + (3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]
])/(a^(5/2)*Sqrt[c]))/16
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {807, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+cx^4)^3} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{(cx^4+a)^3}dx^2$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{3\int \frac{1}{(cx^4+a)^2}dx^2}{4a}+\frac{x^2}{4a(a+cx^4)^2}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{\int \frac{1}{cx^4+a}dx^2}{2a}+\frac{x^2}{2a(a+cx^4)}\right)}{4a}+\frac{x^2}{4a(a+cx^4)^2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x^2}{2a(a+cx^4)}\right)}{4a}+\frac{x^2}{4a(a+cx^4)^2}\right)$$

input
```
Int[x/(a + c*x^4)^3,x]
```

output
```
(x^2/(4*a*(a + c*x^4)^2) + (3*(x^2/(2*a*(a + c*x^4)) + ArcTan[(Sqrt[c]*x^2
)/Sqrt[a]]/(2*a^(3/2)*Sqrt[c])))/(4*a))/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{x^{2}}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{3x^{2}}{16a\left(c\,x^{4}+a\right)}+\frac{3\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{16a\sqrt{ac}}}{a}$	63
risch	$\frac{\frac{3c\,x^{6}}{16a^{2}}+\frac{5x^{2}}{16a}}{\left(c\,x^{4}+a\right)^{2}}-\frac{3\ln\left(x^{2}\sqrt{-ac}-a\right)}{32\sqrt{-ac}\,a^{2}}+\frac{3\ln\left(x^{2}\sqrt{-ac}+a\right)}{32\sqrt{-ac}\,a^{2}}$	80

input
```
int(x/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x^2/a/(c*x^4+a)^2+3/8/a*(1/2*x^2/a/(c*x^4+a)+1/2/a/(a*c)^(1/2)*arctan(
c*x^2/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.88

$$\int \frac{x}{\left(a+cx^4\right)^3}\,dx$$
$$= \left[\frac{6\,ac^2x^6+10\,a^2cx^2-3\,(c^2x^8+2\,acx^4+a^2)\sqrt{-ac}\log\left(\frac{cx^4-2\,\sqrt{-ac}x^2-a}{cx^4+a}\right)}{32\,(a^3c^3x^8+2\,a^4c^2x^4+a^5c)}, \frac{3\,ac^2x^6+5\,a^2cx^2-3\,(c^2x^8+2}{16\,(a^3c^3x^8+2}\right.$$

input
```
integrate(x/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
[1/32*(6*a*c^2*x^6 + 10*a^2*c*x^2 - 3*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(-a*
c)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a^3*c^3*x^8 + 2*a^4*c
^2*x^4 + a^5*c), 1/16*(3*a*c^2*x^6 + 5*a^2*c*x^2 - 3*(c^2*x^8 + 2*a*c*x^4
+ a^2)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2)))/(a^3*c^3*x^8 + 2*a^4*c^2*x^4 +
 a^5*c)]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.62

$$\int \frac{x}{\left(a+cx^4\right)^3}\,dx = -\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x^2\right)}{32} + \frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x^2\right)}{32} + \frac{5ax^2+3cx^6}{16a^4+32a^3cx^4+16a^2c^2x^8}$$

input
```
integrate(x/(c*x**4+a)**3,x)
```

output
```
-3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x**2)/32 + 3*sqrt(-1/(a
**5*c))*log(a**3*sqrt(-1/(a**5*c)) + x**2)/32 + (5*a*x**2 + 3*c*x**6)/(16*
a**4 + 32*a**3*c*x**4 + 16*a**2*c**2*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a+cx^4)^3}\,dx = \frac{3\,cx^6+5\,ax^2}{16\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^2}$$

input
```
integrate(x/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/16*(3*c*x^6 + 5*a*x^2)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 3/16*arctan(c
*x^2/sqrt(a*c))/(sqrt(a*c)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{x}{(a+cx^4)^3}\,dx = \frac{3\,\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^2} + \frac{3\,cx^6+5\,ax^2}{16\,(cx^4+a)^2a^2}$$

input
```
integrate(x/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
3/16*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/16*(3*c*x^6 + 5*a*x^2)/((
c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{x}{(a+cx^4)^3}\,dx = \frac{\frac{5\,x^2}{16\,a}+\frac{3\,c\,x^6}{16\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,a^{5/2}\,\sqrt{c}}$$

input
```
int(x/(a + c*x^4)^3,x)
```

output
```
((5*x^2)/(16*a) + (3*c*x^6)/(16*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (3*ata
n((c^(1/2)*x^2)/a^(1/2)))/(16*a^(5/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.01

$$\int \frac{x}{(a+cx^4)^3}\,dx$$
$$= \frac{-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2-6\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)ac\,x^4-3\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input
```
int(x/(c*x^4+a)^3,x)
```

output
```
( - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*a**2 - 6*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 - 3*sqrt(c)*sq
rt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sq
rt(2)))*c**2*x**8 - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*
sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 6*sqrt(c)*sqrt(a)*atan((c**
(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**
4 - 3*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(
1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 5*a**2*c*x**2 + 3*a*c**2*x**6)/(16*a**
3*c*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.95 $\int \frac{1}{x^3(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{1}{x^3(a+cx^4)^3}\,dx = -\frac{1}{2a^3x^2}-\frac{cx^2}{8a^2(a+cx^4)^2}-\frac{7cx^2}{16a^3(a+cx^4)}-\frac{15\sqrt{c}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16a^{7/2}}$$

output
```
-1/2/a^3/x^2-1/8*c*x^2/a^2/(c*x^4+a)^2-7/16*c*x^2/a^3/(c*x^4+a)-15/16*c^(1
/2)*arctan(c^(1/2)*x^2/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^3(a+cx^4)^3}\,dx = \frac{-\frac{\sqrt{a}(8a^2+25acx^4+15c^2x^8)}{x^2(a+cx^4)^2}+15\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+15\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{16a^{7/2}}$$

input
```
Integrate[1/(x^3*(a + c*x^4)^3),x]
```

output
```
(-((Sqrt[a]*(8*a^2 + 25*a*c*x^4 + 15*c^2*x^8))/(x^2*(a + c*x^4)^2)) + 15*S
qrt[c]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 15*Sqrt[c]*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(16*a^(7/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {807, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+cx^4)^3} \, dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2}\int \frac{1}{x^4 (cx^4+a)^3}dx^2$$

$$\Big\downarrow \; 253$$

$$\frac{1}{2}\left(\frac{5\int \frac{1}{x^4(cx^4+a)^2}dx^2}{4a}+\frac{1}{4ax^2 (a+cx^4)^2}\right)$$

$$\Big\downarrow \; 253$$

$$\frac{1}{2}\left(\frac{5\left(\frac{3\int \frac{1}{x^4\left(cx^4+a\right)}dx^2}{2a}+\frac{1}{2ax^2(a+cx^4)}\right)}{4a}+\frac{1}{4ax^2 (a+cx^4)^2}\right)$$

$$\Big\downarrow \; 264$$

$$\frac{1}{2}\left(\frac{5\left(\frac{3\left(-\frac{c\int \frac{1}{cx^4+a}dx^2}{a}-\frac{1}{ax^2}\right)}{2a}+\frac{1}{2ax^2(a+cx^4)}\right)}{4a}+\frac{1}{4ax^2 (a+cx^4)^2}\right)$$

$$\downarrow 218$$

$$\frac{1}{2}\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax^2}\right)}{2a}+\frac{1}{2ax^2(a+cx^4)}\right)}{4a}+\frac{1}{4ax^2\left(a+cx^4\right)^2}\right)$$

input
```
Int[1/(x^3*(a + c*x^4)^3),x]
```

output
```
(1/(4*a*x^2*(a + c*x^4)^2) + (5*(1/(2*a*x^2*(a + c*x^4)) + (3*(-(1/(a*x^2)
) - (Sqrt[c]*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a))/2
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

method	result	size
default	$-\frac{1}{2a^{3}x^{2}}-\frac{c\left(\frac{\frac{7}{8}c\,x^{6}+\frac{9}{8}a\,x^{2}}{\left(c\,x^{4}+a\right)^{2}}+\frac{15\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{8\sqrt{ac}}\right)}{2a^{3}}$	58
risch	$\frac{-\frac{15c^{2}x^{8}}{16a^{3}}-\frac{25c\,x^{4}}{16a^{2}}-\frac{1}{2a}}{x^{2}\left(c\,x^{4}+a\right)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}\_Z^{2}+c\right)}\_R\ln\left(\left(-5a^{7}\_R^{2}-4c\right)x^{2}-a^{4}\_R\right)\right)}{32}$	82

input
```
int(1/x^3/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3/x^2-1/2/a^3*c*((7/8*c*x^6+9/8*a*x^2)/(c*x^4+a)^2+15/8/(a*c)^(1/2)
*arctan(c*x^2/(a*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^3\left(a+cx^4\right)^3}\,dx$$

$$=\left[-\frac{30\,c^2x^8+50\,acx^4-15\,(c^2x^{10}+2\,acx^6+a^2x^2)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+16\,a^2}{32\,(a^3c^2x^{10}+2\,a^4cx^6+a^5x^2)},\right.$$

$$\left.-\frac{15\,c^2x^8+25\,acx^4+15\,(c^2x^{10}+2\,acx^6+a^2x^2)\sqrt{\frac{c}{a}}\arctan\left(x^2\sqrt{\frac{c}{a}}\right)+8\,a^2}{16\,(a^3c^2x^{10}+2\,a^4cx^6+a^5x^2)}\right]$$

input
```
integrate(1/x^3/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
[-1/32*(30*c^2*x^8 + 50*a*c*x^4 - 15*(c^2*x^10 + 2*a*c*x^6 + a^2*x^2)*sqrt
(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) + 16*a^2)/(a^3*c^
2*x^10 + 2*a^4*c*x^6 + a^5*x^2), -1/16*(15*c^2*x^8 + 25*a*c*x^4 + 15*(c^2*
x^10 + 2*a*c*x^6 + a^2*x^2)*sqrt(c/a)*arctan(x^2*sqrt(c/a)) + 8*a^2)/(a^3*
c^2*x^10 + 2*a^4*c*x^6 + a^5*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^3 (a+cx^4)^3}\,dx = \frac{15\sqrt{-\frac{c}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{c}{a^7}}}{c}+x^2\right)}{32} - \frac{15\sqrt{-\frac{c}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{c}{a^7}}}{c}+x^2\right)}{32} + \frac{-8a^2-25acx^4-15c^2x^8}{16a^5x^2+32a^4cx^6+16a^3c^2x^{10}}$$

input
```
integrate(1/x**3/(c*x**4+a)**3,x)
```

output
```
15*sqrt(-c/a**7)*log(-a**4*sqrt(-c/a**7)/c + x**2)/32 - 15*sqrt(-c/a**7)*l
og(a**4*sqrt(-c/a**7)/c + x**2)/32 + (-8*a**2 - 25*a*c*x**4 - 15*c**2*x**8
)/(16*a**5*x**2 + 32*a**4*c*x**6 + 16*a**3*c**2*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a+cx^4)^3}\,dx = -\frac{15\,c^2x^8+25\,acx^4+8\,a^2}{16\,(a^3c^2x^{10}+2\,a^4cx^6+a^5x^2)} - \frac{15\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3}$$

input
```
integrate(1/x^3/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(15*c^2*x^8 + 25*a*c*x^4 + 8*a^2)/(a^3*c^2*x^10 + 2*a^4*c*x^6 + a^5*
x^2) - 15/16*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\left(a+cx^4\right)^3}\,dx = -\frac{15\,c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{16\,\sqrt{ac}a^3} - \frac{7\,c^2x^6+9\,acx^2}{16\,\left(cx^4+a\right)^2a^3} - \frac{1}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-15/16*c*arctan(c*x^2/sqrt(a*c))/(sqrt(a*c)*a^3) - 1/16*(7*c^2*x^6 + 9*a*c
*x^2)/((c*x^4 + a)^2*a^3) - 1/2/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3\left(a+cx^4\right)^3}\,dx = -\frac{\frac{1}{2\,a}+\frac{25\,c\,x^4}{16\,a^2}+\frac{15\,c^2\,x^8}{16\,a^3}}{a^2\,x^2+2\,a\,c\,x^6+c^2\,x^{10}} - \frac{15\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{16\,a^{7/2}}$$

input
```
int(1/(x^3*(a + c*x^4)^3),x)
```

output
```
- (1/(2*a) + (25*c*x^4)/(16*a^2) + (15*c^2*x^8)/(16*a^3))/(a^2*x^2 + c^2*x
^10 + 2*a*c*x^6) - (15*c^(1/2)*atan((c^(1/2)*x^2)/a^(1/2)))/(16*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.55

$$\int \frac{1}{x^3 (a+cx^4)^3} \, dx = \frac{15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)a^2x^2 + 30\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}\,x}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)ac\,x^6 + 15\sqrt{c}\,\sqrt{a}\,atan\left(\frac{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{c}}{c^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right.}{}$$

input
```
int(1/x^3/(c*x^4+a)^3,x)
```

output
```
(15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/
4)*a**(1/4)*sqrt(2)))*a**2*x**2 + 30*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/
4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**6 + 15*sqrt(
c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/
4)*sqrt(2)))*c**2*x**10 + 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*x**2 + 30*sqrt(c)*sqrt
(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt
(2)))*a*c*x**6 + 15*sqrt(c)*sqrt(a)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sq
rt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**10 - 8*a**3 - 25*a**2*c*x**4
 - 15*a*c**2*x**8)/(16*a**4*x**2*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.96 $\int \frac{x^{10}}{(a+cx^4)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 172

$$\int \frac{x^{10}}{(a+cx^4)^3} \, dx = -\frac{x^7}{8c(a+cx^4)^2} - \frac{7x^3}{32c^2(a+cx^4)} - \frac{21 \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\sqrt[4]{a}c^{11/4}} + \frac{21 \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}\sqrt[4]{a}c^{11/4}} - \frac{21 \text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}\sqrt[4]{a}c^{11/4}}$$

output
```
-1/8*x^7/c/(c*x^4+a)^2-7/32*x^3/c^2/(c*x^4+a)+21/128*arctan(-1+2^(1/2)*c^(
1/4)*x/a^(1/4))*2^(1/2)/a^(1/4)/c^(11/4)+21/128*arctan(1+2^(1/2)*c^(1/4)*x
/a^(1/4))*2^(1/2)/a^(1/4)/c^(11/4)-21/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*
x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(1/4)/c^(11/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = \frac{\frac{32ac^{3/4}x^3}{(a+cx^4)^2}-\frac{88c^{3/4}x^3}{a+cx^4}-\frac{42\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}+\frac{42\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}+\frac{21\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{a}}-\frac{21\sqrt{}}{}}{256c^{11/4}}$$

input
```
Integrate[x^10/(a + c*x^4)^3,x]
```

output
```
((32*a*c^(3/4)*x^3)/(a + c*x^4)^2 - (88*c^(3/4)*x^3)/(a + c*x^4) - (42*Sqr
t[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(1/4) + (42*Sqrt[2]*ArcTan
[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(1/4))/(256*c^(11/4))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 817, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7\int \frac{x^6}{(cx^4+a)^2}dx}{8c}-\frac{x^7}{8c\,(a+cx^4)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{7\left(\frac{3\int \frac{x^2}{cx^4+a}dx}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 826

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\,(a+cx^4)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^3}{4c(a+cx^4)}\right)}{8c}-\frac{x^7}{8c\,(a+cx^4)^2}$$

input
```
Int[x^10/(a + c*x^4)^3,x]
```

output
```
-1/8*x^7/(c*(a + c*x^4)^2) + (7*(-1/4*x^3/(c*(a + c*x^4)) + (3*((-(ArcTan[
1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c
^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2
]*a^(1/4)*c^(1/4)))/(2*Sqrt[c])))/(4*c)))/(8*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

method	result	size
risch	$\frac{-\frac{11x^7}{32c}-\frac{7a\,x^3}{32c^2}}{(c\,x^4+a)^2}+\frac{21\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{128c^3}$	56
default	$\frac{-\frac{11x^7}{32c}-\frac{7a\,x^3}{32c^2}}{(c\,x^4+a)^2}+\frac{21\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c^3\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	131

input
```
int(x^10/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-11/32/c*x^7-7/32/c^2*a*x^3)/(c*x^4+a)^2+21/128/c^3*sum(1/_R*ln(x-_R),_R=
RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.53

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = -\frac{44\,cx^7+28\,ax^3-21\,(c^4x^8+2\,ac^3x^4+a^2c^2)\left(-\frac{1}{ac^{11}}\right)^{\frac{1}{4}}\log\left(ac^8\left(-\frac{1}{ac^{11}}\right)^{\frac{3}{4}}+x\right)+21\,(i\,c^4x^8+2i\,ac^3x^4+\cdots}{}$$

input
```
integrate(x^10/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(44*c*x^7 + 28*a*x^3 - 21*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a*
c^11))^(1/4)*log(a*c^8*(-1/(a*c^11))^(3/4) + x) + 21*(I*c^4*x^8 + 2*I*a*c^
3*x^4 + I*a^2*c^2)*(-1/(a*c^11))^(1/4)*log(I*a*c^8*(-1/(a*c^11))^(3/4) + x
) + 21*(-I*c^4*x^8 - 2*I*a*c^3*x^4 - I*a^2*c^2)*(-1/(a*c^11))^(1/4)*log(-I
*a*c^8*(-1/(a*c^11))^(3/4) + x) + 21*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1
/(a*c^11))^(1/4)*log(-a*c^8*(-1/(a*c^11))^(3/4) + x))/(c^4*x^8 + 2*a*c^3*x
^4 + a^2*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.41

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = \frac{-7ax^3-11cx^7}{32a^2c^2+64ac^3x^4+32c^4x^8} + \operatorname{RootSum}\left(268435456t^4ac^{11}+194481,\left(t\mapsto t\log\left(\frac{2097152t^3ac^8}{9261}+x\right)\right)\right)$$

input
```
integrate(x**10/(c*x**4+a)**3,x)
```

output
```
(-7*a*x**3 - 11*c*x**7)/(32*a**2*c**2 + 64*a*c**3*x**4 + 32*c**4*x**8) + R
ootSum(268435456*_t**4*a*c**11 + 194481, Lambda(_t, _t*log(2097152*_t**3*a
*c**8/9261 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.25

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = -\frac{11\,cx^7+7\,ax^3}{32\,(c^4x^8+2\,ac^3x^4+a^2c^2)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x\right.}{}\right.}{256\,c^2}$$

input
```
integrate(x^10/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(11*c*x^7 + 7*a*x^3)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2) + 21/256*(2*s
qrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqr
t(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt
(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(s
qrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/
4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1
/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = -\frac{11\,cx^7+7\,ax^3}{32\,(cx^4+a)^2c^2} + \frac{21\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^5} + \frac{21\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^5} - \frac{21\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,ac^5} + \frac{21\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,ac^5}$$

input
```
integrate(x^10/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-1/32*(11*c*x^7 + 7*a*x^3)/((c*x^4 + a)^2*c^2) + 21/128*sqrt(2)*(a*c^3)^(3
/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^5) +
21/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(a*c^5) - 21/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*
(a/c)^(1/4) + sqrt(a/c))/(a*c^5) + 21/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 -
sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^5)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.48

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = \frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{1/4}\,c^{11/4}} - \frac{\frac{11\,x^7}{32\,c}+\frac{7\,a\,x^3}{32\,c^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{1/4}\,c^{11/4}}$$

input
```
int(x^10/(a + c*x^4)^3,x)
```

output
```
(21*atan((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(1/4)*c^(11/4)) - ((11*x^7)/(32
*c) + (7*a*x^3)/(32*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*atanh((c^(1/4)
*x)/(-a)^(1/4)))/(64*(-a)^(1/4)*c^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.77

$$\int \frac{x^{10}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^10/(c*x^4+a)^3,x)
```

output
```
( - 42*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 84*c**(1/4)*a**(3/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c*x**4 - 42*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 42*c**(1/4)*a**(3/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*a**2 + 84*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 42*c**(1/4)*a**
(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*c**2*x**8 + 21*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a
**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 42*c**(1/4)*a**(3/4)*sq
rt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**
4 + 21*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt
(a) + sqrt(c)*x**2)*c**2*x**8 - 21*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 42*c**(1/4)*a**(3/4)*s
qrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4
- 21*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(c)*x**2)*c**2*x**8 - 56*a**2*c*x**3 - 88*a*c**2*x**7)/(256*a*c**3*(a
**2 + 2*a*c*x**4 + c**2*x**8))
```

3.97 $\int \frac{x^8}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 170

$$\int \frac{x^8}{(a+cx^4)^3}\,dx = -\frac{x^5}{8c(a+cx^4)^2} - \frac{5x}{32c^2(a+cx^4)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{3/4}c^{9/4}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{3/4}c^{9/4}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{3/4}c^{9/4}}$$

output

```
-1/8*x^5/c/(c*x^4+a)^2-5/32*x/c^2/(c*x^4+a)+5/128*arctan(-1+2^(1/2)*c^(1/4
)*x/a^(1/4))*2^(1/2)/a^(3/4)/c^(9/4)+5/128*arctan(1+2^(1/2)*c^(1/4)*x/a^(1
/4))*2^(1/2)/a^(3/4)/c^(9/4)+5/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1
/2)+c^(1/2)*x^2))*2^(1/2)/a^(3/4)/c^(9/4)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18

$$\int \frac{x^8}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{32a\sqrt[4]{c}x}{(a+cx^4)^2} - \frac{72\sqrt[4]{c}x}{a+cx^4} - \frac{10\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{10\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{5\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{3/4}} + \frac{5\sqrt{2}\log}{}}{256c^{9/4}}$$

input
```
Integrate[x^8/(a + c*x^4)^3,x]
```

output
```
((32*a*c^(1/4)*x)/(a + c*x^4)^2 - (72*c^(1/4)*x)/(a + c*x^4) - (10*Sqrt[2]
*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (10*Sqrt[2]*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) - (5*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4) + (5*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(256*c^(9/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{817}$$
$$\frac{5\int \frac{x^4}{(cx^4+a)^2}dx}{8c} - \frac{x^5}{8c\,(a+cx^4)^2}$$
$$\Big\downarrow \text{817}$$

$$\frac{5\left(\frac{\int \frac{1}{cx^4+a}dx}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{5\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}}{4c}-\frac{x}{4c(a+cx^4)}\right)}{8c}-\frac{x^5}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^8/(a + c*x^4)^3,x]
```

output
```
-1/8*x^5/(c*(a + c*x^4)^2) + (5*(-1/4*x/(c*(a + c*x^4)) + ((-(ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[
2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4
)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(
1/4)*c^(1/4)))/(2*Sqrt[a]))/(4*c)))/(8*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.32

method	result	size
risch	$\frac{-\frac{9x^5}{32c}-\frac{5ax}{32c^2}}{(cx^4+a)^2}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{128c^3}$	54
default	$\frac{-\frac{9x^5}{32c}-\frac{5ax}{32c^2}}{(cx^4+a)^2}+\frac{5\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c^2a}$	132

input
```
int(x^8/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-9/32/c*x^5-5/32/c^2*a*x)/(c*x^4+a)^2+5/128/c^3*sum(1/_R^3*ln(x-_R),_R=Ro
otOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.54

$$\int \frac{x^8}{(a+cx^4)^3}\,dx =$$

$$-\frac{36\,cx^5-5\left(c^4x^8+2\,ac^3x^4+a^2c^2\right)\left(-\frac{1}{a^3c^9}\right)^{\frac{1}{4}}\log\left(ac^2\left(-\frac{1}{a^3c^9}\right)^{\frac{1}{4}}+x\right)+5\left(-i\,c^4x^8-2i\,ac^3x^4-i\,a^2c^2\right)\left(-\right.}{}$$

input
```
integrate(x^8/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(36*c*x^5 - 5*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^3*c^9))^(1/4
)*log(a*c^2*(-1/(a^3*c^9))^(1/4) + x) + 5*(-I*c^4*x^8 - 2*I*a*c^3*x^4 - I*
a^2*c^2)*(-1/(a^3*c^9))^(1/4)*log(I*a*c^2*(-1/(a^3*c^9))^(1/4) + x) + 5*(I
*c^4*x^8 + 2*I*a*c^3*x^4 + I*a^2*c^2)*(-1/(a^3*c^9))^(1/4)*log(-I*a*c^2*(-
1/(a^3*c^9))^(1/4) + x) + 5*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^3*c^9
))^(1/4)*log(-a*c^2*(-1/(a^3*c^9))^(1/4) + x) + 20*a*x)/(c^4*x^8 + 2*a*c^3
*x^4 + a^2*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.40

$$\int \frac{x^8}{(a+cx^4)^3}\,dx = \frac{-5ax-9cx^5}{32a^2c^2+64ac^3x^4+32c^4x^8} + \operatorname{RootSum}\left(268435456t^4a^3c^9+625, \left(t\mapsto t\log\left(\frac{128tac^2}{5}+x\right)\right)\right)$$

input
```
integrate(x**8/(c*x**4+a)**3,x)
```

output
```
(-5*a*x - 9*c*x**5)/(32*a**2*c**2 + 64*a*c**3*x**4 + 32*c**4*x**8) + RootS
um(268435456*_t**4*a**3*c**9 + 625, Lambda(_t, _t*log(128*_t*a*c**2/5 + x)
))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.25

$$\int \frac{x^8}{(a+cx^4)^3}\,dx = -\frac{9\,cx^5+5\,ax}{32\,(c^4x^8+2\,ac^3x^4+a^2c^2)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac14}c^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}c^{\frac14}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2 \cdots\right.}{\cdots}\right.}{256\,c^2}$$

input
```
integrate(x^8/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(9*c*x^5 + 5*a*x)/(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2) + 5/256*(2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)
*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x
 + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.20

$$\int \frac{x^8}{(a+cx^4)^3}\,dx = \frac{5\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^3} + \frac{5\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,ac^3} + \frac{5\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,ac^3} - \frac{5\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,ac^3} - \frac{9\,cx^5+5\,ax}{32\,(cx^4+a)^2c^2}$$

input
```
integrate(x^8/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
5/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a*c^3) + 5/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*
x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 5/256*sqrt(2)*(a*c^3)^(1/4
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 5/256*sqrt(2)*(a*
c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/32*(9*
c*x^5 + 5*a*x)/((c*x^4 + a)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.48

$$\int \frac{x^8}{(a+cx^4)^3}\,dx = -\frac{\frac{9\,x^5}{32\,c}+\frac{5\,a\,x}{32\,c^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{3/4}\,c^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{3/4}\,c^{9/4}}$$

input
```
int(x^8/(a + c*x^4)^3,x)
```

output
```
- ((9*x^5)/(32*c) + (5*a*x)/(32*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (5*ata
n((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(3/4)*c^(9/4)) - (5*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(3/4)*c^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.79

$$\int \frac{x^8}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^8/(c*x^4+a)^3,x)
```

output
```
( - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 20*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c*x**4 - 10*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 10*c**(3/4)*a**(1/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*a**2 + 20*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 10*c**(3/4)*a**
(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*c**2*x**8 - 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a*
*(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 10*c**(3/4)*a**(1/4)*sqr
t(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4
 - 5*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a
) + sqrt(c)*x**2)*c**2*x**8 + 5*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**
(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 10*c**(3/4)*a**(1/4)*sqrt
(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 5
*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(c)*x**2)*c**2*x**8 - 40*a**2*c*x - 72*a*c**2*x**5)/(256*a*c**3*(a**2 + 2
*a*c*x**4 + c**2*x**8))
```

3.98 $\int \frac{x^6}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 175

$$\int \frac{x^6}{(a+cx^4)^3}\,dx = -\frac{x^3}{8c(a+cx^4)^2}+\frac{3x^3}{32ac(a+cx^4)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{5/4}c^{7/4}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{5/4}c^{7/4}}-\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{5/4}c^{7/4}}$$

output
```
-1/8*x^3/c/(c*x^4+a)^2+3/32*x^3/a/c/(c*x^4+a)+3/128*arctan(-1+2^(1/2)*c^(1
/4)*x/a^(1/4))*2^(1/2)/a^(5/4)/c^(7/4)+3/128*arctan(1+2^(1/2)*c^(1/4)*x/a^
(1/4))*2^(1/2)/a^(5/4)/c^(7/4)-3/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^
(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(5/4)/c^(7/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.18

$$\int \frac{x^6}{(a+cx^4)^3}\,dx$$
$$= \frac{-\frac{32c^{3/4}x^3}{(a+cx^4)^2}+\frac{24c^{3/4}x^3}{a^2+acx^4}-\frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{5/4}}-\frac{3\sqrt{2}\,\text{lo}}{}}{256c^{7/4}}$$

input
```
Integrate[x^6/(a + c*x^4)^3,x]
```

output
```
((-32*c^(3/4)*x^3)/(a + c*x^4)^2 + (24*c^(3/4)*x^3)/(a^2 + a*c*x^4) - (6*S
qrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(5/4) + (6*Sqrt[2]*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(5/4) + (3*Sqrt[2]*Log[Sqrt[a] - Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(5/4) - (3*Sqrt[2]*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(5/4))/(256*c^(7/4))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {817, 819, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{ 817}$$
$$\frac{3\int \frac{x^2}{(cx^4+a)^2}dx}{8c}-\frac{x^3}{8c\,(a+cx^4)^2}$$
$$\Big\downarrow \text{ 819}$$

$$\frac{3\left(\frac{\int \frac{x^2}{cx^4+a}dx}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8c}-\frac{x^3}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^6/(a + c*x^4)^3,x]
```

output
```
-1/8*x^3/(c*(a + c*x^4)^2) + (3*(x^3/(4*a*(a + c*x^4)) + ((-(ArcTan[1 - (S
qrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)
) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1
/4)*c^(1/4)))/(2*Sqrt[c]))/(4*a)))/(8*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.33

method	result	size
risch	$\frac{\frac{3x^7}{32a}-\frac{x^3}{32c}}{(c\,x^4+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{128a\,c^2}$	58
default	$\frac{\frac{3x^7}{32a}-\frac{x^3}{32c}}{(c\,x^4+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a\,c^2\left(\frac{a}{c}\right)^{\frac{1}{4}}}$	133

input
```
int(x^6/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3/32/a*x^7-1/32*x^3/c)/(c*x^4+a)^2+3/128/a/c^2*sum(1/_R*ln(x-_R),_R=RootO
f(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.58

$$\int \frac{x^6}{(a+cx^4)^3}\,dx$$
$$= \frac{12\,cx^7-4\,ax^3+3\left(ac^3x^8+2\,a^2c^2x^4+a^3c\right)\left(-\frac{1}{a^5c^7}\right)^{\frac{1}{4}}\log\left(a^4c^5\left(-\frac{1}{a^5c^7}\right)^{\frac{3}{4}}+x\right)-3\left(i\,ac^3x^8+2i\,a^2c^2x^4+\cdots\right.}{}$$

input
```
integrate(x^6/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(12*c*x^7 - 4*a*x^3 + 3*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^5
*c^7))^(1/4)*log(a^4*c^5*(-1/(a^5*c^7))^(3/4) + x) - 3*(I*a*c^3*x^8 + 2*I*
a^2*c^2*x^4 + I*a^3*c)*(-1/(a^5*c^7))^(1/4)*log(I*a^4*c^5*(-1/(a^5*c^7))^(
3/4) + x) - 3*(-I*a*c^3*x^8 - 2*I*a^2*c^2*x^4 - I*a^3*c)*(-1/(a^5*c^7))^(1
/4)*log(-I*a^4*c^5*(-1/(a^5*c^7))^(3/4) + x) - 3*(a*c^3*x^8 + 2*a^2*c^2*x^
4 + a^3*c)*(-1/(a^5*c^7))^(1/4)*log(-a^4*c^5*(-1/(a^5*c^7))^(3/4) + x))/(a
*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

$$\int \frac{x^6}{(a+cx^4)^3}\,dx = \frac{-ax^3+3cx^7}{32a^3c+64a^2c^2x^4+32ac^3x^8} + \operatorname{RootSum}\left(268435456t^4a^5c^7+81, \left(t\mapsto t\log\left(\frac{2097152t^3a^4c^5}{27}+x\right)\right)\right)$$

input
```
integrate(x**6/(c*x**4+a)**3,x)
```

output
```
(-a*x**3 + 3*c*x**7)/(32*a**3*c + 64*a**2*c**2*x**4 + 32*a*c**3*x**8) + Ro
otSum(268435456*_t**4*a**5*c**7 + 81, Lambda(_t, _t*log(2097152*_t**3*a**4
*c**5/27 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.25

$$\int \frac{x^6}{(a+cx^4)^3}\,dx = \frac{3\,cx^7-ax^3}{32\,(ac^3x^8+2\,a^2c^2x^4+a^3c)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2\right.}{c}\right.}{256\,ac}$$

input
```
integrate(x^6/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(3*c*x^7 - a*x^3)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 3/256*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt
(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.19

$$\int \frac{x^6}{(a+cx^4)^3}\,dx = \frac{3\,cx^7-ax^3}{32\,(cx^4+a)^2ac} + \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^2c^4} + \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^2c^4} - \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^2c^4} + \frac{3\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^2c^4}$$

input
```
integrate(x^6/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(3*c*x^7 - a*x^3)/((c*x^4 + a)^2*a*c) + 3/128*sqrt(2)*(a*c^3)^(3/4)*a
rctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/1
28*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a^2*c^4) - 3/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/
c)^(1/4) + sqrt(a/c))/(a^2*c^4) + 3/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.46

$$\int \frac{x^6}{(a+cx^4)^3}\,dx = \frac{\frac{3\,x^7}{32\,a}-\frac{x^3}{32\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{5/4}\,c^{7/4}} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{5/4}\,c^{7/4}}$$

input
```
int(x^6/(a + c*x^4)^3,x)
```

output
```
((3*x^7)/(32*a) - x^3/(32*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (3*atan((c^(1/
4)*x)/(-a)^(1/4)))/(64*(-a)^(5/4)*c^(7/4)) + (3*atanh((c^(1/4)*x)/(-a)^(1/
4)))/(64*(-a)^(5/4)*c^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.72

$$\int \frac{x^6}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^6/(c*x^4+a)^3,x)
```

output
```
( - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 12*c**(1/4)*a**(3/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*
c*x**4 - 6*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 6*c**(1/4)*a**(3/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a**2 + 12*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 6*c**(1/4)*a**(3/4
)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*c**2*x**8 + 3*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/
4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 6*c**(1/4)*a**(3/4)*sqrt(2)*
log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 3*
c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + s
qrt(c)*x**2)*c**2*x**8 - 3*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)
*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 6*c**(1/4)*a**(3/4)*sqrt(2)*lo
g(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 - 3*c**(1
/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x
**2)*c**2*x**8 - 8*a**2*c*x**3 + 24*a*c**2*x**7)/(256*a**2*c**2*(a**2 + 2*
a*c*x**4 + c**2*x**8))
```

3.99 $\int \frac{x^4}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 171

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = -\frac{x}{8c(a+cx^4)^2}+\frac{x}{32ac(a+cx^4)}-\frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}c^{5/4}}+\frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{7/4}c^{5/4}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{7/4}c^{5/4}}$$

output
```
-1/8*x/c/(c*x^4+a)^2+1/32*x/a/c/(c*x^4+a)+3/128*arctan(-1+2^(1/2)*c^(1/4)*
x/a^(1/4))*2^(1/2)/a^(7/4)/c^(5/4)+3/128*arctan(1+2^(1/2)*c^(1/4)*x/a^(1/4
))*2^(1/2)/a^(7/4)/c^(5/4)+3/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2
)+c^(1/2)*x^2))*2^(1/2)/a^(7/4)/c^(5/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.19

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = \frac{-\frac{32\sqrt[4]{c}x}{(a+cx^4)^2}+\frac{8\sqrt[4]{c}x}{a^2+acx^4}-\frac{6\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{7/4}}-\frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{a^{7/4}}}{256c^{5/4}}$$

input
```
Integrate[x^4/(a + c*x^4)^3,x]
```

output
```
((-32*c^(1/4)*x)/(a + c*x^4)^2 + (8*c^(1/4)*x)/(a^2 + a*c*x^4) - (6*Sqrt[2
]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) + (6*Sqrt[2]*ArcTan[1 +
 (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(256*c^(5/4))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {817, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \ 817$$

$$\frac{\int \frac{1}{(cx^4+a)^2}dx}{8c}-\frac{x}{8c\,(a+cx^4)^2}$$

$$\Big\downarrow \ 749$$

$$\frac{\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 755

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1476

$$\frac{\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1082

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 217

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1479

$$\frac{\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}}{8c}-\frac{x}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^4/(a + c*x^4)^3,x]
```

output
```
-1/8*x/(c*(a + c*x^4)^2) + (x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*
c^(1/4)))/(2*Sqrt[a])))/(4*a))/(8*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.33

method	result	size
risch	$\frac{\frac{x^5}{32a}-\frac{3x}{32c}}{(c\,x^4+a)^2}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4c+a)}\frac{\ln(x-\_R)}{\_R^3}\right)}{128a\,c^2}$	56
default	$\frac{\frac{x^5}{32a}-\frac{3x}{32c}}{(c\,x^4+a)^2}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^2c}$	131

input
```
int(x^4/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/32/a*x^5-3/32*x/c)/(c*x^4+a)^2+3/128/a/c^2*sum(1/_R^3*ln(x-_R),_R=RootO
f(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.56

$$\int \frac{x^4}{(a+cx^4)^3}\,dx$$

$$= \frac{4\,cx^5+3\,(ac^3x^8+2\,a^2c^2x^4+a^3c)\left(-\frac{1}{a^7c^5}\right)^{\frac{1}{4}}\log\left(a^2c\left(-\frac{1}{a^7c^5}\right)^{\frac{1}{4}}+x\right)-3\,(-i\,ac^3x^8-2i\,a^2c^2x^4-i\,a^3c)\big(-}{}$$

input
```
integrate(x^4/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(4*c*x^5 + 3*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^7*c^5))^(1/4
)*log(a^2*c*(-1/(a^7*c^5))^(1/4) + x) - 3*(-I*a*c^3*x^8 - 2*I*a^2*c^2*x^4
- I*a^3*c)*(-1/(a^7*c^5))^(1/4)*log(I*a^2*c*(-1/(a^7*c^5))^(1/4) + x) - 3*
(I*a*c^3*x^8 + 2*I*a^2*c^2*x^4 + I*a^3*c)*(-1/(a^7*c^5))^(1/4)*log(-I*a^2*
c*(-1/(a^7*c^5))^(1/4) + x) - 3*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a
^7*c^5))^(1/4)*log(-a^2*c*(-1/(a^7*c^5))^(1/4) + x) - 12*a*x)/(a*c^3*x^8 +
 2*a^2*c^2*x^4 + a^3*c)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.39

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = \frac{-3ax+cx^5}{32a^3c+64a^2c^2x^4+32ac^3x^8} + \operatorname{RootSum}\left(268435456t^4a^7c^5+81,\left(t\mapsto t\log\left(\frac{128ta^2c}{3}+x\right)\right)\right)$$

input
```
integrate(x**4/(c*x**4+a)**3,x)
```

output
```
(-3*a*x + c*x**5)/(32*a**3*c + 64*a**2*c**2*x**4 + 32*a*c**3*x**8) + RootS
um(268435456*_t**4*a**7*c**5 + 81, Lambda(_t, _t*log(128*_t*a**2*c/3 + x))
)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.26

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = \frac{cx^5-3\,ax}{32\,(ac^3x^8+2\,a^2c^2x^4+a^3c)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x^2 \cdots\right.}{\cdots}\right)}{256\,ac}$$

input
```
integrate(x^4/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(c*x^5 - 3*a*x)/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 3/256*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(
2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqr
t(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x
+ sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/(a*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^2c^2} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^2c^2} + \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^2c^2} - \frac{3\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^2c^2} + \frac{cx^5-3\,ax}{32\,(cx^4+a)^2ac}$$

input
```
integrate(x^4/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
3/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))
/(a/c)^(1/4))/(a^2*c^2) + 3/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^2) + 3/256*sqrt(2)*(a*c^3)^
(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) - 3/256*sqrt(
2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^2) +
1/32*(c*x^5 - 3*a*x)/((c*x^4 + a)^2*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.47

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{7/4}\,c^{5/4}} - \frac{\frac{3\,x}{32\,c}-\frac{x^5}{32\,a}}{a^2+2\,a\,c\,x^4+c^2\,x^8} + \frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{7/4}\,c^{5/4}}$$

input
```
int(x^4/(a + c*x^4)^3,x)
```

output
```
(3*atan((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(7/4)*c^(5/4)) - ((3*x)/(32*c) -
 x^5/(32*a))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)
))/(64*(-a)^(7/4)*c^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.77

$$\int \frac{x^4}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^4/(c*x^4+a)^3,x)
```

output
```
( - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c
)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 12*c**(3/4)*a**(1/4)*sqrt(2)*atan
((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*
c*x**4 - 6*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 6*c**(3/4)*a**(1/4)*sqr
t(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqr
t(2)))*a**2 + 12*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 6*c**(3/4)*a**(1/4
)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4
)*sqrt(2)))*c**2*x**8 - 3*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/
4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 6*c**(3/4)*a**(1/4)*sqrt(2)*
log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 - 3*
c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + s
qrt(c)*x**2)*c**2*x**8 + 3*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)
*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 6*c**(3/4)*a**(1/4)*sqrt(2)*lo
g(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 + 3*c**(3
/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x
**2)*c**2*x**8 - 24*a**2*c*x + 8*a*c**2*x**5)/(256*a**2*c**2*(a**2 + 2*a*c
*x**4 + c**2*x**8))
```

3.100 $\int \frac{x^2}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 172

$$\int \frac{x^2}{(a+cx^4)^3} dx = \frac{x^3}{8a(a+cx^4)^2} + \frac{5x^3}{32a^2(a+cx^4)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}c^{3/4}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{9/4}c^{3/4}} - \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{9/4}c^{3/4}}$$

output
```
1/8*x^3/a/(c*x^4+a)^2+5/32*x^3/a^2/(c*x^4+a)+5/128*arctan(-1+2^(1/2)*c^(1/
4)*x/a^(1/4))*2^(1/2)/a^(9/4)/c^(3/4)+5/128*arctan(1+2^(1/2)*c^(1/4)*x/a^(
1/4))*2^(1/2)/a^(9/4)/c^(3/4)-5/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^(
1/2)+c^(1/2)*x^2))*2^(1/2)/a^(9/4)/c^(3/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{32a^{5/4}x^3}{(a+cx^4)^2} + \frac{40\sqrt[4]{a}x^3}{a+cx^4} - \frac{10\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{10\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{5\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{c^{3/4}} - \frac{5\sqrt{2}\,\mathrm{lo}}{}}{256a^{9/4}}$$

input
```
Integrate[x^2/(a + c*x^4)^3,x]
```

output
```
((32*a^(5/4)*x^3)/(a + c*x^4)^2 + (40*a^(1/4)*x^3)/(a + c*x^4) - (10*Sqrt[
2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (10*Sqrt[2]*ArcTan[1
 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (5*Sqrt[2]*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) - (5*Sqrt[2]*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(256*a^(9/4))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {819, 819, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5\int \frac{x^2}{(cx^4+a)^2}dx}{8a} + \frac{x^3}{8a\,(a+cx^4)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{5\left(\frac{\int \frac{x^2}{cx^4+a}dx}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 826

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 1476

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 1082

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 217

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 25

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^3}{4a(a+cx^4)}\right)}{8a}+\frac{x^3}{8a\left(a+cx^4\right)^2}$$

input
```
Int[x^2/(a + c*x^4)^3,x]
```

output
```
x^3/(8*a*(a + c*x^4)^2) + (5*(x^3/(4*a*(a + c*x^4)) + ((-(ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c]) - (-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
 Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)
*c^(1/4)))/(2*Sqrt[c]))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\frac{5c\,x^{7}}{32a^{2}}+\frac{9x^{3}}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{128a^{2}c}$	59
default	$\frac{x^{3}}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{5x^{3}}{32a\left(c\,x^{4}+a\right)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256ac\left(\frac{a}{c}\right)^{\frac{1}{4}}}}{a}$	146

input
```
int(x^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(5/32/a^2*c*x^7+9/32/a*x^3)/(c*x^4+a)^2+5/128/a^2/c*sum(1/_R*ln(x-_R),_R=R
ootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.56

$$\int \frac{x^2}{(a+cx^4)^3}\,dx$$
$$= \frac{20\,cx^7+36\,ax^3+5\,(a^2c^2x^8+2\,a^3cx^4+a^4)\left(-\frac{1}{a^9c^3}\right)^{\frac{1}{4}}\log\left(a^7c^2\left(-\frac{1}{a^9c^3}\right)^{\frac{3}{4}}+x\right)-5\,(i\,a^2c^2x^8+2i\,a^3cx^4+}{}$$

input
```
integrate(x^2/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(20*c*x^7 + 36*a*x^3 + 5*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^9*
c^3))^(1/4)*log(a^7*c^2*(-1/(a^9*c^3))^(3/4) + x) - 5*(I*a^2*c^2*x^8 + 2*I
*a^3*c*x^4 + I*a^4)*(-1/(a^9*c^3))^(1/4)*log(I*a^7*c^2*(-1/(a^9*c^3))^(3/4
) + x) - 5*(-I*a^2*c^2*x^8 - 2*I*a^3*c*x^4 - I*a^4)*(-1/(a^9*c^3))^(1/4)*l
og(-I*a^7*c^2*(-1/(a^9*c^3))^(3/4) + x) - 5*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a
^4)*(-1/(a^9*c^3))^(1/4)*log(-a^7*c^2*(-1/(a^9*c^3))^(3/4) + x))/(a^2*c^2*
x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.41

$$\int \frac{x^2}{(a+cx^4)^3}\,dx = \frac{9ax^3+5cx^7}{32a^4+64a^3cx^4+32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^9c^3+625, \left(t\mapsto t\log\left(\frac{2097152t^3a^7c^2}{125}+x\right)\right)\right)$$

input
```
integrate(x**2/(c*x**4+a)**3,x)
```

output
```
(9*a*x**3 + 5*c*x**7)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + Roo
tSum(268435456*_t**4*a**9*c**3 + 625, Lambda(_t, _t*log(2097152*_t**3*a**7
*c**2/125 + x)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{(a+cx^4)^3}\,dx = \frac{5\,cx^7+9\,ax^3}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}x^2 \right.}{}\right.}{256\,a^2}$$

input
```
integrate(x^2/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(5*c*x^7 + 9*a*x^3)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 5/256*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt
(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20

$$\int \frac{x^2}{(a+cx^4)^3}\,dx = \frac{5\,cx^7+9\,ax^3}{32\,(cx^4+a)^2a^2} + \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} + \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c^3} - \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3} + \frac{5\,\sqrt{2}(ac^3)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c^3}$$

input
```
integrate(x^2/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
1/32*(5*c*x^7 + 9*a*x^3)/((c*x^4 + a)^2*a^2) + 5/128*sqrt(2)*(a*c^3)^(3/4)
*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c^3) + 5
/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(a^3*c^3) - 5/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 + sqrt(2)*x*(
a/c)^(1/4) + sqrt(a/c))/(a^3*c^3) + 5/256*sqrt(2)*(a*c^3)^(3/4)*log(x^2 -
sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c^3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{(a+cx^4)^3}\,dx = \frac{\frac{9\,x^3}{32\,a}+\frac{5\,c\,x^7}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{5\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{9/4}\,c^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{9/4}\,c^{3/4}}$$

input
```
int(x^2/(a + c*x^4)^3,x)
```

output
```
((9*x^3)/(32*a) + (5*c*x^7)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (5*ata
n((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(9/4)*c^(3/4)) - (5*atanh((c^(1/4)*x)/
(-a)^(1/4)))/(64*(-a)^(9/4)*c^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.77

$$\int \frac{x^2}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^2/(c*x^4+a)^3,x)
```

output
```
( - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 20*c**(1/4)*a**(3/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c*x**4 - 10*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 10*c**(1/4)*a**(3/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*a**2 + 20*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 10*c**(1/4)*a**
(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*c**2*x**8 + 5*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a*
*(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 10*c**(1/4)*a**(3/4)*sqr
t(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4
 + 5*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a
) + sqrt(c)*x**2)*c**2*x**8 - 5*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**
(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 10*c**(1/4)*a**(3/4)*sqrt
(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4 - 5
*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqr
t(c)*x**2)*c**2*x**8 + 72*a**2*c*x**3 + 40*a*c**2*x**7)/(256*a**3*c*(a**2
+ 2*a*c*x**4 + c**2*x**8))
```

3.101 $\int \frac{1}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 168

$$\int \frac{1}{(a+cx^4)^3} dx = \frac{x}{8a(a+cx^4)^2} + \frac{7x}{32a^2(a+cx^4)} - \frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}} + \frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{c}}$$

output
```
1/8*x/a/(c*x^4+a)^2+7/32*x/a^2/(c*x^4+a)+21/128*arctan(-1+2^(1/2)*c^(1/4)*
x/a^(1/4))*2^(1/2)/a^(11/4)/c^(1/4)+21/128*arctan(1+2^(1/2)*c^(1/4)*x/a^(1
/4))*2^(1/2)/a^(11/4)/c^(1/4)+21/128*arctanh(2^(1/2)*a^(1/4)*c^(1/4)*x/(a^
(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(11/4)/c^(1/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$= \frac{\frac{32a^{7/4}x}{(a+cx^4)^2}+\frac{56a^{3/4}x}{a+cx^4}-\frac{42\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}}-\frac{21\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\cdots}{\cdots}}{256a^{11/4}}$$

input
```
Integrate[(a + c*x^4)^(-3),x]
```

output
```
((32*a^(7/4)*x)/(a + c*x^4)^2 + (56*a^(3/4)*x)/(a + c*x^4) - (42*Sqrt[2]*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/4))/(256*a^(11/4))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {749, 749, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$\Big\downarrow \text{749}$$
$$\frac{7\int \frac{1}{(cx^4+a)^2}dx}{8a}+\frac{x}{8a(a+cx^4)^2}$$
$$\Big\downarrow \text{749}$$

$$\frac{7\left(\frac{3\int \frac{1}{cx^4+a}dx}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\big\downarrow$ 755

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1476

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1082

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 1479}$$

$$7\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right) \over 8a$$ $+$

$$\frac{x}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 25

$$7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right) \over 8a$$ $+$

$$\frac{x}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 27

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{a}}\right)}{4a}+\frac{x}{4a(a+cx^4)}\right)}{8a}+\frac{x}{8a\left(a+cx^4\right)^2}$$

input
```
Int[(a + c*x^4)^(-3),x]
```

output
```
x/(8*a*(a + c*x^4)^2) + (7*(x/(4*a*(a + c*x^4)) + (3*((-(ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2]*a^(1/4)*c^(1/4)) +
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(2*Sqrt[2]*a^(1/4)*
c^(1/4)))/(2*Sqrt[a])))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\frac{7c\,x^{5}}{32a^{2}}+\frac{11x}{32a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{21\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}c+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{128a^{2}c}$	57
default	$\frac{x}{8a\left(c\,x^{4}+a\right)^{2}}+\frac{\frac{7x}{32a\left(c\,x^{4}+a\right)}+\frac{21\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256a^{2}}}{a}$	139

input
```
int(1/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(7/32/a^2*c*x^5+11/32*x/a)/(c*x^4+a)^2+21/128/a^2/c*sum(1/_R^3*ln(x-_R),_R
=RootOf(_Z^4*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.52

$$\int \frac{1}{(a+cx^4)^3}\,dx$$
$$= \frac{28\,cx^5 + 21\,(a^2c^2x^8 + 2\,a^3cx^4 + a^4)\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}c}\right)^{\frac{1}{4}} + x\right) - 21\,(-i\,a^2c^2x^8 - 2i\,a^3cx^4 - i\,a^4)(-}{}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
1/128*(28*c*x^5 + 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)
*log(a^3*(-1/(a^11*c))^(1/4) + x) - 21*(-I*a^2*c^2*x^8 - 2*I*a^3*c*x^4 - I
*a^4)*(-1/(a^11*c))^(1/4)*log(I*a^3*(-1/(a^11*c))^(1/4) + x) - 21*(I*a^2*c
^2*x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^11*c))^(1/4)*log(-I*a^3*(-1/(a^11*c
))^(1/4) + x) - 21*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^11*c))^(1/4)*l
og(-a^3*(-1/(a^11*c))^(1/4) + x) + 44*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^
4)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{11ax + 7cx^5}{32a^4 + 64a^3cx^4 + 32a^2c^2x^8} + \operatorname{RootSum}\left(268435456t^4a^{11}c + 194481, \left(t \mapsto t\log\left(\frac{128ta^3}{21} + x\right)\right)\right)$$

input
```
integrate(1/(c*x**4+a)**3,x)
```

output
```
(11*a*x + 7*c*x**5)/(32*a**4 + 64*a**3*c*x**4 + 32*a**2*c**2*x**8) + RootS
um(268435456*_t**4*a**11*c + 194481, Lambda(_t, _t*log(128*_t*a**3/21 + x)
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.26

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{7\,cx^5+11\,ax}{32\,(a^2c^2x^8+2\,a^3cx^4+a^4)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}x\right.}{}\right.}{256\,a^2}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/32*(7*c*x^5 + 11*a*x)/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 21/256*(2*sqrt
(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a
)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*
x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\,a^3c} + \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} - \frac{21\sqrt{2}(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\,a^3c} + \frac{7\,cx^5+11\,ax}{32\,(cx^4+a)^2a^2}$$

input
```
integrate(1/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(a^3*c) + 21/128*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^3*c) + 21/256*sqrt(2)*(a*c^3)^(
1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) - 21/256*sqrt(2)
*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^3*c) + 1/32
*(7*c*x^5 + 11*a*x)/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a+cx^4)^3}\,dx = \frac{\frac{11\,x}{32\,a}+\frac{7\,c\,x^5}{32\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{21\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}} - \frac{21\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{(-a)^{1/4}}\right)}{64\,(-a)^{11/4}\,c^{1/4}}$$

input
```
int(1/(a + c*x^4)^3,x)
```

output
```
((11*x)/(32*a) + (7*c*x^5)/(32*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (21*ata
n((c^(1/4)*x)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4)) - (21*atanh((c^(1/4)*x
)/(-a)^(1/4)))/(64*(-a)^(11/4)*c^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.82

$$\int \frac{1}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(c*x^4+a)^3,x)
```

output

```
( - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 84*c**(3/4)*a**(1/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c*x**4 - 42*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**8 + 42*c**(3/4)*a**(1/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*a**2 + 84*c**(3/4)*a**(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**4 + 42*c**(3/4)*a**
(1/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**
(1/4)*sqrt(2)))*c**2*x**8 - 21*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a
**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 - 42*c**(3/4)*a**(1/4)*sq
rt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**
4 - 21*c**(3/4)*a**(1/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt
(a) + sqrt(c)*x**2)*c**2*x**8 + 21*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*
a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2 + 42*c**(3/4)*a**(1/4)*s
qrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*c*x**4
+ 21*c**(3/4)*a**(1/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) +
 sqrt(c)*x**2)*c**2*x**8 + 88*a**2*c*x + 56*a*c**2*x**5)/(256*a**3*c*(a**2
 + 2*a*c*x**4 + c**2*x**8))
```

3.102 $\int \frac{1}{x^2(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 182

$$\int \frac{1}{x^2\left(a+cx^4\right)^3}\,dx = -\frac{1}{a^3x}-\frac{cx^3}{8a^2\left(a+cx^4\right)^2}-\frac{13cx^3}{32a^3\left(a+cx^4\right)}+\frac{45\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{13/4}}-\frac{45\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{64\sqrt{2}a^{13/4}}+\frac{45\sqrt[4]{c}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x}{\sqrt{a}+\sqrt{c}x^2}\right)}{64\sqrt{2}a^{13/4}}$$

output
```
-1/a^3/x-1/8*c*x^3/a^2/(c*x^4+a)^2-13/32*c*x^3/a^3/(c*x^4+a)-45/128*c^(1/4
)*arctan(-1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(13/4)-45/128*c^(1/4)*arc
tan(1+2^(1/2)*c^(1/4)*x/a^(1/4))*2^(1/2)/a^(13/4)+45/128*c^(1/4)*arctanh(2
^(1/2)*a^(1/4)*c^(1/4)*x/(a^(1/2)+c^(1/2)*x^2))*2^(1/2)/a^(13/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2\left(a+cx^4\right)^3}\,dx$$
$$=\frac{-\frac{256\sqrt[4]{a}}{x}-\frac{32a^{5/4}cx^3}{\left(a+cx^4\right)^2}-\frac{104\sqrt[4]{a}cx^3}{a+cx^4}+90\sqrt{2}\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-90\sqrt{2}\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)-45}{256a^{13/4}}$$

input
```
Integrate[1/(x^2*(a + c*x^4)^3),x]
```

output
```
((-256*a^(1/4))/x - (32*a^(5/4)*c*x^3)/(a + c*x^4)^2 - (104*a^(1/4)*c*x^3)
/(a + c*x^4) + 90*Sqrt[2]*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 90*Sqrt[2]*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*
c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 45*Sqrt[2
]*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(256*a^(
13/4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {819, 819, 847, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+cx^4\right)^3}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{9\int \frac{1}{x^2\left(cx^4+a\right)^2}dx}{8a}+\frac{1}{8ax\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{9\left(\frac{5\int \frac{1}{x^2\left(cx^4+a\right)}dx}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+\frac{1}{8ax\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 847

$$\frac{9\left(\frac{5\left(-\frac{c\int \frac{x^2}{cx^4+a}dx}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+\frac{1}{8ax\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 826

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\int \frac{\sqrt{c}x^2+\sqrt{a}}{cx^4+a}dx}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+\frac{1}{8ax\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\int \frac{1}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}+\frac{\int \frac{1}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+\frac{1}{8ax\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)^2-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)^2-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+$$

$$\frac{1}{8ax\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 217

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{cx^4+a}dx}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+\frac{1}{8ax\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1479

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{}{4ax\cdots}\right)}{8a}$$

$$\frac{1}{8ax\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 25$$

$$9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{\sqrt[4]{c}\left(x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}\right)}{\sqrt[4]{c}\left(x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}\right)}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+c}\right.}{8a}$$

$$\frac{1}{8ax\,(a+cx^4)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{c}x}{x^2-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt{2}\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{c}x+\sqrt[4]{a}}{x^2+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+\frac{\sqrt{a}}{\sqrt{c}}}dx}{2\sqrt[4]{a}\sqrt{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}+$$

$$\frac{1}{8ax\left(a+cx^4\right)^2}$$

$\downarrow$ 1103

$$\frac{9\left(\frac{5\left(-\frac{c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{1}{ax}\right)}{4a}+\frac{1}{4ax(a+cx^4)}\right)}{8a}$$

$$\frac{1}{8ax\left(a+cx^4\right)^2}$$

input
```
Int[1/(x^2*(a + c*x^4)^3),x]
```

output
```
1/(8*a*x*(a + c*x^4)^2) + (9*(1/(4*a*x*(a + c*x^4)) + (5*(-(1/(a*x)) - (c*
((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4))) + A
rcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[
c]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(Sqrt[2
]*a^(1/4)*c^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2
]/(2*Sqrt[2]*a^(1/4)*c^(1/4)))/(2*Sqrt[c])))/a))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{-\frac{45c^2x^8}{32a^3}-\frac{81c\,x^4}{32a^2}-\frac{1}{a}}{x(c\,x^4+a)^2}+\frac{45\left(\sum_{\_R=\mathrm{RootOf}\left(a^{13}\_Z^4+c\right)}\_R\ln\left(\left(5\_R^4a^{13}+4c\right)x+a^{10}\_R^3\right)\right)}{128}$	81
default	$-\frac{1}{a^3x}-\frac{c\left(\frac{\frac{13}{32}c\,x^7+\frac{17}{32}a\,x^3}{\left(c\,x^4+a\right)^2}+\frac{45\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{256c\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{a^3}$	141

input
```
int(1/x^2/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-45/32*c^2/a^3*x^8-81/32/a^2*c*x^4-1/a)/x/(c*x^4+a)^2+45/128*sum(_R*ln((5
*_R^4*a^13+4*c)*x+a^10*_R^3),_R=RootOf(_Z^4*a^13+c))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^2 (a+cx^4)^3} \, dx = -\frac{180\, c^2x^8 + 324\, acx^4 + 45\, (a^3c^2x^9 + 2\, a^4cx^5 + a^5x) \left(-\frac{c}{a^{13}}\right)^{\frac{1}{4}} \log\left(91125\, a^{10} \left(-\frac{c}{a^{13}}\right)^{\frac{3}{4}} + 91125\, cx\right) + 45\, (}{}$$

input
```
integrate(1/x^2/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(180*c^2*x^8 + 324*a*c*x^4 + 45*(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*x)
*(-c/a^13)^(1/4)*log(91125*a^10*(-c/a^13)^(3/4) + 91125*c*x) + 45*(-I*a^3*
c^2*x^9 - 2*I*a^4*c*x^5 - I*a^5*x)*(-c/a^13)^(1/4)*log(91125*I*a^10*(-c/a^
13)^(3/4) + 91125*c*x) + 45*(I*a^3*c^2*x^9 + 2*I*a^4*c*x^5 + I*a^5*x)*(-c/
a^13)^(1/4)*log(-91125*I*a^10*(-c/a^13)^(3/4) + 91125*c*x) - 45*(a^3*c^2*x
^9 + 2*a^4*c*x^5 + a^5*x)*(-c/a^13)^(1/4)*log(-91125*a^10*(-c/a^13)^(3/4)
+ 91125*c*x) + 128*a^2)/(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2 (a+cx^4)^3} \, dx = \frac{-32a^2 - 81acx^4 - 45c^2x^8}{32a^5x + 64a^4cx^5 + 32a^3c^2x^9} + \operatorname{RootSum}\left(268435456t^4a^{13} + 4100625c, \left(t \mapsto t \log\left(-\frac{2097152t^3a^{10}}{91125c} + x\right)\right)\right)$$

input
```
integrate(1/x**2/(c*x**4+a)**3,x)
```

output
```
(-32*a**2 - 81*a*c*x**4 - 45*c**2*x**8)/(32*a**5*x + 64*a**4*c*x**5 + 32*a
**3*c**2*x**9) + RootSum(268435456*_t**4*a**13 + 4100625*c, Lambda(_t, _t*
log(-2097152*_t**3*a**10/(91125*c) + x)))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^2(a+cx^4)^3}\,dx = -\frac{45\,c^2x^8+81\,acx^4+32\,a^2}{32\,(a^3c^2x^9+2\,a^4cx^5+a^5x)} - \frac{45\,c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{c}\right.}{}\right.}{256\,a^3}$$

input
```
integrate(1/x^2/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-1/32*(45*c^2*x^8 + 81*a*c*x^4 + 32*a^2)/(a^3*c^2*x^9 + 2*a^4*c*x^5 + a^5*
x) - 45/256*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)
*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)
*sqrt(c)))/(sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*log(sqrt(c)*x^2 + sqr
t(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)) + sqrt(2)*log(sqrt(c)*
x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(1/4)*c^(3/4)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2 (a+cx^4)^3} \, dx = -\frac{45 \sqrt{2}(ac^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\, a^4c^2} - \frac{45 \sqrt{2}(ac^3)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{128\, a^4c^2} + \frac{45 \sqrt{2}(ac^3)^{\frac{3}{4}} \log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\, a^4c^2} - \frac{45 \sqrt{2}(ac^3)^{\frac{3}{4}} \log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{256\, a^4c^2} - \frac{13\, c^2x^7+17\, acx^3}{32\, (cx^4+a)^2a^3} - \frac{1}{a^3x}$$

input
```
integrate(1/x^2/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-45/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4
))/(a/c)^(1/4))/(a^4*c^2) - 45/128*sqrt(2)*(a*c^3)^(3/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^4*c^2) + 45/256*sqrt(2)*(a*c
^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^4*c^2) - 45/256*
sqrt(2)*(a*c^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^4*c^
2) - 1/32*(13*c^2*x^7 + 17*a*c*x^3)/((c*x^4 + a)^2*a^3) - 1/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2 (a+cx^4)^3} \, dx = \frac{45\, (-c)^{1/4}\, \mathrm{atanh}\left(\frac{(-c)^{1/4}\, x}{a^{1/4}}\right)}{64\, a^{13/4}} - \frac{45\, (-c)^{1/4}\, \mathrm{atan}\left(\frac{(-c)^{1/4}\, x}{a^{1/4}}\right)}{64\, a^{13/4}} - \frac{\frac{1}{a}+\frac{81\, c\, x^4}{32\, a^2}+\frac{45\, c^2\, x^8}{32\, a^3}}{a^2\, x+2\, a\, c\, x^5+c^2\, x^9}$$

input
```
int(1/(x^2*(a + c*x^4)^3),x)
```

output
```
(45*(-c)^(1/4)*atanh(((-c)^(1/4)*x)/a^(1/4)))/(64*a^(13/4)) - (45*(-c)^(1/
4)*atan(((-c)^(1/4)*x)/a^(1/4)))/(64*a^(13/4)) - (1/a + (81*c*x^4)/(32*a^2
) + (45*c^2*x^8)/(32*a^3))/(a^2*x + c^2*x^9 + 2*a*c*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.66

$$\int \frac{1}{x^2 (a + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^2/(c*x^4+a)^3,x)
```

output
```
(90*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*
x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a**2*x + 180*c**(1/4)*a**(3/4)*sqrt(2)*ata
n((c**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a
*c*x**5 + 90*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*c**2*x**9 - 90*c**(1/4)*a**(3/4)*
sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*
sqrt(2)))*a**2*x - 180*c**(1/4)*a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*s
qrt(2) + 2*sqrt(c)*x)/(c**(1/4)*a**(1/4)*sqrt(2)))*a*c*x**5 - 90*c**(1/4)*
a**(3/4)*sqrt(2)*atan((c**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(c)*x)/(c**(1/4)*
a**(1/4)*sqrt(2)))*c**2*x**9 - 45*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4
)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2*x - 90*c**(1/4)*a**(3/
4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*
c*x**5 - 45*c**(1/4)*a**(3/4)*sqrt(2)*log( - c**(1/4)*a**(1/4)*sqrt(2)*x +
 sqrt(a) + sqrt(c)*x**2)*c**2*x**9 + 45*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(
1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a**2*x + 90*c**(1/4)*a**
(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sqrt(a) + sqrt(c)*x**2)*a*
c*x**5 + 45*c**(1/4)*a**(3/4)*sqrt(2)*log(c**(1/4)*a**(1/4)*sqrt(2)*x + sq
rt(a) + sqrt(c)*x**2)*c**2*x**9 - 256*a**3 - 648*a**2*c*x**4 - 360*a*c**2*
x**8)/(256*a**4*x*(a**2 + 2*a*c*x**4 + c**2*x**8))
```

3.103 $\int \frac{1}{2a+2b+x^4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{2a+2b+x^4} \, dx = -\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}$$

output
```
-1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)-1/4*arctanh(1
/2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int \frac{1}{2a+2b+x^4} \, dx = \frac{-2\arctan\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)+2\arctan\left(1+\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)-\log\left(2\sqrt{a+b}-2\sqrt[4]{2}\sqrt[4]{a+b}x+\sqrt{2}x^2\right)+\log\left(2 \ldots\right.}{8\sqrt[4]{2}(a+b)^{3/4}}$$

input
```
Integrate[(2*a + 2*b + x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a +
b)^(1/4)] - Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] +
 Log[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(8*2^(1/4)*
(a + b)^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2a+2b+x^4}\,dx$$

$$\Big\downarrow \text{756}$$

$$-\frac{\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx}{2\sqrt{2}\sqrt{-a-b}}-\frac{\int \frac{1}{x^2+\sqrt{2}\sqrt{-a-b}}dx}{2\sqrt{2}\sqrt{-a-b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx}{2\sqrt{2}\sqrt{-a-b}}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}$$

input
```
Int[(2*a + 2*b + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2^(3/4)*(-a - b)^(3/4)) - ArcTanh
[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(3/4)*(-a - b)^(3/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+2a+2b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4}$	27
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}+(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}{x^{2}-(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)\right)}{8(2a+2b)^{\frac{3}{4}}}$	113

input
```
int(1/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+2*a+2*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.46

$$\int \frac{1}{2a+2b+x^4}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}\log\left(2\left(\frac{1}{8}\right)^{\frac{1}{4}}(a+b)\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$
$$-\frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{8}\right)^{\frac{1}{4}}(a+b)\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$
$$-\frac{1}{4}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{8}\right)^{\frac{1}{4}}(i\,a+i\,b)\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$
$$+\frac{1}{4}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{8}\right)^{\frac{1}{4}}(-i\,a-i\,b)\left(-\frac{1}{a^3+3\,a^2b+3\,ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
1/4*(1/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(2*(1/8)^(1/
4)*(a + b)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) - 1/4*(1/8)^(1/
4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(a + b)*(
-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) - 1/4*I*(1/8)^(1/4)*(-1/(a^
3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(I*a + I*b)*(-1/(a^
3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) + 1/4*I*(1/8)^(1/4)*(-1/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(-I*a - I*b)*(-1/(a^3 + 3
*a^2*b + 3*a*b^2 + b^3))^(1/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{1}{2a+2b+x^4}\,dx$$
$$= \text{RootSum}\left(t^4\cdot\left(2048a^3+6144a^2b+6144ab^2+2048b^3\right)+1,\left(t\mapsto t\log\left(8ta+8tb+x\right)\right)\right)$$

input
```
integrate(1/(x**4+2*a+2*b),x)
```

output
```
RootSum(_t**4*(2048*a**3 + 6144*a**2*b + 6144*a*b**2 + 2048*b**3) + 1, Lam
bda(_t, _t*log(8*_t*a + 8*_t*b + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\int \frac{1}{2a+2b+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(2\,a+2\,b)^{\frac{3}{4}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(2\,a+2\,b)^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(2\,a+2\,b)^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(2\,a+2\,b)^{\frac{3}{4}}}$$

input
```
integrate(1/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*
b)^(1/4))/(2*a + 2*b)^(3/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(3/4) + 1/8*sqrt(2)*lo
g(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(3/4) -
 1/8*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a
 + 2*b)^(3/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(61) = 122$.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.77

$$\int \frac{1}{2a+2b+x^4}\,dx = \frac{(2\,a+2\,b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,\left(\sqrt{2}a+\sqrt{2}b\right)} + \frac{(2\,a+2\,b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,\left(\sqrt{2}a+\sqrt{2}b\right)} + \frac{(2\,a+2\,b)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,\left(\sqrt{2}a+\sqrt{2}b\right)} - \frac{(2\,a+2\,b)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,\left(\sqrt{2}a+\sqrt{2}b\right)}$$

input
```
integrate(1/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*(2*a + 2*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))
/(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(1/4)*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)
*a + sqrt(2)*b) + 1/8*(2*a + 2*b)^(1/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4
)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) - 1/8*(2*a + 2*b)^(1/4)*log
(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)
*b)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.53

$$\int \frac{1}{2a+2b+x^4}\,dx = \frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4\,(-a-b)^{3/4}} + \frac{2^{1/4}\,\mathrm{atanh}\left(\frac{2^{1/4}\,x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4\,(-a-b)^{3/4}}$$

input
```
int(1/(2*a + 2*b + x^4),x)
```

output
```
(2^(1/4)*atan((2^(1/4)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a
 - b)^(3/2))*(- a - b)^(3/4))))/(4*(- a - b)^(3/4)) + (2^(1/4)*atanh((2^(1
/4)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a - b)^(3/2))*(- a -
 b)^(3/4))))/(4*(- a - b)^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{1}{2a+2b+x^4}\,dx = \frac{(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}\left(-2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}-2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}+2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\right.}{16a+16b}$$

input
```
int(1/(x^4+2*a+2*b),x)
```

output
```
((a + b)**(1/4)*sqrt(2)*2**(1/4)*( - 2*atan(((a + b)**(1/4)*sqrt(2)*2**(1/
4) - 2*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) + 2*atan(((a + b)**(1/4)*sqrt
(2)*2**(1/4) + 2*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) - log( - (a + b)**(
1/4)*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2) + log((a + b)**(1/4)
*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2)))/(16*(a + b))
```

3.104 $\int \frac{1}{2(a+b)+x^4} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{1}{2(a+b)+x^4} \, dx = -\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}$$

output
```
-1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)-1/4*arctanh(1
/2*x*2^(3/4)/(-a-b)^(1/4))*2^(1/4)/(-a-b)^(3/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int \frac{1}{2(a+b)+x^4} \, dx$$
$$= \frac{-2\arctan\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)+2\arctan\left(1+\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)-\log\left(2\sqrt{a+b}-2\sqrt[4]{2}\sqrt[4]{a+b}x+\sqrt{2}x^2\right)+\log\left(2\right.}{8\sqrt[4]{2}(a+b)^{3/4}}$$

input
```
Integrate[(2*(a + b) + x^4)^(-1),x]
```

output
```
(-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a +
b)^(1/4)] - Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] +
 Log[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(8*2^(1/4)*
(a + b)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2(a+b)+x^4} \, dx$$

$$\Big\downarrow \ 756$$

$$-\frac{\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx}{2\sqrt{2}\sqrt{-a-b}}-\frac{\int \frac{1}{x^2+\sqrt{2}\sqrt{-a-b}}dx}{2\sqrt{2}\sqrt{-a-b}}$$

$$\Big\downarrow \ 216$$

$$-\frac{\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx}{2\sqrt{2}\sqrt{-a-b}}-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\ 2^{3/4}(-a-b)^{3/4}}$$

input
```
Int[(2*(a + b) + x^4)^(-1),x]
```

output
```
-1/2*ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2^(3/4)*(-a - b)^(3/4)) - ArcTanh
[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(3/4)*(-a - b)^(3/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+2a+2b\right)}\frac{\ln\left(x-\_R\right)}{\_R^{3}}\right)}{4}$	27
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}+(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}{x^{2}-(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)\right)}{8(2a+2b)^{\frac{3}{4}}}$	113

input
```
int(1/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R^3*ln(x-_R),_R=RootOf(_Z^4+2*a+2*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.46

$$\int \frac{1}{2(a+b)+x^4}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}\log\left(2\left(\frac{1}{8}\right)^{\frac{1}{4}}(a+b)\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$
$$-\frac{1}{4}\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{8}\right)^{\frac{1}{4}}(a+b)\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$
$$-\frac{1}{4}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{8}\right)^{\frac{1}{4}}(ia+ib)\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$
$$+\frac{1}{4}i\left(\frac{1}{8}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{8}\right)^{\frac{1}{4}}(-ia-ib)\left(-\frac{1}{a^3+3a^2b+3ab^2+b^3}\right)^{\frac{1}{4}}+x\right)$$

input
```
integrate(1/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
1/4*(1/8)^(1/4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(2*(1/8)^(1/
4)*(a + b)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) - 1/4*(1/8)^(1/
4)*(-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(a + b)*(
-1/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) - 1/4*I*(1/8)^(1/4)*(-1/(a^
3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(I*a + I*b)*(-1/(a^
3 + 3*a^2*b + 3*a*b^2 + b^3))^(1/4) + x) + 1/4*I*(1/8)^(1/4)*(-1/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3))^(1/4)*log(-2*(1/8)^(1/4)*(-I*a - I*b)*(-1/(a^3 + 3
*a^2*b + 3*a*b^2 + b^3))^(1/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{1}{2(a+b)+x^4}\,dx$$
$$=\text{RootSum}\left(t^4\cdot\left(2048a^3+6144a^2b+6144ab^2+2048b^3\right)+1,\left(t\mapsto t\log\left(8ta+8tb+x\right)\right)\right)$$

input
```
integrate(1/(x**4+2*a+2*b),x)
```

output
```
RootSum(_t**4*(2048*a**3 + 6144*a**2*b + 6144*a*b**2 + 2048*b**3) + 1, Lam
bda(_t, _t*log(8*_t*a + 8*_t*b + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\int \frac{1}{2(a+b)+x^4}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(2\,a+2\,b)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(2\,a+2\,b)^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(2\,a+2\,b)^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(2\,a+2\,b)^{\frac{3}{4}}}$$

input
```
integrate(1/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*
b)^(1/4))/(2*a + 2*b)^(3/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(3/4) + 1/8*sqrt(2)*lo
g(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(3/4) -
 1/8*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a
 + 2*b)^(3/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.77

$$\int \frac{1}{2(a+b)+x^4}\,dx = \frac{(2\,a+2\,b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)} + \frac{(2\,a+2\,b)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)} + \frac{(2\,a+2\,b)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\left(\sqrt{2}a+\sqrt{2}b\right)} - \frac{(2\,a+2\,b)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\left(\sqrt{2}a+\sqrt{2}b\right)}$$

input
```
integrate(1/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*(2*a + 2*b)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))
/(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(1/4)*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)
*a + sqrt(2)*b) + 1/8*(2*a + 2*b)^(1/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4
)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) - 1/8*(2*a + 2*b)^(1/4)*log
(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)
*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.53

$$\int \frac{1}{2(a+b)+x^4}\,dx = \frac{2^{1/4}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4\,(-a-b)^{3/4}} + \frac{2^{1/4}\,\mathrm{atanh}\left(\frac{2^{1/4}\,x}{\left(\frac{\sqrt{2}\,a}{(-a-b)^{3/2}}+\frac{\sqrt{2}\,b}{(-a-b)^{3/2}}\right)(-a-b)^{3/4}}\right)}{4\,(-a-b)^{3/4}}$$

input
```
int(1/(2*a + 2*b + x^4),x)
```

output
```
(2^(1/4)*atan((2^(1/4)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a
 - b)^(3/2))*(- a - b)^(3/4))))/(4*(- a - b)^(3/4)) + (2^(1/4)*atanh((2^(1
/4)*x)/(((2^(1/2)*a)/(- a - b)^(3/2) + (2^(1/2)*b)/(- a - b)^(3/2))*(- a -
 b)^(3/4))))/(4*(- a - b)^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{1}{2(a+b)+x^4}\,dx = \frac{(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}\left(-2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}-2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}+2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\right.\right.}{16a+16b}$$

input
```
int(1/(x^4+2*a+2*b),x)
```

output
```
((a + b)**(1/4)*sqrt(2)*2**(1/4)*( - 2*atan(((a + b)**(1/4)*sqrt(2)*2**(1/
4) - 2*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) + 2*atan(((a + b)**(1/4)*sqrt
(2)*2**(1/4) + 2*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) - log( - (a + b)**(
1/4)*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2) + log((a + b)**(1/4)
*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2)))/(16*(a + b))
```

3.105 $\int \frac{x}{2a+2b+x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \frac{x}{2a+2b+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

output
```
1/4*arctan(1/2*x^2*2^(1/2)/(a+b)^(1/2))*2^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{2a+2b+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

input
```
Integrate[x/(2*a + 2*b + x^4),x]
```

output
```
ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{2a+2b+x^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^4+2(a+b)}dx^2$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

input
```
Int[x/(2*a + 2*b + x^4),x]
```

output
```
ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^{2}}{\sqrt{2a+2b}}\right)}{2\sqrt{2a+2b}}$	26
risch	$-\frac{\ln\left(x^{2}\sqrt{-2a-2b}-2a-2b\right)}{4\sqrt{-2a-2b}}+\frac{\ln\left(x^{2}\sqrt{-2a-2b}+2a+2b\right)}{4\sqrt{-2a-2b}}$	66

input
```
int(x/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/2/(2*a+2*b)^(1/2)*arctan(x^2/(2*a+2*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.58

$$\int \frac{x}{2a+2b+x^4}\,dx = \left[-\frac{\sqrt{-2\,a-2\,b}\log\left(\frac{x^4-2\,\sqrt{-2\,a-2\,b}x^2-2\,a-2\,b}{x^4+2\,a+2\,b}\right)}{8\,(a+b)},\right. \\ \left. -\frac{\sqrt{2\,a+2\,b}\arctan\left(\frac{\sqrt{2\,a+2\,b}}{x^2}\right)}{4\,(a+b)}\right]$$

input
```
integrate(x/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-2*a - 2*b)*log((x^4 - 2*sqrt(-2*a - 2*b)*x^2 - 2*a - 2*b)/(x^4
 + 2*a + 2*b))/(a + b), -1/4*sqrt(2*a + 2*b)*arctan(sqrt(2*a + 2*b)/x^2)/(
a + b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(31) = 62$.

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.33

$$\int \frac{x}{2a+2b+x^4}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{a+b}}\log\left(-\sqrt{2}a\sqrt{-\frac{1}{a+b}}-\sqrt{2}b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8} + \frac{\sqrt{2}\sqrt{-\frac{1}{a+b}}\log\left(\sqrt{2}a\sqrt{-\frac{1}{a+b}}+\sqrt{2}b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8}$$

input
```
integrate(x/(x**4+2*a+2*b),x)
```

output
```
-sqrt(2)*sqrt(-1/(a + b))*log(-sqrt(2)*a*sqrt(-1/(a + b)) - sqrt(2)*b*sqrt
(-1/(a + b)) + x**2)/8 + sqrt(2)*sqrt(-1/(a + b))*log(sqrt(2)*a*sqrt(-1/(a
 + b)) + sqrt(2)*b*sqrt(-1/(a + b)) + x**2)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{2a+2b+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{\sqrt{2\,a+2\,b}}\right)}{2\sqrt{2\,a+2\,b}}$$

input
```
integrate(x/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/2*arctan(x^2/sqrt(2*a + 2*b))/sqrt(2*a + 2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{x}{2a+2b+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}x^2}{2\sqrt{a+b}}\right)}{4\sqrt{a+b}}$$

input
```
integrate(x/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*x^2/sqrt(a + b))/sqrt(a + b)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x}{2a+2b+x^4}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2\,\sqrt{a+b}}{2\,a+2\,b}\right)}{4\sqrt{a+b}}$$

input
```
int(x/(2*a + 2*b + x^4),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x^2*(a + b)^(1/2))/(2*a + 2*b)))/(4*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \frac{x}{2a+2b+x^4}\,dx = -\frac{\sqrt{a+b}\,\sqrt{2}\left(atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}-2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}+2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)\right)}{4a+4b}$$

input
```
int(x/(x^4+2*a+2*b),x)
```

output
```
( - sqrt(a + b)*sqrt(2)*(atan(((a + b)**(1/4)*sqrt(2)*2**(1/4) - 2*x)/((a
+ b)**(1/4)*sqrt(2)*2**(1/4))) + atan(((a + b)**(1/4)*sqrt(2)*2**(1/4) + 2
*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4)))))/(4*(a + b))
```

3.106 $\int \frac{x}{2(a+b)+x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{x}{2(a+b)+x^4}\,dx=\frac{\arctan\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

output
```
1/4*arctan(1/2*x^2*2^(1/2)/(a+b)^(1/2))*2^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x}{2(a+b)+x^4}\,dx=\frac{\arctan\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

input
```
Integrate[x/(2*(a + b) + x^4),x]
```

output
```
ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{2(a+b)+x^4} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^4+2(a+b)}dx^2$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{x^2}{\sqrt{2}\sqrt{a+b}}\right)}{2\sqrt{2}\sqrt{a+b}}$$

input
```
Int[x/(2*(a + b) + x^4),x]
```

output
```
ArcTan[x^2/(Sqrt[2]*Sqrt[a + b])]/(2*Sqrt[2]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{\arctan\left(\frac{x^2}{\sqrt{2a+2b}}\right)}{2\sqrt{2a+2b}}$	26
risch	$-\frac{\ln\left(x^2\sqrt{-2a-2b}-2a-2b\right)}{4\sqrt{-2a-2b}}+\frac{\ln\left(x^2\sqrt{-2a-2b}+2a+2b\right)}{4\sqrt{-2a-2b}}$	66

input
```
int(x/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/2/(2*a+2*b)^(1/2)*arctan(x^2/(2*a+2*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.58

$$\int \frac{x}{2(a+b)+x^4}\,dx = \left[-\frac{\sqrt{-2\,a-2\,b}\log\left(\frac{x^4-2\,\sqrt{-2\,a-2\,b}x^2-2\,a-2\,b}{x^4+2\,a+2\,b}\right)}{8\,(a+b)}, -\frac{\sqrt{2\,a+2\,b}\arctan\left(\frac{\sqrt{2\,a+2\,b}}{x^2}\right)}{4\,(a+b)}\right]$$

input
```
integrate(x/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-2*a - 2*b)*log((x^4 - 2*sqrt(-2*a - 2*b)*x^2 - 2*a - 2*b)/(x^4
 + 2*a + 2*b))/(a + b), -1/4*sqrt(2*a + 2*b)*arctan(sqrt(2*a + 2*b)/x^2)/(
a + b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(31) = 62$.

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.33

$$\int \frac{x}{2(a+b)+x^4}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{1}{a+b}}\log\left(-\sqrt{2}a\sqrt{-\frac{1}{a+b}}-\sqrt{2}b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8} + \frac{\sqrt{2}\sqrt{-\frac{1}{a+b}}\log\left(\sqrt{2}a\sqrt{-\frac{1}{a+b}}+\sqrt{2}b\sqrt{-\frac{1}{a+b}}+x^2\right)}{8}$$

input
```
integrate(x/(x**4+2*a+2*b),x)
```

output
```
-sqrt(2)*sqrt(-1/(a + b))*log(-sqrt(2)*a*sqrt(-1/(a + b)) - sqrt(2)*b*sqrt
(-1/(a + b)) + x**2)/8 + sqrt(2)*sqrt(-1/(a + b))*log(sqrt(2)*a*sqrt(-1/(a
 + b)) + sqrt(2)*b*sqrt(-1/(a + b)) + x**2)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x}{2(a+b)+x^4}\,dx = \frac{\arctan\left(\frac{x^2}{\sqrt{2\,a+2\,b}}\right)}{2\,\sqrt{2\,a+2\,b}}$$

input
```
integrate(x/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/2*arctan(x^2/sqrt(2*a + 2*b))/sqrt(2*a + 2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{x}{2(a+b)+x^4}\,dx=\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}x^2}{2\sqrt{a+b}}\right)}{4\sqrt{a+b}}$$

input
```
integrate(x/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*x^2/sqrt(a + b))/sqrt(a + b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x}{2(a+b)+x^4}\,dx=\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2\,\sqrt{a+b}}{2\,a+2\,b}\right)}{4\,\sqrt{a+b}}$$

input
```
int(x/(2*a + 2*b + x^4),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x^2*(a + b)^(1/2))/(2*a + 2*b)))/(4*(a + b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \frac{x}{2(a+b)+x^4}\,dx=-\frac{\sqrt{a+b}\,\sqrt{2}\left(atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}-2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}+2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)\right)}{4a+4b}$$

input
```
int(x/(x^4+2*a+2*b),x)
```

output
```
( - sqrt(a + b)*sqrt(2)*(atan(((a + b)**(1/4)*sqrt(2)*2**(1/4) - 2*x)/((a
+ b)**(1/4)*sqrt(2)*2**(1/4))) + atan(((a + b)**(1/4)*sqrt(2)*2**(1/4) + 2
*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4)))))/(4*(a + b))
```

3.107 $\int \frac{x^2}{2a+2b+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{x^2}{2a+2b+x^4}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}$$

output
```
1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)-1/4*arctanh(1/
2*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{2a+2b+x^4}\,dx =\frac{-2\arctan\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)+2\arctan\left(1+\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)+\log\left(2\sqrt{a+b}-2\sqrt[4]{2}\sqrt[4]{a+b}x+\sqrt{2}x^2\right)-\log\left(2\right.}{4\,2^{3/4}\sqrt[4]{a+b}}$$

input
```
Integrate[x^2/(2*a + 2*b + x^4),x]
```

output
```
(-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a +
b)^(1/4)] + Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] -
 Log[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*
(a + b)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2a+2b+x^4}\,dx$$

$$\Big\downarrow \ 827$$

$$\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}\sqrt{-a-b}}dx-\frac{1}{2}\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}-\frac{1}{2}\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx$$

$$\Big\downarrow \ 219$$

$$\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}$$

input
```
Int[x^2/(2*a + 2*b + x^4),x]
```

output
```
ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4)) - ArcTanh[x/
(2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}+2a+2b\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$	27
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}{x^{2}+(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)\right)}{8(2a+2b)^{\frac{1}{4}}}$	113

input
```
int(x^2/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+2*a+2*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

$$\int \frac{x^2}{2a+2b+x^4}\,dx = \frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(2\left(\frac{1}{2}\right)^{\frac{3}{4}}(a+b)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{2}\right)^{\frac{3}{4}}(a+b)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right) + \frac{1}{4}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{2}\right)^{\frac{3}{4}}(ia+ib)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{2}\right)^{\frac{3}{4}}(-ia-ib)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right)$$

input
```
integrate(x^2/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(2*(1/2)^(3/4)*(a + b)*(-1/(a + b))^
(3/4) + x) - 1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(-2*(1/2)^(3/4)*(a + b)
*(-1/(a + b))^(3/4) + x) + 1/4*I*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(-2*(1/
2)^(3/4)*(I*a + I*b)*(-1/(a + b))^(3/4) + x) - 1/4*I*(1/2)^(1/4)*(-1/(a +
b))^(1/4)*log(-2*(1/2)^(3/4)*(-I*a - I*b)*(-1/(a + b))^(3/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{2a+2b+x^4}\,dx = \operatorname{RootSum}\left(t^4\cdot(512a+512b)+1,\left(t\mapsto t\log\left(128t^3a+128t^3b+x\right)\right)\right)$$

input
```
integrate(x**2/(x**4+2*a+2*b),x)
```

output
```
RootSum(_t**4*(512*a + 512*b) + 1, Lambda(_t, _t*log(128*_t**3*a + 128*_t*
*3*b + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\int \frac{x^2}{2a+2b+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{1}{4}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(2a+2b)^{\frac{1}{4}}\right)}{2(2a+2b)^{\frac{1}{4}}}\right)}{4(2a+2b)^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2a+2b)^{\frac{1}{4}}x+\sqrt{2a+2b}\right)}{8(2a+2b)^{\frac{1}{4}}} + \frac{\sqrt{2}\log\left(x^2-\sqrt{2}(2a+2b)^{\frac{1}{4}}x+\sqrt{2a+2b}\right)}{8(2a+2b)^{\frac{1}{4}}}$$

input
```
integrate(x^2/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*
b)^(1/4))/(2*a + 2*b)^(1/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4) - 1/8*sqrt(2)*lo
g(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(1/4) +
 1/8*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a
 + 2*b)^(1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(61) = 122$.

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.77

$$\int \frac{x^2}{2a+2b+x^4}\,dx = \frac{(2\,a+2\,b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(\sqrt{2}a+\sqrt{2}b)} + \frac{(2\,a+2\,b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(\sqrt{2}a+\sqrt{2}b)} - \frac{(2\,a+2\,b)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(\sqrt{2}a+\sqrt{2}b)} + \frac{(2\,a+2\,b)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(\sqrt{2}a+\sqrt{2}b)}$$

input
```
integrate(x^2/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*(2*a + 2*b)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))
/(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(3/4)*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)
*a + sqrt(2)*b) - 1/8*(2*a + 2*b)^(3/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4
)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) + 1/8*(2*a + 2*b)^(3/4)*log
(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)
*b)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{2a+2b+x^4}\,dx = \frac{8^{1/4}\,\mathrm{atan}\left(\frac{8^{1/4}\,x}{2\,(-a-b)^{1/4}}\right) - 8^{1/4}\,\mathrm{atanh}\left(\frac{8^{1/4}\,x}{2\,(-a-b)^{1/4}}\right)}{4\,(-a-b)^{1/4}}$$

input
```
int(x^2/(2*a + 2*b + x^4),x)
```

output
```
(8^(1/4)*atan((8^(1/4)*x)/(2*(- a - b)^(1/4))) - 8^(1/4)*atanh((8^(1/4)*x)
/(2*(- a - b)^(1/4))))/(4*(- a - b)^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{2a+2b+x^4}\,dx = \frac{(a+b)^{\frac{3}{4}}\,2^{\frac{1}{4}}\left(-2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}-2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}+2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{a}\right.\right.}{8a+8b}$$

input
```
int(x^2/(x^4+2*a+2*b),x)
```

output
```
((a + b)**(3/4)*2**(1/4)*( - 2*atan(((a + b)**(1/4)*sqrt(2)*2**(1/4) - 2*x
)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) + 2*atan(((a + b)**(1/4)*sqrt(2)*2**(
1/4) + 2*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) + log( - (a + b)**(1/4)*sqr
t(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2) - log((a + b)**(1/4)*sqrt(2)
*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2)))/(8*(a + b))
```

3.108 $\int \frac{x^2}{2(a+b)+x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{x^2}{2(a+b)+x^4}\,dx=\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}$$

output
```
1/4*arctan(1/2*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)-1/4*arctanh(1/
2*x*2^(3/4)/(-a-b)^(1/4))*2^(3/4)/(-a-b)^(1/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{2(a+b)+x^4}\,dx$$
$$=\frac{-2\arctan\left(1-\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)+2\arctan\left(1+\frac{\sqrt[4]{2}x}{\sqrt[4]{a+b}}\right)+\log\left(2\sqrt{a+b}-2\sqrt[4]{2}\sqrt[4]{a+b}x+\sqrt{2}x^2\right)-\log\left(2\right.}{4\,2^{3/4}\sqrt[4]{a+b}}$$

input
```
Integrate[x^2/(2*(a + b) + x^4),x]
```

output
```
(-2*ArcTan[1 - (2^(1/4)*x)/(a + b)^(1/4)] + 2*ArcTan[1 + (2^(1/4)*x)/(a +
b)^(1/4)] + Log[2*Sqrt[a + b] - 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2] -
 Log[2*Sqrt[a + b] + 2*2^(1/4)*(a + b)^(1/4)*x + Sqrt[2]*x^2])/(4*2^(3/4)*
(a + b)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{827, 216, 219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2(a+b)+x^4} \, dx$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}\sqrt{-a-b}}dx-\frac{1}{2}\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}-\frac{1}{2}\int \frac{1}{\sqrt{2}\sqrt{-a-b}-x^2}dx$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{-a-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{-a-b}}$$

input
```
Int[x^2/(2*(a + b) + x^4),x]
```

output
```
ArcTan[x/(2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4)) - ArcTanh[x/
(2^(1/4)*(-a - b)^(1/4))]/(2*2^(1/4)*(-a - b)^(1/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\_Z^{4}+2a+2b\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$	27
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}{x^{2}+(2a+2b)^{\frac{1}{4}}x\sqrt{2}+\sqrt{2a+2b}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(2a+2b)^{\frac{1}{4}}}-1\right)\right)}{8(2a+2b)^{\frac{1}{4}}}$	113

input
```
int(x^2/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+2*a+2*b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

$$\int \frac{x^2}{2(a+b)+x^4}\,dx = \frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(2\left(\frac{1}{2}\right)^{\frac{3}{4}}(a+b)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{2}\right)^{\frac{3}{4}}(a+b)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right) + \frac{1}{4}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{2}\right)^{\frac{3}{4}}(i\,a+i\,b)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right) - \frac{1}{4}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{a+b}\right)^{\frac{1}{4}}\log\left(-2\left(\frac{1}{2}\right)^{\frac{3}{4}}(-i\,a-i\,b)\left(-\frac{1}{a+b}\right)^{\frac{3}{4}}+x\right)$$

input
```
integrate(x^2/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(2*(1/2)^(3/4)*(a + b)*(-1/(a + b))^
(3/4) + x) - 1/4*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(-2*(1/2)^(3/4)*(a + b)
*(-1/(a + b))^(3/4) + x) + 1/4*I*(1/2)^(1/4)*(-1/(a + b))^(1/4)*log(-2*(1/
2)^(3/4)*(I*a + I*b)*(-1/(a + b))^(3/4) + x) - 1/4*I*(1/2)^(1/4)*(-1/(a +
b))^(1/4)*log(-2*(1/2)^(3/4)*(-I*a - I*b)*(-1/(a + b))^(3/4) + x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{2(a+b)+x^4}\,dx = \operatorname{RootSum}\left(t^4\cdot(512a+512b)+1,\left(t\mapsto t\log\left(128t^3a+128t^3b+x\right)\right)\right)$$

input
```
integrate(x**2/(x**4+2*a+2*b),x)
```

output
```
RootSum(_t**4*(512*a + 512*b) + 1, Lambda(_t, _t*log(128*_t**3*a + 128*_t*
*3*b + x)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(61) = 122.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\int \frac{x^2}{2(a+b)+x^4}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(2\,a+2\,b)^{\frac{1}{4}}} + \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\,(2\,a+2\,b)^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(2\,a+2\,b)^{\frac{1}{4}}} + \frac{\sqrt{2}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\,(2\,a+2\,b)^{\frac{1}{4}}}$$

input
```
integrate(x^2/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*
b)^(1/4))/(2*a + 2*b)^(1/4) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4) - 1/8*sqrt(2)*lo
g(x^2 + sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a + 2*b)^(1/4) +
 1/8*sqrt(2)*log(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(2*a
 + 2*b)^(1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(61) = 122$.

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.77

$$\int \frac{x^2}{2(a+b)+x^4}\,dx = \frac{(2\,a+2\,b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)} + \frac{(2\,a+2\,b)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}\right)}{2\,(2\,a+2\,b)^{\frac{1}{4}}}\right)}{4\left(\sqrt{2}a+\sqrt{2}b\right)} - \frac{(2\,a+2\,b)^{\frac{3}{4}}\log\left(x^2+\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\left(\sqrt{2}a+\sqrt{2}b\right)} + \frac{(2\,a+2\,b)^{\frac{3}{4}}\log\left(x^2-\sqrt{2}(2\,a+2\,b)^{\frac{1}{4}}x+\sqrt{2\,a+2\,b}\right)}{8\left(\sqrt{2}a+\sqrt{2}b\right)}$$

input
```
integrate(x^2/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*(2*a + 2*b)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(2*a + 2*b)^(1/4))
/(2*a + 2*b)^(1/4))/(sqrt(2)*a + sqrt(2)*b) + 1/4*(2*a + 2*b)^(3/4)*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(2*a + 2*b)^(1/4))/(2*a + 2*b)^(1/4))/(sqrt(2)
*a + sqrt(2)*b) - 1/8*(2*a + 2*b)^(3/4)*log(x^2 + sqrt(2)*(2*a + 2*b)^(1/4
)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)*b) + 1/8*(2*a + 2*b)^(3/4)*log
(x^2 - sqrt(2)*(2*a + 2*b)^(1/4)*x + sqrt(2*a + 2*b))/(sqrt(2)*a + sqrt(2)
*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{2(a+b)+x^4}\,dx = \frac{8^{1/4}\,\mathrm{atan}\left(\frac{8^{1/4}\,x}{2\,(-a-b)^{1/4}}\right)-8^{1/4}\,\mathrm{atanh}\left(\frac{8^{1/4}\,x}{2\,(-a-b)^{1/4}}\right)}{4\,(-a-b)^{1/4}}$$

input
```
int(x^2/(2*a + 2*b + x^4),x)
```

output
```
(8^(1/4)*atan((8^(1/4)*x)/(2*(- a - b)^(1/4))) - 8^(1/4)*atanh((8^(1/4)*x)
/(2*(- a - b)^(1/4))))/(4*(- a - b)^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{2(a+b)+x^4}\,dx$$
$$= \frac{(a+b)^{\frac{3}{4}}\,2^{\frac{1}{4}}\left(-2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}-2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}+2x\right)2^{\frac{3}{4}}}{2(a+b)^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{a}\right.\right.}{8a+8b}$$

input
```
int(x^2/(x^4+2*a+2*b),x)
```

output
```
((a + b)**(3/4)*2**(1/4)*( - 2*atan(((a + b)**(1/4)*sqrt(2)*2**(1/4) - 2*x
)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) + 2*atan(((a + b)**(1/4)*sqrt(2)*2**(
1/4) + 2*x)/((a + b)**(1/4)*sqrt(2)*2**(1/4))) + log( - (a + b)**(1/4)*sqr
t(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2) - log((a + b)**(1/4)*sqrt(2)
*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2)))/(8*(a + b))
```

3.109 $\int \frac{x^3}{2a+2b+x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{x^3}{2a+2b+x^4}\,dx=\frac{1}{4}\log\left(2(a+b)+x^4\right)$$

output
```
1/4*ln(x^4+2*a+2*b)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{2a+2b+x^4}\,dx=\frac{1}{4}\log\left(2a+2b+x^4\right)$$

input
```
Integrate[x^3/(2*a + 2*b + x^4),x]
```

output
```
Log[2*a + 2*b + x^4]/4
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{2a+2b+x^4}\,dx$$

$$\Big\downarrow \text{792}$$

$$\frac{1}{4}\log\left(2(a+b)+x^4\right)$$

input
```
Int[x^3/(2*a + 2*b + x^4),x]
```

output
```
Log[2*(a + b) + x^4]/4
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln(x^4+2a+2b)}{4}$	14
default	$\frac{\ln(x^4+2a+2b)}{4}$	14
norman	$\frac{\ln(x^4+2a+2b)}{4}$	14
risch	$\frac{\ln(x^4+2a+2b)}{4}$	14
parallelrisch	$\frac{\ln(x^4+2a+2b)}{4}$	14

input
```
int(x^3/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+2*a+2*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{1}{4}\log\left(x^4+2\,a+2\,b\right)$$

input
```
integrate(x^3/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
1/4*log(x^4 + 2*a + 2*b)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{\log\left(2a+2b+x^4\right)}{4}$$

input
```
integrate(x**3/(x**4+2*a+2*b),x)
```

output
```
log(2*a + 2*b + x**4)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{1}{4}\,\log\left(x^4+2\,a+2\,b\right)$$

input
```
integrate(x^3/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/4*log(x^4 + 2*a + 2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{1}{4}\,\log\left(\left|x^4+2\,a+2\,b\right|\right)$$

input
```
integrate(x^3/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*log(abs(x^4 + 2*a + 2*b))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{\ln\left(x^4+2\,a+2\,b\right)}{4}$$

input
```
int(x^3/(2*a + 2*b + x^4),x)
```

output
```
log(2*a + 2*b + x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.86

$$\int \frac{x^3}{2a+2b+x^4}\,dx = \frac{\log\left(-(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{a+b}\,\sqrt{2}+x^2\right)}{4} + \frac{\log\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{a+b}\,\sqrt{2}+x^2\right)}{4}$$

input
```
int(x^3/(x^4+2*a+2*b),x)
```

output
```
(log( - (a + b)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2) +
log((a + b)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2))/4
```

3.110 $\int \frac{x^3}{2(a+b)+x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{x^3}{2(a+b)+x^4} \, dx = \frac{1}{4} \log \left(2(a+b)+x^4\right)$$

output
```
1/4*ln(x^4+2*a+2*b)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{x^3}{2(a+b)+x^4} \, dx = \frac{1}{4} \log \left(2a+2b+x^4\right)$$

input
```
Integrate[x^3/(2*(a + b) + x^4),x]
```

output
```
Log[2*a + 2*b + x^4]/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{2(a+b)+x^4} \, dx$$

$$\Big\downarrow \text{792}$$

$$\frac{1}{4} \log \left(2(a+b)+x^4\right)$$

input
```
Int[x^3/(2*(a + b) + x^4),x]
```

output
```
Log[2*(a + b) + x^4]/4
```

Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln(x^4+2a+2b)}{4}$	14
default	$\frac{\ln(x^4+2a+2b)}{4}$	14
norman	$\frac{\ln(x^4+2a+2b)}{4}$	14
risch	$\frac{\ln(x^4+2a+2b)}{4}$	14
parallelrisch	$\frac{\ln(x^4+2a+2b)}{4}$	14

input
```
int(x^3/(x^4+2*a+2*b),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(x^4+2*a+2*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{2(a+b)+x^4}\,dx = \frac{1}{4}\,\log\left(x^4+2\,a+2\,b\right)$$

input
```
integrate(x^3/(x^4+2*a+2*b),x, algorithm="fricas")
```

output
```
1/4*log(x^4 + 2*a + 2*b)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{2(a+b)+x^4}\,dx = \frac{\log(2a+2b+x^4)}{4}$$

input
```
integrate(x**3/(x**4+2*a+2*b),x)
```

output
```
log(2*a + 2*b + x**4)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{2(a+b)+x^4}\,dx = \frac{1}{4}\log\left(x^4+2\,a+2\,b\right)$$

input
```
integrate(x^3/(x^4+2*a+2*b),x, algorithm="maxima")
```

output
```
1/4*log(x^4 + 2*a + 2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{2(a+b)+x^4}\,dx = \frac{1}{4}\log\left(\left|x^4+2\,a+2\,b\right|\right)$$

input
```
integrate(x^3/(x^4+2*a+2*b),x, algorithm="giac")
```

output
```
1/4*log(abs(x^4 + 2*a + 2*b))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{2(a+b)+x^4}\,dx=\frac{\ln\left(x^4+2\,a+2\,b\right)}{4}$$

input
```
int(x^3/(2*a + 2*b + x^4),x)
```

output
```
log(2*a + 2*b + x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.86

$$\int \frac{x^3}{2(a+b)+x^4}\,dx=\frac{\log\left(-(a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{a+b}\,\sqrt{2}+x^2\right)}{4}+\frac{\log\left((a+b)^{\frac{1}{4}}\sqrt{2}\,2^{\frac{1}{4}}x+\sqrt{a+b}\,\sqrt{2}+x^2\right)}{4}$$

input
```
int(x^3/(x^4+2*a+2*b),x)
```

output
```
(log( - (a + b)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2) +
log((a + b)**(1/4)*sqrt(2)*2**(1/4)*x + sqrt(a + b)*sqrt(2) + x**2))/4
```

3.111 $\int \frac{x^2}{3+x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 98

$$\int \frac{x^2}{3+x^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\sqrt[4]{3}} + \frac{\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{2\sqrt{2}\sqrt[4]{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{3}x}{\sqrt{3}+x^2}\right)}{2\sqrt{2}\sqrt[4]{3}}$$

output
```
1/12*arctan(-1+1/3*2^(1/2)*x*3^(3/4))*2^(1/2)*3^(3/4)+1/12*arctan(1+1/3*2^
(1/2)*x*3^(3/4))*2^(1/2)*3^(3/4)-1/12*arctanh(2^(1/2)*3^(1/4)*x/(3^(1/2)+x
^2))*2^(1/2)*3^(3/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{3+x^4}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)+2\arctan\left(1+\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)+\log\left(3-\sqrt{2}3^{3/4}x+\sqrt{3}x^2\right)-\log\left(3+\sqrt{2}3^{3/4}x+\sqrt{3}x^2\right)}{4\sqrt{2}\sqrt[4]{3}}$$

input
```
Integrate[x^2/(3 + x^4),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*x)/3^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*x)/3^(1/4)] +
Log[3 - Sqrt[2]*3^(3/4)*x + Sqrt[3]*x^2] - Log[3 + Sqrt[2]*3^(3/4)*x + Sqr
t[3]*x^2])/(4*Sqrt[2]*3^(1/4))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.41, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^4+3}dx$$

$$\Big\downarrow \text{ 826}$$

$$\frac{1}{2}\int \frac{x^2+\sqrt{3}}{x^4+3}dx-\frac{1}{2}\int \frac{\sqrt{3}-x^2}{x^4+3}dx$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^2-\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx+\frac{1}{2}\int \frac{1}{x^2+\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx\right)-\frac{1}{2}\int \frac{\sqrt{3}-x^2}{x^4+3}dx$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)^2-1}d\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\int \frac{1}{-\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)^2-1}d\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)}{\sqrt{2}\sqrt[4]{3}}\right)-\frac{1}{2}\int \frac{\sqrt{3}-x^2}{x^4+3}dx$$

$$\Big\downarrow \text{ 217}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{\sqrt{2}\sqrt[4]{3}}\right)-\frac{1}{2}\int \frac{\sqrt{3}-x^2}{x^4+3}dx$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}\sqrt[4]{3}-2x}{x^2-\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx}{2\sqrt{2}\sqrt[4]{3}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}x+\sqrt[4]{3}\right)}{x^2+\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx}{2\sqrt{2}\sqrt[4]{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{\sqrt{2}\sqrt[4]{3}}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{x^2-\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx}{2\sqrt{2}\sqrt[4]{3}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}x+\sqrt[4]{3}\right)}{x^2+\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx}{2\sqrt{2}\sqrt[4]{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{\sqrt{2}\sqrt[4]{3}}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}\sqrt[4]{3}-2x}{x^2-\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx}{2\sqrt{2}\sqrt[4]{3}}-\frac{\int \frac{\sqrt{2}x+\sqrt[4]{3}}{x^2+\sqrt{2}\sqrt[4]{3}x+\sqrt{3}}dx}{2\sqrt[4]{3}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{\sqrt{2}\sqrt[4]{3}}\right)$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt[4]{3}}+1\right)}{\sqrt{2}\sqrt[4]{3}}-\frac{\arctan\left(1-\frac{\sqrt{2}x}{\sqrt[4]{3}}\right)}{\sqrt{2}\sqrt[4]{3}}\right)+\frac{1}{2}\left(\frac{\log\left(x^2-\sqrt{2}\sqrt[4]{3}x+\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}}-\frac{\log\left(x^2+\sqrt{2}\sqrt[4]{3}x+\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}}\right)$$

input
```
Int[x^2/(3 + x^4),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*x)/3^(1/4)]/(Sqrt[2]*3^(1/4))) + ArcTan[1 + (Sqrt[2
]*x)/3^(1/4)]/(Sqrt[2]*3^(1/4)))/2 + (Log[Sqrt[3] - Sqrt[2]*3^(1/4)*x + x^
2]/(2*Sqrt[2]*3^(1/4)) - Log[Sqrt[3] + Sqrt[2]*3^(1/4)*x + x^2]/(2*Sqrt[2]
*3^(1/4)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.22

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+3\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$
default	$\frac{3^{\frac{3}{4}}\sqrt{2}\left(\ln\left(\frac{x^2-3^{\frac{1}{4}}x\sqrt{2}+\sqrt{3}}{x^2+3^{\frac{1}{4}}x\sqrt{2}+\sqrt{3}}\right)+2\arctan\left(1+\frac{\sqrt{2}\,x\,3^{\frac{3}{4}}}{3}\right)+2\arctan\left(-1+\frac{\sqrt{2}\,x\,3^{\frac{3}{4}}}{3}\right)\right)}{24}$
meijerg	$\frac{3^{\frac{3}{4}}\left(\frac{x^3\sqrt{2}\ln\left(1-\frac{\sqrt{2}\,3^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{3}+\frac{\sqrt{3}\sqrt{x^4}}{3}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\frac{\sqrt{2}\,3^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{6-\sqrt{2}\,3^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\ln\left(1+\frac{\sqrt{2}\,3^{\frac{3}{4}}\left(x^4\right)^{\frac{1}{4}}}{3}+\frac{\sqrt{3}\sqrt{x^4}}{3}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\arctan\left(\ldots\right)}{\left(x^4\right)\ldots}\ldots\right.}{12}$

input
```
int(x^2/(x^4+3),x,method=_RETURNVERBOSE)
```

output
```
1/4*sum(1/_R*ln(x-_R),_R=RootOf(_Z^4+3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{3+x^4}\,dx = \frac{1}{24}\cdot 12^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{3}{4}}x+1\right)+\frac{1}{24}\cdot 12^{\frac{3}{4}}\arctan\left(\frac{1}{6}\cdot 12^{\frac{3}{4}}x-1\right)-\frac{1}{48}\cdot 12^{\frac{3}{4}}\log\left(2\,x^2+2\cdot 12^{\frac{1}{4}}x+2\,\sqrt{3}\right)+\frac{1}{48}\cdot 12^{\frac{3}{4}}\log\left(2\,x^2-2\cdot 12^{\frac{1}{4}}x+2\,\sqrt{3}\right)$$

input
```
integrate(x^2/(x^4+3),x, algorithm="fricas")
```

output
```
1/24*12^(3/4)*arctan(1/6*12^(3/4)*x + 1) + 1/24*12^(3/4)*arctan(1/6*12^(3/
4)*x - 1) - 1/48*12^(3/4)*log(2*x^2 + 2*12^(1/4)*x + 2*sqrt(3)) + 1/48*12^
(3/4)*log(2*x^2 - 2*12^(1/4)*x + 2*sqrt(3))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{3+x^4}\,dx = \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(x^2-\sqrt{2}\cdot\sqrt[4]{3}x+\sqrt{3}\right)}{24}-\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\log\left(x^2+\sqrt{2}\cdot\sqrt[4]{3}x+\sqrt{3}\right)}{24}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}x}{3}-1\right)}{12}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}x}{3}+1\right)}{12}$$

input
```
integrate(x**2/(x**4+3),x)
```

output
```
sqrt(2)*3**(3/4)*log(x**2 - sqrt(2)*3**(1/4)*x + sqrt(3))/24 - sqrt(2)*3**
(3/4)*log(x**2 + sqrt(2)*3**(1/4)*x + sqrt(3))/24 + sqrt(2)*3**(3/4)*atan(
sqrt(2)*3**(3/4)*x/3 - 1)/12 + sqrt(2)*3**(3/4)*atan(sqrt(2)*3**(3/4)*x/3
+ 1)/12
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{3+x^4}\,dx = \frac{1}{12}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x+3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{12}\cdot 3^{\frac{3}{4}}\sqrt{2}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{24}\cdot 3^{\frac{3}{4}}\sqrt{2}\log\left(x^2+3^{\frac{1}{4}}\sqrt{2}x+\sqrt{3}\right)+\frac{1}{24}\cdot 3^{\frac{3}{4}}\sqrt{2}\log\left(x^2-3^{\frac{1}{4}}\sqrt{2}x+\sqrt{3}\right)$$

input
```
integrate(x^2/(x^4+3),x, algorithm="maxima")
```

output
```
1/12*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x + 3^(1/4)*sqrt(2))) +
 1/12*3^(3/4)*sqrt(2)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x - 3^(1/4)*sqrt(2)))
- 1/24*3^(3/4)*sqrt(2)*log(x^2 + 3^(1/4)*sqrt(2)*x + sqrt(3)) + 1/24*3^(3/
4)*sqrt(2)*log(x^2 - 3^(1/4)*sqrt(2)*x + sqrt(3))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{3+x^4}\,dx = \frac{1}{12}\cdot 108^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x+3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{12}\cdot 108^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\,x-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{24}\cdot 108^{\frac{1}{4}}\log\left(x^2+3^{\frac{1}{4}}\sqrt{2}x+\sqrt{3}\right)+\frac{1}{24}\cdot 108^{\frac{1}{4}}\log\left(x^2-3^{\frac{1}{4}}\sqrt{2}x+\sqrt{3}\right)$$

input
```
integrate(x^2/(x^4+3),x, algorithm="giac")
```

output
```
1/12*108^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x + 3^(1/4)*sqrt(2))) + 1/12*
108^(1/4)*arctan(1/6*3^(3/4)*sqrt(2)*(2*x - 3^(1/4)*sqrt(2))) - 1/24*108^(
1/4)*log(x^2 + 3^(1/4)*sqrt(2)*x + sqrt(3)) + 1/24*108^(1/4)*log(x^2 - 3^(
1/4)*sqrt(2)*x + sqrt(3))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{3+x^4}\,dx = \sqrt{2}\,3^{3/4}\,\mathrm{atan}\left(\sqrt{2}\,3^{3/4}\,x\,\left(\frac{1}{6}-\frac{1}{6}\mathrm{i}\right)\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right) + \sqrt{2}\,3^{3/4}\,\mathrm{atan}\left(\sqrt{2}\,3^{3/4}\,x\,\left(\frac{1}{6}+\frac{1}{6}\mathrm{i}\right)\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

input
```
int(x^2/(x^4 + 3),x)
```

output
```
2^(1/2)*3^(3/4)*atan(2^(1/2)*3^(3/4)*x*(1/6 - 1i/6))*(1/12 - 1i/12) + 2^(1
/2)*3^(3/4)*atan(2^(1/2)*3^(3/4)*x*(1/6 + 1i/6))*(1/12 + 1i/12)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{3+x^4}\,dx = \frac{\sqrt{6}\,3^{\frac{1}{4}}\left(-2atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}-2x\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)+2atan\left(\frac{\left(\sqrt{2}\,3^{\frac{1}{4}}+2x\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)+\log\left(-\sqrt{2}\,3^{\frac{1}{4}}x+\sqrt{3}+x^2\right)-\log\left(\sqrt{2}\,3^{\frac{1}{4}}x+\right.\right.}{24}$$

input
```
int(x^2/(x^4+3),x)
```

output
```
(sqrt(6)*3**(1/4)*( - 2*atan((sqrt(2)*3**(1/4) - 2*x)/(sqrt(2)*3**(1/4)))
+ 2*atan((sqrt(2)*3**(1/4) + 2*x)/(sqrt(2)*3**(1/4))) + log( - sqrt(2)*3**
(1/4)*x + sqrt(3) + x**2) - log(sqrt(2)*3**(1/4)*x + sqrt(3) + x**2)))/24
```

3.112 $\int x^{5/2}(a+cx^4)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int x^{5/2}\left(a+cx^4\right)\,dx=\frac{2}{7}ax^{7/2}+\frac{2}{15}cx^{15/2}$$

output
```
2/7*a*x^(7/2)+2/15*c*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{5/2}\left(a+cx^4\right)\,dx=\frac{2}{105}\left(15ax^{7/2}+7cx^{15/2}\right)$$

input
```
Integrate[x^(5/2)*(a + c*x^4),x]
```

output
```
(2*(15*a*x^(7/2) + 7*c*x^(15/2)))/105
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+cx^4\right)\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(ax^{5/2}+cx^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}ax^{7/2}+\frac{2}{15}cx^{15/2}$$

input
```
Int[x^(5/2)*(a + c*x^4),x]
```

output
```
(2*a*x^(7/2))/7 + (2*c*x^(15/2))/15
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{15}{2}}}{15}$	14
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2c\,x^{\frac{15}{2}}}{15}$	14
gosper	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{4}+15a\right)}{105}$	16
trager	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{4}+15a\right)}{105}$	16
risch	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{4}+15a\right)}{105}$	16
orering	$\frac{2x^{\frac{7}{2}}\left(7c\,x^{4}+15a\right)}{105}$	16

input
```
int(x^(5/2)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*x^(7/2)+2/15*c*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{5/2}\left(a+cx^4\right)\,dx=\frac{2}{105}\left(7\,cx^7+15\,ax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+a),x, algorithm="fricas")
```

output
```
2/105*(7*c*x^7 + 15*a*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{5/2}(a+cx^4)\ dx = \frac{2ax^{\frac{7}{2}}}{7} + \frac{2cx^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(5/2)*(c*x**4+a),x)
```

output
```
2*a*x**(7/2)/7 + 2*c*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+cx^4)\ dx = \frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+a),x, algorithm="maxima")
```

output
```
2/15*c*x^(15/2) + 2/7*a*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+cx^4)\ dx = \frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+a),x, algorithm="giac")
```

output
```
2/15*c*x^(15/2) + 2/7*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{5/2}\left(a+cx^4\right)\,dx=\frac{2\,x^{7/2}\,\left(7\,c\,x^4+15\,a\right)}{105}$$

input
```
int(x^(5/2)*(a + c*x^4),x)
```

output
```
(2*x^(7/2)*(15*a + 7*c*x^4))/105
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^3(7c\,x^4+15a)}{105}$$

input
```
int(x^(5/2)*(c*x^4+a),x)
```

output
```
(2*sqrt(x)*x**3*(15*a + 7*c*x**4))/105
```

3.113 $\int x^{3/2}(a+cx^4)\ dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int x^{3/2}(a+cx^4)\ dx=\frac{2}{5}ax^{5/2}+\frac{2}{13}cx^{13/2}$$

output
```
2/5*a*x^(5/2)+2/13*c*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{3/2}(a+cx^4)\ dx=\frac{2}{65}x^{5/2}(13a+5cx^4)$$

input
```
Integrate[x^(3/2)*(a + c*x^4),x]
```

output
```
(2*x^(5/2)*(13*a + 5*c*x^4))/65
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+cx^4\right)\,dx$$

$$\Big\downarrow \ 802$$

$$\int \left(ax^{3/2}+cx^{11/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{5}ax^{5/2}+\frac{2}{13}cx^{13/2}$$

input
```
Int[x^(3/2)*(a + c*x^4),x]
```

output
```
(2*a*x^(5/2))/5 + (2*c*x^(13/2))/13
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{13}{2}}}{13}$	14
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2c\,x^{\frac{13}{2}}}{13}$	14
gosper	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{4}+13a\right)}{65}$	16
trager	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{4}+13a\right)}{65}$	16
risch	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{4}+13a\right)}{65}$	16
orering	$\frac{2x^{\frac{5}{2}}\left(5c\,x^{4}+13a\right)}{65}$	16

input
```
int(x^(3/2)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*x^(5/2)+2/13*c*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{3/2}\left(a+cx^4\right)\,dx=\frac{2}{65}\left(5\,cx^6+13\,ax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+a),x, algorithm="fricas")
```

output
```
2/65*(5*c*x^6 + 13*a*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{3/2}(a+cx^4)\,dx = \frac{2ax^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(3/2)*(c*x**4+a),x)
```

output
```
2*a*x**(5/2)/5 + 2*c*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}(a+cx^4)\,dx = \frac{2}{13}\,cx^{\frac{13}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+a),x, algorithm="maxima")
```

output
```
2/13*c*x^(13/2) + 2/5*a*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}(a+cx^4)\,dx = \frac{2}{13}\,cx^{\frac{13}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+a),x, algorithm="giac")
```

output
```
2/13*c*x^(13/2) + 2/5*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{3/2}\left(a+cx^4\right)\,dx=\frac{2\,x^{5/2}\,\left(5\,c\,x^4+13\,a\right)}{65}$$

input
```
int(x^(3/2)*(a + c*x^4),x)
```

output
```
(2*x^(5/2)*(13*a + 5*c*x^4))/65
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+cx^4\right)\,dx=\frac{2\sqrt{x}\,x^2(5c\,x^4+13a)}{65}$$

input
```
int(x^(3/2)*(c*x^4+a),x)
```

output
```
(2*sqrt(x)*x**2*(13*a + 5*c*x**4))/65
```

3.114 $\int \sqrt{x}(a+cx^4)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \sqrt{x}(a+cx^4)\,dx=\frac{2}{3}ax^{3/2}+\frac{2}{11}cx^{11/2}$$

output
```
2/3*a*x^(3/2)+2/11*c*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{x}(a+cx^4)\,dx=\frac{2}{33}x^{3/2}(11a+3cx^4)$$

input
```
Integrate[Sqrt[x]*(a + c*x^4),x]
```

output
```
(2*x^(3/2)*(11*a + 3*c*x^4))/33
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+cx^4\right) \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a\sqrt{x}+cx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}ax^{3/2}+\frac{2}{11}cx^{11/2}$$

input
```
Int[Sqrt[x]*(a + c*x^4),x]
```

output
```
(2*a*x^(3/2))/3 + (2*c*x^(11/2))/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{11}{2}}}{11}$	14
default	$\frac{2a\,x^{\frac{3}{2}}}{3}+\frac{2c\,x^{\frac{11}{2}}}{11}$	14
gosper	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{4}+11a\right)}{33}$	16
trager	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{4}+11a\right)}{33}$	16
risch	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{4}+11a\right)}{33}$	16
orering	$\frac{2x^{\frac{3}{2}}\left(3c\,x^{4}+11a\right)}{33}$	16

input
```
int(x^(1/2)*(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*x^(3/2)+2/11*c*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt{x}\left(a+cx^4\right)\,dx=\frac{2}{33}\left(3\,cx^5+11\,ax\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+a),x, algorithm="fricas")
```

output
```
2/33*(3*c*x^5 + 11*a*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{x}\left(a+cx^4\right)\,dx=\frac{2ax^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(1/2)*(c*x**4+a),x)
```

output
```
2*a*x**(3/2)/3 + 2*c*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\left(a+cx^4\right)\,dx=\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+a),x, algorithm="maxima")
```

output
```
2/11*c*x^(11/2) + 2/3*a*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}\left(a+cx^4\right)\,dx=\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+a),x, algorithm="giac")
```

output
```
2/11*c*x^(11/2) + 2/3*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{x}(a+cx^4)\ dx = \frac{2\,x^{3/2}\,(3\,c\,x^4+11\,a)}{33}$$

input
```
int(x^(1/2)*(a + c*x^4),x)
```

output
```
(2*x^(3/2)*(11*a + 3*c*x^4))/33
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{x}(a+cx^4)\ dx = \frac{2\sqrt{x}\,x(3c\,x^4+11a)}{33}$$

input
```
int(x^(1/2)*(c*x^4+a),x)
```

output
```
(2*sqrt(x)*x*(11*a + 3*c*x**4))/33
```

3.115 $\int \frac{a+cx^4}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = 2a\sqrt{x}+\frac{2}{9}cx^{9/2}$$

output
```
2*a*x^(1/2)+2/9*c*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = \frac{2}{9}\left(9a\sqrt{x}+cx^{9/2}\right)$$

input
```
Integrate[(a + c*x^4)/Sqrt[x],x]
```

output
```
(2*(9*a*Sqrt[x] + c*x^(9/2)))/9
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{\sqrt{x}}+cx^{7/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\sqrt{x}+\frac{2}{9}cx^{9/2}$$

input
```
Int[(a + c*x^4)/Sqrt[x],x]
```

output
```
2*a*Sqrt[x] + (2*c*x^(9/2))/9
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2a\sqrt{x}+\frac{2c\,x^{\frac{9}{2}}}{9}$	14
default	$2a\sqrt{x}+\frac{2c\,x^{\frac{9}{2}}}{9}$	14
gosper	$\frac{2\sqrt{x}\,(c\,x^{4}+9a)}{9}$	15
trager	$\left(\frac{2c\,x^{4}}{9}+2a\right)\sqrt{x}$	15
risch	$\frac{2\sqrt{x}\,(c\,x^{4}+9a)}{9}$	15
orering	$\frac{2\sqrt{x}\,(c\,x^{4}+9a)}{9}$	15

input
```
int((c*x^4+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*x^(1/2)+2/9*c*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx=\frac{2}{9}\left(cx^4+9\,a\right)\sqrt{x}$$

input
```
integrate((c*x^4+a)/x^(1/2),x, algorithm="fricas")
```

output
```
2/9*(c*x^4 + 9*a)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = 2a\sqrt{x} + \frac{2cx^{\frac{9}{2}}}{9}$$

input
```
integrate((c*x**4+a)/x**(1/2),x)
```

output
```
2*a*sqrt(x) + 2*c*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = \frac{2}{9}\,cx^{\frac{9}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((c*x^4+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2/9*c*x^(9/2) + 2*a*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = \frac{2}{9}\,cx^{\frac{9}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((c*x^4+a)/x^(1/2),x, algorithm="giac")
```

output
```
2/9*c*x^(9/2) + 2*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = \frac{2\,\sqrt{x}\,\left(c\,x^4+9\,a\right)}{9}$$

input
```
int((a + c*x^4)/x^(1/2),x)
```

output
```
(2*x^(1/2)*(9*a + c*x^4))/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^4}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,\left(c\,x^4+9a\right)}{9}$$

input
```
int((c*x^4+a)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(9*a + c*x**4))/9
```

3.116 $\int \frac{a+cx^4}{x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+cx^4}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}}+\frac{2}{7}cx^{7/2}$$

output
```
-2*a/x^(1/2)+2/7*c*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+cx^4}{x^{3/2}}\,dx = -\frac{2(7a-cx^4)}{7\sqrt{x}}$$

input
```
Integrate[(a + c*x^4)/x^(3/2),x]
```

output
```
(-2*(7*a - c*x^4))/(7*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{x^{3/2}}\,dx$$
$$\Big\downarrow \text{802}$$
$$\int \left(\frac{a}{x^{3/2}}+cx^{5/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{7}cx^{7/2}-\frac{2a}{\sqrt{x}}$$

input
```
Int[(a + c*x^4)/x^(3/2),x]
```

output
```
(-2*a)/Sqrt[x] + (2*c*x^(7/2))/7
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2a}{\sqrt{x}}+\frac{2c\,x^{\frac{7}{2}}}{7}$	14
default	$-\frac{2a}{\sqrt{x}}+\frac{2c\,x^{\frac{7}{2}}}{7}$	14
gosper	$-\frac{2\left(-c\,x^{4}+7a\right)}{7\sqrt{x}}$	16
trager	$-\frac{2\left(-c\,x^{4}+7a\right)}{7\sqrt{x}}$	16
risch	$-\frac{2\left(-c\,x^{4}+7a\right)}{7\sqrt{x}}$	16
orering	$-\frac{2\left(-c\,x^{4}+7a\right)}{7\sqrt{x}}$	16

input
```
int((c*x^4+a)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/x^(1/2)+2/7*c*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{a+cx^4}{x^{3/2}}\,dx=\frac{2\,(cx^4-7\,a)}{7\,\sqrt{x}}$$

input
```
integrate((c*x^4+a)/x^(3/2),x, algorithm="fricas")
```

output
```
2/7*(c*x^4 - 7*a)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+cx^4}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+\frac{2cx^{\frac{7}{2}}}{7}$$

input
```
integrate((c*x**4+a)/x**(3/2),x)
```

output
```
-2*a/sqrt(x) + 2*c*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^4}{x^{3/2}}\,dx=\frac{2}{7}\,cx^{\frac{7}{2}}-\frac{2\,a}{\sqrt{x}}$$

input
```
integrate((c*x^4+a)/x^(3/2),x, algorithm="maxima")
```

output
```
2/7*c*x^(7/2) - 2*a/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+cx^4}{x^{3/2}}\,dx=\frac{2}{7}\,cx^{\frac{7}{2}}-\frac{2\,a}{\sqrt{x}}$$

input
```
integrate((c*x^4+a)/x^(3/2),x, algorithm="giac")
```

output
```
2/7*c*x^(7/2) - 2*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a + cx^4}{x^{3/2}} \, dx = -\frac{14 \, a - 2 \, c \, x^4}{7 \, \sqrt{x}}$$

input
```
int((a + c*x^4)/x^(3/2),x)
```

output
```
-(14*a - 2*c*x^4)/(7*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a + cx^4}{x^{3/2}} \, dx = \frac{\frac{2c \, x^4}{7} - 2a}{\sqrt{x}}$$

input
```
int((c*x^4+a)/x^(3/2),x)
```

output
```
(2*( - 7*a + c*x**4))/(7*sqrt(x))
```

3.117 $\int \frac{a+cx^4}{x^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{a+cx^4}{x^{5/2}} \, dx = -\frac{2a}{3x^{3/2}} + \frac{2}{5}cx^{5/2}$$

output
```
-2/3*a/x^(3/2)+2/5*c*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{x^{5/2}} \, dx = -\frac{2(5a-3cx^4)}{15x^{3/2}}$$

input
```
Integrate[(a + c*x^4)/x^(5/2),x]
```

output
```
(-2*(5*a - 3*c*x^4))/(15*x^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^{5/2}}+cx^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}cx^{5/2}-\frac{2a}{3x^{3/2}}$$

input
```
Int[(a + c*x^4)/x^(5/2),x]
```

output
```
(-2*a)/(3*x^(3/2)) + (2*c*x^(5/2))/5
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}+\frac{2c\,x^{\frac{5}{2}}}{5}$	14
default	$-\frac{2a}{3x^{\frac{3}{2}}}+\frac{2c\,x^{\frac{5}{2}}}{5}$	14
gosper	$-\frac{2\left(-3c\,x^{4}+5a\right)}{15x^{\frac{3}{2}}}$	16
trager	$-\frac{2\left(-3c\,x^{4}+5a\right)}{15x^{\frac{3}{2}}}$	16
risch	$-\frac{2\left(-3c\,x^{4}+5a\right)}{15x^{\frac{3}{2}}}$	16
orering	$-\frac{2\left(-3c\,x^{4}+5a\right)}{15x^{\frac{3}{2}}}$	16

input
```
int((c*x^4+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a/x^(3/2)+2/5*c*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a+cx^4}{x^{5/2}}\,dx=\frac{2\left(3\,cx^4-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)/x^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*x^4 - 5*a)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{x^{5/2}}\,dx = -\frac{2a}{3x^{\frac{3}{2}}} + \frac{2cx^{\frac{5}{2}}}{5}$$

input
```
integrate((c*x**4+a)/x**(5/2),x)
```

output
```
-2*a/(3*x**(3/2)) + 2*c*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+cx^4}{x^{5/2}}\,dx = \frac{2}{5}\,cx^{\frac{5}{2}} - \frac{2\,a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)/x^(5/2),x, algorithm="maxima")
```

output
```
2/5*c*x^(5/2) - 2/3*a/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+cx^4}{x^{5/2}}\,dx = \frac{2}{5}\,cx^{\frac{5}{2}} - \frac{2\,a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)/x^(5/2),x, algorithm="giac")
```

output
```
2/5*c*x^(5/2) - 2/3*a/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a+cx^4}{x^{5/2}}\,dx=-\frac{10\,a-6\,c\,x^4}{15\,x^{3/2}}$$

input
```
int((a + c*x^4)/x^(5/2),x)
```

output
```
-(10*a - 6*c*x^4)/(15*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{x^{5/2}}\,dx=\frac{\frac{2c\,x^4}{5}-\frac{2a}{3}}{\sqrt{x}\,x}$$

input
```
int((c*x^4+a)/x^(5/2),x)
```

output
```
(2*( - 5*a + 3*c*x**4))/(15*sqrt(x)*x)
```

3.118 $\int \frac{a+cx^4}{x^{7/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{a+cx^4}{x^{7/2}}\,dx=-\frac{2a}{5x^{5/2}}+\frac{2}{3}cx^{3/2}$$

output
```
-2/5*a/x^(5/2)+2/3*c*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{x^{7/2}}\,dx=-\frac{2(3a-5cx^4)}{15x^{5/2}}$$

input
```
Integrate[(a + c*x^4)/x^(7/2),x]
```

output
```
(-2*(3*a - 5*c*x^4))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+cx^4}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a}{x^{7/2}}+c\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}cx^{3/2}-\frac{2a}{5x^{5/2}}$$

input
```
Int[(a + c*x^4)/x^(7/2),x]
```

output
```
(-2*a)/(5*x^(5/2)) + (2*c*x^(3/2))/3
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{2a}{5x^{\frac{5}{2}}}+\frac{2c\,x^{\frac{3}{2}}}{3}$	14
default	$-\frac{2a}{5x^{\frac{5}{2}}}+\frac{2c\,x^{\frac{3}{2}}}{3}$	14
gosper	$-\frac{2(-5c\,x^{4}+3a)}{15x^{\frac{5}{2}}}$	16
trager	$-\frac{2(-5c\,x^{4}+3a)}{15x^{\frac{5}{2}}}$	16
risch	$-\frac{2(-5c\,x^{4}+3a)}{15x^{\frac{5}{2}}}$	16
orering	$-\frac{2(-5c\,x^{4}+3a)}{15x^{\frac{5}{2}}}$	16

input
```
int((c*x^4+a)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a/x^(5/2)+2/3*c*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a+cx^4}{x^{7/2}}\,dx=\frac{2\left(5\,cx^4-3\,a\right)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*c*x^4 - 3*a)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{x^{7/2}}\,dx = -\frac{2a}{5x^{\frac{5}{2}}} + \frac{2cx^{\frac{3}{2}}}{3}$$

input
```
integrate((c*x**4+a)/x**(7/2),x)
```

output
```
-2*a/(5*x**(5/2)) + 2*c*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+cx^4}{x^{7/2}}\,dx = \frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,a}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*c*x^(3/2) - 2/5*a/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{a+cx^4}{x^{7/2}}\,dx = \frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\,a}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)/x^(7/2),x, algorithm="giac")
```

output
```
2/3*c*x^(3/2) - 2/5*a/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{a+cx^4}{x^{7/2}}\,dx=-\frac{6\,a-10\,c\,x^4}{15\,x^{5/2}}$$

input
```
int((a + c*x^4)/x^(7/2),x)
```

output
```
-(6*a - 10*c*x^4)/(15*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+cx^4}{x^{7/2}}\,dx=\frac{\frac{2c\,x^4}{3}-\frac{2a}{5}}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+a)/x^(7/2),x)
```

output
```
(2*( - 3*a + 5*c*x**4))/(15*sqrt(x)*x**2)
```

3.119 $\int x^{5/2}(a+cx^4)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int x^{5/2}\left(a+cx^4\right)^2\,dx=\frac{2}{7}a^2x^{7/2}+\frac{4}{15}acx^{15/2}+\frac{2}{23}c^2x^{23/2}$$

output
```
2/7*a^2*x^(7/2)+4/15*a*c*x^(15/2)+2/23*c^2*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{5/2}\left(a+cx^4\right)^2\,dx=\frac{2x^{7/2}(345a^2+322acx^4+105c^2x^8)}{2415}$$

input
```
Integrate[x^(5/2)*(a + c*x^4)^2,x]
```

output
```
(2*x^(7/2)*(345*a^2 + 322*a*c*x^4 + 105*c^2*x^8))/2415
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+cx^4\right)^2\,dx$$

$$\Big\downarrow \ 802$$

$$\int \left(a^2x^{5/2}+2acx^{13/2}+c^2x^{21/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{15}acx^{15/2}+\frac{2}{23}c^2x^{23/2}$$

input
```
Int[x^(5/2)*(a + c*x^4)^2,x]
```

output
```
(2*a^2*x^(7/2))/7 + (4*a*c*x^(15/2))/15 + (2*c^2*x^(23/2))/23
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{23}{2}}}{23}$	25
default	$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4ac\,x^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{23}{2}}}{23}$	25
gosper	$\frac{2x^{\frac{7}{2}}\left(105c^2x^8+322a\,x^4c+345a^2\right)}{2415}$	27
trager	$\frac{2x^{\frac{7}{2}}\left(105c^2x^8+322a\,x^4c+345a^2\right)}{2415}$	27
risch	$\frac{2x^{\frac{7}{2}}\left(105c^2x^8+322a\,x^4c+345a^2\right)}{2415}$	27
orering	$\frac{2x^{\frac{7}{2}}\left(105c^2x^8+322a\,x^4c+345a^2\right)}{2415}$	27

input
```
int(x^(5/2)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^2*x^(7/2)+4/15*a*c*x^(15/2)+2/23*c^2*x^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+cx^4\right)^2\,dx=\frac{2}{2415}\left(105\,c^2x^{11}+322\,acx^7+345\,a^2x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
2/2415*(105*c^2*x^11 + 322*a*c*x^7 + 345*a^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{5/2}(a+cx^4)^2\,dx = \frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4acx^{\frac{15}{2}}}{15} + \frac{2c^2x^{\frac{23}{2}}}{23}$$

input
```
integrate(x**(5/2)*(c*x**4+a)**2,x)
```

output
```
2*a**2*x**(7/2)/7 + 4*a*c*x**(15/2)/15 + 2*c**2*x**(23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}(a+cx^4)^2\,dx = \frac{2}{23}\,c^2x^{\frac{23}{2}} + \frac{4}{15}\,acx^{\frac{15}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
2/23*c^2*x^(23/2) + 4/15*a*c*x^(15/2) + 2/7*a^2*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}(a+cx^4)^2\,dx = \frac{2}{23}\,c^2x^{\frac{23}{2}} + \frac{4}{15}\,acx^{\frac{15}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
2/23*c^2*x^(23/2) + 4/15*a*c*x^(15/2) + 2/7*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+cx^4\right)^2\,dx=x^{7/2}\left(\frac{2\,a^2}{7}+\frac{4\,a\,c\,x^4}{15}+\frac{2\,c^2\,x^8}{23}\right)$$

input
```
int(x^(5/2)*(a + c*x^4)^2,x)
```

output
```
x^(7/2)*((2*a^2)/7 + (2*c^2*x^8)/23 + (4*a*c*x^4)/15)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{5/2}\left(a+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^3(105c^2x^8+322ac\,x^4+345a^2)}{2415}$$

input
```
int(x^(5/2)*(c*x^4+a)^2,x)
```

output
```
(2*sqrt(x)*x**3*(345*a**2 + 322*a*c*x**4 + 105*c**2*x**8))/2415
```

3.120 $\int x^{3/2}(a+cx^4)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2}{5}a^2x^{5/2}+\frac{4}{13}acx^{13/2}+\frac{2}{21}c^2x^{21/2}$$

output
```
2/5*a^2*x^(5/2)+4/13*a*c*x^(13/2)+2/21*c^2*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2x^{5/2}(273a^2+210acx^4+65c^2x^8)}{1365}$$

input
```
Integrate[x^(3/2)*(a + c*x^4)^2,x]
```

output
```
(2*x^(5/2)*(273*a^2 + 210*a*c*x^4 + 65*c^2*x^8))/1365
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2x^{3/2}+2acx^{11/2}+c^2x^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{13}acx^{13/2}+\frac{2}{21}c^2x^{21/2}$$

input
```
Int[x^(3/2)*(a + c*x^4)^2,x]
```

output
```
(2*a^2*x^(5/2))/5 + (4*a*c*x^(13/2))/13 + (2*c^2*x^(21/2))/21
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ac\,x^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{21}{2}}}{21}$	25
default	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ac\,x^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{21}{2}}}{21}$	25
gosper	$\frac{2x^{\frac{5}{2}}(65c^2x^8+210a\,x^4c+273a^2)}{1365}$	27
trager	$\frac{2x^{\frac{5}{2}}(65c^2x^8+210a\,x^4c+273a^2)}{1365}$	27
risch	$\frac{2x^{\frac{5}{2}}(65c^2x^8+210a\,x^4c+273a^2)}{1365}$	27
orering	$\frac{2x^{\frac{5}{2}}(65c^2x^8+210a\,x^4c+273a^2)}{1365}$	27

input
```
int(x^(3/2)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*a^2*x^(5/2)+4/13*a*c*x^(13/2)+2/21*c^2*x^(21/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2}{1365}\left(65\,c^2x^{10}+210\,acx^6+273\,a^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
2/1365*(65*c^2*x^10 + 210*a*c*x^6 + 273*a^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(3/2)*(c*x**4+a)**2,x)
```

output
```
2*a**2*x**(5/2)/5 + 4*a*c*x**(13/2)/13 + 2*c**2*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
2/21*c^2*x^(21/2) + 4/13*a*c*x^(13/2) + 2/5*a^2*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
2/21*c^2*x^(21/2) + 4/13*a*c*x^(13/2) + 2/5*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2\,x^{5/2}\,\left(273\,a^2+210\,a\,c\,x^4+65\,c^2\,x^8\right)}{1365}$$

input
```
int(x^(3/2)*(a + c*x^4)^2,x)
```

output
```
(2*x^(5/2)*(273*a^2 + 65*c^2*x^8 + 210*a*c*x^4))/1365
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(a+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x^2(65c^2x^8+210ac\,x^4+273a^2)}{1365}$$

input
```
int(x^(3/2)*(c*x^4+a)^2,x)
```

output
```
(2*sqrt(x)*x**2*(273*a**2 + 210*a*c*x**4 + 65*c**2*x**8))/1365
```

3.121 $\int \sqrt{x}(a+cx^4)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \sqrt{x}(a+cx^4)^2 \, dx = \frac{2}{3}a^2x^{3/2} + \frac{4}{11}acx^{11/2} + \frac{2}{19}c^2x^{19/2}$$

output
```
2/3*a^2*x^(3/2)+4/11*a*c*x^(11/2)+2/19*c^2*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \sqrt{x}(a+cx^4)^2 \, dx = \frac{2}{627}x^{3/2}(209a^2 + 114acx^4 + 33c^2x^8)$$

input
```
Integrate[Sqrt[x]*(a + c*x^4)^2,x]
```

output
```
(2*x^(3/2)*(209*a^2 + 114*a*c*x^4 + 33*c^2*x^8))/627
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+cx^4\right)^2 dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2\sqrt{x}+2acx^{9/2}+c^2x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{11}acx^{11/2}+\frac{2}{19}c^2x^{19/2}$$

input
```
Int[Sqrt[x]*(a + c*x^4)^2,x]
```

output
```
(2*a^2*x^(3/2))/3 + (4*a*c*x^(11/2))/11 + (2*c^2*x^(19/2))/19
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ac\,x^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{19}{2}}}{19}$	25
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ac\,x^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{19}{2}}}{19}$	25
gosper	$\frac{2x^{\frac{3}{2}}\left(33c^2x^8+114a\,x^4c+209a^2\right)}{627}$	27
trager	$\frac{2x^{\frac{3}{2}}\left(33c^2x^8+114a\,x^4c+209a^2\right)}{627}$	27
risch	$\frac{2x^{\frac{3}{2}}\left(33c^2x^8+114a\,x^4c+209a^2\right)}{627}$	27
orering	$\frac{2x^{\frac{3}{2}}\left(33c^2x^8+114a\,x^4c+209a^2\right)}{627}$	27

input
```
int(x^(1/2)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*a^2*x^(3/2)+4/11*a*c*x^(11/2)+2/19*c^2*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\left(a+cx^4\right)^2\,dx=\frac{2}{627}\left(33\,c^2x^9+114\,acx^5+209\,a^2x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
2/627*(33*c^2*x^9 + 114*a*c*x^5 + 209*a^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\left(a+cx^4\right)^2\,dx=\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4acx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(1/2)*(c*x**4+a)**2,x)
```

output
```
2*a**2*x**(3/2)/3 + 4*a*c*x**(11/2)/11 + 2*c**2*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(a+cx^4\right)^2\,dx=\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
2/19*c^2*x^(19/2) + 4/11*a*c*x^(11/2) + 2/3*a^2*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(a+cx^4\right)^2\,dx=\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+a)^2,x, algorithm="giac")
```

output
```
2/19*c^2*x^(19/2) + 4/11*a*c*x^(11/2) + 2/3*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\left(a+cx^4\right)^2\,dx=\frac{2\,x^{3/2}\,\left(209\,a^2+114\,a\,c\,x^4+33\,c^2\,x^8\right)}{627}$$

input
```
int(x^(1/2)*(a + c*x^4)^2,x)
```

output
```
(2*x^(3/2)*(209*a^2 + 33*c^2*x^8 + 114*a*c*x^4))/627
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\left(a+cx^4\right)^2\,dx=\frac{2\sqrt{x}\,x(33c^2x^8+114ac\,x^4+209a^2)}{627}$$

input
```
int(x^(1/2)*(c*x^4+a)^2,x)
```

output
```
(2*sqrt(x)*x*(209*a**2 + 114*a*c*x**4 + 33*c**2*x**8))/627
```

3.122 $\int \frac{(a+cx^4)^2}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(a+cx^4)^2}{\sqrt{x}} \, dx = 2a^2\sqrt{x} + \frac{4}{9}acx^{9/2} + \frac{2}{17}c^2x^{17/2}$$

output
```
2*a^2*x^(1/2)+4/9*a*c*x^(9/2)+2/17*c^2*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^4)^2}{\sqrt{x}} \, dx = \frac{2}{153}\sqrt{x}\left(153a^2 + 34acx^4 + 9c^2x^8\right)$$

input
```
Integrate[(a + c*x^4)^2/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(153*a^2 + 34*a*c*x^4 + 9*c^2*x^8))/153
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{\sqrt{x}}+2acx^{7/2}+c^2x^{15/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^2\sqrt{x}+\frac{4}{9}acx^{9/2}+\frac{2}{17}c^2x^{17/2}$$

input
```
Int[(a + c*x^4)^2/Sqrt[x],x]
```

output
```
2*a^2*Sqrt[x] + (4*a*c*x^(9/2))/9 + (2*c^2*x^(17/2))/17
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2a^2\sqrt{x}+\frac{4ac\,x^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{17}{2}}}{17}$	25
default	$2a^2\sqrt{x}+\frac{4ac\,x^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{17}{2}}}{17}$	25
trager	$\left(\frac{2}{17}c^2x^8+\frac{4}{9}a\,x^4c+2a^2\right)\sqrt{x}$	26
gosper	$\frac{2\sqrt{x}\,(9c^2x^8+34a\,x^4c+153a^2)}{153}$	27
risch	$\frac{2\sqrt{x}\,(9c^2x^8+34a\,x^4c+153a^2)}{153}$	27
orering	$\frac{2\sqrt{x}\,(9c^2x^8+34a\,x^4c+153a^2)}{153}$	27

input
```
int((c*x^4+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2*x^(1/2)+4/9*a*c*x^(9/2)+2/17*c^2*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+cx^4\right)^2}{\sqrt{x}}\,dx=\frac{2}{153}\left(9\,c^2x^8+34\,acx^4+153\,a^2\right)\sqrt{x}$$

input
```
integrate((c*x^4+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/153*(9*c^2*x^8 + 34*a*c*x^4 + 153*a^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^4)^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} + \frac{4acx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

input
```
integrate((c*x**4+a)**2/x**(1/2),x)
```

output
```
2*a**2*sqrt(x) + 4*a*c*x**(9/2)/9 + 2*c**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{9}\,acx^{\frac{9}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate((c*x^4+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/17*c^2*x^(17/2) + 4/9*a*c*x^(9/2) + 2*a^2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^2}{\sqrt{x}}\,dx = \frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{9}\,acx^{\frac{9}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate((c*x^4+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/17*c^2*x^(17/2) + 4/9*a*c*x^(9/2) + 2*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(153\,a^2+34\,a\,c\,x^4+9\,c^2\,x^8)}{153}$$

input
```
int((a + c*x^4)^2/x^(1/2),x)
```

output
```
(2*x^(1/2)*(153*a^2 + 9*c^2*x^8 + 34*a*c*x^4))/153
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{(a+cx^4)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(9c^2x^8+34ac\,x^4+153a^2)}{153}$$

input
```
int((c*x^4+a)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*(153*a**2 + 34*a*c*x**4 + 9*c**2*x**8))/153
```

3.123 $\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + \frac{4}{7}acx^{7/2} + \frac{2}{15}c^2x^{15/2}$$

output
```
-2*a^2/x^(1/2)+4/7*a*c*x^(7/2)+2/15*c^2*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = -\frac{2(105a^2-30acx^4-7c^2x^8)}{105\sqrt{x}}$$

input
```
Integrate[(a + c*x^4)^2/x^(3/2),x]
```

output
```
(-2*(105*a^2 - 30*a*c*x^4 - 7*c^2*x^8))/(105*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{x^{3/2}}+2acx^{5/2}+c^2x^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{7}acx^{7/2}+\frac{2}{15}c^2x^{15/2}$$

input
```
Int[(a + c*x^4)^2/x^(3/2),x]
```

output
```
(-2*a^2)/Sqrt[x] + (4*a*c*x^(7/2))/7 + (2*c^2*x^(15/2))/15
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2a^2}{\sqrt{x}}+\frac{4ac\,x^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{15}{2}}}{15}$	25
default	$-\frac{2a^2}{\sqrt{x}}+\frac{4ac\,x^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{15}{2}}}{15}$	25
gosper	$-\frac{2(-7c^2x^8-30a\,x^4c+105a^2)}{105\sqrt{x}}$	27
trager	$-\frac{2(-7c^2x^8-30a\,x^4c+105a^2)}{105\sqrt{x}}$	27
risch	$-\frac{2(-7c^2x^8-30a\,x^4c+105a^2)}{105\sqrt{x}}$	27
orering	$-\frac{2(-7c^2x^8-30a\,x^4c+105a^2)}{105\sqrt{x}}$	27

input
```
int((c*x^4+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/x^(1/2)+4/7*a*c*x^(7/2)+2/15*c^2*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx=\frac{2\left(7\,c^2x^8+30\,acx^4-105\,a^2\right)}{105\,\sqrt{x}}$$

input
```
integrate((c*x^4+a)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/105*(7*c^2*x^8 + 30*a*c*x^4 - 105*a^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + \frac{4acx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

input
```
integrate((c*x**4+a)**2/x**(3/2),x)
```

output
```
-2*a**2/sqrt(x) + 4*a*c*x**(7/2)/7 + 2*c**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{7}\,acx^{\frac{7}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((c*x^4+a)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/15*c^2*x^(15/2) + 4/7*a*c*x^(7/2) - 2*a^2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = \frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{7}\,acx^{\frac{7}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((c*x^4+a)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/15*c^2*x^(15/2) + 4/7*a*c*x^(7/2) - 2*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = \frac{-2\,a^2+\frac{4\,a\,c\,x^4}{7}+\frac{2\,c^2\,x^8}{15}}{\sqrt{x}}$$

input
```
int((a + c*x^4)^2/x^(3/2),x)
```

output
```
((2*c^2*x^8)/15 - 2*a^2 + (4*a*c*x^4)/7)/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(a+cx^4)^2}{x^{3/2}}\,dx = \frac{\frac{2}{15}c^2x^8+\frac{4}{7}ac\,x^4-2a^2}{\sqrt{x}}$$

input
```
int((c*x^4+a)^2/x^(3/2),x)
```

output
```
(2*( - 105*a**2 + 30*a*c*x**4 + 7*c**2*x**8))/(105*sqrt(x))
```

3.124 $\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{3/2}} + \frac{4}{5}acx^{5/2} + \frac{2}{13}c^2x^{13/2}$$

output
```
-2/3*a^2/x^(3/2)+4/5*a*c*x^(5/2)+2/13*c^2*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = -\frac{2(65a^2-78acx^4-15c^2x^8)}{195x^{3/2}}$$

input
```
Integrate[(a + c*x^4)^2/x^(5/2),x]
```

output
```
(-2*(65*a^2 - 78*a*c*x^4 - 15*c^2*x^8))/(195*x^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^2}{x^{5/2}}+2acx^{3/2}+c^2x^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{3x^{3/2}}+\frac{4}{5}acx^{5/2}+\frac{2}{13}c^2x^{13/2}$$

input
```
Int[(a + c*x^4)^2/x^(5/2),x]
```

output
```
(-2*a^2)/(3*x^(3/2)) + (4*a*c*x^(5/2))/5 + (2*c^2*x^(13/2))/13
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2a^2}{3x^{\frac{3}{2}}}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2c^2x^{\frac{13}{2}}}{13}$	25
default	$-\frac{2a^2}{3x^{\frac{3}{2}}}+\frac{4ac\,x^{\frac{5}{2}}}{5}+\frac{2c^2x^{\frac{13}{2}}}{13}$	25
gosper	$-\frac{2(-15c^2x^8-78a\,x^4c+65a^2)}{195x^{\frac{3}{2}}}$	27
trager	$-\frac{2(-15c^2x^8-78a\,x^4c+65a^2)}{195x^{\frac{3}{2}}}$	27
risch	$-\frac{2(-15c^2x^8-78a\,x^4c+65a^2)}{195x^{\frac{3}{2}}}$	27
orering	$-\frac{2(-15c^2x^8-78a\,x^4c+65a^2)}{195x^{\frac{3}{2}}}$	27

input
```
int((c*x^4+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2/x^(3/2)+4/5*a*c*x^(5/2)+2/13*c^2*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = \frac{2\,(15\,c^2x^8+78\,acx^4-65\,a^2)}{195\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/195*(15*c^2*x^8 + 78*a*c*x^4 - 65*a^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{\frac{3}{2}}}+\frac{4acx^{\frac{5}{2}}}{5}+\frac{2c^2x^{\frac{13}{2}}}{13}$$

input
```
integrate((c*x**4+a)**2/x**(5/2),x)
```

output
```
-2*a**2/(3*x**(3/2)) + 4*a*c*x**(5/2)/5 + 2*c**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{5}\,acx^{\frac{5}{2}}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/13*c^2*x^(13/2) + 4/5*a*c*x^(5/2) - 2/3*a^2/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = \frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{5}\,acx^{\frac{5}{2}}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/13*c^2*x^(13/2) + 4/5*a*c*x^(5/2) - 2/3*a^2/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = \frac{-130\,a^2+156\,a\,c\,x^4+30\,c^2\,x^8}{195\,x^{3/2}}$$

input
```
int((a + c*x^4)^2/x^(5/2),x)
```

output
```
(30*c^2*x^8 - 130*a^2 + 156*a*c*x^4)/(195*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^4)^2}{x^{5/2}}\,dx = \frac{\frac{2}{13}c^2x^8+\frac{4}{5}ac\,x^4-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((c*x^4+a)^2/x^(5/2),x)
```

output
```
(2*( - 65*a**2 + 78*a*c*x**4 + 15*c**2*x**8))/(195*sqrt(x)*x)
```

3.125 $\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = -\frac{2a^2}{5x^{5/2}} + \frac{4}{3}acx^{3/2} + \frac{2}{11}c^2x^{11/2}$$

output
```
-2/5*a^2/x^(5/2)+4/3*a*c*x^(3/2)+2/11*c^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = -\frac{2(33a^2-110acx^4-15c^2x^8)}{165x^{5/2}}$$

input
```
Integrate[(a + c*x^4)^2/x^(7/2),x]
```

output
```
(-2*(33*a^2 - 110*a*c*x^4 - 15*c^2*x^8))/(165*x^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{ 802}$$

$$\int \left(\frac{a^2}{x^{7/2}}+2ac\sqrt{x}+c^2x^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^2}{5x^{5/2}}+\frac{4}{3}acx^{3/2}+\frac{2}{11}c^2x^{11/2}$$

input
```
Int[(a + c*x^4)^2/x^(7/2),x]
```

output
```
(-2*a^2)/(5*x^(5/2)) + (4*a*c*x^(3/2))/3 + (2*c^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{2a^2}{5x^{\frac{5}{2}}}+\frac{4ac\,x^{\frac{3}{2}}}{3}+\frac{2c^2x^{\frac{11}{2}}}{11}$	25
default	$-\frac{2a^2}{5x^{\frac{5}{2}}}+\frac{4ac\,x^{\frac{3}{2}}}{3}+\frac{2c^2x^{\frac{11}{2}}}{11}$	25
gosper	$-\frac{2(-15c^2x^8-110a\,x^4c+33a^2)}{165x^{\frac{5}{2}}}$	27
trager	$-\frac{2(-15c^2x^8-110a\,x^4c+33a^2)}{165x^{\frac{5}{2}}}$	27
risch	$-\frac{2(-15c^2x^8-110a\,x^4c+33a^2)}{165x^{\frac{5}{2}}}$	27
orering	$-\frac{2(-15c^2x^8-110a\,x^4c+33a^2)}{165x^{\frac{5}{2}}}$	27

input
```
int((c*x^4+a)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^2/x^(5/2)+4/3*a*c*x^(3/2)+2/11*c^2*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx=\frac{2\,(15\,c^2x^8+110\,acx^4-33\,a^2)}{165\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/165*(15*c^2*x^8 + 110*a*c*x^4 - 33*a^2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = -\frac{2a^2}{5x^{\frac{5}{2}}} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((c*x**4+a)**2/x**(7/2),x)
```

output
```
-2*a**2/(5*x**(5/2)) + 4*a*c*x**(3/2)/3 + 2*c**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{3}\,acx^{\frac{3}{2}} - \frac{2\,a^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/11*c^2*x^(11/2) + 4/3*a*c*x^(3/2) - 2/5*a^2/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = \frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{3}\,acx^{\frac{3}{2}} - \frac{2\,a^2}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/11*c^2*x^(11/2) + 4/3*a*c*x^(3/2) - 2/5*a^2/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = \frac{-66\,a^2+220\,a\,c\,x^4+30\,c^2\,x^8}{165\,x^{5/2}}$$

input
```
int((a + c*x^4)^2/x^(7/2),x)
```

output
```
(30*c^2*x^8 - 66*a^2 + 220*a*c*x^4)/(165*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{(a+cx^4)^2}{x^{7/2}}\,dx = \frac{\frac{2}{11}c^2x^8+\frac{4}{3}ac\,x^4-\frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+a)^2/x^(7/2),x)
```

output
```
(2*( - 33*a**2 + 110*a*c*x**4 + 15*c**2*x**8))/(165*sqrt(x)*x**2)
```

3.126 $\int x^{5/2}(a+cx^4)^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{5/2}(a+cx^4)^3 \, dx = \frac{2}{7}a^3x^{7/2} + \frac{2}{5}a^2cx^{15/2} + \frac{6}{23}ac^2x^{23/2} + \frac{2}{31}c^3x^{31/2}$$

output
```
2/7*a^3*x^(7/2)+2/5*a^2*c*x^(15/2)+6/23*a*c^2*x^(23/2)+2/31*c^3*x^(31/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int x^{5/2}(a+cx^4)^3 \, dx = \frac{2(3565a^3x^{7/2} + 4991a^2cx^{15/2} + 3255ac^2x^{23/2} + 805c^3x^{31/2})}{24955}$$

input
```
Integrate[x^(5/2)*(a + c*x^4)^3,x]
```

output
```
(2*(3565*a^3*x^(7/2) + 4991*a^2*c*x^(15/2) + 3255*a*c^2*x^(23/2) + 805*c^3
*x^(31/2)))/24955
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^{5/2}+3a^2cx^{13/2}+3ac^2x^{21/2}+c^3x^{29/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{5}a^2cx^{15/2}+\frac{6}{23}ac^2x^{23/2}+\frac{2}{31}c^3x^{31/2}$$

input
```
Int[x^(5/2)*(a + c*x^4)^3,x]
```

output
```
(2*a^3*x^(7/2))/7 + (2*a^2*c*x^(15/2))/5 + (6*a*c^2*x^(23/2))/23 + (2*c^3*
x^(31/2))/31
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{2a^2c\,x^{\frac{15}{2}}}{5}+\frac{6a\,c^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{31}{2}}}{31}$	36
default	$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{2a^2c\,x^{\frac{15}{2}}}{5}+\frac{6a\,c^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{31}{2}}}{31}$	36
gosper	$\frac{2x^{\frac{7}{2}}\left(805c^3x^{12}+3255a\,c^2x^8+4991a^2c\,x^4+3565a^3\right)}{24955}$	38
trager	$\frac{2x^{\frac{7}{2}}\left(805c^3x^{12}+3255a\,c^2x^8+4991a^2c\,x^4+3565a^3\right)}{24955}$	38
risch	$\frac{2x^{\frac{7}{2}}\left(805c^3x^{12}+3255a\,c^2x^8+4991a^2c\,x^4+3565a^3\right)}{24955}$	38
orering	$\frac{2x^{\frac{7}{2}}\left(805c^3x^{12}+3255a\,c^2x^8+4991a^2c\,x^4+3565a^3\right)}{24955}$	38

input
```
int(x^(5/2)*(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^3*x^(7/2)+2/5*a^2*c*x^(15/2)+6/23*a*c^2*x^(23/2)+2/31*c^3*x^(31/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx=\frac{2}{24955}\left(805\,c^3x^{15}+3255\,ac^2x^{11}+4991\,a^2cx^7+3565\,a^3x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
2/24955*(805*c^3*x^15 + 3255*a*c^2*x^11 + 4991*a^2*c*x^7 + 3565*a^3*x^3)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx=\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{2a^2cx^{\frac{15}{2}}}{5}+\frac{6ac^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

input
```
integrate(x**(5/2)*(c*x**4+a)**3,x)
```

output
```
2*a**3*x**(7/2)/7 + 2*a**2*c*x**(15/2)/5 + 6*a*c**2*x**(23/2)/23 + 2*c**3*
x**(31/2)/31
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx=\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{6}{23}\,ac^2x^{\frac{23}{2}}+\frac{2}{5}\,a^2cx^{\frac{15}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
2/31*c^3*x^(31/2) + 6/23*a*c^2*x^(23/2) + 2/5*a^2*c*x^(15/2) + 2/7*a^3*x^(
7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx=\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{6}{23}\,ac^2x^{\frac{23}{2}}+\frac{2}{5}\,a^2cx^{\frac{15}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(c*x^4+a)^3,x, algorithm="giac")
```

output
```
2/31*c^3*x^(31/2) + 6/23*a*c^2*x^(23/2) + 2/5*a^2*c*x^(15/2) + 2/7*a^3*x^(
7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx=\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,c^3\,x^{31/2}}{31}+\frac{2\,a^2\,c\,x^{15/2}}{5}+\frac{6\,a\,c^2\,x^{23/2}}{23}$$

input
```
int(x^(5/2)*(a + c*x^4)^3,x)
```

output
```
(2*a^3*x^(7/2))/7 + (2*c^3*x^(31/2))/31 + (2*a^2*c*x^(15/2))/5 + (6*a*c^2*
x^(23/2))/23
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{5/2}\left(a+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^3(805c^3x^{12}+3255a\,c^2x^8+4991a^2c\,x^4+3565a^3)}{24955}$$

input
```
int(x^(5/2)*(c*x^4+a)^3,x)
```

output
```
(2*sqrt(x)*x**3*(3565*a**3 + 4991*a**2*c*x**4 + 3255*a*c**2*x**8 + 805*c**
3*x**12))/24955
```

3.127 $\int x^{3/2}(a+cx^4)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{3/2}(a+cx^4)^3\,dx=\frac{2}{5}a^3x^{5/2}+\frac{6}{13}a^2cx^{13/2}+\frac{2}{7}ac^2x^{21/2}+\frac{2}{29}c^3x^{29/2}$$

output
```
2/5*a^3*x^(5/2)+6/13*a^2*c*x^(13/2)+2/7*a*c^2*x^(21/2)+2/29*c^3*x^(29/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int x^{3/2}(a+cx^4)^3\,dx=\frac{2(2639a^3x^{5/2}+3045a^2cx^{13/2}+1885ac^2x^{21/2}+455c^3x^{29/2})}{13195}$$

input
```
Integrate[x^(3/2)*(a + c*x^4)^3,x]
```

output
```
(2*(2639*a^3*x^(5/2) + 3045*a^2*c*x^(13/2) + 1885*a*c^2*x^(21/2) + 455*c^3
*x^(29/2)))/13195
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3x^{3/2}+3a^2cx^{11/2}+3ac^2x^{19/2}+c^3x^{27/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{13}a^2cx^{13/2}+\frac{2}{7}ac^2x^{21/2}+\frac{2}{29}c^3x^{29/2}$$

input
```
Int[x^(3/2)*(a + c*x^4)^3,x]
```

output
```
(2*a^3*x^(5/2))/5 + (6*a^2*c*x^(13/2))/13 + (2*a*c^2*x^(21/2))/7 + (2*c^3*
x^(29/2))/29
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2c\,x^{\frac{13}{2}}}{13}+\frac{2a\,c^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{29}{2}}}{29}$	36
default	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2c\,x^{\frac{13}{2}}}{13}+\frac{2a\,c^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{29}{2}}}{29}$	36
gosper	$\frac{2x^{\frac{5}{2}}\left(455c^3x^{12}+1885a\,c^2x^8+3045a^2c\,x^4+2639a^3\right)}{13195}$	38
trager	$\frac{2x^{\frac{5}{2}}\left(455c^3x^{12}+1885a\,c^2x^8+3045a^2c\,x^4+2639a^3\right)}{13195}$	38
risch	$\frac{2x^{\frac{5}{2}}\left(455c^3x^{12}+1885a\,c^2x^8+3045a^2c\,x^4+2639a^3\right)}{13195}$	38
orering	$\frac{2x^{\frac{5}{2}}\left(455c^3x^{12}+1885a\,c^2x^8+3045a^2c\,x^4+2639a^3\right)}{13195}$	38

input
```
int(x^(3/2)*(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*a^3*x^(5/2)+6/13*a^2*c*x^(13/2)+2/7*a*c^2*x^(21/2)+2/29*c^3*x^(29/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx=\frac{2}{13195}\left(455\,c^3x^{14}+1885\,ac^2x^{10}+3045\,a^2cx^6+2639\,a^3x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
2/13195*(455*c^3*x^14 + 1885*a*c^2*x^10 + 3045*a^2*c*x^6 + 2639*a^3*x^2)*s
qrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx=\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{6a^2cx^{\frac{13}{2}}}{13}+\frac{2ac^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

input
```
integrate(x**(3/2)*(c*x**4+a)**3,x)
```

output
```
2*a**3*x**(5/2)/5 + 6*a**2*c*x**(13/2)/13 + 2*a*c**2*x**(21/2)/7 + 2*c**3*
x**(29/2)/29
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx=\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
2/29*c^3*x^(29/2) + 2/7*a*c^2*x^(21/2) + 6/13*a^2*c*x^(13/2) + 2/5*a^3*x^(
5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx=\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(c*x^4+a)^3,x, algorithm="giac")
```

output
```
2/29*c^3*x^(29/2) + 2/7*a*c^2*x^(21/2) + 6/13*a^2*c*x^(13/2) + 2/5*a^3*x^(
5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx=\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,c^3\,x^{29/2}}{29}+\frac{6\,a^2\,c\,x^{13/2}}{13}+\frac{2\,a\,c^2\,x^{21/2}}{7}$$

input
```
int(x^(3/2)*(a + c*x^4)^3,x)
```

output
```
(2*a^3*x^(5/2))/5 + (2*c^3*x^(29/2))/29 + (6*a^2*c*x^(13/2))/13 + (2*a*c^2
*x^(21/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{3/2}\left(a+cx^4\right)^3\,dx=\frac{2\sqrt{x}\,x^2(455c^3x^{12}+1885a\,c^2x^8+3045a^2c\,x^4+2639a^3)}{13195}$$

input
```
int(x^(3/2)*(c*x^4+a)^3,x)
```

output
```
(2*sqrt(x)*x**2*(2639*a**3 + 3045*a**2*c*x**4 + 1885*a*c**2*x**8 + 455*c**
3*x**12))/13195
```

3.128 $\int \sqrt{x}(a+cx^4)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{x}(a+cx^4)^3\,dx=\frac{2}{3}a^3x^{3/2}+\frac{6}{11}a^2cx^{11/2}+\frac{6}{19}ac^2x^{19/2}+\frac{2}{27}c^3x^{27/2}$$

output
```
2/3*a^3*x^(3/2)+6/11*a^2*c*x^(11/2)+6/19*a*c^2*x^(19/2)+2/27*c^3*x^(27/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \sqrt{x}(a+cx^4)^3\,dx=\frac{2x^{3/2}(1881a^3+1539a^2cx^4+891ac^2x^8+209c^3x^{12})}{5643}$$

input
```
Integrate[Sqrt[x]*(a + c*x^4)^3,x]
```

output
```
(2*x^(3/2)*(1881*a^3 + 1539*a^2*c*x^4 + 891*a*c^2*x^8 + 209*c^3*x^12))/564
3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+cx^4\right)^3 \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3\sqrt{x}+3a^2cx^{9/2}+3ac^2x^{17/2}+c^3x^{25/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{11}a^2cx^{11/2}+\frac{6}{19}ac^2x^{19/2}+\frac{2}{27}c^3x^{27/2}$$

input
```
Int[Sqrt[x]*(a + c*x^4)^3,x]
```

output
```
(2*a^3*x^(3/2))/3 + (6*a^2*c*x^(11/2))/11 + (6*a*c^2*x^(19/2))/19 + (2*c^3
*x^(27/2))/27
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2c\,x^{\frac{11}{2}}}{11}+\frac{6a\,c^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{27}{2}}}{27}$	36
default	$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2c\,x^{\frac{11}{2}}}{11}+\frac{6a\,c^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{27}{2}}}{27}$	36
gosper	$\frac{2x^{\frac{3}{2}}\left(209c^3x^{12}+891a\,c^2x^8+1539a^2c\,x^4+1881a^3\right)}{5643}$	38
trager	$\frac{2x^{\frac{3}{2}}\left(209c^3x^{12}+891a\,c^2x^8+1539a^2c\,x^4+1881a^3\right)}{5643}$	38
risch	$\frac{2x^{\frac{3}{2}}\left(209c^3x^{12}+891a\,c^2x^8+1539a^2c\,x^4+1881a^3\right)}{5643}$	38
orering	$\frac{2x^{\frac{3}{2}}\left(209c^3x^{12}+891a\,c^2x^8+1539a^2c\,x^4+1881a^3\right)}{5643}$	38

input
```
int(x^(1/2)*(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*a^3*x^(3/2)+6/11*a^2*c*x^(11/2)+6/19*a*c^2*x^(19/2)+2/27*c^3*x^(27/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\left(a+cx^4\right)^3\,dx=\frac{2}{5643}\left(209\,c^3x^{13}+891\,ac^2x^9+1539\,a^2cx^5+1881\,a^3x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
2/5643*(209*c^3*x^13 + 891*a*c^2*x^9 + 1539*a^2*c*x^5 + 1881*a^3*x)*sqrt(x
)
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \sqrt{x}\left(a+cx^4\right)^3\,dx=\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2cx^{\frac{11}{2}}}{11}+\frac{6ac^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{27}{2}}}{27}$$

input
```
integrate(x**(1/2)*(c*x**4+a)**3,x)
```

output
```
2*a**3*x**(3/2)/3 + 6*a**2*c*x**(11/2)/11 + 6*a*c**2*x**(19/2)/19 + 2*c**3
*x**(27/2)/27
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(a+cx^4\right)^3\,dx=\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{19}\,ac^2x^{\frac{19}{2}}+\frac{6}{11}\,a^2cx^{\frac{11}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
2/27*c^3*x^(27/2) + 6/19*a*c^2*x^(19/2) + 6/11*a^2*c*x^(11/2) + 2/3*a^3*x^
(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(a+cx^4\right)^3\,dx=\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{19}\,ac^2x^{\frac{19}{2}}+\frac{6}{11}\,a^2cx^{\frac{11}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(c*x^4+a)^3,x, algorithm="giac")
```

output
```
2/27*c^3*x^(27/2) + 6/19*a*c^2*x^(19/2) + 6/11*a^2*c*x^(11/2) + 2/3*a^3*x^
(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+cx^4)^3\, dx = \frac{2\,a^3\,x^{3/2}}{3} + \frac{2\,c^3\,x^{27/2}}{27} + \frac{6\,a^2\,c\,x^{11/2}}{11} + \frac{6\,a\,c^2\,x^{19/2}}{19}$$

input
```
int(x^(1/2)*(a + c*x^4)^3,x)
```

output
```
(2*a^3*x^(3/2))/3 + (2*c^3*x^(27/2))/27 + (6*a^2*c*x^(11/2))/11 + (6*a*c^2
*x^(19/2))/19
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \sqrt{x}(a+cx^4)^3\, dx = \frac{2\sqrt{x}\,x(209c^3x^{12} + 891a\,c^2x^8 + 1539a^2c\,x^4 + 1881a^3)}{5643}$$

input
```
int(x^(1/2)*(c*x^4+a)^3,x)
```

output
```
(2*sqrt(x)*x*(1881*a**3 + 1539*a**2*c*x**4 + 891*a*c**2*x**8 + 209*c**3*x*
*12))/5643
```

3.129 $\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x}+\frac{2}{3}a^2cx^{9/2}+\frac{6}{17}ac^2x^{17/2}+\frac{2}{25}c^3x^{25/2}$$

output
```
2*a^3*x^(1/2)+2/3*a^2*c*x^(9/2)+6/17*a*c^2*x^(17/2)+2/25*c^3*x^(25/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}(1275a^3+425a^2cx^4+225ac^2x^8+51c^3x^{12})}{1275}$$

input
```
Integrate[(a + c*x^4)^3/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(1275*a^3 + 425*a^2*c*x^4 + 225*a*c^2*x^8 + 51*c^3*x^12))/1275
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^3}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{\sqrt{x}}+3a^2cx^{7/2}+3ac^2x^{15/2}+c^3x^{23/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a^3\sqrt{x}+\frac{2}{3}a^2cx^{9/2}+\frac{6}{17}ac^2x^{17/2}+\frac{2}{25}c^3x^{25/2}$$

input
```
Int[(a + c*x^4)^3/Sqrt[x],x]
```

output
```
2*a^3*Sqrt[x] + (2*a^2*c*x^(9/2))/3 + (6*a*c^2*x^(17/2))/17 + (2*c^3*x^(25
/2))/25
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$2a^{3}\sqrt{x}+\frac{2a^{2}c\,x^{\frac{9}{2}}}{3}+\frac{6a\,c^{2}x^{\frac{17}{2}}}{17}+\frac{2c^{3}x^{\frac{25}{2}}}{25}$	36
default	$2a^{3}\sqrt{x}+\frac{2a^{2}c\,x^{\frac{9}{2}}}{3}+\frac{6a\,c^{2}x^{\frac{17}{2}}}{17}+\frac{2c^{3}x^{\frac{25}{2}}}{25}$	36
trager	$\left(\frac{2}{25}c^{3}x^{12}+\frac{6}{17}a\,c^{2}x^{8}+\frac{2}{3}a^{2}c\,x^{4}+2a^{3}\right)\sqrt{x}$	37
gosper	$\frac{2\sqrt{x}\,(51c^{3}x^{12}+225a\,c^{2}x^{8}+425a^{2}c\,x^{4}+1275a^{3})}{1275}$	38
risch	$\frac{2\sqrt{x}\,(51c^{3}x^{12}+225a\,c^{2}x^{8}+425a^{2}c\,x^{4}+1275a^{3})}{1275}$	38
orering	$\frac{2\sqrt{x}\,(51c^{3}x^{12}+225a\,c^{2}x^{8}+425a^{2}c\,x^{4}+1275a^{3})}{1275}$	38

input
```
int((c*x^4+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^3*x^(1/2)+2/3*a^2*c*x^(9/2)+6/17*a*c^2*x^(17/2)+2/25*c^3*x^(25/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx=\frac{2}{1275}\left(51\,c^3x^{12}+225\,ac^2x^8+425\,a^2cx^4+1275\,a^3\right)\sqrt{x}$$

input
```
integrate((c*x^4+a)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/1275*(51*c^3*x^12 + 225*a*c^2*x^8 + 425*a^2*c*x^4 + 1275*a^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} + \frac{2a^2cx^{\frac{9}{2}}}{3} + \frac{6ac^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

input
```
integrate((c*x**4+a)**3/x**(1/2),x)
```

output
```
2*a**3*sqrt(x) + 2*a**2*c*x**(9/2)/3 + 6*a*c**2*x**(17/2)/17 + 2*c**3*x**(
25/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{6}{17}\,ac^2x^{\frac{17}{2}} + \frac{2}{3}\,a^2cx^{\frac{9}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((c*x^4+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/25*c^3*x^(25/2) + 6/17*a*c^2*x^(17/2) + 2/3*a^2*c*x^(9/2) + 2*a^3*sqrt(x
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = \frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{6}{17}\,ac^2x^{\frac{17}{2}} + \frac{2}{3}\,a^2cx^{\frac{9}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((c*x^4+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/25*c^3*x^(25/2) + 6/17*a*c^2*x^(17/2) + 2/3*a^2*c*x^(9/2) + 2*a^3*sqrt(x
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = 2\,a^3\,\sqrt{x}+\frac{2\,c^3\,x^{25/2}}{25}+\frac{2\,a^2\,c\,x^{9/2}}{3}+\frac{6\,a\,c^2\,x^{17/2}}{17}$$

input
```
int((a + c*x^4)^3/x^(1/2),x)
```

output
```
2*a^3*x^(1/2) + (2*c^3*x^(25/2))/25 + (2*a^2*c*x^(9/2))/3 + (6*a*c^2*x^(17
/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{(a+cx^4)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(51c^3x^{12}+225a\,c^2x^8+425a^2c\,x^4+1275a^3)}{1275}$$

input
```
int((c*x^4+a)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*(1275*a**3 + 425*a**2*c*x**4 + 225*a*c**2*x**8 + 51*c**3*x**12)
)/1275
```

3.130 $\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + \frac{6}{7}a^2cx^{7/2} + \frac{2}{5}ac^2x^{15/2} + \frac{2}{23}c^3x^{23/2}$$

output
```
-2*a^3/x^(1/2)+6/7*a^2*c*x^(7/2)+2/5*a*c^2*x^(15/2)+2/23*c^3*x^(23/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = -\frac{2(805a^3 - 345a^2cx^4 - 161ac^2x^8 - 35c^3x^{12})}{805\sqrt{x}}$$

input
```
Integrate[(a + c*x^4)^3/x^(3/2),x]
```

output
```
(-2*(805*a^3 - 345*a^2*c*x^4 - 161*a*c^2*x^8 - 35*c^3*x^12))/(805*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^{3/2}}+3a^2cx^{5/2}+3ac^2x^{13/2}+c^3x^{21/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3}{\sqrt{x}}+\frac{6}{7}a^2cx^{7/2}+\frac{2}{5}ac^2x^{15/2}+\frac{2}{23}c^3x^{23/2}$$

input
```
Int[(a + c*x^4)^3/x^(3/2),x]
```

output
```
(-2*a^3)/Sqrt[x] + (6*a^2*c*x^(7/2))/7 + (2*a*c^2*x^(15/2))/5 + (2*c^3*x^(
23/2))/23
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$-\frac{2a^3}{\sqrt{x}}+\frac{6a^2c\,x^{\frac{7}{2}}}{7}+\frac{2a\,c^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{23}{2}}}{23}$	36
default	$-\frac{2a^3}{\sqrt{x}}+\frac{6a^2c\,x^{\frac{7}{2}}}{7}+\frac{2a\,c^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{23}{2}}}{23}$	36
gosper	$-\frac{2(-35c^3x^{12}-161a\,c^2x^8-345a^2c\,x^4+805a^3)}{805\sqrt{x}}$	38
trager	$-\frac{2(-35c^3x^{12}-161a\,c^2x^8-345a^2c\,x^4+805a^3)}{805\sqrt{x}}$	38
risch	$-\frac{2(-35c^3x^{12}-161a\,c^2x^8-345a^2c\,x^4+805a^3)}{805\sqrt{x}}$	38
orering	$-\frac{2(-35c^3x^{12}-161a\,c^2x^8-345a^2c\,x^4+805a^3)}{805\sqrt{x}}$	38

input
```
int((c*x^4+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/x^(1/2)+6/7*a^2*c*x^(7/2)+2/5*a*c^2*x^(15/2)+2/23*c^3*x^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = \frac{2\,(35\,c^3x^{12}+161\,ac^2x^8+345\,a^2cx^4-805\,a^3)}{805\,\sqrt{x}}$$

input
```
integrate((c*x^4+a)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/805*(35*c^3*x^12 + 161*a*c^2*x^8 + 345*a^2*c*x^4 - 805*a^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + \frac{6a^2cx^{\frac{7}{2}}}{7} + \frac{2ac^2x^{\frac{15}{2}}}{5} + \frac{2c^3x^{\frac{23}{2}}}{23}$$

input
```
integrate((c*x**4+a)**3/x**(3/2),x)
```

output
```
-2*a**3/sqrt(x) + 6*a**2*c*x**(7/2)/7 + 2*a*c**2*x**(15/2)/5 + 2*c**3*x**(
23/2)/23
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{2}{5}\,ac^2x^{\frac{15}{2}} + \frac{6}{7}\,a^2cx^{\frac{7}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((c*x^4+a)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/23*c^3*x^(23/2) + 2/5*a*c^2*x^(15/2) + 6/7*a^2*c*x^(7/2) - 2*a^3/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = \frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{2}{5}\,ac^2x^{\frac{15}{2}} + \frac{6}{7}\,a^2cx^{\frac{7}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((c*x^4+a)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/23*c^3*x^(23/2) + 2/5*a*c^2*x^(15/2) + 6/7*a^2*c*x^(7/2) - 2*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = \frac{2\,c^3\,x^{23/2}}{23} - \frac{2\,a^3}{\sqrt{x}} + \frac{6\,a^2\,c\,x^{7/2}}{7} + \frac{2\,a\,c^2\,x^{15/2}}{5}$$

input
```
int((a + c*x^4)^3/x^(3/2),x)
```

output
```
(2*c^3*x^(23/2))/23 - (2*a^3)/x^(1/2) + (6*a^2*c*x^(7/2))/7 + (2*a*c^2*x^(
15/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{(a+cx^4)^3}{x^{3/2}}\,dx = \frac{\frac{2}{23}c^3x^{12} + \frac{2}{5}a\,c^2x^8 + \frac{6}{7}a^2c\,x^4 - 2a^3}{\sqrt{x}}$$

input
```
int((c*x^4+a)^3/x^(3/2),x)
```

output
```
(2*( - 805*a**3 + 345*a**2*c*x**4 + 161*a*c**2*x**8 + 35*c**3*x**12))/(805
*sqrt(x))
```

3.131 $\int \frac{(a+cx^4)^3}{x^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+cx^4)^3}{x^{5/2}} \, dx = -\frac{2a^3}{3x^{3/2}} + \frac{6}{5}a^2cx^{5/2} + \frac{6}{13}ac^2x^{13/2} + \frac{2}{21}c^3x^{21/2}$$

output
```
-2/3*a^3/x^(3/2)+6/5*a^2*c*x^(5/2)+6/13*a*c^2*x^(13/2)+2/21*c^3*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(a+cx^4)^3}{x^{5/2}} \, dx = -\frac{2(455a^3 - 819a^2cx^4 - 315ac^2x^8 - 65c^3x^{12})}{1365x^{3/2}}$$

input
```
Integrate[(a + c*x^4)^3/x^(5/2),x]
```

output
```
(-2*(455*a^3 - 819*a^2*c*x^4 - 315*a*c^2*x^8 - 65*c^3*x^12))/(1365*x^(3/2)
)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^3}{x^{5/2}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^{5/2}}+3a^2cx^{3/2}+3ac^2x^{11/2}+c^3x^{19/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3}{3x^{3/2}}+\frac{6}{5}a^2cx^{5/2}+\frac{6}{13}ac^2x^{13/2}+\frac{2}{21}c^3x^{21/2}$$

input
```
Int[(a + c*x^4)^3/x^(5/2),x]
```

output
```
(-2*a^3)/(3*x^(3/2)) + (6*a^2*c*x^(5/2))/5 + (6*a*c^2*x^(13/2))/13 + (2*c^
3*x^(21/2))/21
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{2a^3}{3x^{\frac{3}{2}}}+\frac{6a^2 c\,x^{\frac{5}{2}}}{5}+\frac{6a\,c^2 x^{\frac{13}{2}}}{13}+\frac{2c^3 x^{\frac{21}{2}}}{21}$	36
default	$-\frac{2a^3}{3x^{\frac{3}{2}}}+\frac{6a^2 c\,x^{\frac{5}{2}}}{5}+\frac{6a\,c^2 x^{\frac{13}{2}}}{13}+\frac{2c^3 x^{\frac{21}{2}}}{21}$	36
gosper	$-\frac{2\left(-65c^3x^{12}-315a\,c^2x^8-819a^2c\,x^4+455a^3\right)}{1365x^{\frac{3}{2}}}$	38
trager	$-\frac{2\left(-65c^3x^{12}-315a\,c^2x^8-819a^2c\,x^4+455a^3\right)}{1365x^{\frac{3}{2}}}$	38
risch	$-\frac{2\left(-65c^3x^{12}-315a\,c^2x^8-819a^2c\,x^4+455a^3\right)}{1365x^{\frac{3}{2}}}$	38
orering	$-\frac{2\left(-65c^3x^{12}-315a\,c^2x^8-819a^2c\,x^4+455a^3\right)}{1365x^{\frac{3}{2}}}$	38

input
```
int((c*x^4+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^3/x^(3/2)+6/5*a^2*c*x^(5/2)+6/13*a*c^2*x^(13/2)+2/21*c^3*x^(21/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{(a+cx^4)^3}{x^{5/2}}\,dx=\frac{2\,(65\,c^3x^{12}+315\,ac^2x^8+819\,a^2cx^4-455\,a^3)}{1365\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/1365*(65*c^3*x^12 + 315*a*c^2*x^8 + 819*a^2*c*x^4 - 455*a^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{(a+cx^4)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{\frac{3}{2}}} + \frac{6a^2cx^{\frac{5}{2}}}{5} + \frac{6ac^2x^{\frac{13}{2}}}{13} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

input
```
integrate((c*x**4+a)**3/x**(5/2),x)
```

output
```
-2*a**3/(3*x**(3/2)) + 6*a**2*c*x**(5/2)/5 + 6*a*c**2*x**(13/2)/13 + 2*c**
3*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{13}\,ac^2x^{\frac{13}{2}} + \frac{6}{5}\,a^2cx^{\frac{5}{2}} - \frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/21*c^3*x^(21/2) + 6/13*a*c^2*x^(13/2) + 6/5*a^2*c*x^(5/2) - 2/3*a^3/x^(3
/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+cx^4)^3}{x^{5/2}}\,dx = \frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{13}\,ac^2x^{\frac{13}{2}} + \frac{6}{5}\,a^2cx^{\frac{5}{2}} - \frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((c*x^4+a)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/21*c^3*x^(21/2) + 6/13*a*c^2*x^(13/2) + 6/5*a^2*c*x^(5/2) - 2/3*a^3/x^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{(a+cx^4)^3}{x^{5/2}}\,dx = \frac{2\,c^3\,x^{21/2}}{21} - \frac{2\,a^3}{3\,x^{3/2}} + \frac{6\,a^2\,c\,x^{5/2}}{5} + \frac{6\,a\,c^2\,x^{13/2}}{13}$$

input
```
int((a + c*x^4)^3/x^(5/2),x)
```

output
```
(2*c^3*x^(21/2))/21 - (2*a^3)/(3*x^(3/2)) + (6*a^2*c*x^(5/2))/5 + (6*a*c^2
*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{(a+cx^4)^3}{x^{5/2}}\,dx = \frac{\frac{2}{21}c^3x^{12} + \frac{6}{13}a\,c^2x^8 + \frac{6}{5}a^2c\,x^4 - \frac{2}{3}a^3}{\sqrt{x}\,x}$$

input
```
int((c*x^4+a)^3/x^(5/2),x)
```

output
```
(2*( - 455*a**3 + 819*a**2*c*x**4 + 315*a*c**2*x**8 + 65*c**3*x**12))/(136
5*sqrt(x)*x)
```

3.132 $\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = -\frac{2a^3}{5x^{5/2}} + 2a^2cx^{3/2} + \frac{6}{11}ac^2x^{11/2} + \frac{2}{19}c^3x^{19/2}$$

output
```
-2/5*a^3/x^(5/2)+2*a^2*c*x^(3/2)+6/11*a*c^2*x^(11/2)+2/19*c^3*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = -\frac{2(209a^3 - 1045a^2cx^4 - 285ac^2x^8 - 55c^3x^{12})}{1045x^{5/2}}$$

input
```
Integrate[(a + c*x^4)^3/x^(7/2),x]
```

output
```
(-2*(209*a^3 - 1045*a^2*c*x^4 - 285*a*c^2*x^8 - 55*c^3*x^12))/(1045*x^(5/2
))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^3}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{a^3}{x^{7/2}}+3a^2c\sqrt{x}+3ac^2x^{9/2}+c^3x^{17/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^3}{5x^{5/2}}+2a^2cx^{3/2}+\frac{6}{11}ac^2x^{11/2}+\frac{2}{19}c^3x^{19/2}$$

input
```
Int[(a + c*x^4)^3/x^(7/2),x]
```

output
```
(-2*a^3)/(5*x^(5/2)) + 2*a^2*c*x^(3/2) + (6*a*c^2*x^(11/2))/11 + (2*c^3*x^
(19/2))/19
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$-\frac{2a^{3}}{5x^{\frac{5}{2}}}+2a^{2}c\,x^{\frac{3}{2}}+\frac{6a\,c^{2}x^{\frac{11}{2}}}{11}+\frac{2c^{3}x^{\frac{19}{2}}}{19}$	36
default	$-\frac{2a^{3}}{5x^{\frac{5}{2}}}+2a^{2}c\,x^{\frac{3}{2}}+\frac{6a\,c^{2}x^{\frac{11}{2}}}{11}+\frac{2c^{3}x^{\frac{19}{2}}}{19}$	36
gosper	$-\frac{2\left(-55c^{3}x^{12}-285a\,c^{2}x^{8}-1045a^{2}c\,x^{4}+209a^{3}\right)}{1045x^{\frac{5}{2}}}$	38
trager	$-\frac{2\left(-55c^{3}x^{12}-285a\,c^{2}x^{8}-1045a^{2}c\,x^{4}+209a^{3}\right)}{1045x^{\frac{5}{2}}}$	38
risch	$-\frac{2\left(-55c^{3}x^{12}-285a\,c^{2}x^{8}-1045a^{2}c\,x^{4}+209a^{3}\right)}{1045x^{\frac{5}{2}}}$	38
orering	$-\frac{2\left(-55c^{3}x^{12}-285a\,c^{2}x^{8}-1045a^{2}c\,x^{4}+209a^{3}\right)}{1045x^{\frac{5}{2}}}$	38

input
```
int((c*x^4+a)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^3/x^(5/2)+2*a^2*c*x^(3/2)+6/11*a*c^2*x^(11/2)+2/19*c^3*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = \frac{2\,(55\,c^3x^{12}+285\,ac^2x^8+1045\,a^2cx^4-209\,a^3)}{1045\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/1045*(55*c^3*x^12 + 285*a*c^2*x^8 + 1045*a^2*c*x^4 - 209*a^3)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = -\frac{2a^3}{5x^{\frac{5}{2}}} + 2a^2cx^{\frac{3}{2}} + \frac{6ac^2x^{\frac{11}{2}}}{11} + \frac{2c^3x^{\frac{19}{2}}}{19}$$

input
```
integrate((c*x**4+a)**3/x**(7/2),x)
```

output
```
-2*a**3/(5*x**(5/2)) + 2*a**2*c*x**(3/2) + 6*a*c**2*x**(11/2)/11 + 2*c**3*
x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{6}{11}\,ac^2x^{\frac{11}{2}} + 2\,a^2cx^{\frac{3}{2}} - \frac{2\,a^3}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/19*c^3*x^(19/2) + 6/11*a*c^2*x^(11/2) + 2*a^2*c*x^(3/2) - 2/5*a^3/x^(5/2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = \frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{6}{11}\,ac^2x^{\frac{11}{2}} + 2\,a^2cx^{\frac{3}{2}} - \frac{2\,a^3}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((c*x^4+a)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/19*c^3*x^(19/2) + 6/11*a*c^2*x^(11/2) + 2*a^2*c*x^(3/2) - 2/5*a^3/x^(5/2
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = \frac{2\,c^3\,x^{19/2}}{19} - \frac{2\,a^3}{5\,x^{5/2}} + 2\,a^2\,c\,x^{3/2} + \frac{6\,a\,c^2\,x^{11/2}}{11}$$

input
```
int((a + c*x^4)^3/x^(7/2),x)
```

output
```
(2*c^3*x^(19/2))/19 - (2*a^3)/(5*x^(5/2)) + 2*a^2*c*x^(3/2) + (6*a*c^2*x^(
11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+cx^4)^3}{x^{7/2}}\,dx = \frac{\frac{2}{19}c^3x^{12} + \frac{6}{11}a\,c^2x^8 + 2a^2c\,x^4 - \frac{2}{5}a^3}{\sqrt{x}\,x^2}$$

input
```
int((c*x^4+a)^3/x^(7/2),x)
```

output
```
(2*( - 209*a**3 + 1045*a**2*c*x**4 + 285*a*c**2*x**8 + 55*c**3*x**12))/(10
45*sqrt(x)*x**2)
```

3.133 $\int \frac{x^{9/2}}{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 240

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = \frac{2x^{3/2}}{3c} + \frac{(-a)^{3/8}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}c^{11/8}} - \frac{(-a)^{3/8}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}c^{11/8}} + \frac{(-a)^{3/8}\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} - \frac{(-a)^{3/8}\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{11/8}} + \frac{(-a)^{3/8}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}c^{11/8}}$$

output
```
2/3*x^(3/2)/c-1/4*(-a)^(3/8)*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))
*2^(1/2)/c^(11/8)-1/4*(-a)^(3/8)*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/
8))*2^(1/2)/c^(11/8)+1/2*(-a)^(3/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(
11/8)-1/2*(-a)^(3/8)*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(11/8)+1/4*(-a)
^(3/8)*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*
2^(1/2)/c^(11/8)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.12

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = \frac{8c^{3/8}x^{3/2} - 3\sqrt{2-\sqrt{2}}a^{3/8}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right) + 3\sqrt{2+\sqrt{2}}a^{3/8}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}}{\sqrt[8]{}}\right.}{}$$

input
```
Integrate[x^(9/2)/(a + c*x^4),x]
```

output
```
(8*c^(3/8)*x^(3/2) - 3*Sqrt[2 - Sqrt[2]]*a^(3/8)*ArcTan[(Sqrt[1 - 1/Sqrt[2
]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + 3*Sqrt[2 + Sqrt[2]]
*a^(3/8)*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/
8)*Sqrt[x])] - 3*Sqrt[2 - Sqrt[2]]*a^(3/8)*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1
/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + 3*Sqrt[2 + Sqrt[2]]*a^(3/8)*
ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1/4)*x)]
)/(12*c^(11/8))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {843, 851, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{a+cx^4}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2x^{3/2}}{3c} - \frac{a\int \frac{\sqrt{x}}{cx^4+a}dx}{c}$$

$$\Big\downarrow \text{851}$$

$$\frac{2x^{3/2}}{3c} - \frac{2a\int \frac{x}{cx^4+a}d\sqrt{x}}{c}$$

$$\downarrow \text{829}$$

$$\frac{2x^{3/2}}{3c}-\frac{2a\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{c}$$

$$\downarrow \text{826}$$

$$\frac{2x^{3/2}}{3c}-\frac{2a\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{c}$$

$$\downarrow \text{827}$$

$$\frac{2x^{3/2}}{3c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{c}$$

$$\downarrow \text{218}$$

$$\frac{2x^{3/2}}{3c}-\frac{2a\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{c}$$

$$\downarrow \text{221}$$

$$\frac{2x^{3/2}}{3c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{c}$$

$$\downarrow \text{1476}$$

$$\frac{2a\left(\frac{2x^{3/2}}{3c}- -\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \; 1082$$

$$\frac{2a\left(\frac{2x^{3/2}}{3c}- -\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \; 217$$

$$\frac{2a\left(\frac{2x^{3/2}}{3c}- -\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{2x^{3/2}}{3c}-2a\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right.}{c}$$

$\downarrow$ 25

$$\frac{\frac{2x^{3/2}}{3c}-2a\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\right.}{c}$$

$\downarrow$ 27

$$\frac{\frac{2x^{3/2}}{3c}-2a\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\text{arctanh}\left(\cdots\right.}{2\sqrt[8]{-a}c^{3/8}\cdots}\right.}{c}$$

$\downarrow$ 1103

$$\frac{2x^{3/2}}{3c} - \frac{2a\left(-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}} - \frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}} - \frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}} - \frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}} - \cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\cdots}{c}$$

input
```
Int[x^(9/2)/(a + c*x^4),x]
```

output
```
(2*x^(3/2))/(3*c) - (2*a*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/
((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)
*c^(3/8)))/Sqrt[-a] - ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]
/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)
^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) -
 Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/
8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*
Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2*Sqrt[-a])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{a\left(\sum_{\_R=\text{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{4c^{2}}$	39
default	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{a\left(\sum_{\_R=\text{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{4c^{2}}$	39
risch	$\frac{2x^{\frac{3}{2}}}{3c}-\frac{a\left(\sum_{\_R=\text{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{4c^{2}}$	39

input
```
int(x^(9/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/c-1/4/c^2*a*sum(1/_R^5*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.31

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = -\frac{-(3i-3)\sqrt{2}c\left(-\frac{a^3}{c^{11}}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}c^4\left(-\frac{a^3}{c^{11}}\right)^{\frac{3}{8}}+a\sqrt{x}\right)+(3i+3)\sqrt{2}c\left(-\frac{a^3}{c^{11}}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\right.}{}$$

input
```
integrate(x^(9/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/24*(-(3*I - 3)*sqrt(2)*c*(-a^3/c^11)^(1/8)*log((1/2*I + 1/2)*sqrt(2)*c^
4*(-a^3/c^11)^(3/8) + a*sqrt(x)) + (3*I + 3)*sqrt(2)*c*(-a^3/c^11)^(1/8)*l
og(-(1/2*I - 1/2)*sqrt(2)*c^4*(-a^3/c^11)^(3/8) + a*sqrt(x)) - (3*I + 3)*s
qrt(2)*c*(-a^3/c^11)^(1/8)*log((1/2*I - 1/2)*sqrt(2)*c^4*(-a^3/c^11)^(3/8)
 + a*sqrt(x)) + (3*I - 3)*sqrt(2)*c*(-a^3/c^11)^(1/8)*log(-(1/2*I + 1/2)*s
qrt(2)*c^4*(-a^3/c^11)^(3/8) + a*sqrt(x)) - 6*c*(-a^3/c^11)^(1/8)*log(c^4*
(-a^3/c^11)^(3/8) + a*sqrt(x)) + 6*I*c*(-a^3/c^11)^(1/8)*log(I*c^4*(-a^3/c
^11)^(3/8) + a*sqrt(x)) - 6*I*c*(-a^3/c^11)^(1/8)*log(-I*c^4*(-a^3/c^11)^(
3/8) + a*sqrt(x)) + 6*c*(-a^3/c^11)^(1/8)*log(-c^4*(-a^3/c^11)^(3/8) + a*s
qrt(x)) - 16*x^(3/2))/c
```

Sympy [A] (verification not implemented)

Time = 54.79 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.25

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} \\ \frac{2x^{\frac{11}{2}}}{11a} \\ \frac{2x^{\frac{3}{2}}}{3c} \\ \frac{2x^{\frac{3}{2}}}{3c} + \frac{\left(-\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4c} - \frac{\left(-\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4c} - \frac{\sqrt{2}\left(-\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{\cdots}\right.}{8c} \end{cases}$$

input
```
integrate(x**(9/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(c, 0)), (2*x**(11/2)/(11*a), Eq(c,
0)), (2*x**(3/2)/(3*c), Eq(a, 0)), (2*x**(3/2)/(3*c) + (-a/c)**(3/8)*log(s
qrt(x) - (-a/c)**(1/8))/(4*c) - (-a/c)**(3/8)*log(sqrt(x) + (-a/c)**(1/8))
/(4*c) - sqrt(2)*(-a/c)**(3/8)*log(-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x
+ 4*(-a/c)**(1/4))/(8*c) + sqrt(2)*(-a/c)**(3/8)*log(4*sqrt(2)*sqrt(x)*(-a
/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8*c) + (-a/c)**(3/8)*atan(sqrt(x)/(-a
/c)**(1/8))/(2*c) - sqrt(2)*(-a/c)**(3/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/
8) - 1)/(4*c) - sqrt(2)*(-a/c)**(3/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) +
 1)/(4*c), True))
```

Maxima [F]

$$\int \frac{x^{9/2}}{a+cx^4}\,dx=\int \frac{x^{\frac{9}{2}}}{cx^4+a}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-a*integrate(sqrt(x)/(c^2*x^4 + a*c), x) + 2/3*x^(3/2)/c
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(159) = 318$.

Time = 0.24 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.89

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = \frac{2\,x^{\frac{3}{2}}}{3\,c} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\,\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,c\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\,\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,c\sqrt{2\,\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\,\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,c\sqrt{-2\,\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\,\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,c\sqrt{-2\,\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,c\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,c\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,c\sqrt{-2\,\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,c\sqrt{-2\,\sqrt{2}+4}}$$

input
```
integrate(x^(9/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
2/3*x^(3/2)/c + 1/2*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2
*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2) + 4)) + 1/2*(
a/c)^(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt
(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2) + 4)) - 1/2*(a/c)^(3/8)*arctan((s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
)/(c*sqrt(-2*sqrt(2) + 4)) - 1/2*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)*(a
/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(-2*sqrt(2
) + 4)) - 1/4*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x +
(a/c)^(1/4))/(c*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(s
qrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(2*sqrt(2) + 4)) + 1/4*(
a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(
c*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = \frac{2\,x^{3/2}}{3\,c}+\frac{(-a)^{3/8}\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{2\,c^{11/8}}+\frac{(-a)^{3/8}\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{2\,c^{11/8}} + \frac{\sqrt{2}\,(-a)^{3/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{c^{11/8}} + \frac{\sqrt{2}\,(-a)^{3/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{c^{11/8}}$$

input
```
int(x^(9/2)/(a + c*x^4),x)
```

output
```
(2*x^(3/2))/(3*c) + ((-a)^(3/8)*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(2*c^(
11/8)) + ((-a)^(3/8)*atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*c^(11/8)
) - (2^(1/2)*(-a)^(3/8)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(
1/8))*(1/4 - 1i/4))/c^(11/8) - (2^(1/2)*(-a)^(3/8)*atan((2^(1/2)*c^(1/8)*x
^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/4 + 1i/4))/c^(11/8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.13

$$\int \frac{x^{9/2}}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(c*x^4+a),x)
```

output

```
( - 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*
sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2)
 + 2))) + 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2))) + 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1
/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqr
t(2) + 2))) - 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8
)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(
2) + 2))) + 6*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8
)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt
( - sqrt(2) + 2))) + 6*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/
8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2))) - 6*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*ata
n((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt( - sqrt(2) + 2))) - 6*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*at
an((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a*
*(1/8)*sqrt( - sqrt(2) + 2))) - 3*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*s
qrt(2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) +
c**(1/4)*x) + 3*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*log(sqrt(x)
*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) - 3*c*...
```

3.134 $\int \frac{x^{7/2}}{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 238

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \frac{2\sqrt{x}}{c}+\frac{\sqrt[8]{-a}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}c^{9/8}} - \frac{\sqrt[8]{-a}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}c^{9/8}} - \frac{\sqrt[8]{-a}\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} - \frac{\sqrt[8]{-a}\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2c^{9/8}} - \frac{\sqrt[8]{-a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}c^{9/8}}$$

output
```
2*x^(1/2)/c-1/4*(-a)^(1/8)*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2
^(1/2)/c^(9/8)-1/4*(-a)^(1/8)*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))
*2^(1/2)/c^(9/8)-1/2*(-a)^(1/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(9/8)
-1/2*(-a)^(1/8)*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/c^(9/8)-1/4*(-a)^(1/8)
*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2
)/c^(9/8)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.12

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \frac{8\sqrt[8]{c}\sqrt{x}+\sqrt{2+\sqrt{2}}\sqrt[8]{a}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)+\sqrt{2-\sqrt{2}}\sqrt[8]{a}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}\ldots\right.}{\sqrt[8]{a}\sqrt[8]{c}\ldots}\right.}{}$$

input
```
Integrate[x^(7/2)/(a + c*x^4),x]
```

output
```
(8*c^(1/8)*Sqrt[x] + Sqrt[2 + Sqrt[2]]*a^(1/8)*ArcTan[(Sqrt[1 - 1/Sqrt[2]]
*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + Sqrt[2 - Sqrt[2]]*a^(
1/8)*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*S
qrt[x])] - Sqrt[2 + Sqrt[2]]*a^(1/8)*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^
(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] - Sqrt[2 - Sqrt[2]]*a^(1/8)*ArcTanh[
(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1/4)*x)])/(4*c^(
9/8))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.46, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {843, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{a+cx^4}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{2\sqrt{x}}{c}-\frac{a\int \frac{1}{\sqrt{x}(cx^4+a)}dx}{c}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\int \frac{1}{cx^4+a}d\sqrt{x}}{c}$$

$$\Big\downarrow \text{758}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \text{755}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \text{756}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{c}$$

$\big\downarrow$ 1082

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{c}$$

$\big\downarrow$ 217

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{c}$$

$\big\downarrow$ 1479

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\cdots\right)}{c}$$

$\downarrow$ 25

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\cdots\right)}{c}$$

$\downarrow$ 27

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)\cdots}{2(-a)^{3/8}\sqrt[8]{c}\cdots}\cdots\right)}{c}$$

$\downarrow$ 1103

$$\frac{2\sqrt{x}}{c}-\frac{2a\left(-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\right)}{c}$$

input
```
Int[x^(7/2)/(a + c*x^4),x]
```

output
```
(2*Sqrt[x])/c - (2*a*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(
3/8)*c^(1/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8
)))/Sqrt[-a] - ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[
2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]
/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqr
t[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))
+ Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt
[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)))/(2*Sqrt[-a])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{c}-\frac{a\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{4c^2}$	39
default	$\frac{2\sqrt{x}}{c}-\frac{a\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{4c^2}$	39
risch	$\frac{2\sqrt{x}}{c}-\frac{a\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{4c^2}$	39

input
```
int(x^(7/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/c-1/4/c^2*a*sum(1/_R^7*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.05

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = -\frac{(i+1)\,\sqrt{2}c\left(-\frac{a}{c^9}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}c\left(-\frac{a}{c^9}\right)^{\frac{1}{8}}+\sqrt{x}\right)-(i-1)\,\sqrt{2}c\left(-\frac{a}{c^9}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}c\left(-\frac{a}{c^9}\right)^{\frac{1}{8}}\right.}{}$$

input
```
integrate(x^(7/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/8*((I + 1)*sqrt(2)*c*(-a/c^9)^(1/8)*log((1/2*I + 1/2)*sqrt(2)*c*(-a/c^9
)^(1/8) + sqrt(x)) - (I - 1)*sqrt(2)*c*(-a/c^9)^(1/8)*log(-(1/2*I - 1/2)*s
qrt(2)*c*(-a/c^9)^(1/8) + sqrt(x)) + (I - 1)*sqrt(2)*c*(-a/c^9)^(1/8)*log(
(1/2*I - 1/2)*sqrt(2)*c*(-a/c^9)^(1/8) + sqrt(x)) - (I + 1)*sqrt(2)*c*(-a/
c^9)^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*c*(-a/c^9)^(1/8) + sqrt(x)) + 2*c*(-
a/c^9)^(1/8)*log(c*(-a/c^9)^(1/8) + sqrt(x)) + 2*I*c*(-a/c^9)^(1/8)*log(I*
c*(-a/c^9)^(1/8) + sqrt(x)) - 2*I*c*(-a/c^9)^(1/8)*log(-I*c*(-a/c^9)^(1/8)
 + sqrt(x)) - 2*c*(-a/c^9)^(1/8)*log(-c*(-a/c^9)^(1/8) + sqrt(x)) - 16*sqr
t(x))/c
```

Sympy [A] (verification not implemented)

Time = 29.23 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.24

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{9}{2}}}{9a} \\ \frac{2\sqrt{x}}{c} \\ \frac{2\sqrt{x}}{c}+\frac{\sqrt[8]{-\frac{a}{c}}\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4c}-\frac{\sqrt[8]{-\frac{a}{c}}\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4c}+\frac{\sqrt{2}\sqrt[8]{-\frac{a}{c}}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{\right.}}{8c} \end{cases}$$

input
```
integrate(x**(7/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(c, 0)), (2*x**(9/2)/(9*a), Eq(c, 0))
, (2*sqrt(x)/c, Eq(a, 0)), (2*sqrt(x)/c + (-a/c)**(1/8)*log(sqrt(x) - (-a/
c)**(1/8))/(4*c) - (-a/c)**(1/8)*log(sqrt(x) + (-a/c)**(1/8))/(4*c) + sqrt
(2)*(-a/c)**(1/8)*log(-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(
1/4))/(8*c) - sqrt(2)*(-a/c)**(1/8)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) +
4*x + 4*(-a/c)**(1/4))/(8*c) - (-a/c)**(1/8)*atan(sqrt(x)/(-a/c)**(1/8))/(
2*c) - sqrt(2)*(-a/c)**(1/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*c)
 - sqrt(2)*(-a/c)**(1/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*c), Tr
ue))
```

Maxima [F]

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \int \frac{x^{\frac{7}{2}}}{cx^4+a}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^(7/2)/(c*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(159) = 318$.

Time = 0.20 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.90

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = -\frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2c\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4c\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4c\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4c\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4c\sqrt{2\sqrt{2}+4}} + \frac{2\sqrt{x}}{c}$$

input
```
integrate(x^(7/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(-2*sqrt(2) + 4)) - 1/2*(a/c)^(1/8)*arc
tan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)
^(1/8)))/(c*sqrt(-2*sqrt(2) + 4)) - 1/2*(a/c)^(1/8)*arctan((sqrt(sqrt(2) +
 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*s
qrt(2) + 4)) - 1/2*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*
sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(c*sqrt(2*sqrt(2) + 4)) - 1/4*(
a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c
*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a
/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(1/8)*lo
g(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(c*sqrt(2*sqrt
(2) + 4)) + 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) +
x + (a/c)^(1/4))/(c*sqrt(2*sqrt(2) + 4)) + 2*sqrt(x)/c
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.53

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \frac{2\sqrt{x}}{c} - \frac{(-a)^{1/8}\,\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2\,c^{9/8}} + \frac{(-a)^{1/8}\,\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{2\,c^{9/8}} + \frac{\sqrt{2}\,(-a)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{c^{9/8}} + \frac{\sqrt{2}\,(-a)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{c^{9/8}}$$

input
```
int(x^(7/2)/(a + c*x^4),x)
```

output
```
(2*x^(1/2))/c - ((-a)^(1/8)*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(2*c^(9/8)
) + ((-a)^(1/8)*atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*c^(9/8)) - (2
^(1/2)*(-a)^(1/8)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*
(1/4 + 1i/4))/c^(9/8) - (2^(1/2)*(-a)^(1/8)*atan((2^(1/2)*c^(1/8)*x^(1/2)*
(1/2 + 1i/2))/(-a)^(1/8))*(1/4 - 1i/4))/c^(9/8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.56

$$\int \frac{x^{7/2}}{a+cx^4}\,dx = \frac{2c^{\frac{7}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}-2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right) - 2c^{\frac{7}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}+2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)}{}$$

input
```
int(x^(7/2)/(c*x^4+a),x)
```

output
```
(2*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqr
t(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2
*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(
2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) + 2*c
**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2)
 + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) - 2*
c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2
) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + c
**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) - c**(7/8)*a**(1/8)*sqrt( - sqr
t(2) + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) +
c**(1/4)*x) + c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*
a**(1/8)*sqrt(sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) - c**(7/8)*a**(1/8)*sq
rt(sqrt(2) + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + a**(1/4)
 + c**(1/4)*x) + 16*sqrt(x)*c)/(8*c**2)
```

3.135 $\int \frac{x^{5/2}}{a+cx^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 228

$$\int \frac{x^{5/2}}{a+cx^4} \, dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\sqrt[8]{-a}c^{7/8}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}\sqrt[8]{-a}c^{7/8}} + \frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{7/8}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{7/8}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}\sqrt[8]{-a}c^{7/8}}$$

output
```
1/4*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(1/8)/c^(7/
8)+1/4*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(1/8)/c^(
7/8)+1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(7/8)-1/2*arctanh
(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^(7/8)-1/4*arctanh(2^(1/2)*(-a)^(
1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(1/8)/c^(7/8)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.04

$$\int \frac{x^{5/2}}{a+cx^4}\,dx = -\frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)+\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)+\sqrt{2+\sqrt{2}}\text{arctanh}\left(\right.}{4\sqrt[8]{a}c^{7/8}}$$

input
```
Integrate[x^(5/2)/(a + c*x^4),x]
```

output
```
-1/4*(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))
/(a^(1/8)*c^(1/8)*Sqrt[x])] + Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]
]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + Sqrt[2 + Sqrt[2]]*Ar
cTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] +
 Sqrt[2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^
(1/4) + c^(1/4)*x)])/(a^(1/8)*c^(7/8))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{a+cx^4}\,dx$$

$$\Big\downarrow \text{851}$$

$$2\int \frac{x^3}{cx^4+a}d\sqrt{x}$$

$$\Big\downarrow \text{830}$$

$$2\left(\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)$$

$\downarrow$ 826

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)$$

$\downarrow$ 827

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}\right)$$

$\downarrow$ 218

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)$$

$\downarrow$ 221

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)$$

$\downarrow$ 1476

$$2\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right.$$

$$\Big\downarrow \ 1082$$

$$2\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \ 217$$

$$2\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \ 1479$$

$$2\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\right.$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{ar}\ldots}{\ldots}\right.$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\ldots}{\sqrt[8]{-\ldots}}\right.}{2\sqrt[8]{-a}c^{3/8}\ldots}\right.$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{ar}\ldots}{\ldots}\right.$$

input
```
Int[x^(5/2)/(a + c*x^4),x]
```

output
```
2*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) +
ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[c] + ((
-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(
1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1
/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1
/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + S
qrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/
8)))/(2*c^(1/4)))/(2*Sqrt[c]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.13

method	result	size
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{4c}$	29
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{4c}$	29

input
```
int(x^(5/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.26

$$\int \frac{x^{5/2}}{a+cx^4}\,dx = -\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) + \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) + \frac{1}{4}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) - \frac{1}{4}i\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(i\,ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) + \frac{1}{4}i\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(-i\,ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right) - \frac{1}{4}\left(-\frac{1}{ac^7}\right)^{\frac{1}{8}}\log\left(-ac^6\left(-\frac{1}{ac^7}\right)^{\frac{7}{8}}+\sqrt{x}\right)$$

input
```
integrate(x^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-(1/8*I - 1/8)*sqrt(2)*(-1/(a*c^7))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a*c^6*
(-1/(a*c^7))^(7/8) + sqrt(x)) + (1/8*I + 1/8)*sqrt(2)*(-1/(a*c^7))^(1/8)*l
og(-(1/2*I - 1/2)*sqrt(2)*a*c^6*(-1/(a*c^7))^(7/8) + sqrt(x)) - (1/8*I + 1
/8)*sqrt(2)*(-1/(a*c^7))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a*c^6*(-1/(a*c^7)
)^(7/8) + sqrt(x)) + (1/8*I - 1/8)*sqrt(2)*(-1/(a*c^7))^(1/8)*log(-(1/2*I
+ 1/2)*sqrt(2)*a*c^6*(-1/(a*c^7))^(7/8) + sqrt(x)) + 1/4*(-1/(a*c^7))^(1/8
)*log(a*c^6*(-1/(a*c^7))^(7/8) + sqrt(x)) - 1/4*I*(-1/(a*c^7))^(1/8)*log(I
*a*c^6*(-1/(a*c^7))^(7/8) + sqrt(x)) + 1/4*I*(-1/(a*c^7))^(1/8)*log(-I*a*c
^6*(-1/(a*c^7))^(7/8) + sqrt(x)) - 1/4*(-1/(a*c^7))^(1/8)*log(-a*c^6*(-1/(
a*c^7))^(7/8) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.27

$$\int \frac{x^{5/2}}{a+cx^4}\,dx=\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{7}{2}}}{7a} \\ -\frac{2}{c\sqrt{x}} \\ \frac{\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4c\sqrt[8]{-\frac{a}{c}}}-\frac{\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4c\sqrt[8]{-\frac{a}{c}}}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{-\frac{a}{c}}\right)}{8c\sqrt[8]{-\frac{a}{c}}}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}\sqrt[8]{-\right.}{8c\sqrt[8]{-}} \end{cases}$$

input
```
integrate(x**(5/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(c, 0)), (2*x**(7/2)/(7*a), Eq(c, 0))
, (-2/(c*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - (-a/c)**(1/8))/(4*c*(-a/c)**(
1/8)) - log(sqrt(x) + (-a/c)**(1/8))/(4*c*(-a/c)**(1/8)) + sqrt(2)*log(-4*
sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8*c*(-a/c)**(1/8))
 - sqrt(2)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8
*c*(-a/c)**(1/8)) + atan(sqrt(x)/(-a/c)**(1/8))/(2*c*(-a/c)**(1/8)) + sqrt
(2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*c*(-a/c)**(1/8)) + sqrt(2)*
atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*c*(-a/c)**(1/8)), True))
```

Maxima [F]

$$\int \frac{x^{5/2}}{a+cx^4}\,dx=\int \frac{x^{\frac{5}{2}}}{cx^4+a}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^(5/2)/(c*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(151) = 302$.

Time = 0.22 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.95

$$\int \frac{x^{5/2}}{a+cx^4}\,dx = \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{2\sqrt{2}+4}}$$

input
```
integrate(x^(5/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(7/8)*arct
an(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^
(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(7/8)*arctan((sqrt(sqrt(2) +
2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sq
rt(2) + 4)) + 1/2*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*s
qrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*(a
/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*
sqrt(-2*sqrt(2) + 4)) + 1/4*(a/c)^(7/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/
c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(7/8)*log
(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*sqrt(
2) + 4)) + 1/4*(a/c)^(7/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x
 + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.52

$$\int \frac{x^{5/2}}{a+cx^4}\,dx = \frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2\,(-a)^{1/8}\,c^{7/8}} + \frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{2\,(-a)^{1/8}\,c^{7/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{(-a)^{1/8}\,c^{7/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{(-a)^{1/8}\,c^{7/8}}$$

input
```
int(x^(5/2)/(a + c*x^4),x)
```

output
```
atan((c^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(1/8)*c^(7/8)) + (atan((c^(1/8)
*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(1/8)*c^(7/8)) + (2^(1/2)*atan((2^(1/
2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 - 1i/4))/((-a)^(1/8)*c^(
7/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(
1/4 + 1i/4))/((-a)^(1/8)*c^(7/8))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.41

$$\int \frac{x^{5/2}}{a+cx^4}\,dx = \frac{-2\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}-2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}+2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)-2\sqrt{-}}{}$$

input
```
int(x^(5/2)/(c*x^4+a),x)
```

output
```
(c**(1/8)*a**(7/8)*( - 2*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
 + 2*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sq
rt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt( - sqrt(2)
 + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**
(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + 2*sqrt( - sqrt(2) + 2)*atan((c**(1
/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2))) + sqrt( - sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/
8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) - sqrt( - sqrt(2) + 2)*lo
g(sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)
+ sqrt(sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + a
**(1/4) + c**(1/4)*x) - sqrt(sqrt(2) + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)))/(8*a*c)
```

3.136 $\int \frac{x^{3/2}}{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 228

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{3/8}c^{5/8}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{3/8}c^{5/8}} - \frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}c^{5/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}(-a)^{3/8}c^{5/8}}$$

output
```
1/4*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(3/8)/c^(5/
8)+1/4*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(3/8)/c^(
5/8)-1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(5/8)-1/2*arctanh
(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^(5/8)+1/4*arctanh(2^(1/2)*(-a)^(
1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(3/8)/c^(5/8)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.05

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)-\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)-\sqrt{2-}}{4a^{3/8}c^{5/8}}$$

input
```
Integrate[x^(3/2)/(a + c*x^4),x]
```

output
```
(Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(
1/8)*c^(1/8)*Sqrt[x])] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^
(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] - Sqrt[2 - Sqrt[2]]*ArcTanh
[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + Sqrt
[2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
 + c^(1/4)*x)])/(4*a^(3/8)*c^(5/8))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.44, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {851, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{a+cx^4}\,dx$$

$$\Big\downarrow \text{851}$$

$$2\int \frac{x^2}{cx^4+a}d\sqrt{x}$$

$$\Big\downarrow \text{830}$$

$$2\left(\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{218}$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{221}$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \ 217$$

$$2\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)$$

$$\Big\downarrow \ 1479$$

$$2\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right.$$

$$\Big\downarrow \ 25$$

$$2\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right.$$

$\downarrow$ 27

$$2\left(\frac{\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}\cdots}{\cdots}\right.$$

$\downarrow$ 1103

$$2\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\text{arc}\cdots}{2\cdots}\right.$$

input
```
Int[x^(3/2)/(a + c*x^4),x]
```

output
```
2*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + Arc
Tanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[c] + ((-(A
rcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8
))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)
*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1
/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + S
qrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/
8)))/(2*(-a)^(1/4)))/(2*Sqrt[c]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.13

method	result	size
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}}{4c}$	29
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}}{4c}$	29

input
```
int(x^(3/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^3*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.33

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = \left(\frac{1}{8}i + \frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) - \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) - \frac{1}{4}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) - \frac{1}{4}i\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(i\,a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) + \frac{1}{4}i\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(-i\,a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right) + \frac{1}{4}\left(-\frac{1}{a^3c^5}\right)^{\frac{1}{8}}\log\left(-a^2c^3\left(-\frac{1}{a^3c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right)$$

input
```
integrate(x^(3/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*(-1/(a^3*c^5))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^2*c
^3*(-1/(a^3*c^5))^(5/8) + sqrt(x)) - (1/8*I - 1/8)*sqrt(2)*(-1/(a^3*c^5))^
(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^2*c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x)) +
 (1/8*I - 1/8)*sqrt(2)*(-1/(a^3*c^5))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^2*
c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x)) - (1/8*I + 1/8)*sqrt(2)*(-1/(a^3*c^5))
^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^2*c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x))
 - 1/4*(-1/(a^3*c^5))^(1/8)*log(a^2*c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x)) - 1
/4*I*(-1/(a^3*c^5))^(1/8)*log(I*a^2*c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x)) +
1/4*I*(-1/(a^3*c^5))^(1/8)*log(-I*a^2*c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x))
+ 1/4*(-1/(a^3*c^5))^(1/8)*log(-a^2*c^3*(-1/(a^3*c^5))^(5/8) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 12.92 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.28

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2x^{\frac{5}{2}}}{5a} \\ -\frac{2}{3cx^{\frac{3}{2}}} \\ \frac{\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4c\left(-\frac{a}{c}\right)^{\frac{3}{8}}} - \frac{\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4c\left(-\frac{a}{c}\right)^{\frac{3}{8}}} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{-\frac{a}{c}}\right)}{8c\left(-\frac{a}{c}\right)^{\frac{3}{8}}} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}\sqrt[8]{-\right.}}{8c\left(-\frac{a}{c}\right.} \end{cases}$$

input
```
integrate(x**(3/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(c, 0)), (2*x**(5/2)/(5*a), Eq(c, 0)
), (-2/(3*c*x**(3/2)), Eq(a, 0)), (log(sqrt(x) - (-a/c)**(1/8))/(4*c*(-a/c
)**(3/8)) - log(sqrt(x) + (-a/c)**(1/8))/(4*c*(-a/c)**(3/8)) - sqrt(2)*log
(-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8*c*(-a/c)**(3
/8)) + sqrt(2)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4)
)/(8*c*(-a/c)**(3/8)) - atan(sqrt(x)/(-a/c)**(1/8))/(2*c*(-a/c)**(3/8)) +
sqrt(2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*c*(-a/c)**(3/8)) + sqrt
(2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*c*(-a/c)**(3/8)), True))
```

Maxima [F]

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = \int \frac{x^{\frac{3}{2}}}{cx^4+a}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(x^(3/2)/(c*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(151) = 302$.

Time = 0.22 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.95

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = -\frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2a\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4a\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4a\sqrt{-2\sqrt{2}+4}}$$

input
```
integrate(x^(3/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/2*(a/c)^(5/8)*arct
an(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^
(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/2*(a/c)^(5/8)*arctan((sqrt(sqrt(2) + 2
)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sq
rt(2) + 4)) + 1/2*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*s
qrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) - 1/4*(
a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a
*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(5/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/
c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) + 1/4*(a/c)^(5/8)*log(
sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(
2) + 4)) - 1/4*(a/c)^(5/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x
 + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.52

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = -\frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2\,(-a)^{3/8}\,c^{5/8}}+\frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{2\,(-a)^{3/8}\,c^{5/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{(-a)^{3/8}\,c^{5/8}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{(-a)^{3/8}\,c^{5/8}}$$

input
```
int(x^(3/2)/(a + c*x^4),x)
```

output
```
(atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(3/8)*c^(5/8)) - atan((
c^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(3/8)*c^(5/8)) + (2^(1/2)*atan((2^(1/
2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 + 1i/4))/((-a)^(3/8)*c^(
5/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(
1/4 - 1i/4))/((-a)^(3/8)*c^(5/8))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.85

$$\int \frac{x^{3/2}}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(c*x^4+a),x)
```

output
```
(c**(3/8)*a**(5/8)*(2*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2))) - 2*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) -
 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt(sqrt(
2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c
**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) + 2*sqrt(sqrt(2) + 2)*atan
((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a
**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8
)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt
( - sqrt(2) + 2))) - 2*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(s
qrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))
) + 2*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + 2*sqr
t( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c*
*(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) - sqrt( - sqrt(2) + 2)*s
qrt(2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) +
c**(1/4)*x) + sqrt( - sqrt(2) + 2)*sqrt(2)*log(sqrt(x)*c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) - sqrt( - sqrt(2) + 2)*log( -
 sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) +
 sqrt( - sqrt(2) + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2...
```

3.137 $\int \frac{\sqrt{x}}{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 228

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{5/8}c^{3/8}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{5/8}c^{3/8}} + \frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{5/8}c^{3/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}(-a)^{5/8}c^{3/8}}$$

output
```
-1/4*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(5/8)/c^(3
/8)-1/4*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(5/8)/c^
(3/8)+1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(3/8)-1/2*arctan
h(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^(3/8)+1/4*arctanh(2^(1/2)*(-a)^
(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(5/8)/c^(3/8)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx$$
$$= \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)-\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)+\sqrt{2-\sqrt{2}}\text{arctanh}\Big(}{4a^{5/8}c^{3/8}}$$

input
```
Integrate[Sqrt[x]/(a + c*x^4),x]
```

output
```
(Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(
1/8)*c^(1/8)*Sqrt[x])] - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^
(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + Sqrt[2 - Sqrt[2]]*ArcTanh
[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] - Sqrt
[2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
 + c^(1/4)*x)])/(4*a^(5/8)*c^(3/8))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.44, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {851, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx$$
$$\Big\downarrow \text{ 851}$$
$$2\int \frac{x}{cx^4+a}d\sqrt{x}$$
$$\Big\downarrow \text{ 829}$$

$$2 \left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c x^2}}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{x}{\sqrt{c x^2}+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)$$

$\big\downarrow$ 826

$$2\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{cx^2}}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)$$

$\big\downarrow$ 827

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)$$

$\big\downarrow$ 218

$$2\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)$$

$\big\downarrow$ 221

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)$$

$\big\downarrow$ 1476

$$2\left(-\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right.$$

$$\Big\downarrow \; 1082$$

$$2\left(-\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)$$

$$\Big\downarrow \; 217$$

$$2\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)$$

$$\Big\downarrow \; 1479$$

$$2\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right.$$

$$\Big\downarrow \; 25$$

$$2\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\right.$$

$\downarrow$ 27

$$2\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\right.$$

$\downarrow$ 1103

$$2\left(-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right.}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt{-a}}\right.$$

input
```
Int[Sqrt[x]/(a + c*x^4),x]
```

output
```
2*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) +
ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[-a] - (
(-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^
(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(
1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(
1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) +
Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1
/8)))/(2*c^(1/4)))/(2*Sqrt[-a]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.13

method	result	size
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}}{4c}$	29
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}}{4c}$	29

input
```
int(x^(1/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^5*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = -\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) + \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) - \frac{1}{4}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) + \frac{1}{4}i\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(i\,a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) - \frac{1}{4}i\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(-i\,a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right) + \frac{1}{4}\left(-\frac{1}{a^5c^3}\right)^{\frac{1}{8}}\log\left(-a^2c\left(-\frac{1}{a^5c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-(1/8*I - 1/8)*sqrt(2)*(-1/(a^5*c^3))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^2*
c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) + (1/8*I + 1/8)*sqrt(2)*(-1/(a^5*c^3))^(
1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) - (1
/8*I + 1/8)*sqrt(2)*(-1/(a^5*c^3))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^2*c*(
-1/(a^5*c^3))^(3/8) + sqrt(x)) + (1/8*I - 1/8)*sqrt(2)*(-1/(a^5*c^3))^(1/8
)*log(-(1/2*I + 1/2)*sqrt(2)*a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) - 1/4*(
-1/(a^5*c^3))^(1/8)*log(a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) + 1/4*I*(-1/
(a^5*c^3))^(1/8)*log(I*a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) - 1/4*I*(-1/(
a^5*c^3))^(1/8)*log(-I*a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x)) + 1/4*(-1/(a^
5*c^3))^(1/8)*log(-a^2*c*(-1/(a^5*c^3))^(3/8) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 8.58 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ -\frac{2}{5cx^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a} \\ -\frac{(-\frac{a}{c})^{\frac{3}{8}}\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4a}+\frac{(-\frac{a}{c})^{\frac{3}{8}}\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4a}+\frac{\sqrt{2}(-\frac{a}{c})^{\frac{3}{8}}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{-\frac{a}{c}}\right)}{8a}-\frac{\sqrt{2}(-\frac{a}{c})^{\frac{3}{8}}\log\left(\right.}{} \end{cases}$$

input
```
integrate(x**(1/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2)), Eq(a, 0
)), (2*x**(3/2)/(3*a), Eq(c, 0)), (-(-a/c)**(3/8)*log(sqrt(x) - (-a/c)**(1
/8))/(4*a) + (-a/c)**(3/8)*log(sqrt(x) + (-a/c)**(1/8))/(4*a) + sqrt(2)*(-
a/c)**(3/8)*log(-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/
(8*a) - sqrt(2)*(-a/c)**(3/8)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x +
4*(-a/c)**(1/4))/(8*a) - (-a/c)**(3/8)*atan(sqrt(x)/(-a/c)**(1/8))/(2*a) +
 sqrt(2)*(-a/c)**(3/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*a) + sqr
t(2)*(-a/c)**(3/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*a), True))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = \int \frac{\sqrt{x}}{cx^4+a}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/(c*x^4 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(151) = 302$.

Time = 0.22 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.95

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = -\frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{-2\sqrt{2}+4}}$$

input
```
integrate(x^(1/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) - 1/2*(a/c)^(3/8)*arct
an(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^
(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/2*(a/c)^(3/8)*arctan((sqrt(sqrt(2) + 2
)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sq
rt(2) + 4)) + 1/2*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*s
qrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(
a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a
*sqrt(2*sqrt(2) + 4)) - 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/
c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4)) - 1/4*(a/c)^(3/8)*log(
sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(
2) + 4)) + 1/4*(a/c)^(3/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x
 + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = \frac{\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2\,(-a)^{5/8}\,c^{3/8}} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{2\,(-a)^{5/8}\,c^{3/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{(-a)^{5/8}\,c^{3/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{(-a)^{5/8}\,c^{3/8}}$$

input
```
int(x^(1/2)/(a + c*x^4),x)
```

output
```
atan((c^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(5/8)*c^(3/8)) + (atan((c^(1/8)
*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(5/8)*c^(3/8)) - (2^(1/2)*atan((2^(1/
2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 - 1i/4))/((-a)^(5/8)*c^(
3/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(
1/4 + 1i/4))/((-a)^(5/8)*c^(3/8))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{x}}{a+cx^4}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(c*x^4+a),x)
```

output
```
(c**(5/8)*a**(3/8)*(2*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2))) - 2*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) -
 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt(sqrt(
2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c
**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) + 2*sqrt(sqrt(2) + 2)*atan
((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a
**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8
)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt
( - sqrt(2) + 2))) - 2*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(s
qrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))
) + 2*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + 2*sqr
t( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c*
*(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + sqrt( - sqrt(2) + 2)*s
qrt(2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) +
c**(1/4)*x) - sqrt( - sqrt(2) + 2)*sqrt(2)*log(sqrt(x)*c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) + sqrt( - sqrt(2) + 2)*log( -
 sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) -
 sqrt( - sqrt(2) + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2...
```

3.138 $\int \frac{1}{\sqrt{x}(a+cx^4)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 228

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx = \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{7/8}\sqrt[8]{c}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{7/8}\sqrt[8]{c}} - \frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{7/8}\sqrt[8]{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}(-a)^{7/8}\sqrt[8]{c}}$$

output
```
-1/4*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(7/8)/c^(1
/8)-1/4*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(7/8)/c^
(1/8)-1/2*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(1/8)-1/2*arctan
h(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(1/8)-1/4*arctanh(2^(1/2)*(-a)^
(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(7/8)/c^(1/8)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx$$
$$= \frac{-\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)-\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)+\sqrt{2+\sqrt{2}}\text{arctanh}}{4a^{7/8}\sqrt[8]{c}}$$

input
```
Integrate[1/(Sqrt[x]*(a + c*x^4)),x]
```

output
```
(-(Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a
^(1/8)*c^(1/8)*Sqrt[x])]) - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*
(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + Sqrt[2 + Sqrt[2]]*ArcT
anh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + S
qrt[2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1
/4) + c^(1/4)*x)])/(4*a^(7/8)*c^(1/8))
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx$$
$$\Big\downarrow \text{ 851}$$
$$2\int \frac{1}{cx^4+a}d\sqrt{x}$$
$$\Big\downarrow \text{ 758}$$

$$2\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)$$

$\downarrow$ 755

$$2\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)$$

$\downarrow$ 756

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)$$

$\downarrow$ 218

$$2\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)$$

$\downarrow$ 221

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)$$

$\downarrow$ 1476

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right.$$

$$\Big\downarrow \text{1082}$$

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(-\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}$$

$$\Big\downarrow \text{25}$$

$$2\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\right.$$

$\Big\downarrow$ 27

$$2\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right.}{2(-a)^{3/8}\sqrt[8]{c}}\right.$$

$\Big\downarrow$ 1103

$$2\left(-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right.}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt{-a}}\right.$$

input
```
Int[1/(Sqrt[x]*(a + c*x^4)),x]
```

output
```
2*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + Arc
Tanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[-a] - ((-(
ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/
8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8
)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(
1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) +
Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1
/8)))/(2*(-a)^(1/4)))/(2*Sqrt[-a]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.13

method	result	size
derivativedivides	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{7}}}{4c}$	29
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{7}}}{4c}$	29

input
```
int(1/x^(1/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/c*sum(1/_R^7*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx = \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) - \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) + \frac{1}{4}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) + \frac{1}{4}i\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(i\,a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) - \frac{1}{4}i\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(-i\,a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right) - \frac{1}{4}\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}\log\left(-a\left(-\frac{1}{a^7c}\right)^{\frac{1}{8}}+\sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*(-1/(a^7*c))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a*(-1/(
a^7*c))^(1/8) + sqrt(x)) - (1/8*I - 1/8)*sqrt(2)*(-1/(a^7*c))^(1/8)*log(-(
1/2*I - 1/2)*sqrt(2)*a*(-1/(a^7*c))^(1/8) + sqrt(x)) + (1/8*I - 1/8)*sqrt(
2)*(-1/(a^7*c))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a*(-1/(a^7*c))^(1/8) + sqr
t(x)) - (1/8*I + 1/8)*sqrt(2)*(-1/(a^7*c))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2
)*a*(-1/(a^7*c))^(1/8) + sqrt(x)) + 1/4*(-1/(a^7*c))^(1/8)*log(a*(-1/(a^7*
c))^(1/8) + sqrt(x)) + 1/4*I*(-1/(a^7*c))^(1/8)*log(I*a*(-1/(a^7*c))^(1/8)
 + sqrt(x)) - 1/4*I*(-1/(a^7*c))^(1/8)*log(-I*a*(-1/(a^7*c))^(1/8) + sqrt(
x)) - 1/4*(-1/(a^7*c))^(1/8)*log(-a*(-1/(a^7*c))^(1/8) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 12.23 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{7cx^{\frac{7}{2}}} \\ \frac{2\sqrt{x}}{a} \\ -\frac{\sqrt[8]{-\frac{a}{c}}\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4a}+\frac{\sqrt[8]{-\frac{a}{c}}\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4a}-\frac{\sqrt{2}\sqrt[8]{-\frac{a}{c}}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{-\frac{a}{c}}\right)}{8a}+\frac{\sqrt{2}\sqrt[8]{-\frac{a}{c}}\log \end{cases}$$

input
```
integrate(1/x**(1/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(c, 0)), (-2/(7*c*x**(7/2)), Eq(a, 0
)), (2*sqrt(x)/a, Eq(c, 0)), (-(-a/c)**(1/8)*log(sqrt(x) - (-a/c)**(1/8))/
(4*a) + (-a/c)**(1/8)*log(sqrt(x) + (-a/c)**(1/8))/(4*a) - sqrt(2)*(-a/c)*
*(1/8)*log(-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8*a)
 + sqrt(2)*(-a/c)**(1/8)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a
/c)**(1/4))/(8*a) + (-a/c)**(1/8)*atan(sqrt(x)/(-a/c)**(1/8))/(2*a) + sqrt
(2)*(-a/c)**(1/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*a) + sqrt(2)*
(-a/c)**(1/8)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*a), True))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx = \int \frac{1}{(cx^4+a)\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-c*integrate(x^(7/2)/(a*c*x^4 + a^2), x) + 2*sqrt(x)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(151) = 302$.

Time = 0.17 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx = \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a\sqrt{2\sqrt{2}+4}}$$

input
```
integrate(1/x^(1/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(
sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(1/8)*arct
an(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^
(1/8)))/(a*sqrt(-2*sqrt(2) + 4)) + 1/2*(a/c)^(1/8)*arctan((sqrt(sqrt(2) +
2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sq
rt(2) + 4)) + 1/2*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*s
qrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*sqrt(2*sqrt(2) + 4)) + 1/4*(a
/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*
sqrt(-2*sqrt(2) + 4)) - 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/
c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(-2*sqrt(2) + 4)) + 1/4*(a/c)^(1/8)*log
(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*sqrt(2*sqrt(
2) + 4)) - 1/4*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x
 + (a/c)^(1/4))/(a*sqrt(2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx = -\frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}}{(-a)^{1/8}}\right)}{2\,(-a)^{7/8}\,c^{1/8}} + \frac{\operatorname{atan}\left(\frac{c^{1/8}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{2\,(-a)^{7/8}\,c^{1/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{(-a)^{7/8}\,c^{1/8}} + \frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,c^{1/8}\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{(-a)^{7/8}\,c^{1/8}}$$

input
```
int(1/(x^(1/2)*(a + c*x^4)),x)
```

output
```
(atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(2*(-a)^(7/8)*c^(1/8)) - atan((
c^(1/8)*x^(1/2))/(-a)^(1/8))/(2*(-a)^(7/8)*c^(1/8)) - (2^(1/2)*atan((2^(1/
2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(1/4 + 1i/4))/((-a)^(7/8)*c^(
1/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(
1/4 - 1i/4))/((-a)^(7/8)*c^(1/8))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)}\,dx$$
$$= \frac{-2\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}-2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)+2\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}+2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)-2\sqrt{-\sqrt{2}+2}\,atan\left(\right.}{}$$

input
```
int(1/x^(1/2)/(c*x^4+a),x)
```

output
```
(c**(7/8)*a**(1/8)*( - 2*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
 + 2*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sq
rt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))) - 2*sqrt( - sqrt(2)
 + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**
(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2))) + 2*sqrt( - sqrt(2) + 2)*atan((c**(1
/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2))) - sqrt( - sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/
8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) + sqrt( - sqrt(2) + 2)*lo
g(sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)
- sqrt(sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + a
**(1/4) + c**(1/4)*x) + sqrt(sqrt(2) + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)))/(8*a*c)
```

3.139 $\int \frac{1}{x^{3/2}(a+cx^4)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 238

$$\int \frac{1}{x^{3/2}(a+cx^4)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{\sqrt[8]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{9/8}} + \frac{\sqrt[8]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{9/8}} + \frac{\sqrt[8]{c}\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} - \frac{\sqrt[8]{c}\,\mathrm{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{9/8}} - \frac{\sqrt[8]{c}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}(-a)^{9/8}}$$

output
```
-2/a/x^(1/2)+1/4*c^(1/8)*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(
1/2)/(-a)^(9/8)+1/4*c^(1/8)*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2
^(1/2)/(-a)^(9/8)+1/2*c^(1/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8
)-1/2*c^(1/8)*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)-1/4*c^(1/8)*a
rctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/
(-a)^(9/8)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{3/2}(a+cx^4)}\,dx = \frac{-8\sqrt[8]{a}+\sqrt{2+\sqrt{2}}\sqrt[8]{c}\sqrt{x}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)+\sqrt[8]{c}\sqrt{-\left(\left(-2+\sqrt{2}\right)x\right)}\,\mathrm{ar}}{}$$

input
```
Integrate[1/(x^(3/2)*(a + c*x^4)),x]
```

output
```
(-8*a^(1/8) + Sqrt[2 + Sqrt[2]]*c^(1/8)*Sqrt[x]*ArcTan[(Sqrt[1 - 1/Sqrt[2]
]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + c^(1/8)*Sqrt[-((-2 +
 Sqrt[2])*x)]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*
c^(1/8)*Sqrt[x])] + Sqrt[2 + Sqrt[2]]*c^(1/8)*Sqrt[x]*ArcTanh[(Sqrt[2 + Sq
rt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + c^(1/8)*Sqrt[-((-
2 + Sqrt[2])*x)]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1
/4) + c^(1/4)*x)])/(4*a^(9/8)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.42, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {847, 851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+cx^4)}\,dx$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{c\int \frac{x^{5/2}}{cx^4+a}dx}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{ 851}$$

$$-\frac{2c\int \frac{x^3}{cx^4+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{830}$$

$$-\frac{2c\left(\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{1476}$$

$$-\frac{2c\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ \textbf{1082}$$

$$-\frac{2c\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ \textbf{217}$$

$$-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ \textbf{1479}$$

$$-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}} \cdots\right)}{a}$$

$$\frac{2}{a\sqrt{x}}$$

$\downarrow$ 25

$$-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}} - \cdots\right)}{a}$$

$$\frac{2}{a\sqrt{x}}$$

$\downarrow$ 27

$$-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}} - \frac{\text{arctanh}\left(\cdots\right)}{2\sqrt[8]{-a}c^{3/8}} \cdots\right)}{a}$$

$$\frac{2}{a\sqrt{x}}$$

$\Big\downarrow$ 1103

$$-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\cdots\right)}{a}-\frac{2}{a\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(a + c*x^4)),x]
```

output
```
-2/(a*Sqrt[x]) - (2*c*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-
a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^
(3/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sq
rt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/
8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqr
t[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))
+ Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt
[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2*Sqrt[c])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result	size
derivativedivides	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{4a}-\frac{2}{a\sqrt{x}}$	38
default	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{4a}-\frac{2}{a\sqrt{x}}$	38
risch	$-\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{4a}-\frac{2}{a\sqrt{x}}$	38

input
```
int(1/x^(3/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*sum(1/_R*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))-2/a/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}(a+cx^4)}\,dx =$$
$$-\frac{-(i-1)\sqrt{2}ax\left(-\frac{c}{a^9}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^8\left(-\frac{c}{a^9}\right)^{\frac{7}{8}}+c\sqrt{x}\right)+(i+1)\sqrt{2}ax\left(-\frac{c}{a^9}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}a\right.}{}$$

input
```
integrate(1/x^(3/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/8*(-(I - 1)*sqrt(2)*a*x*(-c/a^9)^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^8*(-
c/a^9)^(7/8) + c*sqrt(x)) + (I + 1)*sqrt(2)*a*x*(-c/a^9)^(1/8)*log(-(1/2*I
 - 1/2)*sqrt(2)*a^8*(-c/a^9)^(7/8) + c*sqrt(x)) - (I + 1)*sqrt(2)*a*x*(-c/
a^9)^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^8*(-c/a^9)^(7/8) + c*sqrt(x)) + (I
- 1)*sqrt(2)*a*x*(-c/a^9)^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^8*(-c/a^9)^(7
/8) + c*sqrt(x)) + 2*a*x*(-c/a^9)^(1/8)*log(a^8*(-c/a^9)^(7/8) + c*sqrt(x)
) - 2*I*a*x*(-c/a^9)^(1/8)*log(I*a^8*(-c/a^9)^(7/8) + c*sqrt(x)) + 2*I*a*x
*(-c/a^9)^(1/8)*log(-I*a^8*(-c/a^9)^(7/8) + c*sqrt(x)) - 2*a*x*(-c/a^9)^(1
/8)*log(-a^8*(-c/a^9)^(7/8) + c*sqrt(x)) + 16*sqrt(x))/(a*x)
```

Sympy [A] (verification not implemented)

Time = 21.17 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} \\ -\frac{2}{9cx^{\frac{9}{2}}} \\ -\frac{2}{a\sqrt{x}} \\ -\frac{\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4a\sqrt[8]{-\frac{a}{c}}}+\frac{\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4a\sqrt[8]{-\frac{a}{c}}}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{-\frac{a}{c}}\right)}{8a\sqrt[8]{-\frac{a}{c}}}+\frac{\sqrt{2}\log\left(4\sqrt{}\right.}{} \end{cases}$$

input
```
integrate(1/x**(3/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(c, 0)), (-2/(9*c*x**(9/2)), Eq(a, 0
)), (-2/(a*sqrt(x)), Eq(c, 0)), (-log(sqrt(x) - (-a/c)**(1/8))/(4*a*(-a/c)
**(1/8)) + log(sqrt(x) + (-a/c)**(1/8))/(4*a*(-a/c)**(1/8)) - sqrt(2)*log(
-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8*a*(-a/c)**(1/
8)) + sqrt(2)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))
/(8*a*(-a/c)**(1/8)) - atan(sqrt(x)/(-a/c)**(1/8))/(2*a*(-a/c)**(1/8)) - s
qrt(2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*a*(-a/c)**(1/8)) - sqrt(
2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*a*(-a/c)**(1/8)) - 2/(a*sqrt
(x)), True))
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx = \int \frac{1}{(cx^4+a)x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-c*integrate(x^(5/2)/(a*c*x^4 + a^2), x) - 2/(a*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.94

$$\int \frac{1}{x^{3/2}(a+cx^4)}\,dx = -\frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{2\sqrt{2}+4}} + \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{2\sqrt{2}+4}} - \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
-1/2*c*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sq
rt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/2*c*(a/c)^(7/
8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)
*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/2*c*(a/c)^(7/8)*arctan((sqrt
(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(
a^2*sqrt(2*sqrt(2) + 4)) - 1/2*c*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a
/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(
2) + 4)) + 1/4*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x
 + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 1/4*c*(a/c)^(7/8)*log(-sqrt(x
)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) +
4)) + 1/4*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (
a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/4*c*(a/c)^(7/8)*log(-sqrt(x)*sqr
t(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) -
 2/(a*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{(-c)^{1/8}\,\mathrm{atan}\left(\frac{(-c)^{1/8}\sqrt{x}}{a^{1/8}}\right)}{2\,a^{9/8}} - \frac{(-c)^{1/8}\,\mathrm{atan}\left(\frac{(-c)^{1/8}\sqrt{x}\,1\mathrm{i}}{a^{1/8}}\right)\,1\mathrm{i}}{2\,a^{9/8}} + \frac{\sqrt{2}\,(-c)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a^{9/8}} + \frac{\sqrt{2}\,(-c)^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{a^{9/8}}$$

input
```
int(1/(x^(3/2)*(a + c*x^4)),x)
```

output
```
- 2/(a*x^(1/2)) - ((-c)^(1/8)*atan(((-c)^(1/8)*x^(1/2))/a^(1/8)))/(2*a^(9/
8)) - ((-c)^(1/8)*atan(((-c)^(1/8)*x^(1/2)*1i)/a^(1/8))*1i)/(2*a^(9/8)) -
(2^(1/2)*(-c)^(1/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)*(1/2 - 1i/2))/a^(1/8)
)*(1/4 - 1i/4))/a^(9/8) - (2^(1/2)*(-c)^(1/8)*atan((2^(1/2)*(-c)^(1/8)*x^(
1/2)*(1/2 + 1i/2))/a^(1/8))*(1/4 + 1i/4))/a^(9/8)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)}\,dx = \frac{2\sqrt{x}\,c^{\frac{1}{8}}a^{\frac{7}{8}}\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-\sqrt{2}+2}-2\sqrt{x}\,c^{\frac{1}{4}}}{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{\sqrt{2}+2}}\right)-2\sqrt{x}\,c^{\frac{1}{8}}a^{\frac{7}{8}}\sqrt{\sqrt{2}+2}\,atan\left(\frac{c^{\frac{1}{8}}a^{\frac{1}{8}}\sqrt{-}}{c^{\frac{1}{8}}}\right.}{}$$

input
```
int(1/x^(3/2)/(c*x^4+a),x)
```

output
```
(2*sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2))) - 2*sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/
8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt
(2) + 2))) + 2*sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/
8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2))) - 2*sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*ata
n((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt( - sqrt(2) + 2))) - sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) +
 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**
(1/4)*x) + sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*log(sqrt(x)*c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x) - sqrt(x)*c**(
1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt(sqrt
(2) + 2) + a**(1/4) + c**(1/4)*x) + sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2)
 + 2)*log(sqrt(x)*c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + a**(1/4) + c**(1/4
)*x) - 16*a)/(8*sqrt(x)*a**2)
```

3.140 $\int \frac{1}{x^{5/2}(a+cx^4)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 240

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx = -\frac{2}{3ax^{3/2}}-\frac{c^{3/8}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{11/8}}+\frac{c^{3/8}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt{2}(-a)^{11/8}}-\frac{c^{3/8}\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}}-\frac{c^{3/8}\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{11/8}}+\frac{c^{3/8}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{2\sqrt{2}(-a)^{11/8}}$$

output
```
-2/3/a/x^(3/2)+1/4*c^(3/8)*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2
^(1/2)/(-a)^(11/8)+1/4*c^(3/8)*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8)
)*2^(1/2)/(-a)^(11/8)-1/2*c^(3/8)*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^
(11/8)-1/2*c^(3/8)*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)+1/4*c^(
3/8)*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^
(1/2)/(-a)^(11/8)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^{5/2}(a+cx^4)}\,dx =$$

$$-\frac{8a^{3/8}+3\sqrt{2-\sqrt{2}}c^{3/8}x^{3/2}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)-3\sqrt{2+\sqrt{2}}c^{3/8}x^{3/2}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{12a^{1}}$$

input
```
Integrate[1/(x^(5/2)*(a + c*x^4)),x]
```

output
```
-1/12*(8*a^(3/8) + 3*Sqrt[2 - Sqrt[2]]*c^(3/8)*x^(3/2)*ArcTan[(Sqrt[1 - 1/
Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] - 3*Sqrt[2 + Sq
rt[2]]*c^(3/8)*x^(3/2)*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/
(a^(1/8)*c^(1/8)*Sqrt[x])] - 3*Sqrt[2 - Sqrt[2]]*c^(3/8)*x^(3/2)*ArcTanh[(
Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + 3*Sqrt
[2 + Sqrt[2]]*c^(3/8)*x^(3/2)*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2
])*x)])/(a^(1/4) + c^(1/4)*x)])/(a^(11/8)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {847, 851, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+cx^4)}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{c\int \frac{x^{3/2}}{cx^4+a}dx}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$-\frac{2c\int \frac{x^2}{cx^4+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}$$

$\Big\downarrow$ 830

$$-\frac{2c\left(\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\Big\downarrow$ 755

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\Big\downarrow$ 756

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\Big\downarrow$ 218

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\Big\downarrow$ 221

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\Big\downarrow$ 1476

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\downarrow$ 1082

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\downarrow$ 217

$$-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$\downarrow$ 1479

$$-\frac{2c\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\cdots\right)}{a}\cdots$$

$$\frac{2}{3ax^{3/2}}$$

$\big\downarrow$ 25

$$-\frac{2c\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{a}\cdots$$

$$\frac{2}{3ax^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{2c\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)\cdots}{2(-a)^{3/8}\sqrt[8]{c}}\cdots\right)}{a}\cdots$$

$$\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \ 1103$$

$$-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{a}$$

$$\frac{2}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a + c*x^4)),x]
```

output
```
-2/(3*a*x^(3/2)) - (2*c*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a
)^(3/8)*c^(1/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(
1/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqr
t[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8
)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - S
qrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)
) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sq
rt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)))/(2*Sqrt[c])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

method	result	size
derivativedivides	$-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}}{4a}$	38
default	$-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}}{4a}$	38
risch	$-\frac{2}{3a\,x^{\frac{3}{2}}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}}{4a}$	38

input
```
int(1/x^(5/2)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/x^(3/2)-1/4/a*sum(1/_R^3*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^{5/2}(a+cx^4)}\,dx = -\frac{(3i+3)\sqrt{2}ax^2\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^7\left(-\frac{c^3}{a^{11}}\right)^{\frac{5}{8}}+c^2\sqrt{x}\right)-(3i-3)\sqrt{2}ax^2\left(-\frac{c^3}{a^{11}}\right)^{\frac{1}{8}}\log\left(-\left(\frac{1}{2}i-\right.\right.}{}$$

input
```
integrate(1/x^(5/2)/(c*x^4+a),x, algorithm="fricas")
```

output
```
-1/24*((3*I + 3)*sqrt(2)*a*x^2*(-c^3/a^11)^(1/8)*log((1/2*I + 1/2)*sqrt(2)
*a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) - (3*I - 3)*sqrt(2)*a*x^2*(-c^3/a^11
)^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) +
(3*I - 3)*sqrt(2)*a*x^2*(-c^3/a^11)^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^7*(-
c^3/a^11)^(5/8) + c^2*sqrt(x)) - (3*I + 3)*sqrt(2)*a*x^2*(-c^3/a^11)^(1/8)
*log(-(1/2*I + 1/2)*sqrt(2)*a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) - 6*a*x^2
*(-c^3/a^11)^(1/8)*log(a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) - 6*I*a*x^2*(-
c^3/a^11)^(1/8)*log(I*a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) + 6*I*a*x^2*(-c
^3/a^11)^(1/8)*log(-I*a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) + 6*a*x^2*(-c^3
/a^11)^(1/8)*log(-a^7*(-c^3/a^11)^(5/8) + c^2*sqrt(x)) + 16*sqrt(x))/(a*x^
2)
```

Sympy [A] (verification not implemented)

Time = 42.66 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} \\ -\frac{2}{11cx^{\frac{11}{2}}} \\ -\frac{2}{3ax^{\frac{3}{2}}} \\ -\frac{\log\left(\sqrt{x}-\sqrt[8]{-\frac{a}{c}}\right)}{4a\left(-\frac{a}{c}\right)^{\frac{3}{8}}}+\frac{\log\left(\sqrt{x}+\sqrt[8]{-\frac{a}{c}}\right)}{4a\left(-\frac{a}{c}\right)^{\frac{3}{8}}}+\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}\sqrt[8]{-\frac{a}{c}}+4x+4\sqrt[4]{-\frac{a}{c}}\right)}{8a\left(-\frac{a}{c}\right)^{\frac{3}{8}}}-\frac{\sqrt{2}\log\left(4\sqrt{}\right.}{} \end{cases}$$

input
```
integrate(1/x**(5/2)/(c*x**4+a),x)
```

output
```
Piecewise((zoo/x**(11/2), Eq(a, 0) & Eq(c, 0)), (-2/(11*c*x**(11/2)), Eq(a
, 0)), (-2/(3*a*x**(3/2)), Eq(c, 0)), (-log(sqrt(x) - (-a/c)**(1/8))/(4*a*
(-a/c)**(3/8)) + log(sqrt(x) + (-a/c)**(1/8))/(4*a*(-a/c)**(3/8)) + sqrt(2
)*log(-4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**(1/4))/(8*a*(-a/c
)**(3/8)) - sqrt(2)*log(4*sqrt(2)*sqrt(x)*(-a/c)**(1/8) + 4*x + 4*(-a/c)**
(1/4))/(8*a*(-a/c)**(3/8)) + atan(sqrt(x)/(-a/c)**(1/8))/(2*a*(-a/c)**(3/8
)) - sqrt(2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) - 1)/(4*a*(-a/c)**(3/8)) -
 sqrt(2)*atan(sqrt(2)*sqrt(x)/(-a/c)**(1/8) + 1)/(4*a*(-a/c)**(3/8)) - 2/(
3*a*x**(3/2)), True))
```

Maxima [F]

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx = \int \frac{1}{(cx^4+a)x^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^(5/2)/(c*x^4+a),x, algorithm="maxima")
```

output
```
-c*integrate(x^(3/2)/(a*c*x^4 + a^2), x) - 2/3/(a*x^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(159) = 318$.

Time = 0.22 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx = \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{2\sqrt{2}+4}} + \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{2\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{2\sqrt{2}+4}} - \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{c\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{4\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{2}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(c*x^4+a),x, algorithm="giac")
```

output
```
1/2*c*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqr
t(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/2*c*(a/c)^(5/8)
*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(
a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/2*c*(a/c)^(5/8)*arctan((sqrt(sq
rt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2
*sqrt(-2*sqrt(2) + 4)) - 1/2*c*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c
)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2
) + 4)) + 1/4*c*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x
+ (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/4*c*(a/c)^(5/8)*log(-sqrt(x)*
sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4))
 - 1/4*c*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c
)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/4*c*(a/c)^(5/8)*log(-sqrt(x)*sqrt(
-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) -
2/3/(a*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)}\,dx=\frac{(-c)^{3/8}\,\mathrm{atan}\left(\frac{(-c)^{1/8}\sqrt{x}}{a^{1/8}}\right)}{2\,a^{11/8}}-\frac{2}{3\,a\,x^{3/2}}-\frac{(-c)^{3/8}\,\mathrm{atan}\left(\frac{(-c)^{1/8}\sqrt{x}\,1\mathrm{i}}{a^{1/8}}\right)\,1\mathrm{i}}{2\,a^{11/8}}+\frac{\sqrt{2}\,(-c)^{3/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)}{a^{11/8}}+\frac{\sqrt{2}\,(-c)^{3/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,(-c)^{1/8}\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{a^{1/8}}\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)}{a^{11/8}}$$

input
```
int(1/(x^(5/2)*(a + c*x^4)),x)
```

output
```
((-c)^(3/8)*atan(((-c)^(1/8)*x^(1/2))/a^(1/8)))/(2*a^(11/8)) - 2/(3*a*x^(3
/2)) - ((-c)^(3/8)*atan(((-c)^(1/8)*x^(1/2)*1i)/a^(1/8))*1i)/(2*a^(11/8))
- (2^(1/2)*(-c)^(3/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)*(1/2 - 1i/2))/a^(1/
8))*(1/4 + 1i/4))/a^(11/8) - (2^(1/2)*(-c)^(3/8)*atan((2^(1/2)*(-c)^(1/8)*
x^(1/2)*(1/2 + 1i/2))/a^(1/8))*(1/4 - 1i/4))/a^(11/8)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 804, normalized size of antiderivative = 3.35

$$\int \frac{1}{x^{5/2}\left(a+cx^4\right)} \, dx = \text{Too large to display}$$

input
```
int(1/x^(5/2)/(c*x^4+a),x)
```

output
```
( - 6*sqrt(x)*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a
**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt
(sqrt(2) + 2)))*x + 6*sqrt(x)*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*atan((c*
*(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1
/8)*sqrt(sqrt(2) + 2)))*x + 6*sqrt(x)*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*
sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))
/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*x - 6*sqrt(x)*c**(3/8)*a**(5/8)*sq
rt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c
**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*x + 6*sqrt(x)*c**(3/8)*a**
(5/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*x + 6*s
qrt(x)*c**(3/8)*a**(5/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt
(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2
)))*x - 6*sqrt(x)*c**(3/8)*a**(5/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**
(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*
sqrt( - sqrt(2) + 2)))*x - 6*sqrt(x)*c**(3/8)*a**(5/8)*sqrt( - sqrt(2) + 2
)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8
)*a**(1/8)*sqrt( - sqrt(2) + 2)))*x + 3*sqrt(x)*c**(3/8)*a**(5/8)*sqrt( -
sqrt(2) + 2)*sqrt(2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)
 + a**(1/4) + c**(1/4)*x)*x - 3*sqrt(x)*c**(3/8)*a**(5/8)*sqrt( - sqrt(...
```

3.141 $\int \frac{x^{13/2}}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\begin{aligned}\int \frac{x^{13/2}}{(a+cx^4)^2} dx =& -\frac{x^{7/2}}{4c(a+cx^4)} - \frac{7\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\sqrt[8]{-a}c^{15/8}} \\ &+ \frac{7\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}\sqrt[8]{-a}c^{15/8}} + \frac{7\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}c^{15/8}} \\ &- \frac{7\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt[8]{-a}c^{15/8}} - \frac{7\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}\sqrt[8]{-a}c^{15/8}}\end{aligned}$$

output
```
-1/4*x^(7/2)/c/(c*x^4+a)+7/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8)
)*2^(1/2)/(-a)^(1/8)/c^(15/8)+7/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(
1/8))*2^(1/2)/(-a)^(1/8)/c^(15/8)+7/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/
(-a)^(1/8)/c^(15/8)-7/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(1/8)/c^
(15/8)-7/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)
*x))*2^(1/2)/(-a)^(1/8)/c^(15/8)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{x^{13/2}}{(a+cx^4)^2}\,dx = \frac{-\frac{8c^{7/8}x^{7/2}}{a+cx^4}-\frac{7\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{\sqrt[8]{a}}-\frac{7\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{\sqrt[8]{a}}-\frac{7\sqrt{\cdots}}{\cdots}}{32c^{15/8}}$$

input
```
Integrate[x^(13/2)/(a + c*x^4)^2,x]
```

output
```
((-8*c^(7/8)*x^(7/2))/(a + c*x^4) - (7*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 -
1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/a^(1/8) - (7
*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(
1/8)*c^(1/8)*Sqrt[x])])/a^(1/8) - (7*Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2 + S
qrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/a^(1/8) - (7*Sqrt
[2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
 + c^(1/4)*x)])/a^(1/8))/(32*c^(15/8))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {817, 851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(a+cx^4)^2}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7\int \frac{x^{5/2}}{cx^4+a}dx}{8c}-\frac{x^{7/2}}{4c(a+cx^4)}$$

$$\Big\downarrow \text{851}$$

$$\frac{7\int \frac{x^3}{cx^4+a}d\sqrt{x}}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 830

$$\frac{7\left(\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 826

$$\frac{7\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 827

$$\frac{7\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{7\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 221

$$\frac{7\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{7\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{7\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\cdots\right)}{4c}$$

$$\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\big\downarrow$ 25

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arct}\cdots}{\cdots}\right)}{4c}$$

$$\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$\big\downarrow$ 27

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\cdots\right)}{2\sqrt[8]{-a}c^{3/8}}\cdots}{2\cdots}\right)}{4c}$$

$$\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arct}\cdots}{\cdots}\right.}{4c}$$

$$\frac{x^{7/2}}{4c\left(a+cx^4\right)}$$

input
```
Int[x^(13/2)/(a + c*x^4)^2,x]
```

output
```
-1/4*x^(7/2)/(c*(a + c*x^4)) + (7*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a
)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(
-a)^(1/8)*c^(3/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a
)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[
x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)
^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/
8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4
)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2*Sqrt[c])))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

method	result	size
derivativedivides	$-\frac{x^{\frac{7}{2}}}{4c(c\,x^{4}+a)}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{32c^{2}}$	47
default	$-\frac{x^{\frac{7}{2}}}{4c(c\,x^{4}+a)}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{32c^{2}}$	47

input
```
int(x^(13/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^(7/2)/c/(c*x^4+a)+7/32/c^2*sum(1/_R*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c
+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.64

$$\int \frac{x^{13/2}}{(a+cx^4)^2}\,dx =$$

$$-\frac{16\,x^{\frac{7}{2}} + 7\sqrt{2}((i-1)\,c^2x^4 + (i-1)\,ac)\left(-\frac{1}{ac^{15}}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}ac^{13}\left(-\frac{1}{ac^{15}}\right)^{\frac{7}{8}}+\sqrt{x}\right)+7\sqrt{2}(-(i+1)}{}$$

input
```
integrate(x^(13/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(16*x^(7/2) + 7*sqrt(2)*((I - 1)*c^2*x^4 + (I - 1)*a*c)*(-1/(a*c^15)
)^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)) +
7*sqrt(2)*(-(I + 1)*c^2*x^4 - (I + 1)*a*c)*(-1/(a*c^15))^(1/8)*log(-(1/2*I
 - 1/2)*sqrt(2)*a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)) + 7*sqrt(2)*((I + 1)
*c^2*x^4 + (I + 1)*a*c)*(-1/(a*c^15))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a*c^
13*(-1/(a*c^15))^(7/8) + sqrt(x)) + 7*sqrt(2)*(-(I - 1)*c^2*x^4 - (I - 1)*
a*c)*(-1/(a*c^15))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a*c^13*(-1/(a*c^15))^(
7/8) + sqrt(x)) - 14*(c^2*x^4 + a*c)*(-1/(a*c^15))^(1/8)*log(a*c^13*(-1/(a
*c^15))^(7/8) + sqrt(x)) + 14*(I*c^2*x^4 + I*a*c)*(-1/(a*c^15))^(1/8)*log(
I*a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)) + 14*(-I*c^2*x^4 - I*a*c)*(-1/(a*c
^15))^(1/8)*log(-I*a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)) + 14*(c^2*x^4 + a
*c)*(-1/(a*c^15))^(1/8)*log(-a*c^13*(-1/(a*c^15))^(7/8) + sqrt(x)))/(c^2*x
^4 + a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(a+cx^4)^2} dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}}{(a+cx^4)^2} dx = \int \frac{x^{\frac{13}{2}}}{(cx^4+a)^2} dx$$

input
```
integrate(x^(13/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^(7/2)/(c^2*x^4 + a*c) + 7*integrate(1/8*x^(5/2)/(c^2*x^4 + a*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(168) = 336$.

Time = 0.26 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.95

$$\int \frac{x^{13/2}}{(a+cx^4)^2}\,dx = -\frac{x^{\frac{7}{2}}}{4\,(cx^4+a)c} + \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} - \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}} - \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}}$$

input
```
integrate(x^(13/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*x^(7/2)/((c*x^4 + a)*c) + 7/16*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)
*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sq
rt(2) + 4)) + 7/16*(a/c)^(7/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2
*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 7/
16*(a/c)^(7/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-s
qrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 7/16*(a/c)^(7/8)*arc
tan(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)
^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) - 7/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) + 7/32
*(a/c)^(7/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))
/(a*c*sqrt(-2*sqrt(2) + 4)) - 7/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) +
 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) + 7/32*(a/c)^
(7/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*
sqrt(2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{x^{13/2}}{(a+cx^4)^2}\,dx = \frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{1/8}\,c^{15/8}} - \frac{x^{7/2}}{4\,c\,(c\,x^4+a)} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,7\mathrm{i}}{16\,(-a)^{1/8}\,c^{15/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{7}{32}-\frac{7}{32}\mathrm{i}\right)}{(-a)^{1/8}\,c^{15/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{7}{32}+\frac{7}{32}\mathrm{i}\right)}{(-a)^{1/8}\,c^{15/8}}$$

input
```
int(x^(13/2)/(a + c*x^4)^2,x)
```

output
```
(7*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)^(1/8)*c^(15/8)) - x^(7/2)/
(4*c*(a + c*x^4)) + (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*7i)/(16*(-a)^(1
/8)*c^(15/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^
(1/8))*(7/32 - 7i/32))/((-a)^(1/8)*c^(15/8)) + (2^(1/2)*atan((2^(1/2)*c^(1
/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(7/32 + 7i/32))/((-a)^(1/8)*c^(15/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 775, normalized size of antiderivative = 3.11

$$\int \frac{x^{13/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(13/2)/(c*x^4+a)^2,x)
```

output
```
( - 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
*a - 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(
- sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))
)*c*x**4 + 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*
sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2)
 + 2)))*a + 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)
*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2
) + 2)))*c*x**4 - 14*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)
*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(
 - sqrt(2) + 2)))*a - 14*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2)))*c*x**4 + 14*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*a
tan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a
**(1/8)*sqrt( - sqrt(2) + 2)))*a + 14*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) +
2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/
8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c*x**4 + 7*c**(1/8)*a**(7/8)*sqrt( - sq
rt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4
) + c**(1/4)*x)*a + 7*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*log( - sqrt(x
)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)*c*x**...
```

3.142 $\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = -\frac{x^{5/2}}{4c(a+cx^4)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{3/8}c^{13/8}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{3/8}c^{13/8}} - \frac{5\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} - \frac{5\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{3/8}c^{13/8}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{3/8}c^{13/8}}$$

output

```
-1/4*x^(5/2)/c/(c*x^4+a)+5/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8)
)*2^(1/2)/(-a)^(3/8)/c^(13/8)+5/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(
1/8))*2^(1/2)/(-a)^(3/8)/c^(13/8)-5/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/
(-a)^(3/8)/c^(13/8)-5/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(3/8)/c^
(13/8)+5/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)
*x))*2^(1/2)/(-a)^(3/8)/c^(13/8)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = \frac{-\frac{8c^{5/8}x^{5/2}}{a+cx^4}+\frac{5\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{3/8}}-\frac{5\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{3/8}}-\frac{5\sqrt{}}{}}{32c^{13/8}}$$

input
```
Integrate[x^(11/2)/(a + c*x^4)^2,x]
```

output
```
((-8*c^(5/8)*x^(5/2))/(a + c*x^4) + (5*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 -
1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/a^(3/8) - (5
*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(
1/8)*c^(1/8)*Sqrt[x])])/a^(3/8) - (5*Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[2 + S
qrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/a^(3/8) + (5*Sqrt
[2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
 + c^(1/4)*x)])/a^(3/8))/(32*c^(13/8))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.43, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {817, 851, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \; 817$$

$$\frac{5\int \frac{x^{3/2}}{cx^4+a}dx}{8c}-\frac{x^{5/2}}{4c\,(a+cx^4)}$$

$$\Big\downarrow \; 851$$

$$\frac{5\int \frac{x^2}{cx^4+a}d\sqrt{x}}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 830

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 755

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 756

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 221

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \ \textcolor{blue}{1082}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \ \textcolor{blue}{217}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4c}-\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \ \textcolor{blue}{1479}$$

$$\frac{5\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{4c}$$

$$\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{4c}$$

$$\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\cdots}{\cdots}\right)}{4c}$$

$$\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

$\downarrow$ 1103

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\text{arcta}\cdots}{2(-\cdots}\right.}{4c}$$

$$\frac{x^{5/2}}{4c\left(a+cx^4\right)}$$

input
```
Int[x^(11/2)/(a + c*x^4)^2,x]
```

output
```
-1/4*x^(5/2)/(c*(a + c*x^4)) + (5*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/
8)]/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)
^(3/8)*c^(1/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(
1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])
/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)
^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/
8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4
)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)))/(2*Sqrt[c])))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

method	result	size
derivativedivides	$-\frac{x^{\frac{5}{2}}}{4c(c\,x^{4}+a)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{32c^{2}}$	47
default	$-\frac{x^{\frac{5}{2}}}{4c(c\,x^{4}+a)}+\frac{5\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{32c^{2}}$	47

input
```
int(x^(11/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^(5/2)/c/(c*x^4+a)+5/32/c^2*sum(1/_R^3*ln(x^(1/2)-_R),_R=RootOf(_Z^8
*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.70

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = -\frac{5\sqrt{2}(-(i+1)\ c^2x^4-(i+1)\ ac)\left(-\frac{1}{a^3c^{13}}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^2c^8\left(-\frac{1}{a^3c^{13}}\right)^{\frac{5}{8}}+\sqrt{x}\right)+5\sqrt{2}((i-1)\ c^2x^4\cdots}{\cdots}$$

input
```
integrate(x^(11/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(5*sqrt(2)*(-(I + 1)*c^2*x^4 - (I + 1)*a*c)*(-1/(a^3*c^13))^(1/8)*lo
g((1/2*I + 1/2)*sqrt(2)*a^2*c^8*(-1/(a^3*c^13))^(5/8) + sqrt(x)) + 5*sqrt(
2)*((I - 1)*c^2*x^4 + (I - 1)*a*c)*(-1/(a^3*c^13))^(1/8)*log(-(1/2*I - 1/2
)*sqrt(2)*a^2*c^8*(-1/(a^3*c^13))^(5/8) + sqrt(x)) + 5*sqrt(2)*(-(I - 1)*c
^2*x^4 - (I - 1)*a*c)*(-1/(a^3*c^13))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^2*
c^8*(-1/(a^3*c^13))^(5/8) + sqrt(x)) + 5*sqrt(2)*((I + 1)*c^2*x^4 + (I + 1
)*a*c)*(-1/(a^3*c^13))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^2*c^8*(-1/(a^3*c
^13))^(5/8) + sqrt(x)) + 16*x^(5/2) + 10*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(
1/8)*log(a^2*c^8*(-1/(a^3*c^13))^(5/8) + sqrt(x)) + 10*(I*c^2*x^4 + I*a*c)
*(-1/(a^3*c^13))^(1/8)*log(I*a^2*c^8*(-1/(a^3*c^13))^(5/8) + sqrt(x)) + 10
*(-I*c^2*x^4 - I*a*c)*(-1/(a^3*c^13))^(1/8)*log(-I*a^2*c^8*(-1/(a^3*c^13))
^(5/8) + sqrt(x)) - 10*(c^2*x^4 + a*c)*(-1/(a^3*c^13))^(1/8)*log(-a^2*c^8*
(-1/(a^3*c^13))^(5/8) + sqrt(x)))/(c^2*x^4 + a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^(5/2)/(c^2*x^4 + a*c) + 5*integrate(1/8*x^(3/2)/(c^2*x^4 + a*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(168) = 336$.

Time = 0.26 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.95

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = -\frac{x^{\frac{5}{2}}}{4\,(cx^4+a)c} - \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} - \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} - \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}} - \frac{5\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}}$$

input
```
integrate(x^(11/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*x^(5/2)/((c*x^4 + a)*c) - 5/16*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)
*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqr
t(2) + 4)) - 5/16*(a/c)^(5/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*
sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 5/16
*(a/c)^(5/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqr
t(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 5/16*(a/c)^(5/8)*arct
an(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^
(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) - 5/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) + 5/32*
(a/c)^(5/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/
(a*c*sqrt(2*sqrt(2) + 4)) + 5/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2
)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) - 5/32*(a/c)^(
5/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*s
qrt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = -\frac{x^{5/2}}{4\,c\,(c\,x^4+a)} - \frac{5\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{3/8}\,c^{13/8}} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,5\mathrm{i}}{16\,(-a)^{3/8}\,c^{13/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{5}{32}+\frac{5}{32}\mathrm{i}\right)}{(-a)^{3/8}\,c^{13/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{5}{32}-\frac{5}{32}\mathrm{i}\right)}{(-a)^{3/8}\,c^{13/8}}$$

input
```
int(x^(11/2)/(a + c*x^4)^2,x)
```

output
```
(atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*5i)/(16*(-a)^(3/8)*c^(13/8)) - (5*a
tan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)^(3/8)*c^(13/8)) - x^(5/2)/(4*c
*(a + c*x^4)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^
(1/8))*(5/32 + 5i/32))/((-a)^(3/8)*c^(13/8)) + (2^(1/2)*atan((2^(1/2)*c^(1
/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(5/32 - 5i/32))/((-a)^(3/8)*c^(13/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1555, normalized size of antiderivative = 6.24

$$\int \frac{x^{11/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(11/2)/(c*x^4+a)^2,x)
```

output
```
(10*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2)))*a + 10*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a*
*(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(
sqrt(2) + 2)))*c*x**4 - 10*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*atan((c**(1
/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)
*sqrt(sqrt(2) + 2)))*a - 10*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2)))*c*x**4 - 10*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt
(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c*
*(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a - 10*c**(3/8)*a**(5/8)*sqrt(sqrt(2)
+ 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(
1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 + 10*c**(3/8)*a**(5/8)
*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x
)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a + 10*c**(3/8)*a**(5/8
)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(
x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 - 10*c**(3/8)*a
**(5/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2)
+ 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a - 1
0*c**(3/8)*a**(5/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8...
```

3.143 $\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx = -\frac{x^{3/2}}{4c(a+cx^4)} + \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{5/8}c^{11/8}} - \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{5/8}c^{11/8}} + \frac{3\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} - \frac{3\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{5/8}c^{11/8}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{5/8}c^{11/8}}$$

output
```
-1/4*x^(3/2)/c/(c*x^4+a)-3/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8)
)*2^(1/2)/(-a)^(5/8)/c^(11/8)-3/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(
1/8))*2^(1/2)/(-a)^(5/8)/c^(11/8)+3/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/
(-a)^(5/8)/c^(11/8)-3/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(5/8)/c^
(11/8)+3/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)
*x))*2^(1/2)/(-a)^(5/8)/c^(11/8)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx = \frac{-\frac{8c^{3/8}x^{3/2}}{a+cx^4}+\frac{3\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{5/8}}-\frac{3\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{5/8}}+\frac{3\sqrt{}}{}}{32c^{11/8}}$$

input
```
Integrate[x^(9/2)/(a + c*x^4)^2,x]
```

output
```
((-8*c^(3/8)*x^(3/2))/(a + c*x^4) + (3*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 -
1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/a^(5/8) - (3
*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(
1/8)*c^(1/8)*Sqrt[x])])/a^(5/8) + (3*Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[2 + S
qrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/a^(5/8) - (3*Sqrt
[2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
 + c^(1/4)*x)])/a^(5/8))/(32*c^(11/8))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.43, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {817, 851, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{3\int \frac{\sqrt{x}}{cx^4+a}dx}{8c}-\frac{x^{3/2}}{4c(a+cx^4)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{3\int \frac{x}{cx^4+a}d\sqrt{x}}{4c}-\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 829$$

$$\frac{3\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4c}-\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 826$$

$$\frac{3\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4c}-\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 827$$

$$\frac{3\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4c}-\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{3\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4c}-\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{3\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4c}-\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{3\left(-\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4c}$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1082

$$\frac{3\left(-\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4c}-$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 217

$$\frac{3\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4c}-$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1479

$$\frac{3\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right)}{4c}$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$\downarrow$ 25

$$\frac{3\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right)}{4c}$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$\downarrow$ 27

$$\frac{3\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right)}{4c}$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

$$\downarrow \text{1103}$$

$$\frac{3\left(-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\cdots}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4c}$$

$$\frac{x^{3/2}}{4c\left(a+cx^4\right)}$$

input

```
Int[x^(9/2)/(a + c*x^4)^2,x]
```

output

```
-1/4*x^(3/2)/(c*(a + c*x^4)) + (3*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a
)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(
-a)^(1/8)*c^(3/8)))/Sqrt[-a] - ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-
a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt
[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a
)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1
/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/
4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2*Sqrt[-a])))/(4*c)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{4c(c\,x^{4}+a)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{32c^{2}}$	47
default	$-\frac{x^{\frac{3}{2}}}{4c(c\,x^{4}+a)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{32c^{2}}$	47

input
```
int(x^(9/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^(3/2)/c/(c*x^4+a)+3/32/c^2*sum(1/_R^5*ln(x^(1/2)-_R),_R=RootOf(_Z^8
*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.70

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx =$$
$$-\frac{3\sqrt{2}((i-1)\,c^2x^4+(i-1)\,ac)\left(-\frac{1}{a^5c^{11}}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^2c^4\left(-\frac{1}{a^5c^{11}}\right)^{\frac{3}{8}}+\sqrt{x}\right)+3\sqrt{2}(-(i+1)\,c^2x^4}{}$$

input
```
integrate(x^(9/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(3*sqrt(2)*((I - 1)*c^2*x^4 + (I - 1)*a*c)*(-1/(a^5*c^11))^(1/8)*log
((1/2*I + 1/2)*sqrt(2)*a^2*c^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)) + 3*sqrt(2
)*(-(I + 1)*c^2*x^4 - (I + 1)*a*c)*(-1/(a^5*c^11))^(1/8)*log(-(1/2*I - 1/2
)*sqrt(2)*a^2*c^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)) + 3*sqrt(2)*((I + 1)*c^
2*x^4 + (I + 1)*a*c)*(-1/(a^5*c^11))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^2*c
^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)) + 3*sqrt(2)*(-(I - 1)*c^2*x^4 - (I - 1
)*a*c)*(-1/(a^5*c^11))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^2*c^4*(-1/(a^5*c
^11))^(3/8) + sqrt(x)) + 6*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*log(a^2*c
^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)) + 6*(-I*c^2*x^4 - I*a*c)*(-1/(a^5*c^11
))^(1/8)*log(I*a^2*c^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)) + 6*(I*c^2*x^4 + I
*a*c)*(-1/(a^5*c^11))^(1/8)*log(-I*a^2*c^4*(-1/(a^5*c^11))^(3/8) + sqrt(x)
) - 6*(c^2*x^4 + a*c)*(-1/(a^5*c^11))^(1/8)*log(-a^2*c^4*(-1/(a^5*c^11))^(
3/8) + sqrt(x)) + 16*x^(3/2))/(c^2*x^4 + a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{\left(a+cx^4\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}}{\left(a+cx^4\right)^2}\,dx=\int \frac{x^{\frac{9}{2}}}{\left(cx^4+a\right)^2}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^(3/2)/(c^2*x^4 + a*c) + 3*integrate(1/8*sqrt(x)/(c^2*x^4 + a*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(168) = 336$.

Time = 0.25 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.95

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx = -\frac{x^{\frac{3}{2}}}{4\,(cx^4+a)c} - \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} - \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} - \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} - \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}}$$

input
```
integrate(x^(9/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-1/4*x^(3/2)/((c*x^4 + a)*c) - 3/16*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)
*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqr
t(2) + 4)) - 3/16*(a/c)^(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*
sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) + 4)) + 3/16
*(a/c)^(3/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqr
t(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 3/16*(a/c)^(3/8)*arct
an(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^
(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 3/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) - 3/32*
(a/c)^(3/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/
(a*c*sqrt(2*sqrt(2) + 4)) - 3/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2
)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) + 3/32*(a/c)^(
3/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*s
qrt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{5/8}\,c^{11/8}} - \frac{x^{3/2}}{4\,c\,(c\,x^4+a)} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,3\mathrm{i}}{16\,(-a)^{5/8}\,c^{11/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{3}{32}+\frac{3}{32}\mathrm{i}\right)}{(-a)^{5/8}\,c^{11/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{3}{32}-\frac{3}{32}\mathrm{i}\right)}{(-a)^{5/8}\,c^{11/8}}$$

input
```
int(x^(9/2)/(a + c*x^4)^2,x)
```

output
```
(3*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a)^(5/8)*c^(11/8)) - x^(3/2)/
(4*c*(a + c*x^4)) + (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*3i)/(16*(-a)^(5
/8)*c^(11/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^
(1/8))*(3/32 - 3i/32))/((-a)^(5/8)*c^(11/8)) - (2^(1/2)*atan((2^(1/2)*c^(1
/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(3/32 + 3i/32))/((-a)^(5/8)*c^(11/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1553, normalized size of antiderivative = 6.24

$$\int \frac{x^{9/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(c*x^4+a)^2,x)
```

output
```
(6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a + 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*c*x**4 - 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*a - 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*c*x**4 - 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*a
tan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8
)*a**(1/8)*sqrt(sqrt(2) + 2)))*a - 6*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*s
qrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/
(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 + 6*c**(5/8)*a**(3/8)*sqrt(s
qrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1
/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a + 6*c**(5/8)*a**(3/8)*sqrt(s
qrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1
/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 - 6*c**(5/8)*a**(3/8)*s
qrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*
sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a - 6*c**(5/8)
*a**(3/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqr...
```

3.144 $\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx=-\frac{\sqrt{x}}{4c(a+cx^4)}+\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{7/8}c^{9/8}}-\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{7/8}c^{9/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{7/8}c^{9/8}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{7/8}c^{9/8}}$$

output
```
-1/4*x^(1/2)/c/(c*x^4+a)-1/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8)
)*2^(1/2)/(-a)^(7/8)/c^(9/8)-1/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1
/8))*2^(1/2)/(-a)^(7/8)/c^(9/8)-1/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-
a)^(7/8)/c^(9/8)-1/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(9/
8)-1/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))
*2^(1/2)/(-a)^(7/8)/c^(9/8)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.10

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx = \frac{-\frac{8\sqrt[8]{c}\sqrt{x}}{a+cx^4} - \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{7/8}} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{7/8}} + \frac{\sqrt{2+\sqrt{}}\cdots}{}}{32c^{9/8}}$$

input
```
Integrate[x^(7/2)/(a + c*x^4)^2,x]
```

output
```
((-8*c^(1/8)*Sqrt[x])/(a + c*x^4) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 - 1/
Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/a^(7/8) - (Sqr
t[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)
*c^(1/8)*Sqrt[x])])/a^(7/8) + (Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]
]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/a^(7/8) + (Sqrt[2 - Sqr
t[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1/
4)*x)])/a^(7/8))/(32*c^(9/8))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {817, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{\int \frac{1}{\sqrt{x}(cx^4+a)}dx}{8c} - \frac{\sqrt{x}}{4c\,(a+cx^4)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{\int \frac{1}{cx^4+a}d\sqrt{x}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 758

$$\frac{-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 755

$$\frac{-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 756

$$\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 221

$$\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{-\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\cdots}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan(\cdots)}{2(-a\cdots}\cdots}{\cdots}}{4c}-\frac{\sqrt{x}}{4c\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\cdots}{2\sqrt{-\cdots}}}{4c}$$

$$\frac{\sqrt{x}}{4c\,(a+cx^4)}$$

$\big\downarrow$ 1103

$$-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\cdots\right)}{\cdots}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}$$

$$\frac{\sqrt{x}}{4c\,(a+cx^4)}$$

input
```
Int[x^(7/2)/(a + c*x^4)^2,x]
```

output
```
-1/4*Sqrt[x]/(c*(a + c*x^4)) + (-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]
/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3
/8)*c^(1/8)))/Sqrt[-a] - ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/
8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(
-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(
1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)
*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*
x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)))/(2*Sqrt[-a]))/(4*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.19

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{4c(c\,x^{4}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{7}}}{32c^{2}}$	47
default	$-\frac{\sqrt{x}}{4c(c\,x^{4}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{7}}}{32c^{2}}$	47

input
```
int(x^(7/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^(1/2)/c/(c*x^4+a)+1/32/c^2*sum(1/_R^7*ln(x^(1/2)-_R),_R=RootOf(_Z^8
*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.56

$$\int\frac{x^{7/2}}{(a+cx^4)^2}\,dx=$$

$$-\frac{\sqrt{2}(-(i+1)\;c^2x^4-(i+1)\;ac)\left(-\frac{1}{a^7c^9}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}ac\left(-\frac{1}{a^7c^9}\right)^{\frac{1}{8}}+\sqrt{x}\right)+\sqrt{2}((i-1)\;c^2x^4+(i-}{}$$

input
```
integrate(x^(7/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(sqrt(2)*(-(I + 1)*c^2*x^4 - (I + 1)*a*c)*(-1/(a^7*c^9))^(1/8)*log((
1/2*I + 1/2)*sqrt(2)*a*c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) + sqrt(2)*((I - 1
)*c^2*x^4 + (I - 1)*a*c)*(-1/(a^7*c^9))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a
*c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) + sqrt(2)*(-(I - 1)*c^2*x^4 - (I - 1)*a
*c)*(-1/(a^7*c^9))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a*c*(-1/(a^7*c^9))^(1/8
) + sqrt(x)) + sqrt(2)*((I + 1)*c^2*x^4 + (I + 1)*a*c)*(-1/(a^7*c^9))^(1/8
)*log(-(1/2*I + 1/2)*sqrt(2)*a*c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) - 2*(c^2*
x^4 + a*c)*(-1/(a^7*c^9))^(1/8)*log(a*c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) +
2*(-I*c^2*x^4 - I*a*c)*(-1/(a^7*c^9))^(1/8)*log(I*a*c*(-1/(a^7*c^9))^(1/8)
 + sqrt(x)) + 2*(I*c^2*x^4 + I*a*c)*(-1/(a^7*c^9))^(1/8)*log(-I*a*c*(-1/(a
^7*c^9))^(1/8) + sqrt(x)) + 2*(c^2*x^4 + a*c)*(-1/(a^7*c^9))^(1/8)*log(-a*
c*(-1/(a^7*c^9))^(1/8) + sqrt(x)) + 16*sqrt(x))/(c^2*x^4 + a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^(9/2)/(a*c*x^4 + a^2) - integrate(1/8*x^(7/2)/(a*c*x^4 + a^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(168) = 336$.

Time = 0.22 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.95

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx = \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,ac\sqrt{2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{-2\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,ac\sqrt{2\sqrt{2}+4}} - \frac{\sqrt{x}}{4\,(cx^4+a)c}$$

input
```
integrate(x^(7/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/16*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 1/16*(a/c)^(1/8)*
arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a
/c)^(1/8)))/(a*c*sqrt(-2*sqrt(2) + 4)) + 1/16*(a/c)^(1/8)*arctan((sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*
sqrt(2*sqrt(2) + 4)) + 1/16*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(
1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c*sqrt(2*sqrt(2) +
4)) + 1/32*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/
c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) - 1/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(-2*sqrt(2) + 4)) + 1
/32*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/
4))/(a*c*sqrt(2*sqrt(2) + 4)) - 1/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2
) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c*sqrt(2*sqrt(2) + 4)) - 1/4*sqrt
(x)/((c*x^4 + a)*c)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx = -\frac{\sqrt{x}}{4\,c\,(c\,x^4+a)} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{7/8}\,c^{9/8}} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{16\,(-a)^{7/8}\,c^{9/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)}{(-a)^{7/8}\,c^{9/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{32}+\frac{1}{32}\mathrm{i}\right)}{(-a)^{7/8}\,c^{9/8}}$$

input
```
int(x^(7/2)/(a + c*x^4)^2,x)
```

output
```
(atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(16*(-a)^(7/8)*c^(9/8)) - atan(
(c^(1/8)*x^(1/2))/(-a)^(1/8))/(16*(-a)^(7/8)*c^(9/8)) - x^(1/2)/(4*c*(a +
c*x^4)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))
*(1/32 + 1i/32))/((-a)^(7/8)*c^(9/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(
1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/32 - 1i/32))/((-a)^(7/8)*c^(9/8))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 768, normalized size of antiderivative = 3.08

$$\int \frac{x^{7/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(c*x^4+a)^2,x)
```

output
```
( - 2*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*
a - 2*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*
c*x**4 + 2*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a + 2*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*c*x**4 - 2*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(
1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sq
rt(2) + 2)))*a - 2*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a
**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2)))*c*x**4 + 2*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c*
*(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)
*sqrt( - sqrt(2) + 2)))*a + 2*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan(
(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1
/8)*sqrt( - sqrt(2) + 2)))*c*x**4 - c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)
*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/
4)*x)*a - c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a
**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)*c*x**4 + c**(7/8)...
```

3.145 $\int \frac{x^{5/2}}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\int \frac{x^{5/2}}{(a+cx^4)^2} dx = \frac{x^{7/2}}{4a(a+cx^4)} + \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{9/8}c^{7/8}} - \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{9/8}c^{7/8}} - \frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{9/8}c^{7/8}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{9/8}c^{7/8}}$$

output
```
1/4*x^(7/2)/a/(c*x^4+a)-1/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))
*2^(1/2)/(-a)^(9/8)/c^(7/8)-1/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/
8))*2^(1/2)/(-a)^(9/8)/c^(7/8)-1/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a
)^(9/8)/c^(7/8)+1/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(7/8
)+1/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*
2^(1/2)/(-a)^(9/8)/c^(7/8)
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{x^{5/2}}{(a+cx^4)^2}\,dx = \frac{\frac{8\sqrt[8]{a}x^{7/2}}{a+cx^4} - \frac{\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{7/8}} - \frac{\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{7/8}} - \frac{\sqrt{2+\sqrt{2}}}{}}{32a^{9/8}}$$

input
```
Integrate[x^(5/2)/(a + c*x^4)^2,x]
```

output
```
((8*a^(1/8)*x^(7/2))/(a + c*x^4) - (Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 - 1/S
qrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(7/8) - (Sqrt
[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*
c^(1/8)*Sqrt[x])])/c^(7/8) - (Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]
*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/c^(7/8) - (Sqrt[2 - Sqrt
[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1/4
)*x)])/c^(7/8))/(32*a^(9/8))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {819, 851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{\int \frac{x^{5/2}}{cx^4+a}dx}{8a} + \frac{x^{7/2}}{4a(a+cx^4)}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{\int \frac{x^3}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 830$$

$$\frac{\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 826$$

$$\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 827$$

$$\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+$$

$$\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\text{arc}\ldots}{4a}$$

$$\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arctan}\ldots}{2\sqrt[8]{\ldots}}}{4a}$$

$$\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-}{2\sqrt{c}}}{4a}$$

$$\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1103

$$\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\operatorname{arctanh}}{2\sqrt[8]{}}}{}}{4a}$$

$$\frac{x^{7/2}}{4a\left(a+cx^4\right)}$$

input
```
Int[x^(5/2)/(a + c*x^4)^2,x]
```

output
```
x^(7/2)/(4*a*(a + c*x^4)) + (-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8
)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1
/8)*c^(3/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8
)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-
a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4)
 - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(
1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(
2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2*Sqrt[c]))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.52 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.20

method	result	size
derivativedivides	$\frac{x^{\frac{7}{2}}}{4a(c\,x^{4}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{32ac}$	50
default	$\frac{x^{\frac{7}{2}}}{4a(c\,x^{4}+a)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}}{32ac}$	50

input
```
int(x^(5/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(7/2)/a/(c*x^4+a)+1/32/a/c*sum(1/_R*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+
a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.69

$$\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx=\frac{16\,x^{\frac{7}{2}}-\sqrt{2}((i-1)\;acx^4+(i-1)\;a^2)\left(-\frac{1}{a^9c^7}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\;\sqrt{2}a^8c^6\left(-\frac{1}{a^9c^7}\right)^{\frac{7}{8}}+\sqrt{x}\right)}{}$$

input
```
integrate(x^(5/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
1/64*(16*x^(7/2) - sqrt(2)*((I - 1)*a*c*x^4 + (I - 1)*a^2)*(-1/(a^9*c^7))^
(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)) -
sqrt(2)*(-(I + 1)*a*c*x^4 - (I + 1)*a^2)*(-1/(a^9*c^7))^(1/8)*log(-(1/2*I
- 1/2)*sqrt(2)*a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)) - sqrt(2)*((I + 1)*
a*c*x^4 + (I + 1)*a^2)*(-1/(a^9*c^7))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^8*
c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)) - sqrt(2)*(-(I - 1)*a*c*x^4 - (I - 1)*
a^2)*(-1/(a^9*c^7))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^8*c^6*(-1/(a^9*c^7)
)^(7/8) + sqrt(x)) + 2*(a*c*x^4 + a^2)*(-1/(a^9*c^7))^(1/8)*log(a^8*c^6*(-
1/(a^9*c^7))^(7/8) + sqrt(x)) - 2*(I*a*c*x^4 + I*a^2)*(-1/(a^9*c^7))^(1/8)
*log(I*a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)) - 2*(-I*a*c*x^4 - I*a^2)*(-
1/(a^9*c^7))^(1/8)*log(-I*a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(x)) - 2*(a*c
*x^4 + a^2)*(-1/(a^9*c^7))^(1/8)*log(-a^8*c^6*(-1/(a^9*c^7))^(7/8) + sqrt(
x)))/(a*c*x^4 + a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}}{\left(a+cx^4\right)^2}\,dx = \int \frac{x^{\frac{5}{2}}}{\left(cx^4+a\right)^2}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^(7/2)/(a*c*x^4 + a^2) + integrate(1/8*x^(5/2)/(a*c*x^4 + a^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(168) = 336$.

Time = 0.27 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.86

$$\int \frac{x^{5/2}}{(a+cx^4)^2}\,dx = \frac{x^{\frac{7}{2}}}{4\,(cx^4+a)a} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\,\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\,\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\,\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\,\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\,\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\,\sqrt{2}+4}} - \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\,\sqrt{2}+4}} + \frac{\left(\frac{a}{c}\right)^{\frac{7}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\,\sqrt{2}+4}}$$

input
```
integrate(x^(5/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x^(7/2)/((c*x^4 + a)*a) + 1/16*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqr
t(2) + 4)) + 1/16*(a/c)^(7/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*
sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/1
6*(a/c)^(7/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sq
rt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/16*(a/c)^(7/8)*arct
an(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^
(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) - 1/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt
(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 1/32*
(a/c)^(7/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/
(a^2*sqrt(-2*sqrt(2) + 4)) - 1/32*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) +
2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/32*(a/c)^(
7/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*s
qrt(2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{x^{5/2}}{(a+cx^4)^2}\,dx = \frac{x^{7/2}}{4\,a\,(c\,x^4+a)} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{9/8}\,c^{7/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,1\mathrm{i}}{16\,(-a)^{9/8}\,c^{7/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{32}+\frac{1}{32}\mathrm{i}\right)}{(-a)^{9/8}\,c^{7/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{1}{32}-\frac{1}{32}\mathrm{i}\right)}{(-a)^{9/8}\,c^{7/8}}$$

input
```
int(x^(5/2)/(a + c*x^4)^2,x)
```

output
```
x^(7/2)/(4*a*(a + c*x^4)) - atan((c^(1/8)*x^(1/2))/(-a)^(1/8))/(16*(-a)^(9
/8)*c^(7/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*1i)/(16*(-a)^(9/8)*c
^(7/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))
*(1/32 - 1i/32))/((-a)^(9/8)*c^(7/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(
1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(1/32 + 1i/32))/((-a)^(9/8)*c^(7/8))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 771, normalized size of antiderivative = 3.10

$$\int \frac{x^{5/2}}{(a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^(5/2)/(c*x^4+a)^2,x)
```

output
```
( - 2*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*
a - 2*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*
c*x**4 + 2*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a + 2*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*c*x**4 - 2*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(
1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sq
rt(2) + 2)))*a - 2*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a
**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2)))*c*x**4 + 2*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c*
*(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)
*sqrt( - sqrt(2) + 2)))*a + 2*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan(
(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1
/8)*sqrt( - sqrt(2) + 2)))*c*x**4 + c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)
*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/
4)*x)*a + c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*log( - sqrt(x)*c**(1/8)*a
**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)*c*x**4 - c**(1/8)...
```

3.146 $\int \frac{x^{3/2}}{(a+cx^4)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\begin{aligned}\int \frac{x^{3/2}}{(a+cx^4)^2} \, dx &= \frac{x^{5/2}}{4a(a+cx^4)} + \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{11/8}c^{5/8}} \\ &\quad - \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{11/8}c^{5/8}} + \frac{3\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} \\ &\quad + \frac{3\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{11/8}c^{5/8}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{11/8}c^{5/8}}\end{aligned}$$

output
```
1/4*x^(5/2)/a/(c*x^4+a)-3/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))
*2^(1/2)/(-a)^(11/8)/c^(5/8)-3/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1
/8))*2^(1/2)/(-a)^(11/8)/c^(5/8)+3/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(
-a)^(11/8)/c^(5/8)+3/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^
(5/8)-3/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*
x))*2^(1/2)/(-a)^(11/8)/c^(5/8)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx = \frac{\frac{8a^{3/8}x^{5/2}}{a+cx^4} + \frac{3\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{5/8}} - \frac{3\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{5/8}} - \frac{3\sqrt{2-}\cdots}{\cdots}}{32a^{11/8}}$$

input
```
Integrate[x^(3/2)/(a + c*x^4)^2,x]
```

output
```
((8*a^(3/8)*x^(5/2))/(a + c*x^4) + (3*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 - 1
/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(5/8) - (3*
Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1
/8)*c^(1/8)*Sqrt[x])])/c^(5/8) - (3*Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[2 + Sq
rt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/c^(5/8) + (3*Sqrt[
2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
+ c^(1/4)*x)])/c^(5/8))/(32*a^(11/8))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.43, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {819, 851, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \ 819$$

$$\frac{3\int \frac{x^{3/2}}{cx^4+a}dx}{8a} + \frac{x^{5/2}}{4a(a+cx^4)}$$

$$\Big\downarrow \ 851$$

$$\frac{3\int \frac{x^2}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{830}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{756}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{3\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{4a}$$

$$\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$\downarrow$ 25

$$\frac{3\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{4a}$$

$$\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\cdots}{\cdots}\right)}{4a}$$

$$\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\text{arcta}\cdots}{2(-\cdots}\right)}{4a}$$

$$\frac{x^{5/2}}{4a\left(a+cx^4\right)}$$

input
```
Int[x^(3/2)/(a + c*x^4)^2,x]
```

output
```
x^(5/2)/(4*a*(a + c*x^4)) + (3*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]
/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3
/8)*c^(1/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8
)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-
a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1
/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*
c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x
]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)))/(2*Sqrt[c])))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.20

method	result	size
derivativedivides	$\frac{x^{\frac{5}{2}}}{4a(c\,x^{4}+a)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{32ac}$	50
default	$\frac{x^{\frac{5}{2}}}{4a(c\,x^{4}+a)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{32ac}$	50

input
```
int(x^(3/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(5/2)/a/(c*x^4+a)+3/32/a/c*sum(1/_R^3*ln(x^(1/2)-_R),_R=RootOf(_Z^8*
c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.69

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx =$$
$$-\frac{3\sqrt{2}(-(i+1)\,acx^4-(i+1)\,a^2)\left(-\frac{1}{a^{11}c^5}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^7c^3\left(-\frac{1}{a^{11}c^5}\right)^{\frac{5}{8}}+\sqrt{x}\right)+3\sqrt{2}((i-1)\,acx^4}{}$$

input
```
integrate(x^(3/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(3*sqrt(2)*(-(I + 1)*a*c*x^4 - (I + 1)*a^2)*(-1/(a^11*c^5))^(1/8)*lo
g((1/2*I + 1/2)*sqrt(2)*a^7*c^3*(-1/(a^11*c^5))^(5/8) + sqrt(x)) + 3*sqrt(
2)*((I - 1)*a*c*x^4 + (I - 1)*a^2)*(-1/(a^11*c^5))^(1/8)*log(-(1/2*I - 1/2
)*sqrt(2)*a^7*c^3*(-1/(a^11*c^5))^(5/8) + sqrt(x)) + 3*sqrt(2)*(-(I - 1)*a
*c*x^4 - (I - 1)*a^2)*(-1/(a^11*c^5))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^7*
c^3*(-1/(a^11*c^5))^(5/8) + sqrt(x)) + 3*sqrt(2)*((I + 1)*a*c*x^4 + (I + 1
)*a^2)*(-1/(a^11*c^5))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^7*c^3*(-1/(a^11*
c^5))^(5/8) + sqrt(x)) - 16*x^(5/2) + 6*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1
/8)*log(a^7*c^3*(-1/(a^11*c^5))^(5/8) + sqrt(x)) + 6*(I*a*c*x^4 + I*a^2)*(
-1/(a^11*c^5))^(1/8)*log(I*a^7*c^3*(-1/(a^11*c^5))^(5/8) + sqrt(x)) + 6*(-
I*a*c*x^4 - I*a^2)*(-1/(a^11*c^5))^(1/8)*log(-I*a^7*c^3*(-1/(a^11*c^5))^(5
/8) + sqrt(x)) - 6*(a*c*x^4 + a^2)*(-1/(a^11*c^5))^(1/8)*log(-a^7*c^3*(-1/
(a^11*c^5))^(5/8) + sqrt(x)))/(a*c*x^4 + a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+a)^2}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^(5/2)/(a*c*x^4 + a^2) + 3*integrate(1/8*x^(3/2)/(a*c*x^4 + a^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(168) = 336$.

Time = 0.27 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.86

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx = \frac{x^{\frac{5}{2}}}{4\,(cx^4+a)a} - \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\sqrt{2}+4}} - \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\sqrt{2}+4}} + \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{3\left(\frac{a}{c}\right)^{\frac{5}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\sqrt{2}+4}}$$

input
```
integrate(x^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x^(5/2)/((c*x^4 + a)*a) - 3/16*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt
(2) + 4)) - 3/16*(a/c)^(5/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*s
qrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 3/16*
(a/c)^(5/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt
(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 3/16*(a/c)^(5/8)*arcta
n(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(
1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) - 3/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt
(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 3/32*(
a/c)^(5/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(
a^2*sqrt(2*sqrt(2) + 4)) + 3/32*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)
*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 3/32*(a/c)^(5
/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sq
rt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx = \frac{x^{5/2}}{4\,a\,(c\,x^4+a)} + \frac{3\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{11/8}\,c^{5/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,3\mathrm{i}}{16\,(-a)^{11/8}\,c^{5/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{3}{32}-\frac{3}{32}\mathrm{i}\right)}{(-a)^{11/8}\,c^{5/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{3}{32}+\frac{3}{32}\mathrm{i}\right)}{(-a)^{11/8}\,c^{5/8}}$$

input
```
int(x^(3/2)/(a + c*x^4)^2,x)
```

output
```
x^(5/2)/(4*a*(a + c*x^4)) + (3*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a
)^(11/8)*c^(5/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*3i)/(16*(-a)^(1
1/8)*c^(5/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^
(1/8))*(3/32 + 3i/32))/((-a)^(11/8)*c^(5/8)) - (2^(1/2)*atan((2^(1/2)*c^(1
/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(3/32 - 3i/32))/((-a)^(11/8)*c^(5/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1555, normalized size of antiderivative = 6.24

$$\int \frac{x^{3/2}}{(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(c*x^4+a)^2,x)
```

output
```
(6*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a + 6*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*c*x**4 - 6*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*a - 6*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*c*x**4 - 6*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*a
tan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8
)*a**(1/8)*sqrt(sqrt(2) + 2)))*a - 6*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*s
qrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/
(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 + 6*c**(3/8)*a**(5/8)*sqrt(s
qrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1
/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a + 6*c**(3/8)*a**(5/8)*sqrt(s
qrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1
/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 - 6*c**(3/8)*a**(5/8)*s
qrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*
sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a - 6*c**(3/8)
*a**(5/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqr...
```

3.147 $\int \frac{\sqrt{x}}{(a+cx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+cx^4)^2} dx &= \frac{x^{3/2}}{4a(a+cx^4)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{13/8}c^{3/8}} \\ &+ \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{13/8}c^{3/8}} - \frac{5\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} \\ &+ \frac{5\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{13/8}c^{3/8}} - \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{13/8}c^{3/8}}\end{aligned}$$

output
```
1/4*x^(3/2)/a/(c*x^4+a)+5/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))
*2^(1/2)/(-a)^(13/8)/c^(3/8)+5/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1
/8))*2^(1/2)/(-a)^(13/8)/c^(3/8)-5/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(
-a)^(13/8)/c^(3/8)+5/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^
(3/8)-5/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*
x))*2^(1/2)/(-a)^(13/8)/c^(3/8)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx = \frac{\frac{8a^{5/8}x^{3/2}}{a+cx^4}+\frac{5\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{3/8}}-\frac{5\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{3/8}}+\frac{5\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\ldots}{\sqrt[4]{\ldots}}\right.}{c^{3/8}}}{32a^{13/8}}$$

input
```
Integrate[Sqrt[x]/(a + c*x^4)^2,x]
```

output
```
((8*a^(5/8)*x^(3/2))/(a + c*x^4) + (5*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 - 1
/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(3/8) - (5*
Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1
/8)*c^(1/8)*Sqrt[x])])/c^(3/8) + (5*Sqrt[2 - Sqrt[2]]*ArcTanh[(Sqrt[2 + Sq
rt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/c^(3/8) - (5*Sqrt[
2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
+ c^(1/4)*x)])/c^(3/8))/(32*a^(13/8))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.43, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.933$, Rules used = {819, 851, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx$$

$$\Big\downarrow \; 819$$

$$\frac{5\int \frac{\sqrt{x}}{cx^4+a}dx}{8a}+\frac{x^{3/2}}{4a(a+cx^4)}$$

$$\Big\downarrow \; 851$$

$$\frac{5\int \frac{x}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 829

$$\frac{5\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 826

$$\frac{5\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 827

$$\frac{5\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{5\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 221

$$\frac{5\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{5\left(-\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{5\left(-\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right)}{4a}$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 25

$$\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right)}{4a}$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 27

$$\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\cdots}{\sqrt[8]{-\cdots}}\right)}{2\sqrt[8]{-a}c^{3/8}}\cdots\right)}{4a}$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5\left(-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\text{lo}\cdots}{\cdots}}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\cdots\right)}{4a}$$

$$\frac{x^{3/2}}{4a\left(a+cx^4\right)}$$

input

```
Int[Sqrt[x]/(a + c*x^4)^2,x]
```

output

```
x^(3/2)/(4*a*(a + c*x^4)) + (5*(-1/2*(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(
1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)
^(1/8)*c^(3/8)))/Sqrt[-a] - ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^
(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x]
)/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(
1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)
*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*
x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2*Sqrt[-a])))/(4*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.20

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{4a(c\,x^{4}+a)}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{32ac}$	50
default	$\frac{x^{\frac{3}{2}}}{4a(c\,x^{4}+a)}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{32ac}$	50

input
```
int(x^(1/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(3/2)/a/(c*x^4+a)+5/32/a/c*sum(1/_R^5*ln(x^(1/2)-_R),_R=RootOf(_Z^8*
c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx =$$

$$-\frac{5\sqrt{2}((i-1)\,acx^4+(i-1)\,a^2)\left(-\frac{1}{a^{13}c^3}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}a^5c\left(-\frac{1}{a^{13}c^3}\right)^{\frac{3}{8}}+\sqrt{x}\right)+5\sqrt{2}(-(i+1)\,acx}{}$$

input
```
integrate(x^(1/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(5*sqrt(2)*((I - 1)*a*c*x^4 + (I - 1)*a^2)*(-1/(a^13*c^3))^(1/8)*log
((1/2*I + 1/2)*sqrt(2)*a^5*c*(-1/(a^13*c^3))^(3/8) + sqrt(x)) + 5*sqrt(2)*
(-(I + 1)*a*c*x^4 - (I + 1)*a^2)*(-1/(a^13*c^3))^(1/8)*log(-(1/2*I - 1/2)*
sqrt(2)*a^5*c*(-1/(a^13*c^3))^(3/8) + sqrt(x)) + 5*sqrt(2)*((I + 1)*a*c*x^
4 + (I + 1)*a^2)*(-1/(a^13*c^3))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^5*c*(-1
/(a^13*c^3))^(3/8) + sqrt(x)) + 5*sqrt(2)*(-(I - 1)*a*c*x^4 - (I - 1)*a^2)
*(-1/(a^13*c^3))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^5*c*(-1/(a^13*c^3))^(3
/8) + sqrt(x)) + 10*(a*c*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*log(a^5*c*(-1/(a
^13*c^3))^(3/8) + sqrt(x)) + 10*(-I*a*c*x^4 - I*a^2)*(-1/(a^13*c^3))^(1/8)
*log(I*a^5*c*(-1/(a^13*c^3))^(3/8) + sqrt(x)) + 10*(I*a*c*x^4 + I*a^2)*(-1
/(a^13*c^3))^(1/8)*log(-I*a^5*c*(-1/(a^13*c^3))^(3/8) + sqrt(x)) - 10*(a*c
*x^4 + a^2)*(-1/(a^13*c^3))^(1/8)*log(-a^5*c*(-1/(a^13*c^3))^(3/8) + sqrt(
x)) - 16*x^(3/2))/(a*c*x^4 + a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx = \int \frac{\sqrt{x}}{(cx^4+a)^2}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
1/4*x^(3/2)/(a*c*x^4 + a^2) + 5*integrate(1/8*sqrt(x)/(a*c*x^4 + a^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(168) = 336$.

Time = 0.25 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx = \frac{x^{\frac{3}{2}}}{4\,(cx^4+a)a} - \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\sqrt{2}+4}} - \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\sqrt{2}+4}} - \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\sqrt{2}+4}} - \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{5\left(\frac{a}{c}\right)^{\frac{3}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\sqrt{2}+4}}$$

input

```
integrate(x^(1/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
1/4*x^(3/2)/((c*x^4 + a)*a) - 5/16*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*
(a/c)^(1/8) + 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt
(2) + 4)) - 5/16*(a/c)^(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*s
qrt(x))/(sqrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) + 4)) + 5/16*
(a/c)^(3/8)*arctan((sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt
(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 5/16*(a/c)^(3/8)*arcta
n(-(sqrt(sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(
1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 5/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt
(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) - 5/32*(
a/c)^(3/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(
a^2*sqrt(2*sqrt(2) + 4)) - 5/32*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)
*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 5/32*(a/c)^(3
/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sq
rt(-2*sqrt(2) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{x}}{(a+cx^4)^2}\,dx = \frac{x^{3/2}}{4\,a\,(c\,x^4+a)} - \frac{5\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{13/8}\,c^{3/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,5\mathrm{i}}{16\,(-a)^{13/8}\,c^{3/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{5}{32}-\frac{5}{32}\mathrm{i}\right)}{(-a)^{13/8}\,c^{3/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{5}{32}+\frac{5}{32}\mathrm{i}\right)}{(-a)^{13/8}\,c^{3/8}}$$

input
```
int(x^(1/2)/(a + c*x^4)^2,x)
```

output
```
x^(3/2)/(4*a*(a + c*x^4)) - (5*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a
)^(13/8)*c^(3/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*5i)/(16*(-a)^(1
3/8)*c^(3/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^
(1/8))*(5/32 - 5i/32))/((-a)^(13/8)*c^(3/8)) + (2^(1/2)*atan((2^(1/2)*c^(1
/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(5/32 + 5i/32))/((-a)^(13/8)*c^(3/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1553, normalized size of antiderivative = 6.24

$$\int \frac{\sqrt{x}}{(a+cx^4)^2} \, dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(c*x^4+a)^2,x)
```

output
```
(10*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2)))*a + 10*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a*
*(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(
sqrt(2) + 2)))*c*x**4 - 10*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(1
/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)
*sqrt(sqrt(2) + 2)))*a - 10*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2)))*c*x**4 - 10*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt
(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c*
*(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a - 10*c**(5/8)*a**(3/8)*sqrt(sqrt(2)
+ 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(
1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 + 10*c**(5/8)*a**(3/8)
*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x
)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a + 10*c**(5/8)*a**(3/8
)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(
x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 - 10*c**(5/8)*a
**(3/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2)
+ 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a - 1
0*c**(5/8)*a**(3/8)*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8...
```

3.148 $\int \frac{1}{\sqrt{x}(a+cx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx &= \frac{\sqrt{x}}{4a\,(a+cx^4)} - \frac{7\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{15/8}\sqrt[8]{c}} \\ &\quad + \frac{7\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{15/8}\sqrt[8]{c}} + \frac{7\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} \\ &\quad + \frac{7\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{15/8}\sqrt[8]{c}} + \frac{7\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{15/8}\sqrt[8]{c}}\end{aligned}$$

output
```
1/4*x^(1/2)/a/(c*x^4+a)+7/32*arctan(-1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))
*2^(1/2)/(-a)^(15/8)/c^(1/8)+7/32*arctan(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1
/8))*2^(1/2)/(-a)^(15/8)/c^(1/8)+7/16*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(
-a)^(15/8)/c^(1/8)+7/16*arctanh(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^
(1/8)+7/32*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*
x))*2^(1/2)/(-a)^(15/8)/c^(1/8)
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx$$
$$= \frac{\frac{8a^{7/8}\sqrt{x}}{a+cx^4} - \frac{7\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{\sqrt[8]{c}} - \frac{7\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{\sqrt[8]{c}} + \frac{7\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}\ldots}{\sqrt[4]{a}\ldots}\right.}{\sqrt[8]{c}}\ldots}{32a^{15/8}}$$

input
```
Integrate[1/(Sqrt[x]*(a + c*x^4)^2),x]
```

output
```
((8*a^(7/8)*Sqrt[x])/(a + c*x^4) - (7*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 - 1
/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(1/8) - (7*
Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1
/8)*c^(1/8)*Sqrt[x])])/c^(1/8) + (7*Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2 + Sq
rt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])/c^(1/8) + (7*Sqrt[
2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
+ c^(1/4)*x)])/c^(1/8))/(32*a^(15/8))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {819, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+cx^4\right)^2}\,dx$$
$$\Big\downarrow \; 819$$
$$\frac{7\int \frac{1}{\sqrt{x}\left(cx^4+a\right)}dx}{8a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$
$$\Big\downarrow \; 851$$

$$\frac{7\int \frac{1}{cx^4+a}d\sqrt{x}}{4a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 758

$$\frac{7\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 755

$$\frac{7\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 756

$$\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 218

$$\frac{7\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 221

$$\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{7\left(-\frac{-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}} \cdots\right.}{4a} \cdots$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{7\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\cdots\right.}{4a} \cdots$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}\cdots}{\cdots}\right.}{4a} \cdots$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{7\left(-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\text{lo}}{}}{2\sqrt[4]{-}}}{2\sqrt{-a}}\right)}{4a}$$

$$\frac{\sqrt{x}}{4a\left(a+cx^4\right)}$$

input
```
Int[1/(Sqrt[x]*(a + c*x^4)^2),x]
```

output
```
Sqrt[x]/(4*a*(a + c*x^4)) + (7*(-1/2*(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]
/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3
/8)*c^(1/8)))/Sqrt[-a] - ((-(ArcTan[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/
8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(
-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(
1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)
*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*
x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-a)^(1/4)))/(2*Sqrt[-a])))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.20

method	result	size
derivativedivides	$\frac{\sqrt{x}}{4a(c\,x^4+a)}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{32ac}$	50
default	$\frac{\sqrt{x}}{4a(c\,x^4+a)}+\frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{32ac}$	50

input
```
int(1/x^(1/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(1/2)/a/(c*x^4+a)+7/32/a/c*sum(1/_R^7*ln(x^(1/2)-_R),_R=RootOf(_Z^8*
c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.59

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx =$$
$$-\frac{7\sqrt{2}(-(i+1)\ acx^4-(i+1)\ a^2)\left(-\frac{1}{a^{15}c}\right)^{\frac{1}{8}}\log\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}a^2\left(-\frac{1}{a^{15}c}\right)^{\frac{1}{8}}+\sqrt{x}\right)+7\sqrt{2}((i-1)\ acx^4 -$$

input
```
integrate(1/x^(1/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(7*sqrt(2)*(-(I + 1)*a*c*x^4 - (I + 1)*a^2)*(-1/(a^15*c))^(1/8)*log(
(1/2*I + 1/2)*sqrt(2)*a^2*(-1/(a^15*c))^(1/8) + sqrt(x)) + 7*sqrt(2)*((I -
 1)*a*c*x^4 + (I - 1)*a^2)*(-1/(a^15*c))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*
a^2*(-1/(a^15*c))^(1/8) + sqrt(x)) + 7*sqrt(2)*(-(I - 1)*a*c*x^4 - (I - 1)
*a^2)*(-1/(a^15*c))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^2*(-1/(a^15*c))^(1/8
) + sqrt(x)) + 7*sqrt(2)*((I + 1)*a*c*x^4 + (I + 1)*a^2)*(-1/(a^15*c))^(1/
8)*log(-(1/2*I + 1/2)*sqrt(2)*a^2*(-1/(a^15*c))^(1/8) + sqrt(x)) - 14*(a*c
*x^4 + a^2)*(-1/(a^15*c))^(1/8)*log(a^2*(-1/(a^15*c))^(1/8) + sqrt(x)) + 1
4*(-I*a*c*x^4 - I*a^2)*(-1/(a^15*c))^(1/8)*log(I*a^2*(-1/(a^15*c))^(1/8) +
 sqrt(x)) + 14*(I*a*c*x^4 + I*a^2)*(-1/(a^15*c))^(1/8)*log(-I*a^2*(-1/(a^1
5*c))^(1/8) + sqrt(x)) + 14*(a*c*x^4 + a^2)*(-1/(a^15*c))^(1/8)*log(-a^2*(
-1/(a^15*c))^(1/8) + sqrt(x)) - 16*sqrt(x))/(a*c*x^4 + a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx = \int \frac{1}{(cx^4+a)^2\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-7*c*integrate(1/8*x^(7/2)/(a^2*c*x^4 + a^3), x) + 1/4*(7*c*x^(9/2) + 8*a*
sqrt(x))/(a^2*c*x^4 + a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(168) = 336$.

Time = 0.21 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx = \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}-2\sqrt{x}}{\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}}\right)}{16\,a^2\sqrt{2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\sqrt{2}+4}} - \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{-2\sqrt{2}+4}} + \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\sqrt{2}+4}} - \frac{7\left(\frac{a}{c}\right)^{\frac{1}{8}}\log\left(-\sqrt{x}\sqrt{-\sqrt{2}+2}\left(\frac{a}{c}\right)^{\frac{1}{8}}+x+\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{32\,a^2\sqrt{2\sqrt{2}+4}} + \frac{\sqrt{x}}{4\,(cx^4+a)a}$$

input
```
integrate(1/x^(1/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
7/16*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt
(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 7/16*(a/c)^(1/8)*
arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*(a
/c)^(1/8)))/(a^2*sqrt(-2*sqrt(2) + 4)) + 7/16*(a/c)^(1/8)*arctan((sqrt(sqr
t(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*
sqrt(2*sqrt(2) + 4)) + 7/16*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/c)^(
1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*sqrt(2*sqrt(2) +
4)) + 7/32*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/
c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) - 7/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(-2*sqrt(2) + 4)) + 7
/32*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/
4))/(a^2*sqrt(2*sqrt(2) + 4)) - 7/32*(a/c)^(1/8)*log(-sqrt(x)*sqrt(-sqrt(2
) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*sqrt(2*sqrt(2) + 4)) + 1/4*sqrt
(x)/((c*x^4 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx = \frac{\sqrt{x}}{4\,a\,(c\,x^4+a)} + \frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{16\,(-a)^{15/8}\,c^{1/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,7\mathrm{i}}{16\,(-a)^{15/8}\,c^{1/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{7}{32}+\frac{7}{32}\mathrm{i}\right)}{(-a)^{15/8}\,c^{1/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{7}{32}-\frac{7}{32}\mathrm{i}\right)}{(-a)^{15/8}\,c^{1/8}}$$

input
```
int(1/(x^(1/2)*(a + c*x^4)^2),x)
```

output
```
x^(1/2)/(4*a*(a + c*x^4)) + (7*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(16*(-a
)^(15/8)*c^(1/8)) - (atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*7i)/(16*(-a)^(1
5/8)*c^(1/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^
(1/8))*(7/32 + 7i/32))/((-a)^(15/8)*c^(1/8)) + (2^(1/2)*atan((2^(1/2)*c^(1
/8)*x^(1/2)*(1/2 + 1i/2))/(-a)^(1/8))*(7/32 - 7i/32))/((-a)^(15/8)*c^(1/8)
)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.10

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^2}\,dx = \text{Too large to display}$$

input
```
int(1/x^(1/2)/(c*x^4+a)^2,x)
```

output
```
( - 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
*a - 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(
- sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))
)*c*x**4 + 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*
sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2)
 + 2)))*a + 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)
*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2
) + 2)))*c*x**4 - 14*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)
*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(
 - sqrt(2) + 2)))*a - 14*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2)))*c*x**4 + 14*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*a
tan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a
**(1/8)*sqrt( - sqrt(2) + 2)))*a + 14*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) +
2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/
8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c*x**4 - 7*c**(7/8)*a**(1/8)*sqrt( - sq
rt(2) + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4
) + c**(1/4)*x)*a - 7*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*log( - sqrt(x
)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c**(1/4)*x)*c*x**...
```

3.149 $\int \frac{1}{x^{3/2}(a+cx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 261

$$\int \frac{1}{x^{3/2}(a+cx^4)^2} dx = -\frac{9}{4a^2\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+cx^4)} + \frac{9\sqrt[8]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{17/8}} - \frac{9\sqrt[8]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16\sqrt{2}(-a)^{17/8}} - \frac{9\sqrt[8]{c}\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{17/8}} + \frac{9\sqrt[8]{c}\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{16(-a)^{17/8}} + \frac{9\sqrt[8]{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{16\sqrt{2}(-a)^{17/8}}$$

output
```
-9/4/a^2/x^(1/2)+1/4/a/x^(1/2)/(c*x^4+a)-9/32*c^(1/8)*arctan(-1+2^(1/2)*c^
(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(17/8)-9/32*c^(1/8)*arctan(1+2^(1/2
)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(17/8)-9/16*c^(1/8)*arctan(c^(1
/8)*x^(1/2)/(-a)^(1/8))/(-a)^(17/8)+9/16*c^(1/8)*arctanh(c^(1/8)*x^(1/2)/(
-a)^(1/8))/(-a)^(17/8)+9/32*c^(1/8)*arctanh(2^(1/2)*(-a)^(1/8)*c^(1/8)*x^(
1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(17/8)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{3/2}(a+cx^4)^2}\,dx = \frac{-\frac{8\sqrt[8]{a}(8a+9cx^4)}{\sqrt{x}(a+cx^4)} + 9\sqrt{2+\sqrt{2}}\sqrt[8]{c}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right) + 9\sqrt{2-\sqrt{2}}\sqrt[8]{c}\,\text{arct}}{}$$

input

```
Integrate[1/(x^(3/2)*(a + c*x^4)^2),x]
```

output

```
((-8*a^(1/8)*(8*a + 9*c*x^4))/(Sqrt[x]*(a + c*x^4)) + 9*Sqrt[2 + Sqrt[2]]*
c^(1/8)*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8
)*Sqrt[x])] + 9*Sqrt[2 - Sqrt[2]]*c^(1/8)*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(
1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + 9*Sqrt[2 + Sqrt[2]]*c^(1/8
)*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x
)] + 9*Sqrt[2 - Sqrt[2]]*c^(1/8)*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqr
t[2])*x)])/(a^(1/4) + c^(1/4)*x)])/(32*a^(17/8))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {819, 847, 851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+cx^4)^2}\,dx$$

$$\Big\downarrow \; 819$$

$$\frac{9\int \frac{1}{x^{3/2}(cx^4+a)}dx}{8a} + \frac{1}{4a\sqrt{x}(a+cx^4)}$$

$$\Big\downarrow \; 847$$

$$\frac{9\left(-\frac{c\int \frac{x^{5/2}}{cx^4+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+cx^4)}$$

$\Big\downarrow$ 851

$$\frac{9\left(-\frac{2c\int \frac{x^3}{cx^4+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+cx^4)}$$

$\Big\downarrow$ 830

$$\frac{9\left(-\frac{2c\left(\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+cx^4)}$$

$\Big\downarrow$ 826

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+cx^4)}$$

$\Big\downarrow$ 827

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+cx^4)}$$

$\Big\downarrow$ 218

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$\Big\downarrow$ 221

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$\Big\downarrow$ 1476

$$9\left(-\frac{2c\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}\right.$$

$$\overline{8a}$$

$$\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1082$$

$$9\left(-\frac{2c\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)$$

$$\overline{8a}$$

$$\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int-\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}} \cdots}{2\sqrt{c}} \cdots\right)}{a} \cdots\right)}{8a} \cdots$$

$$\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{9\left(-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\right.}{a}\right.}{8a}$$

$$\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$$\Big\downarrow \; 27$$

$$9\left(-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{\cdots}}{\sqrt[8]{\cdots}}\right.}{2\sqrt[8]{-a}c^{3/\cdots}}\cdots\right)}{a}\cdots}{8a}\cdots$$

$$\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

$\downarrow$ 1103

$$9\left(-\frac{2c\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\cdots\right)}{a}\cdots}{8a}\cdots$$

$$\frac{1}{4a\sqrt{x}\left(a+cx^4\right)}$$

input
```
Int[1/(x^(3/2)*(a + c*x^4)^2),x]
```

output
```
1/(4*a*Sqrt[x]*(a + c*x^4)) + (9*(-2/(a*Sqrt[x]) - (2*c*(-1/2*(-1/2*ArcTan
[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/8)*Sqr
t[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[c] + ((-(ArcTan[1 - (Sqrt[2
]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTan[1 +
(Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(
1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4
)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1/8)*c^
(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4)))/(2
*Sqrt[c])))/a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

method	result	size
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{c\,x^{\frac{7}{2}}}{4a^{2}\left(c\,x^{4}+a\right)}-\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{32a^{2}}$	56
derivativedivides	$-\frac{2c\left(\frac{x^{\frac{7}{2}}}{8c\,x^{4}+8a}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{64c}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	59
default	$-\frac{2c\left(\frac{x^{\frac{7}{2}}}{8c\,x^{4}+8a}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{64c}\right)}{a^{2}}-\frac{2}{a^{2}\sqrt{x}}$	59

input
```
int(1/x^(3/2)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/x^(1/2)-1/4/a^2*c*x^(7/2)/(c*x^4+a)-9/32/a^2*sum(1/_R*ln(x^(1/2)-_R
),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.65

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(c*x^4+a)^2,x, algorithm="fricas")
```

output
```
-1/64*(9*sqrt(2)*(-(I - 1)*a^2*c*x^5 - (I - 1)*a^3*x)*(-c/a^17)^(1/8)*log(
(4782969/2*I + 4782969/2)*sqrt(2)*a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x)
) + 9*sqrt(2)*((I + 1)*a^2*c*x^5 + (I + 1)*a^3*x)*(-c/a^17)^(1/8)*log(-(47
82969/2*I - 4782969/2)*sqrt(2)*a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x)) +
 9*sqrt(2)*(-(I + 1)*a^2*c*x^5 - (I + 1)*a^3*x)*(-c/a^17)^(1/8)*log((47829
69/2*I - 4782969/2)*sqrt(2)*a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x)) + 9*
sqrt(2)*((I - 1)*a^2*c*x^5 + (I - 1)*a^3*x)*(-c/a^17)^(1/8)*log(-(4782969/
2*I + 4782969/2)*sqrt(2)*a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x)) + 18*(a
^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*log(4782969*a^15*(-c/a^17)^(7/8) + 47829
69*c*sqrt(x)) + 18*(-I*a^2*c*x^5 - I*a^3*x)*(-c/a^17)^(1/8)*log(4782969*I*
a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x)) + 18*(I*a^2*c*x^5 + I*a^3*x)*(-c
/a^17)^(1/8)*log(-4782969*I*a^15*(-c/a^17)^(7/8) + 4782969*c*sqrt(x)) - 18
*(a^2*c*x^5 + a^3*x)*(-c/a^17)^(1/8)*log(-4782969*a^15*(-c/a^17)^(7/8) + 4
782969*c*sqrt(x)) + 16*(9*c*x^4 + 8*a)*sqrt(x))/(a^2*c*x^5 + a^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(3/2)/(c*x**4+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx = \int \frac{1}{\left(cx^4+a\right)^2 x^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^(3/2)/(c*x^4+a)^2,x, algorithm="maxima")
```

output
```
-9*c*integrate(1/8*x^(5/2)/(a^2*c*x^4 + a^3), x) - 1/4*(9*c*x^(7/2) + 8*a/
sqrt(x))/(a^2*c*x^4 + a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(176) = 352$.

Time = 0.24 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(3/2)/(c*x^4+a)^2,x, algorithm="giac")
```

output
```
-9/16*c*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(s
qrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/16*c*(a/c)^(
7/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) +
2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/16*c*(a/c)^(7/8)*arctan((s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
)/(a^3*sqrt(2*sqrt(2) + 4)) - 9/16*c*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2
)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*s
qrt(2) + 4)) + 9/32*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 9/32*c*(a/c)^(7/8)*log(-
sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt
(2) + 4)) + 9/32*c*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8)
+ x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 9/32*c*(a/c)^(7/8)*log(-sqr
t(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2)
 + 4)) - 1/4*(9*c*x^4 + 8*a)/((c*x^(9/2) + a*sqrt(x))*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.57

$$\begin{aligned}\int \frac{1}{x^{3/2}\left(a+cx^4\right)^2}\,dx &= -\frac{\frac{2}{a}+\frac{9\,c\,x^4}{4\,a^2}}{a\,\sqrt{x}+c\,x^{9/2}} \\ &\quad -\frac{9\,{\left(-c\right)}^{1/8}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/8}\,\sqrt{x}}{a^{1/8}}\right)}{16\,a^{17/8}}-\frac{{\left(-c\right)}^{1/8}\,\mathrm{atan}\left(\frac{{\left(-c\right)}^{1/8}\,\sqrt{x}\,1{}\mathrm{i}}{a^{1/8}}\right)\,9{}\mathrm{i}}{16\,a^{17/8}} \\ &\quad +\frac{\sqrt{2}\,{\left(-c\right)}^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,{\left(-c\right)}^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)}{a^{1/8}}\right)\,\left(-\frac{9}{32}+\frac{9}{32}{}\mathrm{i}\right)}{a^{17/8}} \\ &\quad +\frac{\sqrt{2}\,{\left(-c\right)}^{1/8}\,\mathrm{atan}\left(\frac{\sqrt{2}\,{\left(-c\right)}^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)}{a^{1/8}}\right)\,\left(-\frac{9}{32}-\frac{9}{32}{}\mathrm{i}\right)}{a^{17/8}}\end{aligned}$$

input
```
int(1/(x^(3/2)*(a + c*x^4)^2),x)
```

output
```
- (2/a + (9*c*x^4)/(4*a^2))/(a*x^(1/2) + c*x^(9/2)) - (9*(-c)^(1/8)*atan((
(-c)^(1/8)*x^(1/2))/a^(1/8)))/(16*a^(17/8)) - ((-c)^(1/8)*atan(((-c)^(1/8)
*x^(1/2)*1i)/a^(1/8))*9i)/(16*a^(17/8)) - (2^(1/2)*(-c)^(1/8)*atan((2^(1/2
)*(-c)^(1/8)*x^(1/2)*(1/2 - 1i/2))/a^(1/8))*(9/32 - 9i/32))/a^(17/8) - (2^
(1/2)*(-c)^(1/8)*atan((2^(1/2)*(-c)^(1/8)*x^(1/2)*(1/2 + 1i/2))/a^(1/8))*(
9/32 + 9i/32))/a^(17/8)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 811, normalized size of antiderivative = 3.11

$$\int \frac{1}{x^{3/2} (a + cx^4)^2} dx = \text{Too large to display}$$

input
```
int(1/x^(3/2)/(c*x^4+a)^2,x)
```

output
```
(18*sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2)))*a + 18*sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a*
*(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(
sqrt(2) + 2)))*c*x**4 - 18*sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*ata
n((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*
a**(1/8)*sqrt(sqrt(2) + 2)))*a - 18*sqrt(x)*c**(1/8)*a**(7/8)*sqrt(sqrt(2)
 + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(
c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c*x**4 + 18*sqrt(x)*c**(1/8)*a**(7/8
)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(
x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a + 18*sqrt(x)*c**(
1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c*x**4
- 18*sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(
2) + 2)))*a - 18*sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2)))*c*x**4 - 9*sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2)
 + 2)*log( - sqrt(x)*c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + a**(1/4) + c
**(1/4)*x)*a - 9*sqrt(x)*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*log( - ...
```

3.150 $\int \frac{x^{15/2}}{\left(a+cx^4\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{x^{15/2}}{\left(a+cx^4\right)^3}\,dx = -\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}-\frac{9\sqrt{x}}{64c^2\left(a+cx^4\right)} + \frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{7/8}c^{17/8}}-\frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{7/8}c^{17/8}} - \frac{9\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{7/8}c^{17/8}}-\frac{9\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{7/8}c^{17/8}}-\frac{9\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{7/8}c^{17/8}}$$

output

```
-1/8*x^(9/2)/c/(c*x^4+a)^2-9/64*x^(1/2)/c^2/(c*x^4+a)-9/512*arctan(-1+2^(1
/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(7/8)/c^(17/8)-9/512*arctan(1
+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(7/8)/c^(17/8)-9/256*arc
tan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(17/8)-9/256*arctanh(c^(1/8)*
x^(1/2)/(-a)^(1/8))/(-a)^(7/8)/c^(17/8)-9/512*arctanh(2^(1/2)*(-a)^(1/8)*c
^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(7/8)/c^(17/8)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx = \frac{-\frac{8\sqrt[8]{c}\sqrt{x}(9a+17cx^4)}{(a+cx^4)^2} - \frac{9\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{7/8}} - \frac{9\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{a^{7/8}}}{512c^{17/8}}$$

input
```
Integrate[x^(15/2)/(a + c*x^4)^3,x]
```

output
```
((-8*c^(1/8)*Sqrt[x]*(9*a + 17*c*x^4))/(a + c*x^4)^2 - (9*Sqrt[2 + Sqrt[2]
]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt
[x])])/a^(7/8) - (9*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4)
 - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/a^(7/8) + (9*Sqrt[2 + Sqrt[2]]*
ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)]
)/a^(7/8) + (9*Sqrt[2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqr
t[2])*x)])/(a^(1/4) + c^(1/4)*x)])/a^(7/8))/(512*c^(17/8))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {817, 817, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{9\int \frac{x^{7/2}}{(cx^4+a)^2}dx}{16c} - \frac{x^{9/2}}{8c\,(a+cx^4)^2}$$

$$\Big\downarrow \text{817}$$

$$\frac{9\left(\frac{\int \frac{1}{\sqrt{x}\left(cx^4+a\right)}dx}{8c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 851$$

$$\frac{9\left(\frac{\int \frac{1}{cx^4+a}d\sqrt{x}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 758$$

$$\frac{9\left(\frac{-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 755$$

$$\frac{9\left(\frac{-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 756$$

$$\frac{9\left(\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{9\left(\frac{-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 221

$$\frac{9\left(\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1476

$$\frac{9\left(\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\cdots}{4\cdots}\right)}{16c}$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1082

$$\frac{9\left(\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(\frac{-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}}{4c}-\frac{\sqrt{x}}{4c(a+cx^4)}\right)}{16c}-$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{9\left(\frac{-\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-}{4c}\right.}{16c}$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 25

$$9\left(\frac{-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\text{arcta}\ldots}{2(\ldots}}{4c}\ldots\right.}{16c}\ldots$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 27

$$9\left(\frac{-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\ldots}{2\sqrt{\ldots}}}{4c}\ldots\right.}{16c}\ldots$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 1103

$$\frac{9\left(\frac{-\frac{-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(\right.}{}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}}{4c}\right.}{16c}$$

$$\frac{x^{9/2}}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^(15/2)/(a + c*x^4)^3,x]
```

output
```
-1/8*x^(9/2)/(c*(a + c*x^4)^2) + (9*(-1/4*Sqrt[x]/(c*(a + c*x^4)) + (-1/2*
(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^
(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[-a] - ((-(ArcTan[1
 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + A
rcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8
)))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqr
t[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*
(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2
*(-a)^(1/4)))/(2*Sqrt[-a]))/(4*c)))/(16*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.22

method	result	size
derivativedivides	$\frac{-\frac{9a\sqrt{x}}{64c^2}-\frac{17x^{\frac{9}{2}}}{64c}}{(c\,x^4+a)^2}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{512c^3}$	59
default	$\frac{-\frac{9a\sqrt{x}}{64c^2}-\frac{17x^{\frac{9}{2}}}{64c}}{(c\,x^4+a)^2}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{512c^3}$	59

input
```
int(x^(15/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-9/128/c^2*a*x^(1/2)-17/128/c*x^(9/2))/(c*x^4+a)^2+9/512/c^3*sum(1/_R^7
*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.98

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(15/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(9*sqrt(2)*(-(I + 1)*c^4*x^8 - (2*I + 2)*a*c^3*x^4 - (I + 1)*a^2*c
^2)*(-1/(a^7*c^17))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a*c^2*(-1/(a^7*c^17))^
(1/8) + sqrt(x)) + 9*sqrt(2)*((I - 1)*c^4*x^8 + (2*I - 2)*a*c^3*x^4 + (I -
 1)*a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a*c^2*(-1/(a
^7*c^17))^(1/8) + sqrt(x)) + 9*sqrt(2)*(-(I - 1)*c^4*x^8 - (2*I - 2)*a*c^3
*x^4 - (I - 1)*a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a*
c^2*(-1/(a^7*c^17))^(1/8) + sqrt(x)) + 9*sqrt(2)*((I + 1)*c^4*x^8 + (2*I +
 2)*a*c^3*x^4 + (I + 1)*a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(-(1/2*I + 1/2)*
sqrt(2)*a*c^2*(-1/(a^7*c^17))^(1/8) + sqrt(x)) - 18*(c^4*x^8 + 2*a*c^3*x^4
 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(a*c^2*(-1/(a^7*c^17))^(1/8) + sqrt(x
)) + 18*(-I*c^4*x^8 - 2*I*a*c^3*x^4 - I*a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log
(I*a*c^2*(-1/(a^7*c^17))^(1/8) + sqrt(x)) + 18*(I*c^4*x^8 + 2*I*a*c^3*x^4
+ I*a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(-I*a*c^2*(-1/(a^7*c^17))^(1/8) + sq
rt(x)) + 18*(c^4*x^8 + 2*a*c^3*x^4 + a^2*c^2)*(-1/(a^7*c^17))^(1/8)*log(-a
*c^2*(-1/(a^7*c^17))^(1/8) + sqrt(x)) + 16*(17*c*x^4 + 9*a)*sqrt(x))/(c^4*
x^8 + 2*a*c^3*x^4 + a^2*c^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(15/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{15}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(15/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(9*c*x^(17/2) + a*x^(9/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) - 9*in
tegrate(1/128*x^(7/2)/(a*c^2*x^4 + a^2*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(185) = 370$.

Time = 0.24 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.84

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(15/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
9/256*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqr
t(sqrt(2) + 2)*(a/c)^(1/8)))/(a*c^2*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(1
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2
)*(a/c)^(1/8)))/(a*c^2*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(1/8)*arctan((s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
)/(a*c^2*sqrt(2*sqrt(2) + 4)) + 9/256*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) +
2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a*c^2*sqrt(
2*sqrt(2) + 4)) + 9/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a*c^2*sqrt(-2*sqrt(2) + 4)) - 9/512*(a/c)^(1/8)*lo
g(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c^2*sqrt(-2
*sqrt(2) + 4)) + 9/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a*c^2*sqrt(2*sqrt(2) + 4)) - 9/512*(a/c)^(1/8)*log
(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a*c^2*sqrt(2*
sqrt(2) + 4)) - 1/64*(17*c*x^(9/2) + 9*a*sqrt(x))/((c*x^4 + a)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.59

$$\begin{aligned}\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx &= -\frac{\frac{17\,x^{9/2}}{64\,c}+\frac{9\,a\,\sqrt{x}}{64\,c^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{9\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{7/8}\,c^{17/8}}\\&\quad+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,9\mathrm{i}}{256\,(-a)^{7/8}\,c^{17/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{9}{512}-\frac{9}{512}\mathrm{i}\right)}{(-a)^{7/8}\,c^{17/8}}\\&\quad+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{9}{512}+\frac{9}{512}\mathrm{i}\right)}{(-a)^{7/8}\,c^{17/8}}\end{aligned}$$

input
```
int(x^(15/2)/(a + c*x^4)^3,x)
```

output
```
(atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*9i)/(256*(-a)^(7/8)*c^(17/8)) - (9*
atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(7/8)*c^(17/8)) - ((17*x^(9/
2))/(64*c) + (9*a*x^(1/2))/(64*c^2))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (2^(1/2
)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(9/512 + 9i/512)
)/((-a)^(7/8)*c^(17/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i
/2))/(-a)^(1/8))*(9/512 - 9i/512))/((-a)^(7/8)*c^(17/8))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.38

$$\int \frac{x^{15/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(15/2)/(c*x^4+a)^3,x)
```

output
```
( - 18*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
*a**2 - 36*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a*c*x**4 - 18*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*c**2*x**8 + 18*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2)))*a**2 + 36*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 + 18*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2
)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 18*c**(7/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/
4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a**2 - 36*c**(7/8)*a**(1/8)*
sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)
*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a*c*x**4 - 18*c**(7/8
)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)
- 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c**2*x**8
+ 18*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr...
```

3.151 $\int \frac{x^{13/2}}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 273

$$\int \frac{x^{13/2}}{(a+cx^4)^3} dx = -\frac{x^{7/2}}{8c(a+cx^4)^2} + \frac{7x^{7/2}}{64ac(a+cx^4)} + \frac{7\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{9/8}c^{15/8}} - \frac{7\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{9/8}c^{15/8}} - \frac{7\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{9/8}c^{15/8}} + \frac{7\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{9/8}c^{15/8}} + \frac{7\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{9/8}c^{15/8}}$$

output
```
-1/8*x^(7/2)/c/(c*x^4+a)^2+7/64*x^(7/2)/a/c/(c*x^4+a)-7/512*arctan(-1+2^(1
/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(9/8)/c^(15/8)-7/512*arctan(1
+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(9/8)/c^(15/8)-7/256*arc
tan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(15/8)+7/256*arctanh(c^(1/8)*
x^(1/2)/(-a)^(1/8))/(-a)^(9/8)/c^(15/8)+7/512*arctanh(2^(1/2)*(-a)^(1/8)*c
^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(9/8)/c^(15/8)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01

$$\int \frac{x^{13/2}}{(a+cx^4)^3}\,dx = \frac{-\frac{8\sqrt[8]{a}c^{7/8}x^{7/2}(a-7cx^4)}{(a+cx^4)^2} - 7\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right) - 7\sqrt{2-\sqrt{2}}\arctan\left(\right.}{}$$

input
```
Integrate[x^(13/2)/(a + c*x^4)^3,x]
```

output
```
((-8*a^(1/8)*c^(7/8)*x^(7/2)*(a - 7*c*x^4))/(a + c*x^4)^2 - 7*Sqrt[2 + Sqr
t[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*
Sqrt[x])] - 7*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(
1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] - 7*Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2
+ Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] - 7*Sqrt[2 - Sq
rt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1
/4)*x)])/(512*a^(9/8)*c^(15/8))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.39, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {817, 819, 851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}}{(a+cx^4)^3}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7\int \frac{x^{5/2}}{(cx^4+a)^2}dx}{16c} - \frac{x^{7/2}}{8c(a+cx^4)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{7\left(\frac{\int \frac{x^{5/2}}{cx^4+a}dx}{8a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{7\left(\frac{\int \frac{x^3}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{830}$$

$$\frac{7\left(\frac{\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{7\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{827}$$

$$\frac{7\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{7\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{7\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{7\left(\frac{\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{\cdots}{4a(\cdots}\right.}{16c}$$

$$\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{7\left(\frac{\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{7\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{7\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\cdots}{4a}\cdots\right)}{16c}$$

$$\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arcta}\cdots}{2\cdots}}{4a}\cdots\right.}{16c}\cdots$$

$$\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}\cdots}{2\sqrt{c}\cdots}}{4a}\cdots\right.}{16c}\cdots$$

$$\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{7\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arcta}\cdots}{2\cdots}}{4a}\cdots\right)}{16c\cdots}$$

$$\frac{x^{7/2}}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^(13/2)/(a + c*x^4)^3,x]
```

output
```
-1/8*x^(7/2)/(c*(a + c*x^4)^2) + (7*(x^(7/2)/(4*a*(a + c*x^4)) + (-1/2*(-1
/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^
(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[c] + ((-(ArcTan[1
- (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + Ar
cTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)
))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x]
 + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)
^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(
1/4)))/(2*Sqrt[c]))/(4*a)))/(16*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.22

method	result	size
derivativedivides	$\frac{-\frac{x^{\frac{7}{2}}}{64c}+\frac{7x^{\frac{15}{2}}}{64a}}{(c\,x^4+a)^2}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{512a\,c^2}$	61
default	$\frac{-\frac{x^{\frac{7}{2}}}{64c}+\frac{7x^{\frac{15}{2}}}{64a}}{(c\,x^4+a)^2}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{512a\,c^2}$	61

input
```
int(x^(13/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/128/c*x^(7/2)+7/128/a*x^(15/2))/(c*x^4+a)^2+7/512/a/c^2*sum(1/_R*ln(
x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.06

$$\int \frac{x^{13/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(13/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(7*sqrt(2)*((I - 1)*a*c^3*x^8 + (2*I - 2)*a^2*c^2*x^4 + (I - 1)*a^
3*c)*(-1/(a^9*c^15))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^8*c^13*(-1/(a^9*c^1
5))^(7/8) + sqrt(x)) + 7*sqrt(2)*(-(I + 1)*a*c^3*x^8 - (2*I + 2)*a^2*c^2*x
^4 - (I + 1)*a^3*c)*(-1/(a^9*c^15))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^8*c
^13*(-1/(a^9*c^15))^(7/8) + sqrt(x)) + 7*sqrt(2)*((I + 1)*a*c^3*x^8 + (2*I
 + 2)*a^2*c^2*x^4 + (I + 1)*a^3*c)*(-1/(a^9*c^15))^(1/8)*log((1/2*I - 1/2)
*sqrt(2)*a^8*c^13*(-1/(a^9*c^15))^(7/8) + sqrt(x)) + 7*sqrt(2)*(-(I - 1)*a
*c^3*x^8 - (2*I - 2)*a^2*c^2*x^4 - (I - 1)*a^3*c)*(-1/(a^9*c^15))^(1/8)*lo
g(-(1/2*I + 1/2)*sqrt(2)*a^8*c^13*(-1/(a^9*c^15))^(7/8) + sqrt(x)) - 14*(a
*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^9*c^15))^(1/8)*log(a^8*c^13*(-1/(
a^9*c^15))^(7/8) + sqrt(x)) + 14*(I*a*c^3*x^8 + 2*I*a^2*c^2*x^4 + I*a^3*c)
*(-1/(a^9*c^15))^(1/8)*log(I*a^8*c^13*(-1/(a^9*c^15))^(7/8) + sqrt(x)) + 1
4*(-I*a*c^3*x^8 - 2*I*a^2*c^2*x^4 - I*a^3*c)*(-1/(a^9*c^15))^(1/8)*log(-I*
a^8*c^13*(-1/(a^9*c^15))^(7/8) + sqrt(x)) + 14*(a*c^3*x^8 + 2*a^2*c^2*x^4
+ a^3*c)*(-1/(a^9*c^15))^(1/8)*log(-a^8*c^13*(-1/(a^9*c^15))^(7/8) + sqrt(
x)) - 16*(7*c*x^7 - a*x^3)*sqrt(x))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{13/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{13}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(13/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(7*c*x^(15/2) - a*x^(7/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 7*in
tegrate(1/128*x^(5/2)/(a*c^2*x^4 + a^2*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(188) = 376$.

Time = 0.28 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.83

$$\int \frac{x^{13/2}}{(a+cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(13/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
7/256*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqr
t(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(7
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2
)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(7/8)*arctan((s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
)/(a^2*c*sqrt(2*sqrt(2) + 4)) + 7/256*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) +
2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(
2*sqrt(2) + 4)) - 7/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/512*(a/c)^(7/8)*lo
g(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2
*sqrt(2) + 4)) - 7/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 7/512*(a/c)^(7/8)*log
(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*
sqrt(2) + 4)) + 1/64*(7*c*x^(15/2) - a*x^(7/2))/((c*x^4 + a)^2*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

$$\int \frac{x^{13/2}}{(a+cx^4)^3}\,dx = \frac{\frac{7\,x^{15/2}}{64\,a}-\frac{x^{7/2}}{64\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{9/8}\,c^{15/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,7\mathrm{i}}{256\,(-a)^{9/8}\,c^{15/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{7}{512}+\frac{7}{512}\mathrm{i}\right)}{(-a)^{9/8}\,c^{15/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{7}{512}-\frac{7}{512}\mathrm{i}\right)}{(-a)^{9/8}\,c^{15/8}}$$

input
```
int(x^(13/2)/(a + c*x^4)^3,x)
```

output
```
((7*x^(15/2))/(64*a) - x^(7/2)/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (7*at
an((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(9/8)*c^(15/8)) - (atan((c^(1/
8)*x^(1/2)*1i)/(-a)^(1/8))*7i)/(256*(-a)^(9/8)*c^(15/8)) - (2^(1/2)*atan((
2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(7/512 - 7i/512))/((-a)^
(9/8)*c^(15/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a
)^(1/8))*(7/512 + 7i/512))/((-a)^(9/8)*c^(15/8))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1185, normalized size of antiderivative = 4.34

$$\int \frac{x^{13/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(13/2)/(c*x^4+a)^3,x)
```

output

```
( - 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
*a**2 - 28*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a*c*x**4 - 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*c**2*x**8 + 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2)))*a**2 + 28*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 + 14*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2
)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 14*c**(1/8)*a**(7/8)*sqrt( -
 sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/
4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a**2 - 28*c**(1/8)*a**(7/8)*
sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)
*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a*c*x**4 - 14*c**(1/8
)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)
- 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c**2*x**8
+ 14*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr...
```

3.152 $\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 273

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = -\frac{x^{5/2}}{8c(a+cx^4)^2}+\frac{5x^{5/2}}{64ac(a+cx^4)} + \frac{15\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{11/8}c^{13/8}} - \frac{15\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{11/8}c^{13/8}} + \frac{15\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{11/8}c^{13/8}} + \frac{15\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{11/8}c^{13/8}} - \frac{15\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{11/8}c^{13/8}}$$

output
```
-1/8*x^(5/2)/c/(c*x^4+a)^2+5/64*x^(5/2)/a/c/(c*x^4+a)-15/512*arctan(-1+2^(
1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(11/8)/c^(13/8)-15/512*arcta
n(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(11/8)/c^(13/8)+15/25
6*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^(13/8)+15/256*arctanh(c
^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(11/8)/c^(13/8)-15/512*arctanh(2^(1/2)*(-a
)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(11/8)/c^(13/
8)
```

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.01

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \frac{\frac{8a^{3/8}c^{5/8}x^{5/2}(-3a+5cx^4)}{(a+cx^4)^2} + 15\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right) - 15\sqrt{2+\sqrt{2}}\arctan}{}$$

input
```
Integrate[x^(11/2)/(a + c*x^4)^3,x]
```

output
```
((8*a^(3/8)*c^(5/8)*x^(5/2)*(-3*a + 5*c*x^4))/(a + c*x^4)^2 + 15*Sqrt[2 -
Sqrt[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/
8)*Sqrt[x])] - 15*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) -
 c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] - 15*Sqrt[2 - Sqrt[2]]*ArcTanh[(Sq
rt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + 15*Sqrt[
2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
+ c^(1/4)*x)])/(512*a^(11/8)*c^(13/8))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {817, 819, 851, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5\int \frac{x^{3/2}}{(cx^4+a)^2}dx}{16c} - \frac{x^{5/2}}{8c\,(a+cx^4)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{5\left(\frac{3\int \frac{x^{3/2}}{cx^4+a}dx}{8a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 851$$

$$\frac{5\left(\frac{3\int \frac{x^2}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 830$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 755$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 756$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\right.}{16c}$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 1082

$$\frac{5\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4}\right.}{16c}$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 217

$$\frac{5\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16c}-$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{5\left(\frac{3\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right)}{4a}\cdots\right)}{16c}$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 25

$$5\left(\frac{3\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\text{arct}\ldots}{2\ldots}\right.}{4a}\right.$$

$$16c$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 27

$$5\left(\frac{3\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\cdots}{4a}\cdots}{16c}\cdots$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$5\left(\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots}{4a}\cdots}{16c}\cdots$$

$$\frac{x^{5/2}}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^(11/2)/(a + c*x^4)^3,x]
```

output
```
-1/8*x^(5/2)/(c*(a + c*x^4)^2) + (5*(x^(5/2)/(4*a*(a + c*x^4)) + (3*(-1/2*
(ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^
(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[c] + ((-(ArcTan[1
- (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + Ar
cTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)
))/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt
[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(
-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*
(-a)^(1/4)))/(2*Sqrt[c])))/(4*a)))/(16*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.22

method	result	size
derivativedivides	$\frac{-\frac{3x^{\frac{5}{2}}}{64c}+\frac{5x^{\frac{13}{2}}}{64a}}{(c\,x^{4}+a)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{512a\,c^{2}}$	61
default	$\frac{-\frac{3x^{\frac{5}{2}}}{64c}+\frac{5x^{\frac{13}{2}}}{64a}}{(c\,x^{4}+a)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{512a\,c^{2}}$	61

input
```
int(x^(11/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-3/128/c*x^(5/2)+5/128/a*x^(13/2))/(c*x^4+a)^2+15/512/a/c^2*sum(1/_R^3*
ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.06

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(11/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(15*sqrt(2)*(-(I + 1)*a*c^3*x^8 - (2*I + 2)*a^2*c^2*x^4 - (I + 1)*
a^3*c)*(-1/(a^11*c^13))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^7*c^8*(-1/(a^11*
c^13))^(5/8) + sqrt(x)) + 15*sqrt(2)*((I - 1)*a*c^3*x^8 + (2*I - 2)*a^2*c^
2*x^4 + (I - 1)*a^3*c)*(-1/(a^11*c^13))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a
^7*c^8*(-1/(a^11*c^13))^(5/8) + sqrt(x)) + 15*sqrt(2)*(-(I - 1)*a*c^3*x^8
- (2*I - 2)*a^2*c^2*x^4 - (I - 1)*a^3*c)*(-1/(a^11*c^13))^(1/8)*log((1/2*I
 - 1/2)*sqrt(2)*a^7*c^8*(-1/(a^11*c^13))^(5/8) + sqrt(x)) + 15*sqrt(2)*((I
 + 1)*a*c^3*x^8 + (2*I + 2)*a^2*c^2*x^4 + (I + 1)*a^3*c)*(-1/(a^11*c^13))^
(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^7*c^8*(-1/(a^11*c^13))^(5/8) + sqrt(x))
 + 30*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^11*c^13))^(1/8)*log(a^7*c
^8*(-1/(a^11*c^13))^(5/8) + sqrt(x)) + 30*(I*a*c^3*x^8 + 2*I*a^2*c^2*x^4 +
 I*a^3*c)*(-1/(a^11*c^13))^(1/8)*log(I*a^7*c^8*(-1/(a^11*c^13))^(5/8) + sq
rt(x)) + 30*(-I*a*c^3*x^8 - 2*I*a^2*c^2*x^4 - I*a^3*c)*(-1/(a^11*c^13))^(1
/8)*log(-I*a^7*c^8*(-1/(a^11*c^13))^(5/8) + sqrt(x)) - 30*(a*c^3*x^8 + 2*a
^2*c^2*x^4 + a^3*c)*(-1/(a^11*c^13))^(1/8)*log(-a^7*c^8*(-1/(a^11*c^13))^(
5/8) + sqrt(x)) - 16*(5*c*x^6 - 3*a*x^2)*sqrt(x))/(a*c^3*x^8 + 2*a^2*c^2*x
^4 + a^3*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{11}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(11/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(5*c*x^(13/2) - 3*a*x^(5/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 15
*integrate(1/128*x^(3/2)/(a*c^2*x^4 + a^2*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(188) = 376$.

Time = 0.29 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.83

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(11/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-15/256*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(s
qrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 15/256*(a/c)^
(5/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) +
 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 15/256*(a/c)^(5/8)*arctan(
(sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8
)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 15/256*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2
) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*s
qrt(-2*sqrt(2) + 4)) - 15/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a
/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 15/512*(a/c)^(5
/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*s
qrt(2*sqrt(2) + 4)) + 15/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a
/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) - 15/512*(a/c)^(
5/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c
*sqrt(-2*sqrt(2) + 4)) + 1/64*(5*c*x^(13/2) - 3*a*x^(5/2))/((c*x^4 + a)^2*
a*c)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \frac{\frac{5\,x^{13/2}}{64\,a}-\frac{3\,x^{5/2}}{64\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{15\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{11/8}\,c^{13/8}} - \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,15\mathrm{i}}{256\,(-a)^{11/8}\,c^{13/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{15}{512}-\frac{15}{512}\mathrm{i}\right)}{(-a)^{11/8}\,c^{13/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{15}{512}+\frac{15}{512}\mathrm{i}\right)}{(-a)^{11/8}\,c^{13/8}}$$

input
```
int(x^(11/2)/(a + c*x^4)^3,x)
```

output
```
((5*x^(13/2))/(64*a) - (3*x^(5/2))/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (
15*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(11/8)*c^(13/8)) - (atan(
(c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*15i)/(256*(-a)^(11/8)*c^(13/8)) - (2^(1/2
)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(15/512 + 15i/51
2))/((-a)^(11/8)*c^(13/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 +
 1i/2))/(-a)^(1/8))*(15/512 - 15i/512))/((-a)^(11/8)*c^(13/8))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 2367, normalized size of antiderivative = 8.67

$$\int \frac{x^{11/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(11/2)/(c*x^4+a)^3,x)
```

output
```
(30*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2)))*a**2 + 60*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*a*c*x**4 + 30*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2
)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 30*c**(3/8)*a**(5/8)*sqrt(sq
rt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/
4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a**2 - 60*c**(3/8)*a**(5/8)*sqr
t(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c*
*(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 - 30*c**(3/8)*a**(
5/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sq
rt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 30*c**(
3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sq
rt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*
*2 - 60*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8
)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(
2) + 2)))*a*c*x**4 - 30*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 + 30*c**(3/8)*a**(5/8)*sqrt(sqrt(2)...
```

3.153 $\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 273

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = -\frac{x^{3/2}}{8c(a+cx^4)^2}+\frac{3x^{3/2}}{64ac(a+cx^4)} - \frac{15\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{13/8}c^{11/8}}+\frac{15\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{13/8}c^{11/8}} - \frac{15\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{13/8}c^{11/8}}+\frac{15\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{13/8}c^{11/8}}-\frac{15\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{13/8}c^{11/8}}$$

output
```
-1/8*x^(3/2)/c/(c*x^4+a)^2+3/64*x^(3/2)/a/c/(c*x^4+a)+15/512*arctan(-1+2^(
1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(13/8)/c^(11/8)+15/512*arcta
n(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(13/8)/c^(11/8)-15/25
6*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^(11/8)+15/256*arctanh(c
^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(13/8)/c^(11/8)-15/512*arctanh(2^(1/2)*(-a
)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(13/8)/c^(11/
8)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.01

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \frac{\frac{8a^{5/8}c^{3/8}x^{3/2}(-5a+3cx^4)}{(a+cx^4)^2} + 15\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right) - 15\sqrt{2+\sqrt{2}}\arctan}{}$$

input
```
Integrate[x^(9/2)/(a + c*x^4)^3,x]
```

output
```
((8*a^(5/8)*c^(3/8)*x^(3/2)*(-5*a + 3*c*x^4))/(a + c*x^4)^2 + 15*Sqrt[2 -
Sqrt[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/
8)*Sqrt[x])] - 15*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) -
 c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + 15*Sqrt[2 - Sqrt[2]]*ArcTanh[(Sq
rt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] - 15*Sqrt[
2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4)
+ c^(1/4)*x)])/(512*a^(13/8)*c^(11/8))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {817, 819, 851, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{3\int \frac{\sqrt{x}}{(cx^4+a)^2}dx}{16c} - \frac{x^{3/2}}{8c(a+cx^4)^2}$$

$$\Big\downarrow \text{ 819}$$

$$\frac{3\left(\frac{5\int\frac{\sqrt{x}}{cx^4+a}dx}{8a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 851

$$\frac{3\left(\frac{5\int\frac{x}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 829

$$\frac{3\left(\frac{5\left(-\frac{\int\frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int\frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 826

$$\frac{3\left(\frac{5\left(-\frac{\int\frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int\frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 827

$$\frac{3\left(\frac{5\left(-\frac{\frac{\int\frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\int\frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int\frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{5\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{3\left(\frac{5\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}-\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{5\left(-\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}\right.}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{3\left(\frac{5\left(-\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx}\right.}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 217

$$\frac{3\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{3\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right.}{4a}\right.}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right.}{4a}\cdots\right.}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$3\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\cdots}{\sqrt[8]{-\cdots}}\right)}{2\sqrt[8]{-a}c^{3/8}}\cdots}{\cdots}\right)}{4a}\cdots\right.}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 1103

$$3\left(\frac{5\left(-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\cdots\right)}{4a}\cdots\right.}{16c}$$

$$\frac{x^{3/2}}{8c\left(a+cx^4\right)^2}$$

input
```
Int[x^(9/2)/(a + c*x^4)^3,x]
```

output
```
-1/8*x^(3/2)/(c*(a + c*x^4)^2) + (3*(x^(3/2)/(4*a*(a + c*x^4)) + (5*(-1/2*
(-1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[
(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[-a] - ((-(ArcTa
n[1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))
+ ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(
1/8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqr
t[x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*
(-a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2
*c^(1/4)))/(2*Sqrt[-a])))/(4*a)))/(16*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.22

method	result	size
derivativedivides	$\frac{-\frac{5x^{\frac{3}{2}}}{64c}+\frac{3x^{\frac{11}{2}}}{64a}}{(c\,x^{4}+a)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{512a\,c^{2}}$	61
default	$\frac{-\frac{5x^{\frac{3}{2}}}{64c}+\frac{3x^{\frac{11}{2}}}{64a}}{(c\,x^{4}+a)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{512a\,c^{2}}$	61

input
```
int(x^(9/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-5/128*x^(3/2)/c+3/128/a*x^(11/2))/(c*x^4+a)^2+15/512/a/c^2*sum(1/_R^5*
ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.05

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(15*sqrt(2)*((I - 1)*a*c^3*x^8 + (2*I - 2)*a^2*c^2*x^4 + (I - 1)*a
^3*c)*(-1/(a^13*c^11))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^5*c^4*(-1/(a^13*c
^11))^(3/8) + sqrt(x)) + 15*sqrt(2)*(-(I + 1)*a*c^3*x^8 - (2*I + 2)*a^2*c^
2*x^4 - (I + 1)*a^3*c)*(-1/(a^13*c^11))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a
^5*c^4*(-1/(a^13*c^11))^(3/8) + sqrt(x)) + 15*sqrt(2)*((I + 1)*a*c^3*x^8 +
 (2*I + 2)*a^2*c^2*x^4 + (I + 1)*a^3*c)*(-1/(a^13*c^11))^(1/8)*log((1/2*I
- 1/2)*sqrt(2)*a^5*c^4*(-1/(a^13*c^11))^(3/8) + sqrt(x)) + 15*sqrt(2)*(-(I
 - 1)*a*c^3*x^8 - (2*I - 2)*a^2*c^2*x^4 - (I - 1)*a^3*c)*(-1/(a^13*c^11))^
(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^5*c^4*(-1/(a^13*c^11))^(3/8) + sqrt(x))
 + 30*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^13*c^11))^(1/8)*log(a^5*c
^4*(-1/(a^13*c^11))^(3/8) + sqrt(x)) + 30*(-I*a*c^3*x^8 - 2*I*a^2*c^2*x^4
- I*a^3*c)*(-1/(a^13*c^11))^(1/8)*log(I*a^5*c^4*(-1/(a^13*c^11))^(3/8) + s
qrt(x)) + 30*(I*a*c^3*x^8 + 2*I*a^2*c^2*x^4 + I*a^3*c)*(-1/(a^13*c^11))^(1
/8)*log(-I*a^5*c^4*(-1/(a^13*c^11))^(3/8) + sqrt(x)) - 30*(a*c^3*x^8 + 2*a
^2*c^2*x^4 + a^3*c)*(-1/(a^13*c^11))^(1/8)*log(-a^5*c^4*(-1/(a^13*c^11))^(
3/8) + sqrt(x)) - 16*(3*c*x^5 - 5*a*x)*sqrt(x))/(a*c^3*x^8 + 2*a^2*c^2*x^4
 + a^3*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{9}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(9/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(3*c*x^(11/2) - 5*a*x^(3/2))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c) + 15
*integrate(1/128*sqrt(x)/(a*c^2*x^4 + a^2*c), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(188) = 376$.

Time = 0.27 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.83

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(9/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-15/256*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(s
qrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 15/256*(a/c)^
(3/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) +
 2)*(a/c)^(1/8)))/(a^2*c*sqrt(2*sqrt(2) + 4)) + 15/256*(a/c)^(3/8)*arctan(
(sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8
)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 15/256*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2
) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*s
qrt(-2*sqrt(2) + 4)) + 15/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a
/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 15/512*(a/c)^(3
/8)*log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*s
qrt(2*sqrt(2) + 4)) - 15/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a
/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 15/512*(a/c)^(
3/8)*log(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c
*sqrt(-2*sqrt(2) + 4)) + 1/64*(3*c*x^(11/2) - 5*a*x^(3/2))/((c*x^4 + a)^2*
a*c)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \frac{\frac{3\,x^{11/2}}{64\,a}-\frac{5\,x^{3/2}}{64\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{15\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{{(-a)}^{1/8}}\right)}{256\,{(-a)}^{13/8}\,c^{11/8}}-\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1{}\mathrm{i}}{{(-a)}^{1/8}}\right)\,15{}\mathrm{i}}{256\,{(-a)}^{13/8}\,c^{11/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)}{{(-a)}^{1/8}}\right)\,\left(\frac{15}{512}-\frac{15}{512}{}\mathrm{i}\right)}{{(-a)}^{13/8}\,c^{11/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)}{{(-a)}^{1/8}}\right)\,\left(\frac{15}{512}+\frac{15}{512}{}\mathrm{i}\right)}{{(-a)}^{13/8}\,c^{11/8}}$$

input
```
int(x^(9/2)/(a + c*x^4)^3,x)
```

output
```
((3*x^(11/2))/(64*a) - (5*x^(3/2))/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) - (
15*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(13/8)*c^(11/8)) - (atan(
(c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*15i)/(256*(-a)^(13/8)*c^(11/8)) + (2^(1/2
)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(15/512 - 15i/51
2))/((-a)^(13/8)*c^(11/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 +
 1i/2))/(-a)^(1/8))*(15/512 + 15i/512))/((-a)^(13/8)*c^(11/8))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2365, normalized size of antiderivative = 8.66

$$\int \frac{x^{9/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(9/2)/(c*x^4+a)^3,x)
```

output
```
(30*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2)))*a**2 + 60*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*a*c*x**4 + 30*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2
)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 30*c**(5/8)*a**(3/8)*sqrt(sq
rt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/
4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a**2 - 60*c**(5/8)*a**(3/8)*sqr
t(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c*
*(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 - 30*c**(5/8)*a**(
3/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sq
rt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 30*c**(
5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sq
rt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*
*2 - 60*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8
)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(
2) + 2)))*a*c*x**4 - 30*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 + 30*c**(5/8)*a**(3/8)*sqrt(sqrt(2)...
```

3.154 $\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 273

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = -\frac{\sqrt{x}}{8c(a+cx^4)^2}+\frac{\sqrt{x}}{64ac(a+cx^4)} - \frac{7\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{15/8}c^{9/8}}+\frac{7\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{15/8}c^{9/8}} + \frac{7\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{15/8}c^{9/8}}+\frac{7\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{15/8}c^{9/8}}+\frac{7\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{15/8}c^{9/8}}$$

output
```
-1/8*x^(1/2)/c/(c*x^4+a)^2+1/64*x^(1/2)/a/c/(c*x^4+a)+7/512*arctan(-1+2^(1
/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(15/8)/c^(9/8)+7/512*arctan(1
+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(15/8)/c^(9/8)+7/256*arc
tan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(9/8)+7/256*arctanh(c^(1/8)*
x^(1/2)/(-a)^(1/8))/(-a)^(15/8)/c^(9/8)+7/512*arctanh(2^(1/2)*(-a)^(1/8)*c
^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(15/8)/c^(9/8)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = \frac{\frac{8a^{7/8}\sqrt[8]{c}\sqrt{x}(-7a+cx^4)}{(a+cx^4)^2} - 7\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right) - 7\sqrt{2-\sqrt{2}}\arctan\left(\right.}{}$$

input
```
Integrate[x^(7/2)/(a + c*x^4)^3,x]
```

output
```
((8*a^(7/8)*c^(1/8)*Sqrt[x]*(-7*a + c*x^4))/(a + c*x^4)^2 - 7*Sqrt[2 + Sqr
t[2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*
Sqrt[x])] - 7*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4) - c^(
1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])] + 7*Sqrt[2 + Sqrt[2]]*ArcTanh[(Sqrt[2
+ Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)] + 7*Sqrt[2 - Sq
rt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1
/4)*x)])/(512*a^(15/8)*c^(9/8))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {817, 819, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+cx^4)^3}dx$$

$$\Big\downarrow \; 817$$

$$\frac{\int \frac{1}{\sqrt{x}(cx^4+a)^2}dx}{16c} - \frac{\sqrt{x}}{8c(a+cx^4)^2}$$

$$\Big\downarrow \; 819$$

$$\frac{\frac{7\int \frac{1}{\sqrt{x}\left(cx^4+a\right)}dx}{8a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 851

$$\frac{\frac{7\int \frac{1}{cx^4+a}d\sqrt{x}}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 758

$$\frac{\frac{7\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 755

$$\frac{\frac{7\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 756

$$\frac{\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 218

$$\frac{\frac{7\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+}{16c}$$

$$\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}$$

$$\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}}{16c}-\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 1479

$$\frac{\frac{7\left(-\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\cdots\right)}{4a}\cdots}{16c}\cdots$$

$$\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$\downarrow$ 25

$$\frac{\frac{7\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\cdots\right)}{4a}\cdots}{16c}\cdots$$

$$\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{7\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\cdots}{2\sqrt{\cdots}}\right.}{4a}}{16c}$$

$$\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{7\left(-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\cdots\right.}{\cdots}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right.}{4a}}{16c}$$

$$\frac{\sqrt{x}}{8c\left(a+cx^4\right)^2}$$

input

```
Int[x^(7/2)/(a + c*x^4)^3,x]
```

output

```
-1/8*Sqrt[x]/(c*(a + c*x^4)^2) + (Sqrt[x]/(4*a*(a + c*x^4)) + (7*(-1/2*(Ar
cTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1/
8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[-a] - ((-(ArcTan[1 -
(Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcT
an[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))
/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x
] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a
)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-
a)^(1/4)))/(2*Sqrt[-a])))/(4*a))/(16*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.22

method	result	size
derivativedivides	$\frac{-\frac{7\sqrt{x}}{64c}+\frac{x^{\frac{9}{2}}}{64a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{7}}\right)}{512a\,c^{2}}$	61
default	$\frac{-\frac{7\sqrt{x}}{64c}+\frac{x^{\frac{9}{2}}}{64a}}{\left(c\,x^{4}+a\right)^{2}}+\frac{7\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{7}}\right)}{512a\,c^{2}}$	61

input
```
int(x^(7/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-7/128*x^(1/2)/c+1/128/a*x^(9/2))/(c*x^4+a)^2+7/512/a/c^2*sum(1/_R^7*ln
(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.99

$$\int \frac{x^{7/2}}{\left(a+cx^4\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(7*sqrt(2)*(-(I + 1)*a*c^3*x^8 - (2*I + 2)*a^2*c^2*x^4 - (I + 1)*a
^3*c)*(-1/(a^15*c^9))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^2*c*(-1/(a^15*c^9)
)^(1/8) + sqrt(x)) + 7*sqrt(2)*((I - 1)*a*c^3*x^8 + (2*I - 2)*a^2*c^2*x^4
+ (I - 1)*a^3*c)*(-1/(a^15*c^9))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^2*c*(-
1/(a^15*c^9))^(1/8) + sqrt(x)) + 7*sqrt(2)*(-(I - 1)*a*c^3*x^8 - (2*I - 2)
*a^2*c^2*x^4 - (I - 1)*a^3*c)*(-1/(a^15*c^9))^(1/8)*log((1/2*I - 1/2)*sqrt
(2)*a^2*c*(-1/(a^15*c^9))^(1/8) + sqrt(x)) + 7*sqrt(2)*((I + 1)*a*c^3*x^8
+ (2*I + 2)*a^2*c^2*x^4 + (I + 1)*a^3*c)*(-1/(a^15*c^9))^(1/8)*log(-(1/2*I
 + 1/2)*sqrt(2)*a^2*c*(-1/(a^15*c^9))^(1/8) + sqrt(x)) - 14*(a*c^3*x^8 + 2
*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^9))^(1/8)*log(a^2*c*(-1/(a^15*c^9))^(1/8
) + sqrt(x)) + 14*(-I*a*c^3*x^8 - 2*I*a^2*c^2*x^4 - I*a^3*c)*(-1/(a^15*c^9
))^(1/8)*log(I*a^2*c*(-1/(a^15*c^9))^(1/8) + sqrt(x)) + 14*(I*a*c^3*x^8 +
2*I*a^2*c^2*x^4 + I*a^3*c)*(-1/(a^15*c^9))^(1/8)*log(-I*a^2*c*(-1/(a^15*c^
9))^(1/8) + sqrt(x)) + 14*(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)*(-1/(a^15*c^
9))^(1/8)*log(-a^2*c*(-1/(a^15*c^9))^(1/8) + sqrt(x)) - 16*(c*x^4 - 7*a)*s
qrt(x))/(a*c^3*x^8 + 2*a^2*c^2*x^4 + a^3*c)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{7}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(7/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(7*c*x^(17/2) + 15*a*x^(9/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) - 7*i
ntegrate(1/128*x^(7/2)/(a^2*c*x^4 + a^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(188) = 376$.

Time = 0.25 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.82

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(7/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
7/256*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqr
t(sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(1
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2
)*(a/c)^(1/8)))/(a^2*c*sqrt(-2*sqrt(2) + 4)) + 7/256*(a/c)^(1/8)*arctan((s
qrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8))
)/(a^2*c*sqrt(2*sqrt(2) + 4)) + 7/256*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2) +
2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^2*c*sqrt(
2*sqrt(2) + 4)) + 7/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2*sqrt(2) + 4)) - 7/512*(a/c)^(1/8)*lo
g(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(-2
*sqrt(2) + 4)) + 7/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1
/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*sqrt(2) + 4)) - 7/512*(a/c)^(1/8)*log
(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^2*c*sqrt(2*
sqrt(2) + 4)) + 1/64*(c*x^(9/2) - 7*a*sqrt(x))/((c*x^4 + a)^2*a*c)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.57

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = \frac{\frac{x^{9/2}}{64\,a}-\frac{7\,\sqrt{x}}{64\,c}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{7\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{15/8}\,c^{9/8}}-\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,7\mathrm{i}}{256\,(-a)^{15/8}\,c^{9/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{7}{512}+\frac{7}{512}\mathrm{i}\right)}{(-a)^{15/8}\,c^{9/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{7}{512}-\frac{7}{512}\mathrm{i}\right)}{(-a)^{15/8}\,c^{9/8}}$$

input
```
int(x^(7/2)/(a + c*x^4)^3,x)
```

output
```
(x^(9/2)/(64*a) - (7*x^(1/2))/(64*c))/(a^2 + c^2*x^8 + 2*a*c*x^4) + (7*ata
n((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(15/8)*c^(9/8)) - (atan((c^(1/8
)*x^(1/2)*1i)/(-a)^(1/8))*7i)/(256*(-a)^(15/8)*c^(9/8)) + (2^(1/2)*atan((2
^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(7/512 + 7i/512))/((-a)^(
15/8)*c^(9/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1i/2))/(-a)
^(1/8))*(7/512 - 7i/512))/((-a)^(15/8)*c^(9/8))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.33

$$\int \frac{x^{7/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(c*x^4+a)^3,x)
```

output
```
( - 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
*a**2 - 28*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a*c*x**4 - 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*c**2*x**8 + 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2)))*a**2 + 28*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 + 14*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2
)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 14*c**(7/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/
4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a**2 - 28*c**(7/8)*a**(1/8)*
sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)
*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a*c*x**4 - 14*c**(7/8
)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)
- 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c**2*x**8
+ 14*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr...
```

3.155 $\int \frac{x^{5/2}}{(a+cx^4)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{x^{5/2}}{(a+cx^4)^3} dx = \frac{x^{7/2}}{8a(a+cx^4)^2} + \frac{9x^{7/2}}{64a^2(a+cx^4)} - \frac{9\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{17/8}c^{7/8}} + \frac{9\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{17/8}c^{7/8}} + \frac{9\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{17/8}c^{7/8}} - \frac{9\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{17/8}c^{7/8}} - \frac{9\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{17/8}c^{7/8}}$$

output
```
1/8*x^(7/2)/a/(c*x^4+a)^2+9/64*x^(7/2)/a^2/(c*x^4+a)+9/512*arctan(-1+2^(1/
2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(17/8)/c^(7/8)+9/512*arctan(1+
2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(17/8)/c^(7/8)+9/256*arct
an(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(17/8)/c^(7/8)-9/256*arctanh(c^(1/8)*x
^(1/2)/(-a)^(1/8))/(-a)^(17/8)/c^(7/8)-9/512*arctanh(2^(1/2)*(-a)^(1/8)*c^
(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(17/8)/c^(7/8)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \frac{\frac{8\sqrt[8]{a}x^{7/2}(17a+9cx^4)}{(a+cx^4)^2} - \frac{9\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{7/8}} - \frac{9\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{7/8}}}{512a^{17/8}}$$

input
```
Integrate[x^(5/2)/(a + c*x^4)^3,x]
```

output
```
((8*a^(1/8)*x^(7/2)*(17*a + 9*c*x^4))/(a + c*x^4)^2 - (9*Sqrt[2 + Sqrt[2]]
*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[
x])])/c^(7/8) - (9*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/4)
- c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(7/8) - (9*Sqrt[2 + Sqrt[2]]*A
rcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*x)])
/c^(7/8) - (9*Sqrt[2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 + Sqrt
[2])*x)])/(a^(1/4) + c^(1/4)*x)])/c^(7/8))/(512*a^(17/8))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {819, 819, 851, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{9\int \frac{x^{5/2}}{(cx^4+a)^2}dx}{16a} + \frac{x^{7/2}}{8a\,(a+cx^4)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{9\left(\frac{\int \frac{x^{5/2}}{cx^4+a}dx}{8a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 851

$$\frac{9\left(\frac{\int \frac{x^3}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 830

$$\frac{9\left(\frac{\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 826

$$\frac{9\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 827

$$\frac{9\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 218

$$\frac{9\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 221

$$\frac{9\left(\frac{\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{}{4a(}\right.}{16a}$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{9\left(\frac{\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}-$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{c}}}{4a}+\frac{x^{7/2}}{4a(a+cx^4)}\right)}{16a}+$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{9\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\cdots}{4a}\cdots\right)}{16a}\cdots$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{9\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arcta}\ldots}{2\ldots}}{\ldots}}{4a}\ldots\right)}{16a\ldots}$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{9\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}\ldots}{2\sqrt{c}\ldots}}{4a}\ldots\right)}{16a\ldots}$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1103$$

$$9\left(\frac{\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{c}}-\frac{\text{arcta}\ldots}{2\ldots}}{4a}\right.$$

$$\overline{\qquad\qquad 16a \qquad\qquad}$$

$$\frac{x^{7/2}}{8a\left(a+cx^4\right)^2}$$

input
```
Int[x^(5/2)/(a + c*x^4)^3,x]
```

output
```
x^(7/2)/(8*a*(a + c*x^4)^2) + (9*(x^(7/2)/(4*a*(a + c*x^4)) + (-1/2*(-1/2*
ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c^(1/
8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[c] + ((-(ArcTan[1 - (
Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcTa
n[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))/
(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x] +
c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a)^(1
/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c^(1/4
)))/(2*Sqrt[c]))/(4*a)))/(16*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

method	result	size
derivativedivides	$\frac{\frac{17x^{\frac{7}{2}}}{64a}+\frac{9c\,x^{\frac{15}{2}}}{64a^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{512a^{2}c}$	62
default	$\frac{\frac{17x^{\frac{7}{2}}}{64a}+\frac{9c\,x^{\frac{15}{2}}}{64a^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{9\left(\sum_{\_R=\operatorname{RootOf}\left(c\,\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R}\right)}{512a^{2}c}$	62

input
```
int(x^(5/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(17/128/a*x^(7/2)+9/128/a^2*c*x^(15/2))/(c*x^4+a)^2+9/512/a^2/c*sum(1/_R
*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.04

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(9*sqrt(2)*((I - 1)*a^2*c^2*x^8 + (2*I - 2)*a^3*c*x^4 + (I - 1)*a^
4)*(-1/(a^17*c^7))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^15*c^6*(-1/(a^17*c^7)
)^(7/8) + sqrt(x)) + 9*sqrt(2)*(-(I + 1)*a^2*c^2*x^8 - (2*I + 2)*a^3*c*x^4
 - (I + 1)*a^4)*(-1/(a^17*c^7))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^15*c^6*
(-1/(a^17*c^7))^(7/8) + sqrt(x)) + 9*sqrt(2)*((I + 1)*a^2*c^2*x^8 + (2*I +
 2)*a^3*c*x^4 + (I + 1)*a^4)*(-1/(a^17*c^7))^(1/8)*log((1/2*I - 1/2)*sqrt(
2)*a^15*c^6*(-1/(a^17*c^7))^(7/8) + sqrt(x)) + 9*sqrt(2)*(-(I - 1)*a^2*c^2
*x^8 - (2*I - 2)*a^3*c*x^4 - (I - 1)*a^4)*(-1/(a^17*c^7))^(1/8)*log(-(1/2*
I + 1/2)*sqrt(2)*a^15*c^6*(-1/(a^17*c^7))^(7/8) + sqrt(x)) - 18*(a^2*c^2*x
^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^17*c^7))^(1/8)*log(a^15*c^6*(-1/(a^17*c^7))
^(7/8) + sqrt(x)) + 18*(I*a^2*c^2*x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^17*c
^7))^(1/8)*log(I*a^15*c^6*(-1/(a^17*c^7))^(7/8) + sqrt(x)) + 18*(-I*a^2*c^
2*x^8 - 2*I*a^3*c*x^4 - I*a^4)*(-1/(a^17*c^7))^(1/8)*log(-I*a^15*c^6*(-1/(
a^17*c^7))^(7/8) + sqrt(x)) + 18*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^
17*c^7))^(1/8)*log(-a^15*c^6*(-1/(a^17*c^7))^(7/8) + sqrt(x)) - 16*(9*c*x^
7 + 17*a*x^3)*sqrt(x))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{5}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(5/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(9*c*x^(15/2) + 17*a*x^(7/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 9*i
ntegrate(1/128*x^(5/2)/(a^2*c*x^4 + a^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(185) = 370$.

Time = 0.28 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.75

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(5/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
9/256*(a/c)^(7/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqr
t(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(7/8
)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2)*
(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 9/256*(a/c)^(7/8)*arctan((sqrt(
sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a
^3*sqrt(2*sqrt(2) + 4)) + 9/256*(a/c)^(7/8)*arctan(-(sqrt(sqrt(2) + 2)*(a/
c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2
) + 4)) - 9/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x
+ (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) + 9/512*(a/c)^(7/8)*log(-sqrt(x)
*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4
)) - 9/512*(a/c)^(7/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a
/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) + 9/512*(a/c)^(7/8)*log(-sqrt(x)*sqrt
(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) +
1/64*(9*c*x^(15/2) + 17*a*x^(7/2))/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \frac{\frac{17\,x^{7/2}}{64\,a}+\frac{9\,c\,x^{15/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{9\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{17/8}\,c^{7/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,9\mathrm{i}}{256\,(-a)^{17/8}\,c^{7/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{9}{512}-\frac{9}{512}\mathrm{i}\right)}{(-a)^{17/8}\,c^{7/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{9}{512}+\frac{9}{512}\mathrm{i}\right)}{(-a)^{17/8}\,c^{7/8}}$$

input
```
int(x^(5/2)/(a + c*x^4)^3,x)
```

output
```
((17*x^(7/2))/(64*a) + (9*c*x^(15/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4
) + (9*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(17/8)*c^(7/8)) + (at
an((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*9i)/(256*(-a)^(17/8)*c^(7/8)) + (2^(1/
2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(9/512 - 9i/512
))/((-a)^(17/8)*c^(7/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 + 1
i/2))/(-a)^(1/8))*(9/512 + 9i/512))/((-a)^(17/8)*c^(7/8))
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1185, normalized size of antiderivative = 4.39

$$\int \frac{x^{5/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(5/2)/(c*x^4+a)^3,x)
```

output
```
( - 18*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( -
 sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))
*a**2 - 36*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a*c*x**4 - 18*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*c**2*x**8 + 18*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((c**(
1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8
)*sqrt(sqrt(2) + 2)))*a**2 + 36*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 + 18*c**(1/8)*a**(7/8)*sqrt(sqrt(2) + 2
)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(
1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 18*c**(1/8)*a**(7/8)*sqrt( -
 sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)*c**(1/
4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a**2 - 36*c**(1/8)*a**(7/8)*
sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)
*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a*c*x**4 - 18*c**(1/8
)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)
- 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c**2*x**8
+ 18*c**(1/8)*a**(7/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqr...
```

3.156 $\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx = \frac{x^{5/2}}{8a(a+cx^4)^2}+\frac{11x^{5/2}}{64a^2(a+cx^4)} - \frac{33\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{19/8}c^{5/8}}+\frac{33\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{19/8}c^{5/8}} - \frac{33\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{19/8}c^{5/8}}-\frac{33\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{19/8}c^{5/8}}+\frac{33\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{19/8}c^{5/8}}$$

output
```
1/8*x^(5/2)/a/(c*x^4+a)^2+11/64*x^(5/2)/a^2/(c*x^4+a)+33/512*arctan(-1+2^(
1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(19/8)/c^(5/8)+33/512*arctan
(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(19/8)/c^(5/8)-33/256*
arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(19/8)/c^(5/8)-33/256*arctanh(c^(1
/8)*x^(1/2)/(-a)^(1/8))/(-a)^(19/8)/c^(5/8)+33/512*arctanh(2^(1/2)*(-a)^(1
/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(19/8)/c^(5/8)
```

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx = \frac{\frac{8a^{3/8}x^{5/2}(19a+11cx^4)}{(a+cx^4)^2} + \frac{33\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{5/8}} - \frac{33\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}\right.}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right.}{c^{5/8}}}{512a^{19}}$$

input
```
Integrate[x^(3/2)/(a + c*x^4)^3,x]
```

output
```
((8*a^(3/8)*x^(5/2)*(19*a + 11*c*x^4))/(a + c*x^4)^2 + (33*Sqrt[2 - Sqrt[2
]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqr
t[x])])/c^(5/8) - (33*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/
4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(5/8) - (33*Sqrt[2 - Sqrt[2
]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*
x)])/c^(5/8) + (33*Sqrt[2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 +
 Sqrt[2])*x)])/(a^(1/4) + c^(1/4)*x)])/c^(5/8))/(512*a^(19/8))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {819, 819, 851, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \; 819$$

$$\frac{11\int \frac{x^{3/2}}{(cx^4+a)^2}dx}{16a} + \frac{x^{5/2}}{8a\,(a+cx^4)^2}$$

$$\Big\downarrow \; 819$$

$$\frac{11\left(\frac{3\int \frac{x^{3/2}}{cx^4+a}dx}{8a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{11\left(\frac{3\int \frac{x^2}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{830}$$

$$\frac{11\left(\frac{3\left(\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{11\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{11\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{11\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{11\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 1476$$

$$11\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}\right.$$

$$\overline{\qquad\qquad 16a \qquad\qquad}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$11\left(\frac{3\left(\frac{\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx\ldots}\right.$$

$$\overline{\qquad\qquad 16a \qquad\qquad}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 217

$$\frac{11\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{c}}\right)}{4a}+\frac{x^{5/2}}{4a(a+cx^4)}\right)}{16a}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1479

$$\frac{11\left(\frac{3\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}\right)}{4a}\right)}{16a}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$11\left(\frac{3\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\cdots\right.}{4a}\cdots\right.}{16a}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$11\left(\frac{3\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}} \cdots\right.}{4a} \cdots\right.}{16a}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1103

$$11\left(\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{c}}- \cdots\right.}{4a} \cdots\right.}{16a}$$

$$\frac{x^{5/2}}{8a\left(a+cx^4\right)^2}$$

input
```
Int[x^(3/2)/(a + c*x^4)^3,x]
```

output
```
x^(5/2)/(8*a*(a + c*x^4)^2) + (11*(x^(5/2)/(4*a*(a + c*x^4)) + (3*(-1/2*(A
rcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1
/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[c] + ((-(ArcTan[1 -
(Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + ArcT
an[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)))
/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[x
] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-a
)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(-
a)^(1/4)))/(2*Sqrt[c])))/(4*a)))/(16*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.48 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

method	result	size
derivativedivides	$\frac{\frac{19x^{\frac{5}{2}}}{64a}+\frac{11c\,x^{\frac{13}{2}}}{64a^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{33\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{512a^{2}c}$	62
default	$\frac{\frac{19x^{\frac{5}{2}}}{64a}+\frac{11c\,x^{\frac{13}{2}}}{64a^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{33\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{3}}\right)}{512a^{2}c}$	62

input
```
int(x^(3/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(19/128/a*x^(5/2)+11/128/a^2*c*x^(13/2))/(c*x^4+a)^2+33/512/a^2/c*sum(1/
_R^3*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.04

$$\int \frac{x^{3/2}}{(a + cx^4)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(33*sqrt(2)*(-(I + 1)*a^2*c^2*x^8 - (2*I + 2)*a^3*c*x^4 - (I + 1)*
a^4)*(-1/(a^19*c^5))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^12*c^3*(-1/(a^19*c^
5))^(5/8) + sqrt(x)) + 33*sqrt(2)*((I - 1)*a^2*c^2*x^8 + (2*I - 2)*a^3*c*x
^4 + (I - 1)*a^4)*(-1/(a^19*c^5))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^12*c^
3*(-1/(a^19*c^5))^(5/8) + sqrt(x)) + 33*sqrt(2)*(-(I - 1)*a^2*c^2*x^8 - (2
*I - 2)*a^3*c*x^4 - (I - 1)*a^4)*(-1/(a^19*c^5))^(1/8)*log((1/2*I - 1/2)*s
qrt(2)*a^12*c^3*(-1/(a^19*c^5))^(5/8) + sqrt(x)) + 33*sqrt(2)*((I + 1)*a^2
*c^2*x^8 + (2*I + 2)*a^3*c*x^4 + (I + 1)*a^4)*(-1/(a^19*c^5))^(1/8)*log(-(
1/2*I + 1/2)*sqrt(2)*a^12*c^3*(-1/(a^19*c^5))^(5/8) + sqrt(x)) + 66*(a^2*c
^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^19*c^5))^(1/8)*log(a^12*c^3*(-1/(a^19*c
^5))^(5/8) + sqrt(x)) + 66*(I*a^2*c^2*x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^
19*c^5))^(1/8)*log(I*a^12*c^3*(-1/(a^19*c^5))^(5/8) + sqrt(x)) + 66*(-I*a^
2*c^2*x^8 - 2*I*a^3*c*x^4 - I*a^4)*(-1/(a^19*c^5))^(1/8)*log(-I*a^12*c^3*(
-1/(a^19*c^5))^(5/8) + sqrt(x)) - 66*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1
/(a^19*c^5))^(1/8)*log(-a^12*c^3*(-1/(a^19*c^5))^(5/8) + sqrt(x)) - 16*(11
*c*x^6 + 19*a*x^2)*sqrt(x))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{(a + cx^4)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**(3/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx = \int \frac{x^{\frac{3}{2}}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(3/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(11*c*x^(13/2) + 19*a*x^(5/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 33
*integrate(1/128*x^(3/2)/(a^2*c*x^4 + a^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(185) = 370$.

Time = 0.27 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.75

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-33/256*(a/c)^(5/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(s
qrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) - 33/256*(a/c)^(5
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2
)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) + 33/256*(a/c)^(5/8)*arctan((sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/
(a^3*sqrt(-2*sqrt(2) + 4)) + 33/256*(a/c)^(5/8)*arctan(-(sqrt(sqrt(2) + 2)
*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*s
qrt(2) + 4)) - 33/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) + 33/512*(a/c)^(5/8)*log(-s
qrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2
) + 4)) + 33/512*(a/c)^(5/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) +
x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 33/512*(a/c)^(5/8)*log(-sqrt
(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2)
 + 4)) + 1/64*(11*c*x^(13/2) + 19*a*x^(5/2))/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx = \frac{\frac{19\,x^{5/2}}{64\,a}+\frac{11\,c\,x^{13/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}-\frac{33\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{19/8}\,c^{5/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,33\mathrm{i}}{256\,(-a)^{19/8}\,c^{5/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{33}{512}+\frac{33}{512}\mathrm{i}\right)}{(-a)^{19/8}\,c^{5/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(\frac{33}{512}-\frac{33}{512}\mathrm{i}\right)}{(-a)^{19/8}\,c^{5/8}}$$

input
```
int(x^(3/2)/(a + c*x^4)^3,x)
```

output
```
((19*x^(5/2))/(64*a) + (11*c*x^(13/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^
4) - (33*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(19/8)*c^(5/8)) + (
atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*33i)/(256*(-a)^(19/8)*c^(5/8)) + (2^
(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(33/512 + 33
i/512))/((-a)^(19/8)*c^(5/8)) + (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/
2 + 1i/2))/(-a)^(1/8))*(33/512 - 33i/512))/((-a)^(19/8)*c^(5/8))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 2367, normalized size of antiderivative = 8.77

$$\int \frac{x^{3/2}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(c*x^4+a)^3,x)
```

output
```
(66*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sq
rt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
 2)))*a**2 + 132*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8
)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*s
qrt(sqrt(2) + 2)))*a*c*x**4 + 66*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(
2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**
(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 66*c**(3/8)*a**(5/8)*sqrt(s
qrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1
/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a**2 - 132*c**(3/8)*a**(5/8)*s
qrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*
c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 - 66*c**(3/8)*a*
*(5/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*
sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 66*c*
*(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqrt( -
sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*
a**2 - 132*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(
1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sq
rt(2) + 2)))*a*c*x**4 - 66*c**(3/8)*a**(5/8)*sqrt(sqrt(2) + 2)*sqrt(2)*ata
n((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*
a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 + 66*c**(3/8)*a**(5/8)*sqrt(sqrt...
```

3.157 $\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\begin{aligned}\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx &= \frac{x^{3/2}}{8a(a+cx^4)^2}+\frac{13x^{3/2}}{64a^2(a+cx^4)}+\frac{65\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{21/8}c^{3/8}}\\&\quad-\frac{65\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{21/8}c^{3/8}}+\frac{65\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{21/8}c^{3/8}}\\&\quad-\frac{65\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{21/8}c^{3/8}}+\frac{65\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{21/8}c^{3/8}}\end{aligned}$$

output
```
1/8*x^(3/2)/a/(c*x^4+a)^2+13/64*x^(3/2)/a^2/(c*x^4+a)-65/512*arctan(-1+2^(
1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(21/8)/c^(3/8)-65/512*arctan
(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(21/8)/c^(3/8)+65/256*
arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(21/8)/c^(3/8)-65/256*arctanh(c^(1
/8)*x^(1/2)/(-a)^(1/8))/(-a)^(21/8)/c^(3/8)+65/512*arctanh(2^(1/2)*(-a)^(1
/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(21/8)/c^(3/8)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx$$

$$= \frac{\frac{8a^{5/8}x^{3/2}(21a+13cx^4)}{(a+cx^4)^2} + \frac{65\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{3/8}} - \frac{65\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{c^{3/8}} + \frac{65\sqrt{2-\sqrt{2}}\mathrm{ar}}{}}{512a^{21/8}}$$

input
```
Integrate[Sqrt[x]/(a + c*x^4)^3,x]
```

output
```
((8*a^(5/8)*x^(3/2)*(21*a + 13*c*x^4))/(a + c*x^4)^2 + (65*Sqrt[2 - Sqrt[2
]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqr
t[x])])/c^(3/8) - (65*Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(1/
4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(3/8) + (65*Sqrt[2 - Sqrt[2
]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/4)*
x)])/c^(3/8) - (65*Sqrt[2 + Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((-2 +
 Sqrt[2])*x)])/(a^(1/4) + c^(1/4)*x)])/c^(3/8))/(512*a^(21/8))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.42, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {819, 819, 851, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{13\int \frac{\sqrt{x}}{(cx^4+a)^2}dx}{16a}+\frac{x^{3/2}}{8a\,(a+cx^4)^2}$$

$$\Big\downarrow \text{819}$$

$$\frac{13\left(\frac{5\int \frac{\sqrt{x}}{cx^4+a}dx}{8a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{13\left(\frac{5\int \frac{x}{cx^4+a}d\sqrt{x}}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{829}$$

$$\frac{13\left(\frac{5\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{13\left(\frac{5\left(-\frac{\int \frac{x}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{827}$$

$$\frac{13\left(\frac{5\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{13\left(\frac{5\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{13\left(\frac{5\left(-\frac{\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)}{16a}+\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1476

$$13\left(\frac{5\left(-\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}\right.$$

$$\overline{\qquad\qquad\qquad 16a \qquad\qquad\qquad}$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$13\left(\frac{5\left(-\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/}}{4a(a+}\right.$$

$$\overline{\qquad\qquad\qquad 16a \qquad\qquad\qquad}$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 217

$$13\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}\right)}{4a}+\frac{x^{3/2}}{4a(a+cx^4)}\right)$$

$$\overline{\qquad\qquad 16a \qquad\qquad}$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$\big\downarrow$ 1479

$$13\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}\cdots\right)}{4a}\cdots\right)$$

$$\overline{\qquad\qquad 16a \qquad\qquad}$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{13\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\cdots\right)}{4a}\cdots\right)}{16a}$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{13\left(\frac{5\left(-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{c}}}{2\sqrt{-a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{\cdots}}{\sqrt[8]{\cdots}}\cdots\right)}{2\sqrt[8]{-a}c^{3/\cdots}}\cdots\right)}{4a}\cdots\right)}{16a}\cdots$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

$\downarrow$ 1103

$$\frac{13\left(\frac{5\left(-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2\sqrt[8]{-a}c^{3/8}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{c}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\cdots\right)}{4a}\cdots\right)}{16a}\cdots$$

$$\frac{x^{3/2}}{8a\left(a+cx^4\right)^2}$$

input
```
Int[Sqrt[x]/(a + c*x^4)^3,x]
```

output
```
x^(3/2)/(8*a*(a + c*x^4)^2) + (13*(x^(3/2)/(4*a*(a + c*x^4)) + (5*(-1/2*(-
1/2*ArcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/((-a)^(1/8)*c^(3/8)) + ArcTanh[(c
^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(1/8)*c^(3/8)))/Sqrt[-a] - ((-(ArcTan[
1 - (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) +
ArcTan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/
8)))/(2*c^(1/4)) - (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[
x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-
a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*c
^(1/4)))/(2*Sqrt[-a])))/(4*a)))/(16*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

method	result	size
derivativedivides	$\frac{\frac{21x^{\frac{3}{2}}}{64a}+\frac{13c\,x^{\frac{11}{2}}}{64a^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{65\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{512a^{2}c}$	62
default	$\frac{\frac{21x^{\frac{3}{2}}}{64a}+\frac{13c\,x^{\frac{11}{2}}}{64a^{2}}}{\left(c\,x^{4}+a\right)^{2}}+\frac{65\left(\sum_{\_R=\operatorname{RootOf}\left(c\_Z^{8}+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^{5}}\right)}{512a^{2}c}$	62

input
```
int(x^(1/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(21/128/a*x^(3/2)+13/128/a^2*c*x^(11/2))/(c*x^4+a)^2+65/512/a^2/c*sum(1/
_R^5*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(65*sqrt(2)*((I - 1)*a^2*c^2*x^8 + (2*I - 2)*a^3*c*x^4 + (I - 1)*a
^4)*(-1/(a^21*c^3))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^8*c*(-1/(a^21*c^3))^
(3/8) + sqrt(x)) + 65*sqrt(2)*(-(I + 1)*a^2*c^2*x^8 - (2*I + 2)*a^3*c*x^4
- (I + 1)*a^4)*(-1/(a^21*c^3))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^8*c*(-1/
(a^21*c^3))^(3/8) + sqrt(x)) + 65*sqrt(2)*((I + 1)*a^2*c^2*x^8 + (2*I + 2)
*a^3*c*x^4 + (I + 1)*a^4)*(-1/(a^21*c^3))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*
a^8*c*(-1/(a^21*c^3))^(3/8) + sqrt(x)) + 65*sqrt(2)*(-(I - 1)*a^2*c^2*x^8
- (2*I - 2)*a^3*c*x^4 - (I - 1)*a^4)*(-1/(a^21*c^3))^(1/8)*log(-(1/2*I + 1
/2)*sqrt(2)*a^8*c*(-1/(a^21*c^3))^(3/8) + sqrt(x)) + 130*(a^2*c^2*x^8 + 2*
a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8)*log(a^8*c*(-1/(a^21*c^3))^(3/8) + s
qrt(x)) + 130*(-I*a^2*c^2*x^8 - 2*I*a^3*c*x^4 - I*a^4)*(-1/(a^21*c^3))^(1/
8)*log(I*a^8*c*(-1/(a^21*c^3))^(3/8) + sqrt(x)) + 130*(I*a^2*c^2*x^8 + 2*I
*a^3*c*x^4 + I*a^4)*(-1/(a^21*c^3))^(1/8)*log(-I*a^8*c*(-1/(a^21*c^3))^(3/
8) + sqrt(x)) - 130*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1/(a^21*c^3))^(1/8
)*log(-a^8*c*(-1/(a^21*c^3))^(3/8) + sqrt(x)) - 16*(13*c*x^5 + 21*a*x)*sqr
t(x))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(1/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx = \int \frac{\sqrt{x}}{(cx^4+a)^3}\,dx$$

input
```
integrate(x^(1/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
1/64*(13*c*x^(11/2) + 21*a*x^(3/2))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4) + 65
*integrate(1/128*sqrt(x)/(a^2*c*x^4 + a^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(185) = 370$.

Time = 0.27 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{x}}{(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^(1/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
-65/256*(a/c)^(3/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(s
qrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) - 65/256*(a/c)^(3
/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) + 2
)*(a/c)^(1/8)))/(a^3*sqrt(2*sqrt(2) + 4)) + 65/256*(a/c)^(3/8)*arctan((sqr
t(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/
(a^3*sqrt(-2*sqrt(2) + 4)) + 65/256*(a/c)^(3/8)*arctan(-(sqrt(sqrt(2) + 2)
*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*s
qrt(2) + 4)) + 65/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8
) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 65/512*(a/c)^(3/8)*log(-s
qrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2
) + 4)) - 65/512*(a/c)^(3/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) +
x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) + 65/512*(a/c)^(3/8)*log(-sqrt
(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2)
 + 4)) + 1/64*(13*c*x^(11/2) + 21*a*x^(3/2))/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{x}}{\left(a+cx^4\right)^3}\,dx=\frac{\frac{21\,x^{3/2}}{64\,a}+\frac{13\,c\,x^{11/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8}+\frac{65\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{{\left(-a\right)}^{1/8}}\right)}{256\,{\left(-a\right)}^{21/8}\,c^{3/8}}+\frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/8}}\right)\,65{}\mathrm{i}}{256\,{\left(-a\right)}^{21/8}\,c^{3/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}{}\mathrm{i}\right)}{{\left(-a\right)}^{1/8}}\right)\,\left(-\frac{65}{512}+\frac{65}{512}{}\mathrm{i}\right)}{{\left(-a\right)}^{21/8}\,c^{3/8}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}{}\mathrm{i}\right)}{{\left(-a\right)}^{1/8}}\right)\,\left(-\frac{65}{512}-\frac{65}{512}{}\mathrm{i}\right)}{{\left(-a\right)}^{21/8}\,c^{3/8}}$$

input
```
int(x^(1/2)/(a + c*x^4)^3,x)
```

output
```
((21*x^(3/2))/(64*a) + (13*c*x^(11/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^
4) + (65*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(21/8)*c^(3/8)) + (
atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*65i)/(256*(-a)^(21/8)*c^(3/8)) - (2^
(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(65/512 - 65
i/512))/((-a)^(21/8)*c^(3/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/
2 + 1i/2))/(-a)^(1/8))*(65/512 + 65i/512))/((-a)^(21/8)*c^(3/8))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2365, normalized size of antiderivative = 8.76

$$\int \frac{\sqrt{x}}{\left(a+cx^4\right)^3}\,dx=\text{Too large to display}$$

input
```
int(x^(1/2)/(c*x^4+a)^3,x)
```

output

```
(130*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2)
+ 2)))*a**2 + 260*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/
8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*
sqrt(sqrt(2) + 2)))*a*c*x**4 + 130*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqr
t(2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c
**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 130*c**(5/8)*a**(3/8)*sqr
t(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c*
*(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a**2 - 260*c**(5/8)*a**(3/8
)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(
x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 - 130*c**(5/8
)*a**(3/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)
- 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 1
30*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)*a**(1/8)*sqr
t( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) +
2)))*a**2 - 260*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(2)*atan((c**(1/8)
*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sq
rt(sqrt(2) + 2)))*a*c*x**4 - 130*c**(5/8)*a**(3/8)*sqrt(sqrt(2) + 2)*sqrt(
2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**
(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 + 130*c**(5/8)*a**(3/8)*sq...
```

3.158 $\int \frac{1}{\sqrt{x}(a+cx^4)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 270

$$\int \frac{1}{\sqrt{x}(a+cx^4)^3} dx = \frac{\sqrt{x}}{8a(a+cx^4)^2} + \frac{15\sqrt{x}}{64a^2(a+cx^4)} + \frac{105\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{23/8}\sqrt[8]{c}} - \frac{105\arctan\left(1+\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256\sqrt{2}(-a)^{23/8}\sqrt[8]{c}} - \frac{105\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{23/8}\sqrt[8]{c}} - \frac{105\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{256(-a)^{23/8}\sqrt[8]{c}} - \frac{105\text{arctanh}\left(\frac{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}}{\sqrt[4]{-a}+\sqrt[4]{c}x}\right)}{256\sqrt{2}(-a)^{23/8}\sqrt[8]{c}}$$

output
```
1/8*x^(1/2)/a/(c*x^4+a)^2+15/64*x^(1/2)/a^2/(c*x^4+a)-105/512*arctan(-1+2^
(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(23/8)/c^(1/8)-105/512*arct
an(1+2^(1/2)*c^(1/8)*x^(1/2)/(-a)^(1/8))*2^(1/2)/(-a)^(23/8)/c^(1/8)-105/2
56*arctan(c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(23/8)/c^(1/8)-105/256*arctanh(
c^(1/8)*x^(1/2)/(-a)^(1/8))/(-a)^(23/8)/c^(1/8)-105/512*arctanh(2^(1/2)*(-
a)^(1/8)*c^(1/8)*x^(1/2)/((-a)^(1/4)+c^(1/4)*x))*2^(1/2)/(-a)^(23/8)/c^(1/
8)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \frac{\frac{8a^{7/8}\sqrt{x}(23a+15cx^4)}{(a+cx^4)^2} - \frac{105\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{1-\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{\sqrt[8]{c}} - \frac{105\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{1+\frac{1}{\sqrt{2}}}\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{\sqrt[8]{a}\sqrt[8]{c}\sqrt{x}}\right)}{\sqrt[8]{c}} + 105\sqrt{2+\sqrt{2}}\cdots}{512a^{23/8}}$$

input
```
Integrate[1/(Sqrt[x]*(a + c*x^4)^3),x]
```

output
```
((8*a^(7/8)*Sqrt[x]*(23*a + 15*c*x^4))/(a + c*x^4)^2 - (105*Sqrt[2 + Sqrt[
2]]*ArcTan[(Sqrt[1 - 1/Sqrt[2]]*(a^(1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sq
rt[x])])/c^(1/8) - (105*Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[1 + 1/Sqrt[2]]*(a^(
1/4) - c^(1/4)*x))/(a^(1/8)*c^(1/8)*Sqrt[x])])/c^(1/8) + (105*Sqrt[2 + Sqr
t[2]]*ArcTanh[(Sqrt[2 + Sqrt[2]]*a^(1/8)*c^(1/8)*Sqrt[x])/(a^(1/4) + c^(1/
4)*x)])/c^(1/8) + (105*Sqrt[2 - Sqrt[2]]*ArcTanh[(a^(1/8)*c^(1/8)*Sqrt[-((
-2 + Sqrt[2])*x)])/(a^(1/4) + c^(1/4)*x)])/c^(1/8))/(512*a^(23/8))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.44, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {819, 819, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx$$

$$\Big\downarrow \; 819$$

$$\frac{15\int \frac{1}{\sqrt{x}(cx^4+a)^2}dx}{16a}+\frac{\sqrt{x}}{8a\,(a+cx^4)^2}$$

$$\Big\downarrow \ 819$$

$$\frac{15\left(\frac{7\int \frac{1}{\sqrt{x}\left(cx^4+a\right)}dx}{8a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 851$$

$$\frac{15\left(\frac{7\int \frac{1}{cx^4+a}d\sqrt{x}}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 758$$

$$\frac{15\left(\frac{7\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\int \frac{1}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 755$$

$$\frac{15\left(\frac{7\left(-\frac{\int \frac{1}{\sqrt{-a}-\sqrt{c}x^2}d\sqrt{x}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \ 756$$

$$\frac{15\left(\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{1}{\sqrt[4]{c}x+\sqrt[4]{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{15\left(\frac{7\left(-\frac{\frac{\int \frac{1}{\sqrt[4]{-a}-\sqrt[4]{c}x}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{15\left(\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\int \frac{\sqrt[4]{c}x+\sqrt[4]{-a}}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}+\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{15\left(\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[4]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\cdots}{4a}\cdots}{16a}$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{15\left(\frac{7\left(-\frac{\frac{\int \frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+\cdots}\cdots}{16a}$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 217

$$\frac{15\left(\frac{7\left(-\frac{\frac{\int\frac{\sqrt[4]{-a}-\sqrt[4]{c}x}{\sqrt{c}x^2+\sqrt{-a}}d\sqrt{x}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}\right)}{4a}+\frac{\sqrt{x}}{4a(a+cx^4)}\right)}{16a}$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{15\left(\frac{7\left(-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right.}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}\right.}{4a}\right.}{16a}$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow 25$$

$$\frac{15\left(\frac{7\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{c}\left(x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}\right)}{\sqrt[8]{c}\left(x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\cdots\right)}{4a}\cdots\right)}{16a}$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{15\left(\frac{7\left(-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[8]{-a}-2\sqrt[8]{c}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt{2}\sqrt[8]{-a}\sqrt[4]{c}}+\frac{\int \frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}+\sqrt[8]{-a}}{x+\frac{\sqrt{2}\sqrt[8]{-a}\sqrt{x}}{\sqrt[8]{c}}+\frac{\sqrt[4]{-a}}{\sqrt[4]{c}}}d\sqrt{x}}{2\sqrt[8]{-a}\sqrt[4]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}}{2\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}\cdots\right)}{4a}\cdots\right)}{16a}\cdots$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{15\left(\frac{7\left(-\frac{\frac{\arctan\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{2(-a)^{3/8}\sqrt[8]{c}}}{2\sqrt{-a}}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}+1\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[8]{c}\sqrt{x}}{\sqrt[8]{-a}}\right)}{\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}}{2\sqrt[4]{-a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}\sqrt{x}+\sqrt[4]{-a}+\sqrt[4]{c}x\right)}{2\sqrt{2}\sqrt[8]{-a}\sqrt[8]{c}}-\cdots}{2\sqrt[4]{\cdots}}}{2\sqrt{-a}}\cdots\right)}{4a}\cdots\right)}{16a}\cdots$$

$$\frac{\sqrt{x}}{8a\left(a+cx^4\right)^2}$$

input
```
Int[1/(Sqrt[x]*(a + c*x^4)^3),x]
```

output
```
Sqrt[x]/(8*a*(a + c*x^4)^2) + (15*(Sqrt[x]/(4*a*(a + c*x^4)) + (7*(-1/2*(A
rcTan[(c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)) + ArcTanh[(c^(1
/8)*Sqrt[x])/(-a)^(1/8)]/(2*(-a)^(3/8)*c^(1/8)))/Sqrt[-a] - ((-(ArcTan[1 -
 (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))) + Arc
Tan[1 + (Sqrt[2]*c^(1/8)*Sqrt[x])/(-a)^(1/8)]/(Sqrt[2]*(-a)^(1/8)*c^(1/8))
)/(2*(-a)^(1/4)) + (-1/2*Log[(-a)^(1/4) - Sqrt[2]*(-a)^(1/8)*c^(1/8)*Sqrt[
x] + c^(1/4)*x]/(Sqrt[2]*(-a)^(1/8)*c^(1/8)) + Log[(-a)^(1/4) + Sqrt[2]*(-
a)^(1/8)*c^(1/8)*Sqrt[x] + c^(1/4)*x]/(2*Sqrt[2]*(-a)^(1/8)*c^(1/8)))/(2*(
-a)^(1/4)))/(2*Sqrt[-a])))/(4*a)))/(16*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.55 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

method	result	size
derivativedivides	$\frac{\frac{23\sqrt{x}}{64a}+\frac{15c\,x^{\frac{9}{2}}}{64a^2}}{(c\,x^4+a)^2}+\frac{105\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{512a^2c}$	62
default	$\frac{\frac{23\sqrt{x}}{64a}+\frac{15c\,x^{\frac{9}{2}}}{64a^2}}{(c\,x^4+a)^2}+\frac{105\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+a\right)}\frac{\ln\left(\sqrt{x}-\_R\right)}{\_R^7}\right)}{512a^2c}$	62

input
```
int(1/x^(1/2)/(c*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(23/128/a*x^(1/2)+15/128/a^2*c*x^(9/2))/(c*x^4+a)^2+105/512/a^2/c*sum(1/
_R^7*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(1/2)/(c*x^4+a)^3,x, algorithm="fricas")
```

output
```
-1/1024*(105*sqrt(2)*(-(I + 1)*a^2*c^2*x^8 - (2*I + 2)*a^3*c*x^4 - (I + 1)
*a^4)*(-1/(a^23*c))^(1/8)*log((1/2*I + 1/2)*sqrt(2)*a^3*(-1/(a^23*c))^(1/8
) + sqrt(x)) + 105*sqrt(2)*((I - 1)*a^2*c^2*x^8 + (2*I - 2)*a^3*c*x^4 + (I
 - 1)*a^4)*(-1/(a^23*c))^(1/8)*log(-(1/2*I - 1/2)*sqrt(2)*a^3*(-1/(a^23*c)
)^(1/8) + sqrt(x)) + 105*sqrt(2)*(-(I - 1)*a^2*c^2*x^8 - (2*I - 2)*a^3*c*x
^4 - (I - 1)*a^4)*(-1/(a^23*c))^(1/8)*log((1/2*I - 1/2)*sqrt(2)*a^3*(-1/(a
^23*c))^(1/8) + sqrt(x)) + 105*sqrt(2)*((I + 1)*a^2*c^2*x^8 + (2*I + 2)*a^
3*c*x^4 + (I + 1)*a^4)*(-1/(a^23*c))^(1/8)*log(-(1/2*I + 1/2)*sqrt(2)*a^3*
(-1/(a^23*c))^(1/8) + sqrt(x)) - 210*(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)*(-1
/(a^23*c))^(1/8)*log(a^3*(-1/(a^23*c))^(1/8) + sqrt(x)) + 210*(-I*a^2*c^2*
x^8 - 2*I*a^3*c*x^4 - I*a^4)*(-1/(a^23*c))^(1/8)*log(I*a^3*(-1/(a^23*c))^(
1/8) + sqrt(x)) + 210*(I*a^2*c^2*x^8 + 2*I*a^3*c*x^4 + I*a^4)*(-1/(a^23*c)
)^(1/8)*log(-I*a^3*(-1/(a^23*c))^(1/8) + sqrt(x)) + 210*(a^2*c^2*x^8 + 2*a
^3*c*x^4 + a^4)*(-1/(a^23*c))^(1/8)*log(-a^3*(-1/(a^23*c))^(1/8) + sqrt(x)
) - 16*(15*c*x^4 + 23*a)*sqrt(x))/(a^2*c^2*x^8 + 2*a^3*c*x^4 + a^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(1/2)/(c*x**4+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \int \frac{1}{(cx^4+a)^3\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(c*x^4+a)^3,x, algorithm="maxima")
```

output
```
-105*c*integrate(1/128*x^(7/2)/(a^3*c*x^4 + a^4), x) + 1/64*(105*c^2*x^(17
/2) + 225*a*c*x^(9/2) + 128*a^2*sqrt(x))/(a^3*c^2*x^8 + 2*a^4*c*x^4 + a^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(185) = 370$.

Time = 0.23 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^(1/2)/(c*x^4+a)^3,x, algorithm="giac")
```

output
```
105/256*(a/c)^(1/8)*arctan((sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(s
qrt(sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 105/256*(a/c)^
(1/8)*arctan(-(sqrt(-sqrt(2) + 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(sqrt(2) +
 2)*(a/c)^(1/8)))/(a^3*sqrt(-2*sqrt(2) + 4)) + 105/256*(a/c)^(1/8)*arctan(
(sqrt(sqrt(2) + 2)*(a/c)^(1/8) + 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8
)))/(a^3*sqrt(2*sqrt(2) + 4)) + 105/256*(a/c)^(1/8)*arctan(-(sqrt(sqrt(2)
+ 2)*(a/c)^(1/8) - 2*sqrt(x))/(sqrt(-sqrt(2) + 2)*(a/c)^(1/8)))/(a^3*sqrt(
2*sqrt(2) + 4)) + 105/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^
(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2*sqrt(2) + 4)) - 105/512*(a/c)^(1/8)*
log(-sqrt(x)*sqrt(sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(-2
*sqrt(2) + 4)) + 105/512*(a/c)^(1/8)*log(sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^
(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*sqrt(2) + 4)) - 105/512*(a/c)^(1/8)*l
og(-sqrt(x)*sqrt(-sqrt(2) + 2)*(a/c)^(1/8) + x + (a/c)^(1/4))/(a^3*sqrt(2*
sqrt(2) + 4)) + 1/64*(15*c*x^(9/2) + 23*a*sqrt(x))/((c*x^4 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \frac{\frac{23\sqrt{x}}{64\,a}+\frac{15\,c\,x^{9/2}}{64\,a^2}}{a^2+2\,a\,c\,x^4+c^2\,x^8} - \frac{105\,\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}}{(-a)^{1/8}}\right)}{256\,(-a)^{23/8}\,c^{1/8}} + \frac{\mathrm{atan}\left(\frac{c^{1/8}\,\sqrt{x}\,1\mathrm{i}}{(-a)^{1/8}}\right)\,105\mathrm{i}}{256\,(-a)^{23/8}\,c^{1/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{105}{512}-\frac{105}{512}\mathrm{i}\right)}{(-a)^{23/8}\,c^{1/8}} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,c^{1/8}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)}{(-a)^{1/8}}\right)\,\left(-\frac{105}{512}+\frac{105}{512}\mathrm{i}\right)}{(-a)^{23/8}\,c^{1/8}}$$

input
```
int(1/(x^(1/2)*(a + c*x^4)^3),x)
```

output
```
((23*x^(1/2))/(64*a) + (15*c*x^(9/2))/(64*a^2))/(a^2 + c^2*x^8 + 2*a*c*x^4
) - (105*atan((c^(1/8)*x^(1/2))/(-a)^(1/8)))/(256*(-a)^(23/8)*c^(1/8)) + (
atan((c^(1/8)*x^(1/2)*1i)/(-a)^(1/8))*105i)/(256*(-a)^(23/8)*c^(1/8)) - (2
^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*(1/2 - 1i/2))/(-a)^(1/8))*(105/512 +
105i/512))/((-a)^(23/8)*c^(1/8)) - (2^(1/2)*atan((2^(1/2)*c^(1/8)*x^(1/2)*
(1/2 + 1i/2))/(-a)^(1/8))*(105/512 - 105i/512))/((-a)^(23/8)*c^(1/8))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1182, normalized size of antiderivative = 4.38

$$\int \frac{1}{\sqrt{x}\,(a+cx^4)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x^(1/2)/(c*x^4+a)^3,x)
```

output

```
( - 210*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(
- sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2))
)*a**2 - 420*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*s
qrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt(sqrt(2)
+ 2)))*a*c*x**4 - 210*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((c**(1/8)*a
**(1/8)*sqrt( - sqrt(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt
(sqrt(2) + 2)))*c**2*x**8 + 210*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*atan((
c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**
(1/8)*sqrt(sqrt(2) + 2)))*a**2 + 420*c**(7/8)*a**(1/8)*sqrt(sqrt(2) + 2)*a
tan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))/(c**(1/8
)*a**(1/8)*sqrt(sqrt(2) + 2)))*a*c*x**4 + 210*c**(7/8)*a**(1/8)*sqrt(sqrt(
2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2) + 2*sqrt(x)*c**(1/4))
/(c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2)))*c**2*x**8 - 210*c**(7/8)*a**(1/8)*
sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2*sqrt(x)
*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a**2 - 420*c**(7/8)*a
**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt(2) + 2) - 2
*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*a*c*x**4 - 21
0*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a**(1/8)*sqrt(sqrt
(2) + 2) - 2*sqrt(x)*c**(1/4))/(c**(1/8)*a**(1/8)*sqrt( - sqrt(2) + 2)))*c
**2*x**8 + 210*c**(7/8)*a**(1/8)*sqrt( - sqrt(2) + 2)*atan((c**(1/8)*a*...
```

3.159 $\int (cx)^m \left(a+bx^4\right)^3 dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int (cx)^m \left(a+bx^4\right)^3 dx = \frac{a^3(cx)^{1+m}}{c(1+m)} + \frac{3a^2b(cx)^{5+m}}{c^5(5+m)} + \frac{3ab^2(cx)^{9+m}}{c^9(9+m)} + \frac{b^3(cx)^{13+m}}{c^{13}(13+m)}$$

output
```
a^3*(c*x)^(1+m)/c/(1+m)+3*a^2*b*(c*x)^(5+m)/c^5/(5+m)+3*a*b^2*(c*x)^(9+m)/
c^9/(9+m)+b^3*(c*x)^(13+m)/c^13/(13+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int (cx)^m \left(a+bx^4\right)^3 dx = x(cx)^m \left(\frac{a^3}{1+m} + \frac{3a^2bx^4}{5+m} + \frac{3ab^2x^8}{9+m} + \frac{b^3x^{12}}{13+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^4)^3,x]
```

output
```
x*(c*x)^m*(a^3/(1 + m) + (3*a^2*b*x^4)/(5 + m) + (3*a*b^2*x^8)/(9 + m) + (
b^3*x^12)/(13 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)^3 (cx)^m \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^3(cx)^m+\frac{3a^2b(cx)^{m+4}}{c^4}+\frac{3ab^2(cx)^{m+8}}{c^8}+\frac{b^3(cx)^{m+12}}{c^{12}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(cx)^{m+1}}{c(m+1)}+\frac{3a^2b(cx)^{m+5}}{c^5(m+5)}+\frac{3ab^2(cx)^{m+9}}{c^9(m+9)}+\frac{b^3(cx)^{m+13}}{c^{13}(m+13)}$$

input
```
Int[(c*x)^m*(a + b*x^4)^3,x]
```

output
```
(a^3*(c*x)^(1 + m))/(c*(1 + m)) + (3*a^2*b*(c*x)^(5 + m))/(c^5*(5 + m)) +
(3*a*b^2*(c*x)^(9 + m))/(c^9*(9 + m)) + (b^3*(c*x)^(13 + m))/(c^13*(13 + m
))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(81) = 162$.

Time = 0.58 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

method	result
gosper	$\frac{x(b^3m^3x^{12}+15b^3m^2x^{12}+59m\,x^{12}b^3+45b^3x^{12}+3a\,b^2m^3x^8+57a\,b^2m^2x^8+249m\,x^8a\,b^2+195a\,b^2x^8+3a^2b\,m^3x^4+69a^2b\,m^2x}{(13+m)(9+m)(5+m)(1+m)}$
risch	$\frac{x(b^3m^3x^{12}+15b^3m^2x^{12}+59m\,x^{12}b^3+45b^3x^{12}+3a\,b^2m^3x^8+57a\,b^2m^2x^8+249m\,x^8a\,b^2+195a\,b^2x^8+3a^2b\,m^3x^4+69a^2b\,m^2x}{(13+m)(9+m)(5+m)(1+m)}$
orering	$\frac{x(b^3m^3x^{12}+15b^3m^2x^{12}+59m\,x^{12}b^3+45b^3x^{12}+3a\,b^2m^3x^8+57a\,b^2m^2x^8+249m\,x^8a\,b^2+195a\,b^2x^8+3a^2b\,m^3x^4+69a^2b\,m^2x}{(13+m)(9+m)(5+m)(1+m)}$
parallelrisch	$x^{13}(cx)^m b^3m^3+15x^{13}(cx)^m b^3m^2+59x^{13}(cx)^m b^3m+45x^{13}(cx)^m b^3+3x^9(cx)^m a\,b^2m^3+57x^9(cx)^m a\,b^2m^2+249x^9(cx)^m a\,b$

input
```
int((c*x)^m*(b*x^4+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x*(b^3*m^3*x^12+15*b^3*m^2*x^12+59*b^3*m*x^12+45*b^3*x^12+3*a*b^2*m^3*x^8+
57*a*b^2*m^2*x^8+249*a*b^2*m*x^8+195*a*b^2*x^8+3*a^2*b*m^3*x^4+69*a^2*b*m^
2*x^4+417*a^2*b*m*x^4+351*a^2*b*x^4+a^3*m^3+27*a^3*m^2+227*a^3*m+585*a^3)*
(c*x)^m/(13+m)/(9+m)/(5+m)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.96

$$\int (cx)^m \left(a+bx^4\right)^3 \, dx = \frac{((b^3m^3+15\,b^3m^2+59\,b^3m+45\,b^3)x^{13}+3\,(ab^2m^3+19\,ab^2m^2+83\,ab^2m+65\,ab^2)x^9+3\,(a^2bm^3+23\,a}{m^4+28\,m^3+254\,m^2+812\,m}$$

input
```
integrate((c*x)^m*(b*x^4+a)^3,x, algorithm="fricas")
```

output
```
((b^3*m^3 + 15*b^3*m^2 + 59*b^3*m + 45*b^3)*x^13 + 3*(a*b^2*m^3 + 19*a*b^2
*m^2 + 83*a*b^2*m + 65*a*b^2)*x^9 + 3*(a^2*b*m^3 + 23*a^2*b*m^2 + 139*a^2*
b*m + 117*a^2*b)*x^5 + (a^3*m^3 + 27*a^3*m^2 + 227*a^3*m + 585*a^3)*x)*(c*
x)^m/(m^4 + 28*m^3 + 254*m^2 + 812*m + 585)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(71) = 142$.

Time = 0.75 (sec) , antiderivative size = 722, normalized size of antiderivative = 8.91

$$\int (cx)^m \left(a+bx^4\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((c*x)**m*(b*x**4+a)**3,x)
```

output
```
Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(8*x**8) - 3*a*b**2/(4*x**4) + b**
3*log(x))/c**13, Eq(m, -13)), ((-a**3/(8*x**8) - 3*a**2*b/(4*x**4) + 3*a*b
**2*log(x) + b**3*x**4/4)/c**9, Eq(m, -9)), ((-a**3/(4*x**4) + 3*a**2*b*lo
g(x) + 3*a*b**2*x**4/4 + b**3*x**8/8)/c**5, Eq(m, -5)), ((a**3*log(x) + 3*
a**2*b*x**4/4 + 3*a*b**2*x**8/8 + b**3*x**12/12)/c, Eq(m, -1)), (a**3*m**3
*x*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 27*a**3*m**2*x*(c*
x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 227*a**3*m*x*(c*x)**m/(m
**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 585*a**3*x*(c*x)**m/(m**4 + 28*m
**3 + 254*m**2 + 812*m + 585) + 3*a**2*b*m**3*x**5*(c*x)**m/(m**4 + 28*m**
3 + 254*m**2 + 812*m + 585) + 69*a**2*b*m**2*x**5*(c*x)**m/(m**4 + 28*m**3
 + 254*m**2 + 812*m + 585) + 417*a**2*b*m*x**5*(c*x)**m/(m**4 + 28*m**3 +
254*m**2 + 812*m + 585) + 351*a**2*b*x**5*(c*x)**m/(m**4 + 28*m**3 + 254*m
**2 + 812*m + 585) + 3*a*b**2*m**3*x**9*(c*x)**m/(m**4 + 28*m**3 + 254*m**
2 + 812*m + 585) + 57*a*b**2*m**2*x**9*(c*x)**m/(m**4 + 28*m**3 + 254*m**2
 + 812*m + 585) + 249*a*b**2*m*x**9*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 +
812*m + 585) + 195*a*b**2*x**9*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m
 + 585) + b**3*m**3*x**13*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 58
5) + 15*b**3*m**2*x**13*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585)
 + 59*b**3*m*x**13*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585) + 45
*b**3*x**13*(c*x)**m/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int (cx)^m \left(a+bx^4\right)^3 dx = \frac{b^3c^mx^{13}x^m}{m+13} + \frac{3\,ab^2c^mx^9x^m}{m+9} + \frac{3\,a^2bc^mx^5x^m}{m+5} + \frac{(cx)^{m+1}\,a^3}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^4+a)^3,x, algorithm="maxima")
```

output
```
b^3*c^m*x^13*x^m/(m + 13) + 3*a*b^2*c^m*x^9*x^m/(m + 9) + 3*a^2*b*c^m*x^5*
x^m/(m + 5) + (c*x)^(m + 1)*a^3/(c*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(81) = 162.

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.16

$$\int (cx)^m \left(a+bx^4\right)^3 dx$$
$$= \frac{(cx)^m\,b^3m^3x^{13} + 15\,(cx)^m\,b^3m^2x^{13} + 59\,(cx)^m\,b^3mx^{13} + 45\,(cx)^m\,b^3x^{13} + 3\,(cx)^m\,ab^2m^3x^9 + 57\,(cx)^m\,ab}{}$$

input
```
integrate((c*x)^m*(b*x^4+a)^3,x, algorithm="giac")
```

output
```
((c*x)^m*b^3*m^3*x^13 + 15*(c*x)^m*b^3*m^2*x^13 + 59*(c*x)^m*b^3*m*x^13 +
45*(c*x)^m*b^3*x^13 + 3*(c*x)^m*a*b^2*m^3*x^9 + 57*(c*x)^m*a*b^2*m^2*x^9 +
 249*(c*x)^m*a*b^2*m*x^9 + 195*(c*x)^m*a*b^2*x^9 + 3*(c*x)^m*a^2*b*m^3*x^5
 + 69*(c*x)^m*a^2*b*m^2*x^5 + 417*(c*x)^m*a^2*b*m*x^5 + 351*(c*x)^m*a^2*b*
x^5 + (c*x)^m*a^3*m^3*x + 27*(c*x)^m*a^3*m^2*x + 227*(c*x)^m*a^3*m*x + 585
*(c*x)^m*a^3*x)/(m^4 + 28*m^3 + 254*m^2 + 812*m + 585)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.09

$$\int (cx)^m \left(a+bx^4\right)^3 \, dx = (c\,x)^m \left(\frac{a^3\,x\,\left(m^3+27\,m^2+227\,m+585\right)}{m^4+28\,m^3+254\,m^2+812\,m+585} + \frac{b^3\,x^{13}\,\left(m^3+15\,m^2+59\,m+45\right)}{m^4+28\,m^3+254\,m^2+812\,m+585} + \frac{3\,a\,b^2\,x^9\,\left(m^3+19\,m^2+83\,m+65\right)}{m^4+28\,m^3+254\,m^2+812\,m+585} + \frac{3\,a^2\,b\,x^5\,\left(m^3+23\,m^2+139\,m+117\right)}{m^4+28\,m^3+254\,m^2+812\,m+585}\right)$$

input
```
int((c*x)^m*(a + b*x^4)^3,x)
```

output
```
(c*x)^m*((a^3*x*(227*m + 27*m^2 + m^3 + 585))/(812*m + 254*m^2 + 28*m^3 +
m^4 + 585) + (b^3*x^13*(59*m + 15*m^2 + m^3 + 45))/(812*m + 254*m^2 + 28*m
^3 + m^4 + 585) + (3*a*b^2*x^9*(83*m + 19*m^2 + m^3 + 65))/(812*m + 254*m^
2 + 28*m^3 + m^4 + 585) + (3*a^2*b*x^5*(139*m + 23*m^2 + m^3 + 117))/(812*
m + 254*m^2 + 28*m^3 + m^4 + 585))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.21

$$\int (cx)^m \left(a+bx^4\right)^3 \, dx = \frac{x^m c^m x(b^3m^3x^{12}+15b^3m^2x^{12}+59b^3m\,x^{12}+45b^3x^{12}+3a\,b^2m^3x^8+57a\,b^2m^2x^8+249a\,b^2m\,x^8+195a\,b}{m^4+28m^3+254m^2}$$

input
```
int((c*x)^m*(b*x^4+a)^3,x)
```

output
```
(x**m*c**m*x*(a**3*m**3 + 27*a**3*m**2 + 227*a**3*m + 585*a**3 + 3*a**2*b*
m**3*x**4 + 69*a**2*b*m**2*x**4 + 417*a**2*b*m*x**4 + 351*a**2*b*x**4 + 3*
a*b**2*m**3*x**8 + 57*a*b**2*m**2*x**8 + 249*a*b**2*m*x**8 + 195*a*b**2*x*
*8 + b**3*m**3*x**12 + 15*b**3*m**2*x**12 + 59*b**3*m*x**12 + 45*b**3*x**1
2))/(m**4 + 28*m**3 + 254*m**2 + 812*m + 585)
```

3.160 $\int (cx)^m \left(a+bx^4\right)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int (cx)^m \left(a+bx^4\right)^2 \, dx=\frac{a^2(cx)^{1+m}}{c(1+m)}+\frac{2ab(cx)^{5+m}}{c^5(5+m)}+\frac{b^2(cx)^{9+m}}{c^9(9+m)}$$

output
```
a^2*(c*x)^(1+m)/c/(1+m)+2*a*b*(c*x)^(5+m)/c^5/(5+m)+b^2*(c*x)^(9+m)/c^9/(9
+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int (cx)^m \left(a+bx^4\right)^2 \, dx=x(cx)^m \left(\frac{a^2}{1+m}+\frac{2abx^4}{5+m}+\frac{b^2x^8}{9+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^4)^2,x]
```

output
```
x*(c*x)^m*(a^2/(1 + m) + (2*a*b*x^4)/(5 + m) + (b^2*x^8)/(9 + m))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^2 (cx)^m \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a^2 (cx)^m+\frac{2ab(cx)^{m+4}}{c^4}+\frac{b^2 (cx)^{m+8}}{c^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2 (cx)^{m+1}}{c(m+1)}+\frac{2ab(cx)^{m+5}}{c^5 (m+5)}+\frac{b^2 (cx)^{m+9}}{c^9 (m+9)}$$

input
```
Int[(c*x)^m*(a + b*x^4)^2,x]
```

output
```
(a^2*(c*x)^(1 + m))/(c*(1 + m)) + (2*a*b*(c*x)^(5 + m))/(c^5*(5 + m)) + (b
^2*(c*x)^(9 + m))/(c^9*(9 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

method	result
gosper	$\frac{x(b^{2}m^{2}x^{8}+6m\,x^{8}b^{2}+5b^{2}x^{8}+2ab\,m^{2}x^{4}+20m\,x^{4}ab+18ab\,x^{4}+a^{2}m^{2}+14m\,a^{2}+45a^{2})(cx)^{m}}{(9+m)(5+m)(1+m)}$
risch	$\frac{x(b^{2}m^{2}x^{8}+6m\,x^{8}b^{2}+5b^{2}x^{8}+2ab\,m^{2}x^{4}+20m\,x^{4}ab+18ab\,x^{4}+a^{2}m^{2}+14m\,a^{2}+45a^{2})(cx)^{m}}{(9+m)(5+m)(1+m)}$
orering	$\frac{x(b^{2}m^{2}x^{8}+6m\,x^{8}b^{2}+5b^{2}x^{8}+2ab\,m^{2}x^{4}+20m\,x^{4}ab+18ab\,x^{4}+a^{2}m^{2}+14m\,a^{2}+45a^{2})(cx)^{m}}{(9+m)(5+m)(1+m)}$
parallelrisch	$\frac{x^{9}(cx)^{m}b^{2}m^{2}+6x^{9}(cx)^{m}b^{2}m+5x^{9}(cx)^{m}b^{2}+2x^{5}(cx)^{m}ab\,m^{2}+20x^{5}(cx)^{m}abm+18x^{5}(cx)^{m}ab+x(cx)^{m}a^{2}m^{2}+14x(cx)^{m}a^{2}m}{(9+m)(5+m)(1+m)}$

input
```
int((c*x)^m*(b*x^4+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x*(b^2*m^2*x^8+6*b^2*m*x^8+5*b^2*x^8+2*a*b*m^2*x^4+20*a*b*m*x^4+18*a*b*x^4
+a^2*m^2+14*a^2*m+45*a^2)*(c*x)^m/(9+m)/(5+m)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int (cx)^m \left(a+bx^4\right)^2 \, dx = \frac{((b^2m^2+6\,b^2m+5\,b^2)x^9+2\,(abm^2+10\,abm+9\,ab)x^5+(a^2m^2+14\,a^2m+45\,a^2)x)(cx)^m}{m^3+15\,m^2+59\,m+45}$$

input
```
integrate((c*x)^m*(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 6*b^2*m + 5*b^2)*x^9 + 2*(a*b*m^2 + 10*a*b*m + 9*a*b)*x^5 + (a
^2*m^2 + 14*a^2*m + 45*a^2)*x)*(c*x)^m/(m^3 + 15*m^2 + 59*m + 45)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(49) = 98$.

Time = 0.57 (sec) , antiderivative size = 333, normalized size of antiderivative = 5.74

$$\int (cx)^m \left(a+bx^4\right)^2 \, dx = \begin{cases} \frac{-\frac{a^2}{8x^8}-\frac{ab}{2x^4}+b^2\log(x)}{c^9} \\ \frac{-\frac{a^2}{4x^4}+2ab\log(x)+\frac{b^2x^4}{4}}{c^5} \\ \frac{a^2\log(x)+\frac{abx^4}{2}+\frac{b^2x^8}{8}}{c} \\ \frac{a^2m^2x(cx)^m}{m^3+15m^2+59m+45} + \frac{14a^2mx(cx)^m}{m^3+15m^2+59m+45} + \frac{45a^2x(cx)^m}{m^3+15m^2+59m+45} + \frac{2abm^2x^5(cx)^m}{m^3+15m^2+59m+45} + \frac{20abmx^5(cx)^m}{m^3+15m^2+59m+45} + \frac{18abx^5}{m^3+15m^2} \end{cases}$$

input
```
integrate((c*x)**m*(b*x**4+a)**2,x)
```

output
```
Piecewise(((-a**2/(8*x**8) - a*b/(2*x**4) + b**2*log(x))/c**9, Eq(m, -9)),
 ((-a**2/(4*x**4) + 2*a*b*log(x) + b**2*x**4/4)/c**5, Eq(m, -5)), ((a**2*l
og(x) + a*b*x**4/2 + b**2*x**8/8)/c, Eq(m, -1)), (a**2*m**2*x*(c*x)**m/(m*
*3 + 15*m**2 + 59*m + 45) + 14*a**2*m*x*(c*x)**m/(m**3 + 15*m**2 + 59*m +
45) + 45*a**2*x*(c*x)**m/(m**3 + 15*m**2 + 59*m + 45) + 2*a*b*m**2*x**5*(c
*x)**m/(m**3 + 15*m**2 + 59*m + 45) + 20*a*b*m*x**5*(c*x)**m/(m**3 + 15*m*
*2 + 59*m + 45) + 18*a*b*x**5*(c*x)**m/(m**3 + 15*m**2 + 59*m + 45) + b**2
*m**2*x**9*(c*x)**m/(m**3 + 15*m**2 + 59*m + 45) + 6*b**2*m*x**9*(c*x)**m/
(m**3 + 15*m**2 + 59*m + 45) + 5*b**2*x**9*(c*x)**m/(m**3 + 15*m**2 + 59*m
 + 45), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(a+bx^4\right)^2 \, dx = \frac{b^2c^mx^9x^m}{m+9} + \frac{2\,abc^mx^5x^m}{m+5} + \frac{(cx)^{m+1}a^2}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
b^2*c^m*x^9*x^m/(m + 9) + 2*a*b*c^m*x^5*x^m/(m + 5) + (c*x)^(m + 1)*a^2/(c
*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.33

$$\int (cx)^m \left(a+bx^4\right)^2 dx$$
$$= \frac{(cx)^m b^2m^2x^9 + 6\,(cx)^m b^2mx^9 + 5\,(cx)^m b^2x^9 + 2\,(cx)^m abm^2x^5 + 20\,(cx)^m abmx^5 + 18\,(cx)^m abx^5 + (c}{m^3+15\,m^2+59\,m+45}$$

input
```
integrate((c*x)^m*(b*x^4+a)^2,x, algorithm="giac")
```

output
```
((c*x)^m*b^2*m^2*x^9 + 6*(c*x)^m*b^2*m*x^9 + 5*(c*x)^m*b^2*x^9 + 2*(c*x)^m
*a*b*m^2*x^5 + 20*(c*x)^m*a*b*m*x^5 + 18*(c*x)^m*a*b*x^5 + (c*x)^m*a^2*m^2
*x + 14*(c*x)^m*a^2*m*x + 45*(c*x)^m*a^2*x)/(m^3 + 15*m^2 + 59*m + 45)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (cx)^m \left(a+bx^4\right)^2 dx = (c\,x)^m \left(\frac{a^2\,x\,(m^2+14\,m+45)}{m^3+15\,m^2+59\,m+45} + \frac{b^2\,x^9\,(m^2+6\,m+5)}{m^3+15\,m^2+59\,m+45} + \frac{2\,a\,b\,x^5\,(m^2+10\,m+9)}{m^3+15\,m^2+59\,m+45}\right)$$

input
```
int((c*x)^m*(a + b*x^4)^2,x)
```

output
```
(c*x)^m*((a^2*x*(14*m + m^2 + 45))/(59*m + 15*m^2 + m^3 + 45) + (b^2*x^9*(
6*m + m^2 + 5))/(59*m + 15*m^2 + m^3 + 45) + (2*a*b*x^5*(10*m + m^2 + 9))/
(59*m + 15*m^2 + m^3 + 45))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

$$\int (cx)^m \left(a+bx^4\right)^2 \, dx = \frac{x^m c^m x(b^2m^2x^8+6b^2m\,x^8+5b^2x^8+2ab\,m^2x^4+20abm\,x^4+18ab\,x^4+a^2m^2+14a^2m+45a^2)}{m^3+15m^2+59m+45}$$

input
```
int((c*x)^m*(b*x^4+a)^2,x)
```

output
```
(x**m*c**m*x*(a**2*m**2 + 14*a**2*m + 45*a**2 + 2*a*b*m**2*x**4 + 20*a*b*m
*x**4 + 18*a*b*x**4 + b**2*m**2*x**8 + 6*b**2*m*x**8 + 5*b**2*x**8))/(m**3
 + 15*m**2 + 59*m + 45)
```

3.161 $\int (cx)^m (a+bx^4) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int (cx)^m (a+bx^4) \, dx = \frac{a(cx)^{1+m}}{c(1+m)} + \frac{b(cx)^{5+m}}{c^5(5+m)}$$

output
```
a*(c*x)^(1+m)/c/(1+m)+b*(c*x)^(5+m)/c^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int (cx)^m (a+bx^4) \, dx = x(cx)^m \left(\frac{a}{1+m} + \frac{bx^4}{5+m}\right)$$

input
```
Integrate[(c*x)^m*(a + b*x^4),x]
```

output
```
x*(c*x)^m*(a/(1 + m) + (b*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^4)(cx)^m\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(a(cx)^m+\frac{b(cx)^{m+4}}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(cx)^{m+1}}{c(m+1)}+\frac{b(cx)^{m+5}}{c^5(m+5)}$$

input
```
Int[(c*x)^m*(a + b*x^4),x]
```

output
```
(a*(c*x)^(1 + m))/(c*(1 + m)) + (b*(c*x)^(5 + m))/(c^5*(5 + m))
```

Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{a x \,{\mathrm e}^{m \ln \left(c x \right)}}{1+m}+\frac{b \,x^{5} {\mathrm e}^{m \ln \left(c x \right)}}{5+m}$	34
gosper	$\frac{x \left(b m \,x^{4}+b \,x^{4}+a m +5 a \right) \left(c x \right)^{m}}{\left(5+m \right) \left(1+m \right)}$	36
risch	$\frac{x \left(b m \,x^{4}+b \,x^{4}+a m +5 a \right) \left(c x \right)^{m}}{\left(5+m \right) \left(1+m \right)}$	36
orering	$\frac{x \left(b m \,x^{4}+b \,x^{4}+a m +5 a \right) \left(c x \right)^{m}}{\left(5+m \right) \left(1+m \right)}$	36
parallelrisch	$\frac{x^{5} \left(c x \right)^{m} b m +x^{5} \left(c x \right)^{m} b +x \left(c x \right)^{m} a m +5 x \left(c x \right)^{m} a}{\left(5+m \right) \left(1+m \right)}$	52

input
```
int((c*x)^m*(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*exp(m*ln(c*x))+b/(5+m)*x^5*exp(m*ln(c*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (cx)^m \left(a+bx^4\right) \, dx=\frac{((bm+b)x^5+(am+5\,a)x)(cx)^m}{m^2+6\,m+5}$$

input
```
integrate((c*x)^m*(b*x^4+a),x, algorithm="fricas")
```

output
```
((b*m + b)*x^5 + (a*m + 5*a)*x)*(c*x)^m/(m^2 + 6*m + 5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(27) = 54$.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int (cx)^m \left(a+bx^4\right)\,dx = \begin{cases} \frac{-\frac{a}{4x^4}+b\log(x)}{c^5} & \text{for}\ m=-5 \\ \frac{a\log(x)+\frac{bx^4}{4}}{c} & \text{for}\ m=-1 \\ \frac{amx(cx)^m}{m^2+6m+5}+\frac{5ax(cx)^m}{m^2+6m+5}+\frac{bmx^5(cx)^m}{m^2+6m+5}+\frac{bx^5(cx)^m}{m^2+6m+5} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x)**m*(b*x**4+a),x)
```

output
```
Piecewise(((-a/(4*x**4) + b*log(x))/c**5, Eq(m, -5)), ((a*log(x) + b*x**4/
4)/c, Eq(m, -1)), (a*m*x*(c*x)**m/(m**2 + 6*m + 5) + 5*a*x*(c*x)**m/(m**2
+ 6*m + 5) + b*m*x**5*(c*x)**m/(m**2 + 6*m + 5) + b*x**5*(c*x)**m/(m**2 +
6*m + 5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(a+bx^4\right)\,dx = \frac{bc^m x^5 x^m}{m+5} + \frac{(cx)^{m+1}a}{c(m+1)}$$

input
```
integrate((c*x)^m*(b*x^4+a),x, algorithm="maxima")
```

output
```
b*c^m*x^5*x^m/(m + 5) + (c*x)^(m + 1)*a/(c*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int (cx)^m \left(a+bx^4\right) dx = \frac{(cx)^m bmx^5 + (cx)^m bx^5 + (cx)^m amx + 5\,(cx)^m ax}{m^2+6\,m+5}$$

input
```
integrate((c*x)^m*(b*x^4+a),x, algorithm="giac")
```

output
```
((c*x)^m*b*m*x^5 + (c*x)^m*b*x^5 + (c*x)^m*a*m*x + 5*(c*x)^m*a*x)/(m^2 + 6
*m + 5)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (cx)^m \left(a+bx^4\right) dx = \frac{x\,(c\,x)^m\,\left(5\,a+a\,m+b\,x^4+b\,m\,x^4\right)}{m^2+6\,m+5}$$

input
```
int((c*x)^m*(a + b*x^4),x)
```

output
```
(x*(c*x)^m*(5*a + a*m + b*x^4 + b*m*x^4))/(6*m + m^2 + 5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int (cx)^m \left(a+bx^4\right) dx = \frac{x^m c^m x(bm\,x^4+b\,x^4+am+5a)}{m^2+6m+5}$$

input
```
int((c*x)^m*(b*x^4+a),x)
```

output
```
(x**m*c**m*x*(a*m + 5*a + b*m*x**4 + b*x**4))/(m**2 + 6*m + 5)
```

3.162 $\int \frac{(cx)^m}{a+bx^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(cx)^m}{a+bx^4} dx = \frac{(cx)^{1+m} \text{Hypergeometric2F1}\left(1, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{bx^4}{a}\right)}{ac(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/a/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{a+bx^4} dx = \frac{x(cx)^m \text{Hypergeometric2F1}\left(1, \frac{1+m}{4}, 1 + \frac{1+m}{4}, -\frac{bx^4}{a}\right)}{a(1+m)}$$

input
```
Integrate[(c*x)^m/(a + b*x^4),x]
```

output
```
(x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/4, 1 + (1 + m)/4, -((b*x^4)/a)])/(
a*(1 + m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{a+bx^4}\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{(cx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-\frac{bx^4}{a}\right)}{ac(m+1)}$$

input
```
Int[(c*x)^m/(a + b*x^4),x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -((b*x^4)/a)])/(
a*c*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^m}{b\,x^4+a}dx$$

input
```
int((c*x)^m/(b*x^4+a),x)
```

output
```
int((c*x)^m/(b*x^4+a),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{a+bx^4}\,dx = \int \frac{(cx)^m}{bx^4+a}\,dx$$

input
```
integrate((c*x)^m/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral((c*x)^m/(b*x^4 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{(cx)^m}{a+bx^4}\,dx = \frac{c^m m x^{m+1}\Phi\left(\frac{bx^4e^{i\pi}}{a},1,\frac{m}{4}+\frac{1}{4}\right)\Gamma\left(\frac{m}{4}+\frac{1}{4}\right)}{16a\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{c^m x^{m+1}\Phi\left(\frac{bx^4e^{i\pi}}{a},1,\frac{m}{4}+\frac{1}{4}\right)\Gamma\left(\frac{m}{4}+\frac{1}{4}\right)}{16a\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

input
```
integrate((c*x)**m/(b*x**4+a),x)
```

output
```
c**m*m*x**(m + 1)*lerchphi(b*x**4*exp_polar(I*pi)/a, 1, m/4 + 1/4)*gamma(m
/4 + 1/4)/(16*a*gamma(m/4 + 5/4)) + c**m*x**(m + 1)*lerchphi(b*x**4*exp_po
lar(I*pi)/a, 1, m/4 + 1/4)*gamma(m/4 + 1/4)/(16*a*gamma(m/4 + 5/4))
```

Maxima [F]

$$\int \frac{(cx)^m}{a+bx^4}\,dx=\int \frac{(cx)^m}{bx^4+a}\,dx$$

input
```
integrate((c*x)^m/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{(cx)^m}{a+bx^4}\,dx=\int \frac{(cx)^m}{bx^4+a}\,dx$$

input
```
integrate((c*x)^m/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{a+bx^4}\,dx=\int \frac{(c\,x)^m}{b\,x^4+a}\,dx$$

input
```
int((c*x)^m/(a + b*x^4),x)
```

output
```
int((c*x)^m/(a + b*x^4), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{a+bx^4}\,dx = c^m\left(\int \frac{x^m}{b\,x^4+a}dx\right)$$

input
```
int((c*x)^m/(b*x^4+a),x)
```

output
```
c**m*int(x**m/(a + b*x**4),x)
```

3.163 $\int \frac{(cx)^m}{(a+bx^4)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(cx)^m}{(a+bx^4)^2} dx = \frac{(cx)^{1+m} \text{Hypergeometric2F1}\left(2, \frac{1+m}{4}, \frac{5+m}{4}, -\frac{bx^4}{a}\right)}{a^2c(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],-b*x^4/a)/a^2/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{(a+bx^4)^2} dx = \frac{x(cx)^m \text{Hypergeometric2F1}\left(2, \frac{1+m}{4}, 1+\frac{1+m}{4}, -\frac{bx^4}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[(c*x)^m/(a + b*x^4)^2,x]
```

output
```
(x*(c*x)^m*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -((b*x^4)/a)])/(
a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx^4)^2} \, dx$$

$$\Big\downarrow \; 888$$

$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(2, \frac{m+1}{4}, \frac{m+5}{4}, -\frac{bx^4}{a}\right)}{a^2c(m+1)}$$

input
```
Int[(c*x)^m/(a + b*x^4)^2,x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -((b*x^4)/a)])/(
a^2*c*(1 + m))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^4+a)^2}dx$$

input
```
int((c*x)^m/(b*x^4+a)^2,x)
```

output
```
int((c*x)^m/(b*x^4+a)^2,x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx^4)^2}\,dx = \int \frac{(cx)^m}{(bx^4+a)^2}\,dx$$

input
```
integrate((c*x)^m/(b*x^4+a)^2,x, algorithm="fricas")
```

output
```
integral((c*x)^m/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.63 (sec) , antiderivative size = 551, normalized size of antiderivative = 12.52

$$\int \frac{(cx)^m}{(a+bx^4)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x)**m/(b*x**4+a)**2,x)
```

output
```
-a*c**m*m**2*x**(m + 1)*lerchphi(b*x**4*exp_polar(I*pi)/a, 1, m/4 + 1/4)*g
amma(m/4 + 1/4)/(64*a**3*gamma(m/4 + 5/4) + 64*a**2*b*x**4*gamma(m/4 + 5/4
)) + 2*a*c**m*m*x**(m + 1)*lerchphi(b*x**4*exp_polar(I*pi)/a, 1, m/4 + 1/4
)*gamma(m/4 + 1/4)/(64*a**3*gamma(m/4 + 5/4) + 64*a**2*b*x**4*gamma(m/4 +
5/4)) + 4*a*c**m*m*x**(m + 1)*gamma(m/4 + 1/4)/(64*a**3*gamma(m/4 + 5/4) +
 64*a**2*b*x**4*gamma(m/4 + 5/4)) + 3*a*c**m*x**(m + 1)*lerchphi(b*x**4*ex
p_polar(I*pi)/a, 1, m/4 + 1/4)*gamma(m/4 + 1/4)/(64*a**3*gamma(m/4 + 5/4)
+ 64*a**2*b*x**4*gamma(m/4 + 5/4)) + 4*a*c**m*x**(m + 1)*gamma(m/4 + 1/4)/
(64*a**3*gamma(m/4 + 5/4) + 64*a**2*b*x**4*gamma(m/4 + 5/4)) - b*c**m*m**2
*x**4*x**(m + 1)*lerchphi(b*x**4*exp_polar(I*pi)/a, 1, m/4 + 1/4)*gamma(m/
4 + 1/4)/(64*a**3*gamma(m/4 + 5/4) + 64*a**2*b*x**4*gamma(m/4 + 5/4)) + 2*
b*c**m*m*x**4*x**(m + 1)*lerchphi(b*x**4*exp_polar(I*pi)/a, 1, m/4 + 1/4)*
gamma(m/4 + 1/4)/(64*a**3*gamma(m/4 + 5/4) + 64*a**2*b*x**4*gamma(m/4 + 5/
4)) + 3*b*c**m*x**4*x**(m + 1)*lerchphi(b*x**4*exp_polar(I*pi)/a, 1, m/4 +
 1/4)*gamma(m/4 + 1/4)/(64*a**3*gamma(m/4 + 5/4) + 64*a**2*b*x**4*gamma(m/
4 + 5/4))
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx^4)^2}\,dx = \int \frac{(cx)^m}{(bx^4+a)^2}\,dx$$

input
```
integrate((c*x)^m/(b*x^4+a)^2,x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x^4 + a)^2, x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx^4)^2}\,dx = \int \frac{(cx)^m}{(bx^4+a)^2}\,dx$$

input
```
integrate((c*x)^m/(b*x^4+a)^2,x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x^4 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx^4)^2}\,dx=\int \frac{(c\,x)^m}{(b\,x^4+a)^2}\,dx$$

input
```
int((c*x)^m/(a + b*x^4)^2,x)
```

output
```
int((c*x)^m/(a + b*x^4)^2, x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx^4)^2}\,dx=c^m\left(\int \frac{x^m}{b^2x^8+2ab\,x^4+a^2}dx\right)$$

input
```
int((c*x)^m/(b*x^4+a)^2,x)
```

output
```
c**m*int(x**m/(a**2 + 2*a*b*x**4 + b**2*x**8),x)
```

3.164 $\int x^{11}\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int x^{11}\sqrt{a-bx^4}\,dx=-\frac{a^2(a-bx^4)^{3/2}}{6b^3}+\frac{a(a-bx^4)^{5/2}}{5b^3}-\frac{(a-bx^4)^{7/2}}{14b^3}$$

output
```
-1/6*a^2*(-b*x^4+a)^(3/2)/b^3+1/5*a*(-b*x^4+a)^(5/2)/b^3-1/14*(-b*x^4+a)^(
7/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int x^{11}\sqrt{a-bx^4}\,dx=\frac{\sqrt{a-bx^4}(-8a^3-4a^2bx^4-3ab^2x^8+15b^3x^{12})}{210b^3}$$

input
```
Integrate[x^11*Sqrt[a - b*x^4],x]
```

output
```
(Sqrt[a - b*x^4]*(-8*a^3 - 4*a^2*b*x^4 - 3*a*b^2*x^8 + 15*b^3*x^12))/(210*
b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^8\sqrt{a-bx^4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(a-bx^4\right)^{5/2}}{b^2}-\frac{2a\left(a-bx^4\right)^{3/2}}{b^2}+\frac{a^2\sqrt{a-bx^4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{2a^2\left(a-bx^4\right)^{3/2}}{3b^3}-\frac{2\left(a-bx^4\right)^{7/2}}{7b^3}+\frac{4a\left(a-bx^4\right)^{5/2}}{5b^3}\right)$$

input
```
Int[x^11*Sqrt[a - b*x^4],x]
```

output
```
((-2*a^2*(a - b*x^4)^(3/2))/(3*b^3) + (4*a*(a - b*x^4)^(5/2))/(5*b^3) - (2
*(a - b*x^4)^(7/2))/(7*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{8}+12 a b\,x^{4}+8 a^{2}\right)}{210 b^{3}}$	37
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{8}+12 a b\,x^{4}+8 a^{2}\right)}{210 b^{3}}$	37
elliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{8}+12 a b\,x^{4}+8 a^{2}\right)}{210 b^{3}}$	37
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{8}+12 a b\,x^{4}+8 a^{2}\right)}{210 b^{3}}$	37
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{8}+12 a b\,x^{4}+8 a^{2}\right)}{210 b^{3}}$	37
trager	$-\frac{\left(-15 b^{3} x^{12}+3 a\,b^{2} x^{8}+4 a^{2} b\,x^{4}+8 a^{3}\right) \sqrt{-b\,x^{4}+a}}{210 b^{3}}$	48
risch	$-\frac{\left(-15 b^{3} x^{12}+3 a\,b^{2} x^{8}+4 a^{2} b\,x^{4}+8 a^{3}\right) \sqrt{-b\,x^{4}+a}}{210 b^{3}}$	48

input
```
int(x^11*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/210*(-b*x^4+a)^(3/2)*(15*b^2*x^8+12*a*b*x^4+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int x^{11}\sqrt{a-bx^4}\,dx = \frac{(15\,b^3x^{12}-3\,ab^2x^8-4\,a^2bx^4-8\,a^3)\sqrt{-bx^4+a}}{210\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/210*(15*b^3*x^12 - 3*a*b^2*x^8 - 4*a^2*b*x^4 - 8*a^3)*sqrt(-b*x^4 + a)/b
^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int x^{11}\sqrt{a-bx^4}\,dx = \begin{cases} -\frac{4a^3\sqrt{a-bx^4}}{105b^3} - \frac{2a^2x^4\sqrt{a-bx^4}}{105b^2} - \frac{ax^8\sqrt{a-bx^4}}{70b} + \frac{x^{12}\sqrt{a-bx^4}}{14} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-4*a**3*sqrt(a - b*x**4)/(105*b**3) - 2*a**2*x**4*sqrt(a - b*x*
*4)/(105*b**2) - a*x**8*sqrt(a - b*x**4)/(70*b) + x**12*sqrt(a - b*x**4)/1
4, Ne(b, 0)), (sqrt(a)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int x^{11}\sqrt{a-bx^4}\,dx = -\frac{(-bx^4+a)^{\frac{7}{2}}}{14\,b^3} + \frac{(-bx^4+a)^{\frac{5}{2}}a}{5\,b^3} - \frac{(-bx^4+a)^{\frac{3}{2}}a^2}{6\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/14*(-b*x^4 + a)^(7/2)/b^3 + 1/5*(-b*x^4 + a)^(5/2)*a/b^3 - 1/6*(-b*x^4
+ a)^(3/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int x^{11}\sqrt{a-bx^4}\,dx = \frac{15\,(bx^4-a)^3\sqrt{-bx^4+a}+42\,(bx^4-a)^2\sqrt{-bx^4+a}a-35\,(-bx^4+a)^{\frac{3}{2}}a^2}{210\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/210*(15*(b*x^4 - a)^3*sqrt(-b*x^4 + a) + 42*(b*x^4 - a)^2*sqrt(-b*x^4 +
a)*a - 35*(-b*x^4 + a)^(3/2)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int x^{11}\sqrt{a-bx^4}\,dx = -\sqrt{a-b\,x^4}\left(\frac{4\,a^3}{105\,b^3}-\frac{x^{12}}{14}+\frac{a\,x^8}{70\,b}+\frac{2\,a^2\,x^4}{105\,b^2}\right)$$

input
```
int(x^11*(a - b*x^4)^(1/2),x)
```

output
```
-(a - b*x^4)^(1/2)*((4*a^3)/(105*b^3) - x^12/14 + (a*x^8)/(70*b) + (2*a^2*
x^4)/(105*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int x^{11}\sqrt{a-bx^4}\,dx=\frac{\sqrt{-b\,x^4+a}\,(15b^3x^{12}-3a\,b^2x^8-4a^2b\,x^4-8a^3)}{210b^3}$$

input
```
int(x^11*(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 8*a**3 - 4*a**2*b*x**4 - 3*a*b**2*x**8 + 15*b**3*x**
12))/(210*b**3)
```

3.165 $\int x^7\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int x^7\sqrt{a-bx^4}\,dx=-\frac{a(a-bx^4)^{3/2}}{6b^2}+\frac{(a-bx^4)^{5/2}}{10b^2}$$

output
```
-1/6*a*(-b*x^4+a)^(3/2)/b^2+1/10*(-b*x^4+a)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x^7\sqrt{a-bx^4}\,dx=\frac{\sqrt{a-bx^4}(-2a^2-abx^4+3b^2x^8)}{30b^2}$$

input
```
Integrate[x^7*Sqrt[a - b*x^4],x]
```

output
```
(Sqrt[a - b*x^4]*(-2*a^2 - a*b*x^4 + 3*b^2*x^8))/(30*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^4\sqrt{a-bx^4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a\sqrt{a-bx^4}}{b}-\frac{\left(a-bx^4\right)^{3/2}}{b}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2\left(a-bx^4\right)^{5/2}}{5b^2}-\frac{2a\left(a-bx^4\right)^{3/2}}{3b^2}\right)$$

input
```
Int[x^7*Sqrt[a - b*x^4],x]
```

output
```
((-2*a*(a - b*x^4)^(3/2))/(3*b^2) + (2*(a - b*x^4)^(5/2))/(5*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(3b\,x^{4}+2a\right)}{30b^{2}}$	26
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(3b\,x^{4}+2a\right)}{30b^{2}}$	26
elliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(3b\,x^{4}+2a\right)}{30b^{2}}$	26
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(3b\,x^{4}+2a\right)}{30b^{2}}$	26
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(3b\,x^{4}+2a\right)}{30b^{2}}$	26
trager	$-\frac{\left(-3b^{2}x^{8}+ab\,x^{4}+2a^{2}\right)\sqrt{-b\,x^{4}+a}}{30b^{2}}$	36
risch	$-\frac{\left(-3b^{2}x^{8}+ab\,x^{4}+2a^{2}\right)\sqrt{-b\,x^{4}+a}}{30b^{2}}$	36

input
```
int(x^7*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(-b*x^4+a)^(3/2)*(3*b*x^4+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int x^7\sqrt{a-bx^4}\,dx = \frac{(3\,b^2x^8-abx^4-2\,a^2)\sqrt{-bx^4+a}}{30\,b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(3*b^2*x^8 - a*b*x^4 - 2*a^2)*sqrt(-b*x^4 + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int x^7\sqrt{a-bx^4}\,dx = \begin{cases} -\frac{a^2\sqrt{a-bx^4}}{15b^2}-\frac{ax^4\sqrt{a-bx^4}}{30b}+\frac{x^8\sqrt{a-bx^4}}{10} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-a**2*sqrt(a - b*x**4)/(15*b**2) - a*x**4*sqrt(a - b*x**4)/(30*
b) + x**8*sqrt(a - b*x**4)/10, Ne(b, 0)), (sqrt(a)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^7\sqrt{a-bx^4}\,dx = \frac{(-bx^4+a)^{\frac{5}{2}}}{10\,b^2}-\frac{(-bx^4+a)^{\frac{3}{2}}a}{6\,b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/10*(-b*x^4 + a)^(5/2)/b^2 - 1/6*(-b*x^4 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int x^7\sqrt{a-bx^4}\,dx=\frac{3\,(bx^4-a)^2\sqrt{-bx^4+a}-5\,(-bx^4+a)^{\frac{3}{2}}a}{30\,b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/30*(3*(b*x^4 - a)^2*sqrt(-b*x^4 + a) - 5*(-b*x^4 + a)^(3/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x^7\sqrt{a-bx^4}\,dx=-\sqrt{a-b\,x^4}\,\left(\frac{a^2}{15\,b^2}-\frac{x^8}{10}+\frac{a\,x^4}{30\,b}\right)$$

input
```
int(x^7*(a - b*x^4)^(1/2),x)
```

output
```
-(a - b*x^4)^(1/2)*(a^2/(15*b^2) - x^8/10 + (a*x^4)/(30*b))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x^7\sqrt{a-bx^4}\,dx=\frac{\sqrt{-b\,x^4+a}\,(3b^2x^8-ab\,x^4-2a^2)}{30b^2}$$

input
```
int(x^7*(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 2*a**2 - a*b*x**4 + 3*b**2*x**8))/(30*b**2)
```

3.166 $\int x^3\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int x^3\sqrt{a-bx^4}\,dx=-\frac{\left(a-bx^4\right)^{3/2}}{6b}$$

output
```
-1/6*(-b*x^4+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{a-bx^4}\,dx=-\frac{\left(a-bx^4\right)^{3/2}}{6b}$$

input
```
Integrate[x^3*Sqrt[a - b*x^4],x]
```

output
```
-1/6*(a - b*x^4)^(3/2)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{\left(a-bx^4\right)^{3/2}}{6b}$$

input
```
Int[x^3*Sqrt[a - b*x^4],x]
```

output
```
-1/6*(a - b*x^4)^(3/2)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16
derivativedivides	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16
default	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16
trager	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16
risch	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16
pseudoelliptic	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16
orering	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}}{6b}$	16

input
```
int(x^3*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b*x^4+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int x^3\sqrt{a-bx^4}\,dx=\frac{(bx^4-a)\sqrt{-bx^4+a}}{6\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(b*x^4 - a)*sqrt(-b*x^4 + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int x^3\sqrt{a-bx^4}\,dx = \begin{cases} -\frac{a\sqrt{a-bx^4}}{6b} + \frac{x^4\sqrt{a-bx^4}}{6} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-a*sqrt(a - b*x**4)/(6*b) + x**4*sqrt(a - b*x**4)/6, Ne(b, 0)),
 (sqrt(a)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{a-bx^4}\,dx = -\frac{(-bx^4+a)^{\frac{3}{2}}}{6\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(-b*x^4 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{a-bx^4}\,dx = -\frac{(-bx^4+a)^{\frac{3}{2}}}{6\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(-b*x^4 + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{a-bx^4}\,dx=-\frac{\left(a-b\,x^4\right)^{3/2}}{6\,b}$$

input
```
int(x^3*(a - b*x^4)^(1/2),x)
```

output
```
-(a - b*x^4)^(3/2)/(6*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int x^3\sqrt{a-bx^4}\,dx=\frac{\sqrt{-b\,x^4+a}\,(b\,x^4-a)}{6b}$$

input
```
int(x^3*(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - a + b*x**4))/(6*b)
```

3.167 $\int \frac{\sqrt{a-bx^4}}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{\sqrt{a-bx^4}}{x} \, dx = \frac{1}{2}\sqrt{a-bx^4} - \frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)$$

output
```
1/2*(-b*x^4+a)^(1/2)-1/2*a^(1/2)*arctanh((-b*x^4+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^4}}{x} \, dx = \frac{1}{2}\sqrt{a-bx^4} - \frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a - b*x^4]/x,x]
```

output
```
Sqrt[a - b*x^4]/2 - (Sqrt[a]*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{\sqrt{a-bx^4}}{x^4} dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4} \left(a \int \frac{1}{x^4\sqrt{a-bx^4}} dx^4 + 2\sqrt{a-bx^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(2\sqrt{a-bx^4} - \frac{2a \int \frac{1}{\frac{a}{b}-\frac{x^8}{b}} d\sqrt{a-bx^4}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4} \left(2\sqrt{a-bx^4} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a - b*x^4]/x,x]
```

output
```
(2*Sqrt[a - b*x^4] - 2*Sqrt[a]*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{\sqrt{-b\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right)}{2}$	34
default	$\frac{\sqrt{-b\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2}$	43
elliptic	$\frac{\sqrt{-b\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2}$	43

input
```
int((-b*x^4+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x^4+a)^(1/2)-1/2*a^(1/2)*arctanh((-b*x^4+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a-bx^4}}{x}\,dx = \left[\frac{1}{4}\sqrt{a}\log\left(\frac{bx^4+2\sqrt{-bx^4+a}\sqrt{a}-2\,a}{x^4}\right)+\frac{1}{2}\sqrt{-bx^4+a},\ -\frac{1}{2}\sqrt{-a}\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)+\frac{1}{2}\sqrt{-bx^4+a}\right]$$

input
```
integrate((-b*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4) + 1/2*sqr
t(-b*x^4 + a), -1/2*sqrt(-a)*arctan(sqrt(-b*x^4 + a)*sqrt(-a)/(b*x^4 - a))
 + 1/2*sqrt(-b*x^4 + a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.24

$$\int \frac{\sqrt{a-bx^4}}{x}\,dx = \begin{cases} -\frac{\sqrt{a}\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{a}{2\sqrt{b}x^2\sqrt{\frac{a}{bx^4}-1}}-\frac{\sqrt{b}x^2}{2\sqrt{\frac{a}{bx^4}-1}} & \text{for}\ \left|\frac{a}{bx^4}\right|>1 \\ \frac{i\sqrt{a}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}-\frac{ia}{2\sqrt{b}x^2\sqrt{-\frac{a}{bx^4}+1}}+\frac{i\sqrt{b}x^2}{2\sqrt{-\frac{a}{bx^4}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x,x)
```

output
```
Piecewise((-sqrt(a)*acosh(sqrt(a)/(sqrt(b)*x**2))/2 + a/(2*sqrt(b)*x**2*sq
rt(a/(b*x**4) - 1)) - sqrt(b)*x**2/(2*sqrt(a/(b*x**4) - 1)), Abs(a/(b*x**4
)) > 1), (I*sqrt(a)*asin(sqrt(a)/(sqrt(b)*x**2))/2 - I*a/(2*sqrt(b)*x**2*s
qrt(-a/(b*x**4) + 1)) + I*sqrt(b)*x**2/(2*sqrt(-a/(b*x**4) + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{a-bx^4}}{x}\,dx = \frac{1}{4}\sqrt{a}\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)+\frac{1}{2}\sqrt{-bx^4+a}$$

input
```
integrate((-b*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/4*sqrt(a)*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))
 + 1/2*sqrt(-b*x^4 + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a-bx^4}}{x}\,dx = \frac{a\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}+\frac{1}{2}\sqrt{-bx^4+a}$$

input
```
integrate((-b*x^4+a)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*a*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a) + 1/2*sqrt(-b*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a-bx^4}}{x}\,dx = \frac{\sqrt{a-b\,x^4}}{2}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{2}$$

input
```
int((a - b*x^4)^(1/2)/x,x)
```

output
```
(a - b*x^4)^(1/2)/2 - (a^(1/2)*atanh((a - b*x^4)^(1/2)/a^(1/2)))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a-bx^4}}{x}\,dx = \frac{\sqrt{-b\,x^4+a}}{2}+\frac{\sqrt{a}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)}{2}-\frac{\sqrt{a}}{2}$$

input
```
int((-b*x^4+a)^(1/2)/x,x)
```

output
```
(sqrt(a - b*x**4) + sqrt(a)*log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2)) - sqr
t(a))/2
```

3.168 $\int \frac{\sqrt{a-bx^4}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = -\frac{\sqrt{a-bx^4}}{4x^4} + \frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
-1/4*(-b*x^4+a)^(1/2)/x^4+1/4*b*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = -\frac{\sqrt{a-bx^4}}{4x^4} + \frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^5,x]
```

output
```
-1/4*Sqrt[a - b*x^4]/x^4 + (b*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/(4*Sqrt[a]
)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^5}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{\sqrt{a-bx^4}}{x^8}dx^4$$

$$\Big\downarrow \ 51$$

$$\frac{1}{4}\left(-\frac{1}{2}b\int \frac{1}{x^4\sqrt{a-bx^4}}dx^4-\frac{\sqrt{a-bx^4}}{x^4}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}}d\sqrt{a-bx^4}-\frac{\sqrt{a-bx^4}}{x^4}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a-bx^4}}{x^4}\right)$$

input
```
Int[Sqrt[a - b*x^4]/x^5,x]
```

output
```
(-(Sqrt[a - b*x^4]/x^4) + (b*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/Sqrt[a])/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$-\frac{b\left(\frac{\sqrt{-b\,x^4+a}}{x^4 b}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^4+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{4}$	42
risch	$-\frac{\sqrt{-b\,x^4+a}}{4x^4}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^4+a}}{x^2}\right)}{4\sqrt{a}}$	47
default	$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{2}}}{4a\,x^4}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^4+a}}{x^2}\right)}{4\sqrt{a}}-\frac{b\sqrt{-b\,x^4+a}}{4a}$	66
elliptic	$-\frac{\left(-b\,x^4+a\right)^{\frac{3}{2}}}{4a\,x^4}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^4+a}}{x^2}\right)}{4\sqrt{a}}-\frac{b\sqrt{-b\,x^4+a}}{4a}$	66

input
```
int((-b*x^4+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*((-b*x^4+a)^(1/2)/x^4/b-arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.45

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = \left[\frac{\sqrt{a}bx^4\log\left(\frac{bx^4-2\sqrt{-bx^4+a}\sqrt{a}-2a}{x^4}\right)-2\sqrt{-bx^4+a}a}{8ax^4}, \frac{\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)-\sqrt{-bx^4+a}a}{4ax^4}\right]$$

input
```
integrate((-b*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*x^4*log((b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4) -
2*sqrt(-b*x^4 + a)*a)/(a*x^4), 1/4*(sqrt(-a)*b*x^4*arctan(sqrt(-b*x^4 + a)
*sqrt(-a)/(b*x^4 - a)) - sqrt(-b*x^4 + a)*a)/(a*x^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = \begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{4x^2}+\frac{b\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ \frac{ia}{4\sqrt{b}x^6\sqrt{-\frac{a}{bx^4}+1}}-\frac{i\sqrt{b}}{4x^2\sqrt{-\frac{a}{bx^4}+1}}-\frac{ib\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**5,x)
```

output
```
Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(4*x**2) + b*acosh(sqrt(a)/(sqrt(
b)*x**2))/(4*sqrt(a)), Abs(a/(b*x**4)) > 1), (I*a/(4*sqrt(b)*x**6*sqrt(-a/
(b*x**4) + 1)) - I*sqrt(b)/(4*x**2*sqrt(-a/(b*x**4) + 1)) - I*b*asin(sqrt(
a)/(sqrt(b)*x**2))/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = -\frac{b\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{8\sqrt{a}} - \frac{\sqrt{-bx^4+a}}{4\,x^4}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-1/8*b*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/sqrt
(a) - 1/4*sqrt(-b*x^4 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = -\frac{\frac{b^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\sqrt{-bx^4+a}b}{x^4}}{4\,b}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*(b^2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a) + sqrt(-b*x^4 + a)*b/
x^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{4\,\sqrt{a}} - \frac{\sqrt{a-b\,x^4}}{4\,x^4}$$

input
```
int((a - b*x^4)^(1/2)/x^5,x)
```

output
```
(b*atanh((a - b*x^4)^(1/2)/a^(1/2)))/(4*a^(1/2)) - (a - b*x^4)^(1/2)/(4*x^
4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a-bx^4}}{x^5}\,dx = \frac{-\sqrt{a}\,\sqrt{-b\,x^4+a}-\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)b\,x^4}{4\sqrt{a}\,x^4}$$

input
```
int((-b*x^4+a)^(1/2)/x^5,x)
```

output
```
( - (sqrt(a)*sqrt(a - b*x**4) + log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2))*b
*x**4))/(4*sqrt(a)*x**4)
```

3.169 $\int \frac{\sqrt{a-bx^4}}{x^9} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int \frac{\sqrt{a-bx^4}}{x^9} \, dx = -\frac{\sqrt{a-bx^4}}{8x^8} + \frac{b\sqrt{a-bx^4}}{16ax^4} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
-1/8*(-b*x^4+a)^(1/2)/x^8+1/16*b*(-b*x^4+a)^(1/2)/a/x^4+1/16*b^2*arctanh((
-b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a-bx^4}}{x^9} \, dx = \frac{\sqrt{a-bx^4}(-2a+bx^4)}{16ax^8} + \frac{b^2 \text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{16a^{3/2}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^9,x]
```

output
```
(Sqrt[a - b*x^4]*(-2*a + b*x^4))/(16*a*x^8) + (b^2*ArcTanh[Sqrt[a - b*x^4]
/Sqrt[a]])/(16*a^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^9} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{\sqrt{a-bx^4}}{x^{12}} dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4} \left(-\frac{1}{4} b \int \frac{1}{x^8 \sqrt{a-bx^4}} dx^4 - \frac{\sqrt{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{1}{4} b \left(\frac{b \int \frac{1}{x^4 \sqrt{a-bx^4}} dx^4}{2a} - \frac{\sqrt{a-bx^4}}{ax^4}\right) - \frac{\sqrt{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(-\frac{1}{4} b \left(-\frac{\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}} d\sqrt{a-bx^4}}{a} - \frac{\sqrt{a-bx^4}}{ax^4}\right) - \frac{\sqrt{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4} \left(-\frac{1}{4} b \left(-\frac{b \text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a-bx^4}}{ax^4}\right) - \frac{\sqrt{a-bx^4}}{2x^8}\right)$$

input
```
Int[Sqrt[a - b*x^4]/x^9,x]
```

output
```
(-1/2*Sqrt[a - b*x^4]/x^8 - (b*(-(Sqrt[a - b*x^4]/(a*x^4)) - (b*ArcTanh[Sq
rt[a - b*x^4]/Sqrt[a]])/a^(3/2)))/4)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{4}\sqrt{a}-2a^{\frac{3}{2}}\right)\sqrt{-b\,x^{4}+a}+\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right)b^{2}x^{8}}{16a^{\frac{3}{2}}x^{8}}$	57
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-b\,x^{4}+2a\right)}{16x^{8}a}+\frac{b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}$	62
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{8a\,x^{8}}-\frac{b\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{16a^{2}x^{4}}+\frac{b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}-\frac{b^{2}\sqrt{-b\,x^{4}+a}}{16a^{2}}$	89
elliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{8a\,x^{8}}-\frac{b\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{16a^{2}x^{4}}+\frac{b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{16a^{\frac{3}{2}}}-\frac{b^{2}\sqrt{-b\,x^{4}+a}}{16a^{2}}$	89

input
```
int((-b*x^4+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/16*((b*x^4*a^(1/2)-2*a^(3/2))*(-b*x^4+a)^(1/2)+arctanh((-b*x^4+a)^(1/2)/
a^(1/2))*b^2*x^8)/a^(3/2)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{a-bx^4}}{x^9}\,dx = \left[\frac{\sqrt{a}b^2x^8\log\left(\frac{bx^4-2\sqrt{-bx^4+a}\sqrt{a}-2\,a}{x^4}\right)+2\,(abx^4-2\,a^2)\sqrt{-bx^4+a}}{32\,a^2x^8}, \frac{\sqrt{-a}b^2x^8\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)+(abx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
[1/32*(sqrt(a)*b^2*x^8*log((b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4)
 + 2*(a*b*x^4 - 2*a^2)*sqrt(-b*x^4 + a))/(a^2*x^8), 1/16*(sqrt(-a)*b^2*x^8
*arctan(sqrt(-b*x^4 + a)*sqrt(-a)/(b*x^4 - a)) + (a*b*x^4 - 2*a^2)*sqrt(-b
*x^4 + a))/(a^2*x^8)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.78

$$\int \frac{\sqrt{a-bx^4}}{x^9}\,dx = \begin{cases} -\frac{a}{8\sqrt{b}x^{10}\sqrt{\frac{a}{bx^4}-1}} + \frac{3\sqrt{b}}{16x^6\sqrt{\frac{a}{bx^4}-1}} - \frac{b^{\frac{3}{2}}}{16ax^2\sqrt{\frac{a}{bx^4}-1}} + \frac{b^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{ia}{8\sqrt{b}x^{10}\sqrt{-\frac{a}{bx^4}+1}} - \frac{3i\sqrt{b}}{16x^6\sqrt{-\frac{a}{bx^4}+1}} + \frac{ib^{\frac{3}{2}}}{16ax^2\sqrt{-\frac{a}{bx^4}+1}} - \frac{ib^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**9,x)
```

output
```
Piecewise((-a/(8*sqrt(b)*x**10*sqrt(a/(b*x**4) - 1)) + 3*sqrt(b)/(16*x**6*
sqrt(a/(b*x**4) - 1)) - b**(3/2)/(16*a*x**2*sqrt(a/(b*x**4) - 1)) + b**2*a
cosh(sqrt(a)/(sqrt(b)*x**2))/(16*a**(3/2)), Abs(a/(b*x**4)) > 1), (I*a/(8*
sqrt(b)*x**10*sqrt(-a/(b*x**4) + 1)) - 3*I*sqrt(b)/(16*x**6*sqrt(-a/(b*x**
4) + 1)) + I*b**(3/2)/(16*a*x**2*sqrt(-a/(b*x**4) + 1)) - I*b**2*asin(sqrt
(a)/(sqrt(b)*x**2))/(16*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a-bx^4}}{x^9}\,dx = -\frac{b^2\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{32\,a^{\frac{3}{2}}} - \frac{(-bx^4+a)^{\frac{3}{2}}b^2 + \sqrt{-bx^4+a}\,ab^2}{16\left((bx^4-a)^2a + 2\,(bx^4-a)a^2 + a^3\right)}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/32*b^2*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/a
^(3/2) - 1/16*((-b*x^4 + a)^(3/2)*b^2 + sqrt(-b*x^4 + a)*a*b^2)/((b*x^4 -
a)^2*a + 2*(b*x^4 - a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a-bx^4}}{x^9}\,dx = -\frac{\frac{b^3\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{(-bx^4+a)^{\frac{3}{2}}b^3+\sqrt{-bx^4+a}ab^3}{ab^2x^8}}{16\,b}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
-1/16*(b^3*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + ((-b*x^4 + a)^
(3/2)*b^3 + sqrt(-b*x^4 + a)*a*b^3)/(a*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a-bx^4}}{x^9}\,dx = \frac{b^2\operatorname{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{16\,a^{3/2}} - \frac{\sqrt{a-b\,x^4}}{16\,x^8} - \frac{\left(a-b\,x^4\right)^{3/2}}{16\,a\,x^8}$$

input
```
int((a - b*x^4)^(1/2)/x^9,x)
```

output
```
(b^2*atanh((a - b*x^4)^(1/2)/a^(1/2)))/(16*a^(3/2)) - (a - b*x^4)^(1/2)/(1
6*x^8) - (a - b*x^4)^(3/2)/(16*a*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{a-bx^4}}{x^9}\,dx = \frac{\sqrt{a}\,b^2\left(-8\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)^4+\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)^8-1\right)}{128\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)^4 a^2}$$

input
```
int((-b*x^4+a)^(1/2)/x^9,x)
```

output
```
(sqrt(a)*b**2*( - 8*log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2))*tan(asin((sqr
t(b)*x**2)/sqrt(a))/2)**4 + tan(asin((sqrt(b)*x**2)/sqrt(a))/2)**8 - 1))/(
128*tan(asin((sqrt(b)*x**2)/sqrt(a))/2)**4*a**2)
```

3.170 $\int x^5\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int x^5\sqrt{a-bx^4}\,dx=-\frac{ax^2\sqrt{a-bx^4}}{16b}+\frac{1}{8}x^6\sqrt{a-bx^4}+\frac{a^2\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{16b^{3/2}}$$

output
```
-1/16*a*x^2*(-b*x^4+a)^(1/2)/b+1/8*x^6*(-b*x^4+a)^(1/2)+1/16*a^2*arctan(b^
(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int x^5\sqrt{a-bx^4}\,dx=\frac{x^2\sqrt{a-bx^4}(-a+2bx^4)}{16b}-\frac{ia^2\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{16b^{3/2}}$$

input
```
Integrate[x^5*Sqrt[a - b*x^4],x]
```

output
```
(x^2*Sqrt[a - b*x^4]*(-a + 2*b*x^4))/(16*b) - ((I/16)*a^2*Log[I*Sqrt[b]*x^
2 + Sqrt[a - b*x^4]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {807, 248, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^4\sqrt{a-bx^4}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{1}{4}a\int \frac{x^4}{\sqrt{a-bx^4}}dx^2+\frac{1}{4}x^6\sqrt{a-bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{4}a\left(\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx^2}{2b}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)+\frac{1}{4}x^6\sqrt{a-bx^4}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{4}a\left(\frac{a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}}{2b}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)+\frac{1}{4}x^6\sqrt{a-bx^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2b^{3/2}}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)+\frac{1}{4}x^6\sqrt{a-bx^4}\right)$$

input
```
Int[x^5*Sqrt[a - b*x^4],x]
```

output
```
((x^6*Sqrt[a - b*x^4])/4 + (a*(-1/2*(x^2*Sqrt[a - b*x^4])/b + (a*ArcTan[(S
qrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*b^(3/2))))/4)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{x^{2}\left(-2b\,x^{4}+a\right)\sqrt{-b\,x^{4}+a}}{16b}+\frac{a^{2}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{16b^{\frac{3}{2}}}$	54
default	$-\frac{a\,x^{2}\sqrt{-b\,x^{4}+a}}{16b}+\frac{x^{6}\sqrt{-b\,x^{4}+a}}{8}+\frac{a^{2}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{16b^{\frac{3}{2}}}$	62
elliptic	$-\frac{a\,x^{2}\sqrt{-b\,x^{4}+a}}{16b}+\frac{x^{6}\sqrt{-b\,x^{4}+a}}{8}+\frac{a^{2}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{16b^{\frac{3}{2}}}$	62
pseudoelliptic	$\frac{2\sqrt{-b\,x^{4}+a}\,b^{\frac{3}{2}}x^{6}-a\,x^{2}\sqrt{-b\,x^{4}+a}\,\sqrt{b}-\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)a^{2}}{16b^{\frac{3}{2}}}$	67

input
```
int(x^5*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*x^2*(-2*b*x^4+a)/b*(-b*x^4+a)^(1/2)+1/16*a^2*arctan(b^(1/2)*x^2/(-b*
x^4+a)^(1/2))/b^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int x^5\sqrt{a-bx^4}\,dx$$
$$=\left[-\frac{a^{2}\sqrt{-b}\log\left(2\,bx^{4}-2\,\sqrt{-bx^{4}+a}\sqrt{-b}x^{2}-a\right)-2\left(2\,b^{2}x^{6}-abx^{2}\right)\sqrt{-bx^{4}+a}}{32\,b^{2}},\right.$$
$$\left.-\frac{a^{2}\sqrt{b}\arctan\left(\frac{\sqrt{-bx^{4}+a}}{\sqrt{b}x^{2}}\right)-\left(2\,b^{2}x^{6}-abx^{2}\right)\sqrt{-bx^{4}+a}}{16\,b^{2}}\right]$$

input
```
integrate(x^5*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*(a^2*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a) -
2*(2*b^2*x^6 - a*b*x^2)*sqrt(-b*x^4 + a))/b^2, -1/16*(a^2*sqrt(b)*arctan(s
qrt(-b*x^4 + a)/(sqrt(b)*x^2)) - (2*b^2*x^6 - a*b*x^2)*sqrt(-b*x^4 + a))/b
^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.68

$$\int x^5\sqrt{a-bx^4}\,dx = \begin{cases} \frac{ia^{\frac{3}{2}}x^2}{16b\sqrt{-1+\frac{bx^4}{a}}} - \frac{3i\sqrt{a}x^6}{16\sqrt{-1+\frac{bx^4}{a}}} - \frac{ia^2\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + \frac{ibx^{10}}{8\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{a^{\frac{3}{2}}x^2}{16b\sqrt{1-\frac{bx^4}{a}}} + \frac{3\sqrt{a}x^6}{16\sqrt{1-\frac{bx^4}{a}}} + \frac{a^2\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} - \frac{bx^{10}}{8\sqrt{a}\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((I*a**(3/2)*x**2/(16*b*sqrt(-1 + b*x**4/a)) - 3*I*sqrt(a)*x**6/(
16*sqrt(-1 + b*x**4/a)) - I*a**2*acosh(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2))
 + I*b*x**10/(8*sqrt(a)*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a) > 1), (-a**(3/
2)*x**2/(16*b*sqrt(1 - b*x**4/a)) + 3*sqrt(a)*x**6/(16*sqrt(1 - b*x**4/a))
 + a**2*asin(sqrt(b)*x**2/sqrt(a))/(16*b**(3/2)) - b*x**10/(8*sqrt(a)*sqrt
(1 - b*x**4/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.36

$$\int x^5\sqrt{a-bx^4}\,dx = -\frac{a^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{16\,b^{\frac{3}{2}}} + \frac{\frac{\sqrt{-bx^4+a}a^2b}{x^2} - \frac{(-bx^4+a)^{\frac{3}{2}}a^2}{x^6}}{16\left(b^3 - \frac{2\,(bx^4-a)b^2}{x^4} + \frac{(bx^4-a)^2b}{x^8}\right)}$$

input
```
integrate(x^5*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*a^2*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/b^(3/2) + 1/16*(sqrt(-b*x
^4 + a)*a^2*b/x^2 - (-b*x^4 + a)^(3/2)*a^2/x^6)/(b^3 - 2*(b*x^4 - a)*b^2/x
^4 + (b*x^4 - a)^2*b/x^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int x^5\sqrt{a-bx^4}\,dx = \frac{1}{16}\sqrt{-bx^4+a}\left(2\,x^4-\frac{a}{b}\right)x^2-\frac{a^2\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{16\sqrt{-b}b}$$

input
```
integrate(x^5*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(-b*x^4 + a)*(2*x^4 - a/b)*x^2 - 1/16*a^2*log(abs(-sqrt(-b)*x^2 +
 sqrt(-b*x^4 + a)))/(sqrt(-b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a-bx^4}\,dx = \int x^5\,\sqrt{a-b\,x^4}\,dx$$

input
```
int(x^5*(a - b*x^4)^(1/2),x)
```

output
```
int(x^5*(a - b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int x^5\sqrt{a-bx^4}\,dx = \frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)a^2-\sqrt{b}\,\sqrt{-b\,x^4+a}\,a\,x^2+2\sqrt{b}\,\sqrt{-b\,x^4+a}\,b\,x^6}{16\sqrt{b}\,b}$$

input
```
int(x^5*(-b*x^4+a)^(1/2),x)
```

output
```
(asin((sqrt(b)*x**2)/sqrt(a))*a**2 - sqrt(b)*sqrt(a - b*x**4)*a*x**2 + 2*s
qrt(b)*sqrt(a - b*x**4)*b*x**6)/(16*sqrt(b)*b)
```

3.171 $\int x\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int x\sqrt{a-bx^4}\,dx=\frac{1}{4}x^2\sqrt{a-bx^4}+\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{4\sqrt{b}}$$

output
```
1/4*x^2*(-b*x^4+a)^(1/2)+1/4*a*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int x\sqrt{a-bx^4}\,dx=\frac{1}{4}x^2\sqrt{a-bx^4}-\frac{ia\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{4\sqrt{b}}$$

input
```
Integrate[x*Sqrt[a - b*x^4],x]
```

output
```
(x^2*Sqrt[a - b*x^4])/4 - ((I/4)*a*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/S
qrt[b]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {807, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \sqrt{a-bx^4}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{1}{\sqrt{a-bx^4}}dx^2+\frac{1}{2}x^2\sqrt{a-bx^4}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}+\frac{1}{2}x^2\sqrt{a-bx^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{1}{2}x^2\sqrt{a-bx^4}\right)$$

input
```
Int[x*Sqrt[a - b*x^4],x]
```

output
```
((x^2*Sqrt[a - b*x^4])/2 + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sq
rt[b]))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4}+\frac{a \arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4 \sqrt{b}}$	41
risch	$\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4}+\frac{a \arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4 \sqrt{b}}$	41
elliptic	$\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4}+\frac{a \arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4 \sqrt{b}}$	41
pseudoelliptic	$\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4}-\frac{a \arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)}{4 \sqrt{b}}$	41

input
```
int(x*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(-b*x^4+a)^(1/2)+1/4*a*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.06

$$\int x\sqrt{a-bx^4}\,dx = \left[\frac{2\sqrt{-bx^4+a}bx^2-a\sqrt{-b}\log\left(2\,bx^4-2\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)}{8\,b},\ \frac{\sqrt{-bx^4+a}bx^2-a\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{4\,b}\right]$$

input
```
integrate(x*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(-b*x^4 + a)*b*x^2 - a*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 +
a)*sqrt(-b)*x^2 - a))/b, 1/4*(sqrt(-b*x^4 + a)*b*x^2 - a*sqrt(b)*arctan(sq
rt(-b*x^4 + a)/(sqrt(b)*x^2)))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.42

$$\int x\sqrt{a-bx^4}\,dx = \begin{cases} -\frac{i\sqrt{a}x^2}{4\sqrt{-1+\frac{bx^4}{a}}} - \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} + \frac{ibx^6}{4\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for}\ \left|\frac{bx^4}{a}\right| > 1 \\ \frac{\sqrt{a}x^2\sqrt{1-\frac{bx^4}{a}}}{4} + \frac{a\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(a)*x**2/(4*sqrt(-1 + b*x**4/a)) - I*a*acosh(sqrt(b)*x**
2/sqrt(a))/(4*sqrt(b)) + I*b*x**6/(4*sqrt(a)*sqrt(-1 + b*x**4/a)), Abs(b*x
**4/a) > 1), (sqrt(a)*x**2*sqrt(1 - b*x**4/a)/4 + a*asin(sqrt(b)*x**2/sqrt
(a))/(4*sqrt(b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int x\sqrt{a-bx^4}\,dx = -\frac{a\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{4\sqrt{b}} + \frac{\sqrt{-bx^4+a}a}{4\left(b-\frac{bx^4-a}{x^4}\right)x^2}$$

input
```
integrate(x*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/sqrt(b) + 1/4*sqrt(-b*x^4 +
a)*a/((b - (b*x^4 - a)/x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int x\sqrt{a-bx^4}\,dx = \frac{1}{4}\sqrt{-bx^4+a}x^2 - \frac{a\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{4\sqrt{-b}}$$

input
```
integrate(x*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-b*x^4 + a)*x^2 - 1/4*a*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 + a))
)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a-bx^4}\,dx = \int x\sqrt{a-b\,x^4}\,dx$$

input
```
int(x*(a - b*x^4)^(1/2),x)
```

output
```
int(x*(a - b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int x\sqrt{a-bx^4}\,dx = \frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)a+\sqrt{b}\,\sqrt{-b\,x^4+a}\,x^2}{4\sqrt{b}}$$

input
```
int(x*(-b*x^4+a)^(1/2),x)
```

output
```
(asin((sqrt(b)*x**2)/sqrt(a))*a + sqrt(b)*sqrt(a - b*x**4)*x**2)/(4*sqrt(b
))
```

3.172 $\int \frac{\sqrt{a-bx^4}}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{\sqrt{a-bx^4}}{x^3} \, dx = -\frac{\sqrt{a-bx^4}}{2x^2} - \frac{1}{2}\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)$$

output
```
-1/2*(-b*x^4+a)^(1/2)/x^2-1/2*b^(1/2)*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a-bx^4}}{x^3} \, dx = -\frac{\sqrt{a-bx^4}}{2x^2} + \frac{1}{2}i\sqrt{b}\log\left(i\sqrt{b}x^2 + \sqrt{a-bx^4}\right)$$

input
```
Integrate[Sqrt[a - b*x^4]/x^3,x]
```

output
```
-1/2*Sqrt[a - b*x^4]/x^2 + (I/2)*Sqrt[b]*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^
4]]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {807, 247, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt{a-bx^4}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(-b\int \frac{1}{\sqrt{a-bx^4}}dx^2-\frac{\sqrt{a-bx^4}}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(-b\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x^2}\right)$$

input
```
Int[Sqrt[a - b*x^4]/x^3,x]
```

output
```
(-(Sqrt[a - b*x^4]/x^2) - Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{2x^{2}}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2}$	40
pseudoelliptic	$\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)x^{2}-\sqrt{-b\,x^{4}+a}}{2x^{2}}$	44
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}-\frac{b\,x^{2}\sqrt{-b\,x^{4}+a}}{2a}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2}$	62
elliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}-\frac{b\,x^{2}\sqrt{-b\,x^{4}+a}}{2a}-\frac{\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2}$	62

input
```
int((-b*x^4+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^4+a)^(1/2)/x^2-1/2*b^(1/2)*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a-bx^4}}{x^3}\,dx = \left[\frac{\sqrt{-b}x^2\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)-2\,\sqrt{-bx^4+a}}{4\,x^2}, \frac{\sqrt{b}x^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)-\sqrt{-bx^4+a}}{2\,x^2}\right]$$

input
```
integrate((-b*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(-b)*x^2*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a) - 2*
sqrt(-b*x^4 + a))/x^2, 1/2*(sqrt(b)*x^2*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x
^2)) - sqrt(-b*x^4 + a))/x^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{a-bx^4}}{x^3}\,dx = \begin{cases} \frac{i\sqrt{a}}{2x^2\sqrt{-1+\frac{bx^4}{a}}}+\frac{i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}-\frac{ibx^2}{2\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right|>1 \\ -\frac{\sqrt{a}}{2x^2\sqrt{1-\frac{bx^4}{a}}}-\frac{\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2}+\frac{bx^2}{2\sqrt{a}\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**3,x)
```

output
```
Piecewise((I*sqrt(a)/(2*x**2*sqrt(-1 + b*x**4/a)) + I*sqrt(b)*acosh(sqrt(b
)*x**2/sqrt(a))/2 - I*b*x**2/(2*sqrt(a)*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a
) > 1), (-sqrt(a)/(2*x**2*sqrt(1 - b*x**4/a)) - sqrt(b)*asin(sqrt(b)*x**2/
sqrt(a))/2 + b*x**2/(2*sqrt(a)*sqrt(1 - b*x**4/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a-bx^4}}{x^3}\,dx = \frac{1}{2}\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right) - \frac{\sqrt{-bx^4+a}}{2\,x^2}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(b)*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)) - 1/2*sqrt(-b*x^4 + a)/
x^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{a-bx^4}}{x^3}\,dx = -\frac{1}{4}\sqrt{-b}\log\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2\right) + \frac{a\sqrt{-b}}{\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*sqrt(-b)*log((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2) + a*sqrt(-b)/((sqrt
(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{x^3}\,dx = \int \frac{\sqrt{a-b\,x^4}}{x^3}\,dx$$

input
```
int((a - b*x^4)^(1/2)/x^3,x)
```

output
```
int((a - b*x^4)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a-bx^4}}{x^3}\,dx = \frac{-\sqrt{b}\,asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)x^2-\sqrt{-b\,x^4+a}}{2x^2}$$

input
```
int((-b*x^4+a)^(1/2)/x^3,x)
```

output
```
( - (sqrt(b)*asin((sqrt(b)*x**2)/sqrt(a))*x**2 + sqrt(a - b*x**4)))/(2*x**
2)
```

3.173 $\int \frac{\sqrt{a-bx^4}}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt{a-bx^4}}{x^7} dx = -\frac{(a-bx^4)^{3/2}}{6ax^6}$$

output
```
-1/6*(-b*x^4+a)^(3/2)/a/x^6
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^4}}{x^7} dx = -\frac{(a-bx^4)^{3/2}}{6ax^6}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^7,x]
```

output
```
-1/6*(a - b*x^4)^(3/2)/(a*x^6)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\left(a-bx^4\right)^{3/2}}{6ax^6}$$

input
```
Int[Sqrt[a - b*x^4]/x^7,x]
```

output
```
-1/6*(a - b*x^4)^(3/2)/(a*x^6)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19
elliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	19

input
```
int((-b*x^4+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b*x^4+a)^(3/2)/a/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx=\frac{(bx^4-a)\sqrt{-bx^4+a}}{6\,ax^6}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
1/6*(b*x^4 - a)*sqrt(-b*x^4 + a)/(a*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.27

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx = \begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{6x^4} + \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{6a} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ -\frac{i\sqrt{b}\sqrt{-\frac{a}{bx^4}+1}}{6x^4} + \frac{ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{6a} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**7,x)
```

output
```
Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(6*x**4) + b**(3/2)*sqrt(a/(b*x**
4) - 1)/(6*a), Abs(a/(b*x**4)) > 1), (-I*sqrt(b)*sqrt(-a/(b*x**4) + 1)/(6*
x**4) + I*b**(3/2)*sqrt(-a/(b*x**4) + 1)/(6*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx = -\frac{(-bx^4+a)^{\frac{3}{2}}}{6\,ax^6}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(-b*x^4 + a)^(3/2)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(18) = 36$.

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.41

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx = -\frac{3\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^4\sqrt{-b}b+a^2\sqrt{-b}b}{3\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^3}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/3*(3*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^4*sqrt(-b)*b + a^2*sqrt(-b)*b)/(
(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx = -\frac{\left(a-b\,x^4\right)^{3/2}}{6\,a\,x^6}$$

input
```
int((a - b*x^4)^(1/2)/x^7,x)
```

output
```
-(a - b*x^4)^(3/2)/(6*a*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a-bx^4}}{x^7}\,dx = \frac{\sqrt{-b\,x^4+a}\,\left(b\,x^4-a\right)}{6a\,x^6}$$

input
```
int((-b*x^4+a)^(1/2)/x^7,x)
```

output
```
(sqrt(a - b*x**4)*( - a + b*x**4))/(6*a*x**6)
```

3.174 $\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = -\frac{(a-bx^4)^{3/2}}{10ax^{10}} - \frac{b(a-bx^4)^{3/2}}{15a^2x^6}$$

output
```
-1/10*(-b*x^4+a)^(3/2)/a/x^10-1/15*b*(-b*x^4+a)^(3/2)/a^2/x^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = \frac{\sqrt{a-bx^4}(-3a^2+abx^4+2b^2x^8)}{30a^2x^{10}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^11,x]
```

output
```
(Sqrt[a - b*x^4]*(-3*a^2 + a*b*x^4 + 2*b^2*x^8))/(30*a^2*x^10)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx$$

$$\Big\downarrow \ 803$$

$$\frac{2b\int \frac{\sqrt{a-bx^4}}{x^7}dx}{5a}-\frac{\left(a-bx^4\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \ 796$$

$$-\frac{b\left(a-bx^4\right)^{3/2}}{15a^2x^6}-\frac{\left(a-bx^4\right)^{3/2}}{10ax^{10}}$$

input
```
Int[Sqrt[a - b*x^4]/x^11,x]
```

output
```
-1/10*(a - b*x^4)^(3/2)/(a*x^10) - (b*(a - b*x^4)^(3/2))/(15*a^2*x^6)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}(2b\,x^4+3a)}{30x^{10}a^2}$	29
default	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}(2b\,x^4+3a)}{30x^{10}a^2}$	29
elliptic	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}(2b\,x^4+3a)}{30x^{10}a^2}$	29
pseudoelliptic	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}(2b\,x^4+3a)}{30x^{10}a^2}$	29
orering	$-\frac{(-b\,x^4+a)^{\frac{3}{2}}(2b\,x^4+3a)}{30x^{10}a^2}$	29
trager	$-\frac{(-2b^2x^8-ab\,x^4+3a^2)\sqrt{-b\,x^4+a}}{30x^{10}a^2}$	40
risch	$-\frac{(-2b^2x^8-ab\,x^4+3a^2)\sqrt{-b\,x^4+a}}{30x^{10}a^2}$	40

input
```
int((-b*x^4+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(-b*x^4+a)^(3/2)*(2*b*x^4+3*a)/x^10/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = \frac{(2\,b^2x^8+abx^4-3\,a^2)\sqrt{-bx^4+a}}{30\,a^2x^{10}}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
1/30*(2*b^2*x^8 + a*b*x^4 - 3*a^2)*sqrt(-b*x^4 + a)/(a^2*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 260, normalized size of antiderivative = 5.65

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx$$
$$= \begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{10x^8} + \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{30ax^4} + \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^4}-1}}{15a^2} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{3ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{x^4(-30a^3bx^4+30a^2b^2x^8)} - \frac{4ia^2b^{\frac{5}{2}}\sqrt{-\frac{a}{bx^4}+1}}{-30a^3bx^4+30a^2b^2x^8} - \frac{iab^{\frac{7}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{-30a^3bx^4+30a^2b^2x^8} + \frac{2ib^{\frac{9}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{-30a^3bx^4+30a^2b^2x^8} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**11,x)
```

output
```
Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(10*x**8) + b**(3/2)*sqrt(a/(b*x*
*4) - 1)/(30*a*x**4) + b**(5/2)*sqrt(a/(b*x**4) - 1)/(15*a**2), Abs(a/(b*x
**4)) > 1), (3*I*a**3*b**(3/2)*sqrt(-a/(b*x**4) + 1)/(x**4*(-30*a**3*b*x**
4 + 30*a**2*b**2*x**8)) - 4*I*a**2*b**(5/2)*sqrt(-a/(b*x**4) + 1)/(-30*a**
3*b*x**4 + 30*a**2*b**2*x**8) - I*a*b**(7/2)*x**4*sqrt(-a/(b*x**4) + 1)/(-
30*a**3*b*x**4 + 30*a**2*b**2*x**8) + 2*I*b**(9/2)*x**8*sqrt(-a/(b*x**4) +
 1)/(-30*a**3*b*x**4 + 30*a**2*b**2*x**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = -\frac{\frac{5\,(-bx^4+a)^{\frac{3}{2}}b}{x^6} + \frac{3\,(-bx^4+a)^{\frac{5}{2}}}{x^{10}}}{30\,a^2}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
-1/30*(5*(-b*x^4 + a)^(3/2)*b/x^6 + 3*(-b*x^4 + a)^(5/2)/x^10)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(38) = 76$.

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = \frac{2\left(15\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^6\sqrt{-b}b^2+5\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^4a\sqrt{-b}b^2+5\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2}{15\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^5}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^11,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^6*sqrt(-b)*b^2 + 5*(sqrt(-b)*x^
2 - sqrt(-b*x^4 + a))^4*a*sqrt(-b)*b^2 + 5*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a
))^2*a^2*sqrt(-b)*b^2 - a^3*sqrt(-b)*b^2)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a
))^2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = \frac{\sqrt{a-b\,x^4}\,\left(-3\,a^2+a\,b\,x^4+2\,b^2\,x^8\right)}{30\,a^2\,x^{10}}$$

input
```
int((a - b*x^4)^(1/2)/x^11,x)
```

output
```
((a - b*x^4)^(1/2)*(2*b^2*x^8 - 3*a^2 + a*b*x^4))/(30*a^2*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a-bx^4}}{x^{11}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(2b^2x^8+ab\,x^4-3a^2)}{30a^2x^{10}}$$

input
```
int((-b*x^4+a)^(1/2)/x^11,x)
```

output
```
(sqrt(a - b*x**4)*( - 3*a**2 + a*b*x**4 + 2*b**2*x**8))/(30*a**2*x**10)
```

3.175 $\int \frac{\sqrt{a-bx^4}}{x^{15}} dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{\sqrt{a-bx^4}}{x^{15}} dx = -\frac{(a-bx^4)^{3/2}}{14ax^{14}} - \frac{2b(a-bx^4)^{3/2}}{35a^2x^{10}} - \frac{4b^2(a-bx^4)^{3/2}}{105a^3x^6}$$

output
```
-1/14*(-b*x^4+a)^(3/2)/a/x^14-2/35*b*(-b*x^4+a)^(3/2)/a^2/x^10-4/105*b^2*(
-b*x^4+a)^(3/2)/a^3/x^6
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a-bx^4}}{x^{15}} dx = \frac{\sqrt{a-bx^4}(-15a^3+3a^2bx^4+4ab^2x^8+8b^3x^{12})}{210a^3x^{14}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^15,x]
```

output
```
(Sqrt[a - b*x^4]*(-15*a^3 + 3*a^2*b*x^4 + 4*a*b^2*x^8 + 8*b^3*x^12))/(210*
a^3*x^14)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^{15}} dx$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\int \frac{\sqrt{a-bx^4}}{x^{11}}dx}{7a}-\frac{\left(a-bx^4\right)^{3/2}}{14ax^{14}}$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\left(\frac{2b\int \frac{\sqrt{a-bx^4}}{x^7}dx}{5a}-\frac{\left(a-bx^4\right)^{3/2}}{10ax^{10}}\right)}{7a}-\frac{\left(a-bx^4\right)^{3/2}}{14ax^{14}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4b\left(-\frac{b\left(a-bx^4\right)^{3/2}}{15a^2x^6}-\frac{\left(a-bx^4\right)^{3/2}}{10ax^{10}}\right)}{7a}-\frac{\left(a-bx^4\right)^{3/2}}{14ax^{14}}$$

input
```
Int[Sqrt[a - b*x^4]/x^15,x]
```

output
```
-1/14*(a - b*x^4)^(3/2)/(a*x^14) + (4*b*(-1/10*(a - b*x^4)^(3/2)/(a*x^10)
- (b*(a - b*x^4)^(3/2))/(15*a^2*x^6)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(8 b^{2} x^{8}+12 a b\,x^{4}+15 a^{2}\right)}{210 x^{14} a^{3}}$	40
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(8 b^{2} x^{8}+12 a b\,x^{4}+15 a^{2}\right)}{210 x^{14} a^{3}}$	40
elliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(8 b^{2} x^{8}+12 a b\,x^{4}+15 a^{2}\right)}{210 x^{14} a^{3}}$	40
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(8 b^{2} x^{8}+12 a b\,x^{4}+15 a^{2}\right)}{210 x^{14} a^{3}}$	40
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{2}} \left(8 b^{2} x^{8}+12 a b\,x^{4}+15 a^{2}\right)}{210 x^{14} a^{3}}$	40
trager	$-\frac{\left(-8 b^{3} x^{12}-4 a\,b^{2} x^{8}-3 a^{2} b\,x^{4}+15 a^{3}\right) \sqrt{-b\,x^{4}+a}}{210 x^{14} a^{3}}$	51
risch	$-\frac{\left(-8 b^{3} x^{12}-4 a\,b^{2} x^{8}-3 a^{2} b\,x^{4}+15 a^{3}\right) \sqrt{-b\,x^{4}+a}}{210 x^{14} a^{3}}$	51

input
```
int((-b*x^4+a)^(1/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/210*(-b*x^4+a)^(3/2)*(8*b^2*x^8+12*a*b*x^4+15*a^2)/x^14/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a-bx^4}}{x^{15}}\,dx = \frac{(8\,b^3x^{12}+4\,ab^2x^8+3\,a^2bx^4-15\,a^3)\sqrt{-bx^4+a}}{210\,a^3x^{14}}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^15,x, algorithm="fricas")
```

output
```
1/210*(8*b^3*x^12 + 4*a*b^2*x^8 + 3*a^2*b*x^4 - 15*a^3)*sqrt(-b*x^4 + a)/(
a^3*x^14)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.80 (sec) , antiderivative size = 733, normalized size of antiderivative = 10.32

$$\int \frac{\sqrt{a-bx^4}}{x^{15}}\,dx$$
$$= \begin{cases} -\frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}-1}}{210a^5b^4x^{12}-420a^4b^5x^{16}+210a^3b^6x^{20}} + \frac{33a^4b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}-1}}{210a^5b^4x^{12}-420a^4b^5x^{16}+210a^3b^6x^{20}} - \frac{17a^3b^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}-1}}{210a^5b^4x^{12}-420a^4b^5x^{16}+210a^3b^6x^{20}} + \frac{\cdots}{210a^5b^4\cdots} \\ -\frac{15ia^5b^{\frac{9}{2}}\sqrt{-\frac{a}{bx^4}+1}}{210a^5b^4x^{12}-420a^4b^5x^{16}+210a^3b^6x^{20}} + \frac{33ia^4b^{\frac{11}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{210a^5b^4x^{12}-420a^4b^5x^{16}+210a^3b^6x^{20}} - \frac{17ia^3b^{\frac{13}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{210a^5b^4x^{12}-420a^4b^5x^{16}+210a^3b^6x^{20}} + \frac{\cdots}{210a^5b^4\cdots} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**15,x)
```

output
```
Piecewise((-15*a**5*b**(9/2)*sqrt(a/(b*x**4) - 1)/(210*a**5*b**4*x**12 - 4
20*a**4*b**5*x**16 + 210*a**3*b**6*x**20) + 33*a**4*b**(11/2)*x**4*sqrt(a/
(b*x**4) - 1)/(210*a**5*b**4*x**12 - 420*a**4*b**5*x**16 + 210*a**3*b**6*x
**20) - 17*a**3*b**(13/2)*x**8*sqrt(a/(b*x**4) - 1)/(210*a**5*b**4*x**12 -
 420*a**4*b**5*x**16 + 210*a**3*b**6*x**20) + 3*a**2*b**(15/2)*x**12*sqrt(
a/(b*x**4) - 1)/(210*a**5*b**4*x**12 - 420*a**4*b**5*x**16 + 210*a**3*b**6
*x**20) - 12*a*b**(17/2)*x**16*sqrt(a/(b*x**4) - 1)/(210*a**5*b**4*x**12 -
 420*a**4*b**5*x**16 + 210*a**3*b**6*x**20) + 8*b**(19/2)*x**20*sqrt(a/(b*
x**4) - 1)/(210*a**5*b**4*x**12 - 420*a**4*b**5*x**16 + 210*a**3*b**6*x**2
0), Abs(a/(b*x**4)) > 1), (-15*I*a**5*b**(9/2)*sqrt(-a/(b*x**4) + 1)/(210*
a**5*b**4*x**12 - 420*a**4*b**5*x**16 + 210*a**3*b**6*x**20) + 33*I*a**4*b
**(11/2)*x**4*sqrt(-a/(b*x**4) + 1)/(210*a**5*b**4*x**12 - 420*a**4*b**5*x
**16 + 210*a**3*b**6*x**20) - 17*I*a**3*b**(13/2)*x**8*sqrt(-a/(b*x**4) +
1)/(210*a**5*b**4*x**12 - 420*a**4*b**5*x**16 + 210*a**3*b**6*x**20) + 3*I
*a**2*b**(15/2)*x**12*sqrt(-a/(b*x**4) + 1)/(210*a**5*b**4*x**12 - 420*a**
4*b**5*x**16 + 210*a**3*b**6*x**20) - 12*I*a*b**(17/2)*x**16*sqrt(-a/(b*x*
*4) + 1)/(210*a**5*b**4*x**12 - 420*a**4*b**5*x**16 + 210*a**3*b**6*x**20)
 + 8*I*b**(19/2)*x**20*sqrt(-a/(b*x**4) + 1)/(210*a**5*b**4*x**12 - 420*a*
*4*b**5*x**16 + 210*a**3*b**6*x**20), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a-bx^4}}{x^{15}}\,dx = -\frac{\frac{35\,(-bx^4+a)^{\frac{3}{2}}b^2}{x^6}+\frac{42\,(-bx^4+a)^{\frac{5}{2}}b}{x^{10}}+\frac{15\,(-bx^4+a)^{\frac{7}{2}}}{x^{14}}}{210\,a^3}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^15,x, algorithm="maxima")
```

output
```
-1/210*(35*(-b*x^4 + a)^(3/2)*b^2/x^6 + 42*(-b*x^4 + a)^(5/2)*b/x^10 + 15*
(-b*x^4 + a)^(7/2)/x^14)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(59) = 118$.

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{a-bx^4}}{x^{15}}\,dx =$$

$$-\frac{8\left(70\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^8\sqrt{-b}b^3+35\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^6a\sqrt{-b}b^3+21\left(\sqrt{-b}x^2-\sqrt{-bx^4+\cdots}\right.\right.}{105\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^7}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^15,x, algorithm="giac")
```

output
```
-8/105*(70*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^8*sqrt(-b)*b^3 + 35*(sqrt(-b)
*x^2 - sqrt(-b*x^4 + a))^6*a*sqrt(-b)*b^3 + 21*(sqrt(-b)*x^2 - sqrt(-b*x^4
 + a))^4*a^2*sqrt(-b)*b^3 - 7*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2*a^3*sqrt
(-b)*b^3 + a^4*sqrt(-b)*b^3)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a-bx^4}}{x^{15}}\,dx = \frac{b\sqrt{a-b\,x^4}}{70\,a\,x^{10}} - \frac{\sqrt{a-b\,x^4}}{14\,x^{14}} + \frac{4\,b^3\sqrt{a-b\,x^4}}{105\,a^3\,x^2} + \frac{2\,b^2\sqrt{a-b\,x^4}}{105\,a^2\,x^6}$$

input
```
int((a - b*x^4)^(1/2)/x^15,x)
```

output
```
(b*(a - b*x^4)^(1/2))/(70*a*x^10) - (a - b*x^4)^(1/2)/(14*x^14) + (4*b^3*(
a - b*x^4)^(1/2))/(105*a^3*x^2) + (2*b^2*(a - b*x^4)^(1/2))/(105*a^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a-bx^4}}{x^{15}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(8b^3x^{12}+4a\,b^2x^8+3a^2b\,x^4-15a^3)}{210a^3x^{14}}$$

input
```
int((-b*x^4+a)^(1/2)/x^15,x)
```

output
```
(sqrt(a - b*x**4)*( - 15*a**3 + 3*a**2*b*x**4 + 4*a*b**2*x**8 + 8*b**3*x**
12))/(210*a**3*x**14)
```

3.176 $\int x^4\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int x^4\sqrt{a-bx^4}\,dx = -\frac{2ax\sqrt{a-bx^4}}{21b}+\frac{1}{7}x^5\sqrt{a-bx^4} + \frac{2a^{9/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21b^{5/4}\sqrt{a-bx^4}}$$

output
```
-2/21*a*x*(-b*x^4+a)^(1/2)/b+1/7*x^5*(-b*x^4+a)^(1/2)+2/21*a^(9/4)*(1-b*x^
4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(5/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66

$$\int x^4\sqrt{a-bx^4}\,dx = \frac{x\sqrt{a-bx^4}\left(-a+bx^4+\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{1-\frac{bx^4}{a}}}\right)}{7b}$$

input
```
Integrate[x^4*Sqrt[a - b*x^4],x]
```

output
```
(x*Sqrt[a - b*x^4]*(-a + b*x^4 + (a*Hypergeometric2F1[-1/2, 1/4, 5/4, (b*x
^4)/a])/Sqrt[1 - (b*x^4)/a]))/(7*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {811, 843, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{2}{7}a\int \frac{x^4}{\sqrt{a-bx^4}}dx+\frac{1}{7}x^5\sqrt{a-bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{7}a\left(\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx}{3b}-\frac{x\sqrt{a-bx^4}}{3b}\right)+\frac{1}{7}x^5\sqrt{a-bx^4}$$

$$\Big\downarrow \text{765}$$

$$\frac{2}{7}a\left(\frac{a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3b\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)+\frac{1}{7}x^5\sqrt{a-bx^4}$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{7}a\left(\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)+\frac{1}{7}x^5\sqrt{a-bx^4}$$

input
```
Int[x^4*Sqrt[a - b*x^4],x]
```

output
```
(x^5*Sqrt[a - b*x^4])/7 + (2*a*(-1/3*(x*Sqrt[a - b*x^4])/b + (a^(5/4)*Sqrt
[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(5/4)*Sqr
t[a - b*x^4])))/7
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{x(-3b\,x^4+2a)\sqrt{-b\,x^4+a}}{21b}+\frac{2a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	98
default	$\frac{x^5\sqrt{-b\,x^4+a}}{7}-\frac{2ax\sqrt{-b\,x^4+a}}{21b}+\frac{2a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	104
elliptic	$\frac{x^5\sqrt{-b\,x^4+a}}{7}-\frac{2ax\sqrt{-b\,x^4+a}}{21b}+\frac{2a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	104

input
```
int(x^4*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(-3*b*x^4+2*a)*(-b*x^4+a)^(1/2)/b+2/21/b*a^2/(1/a^(1/2)*b^(1/2))^(
1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a
)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int x^4\sqrt{a-bx^4}\,dx=\frac{2\,a\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(3\,bx^5-2\,ax)\sqrt{-bx^4+a}}{21\,b}$$

input
```
integrate(x^4*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(2*a*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + (3*
b*x^5 - 2*a*x)*sqrt(-b*x^4 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int x^4\sqrt{a-bx^4}\,dx=\frac{\sqrt{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(-b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4\sqrt{a-bx^4}\,dx=\int\sqrt{-bx^4+a}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a-bx^4}\,dx=\int\sqrt{-bx^4+a}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a-bx^4}\,dx=\int x^4\,\sqrt{a-b\,x^4}\,dx$$

input
```
int(x^4*(a - b*x^4)^(1/2),x)
```

output
```
int(x^4*(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a-bx^4}\,dx=\frac{-2\sqrt{-b\,x^4+a}\,ax+3\sqrt{-b\,x^4+a}\,b\,x^5+2\left(\int\frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)a^2}{21b}$$

input
```
int(x^4*(-b*x^4+a)^(1/2),x)
```

output
```
( - 2*sqrt(a - b*x**4)*a*x + 3*sqrt(a - b*x**4)*b*x**5 + 2*int(sqrt(a - b*
x**4)/(a - b*x**4),x)*a**2)/(21*b)
```

3.177 $\int \sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \sqrt{a-bx^4}\,dx=\frac{1}{3}x\sqrt{a-bx^4}+\frac{2a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
1/3*x*(-b*x^4+a)^(1/2)+2/3*a^(5/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a
^(1/4),I)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \sqrt{a-bx^4}\,dx=\frac{x(a-bx^4)-\frac{2ia\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}}{3\sqrt{a-bx^4}}$$

input
```
Integrate[Sqrt[a - b*x^4],x]
```

output
```
(x*(a - b*x^4) - ((2*I)*a*Sqrt[1 - (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(S
qrt[b]/Sqrt[a])]*x], -1])/Sqrt[-(Sqrt[b]/Sqrt[a])])/(3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{ 748}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{1}{3}x\sqrt{a-bx^4}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{2a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3\sqrt{a-bx^4}}+\frac{1}{3}x\sqrt{a-bx^4}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{1}{3}x\sqrt{a-bx^4}$$

input
```
Int[Sqrt[a - b*x^4],x]
```

output
```
(x*Sqrt[a - b*x^4])/3 + (2*a^(5/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/(3*b^(1/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{x\sqrt{-b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	80
risch	$\frac{x\sqrt{-b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	80
elliptic	$\frac{x\sqrt{-b\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	80

input
```
int((-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-b*x^4+a)^(1/2)+2/3*a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1
/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a
^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \sqrt{a-bx^4}\,dx = \frac{2}{3}\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) | -1\right) + \frac{1}{3}\sqrt{-bx^4+a}\,x$$

input
```
integrate((-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + 1/3*sqrt(
-b*x^4 + a)*x
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \sqrt{a-bx^4}\,dx = \frac{\sqrt{a}x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)/a
)/(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt{a-bx^4}\,dx = \int \sqrt{-bx^4+a}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \sqrt{a-bx^4}\,dx=\int \sqrt{-bx^4+a}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.51

$$\int \sqrt{a-bx^4}\,dx=\frac{x\sqrt{a-b\,x^4}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{\sqrt{1-\frac{b\,x^4}{a}}}$$

input
```
int((a - b*x^4)^(1/2),x)
```

output
```
(x*(a - b*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, (b*x^4)/a))/(1 - (b*x^4)/
a)^(1/2)
```

Reduce [F]

$$\int \sqrt{a-bx^4}\,dx=\frac{\sqrt{-b\,x^4+a}\,x}{3}+\frac{2\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)a}{3}$$

input
```
int((-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*x + 2*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a)/3
```

3.178 $\int \frac{\sqrt{a-bx^4}}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{\sqrt{a-bx^4}}{x^4} dx = -\frac{\sqrt{a-bx^4}}{3x^3} - \frac{2\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt{a-bx^4}}$$

output
```
-1/3*(-b*x^4+a)^(1/2)/x^3-2/3*a^(1/4)*b^(3/4)*(1-b*x^4/a)^(1/2)*EllipticF(
b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a-bx^4}}{x^4} dx = -\frac{\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},\frac{bx^4}{a}\right)}{3x^3\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^4,x]
```

output
```
-1/3*(Sqrt[a - b*x^4]*Hypergeometric2F1[-3/4, -1/2, 1/4, (b*x^4)/a])/(x^3*
Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {809, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx$$

$$\Big\downarrow \ 809$$

$$-\frac{2}{3}b\int \frac{1}{\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}}{3x^3}$$

$$\Big\downarrow \ 765$$

$$-\frac{2b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3x^3}$$

$$\Big\downarrow \ 762$$

$$-\frac{2\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3x^3}$$

input
```
Int[Sqrt[a - b*x^4]/x^4,x]
```

output
```
-1/3*Sqrt[a - b*x^4]/x^3 - (2*a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*Elliptic
F[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\sqrt{-b\,x^{4}+a}}{3x^{3}}-\frac{2b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	82
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{3x^{3}}-\frac{2b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	82
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{3x^{3}}-\frac{2b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	82

input
```
int((-b*x^4+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^4+a)^(1/2)/x^3-2/3*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a
^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(
1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx = -\frac{2\sqrt{a}x^3\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)+\sqrt{-bx^4+a}}{3\,x^3}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(a)*x^3*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + sq
rt(-b*x^4 + a))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx = \frac{i\sqrt{b}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**4,x)
```

output
```
I*sqrt(b)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), a/(b*x**4))/(4*x*gamma(3/
4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx=\int \frac{\sqrt{-bx^4+a}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx=\int \frac{\sqrt{-bx^4+a}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx=\int \frac{\sqrt{a-b\,x^4}}{x^4}\,dx$$

input
```
int((a - b*x^4)^(1/2)/x^4,x)
```

output
```
int((a - b*x^4)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{x^4}\,dx = \frac{-\sqrt{-b\,x^4+a}-2\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^8+a\,x^4}dx\right)a\,x^3}{x^3}$$

input
```
int((-b*x^4+a)^(1/2)/x^4,x)
```

output
```
( - sqrt(a - b*x**4) - 2*int(sqrt(a - b*x**4)/(a*x**4 - b*x**8),x)*a*x**3)
/x**3
```

3.179 $\int \frac{\sqrt{a-bx^4}}{x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx=-\frac{\sqrt{a-bx^4}}{7x^7}+\frac{2b\sqrt{a-bx^4}}{21ax^3}-\frac{2b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21a^{3/4}\sqrt{a-bx^4}}$$

output
```
-1/7*(-b*x^4+a)^(1/2)/x^7+2/21*b*(-b*x^4+a)^(1/2)/a/x^3-2/21*b^(7/4)*(1-b*
x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx=-\frac{\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},\frac{bx^4}{a}\right)}{7x^7\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^8,x]
```

output
```
-1/7*(Sqrt[a - b*x^4]*Hypergeometric2F1[-7/4, -1/2, -3/4, (b*x^4)/a])/(x^7
*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {809, 847, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{2}{7}b\int \frac{1}{x^4\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2}{7}b\left(\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{3a}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)-\frac{\sqrt{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{765}$$

$$-\frac{2}{7}b\left(\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3a\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)-\frac{\sqrt{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{762}$$

$$-\frac{2}{7}b\left(\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)-\frac{\sqrt{a-bx^4}}{7x^7}$$

input
```
Int[Sqrt[a - b*x^4]/x^8,x]
```

output
```
-1/7*Sqrt[a - b*x^4]/x^7 - (2*b*(-1/3*Sqrt[a - b*x^4]/(a*x^3) + (b^(3/4)*S
qrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)*
Sqrt[a - b*x^4])))/7
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-2b\,x^{4}+3a\right)}{21x^{7}a}-\frac{2b^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	100
default	$-\frac{\sqrt{-b\,x^{4}+a}}{7x^{7}}+\frac{2b\sqrt{-b\,x^{4}+a}}{21a\,x^{3}}-\frac{2b^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	106
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{7x^{7}}+\frac{2b\sqrt{-b\,x^{4}+a}}{21a\,x^{3}}-\frac{2b^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	106

input
```
int((-b*x^4+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-b*x^4+a)^(1/2)*(-2*b*x^4+3*a)/x^7/a-2/21/a*b^2/(1/a^(1/2)*b^(1/2))
^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4
+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx=-\frac{2\,\sqrt{a}bx^7\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-\left(2\,bx^4-3\,a\right)\sqrt{-bx^4+a}}{21\,ax^7}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/21*(2*sqrt(a)*b*x^7*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) -
 (2*b*x^4 - 3*a)*sqrt(-b*x^4 + a))/(a*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**8,x)
```

output
```
sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**4*exp_polar(2*I*pi)/
a)/(4*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx = \int \frac{\sqrt{-bx^4+a}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx = \int \frac{\sqrt{-bx^4+a}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx = \int \frac{\sqrt{a-b\,x^4}}{x^8}\,dx$$

input
```
int((a - b*x^4)^(1/2)/x^8,x)
```

output
```
int((a - b*x^4)^(1/2)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{x^8}\,dx = \frac{-\sqrt{-b\,x^4+a}-2\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{12}+a\,x^8}dx\right)a\,x^7}{5x^7}$$

input
```
int((-b*x^4+a)^(1/2)/x^8,x)
```

output
```
( - sqrt(a - b*x**4) - 2*int(sqrt(a - b*x**4)/(a*x**8 - b*x**12),x)*a*x**7
)/(5*x**7)
```

3.180 $\int x^2\sqrt{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 132

$$\int x^2\sqrt{a-bx^4}\,dx=\frac{1}{5}x^3\sqrt{a-bx^4}+\frac{2a^{7/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{3/4}\sqrt{a-bx^4}}-\frac{2a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{5b^{3/4}\sqrt{a-bx^4}}$$

output
```
1/5*x^3*(-b*x^4+a)^(1/2)+2/5*a^(7/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x
/a^(1/4),I)/b^(3/4)/(-b*x^4+a)^(1/2)-2/5*a^(7/4)*(1-b*x^4/a)^(1/2)*Ellipti
cF(b^(1/4)*x/a^(1/4),I)/b^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int x^2\sqrt{a-bx^4}\,dx=\frac{x^3\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)}{3\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^2*Sqrt[a - b*x^4],x]
```

output
```
(x^3*Sqrt[a - b*x^4]*Hypergeometric2F1[-1/2, 3/4, 7/4, (b*x^4)/a])/(3*Sqrt
[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {811, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{2}{5}a\int \frac{x^2}{\sqrt{a-bx^4}}dx+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$$\Big\downarrow \text{836}$$

$$\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$$\Big\downarrow \text{765}$$

$$\frac{2}{5}a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{5}a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$\Big\downarrow$ 1390

$$\frac{2}{5}a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$\Big\downarrow$ 1389

$$\frac{2}{5}a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

$\Big\downarrow$ 327

$$\frac{2}{5}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}$$

input
```
Int[x^2*Sqrt[a - b*x^4],x]
```

output
```
(x^3*Sqrt[a - b*x^4])/5 + (2*a*((a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1
 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a
- b*x^4])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{x^{3}\sqrt{-b\,x^{4}+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$	104
risch	$\frac{x^{3}\sqrt{-b\,x^{4}+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$	104
elliptic	$\frac{x^{3}\sqrt{-b\,x^{4}+a}}{5}-\frac{2a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$	104

input
```
int(x^2*(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(-b*x^4+a)^(1/2)-2/5*a^(3/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*
x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*
(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^
(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{a-bx^4}\,dx =$$

$$-\frac{2\,a\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-2\,a\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-(bx^4-2\,a)\sqrt{-bx^4+}}{5\,bx}$$

input
```
integrate(x^2*(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(2*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1) - 2
*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - (b*x^4 -
 2*a)*sqrt(-b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.31

$$\int x^2\sqrt{a-bx^4}\,dx = \frac{\sqrt{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(-b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi
)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int x^2\sqrt{a-bx^4}\,dx = \int \sqrt{-bx^4+a}x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a-bx^4}\,dx=\int \sqrt{-bx^4+a}x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a-bx^4}\,dx=\int x^2\,\sqrt{a-b\,x^4}\,dx$$

input
```
int(x^2*(a - b*x^4)^(1/2),x)
```

output
```
int(x^2*(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a-bx^4}\,dx=\frac{\sqrt{-b\,x^4+a}\,x^3}{5}+\frac{2\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)a}{5}$$

input
```
int(x^2*(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*x**3 + 2*int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)*a)/
5
```

3.181 $\int \frac{\sqrt{a-bx^4}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx = -\frac{\sqrt{a-bx^4}}{x} - \frac{2a^{3/4}\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a-bx^4}} + \frac{2a^{3/4}\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{a-bx^4}}$$

output

```
-(-b*x^4+a)^(1/2)/x-2*a^(3/4)*b^(1/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*
x/a^(1/4),I)/(-b*x^4+a)^(1/2)+2*a^(3/4)*b^(1/4)*(1-b*x^4/a)^(1/2)*Elliptic
F(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx = -\frac{\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},\frac{bx^4}{a}\right)}{x\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^2,x]
```

output
```
-((Sqrt[a - b*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, (b*x^4)/a])/(x*Sqrt[
1 - (b*x^4)/a]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {809, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^2}dx$$

$$\Big\downarrow \text{809}$$

$$-2b\int \frac{x^2}{\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{836}$$

$$-2b\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{27}$$

$$-2b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{765}$$

$$-2b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{762}$$

$$-2b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{1390}$$

$$-2b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{1389}$$

$$-2b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

$$\Big\downarrow \text{327}$$

$$-2b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}$$

input
```
Int[Sqrt[a - b*x^4]/x^2,x]
```

output
```
-(Sqrt[a - b*x^4]/x) - 2*b*((a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (
b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*
x^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\sqrt{-b\,x^{4}+a}}{x}+\frac{2\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	104
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{x}+\frac{2\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	104
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{x}+\frac{2\sqrt{b}\,\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	104

input
```
int((-b*x^4+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-b*x^4+a)^(1/2)/x+2*b^(1/2)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)
*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(Ellipt
icF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I
))
```

Fricas [F]

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx=\int \frac{\sqrt{-bx^4+a}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(-b*x^4 + a)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx=\frac{i\sqrt{b}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**2,x)
```

output
```
I*sqrt(b)*x*gamma(1/4)*hyper((-1/2, -1/4), (3/4,), a/(b*x**4))/(4*gamma(5/
4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx=\int \frac{\sqrt{-bx^4+a}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx = \int \frac{\sqrt{-bx^4+a}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx = \frac{\sqrt{a-b\,x^4}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ \frac{a}{b\,x^4}\right)}{x\,\sqrt{1-\frac{a}{b\,x^4}}}$$

input
```
int((a - b*x^4)^(1/2)/x^2,x)
```

output
```
((a - b*x^4)^(1/2)*hypergeom([-1/2, -1/4], 3/4, a/(b*x^4)))/(x*(1 - a/(b*x
^4))^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{x^2}\,dx = \frac{\sqrt{-b\,x^4+a}+2\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx\right)ax}{x}$$

input
```
int((-b*x^4+a)^(1/2)/x^2,x)
```

output
```
(sqrt(a - b*x**4) + 2*int(sqrt(a - b*x**4)/(a*x**2 - b*x**6),x)*a*x)/x
```

3.182 $\int \frac{\sqrt{a-bx^4}}{x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 155

$$\int \frac{\sqrt{a-bx^4}}{x^6} dx = -\frac{\sqrt{a-bx^4}}{5x^5} + \frac{2b\sqrt{a-bx^4}}{5ax} + \frac{2b^{5/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt[4]{a}\sqrt{a-bx^4}} - \frac{2b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{5\sqrt[4]{a}\sqrt{a-bx^4}}$$

output
```
-1/5*(-b*x^4+a)^(1/2)/x^5+2/5*b*(-b*x^4+a)^(1/2)/a/x+2/5*b^(5/4)*(1-b*x^4/
a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/a^(1/4)/(-b*x^4+a)^(1/2)-2/5*b^(5/
4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(1/4)/(-b*x^4+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = -\frac{\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},\frac{bx^4}{a}\right)}{5x^5\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[Sqrt[a - b*x^4]/x^6,x]
```

output
```
-1/5*(Sqrt[a - b*x^4]*Hypergeometric2F1[-5/4, -1/2, -1/4, (b*x^4)/a])/(x^5
*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {809, 847, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^4}}{x^6}dx$$

$$\Big\downarrow \text{ 809}$$

$$-\frac{2}{5}b\int \frac{1}{x^2\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}}{5x^5}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{2}{5}b\left(-\frac{b\int \frac{x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$$\Big\downarrow \text{ 836}$$

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$\Big\downarrow$ 27

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$\Big\downarrow$ 765

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$\Big\downarrow$ 762

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$\Big\downarrow$ 1390

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$\Big\downarrow$ 1389

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

$$\Big\downarrow \ 327$$

$$-\frac{2}{5}b\left(-\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)-\frac{\sqrt{a-bx^4}}{5x^5}$$

input
```
Int[Sqrt[a - b*x^4]/x^6,x]
```

output
```
-1/5*Sqrt[a - b*x^4]/x^5 - (2*b*(-(Sqrt[a - b*x^4]/(a*x)) - (b*((a^(3/4)*S
qrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sq
rt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)
/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/a))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{5x^{5}a}-\frac{2b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	115
default	$-\frac{\sqrt{-b\,x^{4}+a}}{5x^{5}}+\frac{2b\sqrt{-b\,x^{4}+a}}{5ax}-\frac{2b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	123
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{5x^{5}}+\frac{2b\sqrt{-b\,x^{4}+a}}{5ax}-\frac{2b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	123

input
```
int((-b*x^4+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/2)*(-2*b*x^4+a)/x^5/a-2/5/a^(1/2)*b^(3/2)/(1/a^(1/2)*b^
(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(
-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a
^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = \frac{2\,\sqrt{a}bx^5\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1) - 2\,\sqrt{a}bx^5\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1) + (2\,bx^4-a)\sqrt{-bx^4+a}}{5\,ax^5}$$

input
```
integrate((-b*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/5*(2*sqrt(a)*b*x^5*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1/4)), -1) - 2
*sqrt(a)*b*x^5*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + (2*b*x^
4 - a)*sqrt(-b*x^4 + a))/(a*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = \frac{i\sqrt{b}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/2)/x**6,x)
```

output
```
I*sqrt(b)*gamma(-3/4)*hyper((-1/2, 3/4), (7/4,), a/(b*x**4))/(4*x**3*gamma
(1/4))
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = \int \frac{\sqrt{-bx^4+a}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = \int \frac{\sqrt{-bx^4+a}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^4 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = \int \frac{\sqrt{a-b\,x^4}}{x^6}\,dx$$

input
```
int((a - b*x^4)^(1/2)/x^6,x)
```

output
```
int((a - b*x^4)^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^4}}{x^6}\,dx = \frac{-\sqrt{-b\,x^4+a}-2\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{10}+a\,x^6}dx\right)a\,x^5}{3x^5}$$

input
```
int((-b*x^4+a)^(1/2)/x^6,x)
```

output
```
( - sqrt(a - b*x**4) - 2*int(sqrt(a - b*x**4)/(a*x**6 - b*x**10),x)*a*x**5
)/(3*x**5)
```

3.183 $\int x^{11}(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int x^{11}(a-bx^4)^{3/2}\,dx=-\frac{a^2(a-bx^4)^{5/2}}{10b^3}+\frac{a(a-bx^4)^{7/2}}{7b^3}-\frac{(a-bx^4)^{9/2}}{18b^3}$$

output
```
-1/10*a^2*(-b*x^4+a)^(5/2)/b^3+1/7*a*(-b*x^4+a)^(7/2)/b^3-1/18*(-b*x^4+a)^
(9/2)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int x^{11}(a-bx^4)^{3/2}\,dx=-\frac{(a-bx^4)^{5/2}\,(8a^2+20abx^4+35b^2x^8)}{630b^3}$$

input
```
Integrate[x^11*(a - b*x^4)^(3/2),x]
```

output
```
-1/630*((a - b*x^4)^(5/2)*(8*a^2 + 20*a*b*x^4 + 35*b^2*x^8))/b^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a-bx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^8\left(a-bx^4\right)^{3/2}\,dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(a-bx^4\right)^{7/2}}{b^2}-\frac{2a\left(a-bx^4\right)^{5/2}}{b^2}+\frac{a^2\left(a-bx^4\right)^{3/2}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{2a^2\left(a-bx^4\right)^{5/2}}{5b^3}-\frac{2\left(a-bx^4\right)^{9/2}}{9b^3}+\frac{4a\left(a-bx^4\right)^{7/2}}{7b^3}\right)$$

input
```
Int[x^11*(a - b*x^4)^(3/2),x]
```

output
```
((-2*a^2*(a - b*x^4)^(5/2))/(5*b^3) + (4*a*(a - b*x^4)^(7/2))/(7*b^3) - (2
*(a - b*x^4)^(9/2))/(9*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(35b^{2}x^{8}+20ab\,x^{4}+8a^{2}\right)}{630b^{3}}$	37
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(35b^{2}x^{8}+20ab\,x^{4}+8a^{2}\right)}{630b^{3}}$	37
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(35b^{2}x^{8}+20ab\,x^{4}+8a^{2}\right)}{630b^{3}}$	37
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(35b^{2}x^{8}+20ab\,x^{4}+8a^{2}\right)\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)}{630b^{3}}$	55
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(35b^{2}x^{8}+20ab\,x^{4}+8a^{2}\right)\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)}{630b^{3}}$	55
trager	$-\frac{\left(35b^{4}x^{16}-50a\,b^{3}x^{12}+3a^{2}b^{2}x^{8}+4a^{3}b\,x^{4}+8a^{4}\right)\sqrt{-b\,x^{4}+a}}{630b^{3}}$	59
risch	$-\frac{\left(35b^{4}x^{16}-50a\,b^{3}x^{12}+3a^{2}b^{2}x^{8}+4a^{3}b\,x^{4}+8a^{4}\right)\sqrt{-b\,x^{4}+a}}{630b^{3}}$	59

input
```
int(x^11*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/630*(-b*x^4+a)^(5/2)*(35*b^2*x^8+20*a*b*x^4+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int x^{11}\left(a-bx^4\right)^{3/2}\,dx=-\frac{\left(35\,b^4x^{16}-50\,ab^3x^{12}+3\,a^2b^2x^8+4\,a^3bx^4+8\,a^4\right)\sqrt{-bx^4+a}}{630\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/630*(35*b^4*x^16 - 50*a*b^3*x^12 + 3*a^2*b^2*x^8 + 4*a^3*b*x^4 + 8*a^4)
*sqrt(-b*x^4 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(49) = 98.

Time = 0.65 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int x^{11}\left(a-bx^4\right)^{3/2}\,dx=\begin{cases}-\frac{4a^4\sqrt{a-bx^4}}{315b^3}-\frac{2a^3x^4\sqrt{a-bx^4}}{315b^2}-\frac{a^2x^8\sqrt{a-bx^4}}{210b}+\frac{5ax^{12}\sqrt{a-bx^4}}{63}-\frac{bx^{16}\sqrt{a-bx^4}}{18} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{2}}x^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-4*a**4*sqrt(a - b*x**4)/(315*b**3) - 2*a**3*x**4*sqrt(a - b*x*
*4)/(315*b**2) - a**2*x**8*sqrt(a - b*x**4)/(210*b) + 5*a*x**12*sqrt(a - b
*x**4)/63 - b*x**16*sqrt(a - b*x**4)/18, Ne(b, 0)), (a**(3/2)*x**12/12, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int x^{11}(a-bx^4)^{3/2}\,dx = -\frac{(-bx^4+a)^{\frac{9}{2}}}{18\,b^3} + \frac{(-bx^4+a)^{\frac{7}{2}}a}{7\,b^3} - \frac{(-bx^4+a)^{\frac{5}{2}}a^2}{10\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(-b*x^4 + a)^(9/2)/b^3 + 1/7*(-b*x^4 + a)^(7/2)*a/b^3 - 1/10*(-b*x^4
 + a)^(5/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int x^{11}(a-bx^4)^{3/2}\,dx = -\frac{35\,(bx^4-a)^4\sqrt{-bx^4+a}+90\,(bx^4-a)^3\sqrt{-bx^4+a}a+63\,(bx^4-a)^2\sqrt{-bx^4+a}a^2}{630\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/630*(35*(b*x^4 - a)^4*sqrt(-b*x^4 + a) + 90*(b*x^4 - a)^3*sqrt(-b*x^4 +
 a)*a + 63*(b*x^4 - a)^2*sqrt(-b*x^4 + a)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int x^{11}(a-bx^4)^{3/2}\,dx = -\sqrt{a-b\,x^4}\left(\frac{b\,x^{16}}{18}-\frac{5\,a\,x^{12}}{63}+\frac{4\,a^4}{315\,b^3}+\frac{2\,a^3\,x^4}{315\,b^2}+\frac{a^2\,x^8}{210\,b}\right)$$

input
```
int(x^11*(a - b*x^4)^(3/2),x)
```

output
```
-(a - b*x^4)^(1/2)*((b*x^16)/18 - (5*a*x^12)/63 + (4*a^4)/(315*b^3) + (2*a
^3*x^4)/(315*b^2) + (a^2*x^8)/(210*b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x^{11}\left(a-bx^4\right)^{3/2}\,dx=\frac{\sqrt{-b\,x^4+a}\,(-35b^4x^{16}+50a\,b^3x^{12}-3a^2b^2x^8-4a^3b\,x^4-8a^4)}{630b^3}$$

input
```
int(x^11*(-b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 8*a**4 - 4*a**3*b*x**4 - 3*a**2*b**2*x**8 + 50*a*b**
3*x**12 - 35*b**4*x**16))/(630*b**3)
```

3.184 $\int x^7(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int x^7(a-bx^4)^{3/2}\,dx=-\frac{a(a-bx^4)^{5/2}}{10b^2}+\frac{(a-bx^4)^{7/2}}{14b^2}$$

output
```
-1/10*a*(-b*x^4+a)^(5/2)/b^2+1/14*(-b*x^4+a)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int x^7(a-bx^4)^{3/2}\,dx=-\frac{(a-bx^4)^{5/2}(2a+5bx^4)}{70b^2}$$

input
```
Integrate[x^7*(a - b*x^4)^(3/2),x]
```

output
```
-1/70*((a - b*x^4)^(5/2)*(2*a + 5*b*x^4))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a-bx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^4 \left(a-bx^4\right)^{3/2}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a\left(a-bx^4\right)^{3/2}}{b}-\frac{\left(a-bx^4\right)^{5/2}}{b}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2\left(a-bx^4\right)^{7/2}}{7b^2}-\frac{2a\left(a-bx^4\right)^{5/2}}{5b^2}\right)$$

input
```
Int[x^7*(a - b*x^4)^(3/2),x]
```

output
```
((-2*a*(a - b*x^4)^(5/2))/(5*b^2) + (2*(a - b*x^4)^(7/2))/(7*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(5b\,x^{4}+2a\right)}{70b^{2}}$	26
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(5b\,x^{4}+2a\right)}{70b^{2}}$	26
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(5b\,x^{4}+2a\right)}{70b^{2}}$	26
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(5b\,x^{4}+2a\right)\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)}{70b^{2}}$	44
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(5b\,x^{4}+2a\right)\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)}{70b^{2}}$	44
trager	$-\frac{\left(5b^{3}x^{12}-8a\,b^{2}x^{8}+a^{2}b\,x^{4}+2a^{3}\right)\sqrt{-b\,x^{4}+a}}{70b^{2}}$	47
risch	$-\frac{\left(5b^{3}x^{12}-8a\,b^{2}x^{8}+a^{2}b\,x^{4}+2a^{3}\right)\sqrt{-b\,x^{4}+a}}{70b^{2}}$	47

input
```
int(x^7*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/70*(-b*x^4+a)^(5/2)*(5*b*x^4+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int x^7 (a - bx^4)^{3/2} \, dx = -\frac{(5\, b^3 x^{12} - 8\, ab^2 x^8 + a^2 b x^4 + 2\, a^3)\sqrt{-bx^4 + a}}{70\, b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/70*(5*b^3*x^12 - 8*a*b^2*x^8 + a^2*b*x^4 + 2*a^3)*sqrt(-b*x^4 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(31) = 62.

Time = 0.44 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.08

$$\int x^7 (a - bx^4)^{3/2} \, dx = \begin{cases} -\frac{a^3\sqrt{a-bx^4}}{35b^2} - \frac{a^2x^4\sqrt{a-bx^4}}{70b} + \frac{4ax^8\sqrt{a-bx^4}}{35} - \frac{bx^{12}\sqrt{a-bx^4}}{14} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-a**3*sqrt(a - b*x**4)/(35*b**2) - a**2*x**4*sqrt(a - b*x**4)/(
70*b) + 4*a*x**8*sqrt(a - b*x**4)/35 - b*x**12*sqrt(a - b*x**4)/14, Ne(b,
0)), (a**(3/2)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^7 (a - bx^4)^{3/2} \, dx = \frac{(-bx^4 + a)^{\frac{7}{2}}}{14\, b^2} - \frac{(-bx^4 + a)^{\frac{5}{2}} a}{10\, b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/14*(-b*x^4 + a)^(7/2)/b^2 - 1/10*(-b*x^4 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int x^7 (a - bx^4)^{3/2} \, dx = -\frac{5 (bx^4 - a)^3 \sqrt{-bx^4 + a} + 7 (bx^4 - a)^2 \sqrt{-bx^4 + a}a}{70 \, b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/70*(5*(b*x^4 - a)^3*sqrt(-b*x^4 + a) + 7*(b*x^4 - a)^2*sqrt(-b*x^4 + a)
*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int x^7 (a - bx^4)^{3/2} \, dx = -\sqrt{a - b\, x^4} \left(\frac{b\, x^{12}}{14} - \frac{4\, a\, x^8}{35} + \frac{a^3}{35\, b^2} + \frac{a^2\, x^4}{70\, b} \right)$$

input
```
int(x^7*(a - b*x^4)^(3/2),x)
```

output
```
-(a - b*x^4)^(1/2)*((b*x^12)/14 - (4*a*x^8)/35 + a^3/(35*b^2) + (a^2*x^4)/
(70*b))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int x^7 \left(a - b x^4\right)^{3/2} \, dx = \frac{\sqrt{-b\,x^4 + a}\,\left(-5b^3 x^{12} + 8a\,b^2 x^8 - a^2 b\,x^4 - 2a^3\right)}{70b^2}$$

input
```
int(x^7*(-b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 2*a**3 - a**2*b*x**4 + 8*a*b**2*x**8 - 5*b**3*x**12)
)/(70*b**2)
```

3.185 $\int x^3(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int x^3\left(a-bx^4\right)^{3/2}\,dx=-\frac{\left(a-bx^4\right)^{5/2}}{10b}$$

output
```
-1/10*(-b*x^4+a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^3\left(a-bx^4\right)^{3/2}\,dx=-\frac{\left(a-bx^4\right)^{5/2}}{10b}$$

input
```
Integrate[x^3*(a - b*x^4)^(3/2),x]
```

output
```
-1/10*(a - b*x^4)^(5/2)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a - bx^4)^{3/2} \, dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{(a - bx^4)^{5/2}}{10b}$$

input
```
Int[x^3*(a - b*x^4)^(3/2),x]
```

output
```
-1/10*(a - b*x^4)^(5/2)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}}{10b}$	16
derivativedivides	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}}{10b}$	16
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}}{10b}$	16
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}}{10b}$	16
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}}{10b}$	16
trager	$-\frac{\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)\sqrt{-b\,x^{4}+a}}{10b}$	34
risch	$-\frac{\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)\sqrt{-b\,x^{4}+a}}{10b}$	34

input
```
int(x^3*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-b*x^4+a)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int x^3\left(a-bx^4\right)^{3/2}\,dx=-\frac{\left(b^2x^8-2\,abx^4+a^2\right)\sqrt{-bx^4+a}}{10\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(b^2*x^8 - 2*a*b*x^4 + a^2)*sqrt(-b*x^4 + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

$$\int x^3 (a - bx^4)^{3/2}\, dx = \begin{cases} -\frac{a^2\sqrt{a-bx^4}}{10b} + \frac{ax^4\sqrt{a-bx^4}}{5} - \frac{bx^8\sqrt{a-bx^4}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-a**2*sqrt(a - b*x**4)/(10*b) + a*x**4*sqrt(a - b*x**4)/5 - b*x
**8*sqrt(a - b*x**4)/10, Ne(b, 0)), (a**(3/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 (a - bx^4)^{3/2}\, dx = -\frac{(-bx^4 + a)^{\frac{5}{2}}}{10\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/10*(-b*x^4 + a)^(5/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int x^3 (a - bx^4)^{3/2}\, dx = -\frac{(bx^4 - a)^2\sqrt{-bx^4 + a}}{10\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/10*(b*x^4 - a)^2*sqrt(-b*x^4 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \left(a - bx^4\right)^{3/2} \, dx = -\frac{\left(a - b\,x^4\right)^{5/2}}{10\,b}$$

input
```
int(x^3*(a - b*x^4)^(3/2),x)
```

output
```
-(a - b*x^4)^(5/2)/(10*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int x^3 \left(a - bx^4\right)^{3/2} \, dx = \frac{\sqrt{-b\,x^4 + a}\,\left(-b^2 x^8 + 2ab\,x^4 - a^2\right)}{10b}$$

input
```
int(x^3*(-b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - b*x**4)*( - a**2 + 2*a*b*x**4 - b**2*x**8))/(10*b)
```

3.186 $\int \frac{(a-bx^4)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx = \frac{1}{2}a\sqrt{a-bx^4} + \frac{1}{6}(a-bx^4)^{3/2} - \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)$$

output
```
1/2*a*(-b*x^4+a)^(1/2)+1/6*(-b*x^4+a)^(3/2)-1/2*a^(3/2)*arctanh((-b*x^4+a)
^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx = \frac{1}{6}\sqrt{a-bx^4}(4a-bx^4) - \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[(a - b*x^4)^(3/2)/x,x]
```

output
```
(Sqrt[a - b*x^4]*(4*a - b*x^4))/6 - (a^(3/2)*ArcTanh[Sqrt[a - b*x^4]/Sqrt[
a]])/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(a-bx^4)^{3/2}}{x^4}dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\int \frac{\sqrt{a-bx^4}}{x^4}dx^4+\frac{2}{3}(a-bx^4)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\left(a\int \frac{1}{x^4\sqrt{a-bx^4}}dx^4+2\sqrt{a-bx^4}\right)+\frac{2}{3}(a-bx^4)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(a\left(2\sqrt{a-bx^4}-\frac{2a\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}}d\sqrt{a-bx^4}}{b}\right)+\frac{2}{3}(a-bx^4)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(a\left(2\sqrt{a-bx^4}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a-bx^4)^{3/2}\right)$$

input
```
Int[(a - b*x^4)^(3/2)/x,x]
```

output
```
((2*(a - b*x^4)^(3/2))/3 + a*(2*Sqrt[a - b*x^4] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 - b*x^4]/Sqrt[a]]))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right)}{2}+\frac{\sqrt{-b\,x^{4}+a}\,\left(-b\,x^{4}+4a\right)}{6}$	44
default	$-\frac{a^{\frac{3}{2}} \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2}-\frac{x^{4} b\sqrt{-b\,x^{4}+a}}{6}+\frac{2a\sqrt{-b\,x^{4}+a}}{3}$	60
elliptic	$-\frac{a^{\frac{3}{2}} \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2}-\frac{x^{4} b\sqrt{-b\,x^{4}+a}}{6}+\frac{2a\sqrt{-b\,x^{4}+a}}{3}$	60

input
```
int((-b*x^4+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^(3/2)*arctanh((-b*x^4+a)^(1/2)/a^(1/2))+1/6*(-b*x^4+a)^(1/2)*(-b*x^
4+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.85

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx = \left[\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{bx^4+2\,\sqrt{-bx^4+a}\sqrt{a}-2\,a}{x^4}\right) - \frac{1}{6}\left(bx^4-4\,a\right)\sqrt{-bx^4+a}, -\frac{1}{2}\,\sqrt{-a}a\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right) - \frac{1}{6}\left(bx^4-4\,a\right)\sqrt{-bx^4+a}\right]$$

input
```
integrate((-b*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/4*a^(3/2)*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4) - 1/6*(b*
x^4 - 4*a)*sqrt(-b*x^4 + a), -1/2*sqrt(-a)*a*arctan(sqrt(-b*x^4 + a)*sqrt(
-a)/(b*x^4 - a)) - 1/6*(b*x^4 - 4*a)*sqrt(-b*x^4 + a)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.16

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx = \begin{cases} \frac{2ia^{\frac{3}{2}}\sqrt{-1+\frac{bx^4}{a}}}{3} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^4}{a}\right)}{4} - \frac{a^{\frac{3}{2}}\log\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} + \frac{ia^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2} - \frac{i\sqrt{a}bx^4\sqrt{-1+\frac{bx^4}{a}}}{6} & \text{for } \left|\frac{bx^4}{a}\right| \\ \frac{2a^{\frac{3}{2}}\sqrt{1-\frac{bx^4}{a}}}{3} + \frac{a^{\frac{3}{2}}\log\left(\frac{bx^4}{a}\right)}{4} - \frac{a^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx^4}{a}}+1\right)}{2} - \frac{\sqrt{a}bx^4\sqrt{1-\frac{bx^4}{a}}}{6} & \text{otherwis} \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x,x)
```

output
```
Piecewise((2*I*a**(3/2)*sqrt(-1 + b*x**4/a)/3 + a**(3/2)*log(b*x**4/a)/4 -
 a**(3/2)*log(sqrt(b)*x**2/sqrt(a))/2 + I*a**(3/2)*asin(sqrt(a)/(sqrt(b)*x
**2))/2 - I*sqrt(a)*b*x**4*sqrt(-1 + b*x**4/a)/6, Abs(b*x**4/a) > 1), (2*a
**(3/2)*sqrt(1 - b*x**4/a)/3 + a**(3/2)*log(b*x**4/a)/4 - a**(3/2)*log(sqr
t(1 - b*x**4/a) + 1)/2 - sqrt(a)*b*x**4*sqrt(1 - b*x**4/a)/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx = \frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)+\frac{1}{6}\left(-bx^4+a\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-bx^4+a}a$$

input
```
integrate((-b*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*a^(3/2)*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))
 + 1/6*(-b*x^4 + a)^(3/2) + 1/2*sqrt(-b*x^4 + a)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}+\frac{1}{6}\left(-bx^4+a\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-bx^4+a}a$$

input
```
integrate((-b*x^4+a)^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*a^2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a) + 1/6*(-b*x^4 + a)^(3/2
) + 1/2*sqrt(-b*x^4 + a)*a
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx=\frac{a\,\sqrt{a-b\,x^4}}{2}-\frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{2}+\frac{\left(a-b\,x^4\right)^{3/2}}{6}$$

input
```
int((a - b*x^4)^(3/2)/x,x)
```

output
```
(a*(a - b*x^4)^(1/2))/2 - (a^(3/2)*atanh((a - b*x^4)^(1/2)/a^(1/2)))/2 + (
a - b*x^4)^(3/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{(a-bx^4)^{3/2}}{x}\,dx=\frac{2\sqrt{-b\,x^4+a}\,a}{3}-\frac{\sqrt{-b\,x^4+a}\,b\,x^4}{6}+\frac{\sqrt{a}\,\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)a}{2}-\frac{2\sqrt{a}\,a}{3}$$

input
```
int((-b*x^4+a)^(3/2)/x,x)
```

output
```
(4*sqrt(a - b*x**4)*a - sqrt(a - b*x**4)*b*x**4 + 3*sqrt(a)*log(tan(asin((
sqrt(b)*x**2)/sqrt(a))/2))*a - 4*sqrt(a)*a)/6
```

3.187 $\int \frac{(a-bx^4)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{(a-bx^4)^{3/2}}{x^5} dx = -\frac{1}{2}b\sqrt{a-bx^4} - \frac{a\sqrt{a-bx^4}}{4x^4} + \frac{3}{4}\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)$$

output
```
-1/2*b*(-b*x^4+a)^(1/2)-1/4*a*(-b*x^4+a)^(1/2)/x^4+3/4*a^(1/2)*b*arctanh((
-b*x^4+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a-bx^4)^{3/2}}{x^5} dx = \frac{(-a-2bx^4)\sqrt{a-bx^4}}{4x^4} + \frac{3}{4}\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^5,x]
```

output
```
((-a - 2*b*x^4)*Sqrt[a - b*x^4])/(4*x^4) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a - b
*x^4]/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^5} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(a-bx^4)^{3/2}}{x^8}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(-\frac{3}{2}b\int \frac{\sqrt{a-bx^4}}{x^4}dx^4-\frac{(a-bx^4)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(-\frac{3}{2}b\left(a\int \frac{1}{x^4\sqrt{a-bx^4}}dx^4+2\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{3}{2}b\left(2\sqrt{a-bx^4}-\frac{2a\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}}d\sqrt{a-bx^4}}{b}\right)-\frac{(a-bx^4)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{3}{2}b\left(2\sqrt{a-bx^4}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)\right)-\frac{(a-bx^4)^{3/2}}{x^4}\right)$$

input
```
Int[(a - b*x^4)^(3/2)/x^5,x]
```

output
```
(-((a - b*x^4)^(3/2)/x^4) - (3*b*(2*Sqrt[a - b*x^4] - 2*Sqrt[a]*ArcTanh[Sq
rt[a - b*x^4]/Sqrt[a]]))/2)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{3\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right) b\,x^{4}-\sqrt{-b\,x^{4}+a}\,\left(2 b\,x^{4}+a \right)}{4 x^{4}}$	51
default	$-\frac{b\sqrt{-b\,x^{4}+a}}{2}-\frac{a\sqrt{-b\,x^{4}+a}}{4 x^{4}}+\frac{3\sqrt{a}\,b\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{4}$	61
risch	$-\frac{b\sqrt{-b\,x^{4}+a}}{2}-\frac{a\sqrt{-b\,x^{4}+a}}{4 x^{4}}+\frac{3\sqrt{a}\,b\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{4}$	61
elliptic	$-\frac{b\sqrt{-b\,x^{4}+a}}{2}-\frac{a\sqrt{-b\,x^{4}+a}}{4 x^{4}}+\frac{3\sqrt{a}\,b\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{4}$	61

input
```
int((-b*x^4+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(3*a^(1/2)*arctanh((-b*x^4+a)^(1/2)/a^(1/2))*b*x^4-(-b*x^4+a)^(1/2)*(2
*b*x^4+a))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.94

$$\int \frac{(a-bx^4)^{3/2}}{x^5}\,dx = \left[\frac{3\,\sqrt{a}bx^4\log\left(\frac{bx^4-2\,\sqrt{-bx^4+a}\sqrt{a}-2\,a}{x^4}\right)-2\,(2\,bx^4+a)\sqrt{-bx^4+a}}{8\,x^4},\ \frac{3\,\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{-}\right.}{}}{}\right.$$

input
```
integrate((-b*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b*x^4*log((b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4)
- 2*(2*b*x^4 + a)*sqrt(-b*x^4 + a))/x^4, 1/4*(3*sqrt(-a)*b*x^4*arctan(sqrt
(-b*x^4 + a)*sqrt(-a)/(b*x^4 - a)) - (2*b*x^4 + a)*sqrt(-b*x^4 + a))/x^4]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.07

$$\int \frac{(a-bx^4)^{3/2}}{x^5}\,dx = \begin{cases} \frac{3\sqrt{a}b\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} - \frac{a^2}{4\sqrt{b}x^6\sqrt{\frac{a}{bx^4}-1}} - \frac{a\sqrt{b}}{4x^2\sqrt{\frac{a}{bx^4}-1}} + \frac{b^{\frac{3}{2}}x^2}{2\sqrt{\frac{a}{bx^4}-1}} & \text{for}\ \left|\frac{a}{bx^4}\right| > 1 \\ -\frac{3i\sqrt{a}b\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4} + \frac{ia^2}{4\sqrt{b}x^6\sqrt{-\frac{a}{bx^4}+1}} + \frac{ia\sqrt{b}}{4x^2\sqrt{-\frac{a}{bx^4}+1}} - \frac{ib^{\frac{3}{2}}x^2}{2\sqrt{-\frac{a}{bx^4}+1}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**5,x)
```

output
```
Piecewise((3*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*x**2))/4 - a**2/(4*sqrt(b)*x
**6*sqrt(a/(b*x**4) - 1)) - a*sqrt(b)/(4*x**2*sqrt(a/(b*x**4) - 1)) + b**(
3/2)*x**2/(2*sqrt(a/(b*x**4) - 1)), Abs(a/(b*x**4)) > 1), (-3*I*sqrt(a)*b*
asin(sqrt(a)/(sqrt(b)*x**2))/4 + I*a**2/(4*sqrt(b)*x**6*sqrt(-a/(b*x**4) +
 1)) + I*a*sqrt(b)/(4*x**2*sqrt(-a/(b*x**4) + 1)) - I*b**(3/2)*x**2/(2*sqr
t(-a/(b*x**4) + 1)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

$$\int \frac{(a-bx^4)^{3/2}}{x^5}\,dx = -\frac{3}{8}\sqrt{a}b\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right) - \frac{1}{2}\sqrt{-bx^4+a}b - \frac{\sqrt{-bx^4+a}a}{4\,x^4}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-3/8*sqrt(a)*b*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a
))) - 1/2*sqrt(-b*x^4 + a)*b - 1/4*sqrt(-b*x^4 + a)*a/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{(a-bx^4)^{3/2}}{x^5}\,dx = -\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{-bx^4+a}b^2 + \frac{\sqrt{-bx^4+a}ab}{x^4}}{4\,b}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-1/4*(3*a*b^2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(-b*x^4 +
 a)*b^2 + sqrt(-b*x^4 + a)*a*b/x^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{(a-bx^4)^{3/2}}{x^5}\,dx = \frac{3\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{4} - \frac{a\sqrt{a-b\,x^4}}{4\,x^4} - \frac{b\sqrt{a-b\,x^4}}{2}$$

input
```
int((a - b*x^4)^(3/2)/x^5,x)
```

output
```
(3*a^(1/2)*b*atanh((a - b*x^4)^(1/2)/a^(1/2)))/4 - (a*(a - b*x^4)^(1/2))/(
4*x^4) - (b*(a - b*x^4)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{(a-bx^4)^{3/2}}{x^5}\,dx = \frac{-4\sqrt{-b\,x^4+a}\,a - 8\sqrt{-b\,x^4+a}\,b\,x^4 - 12\sqrt{a}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)b\,x^4 + 9\sqrt{a}\,b\,x^4}{16x^4}$$

input
```
int((-b*x^4+a)^(3/2)/x^5,x)
```

output
```
( - 4*sqrt(a - b*x**4)*a - 8*sqrt(a - b*x**4)*b*x**4 - 12*sqrt(a)*log(tan(
asin((sqrt(b)*x**2)/sqrt(a))/2))*b*x**4 + 9*sqrt(a)*b*x**4)/(16*x**4)
```

3.188 $\int \frac{(a-bx^4)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{(a-bx^4)^{3/2}}{x^9} dx = -\frac{a\sqrt{a-bx^4}}{8x^8} + \frac{5b\sqrt{a-bx^4}}{16x^4} - \frac{3b^2 \text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

output
```
-1/8*a*(-b*x^4+a)^(1/2)/x^8+5/16*b*(-b*x^4+a)^(1/2)/x^4-3/16*b^2*arctanh((
-b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int \frac{(a-bx^4)^{3/2}}{x^9} dx = \frac{\sqrt{a-bx^4}(-2a+5bx^4)}{16x^8} - \frac{3b^2 \text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^9,x]
```

output
```
(Sqrt[a - b*x^4]*(-2*a + 5*b*x^4))/(16*x^8) - (3*b^2*ArcTanh[Sqrt[a - b*x^
4]/Sqrt[a]])/(16*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {798, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^9} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(a-bx^4)^{3/2}}{x^{12}}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(-\frac{3}{4}b\int \frac{\sqrt{a-bx^4}}{x^8}dx^4-\frac{(a-bx^4)^{3/2}}{2x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(-\frac{3}{4}b\left(-\frac{1}{2}b\int \frac{1}{x^4\sqrt{a-bx^4}}dx^4-\frac{\sqrt{a-bx^4}}{x^4}\right)-\frac{(a-bx^4)^{3/2}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{3}{4}b\left(\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}}d\sqrt{a-bx^4}-\frac{\sqrt{a-bx^4}}{x^4}\right)-\frac{(a-bx^4)^{3/2}}{2x^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{3}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a-bx^4}}{x^4}\right)-\frac{(a-bx^4)^{3/2}}{2x^8}\right)$$

input
```
Int[(a - b*x^4)^(3/2)/x^9,x]
```

output
```
(-1/2*(a - b*x^4)^(3/2)/x^8 - (3*b*(-(Sqrt[a - b*x^4]/x^4) + (b*ArcTanh[Sq
rt[a - b*x^4]/Sqrt[a]])/Sqrt[a]))/4)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-5b\,x^{4}+2a\right)}{16x^{8}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}$	59
default	$-\frac{a\sqrt{-b\,x^{4}+a}}{8x^{8}}+\frac{5b\sqrt{-b\,x^{4}+a}}{16x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}$	66
elliptic	$-\frac{a\sqrt{-b\,x^{4}+a}}{8x^{8}}+\frac{5b\sqrt{-b\,x^{4}+a}}{16x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}$	66
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right)b^{2}x^{8}+5b\,x^{4}\sqrt{-b\,x^{4}+a}\,\sqrt{a}-2\sqrt{-b\,x^{4}+a}\,a^{\frac{3}{2}}}{16x^{8}\sqrt{a}}$	67

input
```
int((-b*x^4+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-b*x^4+a)^(1/2)*(-5*b*x^4+2*a)/x^8-3/16*b^2/a^(1/2)*ln((2*a+2*a^(1/
2)*(-b*x^4+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.08

$$\int \frac{(a-bx^4)^{3/2}}{x^9}\,dx = \left[\frac{3\sqrt{a}b^2x^8\log\left(\frac{bx^4+2\sqrt{-bx^4+a}\sqrt{a}-2a}{x^4}\right)+2\left(5abx^4-2a^2\right)\sqrt{-bx^4+a}}{32ax^8},\right.$$
$$\left.-\frac{3\sqrt{-a}b^2x^8\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)-\left(5abx^4-2a^2\right)\sqrt{-bx^4+a}}{16ax^8}\right]$$

input
```
integrate((-b*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/32*(3*sqrt(a)*b^2*x^8*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^
4) + 2*(5*a*b*x^4 - 2*a^2)*sqrt(-b*x^4 + a))/(a*x^8), -1/16*(3*sqrt(-a)*b^
2*x^8*arctan(sqrt(-b*x^4 + a)*sqrt(-a)/(b*x^4 - a)) - (5*a*b*x^4 - 2*a^2)*
sqrt(-b*x^4 + a))/(a*x^8)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.62

$$\int \frac{(a-bx^4)^{3/2}}{x^9}\,dx = \begin{cases} -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{8x^6}+\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{16x^2}-\frac{3b^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ \frac{ia^2}{8\sqrt{b}x^{10}\sqrt{-\frac{a}{bx^4}+1}}-\frac{7ia\sqrt{b}}{16x^6\sqrt{-\frac{a}{bx^4}+1}}+\frac{5ib^{\frac{3}{2}}}{16x^2\sqrt{-\frac{a}{bx^4}+1}}+\frac{3ib^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{16\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**9,x)
```

output
```
Piecewise((-a*sqrt(b)*sqrt(a/(b*x**4) - 1)/(8*x**6) + 5*b**(3/2)*sqrt(a/(b
*x**4) - 1)/(16*x**2) - 3*b**2*acosh(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)),
 Abs(a/(b*x**4)) > 1), (I*a**2/(8*sqrt(b)*x**10*sqrt(-a/(b*x**4) + 1)) - 7
*I*a*sqrt(b)/(16*x**6*sqrt(-a/(b*x**4) + 1)) + 5*I*b**(3/2)/(16*x**2*sqrt(
-a/(b*x**4) + 1)) + 3*I*b**2*asin(sqrt(a)/(sqrt(b)*x**2))/(16*sqrt(a)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int \frac{(a-bx^4)^{3/2}}{x^9}\,dx = \frac{3\,b^2\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{32\,\sqrt{a}} - \frac{5\,(-bx^4+a)^{\frac{3}{2}}b^2 - 3\,\sqrt{-bx^4+a}ab^2}{16\,\left((bx^4-a)^2+2\,(bx^4-a)a+a^2\right)}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
3/32*b^2*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/sq
rt(a) - 1/16*(5*(-b*x^4 + a)^(3/2)*b^2 - 3*sqrt(-b*x^4 + a)*a*b^2)/((b*x^4
 - a)^2 + 2*(b*x^4 - a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{(a-bx^4)^{3/2}}{x^9}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5\,(-bx^4+a)^{\frac{3}{2}}b^3 - 3\,\sqrt{-bx^4+a}ab^3}{b^2x^8}}{16\,b}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

output
```
1/16*(3*b^3*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a) - (5*(-b*x^4 + a)^(
3/2)*b^3 - 3*sqrt(-b*x^4 + a)*a*b^3)/(b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{(a-bx^4)^{3/2}}{x^9}\,dx = \frac{3\,a\,\sqrt{a-b\,x^4}}{16\,x^8} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{16\,\sqrt{a}} - \frac{5\,(a-b\,x^4)^{3/2}}{16\,x^8}$$

input
```
int((a - b*x^4)^(3/2)/x^9,x)
```

output
```
(3*a*(a - b*x^4)^(1/2))/(16*x^8) - (3*b^2*atanh((a - b*x^4)^(1/2)/a^(1/2))
)/(16*a^(1/2)) - (5*(a - b*x^4)^(3/2))/(16*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{(a-bx^4)^{3/2}}{x^9}\,dx = \frac{-2\sqrt{a}\,\sqrt{-b\,x^4+a}\,a+5\sqrt{a}\,\sqrt{-b\,x^4+a}\,b\,x^4+3\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)b^2x^8}{16\sqrt{a}\,x^8}$$

input
```
int((-b*x^4+a)^(3/2)/x^9,x)
```

output
```
( - 2*sqrt(a)*sqrt(a - b*x**4)*a + 5*sqrt(a)*sqrt(a - b*x**4)*b*x**4 + 3*l
og(tan(asin((sqrt(b)*x**2)/sqrt(a))/2))*b**2*x**8)/(16*sqrt(a)*x**8)
```

3.189 $\int x^5(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int x^5(a-bx^4)^{3/2}\,dx=-\frac{a^2x^2\sqrt{a-bx^4}}{32b}+\frac{1}{16}ax^6\sqrt{a-bx^4}+\frac{1}{12}x^6(a-bx^4)^{3/2}+\frac{a^3\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{32b^{3/2}}$$

output
```
-1/32*a^2*x^2*(-b*x^4+a)^(1/2)/b+1/16*a*x^6*(-b*x^4+a)^(1/2)+1/12*x^6*(-b*
x^4+a)^(3/2)+1/32*a^3*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int x^5(a-bx^4)^{3/2}\,dx=-\frac{x^2\sqrt{a-bx^4}(3a^2-14abx^4+8b^2x^8)}{96b}-\frac{ia^3\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{32b^{3/2}}$$

input
```
Integrate[x^5*(a - b*x^4)^(3/2),x]
```

output
```
-1/96*(x^2*Sqrt[a - b*x^4]*(3*a^2 - 14*a*b*x^4 + 8*b^2*x^8))/b - ((I/32)*a
^3*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {807, 248, 248, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a-bx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int x^4 \left(a-bx^4\right)^{3/2} dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} \left(\frac{1}{2} a \int x^4 \sqrt{a-bx^4} dx^2 + \frac{1}{6} x^6 \left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \int \frac{x^4}{\sqrt{a-bx^4}} dx^2 + \frac{1}{4} x^6 \sqrt{a-bx^4}\right) + \frac{1}{6} x^6 \left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{a \int \frac{1}{\sqrt{a-bx^4}} dx^2}{2b} - \frac{x^2 \sqrt{a-bx^4}}{2b}\right) + \frac{1}{4} x^6 \sqrt{a-bx^4}\right) + \frac{1}{6} x^6 \left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{a \int \frac{1}{bx^4+1} d\frac{x^2}{\sqrt{a-bx^4}}}{2b} - \frac{x^2 \sqrt{a-bx^4}}{2b}\right) + \frac{1}{4} x^6 \sqrt{a-bx^4}\right) + \frac{1}{6} x^6 \left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{a \arctan\left(\frac{\sqrt{b} x^2}{\sqrt{a-bx^4}}\right)}{2b^{3/2}} - \frac{x^2 \sqrt{a-bx^4}}{2b}\right) + \frac{1}{4} x^6 \sqrt{a-bx^4}\right) + \frac{1}{6} x^6 \left(a-bx^4\right)^{3/2}\right)$$

input
```
Int[x^5*(a - b*x^4)^(3/2),x]
```

output
```
((x^6*(a - b*x^4)^(3/2))/6 + (a*((x^6*Sqrt[a - b*x^4])/4 + (a*(-1/2*(x^2*S
qrt[a - b*x^4])/b + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*b^(3/2)))
)/4))/2)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{x^{2}\left(8b^{2}x^{8}-14ab\,x^{4}+3a^{2}\right)\sqrt{-b\,x^{4}+a}}{96b}+\frac{a^{3}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{32b^{\frac{3}{2}}}$	67
default	$-\frac{x^{10}\sqrt{-b\,x^{4}+a}\,b}{12}+\frac{7a\,x^{6}\sqrt{-b\,x^{4}+a}}{48}-\frac{a^{2}x^{2}\sqrt{-b\,x^{4}+a}}{32b}+\frac{a^{3}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{32b^{\frac{3}{2}}}$	81
elliptic	$-\frac{x^{10}\sqrt{-b\,x^{4}+a}\,b}{12}+\frac{7a\,x^{6}\sqrt{-b\,x^{4}+a}}{48}-\frac{a^{2}x^{2}\sqrt{-b\,x^{4}+a}}{32b}+\frac{a^{3}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{32b^{\frac{3}{2}}}$	81
pseudoelliptic	$\frac{-8\sqrt{-b\,x^{4}+a}\,b^{\frac{5}{2}}x^{10}+14a\,b^{\frac{3}{2}}x^{6}\sqrt{-b\,x^{4}+a}-3a^{2}x^{2}\sqrt{-b\,x^{4}+a}\,\sqrt{b}-3\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)a^{3}}{96b^{\frac{3}{2}}}$	88

input
```
int(x^5*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/96*x^2*(8*b^2*x^8-14*a*b*x^4+3*a^2)/b*(-b*x^4+a)^(1/2)+1/32*a^3*arctan(
b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.59

$$\int x^5(a-bx^4)^{3/2}\,dx=\left[-\frac{3\,a^3\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)+2\left(8\,b^3x^{10}-14\,ab^2x^6+3\,a^2bx^2\right)\sqrt{-bx}}{192\,b^2}\right.$$
$$\left.-\frac{3\,a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)+\left(8\,b^3x^{10}-14\,ab^2x^6+3\,a^2bx^2\right)\sqrt{-bx^4+a}}{96\,b^2}\right]$$

input
```
integrate(x^5*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*a^3*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a)
 + 2*(8*b^3*x^10 - 14*a*b^2*x^6 + 3*a^2*b*x^2)*sqrt(-b*x^4 + a))/b^2, -1/9
6*(3*a^3*sqrt(b)*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)) + (8*b^3*x^10 - 14
*a*b^2*x^6 + 3*a^2*b*x^2)*sqrt(-b*x^4 + a))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.68 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.65

$$\int x^5(a-bx^4)^{3/2}\,dx=\begin{cases}\frac{ia^{\frac{5}{2}}x^2}{32b\sqrt{-1+\frac{bx^4}{a}}}-\frac{17ia^{\frac{3}{2}}x^6}{96\sqrt{-1+\frac{bx^4}{a}}}+\frac{11i\sqrt{a}bx^{10}}{48\sqrt{-1+\frac{bx^4}{a}}}-\frac{ia^3\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}-\frac{ib^2x^{14}}{12\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right|>1\\ -\frac{a^{\frac{5}{2}}x^2}{32b\sqrt{1-\frac{bx^4}{a}}}+\frac{17a^{\frac{3}{2}}x^6}{96\sqrt{1-\frac{bx^4}{a}}}-\frac{11\sqrt{a}bx^{10}}{48\sqrt{1-\frac{bx^4}{a}}}+\frac{a^3\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{32b^{\frac{3}{2}}}+\frac{b^2x^{14}}{12\sqrt{a}\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((I*a**(5/2)*x**2/(32*b*sqrt(-1 + b*x**4/a)) - 17*I*a**(3/2)*x**6
/(96*sqrt(-1 + b*x**4/a)) + 11*I*sqrt(a)*b*x**10/(48*sqrt(-1 + b*x**4/a))
- I*a**3*acosh(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)) - I*b**2*x**14/(12*sqrt
(a)*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a) > 1), (-a**(5/2)*x**2/(32*b*sqrt(1
 - b*x**4/a)) + 17*a**(3/2)*x**6/(96*sqrt(1 - b*x**4/a)) - 11*sqrt(a)*b*x*
*10/(48*sqrt(1 - b*x**4/a)) + a**3*asin(sqrt(b)*x**2/sqrt(a))/(32*b**(3/2)
) + b**2*x**14/(12*sqrt(a)*sqrt(1 - b*x**4/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.48

$$\int x^5(a-bx^4)^{3/2}\,dx=-\frac{a^3\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{32\,b^{\frac{3}{2}}}+\frac{\frac{3\sqrt{-bx^4+a}a^3b^2}{x^2}+\frac{8\left(-bx^4+a\right)^{\frac{3}{2}}a^3b}{x^6}-\frac{3\left(-bx^4+a\right)^{\frac{5}{2}}a^3}{x^{10}}}{96\left(b^4-\frac{3\left(bx^4-a\right)b^3}{x^4}+\frac{3\left(bx^4-a\right)^2b^2}{x^8}-\frac{\left(bx^4-a\right)^3b}{x^{12}}\right)}$$

input
```
integrate(x^5*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/32*a^3*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/b^(3/2) + 1/96*(3*sqrt(-b
*x^4 + a)*a^3*b^2/x^2 + 8*(-b*x^4 + a)^(3/2)*a^3*b/x^6 - 3*(-b*x^4 + a)^(5
/2)*a^3/x^10)/(b^4 - 3*(b*x^4 - a)*b^3/x^4 + 3*(b*x^4 - a)^2*b^2/x^8 - (b*
x^4 - a)^3*b/x^12)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.48

$$\int x^5(a-bx^4)^{3/2}\,dx = \frac{1}{16}\left(\sqrt{-bx^4+a}\left(2\,x^4-\frac{a}{b}\right)x^2-\frac{a^2\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{\sqrt{-b}b}\right)a - \frac{1}{96}\left(\left(2\left(4\,x^4-\frac{a}{b}\right)x^4-\frac{3\,a^2}{b^2}\right)\sqrt{-bx^4+a}x^2-\frac{3\,a^3\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{\sqrt{-b}b^2}\right)b$$

input
```
integrate(x^5*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/16*(sqrt(-b*x^4 + a)*(2*x^4 - a/b)*x^2 - a^2*log(abs(-sqrt(-b)*x^2 + sqr
t(-b*x^4 + a)))/(sqrt(-b)*b))*a - 1/96*((2*(4*x^4 - a/b)*x^4 - 3*a^2/b^2)*
sqrt(-b*x^4 + a)*x^2 - 3*a^3*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 + a)))/(s
qrt(-b)*b^2))*b
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a-bx^4)^{3/2}\,dx = \int x^5\,\left(a-b\,x^4\right)^{3/2}\,dx$$

input
```
int(x^5*(a - b*x^4)^(3/2),x)
```

output
```
int(x^5*(a - b*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int x^5 (a - bx^4)^{3/2} \, dx = \frac{3asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right) a^3 - 3\sqrt{b}\,\sqrt{-b\,x^4+a}\,a^2x^2 + 14\sqrt{b}\,\sqrt{-b\,x^4+a}\,ab\,x^6 - 8\sqrt{b}\,\sqrt{-b\,x^4+a}\,b^2x^{10}}{96\sqrt{b}\,b}$$

input
```
int(x^5*(-b*x^4+a)^(3/2),x)
```

output
```
(3*asin((sqrt(b)*x**2)/sqrt(a))*a**3 - 3*sqrt(b)*sqrt(a - b*x**4)*a**2*x**
2 + 14*sqrt(b)*sqrt(a - b*x**4)*a*b*x**6 - 8*sqrt(b)*sqrt(a - b*x**4)*b**2
*x**10)/(96*sqrt(b)*b)
```

3.190 $\int x(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int x(a-bx^4)^{3/2}\,dx=\frac{3}{16}ax^2\sqrt{a-bx^4}+\frac{1}{8}x^2(a-bx^4)^{3/2}+\frac{3a^2\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{16\sqrt{b}}$$

output
```
3/16*a*x^2*(-b*x^4+a)^(1/2)+1/8*x^2*(-b*x^4+a)^(3/2)+3/16*a^2*arctan(b^(1/
2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int x(a-bx^4)^{3/2}\,dx=-\frac{1}{16}x^2\sqrt{a-bx^4}(-5a+2bx^4)-\frac{3ia^2\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{16\sqrt{b}}$$

input
```
Integrate[x*(a - b*x^4)^(3/2),x]
```

output
```
-1/16*(x^2*Sqrt[a - b*x^4]*(-5*a + 2*b*x^4)) - (((3*I)/16)*a^2*Log[I*Sqrt[
b]*x^2 + Sqrt[a - b*x^4]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {807, 211, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-bx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \left(a-bx^4\right)^{3/2}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{4}a\int \sqrt{a-bx^4}dx^2+\frac{1}{4}x^2\left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{a-bx^4}}dx^2+\frac{1}{2}x^2\sqrt{a-bx^4}\right)+\frac{1}{4}x^2\left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}+\frac{1}{2}x^2\sqrt{a-bx^4}\right)+\frac{1}{4}x^2\left(a-bx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{3}{4}a\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{1}{2}x^2\sqrt{a-bx^4}\right)+\frac{1}{4}x^2\left(a-bx^4\right)^{3/2}\right)$$

input
```
Int[x*(a - b*x^4)^(3/2),x]
```

output
```
((x^2*(a - b*x^4)^(3/2))/4 + (3*a*((x^2*Sqrt[a - b*x^4])/2 + (a*ArcTan[(Sq
rt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b])))/4)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x^{2}\left(-2 b \,x^{4}+5 a \right) \sqrt{-b \,x^{4}+a}}{16}+\frac{3 a^{2} \arctan \left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b \,x^{4}+a}}\right)}{16 \sqrt{b}}$	53
default	$\frac{3 a^{2} \arctan \left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b \,x^{4}+a}}\right)}{16 \sqrt{b}}-\frac{x^{6} \sqrt{-b \,x^{4}+a}\, b}{8}+\frac{5 a \,x^{2} \sqrt{-b \,x^{4}+a}}{16}$	60
elliptic	$\frac{3 a^{2} \arctan \left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b \,x^{4}+a}}\right)}{16 \sqrt{b}}-\frac{x^{6} \sqrt{-b \,x^{4}+a}\, b}{8}+\frac{5 a \,x^{2} \sqrt{-b \,x^{4}+a}}{16}$	60
pseudoelliptic	$-\frac{3 \arctan \left(\frac{\sqrt{-b \,x^{4}+a}}{\sqrt{b}\, x^{2}}\right) a^{2}}{16 \sqrt{b}}+\frac{5 x^{2} \left(-\frac{2 b^{\frac{3}{2}} x^{4}}{5}+a \sqrt{b}\right) \sqrt{-b \,x^{4}+a}}{16 \sqrt{b}}$	60

input
```
int(x*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*x^2*(-2*b*x^4+5*a)*(-b*x^4+a)^(1/2)+3/16*a^2*arctan(b^(1/2)*x^2/(-b*x
^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.82

$$\int x(a-bx^4)^{3/2}\,dx = \left[-\frac{3\,a^2\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)+2\,(2\,b^2x^6-5\,abx^2)\sqrt{-bx^4+a}}{32\,b}, -\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)+(2\,b^2x^6-5\,abx^2)\sqrt{-bx^4+a}}{16\,b}\right]$$

input
```
integrate(x*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/32*(3*a^2*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a)
+ 2*(2*b^2*x^6 - 5*a*b*x^2)*sqrt(-b*x^4 + a))/b, -1/16*(3*a^2*sqrt(b)*arct
an(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)) + (2*b^2*x^6 - 5*a*b*x^2)*sqrt(-b*x^4 +
 a))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.55

$$\int x(a-bx^4)^{3/2}\,dx = \begin{cases} -\frac{5ia^{\frac{3}{2}}x^2}{16\sqrt{-1+\frac{bx^4}{a}}}+\frac{7i\sqrt{a}bx^6}{16\sqrt{-1+\frac{bx^4}{a}}}-\frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}}-\frac{ib^2x^{10}}{8\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right|>1 \\ \frac{5a^{\frac{3}{2}}x^2\sqrt{1-\frac{bx^4}{a}}}{16}-\frac{\sqrt{a}bx^6\sqrt{1-\frac{bx^4}{a}}}{8}+\frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{16\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-5*I*a**(3/2)*x**2/(16*sqrt(-1 + b*x**4/a)) + 7*I*sqrt(a)*b*x**
6/(16*sqrt(-1 + b*x**4/a)) - 3*I*a**2*acosh(sqrt(b)*x**2/sqrt(a))/(16*sqrt
(b)) - I*b**2*x**10/(8*sqrt(a)*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a) > 1), (
5*a**(3/2)*x**2*sqrt(1 - b*x**4/a)/16 - sqrt(a)*b*x**6*sqrt(1 - b*x**4/a)/
8 + 3*a**2*asin(sqrt(b)*x**2/sqrt(a))/(16*sqrt(b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int x(a-bx^4)^{3/2}\,dx = -\frac{3\,a^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{16\,\sqrt{b}} + \frac{\frac{3\,\sqrt{-bx^4+a}a^2b}{x^2} + \frac{5\,(-bx^4+a)^{\frac{3}{2}}a^2}{x^6}}{16\left(b^2 - \frac{2\,(bx^4-a)b}{x^4} + \frac{(bx^4-a)^2}{x^8}\right)}$$

input
```
integrate(x*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-3/16*a^2*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/sqrt(b) + 1/16*(3*sqrt(-b
*x^4 + a)*a^2*b/x^2 + 5*(-b*x^4 + a)^(3/2)*a^2/x^6)/(b^2 - 2*(b*x^4 - a)*b
/x^4 + (b*x^4 - a)^2/x^8)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

$$\int x(a-bx^4)^{3/2}\,dx = \frac{1}{4}\left(\sqrt{-bx^4+a}x^2 - \frac{a\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{\sqrt{-b}}\right)a - \frac{1}{16}\left(\sqrt{-bx^4+a}\left(2\,x^4-\frac{a}{b}\right)x^2 - \frac{a^2\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{\sqrt{-b}b}\right)b$$

input
```
integrate(x*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(-b*x^4 + a)*x^2 - a*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 + a)))/s
qrt(-b))*a - 1/16*(sqrt(-b*x^4 + a)*(2*x^4 - a/b)*x^2 - a^2*log(abs(-sqrt(
-b)*x^2 + sqrt(-b*x^4 + a)))/(sqrt(-b)*b))*b
```

Mupad [F(-1)]

Timed out.

$$\int x(a-bx^4)^{3/2}\,dx=\int x\,(a-b\,x^4)^{3/2}\,dx$$

input
```
int(x*(a - b*x^4)^(3/2),x)
```

output
```
int(x*(a - b*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int x(a-bx^4)^{3/2}\,dx=\frac{3asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)a^2+5\sqrt{b}\,\sqrt{-b\,x^4+a}\,a\,x^2-2\sqrt{b}\,\sqrt{-b\,x^4+a}\,b\,x^6}{16\sqrt{b}}$$

input
```
int(x*(-b*x^4+a)^(3/2),x)
```

output
```
(3*asin((sqrt(b)*x**2)/sqrt(a))*a**2 + 5*sqrt(b)*sqrt(a - b*x**4)*a*x**2 -
 2*sqrt(b)*sqrt(a - b*x**4)*b*x**6)/(16*sqrt(b))
```

3.191 $\int \frac{(a-bx^4)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{(a-bx^4)^{3/2}}{x^3} \, dx = -\frac{a\sqrt{a-bx^4}}{2x^2} - \frac{1}{4}bx^2\sqrt{a-bx^4} - \frac{3}{4}a\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)$$

output
```
-1/2*a*(-b*x^4+a)^(1/2)/x^2-1/4*b*x^2*(-b*x^4+a)^(1/2)-3/4*a*b^(1/2)*arcta
n(b^(1/2)*x^2/(-b*x^4+a)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{(a-bx^4)^{3/2}}{x^3} \, dx = \frac{(-2a-bx^4)\sqrt{a-bx^4}}{4x^2} + \frac{3}{4}ia\sqrt{b}\log\left(i\sqrt{b}x^2 + \sqrt{a-bx^4}\right)$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^3,x]
```

output
```
((-2*a - b*x^4)*Sqrt[a - b*x^4])/(4*x^2) + ((3*I)/4)*a*Sqrt[b]*Log[I*Sqrt[
b]*x^2 + Sqrt[a - b*x^4]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 247, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{(a-bx^4)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(-3b\int \sqrt{a-bx^4}dx^2-\frac{(a-bx^4)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(-3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{a-bx^4}}dx^2+\frac{1}{2}x^2\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(-3b\left(\frac{1}{2}a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}+\frac{1}{2}x^2\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(-3b\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}+\frac{1}{2}x^2\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{x^2}\right)$$

input
```
Int[(a - b*x^4)^(3/2)/x^3,x]
```

output
```
(-((a - b*x^4)^(3/2)/x^2) - 3*b*((x^2*Sqrt[a - b*x^4])/2 + (a*ArcTan[(Sqrt
[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b])))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{\sqrt{-b\,x^4+a}\,(b\,x^4+2a)}{4x^2}-\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,x^2}{\sqrt{-b\,x^4+a}}\right)}{4}$	50
pseudoelliptic	$\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{-b\,x^4+a}}{\sqrt{b}\,x^2}\right)x^2-(b\,x^4+2a)\sqrt{-b\,x^4+a}}{4x^2}$	55
default	$-\frac{a\sqrt{-b\,x^4+a}}{2x^2}-\frac{b\,x^2\sqrt{-b\,x^4+a}}{4}-\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,x^2}{\sqrt{-b\,x^4+a}}\right)}{4}$	58
elliptic	$-\frac{a\sqrt{-b\,x^4+a}}{2x^2}-\frac{b\,x^2\sqrt{-b\,x^4+a}}{4}-\frac{3a\sqrt{b}\,\arctan\left(\frac{\sqrt{b}\,x^2}{\sqrt{-b\,x^4+a}}\right)}{4}$	58

input
```
int((-b*x^4+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-b*x^4+a)^(1/2)*(b*x^4+2*a)/x^2-3/4*a*b^(1/2)*arctan(b^(1/2)*x^2/(-b
*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{(a-bx^4)^{3/2}}{x^3}\,dx=\left[\frac{3\,a\sqrt{-b}x^2\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)-2\,(bx^4+2\,a)\sqrt{-bx^4+a}}{8\,x^2},\ \frac{3\,a\sqrt{b}x\cdots}{}\right.$$

input
```
integrate((-b*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*a*sqrt(-b)*x^2*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a)
- 2*(b*x^4 + 2*a)*sqrt(-b*x^4 + a))/x^2, 1/4*(3*a*sqrt(b)*x^2*arctan(sqrt(
-b*x^4 + a)/(sqrt(b)*x^2)) - (b*x^4 + 2*a)*sqrt(-b*x^4 + a))/x^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.82

$$\int \frac{(a-bx^4)^{3/2}}{x^3}\,dx = \begin{cases} \frac{ia^{\frac{3}{2}}}{2x^2\sqrt{-1+\frac{bx^4}{a}}} - \frac{i\sqrt{a}bx^2}{4\sqrt{-1+\frac{bx^4}{a}}} + \frac{3ia\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} - \frac{ib^2x^6}{4\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{a^{\frac{3}{2}}}{2x^2\sqrt{1-\frac{bx^4}{a}}} + \frac{\sqrt{a}bx^2}{4\sqrt{1-\frac{bx^4}{a}}} - \frac{3a\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4} + \frac{b^2x^6}{4\sqrt{a}\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**3,x)
```

output
```
Piecewise((I*a**(3/2)/(2*x**2*sqrt(-1 + b*x**4/a)) - I*sqrt(a)*b*x**2/(4*s
qrt(-1 + b*x**4/a)) + 3*I*a*sqrt(b)*acosh(sqrt(b)*x**2/sqrt(a))/4 - I*b**2
*x**6/(4*sqrt(a)*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a) > 1), (-a**(3/2)/(2*x
**2*sqrt(1 - b*x**4/a)) + sqrt(a)*b*x**2/(4*sqrt(1 - b*x**4/a)) - 3*a*sqrt
(b)*asin(sqrt(b)*x**2/sqrt(a))/4 + b**2*x**6/(4*sqrt(a)*sqrt(1 - b*x**4/a)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{(a-bx^4)^{3/2}}{x^3}\,dx = \frac{3}{4}\,a\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right) - \frac{\sqrt{-bx^4+a}a}{2\,x^2} - \frac{\sqrt{-bx^4+a}ab}{4\left(b-\frac{bx^4-a}{x^4}\right)x^2}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/4*a*sqrt(b)*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)) - 1/2*sqrt(-b*x^4 + a
)*a/x^2 - 1/4*sqrt(-b*x^4 + a)*a*b/((b - (b*x^4 - a)/x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{(a-bx^4)^{3/2}}{x^3}\,dx = -\frac{1}{4}\sqrt{-bx^4+a}bx^2 - \frac{3}{8}a\sqrt{-b}\log\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2\right) + \frac{a^2\sqrt{-b}}{\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*sqrt(-b*x^4 + a)*b*x^2 - 3/8*a*sqrt(-b)*log((sqrt(-b)*x^2 - sqrt(-b*x
^4 + a))^2) + a^2*sqrt(-b)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{x^3}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{x^3}\,dx$$

input
```
int((a - b*x^4)^(3/2)/x^3,x)
```

output
```
int((a - b*x^4)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.71

$$\int \frac{(a-bx^4)^{3/2}}{x^3}\,dx = \frac{-3\sqrt{b}\,asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)a\,x^2-2\sqrt{-b\,x^4+a}\,a-\sqrt{-b\,x^4+a}\,b\,x^4}{4x^2}$$

input
```
int((-b*x^4+a)^(3/2)/x^3,x)
```

output
```
( - 3*sqrt(b)*asin((sqrt(b)*x**2)/sqrt(a))*a*x**2 - 2*sqrt(a - b*x**4)*a -
 sqrt(a - b*x**4)*b*x**4)/(4*x**2)
```

3.192 $\int \frac{(a-bx^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{(a-bx^4)^{3/2}}{x^7} dx = -\frac{a\sqrt{a-bx^4}}{6x^6} + \frac{2b\sqrt{a-bx^4}}{3x^2} + \frac{1}{2}b^{3/2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)$$

output
```
-1/6*a*(-b*x^4+a)^(1/2)/x^6+2/3*b*(-b*x^4+a)^(1/2)/x^2+1/2*b^(3/2)*arctan(
b^(1/2)*x^2/(-b*x^4+a)^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{(a-bx^4)^{3/2}}{x^7} dx = \frac{\sqrt{a-bx^4}(-a+4bx^4)}{6x^6} - \frac{1}{2}ib^{3/2}\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^7,x]
```

output
```
(Sqrt[a - b*x^4]*(-a + 4*b*x^4))/(6*x^6) - (I/2)*b^(3/2)*Log[I*Sqrt[b]*x^2
 + Sqrt[a - b*x^4]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 247, 247, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^7} \, dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2} \int \frac{(a-bx^4)^{3/2}}{x^8} dx^2$$

$$\Big\downarrow \; 247$$

$$\frac{1}{2} \left(-b \int \frac{\sqrt{a-bx^4}}{x^4} dx^2 - \frac{(a-bx^4)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \; 247$$

$$\frac{1}{2} \left(-b \left(-b \int \frac{1}{\sqrt{a-bx^4}} dx^2 - \frac{\sqrt{a-bx^4}}{x^2} \right) - \frac{(a-bx^4)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2} \left(-b \left(-b \int \frac{1}{bx^4+1} d\frac{x^2}{\sqrt{a-bx^4}} - \frac{\sqrt{a-bx^4}}{x^2} \right) - \frac{(a-bx^4)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2} \left(-b \left(-\sqrt{b} \arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}} \right) - \frac{\sqrt{a-bx^4}}{x^2} \right) - \frac{(a-bx^4)^{3/2}}{3x^6} \right)$$

input
```
Int[(a - b*x^4)^(3/2)/x^7,x]
```

output
```
(-1/3*(a - b*x^4)^(3/2)/x^6 - b*(-(Sqrt[a - b*x^4]/x^2) - Sqrt[b]*ArcTan[(
Sqrt[b]*x^2)/Sqrt[a - b*x^4]]))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-4b\,x^{4}+a\right)}{6x^{6}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2}$	48
pseudoelliptic	$\frac{-3b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)x^{6}-\left(-4b\,x^{4}+a\right)\sqrt{-b\,x^{4}+a}}{6x^{6}}$	53
default	$-\frac{a\sqrt{-b\,x^{4}+a}}{6x^{6}}+\frac{2b\sqrt{-b\,x^{4}+a}}{3x^{2}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2}$	57
elliptic	$-\frac{a\sqrt{-b\,x^{4}+a}}{6x^{6}}+\frac{2b\sqrt{-b\,x^{4}+a}}{3x^{2}}+\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2}$	57

input
```
int((-b*x^4+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b*x^4+a)^(1/2)*(-4*b*x^4+a)/x^6+1/2*b^(3/2)*arctan(b^(1/2)*x^2/(-b*
x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.74

$$\int \frac{(a-bx^4)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{-b}bx^6\log\left(2\,bx^4+2\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)+2\left(4\,bx^4-a\right)\sqrt{-bx^4+a}}{12\,x^6}, -\frac{3\,b^{\frac{3}{2}}x^6\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)-\left(4\,bx^4-a\right)\sqrt{-bx^4+a}}{6\,x^6}\right]$$

input
```
integrate((-b*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(-b)*b*x^6*log(2*b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a)
 + 2*(4*b*x^4 - a)*sqrt(-b*x^4 + a))/x^6, -1/6*(3*b^(3/2)*x^6*arctan(sqrt(
-b*x^4 + a)/(sqrt(b)*x^2)) - (4*b*x^4 - a)*sqrt(-b*x^4 + a))/x^6]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.79

$$\int \frac{(a-bx^4)^{3/2}}{x^7}\,dx = \begin{cases} -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{6x^4}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{3}-\frac{ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2}+\frac{ib^{\frac{3}{2}}\log\left(\frac{a}{bx^4}\right)}{4}+\frac{b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ -\frac{ia\sqrt{b}\sqrt{-\frac{a}{bx^4}+1}}{6x^4}+\frac{2ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{3}+\frac{ib^{\frac{3}{2}}\log\left(\frac{a}{bx^4}\right)}{4}-\frac{ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx^4}+1}+1\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**7,x)
```

output
```
Piecewise((-a*sqrt(b)*sqrt(a/(b*x**4) - 1)/(6*x**4) + 2*b**(3/2)*sqrt(a/(b
*x**4) - 1)/3 - I*b**(3/2)*log(sqrt(a)/(sqrt(b)*x**2))/2 + I*b**(3/2)*log(
a/(b*x**4))/4 + b**(3/2)*asin(sqrt(b)*x**2/sqrt(a))/2, Abs(a/(b*x**4)) > 1
), (-I*a*sqrt(b)*sqrt(-a/(b*x**4) + 1)/(6*x**4) + 2*I*b**(3/2)*sqrt(-a/(b*
x**4) + 1)/3 + I*b**(3/2)*log(a/(b*x**4))/4 - I*b**(3/2)*log(sqrt(-a/(b*x*
*4) + 1) + 1)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int \frac{(a-bx^4)^{3/2}}{x^7}\,dx = -\frac{1}{2}\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)+\frac{\sqrt{-bx^4+a}b}{2\,x^2}-\frac{(-bx^4+a)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/2*b^(3/2)*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)) + 1/2*sqrt(-b*x^4 + a)
*b/x^2 - 1/6*(-b*x^4 + a)^(3/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(56) = 112$.

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.03

$$\int \frac{(a-bx^4)^{3/2}}{x^7}\,dx = \frac{1}{4}\sqrt{-b}b\log\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2\right) - \frac{2\left(3\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^4a\sqrt{-b}b-3\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2a^2\sqrt{-b}b+2\,a^3\sqrt{-b}b\right)}{3\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^3}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
1/4*sqrt(-b)*b*log((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2) - 2/3*(3*(sqrt(-b)
*x^2 - sqrt(-b*x^4 + a))^4*a*sqrt(-b)*b - 3*(sqrt(-b)*x^2 - sqrt(-b*x^4 +
a))^2*a^2*sqrt(-b)*b + 2*a^3*sqrt(-b)*b)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a)
)^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{x^7}\,dx=\int \frac{(a-b\,x^4)^{3/2}}{x^7}\,dx$$

input
```
int((a - b*x^4)^(3/2)/x^7,x)
```

output
```
int((a - b*x^4)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.72

$$\int \frac{(a-bx^4)^{3/2}}{x^7}\,dx=\frac{3\sqrt{b}\,asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)b\,x^6-\sqrt{-b\,x^4+a}\,a+4\sqrt{-b\,x^4+a}\,b\,x^4}{6x^6}$$

input
```
int((-b*x^4+a)^(3/2)/x^7,x)
```

output
```
(3*sqrt(b)*asin((sqrt(b)*x**2)/sqrt(a))*b*x**6 - sqrt(a - b*x**4)*a + 4*sq
rt(a - b*x**4)*b*x**4)/(6*x**6)
```

3.193 $\int \frac{(a-bx^4)^{3/2}}{x^{11}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}} \, dx = -\frac{(a-bx^4)^{5/2}}{10ax^{10}}$$

output
```
-1/10*(-b*x^4+a)^(5/2)/a/x^10
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}} \, dx = -\frac{(a-bx^4)^{5/2}}{10ax^{10}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^11,x]
```

output
```
-1/10*(a - b*x^4)^(5/2)/(a*x^10)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a-bx^4)^{5/2}}{10ax^{10}}$$

input
```
Int[(a - b*x^4)^(3/2)/x^11,x]
```

output
```
-1/10*(a - b*x^4)^(5/2)/(a*x^10)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(-b\,x^4+a)^{\frac{5}{2}}}{10a\,x^{10}}$	19
pseudoelliptic	$-\frac{(-b\,x^4+a)^{\frac{5}{2}}}{10a\,x^{10}}$	19
orering	$-\frac{(-b\,x^4+a)^{\frac{5}{2}}}{10a\,x^{10}}$	19
default	$-\frac{\sqrt{-b\,x^4+a}\,(b^2x^8-2ab\,x^4+a^2)}{10x^{10}a}$	37
trager	$-\frac{\sqrt{-b\,x^4+a}\,(b^2x^8-2ab\,x^4+a^2)}{10x^{10}a}$	37
risch	$-\frac{\sqrt{-b\,x^4+a}\,(b^2x^8-2ab\,x^4+a^2)}{10x^{10}a}$	37
elliptic	$-\frac{\sqrt{-b\,x^4+a}\,(b^2x^8-2ab\,x^4+a^2)}{10x^{10}a}$	37

input
```
int((-b*x^4+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-b*x^4+a)^(5/2)/a/x^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx = -\frac{(b^2x^8-2\,abx^4+a^2)\sqrt{-bx^4+a}}{10\,ax^{10}}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/10*(b^2*x^8 - 2*a*b*x^4 + a^2)*sqrt(-b*x^4 + a)/(a*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 6.50

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx = \begin{cases} -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{10x^8} + \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{5x^4} - \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^4}-1}}{10a} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ -\frac{ia\sqrt{b}\sqrt{-\frac{a}{bx^4}+1}}{10x^8} + \frac{ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{5x^4} - \frac{ib^{\frac{5}{2}}\sqrt{-\frac{a}{bx^4}+1}}{10a} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**11,x)
```

output
```
Piecewise((-a*sqrt(b)*sqrt(a/(b*x**4) - 1)/(10*x**8) + b**(3/2)*sqrt(a/(b*
x**4) - 1)/(5*x**4) - b**(5/2)*sqrt(a/(b*x**4) - 1)/(10*a), Abs(a/(b*x**4)
) > 1), (-I*a*sqrt(b)*sqrt(-a/(b*x**4) + 1)/(10*x**8) + I*b**(3/2)*sqrt(-a
/(b*x**4) + 1)/(5*x**4) - I*b**(5/2)*sqrt(-a/(b*x**4) + 1)/(10*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx = -\frac{(-bx^4+a)^{\frac{5}{2}}}{10\,ax^{10}}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/10*(-b*x^4 + a)^(5/2)/(a*x^10)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(18) = 36$.

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.27

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx = \frac{5\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^8\sqrt{-b}b^2+10\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^4a^2\sqrt{-b}b^2+a^4\sqrt{-b}b^2}{5\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^5}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

output
```
1/5*(5*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^8*sqrt(-b)*b^2 + 10*(sqrt(-b)*x^2
 - sqrt(-b*x^4 + a))^4*a^2*sqrt(-b)*b^2 + a^4*sqrt(-b)*b^2)/((sqrt(-b)*x^2
 - sqrt(-b*x^4 + a))^2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx = -\frac{(a-b\,x^4)^{5/2}}{10\,a\,x^{10}}$$

input
```
int((a - b*x^4)^(3/2)/x^11,x)
```

output
```
-(a - b*x^4)^(5/2)/(10*a*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{(a-bx^4)^{3/2}}{x^{11}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-b^2x^8+2ab\,x^4-a^2)}{10a\,x^{10}}$$

input
```
int((-b*x^4+a)^(3/2)/x^11,x)
```

output
```
(sqrt(a - b*x**4)*( - a**2 + 2*a*b*x**4 - b**2*x**8))/(10*a*x**10)
```

3.194 $\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = -\frac{(a-bx^4)^{5/2}}{14ax^{14}} - \frac{b(a-bx^4)^{5/2}}{35a^2x^{10}}$$

output
```
-1/14*(-b*x^4+a)^(5/2)/a/x^14-1/35*b*(-b*x^4+a)^(5/2)/a^2/x^10
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = \frac{(-5a-2bx^4)\,(a-bx^4)^{5/2}}{70a^2x^{14}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^15,x]
```

output
```
((-5*a - 2*b*x^4)*(a - b*x^4)^(5/2))/(70*a^2*x^14)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}dx$$
$$\Big\downarrow \text{803}$$
$$\frac{2b\int \frac{(a-bx^4)^{3/2}}{x^{11}}dx}{7a}-\frac{(a-bx^4)^{5/2}}{14ax^{14}}$$
$$\Big\downarrow \text{796}$$
$$-\frac{b(a-bx^4)^{5/2}}{35a^2x^{10}}-\frac{(a-bx^4)^{5/2}}{14ax^{14}}$$

input
```
Int[(a - b*x^4)^(3/2)/x^15,x]
```

output
```
-1/14*(a - b*x^4)^(5/2)/(a*x^14) - (b*(a - b*x^4)^(5/2))/(35*a^2*x^10)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(2b\,x^{4}+5a\right)}{70x^{14}a^{2}}$	29
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(2b\,x^{4}+5a\right)}{70x^{14}a^{2}}$	29
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}}\left(2b\,x^{4}+5a\right)}{70x^{14}a^{2}}$	29
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+5a\right)\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)}{70a^{2}x^{14}}$	47
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+5a\right)\left(b^{2}x^{8}-2ab\,x^{4}+a^{2}\right)}{70a^{2}x^{14}}$	47
trager	$-\frac{\left(2b^{3}x^{12}+a\,b^{2}x^{8}-8a^{2}b\,x^{4}+5a^{3}\right)\sqrt{-b\,x^{4}+a}}{70x^{14}a^{2}}$	50
risch	$-\frac{\left(2b^{3}x^{12}+a\,b^{2}x^{8}-8a^{2}b\,x^{4}+5a^{3}\right)\sqrt{-b\,x^{4}+a}}{70x^{14}a^{2}}$	50

input
```
int((-b*x^4+a)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/70*(-b*x^4+a)^(5/2)*(2*b*x^4+5*a)/x^14/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = -\frac{(2\,b^3x^{12}+ab^2x^8-8\,a^2bx^4+5\,a^3)\sqrt{-bx^4+a}}{70\,a^2x^{14}}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^15,x, algorithm="fricas")
```

output
```
-1/70*(2*b^3*x^12 + a*b^2*x^8 - 8*a^2*b*x^4 + 5*a^3)*sqrt(-b*x^4 + a)/(a^2
*x^14)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 7.28

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = \begin{cases} -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{14x^{12}} + \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{35x^8} - \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^4}-1}}{70ax^4} - \frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^4}-1}}{35a^2} \\ \frac{5ia^4b^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{x^8(-70a^3bx^4+70a^2b^2x^8)} - \frac{13ia^3b^{\frac{5}{2}}\sqrt{-\frac{a}{bx^4}+1}}{x^4(-70a^3bx^4+70a^2b^2x^8)} + \frac{9ia^2b^{\frac{7}{2}}\sqrt{-\frac{a}{bx^4}+1}}{-70a^3bx^4+70a^2b^2x^8} + \frac{iab^{\frac{9}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{-70a^3bx^4+70a^2b^2x^8} - \end{cases}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**15,x)
```

output
```
Piecewise((-a*sqrt(b)*sqrt(a/(b*x**4) - 1)/(14*x**12) + 4*b**(3/2)*sqrt(a/
(b*x**4) - 1)/(35*x**8) - b**(5/2)*sqrt(a/(b*x**4) - 1)/(70*a*x**4) - b**(
7/2)*sqrt(a/(b*x**4) - 1)/(35*a**2), Abs(a/(b*x**4)) > 1), (5*I*a**4*b**(3
/2)*sqrt(-a/(b*x**4) + 1)/(x**8*(-70*a**3*b*x**4 + 70*a**2*b**2*x**8)) - 1
3*I*a**3*b**(5/2)*sqrt(-a/(b*x**4) + 1)/(x**4*(-70*a**3*b*x**4 + 70*a**2*b
**2*x**8)) + 9*I*a**2*b**(7/2)*sqrt(-a/(b*x**4) + 1)/(-70*a**3*b*x**4 + 70
*a**2*b**2*x**8) + I*a*b**(9/2)*x**4*sqrt(-a/(b*x**4) + 1)/(-70*a**3*b*x**
4 + 70*a**2*b**2*x**8) - 2*I*b**(11/2)*x**8*sqrt(-a/(b*x**4) + 1)/(-70*a**
3*b*x**4 + 70*a**2*b**2*x**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = -\frac{\frac{7\,(-bx^4+a)^{\frac{5}{2}}b}{x^{10}} + \frac{5\,(-bx^4+a)^{\frac{7}{2}}}{x^{14}}}{70\,a^2}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^15,x, algorithm="maxima")
```

output
```
-1/70*(7*(-b*x^4 + a)^(5/2)*b/x^10 + 5*(-b*x^4 + a)^(7/2)/x^14)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(38) = 76$.

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 4.91

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx =$$

$$-\frac{2\left(35\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^{10}\sqrt{-b}b^3+35\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^{8}a\sqrt{-b}b^3+70\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right.\right.}{35\left(\left(\sqrt{-b}x^2-\sqrt{-}\right.\right.}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^15,x, algorithm="giac")
```

output
```
-2/35*(35*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^10*sqrt(-b)*b^3 + 35*(sqrt(-b)
*x^2 - sqrt(-b*x^4 + a))^8*a*sqrt(-b)*b^3 + 70*(sqrt(-b)*x^2 - sqrt(-b*x^4
 + a))^6*a^2*sqrt(-b)*b^3 + 14*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^4*a^3*sqr
t(-b)*b^3 + 7*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2*a^4*sqrt(-b)*b^3 - a^5*s
qrt(-b)*b^3)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = \frac{4\,b\,\sqrt{a-b\,x^4}}{35\,x^{10}} - \frac{a\,\sqrt{a-b\,x^4}}{14\,x^{14}} - \frac{b^3\,\sqrt{a-b\,x^4}}{35\,a^2\,x^2} - \frac{b^2\,\sqrt{a-b\,x^4}}{70\,a\,x^6}$$

input
```
int((a - b*x^4)^(3/2)/x^15,x)
```

output
```
(4*b*(a - b*x^4)^(1/2))/(35*x^10) - (a*(a - b*x^4)^(1/2))/(14*x^14) - (b^3
*(a - b*x^4)^(1/2))/(35*a^2*x^2) - (b^2*(a - b*x^4)^(1/2))/(70*a*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{(a-bx^4)^{3/2}}{x^{15}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-2b^3x^{12}-a\,b^2x^8+8a^2b\,x^4-5a^3)}{70a^2x^{14}}$$

input
```
int((-b*x^4+a)^(3/2)/x^15,x)
```

output
```
(sqrt(a - b*x**4)*( - 5*a**3 + 8*a**2*b*x**4 - a*b**2*x**8 - 2*b**3*x**12)
)/(70*a**2*x**14)
```

3.195 $\int \frac{(a-bx^4)^{3/2}}{x^{19}} dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}} dx = -\frac{(a-bx^4)^{5/2}}{18ax^{18}} - \frac{2b(a-bx^4)^{5/2}}{63a^2x^{14}} - \frac{4b^2(a-bx^4)^{5/2}}{315a^3x^{10}}$$

output
```
-1/18*(-b*x^4+a)^(5/2)/a/x^18-2/63*b*(-b*x^4+a)^(5/2)/a^2/x^14-4/315*b^2*(
-b*x^4+a)^(5/2)/a^3/x^10
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}} dx = \frac{(a-bx^4)^{5/2}(-35a^2-20abx^4-8b^2x^8)}{630a^3x^{18}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^19,x]
```

output
```
((a - b*x^4)^(5/2)*(-35*a^2 - 20*a*b*x^4 - 8*b^2*x^8))/(630*a^3*x^18)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}} \, dx$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\int \frac{(a-bx^4)^{3/2}}{x^{15}}dx}{9a}-\frac{(a-bx^4)^{5/2}}{18ax^{18}}$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\left(\frac{2b\int \frac{(a-bx^4)^{3/2}}{x^{11}}dx}{7a}-\frac{(a-bx^4)^{5/2}}{14ax^{14}}\right)}{9a}-\frac{(a-bx^4)^{5/2}}{18ax^{18}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4b\left(-\frac{b(a-bx^4)^{5/2}}{35a^2x^{10}}-\frac{(a-bx^4)^{5/2}}{14ax^{14}}\right)}{9a}-\frac{(a-bx^4)^{5/2}}{18ax^{18}}$$

input
```
Int[(a - b*x^4)^(3/2)/x^19,x]
```

output
```
-1/18*(a - b*x^4)^(5/2)/(a*x^18) + (4*b*(-1/14*(a - b*x^4)^(5/2)/(a*x^14)
- (b*(a - b*x^4)^(5/2))/(35*a^2*x^10)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}} \left(8b^{2} x^{8}+20ab\,x^{4}+35a^{2}\right)}{630x^{18} a^{3}}$	40
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}} \left(8b^{2} x^{8}+20ab\,x^{4}+35a^{2}\right)}{630x^{18} a^{3}}$	40
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{2}} \left(8b^{2} x^{8}+20ab\,x^{4}+35a^{2}\right)}{630x^{18} a^{3}}$	40
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2} x^{8}+20ab\,x^{4}+35a^{2}\right) \left(b^{2} x^{8}-2ab\,x^{4}+a^{2}\right)}{630a^{3} x^{18}}$	58
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2} x^{8}+20ab\,x^{4}+35a^{2}\right) \left(b^{2} x^{8}-2ab\,x^{4}+a^{2}\right)}{630a^{3} x^{18}}$	58
trager	$-\frac{\left(8b^{4} x^{16}+4a\,b^{3} x^{12}+3a^{2} b^{2} x^{8}-50a^{3} b\,x^{4}+35a^{4}\right) \sqrt{-b\,x^{4}+a}}{630x^{18} a^{3}}$	62
risch	$-\frac{\left(8b^{4} x^{16}+4a\,b^{3} x^{12}+3a^{2} b^{2} x^{8}-50a^{3} b\,x^{4}+35a^{4}\right) \sqrt{-b\,x^{4}+a}}{630x^{18} a^{3}}$	62

input
```
int((-b*x^4+a)^(3/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/630*(-b*x^4+a)^(5/2)*(8*b^2*x^8+20*a*b*x^4+35*a^2)/x^18/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}}\,dx = -\frac{(8\,b^4x^{16}+4\,ab^3x^{12}+3\,a^2b^2x^8-50\,a^3bx^4+35\,a^4)\sqrt{-bx^4+a}}{630\,a^3x^{18}}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^19,x, algorithm="fricas")
```

output
```
-1/630*(8*b^4*x^16 + 4*a*b^3*x^12 + 3*a^2*b^2*x^8 - 50*a^3*b*x^4 + 35*a^4)
*sqrt(-b*x^4 + a)/(a^3*x^18)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 857, normalized size of antiderivative = 12.07

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**19,x)
```

output
```
Piecewise((-35*a**6*b**(9/2)*sqrt(a/(b*x**4) - 1)/(630*a**5*b**4*x**16 - 1
260*a**4*b**5*x**20 + 630*a**3*b**6*x**24) + 120*a**5*b**(11/2)*x**4*sqrt(
a/(b*x**4) - 1)/(630*a**5*b**4*x**16 - 1260*a**4*b**5*x**20 + 630*a**3*b**
6*x**24) - 138*a**4*b**(13/2)*x**8*sqrt(a/(b*x**4) - 1)/(630*a**5*b**4*x**
16 - 1260*a**4*b**5*x**20 + 630*a**3*b**6*x**24) + 52*a**3*b**(15/2)*x**12
*sqrt(a/(b*x**4) - 1)/(630*a**5*b**4*x**16 - 1260*a**4*b**5*x**20 + 630*a*
*3*b**6*x**24) - 3*a**2*b**(17/2)*x**16*sqrt(a/(b*x**4) - 1)/(630*a**5*b**
4*x**16 - 1260*a**4*b**5*x**20 + 630*a**3*b**6*x**24) + 12*a*b**(19/2)*x**
20*sqrt(a/(b*x**4) - 1)/(630*a**5*b**4*x**16 - 1260*a**4*b**5*x**20 + 630*
a**3*b**6*x**24) - 8*b**(21/2)*x**24*sqrt(a/(b*x**4) - 1)/(630*a**5*b**4*x
**16 - 1260*a**4*b**5*x**20 + 630*a**3*b**6*x**24), Abs(a/(b*x**4)) > 1),
(-35*I*a**6*b**(9/2)*sqrt(-a/(b*x**4) + 1)/(630*a**5*b**4*x**16 - 1260*a**
4*b**5*x**20 + 630*a**3*b**6*x**24) + 120*I*a**5*b**(11/2)*x**4*sqrt(-a/(b
*x**4) + 1)/(630*a**5*b**4*x**16 - 1260*a**4*b**5*x**20 + 630*a**3*b**6*x*
*24) - 138*I*a**4*b**(13/2)*x**8*sqrt(-a/(b*x**4) + 1)/(630*a**5*b**4*x**1
6 - 1260*a**4*b**5*x**20 + 630*a**3*b**6*x**24) + 52*I*a**3*b**(15/2)*x**1
2*sqrt(-a/(b*x**4) + 1)/(630*a**5*b**4*x**16 - 1260*a**4*b**5*x**20 + 630*
a**3*b**6*x**24) - 3*I*a**2*b**(17/2)*x**16*sqrt(-a/(b*x**4) + 1)/(630*a**
5*b**4*x**16 - 1260*a**4*b**5*x**20 + 630*a**3*b**6*x**24) + 12*I*a*b**(19
/2)*x**20*sqrt(-a/(b*x**4) + 1)/(630*a**5*b**4*x**16 - 1260*a**4*b**5*x...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}}\,dx = -\frac{\frac{63\,(-bx^4+a)^{\frac{5}{2}}b^2}{x^{10}} + \frac{90\,(-bx^4+a)^{\frac{7}{2}}b}{x^{14}} + \frac{35\,(-bx^4+a)^{\frac{9}{2}}}{x^{18}}}{630\,a^3}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^19,x, algorithm="maxima")
```

output
```
-1/630*(63*(-b*x^4 + a)^(5/2)*b^2/x^10 + 90*(-b*x^4 + a)^(7/2)*b/x^14 + 35
*(-b*x^4 + a)^(9/2)/x^18)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(59) = 118$.

Time = 0.15 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.69

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}}\,dx = \frac{8\left(210\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^{12}\sqrt{-b}b^4+315\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^{10}a\sqrt{-b}b^4+44\right.}{}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^19,x, algorithm="giac")
```

output
```
8/315*(210*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^12*sqrt(-b)*b^4 + 315*(sqrt(-
b)*x^2 - sqrt(-b*x^4 + a))^10*a*sqrt(-b)*b^4 + 441*(sqrt(-b)*x^2 - sqrt(-b
*x^4 + a))^8*a^2*sqrt(-b)*b^4 + 126*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^6*a^
3*sqrt(-b)*b^4 + 36*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^4*a^4*sqrt(-b)*b^4 -
 9*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2*a^5*sqrt(-b)*b^4 + a^6*sqrt(-b)*b^4
)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^9
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.35

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}}\,dx = \frac{5\,b\,\sqrt{a-b\,x^4}}{63\,x^{14}} - \frac{a\,\sqrt{a-b\,x^4}}{18\,x^{18}} - \frac{4\,b^4\,\sqrt{a-b\,x^4}}{315\,a^3\,x^2} - \frac{2\,b^3\,\sqrt{a-b\,x^4}}{315\,a^2\,x^6} - \frac{b^2\,\sqrt{a-b\,x^4}}{210\,a\,x^{10}}$$

input
```
int((a - b*x^4)^(3/2)/x^19,x)
```

output
```
(5*b*(a - b*x^4)^(1/2))/(63*x^14) - (a*(a - b*x^4)^(1/2))/(18*x^18) - (4*b
^4*(a - b*x^4)^(1/2))/(315*a^3*x^2) - (2*b^3*(a - b*x^4)^(1/2))/(315*a^2*x
^6) - (b^2*(a - b*x^4)^(1/2))/(210*a*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{(a-bx^4)^{3/2}}{x^{19}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-8b^4x^{16}-4a\,b^3x^{12}-3a^2b^2x^8+50a^3b\,x^4-35a^4)}{630a^3x^{18}}$$

input
```
int((-b*x^4+a)^(3/2)/x^19,x)
```

output
```
(sqrt(a - b*x**4)*( - 35*a**4 + 50*a**3*b*x**4 - 3*a**2*b**2*x**8 - 4*a*b*
*3*x**12 - 8*b**4*x**16))/(630*a**3*x**18)
```

3.196 $\int x^4(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int x^4(a-bx^4)^{3/2}\,dx=-\frac{4a^2x\sqrt{a-bx^4}}{77b}+\frac{6}{77}ax^5\sqrt{a-bx^4}+\frac{1}{11}x^5(a-bx^4)^{3/2}+\frac{4a^{13/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{77b^{5/4}\sqrt{a-bx^4}}$$

output
```
-4/77*a^2*x*(-b*x^4+a)^(1/2)/b+6/77*a*x^5*(-b*x^4+a)^(1/2)+1/11*x^5*(-b*x^
4+a)^(3/2)+4/77*a^(13/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/
b^(5/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int x^4(a-bx^4)^{3/2}\,dx=\frac{x\sqrt{a-bx^4}\left(-(a-bx^4)^2+\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{1-\frac{bx^4}{a}}}\right)}{11b}$$

input
```
Integrate[x^4*(a - b*x^4)^(3/2),x]
```

output
```
(x*Sqrt[a - b*x^4]*(-(a - b*x^4)^2 + (a^2*Hypergeometric2F1[-3/2, 1/4, 5/4
, (b*x^4)/a])/Sqrt[1 - (b*x^4)/a]))/(11*b)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {811, 811, 843, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a-bx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{811}$$

$$\frac{6}{11} a \int x^4 \sqrt{a-bx^4}dx+\frac{1}{11} x^5 \left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{811}$$

$$\frac{6}{11} a \left(\frac{2}{7} a \int \frac{x^4}{\sqrt{a-bx^4}}dx+\frac{1}{7} x^5 \sqrt{a-bx^4}\right)+\frac{1}{11} x^5 \left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{843}$$

$$\frac{6}{11} a \left(\frac{2}{7} a \left(\frac{a \int \frac{1}{\sqrt{a-bx^4}}dx}{3b}-\frac{x\sqrt{a-bx^4}}{3b}\right)+\frac{1}{7} x^5 \sqrt{a-bx^4}\right)+\frac{1}{11} x^5 \left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{765}$$

$$\frac{6}{11} a \left(\frac{2}{7} a \left(\frac{a\sqrt{1-\frac{bx^4}{a}} \int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3b\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)+\frac{1}{7} x^5 \sqrt{a-bx^4}\right)+\frac{1}{11} x^5 \left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{762}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)+\frac{1}{7}x^5\sqrt{a-bx^4}\right)+\frac{1}{11}x^5\left(a-bx^4\right)^{3/2}$$

input
```
Int[x^4*(a - b*x^4)^(3/2),x]
```

output
```
(x^5*(a - b*x^4)^(3/2))/11 + (6*a*((x^5*Sqrt[a - b*x^4])/7 + (2*a*(-1/3*(x
*Sqrt[a - b*x^4])/b + (a^(5/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(3*b^(5/4)*Sqrt[a - b*x^4])))/7))/11
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{x(7b^2x^8-13ab\,x^4+4a^2)\sqrt{-b\,x^4+a}}{77b}+\frac{4a^3\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{77b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	109
default	$-\frac{b\,x^9\sqrt{-b\,x^4+a}}{11}+\frac{13a\,x^5\sqrt{-b\,x^4+a}}{77}-\frac{4a^2x\sqrt{-b\,x^4+a}}{77b}+\frac{4a^3\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{77b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	123
elliptic	$-\frac{b\,x^9\sqrt{-b\,x^4+a}}{11}+\frac{13a\,x^5\sqrt{-b\,x^4+a}}{77}-\frac{4a^2x\sqrt{-b\,x^4+a}}{77b}+\frac{4a^3\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{77b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	123

input
```
int(x^4*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/77*x*(7*b^2*x^8-13*a*b*x^4+4*a^2)*(-b*x^4+a)^(1/2)/b+4/77/b*a^3/(1/a^(1
/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

$$\int x^4\left(a-bx^4\right)^{3/2}\,dx=\frac{4\,a^2\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\middle|\,-1\right)-\left(7\,b^2x^9-13\,abx^5+4\,a^2x\right)\sqrt{-bx^4+a}}{77\,b}$$

input
```
integrate(x^4*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/77*(4*a^2*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - (
7*b^2*x^9 - 13*a*b*x^5 + 4*a^2*x)*sqrt(-b*x^4 + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.34

$$\int x^4 (a-bx^4)^{3/2}\,dx = \frac{a^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(-b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x**5*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4 (a-bx^4)^{3/2}\,dx = \int (-bx^4+a)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4 (a-bx^4)^{3/2}\,dx = \int (-bx^4+a)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a - bx^4)^{3/2} \, dx = \int x^4 \left(a - b\, x^4\right)^{3/2} \, dx$$

input
```
int(x^4*(a - b*x^4)^(3/2),x)
```

output
```
int(x^4*(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^4 (a - bx^4)^{3/2} \, dx = \frac{-4\sqrt{-b\, x^4 + a}\, a^2 x + 13\sqrt{-b\, x^4 + a}\, ab\, x^5 - 7\sqrt{-b\, x^4 + a}\, b^2 x^9 + 4\left(\int \frac{\sqrt{-b\, x^4 + a}}{-b\, x^4 + a} dx\right) a^3}{77b}$$

input
```
int(x^4*(-b*x^4+a)^(3/2),x)
```

output
```
( - 4*sqrt(a - b*x**4)*a**2*x + 13*sqrt(a - b*x**4)*a*b*x**5 - 7*sqrt(a -
b*x**4)*b**2*x**9 + 4*int(sqrt(a - b*x**4)/(a - b*x**4),x)*a**3)/(77*b)
```

3.197 $\int (a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int (a-bx^4)^{3/2}\,dx = \frac{2}{7}ax\sqrt{a-bx^4} + \frac{1}{7}x(a-bx^4)^{3/2} + \frac{4a^{9/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{7\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
2/7*a*x*(-b*x^4+a)^(1/2)+1/7*x*(-b*x^4+a)^(3/2)+4/7*a^(9/4)*(1-b*x^4/a)^(1
/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int (a-bx^4)^{3/2}\,dx = \frac{ax\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(3/2),x]
```

output
```
(a*x*Sqrt[a - b*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, (b*x^4)/a])/Sqrt[1
- (b*x^4)/a]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {748, 748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{748}$$

$$\frac{6}{7}a\int \sqrt{a-bx^4}dx+\frac{1}{7}x\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{748}$$

$$\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{1}{3}x\sqrt{a-bx^4}\right)+\frac{1}{7}x\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{765}$$

$$\frac{6}{7}a\left(\frac{2a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3\sqrt{a-bx^4}}+\frac{1}{3}x\sqrt{a-bx^4}\right)+\frac{1}{7}x\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{762}$$

$$\frac{6}{7}a\left(\frac{2a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{1}{3}x\sqrt{a-bx^4}\right)+\frac{1}{7}x\left(a-bx^4\right)^{3/2}$$

input
```
Int[(a - b*x^4)^(3/2),x]
```

output
```
(x*(a - b*x^4)^(3/2))/7 + (6*a*((x*Sqrt[a - b*x^4])/3 + (2*a^(5/4)*Sqrt[1
- (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(1/4)*Sqrt[a
 - b*x^4])))/7
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{x\left(-b\,x^{4}+3a\right)\sqrt{-b\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	92
default	$-\frac{b\,x^{5}\sqrt{-b\,x^{4}+a}}{7}+\frac{3ax\sqrt{-b\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	99
elliptic	$-\frac{b\,x^{5}\sqrt{-b\,x^{4}+a}}{7}+\frac{3ax\sqrt{-b\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	99

input
```
int((-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(-b*x^4+3*a)*(-b*x^4+a)^(1/2)+4/7*a^2/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b
^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*E
llipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

$$\int \left(a-bx^4\right)^{3/2}\,dx=\frac{4}{7}\,a\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-\frac{1}{7}\left(bx^5-3\,ax\right)\sqrt{-bx^4+a}$$

input
```
integrate((-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
4/7*a*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - 1/7*(b*
x^5 - 3*a*x)*sqrt(-b*x^4 + a)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \left(a-bx^4\right)^{3/2}\,dx=\frac{a^{\frac{3}{2}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)/
a)/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(a-bx^4\right)^{3/2}\,dx=\int \left(-bx^4+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a-bx^4\right)^{3/2}\,dx=\int \left(-bx^4+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.41

$$\int \left(a-bx^4\right)^{3/2}\,dx=\frac{x\left(a-b\,x^4\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{\left(1-\frac{b\,x^4}{a}\right)^{3/2}}$$

input
```
int((a - b*x^4)^(3/2),x)
```

output
```
(x*(a - b*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, (b*x^4)/a))/(1 - (b*x^4)/
a)^(3/2)
```

Reduce [F]

$$\int \left(a-bx^4\right)^{3/2} dx = \frac{3\sqrt{-b\,x^4+a}\,ax}{7} - \frac{\sqrt{-b\,x^4+a}\,b\,x^5}{7} + \frac{4\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)a^2}{7}$$

input
```
int((-b*x^4+a)^(3/2),x)
```

output
```
(3*sqrt(a - b*x**4)*a*x - sqrt(a - b*x**4)*b*x**5 + 4*int(sqrt(a - b*x**4)
/(a - b*x**4),x)*a**2)/7
```

3.198 $\int \frac{(a-bx^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int \frac{(a-bx^4)^{3/2}}{x^4} dx = -\frac{a\sqrt{a-bx^4}}{3x^3} - \frac{1}{3}bx\sqrt{a-bx^4} - \frac{4a^{5/4}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt{a-bx^4}}$$

output
```
-1/3*a*(-b*x^4+a)^(1/2)/x^3-1/3*b*x*(-b*x^4+a)^(1/2)-4/3*a^(5/4)*b^(3/4)*(
1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{(a-bx^4)^{3/2}}{x^4} dx = -\frac{a\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},\frac{bx^4}{a}\right)}{3x^3\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^4,x]
```

output
```
-1/3*(a*Sqrt[a - b*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, (b*x^4)/a])/(x^
3*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {809, 748, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{809}$$

$$-2b\int \sqrt{a-bx^4}dx-\frac{(a-bx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{748}$$

$$-2b\left(\frac{2}{3}a\int \frac{1}{\sqrt{a-bx^4}}dx+\frac{1}{3}x\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{765}$$

$$-2b\left(\frac{2a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3\sqrt{a-bx^4}}+\frac{1}{3}x\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{762}$$

$$-2b\left(\frac{2a^{5/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{1}{3}x\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{3x^3}$$

input
```
Int[(a - b*x^4)^(3/2)/x^4,x]
```

output
```
-1/3*(a - b*x^4)^(3/2)/x^3 - 2*b*((x*Sqrt[a - b*x^4])/3 + (2*a^(5/4)*Sqrt[
1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(1/4)*Sqrt
[a - b*x^4]))
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+a\right)}{3x^{3}}-\frac{4ab\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	90
default	$-\frac{a\sqrt{-b\,x^{4}+a}}{3x^{3}}-\frac{bx\sqrt{-b\,x^{4}+a}}{3}-\frac{4ab\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	98
elliptic	$-\frac{a\sqrt{-b\,x^{4}+a}}{3x^{3}}-\frac{bx\sqrt{-b\,x^{4}+a}}{3}-\frac{4ab\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	98

input
```
int((-b*x^4+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^4+a)^(1/2)*(b*x^4+a)/x^3-4/3*a*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b
^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*E
llipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [F]

$$\int \frac{\left(a-bx^4\right)^{3/2}}{x^4}\,dx=\int \frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(3/2)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int \frac{(a-bx^4)^{3/2}}{x^4}\,dx = -\frac{ib^{\frac{3}{2}}x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**4,x)
```

output
```
-I*b**(3/2)*x**3*gamma(3/4)*hyper((-3/2, -3/4), (1/4,), a/(b*x**4))/(4*gam
ma(7/4))
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^4}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^4}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{x^4}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{x^4}\,dx$$

input
```
int((a - b*x^4)^(3/2)/x^4,x)
```

output
```
int((a - b*x^4)^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^4}\,dx = \frac{-5\sqrt{-b\,x^4+a}\,a-\sqrt{-b\,x^4+a}\,b\,x^4-12\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^8+a\,x^4}dx\right)a^2x^3}{3x^3}$$

input
```
int((-b*x^4+a)^(3/2)/x^4,x)
```

output
```
( - 5*sqrt(a - b*x**4)*a - sqrt(a - b*x**4)*b*x**4 - 12*int(sqrt(a - b*x**
4)/(a*x**4 - b*x**8),x)*a**2*x**3)/(3*x**3)
```

3.199 $\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = -\frac{a\sqrt{a-bx^4}}{7x^7}+\frac{3b\sqrt{a-bx^4}}{7x^3}+\frac{4\sqrt[4]{a}b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{7\sqrt{a-bx^4}}$$

output
```
-1/7*a*(-b*x^4+a)^(1/2)/x^7+3/7*b*(-b*x^4+a)^(1/2)/x^3+4/7*a^(1/4)*b^(7/4)
*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.55

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = -\frac{a\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},\frac{bx^4}{a}\right)}{7x^7\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^8,x]
```

output
```
-1/7*(a*Sqrt[a - b*x^4]*Hypergeometric2F1[-7/4, -3/2, -3/4, (b*x^4)/a])/(x
^7*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {809, 809, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^4\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{6}{7}b\int \frac{\sqrt{a-bx^4}}{x^4}dx-\frac{\left(a-bx^4\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{809}$$

$$-\frac{6}{7}b\left(-\frac{2}{3}b\int \frac{1}{\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}}{3x^3}\right)-\frac{\left(a-bx^4\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{765}$$

$$-\frac{6}{7}b\left(-\frac{2b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3x^3}\right)-\frac{\left(a-bx^4\right)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{762}$$

$$-\frac{6}{7}b\left(-\frac{2\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3x^3}\right)-\frac{\left(a-bx^4\right)^{3/2}}{7x^7}$$

input
```
Int[(a - b*x^4)^(3/2)/x^8,x]
```

output
```
-1/7*(a - b*x^4)^(3/2)/x^7 - (6*b*(-1/3*Sqrt[a - b*x^4]/x^3 - (2*a^(1/4)*b
^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*
Sqrt[a - b*x^4])))/7
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-3 b\,x^{4}+a \right)}{7 x^{7}}+\frac{4 b^{2} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	92
default	$-\frac{a\sqrt{-b\,x^{4}+a}}{7 x^{7}}+\frac{3 b\sqrt{-b\,x^{4}+a}}{7 x^{3}}+\frac{4 b^{2} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	101
elliptic	$-\frac{a\sqrt{-b\,x^{4}+a}}{7 x^{7}}+\frac{3 b\sqrt{-b\,x^{4}+a}}{7 x^{3}}+\frac{4 b^{2} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	101

input
```
int((-b*x^4+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(-b*x^4+a)^(1/2)*(-3*b*x^4+a)/x^7+4/7*b^2/(1/a^(1/2)*b^(1/2))^(1/2)*(
1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2
)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.57

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = \frac{4\,\sqrt{a}bx^7\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+(3\,bx^4-a)\sqrt{-bx^4+a}}{7\,x^7}$$

input
```
integrate((-b*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
1/7*(4*sqrt(a)*b*x^7*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + (
3*b*x^4 - a)*sqrt(-b*x^4 + a))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = -\frac{ib^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**8,x)
```

output
```
-I*b**(3/2)*gamma(-1/4)*hyper((-3/2, 1/4), (5/4,), a/(b*x**4))/(4*x*gamma(
3/4))
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{x^8}\,dx$$

input
```
int((a - b*x^4)^(3/2)/x^8,x)
```

output
```
int((a - b*x^4)^(3/2)/x^8, x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{-b\,x^4+a}\,a+5\sqrt{-b\,x^4+a}\,b\,x^4+12\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{12}+a\,x^8}dx\right)a^2x^7}{5x^7}$$

input
```
int((-b*x^4+a)^(3/2)/x^8,x)
```

output
```
(sqrt(a - b*x**4)*a + 5*sqrt(a - b*x**4)*b*x**4 + 12*int(sqrt(a - b*x**4)/
(a*x**8 - b*x**12),x)*a**2*x**7)/(5*x**7)
```

3.200 $\int x^2(a-bx^4)^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 152

$$\int x^2\left(a-bx^4\right)^{3/2}\,dx=\frac{2}{15}ax^3\sqrt{a-bx^4}+\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}+\frac{4a^{11/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{3/4}\sqrt{a-bx^4}}-\frac{4a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{15b^{3/4}\sqrt{a-bx^4}}$$

output
```
2/15*a*x^3*(-b*x^4+a)^(1/2)+1/9*x^3*(-b*x^4+a)^(3/2)+4/15*a^(11/4)*(1-b*x^
4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(3/4)/(-b*x^4+a)^(1/2)-4/15*a^
(11/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(3/4)/(-b*x^4+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.94 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

$$\int x^2 (a - bx^4)^{3/2} \, dx = \frac{ax^3\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)}{3\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^2*(a - b*x^4)^(3/2),x]
```

output
```
(a*x^3*Sqrt[a - b*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, (b*x^4)/a])/(3*Sq
rt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {811, 811, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a - bx^4)^{3/2} \, dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{2}{3}a\int x^2\sqrt{a-bx^4}dx+\frac{1}{9}x^3(a-bx^4)^{3/2}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\int\frac{x^2}{\sqrt{a-bx^4}}dx+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3(a-bx^4)^{3/2}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3(a-bx^4)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{765}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{762}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{1390}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{1389}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)+\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a}\right.$$
$$\frac{1}{9}x^3\left(a-bx^4\right)^{3/2}$$

input
```
Int[x^2*(a - b*x^4)^(3/2),x]
```

output
```
(x^3*(a - b*x^4)^(3/2))/9 + (2*a*((x^3*Sqrt[a - b*x^4])/5 + (2*a*((a^(3/4)
*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*
Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*
x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/5))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{x^{3}\left(-5 b \,x^{4}+11 a \right) \sqrt{-b \,x^{4}+a}}{45}-\frac{4 a^{\frac{5}{2}} \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}\, \sqrt{b}}$	114
default	$-\frac{x^{7} \sqrt{-b \,x^{4}+a}\, b}{9}+\frac{11 a \,x^{3} \sqrt{-b \,x^{4}+a}}{45}-\frac{4 a^{\frac{5}{2}} \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}\, \sqrt{b}}$	121
elliptic	$-\frac{x^{7} \sqrt{-b \,x^{4}+a}\, b}{9}+\frac{11 a \,x^{3} \sqrt{-b \,x^{4}+a}}{45}-\frac{4 a^{\frac{5}{2}} \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}\, \sqrt{b}}$	121

input
```
int(x^2*(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3*(-5*b*x^4+11*a)*(-b*x^4+a)^(1/2)-4/15*a^(5/2)/(1/a^(1/2)*b^(1/2))
^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4
+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1
/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.69

$$\int x^2(a-bx^4)^{3/2}\,dx =$$
$$-\frac{12\,a^2\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)-12\,a^2\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1)+(5\,b^2x^8-11\,abx^4-}{45\,bx}$$

input
```
integrate(x^2*(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/45*(12*a^2*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1)
 - 12*a^2*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + (
5*b^2*x^8 - 11*a*b*x^4 + 12*a^2)*sqrt(-b*x^4 + a))/(b*x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int x^2(a-bx^4)^{3/2}\,dx = \frac{a^{\frac{3}{2}}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(-b*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x**3*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int x^2(a-bx^4)^{3/2}\,dx=\int(-bx^4+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2(a-bx^4)^{3/2}\,dx=\int(-bx^4+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a-bx^4)^{3/2}\,dx=\int x^2\,(a-b\,x^4)^{3/2}\,dx$$

input
```
int(x^2*(a - b*x^4)^(3/2),x)
```

output
```
int(x^2*(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^2\left(a-bx^4\right)^{3/2}\,dx=\frac{11\sqrt{-b\,x^4+a}\,a\,x^3}{45}-\frac{\sqrt{-b\,x^4+a}\,b\,x^7}{9}+\frac{4\left(\int\frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)a^2}{15}$$

input
```
int(x^2*(-b*x^4+a)^(3/2),x)
```

output
```
(11*sqrt(a - b*x**4)*a*x**3 - 5*sqrt(a - b*x**4)*b*x**7 + 12*int((sqrt(a -
 b*x**4)*x**2)/(a - b*x**4),x)*a**2)/45
```

3.201 $\int \frac{(a-bx^4)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{(a-bx^4)^{3/2}}{x^2} dx = -\frac{a\sqrt{a-bx^4}}{x} - \frac{1}{5}bx^3\sqrt{a-bx^4} - \frac{12a^{7/4}\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt{a-bx^4}} + \frac{12a^{7/4}\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{5\sqrt{a-bx^4}}$$

output
```
-a*(-b*x^4+a)^(1/2)/x-1/5*b*x^3*(-b*x^4+a)^(1/2)-12/5*a^(7/4)*b^(1/4)*(1-b
*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)+12/5*a^(7/4)
*b^(1/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{(a-bx^4)^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},\frac{bx^4}{a}\right)}{x\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a - b*x^4]*Hypergeometric2F1[-3/2, -1/4, 3/4, (b*x^4)/a])/(x*Sqr
t[1 - (b*x^4)/a]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {809, 811, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{809}$$

$$-6b\int x^2\sqrt{a-bx^4}dx-\frac{(a-bx^4)^{3/2}}{x}$$

$$\Big\downarrow \text{811}$$

$$-6b\left(\frac{2}{5}a\int \frac{x^2}{\sqrt{a-bx^4}}dx+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{(a-bx^4)^{3/2}}{x}$$

$$\Big\downarrow \text{836}$$

$$-6b\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{\left(a-bx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{27}$$

$$-6b\left(\frac{2}{5}a\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{\left(a-bx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{765}$$

$$-6b\left(\frac{2}{5}a\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{\left(a-bx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{762}$$

$$-6b\left(\frac{2}{5}a\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{\left(a-bx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1390}$$

$$-6b\left(\frac{2}{5}a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{\left(a-bx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1389}$$

$$-6b\left(\frac{2}{5}a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a-bx^4}\right)-\frac{\left(a-bx^4\right)^{3/2}}{x}$$

$$\Big\downarrow 327$$

$$-6b\left(\frac{2}{5}a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)+\frac{1}{5}x^3\sqrt{a}\right.$$

$$\frac{\left(a-bx^4\right)^{3/2}}{x}$$

input
```
Int[(a - b*x^4)^(3/2)/x^2,x]
```

output
```
-((a - b*x^4)^(3/2)/x) - 6*b*((x^3*Sqrt[a - b*x^4])/5 + (2*a*((a^(3/4)*Sqr
t[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt
[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a
^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,(b\,x^{4}+5a)}{5x}+\frac{12a^{\frac{3}{2}}\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	113
default	$-\frac{a\sqrt{-b\,x^{4}+a}}{x}-\frac{b\,x^{3}\sqrt{-b\,x^{4}+a}}{5}+\frac{12a^{\frac{3}{2}}\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	121
elliptic	$-\frac{a\sqrt{-b\,x^{4}+a}}{x}-\frac{b\,x^{3}\sqrt{-b\,x^{4}+a}}{5}+\frac{12a^{\frac{3}{2}}\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	121

input
```
int((-b*x^4+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/2)*(b*x^4+5*a)/x+12/5*a^(3/2)*b^(1/2)/(1/a^(1/2)*b^(1/2
))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x
^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/
2)*b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{\left(a-bx^4\right)^{3/2}}{x^2}\,dx=\int \frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(3/2)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.28

$$\int \frac{(a-bx^4)^{3/2}}{x^2}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**2,x)
```

output
```
a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*x**4*exp_polar(2*I*pi)/
a)/(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^2}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^2}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.27

$$\int \frac{(a-bx^4)^{3/2}}{x^2}\,dx=\frac{(a-b\,x^4)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ \frac{a}{b\,x^4}\right)}{5\,x\,\left(1-\frac{a}{b\,x^4}\right)^{3/2}}$$

input
```
int((a - b*x^4)^(3/2)/x^2,x)
```

output
```
((a - b*x^4)^(3/2)*hypergeom([-3/2, -5/4], -1/4, a/(b*x^4)))/(5*x*(1 - a/(
b*x^4))^(3/2))
```

Reduce [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^2}\,dx=\frac{7\sqrt{-b\,x^4+a}\,a-\sqrt{-b\,x^4+a}\,b\,x^4+12\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx\right)a^2x}{5x}$$

input
```
int((-b*x^4+a)^(3/2)/x^2,x)
```

output
```
(7*sqrt(a - b*x**4)*a - sqrt(a - b*x**4)*b*x**4 + 12*int(sqrt(a - b*x**4)/
(a*x**2 - b*x**6),x)*a**2*x)/(5*x)
```

3.202 $\int \frac{(a-bx^4)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 153

$$\int \frac{(a-bx^4)^{3/2}}{x^6} dx = -\frac{a\sqrt{a-bx^4}}{5x^5}+\frac{7b\sqrt{a-bx^4}}{5x} + \frac{12a^{3/4}b^{5/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5\sqrt{a-bx^4}} - \frac{12a^{3/4}b^{5/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{5\sqrt{a-bx^4}}$$

output
```
-1/5*a*(-b*x^4+a)^(1/2)/x^5+7/5*b*(-b*x^4+a)^(1/2)/x+12/5*a^(3/4)*b^(5/4)*
(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(1/2)-12/5*a^(
3/4)*b^(5/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/(-b*x^4+a)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = -\frac{a\sqrt{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},\frac{bx^4}{a}\right)}{5x^5\sqrt{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(3/2)/x^6,x]
```

output
```
-1/5*(a*Sqrt[a - b*x^4]*Hypergeometric2F1[-3/2, -5/4, -1/4, (b*x^4)/a])/(x
^5*Sqrt[1 - (b*x^4)/a])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {809, 809, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{6}{5}b\int \frac{\sqrt{a-bx^4}}{x^2}dx-\frac{(a-bx^4)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{809}$$

$$-\frac{6}{5}b\left(-2b\int \frac{x^2}{\sqrt{a-bx^4}}dx-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{(a-bx^4)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{836}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \textbf{765}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \textbf{762}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \textbf{1390}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \textbf{1389}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-bx^4}}{x}\right)-\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{327}$$

$$-\frac{6}{5}b\left(-2b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)-\frac{\sqrt{a-}}{x}$$

$$\frac{\left(a-bx^4\right)^{3/2}}{5x^5}$$

input
```
Int[(a - b*x^4)^(3/2)/x^6,x]
```

output
```
-1/5*(a - b*x^4)^(3/2)/x^5 - (6*b*(-(Sqrt[a - b*x^4]/x) - 2*b*((a^(3/4)*Sq
rt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqr
t[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/
a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]))))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] && !GtQ[a, 0]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(-7b\,x^{4}+a\right)}{5x^{5}}-\frac{12b^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	112
default	$-\frac{a\sqrt{-b\,x^{4}+a}}{5x^{5}}+\frac{7b\sqrt{-b\,x^{4}+a}}{5x}-\frac{12b^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	121
elliptic	$-\frac{a\sqrt{-b\,x^{4}+a}}{5x^{5}}+\frac{7b\sqrt{-b\,x^{4}+a}}{5x}-\frac{12b^{\frac{3}{2}}\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	121

input
```
int((-b*x^4+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/2)*(-7*b*x^4+a)/x^5-12/5*b^(3/2)*a^(1/2)/(1/a^(1/2)*b^(
1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-
b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^
(1/2)*b^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(3/2)/x^6, x)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.24

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = -\frac{ib^{\frac{3}{2}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(3/2)/x**6,x)
```

output
```
-I*b**(3/2)*x*gamma(1/4)*hyper((-3/2, -1/4), (3/4,), a/(b*x**4))/(4*gamma(
5/4))
```

Maxima [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = \int \frac{(-bx^4+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = \int \frac{(a-b\,x^4)^{3/2}}{x^6}\,dx$$

input
```
int((a - b*x^4)^(3/2)/x^6,x)
```

output
```
int((a - b*x^4)^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{(a-bx^4)^{3/2}}{x^6}\,dx = \frac{-\sqrt{-b\,x^4+a}\,a-\sqrt{-b\,x^4+a}\,b\,x^4-4\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{10}+a\,x^6}dx\right)a^2x^5}{x^5}$$

input
```
int((-b*x^4+a)^(3/2)/x^6,x)
```

output
```
( - sqrt(a - b*x**4)*a - sqrt(a - b*x**4)*b*x**4 - 4*int(sqrt(a - b*x**4)/
(a*x**6 - b*x**10),x)*a**2*x**5)/x**5
```

3.203 $\int x\sqrt{-2+x^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int x\sqrt{-2+x^4}\,dx=\frac{1}{4}x^2\sqrt{-2+x^4}-\frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{-2+x^4}}\right)$$

output
```
1/4*x^2*(x^4-2)^(1/2)-1/2*arctanh(x^2/(x^4-2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x\sqrt{-2+x^4}\,dx=\frac{1}{4}x^2\sqrt{-2+x^4}-\frac{1}{2}\log\left(x^2+\sqrt{-2+x^4}\right)$$

input
```
Integrate[x*Sqrt[-2 + x^4],x]
```

output
```
(x^2*Sqrt[-2 + x^4])/4 - Log[x^2 + Sqrt[-2 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {807, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^4-2}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \sqrt{x^4-2}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\sqrt{x^4-2}-\int \frac{1}{\sqrt{x^4-2}}dx^2\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\sqrt{x^4-2}-\int \frac{1}{1-x^4}d\frac{x^2}{\sqrt{x^4-2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\sqrt{x^4-2}-\text{arctanh}\left(\frac{x^2}{\sqrt{x^4-2}}\right)\right)$$

input
```
Int[x*Sqrt[-2 + x^4],x]
```

output
```
((x^2*Sqrt[-2 + x^4])/2 - ArcTanh[x^2/Sqrt[-2 + x^4]])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^{2} \sqrt{x^{4}-2}}{4}-\frac{\ln\left(x^{2}+\sqrt{x^{4}-2}\right)}{2}$	28
trager	$\frac{x^{2} \sqrt{x^{4}-2}}{4}-\frac{\ln\left(x^{2}+\sqrt{x^{4}-2}\right)}{2}$	28
risch	$\frac{x^{2} \sqrt{x^{4}-2}}{4}-\frac{\ln\left(x^{2}+\sqrt{x^{4}-2}\right)}{2}$	28
elliptic	$\frac{x^{2} \sqrt{x^{4}-2}}{4}-\frac{\ln\left(x^{2}+\sqrt{x^{4}-2}\right)}{2}$	28
pseudoelliptic	$\frac{x^{2} \sqrt{x^{4}-2}}{4}-\frac{\ln\left(x^{2}+\sqrt{x^{4}-2}\right)}{2}$	28
meijerg	$\frac{i \sqrt{\operatorname{signum}\left(-1+\frac{x^{4}}{2}\right)}\, \left(-i \sqrt{\pi}\, \sqrt{2}\, x^{2} \sqrt{-\frac{x^{4}}{2}+1}-2 i \sqrt{\pi}\, \arcsin\left(\frac{\sqrt{2}\, x^{2}}{2}\right)\right)}{4 \sqrt{\pi}\, \sqrt{-\operatorname{signum}\left(-1+\frac{x^{4}}{2}\right)}}$	66

input
```
int(x*(x^4-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(x^4-2)^(1/2)-1/2*ln(x^2+(x^4-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\sqrt{-2+x^4}\,dx = \frac{1}{4}\sqrt{x^4-2}x^2 + \frac{1}{2}\log\left(-x^2+\sqrt{x^4-2}\right)$$

input
```
integrate(x*(x^4-2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^4 - 2)*x^2 + 1/2*log(-x^2 + sqrt(x^4 - 2))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.51

$$\int x\sqrt{-2+x^4}\,dx = \begin{cases} \frac{x^6}{4\sqrt{x^4-2}} - \frac{x^2}{2\sqrt{x^4-2}} - \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^2}{2}\right)}{2} & \text{for } |x^4| > 2 \\ -\frac{ix^6}{4\sqrt{2-x^4}} + \frac{ix^2}{2\sqrt{2-x^4}} + \frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^2}{2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(x**4-2)**(1/2),x)
```

output
```
Piecewise((x**6/(4*sqrt(x**4 - 2)) - x**2/(2*sqrt(x**4 - 2)) - acosh(sqrt(
2)*x**2/2)/2, Abs(x**4) > 2), (-I*x**6/(4*sqrt(2 - x**4)) + I*x**2/(2*sqrt
(2 - x**4)) + I*asin(sqrt(2)*x**2/2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int x\sqrt{-2+x^4}\,dx = -\frac{\sqrt{x^4-2}}{2\,x^2\left(\frac{x^4-2}{x^4}-1\right)} - \frac{1}{4}\log\left(\frac{\sqrt{x^4-2}}{x^2}+1\right) + \frac{1}{4}\log\left(\frac{\sqrt{x^4-2}}{x^2}-1\right)$$

input
```
integrate(x*(x^4-2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(x^4 - 2)/(x^2*((x^4 - 2)/x^4 - 1)) - 1/4*log(sqrt(x^4 - 2)/x^2 +
 1) + 1/4*log(sqrt(x^4 - 2)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x\sqrt{-2+x^4}\,dx = \frac{1}{4}\sqrt{x^4-2}x^2 + \frac{1}{2}\log\left(x^2-\sqrt{x^4-2}\right)$$

input
```
integrate(x*(x^4-2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 - 2)*x^2 + 1/2*log(x^2 - sqrt(x^4 - 2))
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{-2+x^4}\,dx = \int x\,\sqrt{x^4-2}\,dx$$

input
```
int(x*(x^4 - 2)^(1/2),x)
```

output
```
int(x*(x^4 - 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.34

$$\int x\sqrt{-2+x^4}\,dx$$
$$= \frac{-2\sqrt{x^4-2}\log\left(\frac{\sqrt{x^4-2}+x^2}{\sqrt{2}}\right)x^2+\sqrt{x^4-2}\,x^6-\sqrt{x^4-2}\,x^2-2\log\left(\frac{\sqrt{x^4-2}+x^2}{\sqrt{2}}\right)x^4+2\log\left(\frac{\sqrt{x^4-2}+x^2}{\sqrt{2}}\right)+x}{4\sqrt{x^4-2}\,x^2+4x^4-4}$$

input
```
int(x*(x^4-2)^(1/2),x)
```

output
```
( - 2*sqrt(x**4 - 2)*log((sqrt(x**4 - 2) + x**2)/sqrt(2))*x**2 + sqrt(x**4
 - 2)*x**6 - sqrt(x**4 - 2)*x**2 - 2*log((sqrt(x**4 - 2) + x**2)/sqrt(2))*
x**4 + 2*log((sqrt(x**4 - 2) + x**2)/sqrt(2)) + x**8 - 2*x**4)/(4*(sqrt(x*
*4 - 2)*x**2 + x**4 - 1))
```

3.204 $\int \frac{x^{11}}{\sqrt{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{\sqrt{a-bx^4}} \, dx = -\frac{a^2\sqrt{a-bx^4}}{2b^3} + \frac{a(a-bx^4)^{3/2}}{3b^3} - \frac{(a-bx^4)^{5/2}}{10b^3}$$

output
```
-1/2*a^2*(-b*x^4+a)^(1/2)/b^3+1/3*a*(-b*x^4+a)^(3/2)/b^3-1/10*(-b*x^4+a)^(
5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x^{11}}{\sqrt{a-bx^4}} \, dx = \frac{\sqrt{a-bx^4}(-8a^2-4abx^4-3b^2x^8)}{30b^3}$$

input
```
Integrate[x^11/Sqrt[a - b*x^4],x]
```

output
```
(Sqrt[a - b*x^4]*(-8*a^2 - 4*a*b*x^4 - 3*b^2*x^8))/(30*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{x^8}{\sqrt{a-bx^4}}dx^4$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{4}\int \left(\frac{a^2}{b^2\sqrt{a-bx^4}}-\frac{2\sqrt{a-bx^4}a}{b^2}+\frac{\left(a-bx^4\right)^{3/2}}{b^2}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(-\frac{2a^2\sqrt{a-bx^4}}{b^3}-\frac{2\left(a-bx^4\right)^{5/2}}{5b^3}+\frac{4a\left(a-bx^4\right)^{3/2}}{3b^3}\right)$$

input
```
Int[x^11/Sqrt[a - b*x^4],x]
```

output
```
((-2*a^2*Sqrt[a - b*x^4])/b^3 + (4*a*(a - b*x^4)^(3/2))/(3*b^3) - (2*(a -
b*x^4)^(5/2))/(5*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37
trager	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37
pseudoelliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37
orering	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(3 b^{2} x^{8}+4 a b\,x^{4}+8 a^{2}\right)}{30 b^{3}}$	37

input
```
int(x^11/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(-b*x^4+a)^(1/2)*(3*b^2*x^8+4*a*b*x^4+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx = -\frac{(3\,b^2x^8+4\,abx^4+8\,a^2)\sqrt{-bx^4+a}}{30\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(3*b^2*x^8 + 4*a*b*x^4 + 8*a^2)*sqrt(-b*x^4 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{4a^2\sqrt{a-bx^4}}{15b^3} - \frac{2ax^4\sqrt{a-bx^4}}{15b^2} - \frac{x^8\sqrt{a-bx^4}}{10b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-4*a**2*sqrt(a - b*x**4)/(15*b**3) - 2*a*x**4*sqrt(a - b*x**4)/
(15*b**2) - x**8*sqrt(a - b*x**4)/(10*b), Ne(b, 0)), (x**12/(12*sqrt(a)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{5}{2}}}{10\,b^3} + \frac{(-bx^4+a)^{\frac{3}{2}}a}{3\,b^3} - \frac{\sqrt{-bx^4+a}a^2}{2\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/10*(-b*x^4 + a)^(5/2)/b^3 + 1/3*(-b*x^4 + a)^(3/2)*a/b^3 - 1/2*sqrt(-b*
x^4 + a)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-bx^4+a}a^2}{2\,b^3} - \frac{3\,(bx^4-a)^2\sqrt{-bx^4+a} - 10\,(-bx^4+a)^{\frac{3}{2}}a}{30\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)*a^2/b^3 - 1/30*(3*(b*x^4 - a)^2*sqrt(-b*x^4 + a) - 1
0*(-b*x^4 + a)^(3/2)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx = -\sqrt{a-b\,x^4}\left(\frac{4\,a^2}{15\,b^3} + \frac{x^8}{10\,b} + \frac{2\,a\,x^4}{15\,b^2}\right)$$

input
```
int(x^11/(a - b*x^4)^(1/2),x)
```

output
```
-(a - b*x^4)^(1/2)*((4*a^2)/(15*b^3) + x^8/(10*b) + (2*a*x^4)/(15*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.56

$$\int \frac{x^{11}}{\sqrt{a-bx^4}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-3b^2x^8-4ab\,x^4-8a^2)}{30b^3}$$

input
```
int(x^11/(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 8*a**2 - 4*a*b*x**4 - 3*b**2*x**8))/(30*b**3)
```

3.205 $\int \frac{x^7}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = -\frac{a\sqrt{a-bx^4}}{2b^2}+\frac{(a-bx^4)^{3/2}}{6b^2}$$

output
```
-1/2*a*(-b*x^4+a)^(1/2)/b^2+1/6*(-b*x^4+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = \frac{(-2a-bx^4)\sqrt{a-bx^4}}{6b^2}$$

input
```
Integrate[x^7/Sqrt[a - b*x^4],x]
```

output
```
((-2*a - b*x^4)*Sqrt[a - b*x^4])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt{a-bx^4}}dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4}\int \left(\frac{a}{b\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{b}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{2\left(a-bx^4\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a-bx^4}}{b^2}\right)$$

input
```
Int[x^7/Sqrt[a - b*x^4],x]
```

output
```
((-2*a*Sqrt[a - b*x^4])/b^2 + (2*(a - b*x^4)^(3/2))/(3*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25
trager	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25
pseudoelliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25
orering	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(b\,x^{4}+2a\right)}{6b^{2}}$	25

input
```
int(x^7/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b*x^4+a)^(1/2)*(b*x^4+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx=-\frac{(bx^4+2\,a)\sqrt{-bx^4+a}}{6\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(b*x^4 + 2*a)*sqrt(-b*x^4 + a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{a\sqrt{a-bx^4}}{3b^2} - \frac{x^4\sqrt{a-bx^4}}{6b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-a*sqrt(a - b*x**4)/(3*b**2) - x**4*sqrt(a - b*x**4)/(6*b), Ne(
b, 0)), (x**8/(8*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = \frac{(-bx^4+a)^{\frac{3}{2}}}{6\,b^2} - \frac{\sqrt{-bx^4+a}a}{2\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(-b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(-b*x^4 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = \frac{(-bx^4+a)^{\frac{3}{2}}}{6\,b^2} - \frac{\sqrt{-bx^4+a}a}{2\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*(-b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(-b*x^4 + a)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-b\,x^4}\,(b\,x^4+2\,a)}{6\,b^2}$$

input
```
int(x^7/(a - b*x^4)^(1/2),x)
```

output
```
-((a - b*x^4)^(1/2)*(2*a + b*x^4))/(6*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{\sqrt{a-bx^4}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-b\,x^4-2a)}{6b^2}$$

input
```
int(x^7/(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 2*a - b*x**4))/(6*b**2)
```

3.206 $\int \frac{x^3}{\sqrt{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{x^3}{\sqrt{a-bx^4}} \, dx = -\frac{\sqrt{a-bx^4}}{2b}$$

output
```
-1/2*(-b*x^4+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a-bx^4}} \, dx = -\frac{\sqrt{a-bx^4}}{2b}$$

input
```
Integrate[x^3/Sqrt[a - b*x^4],x]
```

output
```
-1/2*Sqrt[a - b*x^4]/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a-bx^4}} \, dx$$

$$\Big\downarrow \; 793$$

$$-\frac{\sqrt{a-bx^4}}{2b}$$

input
```
Int[x^3/Sqrt[a - b*x^4],x]
```

output
```
-1/2*Sqrt[a - b*x^4]/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
derivativedivides	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
default	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
trager	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
pseudoelliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16
orering	$-\frac{\sqrt{-b\,x^{4}+a}}{2b}$	16

input
```
int(x^3/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^4+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{-bx^4+a}}{2\,b}$$

input
```
integrate(x^3/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^4 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{\sqrt{a-bx^4}}{2b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-sqrt(a - b*x**4)/(2*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-bx^4+a}}{2\,b}$$

input
```
integrate(x^3/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-b*x^4 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-bx^4+a}}{2\,b}$$

input
```
integrate(x^3/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{a-b\,x^4}}{2\,b}$$

input
```
int(x^3/(a - b*x^4)^(1/2),x)
```

output
```
-(a - b*x^4)^(1/2)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{-b\,x^4+a}}{2b}$$

input
```
int(x^3/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - b*x**4))/(2*b)
```

3.207 $\int \frac{1}{x\sqrt{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{1}{x\sqrt{a-bx^4}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-bx^4}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a - b*x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]]/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{1}{x^4\sqrt{a-bx^4}}dx^4$$

$$\Big\downarrow \; 73$$

$$-\frac{\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}}d\sqrt{a-bx^4}}{2b}$$

$$\Big\downarrow \; 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a - b*x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]]/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$	21
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}$	30
elliptic	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}$	30

input
```
int(1/x/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx = \left[\frac{\log\left(\frac{bx^4+2\,\sqrt{-bx^4+a}\sqrt{a}-2\,a}{x^4}\right)}{4\,\sqrt{a}}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)}{2\,a}\right]$$

input
```
integrate(1/x/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4)/sqrt(a), -1/2*sqr
t(-a)*arctan(sqrt(-b*x^4 + a)*sqrt(-a)/(b*x^4 - a))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{i\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-acosh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)), Abs(a/(b*x**4)) > 1
), (I*asin(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx = \frac{\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
integrate(1/x/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

input
```
integrate(1/x/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
int(1/(x*(a - b*x^4)^(1/2)),x)
```

output
```
-atanh((a - b*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\sqrt{a-bx^4}}\,dx = \frac{\sqrt{a}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)}{2a}$$

input
```
int(1/x/(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2)))/(2*a)
```

3.208 $\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{4ax^4} - \frac{b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
-1/4*(-b*x^4+a)^(1/2)/a/x^4-1/4*b*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(3/2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{4ax^4} - \frac{b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[1/(x^5*Sqrt[a - b*x^4]),x]
```

output
```
-1/4*Sqrt[a - b*x^4]/(a*x^4) - (b*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/(4*a^(
3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8\sqrt{a-bx^4}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{x^4\sqrt{a-bx^4}}dx^4}{2a}-\frac{\sqrt{a-bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\int \frac{1}{\frac{a}{b}-\frac{x^8}{b}}d\sqrt{a-bx^4}}{a}-\frac{\sqrt{a-bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a-bx^4}}{ax^4}\right)$$

input
```
Int[1/(x^5*Sqrt[a - b*x^4]),x]
```

output
```
(-(Sqrt[a - b*x^4]/(a*x^4)) - (b*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/a^(3/2)
)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right) b\,x^{4}+\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{4a^{\frac{3}{2}}\,x^{4}}$	44
default	$-\frac{\sqrt{-b\,x^{4}+a}}{4a\,x^{4}}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	50
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{4a\,x^{4}}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	50
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{4a\,x^{4}}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{3}{2}}}$	50

input
```
int(1/x^5/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(arctanh((-b*x^4+a)^(1/2)/a^(1/2))*b*x^4+a^(1/2)*(-b*x^4+a)^(1/2))/a^
(3/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.29

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx = \left[\frac{\sqrt{a}bx^4\log\left(\frac{bx^4+2\sqrt{-bx^4+a}\sqrt{a}-2a}{x^4}\right)-2\sqrt{-bx^4+a}a}{8a^2x^4},\ -\frac{\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)+\sqrt{-bx^4+a}a}{4a^2x^4}\right]$$

input
```
integrate(1/x^5/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*x^4*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x^4) -
2*sqrt(-b*x^4 + a)*a)/(a^2*x^4), -1/4*(sqrt(-a)*b*x^4*arctan(sqrt(-b*x^4 +
 a)*sqrt(-a)/(b*x^4 - a)) + sqrt(-b*x^4 + a)*a)/(a^2*x^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{4ax^2}-\frac{b\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ \frac{i}{4\sqrt{b}x^6\sqrt{-\frac{a}{bx^4}+1}}-\frac{i\sqrt{b}}{4ax^2\sqrt{-\frac{a}{bx^4}+1}}+\frac{ib\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(4*a*x**2) - b*acosh(sqrt(a)/(sqr
t(b)*x**2))/(4*a**(3/2)), Abs(a/(b*x**4)) > 1), (I/(4*sqrt(b)*x**6*sqrt(-a
/(b*x**4) + 1)) - I*sqrt(b)/(4*a*x**2*sqrt(-a/(b*x**4) + 1)) + I*b*asin(sq
rt(a)/(sqrt(b)*x**2))/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-bx^4+a}b}{4\left((bx^4-a)a+a^2\right)} + \frac{b\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-b*x^4 + a)*b/((b*x^4 - a)*a + a^2) + 1/8*b*log((sqrt(-b*x^4 + a
) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx = \frac{\frac{b^2\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\sqrt{-bx^4+a}b}{ax^4}}{4\,b}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(b^2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt(-b*x^4 + a)
*b/(a*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{a-b\,x^4}}{4\,a\,x^4}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{4\,a^{3/2}}$$

input
```
int(1/(x^5*(a - b*x^4)^(1/2)),x)
```

output
```
- (a - b*x^4)^(1/2)/(4*a*x^4) - (b*atanh((a - b*x^4)^(1/2)/a^(1/2)))/(4*a^
(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^5\sqrt{a-bx^4}}\,dx=\frac{-\sqrt{a}\,\sqrt{-b\,x^4+a}+\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)b\,x^4}{4\sqrt{a}\,a\,x^4}$$

input
```
int(1/x^5/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a)*sqrt(a - b*x**4) + log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2))*b*
x**4)/(4*sqrt(a)*a*x**4)
```

3.209 $\int \frac{x^5}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = -\frac{x^2\sqrt{a-bx^4}}{4b}+\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{4b^{3/2}}$$

output
```
-1/4*x^2*(-b*x^4+a)^(1/2)/b+1/4*a*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(
3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = -\frac{x^2\sqrt{a-bx^4}}{4b}-\frac{ia\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{4b^{3/2}}$$

input
```
Integrate[x^5/Sqrt[a - b*x^4],x]
```

output
```
-1/4*(x^2*Sqrt[a - b*x^4])/b - ((I/4)*a*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4
]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {807, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a-bx^4}}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{x^4}{\sqrt{a-bx^4}}dx^2$$
$$\Big\downarrow \text{262}$$
$$\frac{1}{2}\left(\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx^2}{2b}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)$$
$$\Big\downarrow \text{224}$$
$$\frac{1}{2}\left(\frac{a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}}{2b}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2b^{3/2}}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)$$

input
```
Int[x^5/Sqrt[a - b*x^4],x]
```

output
```
(-1/2*(x^2*Sqrt[a - b*x^4])/b + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/
(2*b^(3/2)))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4 b}+\frac{a \arctan\left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4 b^{\frac{3}{2}}}$	44
risch	$-\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4 b}+\frac{a \arctan\left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4 b^{\frac{3}{2}}}$	44
elliptic	$-\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4 b}+\frac{a \arctan\left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4 b^{\frac{3}{2}}}$	44
pseudoelliptic	$-\frac{x^{2} \sqrt{-b\,x^{4}+a}}{4 b}-\frac{a \arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\, x^{2}}\right)}{4 b^{\frac{3}{2}}}$	44

input
```
int(x^5/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*(-b*x^4+a)^(1/2)/b+1/4*a*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(
3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = \left[-\frac{2\sqrt{-bx^4+a}bx^2 + a\sqrt{-b}\log\left(2\,bx^4 - 2\sqrt{-bx^4+a}\sqrt{-b}x^2 - a\right)}{8\,b^2}, -\frac{\sqrt{-bx^4+a}bx^2 + a\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{4\,b^2}\right]$$

input
```
integrate(x^5/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(2*sqrt(-b*x^4 + a)*b*x^2 + a*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 +
 a)*sqrt(-b)*x^2 - a))/b^2, -1/4*(sqrt(-b*x^4 + a)*b*x^2 + a*sqrt(b)*arcta
n(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)))/b^2]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.33

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = \begin{cases} \frac{i\sqrt{a}x^2}{4b\sqrt{-1+\frac{bx^4}{a}}} - \frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} - \frac{ix^6}{4\sqrt{a}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{\sqrt{a}x^2\sqrt{1-\frac{bx^4}{a}}}{4b} + \frac{a\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((I*sqrt(a)*x**2/(4*b*sqrt(-1 + b*x**4/a)) - I*a*acosh(sqrt(b)*x*
*2/sqrt(a))/(4*b**(3/2)) - I*x**6/(4*sqrt(a)*sqrt(-1 + b*x**4/a)), Abs(b*x
**4/a) > 1), (-sqrt(a)*x**2*sqrt(1 - b*x**4/a)/(4*b) + a*asin(sqrt(b)*x**2
/sqrt(a))/(4*b**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = -\frac{a \arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{4\,b^{\frac{3}{2}}} - \frac{\sqrt{-bx^4+a}a}{4\left(b^2-\frac{(bx^4-a)b}{x^4}\right)x^2}$$

input
```
integrate(x^5/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/b^(3/2) - 1/4*sqrt(-b*x^4 +
a)*a/((b^2 - (b*x^4 - a)*b/x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-bx^4+a}x^2}{4\,b} - \frac{a \log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{4\,\sqrt{-b}b}$$

input
```
integrate(x^5/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(-b*x^4 + a)*x^2/b - 1/4*a*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 +
a)))/(sqrt(-b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = \int \frac{x^5}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x^5/(a - b*x^4)^(1/2),x)
```

output
```
int(x^5/(a - b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{a-bx^4}}\,dx = \frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)a-\sqrt{b}\,\sqrt{-b\,x^4+a}\,x^2}{4\sqrt{b}\,b}$$

input
```
int(x^5/(-b*x^4+a)^(1/2),x)
```

output
```
(asin((sqrt(b)*x**2)/sqrt(a))*a - sqrt(b)*sqrt(a - b*x**4)*x**2)/(4*sqrt(b
)*b)
```

3.210 $\int \frac{x}{\sqrt{a-bx^4}} dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{x}{\sqrt{a-bx^4}} dx = \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

output
```
1/2*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{x}{\sqrt{a-bx^4}} dx = -\frac{i\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{2\sqrt{b}}$$

input
```
Integrate[x/Sqrt[a - b*x^4],x]
```

output
```
((-1/2*I)*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {807, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2}\int \frac{1}{\sqrt{a-bx^4}}dx^2$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[x/Sqrt[a - b*x^4],x]
```

output
```
ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]]/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2\sqrt{b}}$	24
elliptic	$\frac{\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{2\sqrt{b}}$	24
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)}{2\sqrt{b}}$	24

input
```
int(x/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{x}{\sqrt{a-bx^4}}\,dx = \left[-\frac{\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)}{4\,b}, -\frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{2\,\sqrt{b}}\right]$$

input
```
integrate(x/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b*x^4 + a)*sqrt(-b)*x^2 - a)/b, -1/2*
arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{x}{\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{i\,\text{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ \frac{\text{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), Abs(b*x**4/a) > 1),
 (asin(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{a-bx^4}}\,dx = -\frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{2\sqrt{b}}$$

input
```
integrate(x/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{x}{\sqrt{a-bx^4}}\,dx = -\frac{\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{2\sqrt{-b}}$$

input
```
integrate(x/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 + a)))/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-bx^4}}\,dx = \int \frac{x}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x/(a - b*x^4)^(1/2),x)
```

output
```
int(x/(a - b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x}{\sqrt{a-bx^4}}\,dx = \frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
int(x/(-b*x^4+a)^(1/2),x)
```

output
```
asin((sqrt(b)*x**2)/sqrt(a))/(2*sqrt(b))
```

3.211 $\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{2ax^2}$$

output
```
-1/2*(-b*x^4+a)^(1/2)/a/x^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{2ax^2}$$

input
```
Integrate[1/(x^3*Sqrt[a - b*x^4]),x]
```

output
```
-1/2*Sqrt[a - b*x^4]/(a*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\sqrt{a-bx^4}}{2ax^2}$$

input
```
Int[1/(x^3*Sqrt[a - b*x^4]),x]
```

output
```
-1/2*Sqrt[a - b*x^4]/(a*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19
default	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19
trager	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19
pseudoelliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19
orering	$-\frac{\sqrt{-b\,x^{4}+a}}{2a\,x^{2}}$	19

input
```
int(1/x^3/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^4+a)^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{-bx^4+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^4 + a)/(a*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{2a} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ -\frac{i\sqrt{b}\sqrt{-\frac{a}{bx^4}+1}}{2a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(2*a), Abs(a/(b*x**4)) > 1), (-I*
sqrt(b)*sqrt(-a/(b*x**4) + 1)/(2*a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-bx^4+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-b*x^4 + a)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = \frac{\sqrt{-b}}{\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a}$$

input
```
integrate(1/x^3/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(-b)/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-b\,x^4}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a - b*x^4)^(1/2)),x)
```

output
```
-(a - b*x^4)^(1/2)/(2*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-b\,x^4+a}}{2a\,x^2}$$

input
```
int(1/x^3/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - b*x**4))/(2*a*x**2)
```

3.212 $\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{6ax^6}-\frac{b\sqrt{a-bx^4}}{3a^2x^2}$$

output
```
-1/6*(-b*x^4+a)^(1/2)/a/x^6-1/3*b*(-b*x^4+a)^(1/2)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = \frac{(-a-2bx^4)\sqrt{a-bx^4}}{6a^2x^6}$$

input
```
Integrate[1/(x^7*Sqrt[a - b*x^4]),x]
```

output
```
((-a - 2*b*x^4)*Sqrt[a - b*x^4])/(6*a^2*x^6)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{2b\int \frac{1}{x^3\sqrt{a-bx^4}}dx}{3a}-\frac{\sqrt{a-bx^4}}{6ax^6}$$

$$\Big\downarrow \text{796}$$

$$-\frac{b\sqrt{a-bx^4}}{3a^2x^2}-\frac{\sqrt{a-bx^4}}{6ax^6}$$

input
```
Int[1/(x^7*Sqrt[a - b*x^4]),x]
```

output
```
-1/6*Sqrt[a - b*x^4]/(a*x^6) - (b*Sqrt[a - b*x^4])/(3*a^2*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27
trager	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27
pseudoelliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27
orering	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	27

input
```
int(1/x^7/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-b*x^4+a)^(1/2)*(2*b*x^4+a)/a^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = -\frac{(2\,bx^4+a)\sqrt{-bx^4+a}}{6\,a^2x^6}$$

input
```
integrate(1/x^7/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*b*x^4 + a)*sqrt(-b*x^4 + a)/(a^2*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.11

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = \begin{cases} -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}-1}}{6ax^4} - \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}-1}}{3a^2} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{-6a^3bx^4+6a^2b^2x^8} + \frac{iab^{\frac{5}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{-6a^3bx^4+6a^2b^2x^8} - \frac{2ib^{\frac{7}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{-6a^3bx^4+6a^2b^2x^8} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**7/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-sqrt(b)*sqrt(a/(b*x**4) - 1)/(6*a*x**4) - b**(3/2)*sqrt(a/(b*x
**4) - 1)/(3*a**2), Abs(a/(b*x**4)) > 1), (I*a**2*b**(3/2)*sqrt(-a/(b*x**4
) + 1)/(-6*a**3*b*x**4 + 6*a**2*b**2*x**8) + I*a*b**(5/2)*x**4*sqrt(-a/(b*
x**4) + 1)/(-6*a**3*b*x**4 + 6*a**2*b**2*x**8) - 2*I*b**(7/2)*x**8*sqrt(-a
/(b*x**4) + 1)/(-6*a**3*b*x**4 + 6*a**2*b**2*x**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = -\frac{\frac{3\sqrt{-bx^4+a}\,b}{x^2} + \frac{(-bx^4+a)^{\frac{3}{2}}}{x^6}}{6\,a^2}$$

input
```
integrate(1/x^7/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(-b*x^4 + a)*b/x^2 + (-b*x^4 + a)^(3/2)/x^6)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = -\frac{2\left(3\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)\sqrt{-b}b}{3\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^3}$$

input
```
integrate(1/x^7/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)*sqrt(-b)*b/((sqrt(-b)*x^2
 - sqrt(-b*x^4 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-b\,x^4}\,\left(2\,b\,x^4+a\right)}{6\,a^2\,x^6}$$

input
```
int(1/(x^7*(a - b*x^4)^(1/2)),x)
```

output
```
-((a - b*x^4)^(1/2)*(a + 2*b*x^4))/(6*a^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^7\sqrt{a-bx^4}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-2b\,x^4-a)}{6a^2x^6}$$

input
```
int(1/x^7/(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - a - 2*b*x**4))/(6*a**2*x**6)
```

3.213 $\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{10ax^{10}} - \frac{2b\sqrt{a-bx^4}}{15a^2x^6} - \frac{4b^2\sqrt{a-bx^4}}{15a^3x^2}$$

output
```
-1/10*(-b*x^4+a)^(1/2)/a/x^10-2/15*b*(-b*x^4+a)^(1/2)/a^2/x^6-4/15*b^2*(-b
*x^4+a)^(1/2)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = \frac{\sqrt{a-bx^4}(-3a^2-4abx^4-8b^2x^8)}{30a^3x^{10}}$$

input
```
Integrate[1/(x^11*Sqrt[a - b*x^4]),x]
```

output
```
(Sqrt[a - b*x^4]*(-3*a^2 - 4*a*b*x^4 - 8*b^2*x^8))/(30*a^3*x^10)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx$$
$$\Big\downarrow \text{ 803}$$
$$\frac{4b\int \frac{1}{x^7\sqrt{a-bx^4}}dx}{5a}-\frac{\sqrt{a-bx^4}}{10ax^{10}}$$
$$\Big\downarrow \text{ 803}$$
$$\frac{4b\left(\frac{2b\int \frac{1}{x^3\sqrt{a-bx^4}}dx}{3a}-\frac{\sqrt{a-bx^4}}{6ax^6}\right)}{5a}-\frac{\sqrt{a-bx^4}}{10ax^{10}}$$
$$\Big\downarrow \text{ 796}$$
$$\frac{4b\left(-\frac{b\sqrt{a-bx^4}}{3a^2x^2}-\frac{\sqrt{a-bx^4}}{6ax^6}\right)}{5a}-\frac{\sqrt{a-bx^4}}{10ax^{10}}$$

input
```
Int[1/(x^11*Sqrt[a - b*x^4]),x]
```

output
```
-1/10*Sqrt[a - b*x^4]/(a*x^10) + (4*b*(-1/6*Sqrt[a - b*x^4]/(a*x^6) - (b*S
qrt[a - b*x^4])/(3*a^2*x^2)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40
default	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40
trager	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40
pseudoelliptic	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40
orering	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(8b^{2}x^{8}+4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	40

input
```
int(1/x^11/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(-b*x^4+a)^(1/2)*(8*b^2*x^8+4*a*b*x^4+3*a^2)/a^3/x^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = -\frac{(8\,b^2x^8+4\,abx^4+3\,a^2)\sqrt{-bx^4+a}}{30\,a^3x^{10}}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(8*b^2*x^8 + 4*a*b*x^4 + 3*a^2)*sqrt(-b*x^4 + a)/(a^3*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 609, normalized size of antiderivative = 8.58

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx$$
$$= \begin{cases} -\frac{3a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} + \frac{2a^3b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} - \frac{3a^2b^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}-1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} + \frac{12ab^{\frac{15}{2}}x^{12}\sqrt{\frac{a}{b\cdots}}}{30a^5b^4x^8-60a^4b^5x^{12}\cdots} \\ -\frac{3ia^4b^{\frac{9}{2}}\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} + \frac{2ia^3b^{\frac{11}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} - \frac{3ia^2b^{\frac{13}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{30a^5b^4x^8-60a^4b^5x^{12}+30a^3b^6x^{16}} + \frac{12iab^{\frac{15}{2}}x^{12}\sqrt{-\cdots}}{30a^5b^4x^8-60a^4b^5x^{12}\cdots} \end{cases}$$

input
```
integrate(1/x**11/(-b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-3*a**4*b**(9/2)*sqrt(a/(b*x**4) - 1)/(30*a**5*b**4*x**8 - 60*a
**4*b**5*x**12 + 30*a**3*b**6*x**16) + 2*a**3*b**(11/2)*x**4*sqrt(a/(b*x**
4) - 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 3*
a**2*b**(13/2)*x**8*sqrt(a/(b*x**4) - 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5
*x**12 + 30*a**3*b**6*x**16) + 12*a*b**(15/2)*x**12*sqrt(a/(b*x**4) - 1)/(
30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 8*b**(17/2)
*x**16*sqrt(a/(b*x**4) - 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a
**3*b**6*x**16), Abs(a/(b*x**4)) > 1), (-3*I*a**4*b**(9/2)*sqrt(-a/(b*x**4
) + 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) + 2*I
*a**3*b**(11/2)*x**4*sqrt(-a/(b*x**4) + 1)/(30*a**5*b**4*x**8 - 60*a**4*b*
*5*x**12 + 30*a**3*b**6*x**16) - 3*I*a**2*b**(13/2)*x**8*sqrt(-a/(b*x**4)
+ 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) + 12*I*
a*b**(15/2)*x**12*sqrt(-a/(b*x**4) + 1)/(30*a**5*b**4*x**8 - 60*a**4*b**5*
x**12 + 30*a**3*b**6*x**16) - 8*I*b**(17/2)*x**16*sqrt(-a/(b*x**4) + 1)/(3
0*a**5*b**4*x**8 - 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = -\frac{\frac{15\sqrt{-bx^4+a}b^2}{x^2}+\frac{10\,(-bx^4+a)^{\frac{3}{2}}b}{x^6}+\frac{3\,(-bx^4+a)^{\frac{5}{2}}}{x^{10}}}{30\,a^3}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/30*(15*sqrt(-b*x^4 + a)*b^2/x^2 + 10*(-b*x^4 + a)^(3/2)*b/x^6 + 3*(-b*x
^4 + a)^(5/2)/x^10)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = \frac{8\left(10\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^4-5\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2a+a^2\right)\sqrt{-b}b^2}{15\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)^5}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
8/15*(10*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^4 - 5*(sqrt(-b)*x^2 - sqrt(-b*x
^4 + a))^2*a + a^2)*sqrt(-b)*b^2/((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)
^5
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-b\,x^4}\,\left(3\,a^2+4\,a\,b\,x^4+8\,b^2\,x^8\right)}{30\,a^3\,x^{10}}$$

input
```
int(1/(x^11*(a - b*x^4)^(1/2)),x)
```

output
```
-((a - b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^8 + 4*a*b*x^4))/(30*a^3*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{11}\sqrt{a-bx^4}}\,dx = \frac{\sqrt{-b\,x^4+a}\,(-8b^2x^8-4ab\,x^4-3a^2)}{30a^3x^{10}}$$

input
```
int(1/x^11/(-b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a - b*x**4)*( - 3*a**2 - 4*a*b*x**4 - 8*b**2*x**8))/(30*a**3*x**10)
```

3.214 $\int \frac{x^8}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = -\frac{5ax\sqrt{a-bx^4}}{21b^2}-\frac{x^5\sqrt{a-bx^4}}{7b}+\frac{5a^{9/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21b^{9/4}\sqrt{a-bx^4}}$$

output
```
-5/21*a*x*(-b*x^4+a)^(1/2)/b^2-1/7*x^5*(-b*x^4+a)^(1/2)/b+5/21*a^(9/4)*(1-
b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(9/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \frac{-5a^2x+2abx^5+3b^2x^9+5a^2x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{21b^2\sqrt{a-bx^4}}$$

input
```
Integrate[x^8/Sqrt[a - b*x^4],x]
```

output
```
(-5*a^2*x + 2*a*b*x^5 + 3*b^2*x^9 + 5*a^2*x*Sqrt[1 - (b*x^4)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/(21*b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {843, 843, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5a\int \frac{x^4}{\sqrt{a-bx^4}}dx}{7b}-\frac{x^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5a\left(\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx}{3b}-\frac{x\sqrt{a-bx^4}}{3b}\right)}{7b}-\frac{x^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{5a\left(\frac{a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3b\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)}{7b}-\frac{x^5\sqrt{a-bx^4}}{7b}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{5a\left(\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)}{7b}-\frac{x^5\sqrt{a-bx^4}}{7b}$$

input
```
Int[x^8/Sqrt[a - b*x^4],x]
```

output
```
-1/7*(x^5*Sqrt[a - b*x^4])/b + (5*a*(-1/3*(x*Sqrt[a - b*x^4])/b + (a^(5/4)
*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(5/4
)*Sqrt[a - b*x^4])))/(7*b)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{x(3b\,x^4+5a)\sqrt{-b\,x^4+a}}{21b^2}+\frac{5a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	98
default	$-\frac{x^5\sqrt{-b\,x^4+a}}{7b}-\frac{5ax\sqrt{-b\,x^4+a}}{21b^2}+\frac{5a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	107
elliptic	$-\frac{x^5\sqrt{-b\,x^4+a}}{7b}-\frac{5ax\sqrt{-b\,x^4+a}}{21b^2}+\frac{5a^2\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{21b^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	107

input
```
int(x^8/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(3*b*x^4+5*a)*(-b*x^4+a)^(1/2)/b^2+5/21*a^2/b^2/(1/a^(1/2)*b^(1/2)
)^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^
4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \frac{5\,a\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)-(3\,bx^5+5\,ax)\sqrt{-bx^4+a}}{21\,b^2}$$

input
```
integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(5*a*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - (3*
b*x^5 + 5*a*x)*sqrt(-b*x^4 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(-b*x**4+a)**(1/2),x)
```

output
```
x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \int \frac{x^8}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \int \frac{x^8}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \int \frac{x^8}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x^8/(a - b*x^4)^(1/2),x)
```

output
```
int(x^8/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a-bx^4}}\,dx = \frac{-5\sqrt{-b\,x^4+a}\,ax-3\sqrt{-b\,x^4+a}\,b\,x^5+5\left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)a^2}{21b^2}$$

input
```
int(x^8/(-b*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a - b*x**4)*a*x - 3*sqrt(a - b*x**4)*b*x**5 + 5*int(sqrt(a - b*
x**4)/(a - b*x**4),x)*a**2)/(21*b**2)
```

3.215 $\int \frac{x^4}{\sqrt{a-bx^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^4}{\sqrt{a-bx^4}} dx = -\frac{x\sqrt{a-bx^4}}{3b} + \frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}$$

output
```
-1/3*x*(-b*x^4+a)^(1/2)/b+1/3*a^(5/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*
x/a^(1/4),I)/b^(5/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\sqrt{a-bx^4}} dx = \frac{x\left(-a+bx^4+a\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)\right)}{3b\sqrt{a-bx^4}}$$

input
```
Integrate[x^4/Sqrt[a - b*x^4],x]
```

output
```
(x*(-a + b*x^4 + a*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b
*x^4)/a]))/(3*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {843, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx}{3b}-\frac{x\sqrt{a-bx^4}}{3b}$$

$$\Big\downarrow \text{765}$$

$$\frac{a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3b\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}$$

$$\Big\downarrow \text{762}$$

$$\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}$$

input
```
Int[x^4/Sqrt[a - b*x^4],x]
```

output
```
-1/3*(x*Sqrt[a - b*x^4])/b + (a^(5/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin
[(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(5/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{x\sqrt{-b\,x^4+a}}{3b}+\frac{a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	86
risch	$-\frac{x\sqrt{-b\,x^4+a}}{3b}+\frac{a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	86
elliptic	$-\frac{x\sqrt{-b\,x^4+a}}{3b}+\frac{a\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$	86

input
```
int(x^4/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x*(-b*x^4+a)^(1/2)/b+1/3*a/b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2
/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x
*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx = \frac{\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - \sqrt{-bx^4+a}\,x}{3\,b}$$

input
```
integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) - sqrt(-b*
x^4 + a)*x)/b
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(-b*x**4+a)**(1/2),x)
```

output
```
x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*s
qrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx = \int \frac{x^4}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx = \int \frac{x^4}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a-bx^4}}\,dx = \int \frac{x^4}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x^4/(a - b*x^4)^(1/2),x)
```

output
```
int(x^4/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a - bx^4}}\,dx = \frac{-\sqrt{-b\,x^4 + a}\,x + \left(\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx\right)a}{3b}$$

input
```
int(x^4/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - b*x**4)*x + int(sqrt(a - b*x**4)/(a - b*x**4),x)*a)/(3*b)
```

3.216 $\int \frac{1}{\sqrt{a-bx^4}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{1}{\sqrt{a-bx^4}} \, dx = \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
a^(1/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(1/4)/(-b*x^4+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a-bx^4}} \, dx = -\frac{i\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{a-bx^4}}$$

input
```
Integrate[1/Sqrt[a - b*x^4],x]
```

output
```
((-I)*Sqrt[1 - (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[b]/Sqrt[a])]*x],
 -1])/(Sqrt[-(Sqrt[b]/Sqrt[a])]*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{ 765}$$

$$\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^4}}$$

input
```
Int[1/Sqrt[a - b*x^4],x]
```

output
```
(a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(
b^(1/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	64
elliptic	$\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	64

input
```
int(1/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a
^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a-bx^4}}\,dx=\frac{\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)}{b}$$

input
```
integrate(1/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1)/b
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a-bx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-b*x**4+a)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt
(a)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^4}}\,dx=\int \frac{1}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{a-bx^4}}\,dx=\frac{x\sqrt{1-\frac{b\,x^4}{a}}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{\sqrt{a-b\,x^4}}$$

input
```
int(1/(a - b*x^4)^(1/2),x)
```

output
```
(x*(1 - (b*x^4)/a)^(1/2)*hypergeom([1/4, 1/2], 5/4, (b*x^4)/a))/(a - b*x^4
)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a-bx^4}}\,dx=\int \frac{\sqrt{-b\,x^4+a}}{-b\,x^4+a}dx$$

input
```
int(1/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a - b*x**4),x)
```

3.217 $\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{a-bx^4}}{3ax^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt{a-bx^4}}$$

output
```
-1/3*(-b*x^4+a)^(1/2)/a/x^3+1/3*b^(3/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4
)*x/a^(1/4),I)/a^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx=-\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\frac{bx^4}{a}\right)}{3x^3\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^4*Sqrt[a - b*x^4]),x]
```

output
```
-1/3*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, (b*x^4)/a])/(x
^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {847, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{3a}-\frac{\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{765}$$

$$\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3a\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{762}$$

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a - b*x^4]),x]
```

output
```
-1/3*Sqrt[a - b*x^4]/(a*x^3) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcS
in[(b^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

method	result	size
default	$-\frac{\sqrt{-b\,x^{4}+a}}{3a\,x^{3}}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	88
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{3a\,x^{3}}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	88
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{3a\,x^{3}}+\frac{b\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	88

input
```
int(1/x^4/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^4+a)^(1/2)/a/x^3+1/3*b/a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x
^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx = \frac{\sqrt{a}x^3\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)-\sqrt{-bx^4+a}}{3\,ax^3}$$

input
```
integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(a)*x^3*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) - sqrt(
-b*x^4 + a))/(a*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(-b*x**4+a)**(1/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt
(a)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+ax^4}}\,dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+ax^4}}\,dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx = \int \frac{1}{x^4\,\sqrt{a-b\,x^4}}\,dx$$

input
```
int(1/(x^4*(a - b*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a - b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a-bx^4}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-b\,x^8+a\,x^4}dx$$

input
```
int(1/x^4/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**4 - b*x**8),x)
```

3.218 $\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{7ax^7}-\frac{5b\sqrt{a-bx^4}}{21a^2x^3}+\frac{5b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21a^{7/4}\sqrt{a-bx^4}}$$

output
```
-1/7*(-b*x^4+a)^(1/2)/a/x^7-5/21*b*(-b*x^4+a)^(1/2)/a^2/x^3+5/21*b^(7/4)*(
1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},\frac{bx^4}{a}\right)}{7x^7\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^8*Sqrt[a - b*x^4]),x]
```

output
```
-1/7*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-7/4, 1/2, -3/4, (b*x^4)/a])/(
x^7*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {847, 847, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{5b\int \frac{1}{x^4\sqrt{a-bx^4}}dx}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$\frac{5b\left(\frac{b\int \frac{1}{\sqrt{a-bx^4}}dx}{3a}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \text{765}$$

$$\frac{5b\left(\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3a\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \text{762}$$

$$\frac{5b\left(\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}$$

input
```
Int[1/(x^8*Sqrt[a - b*x^4]),x]
```

output
```
-1/7*Sqrt[a - b*x^4]/(a*x^7) + (5*b*(-1/3*Sqrt[a - b*x^4]/(a*x^3) + (b^(3/
4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*a^(3
/4)*Sqrt[a - b*x^4])))/(7*a)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(5 b\,x^{4}+3 a\right)}{21 a^{2} x^{7}}+\frac{5 b^{2} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{21 a^{2} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	100
default	$-\frac{\sqrt{-b\,x^{4}+a}}{7 a\,x^{7}}-\frac{5 b \sqrt{-b\,x^{4}+a}}{21 a^{2} x^{3}}+\frac{5 b^{2} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{21 a^{2} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	109
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{7 a\,x^{7}}-\frac{5 b \sqrt{-b\,x^{4}+a}}{21 a^{2} x^{3}}+\frac{5 b^{2} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)}{21 a^{2} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	109

input
```
int(1/x^8/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-b*x^4+a)^(1/2)*(5*b*x^4+3*a)/a^2/x^7+5/21*b^2/a^2/(1/a^(1/2)*b^(1/
2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = \frac{5\,\sqrt{a}bx^7\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-(5\,bx^4+3\,a)\sqrt{-bx^4+a}}{21\,a^2x^7}$$

input
```
integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(5*sqrt(a)*b*x^7*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) -
(5*b*x^4 + 3*a)*sqrt(-b*x^4 + a))/(a^2*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(-b*x**4+a)**(1/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqr
t(a)*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+ax^8}}\,dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+ax^8}}\,dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = \int \frac{1}{x^8\,\sqrt{a-b\,x^4}}\,dx$$

input
```
int(1/(x^8*(a - b*x^4)^(1/2)),x)
```

output
```
int(1/(x^8*(a - b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{a-bx^4}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{12}+a\,x^8}dx$$

input
```
int(1/x^8/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**8 - b*x**12),x)
```

3.219 $\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 158

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = -\frac{7ax^3\sqrt{a-bx^4}}{45b^2}-\frac{x^7\sqrt{a-bx^4}}{9b}+\frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{15b^{11/4}\sqrt{a-bx^4}}-\frac{7a^{11/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{15b^{11/4}\sqrt{a-bx^4}}$$

output
```
-7/45*a*x^3*(-b*x^4+a)^(1/2)/b^2-1/9*x^7*(-b*x^4+a)^(1/2)/b+7/15*a^(11/4)*
(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(11/4)/(-b*x^4+a)^(1/2)
-7/15*a^(11/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(11/4)/(
-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = \frac{x^3\left(-7a^2+2abx^4+5b^2x^8+7a^2\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)\right)}{45b^2\sqrt{a-bx^4}}$$

input
```
Integrate[x^10/Sqrt[a - b*x^4],x]
```

output
```
(x^3*(-7*a^2 + 2*a*b*x^4 + 5*b^2*x^8 + 7*a^2*Sqrt[1 - (b*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, (b*x^4)/a]))/(45*b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {843, 843, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{7a\int \frac{x^6}{\sqrt{a-bx^4}}dx}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \text{843}$$

$$\frac{7a\left(\frac{3a\int \frac{x^2}{\sqrt{a-bx^4}}dx}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 836$$

$$\frac{7a\left(\frac{3a\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 27$$

$$\frac{7a\left(\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 765$$

$$\frac{7a\left(\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 762$$

$$\frac{7a\left(\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 1390$$

$$\frac{7a\left(\frac{3a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 1389$$

$$\frac{7a\left(\frac{3a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

$$\Big\downarrow \ 327$$

$$\frac{7a\left(\frac{3a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{9b}-\frac{x^7\sqrt{a-bx^4}}{9b}$$

input
```
Int[x^10/Sqrt[a - b*x^4],x]
```

output
```
-1/9*(x^7*Sqrt[a - b*x^4])/b + (7*a*(-1/5*(x^3*Sqrt[a - b*x^4])/b + (3*a*(
(a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(
b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(
b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/(5*b)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{x^{3}\left(5 b \,x^{4}+7 a \right) \sqrt{-b \,x^{4}+a}}{45 b^{2}}-\frac{7 a^{\frac{5}{2}} \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 b^{\frac{5}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}}$	117
default	$-\frac{x^{7} \sqrt{-b \,x^{4}+a}}{9 b}-\frac{7 a \,x^{3} \sqrt{-b \,x^{4}+a}}{45 b^{2}}-\frac{7 a^{\frac{5}{2}} \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 b^{\frac{5}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}}$	126
elliptic	$-\frac{x^{7} \sqrt{-b \,x^{4}+a}}{9 b}-\frac{7 a \,x^{3} \sqrt{-b \,x^{4}+a}}{45 b^{2}}-\frac{7 a^{\frac{5}{2}} \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 b^{\frac{5}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}}$	126

input
```
int(x^10/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*x^3*(5*b*x^4+7*a)*(-b*x^4+a)^(1/2)/b^2-7/15*a^(5/2)/b^(5/2)/(1/a^(1/
2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x
*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = -\frac{21\,a^{2}\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-21\,a^{2}\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(5\,b^{2}x^{8}+7\,abx^{4}}{45\,b^{3}x}$$

input
```
integrate(x^10/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/45*(21*a^2*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1)
 - 21*a^2*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + (
5*b^2*x^8 + 7*a*b*x^4 + 21*a^2)*sqrt(-b*x^4 + a))/(b^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.25

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(-b*x**4+a)**(1/2),x)
```

output
```
x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/
(4*sqrt(a)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = \int \frac{x^{10}}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^10/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = \int \frac{x^{10}}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^10/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = \int \frac{x^{10}}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x^10/(a - b*x^4)^(1/2),x)
```

output
```
int(x^10/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{a-bx^4}}\,dx = \frac{-7\sqrt{-b\,x^4+a}\,a\,x^3-5\sqrt{-b\,x^4+a}\,b\,x^7+21\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)a^2}{45b^2}$$

input
```
int(x^10/(-b*x^4+a)^(1/2),x)
```

output
```
( - 7*sqrt(a - b*x**4)*a*x**3 - 5*sqrt(a - b*x**4)*b*x**7 + 21*int((sqrt(a
 - b*x**4)*x**2)/(a - b*x**4),x)*a**2)/(45*b**2)
```

3.220 $\int \frac{x^6}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx = -\frac{x^3\sqrt{a-bx^4}}{5b} + \frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5b^{7/4}\sqrt{a-bx^4}} - \frac{3a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{5b^{7/4}\sqrt{a-bx^4}}$$

output
```
-1/5*x^3*(-b*x^4+a)^(1/2)/b+3/5*a^(7/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4
)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)-3/5*a^(7/4)*(1-b*x^4/a)^(1/2)*Elli
pticF(b^(1/4)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.49

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx = \frac{x^3\left(-a+bx^4+a\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)\right)}{5b\sqrt{a-bx^4}}$$

input
```
Integrate[x^6/Sqrt[a - b*x^4],x]
```

output
```
(x^3*(-a + b*x^4 + a*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
(b*x^4)/a]))/(5*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {843, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{3a\int \frac{x^2}{\sqrt{a-bx^4}}dx}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{836}$$

$$\frac{3a\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{765}$$

$$\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{762}$$

$$\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{3a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{3a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{3a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}$$

input
```
Int[x^6/Sqrt[a - b*x^4],x]
```

output
```
-1/5*(x^3*Sqrt[a - b*x^4])/b + (3*a*((a^(3/4)*Sqrt[1 - (b*x^4)/a]*Elliptic
E[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*S
qrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sq
rt[a - b*x^4])))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{x^3\sqrt{-b x^4+a}}{5b}-\frac{3a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}$	107
risch	$-\frac{x^3\sqrt{-b x^4+a}}{5b}-\frac{3a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}$	107
elliptic	$-\frac{x^3\sqrt{-b x^4+a}}{5b}-\frac{3a^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5b^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b x^4+a}}$	107

input
```
int(x^6/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x^3*(-b*x^4+a)^(1/2)/b-3/5*a^(3/2)/b^(3/2)/(1/a^(1/2)*b^(1/2))^(1/2)*
(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/
2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2
))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx =$$
$$-\frac{3\,a\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-3\,a\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+(bx^4+3\,a)\sqrt{-bx^4+}}{5\,b^2x}$$

input
```
integrate(x^6/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1) - 3
*a*sqrt(-b)*x*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + (b*x^4 +
 3*a)*sqrt(-b*x^4 + a))/(b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(-b*x**4+a)**(1/2),x)
```

output
```
x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx = \int \frac{x^6}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx=\int \frac{x^6}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx=\int \frac{x^6}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x^6/(a - b*x^4)^(1/2),x)
```

output
```
int(x^6/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a-bx^4}}\,dx=\frac{-\sqrt{-b\,x^4+a}\,x^3+3\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx\right)a}{5b}$$

input
```
int(x^6/(-b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a - b*x**4)*x**3 + 3*int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)*
a)/(5*b)
```

3.221 $\int \frac{x^2}{\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx = \frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}} - \frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

output
```
a^(3/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(3/4)/(-b*x^4+a
)^(1/2)-a^(3/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(3/4)/(
-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx = \frac{x^3\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)}{3\sqrt{a-bx^4}}$$

input
```
Integrate[x^2/Sqrt[a - b*x^4],x]
```

output
```
(x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (b*x^4)/a])/(3*S
qrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx$$

$$\Big\downarrow \text{836}$$

$$\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{1390}$$

$$\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{1389}$$

$$\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}$$

input
```
Int[x^2/Sqrt[a - b*x^4],x]
```

output
```
(a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(
b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(
b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$	88
elliptic	$-\frac{\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}$	88

input
```
int(x^2/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2
)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1
/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}} E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) | -1) - \sqrt{-b}x\left(\frac{a}{b}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) | -1) + \sqrt{-bx^4+a}}{bx}$$

input
```
integrate(x^2/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-b)*x*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x), -1) - sqrt(-b)*
x*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x), -1) + sqrt(-b*x^4 + a))/(b
*x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{3}{4}\\ \frac{7}{4}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-b*x**4+a)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*s
qrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx=\int \frac{x^2}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx=\int \frac{x^2}{\sqrt{-bx^4+a}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx=\int \frac{x^2}{\sqrt{a-b\,x^4}}\,dx$$

input
```
int(x^2/(a - b*x^4)^(1/2),x)
```

output
```
int(x^2/(a - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a-bx^4}}\,dx = \int \frac{\sqrt{-b\,x^4+a}\,x^2}{-b\,x^4+a}dx$$

input
```
int(x^2/(-b*x^4+a)^(1/2),x)
```

output
```
int((sqrt(a - b*x**4)*x**2)/(a - b*x**4),x)
```

3.222 $\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{ax} - \frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}} + \frac{\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}$$

output
```
-(-b*x^4+a)^(1/2)/a/x-b^(1/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4
),I)/a^(1/4)/(-b*x^4+a)^(1/2)+b^(1/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*
x/a^(1/4),I)/a^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{bx^4}{a}\right)}{x\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^2*Sqrt[a - b*x^4]),x]
```

output
```
-((Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, (b*x^4)/a])/(x*Sq
rt[a - b*x^4]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {847, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{b\int \frac{x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{836}$$

$$-\frac{b\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{765}$$

$$-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{762}$$

$$-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{1390}$$

$$-\frac{b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{1389}$$

$$-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

$$\Big\downarrow \text{327}$$

$$-\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a - b*x^4]),x]
```

output
```
-(Sqrt[a - b*x^4]/(a*x)) - (b*((a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcS
in[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1
- (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a -
 b*x^4])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\sqrt{-b\,x^{4}+a}}{a x}+\frac{\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	106
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{a x}+\frac{\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	106
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{a x}+\frac{\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{a}\,\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	106

input
```
int(1/x^2/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-b*x^4+a)^(1/2)/a/x+b^(1/2)/a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)
*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(Ellipt
icF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a}x\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-\sqrt{a}x\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)+\sqrt{-bx^4+a}}{ax}$$

input
```
integrate(1/x^2/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*x*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1/4)), -1) - sqrt(a)*x*
(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + sqrt(-b*x^4 + a))/(a*x
)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(-b*x**4+a)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*sqrt
(a)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+a}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+ax^2}}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{1-\frac{a}{b\,x^4}}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ \frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{a-b\,x^4}}$$

input
```
int(1/(x^2*(a - b*x^4)^(1/2)),x)
```

output
```
-((1 - a/(b*x^4))^(1/2)*hypergeom([1/2, 3/4], 7/4, a/(b*x^4)))/(3*x*(a - b
*x^4)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a-bx^4}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-b\,x^6+a\,x^2}dx$$

input
```
int(1/x^2/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**2 - b*x**6),x)
```

3.223 $\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 158

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{a-bx^4}}{5ax^5}-\frac{3b\sqrt{a-bx^4}}{5a^2x} - \frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5a^{5/4}\sqrt{a-bx^4}} + \frac{3b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{5a^{5/4}\sqrt{a-bx^4}}$$

output
```
-1/5*(-b*x^4+a)^(1/2)/a/x^5-3/5*b*(-b*x^4+a)^(1/2)/a^2/x-3/5*b^(5/4)*(1-b*
x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^4+a)^(1/2)+3/5*b
^(5/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^4+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},\frac{bx^4}{a}\right)}{5x^5\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^6*Sqrt[a - b*x^4]),x]
```

output
```
-1/5*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, (b*x^4)/a])/(
x^5*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {847, 847, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a-bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{3b\int \frac{1}{x^2\sqrt{a-bx^4}}dx}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{3b\left(-\frac{b\int \frac{x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{836}$$

$$\frac{3b\left(-\frac{b\left(\frac{\sqrt{a}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 27

$$\frac{3b\left(-\frac{b\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 765

$$\frac{3b\left(-\frac{b\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 762

$$\frac{3b\left(-\frac{b\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$\downarrow$ 1390

$$\frac{3b\left(-\frac{b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{1389}$$

$$\frac{3b\left(-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

$$\Big\downarrow \text{327}$$

$$\frac{3b\left(-\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}$$

input
```
Int[1/(x^6*Sqrt[a - b*x^4]),x]
```

output
```
-1/5*Sqrt[a - b*x^4]/(a*x^5) + (3*b*(-(Sqrt[a - b*x^4]/(a*x)) - (b*((a^(3/
4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4
)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4
)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,(3b\,x^{4}+a)}{5a^{2}x^{5}}+\frac{3b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	115
default	$-\frac{\sqrt{-b\,x^{4}+a}}{5a\,x^{5}}-\frac{3b\sqrt{-b\,x^{4}+a}}{5a^{2}x}+\frac{3b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	126
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{5a\,x^{5}}-\frac{3b\sqrt{-b\,x^{4}+a}}{5a^{2}x}+\frac{3b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	126

input
```
int(1/x^6/(-b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/2)*(3*b*x^4+a)/a^2/x^5+3/5*b^(3/2)/a^(3/2)/(1/a^(1/2)*b
^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/
(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/
a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx =$$
$$-\frac{3\,\sqrt{a}bx^5\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)-3\,\sqrt{a}bx^5\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1)+(3\,bx^4+a)\sqrt{-bx^4+}}{5\,a^2x^5}$$

input
```
integrate(1/x^6/(-b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*sqrt(a)*b*x^5*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1/4)), -1) -
3*sqrt(a)*b*x^5*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1) + (3*b*x
^4 + a)*sqrt(-b*x^4 + a))/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = -\frac{i\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\sqrt{b}x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**6/(-b*x**4+a)**(1/2),x)
```

output
```
-I*gamma(-7/4)*hyper((1/2, 7/4), (11/4,), a/(b*x**4))/(4*sqrt(b)*x**7*gamm
a(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+a}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = \int \frac{1}{\sqrt{-bx^4+a}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-b*x^4 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = \int \frac{1}{x^6\,\sqrt{a-b\,x^4}}\,dx$$

input
```
int(1/(x^6*(a - b*x^4)^(1/2)),x)
```

output
```
int(1/(x^6*(a - b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a-bx^4}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{-b\,x^{10}+a\,x^6}dx$$

input
```
int(1/x^6/(-b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a*x**6 - b*x**10),x)
```

3.224 $\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 59

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = \frac{a^2}{2b^3\sqrt{a-bx^4}} + \frac{a\sqrt{a-bx^4}}{b^3} - \frac{(a-bx^4)^{3/2}}{6b^3}$$

output
```
1/2*a^2/b^3/(-b*x^4+a)^(1/2)+a*(-b*x^4+a)^(1/2)/b^3-1/6*(-b*x^4+a)^(3/2)/b
^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = -\frac{-8a^2+4abx^4+b^2x^8}{6b^3\sqrt{a-bx^4}}$$

input
```
Integrate[x^11/(a - b*x^4)^(3/2),x]
```

output
```
-1/6*(-8*a^2 + 4*a*b*x^4 + b^2*x^8)/(b^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(a-bx^4)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2(a-bx^4)^{3/2}}-\frac{2a}{b^2\sqrt{a-bx^4}}+\frac{\sqrt{a-bx^4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2a^2}{b^3\sqrt{a-bx^4}}+\frac{4a\sqrt{a-bx^4}}{b^3}-\frac{2(a-bx^4)^{3/2}}{3b^3}\right)$$

input
```
Int[x^11/(a - b*x^4)^(3/2),x]
```

output
```
((2*a^2)/(b^3*Sqrt[a - b*x^4]) + (4*a*Sqrt[a - b*x^4])/b^3 - (2*(a - b*x^4
)^(3/2))/(3*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{-b^{2}x^{8}-4ab\,x^{4}+8a^{2}}{6\sqrt{-b\,x^{4}+a}\,b^{3}}$	37
default	$\frac{-b^{2}x^{8}-4ab\,x^{4}+8a^{2}}{6\sqrt{-b\,x^{4}+a}\,b^{3}}$	37
trager	$\frac{-b^{2}x^{8}-4ab\,x^{4}+8a^{2}}{6\sqrt{-b\,x^{4}+a}\,b^{3}}$	37
elliptic	$\frac{-b^{2}x^{8}-4ab\,x^{4}+8a^{2}}{6\sqrt{-b\,x^{4}+a}\,b^{3}}$	37
pseudoelliptic	$\frac{-b^{2}x^{8}-4ab\,x^{4}+8a^{2}}{6\sqrt{-b\,x^{4}+a}\,b^{3}}$	37
orering	$\frac{-b^{2}x^{8}-4ab\,x^{4}+8a^{2}}{6\sqrt{-b\,x^{4}+a}\,b^{3}}$	37
risch	$\frac{(b\,x^{4}+5a)\sqrt{-b\,x^{4}+a}}{6b^{3}}+\frac{a^{2}}{2b^{3}\sqrt{-b\,x^{4}+a}}$	44

input
```
int(x^11/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-b^2*x^8-4*a*b*x^4+8*a^2)/(-b*x^4+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = \frac{(b^2x^8+4\,abx^4-8\,a^2)\sqrt{-bx^4+a}}{6\,(b^4x^4-ab^3)}$$

input
```
integrate(x^11/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^8 + 4*a*b*x^4 - 8*a^2)*sqrt(-b*x^4 + a)/(b^4*x^4 - a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = \begin{cases} \frac{4a^2}{3b^3\sqrt{a-bx^4}} - \frac{2ax^4}{3b^2\sqrt{a-bx^4}} - \frac{x^8}{6b\sqrt{a-bx^4}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((4*a**2/(3*b**3*sqrt(a - b*x**4)) - 2*a*x**4/(3*b**2*sqrt(a - b*
x**4)) - x**8/(6*b*sqrt(a - b*x**4)), Ne(b, 0)), (x**12/(12*a**(3/2)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = -\frac{(-bx^4+a)^{\frac{3}{2}}}{6\,b^3} + \frac{\sqrt{-bx^4+a}a}{b^3} + \frac{a^2}{2\sqrt{-bx^4+a}b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/6*(-b*x^4 + a)^(3/2)/b^3 + sqrt(-b*x^4 + a)*a/b^3 + 1/2*a^2/(sqrt(-b*x^
4 + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = \frac{a^2}{2\sqrt{-bx^4+a}b^3} - \frac{(-bx^4+a)^{\frac{3}{2}}b^6 - 6\sqrt{-bx^4+a}ab^6}{6\,b^9}$$

input
```
integrate(x^11/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*a^2/(sqrt(-b*x^4 + a)*b^3) - 1/6*((-b*x^4 + a)^(3/2)*b^6 - 6*sqrt(-b*x
^4 + a)*a*b^6)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = \frac{6\,a\,(a-b\,x^4) - (a-b\,x^4)^2 + 3\,a^2}{6\,b^3\,\sqrt{a-b\,x^4}}$$

input
```
int(x^11/(a - b*x^4)^(3/2),x)
```

output
```
(6*a*(a - b*x^4) - (a - b*x^4)^2 + 3*a^2)/(6*b^3*(a - b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{x^{11}}{(a-bx^4)^{3/2}}\,dx = \frac{-b^2x^8-4ab\,x^4+8a^2}{6\sqrt{-b\,x^4+a}\,b^3}$$

input
```
int(x^11/(-b*x^4+a)^(3/2),x)
```

output
```
(8*a**2 - 4*a*b*x**4 - b**2*x**8)/(6*sqrt(a - b*x**4)*b**3)
```

3.225 $\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx=\frac{a}{2b^2\sqrt{a-bx^4}}+\frac{\sqrt{a-bx^4}}{2b^2}$$

output
```
1/2*a/b^2/(-b*x^4+a)^(1/2)+1/2*(-b*x^4+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx=\frac{2a-bx^4}{2b^2\sqrt{a-bx^4}}$$

input
```
Integrate[x^7/(a - b*x^4)^(3/2),x]
```

output
```
(2*a - b*x^4)/(2*b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a-bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(a-bx^4)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a}{b(a-bx^4)^{3/2}}-\frac{1}{b\sqrt{a-bx^4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2a}{b^2\sqrt{a-bx^4}}+\frac{2\sqrt{a-bx^4}}{b^2}\right)$$

input
```
Int[x^7/(a - b*x^4)^(3/2),x]
```

output
```
((2*a)/(b^2*Sqrt[a - b*x^4]) + (2*Sqrt[a - b*x^4])/b^2)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{-b\,x^{4}+2a}{2\sqrt{-b\,x^{4}+a}\,b^{2}}$	26
default	$\frac{-b\,x^{4}+2a}{2\sqrt{-b\,x^{4}+a}\,b^{2}}$	26
trager	$\frac{-b\,x^{4}+2a}{2\sqrt{-b\,x^{4}+a}\,b^{2}}$	26
elliptic	$\frac{-b\,x^{4}+2a}{2\sqrt{-b\,x^{4}+a}\,b^{2}}$	26
pseudoelliptic	$\frac{-b\,x^{4}+2a}{2\sqrt{-b\,x^{4}+a}\,b^{2}}$	26
orering	$\frac{-b\,x^{4}+2a}{2\sqrt{-b\,x^{4}+a}\,b^{2}}$	26
risch	$\frac{a}{2b^{2}\sqrt{-b\,x^{4}+a}}+\frac{\sqrt{-b\,x^{4}+a}}{2b^{2}}$	33

input
```
int(x^7/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*x^4+2*a)/(-b*x^4+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx=\frac{(bx^4-2\,a)\sqrt{-bx^4+a}}{2\,(b^3x^4-ab^2)}$$

input
```
integrate(x^7/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*x^4 - 2*a)*sqrt(-b*x^4 + a)/(b^3*x^4 - a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx = \begin{cases} \frac{a}{b^2\sqrt{a-bx^4}} - \frac{x^4}{2b\sqrt{a-bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((a/(b**2*sqrt(a - b*x**4)) - x**4/(2*b*sqrt(a - b*x**4)), Ne(b,
0)), (x**8/(8*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx = \frac{\sqrt{-bx^4+a}}{2\,b^2} + \frac{a}{2\,\sqrt{-bx^4+a}b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-b*x^4 + a)/b^2 + 1/2*a/(sqrt(-b*x^4 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx = \frac{\frac{\sqrt{-bx^4+a}}{b} + \frac{a}{\sqrt{-bx^4+a}b}}{2\,b}$$

input
```
integrate(x^7/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(-b*x^4 + a)/b + a/(sqrt(-b*x^4 + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx = \frac{a-\frac{b\,x^4}{2}}{b^2\,\sqrt{a-b\,x^4}}$$

input
```
int(x^7/(a - b*x^4)^(3/2),x)
```

output
```
(a - (b*x^4)/2)/(b^2*(a - b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int \frac{x^7}{(a-bx^4)^{3/2}}\,dx = \frac{-b\,x^4+2a}{2\sqrt{-b\,x^4+a}\,b^2}$$

input
```
int(x^7/(-b*x^4+a)^(3/2),x)
```

output
```
(2*a - b*x**4)/(2*sqrt(a - b*x**4)*b**2)
```

3.226 $\int \frac{x^3}{\left(a-bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{x^3}{\left(a-bx^4\right)^{3/2}}\,dx = \frac{1}{2b\sqrt{a-bx^4}}$$

output
```
1/2/b/(-b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a-bx^4\right)^{3/2}}\,dx = \frac{1}{2b\sqrt{a-bx^4}}$$

input
```
Integrate[x^3/(a - b*x^4)^(3/2),x]
```

output
```
1/(2*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{2b\sqrt{a-bx^4}}$$

input
```
Int[x^3/(a - b*x^4)^(3/2),x]
```

output
```
1/(2*b*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16
derivativedivides	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16
default	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16
trager	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16
elliptic	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16
pseudoelliptic	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16
orering	$\frac{1}{2b\sqrt{-b\,x^{4}+a}}$	16

input
```
int(x^3/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(-b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{x^3}{\left(a-bx^4\right)^{3/2}}\,dx=-\frac{\sqrt{-bx^4+a}}{2\,(b^2x^4-ab)}$$

input
```
integrate(x^3/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^4 + a)/(b^2*x^4 - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{(a-bx^4)^{3/2}}\,dx = \begin{cases} \frac{1}{2b\sqrt{a-bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((1/(2*b*sqrt(a - b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a-bx^4)^{3/2}}\,dx = \frac{1}{2\sqrt{-bx^4+a}b}$$

input
```
integrate(x^3/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2/(sqrt(-b*x^4 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a-bx^4)^{3/2}}\,dx = \frac{1}{2\sqrt{-bx^4+a}b}$$

input
```
integrate(x^3/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2/(sqrt(-b*x^4 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a-bx^4)^{3/2}}\,dx = \frac{1}{2\,b\,\sqrt{a-b\,x^4}}$$

input
```
int(x^3/(a - b*x^4)^(3/2),x)
```

output
```
1/(2*b*(a - b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{(a-bx^4)^{3/2}}\,dx = \frac{1}{2\sqrt{-b\,x^4+a}\,b}$$

input
```
int(x^3/(-b*x^4+a)^(3/2),x)
```

output
```
1/(2*sqrt(a - b*x**4)*b)
```

3.227 $\int \frac{1}{x(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x(a-bx^4)^{3/2}}\,dx = \frac{1}{2a\sqrt{a-bx^4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/2/a/(-b*x^4+a)^(1/2)-1/2*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a-bx^4)^{3/2}}\,dx = \frac{1}{2a\sqrt{a-bx^4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[1/(x*(a - b*x^4)^(3/2)),x]
```

output
```
1/(2*a*Sqrt[a - b*x^4]) - ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]]/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a-bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^4 (a-bx^4)^{3/2}} dx^4$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(\frac{\int \frac{1}{x^4 \sqrt{a-bx^4}} dx^4}{a} + \frac{2}{a\sqrt{a-bx^4}} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(\frac{2}{a\sqrt{a-bx^4}} - \frac{2 \int \frac{1}{\frac{a}{b}-\frac{x^8}{b}} d\sqrt{a-bx^4}}{ab} \right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4} \left(\frac{2}{a\sqrt{a-bx^4}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{a^{3/2}} \right)$$

input
```
Int[1/(x*(a - b*x^4)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a - b*x^4]) - (2*ArcTanh[Sqrt[a - b*x^4]/Sqrt[a]])/a^(3/2))/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{1}{2a\sqrt{-b\,x^{4}+a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$	37
default	$\frac{1}{2a\sqrt{-b\,x^{4}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	46
elliptic	$\frac{1}{2a\sqrt{-b\,x^{4}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^{4}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	46

input
```
int(1/x/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(-b*x^4+a)^(1/2)-1/2*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.10

$$\int \frac{1}{x\left(a-bx^4\right)^{3/2}}\,dx = \left[\frac{(bx^4-a)\sqrt{a}\log\left(\frac{bx^4+2\sqrt{-bx^4+a}\sqrt{a}-2a}{x^4}\right)-2\sqrt{-bx^4+a}a}{4\left(a^2bx^4-a^3\right)},\ -\frac{(bx^4-a)\sqrt{-a}\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)+\sqrt{-bx^4+a}a}{2\left(a^2bx^4-a^3\right)}\right]$$

input
```
integrate(1/x/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*x^4 - a)*sqrt(a)*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a) - 2*a)/x
^4) - 2*sqrt(-b*x^4 + a)*a)/(a^2*b*x^4 - a^3), -1/2*((b*x^4 - a)*sqrt(-a)*
arctan(sqrt(-b*x^4 + a)*sqrt(-a)/(b*x^4 - a)) + sqrt(-b*x^4 + a)*a)/(a^2*b
*x^4 - a^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 520, normalized size of antiderivative = 10.83

$$\int \frac{1}{x\left(a-bx^4\right)^{3/2}}\,dx = \begin{cases} -\frac{2ia^3\sqrt{-1+\frac{bx^4}{a}}}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{a^3\log\left(\frac{bx^4}{a}\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{2a^3\log\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2ia^3\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2a^2b}{-4} \\ -\frac{2a^3\sqrt{1-\frac{bx^4}{a}}}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{a^3\log\left(\frac{bx^4}{a}\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{2a^3\log\left(\sqrt{1-\frac{bx^4}{a}}+1\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{i\pi a^3}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} + \frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{-4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \end{cases}$$

input
```
integrate(1/x/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-2*I*a**3*sqrt(-1 + b*x**4/a)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4)
 - a**3*log(b*x**4/a)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) + 2*a**3*log(sqrt(
b)*x**2/sqrt(a))/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*I*a**3*asin(sqrt(a)
/(sqrt(b)*x**2))/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**
4/a)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(b)*x**2/sq
rt(a))/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) + 2*I*a**2*b*x**4*asin(sqrt(a)/(s
qrt(b)*x**2))/(-4*a**(9/2) + 4*a**(7/2)*b*x**4), Abs(b*x**4/a) > 1), (-2*a
**3*sqrt(1 - b*x**4/a)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) - a**3*log(b*x**4
/a)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) + 2*a**3*log(sqrt(1 - b*x**4/a) + 1)
/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) - I*pi*a**3/(-4*a**(9/2) + 4*a**(7/2)*b
*x**4) + a**2*b*x**4*log(b*x**4/a)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a
**2*b*x**4*log(sqrt(1 - b*x**4/a) + 1)/(-4*a**(9/2) + 4*a**(7/2)*b*x**4) +
 I*pi*a**2*b*x**4/(-4*a**(9/2) + 4*a**(7/2)*b*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{x\left(a-bx^4\right)^{3/2}}\,dx = \frac{\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}} + \frac{1}{2\sqrt{-bx^4+a}a}$$

input
```
integrate(1/x/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a)))/a^(3/2)
 + 1/2/(sqrt(-b*x^4 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a-bx^4\right)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a} + \frac{1}{2\sqrt{-bx^4+a}a}$$

input
```
integrate(1/x/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/2/(sqrt(-b*x^4 + a)
*a)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(a-bx^4\right)^{3/2}}\,dx = \frac{1}{2\,a\,\sqrt{a-b\,x^4}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{a-b\,x^4}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int(1/(x*(a - b*x^4)^(3/2)),x)
```

output
```
1/(2*a*(a - b*x^4)^(1/2)) - atanh((a - b*x^4)^(1/2)/a^(1/2))/(2*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(a-bx^4\right)^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{-b\,x^4+a}\,\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right) - \sqrt{a}\,\sqrt{-b\,x^4+a}+a}{2\sqrt{-b\,x^4+a}\,a^2}$$

input
```
int(1/x/(-b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(a - b*x**4)*log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2)) - sqrt(
a)*sqrt(a - b*x**4) + a)/(2*sqrt(a - b*x**4)*a**2)
```

3.228 $\int \frac{1}{x^5(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 72

$$\int \frac{1}{x^5\left(a-bx^4\right)^{3/2}}\,dx = \frac{3b}{4a^2\sqrt{a-bx^4}} - \frac{1}{4ax^4\sqrt{a-bx^4}} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
3/4*b/a^2/(-b*x^4+a)^(1/2)-1/4/a/x^4/(-b*x^4+a)^(1/2)-3/4*b*arctanh((-b*x^
4+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\left(a-bx^4\right)^{3/2}}\,dx = \frac{-a+3bx^4}{4a^2x^4\sqrt{a-bx^4}} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[1/(x^5*(a - b*x^4)^(3/2)),x]
```

output
```
(-a + 3*b*x^4)/(4*a^2*x^4*Sqrt[a - b*x^4]) - (3*b*ArcTanh[Sqrt[a - b*x^4]/
Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {798, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a-bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^8 (a-bx^4)^{3/2}} dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(\frac{3b \int \frac{1}{x^4 (a-bx^4)^{3/2}} dx^4}{2a} - \frac{1}{a x^4 \sqrt{a-bx^4}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(\frac{3b \left(\frac{\int \frac{1}{x^4 \sqrt{a-bx^4}} dx^4}{a} + \frac{2}{a\sqrt{a-bx^4}} \right)}{2a} - \frac{1}{a x^4 \sqrt{a-bx^4}} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(\frac{3b \left(\frac{2}{a\sqrt{a-bx^4}} - \frac{2 \int \frac{1}{\frac{a}{b} - \frac{x^8}{b}} d\sqrt{a-bx^4}}{ab} \right)}{2a} - \frac{1}{a x^4 \sqrt{a-bx^4}} \right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4} \left(\frac{3b \left(\frac{2}{a\sqrt{a-bx^4}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}} \right)}{a^{3/2}} \right)}{2a} - \frac{1}{a x^4 \sqrt{a-bx^4}} \right)$$

input
```
Int[1/(x^5*(a - b*x^4)^(3/2)),x]
```

output
```
(-(1/(a*x^4*Sqrt[a - b*x^4])) + (3*b*(2/(a*Sqrt[a - b*x^4]) - (2*ArcTanh[S
qrt[a - b*x^4]/Sqrt[a]])/a^(3/2)))/(2*a))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{b\left(\frac{\sqrt{-b\,x^4+a}}{x^4 b}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{-b\,x^4+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2}{\sqrt{-b\,x^4+a}}\right)}{4a^2}$
risch	$-\frac{\sqrt{-b\,x^4+a}}{4a^2x^4}-\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^4+a}}{x^2}\right)}{4a^{\frac{5}{2}}}+\frac{b}{2a^2\sqrt{-b\,x^4+a}}$
default	$-\frac{\sqrt{-b\,x^4+a}}{4a^2x^4}-\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^4+a}}{x^2}\right)}{4a^{\frac{5}{2}}}-\frac{b\sqrt{-\left(x^2-\frac{\sqrt{ab}}{b}\right)^2 b-2\sqrt{ab}\left(x^2-\frac{\sqrt{ab}}{b}\right)}}{4a^2\sqrt{ab}\left(x^2-\frac{\sqrt{ab}}{b}\right)}+\frac{b\sqrt{-\left(x^2+\frac{\sqrt{ab}}{b}\right)^2 b+2\sqrt{ab}\left(x^2\right.}}{4a^2\sqrt{ab}\left(x^2+\frac{\sqrt{ab}}{b}\right)}$
elliptic	$-\frac{\sqrt{-b\,x^4+a}}{4a^2x^4}-\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{-b\,x^4+a}}{x^2}\right)}{4a^{\frac{5}{2}}}-\frac{b\sqrt{-\left(x^2-\frac{\sqrt{ab}}{b}\right)^2 b-2\sqrt{ab}\left(x^2-\frac{\sqrt{ab}}{b}\right)}}{4a^2\sqrt{ab}\left(x^2-\frac{\sqrt{ab}}{b}\right)}+\frac{b\sqrt{-\left(x^2+\frac{\sqrt{ab}}{b}\right)^2 b+2\sqrt{ab}\left(x^2\right.}}{4a^2\sqrt{ab}\left(x^2+\frac{\sqrt{ab}}{b}\right)}$

input
```
int(1/x^5/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b/a^2*((-b*x^4+a)^(1/2)/x^4/b+3*arctanh((-b*x^4+a)^(1/2)/a^(1/2))/a^(
1/2)-2/(-b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^5\,(a-bx^4)^{3/2}}\,dx = \left[\frac{3\,(b^2x^8-abx^4)\sqrt{a}\log\left(\frac{bx^4+2\,\sqrt{-bx^4+a}\sqrt{a}-2\,a}{x^4}\right)-2\,(3\,abx^4-a^2)\sqrt{-bx^4+a}}{8\,(a^3bx^8-a^4x^4)},\right.$$
$$\left.-\frac{3\,(b^2x^8-abx^4)\sqrt{-a}\arctan\left(\frac{\sqrt{-bx^4+a}\sqrt{-a}}{bx^4-a}\right)+(3\,abx^4-a^2)\sqrt{-bx^4+a}}{4\,(a^3bx^8-a^4x^4)}\right]$$

input
```
integrate(1/x^5/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(b^2*x^8 - a*b*x^4)*sqrt(a)*log((b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(a
) - 2*a)/x^4) - 2*(3*a*b*x^4 - a^2)*sqrt(-b*x^4 + a))/(a^3*b*x^8 - a^4*x^4
), -1/4*(3*(b^2*x^8 - a*b*x^4)*sqrt(-a)*arctan(sqrt(-b*x^4 + a)*sqrt(-a)/(
b*x^4 - a)) + (3*a*b*x^4 - a^2)*sqrt(-b*x^4 + a))/(a^3*b*x^8 - a^4*x^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.29

$$\int \frac{1}{x^5 (a - bx^4)^{3/2}} dx = \begin{cases} -\frac{1}{4a\sqrt{b}x^6\sqrt{\frac{a}{bx^4}-1}} + \frac{3\sqrt{b}}{4a^2x^2\sqrt{\frac{a}{bx^4}-1}} - \frac{3b\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{i}{4a\sqrt{b}x^6\sqrt{-\frac{a}{bx^4}+1}} - \frac{3i\sqrt{b}}{4a^2x^2\sqrt{-\frac{a}{bx^4}+1}} + \frac{3ib\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-1/(4*a*sqrt(b)*x**6*sqrt(a/(b*x**4) - 1)) + 3*sqrt(b)/(4*a**2*
x**2*sqrt(a/(b*x**4) - 1)) - 3*b*acosh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(5/2)
), Abs(a/(b*x**4)) > 1), (I/(4*a*sqrt(b)*x**6*sqrt(-a/(b*x**4) + 1)) - 3*I
*sqrt(b)/(4*a**2*x**2*sqrt(-a/(b*x**4) + 1)) + 3*I*b*asin(sqrt(a)/(sqrt(b)
*x**2))/(4*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^5 (a - bx^4)^{3/2}} dx = -\frac{3(bx^4 - a)b + 2ab}{4\left((-bx^4 + a)^{\frac{3}{2}}a^2 - \sqrt{-bx^4 + a}a^3\right)} + \frac{3b\log\left(\frac{\sqrt{-bx^4+a}-\sqrt{a}}{\sqrt{-bx^4+a}+\sqrt{a}}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*(b*x^4 - a)*b + 2*a*b)/((-b*x^4 + a)^(3/2)*a^2 - sqrt(-b*x^4 + a)*
a^3) + 3/8*b*log((sqrt(-b*x^4 + a) - sqrt(a))/(sqrt(-b*x^4 + a) + sqrt(a))
)/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5 (a-bx^4)^{3/2}} \, dx = \frac{3 \, b \arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{-a}}\right)}{4 \, \sqrt{-a}a^2} - \frac{3 \, (bx^4-a)b + 2 \, ab}{4 \left((-bx^4+a)^{\frac{3}{2}} - \sqrt{-bx^4+a}a\right)a^2}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
3/4*b*arctan(sqrt(-b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(b*x^4 - a
)*b + 2*a*b)/(((-b*x^4 + a)^(3/2) - sqrt(-b*x^4 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^5 (a-bx^4)^{3/2}} \, dx = \frac{3 \, b}{4 \, a^2 \, \sqrt{a-b \, x^4}} - \frac{1}{4 \, a \, x^4 \, \sqrt{a-b \, x^4}} - \frac{3 \, b \, \mathrm{atanh}\left(\frac{\sqrt{a-b \, x^4}}{\sqrt{a}}\right)}{4 \, a^{5/2}}$$

input
```
int(1/(x^5*(a - b*x^4)^(3/2)),x)
```

output
```
(3*b)/(4*a^2*(a - b*x^4)^(1/2)) - 1/(4*a*x^4*(a - b*x^4)^(1/2)) - (3*b*ata
nh((a - b*x^4)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^5 (a-bx^4)^{3/2}}\,dx = \frac{12\sqrt{a}\,\sqrt{-b\,x^4+a}\log\left(\tan\left(\frac{asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2}\right)\right)b\,x^4-9\sqrt{a}\,\sqrt{-b\,x^4+a}\,b\,x^4-4a^2+12a}{16\sqrt{-b\,x^4+a}\,a^3x^4}$$

input
```
int(1/x^5/(-b*x^4+a)^(3/2),x)
```

output
```
(12*sqrt(a)*sqrt(a - b*x**4)*log(tan(asin((sqrt(b)*x**2)/sqrt(a))/2))*b*x*
*4 - 9*sqrt(a)*sqrt(a - b*x**4)*b*x**4 - 4*a**2 + 12*a*b*x**4)/(16*sqrt(a
- b*x**4)*a**3*x**4)
```

3.229 $\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx=\frac{x^6}{2b\sqrt{a-bx^4}}+\frac{3x^2\sqrt{a-bx^4}}{4b^2}-\frac{3a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{4b^{5/2}}$$

output
```
1/2*x^6/b/(-b*x^4+a)^(1/2)+3/4*x^2*(-b*x^4+a)^(1/2)/b^2-3/4*a*arctan(b^(1/
2)*x^2/(-b*x^4+a)^(1/2))/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx=-\frac{-3ax^2+bx^6}{4b^2\sqrt{a-bx^4}}+\frac{3ia\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{4b^{5/2}}$$

input
```
Integrate[x^9/(a - b*x^4)^(3/2),x]
```

output
```
-1/4*(-3*a*x^2 + b*x^6)/(b^2*Sqrt[a - b*x^4]) + (((3*I)/4)*a*Log[I*Sqrt[b]
*x^2 + Sqrt[a - b*x^4]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 252, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a-bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(a-bx^4)^{3/2}}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{x^6}{b\sqrt{a-bx^4}}-\frac{3\int \frac{x^4}{\sqrt{a-bx^4}}dx^2}{b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{x^6}{b\sqrt{a-bx^4}}-\frac{3\left(\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx^2}{2b}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{x^6}{b\sqrt{a-bx^4}}-\frac{3\left(\frac{a\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}}{2b}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)}{b}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{x^6}{b\sqrt{a-bx^4}}-\frac{3\left(\frac{a\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2b^{3/2}}-\frac{x^2\sqrt{a-bx^4}}{2b}\right)}{b}\right)$$

input
```
Int[x^9/(a - b*x^4)^(3/2),x]
```

output
```
(x^6/(b*Sqrt[a - b*x^4]) - (3*(-1/2*(x^2*Sqrt[a - b*x^4])/b + (a*ArcTan[(S
qrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*b^(3/2))))/b)/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{x^{6}}{4b\sqrt{-b\,x^{4}+a}}+\frac{3a\,x^{2}}{4b^{2}\sqrt{-b\,x^{4}+a}}-\frac{3a\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4b^{\frac{5}{2}}}$	63
risch	$\frac{x^{2}\sqrt{-b\,x^{4}+a}}{4b^{2}}-\frac{3a\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4b^{\frac{5}{2}}}+\frac{a\,x^{2}}{2b^{2}\sqrt{-b\,x^{4}+a}}$	63
elliptic	$-\frac{x^{6}}{4b\sqrt{-b\,x^{4}+a}}+\frac{3a\,x^{2}}{4b^{2}\sqrt{-b\,x^{4}+a}}-\frac{3a\arctan\left(\frac{\sqrt{b}\,x^{2}}{\sqrt{-b\,x^{4}+a}}\right)}{4b^{\frac{5}{2}}}$	63
pseudoelliptic	$-\frac{-3a\arctan\left(\frac{\sqrt{-b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)b^{2}\sqrt{-b\,x^{4}+a}+b^{\frac{5}{2}}x^{2}\left(b\,x^{4}-3a\right)}{4\sqrt{-b\,x^{4}+a}\,b^{\frac{9}{2}}}$	67

input
```
int(x^9/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^6/b/(-b*x^4+a)^(1/2)+3/4*a/b^2*x^2/(-b*x^4+a)^(1/2)-3/4*a*arctan(b^
(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.30

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx=\left[-\frac{3\,(abx^4-a^2)\sqrt{-b}\log\left(2\,bx^4+2\,\sqrt{-bx^4+a}\sqrt{-b}x^2-a\right)-2\,(b^2x^6-3\,abx^2)\sqrt{-bx}}{8\,(b^4x^4-ab^3)}\right.$$

input
```
integrate(x^9/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(a*b*x^4 - a^2)*sqrt(-b)*log(2*b*x^4 + 2*sqrt(-b*x^4 + a)*sqrt(-b
)*x^2 - a) - 2*(b^2*x^6 - 3*a*b*x^2)*sqrt(-b*x^4 + a))/(b^4*x^4 - a*b^3),
1/4*(3*(a*b*x^4 - a^2)*sqrt(b)*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)) + (b
^2*x^6 - 3*a*b*x^2)*sqrt(-b*x^4 + a))/(b^4*x^4 - a*b^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.12

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx = \begin{cases} -\frac{3i\sqrt{a}x^2}{4b^2\sqrt{-1+\frac{bx^4}{a}}} + \frac{3ia\operatorname{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{ix^6}{4\sqrt{a}b\sqrt{-1+\frac{bx^4}{a}}} & \text{for}\ \left|\frac{bx^4}{a}\right| > 1 \\ \frac{3\sqrt{a}x^2}{4b^2\sqrt{1-\frac{bx^4}{a}}} - \frac{3a\operatorname{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} - \frac{x^6}{4\sqrt{a}b\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**9/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-3*I*sqrt(a)*x**2/(4*b**2*sqrt(-1 + b*x**4/a)) + 3*I*a*acosh(sq
rt(b)*x**2/sqrt(a))/(4*b**(5/2)) + I*x**6/(4*sqrt(a)*b*sqrt(-1 + b*x**4/a)
), Abs(b*x**4/a) > 1), (3*sqrt(a)*x**2/(4*b**2*sqrt(1 - b*x**4/a)) - 3*a*a
sin(sqrt(b)*x**2/sqrt(a))/(4*b**(5/2)) - x**6/(4*sqrt(a)*b*sqrt(1 - b*x**4
/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx = \frac{2\,ab - \frac{3\,(bx^4-a)a}{x^4}}{4\left(\frac{\sqrt{-bx^4+a}b^3}{x^2} + \frac{(-bx^4+a)^{\frac{3}{2}}b^2}{x^6}\right)} + \frac{3\,a\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{4\,b^{\frac{5}{2}}}$$

input
```
integrate(x^9/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*a*b - 3*(b*x^4 - a)*a/x^4)/(sqrt(-b*x^4 + a)*b^3/x^2 + (-b*x^4 + a)
^(3/2)*b^2/x^6) + 3/4*a*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx = \frac{\sqrt{-bx^4+a}\left(\frac{x^4}{b}-\frac{3\,a}{b^2}\right)x^2}{4\,(bx^4-a)} + \frac{3\,a\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{4\,\sqrt{-b}b^2}$$

input
```
integrate(x^9/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-b*x^4 + a)*(x^4/b - 3*a/b^2)*x^2/(b*x^4 - a) + 3/4*a*log(abs(-sq
rt(-b)*x^2 + sqrt(-b*x^4 + a)))/(sqrt(-b)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^9}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^9/(a - b*x^4)^(3/2),x)
```

output
```
int(x^9/(a - b*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{x^9}{(a-bx^4)^{3/2}}\,dx = \frac{-3\sqrt{-b\,x^4+a}\,asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)a+3\sqrt{b}\,a\,x^2-\sqrt{b}\,b\,x^6}{4\sqrt{b}\,\sqrt{-b\,x^4+a}\,b^2}$$

input
```
int(x^9/(-b*x^4+a)^(3/2),x)
```

output
```
( - 3*sqrt(a - b*x**4)*asin((sqrt(b)*x**2)/sqrt(a))*a + 3*sqrt(b)*a*x**2 -
 sqrt(b)*b*x**6)/(4*sqrt(b)*sqrt(a - b*x**4)*b**2)
```

3.230 $\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \frac{x^2}{2b\sqrt{a-bx^4}} - \frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2b^{3/2}}$$

output
```
1/2*x^2/b/(-b*x^4+a)^(1/2)-1/2*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(3/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \frac{x^2}{2b\sqrt{a-bx^4}} + \frac{i\log\left(i\sqrt{b}x^2+\sqrt{a-bx^4}\right)}{2b^{3/2}}$$

input
```
Integrate[x^5/(a - b*x^4)^(3/2),x]
```

output
```
x^2/(2*b*Sqrt[a - b*x^4]) + ((I/2)*Log[I*Sqrt[b]*x^2 + Sqrt[a - b*x^4]])/b
^(3/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {807, 252, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(a-bx^4)^{3/2}}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2}\left(\frac{x^2}{b\sqrt{a-bx^4}}-\frac{\int \frac{1}{\sqrt{a-bx^4}}dx^2}{b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{x^2}{b\sqrt{a-bx^4}}-\frac{\int \frac{1}{bx^4+1}d\frac{x^2}{\sqrt{a-bx^4}}}{b}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{x^2}{b\sqrt{a-bx^4}}-\frac{\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{b^{3/2}}\right)$$

input
```
Int[x^5/(a - b*x^4)^(3/2),x]
```

output
```
(x^2/(b*Sqrt[a - b*x^4]) - ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]]/b^(3/2))/
2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^{2}}{2 b \sqrt{-b \,x^{4}+a}}-\frac{\arctan\left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b \,x^{4}+a}}\right)}{2 b^{\frac{3}{2}}}$	43
elliptic	$\frac{x^{2}}{2 b \sqrt{-b \,x^{4}+a}}-\frac{\arctan\left(\frac{\sqrt{b}\, x^{2}}{\sqrt{-b \,x^{4}+a}}\right)}{2 b^{\frac{3}{2}}}$	43
pseudoelliptic	$\frac{x^{2}}{2 b \sqrt{-b \,x^{4}+a}}+\frac{\arctan\left(\frac{\sqrt{-b \,x^{4}+a}}{\sqrt{b}\, x^{2}}\right)}{2 b^{\frac{3}{2}}}$	43

input
```
int(x^5/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b/(-b*x^4+a)^(1/2)-1/2*arctan(b^(1/2)*x^2/(-b*x^4+a)^(1/2))/b^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \left[-\frac{2\sqrt{-bx^4+a}\,bx^2 + (bx^4-a)\sqrt{-b}\log\left(2\,bx^4 + 2\sqrt{-bx^4+a}\sqrt{-b}x^2 - a\right)}{4\,(b^3x^4 - ab^2)}, -\frac{\sqrt{-bx^4+a}\,bx^2 - (bx^4-a)\sqrt{b}\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{2\,(b^3x^4 - ab^2)}\right]$$

input
```
integrate(x^5/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(-b*x^4 + a)*b*x^2 + (b*x^4 - a)*sqrt(-b)*log(2*b*x^4 + 2*sqr
t(-b*x^4 + a)*sqrt(-b)*x^2 - a))/(b^3*x^4 - a*b^2), -1/2*(sqrt(-b*x^4 + a)
*b*x^2 - (b*x^4 - a)*sqrt(b)*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2)))/(b^3*
x^4 - a*b^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \begin{cases} \frac{i\,\mathrm{acosh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{ix^2}{2\sqrt{a}b\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ -\frac{\mathrm{asin}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} + \frac{x^2}{2\sqrt{a}b\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((I*acosh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - I*x**2/(2*sqrt(a)*
b*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a) > 1), (-asin(sqrt(b)*x**2/sqrt(a))/(
2*b**(3/2)) + x**2/(2*sqrt(a)*b*sqrt(1 - b*x**4/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-bx^4+ab}} + \frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{\sqrt{b}x^2}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^5/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/(sqrt(-b*x^4 + a)*b) + 1/2*arctan(sqrt(-b*x^4 + a)/(sqrt(b)*x^2))/
b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = -\frac{\sqrt{-bx^4+a}x^2}{2\,(bx^4-a)b} + \frac{\log\left(\left|-\sqrt{-b}x^2+\sqrt{-bx^4+a}\right|\right)}{2\,\sqrt{-b}b}$$

input
```
integrate(x^5/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)*x^2/((b*x^4 - a)*b) + 1/2*log(abs(-sqrt(-b)*x^2 + sq
rt(-b*x^4 + a)))/(sqrt(-b)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^5}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^5/(a - b*x^4)^(3/2),x)
```

output
```
int(x^5/(a - b*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(a-bx^4)^{3/2}}\,dx = \frac{-\sqrt{-b\,x^4+a}\,asin\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)+\sqrt{b}\,x^2}{2\sqrt{b}\,\sqrt{-b\,x^4+a}\,b}$$

input
```
int(x^5/(-b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a - b*x**4)*asin((sqrt(b)*x**2)/sqrt(a)) + sqrt(b)*x**2)/(2*sqrt(
b)*sqrt(a - b*x**4)*b)
```

3.231 $\int \frac{x}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx = \frac{x^2}{2a\sqrt{a-bx^4}}$$

output
```
1/2*x^2/a/(-b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx = \frac{x^2}{2a\sqrt{a-bx^4}}$$

input
```
Integrate[x/(a - b*x^4)^(3/2),x]
```

output
```
x^2/(2*a*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{796}$$

$$\frac{x^2}{2a\sqrt{a-bx^4}}$$

input
```
Int[x/(a - b*x^4)^(3/2),x]
```

output
```
x^2/(2*a*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^2}{2a\sqrt{-b\,x^4+a}}$	19
default	$\frac{x^2}{2a\sqrt{-b\,x^4+a}}$	19
trager	$\frac{x^2}{2a\sqrt{-b\,x^4+a}}$	19
elliptic	$\frac{x^2}{2a\sqrt{-b\,x^4+a}}$	19
pseudoelliptic	$\frac{x^2}{2a\sqrt{-b\,x^4+a}}$	19
orering	$\frac{x^2}{2a\sqrt{-b\,x^4+a}}$	19

input
```
int(x/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a/(-b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx = -\frac{\sqrt{-bx^4+a}x^2}{2\left(abx^4-a^2\right)}$$

input
```
integrate(x/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^4 + a)*x^2/(a*b*x^4 - a^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx = \begin{cases} -\frac{ix^2}{2a^{\frac{3}{2}}\sqrt{-1+\frac{bx^4}{a}}} & \text{for } \left|\frac{bx^4}{a}\right| > 1 \\ \frac{x^2}{2a^{\frac{3}{2}}\sqrt{1-\frac{bx^4}{a}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-I*x**2/(2*a**(3/2)*sqrt(-1 + b*x**4/a)), Abs(b*x**4/a) > 1), (
x**2/(2*a**(3/2)*sqrt(1 - b*x**4/a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-bx^4+a}a}$$

input
```
integrate(x/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/(sqrt(-b*x^4 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx = -\frac{\sqrt{-bx^4+a}x^2}{2(bx^4-a)a}$$

input
```
integrate(x/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)*x^2/((b*x^4 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx=\frac{x^2}{2\,a\,\sqrt{a-b\,x^4}}$$

input
```
int(x/(a - b*x^4)^(3/2),x)
```

output
```
x^2/(2*a*(a - b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{x}{(a-bx^4)^{3/2}}\,dx=\frac{x^2}{2\sqrt{-b\,x^4+a}\,a}$$

input
```
int(x/(-b*x^4+a)^(3/2),x)
```

output
```
x**2/(2*sqrt(a - b*x**4)*a)
```

3.232 $\int \frac{1}{x^3(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{1}{x^3\left(a-bx^4\right)^{3/2}}\,dx=\frac{1}{2ax^2\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{a^2x^2}$$

output
```
1/2/a/x^2/(-b*x^4+a)^(1/2)-(-b*x^4+a)^(1/2)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^3\left(a-bx^4\right)^{3/2}}\,dx=-\frac{a-2bx^4}{2a^2x^2\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^3*(a - b*x^4)^(3/2)),x]
```

output
```
-1/2*(a - 2*b*x^4)/(a^2*x^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a-b x^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{2 b \int \frac{x}{\left(a-b x^4\right)^{3/2}}dx}{a}-\frac{1}{2 a x^2 \sqrt{a-b x^4}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{b x^2}{a^2 \sqrt{a-b x^4}}-\frac{1}{2 a x^2 \sqrt{a-b x^4}}$$

input
```
Int[1/(x^3*(a - b*x^4)^(3/2)),x]
```

output
```
-1/2*1/(a*x^2*Sqrt[a - b*x^4]) + (b*x^2)/(a^2*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{-2b\,x^{4}+a}{2x^{2}\sqrt{-b\,x^{4}+a}\,a^{2}}$	27
default	$-\frac{-2b\,x^{4}+a}{2x^{2}\sqrt{-b\,x^{4}+a}\,a^{2}}$	27
trager	$-\frac{-2b\,x^{4}+a}{2x^{2}\sqrt{-b\,x^{4}+a}\,a^{2}}$	27
elliptic	$-\frac{-2b\,x^{4}+a}{2x^{2}\sqrt{-b\,x^{4}+a}\,a^{2}}$	27
pseudoelliptic	$-\frac{-2b\,x^{4}+a}{2x^{2}\sqrt{-b\,x^{4}+a}\,a^{2}}$	27
orering	$-\frac{-2b\,x^{4}+a}{2x^{2}\sqrt{-b\,x^{4}+a}\,a^{2}}$	27
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{2a^{2}x^{2}}+\frac{x^{2}b}{2\sqrt{-b\,x^{4}+a}\,a^{2}}$	39

input
```
int(1/x^3/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2*b*x^4+a)/x^2/(-b*x^4+a)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\left(a-bx^4\right)^{3/2}}\,dx=-\frac{\left(2\,bx^4-a\right)\sqrt{-bx^4+a}}{2\left(a^2bx^6-a^3x^2\right)}$$

input
```
integrate(1/x^3/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x^4 - a)*sqrt(-b*x^4 + a)/(a^2*b*x^6 - a^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.09

$$\int \frac{1}{x^3 (a - bx^4)^{3/2}} \, dx = \begin{cases} -\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}-1}} + \frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}-1}} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx^4}+1}}{-2a^3b+2a^2b^2x^4} - \frac{2ib^{\frac{5}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{-2a^3b+2a^2b^2x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) - 1)) + sqrt(b)/(a**2*sqrt
(a/(b*x**4) - 1)), Abs(a/(b*x**4)) > 1), (I*a*b**(3/2)*sqrt(-a/(b*x**4) +
1)/(-2*a**3*b + 2*a**2*b**2*x**4) - 2*I*b**(5/2)*x**4*sqrt(-a/(b*x**4) + 1
)/(-2*a**3*b + 2*a**2*b**2*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a - bx^4)^{3/2}} \, dx = \frac{bx^2}{2\sqrt{-bx^4 + a}a^2} - \frac{\sqrt{-bx^4 + a}}{2a^2x^2}$$

input
```
integrate(1/x^3/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b*x^2/(sqrt(-b*x^4 + a)*a^2) - 1/2*sqrt(-b*x^4 + a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^3 (a-bx^4)^{3/2}}\,dx = -\frac{\sqrt{-bx^4+a}bx^2}{2(bx^4-a)a^2} + \frac{\sqrt{-b}}{\left(\left(\sqrt{-b}x^2-\sqrt{-bx^4+a}\right)^2-a\right)a}$$

input
```
integrate(1/x^3/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)*b*x^2/((b*x^4 - a)*a^2) + sqrt(-b)/(((sqrt(-b)*x^2 -
 sqrt(-b*x^4 + a))^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^3 (a-bx^4)^{3/2}}\,dx = -\frac{a-2\,b\,x^4}{2\,a^2\,x^2\,\sqrt{a-b\,x^4}}$$

input
```
int(1/(x^3*(a - b*x^4)^(3/2)),x)
```

output
```
-(a - 2*b*x^4)/(2*a^2*x^2*(a - b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^3 (a-bx^4)^{3/2}}\,dx = \frac{2b\,x^4-a}{2\sqrt{-b\,x^4+a}\,a^2x^2}$$

input
```
int(1/x^3/(-b*x^4+a)^(3/2),x)
```

output
```
( - a + 2*b*x**4)/(2*sqrt(a - b*x**4)*a**2*x**2)
```

3.233 $\int \frac{1}{x^7(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{1}{x^7 (a-bx^4)^{3/2}}\,dx = \frac{1}{2ax^6\sqrt{a-bx^4}} - \frac{2\sqrt{a-bx^4}}{3a^2x^6} - \frac{4b\sqrt{a-bx^4}}{3a^3x^2}$$

output
```
1/2/a/x^6/(-b*x^4+a)^(1/2)-2/3*(-b*x^4+a)^(1/2)/a^2/x^6-4/3*b*(-b*x^4+a)^(
1/2)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^7 (a-bx^4)^{3/2}}\,dx = -\frac{a^2+4abx^4-8b^2x^8}{6a^3x^6\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^7*(a - b*x^4)^(3/2)),x]
```

output
```
-1/6*(a^2 + 4*a*b*x^4 - 8*b^2*x^8)/(a^3*x^6*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a-bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\int \frac{1}{x^3(a-bx^4)^{3/2}}dx}{3a}-\frac{1}{6ax^6\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{803}$$

$$\frac{4b\left(\frac{2b\int \frac{x}{\left(a-bx^4\right)^{3/2}}dx}{a}-\frac{1}{2ax^2\sqrt{a-bx^4}}\right)}{3a}-\frac{1}{6ax^6\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4b\left(\frac{bx^2}{a^2\sqrt{a-bx^4}}-\frac{1}{2ax^2\sqrt{a-bx^4}}\right)}{3a}-\frac{1}{6ax^6\sqrt{a-bx^4}}$$

input
```
Int[1/(x^7*(a - b*x^4)^(3/2)),x]
```

output
```
-1/6*1/(a*x^6*Sqrt[a - b*x^4]) + (4*b*(-1/2*1/(a*x^2*Sqrt[a - b*x^4]) + (b
*x^2)/(a^2*Sqrt[a - b*x^4])))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{-8b^{2}x^{8}+4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{-b\,x^{4}+a}\,a^{3}}$	38
default	$-\frac{-8b^{2}x^{8}+4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{-b\,x^{4}+a}\,a^{3}}$	38
trager	$-\frac{-8b^{2}x^{8}+4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{-b\,x^{4}+a}\,a^{3}}$	38
elliptic	$-\frac{-8b^{2}x^{8}+4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{-b\,x^{4}+a}\,a^{3}}$	38
pseudoelliptic	$-\frac{-8b^{2}x^{8}+4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{-b\,x^{4}+a}\,a^{3}}$	38
orering	$-\frac{-8b^{2}x^{8}+4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{-b\,x^{4}+a}\,a^{3}}$	38
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(5b\,x^{4}+a\right)}{6a^{3}x^{6}}+\frac{x^{2}b^{2}}{2\sqrt{-b\,x^{4}+a}\,a^{3}}$	49

input
```
int(1/x^7/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-8*b^2*x^8+4*a*b*x^4+a^2)/x^6/(-b*x^4+a)^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^7 \left(a-b x^4\right)^{3/2}}\,dx=-\frac{\left(8\,b^2x^8-4\,abx^4-a^2\right)\sqrt{-bx^4+a}}{6\left(a^3bx^{10}-a^4x^6\right)}$$

input
```
integrate(1/x^7/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(8*b^2*x^8 - 4*a*b*x^4 - a^2)*sqrt(-b*x^4 + a)/(a^3*b*x^10 - a^4*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 481, normalized size of antiderivative = 7.07

$$\int \frac{1}{x^7 \left(a-b x^4\right)^{3/2}}\,dx=\begin{cases} -\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}-1}}{6a^5b^4x^4-12a^4b^5x^8+6a^3b^6x^{12}}-\frac{3a^2b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}-1}}{6a^5b^4x^4-12a^4b^5x^8+6a^3b^6x^{12}}+\frac{12ab^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}-1}}{6a^5b^4x^4-12a^4b^5x^8+6a^3b^6x^{12}}-\cdots \\ -\frac{ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx^4}+1}}{6a^5b^4x^4-12a^4b^5x^8+6a^3b^6x^{12}}-\frac{3ia^2b^{\frac{11}{2}}x^4\sqrt{-\frac{a}{bx^4}+1}}{6a^5b^4x^4-12a^4b^5x^8+6a^3b^6x^{12}}+\frac{12iab^{\frac{13}{2}}x^8\sqrt{-\frac{a}{bx^4}+1}}{6a^5b^4x^4-12a^4b^5x^8+6a^3b^6x^{12}}-\cdots \end{cases}$$

input
```
integrate(1/x**7/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-a**3*b**(9/2)*sqrt(a/(b*x**4) - 1)/(6*a**5*b**4*x**4 - 12*a**4
*b**5*x**8 + 6*a**3*b**6*x**12) - 3*a**2*b**(11/2)*x**4*sqrt(a/(b*x**4) -
1)/(6*a**5*b**4*x**4 - 12*a**4*b**5*x**8 + 6*a**3*b**6*x**12) + 12*a*b**(1
3/2)*x**8*sqrt(a/(b*x**4) - 1)/(6*a**5*b**4*x**4 - 12*a**4*b**5*x**8 + 6*a
**3*b**6*x**12) - 8*b**(15/2)*x**12*sqrt(a/(b*x**4) - 1)/(6*a**5*b**4*x**4
 - 12*a**4*b**5*x**8 + 6*a**3*b**6*x**12), Abs(a/(b*x**4)) > 1), (-I*a**3*
b**(9/2)*sqrt(-a/(b*x**4) + 1)/(6*a**5*b**4*x**4 - 12*a**4*b**5*x**8 + 6*a
**3*b**6*x**12) - 3*I*a**2*b**(11/2)*x**4*sqrt(-a/(b*x**4) + 1)/(6*a**5*b*
*4*x**4 - 12*a**4*b**5*x**8 + 6*a**3*b**6*x**12) + 12*I*a*b**(13/2)*x**8*s
qrt(-a/(b*x**4) + 1)/(6*a**5*b**4*x**4 - 12*a**4*b**5*x**8 + 6*a**3*b**6*x
**12) - 8*I*b**(15/2)*x**12*sqrt(-a/(b*x**4) + 1)/(6*a**5*b**4*x**4 - 12*a
**4*b**5*x**8 + 6*a**3*b**6*x**12), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7 (a - bx^4)^{3/2}} \, dx = \frac{b^2x^2}{2\sqrt{-bx^4 + a}a^3} - \frac{\frac{6\sqrt{-bx^4+a}b}{x^2} + \frac{(-bx^4+a)^{\frac{3}{2}}}{x^6}}{6\,a^3}$$

input
```
integrate(1/x^7/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b^2*x^2/(sqrt(-b*x^4 + a)*a^3) - 1/6*(6*sqrt(-b*x^4 + a)*b/x^2 + (-b*x
^4 + a)^(3/2)/x^6)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(56) = 112.

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.13

$$\int \frac{1}{x^7 (a - bx^4)^{3/2}} \, dx = -\frac{\sqrt{-bx^4 + a}b^2x^2}{2\,(bx^4 - a)a^3} + \frac{3\left(\sqrt{-b}x^2 - \sqrt{-bx^4 + a}\right)^4\sqrt{-b}b - 12\left(\sqrt{-b}x^2 - \sqrt{-bx^4 + a}\right)^2 a\sqrt{-b}b + 5\,a^2\sqrt{-b}b}{3\left(\left(\sqrt{-b}x^2 - \sqrt{-bx^4 + a}\right)^2 - a\right)^3 a^2}$$

input
```
integrate(1/x^7/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)*b^2*x^2/((b*x^4 - a)*a^3) + 1/3*(3*(sqrt(-b)*x^2 - s
qrt(-b*x^4 + a))^4*sqrt(-b)*b - 12*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2*a*s
qrt(-b)*b + 5*a^2*sqrt(-b)*b)/(((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^3
*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^7 (a-bx^4)^{3/2}} dx = \frac{8 (a-b x^4)^2 - 12 a (a-b x^4) + 3 a^2}{\left(\frac{6 a^4 x^2}{b} - \frac{6 a^3 x^2 (a-b x^4)}{b}\right) \sqrt{a-b x^4}}$$

input
```
int(1/(x^7*(a - b*x^4)^(3/2)),x)
```

output
```
(8*(a - b*x^4)^2 - 12*a*(a - b*x^4) + 3*a^2)/(((6*a^4*x^2)/b - (6*a^3*x^2*
(a - b*x^4))/b)*(a - b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^7 (a-bx^4)^{3/2}} dx = \frac{8b^2x^8 - 4ab x^4 - a^2}{6\sqrt{-b x^4 + a}\, a^3x^6}$$

input
```
int(1/x^7/(-b*x^4+a)^(3/2),x)
```

output
```
( - a**2 - 4*a*b*x**4 + 8*b**2*x**8)/(6*sqrt(a - b*x**4)*a**3*x**6)
```

3.234 $\int \frac{1}{x^{11}(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 93

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/2}}\,dx = \frac{1}{2ax^{10}\sqrt{a-bx^4}}-\frac{3\sqrt{a-bx^4}}{5a^2x^{10}}-\frac{4b\sqrt{a-bx^4}}{5a^3x^6}-\frac{8b^2\sqrt{a-bx^4}}{5a^4x^2}$$

output
```
1/2/a/x^10/(-b*x^4+a)^(1/2)-3/5*(-b*x^4+a)^(1/2)/a^2/x^10-4/5*b*(-b*x^4+a)
^(1/2)/a^3/x^6-8/5*b^2*(-b*x^4+a)^(1/2)/a^4/x^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/2}}\,dx = -\frac{a^3+2a^2bx^4+8ab^2x^8-16b^3x^{12}}{10a^4x^{10}\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^11*(a - b*x^4)^(3/2)),x]
```

output
```
-1/10*(a^3 + 2*a^2*b*x^4 + 8*a*b^2*x^8 - 16*b^3*x^12)/(a^4*x^10*Sqrt[a - b
*x^4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11} (a-bx^4)^{3/2}} dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{6b \int \frac{1}{x^7 (a-bx^4)^{3/2}}dx}{5a}-\frac{1}{10ax^{10}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{6b \left(\frac{4b \int \frac{1}{x^3 \left(a-bx^4\right)^{3/2}}dx}{3a}-\frac{1}{6ax^6\sqrt{a-bx^4}}\right)}{5a}-\frac{1}{10ax^{10}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{6b \left(\frac{4b \left(\frac{2b \int \frac{x}{\left(a-bx^4\right)^{3/2}}dx}{a}-\frac{1}{2ax^2\sqrt{a-bx^4}}\right)}{3a}-\frac{1}{6ax^6\sqrt{a-bx^4}}\right)}{5a}-\frac{1}{10ax^{10}\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{6b \left(\frac{4b \left(\frac{bx^2}{a^2\sqrt{a-bx^4}}-\frac{1}{2ax^2\sqrt{a-bx^4}}\right)}{3a}-\frac{1}{6ax^6\sqrt{a-bx^4}}\right)}{5a}-\frac{1}{10ax^{10}\sqrt{a-bx^4}}$$

input
```
Int[1/(x^11*(a - b*x^4)^(3/2)),x]
```

output
```
-1/10*1/(a*x^10*Sqrt[a - b*x^4]) + (6*b*(-1/6*1/(a*x^6*Sqrt[a - b*x^4]) +
(4*b*(-1/2*1/(a*x^2*Sqrt[a - b*x^4]) + (b*x^2)/(a^2*Sqrt[a - b*x^4])))/(3*
a)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{-16b^{3}x^{12}+8a\,b^{2}x^{8}+2a^{2}b\,x^{4}+a^{3}}{10x^{10}\sqrt{-b\,x^{4}+a}\,a^{4}}$	49
default	$-\frac{-16b^{3}x^{12}+8a\,b^{2}x^{8}+2a^{2}b\,x^{4}+a^{3}}{10x^{10}\sqrt{-b\,x^{4}+a}\,a^{4}}$	49
trager	$-\frac{-16b^{3}x^{12}+8a\,b^{2}x^{8}+2a^{2}b\,x^{4}+a^{3}}{10x^{10}\sqrt{-b\,x^{4}+a}\,a^{4}}$	49
elliptic	$-\frac{-16b^{3}x^{12}+8a\,b^{2}x^{8}+2a^{2}b\,x^{4}+a^{3}}{10x^{10}\sqrt{-b\,x^{4}+a}\,a^{4}}$	49
pseudoelliptic	$-\frac{-16b^{3}x^{12}+8a\,b^{2}x^{8}+2a^{2}b\,x^{4}+a^{3}}{10x^{10}\sqrt{-b\,x^{4}+a}\,a^{4}}$	49
orering	$-\frac{-16b^{3}x^{12}+8a\,b^{2}x^{8}+2a^{2}b\,x^{4}+a^{3}}{10x^{10}\sqrt{-b\,x^{4}+a}\,a^{4}}$	49
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,(11b^{2}x^{8}+3ab\,x^{4}+a^{2})}{10a^{4}x^{10}}+\frac{x^{2}b^{3}}{2\sqrt{-b\,x^{4}+a}\,a^{4}}$	60

input
```
int(1/x^11/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-16*b^3*x^12+8*a*b^2*x^8+2*a^2*b*x^4+a^3)/x^10/(-b*x^4+a)^(1/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^{11} \left(a - bx^4\right)^{3/2}}\, dx = -\frac{\left(16\, b^3 x^{12} - 8\, a b^2 x^8 - 2\, a^2 b x^4 - a^3\right)\sqrt{-bx^4 + a}}{10\left(a^4 b x^{14} - a^5 x^{10}\right)}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(16*b^3*x^12 - 8*a*b^2*x^8 - 2*a^2*b*x^4 - a^3)*sqrt(-b*x^4 + a)/(a^
4*b*x^14 - a^5*x^10)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 724, normalized size of antiderivative = 7.78

$$\int \frac{1}{x^{11} \left(a - bx^4\right)^{3/2}}\, dx = \begin{cases} \frac{a^5 b^{\frac{19}{2}} \sqrt{\frac{a}{bx^4} - 1}}{-10a^7b^9x^8 + 30a^6b^{10}x^{12} - 30a^5b^{11}x^{16} + 10a^4b^{12}x^{20}} + \frac{5a^3 b^{\frac{23}{2}} x^8 \sqrt{\frac{a}{bx^4} - 1}}{-10a^7b^9x^8 + 30a^6b^{10}x^{12} - 30a^5b^{11}x^{16} + 10a^4b^{12}x^{20}} - \\ \frac{i a^5 b^{\frac{19}{2}} \sqrt{-\frac{a}{bx^4} + 1}}{-10a^7b^9x^8 + 30a^6b^{10}x^{12} - 30a^5b^{11}x^{16} + 10a^4b^{12}x^{20}} + \frac{5 i a^3 b^{\frac{23}{2}} x^8 \sqrt{-\frac{a}{bx^4} + 1}}{-10a^7b^9x^8 + 30a^6b^{10}x^{12} - 30a^5b^{11}x^{16} + 10a^4b^{12}x^{20}} - \end{cases}$$

input
```
integrate(1/x**11/(-b*x**4+a)**(3/2),x)
```

output
```
Piecewise((a**5*b**(19/2)*sqrt(a/(b*x**4) - 1)/(-10*a**7*b**9*x**8 + 30*a*
*6*b**10*x**12 - 30*a**5*b**11*x**16 + 10*a**4*b**12*x**20) + 5*a**3*b**(2
3/2)*x**8*sqrt(a/(b*x**4) - 1)/(-10*a**7*b**9*x**8 + 30*a**6*b**10*x**12 -
 30*a**5*b**11*x**16 + 10*a**4*b**12*x**20) - 30*a**2*b**(25/2)*x**12*sqrt
(a/(b*x**4) - 1)/(-10*a**7*b**9*x**8 + 30*a**6*b**10*x**12 - 30*a**5*b**11
*x**16 + 10*a**4*b**12*x**20) + 40*a*b**(27/2)*x**16*sqrt(a/(b*x**4) - 1)/
(-10*a**7*b**9*x**8 + 30*a**6*b**10*x**12 - 30*a**5*b**11*x**16 + 10*a**4*
b**12*x**20) - 16*b**(29/2)*x**20*sqrt(a/(b*x**4) - 1)/(-10*a**7*b**9*x**8
 + 30*a**6*b**10*x**12 - 30*a**5*b**11*x**16 + 10*a**4*b**12*x**20), Abs(a
/(b*x**4)) > 1), (I*a**5*b**(19/2)*sqrt(-a/(b*x**4) + 1)/(-10*a**7*b**9*x*
*8 + 30*a**6*b**10*x**12 - 30*a**5*b**11*x**16 + 10*a**4*b**12*x**20) + 5*
I*a**3*b**(23/2)*x**8*sqrt(-a/(b*x**4) + 1)/(-10*a**7*b**9*x**8 + 30*a**6*
b**10*x**12 - 30*a**5*b**11*x**16 + 10*a**4*b**12*x**20) - 30*I*a**2*b**(2
5/2)*x**12*sqrt(-a/(b*x**4) + 1)/(-10*a**7*b**9*x**8 + 30*a**6*b**10*x**12
 - 30*a**5*b**11*x**16 + 10*a**4*b**12*x**20) + 40*I*a*b**(27/2)*x**16*sqr
t(-a/(b*x**4) + 1)/(-10*a**7*b**9*x**8 + 30*a**6*b**10*x**12 - 30*a**5*b**
11*x**16 + 10*a**4*b**12*x**20) - 16*I*b**(29/2)*x**20*sqrt(-a/(b*x**4) +
1)/(-10*a**7*b**9*x**8 + 30*a**6*b**10*x**12 - 30*a**5*b**11*x**16 + 10*a*
*4*b**12*x**20), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/2}}\,dx = \frac{b^3x^2}{2\sqrt{-bx^4+a}a^4} - \frac{\frac{15\sqrt{-bx^4+a}b^2}{x^2} + \frac{5\left(-bx^4+a\right)^{\frac{3}{2}}b}{x^6} + \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{x^{10}}}{10\,a^4}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b^3*x^2/(sqrt(-b*x^4 + a)*a^4) - 1/10*(15*sqrt(-b*x^4 + a)*b^2/x^2 + 5
*(-b*x^4 + a)^(3/2)*b/x^6 + (-b*x^4 + a)^(5/2)/x^10)/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.42

$$\int \frac{1}{x^{11} (a - bx^4)^{3/2}} \, dx = -\frac{\sqrt{-bx^4 + a} b^3 x^2}{2 (bx^4 - a) a^4} + \frac{5 \left(\sqrt{-b} x^2 - \sqrt{-bx^4 + a}\right)^8 \sqrt{-b} b^2 - 30 \left(\sqrt{-b} x^2 - \sqrt{-bx^4 + a}\right)^6 a \sqrt{-b} b^2 + 80 \left(\sqrt{-b} x^2 - \sqrt{-bx^4 + a}\right)^4 a^2}{5 \left(\left(\sqrt{-b} x^2 - \sqrt{-bx^4 + a}\right)^2 - a\right)^5 a^3}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-b*x^4 + a)*b^3*x^2/((b*x^4 - a)*a^4) + 1/5*(5*(sqrt(-b)*x^2 - s
qrt(-b*x^4 + a))^8*sqrt(-b)*b^2 - 30*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^6*a
*sqrt(-b)*b^2 + 80*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^4*a^2*sqrt(-b)*b^2 -
50*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2*a^3*sqrt(-b)*b^2 + 11*a^4*sqrt(-b)*
b^2)/(((sqrt(-b)*x^2 - sqrt(-b*x^4 + a))^2 - a)^5*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{11} (a - bx^4)^{3/2}} \, dx = \frac{b^3 x^2}{2 a^4 \sqrt{a - b x^4}} - \frac{3 b \sqrt{a - b x^4}}{10 a^3 x^6} - \frac{11 b^2 \sqrt{a - b x^4}}{10 a^4 x^2} - \frac{\sqrt{a - b x^4}}{10 a^2 x^{10}}$$

input
```
int(1/(x^11*(a - b*x^4)^(3/2)),x)
```

output
```
(b^3*x^2)/(2*a^4*(a - b*x^4)^(1/2)) - (3*b*(a - b*x^4)^(1/2))/(10*a^3*x^6)
 - (11*b^2*(a - b*x^4)^(1/2))/(10*a^4*x^2) - (a - b*x^4)^(1/2)/(10*a^2*x^1
0)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{11} \left(a-bx^4\right)^{3/2}}\,dx = \frac{16b^3x^{12}-8a\,b^2x^8-2a^2b\,x^4-a^3}{10\sqrt{-b\,x^4+a}\,a^4x^{10}}$$

input
```
int(1/x^11/(-b*x^4+a)^(3/2),x)
```

output
```
( - a**3 - 2*a**2*b*x**4 - 8*a*b**2*x**8 + 16*b**3*x**12)/(10*sqrt(a - b*x
**4)*a**4*x**10)
```

3.235 $\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx = \frac{x^5}{2b\sqrt{a-bx^4}}+\frac{5x\sqrt{a-bx^4}}{6b^2} - \frac{5a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{6b^{9/4}\sqrt{a-bx^4}}$$

output
```
1/2*x^5/b/(-b*x^4+a)^(1/2)+5/6*x*(-b*x^4+a)^(1/2)/b^2-5/6*a^(5/4)*(1-b*x^4
/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(9/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx = \frac{5ax-2bx^5-5ax\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{6b^2\sqrt{a-bx^4}}$$

input
```
Integrate[x^8/(a - b*x^4)^(3/2),x]
```

output
```
(5*a*x - 2*b*x^5 - 5*a*x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5
/4, (b*x^4)/a])/(6*b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {817, 843, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^5}{2b\sqrt{a-bx^4}}-\frac{5\int \frac{x^4}{\sqrt{a-bx^4}}dx}{2b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5}{2b\sqrt{a-bx^4}}-\frac{5\left(\frac{a\int \frac{1}{\sqrt{a-bx^4}}dx}{3b}-\frac{x\sqrt{a-bx^4}}{3b}\right)}{2b}$$

$$\Big\downarrow \text{765}$$

$$\frac{x^5}{2b\sqrt{a-bx^4}}-\frac{5\left(\frac{a\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3b\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)}{2b}$$

$$\Big\downarrow \text{762}$$

$$\frac{x^5}{2b\sqrt{a-bx^4}}-\frac{5\left(\frac{a^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3b^{5/4}\sqrt{a-bx^4}}-\frac{x\sqrt{a-bx^4}}{3b}\right)}{2b}$$

input
```
Int[x^8/(a - b*x^4)^(3/2),x]
```

output
```
x^5/(2*b*Sqrt[a - b*x^4]) - (5*(-1/3*(x*Sqrt[a - b*x^4])/b + (a^(5/4)*Sqrt
[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*b^(5/4)*Sqr
t[a - b*x^4])))/(2*b)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

method	result
default	$\frac{xa}{2b^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{x\sqrt{-b\,x^{4}+a}}{3b^{2}}-\frac{5a\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{6b^{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
elliptic	$\frac{xa}{2b^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{x\sqrt{-b\,x^{4}+a}}{3b^{2}}-\frac{5a\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{6b^{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
risch	$\frac{x\sqrt{-b\,x^{4}+a}}{3b^{2}}-\frac{a\left(\frac{4\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+3a\left(-\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right)\right)}{3b^{2}}$

input
```
int(x^8/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*x*a/(-(x^4-a/b)*b)^(1/2)+1/3*x*(-b*x^4+a)^(1/2)/b^2-5/6*a/b^2/(1/a
^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2)
)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx = -\frac{5\,(bx^4-a)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,\middle|\,-1\right)-(2\,bx^5-5\,ax)\sqrt{-bx^4+a}}{6\,(b^3x^4-ab^2)}$$

input
```
integrate(x^8/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(5*(b*x^4 - a)*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)/x),
 -1) - (2*b*x^5 - 5*a*x)*sqrt(-b*x^4 + a))/(b^3*x^4 - a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(-b*x**4+a)**(3/2),x)
```

output
```
x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^8}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^8}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx=\int \frac{x^8}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^8/(a - b*x^4)^(3/2),x)
```

output
```
int(x^8/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8}{(a-bx^4)^{3/2}}\,dx=\frac{5\sqrt{-b\,x^4+a}\,ax-\sqrt{-b\,x^4+a}\,b\,x^5-5\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)a^3+5\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right.}{3b^2\left(-b\,x^4+a\right)}$$

input
```
int(x^8/(-b*x^4+a)^(3/2),x)
```

output
```
(5*sqrt(a - b*x**4)*a*x - sqrt(a - b*x**4)*b*x**5 - 5*int(sqrt(a - b*x**4)
/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**3 + 5*int(sqrt(a - b*x**4)/(a**2 -
2*a*b*x**4 + b**2*x**8),x)*a**2*b*x**4)/(3*b**2*(a - b*x**4))
```

3.236 $\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx = \frac{x}{2b\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2b^{5/4}\sqrt{a-bx^4}}$$

output
```
1/2*x/b/(-b*x^4+a)^(1/2)-1/2*a^(1/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x
/a^(1/4),I)/b^(5/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx = \frac{x-x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{2b\sqrt{a-bx^4}}$$

input
```
Integrate[x^4/(a - b*x^4)^(3/2),x]
```

output
```
(x - x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/(2
*b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {817, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x}{2b\sqrt{a-bx^4}}-\frac{\int \frac{1}{\sqrt{a-bx^4}}dx}{2b}$$

$$\Big\downarrow \text{765}$$

$$\frac{x}{2b\sqrt{a-bx^4}}-\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{2b\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{x}{2b\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2b^{5/4}\sqrt{a-bx^4}}$$

input
```
Int[x^4/(a - b*x^4)^(3/2),x]
```

output
```
x/(2*b*Sqrt[a - b*x^4]) - (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/(2*b^(5/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{x}{2b\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	90
elliptic	$\frac{x}{2b\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	90

input
```
int(x^4/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*x/(-(x^4-a/b)*b)^(1/2)-1/2/b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^
2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(
x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx = -\frac{(bx^4-a)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right) + \sqrt{-bx^4+a}\,bx}{2\left(b^3x^4-ab^2\right)}$$

input
```
integrate(x^4/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((b*x^4 - a)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1
) + sqrt(-b*x^4 + a)*b*x)/(b^3*x^4 - a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(-b*x**4+a)**(3/2),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^4}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx=\int \frac{x^4}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx=\int \frac{x^4}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^4/(a - b*x^4)^(3/2),x)
```

output
```
int(x^4/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^4)^{3/2}}\,dx=\frac{\sqrt{-b\,x^4+a}\,x-\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)a^2+\left(\int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx\right)ab\,x^4}{b\,(-b\,x^4+a)}$$

input
```
int(x^4/(-b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a - b*x**4)*x - int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8)
,x)*a**2 + int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a*b*x**
4)/(b*(a - b*x**4))
```

3.237 $\int \frac{1}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \frac{x}{2a\sqrt{a-bx^4}} + \frac{\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a-bx^4}}$$

output
```
1/2*x/a/(-b*x^4+a)^(1/2)+1/2*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4)
,I)/a^(3/4)/b^(1/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \frac{x+x\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{bx^4}{a}\right)}{2a\sqrt{a-bx^4}}$$

input
```
Integrate[(a - b*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (b*x^4)/a])/(2
*a*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {749, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 749$$

$$\frac{\int \frac{1}{\sqrt{a-bx^4}}dx}{2a}+\frac{x}{2a\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{2a\sqrt{a-bx^4}}+\frac{x}{2a\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 762$$

$$\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt[4]{b}\sqrt{a-bx^4}}+\frac{x}{2a\sqrt{a-bx^4}}$$

input
```
Int[(a - b*x^4)^(-3/2),x]
```

output
```
x/(2*a*Sqrt[a - b*x^4]) + (Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(2*a^(3/4)*b^(1/4)*Sqrt[a - b*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	90
elliptic	$\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$	90

input
```
int(1/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(-(x^4-a/b)*b)^(1/2)+1/2/a/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^
2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(
x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \frac{(bx^4-a)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-\sqrt{-bx^4+a}\,abx}{2\,(ab^2x^4-a^2b)}$$

input
```
integrate(1/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((b*x^4 - a)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)), -1)
 - sqrt(-b*x^4 + a)*b*x)/(a*b^2*x^4 - a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.48

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-b*x**4+a)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(
3/2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \frac{x\left(1-\frac{b\,x^4}{a}\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{(a-b\,x^4)^{3/2}}$$

input
```
int(1/(a - b*x^4)^(3/2),x)
```

output
```
(x*(1 - (b*x^4)/a)^(3/2)*hypergeom([1/4, 3/2], 5/4, (b*x^4)/a))/(a - b*x^4
)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a-bx^4)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^2x^8-2ab\,x^4+a^2}dx$$

input
```
int(1/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)
```

3.238 $\int \frac{1}{x^4(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{1}{x^4\left(a-bx^4\right)^{3/2}}\,dx = \frac{1}{2ax^3\sqrt{a-bx^4}} - \frac{5\sqrt{a-bx^4}}{6a^2x^3} + \frac{5b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{6a^{7/4}\sqrt{a-bx^4}}$$

output
```
1/2/a/x^3/(-b*x^4+a)^(1/2)-5/6*(-b*x^4+a)^(1/2)/a^2/x^3+5/6*b^(3/4)*(1-b*x
^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^4\left(a-bx^4\right)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},\frac{bx^4}{a}\right)}{3ax^3\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^4*(a - b*x^4)^(3/2)),x]
```

output
```
-1/3*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, (b*x^4)/a])/(a
*x^3*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {819, 847, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a-b x^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{5 \int \frac{1}{x^4 \sqrt{a-b x^4}}dx}{2 a}+\frac{1}{2 a x^3 \sqrt{a-b x^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{5 \left(\frac{b \int \frac{1}{\sqrt{a-b x^4}}dx}{3 a}-\frac{\sqrt{a-b x^4}}{3 a x^3}\right)}{2 a}+\frac{1}{2 a x^3 \sqrt{a-b x^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{5 \left(\frac{b \sqrt{1-\frac{b x^4}{a}} \int \frac{1}{\sqrt{1-\frac{b x^4}{a}}}dx}{3 a \sqrt{a-b x^4}}-\frac{\sqrt{a-b x^4}}{3 a x^3}\right)}{2 a}+\frac{1}{2 a x^3 \sqrt{a-b x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{5 \left(\frac{b^{3/4} \sqrt{1-\frac{b x^4}{a}} \text{EllipticF}\left(\arcsin \left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a}}\right),-1\right)}{3 a^{3/4} \sqrt{a-b x^4}}-\frac{\sqrt{a-b x^4}}{3 a x^3}\right)}{2 a}+\frac{1}{2 a x^3 \sqrt{a-b x^4}}$$

input
```
Int[1/(x^4*(a - b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x^3*Sqrt[a - b*x^4]) + (5*(-1/3*Sqrt[a - b*x^4]/(a*x^3) + (b^(3/4)*
Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)
*Sqrt[a - b*x^4])))/(2*a)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

method	result
default	$-\frac{\sqrt{-b\,x^4+a}}{3a^2x^3}+\frac{bx}{2a^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{5b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
elliptic	$-\frac{\sqrt{-b\,x^4+a}}{3a^2x^3}+\frac{bx}{2a^2\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{5b\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{6a^2\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
risch	$-\frac{\sqrt{-b\,x^4+a}}{3a^2x^3}+\frac{b\left(\frac{\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}-3a\left(-\frac{x}{2a\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}\right)\right)}{3a^2}$

input
```
int(1/x^4/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^4+a)^(1/2)/a^2/x^3+1/2*b/a^2*x/(-(x^4-a/b)*b)^(1/2)+5/6*b/a^2/(
1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1
/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^4\left(a-bx^4\right)^{3/2}}\,dx=\frac{5\left(bx^7-ax^3\right)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)-\left(5\,bx^4-2\,a\right)\sqrt{-bx^4+a}}{6\left(a^2bx^7-a^3x^3\right)}$$

input
```
integrate(1/x^4/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(5*(b*x^7 - a*x^3)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4)
), -1) - (5*b*x^4 - 2*a)*sqrt(-b*x^4 + a))/(a^2*b*x^7 - a^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^4 \left(a-b x^4\right)^{3/2}}\, dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\right| \frac{b x^4 e^{2i\pi}}{a}\right)}{4 a^{\frac{3}{2}} x^3 \Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(-b*x**4+a)**(3/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(
3/2)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4 \left(a-b x^4\right)^{3/2}}\, dx = \int \frac{1}{\left(-b x^4+a\right)^{\frac{3}{2}} x^4}\, dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(a-b x^4\right)^{3/2}}\, dx = \int \frac{1}{\left(-b x^4+a\right)^{\frac{3}{2}} x^4}\, dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^4)^{3/2}}\,dx = \int \frac{1}{x^4 \left(a-b\,x^4\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a - b*x^4)^(3/2)),x)
```

output
```
int(1/(x^4*(a - b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^4)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^2x^{12}-2ab\,x^8+a^2x^4}dx$$

input
```
int(1/x^4/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2*x**4 - 2*a*b*x**8 + b**2*x**12),x)
```

3.239 $\int \frac{1}{x^8 (a-bx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{1}{x^8 (a-bx^4)^{3/2}} \, dx = \frac{1}{2ax^7\sqrt{a-bx^4}} - \frac{9\sqrt{a-bx^4}}{14a^2x^7} - \frac{15b\sqrt{a-bx^4}}{14a^3x^3} + \frac{15b^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{14a^{11/4}\sqrt{a-bx^4}}$$

output
```
1/2/a/x^7/(-b*x^4+a)^(1/2)-9/14*(-b*x^4+a)^(1/2)/a^2/x^7-15/14*b*(-b*x^4+a
)^(1/2)/a^3/x^3+15/14*b^(7/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4
),I)/a^(11/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^8 (a-bx^4)^{3/2}} \, dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},-\frac{3}{4},\frac{bx^4}{a}\right)}{7ax^7\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^8*(a - b*x^4)^(3/2)),x]
```

output
```
-1/7*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-7/4, 3/2, -3/4, (b*x^4)/a])/(
a*x^7*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {819, 847, 847, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a-bx^4\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{9 \int \frac{1}{x^8 \sqrt{a-bx^4}}dx}{2a}+\frac{1}{2ax^7 \sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{9 \left(\frac{5b \int \frac{1}{x^4 \sqrt{a-bx^4}}dx}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7 \sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{9 \left(\frac{5b \left(\frac{b \int \frac{1}{\sqrt{a-bx^4}}dx}{3a}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7 \sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{9\left(\frac{5b\left(\frac{b\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{3a\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{9\left(\frac{5b\left(\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3a^{3/4}\sqrt{a-bx^4}}-\frac{\sqrt{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a-bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7\sqrt{a-bx^4}}$$

input
```
Int[1/(x^8*(a - b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x^7*Sqrt[a - b*x^4]) + (9*(-1/7*Sqrt[a - b*x^4]/(a*x^7) + (5*b*(-1/
3*Sqrt[a - b*x^4]/(a*x^3) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(3*a^(3/4)*Sqrt[a - b*x^4])))/(7*a)))/(2*a)
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07

method	result
default	$-\frac{\sqrt{-b\,x^{4}+a}}{7a^{2}x^{7}}-\frac{4b\sqrt{-b\,x^{4}+a}}{7a^{3}x^{3}}+\frac{b^{2}x}{2a^{3}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{15b^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{14a^{3}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
elliptic	$-\frac{\sqrt{-b\,x^{4}+a}}{7a^{2}x^{7}}-\frac{4b\sqrt{-b\,x^{4}+a}}{7a^{3}x^{3}}+\frac{b^{2}x}{2a^{3}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}+\frac{15b^{2}\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{14a^{3}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
risch	$-\frac{\sqrt{-b\,x^{4}+a}\,\left(4b\,x^{4}+a\right)}{7a^{3}x^{7}}+\frac{b^{2}\left(\frac{4\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\;\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}-7a\left(-\frac{x}{2a\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\;\mathrm{EllipticF}}{2a\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}\right.\right.}{7a^{3}}$

input
```
int(1/x^8/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*(-b*x^4+a)^(1/2)/a^2/x^7-4/7*b*(-b*x^4+a)^(1/2)/a^3/x^3+1/2*b^2/a^3*x
/(-(x^4-a/b)*b)^(1/2)+15/14*b^2/a^3/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x
^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF
(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^8 (a-bx^4)^{3/2}}\,dx = \frac{15\,(b^2x^{11}-abx^7)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}} F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right) | -1) - (15\,b^2x^8 - 6\,abx^4 - 2\,a^2)\sqrt{-b}}{14\,(a^3bx^{11}-a^4x^7)}$$

input
```
integrate(1/x^8/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/14*(15*(b^2*x^11 - a*b*x^7)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a
)^(1/4)), -1) - (15*b^2*x^8 - 6*a*b*x^4 - 2*a^2)*sqrt(-b*x^4 + a))/(a^3*b*
x^11 - a^4*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^8 (a-bx^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(-b*x**4+a)**(3/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**
(3/2)*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8 (a - bx^4)^{3/2}} \, dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{2}} x^8} \, dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (a - bx^4)^{3/2}} \, dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{2}} x^8} \, dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (a - bx^4)^{3/2}} \, dx = \int \frac{1}{x^8 (a - b x^4)^{3/2}} \, dx$$

input
```
int(1/(x^8*(a - b*x^4)^(3/2)),x)
```

output
```
int(1/(x^8*(a - b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 \left(a-bx^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^2x^{16}-2ab\,x^{12}+a^2x^8}dx$$

input
```
int(1/x^8/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2*x**8 - 2*a*b*x**12 + b**2*x**16),x)
```

3.240 $\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 157

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \frac{x^7}{2b\sqrt{a-bx^4}}+\frac{7x^3\sqrt{a-bx^4}}{10b^2} - \frac{21a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{10b^{11/4}\sqrt{a-bx^4}} + \frac{21a^{7/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{10b^{11/4}\sqrt{a-bx^4}}$$

output
```
1/2*x^7/b/(-b*x^4+a)^(1/2)+7/10*x^3*(-b*x^4+a)^(1/2)/b^2-21/10*a^(7/4)*(1-
b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/b^(11/4)/(-b*x^4+a)^(1/2)+21
/10*a^(7/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/b^(11/4)/(-b*
x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.43

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = -\frac{x^3\left(7a+bx^4-7a\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{bx^4}{a}\right)\right)}{5b^2\sqrt{a-bx^4}}$$

input
```
Integrate[x^10/(a - b*x^4)^(3/2),x]
```

output
```
-1/5*(x^3*(7*a + b*x^4 - 7*a*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[3/4, 3/
2, 7/4, (b*x^4)/a]))/(b^2*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {817, 843, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\int \frac{x^6}{\sqrt{a-bx^4}}dx}{2b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\int \frac{x^2}{\sqrt{a-bx^4}}dx}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \text{836}$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \ 27$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \ 765$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \ 762$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \ 1390$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \textbf{1389}$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

$$\Big\downarrow \textbf{327}$$

$$\frac{x^7}{2b\sqrt{a-bx^4}}-\frac{7\left(\frac{3a\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{5b}-\frac{x^3\sqrt{a-bx^4}}{5b}\right)}{2b}$$

input

```
Int[x^10/(a - b*x^4)^(3/2),x]
```

output

```
x^7/(2*b*Sqrt[a - b*x^4]) - (7*(-1/5*(x^3*Sqrt[a - b*x^4])/b + (3*a*((a^(3
/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/
4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/
4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/(5*b)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

method	result
default	$\frac{x^{3} a}{2 b^{2} \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{x^{3} \sqrt{-b\,x^{4}+a}}{5 b^{2}}+\frac{21 a^{\frac{3}{2}} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{10 b^{\frac{5}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
elliptic	$\frac{x^{3} a}{2 b^{2} \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{x^{3} \sqrt{-b\,x^{4}+a}}{5 b^{2}}+\frac{21 a^{\frac{3}{2}} \sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{10 b^{\frac{5}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
risch	$\frac{x^{3} \sqrt{-b\,x^{4}+a}}{5 b^{2}}-\frac{a \left(-\frac{8 \sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}\,\sqrt{b}}+5 a \left(-\frac{x^{3}}{2 a \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1}}{}\right.\right.}{5 b^{2}}$

input
```
int(x^10/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*x^3*a/(-(x^4-a/b)*b)^(1/2)+1/5*x^3*(-b*x^4+a)^(1/2)/b^2+21/10*a^(3
/2)/b^(5/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(
1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^
(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \frac{21\,(abx^5-a^2x)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}} E\left(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - 21\,(abx^5-a^2x)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}} F(\arcsin}{10\,(b^4x^5-ab^3x)}$$

input
```
integrate(x^10/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(21*(a*b*x^5 - a^2*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1
/4)/x), -1) - 21*(a*b*x^5 - a^2*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin(
(a/b)^(1/4)/x), -1) + (2*b^2*x^8 + 14*a*b*x^4 - 21*a^2)*sqrt(-b*x^4 + a))/
(b^4*x^5 - a*b^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.25

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{3}{2}, \frac{11}{4}\\ \frac{15}{4}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(-b*x**4+a)**(3/2),x)
```

output
```
x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/
(4*a**(3/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^{10}}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^10/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^{10}}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^10/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^{10}}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^10/(a - b*x^4)^(3/2),x)
```

output
```
int(x^10/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a-bx^4)^{3/2}}\,dx = \frac{-7\sqrt{-b\,x^4+a}\,a\,x^3-\sqrt{-b\,x^4+a}\,b\,x^7+21\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{b^2x^8-2ab\,x^4+a^2}dx\right)a^3-21\left(\int \frac{\sqrt{-b\,x^4+a}\,x}{b^2x^8-2ab\,x^4-}\right.}{5b^2\left(-b\,x^4+a\right)}$$

input
```
int(x^10/(-b*x^4+a)^(3/2),x)
```

output
```
( - 7*sqrt(a - b*x**4)*a*x**3 - sqrt(a - b*x**4)*b*x**7 + 21*int((sqrt(a -
 b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**3 - 21*int((sqrt(a -
b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)*a**2*b*x**4)/(5*b**2*(a -
 b*x**4))
```

3.241 $\int \frac{x^6}{(a-bx^4)^{3/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{x^6}{(a-bx^4)^{3/2}} dx = \frac{x^3}{2b\sqrt{a-bx^4}} - \frac{3a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2b^{7/4}\sqrt{a-bx^4}} + \frac{3a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2b^{7/4}\sqrt{a-bx^4}}$$

output
```
1/2*x^3/b/(-b*x^4+a)^(1/2)-3/2*a^(3/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)
*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)+3/2*a^(3/4)*(1-b*x^4/a)^(1/2)*Ellip
ticF(b^(1/4)*x/a^(1/4),I)/b^(7/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.41

$$\int \frac{x^6}{(a-bx^4)^{3/2}} dx = \frac{x^3\left(-1+\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{bx^4}{a}\right)\right)}{b\sqrt{a-bx^4}}$$

input
```
Integrate[x^6/(a - b*x^4)^(3/2),x]
```

output
```
(x^3*(-1 + Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (b*x^4)/a]
))/(b*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {817, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\int \frac{x^2}{\sqrt{a-bx^4}}dx}{2b}$$

$$\Big\downarrow \text{836}$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{2b}$$

$$\Big\downarrow \text{765}$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{2b}$$

$$\Big\downarrow \; 762$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{2b}$$

$$\Big\downarrow \; 1390$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{2b}$$

$$\Big\downarrow \; 1389$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{2b}$$

$$\Big\downarrow \; 327$$

$$\frac{x^3}{2b\sqrt{a-bx^4}}-\frac{3\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{2b}$$

input

```
Int[x^6/(a - b*x^4)^(3/2),x]
```

output

```
x^3/(2*b*Sqrt[a - b*x^4]) - (3*((a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1
 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a
- b*x^4])))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^{3}}{2 b \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{3 \sqrt{a}\, \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 b^{\frac{3}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}}$	112
elliptic	$\frac{x^{3}}{2 b \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{3 \sqrt{a}\, \sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 b^{\frac{3}{2}} \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b \,x^{4}+a}}$	112

input
```
int(x^6/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*x^3/(-(x^4-a/b)*b)^(1/2)+3/2/b^(3/2)*a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/
2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^
(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(
1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{x^6}{\left(a-bx^4\right)^{3/2}}\,dx=\frac{3\,(bx^5-ax)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-3\,(bx^5-ax)\sqrt{-b}\left(\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(\frac{a}{b}\right.}{x}\right.}{2\,(b^3x^5-ab^2x)}$$

input
```
integrate(x^6/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*(b*x^5 - a*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_e(arcsin((a/b)^(1/4)/x)
, -1) - 3*(b*x^5 - a*x)*sqrt(-b)*(a/b)^(3/4)*elliptic_f(arcsin((a/b)^(1/4)
/x), -1) + (2*b*x^4 - 3*a)*sqrt(-b*x^4 + a))/(b^3*x^5 - a*b^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{x^6}{(a-bx^4)^{3/2}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(-b*x**4+a)**(3/2),x)
```

output
```
x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^6}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^6}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^6}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^6/(a - b*x^4)^(3/2),x)
```

output
```
int(x^6/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^4)^{3/2}}\,dx = \frac{-\sqrt{-b\,x^4+a}\,x^3+3\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{b^2x^8-2ab\,x^4+a^2}dx\right)a^2-3\left(\int \frac{\sqrt{-b\,x^4+a}\,x^2}{b^2x^8-2ab\,x^4+a^2}dx\right)ab\,x^4}{b\,(-b\,x^4+a)}$$

input
```
int(x^6/(-b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a - b*x**4)*x**3 + 3*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x*
*4 + b**2*x**8),x)*a**2 - 3*int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4
 + b**2*x**8),x)*a*b*x**4)/(b*(a - b*x**4))
```

3.242 $\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \frac{x^3}{2a\sqrt{a-bx^4}} - \frac{\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a-bx^4}} + \frac{\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2\sqrt[4]{a}b^{3/4}\sqrt{a-bx^4}}$$

output
```
1/2*x^3/a/(-b*x^4+a)^(1/2)-1/2*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/
4),I)/a^(1/4)/b^(3/4)/(-b*x^4+a)^(1/2)+1/2*(1-b*x^4/a)^(1/2)*EllipticF(b^(
1/4)*x/a^(1/4),I)/a^(1/4)/b^(3/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.41

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \frac{x^3\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{bx^4}{a}\right)}{3a\sqrt{a-bx^4}}$$

input
```
Integrate[x^2/(a - b*x^4)^(3/2),x]
```

output
```
(x^3*Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, (b*x^4)/a])/(3*a
*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {819, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\int \frac{x^2}{\sqrt{a-bx^4}}dx}{2a}$$

$$\Big\downarrow \text{ 836}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{2a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}}{2a}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}}{2a}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2a}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2a}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2a}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{x^3}{2a\sqrt{a-bx^4}}-\frac{\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}}{2a}$$

input
```
Int[x^2/(a - b*x^4)^(3/2),x]
```

output
```
x^3/(2*a*Sqrt[a - b*x^4]) - ((a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin
[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 -
(b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b
*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^{3}}{2 a \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 \sqrt{a}\, \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\, x^{4}+a}\, \sqrt{b}}$	112
elliptic	$\frac{x^{3}}{2 a \sqrt{-\left(x^{4}-\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 \sqrt{a}\, \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\, \sqrt{-b\, x^{4}+a}\, \sqrt{b}}$	112

input
```
int(x^2/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3/a/(-(x^4-a/b)*b)^(1/2)+1/2/a^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^
(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)/b^
(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(
1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx =$$
$$-\frac{\sqrt{-bx^4+a}bx^3+(bx^4-a)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-(bx^4-a)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1}{2(ab^2x^4-a^2b)}$$

input
```
integrate(x^2/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-b*x^4 + a)*b*x^3 + (b*x^4 - a)*sqrt(a)*(b/a)^(3/4)*elliptic_e(
arcsin(x*(b/a)^(1/4)), -1) - (b*x^4 - a)*sqrt(a)*(b/a)^(3/4)*elliptic_f(ar
csin(x*(b/a)^(1/4)), -1))/(a*b^2*x^4 - a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-b*x**4+a)**(3/2),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \int \frac{x^2}{(a-b\,x^4)^{3/2}}\,dx$$

input
```
int(x^2/(a - b*x^4)^(3/2),x)
```

output
```
int(x^2/(a - b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^4)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^4+a}\,x^2}{b^2x^8-2ab\,x^4+a^2}dx$$

input
```
int(x^2/(-b*x^4+a)^(3/2),x)
```

output
```
int((sqrt(a - b*x**4)*x**2)/(a**2 - 2*a*b*x**4 + b**2*x**8),x)
```

3.243 $\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 157

$$\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx = \frac{1}{2ax\sqrt{a-bx^4}}-\frac{3\sqrt{a-bx^4}}{2a^2x} - \frac{3\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{5/4}\sqrt{a-bx^4}} + \frac{3\sqrt[4]{b}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{5/4}\sqrt{a-bx^4}}$$

output
```
1/2/a/x/(-b*x^4+a)^(1/2)-3/2*(-b*x^4+a)^(1/2)/a^2/x-3/2*b^(1/4)*(1-b*x^4/a
)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^4+a)^(1/2)+3/2*b^(1/4
)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1/4)*x/a^(1/4),I)/a^(5/4)/(-b*x^4+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},\frac{bx^4}{a}\right)}{ax\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^2*(a - b*x^4)^(3/2)),x]
```

output
```
-((Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, (b*x^4)/a])/(a*x*
Sqrt[a - b*x^4]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {819, 847, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{3\int \frac{1}{x^2\sqrt{a-bx^4}}dx}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3\left(-\frac{b\int \frac{x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{836}$$

$$\frac{3\left(-\frac{b\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 765$$

$$\frac{3\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 762$$

$$\frac{3\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \ 1390$$

$$\frac{3\left(-\frac{b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{3\left(-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{3\left(-\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a-bx^4}}$$

input
```
Int[1/(x^2*(a - b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x*Sqrt[a - b*x^4]) + (3*(-(Sqrt[a - b*x^4]/(a*x)) - (b*((a^(3/4)*Sq
rt[1 - (b*x^4)/a]*EllipticE[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqr
t[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/
a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4])))/a))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83

method	result
default	$\frac{b\,x^{3}}{2a^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{-b\,x^{4}+a}}{a^{2}x}+\frac{3\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
elliptic	$\frac{b\,x^{3}}{2a^{2}\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{-b\,x^{4}+a}}{a^{2}x}+\frac{3\sqrt{b}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2a^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}$
risch	$-\frac{\sqrt{-b\,x^{4}+a}}{a^{2}x}-\frac{b^{2}\left(-\frac{\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{b^{\frac{3}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^{4}+a}}+\frac{a\left(-\frac{x^{3}}{2a\sqrt{-\left(x^{4}-\frac{a}{b}\right)b}}-\frac{\sqrt{1-\frac{\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\cdots}}{\cdots}\right.}{\cdots}\right.}{a^{2}}$

input
```
int(1/x^2/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*b/a^2*x^3/(-(x^4-a/b)*b)^(1/2)-(-b*x^4+a)^(1/2)/a^2/x+3/2*b^(1/2)/a^(3
/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)
-EllipticE(x*(1/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^2 (a - bx^4)^{3/2}} \, dx =$$

$$-\frac{3 (bx^5 - ax)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}} E(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right) | -1) - 3 (bx^5 - ax)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}} F(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right) | -1) + (3\, bx^4 -}{2\,(a^2bx^5 - a^3x)}$$

input
```
integrate(1/x^2/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*(b*x^5 - a*x)*sqrt(a)*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a)^(1/4))
, -1) - 3*(b*x^5 - a*x)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcsin(x*(b/a)^(1/4
)), -1) + (3*b*x^4 - 2*a)*sqrt(-b*x^4 + a))/(a^2*b*x^5 - a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2 (a - bx^4)^{3/2}} \, dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix} \middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}} x \Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(-b*x**4+a)**(3/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(
3/2)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2(a-bx^4)^{3/2}}\,dx = -\frac{\left(1-\frac{a}{b\,x^4}\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ \frac{a}{b\,x^4}\right)}{7\,x\,(a-b\,x^4)^{3/2}}$$

input
```
int(1/(x^2*(a - b*x^4)^(3/2)),x)
```

output
```
-((1 - a/(b*x^4))^(3/2)*hypergeom([3/2, 7/4], 11/4, a/(b*x^4)))/(7*x*(a -
b*x^4)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x^2 (a-bx^4)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^2x^{10}-2ab\,x^6+a^2x^2}dx$$

input
```
int(1/x^2/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2*x**2 - 2*a*b*x**6 + b**2*x**10),x)
```

3.244 $\int \frac{1}{x^6(a-bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 180

$$\int \frac{1}{x^6(a-bx^4)^{3/2}}\,dx = \frac{1}{2ax^5\sqrt{a-bx^4}} - \frac{7\sqrt{a-bx^4}}{10a^2x^5} - \frac{21b\sqrt{a-bx^4}}{10a^3x} - \frac{21b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{10a^{9/4}\sqrt{a-bx^4}} + \frac{21b^{5/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{10a^{9/4}\sqrt{a-bx^4}}$$

output
```
1/2/a/x^5/(-b*x^4+a)^(1/2)-7/10*(-b*x^4+a)^(1/2)/a^2/x^5-21/10*b*(-b*x^4+a
)^(1/2)/a^3/x-21/10*b^(5/4)*(1-b*x^4/a)^(1/2)*EllipticE(b^(1/4)*x/a^(1/4),
I)/a^(9/4)/(-b*x^4+a)^(1/2)+21/10*b^(5/4)*(1-b*x^4/a)^(1/2)*EllipticF(b^(1
/4)*x/a^(1/4),I)/a^(9/4)/(-b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx = -\frac{\sqrt{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},\frac{bx^4}{a}\right)}{5ax^5\sqrt{a-bx^4}}$$

input
```
Integrate[1/(x^6*(a - b*x^4)^(3/2)),x]
```

output
```
-1/5*(Sqrt[1 - (b*x^4)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, (b*x^4)/a])/(
a*x^5*Sqrt[a - b*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {819, 847, 847, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{7\int \frac{1}{x^6\sqrt{a-bx^4}}dx}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{7\left(\frac{3b\int \frac{1}{x^2\sqrt{a-bx^4}}dx}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{7\left(\frac{3b\left(-\frac{b\int \frac{x^2}{\sqrt{a-bx^4}}dx}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 836$$

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{\sqrt{a}\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a}\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{\int \frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{1}{\sqrt{a-bx^4}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 765$$

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{1}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$\Big\downarrow$ 762

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{a-bx^4}}dx}{\sqrt{b}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$\Big\downarrow$ 1390

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{b}x^2+\sqrt{a}}{\sqrt{1-\frac{bx^4}{a}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 1389$$

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{\sqrt{a}\sqrt{1-\frac{bx^4}{a}}\int\frac{\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}}dx}{\sqrt{b}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

$$\Big\downarrow \; 327$$

$$\frac{7\left(\frac{3b\left(-\frac{b\left(\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^4}}-\frac{a^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{b^{3/4}\sqrt{a-bx^4}}\right)}{a}-\frac{\sqrt{a-bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a-bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a-bx^4}}$$

input
```
Int[1/(x^6*(a - b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x^5*Sqrt[a - b*x^4]) + (7*(-1/5*Sqrt[a - b*x^4]/(a*x^5) + (3*b*(-(S
qrt[a - b*x^4]/(a*x)) - (b*((a^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticE[ArcSin[
(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*x^4]) - (a^(3/4)*Sqrt[1 - (
b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(b^(3/4)*Sqrt[a - b*
x^4])))/a))/(5*a)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{-b\,x^4+a}}{5a^2x^5}-\frac{8b\sqrt{-b\,x^4+a}}{5a^3x}+\frac{b^2x^3}{2a^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{21b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right.\right.}{10a^{\frac{5}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
elliptic	$-\frac{\sqrt{-b\,x^4+a}}{5a^2x^5}-\frac{8b\sqrt{-b\,x^4+a}}{5a^3x}+\frac{b^2x^3}{2a^3\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\frac{21b^{\frac{3}{2}}\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right.\right.}{10a^{\frac{5}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}}$
risch	$-\frac{\sqrt{-b\,x^4+a}\,(8b\,x^4+a)}{5a^3x^5}-\frac{b^2\left(-\frac{8\sqrt{a}\,\sqrt{1-\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\,\sqrt{-b\,x^4+a}\,\sqrt{b}}+5a\left(-\frac{x^3}{2a\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\right.\right.}{5a^3}$

input
```
int(1/x^6/(-b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/2)/a^2/x^5-8/5*b*(-b*x^4+a)^(1/2)/a^3/x+1/2*b^2/a^3*x^3
/(-(x^4-a/b)*b)^(1/2)+21/10*b^(3/2)/a^(5/2)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-b
^(1/2)*x^2/a^(1/2))^(1/2)*(1+b^(1/2)*x^2/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*(
EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*b^(1/2))^(
1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^6\left(a-bx^4\right)^{3/2}}\,dx =$$
$$-\frac{21\left(b^2x^9-abx^5\right)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-21\left(b^2x^9-abx^5\right)\sqrt{a}\left(\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)+}{10\left(a^3bx^9-a^4x^5\right)}$$

input
```
integrate(1/x^6/(-b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(21*(b^2*x^9 - a*b*x^5)*sqrt(a)*(b/a)^(3/4)*elliptic_e(arcsin(x*(b/a
)^(1/4)), -1) - 21*(b^2*x^9 - a*b*x^5)*sqrt(a)*(b/a)^(3/4)*elliptic_f(arcs
in(x*(b/a)^(1/4)), -1) + (21*b^2*x^8 - 14*a*b*x^4 - 2*a^2)*sqrt(-b*x^4 + a
))/(a^3*b*x^9 - a^4*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{2}}x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(-b*x**4+a)**(3/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**
(3/2)*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{2}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx = \int \frac{1}{x^6 (a-b\,x^4)^{3/2}}\,dx$$

input
```
int(1/(x^6*(a - b*x^4)^(3/2)),x)
```

output
```
int(1/(x^6*(a - b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a-bx^4)^{3/2}}\,dx = \int \frac{\sqrt{-b\,x^4+a}}{b^2x^{14}-2ab\,x^{10}+a^2x^6}dx$$

input
```
int(1/x^6/(-b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a - b*x**4)/(a**2*x**6 - 2*a*b*x**10 + b**2*x**14),x)
```

3.245 $\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\sqrt{1-x^4}+\frac{1}{3}\left(1-x^4\right)^{3/2}-\frac{1}{10}\left(1-x^4\right)^{5/2}$$

output
```
-1/2*(-x^4+1)^(1/2)+1/3*(-x^4+1)^(3/2)-1/10*(-x^4+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = \frac{1}{30}\sqrt{1-x^4}\left(-8-4x^4-3x^8\right)$$

input
```
Integrate[x^11/Sqrt[1 - x^4],x]
```

output
```
(Sqrt[1 - x^4]*(-8 - 4*x^4 - 3*x^8))/30
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{x^8}{\sqrt{1-x^4}}dx^4$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{4}\int \left(\left(1-x^4\right)^{3/2}-2\sqrt{1-x^4}+\frac{1}{\sqrt{1-x^4}}\right)dx^4$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(-\frac{2}{5}\left(1-x^4\right)^{5/2}+\frac{4}{3}\left(1-x^4\right)^{3/2}-2\sqrt{1-x^4}\right)$$

input
```
Int[x^11/Sqrt[1 - x^4],x]
```

output
```
(-2*Sqrt[1 - x^4] + (4*(1 - x^4)^(3/2))/3 - (2*(1 - x^4)^(5/2))/5)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\left(-\frac{1}{10}x^{8}-\frac{2}{15}x^{4}-\frac{4}{15}\right)\sqrt{-x^{4}+1}$	23
pseudoelliptic	$-\frac{\sqrt{-x^{4}+1}\,\left(3x^{8}+4x^{4}+8\right)}{30}$	24
risch	$\frac{\left(3x^{8}+4x^{4}+8\right)\left(x^{4}-1\right)}{30\sqrt{-x^{4}+1}}$	29
default	$\frac{\left(x^{2}+1\right)\left(x^{2}-1\right)\left(3x^{8}+4x^{4}+8\right)}{30\sqrt{-x^{4}+1}}$	34
elliptic	$\frac{\left(x^{2}+1\right)\left(x^{2}-1\right)\left(3x^{8}+4x^{4}+8\right)}{30\sqrt{-x^{4}+1}}$	34
gosper	$\frac{(-1+x)(1+x)\left(x^{2}+1\right)\left(3x^{8}+4x^{4}+8\right)}{30\sqrt{-x^{4}+1}}$	35
orering	$\frac{(-1+x)(1+x)\left(x^{2}+1\right)\left(3x^{8}+4x^{4}+8\right)}{30\sqrt{-x^{4}+1}}$	35
meijerg	$-\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,\left(6x^{8}+8x^{4}+16\right)\sqrt{-x^{4}+1}}{15}}{4\sqrt{\pi}}$	38

input
```
int(x^11/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/10*x^8-2/15*x^4-4/15)*(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = -\frac{1}{30}\left(3\,x^8+4\,x^4+8\right)\sqrt{-x^4+1}$$

input
```
integrate(x^11/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(3*x^8 + 4*x^4 + 8)*sqrt(-x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = -\frac{x^8\sqrt{1-x^4}}{10}-\frac{2x^4\sqrt{1-x^4}}{15}-\frac{4\sqrt{1-x^4}}{15}$$

input
```
integrate(x**11/(-x**4+1)**(1/2),x)
```

output
```
-x**8*sqrt(1 - x**4)/10 - 2*x**4*sqrt(1 - x**4)/15 - 4*sqrt(1 - x**4)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = -\frac{1}{10}\left(-x^4+1\right)^{\frac{5}{2}}+\frac{1}{3}\left(-x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{-x^4+1}$$

input
```
integrate(x^11/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/10*(-x^4 + 1)^(5/2) + 1/3*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = -\frac{1}{10}\left(x^4-1\right)^2\sqrt{-x^4+1}+\frac{1}{3}\left(-x^4+1\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{-x^4+1}$$

input
```
integrate(x^11/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/10*(x^4 - 1)^2*sqrt(-x^4 + 1) + 1/3*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 +
1)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = -\sqrt{1-x^4}\left(\frac{x^8}{10}+\frac{2\,x^4}{15}+\frac{4}{15}\right)$$

input
```
int(x^11/(1 - x^4)^(1/2),x)
```

output
```
-(1 - x^4)^(1/2)*((2*x^4)/15 + x^8/10 + 4/15)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^{11}}{\sqrt{1-x^4}}\,dx = \frac{\sqrt{-x^4+1}\,(-3x^8-4x^4-8)}{30}$$

input
```
int(x^11/(-x^4+1)^(1/2),x)
```

output
```
(sqrt( - x**4 + 1)*( - 3*x**8 - 4*x**4 - 8))/30
```

3.246 $\int \frac{x^7}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\sqrt{1-x^4}+\frac{1}{6}\left(1-x^4\right)^{3/2}$$

output
```
-1/2*(-x^4+1)^(1/2)+1/6*(-x^4+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = \frac{1}{6}\left(-2-x^4\right)\sqrt{1-x^4}$$

input
```
Integrate[x^7/Sqrt[1 - x^4],x]
```

output
```
((-2 - x^4)*Sqrt[1 - x^4])/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{1-x^4}} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt{1-x^4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{1}{\sqrt{1-x^4}}-\sqrt{1-x^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2}{3}\left(1-x^4\right)^{3/2}-2\sqrt{1-x^4}\right)$$

input
```
Int[x^7/Sqrt[1 - x^4],x]
```

output
```
(-2*Sqrt[1 - x^4] + (2*(1 - x^4)^(3/2))/3)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

method	result	size
pseudoelliptic	$-\frac{\sqrt{-x^{4}+1}\,\left(x^{4}+2\right)}{6}$	17
trager	$\left(-\frac{x^{4}}{6}-\frac{1}{3}\right)\sqrt{-x^{4}+1}$	18
risch	$\frac{\left(x^{4}+2\right)\left(x^{4}-1\right)}{6\sqrt{-x^{4}+1}}$	22
default	$\frac{\left(x^{2}+1\right)\left(x^{2}-1\right)\left(x^{4}+2\right)}{6\sqrt{-x^{4}+1}}$	27
elliptic	$\frac{\left(x^{2}+1\right)\left(x^{2}-1\right)\left(x^{4}+2\right)}{6\sqrt{-x^{4}+1}}$	27
gosper	$\frac{\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)\left(x^{4}+2\right)}{6\sqrt{-x^{4}+1}}$	28
orering	$\frac{\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)\left(x^{4}+2\right)}{6\sqrt{-x^{4}+1}}$	28
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,\left(4x^{4}+8\right)\sqrt{-x^{4}+1}}{6}}{4\sqrt{\pi}}$	33

input
```
int(x^7/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-x^4+1)^(1/2)*(x^4+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = -\frac{1}{6}\left(x^4+2\right)\sqrt{-x^4+1}$$

input
```
integrate(x^7/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(x^4 + 2)*sqrt(-x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = -\frac{x^4\sqrt{1-x^4}}{6} - \frac{\sqrt{1-x^4}}{3}$$

input
```
integrate(x**7/(-x**4+1)**(1/2),x)
```

output
```
-x**4*sqrt(1 - x**4)/6 - sqrt(1 - x**4)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = \frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-x^4+1}$$

input
```
integrate(x^7/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = \frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-x^4+1}$$

input
```
integrate(x^7/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/6*(-x^4 + 1)^(3/2) - 1/2*sqrt(-x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}\,(x^4+2)}{6}$$

input
```
int(x^7/(1 - x^4)^(1/2),x)
```

output
```
-((1 - x^4)^(1/2)*(x^4 + 2))/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x^7}{\sqrt{1-x^4}}\,dx = \frac{\sqrt{-x^4+1}\,(-x^4-2)}{6}$$

input
```
int(x^7/(-x^4+1)^(1/2),x)
```

output
```
(sqrt( - x**4 + 1)*( - x**4 - 2))/6
```

3.247 $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\sqrt{1-x^4}$$

output
```
-1/2*(-x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\sqrt{1-x^4}$$

input
```
Integrate[x^3/Sqrt[1 - x^4],x]
```

output
```
-1/2*Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{1}{2}\sqrt{1-x^4}$$

input
```
Int[x^3/Sqrt[1 - x^4],x]
```

output
```
-1/2*Sqrt[1 - x^4]
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{-x^4+1}}{2}$	12
default	$-\frac{\sqrt{-x^4+1}}{2}$	12
trager	$-\frac{\sqrt{-x^4+1}}{2}$	12
pseudoelliptic	$-\frac{\sqrt{-x^4+1}}{2}$	12
risch	$\frac{x^4-1}{2\sqrt{-x^4+1}}$	17
elliptic	$\frac{(x^2+1)(x^2-1)}{2\sqrt{-x^4+1}}$	22
gosper	$\frac{(-1+x)(1+x)(x^2+1)}{2\sqrt{-x^4+1}}$	23
orering	$\frac{(-1+x)(1+x)(x^2+1)}{2\sqrt{-x^4+1}}$	23
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^4+1}}{4\sqrt{\pi}}$	26

input
```
int(x^3/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\sqrt{-x^4+1}$$

input
```
integrate(x^3/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{2}$$

input
```
integrate(x**3/(-x**4+1)**(1/2),x)
```

output
```
-sqrt(1 - x**4)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\,\sqrt{-x^4+1}$$

input
```
integrate(x^3/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\,\sqrt{-x^4+1}$$

input
```
integrate(x^3/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{2}$$

input
```
int(x^3/(1 - x^4)^(1/2),x)
```

output
```
-(1 - x^4)^(1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{2}$$

input
```
int(x^3/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1))/2
```

3.248 $\int \frac{1}{x\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = -\frac{1}{2}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

output
```
-1/2*arctanh((-x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = -\frac{1}{2}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

input
```
Integrate[1/(x*Sqrt[1 - x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[1 - x^4]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{1-x^4}}dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{1}{4}\int \frac{1}{x^4\sqrt{1-x^4}}dx^4$$
$$\Big\downarrow \text{ 73}$$
$$-\frac{1}{2}\int \frac{1}{1-x^8}d\sqrt{1-x^4}$$
$$\Big\downarrow \text{ 219}$$
$$-\frac{1}{2}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

input
```
Int[1/(x*Sqrt[1 - x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[1 - x^4]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}$	13
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}$	13
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}$	13
trager	$-\frac{\ln\left(\frac{1+\sqrt{-x^{4}+1}}{x^{2}}\right)}{2}$	19
meijerg	$\frac{(-2\ln(2)+4\ln(x)+i\pi)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{4}+1}}{2}\right)}{4\sqrt{\pi}}$	43

input
```
int(1/x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/(-x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{-x^4+1}-1\right)$$

input
```
integrate(1/x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(sqrt(-x^4 + 1) - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^2}\right)}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-acosh(x**(-2))/2, 1/Abs(x**4) > 1), (I*asin(x**(-2))/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{-x^4+1}-1\right)$$

input
```
integrate(1/x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(sqrt(-x^4 + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right)+\frac{1}{4}\log\left(-\sqrt{-x^4+1}+1\right)$$

input
```
integrate(1/x/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(-sqrt(-x^4 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = -\frac{\operatorname{atanh}\left(\sqrt{1-x^4}\right)}{2}$$

input
```
int(1/(x*(1 - x^4)^(1/2)),x)
```

output
```
-atanh((1 - x^4)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\sqrt{1-x^4}}\,dx = \frac{\log\left(\tan\left(\frac{asin(x^2)}{2}\right)\right)}{2}$$

input
```
int(1/x/(-x^4+1)^(1/2),x)
```

output
```
log(tan(asin(x**2)/2))/2
```

3.249 $\int \frac{1}{x^5\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{4x^4} - \frac{1}{4}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

output
```
-1/4*(-x^4+1)^(1/2)/x^4-1/4*arctanh((-x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{4x^4} - \frac{1}{4}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

input
```
Integrate[1/(x^5*Sqrt[1 - x^4]),x]
```

output
```
-1/4*Sqrt[1 - x^4]/x^4 - ArcTanh[Sqrt[1 - x^4]]/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8\sqrt{1-x^4}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^4\sqrt{1-x^4}}dx^4-\frac{\sqrt{1-x^4}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\int \frac{1}{1-x^8}d\sqrt{1-x^4}-\frac{\sqrt{1-x^4}}{x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\text{arctanh}\left(\sqrt{1-x^4}\right)-\frac{\sqrt{1-x^4}}{x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[1 - x^4]),x]
```

output
```
(-(Sqrt[1 - x^4]/x^4) - ArcTanh[Sqrt[1 - x^4]])/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\sqrt{-x^4+1}}{4x^4}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}$	28
elliptic	$-\frac{\sqrt{-x^4+1}}{4x^4}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}$	28
risch	$\frac{x^4-1}{4x^4\sqrt{-x^4+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}$	33
pseudoelliptic	$\frac{-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)x^4-\sqrt{-x^4+1}}{4x^4}$	33
trager	$-\frac{\sqrt{-x^4+1}}{4x^4}-\frac{\ln\left(\frac{1+\sqrt{-x^4+1}}{x^2}\right)}{4}$	34
meijerg	$-\frac{\frac{\sqrt{\pi}}{x^4}-\frac{(1-2\ln(2)+4\ln(x)+i\pi)\sqrt{\pi}}{2}-\frac{\sqrt{\pi}\,(-4x^4+8)}{8x^4}+\frac{\sqrt{\pi}\,\sqrt{-x^4+1}}{x^4}+\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^4+1}}{2}\right)}{4\sqrt{\pi}}$	82

input
```
int(1/x^5/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-x^4+1)^(1/2)/x^4-1/4*arctanh(1/(-x^4+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = -\frac{x^4\log\left(\sqrt{-x^4+1}+1\right)-x^4\log\left(\sqrt{-x^4+1}-1\right)+2\sqrt{-x^4+1}}{8\,x^4}$$

input
```
integrate(1/x^5/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(x^4*log(sqrt(-x^4 + 1) + 1) - x^4*log(sqrt(-x^4 + 1) - 1) + 2*sqrt(-
x^4 + 1))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x^2}\right)}{4} + \frac{1}{4x^2\sqrt{-1+\frac{1}{x^4}}} - \frac{1}{4x^6\sqrt{-1+\frac{1}{x^4}}} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{1}{x^2}\right)}{4} - \frac{i\sqrt{1-\frac{1}{x^4}}}{4x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-acosh(x**(-2))/4 + 1/(4*x**2*sqrt(-1 + x**(-4))) - 1/(4*x**6*s
qrt(-1 + x**(-4))), 1/Abs(x**4) > 1), (I*asin(x**(-2))/4 - I*sqrt(1 - 1/x*
*4)/(4*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{4\,x^4} - \frac{1}{8}\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{8}\log\left(\sqrt{-x^4+1}-1\right)$$

input
```
integrate(1/x^5/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-x^4 + 1)/x^4 - 1/8*log(sqrt(-x^4 + 1) + 1) + 1/8*log(sqrt(-x^4
+ 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{4\,x^4} - \frac{1}{8}\,\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{8}\,\log\left(-\sqrt{-x^4+1}+1\right)$$

input
```
integrate(1/x^5/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(-x^4 + 1)/x^4 - 1/8*log(sqrt(-x^4 + 1) + 1) + 1/8*log(-sqrt(-x^4
 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = -\frac{\mathrm{atanh}\left(\sqrt{1-x^4}\right)}{4} - \frac{\sqrt{1-x^4}}{4\,x^4}$$

input
```
int(1/(x^5*(1 - x^4)^(1/2)),x)
```

output
```
- atanh((1 - x^4)^(1/2))/4 - (1 - x^4)^(1/2)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5\sqrt{1-x^4}}\,dx = \frac{-\sqrt{-x^4+1}+\log\left(\tan\left(\frac{asin(x^2)}{2}\right)\right)x^4}{4x^4}$$

input
```
int(1/x^5/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1) + log(tan(asin(x**2)/2))*x**4)/(4*x**4)
```

3.250 $\int \frac{x^9}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^9}{\sqrt{1-x^4}} \, dx = -\frac{3}{16}x^2\sqrt{1-x^4} - \frac{1}{8}x^6\sqrt{1-x^4} + \frac{3\arcsin\left(x^2\right)}{16}$$

output
```
-3/16*x^2*(-x^4+1)^(1/2)-1/8*x^6*(-x^4+1)^(1/2)+3/16*arcsin(x^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{x^9}{\sqrt{1-x^4}} \, dx = \frac{1}{16}\left(-x^2\sqrt{1-x^4}\left(3+2x^4\right) + 3\arcsin\left(x^2\right)\right)$$

input
```
Integrate[x^9/Sqrt[1 - x^4],x]
```

output
```
(-(x^2*Sqrt[1 - x^4]*(3 + 2*x^4)) + 3*ArcSin[x^2])/16
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {807, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\sqrt{1-x^4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^8}{\sqrt{1-x^4}} dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{3}{4} \int \frac{x^4}{\sqrt{1-x^4}} dx^2 - \frac{1}{4} x^6 \sqrt{1-x^4} \right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{3}{4} \left(\frac{1}{2} \int \frac{1}{\sqrt{1-x^4}} dx^2 - \frac{1}{2} x^2 \sqrt{1-x^4} \right) - \frac{1}{4} x^6 \sqrt{1-x^4} \right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2} \left(\frac{3}{4} \left(\frac{\arcsin(x^2)}{2} - \frac{1}{2} x^2 \sqrt{1-x^4} \right) - \frac{1}{4} x^6 \sqrt{1-x^4} \right)$$

input
```
Int[x^9/Sqrt[1 - x^4],x]
```

output
```
(-1/4*(x^6*Sqrt[1 - x^4]) + (3*(-1/2*(x^2*Sqrt[1 - x^4]) + ArcSin[x^2]/2))
/4)/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\left(-2 x^{6}-3 x^{2}\right) \sqrt{-x^{4}+1}}{16}+\frac{3 \arcsin \left(x^{2}\right)}{16}$	30
risch	$\frac{x^{2} \left(2 x^{4}+3\right) \left(x^{4}-1\right)}{16 \sqrt{-x^{4}+1}}+\frac{3 \arcsin \left(x^{2}\right)}{16}$	34
default	$-\frac{3 x^{2} \sqrt{-x^{4}+1}}{16}-\frac{x^{6} \sqrt{-x^{4}+1}}{8}+\frac{3 \arcsin \left(x^{2}\right)}{16}$	36
elliptic	$-\frac{3 x^{2} \sqrt{-x^{4}+1}}{16}-\frac{x^{6} \sqrt{-x^{4}+1}}{8}+\frac{3 \arcsin \left(x^{2}\right)}{16}$	36
meijerg	$-\frac{i \left(-\frac{i \sqrt{\pi}\, x^{2} \left(10 x^{4}+15\right) \sqrt{-x^{4}+1}}{20}+\frac{3 i \sqrt{\pi}\, \arcsin \left(x^{2}\right)}{4}\right)}{4 \sqrt{\pi}}$	43
trager	$-\frac{x^{2} \left(2 x^{4}+3\right) \sqrt{-x^{4}+1}}{16}+\frac{3 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln \left(\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-x^{4}+1}+x^{2}\right)}{16}$	52

input
```
int(x^9/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-2*x^6-3*x^2)*(-x^4+1)^(1/2)+3/16*arcsin(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^9}{\sqrt{1-x^4}}\,dx = -\frac{1}{16}\left(2\,x^6+3\,x^2\right)\sqrt{-x^4+1}-\frac{3}{8}\,\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

input
```
integrate(x^9/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/16*(2*x^6 + 3*x^2)*sqrt(-x^4 + 1) - 3/8*arctan((sqrt(-x^4 + 1) - 1)/x^2
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.44

$$\int \frac{x^9}{\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{ix^{10}}{8\sqrt{x^4-1}}-\frac{ix^6}{16\sqrt{x^4-1}}+\frac{3ix^2}{16\sqrt{x^4-1}}-\frac{3i\,\mathrm{acosh}\,(x^2)}{16} & \text{for } |x^4|>1 \\ \frac{x^{10}}{8\sqrt{1-x^4}}+\frac{x^6}{16\sqrt{1-x^4}}-\frac{3x^2}{16\sqrt{1-x^4}}+\frac{3\,\mathrm{asin}\,(x^2)}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**9/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-I*x**10/(8*sqrt(x**4 - 1)) - I*x**6/(16*sqrt(x**4 - 1)) + 3*I*
x**2/(16*sqrt(x**4 - 1)) - 3*I*acosh(x**2)/16, Abs(x**4) > 1), (x**10/(8*s
qrt(1 - x**4)) + x**6/(16*sqrt(1 - x**4)) - 3*x**2/(16*sqrt(1 - x**4)) + 3
*asin(x**2)/16, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(35) = 70.

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \frac{x^9}{\sqrt{1-x^4}}\,dx = \frac{\frac{5\sqrt{-x^4+1}}{x^2}+\frac{3\left(-x^4+1\right)^{\frac{3}{2}}}{x^6}}{16\left(\frac{2\left(x^4-1\right)}{x^4}-\frac{\left(x^4-1\right)^2}{x^8}-1\right)}-\frac{3}{16}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(x^9/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(5*sqrt(-x^4 + 1)/x^2 + 3*(-x^4 + 1)^(3/2)/x^6)/(2*(x^4 - 1)/x^4 - (x
^4 - 1)^2/x^8 - 1) - 3/16*arctan(sqrt(-x^4 + 1)/x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.62

$$\int \frac{x^9}{\sqrt{1-x^4}}\,dx = -\frac{1}{16}\left(2\,x^4+3\right)\sqrt{-x^4+1}x^2+\frac{3}{16}\arcsin\left(x^2\right)$$

input
```
integrate(x^9/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/16*(2*x^4 + 3)*sqrt(-x^4 + 1)*x^2 + 3/16*arcsin(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\sqrt{1-x^4}}\,dx = \int \frac{x^9}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^9/(1 - x^4)^(1/2),x)
```

output
```
int(x^9/(1 - x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{\sqrt{1-x^4}}\,dx = \frac{3asin(x^2)}{16} - \frac{\sqrt{-x^4+1}\,x^6}{8} - \frac{3\sqrt{-x^4+1}\,x^2}{16}$$

input
```
int(x^9/(-x^4+1)^(1/2),x)
```

output
```
(3*asin(x**2) - 2*sqrt( - x**4 + 1)*x**6 - 3*sqrt( - x**4 + 1)*x**2)/16
```

3.251 $\int \frac{x^5}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx=-\frac{1}{4}x^2\sqrt{1-x^4}+\frac{\arcsin\left(x^2\right)}{4}$$

output
```
-1/4*x^2*(-x^4+1)^(1/2)+1/4*arcsin(x^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx=-\frac{1}{4}x^2\sqrt{1-x^4}+\frac{\arcsin\left(x^2\right)}{4}$$

input
```
Integrate[x^5/Sqrt[1 - x^4],x]
```

output
```
-1/4*(x^2*Sqrt[1 - x^4]) + ArcSin[x^2]/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {807, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{1-x^4}}dx^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}dx^2-\frac{1}{2}x^2\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2}\left(\frac{\arcsin\left(x^2\right)}{2}-\frac{1}{2}x^2\sqrt{1-x^4}\right)$$

input
```
Int[x^5/Sqrt[1 - x^4],x]
```

output
```
(-1/2*(x^2*Sqrt[1 - x^4]) + ArcSin[x^2]/2)/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x^{2}\sqrt{-x^{4}+1}}{4}+\frac{\arcsin\left(x^{2}\right)}{4}$	22
elliptic	$-\frac{x^{2}\sqrt{-x^{4}+1}}{4}+\frac{\arcsin\left(x^{2}\right)}{4}$	22
pseudoelliptic	$-\frac{x^{2}\sqrt{-x^{4}+1}}{4}+\frac{\arcsin\left(x^{2}\right)}{4}$	22
risch	$\frac{x^{2}\left(x^{4}-1\right)}{4\sqrt{-x^{4}+1}}+\frac{\arcsin\left(x^{2}\right)}{4}$	27
meijerg	$\frac{i\left(i\sqrt{\pi}\, x^{2}\sqrt{-x^{4}+1}-i\sqrt{\pi}\, \arcsin\left(x^{2}\right)\right)}{4\sqrt{\pi}}$	36
trager	$-\frac{x^{2}\sqrt{-x^{4}+1}}{4}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{4}+1}+x^{2}\right)}{4}$	45

input
```
int(x^5/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*(-x^4+1)^(1/2)+1/4*arcsin(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx = -\frac{1}{4}\sqrt{-x^4+1}x^2 - \frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

input
```
integrate(x^5/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-x^4 + 1)*x^2 - 1/2*arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{ix^2\sqrt{x^4-1}}{4} - \frac{i\operatorname{acosh}(x^2)}{4} & \text{for } |x^4| > 1 \\ \frac{x^6}{4\sqrt{1-x^4}} - \frac{x^2}{4\sqrt{1-x^4}} + \frac{\operatorname{asin}(x^2)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-I*x**2*sqrt(x**4 - 1)/4 - I*acosh(x**2)/4, Abs(x**4) > 1), (x*
*6/(4*sqrt(1 - x**4)) - x**2/(4*sqrt(1 - x**4)) + asin(x**2)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(21) = 42$.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx = \frac{\sqrt{-x^4+1}}{4\,x^2\left(\frac{x^4-1}{x^4}-1\right)} - \frac{1}{4}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(x^5/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(-x^4 + 1)/(x^2*((x^4 - 1)/x^4 - 1)) - 1/4*arctan(sqrt(-x^4 + 1)/x
^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx = -\frac{1}{4}\sqrt{-x^4+1}x^2 + \frac{1}{4}\arcsin\left(x^2\right)$$

input
```
integrate(x^5/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(-x^4 + 1)*x^2 + 1/4*arcsin(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx = \int \frac{x^5}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^5/(1 - x^4)^(1/2),x)
```

output
```
int(x^5/(1 - x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{1-x^4}}\,dx = \frac{asin(x^2)}{4} - \frac{\sqrt{-x^4+1}\,x^2}{4}$$

input
```
int(x^5/(-x^4+1)^(1/2),x)
```

output
```
(asin(x**2) - sqrt( - x**4 + 1)*x**2)/4
```

3.252 $\int \frac{x}{\sqrt{1-x^4}} dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{\arcsin(x^2)}{2}$$

output
```
1/2*arcsin(x^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{\arcsin(x^2)}{2}$$

input
```
Integrate[x/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x^2]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^4}}\,dx$$
$$\Big\downarrow \; 807$$
$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}dx^2$$
$$\Big\downarrow \; 223$$
$$\frac{\arcsin\left(x^2\right)}{2}$$

input
```
Int[x/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x^2]/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\arcsin(x^2)}{2}$	7
meijerg	$\frac{\arcsin(x^2)}{2}$	7
elliptic	$\frac{\arcsin(x^2)}{2}$	7
pseudoelliptic	$\frac{\arcsin(x^2)}{2}$	7
trager	$\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^4+1}+x^2\right)}{2}$	30

input
```
int(x/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

input
```
integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{i \operatorname{acosh}\left(x^{2}\right)}{2} & \text{for}\: \left|{x^{4}}\right| > 1 \\ \frac{\operatorname{asin}\left(x^{2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\arcsin\left(x^2\right)$$

input
```
integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*arcsin(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1-x^4}} \, dx = \frac{\operatorname{atan}\left(\frac{x^2}{\sqrt{1-x^4}}\right)}{2}$$

input
```
int(x/(1 - x^4)^(1/2),x)
```

output
```
atan(x^2/(1 - x^4)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{1-x^4}} \, dx = \frac{asin(x^2)}{2}$$

input
```
int(x/(-x^4+1)^(1/2),x)
```

output
```
asin(x**2)/2
```

3.253 $\int \frac{1}{x^3\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{2x^2}$$

output
```
-1/2*(-x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{2x^2}$$

input
```
Integrate[1/(x^3*Sqrt[1 - x^4]),x]
```

output
```
-1/2*Sqrt[1 - x^4]/x^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{\sqrt{1-x^4}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 - x^4]),x]
```

output
```
-1/2*Sqrt[1 - x^4]/x^2
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
trager	$-\frac{\sqrt{-x^4+1}}{2x^2}$	15
meijerg	$-\frac{\sqrt{-x^4+1}}{2x^2}$	15
pseudoelliptic	$-\frac{\sqrt{-x^4+1}}{2x^2}$	15
risch	$\frac{x^4-1}{2x^2\sqrt{-x^4+1}}$	20
default	$\frac{(x^2-1)(x^2+1)}{2x^2\sqrt{-x^4+1}}$	25
elliptic	$\frac{(x^2-1)(x^2+1)}{2x^2\sqrt{-x^4+1}}$	25
gosper	$\frac{(-1+x)(1+x)(x^2+1)}{2x^2\sqrt{-x^4+1}}$	26
orering	$\frac{(-1+x)(1+x)(x^2+1)}{2x^2\sqrt{-x^4+1}}$	26

input
```
int(1/x^3/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^4+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^4 + 1)/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{i\sqrt{x^4-1}}{2x^2} & \text{for } |x^4| > 1 \\ -\frac{\sqrt{1-x^4}}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(x**4 - 1)/(2*x**2), Abs(x**4) > 1), (-sqrt(1 - x**4)/(2
*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^4 + 1)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = \frac{x^2}{4\left(\sqrt{-x^4+1}-1\right)} - \frac{\sqrt{-x^4+1}-1}{4\,x^2}$$

input
```
integrate(1/x^3/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*x^2/(sqrt(-x^4 + 1) - 1) - 1/4*(sqrt(-x^4 + 1) - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{2\,x^2}$$

input
```
int(1/(x^3*(1 - x^4)^(1/2)),x)
```

output
```
-(1 - x^4)^(1/2)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{2x^2}$$

input
```
int(1/x^3/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1))/(2*x**2)
```

3.254 $\int \frac{1}{x^7\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{6x^6} - \frac{\sqrt{1-x^4}}{3x^2}$$

output
```
-1/6*(-x^4+1)^(1/2)/x^6-1/3*(-x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = \frac{(-1-2x^4)\sqrt{1-x^4}}{6x^6}$$

input
```
Integrate[1/(x^7*Sqrt[1 - x^4]),x]
```

output
```
((-1 - 2*x^4)*Sqrt[1 - x^4])/(6*x^6)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{1-x^4}}dx$$
$$\Big\downarrow \text{ 803}$$
$$\frac{2}{3}\int \frac{1}{x^3\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{6x^6}$$
$$\Big\downarrow \text{ 796}$$
$$-\frac{\sqrt{1-x^4}}{6x^6}-\frac{\sqrt{1-x^4}}{3x^2}$$

input
```
Int[1/(x^7*Sqrt[1 - x^4]),x]
```

output
```
-1/6*Sqrt[1 - x^4]/x^6 - Sqrt[1 - x^4]/(3*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
trager	$-\frac{(2x^4+1)\sqrt{-x^4+1}}{6x^6}$	22
meijerg	$-\frac{(2x^4+1)\sqrt{-x^4+1}}{6x^6}$	22
pseudoelliptic	$-\frac{(2x^4+1)\sqrt{-x^4+1}}{6x^6}$	22
risch	$\frac{2x^8-x^4-1}{6x^6\sqrt{-x^4+1}}$	27
default	$\frac{(x^2+1)(x^2-1)(2x^4+1)}{6x^6\sqrt{-x^4+1}}$	32
elliptic	$\frac{(x^2+1)(x^2-1)(2x^4+1)}{6x^6\sqrt{-x^4+1}}$	32
gosper	$\frac{(-1+x)(1+x)(x^2+1)(2x^4+1)}{6x^6\sqrt{-x^4+1}}$	33
orering	$\frac{(-1+x)(1+x)(x^2+1)(2x^4+1)}{6x^6\sqrt{-x^4+1}}$	33

input
```
int(1/x^7/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2*x^4+1)/x^6*(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = -\frac{(2\,x^4+1)\sqrt{-x^4+1}}{6\,x^6}$$

input
```
integrate(1/x^7/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(2*x^4 + 1)*sqrt(-x^4 + 1)/x^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{i\sqrt{x^4-1}}{3x^2} - \frac{i\sqrt{x^4-1}}{6x^6} & \text{for } |x^4| > 1 \\ -\frac{\sqrt{1-x^4}}{3x^2} - \frac{\sqrt{1-x^4}}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**7/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(x**4 - 1)/(3*x**2) - I*sqrt(x**4 - 1)/(6*x**6), Abs(x**
4) > 1), (-sqrt(1 - x**4)/(3*x**2) - sqrt(1 - x**4)/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{2\,x^2} - \frac{(-x^4+1)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate(1/x^7/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^4 + 1)/x^2 - 1/6*(-x^4 + 1)^(3/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = \frac{x^6\left(\frac{9\left(\sqrt{-x^4+1}-1\right)^2}{x^4}+1\right)}{48\left(\sqrt{-x^4+1}-1\right)^3} - \frac{3\left(\sqrt{-x^4+1}-1\right)}{16\,x^2} - \frac{\left(\sqrt{-x^4+1}-1\right)^3}{48\,x^6}$$

input
```
integrate(1/x^7/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/48*x^6*(9*(sqrt(-x^4 + 1) - 1)^2/x^4 + 1)/(sqrt(-x^4 + 1) - 1)^3 - 3/16*
(sqrt(-x^4 + 1) - 1)/x^2 - 1/48*(sqrt(-x^4 + 1) - 1)^3/x^6
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}\,(2\,x^4+1)}{6\,x^6}$$

input
```
int(1/(x^7*(1 - x^4)^(1/2)),x)
```

output
```
-((1 - x^4)^(1/2)*(2*x^4 + 1))/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^7\sqrt{1-x^4}}\,dx = \frac{\sqrt{-x^4+1}\,(-2x^4-1)}{6x^6}$$

input
```
int(1/x^7/(-x^4+1)^(1/2),x)
```

output
```
(sqrt( - x**4 + 1)*( - 2*x**4 - 1))/(6*x**6)
```

3.255 $\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{10x^{10}} - \frac{2\sqrt{1-x^4}}{15x^6} - \frac{4\sqrt{1-x^4}}{15x^2}$$

output
```
-1/10*(-x^4+1)^(1/2)/x^10-2/15*(-x^4+1)^(1/2)/x^6-4/15*(-x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = \frac{\sqrt{1-x^4}(-3-4x^4-8x^8)}{30x^{10}}$$

input
```
Integrate[1/(x^11*Sqrt[1 - x^4]),x]
```

output
```
(Sqrt[1 - x^4]*(-3 - 4*x^4 - 8*x^8))/(30*x^10)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{4}{5}\int \frac{1}{x^7\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{10x^{10}}$$

$$\Big\downarrow \text{803}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{x^3\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{6x^6}\right)-\frac{\sqrt{1-x^4}}{10x^{10}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4}{5}\left(-\frac{\sqrt{1-x^4}}{6x^6}-\frac{\sqrt{1-x^4}}{3x^2}\right)-\frac{\sqrt{1-x^4}}{10x^{10}}$$

input
```
Int[1/(x^11*Sqrt[1 - x^4]),x]
```

output
```
-1/10*Sqrt[1 - x^4]/x^10 + (4*(-1/6*Sqrt[1 - x^4]/x^6 - Sqrt[1 - x^4]/(3*x
^2)))/5
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.49

method	result	size
trager	$-\frac{\left(8 x^{8}+4 x^{4}+3\right) \sqrt{-x^{4}+1}}{30 x^{10}}$	27
meijerg	$-\frac{\left(\frac{8}{3} x^{8}+\frac{4}{3} x^{4}+1\right) \sqrt{-x^{4}+1}}{10 x^{10}}$	27
pseudoelliptic	$-\frac{\left(8 x^{8}+4 x^{4}+3\right) \sqrt{-x^{4}+1}}{30 x^{10}}$	27
risch	$\frac{8 x^{12}-4 x^{8}-x^{4}-3}{30 x^{10} \sqrt{-x^{4}+1}}$	32
default	$\frac{\left(x^{2}-1\right) \left(x^{2}+1\right) \left(8 x^{8}+4 x^{4}+3\right)}{30 x^{10} \sqrt{-x^{4}+1}}$	37
elliptic	$\frac{\left(x^{2}-1\right) \left(x^{2}+1\right) \left(8 x^{8}+4 x^{4}+3\right)}{30 x^{10} \sqrt{-x^{4}+1}}$	37
gosper	$\frac{\left(-1+x \right) \left(1+x \right) \left(x^{2}+1\right) \left(8 x^{8}+4 x^{4}+3\right)}{30 x^{10} \sqrt{-x^{4}+1}}$	38
orering	$\frac{\left(-1+x \right) \left(1+x \right) \left(x^{2}+1\right) \left(8 x^{8}+4 x^{4}+3\right)}{30 x^{10} \sqrt{-x^{4}+1}}$	38

input
```
int(1/x^11/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(8*x^8+4*x^4+3)/x^10*(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = -\frac{(8\,x^8+4\,x^4+3)\sqrt{-x^4+1}}{30\,x^{10}}$$

input
```
integrate(1/x^11/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(8*x^8 + 4*x^4 + 3)*sqrt(-x^4 + 1)/x^10
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = \begin{cases} -\frac{4\sqrt{-1+\frac{1}{x^4}}}{15} - \frac{2\sqrt{-1+\frac{1}{x^4}}}{15x^4} - \frac{\sqrt{-1+\frac{1}{x^4}}}{10x^8} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{4i\sqrt{1-\frac{1}{x^4}}}{15} - \frac{2i\sqrt{1-\frac{1}{x^4}}}{15x^4} - \frac{i\sqrt{1-\frac{1}{x^4}}}{10x^8} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**11/(-x**4+1)**(1/2),x)
```

output
```
Piecewise((-4*sqrt(-1 + x**(-4))/15 - 2*sqrt(-1 + x**(-4))/(15*x**4) - sqr
t(-1 + x**(-4))/(10*x**8), 1/Abs(x**4) > 1), (-4*I*sqrt(1 - 1/x**4)/15 - 2
*I*sqrt(1 - 1/x**4)/(15*x**4) - I*sqrt(1 - 1/x**4)/(10*x**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}}{2\,x^2} - \frac{(-x^4+1)^{\frac{3}{2}}}{3\,x^6} - \frac{(-x^4+1)^{\frac{5}{2}}}{10\,x^{10}}$$

input
```
integrate(1/x^11/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^4 + 1)/x^2 - 1/3*(-x^4 + 1)^(3/2)/x^6 - 1/10*(-x^4 + 1)^(5/2)
/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.98

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = \frac{x^{10}\left(\frac{25\left(\sqrt{-x^4+1}-1\right)^2}{x^4} + \frac{150\left(\sqrt{-x^4+1}-1\right)^4}{x^8} + 3\right)}{960\left(\sqrt{-x^4+1}-1\right)^5} - \frac{5\left(\sqrt{-x^4+1}-1\right)}{32\,x^2} - \frac{5\left(\sqrt{-x^4+1}-1\right)^3}{192\,x^6} - \frac{\left(\sqrt{-x^4+1}-1\right)^5}{320\,x^{10}}$$

input
```
integrate(1/x^11/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/960*x^10*(25*(sqrt(-x^4 + 1) - 1)^2/x^4 + 150*(sqrt(-x^4 + 1) - 1)^4/x^8
 + 3)/(sqrt(-x^4 + 1) - 1)^5 - 5/32*(sqrt(-x^4 + 1) - 1)/x^2 - 5/192*(sqrt
(-x^4 + 1) - 1)^3/x^6 - 1/320*(sqrt(-x^4 + 1) - 1)^5/x^10
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}\,(8\,x^8+4\,x^4+3)}{30\,x^{10}}$$

input
```
int(1/(x^11*(1 - x^4)^(1/2)),x)
```

output
```
-((1 - x^4)^(1/2)*(4*x^4 + 8*x^8 + 3))/(30*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^{11}\sqrt{1-x^4}}\,dx = \frac{\sqrt{-x^4+1}\,(-8x^8-4x^4-3)}{30x^{10}}$$

input
```
int(1/x^11/(-x^4+1)^(1/2),x)
```

output
```
(sqrt( - x**4 + 1)*( - 8*x**8 - 4*x**4 - 3))/(30*x**10)
```

3.256 $\int \frac{x^8}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx = -\frac{5}{21}x\sqrt{1-x^4}-\frac{1}{7}x^5\sqrt{1-x^4}+\frac{5}{21}\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
-5/21*x*(-x^4+1)^(1/2)-1/7*x^5*(-x^4+1)^(1/2)+5/21*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx = \frac{1}{21}\left(-x\sqrt{1-x^4}(5+3x^4)+5x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)$$

input
```
Integrate[x^8/Sqrt[1 - x^4],x]
```

output
```
(-(x*Sqrt[1 - x^4]*(5 + 3*x^4)) + 5*x*Hypergeometric2F1[1/4, 1/2, 5/4, x^4
])/21
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {843, 843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5}{7}\int \frac{x^4}{\sqrt{1-x^4}}dx-\frac{1}{7}x^5\sqrt{1-x^4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}dx-\frac{1}{3}x\sqrt{1-x^4}\right)-\frac{1}{7}x^5\sqrt{1-x^4}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{5}{7}\left(\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)-\frac{1}{3}x\sqrt{1-x^4}\right)-\frac{1}{7}x^5\sqrt{1-x^4}$$

input
```
Int[x^8/Sqrt[1 - x^4],x]
```

output
```
-1/7*(x^5*Sqrt[1 - x^4]) + (5*(-1/3*(x*Sqrt[1 - x^4]) + EllipticF[ArcSin[x
], -1]/3))/7
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.35

method	result	size
meijerg	$\frac{x^9 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{9}{4}\right],\left[\frac{13}{4}\right],x^4\right)}{9}$	15
risch	$\frac{x(3x^4+5)(x^4-1)}{21\sqrt{-x^4+1}}+\frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\operatorname{EllipticF}(x,i)}{21\sqrt{-x^4+1}}$	57
default	$-\frac{x^5\sqrt{-x^4+1}}{7}-\frac{5x\sqrt{-x^4+1}}{21}+\frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\operatorname{EllipticF}(x,i)}{21\sqrt{-x^4+1}}$	59
elliptic	$-\frac{x^5\sqrt{-x^4+1}}{7}-\frac{5x\sqrt{-x^4+1}}{21}+\frac{5\sqrt{-x^2+1}\,\sqrt{x^2+1}\,\operatorname{EllipticF}(x,i)}{21\sqrt{-x^4+1}}$	59

input
```
int(x^8/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*hypergeom([1/2,9/4],[13/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx = -\frac{1}{21}\left(3\,x^5+5\,x\right)\sqrt{-x^4+1}+\frac{5}{21}i\,F(\arcsin\left(\frac{1}{x}\right)\,|-1)$$

input
```
integrate(x^8/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/21*(3*x^5 + 5*x)*sqrt(-x^4 + 1) + 5/21*I*elliptic_f(arcsin(1/x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(-x**4+1)**(1/2),x)
```

output
```
x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), x**4*exp_polar(2*I*pi))/(4*gamm
a(13/4))
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx = \int \frac{x^8}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^8/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx = \int \frac{x^8}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^8/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx=\int \frac{x^8}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^8/(1 - x^4)^(1/2),x)
```

output
```
int(x^8/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{1-x^4}}\,dx=-\frac{\sqrt{-x^4+1}\,x^5}{7}-\frac{5\sqrt{-x^4+1}\,x}{21}-\frac{5\left(\int \frac{\sqrt{-x^4+1}}{x^4-1}dx\right)}{21}$$

input
```
int(x^8/(-x^4+1)^(1/2),x)
```

output
```
( - 3*sqrt( - x**4 + 1)*x**5 - 5*sqrt( - x**4 + 1)*x - 5*int(sqrt( - x**4
+ 1)/(x**4 - 1),x))/21
```

3.257 $\int \frac{x^4}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = -\frac{1}{3}x\sqrt{1-x^4}+\frac{1}{3}\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
-1/3*x*(-x^4+1)^(1/2)+1/3*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = \frac{1}{3}x\left(-\sqrt{1-x^4}+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)$$

input
```
Integrate[x^4/Sqrt[1 - x^4],x]
```

output
```
(x*(-Sqrt[1 - x^4] + Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \ 843$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}dx-\frac{1}{3}x\sqrt{1-x^4}$$

$$\Big\downarrow \ 762$$

$$\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)-\frac{1}{3}x\sqrt{1-x^4}$$

input
```
Int[x^4/Sqrt[1 - x^4],x]
```

output
```
-1/3*(x*Sqrt[1 - x^4]) + EllipticF[ArcSin[x], -1]/3
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

method	result	size
meijerg	$\frac{x^{5} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{5}{4}\right],\left[\frac{9}{4}\right], x^{4}\right)}{5}$	15
default	$-\frac{x \sqrt{-x^{4}+1}}{3}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}(x, i)}{3 \sqrt{-x^{4}+1}}$	45
elliptic	$-\frac{x \sqrt{-x^{4}+1}}{3}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}(x, i)}{3 \sqrt{-x^{4}+1}}$	45
risch	$\frac{x\left(x^{4}-1\right)}{3 \sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}(x, i)}{3 \sqrt{-x^{4}+1}}$	50

input
```
int(x^4/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1/2,5/4],[9/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = -\frac{1}{3}\,\sqrt{-x^4+1}x + \frac{1}{3}i\,F(\arcsin\left(\frac{1}{x}\right)\,|-1)$$

input
```
integrate(x^4/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-x^4 + 1)*x + 1/3*I*elliptic_f(arcsin(1/x), -1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(-x**4+1)**(1/2),x)
```

output
```
x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(9/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = \int \frac{x^4}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^4/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = \int \frac{x^4}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^4/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = \int \frac{x^4}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^4/(1 - x^4)^(1/2),x)
```

output
```
int(x^4/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}\,x}{3} - \frac{\left(\int \frac{\sqrt{-x^4+1}}{x^4-1}dx\right)}{3}$$

input
```
int(x^4/(-x^4+1)^(1/2),x)
```

output
```
( - (sqrt( - x**4 + 1)*x + int(sqrt( - x**4 + 1)/(x**4 - 1),x)))/3
```

3.258 $\int \frac{1}{\sqrt{1-x^4}} dx$

Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \frac{1}{\sqrt{1-x^4}} dx = \text{EllipticF}(\arcsin(x), -1)$$

output
```
EllipticF(x,I)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^4}} dx = \text{EllipticF}(\arcsin(x), -1)$$

input
```
Integrate[1/Sqrt[1 - x^4],x]
```

output
```
EllipticF[ArcSin[x], -1]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^4}} \, dx$$

$$\Big\downarrow \text{762}$$

$$\text{EllipticF}(\arcsin(x),-1)$$

input
```
Int[1/Sqrt[1 - x^4],x]
```

output
```
EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

method	result	size
meijerg	$x \,\text{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^{4}\right)$	12
default	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \text{EllipticF}(x, i)}{\sqrt{-x^{4}+1}}$	31
elliptic	$\frac{\sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \text{EllipticF}(x, i)}{\sqrt{-x^{4}+1}}$	31

input
```
int(1/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,1/2],[5/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = F(\arcsin{(x)}\,|{-1})$$

input
```
integrate(1/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
elliptic_f(arcsin(x), -1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(2) = 4.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\,x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-x**4+1)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/
4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(1/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(1/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ x^4\right)$$

input
```
int(1/(1 - x^4)^(1/2),x)
```

output
```
x*hypergeom([1/4, 1/2], 5/4, x^4)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^4-1}dx\right)$$

input
```
int(1/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**4 - 1),x)
```

3.259 $\int \frac{1}{x^4\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{3x^3}+\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)$$

output
```
-1/3*(-x^4+1)^(1/2)/x^3+1/3*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},x^4\right)}{3x^3}$$

input
```
Integrate[1/(x^4*Sqrt[1 - x^4]),x]
```

output
```
-1/3*Hypergeometric2F1[-3/4, 1/2, 1/4, x^4]/x^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{1-x^4}}dx$$
$$\Big\downarrow \text{847}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{3x^3}$$
$$\Big\downarrow \text{762}$$
$$\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)-\frac{\sqrt{1-x^4}}{3x^3}$$

input
```
Int[1/(x^4*Sqrt[1 - x^4]),x]
```

output
```
-1/3*Sqrt[1 - x^4]/x^3 + EllipticF[ArcSin[x], -1]/3
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{3}{4},\frac{1}{2}\right],\left[\frac{1}{4}\right],x^{4}\right)}{3x^{3}}$	15
default	$-\frac{\sqrt{-x^{4}+1}}{3x^{3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\text{EllipticF}(x,i)}{3\sqrt{-x^{4}+1}}$	47
elliptic	$-\frac{\sqrt{-x^{4}+1}}{3x^{3}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\text{EllipticF}(x,i)}{3\sqrt{-x^{4}+1}}$	47
risch	$\frac{x^{4}-1}{3x^{3}\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\text{EllipticF}(x,i)}{3\sqrt{-x^{4}+1}}$	52

input
```
int(1/x^4/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*hypergeom([-3/4,1/2],[1/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx=\frac{x^3F(\arcsin{(x)}\mid -1)-\sqrt{-x^4+1}}{3\,x^3}$$

input
```
integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^3*elliptic_f(arcsin(x), -1) - sqrt(-x^4 + 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(-x**4+1)**(1/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(2*I*pi))/(4*x**3*gam
ma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = \int \frac{1}{x^4\,\sqrt{1-x^4}}\,dx$$

input
```
int(1/(x^4*(1 - x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^8-x^4}dx\right)$$

input
```
int(1/x^4/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**8 - x**4),x)
```

3.260 $\int \frac{1}{x^8\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{7x^7}-\frac{5\sqrt{1-x^4}}{21x^3}+\frac{5}{21}\text{EllipticF}(\arcsin(x),-1)$$

output
```
-1/7*(-x^4+1)^(1/2)/x^7-5/21*(-x^4+1)^(1/2)/x^3+5/21*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},x^4\right)}{7x^7}$$

input
```
Integrate[1/(x^8*Sqrt[1 - x^4]),x]
```

output
```
-1/7*Hypergeometric2F1[-7/4, 1/2, -3/4, x^4]/x^7
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{7}\int \frac{1}{x^4\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{3x^3}\right)-\frac{\sqrt{1-x^4}}{7x^7}$$

$$\Big\downarrow \text{762}$$

$$\frac{5}{7}\left(\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)-\frac{\sqrt{1-x^4}}{3x^3}\right)-\frac{\sqrt{1-x^4}}{7x^7}$$

input
```
Int[1/(x^8*Sqrt[1 - x^4]),x]
```

output
```
-1/7*Sqrt[1 - x^4]/x^7 + (5*(-1/3*Sqrt[1 - x^4]/x^3 + EllipticF[ArcSin[x],
 -1]/3))/7
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.84 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{7}{4}, \frac{1}{2}\right], \left[-\frac{3}{4}\right], x^{4}\right)}{7 x^{7}}$	15
risch	$\frac{5 x^{8}-2 x^{4}-3}{21 x^{7} \sqrt{-x^{4}+1}}+\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{21 \sqrt{-x^{4}+1}}$	59
default	$-\frac{\sqrt{-x^{4}+1}}{7 x^{7}}-\frac{5 \sqrt{-x^{4}+1}}{21 x^{3}}+\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{21 \sqrt{-x^{4}+1}}$	61
elliptic	$-\frac{\sqrt{-x^{4}+1}}{7 x^{7}}-\frac{5 \sqrt{-x^{4}+1}}{21 x^{3}}+\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{21 \sqrt{-x^{4}+1}}$	61

input
```
int(1/x^8/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7*hypergeom([-7/4,1/2],[-3/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^8\sqrt{1-x^4}}\, dx = \frac{5\, x^7 F(\arcsin{(x)}\,|-1) - (5\, x^4 + 3)\sqrt{-x^4 + 1}}{21\, x^7}$$

input
```
integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(5*x^7*elliptic_f(arcsin(x), -1) - (5*x^4 + 3)*sqrt(-x^4 + 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ -\frac{3}{4} \end{matrix}\right| x^4 e^{2i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(-x**4+1)**(1/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), x**4*exp_polar(2*I*pi))/(4*x**7*ga
mma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^8}\,dx$$

input
```
integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^8}\,dx$$

input
```
integrate(1/x^8/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = \int \frac{1}{x^8\,\sqrt{1-x^4}}\,dx$$

input
```
int(1/(x^8*(1 - x^4)^(1/2)),x)
```

output
```
int(1/(x^8*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^{12}-x^8}dx\right)$$

input
```
int(1/x^8/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**12 - x**8),x)
```

3.261 $\int \frac{x^{10}}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{x^{10}}{\sqrt{1-x^4}} \, dx = -\frac{7}{45}x^3\sqrt{1-x^4} - \frac{1}{9}x^7\sqrt{1-x^4} + \frac{7}{15}E(\arcsin(x)|-1) - \frac{7}{15}\operatorname{EllipticF}(\arcsin(x),-1)$$

output
```
-7/45*x^3*(-x^4+1)^(1/2)-1/9*x^7*(-x^4+1)^(1/2)+7/15*EllipticE(x,I)-7/15*E
llipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{x^{10}}{\sqrt{1-x^4}} \, dx = \frac{1}{45}x^3\left(-\sqrt{1-x^4}\left(7+5x^4\right) + 7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)\right)$$

input
```
Integrate[x^10/Sqrt[1 - x^4],x]
```

output
```
(x^3*(-(Sqrt[1 - x^4]*(7 + 5*x^4)) + 7*Hypergeometric2F1[1/2, 3/4, 7/4, x^
4]))/45
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {843, 843, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{7}{9}\int \frac{x^6}{\sqrt{1-x^4}}dx-\frac{1}{9}x^7\sqrt{1-x^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{7}{9}\left(\frac{3}{5}\int \frac{x^2}{\sqrt{1-x^4}}dx-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}$$

$$\Big\downarrow \text{836}$$

$$\frac{7}{9}\left(\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\int \frac{1}{\sqrt{1-x^4}}dx\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}$$

$$\Big\downarrow \text{762}$$

$$\frac{7}{9}\left(\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}$$

$$\Big\downarrow \text{1388}$$

$$\frac{7}{9}\left(\frac{3}{5}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}$$

$$\Big\downarrow \text{327}$$

$$\frac{7}{9}\left(\frac{3}{5}(E(\arcsin(x)|-1)-\text{EllipticF}(\arcsin(x),-1))-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}$$

input
```
Int[x^10/Sqrt[1 - x^4],x]
```

output
```
-1/9*(x^7*Sqrt[1 - x^4]) + (7*(-1/5*(x^3*Sqrt[1 - x^4]) + (3*(EllipticE[Ar
cSin[x], -1] - EllipticF[ArcSin[x], -1]))/5))/9
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.28

method	result	size
meijerg	$\frac{x^{11} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{11}{4}\right], \left[\frac{15}{4}\right], x^{4}\right)}{11}$	15
risch	$\frac{x^{3}\left(5 x^{4}+7\right)\left(x^{4}-1\right)}{45 \sqrt{-x^{4}+1}}-\frac{7 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, (\operatorname{EllipticF}(x, i)-\operatorname{EllipticE}(x, i))}{15 \sqrt{-x^{4}+1}}$	66
default	$-\frac{x^{7} \sqrt{-x^{4}+1}}{9}-\frac{7 x^{3} \sqrt{-x^{4}+1}}{45}-\frac{7 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, (\operatorname{EllipticF}(x, i)-\operatorname{EllipticE}(x, i))}{15 \sqrt{-x^{4}+1}}$	68
elliptic	$-\frac{x^{7} \sqrt{-x^{4}+1}}{9}-\frac{7 x^{3} \sqrt{-x^{4}+1}}{45}-\frac{7 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, (\operatorname{EllipticF}(x, i)-\operatorname{EllipticE}(x, i))}{15 \sqrt{-x^{4}+1}}$	68

input
```
int(x^10/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11*hypergeom([1/2,11/4],[15/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx = \frac{-21 i\, x E\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right) + 21 i\, x F\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right) - (5\,x^{8} + 7\,x^{4} + 21)\sqrt{-x^{4}+1}}{45\,x}$$

input
```
integrate(x^10/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/45*(-21*I*x*elliptic_e(arcsin(1/x), -1) + 21*I*x*elliptic_f(arcsin(1/x),
 -1) - (5*x^8 + 7*x^4 + 21)*sqrt(-x^4 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(-x**4+1)**(1/2),x)
```

output
```
x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), x**4*exp_polar(2*I*pi))/(4*g
amma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx = \int \frac{x^{10}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^10/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^10/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx = \int \frac{x^{10}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^10/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^10/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx=\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^10/(1 - x^4)^(1/2),x)
```

output
```
int(x^10/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{1-x^4}}\,dx=-\frac{\sqrt{-x^4+1}\,x^7}{9}-\frac{7\sqrt{-x^4+1}\,x^3}{45}-\frac{7\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^4-1}dx\right)}{15}$$

input
```
int(x^10/(-x^4+1)^(1/2),x)
```

output
```
( - 5*sqrt( - x**4 + 1)*x**7 - 7*sqrt( - x**4 + 1)*x**3 - 21*int((sqrt( -
x**4 + 1)*x**2)/(x**4 - 1),x))/45
```

3.262 $\int \frac{x^6}{\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = -\frac{1}{5}x^3\sqrt{1-x^4}+\frac{3}{5}E(\arcsin(x)|-1)-\frac{3}{5}\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
-1/5*x^3*(-x^4+1)^(1/2)+3/5*EllipticE(x,I)-3/5*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = \frac{1}{5}x^3\left(-\sqrt{1-x^4}+\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)\right)$$

input
```
Integrate[x^6/Sqrt[1 - x^4],x]
```

output
```
(x^3*(-Sqrt[1 - x^4] + Hypergeometric2F1[1/2, 3/4, 7/4, x^4]))/5
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {843, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{5}\int \frac{x^2}{\sqrt{1-x^4}}dx-\frac{1}{5}x^3\sqrt{1-x^4}$$

$$\Big\downarrow \text{836}$$

$$\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\int \frac{1}{\sqrt{1-x^4}}dx\right)-\frac{1}{5}x^3\sqrt{1-x^4}$$

$$\Big\downarrow \text{762}$$

$$\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}$$

$$\Big\downarrow \text{1388}$$

$$\frac{3}{5}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{5}(E(\arcsin(x)|-1)-\text{EllipticF}(\arcsin(x),-1))-\frac{1}{5}x^3\sqrt{1-x^4}$$

input
```
Int[x^6/Sqrt[1 - x^4],x]
```

output
```
-1/5*(x^3*Sqrt[1 - x^4]) + (3*(EllipticE[ArcSin[x], -1] - EllipticF[ArcSin
[x], -1]))/5
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

method	result	size
meijerg	$\frac{x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{7}{4}\right], \left[\frac{11}{4}\right], x^{4}\right)}{7}$	15
default	$-\frac{x^{3} \sqrt{-x^{4}+1}}{5}-\frac{3 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{5 \sqrt{-x^{4}+1}}$	54
elliptic	$-\frac{x^{3} \sqrt{-x^{4}+1}}{5}-\frac{3 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{5 \sqrt{-x^{4}+1}}$	54
risch	$\frac{x^{3} \left(x^{4}-1\right)}{5 \sqrt{-x^{4}+1}}-\frac{3 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{5 \sqrt{-x^{4}+1}}$	59

input
```
int(x^6/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*hypergeom([1/2,7/4],[11/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int \frac{x^6}{\sqrt{1-x^4}}\, dx = \frac{-3i\, xE\left(\arcsin\left(\frac{1}{x}\right)\mid -1\right) + 3i\, xF\left(\arcsin\left(\frac{1}{x}\right)\mid -1\right) - (x^4+3)\sqrt{-x^4+1}}{5\, x}$$

input
```
integrate(x^6/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*x*elliptic_e(arcsin(1/x), -1) + 3*I*x*elliptic_f(arcsin(1/x), -1
) - (x^4 + 3)*sqrt(-x^4 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(-x**4+1)**(1/2),x)
```

output
```
x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(2*I*pi))/(4*gamm
a(11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = \int \frac{x^6}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^6/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = \int \frac{x^6}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^6/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = \int \frac{x^6}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^6/(1 - x^4)^(1/2),x)
```

output
```
int(x^6/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{1-x^4}}\,dx = -\frac{\sqrt{-x^4+1}\,x^3}{5} - \frac{3\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^4-1}dx\right)}{5}$$

input
```
int(x^6/(-x^4+1)^(1/2),x)
```

output
```
( - sqrt( - x**4 + 1)*x**3 - 3*int((sqrt( - x**4 + 1)*x**2)/(x**4 - 1),x))
/5
```

3.263 $\int \frac{x^2}{\sqrt{1-x^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{x^2}{\sqrt{1-x^4}} \, dx = E(\arcsin(x)|-1) - \text{EllipticF}(\arcsin(x), -1)$$

output
```
EllipticE(x,I)-EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{x^2}{\sqrt{1-x^4}} \, dx = \frac{1}{3}x^3 \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, x^4\right)$$

input
```
Integrate[x^2/Sqrt[1 - x^4],x]
```

output
```
(x^3*Hypergeometric2F1[1/2, 3/4, 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx$$

$$\Big\downarrow \text{836}$$

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\int \frac{1}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{762}$$

$$\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)$$

$$\Big\downarrow \text{327}$$

$$E(\arcsin(x)|-1)-\text{EllipticF}(\arcsin(x),-1)$$

input
```
Int[x^2/Sqrt[1 - x^4],x]
```

output
```
EllipticE[ArcSin[x], -1] - EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
meijerg	$\frac{x^3 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^4\right)}{3}$	15
default	$-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	39
elliptic	$-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{\sqrt{-x^4+1}}$	39

input
```
int(x^2/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,3/4],[7/4],x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx = \frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)-\sqrt{-x^4+1}}{x}$$

input
```
integrate(x^2/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*x*elliptic_e(arcsin(1/x), -1) + I*x*elliptic_f(arcsin(1/x), -1) - sqrt
(-x^4 + 1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(5) = 10.

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-x**4+1)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^2/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate(x^2/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx = \int \frac{x^2}{\sqrt{1-x^4}}\,dx$$

input
```
int(x^2/(1 - x^4)^(1/2),x)
```

output
```
int(x^2/(1 - x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^4-1}dx\right)$$

input
```
int(x^2/(-x^4+1)^(1/2),x)
```

output
```
 - int((sqrt( - x**4 + 1)*x**2)/(x**4 - 1),x)
```

3.264 $\int \frac{1}{x^2\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{x} - E(\arcsin(x)|-1) + \text{EllipticF}(\arcsin(x), -1)$$

output
```
-(-x^4+1)^(1/2)/x-EllipticE(x,I)+EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},x^4\right)}{x}$$

input
```
Integrate[1/(x^2*Sqrt[1 - x^4]),x]
```

output
```
-(Hypergeometric2F1[-1/4, 1/2, 3/4, x^4]/x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{847}$$

$$-\int \frac{x^2}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{x}$$

$$\Big\downarrow \text{836}$$

$$\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{x}$$

$$\Big\downarrow \text{762}$$

$$-\int \frac{x^2+1}{\sqrt{1-x^4}}dx+\text{EllipticF}(\arcsin(x),-1)-\frac{\sqrt{1-x^4}}{x}$$

$$\Big\downarrow \text{1388}$$

$$-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx+\text{EllipticF}(\arcsin(x),-1)-\frac{\sqrt{1-x^4}}{x}$$

$$\Big\downarrow \text{327}$$

$$\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)-\frac{\sqrt{1-x^4}}{x}$$

input
```
Int[1/(x^2*Sqrt[1 - x^4]),x]
```

output
```
-(Sqrt[1 - x^4]/x) - EllipticE[ArcSin[x], -1] + EllipticF[ArcSin[x], -1]
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{4}\right],x^{4}\right)}{x}$	15
default	$-\frac{\sqrt{-x^{4}+1}}{x}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^{4}+1}}$	53
elliptic	$-\frac{\sqrt{-x^{4}+1}}{x}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^{4}+1}}$	53
risch	$\frac{x^{4}-1}{x\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{\sqrt{-x^{4}+1}}$	57

input
```
int(1/x^2/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x*hypergeom([-1/4,1/2],[3/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = -\frac{xE(\arcsin{(x)}\mid -1) - xF(\arcsin{(x)}\mid -1) + \sqrt{-x^4+1}}{x}$$

input
```
integrate(1/x^2/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-(x*elliptic_e(arcsin(x), -1) - x*elliptic_f(arcsin(x), -1) + sqrt(-x^4 +
1))/x
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(-x**4+1)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(2*I*pi))/(4*x*gamma(
3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = -\frac{{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ x^4\right)}{x}$$

input
```
int(1/(x^2*(1 - x^4)^(1/2)),x)
```

output
```
-hypergeom([-1/4, 1/2], 3/4, x^4)/x
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^6-x^2}dx\right)$$

input
```
int(1/x^2/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**6 - x**2),x)
```

3.265 $\int \frac{1}{x^6\sqrt{1-x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = -\frac{\sqrt{1-x^4}}{5x^5}-\frac{3\sqrt{1-x^4}}{5x}-\frac{3}{5}E(\arcsin(x)|-1)+\frac{3}{5}\text{EllipticF}(\arcsin(x),-1)$$

output
```
-1/5*(-x^4+1)^(1/2)/x^5-3/5*(-x^4+1)^(1/2)/x-3/5*EllipticE(x,I)+3/5*Ellipt
icF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},x^4\right)}{5x^5}$$

input
```
Integrate[1/(x^6*Sqrt[1 - x^4]),x]
```

output
```
-1/5*Hypergeometric2F1[-5/4, 1/2, -1/4, x^4]/x^5
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {847, 847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{5}\int \frac{1}{x^2\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{5}\left(-\int \frac{x^2}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{x}\right)-\frac{\sqrt{1-x^4}}{5x^5}$$

$$\Big\downarrow \text{836}$$

$$\frac{3}{5}\left(\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{x}\right)-\frac{\sqrt{1-x^4}}{5x^5}$$

$$\Big\downarrow \text{762}$$

$$\frac{3}{5}\left(-\int \frac{x^2+1}{\sqrt{1-x^4}}dx+\text{EllipticF}(\arcsin(x),-1)-\frac{\sqrt{1-x^4}}{x}\right)-\frac{\sqrt{1-x^4}}{5x^5}$$

$$\Big\downarrow \text{1388}$$

$$\frac{3}{5}\left(-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx+\text{EllipticF}(\arcsin(x),-1)-\frac{\sqrt{1-x^4}}{x}\right)-\frac{\sqrt{1-x^4}}{5x^5}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{5}\left(\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)-\frac{\sqrt{1-x^4}}{x}\right)-\frac{\sqrt{1-x^4}}{5x^5}$$

input
```
Int[1/(x^6*Sqrt[1 - x^4]),x]
```

output
```
-1/5*Sqrt[1 - x^4]/x^5 + (3*(-(Sqrt[1 - x^4]/x) - EllipticE[ArcSin[x], -1]
 + EllipticF[ArcSin[x], -1]))/5
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.28

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{2}\right],\left[-\frac{1}{4}\right],x^4\right)}{5x^5}$	15
risch	$\frac{3x^8-2x^4-1}{5x^5\sqrt{-x^4+1}}+\frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{5\sqrt{-x^4+1}}$	66
default	$-\frac{\sqrt{-x^4+1}}{5x^5}-\frac{3\sqrt{-x^4+1}}{5x}+\frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{5\sqrt{-x^4+1}}$	68
elliptic	$-\frac{\sqrt{-x^4+1}}{5x^5}-\frac{3\sqrt{-x^4+1}}{5x}+\frac{3\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{5\sqrt{-x^4+1}}$	68

input
```
int(1/x^6/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5*hypergeom([-5/4,1/2],[-1/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = -\frac{3\,x^5E(\arcsin{(x)}\,|-1)-3\,x^5F(\arcsin{(x)}\,|-1)+(3\,x^4+1)\sqrt{-x^4+1}}{5\,x^5}$$

input
```
integrate(1/x^6/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*x^5*elliptic_e(arcsin(x), -1) - 3*x^5*elliptic_f(arcsin(x), -1) +
(3*x^4 + 1)*sqrt(-x^4 + 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4},\frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| x^4e^{2i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(-x**4+1)**(1/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), x**4*exp_polar(2*I*pi))/(4*x**5*ga
mma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+1}x^6}\,dx$$

input
```
integrate(1/x^6/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^4 + 1)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = \int \frac{1}{x^6\,\sqrt{1-x^4}}\,dx$$

input
```
int(1/(x^6*(1 - x^4)^(1/2)),x)
```

output
```
int(1/(x^6*(1 - x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{1-x^4}}\,dx = -\left(\int \frac{\sqrt{-x^4+1}}{x^{10}-x^6}dx\right)$$

input
```
int(1/x^6/(-x^4+1)^(1/2),x)
```

output
```
- int(sqrt( - x**4 + 1)/(x**10 - x**6),x)
```

3.266 $\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}} + \sqrt{1-x^4} - \frac{1}{6}(1-x^4)^{3/2}$$

output
```
1/2/(-x^4+1)^(1/2)+(-x^4+1)^(1/2)-1/6*(-x^4+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = -\frac{-8+4x^4+x^8}{6\sqrt{1-x^4}}$$

input
```
Integrate[x^11/(1 - x^4)^(3/2),x]
```

output
```
-1/6*(-8 + 4*x^4 + x^8)/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(1-x^4)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\sqrt{1-x^4}-\frac{2}{\sqrt{1-x^4}}+\frac{1}{(1-x^4)^{3/2}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{2}{3}(1-x^4)^{3/2}+4\sqrt{1-x^4}+\frac{2}{\sqrt{1-x^4}}\right)$$

input
```
Int[x^11/(1 - x^4)^(3/2),x]
```

output
```
(2/Sqrt[1 - x^4] + 4*Sqrt[1 - x^4] - (2*(1 - x^4)^(3/2))/3)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

method	result	size
default	$-\frac{x^{8}+4x^{4}-8}{6\sqrt{-x^{4}+1}}$	22
risch	$-\frac{x^{8}+4x^{4}-8}{6\sqrt{-x^{4}+1}}$	22
elliptic	$-\frac{x^{8}+4x^{4}-8}{6\sqrt{-x^{4}+1}}$	22
pseudoelliptic	$-\frac{x^{8}+4x^{4}-8}{6\sqrt{-x^{4}+1}}$	22
trager	$\frac{\left(x^{8}+4x^{4}-8\right)\sqrt{-x^{4}+1}}{6x^{4}-6}$	29
gosper	$\frac{(-1+x)(1+x)\left(x^{2}+1\right)\left(x^{8}+4x^{4}-8\right)}{6(-x^{4}+1)^{\frac{3}{2}}}$	33
orering	$\frac{(-1+x)(1+x)\left(x^{2}+1\right)\left(x^{8}+4x^{4}-8\right)}{6(-x^{4}+1)^{\frac{3}{2}}}$	33
meijerg	$-\frac{\frac{8\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-2x^{8}-8x^{4}+16\right)}{6\sqrt{-x^{4}+1}}}{2\sqrt{\pi}}$	38

input
```
int(x^11/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(x^8+4*x^4-8)/(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = \frac{(x^8+4\,x^4-8)\sqrt{-x^4+1}}{6\,(x^4-1)}$$

input
```
integrate(x^11/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(x^8 + 4*x^4 - 8)*sqrt(-x^4 + 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = \frac{x^8\sqrt{1-x^4}}{6x^4-6} + \frac{4x^4\sqrt{1-x^4}}{6x^4-6} - \frac{8\sqrt{1-x^4}}{6x^4-6}$$

input
```
integrate(x**11/(-x**4+1)**(3/2),x)
```

output
```
x**8*sqrt(1 - x**4)/(6*x**4 - 6) + 4*x**4*sqrt(1 - x**4)/(6*x**4 - 6) - 8*
sqrt(1 - x**4)/(6*x**4 - 6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = -\frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}} + \sqrt{-x^4+1} + \frac{1}{2\sqrt{-x^4+1}}$$

input
```
integrate(x^11/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/6*(-x^4 + 1)^(3/2) + sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = -\frac{1}{6}\left(-x^4+1\right)^{\frac{3}{2}} + \sqrt{-x^4+1} + \frac{1}{2\sqrt{-x^4+1}}$$

input
```
integrate(x^11/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/6*(-x^4 + 1)^(3/2) + sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = -\frac{(x^4-1)^2 + 6\,x^4 - 9}{6\sqrt{1-x^4}}$$

input
```
int(x^11/(1 - x^4)^(3/2),x)
```

output
```
-((x^4 - 1)^2 + 6*x^4 - 9)/(6*(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{x^{11}}{(1-x^4)^{3/2}}\,dx = \frac{-x^8-4x^4+8}{6\sqrt{-x^4+1}}$$

input
```
int(x^11/(-x^4+1)^(3/2),x)
```

output
```
( - x**8 - 4*x**4 + 8)/(6*sqrt( - x**4 + 1))
```

3.267 $\int \frac{x^7}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^7}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}} + \frac{\sqrt{1-x^4}}{2}$$

output
```
1/2/(-x^4+1)^(1/2)+1/2*(-x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{x^7}{(1-x^4)^{3/2}}\,dx = -\frac{-2+x^4}{2\sqrt{1-x^4}}$$

input
```
Integrate[x^7/(1 - x^4)^(3/2),x]
```

output
```
-1/2*(-2 + x^4)/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(1-x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(1-x^4)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{1}{(1-x^4)^{3/2}}-\frac{1}{\sqrt{1-x^4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(2\sqrt{1-x^4}+\frac{2}{\sqrt{1-x^4}}\right)$$

input
```
Int[x^7/(1 - x^4)^(3/2),x]
```

output
```
(2/Sqrt[1 - x^4] + 2*Sqrt[1 - x^4])/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

method	result	size
default	$-\frac{x^4-2}{2\sqrt{-x^4+1}}$	17
risch	$-\frac{x^4-2}{2\sqrt{-x^4+1}}$	17
elliptic	$-\frac{x^4-2}{2\sqrt{-x^4+1}}$	17
pseudoelliptic	$-\frac{x^4-2}{2\sqrt{-x^4+1}}$	17
trager	$\frac{(x^4-2)\sqrt{-x^4+1}}{2x^4-2}$	24
gosper	$\frac{(-1+x)(1+x)(x^2+1)(x^4-2)}{2(-x^4+1)^{\frac{3}{2}}}$	28
orering	$\frac{(-1+x)(1+x)(x^2+1)(x^4-2)}{2(-x^4+1)^{\frac{3}{2}}}$	28
meijerg	$\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(-4x^4+8\right)}{4\sqrt{-x^4+1}}}{2\sqrt{\pi}}$	33

input
```
int(x^7/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^4-2)/(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(1-x^4)^{3/2}}\,dx = \frac{(x^4-2)\sqrt{-x^4+1}}{2\,(x^4-1)}$$

input
```
integrate(x^7/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(x^4 - 2)*sqrt(-x^4 + 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{x^7}{(1-x^4)^{3/2}}\,dx = \frac{x^4\sqrt{1-x^4}}{2x^4-2} - \frac{2\sqrt{1-x^4}}{2x^4-2}$$

input
```
integrate(x**7/(-x**4+1)**(3/2),x)
```

output
```
x**4*sqrt(1 - x**4)/(2*x**4 - 2) - 2*sqrt(1 - x**4)/(2*x**4 - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(1-x^4)^{3/2}}\,dx = \frac{1}{2}\sqrt{-x^4+1} + \frac{1}{2\sqrt{-x^4+1}}$$

input
```
integrate(x^7/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(1-x^4)^{3/2}} \, dx = \frac{1}{2} \sqrt{-x^4+1} + \frac{1}{2\sqrt{-x^4+1}}$$

input
```
integrate(x^7/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^4 + 1) + 1/2/sqrt(-x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{(1-x^4)^{3/2}} \, dx = -\frac{x^4-2}{2\sqrt{1-x^4}}$$

input
```
int(x^7/(1 - x^4)^(3/2),x)
```

output
```
-(x^4 - 2)/(2*(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{x^7}{(1-x^4)^{3/2}} \, dx = \frac{-x^4+2}{2\sqrt{-x^4+1}}$$

input
```
int(x^7/(-x^4+1)^(3/2),x)
```

output
```
( - x**4 + 2)/(2*sqrt( - x**4 + 1))
```

3.268 $\int \frac{x^3}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}}$$

output
```
1/2/(-x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}}$$

input
```
Integrate[x^3/(1 - x^4)^(3/2),x]
```

output
```
1/(2*Sqrt[1 - x^4])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx$$
$$\Big\downarrow \; 793$$
$$\frac{1}{2\sqrt{1-x^4}}$$

input
```
Int[x^3/(1 - x^4)^(3/2),x]
```

output
```
1/(2*Sqrt[1 - x^4])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{1}{2\sqrt{-x^4+1}}$	12
default	$\frac{1}{2\sqrt{-x^4+1}}$	12
risch	$\frac{1}{2\sqrt{-x^4+1}}$	12
elliptic	$\frac{1}{2\sqrt{-x^4+1}}$	12
pseudoelliptic	$\frac{1}{2\sqrt{-x^4+1}}$	12
trager	$-\frac{\sqrt{-x^4+1}}{2(x^4-1)}$	19
gosper	$-\frac{(-1+x)(1+x)(x^2+1)}{2(-x^4+1)^{\frac{3}{2}}}$	23
orering	$-\frac{(-1+x)(1+x)(x^2+1)}{2(-x^4+1)^{\frac{3}{2}}}$	23
meijerg	$-\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{-x^4+1}}}{2\sqrt{\pi}}$	24

input
```
int(x^3/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}}{2\,(x^4-1)}$$

input
```
integrate(x^3/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^4 + 1)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}}$$

input
```
integrate(x**3/(-x**4+1)**(3/2),x)
```

output
```
1/(2*sqrt(1 - x**4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{-x^4+1}}$$

input
```
integrate(x^3/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2/sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{-x^4+1}}$$

input
```
integrate(x^3/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2/sqrt(-x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}}$$

input
```
int(x^3/(1 - x^4)^(3/2),x)
```

output
```
1/(2*(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(1-x^4)^{3/2}}\,dx = \frac{1}{2\sqrt{-x^4+1}}$$

input
```
int(x^3/(-x^4+1)^(3/2),x)
```

output
```
1/(2*sqrt( - x**4 + 1))
```

3.269 $\int \frac{1}{x(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx=\frac{1}{2\sqrt{1-x^4}}-\frac{1}{2}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

output
```
1/2/(-x^4+1)^(1/2)-1/2*arctanh((-x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx=\frac{1}{2}\left(\frac{1}{\sqrt{1-x^4}}-\text{arctanh}\left(\sqrt{1-x^4}\right)\right)$$

input
```
Integrate[1/(x*(1 - x^4)^(3/2)),x]
```

output
```
(1/Sqrt[1 - x^4] - ArcTanh[Sqrt[1 - x^4]])/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^4 (1-x^4)^{3/2}} dx^4$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(\int \frac{1}{x^4 \sqrt{1-x^4}} dx^4 + \frac{2}{\sqrt{1-x^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(\frac{2}{\sqrt{1-x^4}} - 2 \int \frac{1}{1-x^8} d\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4} \left(\frac{2}{\sqrt{1-x^4}} - 2\text{arctanh}\left(\sqrt{1-x^4}\right)\right)$$

input
```
Int[1/(x*(1 - x^4)^(3/2)),x]
```

output
```
(2/Sqrt[1 - x^4] - 2*ArcTanh[Sqrt[1 - x^4]])/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{1}{2\sqrt{-x^{4}+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}$	25
pseudoelliptic	$\frac{1}{2\sqrt{-x^{4}+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}$	25
trager	$-\frac{\sqrt{-x^{4}+1}}{2(x^{4}-1)}+\frac{\ln\left(\frac{-1+\sqrt{-x^{4}+1}}{x^{2}}\right)}{2}$	38
meijerg	$\frac{\frac{(2-2\ln(2)+4\ln(x)+i\pi)\sqrt{\pi}}{2}-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-x^{4}+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{4}+1}}{2}\right)}{2\sqrt{\pi}}$	63
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}-\frac{\sqrt{-(x^{2}-1)^{2}+2-2x^{2}}}{4(x^{2}-1)}+\frac{\sqrt{-(x^{2}+1)^{2}+2+2x^{2}}}{4x^{2}+4}$	68
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{4}+1}}\right)}{2}-\frac{\sqrt{-(x^{2}-1)^{2}+2-2x^{2}}}{4(x^{2}-1)}+\frac{\sqrt{-(x^{2}+1)^{2}+2+2x^{2}}}{4x^{2}+4}$	68

input
```
int(1/x/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-x^4+1)^(1/2)-1/2*arctanh(1/(-x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.81

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx = -\frac{\left(x^4-1\right)\log\left(\sqrt{-x^4+1}+1\right)-\left(x^4-1\right)\log\left(\sqrt{-x^4+1}-1\right)+2\sqrt{-x^4+1}}{4\left(x^4-1\right)}$$

input
```
integrate(1/x/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*((x^4 - 1)*log(sqrt(-x^4 + 1) + 1) - (x^4 - 1)*log(sqrt(-x^4 + 1) - 1
) + 2*sqrt(-x^4 + 1))/(x^4 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 228, normalized size of antiderivative = 7.12

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx = \begin{cases} -\frac{2x^4\log\left(x^2\right)}{4x^4-4} + \frac{x^4\log\left(x^4\right)}{4x^4-4} + \frac{2ix^4\operatorname{asin}\left(\frac{1}{x^2}\right)}{4x^4-4} - \frac{2i\sqrt{x^4-1}}{4x^4-4} + \frac{2\log\left(x^2\right)}{4x^4-4} - \frac{\log\left(x^4\right)}{4x^4-4} - \frac{2i\operatorname{asin}\left(\frac{1}{x^2}\right)}{4x^4-4} & \text{f} \\ \frac{x^4\log\left(x^4\right)}{4x^4-4} - \frac{2x^4\log\left(\sqrt{1-x^4}+1\right)}{4x^4-4} + \frac{i\pi x^4}{4x^4-4} - \frac{2\sqrt{1-x^4}}{4x^4-4} - \frac{\log\left(x^4\right)}{4x^4-4} + \frac{2\log\left(\sqrt{1-x^4}+1\right)}{4x^4-4} - \frac{i\pi}{4x^4-4} & \text{o} \end{cases}$$

input
```
integrate(1/x/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((-2*x***4*log(x**2)/(4*x***4 - 4) + x***4*log(x***4)/(4*x***4 - 4) +
2*I*x***4*asin(x**(-2))/(4*x***4 - 4) - 2*I*sqrt(x**4 - 1)/(4*x***4 - 4) + 2*
log(x**2)/(4*x***4 - 4) - log(x***4)/(4*x***4 - 4) - 2*I*asin(x**(-2))/(4*x**
4 - 4), Abs(x***4) > 1), (x***4*log(x***4)/(4*x***4 - 4) - 2*x***4*log(sqrt(1 -
 x***4) + 1)/(4*x***4 - 4) + I*pi*x***4/(4*x***4 - 4) - 2*sqrt(1 - x***4)/(4*x*
*4 - 4) - log(x***4)/(4*x***4 - 4) + 2*log(sqrt(1 - x***4) + 1)/(4*x***4 - 4)
- I*pi/(4*x***4 - 4), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx = \frac{1}{2\sqrt{-x^4+1}} - \frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{-x^4+1}-1\right)$$

input
```
integrate(1/x/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2/sqrt(-x^4 + 1) - 1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(sqrt(-x^4 + 1)
- 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx = \frac{1}{2\sqrt{-x^4+1}} - \frac{1}{4}\log\left(\sqrt{-x^4+1}+1\right) + \frac{1}{4}\log\left(-\sqrt{-x^4+1}+1\right)$$

input
```
integrate(1/x/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2/sqrt(-x^4 + 1) - 1/4*log(sqrt(-x^4 + 1) + 1) + 1/4*log(-sqrt(-x^4 + 1)
 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx = \frac{1}{2\sqrt{1-x^4}} - \frac{\operatorname{atanh}\left(\sqrt{1-x^4}\right)}{2}$$

input
```
int(1/(x*(1 - x^4)^(3/2)),x)
```

output
```
1/(2*(1 - x^4)^(1/2)) - atanh((1 - x^4)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(1-x^4\right)^{3/2}}\,dx = \frac{\sqrt{-x^4+1}\log\left(\tan\left(\frac{asin(x^2)}{2}\right)\right) - \sqrt{-x^4+1}+1}{2\sqrt{-x^4+1}}$$

input
```
int(1/x/(-x^4+1)^(3/2),x)
```

output
```
(sqrt( - x**4 + 1)*log(tan(asin(x**2)/2)) - sqrt( - x**4 + 1) + 1)/(2*sqrt
( - x**4 + 1))
```

3.270 $\int \frac{1}{x^5(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{1}{x^5 (1-x^4)^{3/2}}\,dx = \frac{3}{4\sqrt{1-x^4}} - \frac{1}{4x^4\sqrt{1-x^4}} - \frac{3}{4}\text{arctanh}\left(\sqrt{1-x^4}\right)$$

output
```
3/4/(-x^4+1)^(1/2)-1/4/x^4/(-x^4+1)^(1/2)-3/4*arctanh((-x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^5 (1-x^4)^{3/2}}\,dx = \frac{1}{4}\left(\frac{-1+3x^4}{x^4\sqrt{1-x^4}} - 3\text{arctanh}\left(\sqrt{1-x^4}\right)\right)$$

input
```
Integrate[1/(x^5*(1 - x^4)^(3/2)),x]
```

output
```
((-1 + 3*x^4)/(x^4*Sqrt[1 - x^4]) - 3*ArcTanh[Sqrt[1 - x^4]])/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^8 (1-x^4)^{3/2}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(\frac{3}{2} \int \frac{1}{x^4 (1-x^4)^{3/2}}dx^4-\frac{1}{x^4 \sqrt{1-x^4}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(\frac{3}{2} \left(\int \frac{1}{x^4 \sqrt{1-x^4}}dx^4+\frac{2}{\sqrt{1-x^4}}\right)-\frac{1}{x^4 \sqrt{1-x^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(\frac{3}{2} \left(\frac{2}{\sqrt{1-x^4}}-2 \int \frac{1}{1-x^8}d\sqrt{1-x^4}\right)-\frac{1}{x^4 \sqrt{1-x^4}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4} \left(\frac{3}{2} \left(\frac{2}{\sqrt{1-x^4}}-2 \text{arctanh}\left(\sqrt{1-x^4}\right)\right)-\frac{1}{x^4 \sqrt{1-x^4}}\right)$$

input
```
Int[1/(x^5*(1 - x^4)^(3/2)),x]
```

output
```
(-(1/(x^4*Sqrt[1 - x^4])) + (3*(2/Sqrt[1 - x^4] - 2*ArcTanh[Sqrt[1 - x^4]]
))/2)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{3x^4-1}{4x^4\sqrt{-x^4+1}}-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}$	35
pseudoelliptic	$-\frac{1}{4x^4\sqrt{-x^4+1}}+\frac{3}{4\sqrt{-x^4+1}}-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}$	39
trager	$-\frac{(3x^4-1)\sqrt{-x^4+1}}{4(x^4-1)x^4}+\frac{3\ln\left(\frac{-1+\sqrt{-x^4+1}}{x^2}\right)}{4}$	48
default	$-\frac{\sqrt{-x^4+1}}{4x^4}-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}-\frac{\sqrt{-(x^2-1)^2+2-2x^2}}{4(x^2-1)}+\frac{\sqrt{-(x^2+1)^2+2+2x^2}}{4x^2+4}$	82
elliptic	$-\frac{\sqrt{-x^4+1}}{4x^4}-\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{-x^4+1}}\right)}{4}-\frac{\sqrt{-(x^2-1)^2+2-2x^2}}{4(x^2-1)}+\frac{\sqrt{-(x^2+1)^2+2+2x^2}}{4x^2+4}$	82
meijerg	$-\frac{\frac{\sqrt{\pi}}{2x^4}-\frac{3\left(\frac{5}{3}-2\ln(2)+4\ln(x)+i\pi\right)\sqrt{\pi}}{4}-\frac{\sqrt{\pi}\left(-20x^4+8\right)}{16x^4}+\frac{\sqrt{\pi}\left(-24x^4+8\right)}{16x^4\sqrt{-x^4+1}}+\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^4+1}}{2}\right)}{2}}{2\sqrt{\pi}}$	92

input
```
int(1/x^5/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(3*x^4-1)/x^4/(-x^4+1)^(1/2)-3/4*arctanh(1/(-x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(38) = 76.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^5\left(1-x^4\right)^{3/2}}\,dx = -\frac{3\left(x^8-x^4\right)\log\left(\sqrt{-x^4+1}+1\right)-3\left(x^8-x^4\right)\log\left(\sqrt{-x^4+1}-1\right)+2\left(3\,x^4-1\right)\sqrt{-x^4+1}}{8\left(x^8-x^4\right)}$$

input
```
integrate(1/x^5/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(3*(x^8 - x^4)*log(sqrt(-x^4 + 1) + 1) - 3*(x^8 - x^4)*log(sqrt(-x^4
+ 1) - 1) + 2*(3*x^4 - 1)*sqrt(-x^4 + 1))/(x^8 - x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^5 (1-x^4)^{3/2}}\,dx = \begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x^2}\right)}{4} + \frac{3}{4x^2\sqrt{-1+\frac{1}{x^4}}} - \frac{1}{4x^6\sqrt{-1+\frac{1}{x^4}}} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{3i\operatorname{asin}\left(\frac{1}{x^2}\right)}{4} - \frac{3i}{4x^2\sqrt{1-\frac{1}{x^4}}} + \frac{i}{4x^6\sqrt{1-\frac{1}{x^4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((-3*acosh(x**(-2))/4 + 3/(4*x**2*sqrt(-1 + x**(-4))) - 1/(4*x**6
*sqrt(-1 + x**(-4))), 1/Abs(x**4) > 1), (3*I*asin(x**(-2))/4 - 3*I/(4*x**2
*sqrt(1 - 1/x**4)) + I/(4*x**6*sqrt(1 - 1/x**4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^5 (1-x^4)^{3/2}}\,dx = -\frac{3x^4-1}{4\left((-x^4+1)^{\frac{3}{2}} - \sqrt{-x^4+1}\right)} - \frac{3}{8}\log\left(\sqrt{-x^4+1}+1\right) + \frac{3}{8}\log\left(\sqrt{-x^4+1}-1\right)$$

input
```
integrate(1/x^5/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*x^4 - 1)/((-x^4 + 1)^(3/2) - sqrt(-x^4 + 1)) - 3/8*log(sqrt(-x^4 +
 1) + 1) + 3/8*log(sqrt(-x^4 + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^5 (1-x^4)^{3/2}} \, dx = -\frac{3 x^4 - 1}{4 \left((-x^4+1)^{\frac{3}{2}} - \sqrt{-x^4+1} \right)} - \frac{3}{8} \log \left(\sqrt{-x^4+1} + 1 \right) + \frac{3}{8} \log \left(-\sqrt{-x^4+1} + 1 \right)$$

input
```
integrate(1/x^5/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 - 1)/((-x^4 + 1)^(3/2) - sqrt(-x^4 + 1)) - 3/8*log(sqrt(-x^4 +
 1) + 1) + 3/8*log(-sqrt(-x^4 + 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5 (1-x^4)^{3/2}} \, dx = \frac{3}{4 \sqrt{1-x^4}} - \frac{1}{4 x^4 \sqrt{1-x^4}} - \frac{3 \operatorname{atanh}\left(\sqrt{1-x^4}\right)}{4}$$

input
```
int(1/(x^5*(1 - x^4)^(3/2)),x)
```

output
```
3/(4*(1 - x^4)^(1/2)) - 1/(4*x^4*(1 - x^4)^(1/2)) - (3*atanh((1 - x^4)^(1/
2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^5 (1-x^4)^{3/2}} \, dx = \frac{12\sqrt{-x^4+1} \log \left(\tan \left(\frac{asin(x^2)}{2} \right) \right) x^4 - 9\sqrt{-x^4+1}\, x^4 + 12x^4 - 4}{16\sqrt{-x^4+1}\, x^4}$$

input
```
int(1/x^5/(-x^4+1)^(3/2),x)
```

output
```
(12*sqrt( - x**4 + 1)*log(tan(asin(x**2)/2))*x**4 - 9*sqrt( - x**4 + 1)*x*
*4 + 12*x**4 - 4)/(16*sqrt( - x**4 + 1)*x**4)
```

3.271 $\int \frac{x^9}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx=\frac{x^6}{2\sqrt{1-x^4}}+\frac{3}{4}x^2\sqrt{1-x^4}-\frac{3\arcsin(x^2)}{4}$$

output
```
1/2*x^6/(-x^4+1)^(1/2)+3/4*x^2*(-x^4+1)^(1/2)-3/4*arcsin(x^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx=\frac{1}{4}\left(-\frac{x^2(-3+x^4)}{\sqrt{1-x^4}}-3\arcsin(x^2)\right)$$

input
```
Integrate[x^9/(1 - x^4)^(3/2),x]
```

output
```
(-((x^2*(-3 + x^4))/Sqrt[1 - x^4]) - 3*ArcSin[x^2])/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {807, 252, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2} \int \frac{x^8}{(1-x^4)^{3/2}} dx^2$$

$$\Big\downarrow \text{ 252}$$

$$\frac{1}{2} \left(\frac{x^6}{\sqrt{1-x^4}} - 3 \int \frac{x^4}{\sqrt{1-x^4}} dx^2\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2} \left(\frac{x^6}{\sqrt{1-x^4}} - 3 \left(\frac{1}{2} \int \frac{1}{\sqrt{1-x^4}} dx^2 - \frac{1}{2} x^2 \sqrt{1-x^4}\right)\right)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2} \left(\frac{x^6}{\sqrt{1-x^4}} - 3 \left(\frac{\arcsin(x^2)}{2} - \frac{1}{2} x^2 \sqrt{1-x^4}\right)\right)$$

input
```
Int[x^9/(1 - x^4)^(3/2),x]
```

output
```
(x^6/Sqrt[1 - x^4] - 3*(-1/2*(x^2*Sqrt[1 - x^4]) + ArcSin[x^2]/2))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

method	result	size
risch	$-\frac{x^2(x^4-3)}{4\sqrt{-x^4+1}}-\frac{3\arcsin(x^2)}{4}$	27
pseudoelliptic	$-\frac{x^6+3\arcsin(x^2)\sqrt{-x^4+1}-3x^2}{4\sqrt{-x^4+1}}$	36
meijerg	$-\frac{i\left(\frac{i\sqrt{\pi}\,x^2\left(-5x^4+15\right)}{10\sqrt{-x^4+1}}-\frac{3i\sqrt{\pi}\,\arcsin\left(x^2\right)}{2}\right)}{2\sqrt{\pi}}$	43
trager	$\frac{x^2(x^4-3)\sqrt{-x^4+1}}{4x^4-4}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^4+1}+x^2\right)}{4}$	58
default	$-\frac{3\arcsin(x^2)}{4}+\frac{x^2\sqrt{-x^4+1}}{4}-\frac{\sqrt{-(x^2-1)^2+2-2x^2}}{4(x^2-1)}-\frac{\sqrt{-(x^2+1)^2+2+2x^2}}{4(x^2+1)}$	76
elliptic	$-\frac{3\arcsin(x^2)}{4}+\frac{x^2\sqrt{-x^4+1}}{4}-\frac{\sqrt{-(x^2-1)^2+2-2x^2}}{4(x^2-1)}-\frac{\sqrt{-(x^2+1)^2+2+2x^2}}{4(x^2+1)}$	76

input
```
int(x^9/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2*(x^4-3)/(-x^4+1)^(1/2)-3/4*arcsin(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx=\frac{6\,(x^4-1)\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)+(x^6-3\,x^2)\sqrt{-x^4+1}}{4\,(x^4-1)}$$

input
```
integrate(x^9/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(6*(x^4 - 1)*arctan((sqrt(-x^4 + 1) - 1)/x^2) + (x^6 - 3*x^2)*sqrt(-x^
4 + 1))/(x^4 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx = \begin{cases} \frac{ix^6}{4\sqrt{x^4-1}} - \frac{3ix^2}{4\sqrt{x^4-1}} + \frac{3i\operatorname{acosh}(x^2)}{4} & \text{for } |x^4| > 1 \\ -\frac{x^6}{4\sqrt{1-x^4}} + \frac{3x^2}{4\sqrt{1-x^4}} - \frac{3\operatorname{asin}(x^2)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**9/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((I*x**6/(4*sqrt(x**4 - 1)) - 3*I*x**2/(4*sqrt(x**4 - 1)) + 3*I*a
cosh(x**2)/4, Abs(x**4) > 1), (-x**6/(4*sqrt(1 - x**4)) + 3*x**2/(4*sqrt(1
 - x**4)) - 3*asin(x**2)/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx = -\frac{\frac{3(x^4-1)}{x^4} - 2}{4\left(\frac{\sqrt{-x^4+1}}{x^2} + \frac{(-x^4+1)^{\frac{3}{2}}}{x^6}\right)} + \frac{3}{4}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(x^9/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*(x^4 - 1)/x^4 - 2)/(sqrt(-x^4 + 1)/x^2 + (-x^4 + 1)^(3/2)/x^6) + 3
/4*arctan(sqrt(-x^4 + 1)/x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx = \frac{(x^4-3)\sqrt{-x^4+1}x^2}{4\,(x^4-1)} - \frac{3}{4}\arcsin\left(x^2\right)$$

input
```
integrate(x^9/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/4*(x^4 - 3)*sqrt(-x^4 + 1)*x^2/(x^4 - 1) - 3/4*arcsin(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx = \int \frac{x^9}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^9/(1 - x^4)^(3/2),x)
```

output
```
int(x^9/(1 - x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{(1-x^4)^{3/2}}\,dx = \frac{-3\sqrt{-x^4+1}\,asin(x^2)-x^6+3x^2}{4\sqrt{-x^4+1}}$$

input
```
int(x^9/(-x^4+1)^(3/2),x)
```

output
```
( - 3*sqrt( - x**4 + 1)*asin(x**2) - x**6 + 3*x**2)/(4*sqrt( - x**4 + 1))
```

3.272 $\int \frac{x^5}{(1-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^5}{(1-x^4)^{3/2}} \, dx = \frac{x^2}{2\sqrt{1-x^4}} - \frac{\arcsin(x^2)}{2}$$

output
```
1/2*x^2/(-x^4+1)^(1/2)-1/2*arcsin(x^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(1-x^4)^{3/2}} \, dx = \frac{1}{2}\left(\frac{x^2}{\sqrt{1-x^4}} - \arcsin(x^2)\right)$$

input
```
Integrate[x^5/(1 - x^4)^(3/2),x]
```

output
```
(x^2/Sqrt[1 - x^4] - ArcSin[x^2])/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {807, 252, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{(1-x^4)^{3/2}} dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2} \left(\frac{x^2}{\sqrt{1-x^4}} - \int \frac{1}{\sqrt{1-x^4}} dx^2 \right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2} \left(\frac{x^2}{\sqrt{1-x^4}} - \arcsin\left(x^2\right) \right)$$

input
```
Int[x^5/(1 - x^4)^(3/2),x]
```

output
```
(x^2/Sqrt[1 - x^4] - ArcSin[x^2])/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}-\frac{\arcsin\left(x^{2}\right)}{2}$	22
pseudoelliptic	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}-\frac{\arcsin\left(x^{2}\right)}{2}$	22
meijerg	$\frac{i\left(-\frac{i\sqrt{\pi}\,x^{2}}{\sqrt{-x^{4}+1}}+i\sqrt{\pi}\,\arcsin\left(x^{2}\right)\right)}{2\sqrt{\pi}}$	36
trager	$-\frac{x^{2}\sqrt{-x^{4}+1}}{2\left(x^{4}-1\right)}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{4}+1}+x^{2}\right)}{2}$	53
default	$-\frac{\arcsin\left(x^{2}\right)}{2}-\frac{\sqrt{-\left(x^{2}-1\right)^{2}+2-2x^{2}}}{4\left(x^{2}-1\right)}-\frac{\sqrt{-\left(x^{2}+1\right)^{2}+2+2x^{2}}}{4\left(x^{2}+1\right)}$	62
elliptic	$-\frac{\arcsin\left(x^{2}\right)}{2}-\frac{\sqrt{-\left(x^{2}-1\right)^{2}+2-2x^{2}}}{4\left(x^{2}-1\right)}-\frac{\sqrt{-\left(x^{2}+1\right)^{2}+2+2x^{2}}}{4\left(x^{2}+1\right)}$	62

input
```
int(x^5/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(-x^4+1)^(1/2)-1/2*arcsin(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{x^5}{(1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}x^2 - 2\,(x^4-1)\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)}{2\,(x^4-1)}$$

input
```
integrate(x^5/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-x^4 + 1)*x^2 - 2*(x^4 - 1)*arctan((sqrt(-x^4 + 1) - 1)/x^2))/(
x^4 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{x^5}{(1-x^4)^{3/2}}\,dx = \begin{cases} -\frac{ix^2}{2\sqrt{x^4-1}} + \frac{i\operatorname{acosh}(x^2)}{2} & \text{for } |x^4| > 1 \\ \frac{x^2}{2\sqrt{1-x^4}} - \frac{\operatorname{asin}(x^2)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((-I*x**2/(2*sqrt(x**4 - 1)) + I*acosh(x**2)/2, Abs(x**4) > 1), (
x**2/(2*sqrt(1 - x**4)) - asin(x**2)/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x^5}{(1-x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-x^4+1}} + \frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

input
```
integrate(x^5/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(-x^4 + 1) + 1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}x^2}{2(x^4-1)} - \frac{1}{2}\arcsin\left(x^2\right)$$

input
```
integrate(x^5/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1) - 1/2*arcsin(x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(1-x^4)^{3/2}}\,dx = \int \frac{x^5}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^5/(1 - x^4)^(3/2),x)
```

output
```
int(x^5/(1 - x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{(1-x^4)^{3/2}}\,dx = \frac{-\sqrt{-x^4+1}\,asin(x^2)+x^2}{2\sqrt{-x^4+1}}$$

input
```
int(x^5/(-x^4+1)^(3/2),x)
```

output
```
( - sqrt( - x**4 + 1)*asin(x**2) + x**2)/(2*sqrt( - x**4 + 1))
```

3.273 $\int \frac{x}{(1-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x}{(1-x^4)^{3/2}} \, dx = \frac{x^2}{2\sqrt{1-x^4}}$$

output
```
1/2*x^2/(-x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1-x^4)^{3/2}} \, dx = \frac{x^2}{2\sqrt{1-x^4}}$$

input
```
Integrate[x/(1 - x^4)^(3/2),x]
```

output
```
x^2/(2*Sqrt[1 - x^4])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \; 796$$

$$\frac{x^2}{2\sqrt{1-x^4}}$$

input
```
Int[x/(1 - x^4)^(3/2),x]
```

output
```
x^2/(2*Sqrt[1 - x^4])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}$	15
meijerg	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}$	15
risch	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}$	15
elliptic	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}$	15
pseudoelliptic	$\frac{x^{2}}{2\sqrt{-x^{4}+1}}$	15
trager	$-\frac{x^{2}\sqrt{-x^{4}+1}}{2(x^{4}-1)}$	22
gosper	$-\frac{(-1+x)(1+x)(x^{2}+1)x^{2}}{2(-x^{4}+1)^{\frac{3}{2}}}$	26
orering	$-\frac{(-1+x)(1+x)(x^{2}+1)x^{2}}{2(-x^{4}+1)^{\frac{3}{2}}}$	26

input
```
int(x/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(-x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x}{(1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}x^2}{2\,(x^4-1)}$$

input
```
integrate(x/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{x}{(1-x^4)^{3/2}}\,dx = \begin{cases} -\frac{ix^2}{2\sqrt{x^4-1}} & \text{for } |x^4| > 1 \\ \frac{x^2}{2\sqrt{1-x^4}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((-I*x**2/(2*sqrt(x**4 - 1)), Abs(x**4) > 1), (x**2/(2*sqrt(1 - x
**4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1-x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-x^4+1}}$$

input
```
integrate(x/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(-x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x}{(1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}x^2}{2\,(x^4-1)}$$

input
```
integrate(x/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(1-x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{1-x^4}}$$

input
```
int(x/(1 - x^4)^(3/2),x)
```

output
```
x^2/(2*(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x}{(1-x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-x^4+1}}$$

input
```
int(x/(-x^4+1)^(3/2),x)
```

output
```
x**2/(2*sqrt( - x**4 + 1))
```

3.274 $\int \frac{1}{x^3(1-x^4)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{1}{x^3 (1-x^4)^{3/2}} dx = \frac{1}{2x^2\sqrt{1-x^4}} - \frac{\sqrt{1-x^4}}{x^2}$$

output
```
1/2/x^2/(-x^4+1)^(1/2)-(-x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^3 (1-x^4)^{3/2}} dx = \frac{-1+2x^4}{2x^2\sqrt{1-x^4}}$$

input
```
Integrate[1/(x^3*(1 - x^4)^(3/2)),x]
```

output
```
(-1 + 2*x^4)/(2*x^2*Sqrt[1 - x^4])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{803}$$

$$2 \int \frac{x}{(1-x^4)^{3/2}} dx - \frac{1}{2x^2\sqrt{1-x^4}}$$

$$\Big\downarrow \text{796}$$

$$\frac{x^2}{\sqrt{1-x^4}} - \frac{1}{2x^2\sqrt{1-x^4}}$$

input
```
Int[1/(x^3*(1 - x^4)^(3/2)),x]
```

output
```
-1/2*1/(x^2*Sqrt[1 - x^4]) + x^2/Sqrt[1 - x^4]
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{2x^4-1}{2\sqrt{-x^4+1}\,x^2}$	22
meijerg	$-\frac{-2x^4+1}{2x^2\sqrt{-x^4+1}}$	22
risch	$\frac{2x^4-1}{2\sqrt{-x^4+1}\,x^2}$	22
elliptic	$\frac{2x^4-1}{2\sqrt{-x^4+1}\,x^2}$	22
pseudoelliptic	$\frac{2x^4-1}{2\sqrt{-x^4+1}\,x^2}$	22
trager	$-\frac{(2x^4-1)\sqrt{-x^4+1}}{2(x^4-1)x^2}$	29
gosper	$-\frac{(-1+x)(1+x)(x^2+1)(2x^4-1)}{2x^2(-x^4+1)^{\frac{3}{2}}}$	33
orering	$-\frac{(-1+x)(1+x)(x^2+1)(2x^4-1)}{2x^2(-x^4+1)^{\frac{3}{2}}}$	33

input
```
int(1/x^3/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x^4-1)/(-x^4+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3\left(1-x^4\right)^{3/2}}\,dx = -\frac{(2\,x^4-1)\sqrt{-x^4+1}}{2\,(x^6-x^2)}$$

input
```
integrate(1/x^3/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*x^4 - 1)*sqrt(-x^4 + 1)/(x^6 - x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^3 (1-x^4)^{3/2}}\,dx = \begin{cases} -\frac{2ix^4\sqrt{x^4-1}}{2x^6-2x^2} + \frac{i\sqrt{x^4-1}}{2x^6-2x^2} & \text{for } |x^4| > 1 \\ -\frac{2x^4\sqrt{1-x^4}}{2x^6-2x^2} + \frac{\sqrt{1-x^4}}{2x^6-2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((-2*I*x**4*sqrt(x**4 - 1)/(2*x**6 - 2*x**2) + I*sqrt(x**4 - 1)/(
2*x**6 - 2*x**2), Abs(x**4) > 1), (-2*x**4*sqrt(1 - x**4)/(2*x**6 - 2*x**2
) + sqrt(1 - x**4)/(2*x**6 - 2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (1-x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-x^4+1}} - \frac{\sqrt{-x^4+1}}{2x^2}$$

input
```
integrate(1/x^3/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(-x^4 + 1) - 1/2*sqrt(-x^4 + 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^3 (1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}x^2}{2(x^4-1)} + \frac{x^2}{4\left(\sqrt{-x^4+1}-1\right)} - \frac{\sqrt{-x^4+1}-1}{4x^2}$$

input
```
integrate(1/x^3/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^4 + 1)*x^2/(x^4 - 1) + 1/4*x^2/(sqrt(-x^4 + 1) - 1) - 1/4*(sq
rt(-x^4 + 1) - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^3 (1-x^4)^{3/2}} dx = \frac{x^4 - \frac{1}{2}}{x^2 \sqrt{1-x^4}}$$

input
```
int(1/(x^3*(1 - x^4)^(3/2)),x)
```

output
```
(x^4 - 1/2)/(x^2*(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^3 (1-x^4)^{3/2}} dx = \frac{2x^4 - 1}{2\sqrt{-x^4+1}\, x^2}$$

input
```
int(1/x^3/(-x^4+1)^(3/2),x)
```

output
```
(2*x**4 - 1)/(2*sqrt( - x**4 + 1)*x**2)
```

3.275 $\int \frac{1}{x^7(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x^7(1-x^4)^{3/2}}\,dx = \frac{1}{2x^6\sqrt{1-x^4}} - \frac{2\sqrt{1-x^4}}{3x^6} - \frac{4\sqrt{1-x^4}}{3x^2}$$

output
```
1/2/x^6/(-x^4+1)^(1/2)-2/3*(-x^4+1)^(1/2)/x^6-4/3*(-x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^7(1-x^4)^{3/2}}\,dx = \frac{-1-4x^4+8x^8}{6x^6\sqrt{1-x^4}}$$

input
```
Integrate[1/(x^7*(1 - x^4)^(3/2)),x]
```

output
```
(-1 - 4*x^4 + 8*x^8)/(6*x^6*Sqrt[1 - x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (1-x^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{803}$$
$$\frac{4}{3} \int \frac{1}{x^3 (1-x^4)^{3/2}} dx - \frac{1}{6x^6\sqrt{1-x^4}}$$
$$\Big\downarrow \text{803}$$
$$\frac{4}{3} \left(2 \int \frac{x}{(1-x^4)^{3/2}} dx - \frac{1}{2x^2\sqrt{1-x^4}}\right) - \frac{1}{6x^6\sqrt{1-x^4}}$$
$$\Big\downarrow \text{796}$$
$$\frac{4}{3} \left(\frac{x^2}{\sqrt{1-x^4}} - \frac{1}{2x^2\sqrt{1-x^4}}\right) - \frac{1}{6x^6\sqrt{1-x^4}}$$

input
```
Int[1/(x^7*(1 - x^4)^(3/2)),x]
```

output
```
-1/6*1/(x^6*Sqrt[1 - x^4]) + (4*(-1/2*1/(x^2*Sqrt[1 - x^4]) + x^2/Sqrt[1 -
 x^4]))/3
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{8x^{8}-4x^{4}-1}{6\sqrt{-x^{4}+1}\, x^{6}}$	27
meijerg	$-\frac{-8x^{8}+4x^{4}+1}{6x^{6}\sqrt{-x^{4}+1}}$	27
risch	$\frac{8x^{8}-4x^{4}-1}{6\sqrt{-x^{4}+1}\, x^{6}}$	27
elliptic	$\frac{8x^{8}-4x^{4}-1}{6\sqrt{-x^{4}+1}\, x^{6}}$	27
pseudoelliptic	$\frac{8x^{8}-4x^{4}-1}{6\sqrt{-x^{4}+1}\, x^{6}}$	27
trager	$-\frac{\left(8x^{8}-4x^{4}-1\right)\sqrt{-x^{4}+1}}{6\left(x^{4}-1\right)x^{6}}$	34
gosper	$-\frac{\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)\left(8x^{8}-4x^{4}-1\right)}{6x^{6}\left(-x^{4}+1\right)^{\frac{3}{2}}}$	38
orering	$-\frac{\left(-1+x\right)\left(1+x\right)\left(x^{2}+1\right)\left(8x^{8}-4x^{4}-1\right)}{6x^{6}\left(-x^{4}+1\right)^{\frac{3}{2}}}$	38

input
```
int(1/x^7/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(8*x^8-4*x^4-1)/(-x^4+1)^(1/2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^7 \left(1-x^4\right)^{3/2}}\, dx = -\frac{\left(8\, x^{8} - 4\, x^{4} - 1\right)\sqrt{-x^{4} + 1}}{6\,\left(x^{10} - x^{6}\right)}$$

input
```
integrate(1/x^7/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(8*x^8 - 4*x^4 - 1)*sqrt(-x^4 + 1)/(x^10 - x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^7 (1-x^4)^{3/2}}\,dx = \begin{cases} -\frac{8x^8\sqrt{-1+\frac{1}{x^4}}}{6x^8-6x^4} + \frac{4x^4\sqrt{-1+\frac{1}{x^4}}}{6x^8-6x^4} + \frac{\sqrt{-1+\frac{1}{x^4}}}{6x^8-6x^4} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{8ix^8\sqrt{1-\frac{1}{x^4}}}{6x^8-6x^4} + \frac{4ix^4\sqrt{1-\frac{1}{x^4}}}{6x^8-6x^4} + \frac{i\sqrt{1-\frac{1}{x^4}}}{6x^8-6x^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**7/(-x**4+1)**(3/2),x)
```

output
```
Piecewise((-8*x**8*sqrt(-1 + x**(-4))/(6*x**8 - 6*x**4) + 4*x**4*sqrt(-1 +
 x**(-4))/(6*x**8 - 6*x**4) + sqrt(-1 + x**(-4))/(6*x**8 - 6*x**4), 1/Abs(
x**4) > 1), (-8*I*x**8*sqrt(1 - 1/x**4)/(6*x**8 - 6*x**4) + 4*I*x**4*sqrt(
1 - 1/x**4)/(6*x**8 - 6*x**4) + I*sqrt(1 - 1/x**4)/(6*x**8 - 6*x**4), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^7 (1-x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{-x^4+1}} - \frac{\sqrt{-x^4+1}}{x^2} - \frac{(-x^4+1)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate(1/x^7/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(-x^4 + 1) - sqrt(-x^4 + 1)/x^2 - 1/6*(-x^4 + 1)^(3/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(43) = 86$.

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^7 (1-x^4)^{3/2}}\,dx = \frac{x^6\left(\frac{21\left(\sqrt{-x^4+1}-1\right)^2}{x^4}+1\right)}{48\left(\sqrt{-x^4+1}-1\right)^3} - \frac{\sqrt{-x^4+1}x^2}{2(x^4-1)} - \frac{7\left(\sqrt{-x^4+1}-1\right)}{16\,x^2} - \frac{\left(\sqrt{-x^4+1}-1\right)^3}{48\,x^6}$$

input
```
integrate(1/x^7/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/48*x^6*(21*(sqrt(-x^4 + 1) - 1)^2/x^4 + 1)/(sqrt(-x^4 + 1) - 1)^3 - 1/2*
sqrt(-x^4 + 1)*x^2/(x^4 - 1) - 7/16*(sqrt(-x^4 + 1) - 1)/x^2 - 1/48*(sqrt(
-x^4 + 1) - 1)^3/x^6
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^7 (1-x^4)^{3/2}}\,dx = \frac{8\left(x^4-1\right)^2+12\,x^4-9}{6\,x^6\sqrt{1-x^4}}$$

input
```
int(1/(x^7*(1 - x^4)^(3/2)),x)
```

output
```
(8*(x^4 - 1)^2 + 12*x^4 - 9)/(6*x^6*(1 - x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^7 (1-x^4)^{3/2}}\,dx = \frac{8x^8-4x^4-1}{6\sqrt{-x^4+1}\,x^6}$$

input
```
int(1/x^7/(-x^4+1)^(3/2),x)
```

output
```
(8*x**8 - 4*x**4 - 1)/(6*sqrt( - x**4 + 1)*x**6)
```

3.276 $\int \frac{x^{12}}{(1-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{x^{12}}{(1-x^4)^{3/2}} \, dx = \frac{x^9}{2\sqrt{1-x^4}} + \frac{15}{14}x\sqrt{1-x^4} + \frac{9}{14}x^5\sqrt{1-x^4} - \frac{15}{14}\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x^9/(-x^4+1)^(1/2)+15/14*x*(-x^4+1)^(1/2)+9/14*x^5*(-x^4+1)^(1/2)-15/1
4*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{x^{12}}{(1-x^4)^{3/2}} \, dx = -\frac{x\left(-15+6x^4+2x^8+15\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)}{14\sqrt{1-x^4}}$$

input
```
Integrate[x^12/(1 - x^4)^(3/2),x]
```

output
```
-1/14*(x*(-15 + 6*x^4 + 2*x^8 + 15*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/
2, 5/4, x^4]))/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {817, 843, 843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^9}{2\sqrt{1-x^4}}-\frac{9}{2}\int \frac{x^8}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9}{2\sqrt{1-x^4}}-\frac{9}{2}\left(\frac{5}{7}\int \frac{x^4}{\sqrt{1-x^4}}dx-\frac{1}{7}x^5\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9}{2\sqrt{1-x^4}}-\frac{9}{2}\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}dx-\frac{1}{3}x\sqrt{1-x^4}\right)-\frac{1}{7}x^5\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{x^9}{2\sqrt{1-x^4}}-\frac{9}{2}\left(\frac{5}{7}\left(\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)-\frac{1}{3}x\sqrt{1-x^4}\right)-\frac{1}{7}x^5\sqrt{1-x^4}\right)$$

input
```
Int[x^12/(1 - x^4)^(3/2),x]
```

output
```
x^9/(2*Sqrt[1 - x^4]) - (9*(-1/7*(x^5*Sqrt[1 - x^4]) + (5*(-1/3*(x*Sqrt[1
- x^4]) + EllipticF[ArcSin[x], -1]/3))/7))/2
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{x^{13} \operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{13}{4}\right],\left[\frac{17}{4}\right],x^{4}\right)}{13}$	15
risch	$-\frac{x\left(2x^{8}+6x^{4}-15\right)}{14\sqrt{-x^{4}+1}}-\frac{15\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}(x,i)}{14\sqrt{-x^{4}+1}}$	57
default	$\frac{x}{2\sqrt{-x^{4}+1}}+\frac{x^{5}\sqrt{-x^{4}+1}}{7}+\frac{4x\sqrt{-x^{4}+1}}{7}-\frac{15\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}(x,i)}{14\sqrt{-x^{4}+1}}$	71
elliptic	$\frac{x}{2\sqrt{-x^{4}+1}}+\frac{x^{5}\sqrt{-x^{4}+1}}{7}+\frac{4x\sqrt{-x^{4}+1}}{7}-\frac{15\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}(x,i)}{14\sqrt{-x^{4}+1}}$	71

input
```
int(x^12/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/13*x^13*hypergeom([3/2,13/4],[17/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx = -\frac{15\,(i\,x^4-i)F\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)-(2\,x^9+6\,x^5-15\,x)\sqrt{-x^4+1}}{14\,(x^4-1)}$$

input
```
integrate(x^12/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/14*(15*(I*x^4 - I)*elliptic_f(arcsin(1/x), -1) - (2*x^9 + 6*x^5 - 15*x)
*sqrt(-x^4 + 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx = \frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12/(-x**4+1)**(3/2),x)
```

output
```
x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), x**4*exp_polar(2*I*pi))/(4*g
amma(17/4))
```

Maxima [F]

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{12}}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^12/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^12/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{12}}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^12/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^12/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^12/(1 - x^4)^(3/2),x)
```

output
```
int(x^12/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{12}}{(1-x^4)^{3/2}}\,dx = \frac{\sqrt{-x^4+1}\,x^9+3\sqrt{-x^4+1}\,x^5-15\sqrt{-x^4+1}\,x-15\left(\int \frac{\sqrt{-x^4+1}}{x^8-2x^4+1}dx\right)x^4+15\left(\int \frac{\sqrt{-x^4}}{x^8-2x^4}\right.}{7x^4-7}$$

input
```
int(x^12/(-x^4+1)^(3/2),x)
```

output
```
(sqrt( - x**4 + 1)*x**9 + 3*sqrt( - x**4 + 1)*x**5 - 15*sqrt( - x**4 + 1)*
x - 15*int(sqrt( - x**4 + 1)/(x**8 - 2*x**4 + 1),x)*x**4 + 15*int(sqrt( -
x**4 + 1)/(x**8 - 2*x**4 + 1),x))/(7*(x**4 - 1))
```

3.277 $\int \frac{x^8}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = \frac{x^5}{2\sqrt{1-x^4}} + \frac{5}{6}x\sqrt{1-x^4} - \frac{5}{6}\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x^5/(-x^4+1)^(1/2)+5/6*x*(-x^4+1)^(1/2)-5/6*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.60 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = -\frac{x\left(-5+2x^4+5\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)}{6\sqrt{1-x^4}}$$

input
```
Integrate[x^8/(1 - x^4)^(3/2),x]
```

output
```
-1/6*(x*(-5 + 2*x^4 + 5*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4
]))/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {817, 843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^5}{2\sqrt{1-x^4}}-\frac{5}{2}\int \frac{x^4}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5}{2\sqrt{1-x^4}}-\frac{5}{2}\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^4}}dx-\frac{1}{3}x\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{x^5}{2\sqrt{1-x^4}}-\frac{5}{2}\left(\frac{1}{3}\text{EllipticF}(\arcsin(x),-1)-\frac{1}{3}x\sqrt{1-x^4}\right)$$

input
```
Int[x^8/(1 - x^4)^(3/2),x]
```

output
```
x^5/(2*Sqrt[1 - x^4]) - (5*(-1/3*(x*Sqrt[1 - x^4]) + EllipticF[ArcSin[x],
-1]/3))/2
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.35

method	result	size
meijerg	$\frac{x^{9} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{9}{4}\right], \left[\frac{13}{4}\right], x^{4}\right)}{9}$	15
risch	$-\frac{x \left(2 x^{4}-5\right)}{6 \sqrt{-x^{4}+1}}-\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{6 \sqrt{-x^{4}+1}}$	52
default	$\frac{x}{2 \sqrt{-x^{4}+1}}+\frac{x \sqrt{-x^{4}+1}}{3}-\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{6 \sqrt{-x^{4}+1}}$	57
elliptic	$\frac{x}{2 \sqrt{-x^{4}+1}}+\frac{x \sqrt{-x^{4}+1}}{3}-\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{6 \sqrt{-x^{4}+1}}$	57

input
```
int(x^8/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*hypergeom([3/2,9/4],[13/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = -\frac{5\,(i\,x^4-i)F(\arcsin\left(\frac{1}{x}\right)\,|-1)-(2\,x^5-5\,x)\sqrt{-x^4+1}}{6\,(x^4-1)}$$

input
```
integrate(x^8/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(5*(I*x^4 - I)*elliptic_f(arcsin(1/x), -1) - (2*x^5 - 5*x)*sqrt(-x^4
+ 1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(-x**4+1)**(3/2),x)
```

output
```
x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), x**4*exp_polar(2*I*pi))/(4*gamm
a(13/4))
```

Maxima [F]

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = \int \frac{x^8}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = \int \frac{x^8}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = \int \frac{x^8}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^8/(1 - x^4)^(3/2),x)
```

output
```
int(x^8/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8}{(1-x^4)^{3/2}}\,dx = \frac{\sqrt{-x^4+1}\,x^5 - 5\sqrt{-x^4+1}\,x - 5\left(\int \frac{\sqrt{-x^4+1}}{x^8-2x^4+1}dx\right)x^4 + 5\left(\int \frac{\sqrt{-x^4+1}}{x^8-2x^4+1}dx\right)}{3x^4-3}$$

input
```
int(x^8/(-x^4+1)^(3/2),x)
```

output
```
(sqrt( - x**4 + 1)*x**5 - 5*sqrt( - x**4 + 1)*x - 5*int(sqrt( - x**4 + 1)/
(x**8 - 2*x**4 + 1),x)*x**4 + 5*int(sqrt( - x**4 + 1)/(x**8 - 2*x**4 + 1),
x))/(3*(x**4 - 1))
```

3.278 $\int \frac{x^4}{(1-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{x^4}{(1-x^4)^{3/2}} \, dx = \frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}\text{EllipticF}(\arcsin(x), -1)$$

output
```
1/2*x/(-x^4+1)^(1/2)-1/2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{x^4}{(1-x^4)^{3/2}} \, dx = \frac{1}{2}x\left(\frac{1}{\sqrt{1-x^4}} - \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, x^4\right)\right)$$

input
```
Integrate[x^4/(1 - x^4)^(3/2),x]
```

output
```
(x*(1/Sqrt[1 - x^4] - Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {817, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{762}$$

$$\frac{x}{2\sqrt{1-x^4}} - \frac{1}{2}\text{EllipticF}(\arcsin(x), -1)$$

input
```
Int[x^4/(1 - x^4)^(3/2),x]
```

output
```
x/(2*Sqrt[1 - x^4]) - EllipticF[ArcSin[x], -1]/2
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

method	result	size
meijerg	$\frac{x^{5} \operatorname{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],x^{4}\right)}{5}$	15
default	$\frac{x}{2\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(x,i\right)}{2\sqrt{-x^{4}+1}}$	45
risch	$\frac{x}{2\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(x,i\right)}{2\sqrt{-x^{4}+1}}$	45
elliptic	$\frac{x}{2\sqrt{-x^{4}+1}}-\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}\left(x,i\right)}{2\sqrt{-x^{4}+1}}$	45

input
```
int(x^4/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([5/4,3/2],[9/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{x^{4}}{(1-x^{4})^{3/2}}\,dx=-\frac{(x^{4}-1)F(\arcsin(x)\mid -1)+\sqrt{-x^{4}+1}x}{2\,(x^{4}-1)}$$

input
```
integrate(x^4/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((x^4 - 1)*elliptic_f(arcsin(x), -1) + sqrt(-x^4 + 1)*x)/(x^4 - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(15) = 30.

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{(1-x^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(-x**4+1)**(3/2),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(9/4))
```

Maxima [F]

$$\int \frac{x^4}{(1-x^4)^{3/2}}\,dx = \int \frac{x^4}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(1-x^4)^{3/2}}\,dx = \int \frac{x^4}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1-x^4)^{3/2}}\,dx = \int \frac{x^4}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^4/(1 - x^4)^(3/2),x)
```

output
```
int(x^4/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(1-x^4)^{3/2}}\,dx = \frac{-\sqrt{-x^4+1}\,x - \left(\int \frac{\sqrt{-x^4+1}}{x^8-2x^4+1}dx\right)x^4 + \int \frac{\sqrt{-x^4+1}}{x^8-2x^4+1}dx}{x^4-1}$$

input
```
int(x^4/(-x^4+1)^(3/2),x)
```

output
```
( - sqrt( - x**4 + 1)*x - int(sqrt( - x**4 + 1)/(x**8 - 2*x**4 + 1),x)*x**
4 + int(sqrt( - x**4 + 1)/(x**8 - 2*x**4 + 1),x))/(x**4 - 1)
```

3.279 $\int \frac{1}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \frac{x}{2\sqrt{1-x^4}} + \frac{1}{2}\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x/(-x^4+1)^(1/2)+1/2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \frac{1}{2}x\left(\frac{1}{\sqrt{1-x^4}} + \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)$$

input
```
Integrate[(1 - x^4)^(-3/2),x]
```

output
```
(x*(1/Sqrt[1 - x^4] + Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {749, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{749}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}dx+\frac{x}{2\sqrt{1-x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{2}\text{EllipticF}(\arcsin(x),-1)+\frac{x}{2\sqrt{1-x^4}}$$

input
```
Int[(1 - x^4)^(-3/2),x]
```

output
```
x/(2*Sqrt[1 - x^4]) + EllipticF[ArcSin[x], -1]/2
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

method	result	size
meijerg	$x \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{4}\right], x^{4}\right)$	12
default	$\frac{x}{2\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}(x,i)}{2\sqrt{-x^{4}+1}}$	45
risch	$\frac{x}{2\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}(x,i)}{2\sqrt{-x^{4}+1}}$	45
elliptic	$\frac{x}{2\sqrt{-x^{4}+1}}+\frac{\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}(x,i)}{2\sqrt{-x^{4}+1}}$	45

input
```
int(1/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \frac{(x^4-1)F(\arcsin(x)\,|-1)-\sqrt{-x^4+1}x}{2(x^4-1)}$$

input
```
integrate(1/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((x^4 - 1)*elliptic_f(arcsin(x), -1) - sqrt(-x^4 + 1)*x)/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-x**4+1)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(2*I*pi))/(4*gamma(5/
4))
```

Maxima [F]

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 1)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.40

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ x^4\right)$$

input
```
int(1/(1 - x^4)^(3/2),x)
```

output
```
x*hypergeom([1/4, 3/2], 5/4, x^4)
```

Reduce [F]

$$\int \frac{1}{(1-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^8-2x^4+1}dx$$

input
```
int(1/(-x^4+1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**8 - 2*x**4 + 1),x)
```

3.280 $\int \frac{1}{x^4(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{1}{x^4\left(1-x^4\right)^{3/2}}\,dx=\frac{1}{2x^3\sqrt{1-x^4}}-\frac{5\sqrt{1-x^4}}{6x^3}+\frac{5}{6}\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2/x^3/(-x^4+1)^(1/2)-5/6*(-x^4+1)^(1/2)/x^3+5/6*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4\left(1-x^4\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},x^4\right)}{3x^3}$$

input
```
Integrate[1/(x^4*(1 - x^4)^(3/2)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/4, 3/2, 1/4, x^4]/x^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {819, 847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5}{2} \int \frac{1}{x^4\sqrt{1-x^4}}dx + \frac{1}{2x^3\sqrt{1-x^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{2} \left(\frac{1}{3} \int \frac{1}{\sqrt{1-x^4}}dx - \frac{\sqrt{1-x^4}}{3x^3}\right) + \frac{1}{2x^3\sqrt{1-x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{5}{2} \left(\frac{1}{3} \text{EllipticF}(\arcsin(x),-1) - \frac{\sqrt{1-x^4}}{3x^3}\right) + \frac{1}{2x^3\sqrt{1-x^4}}$$

input
```
Int[1/(x^4*(1 - x^4)^(3/2)),x]
```

output
```
1/(2*x^3*Sqrt[1 - x^4]) + (5*(-1/3*Sqrt[1 - x^4]/x^3 + EllipticF[ArcSin[x]
, -1]/3))/2
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{3}{4}, \frac{3}{2}\right], \left[\frac{1}{4}\right], x^{4}\right)}{3 x^{3}}$	15
risch	$\frac{5 x^{4}-2}{6 x^{3} \sqrt{-x^{4}+1}}+\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{6 \sqrt{-x^{4}+1}}$	54
default	$-\frac{\sqrt{-x^{4}+1}}{3 x^{3}}+\frac{x}{2 \sqrt{-x^{4}+1}}+\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{6 \sqrt{-x^{4}+1}}$	59
elliptic	$-\frac{\sqrt{-x^{4}+1}}{3 x^{3}}+\frac{x}{2 \sqrt{-x^{4}+1}}+\frac{5 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \operatorname{EllipticF}\left(x , i\right)}{6 \sqrt{-x^{4}+1}}$	59

input
```
int(1/x^4/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*hypergeom([-3/4,3/2],[1/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx = \frac{5\,(x^7-x^3)F(\arcsin(x)\mid -1)-(5\,x^4-2)\sqrt{-x^4+1}}{6\,(x^7-x^3)}$$

input
```
integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(5*(x^7 - x^3)*elliptic_f(arcsin(x), -1) - (5*x^4 - 2)*sqrt(-x^4 + 1))
/(x^7 - x^3)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(-x**4+1)**(3/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_polar(2*I*pi))/(4*x**3*gam
ma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx = \int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx$$

input
```
int(1/(x^4*(1 - x^4)^(3/2)),x)
```

output
```
int(1/(x^4*(1 - x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (1-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^{12}-2x^8+x^4}dx$$

input
```
int(1/x^4/(-x^4+1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**12 - 2*x**8 + x**4),x)
```

3.281 $\int \frac{1}{x^8(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{1}{x^8(1-x^4)^{3/2}}\,dx = \frac{1}{2x^7\sqrt{1-x^4}} - \frac{9\sqrt{1-x^4}}{14x^7} - \frac{15\sqrt{1-x^4}}{14x^3} + \frac{15}{14}\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2/x^7/(-x^4+1)^(1/2)-9/14*(-x^4+1)^(1/2)/x^7-15/14*(-x^4+1)^(1/2)/x^3+15
/14*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^8(1-x^4)^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},-\frac{3}{4},x^4\right)}{7x^7}$$

input
```
Integrate[1/(x^8*(1 - x^4)^(3/2)),x]
```

output
```
-1/7*Hypergeometric2F1[-7/4, 3/2, -3/4, x^4]/x^7
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {819, 847, 847, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{9}{2} \int \frac{1}{x^8 \sqrt{1-x^4}}dx+\frac{1}{2 x^7 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{9}{2} \left(\frac{5}{7} \int \frac{1}{x^4 \sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{7 x^7}\right)+\frac{1}{2 x^7 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{9}{2} \left(\frac{5}{7} \left(\frac{1}{3} \int \frac{1}{\sqrt{1-x^4}}dx-\frac{\sqrt{1-x^4}}{3 x^3}\right)-\frac{\sqrt{1-x^4}}{7 x^7}\right)+\frac{1}{2 x^7 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{9}{2} \left(\frac{5}{7} \left(\frac{1}{3} \text{EllipticF}(\arcsin (x),-1)-\frac{\sqrt{1-x^4}}{3 x^3}\right)-\frac{\sqrt{1-x^4}}{7 x^7}\right)+\frac{1}{2 x^7 \sqrt{1-x^4}}$$

input
```
Int[1/(x^8*(1 - x^4)^(3/2)),x]
```

output
```
1/(2*x^7*Sqrt[1 - x^4]) + (9*(-1/7*Sqrt[1 - x^4]/x^7 + (5*(-1/3*Sqrt[1 - x
^4]/x^3 + EllipticF[ArcSin[x], -1]/3))/7))/2
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.79 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{7}{4},\frac{3}{2}\right],\left[-\frac{3}{4}\right],x^{4}\right)}{7x^{7}}$	15
risch	$\frac{15x^{8}-6x^{4}-2}{14x^{7}\sqrt{-x^{4}+1}}+\frac{15\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}(x,i)}{14\sqrt{-x^{4}+1}}$	59
default	$-\frac{\sqrt{-x^{4}+1}}{7x^{7}}-\frac{4\sqrt{-x^{4}+1}}{7x^{3}}+\frac{x}{2\sqrt{-x^{4}+1}}+\frac{15\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}(x,i)}{14\sqrt{-x^{4}+1}}$	73
elliptic	$-\frac{\sqrt{-x^{4}+1}}{7x^{7}}-\frac{4\sqrt{-x^{4}+1}}{7x^{3}}+\frac{x}{2\sqrt{-x^{4}+1}}+\frac{15\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\operatorname{EllipticF}(x,i)}{14\sqrt{-x^{4}+1}}$	73

input
```
int(1/x^8/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7*hypergeom([-7/4,3/2],[-3/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^8 (1-x^4)^{3/2}}\,dx = \frac{15\,(x^{11}-x^7)F(\arcsin(x)\mid -1)-(15\,x^8-6\,x^4-2)\sqrt{-x^4+1}}{14\,(x^{11}-x^7)}$$

input
```
integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/14*(15*(x^11 - x^7)*elliptic_f(arcsin(x), -1) - (15*x^8 - 6*x^4 - 2)*sqr
t(-x^4 + 1))/(x^11 - x^7)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^8 (1-x^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{2} \\ -\frac{3}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(-x**4+1)**(3/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), x**4*exp_polar(2*I*pi))/(4*x**7*ga
mma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8 (1-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}}x^8}\,dx$$

input
```
integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (1-x^4)^{3/2}}\, dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}} x^8}\, dx$$

input
```
integrate(1/x^8/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (1-x^4)^{3/2}}\, dx = \int \frac{1}{x^8 (1-x^4)^{3/2}}\, dx$$

input
```
int(1/(x^8*(1 - x^4)^(3/2)),x)
```

output
```
int(1/(x^8*(1 - x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (1-x^4)^{3/2}}\, dx = \int \frac{\sqrt{-x^4+1}}{x^{16}-2x^{12}+x^8} dx$$

input
```
int(1/x^8/(-x^4+1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**16 - 2*x**12 + x**8),x)
```

3.282 $\int \frac{x^{14}}{(1-x^4)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{x^{14}}{(1-x^4)^{3/2}} dx = \frac{x^{11}}{2\sqrt{1-x^4}} + \frac{77}{90}x^3\sqrt{1-x^4} + \frac{11}{18}x^7\sqrt{1-x^4} - \frac{77}{30}E(\arcsin(x)|-1) + \frac{77}{30}\operatorname{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x^11/(-x^4+1)^(1/2)+77/90*x^3*(-x^4+1)^(1/2)+11/18*x^7*(-x^4+1)^(1/2)-
77/30*EllipticE(x,I)+77/30*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{x^{14}}{(1-x^4)^{3/2}} dx = -\frac{x^3\left(77+11x^4+5x^8-77\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},x^4\right)\right)}{45\sqrt{1-x^4}}$$

input
```
Integrate[x^14/(1 - x^4)^(3/2),x]
```

output
```
-1/45*(x^3*(77 + 11*x^4 + 5*x^8 - 77*Sqrt[1 - x^4]*Hypergeometric2F1[3/4,
3/2, 7/4, x^4]))/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {817, 843, 843, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\int \frac{x^{10}}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\left(\frac{7}{9}\int \frac{x^6}{\sqrt{1-x^4}}dx-\frac{1}{9}x^7\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\left(\frac{7}{9}\left(\frac{3}{5}\int \frac{x^2}{\sqrt{1-x^4}}dx-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{836}$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\left(\frac{7}{9}\left(\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\int \frac{1}{\sqrt{1-x^4}}dx\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\left(\frac{7}{9}\left(\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{1388}$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\left(\frac{7}{9}\left(\frac{3}{5}\left(\int\frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}\right)$$

$$\Big\downarrow \; 327$$

$$\frac{x^{11}}{2\sqrt{1-x^4}}-\frac{11}{2}\left(\frac{7}{9}\left(\frac{3}{5}(E(\arcsin(x)|-1)-\text{EllipticF}(\arcsin(x),-1))-\frac{1}{5}x^3\sqrt{1-x^4}\right)-\frac{1}{9}x^7\sqrt{1-x^4}\right)$$

input
```
Int[x^14/(1 - x^4)^(3/2),x]
```

output
```
x^11/(2*Sqrt[1 - x^4]) - (11*(-1/9*(x^7*Sqrt[1 - x^4]) + (7*(-1/5*(x^3*Sqr
t[1 - x^4]) + (3*(EllipticE[ArcSin[x], -1] - EllipticF[ArcSin[x], -1]))/5)
)/9))/2
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.70 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

method	result	size
meijerg	$\frac{x^{15} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{15}{4}\right],\left[\frac{19}{4}\right], x^{4}\right)}{15}$	15
risch	$-\frac{x^{3}\left(10 x^{8}+22 x^{4}-77\right)}{90 \sqrt{-x^{4}+1}}+\frac{77 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}(x, i)-\operatorname{EllipticE}(x, i)\right)}{30 \sqrt{-x^{4}+1}}$	66
default	$\frac{x^{3}}{2 \sqrt{-x^{4}+1}}+\frac{x^{7} \sqrt{-x^{4}+1}}{9}+\frac{16 x^{3} \sqrt{-x^{4}+1}}{45}+\frac{77 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}(x, i)-\operatorname{EllipticE}(x, i)\right)}{30 \sqrt{-x^{4}+1}}$	82
elliptic	$\frac{x^{3}}{2 \sqrt{-x^{4}+1}}+\frac{x^{7} \sqrt{-x^{4}+1}}{9}+\frac{16 x^{3} \sqrt{-x^{4}+1}}{45}+\frac{77 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}(x, i)-\operatorname{EllipticE}(x, i)\right)}{30 \sqrt{-x^{4}+1}}$	82

input
```
int(x^14/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x^15*hypergeom([3/2,15/4],[19/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx =$$
$$-\frac{231\,(-i\,x^5+i\,x)E(\arcsin\left(\frac{1}{x}\right)\,|-1)+231\,(i\,x^5-i\,x)F(\arcsin\left(\frac{1}{x}\right)\,|-1)-(10\,x^{12}+22\,x^8+154\,x^4-23}{90\,(x^5-x)}$$

input
```
integrate(x^14/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/90*(231*(-I*x^5 + I*x)*elliptic_e(arcsin(1/x), -1) + 231*(I*x^5 - I*x)*
elliptic_f(arcsin(1/x), -1) - (10*x^12 + 22*x^8 + 154*x^4 - 231)*sqrt(-x^4
 + 1))/(x^5 - x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.44

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx = \frac{x^{15}\Gamma\left(\frac{15}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\right|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**14/(-x**4+1)**(3/2),x)
```

output
```
x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,), x**4*exp_polar(2*I*pi))/(4*g
amma(19/4))
```

Maxima [F]

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{14}}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^14/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{14}}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^14/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^14/(1 - x^4)^(3/2),x)
```

output
```
int(x^14/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(1-x^4)^{3/2}}\,dx = \frac{5\sqrt{-x^4+1}\,x^{11} + 11\sqrt{-x^4+1}\,x^7 + 77\sqrt{-x^4+1}\,x^3 + 231\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^8-2x^4+1}dx\right)x^4 - 231\left(\int \right.}{45x^4-45}$$

input
```
int(x^14/(-x^4+1)^(3/2),x)
```

output
```
(5*sqrt( - x**4 + 1)*x**11 + 11*sqrt( - x**4 + 1)*x**7 + 77*sqrt( - x**4 +
 1)*x**3 + 231*int((sqrt( - x**4 + 1)*x**2)/(x**8 - 2*x**4 + 1),x)*x**4 -
231*int((sqrt( - x**4 + 1)*x**2)/(x**8 - 2*x**4 + 1),x))/(45*(x**4 - 1))
```

3.283 $\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = \frac{x^7}{2\sqrt{1-x^4}} + \frac{7}{10}x^3\sqrt{1-x^4} - \frac{21}{10}E(\arcsin(x)|-1) + \frac{21}{10}\operatorname{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x^7/(-x^4+1)^(1/2)+7/10*x^3*(-x^4+1)^(1/2)-21/10*EllipticE(x,I)+21/10*
EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = -\frac{x^3\left(7+x^4-7\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},x^4\right)\right)}{5\sqrt{1-x^4}}$$

input
```
Integrate[x^10/(1 - x^4)^(3/2),x]
```

output
```
-1/5*(x^3*(7 + x^4 - 7*Sqrt[1 - x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, x^4]
))/Sqrt[1 - x^4]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {817, 843, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{x^7}{2\sqrt{1-x^4}}-\frac{7}{2}\int \frac{x^6}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^7}{2\sqrt{1-x^4}}-\frac{7}{2}\left(\frac{3}{5}\int \frac{x^2}{\sqrt{1-x^4}}dx-\frac{1}{5}x^3\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{ 836}$$

$$\frac{x^7}{2\sqrt{1-x^4}}-\frac{7}{2}\left(\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\int \frac{1}{\sqrt{1-x^4}}dx\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{ 762}$$

$$\frac{x^7}{2\sqrt{1-x^4}}-\frac{7}{2}\left(\frac{3}{5}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{ 1388}$$

$$\frac{x^7}{2\sqrt{1-x^4}}-\frac{7}{2}\left(\frac{3}{5}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)\right)-\frac{1}{5}x^3\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{ 327}$$

$$\frac{x^7}{2\sqrt{1-x^4}}-\frac{7}{2}\left(\frac{3}{5}(E(\arcsin(x)|-1)-\text{EllipticF}(\arcsin(x),-1))-\frac{1}{5}x^3\sqrt{1-x^4}\right)$$

input
```
Int[x^10/(1 - x^4)^(3/2),x]
```

output
```
x^7/(2*Sqrt[1 - x^4]) - (7*(-1/5*(x^3*Sqrt[1 - x^4]) + (3*(EllipticE[ArcSi
n[x], -1] - EllipticF[ArcSin[x], -1]))/5))/2
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.28

method	result	size
meijerg	$\frac{x^{11}\,\mathrm{hypergeom}\left(\left[\frac{3}{2},\frac{11}{4}\right],\left[\frac{15}{4}\right],x^{4}\right)}{11}$	15
risch	$-\frac{x^{3}\left(2x^{4}-7\right)}{10\sqrt{-x^{4}+1}}+\frac{21\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i)\right)}{10\sqrt{-x^{4}+1}}$	61
default	$\frac{x^{3}}{2\sqrt{-x^{4}+1}}+\frac{x^{3}\sqrt{-x^{4}+1}}{5}+\frac{21\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i)\right)}{10\sqrt{-x^{4}+1}}$	68
elliptic	$\frac{x^{3}}{2\sqrt{-x^{4}+1}}+\frac{x^{3}\sqrt{-x^{4}+1}}{5}+\frac{21\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\mathrm{EllipticF}(x,i)-\mathrm{EllipticE}(x,i)\right)}{10\sqrt{-x^{4}+1}}$	68

input
```
int(x^10/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11*hypergeom([3/2,11/4],[15/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = -\frac{21\,(-i\,x^{5}+i\,x)E(\arcsin\left(\frac{1}{x}\right)\,|-1)+21\,(i\,x^{5}-i\,x)F(\arcsin\left(\frac{1}{x}\right)\,|-1)-(2\,x^{8}+14\,x^{4}-21)\sqrt{-x^{4}+1}}{10\,(x^{5}-x)}$$

input
```
integrate(x^10/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(21*(-I*x^5 + I*x)*elliptic_e(arcsin(1/x), -1) + 21*(I*x^5 - I*x)*el
liptic_f(arcsin(1/x), -1) - (2*x^8 + 14*x^4 - 21)*sqrt(-x^4 + 1))/(x^5 - x
)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(-x**4+1)**(3/2),x)
```

output
```
x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), x**4*exp_polar(2*I*pi))/(4*g
amma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{10}}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^10/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{10}}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^10/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = \int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^10/(1 - x^4)^(3/2),x)
```

output
```
int(x^10/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(1-x^4)^{3/2}}\,dx = \frac{\sqrt{-x^4+1}\,x^7+7\sqrt{-x^4+1}\,x^3+21\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^8-2x^4+1}dx\right)x^4-21\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^8-2x^4+1}dx\right)}{5x^4-5}$$

input
```
int(x^10/(-x^4+1)^(3/2),x)
```

output
```
(sqrt( - x**4 + 1)*x**7 + 7*sqrt( - x**4 + 1)*x**3 + 21*int((sqrt( - x**4
+ 1)*x**2)/(x**8 - 2*x**4 + 1),x)*x**4 - 21*int((sqrt( - x**4 + 1)*x**2)/(
x**8 - 2*x**4 + 1),x))/(5*(x**4 - 1))
```

3.284 $\int \frac{x^6}{(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = \frac{x^3}{2\sqrt{1-x^4}} - \frac{3}{2}E(\arcsin(x)|-1) + \frac{3}{2}\,\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x^3/(-x^4+1)^(1/2)-3/2*EllipticE(x,I)+3/2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = x^3\left(-\frac{1}{\sqrt{1-x^4}} + \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},x^4\right)\right)$$

input
```
Integrate[x^6/(1 - x^4)^(3/2),x]
```

output
```
x^3*(-(1/Sqrt[1 - x^4]) + Hypergeometric2F1[3/4, 3/2, 7/4, x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {817, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{817}$$

$$\frac{x^3}{2\sqrt{1-x^4}}-\frac{3}{2}\int \frac{x^2}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{836}$$

$$\frac{x^3}{2\sqrt{1-x^4}}-\frac{3}{2}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\int \frac{1}{\sqrt{1-x^4}}dx\right)$$

$$\Big\downarrow \text{762}$$

$$\frac{x^3}{2\sqrt{1-x^4}}-\frac{3}{2}\left(\int \frac{x^2+1}{\sqrt{1-x^4}}dx-\text{EllipticF}(\arcsin(x),-1)\right)$$

$$\Big\downarrow \text{1388}$$

$$\frac{x^3}{2\sqrt{1-x^4}}-\frac{3}{2}\left(\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx-\text{EllipticF}(\arcsin(x),-1)\right)$$

$$\Big\downarrow \text{327}$$

$$\frac{x^3}{2\sqrt{1-x^4}}-\frac{3}{2}(E(\arcsin(x)|-1)-\text{EllipticF}(\arcsin(x),-1))$$

input
```
Int[x^6/(1 - x^4)^(3/2),x]
```

output
```
x^3/(2*Sqrt[1 - x^4]) - (3*(EllipticE[ArcSin[x], -1] - EllipticF[ArcSin[x]
, -1]))/2
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

method	result	size
meijerg	$\frac{x^{7} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{7}{4}\right], \left[\frac{11}{4}\right], x^{4}\right)}{7}$	15
default	$\frac{x^{3}}{2 \sqrt{-x^{4}+1}}+\frac{3 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{2 \sqrt{-x^{4}+1}}$	54
risch	$\frac{x^{3}}{2 \sqrt{-x^{4}+1}}+\frac{3 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{2 \sqrt{-x^{4}+1}}$	54
elliptic	$\frac{x^{3}}{2 \sqrt{-x^{4}+1}}+\frac{3 \sqrt{-x^{2}+1}\, \sqrt{x^{2}+1}\, \left(\operatorname{EllipticF}\left(x , i\right)-\operatorname{EllipticE}\left(x , i\right)\right)}{2 \sqrt{-x^{4}+1}}$	54

input
```
int(x^6/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*hypergeom([3/2,7/4],[11/4],x^4)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \frac{x^6}{\left(1-x^4\right)^{3/2}}\,dx = -\frac{3\,(-i\,x^5+i\,x)E(\arcsin\left(\frac{1}{x}\right)\,|-1)+3\,(i\,x^5-i\,x)F(\arcsin\left(\frac{1}{x}\right)\,|-1)-(2\,x^4-3)\sqrt{-x^4+1}}{2\,(x^5-x)}$$

input
```
integrate(x^6/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*(-I*x^5 + I*x)*elliptic_e(arcsin(1/x), -1) + 3*(I*x^5 - I*x)*ellip
tic_f(arcsin(1/x), -1) - (2*x^4 - 3)*sqrt(-x^4 + 1))/(x^5 - x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(-x**4+1)**(3/2),x)
```

output
```
x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(2*I*pi))/(4*gamm
a(11/4))
```

Maxima [F]

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = \int \frac{x^6}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = \int \frac{x^6}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = \int \frac{x^6}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^6/(1 - x^4)^(3/2),x)
```

output
```
int(x^6/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(1-x^4)^{3/2}}\,dx = \frac{\sqrt{-x^4+1}\,x^3+3\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^8-2x^4+1}dx\right)x^4-3\left(\int \frac{\sqrt{-x^4+1}\,x^2}{x^8-2x^4+1}dx\right)}{x^4-1}$$

input
```
int(x^6/(-x^4+1)^(3/2),x)
```

output
```
(sqrt( - x**4 + 1)*x**3 + 3*int((sqrt( - x**4 + 1)*x**2)/(x**8 - 2*x**4 +
1),x)*x**4 - 3*int((sqrt( - x**4 + 1)*x**2)/(x**8 - 2*x**4 + 1),x))/(x**4
- 1)
```

3.285 $\int \frac{x^2}{(1-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^2}{(1-x^4)^{3/2}} \, dx = \frac{x^3}{2\sqrt{1-x^4}} - \frac{1}{2}E(\arcsin(x)|-1) + \frac{1}{2}\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2*x^3/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)+1/2*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{(1-x^4)^{3/2}} \, dx = \frac{1}{3}x^3 \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},x^4\right)$$

input
```
Integrate[x^2/(1 - x^4)^(3/2),x]
```

output
```
(x^3*Hypergeometric2F1[3/4, 3/2, 7/4, x^4])/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {819, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{x^3}{2\sqrt{1-x^4}}-\frac{1}{2}\int \frac{x^2}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{836}$$

$$\frac{1}{2}\left(\int \frac{1}{\sqrt{1-x^4}}dx-\int \frac{x^2+1}{\sqrt{1-x^4}}dx\right)+\frac{x^3}{2\sqrt{1-x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{1}{2}\left(\text{EllipticF}(\arcsin(x),-1)-\int \frac{x^2+1}{\sqrt{1-x^4}}dx\right)+\frac{x^3}{2\sqrt{1-x^4}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{1}{2}\left(\text{EllipticF}(\arcsin(x),-1)-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx\right)+\frac{x^3}{2\sqrt{1-x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{2}(\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1))+\frac{x^3}{2\sqrt{1-x^4}}$$

input
```
Int[x^2/(1 - x^4)^(3/2),x]
```

output
```
x^3/(2*Sqrt[1 - x^4]) + (-EllipticE[ArcSin[x], -1] + EllipticF[ArcSin[x],
-1])/2
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.43

method	result	size
meijerg	$\frac{x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],x^4\right)}{3}$	15
default	$\frac{x^3}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	54
risch	$\frac{x^3}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	54
elliptic	$\frac{x^3}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$	54

input
```
int(x^2/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([3/4,3/2],[7/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(1-x^4)^{3/2}}\,dx = -\frac{\sqrt{-x^4+1}x^3+(x^4-1)E(\arcsin(x)\,|-1)-(x^4-1)F(\arcsin(x)\,|-1)}{2\,(x^4-1)}$$

input
```
integrate(x^2/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(-x^4 + 1)*x^3 + (x^4 - 1)*elliptic_e(arcsin(x), -1) - (x^4 - 1)
*elliptic_f(arcsin(x), -1))/(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1-x^4)^{3/2}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-x**4+1)**(3/2),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(2*I*pi))/(4*gamma
(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(1-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(-x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(1-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(-x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1-x^4)^{3/2}}\,dx = \int \frac{x^2}{(1-x^4)^{3/2}}\,dx$$

input
```
int(x^2/(1 - x^4)^(3/2),x)
```

output
```
int(x^2/(1 - x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(1-x^4)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+1}\,x^2}{x^8-2x^4+1}dx$$

input
```
int(x^2/(-x^4+1)^(3/2),x)
```

output
```
int((sqrt( - x**4 + 1)*x**2)/(x**8 - 2*x**4 + 1),x)
```

3.286 $\int \frac{1}{x^2(1-x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^2 (1-x^4)^{3/2}} \, dx = \frac{1}{2x\sqrt{1-x^4}} - \frac{3\sqrt{1-x^4}}{2x} - \frac{3}{2}E(\arcsin(x)|-1) + \frac{3}{2}\operatorname{EllipticF}(\arcsin(x),-1)$$

output
```
1/2/x/(-x^4+1)^(1/2)-3/2*(-x^4+1)^(1/2)/x-3/2*EllipticE(x,I)+3/2*EllipticF
(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^2 (1-x^4)^{3/2}} \, dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},x^4\right)}{x}$$

input
```
Integrate[1/(x^2*(1 - x^4)^(3/2)),x]
```

output
```
-(Hypergeometric2F1[-1/4, 3/2, 3/4, x^4]/x)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {819, 847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{3}{2} \int \frac{1}{x^2\sqrt{1-x^4}}dx + \frac{1}{2x\sqrt{1-x^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{2} \left(-\int \frac{x^2}{\sqrt{1-x^4}}dx - \frac{\sqrt{1-x^4}}{x}\right) + \frac{1}{2x\sqrt{1-x^4}}$$

$$\Big\downarrow \text{836}$$

$$\frac{3}{2} \left(\int \frac{1}{\sqrt{1-x^4}}dx - \int \frac{x^2+1}{\sqrt{1-x^4}}dx - \frac{\sqrt{1-x^4}}{x}\right) + \frac{1}{2x\sqrt{1-x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{3}{2} \left(-\int \frac{x^2+1}{\sqrt{1-x^4}}dx + \text{EllipticF}(\arcsin(x),-1) - \frac{\sqrt{1-x^4}}{x}\right) + \frac{1}{2x\sqrt{1-x^4}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{3}{2} \left(-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}dx + \text{EllipticF}(\arcsin(x),-1) - \frac{\sqrt{1-x^4}}{x}\right) + \frac{1}{2x\sqrt{1-x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3}{2} \left(\text{EllipticF}(\arcsin(x),-1) - E(\arcsin(x)|-1) - \frac{\sqrt{1-x^4}}{x}\right) + \frac{1}{2x\sqrt{1-x^4}}$$

input
```
Int[1/(x^2*(1 - x^4)^(3/2)),x]
```

output
```
1/(2*x*Sqrt[1 - x^4]) + (3*(-(Sqrt[1 - x^4]/x) - EllipticE[ArcSin[x], -1]
+ EllipticF[ArcSin[x], -1]))/2
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.28

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{3}{2}\right],\left[\frac{3}{4}\right],x^{4}\right)}{x}$	15
risch	$\frac{3x^{4}-2}{2x\sqrt{-x^{4}+1}}+\frac{3\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i)\right)}{2\sqrt{-x^{4}+1}}$	61
default	$-\frac{\sqrt{-x^{4}+1}}{x}+\frac{x^{3}}{2\sqrt{-x^{4}+1}}+\frac{3\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i)\right)}{2\sqrt{-x^{4}+1}}$	68
elliptic	$-\frac{\sqrt{-x^{4}+1}}{x}+\frac{x^{3}}{2\sqrt{-x^{4}+1}}+\frac{3\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}(x,i)-\operatorname{EllipticE}(x,i)\right)}{2\sqrt{-x^{4}+1}}$	68

input
```
int(1/x^2/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x*hypergeom([-1/4,3/2],[3/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (1-x^4)^{3/2}}\,dx = -\frac{3\,(x^5-x)E(\arcsin(x)\,|-1)-3\,(x^5-x)F(\arcsin(x)\,|-1)+(3\,x^4-2)\sqrt{-x^4+1}}{2\,(x^5-x)}$$

input
```
integrate(1/x^2/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*(x^5 - x)*elliptic_e(arcsin(x), -1) - 3*(x^5 - x)*elliptic_f(arcsi
n(x), -1) + (3*x^4 - 2)*sqrt(-x^4 + 1))/(x^5 - x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^2 (1-x^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(-x**4+1)**(3/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(2*I*pi))/(4*x*gamma(
3/4))
```

Maxima [F]

$$\int \frac{1}{x^2 (1-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(1-x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(-x^4+1\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^2\left(1-x^4\right)^{3/2}}\,dx=-\frac{{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ x^4\right)}{x}$$

input
```
int(1/(x^2*(1 - x^4)^(3/2)),x)
```

output
```
-hypergeom([-1/4, 3/2], 3/4, x^4)/x
```

Reduce [F]

$$\int \frac{1}{x^2\left(1-x^4\right)^{3/2}}\,dx=\int \frac{\sqrt{-x^4+1}}{x^{10}-2x^6+x^2}dx$$

input
```
int(1/x^2/(-x^4+1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**10 - 2*x**6 + x**2),x)
```

3.287 $\int \frac{1}{x^6(1-x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{1}{x^6\left(1-x^4\right)^{3/2}}\,dx=\frac{1}{2x^5\sqrt{1-x^4}}-\frac{7\sqrt{1-x^4}}{10x^5}-\frac{21\sqrt{1-x^4}}{10x}-\frac{21}{10}E(\arcsin(x)|-1)+\frac{21}{10}\text{EllipticF}(\arcsin(x),-1)$$

output
```
1/2/x^5/(-x^4+1)^(1/2)-7/10*(-x^4+1)^(1/2)/x^5-21/10*(-x^4+1)^(1/2)/x-21/1
0*EllipticE(x,I)+21/10*EllipticF(x,I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^6\left(1-x^4\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},x^4\right)}{5x^5}$$

input
```
Integrate[1/(x^6*(1 - x^4)^(3/2)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/4, 3/2, -1/4, x^4]/x^5
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {819, 847, 847, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (1-x^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7}{2} \int \frac{1}{x^6 \sqrt{1-x^4}} dx + \frac{1}{2 x^5 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{2} \left(\frac{3}{5} \int \frac{1}{x^2 \sqrt{1-x^4}} dx - \frac{\sqrt{1-x^4}}{5 x^5}\right) + \frac{1}{2 x^5 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{2} \left(\frac{3}{5} \left(-\int \frac{x^2}{\sqrt{1-x^4}} dx - \frac{\sqrt{1-x^4}}{x}\right) - \frac{\sqrt{1-x^4}}{5 x^5}\right) + \frac{1}{2 x^5 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{836}$$

$$\frac{7}{2} \left(\frac{3}{5} \left(\int \frac{1}{\sqrt{1-x^4}} dx - \int \frac{x^2+1}{\sqrt{1-x^4}} dx - \frac{\sqrt{1-x^4}}{x}\right) - \frac{\sqrt{1-x^4}}{5 x^5}\right) + \frac{1}{2 x^5 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{7}{2} \left(\frac{3}{5} \left(-\int \frac{x^2+1}{\sqrt{1-x^4}} dx + \text{EllipticF}(\arcsin(x), -1) - \frac{\sqrt{1-x^4}}{x}\right) - \frac{\sqrt{1-x^4}}{5 x^5}\right) + \frac{1}{2 x^5 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{7}{2} \left(\frac{3}{5} \left(-\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}} dx + \text{EllipticF}(\arcsin(x), -1) - \frac{\sqrt{1-x^4}}{x}\right) - \frac{\sqrt{1-x^4}}{5 x^5}\right) + \frac{1}{2 x^5 \sqrt{1-x^4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{7}{2}\left(\frac{3}{5}\left(\text{EllipticF}(\arcsin(x),-1)-E(\arcsin(x)|-1)-\frac{\sqrt{1-x^4}}{x}\right)-\frac{\sqrt{1-x^4}}{5x^5}\right)+\frac{1}{2x^5\sqrt{1-x^4}}$$

input
```
Int[1/(x^6*(1 - x^4)^(3/2)),x]
```

output
```
1/(2*x^5*Sqrt[1 - x^4]) + (7*(-1/5*Sqrt[1 - x^4]/x^5 + (3*(-(Sqrt[1 - x^4]
/x) - EllipticE[ArcSin[x], -1] + EllipticF[ArcSin[x], -1]))/5))/2
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{5}{4},\frac{3}{2}\right],\left[-\frac{1}{4}\right],x^{4}\right)}{5x^{5}}$	15
risch	$\frac{21x^{8}-14x^{4}-2}{10x^{5}\sqrt{-x^{4}+1}}+\frac{21\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}\left(x,i\right)-\operatorname{EllipticE}\left(x,i\right)\right)}{10\sqrt{-x^{4}+1}}$	66
default	$-\frac{\sqrt{-x^{4}+1}}{5x^{5}}-\frac{8\sqrt{-x^{4}+1}}{5x}+\frac{x^{3}}{2\sqrt{-x^{4}+1}}+\frac{21\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}\left(x,i\right)-\operatorname{EllipticE}\left(x,i\right)\right)}{10\sqrt{-x^{4}+1}}$	82
elliptic	$-\frac{\sqrt{-x^{4}+1}}{5x^{5}}-\frac{8\sqrt{-x^{4}+1}}{5x}+\frac{x^{3}}{2\sqrt{-x^{4}+1}}+\frac{21\sqrt{-x^{2}+1}\,\sqrt{x^{2}+1}\,\left(\operatorname{EllipticF}\left(x,i\right)-\operatorname{EllipticE}\left(x,i\right)\right)}{10\sqrt{-x^{4}+1}}$	82

input
```
int(1/x^6/(-x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5*hypergeom([-5/4,3/2],[-1/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^6 (1-x^4)^{3/2}}\, dx = -\frac{21\,(x^9-x^5)E(\arcsin(x)\,|-1)-21\,(x^9-x^5)F(\arcsin(x)\,|-1)+(21\,x^8-14\,x^4-2)\sqrt{-x^4+1}}{10\,(x^9-x^5)}$$

input
```
integrate(1/x^6/(-x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(21*(x^9 - x^5)*elliptic_e(arcsin(x), -1) - 21*(x^9 - x^5)*elliptic_
f(arcsin(x), -1) + (21*x^8 - 14*x^4 - 2)*sqrt(-x^4 + 1))/(x^9 - x^5)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^6 (1-x^4)^{3/2}}\, dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ -\frac{1}{4} \end{matrix}\middle| x^4 e^{2i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(-x**4+1)**(3/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), x**4*exp_polar(2*I*pi))/(4*x**5*ga
mma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6 (1-x^4)^{3/2}} \, dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate(1/x^6/(-x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (1-x^4)^{3/2}} \, dx = \int \frac{1}{(-x^4+1)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate(1/x^6/(-x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-x^4 + 1)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (1-x^4)^{3/2}} \, dx = \int \frac{1}{x^6 (1-x^4)^{3/2}} \, dx$$

input
```
int(1/(x^6*(1 - x^4)^(3/2)),x)
```

output
```
int(1/(x^6*(1 - x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(1-x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{-x^4+1}}{x^{14}-2x^{10}+x^6}dx$$

input
```
int(1/x^6/(-x^4+1)^(3/2),x)
```

output
```
int(sqrt( - x**4 + 1)/(x**14 - 2*x**10 + x**6),x)
```

3.288 $\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{-1+x^4} + \frac{1}{3}\left(-1+x^4\right)^{3/2} + \frac{1}{10}\left(-1+x^4\right)^{5/2}$$

output
```
1/2*(x^4-1)^(1/2)+1/3*(x^4-1)^(3/2)+1/10*(x^4-1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{1}{30}\sqrt{-1+x^4}\left(8+4x^4+3x^8\right)$$

input
```
Integrate[x^11/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 + x^4]*(8 + 4*x^4 + 3*x^8))/30
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{\sqrt{x^4-1}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\left(x^4-1\right)^{3/2}+2\sqrt{x^4-1}+\frac{1}{\sqrt{x^4-1}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2}{5}\left(x^4-1\right)^{5/2}+\frac{4}{3}\left(x^4-1\right)^{3/2}+2\sqrt{x^4-1}\right)$$

input
```
Int[x^11/Sqrt[-1 + x^4],x]
```

output
```
(2*Sqrt[-1 + x^4] + (4*(-1 + x^4)^(3/2))/3 + (2*(-1 + x^4)^(5/2))/5)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
trager	$\left(\frac{1}{10}x^8+\frac{2}{15}x^4+\frac{4}{15}\right)\sqrt{x^4-1}$	21
risch	$\frac{(3x^8+4x^4+8)\sqrt{x^4-1}}{30}$	22
pseudoelliptic	$\frac{(3x^8+4x^4+8)\sqrt{x^4-1}}{30}$	22
default	$\frac{(x^2+1)(x^2-1)(3x^8+4x^4+8)}{30\sqrt{x^4-1}}$	32
elliptic	$\frac{(x^2+1)(x^2-1)(3x^8+4x^4+8)}{30\sqrt{x^4-1}}$	32
gosper	$\frac{(-1+x)(1+x)(x^2+1)(3x^8+4x^4+8)}{30\sqrt{x^4-1}}$	33
orering	$\frac{(-1+x)(1+x)(x^2+1)(3x^8+4x^4+8)}{30\sqrt{x^4-1}}$	33
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,(6x^8+8x^4+16)\sqrt{-x^4+1}}{15}\right)}{4\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^4-1)}}$	56

input
```
int(x^11/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/10*x^8+2/15*x^4+4/15)*(x^4-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{1}{30}\left(3\,x^8+4\,x^4+8\right)\sqrt{x^4-1}$$

input
```
integrate(x^11/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(3*x^8 + 4*x^4 + 8)*sqrt(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{x^8\sqrt{x^4-1}}{10}+\frac{2x^4\sqrt{x^4-1}}{15}+\frac{4\sqrt{x^4-1}}{15}$$

input
```
integrate(x**11/(x**4-1)**(1/2),x)
```

output
```
x**8*sqrt(x**4 - 1)/10 + 2*x**4*sqrt(x**4 - 1)/15 + 4*sqrt(x**4 - 1)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{1}{10}\left(x^4-1\right)^{\frac{5}{2}}+\frac{1}{3}\left(x^4-1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^11/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/10*(x^4 - 1)^(5/2) + 1/3*(x^4 - 1)^(3/2) + 1/2*sqrt(x^4 - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{1}{10}\left(x^4-1\right)^{\frac{5}{2}} + \frac{1}{3}\left(x^4-1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^11/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/10*(x^4 - 1)^(5/2) + 1/3*(x^4 - 1)^(3/2) + 1/2*sqrt(x^4 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \sqrt{x^4-1}\left(\frac{x^8}{10} + \frac{2\,x^4}{15} + \frac{4}{15}\right)$$

input
```
int(x^11/(x^4 - 1)^(1/2),x)
```

output
```
(x^4 - 1)^(1/2)*((2*x^4)/15 + x^8/10 + 4/15)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.28

$$\int \frac{x^{11}}{\sqrt{-1+x^4}}\,dx = \frac{48\sqrt{x^4-1}\,x^{18} + 4\sqrt{x^4-1}\,x^{14} + 63\sqrt{x^4-1}\,x^{10} - 140\sqrt{x^4-1}\,x^6 + 40\sqrt{x^4-1}\,x^2 + 48x^{20} - 20x^{16} + 55}{480\sqrt{x^4-1}\,x^8 - 360\sqrt{x^4-1}\,x^4 + 30\sqrt{x^4-1} + 480x^{10} - 600x^6 + 150x^2}$$

input
```
int(x^11/(x^4-1)^(1/2),x)
```

output
```
(48*sqrt(x**4 - 1)*x**18 + 4*sqrt(x**4 - 1)*x**14 + 63*sqrt(x**4 - 1)*x**1
0 - 140*sqrt(x**4 - 1)*x**6 + 40*sqrt(x**4 - 1)*x**2 + 48*x**20 - 20*x**16
 + 55*x**12 - 175*x**8 + 100*x**4 - 8)/(30*(16*sqrt(x**4 - 1)*x**8 - 12*sq
rt(x**4 - 1)*x**4 + sqrt(x**4 - 1) + 16*x**10 - 20*x**6 + 5*x**2))
```

3.289 $\int \frac{x^7}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{-1+x^4} + \frac{1}{6}\left(-1+x^4\right)^{3/2}$$

output
```
1/2*(x^4-1)^(1/2)+1/6*(x^4-1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{1}{6}\sqrt{-1+x^4}\left(2+x^4\right)$$

input
```
Integrate[x^7/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 + x^4]*(2 + x^4))/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt{x^4-1}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\sqrt{x^4-1}+\frac{1}{\sqrt{x^4-1}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2}{3}\left(x^4-1\right)^{3/2}+2\sqrt{x^4-1}\right)$$

input
```
Int[x^7/Sqrt[-1 + x^4],x]
```

output
```
(2*Sqrt[-1 + x^4] + (2*(-1 + x^4)^(3/2))/3)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{(x^4+2)\sqrt{x^4-1}}{6}$	15
pseudoelliptic	$\frac{(x^4+2)\sqrt{x^4-1}}{6}$	15
trager	$\left(\frac{x^4}{6}+\frac{1}{3}\right)\sqrt{x^4-1}$	16
default	$\frac{(x^2+1)(x^2-1)(x^4+2)}{6\sqrt{x^4-1}}$	25
elliptic	$\frac{(x^2+1)(x^2-1)(x^4+2)}{6\sqrt{x^4-1}}$	25
gosper	$\frac{(-1+x)(1+x)(x^2+1)(x^4+2)}{6\sqrt{x^4-1}}$	26
orering	$\frac{(-1+x)(1+x)(x^2+1)(x^4+2)}{6\sqrt{x^4-1}}$	26
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(4x^4+8\right)\sqrt{-x^4+1}}{6}\right)}{4\sqrt{\pi}\sqrt{\operatorname{signum}(x^4-1)}}$	51

input
```
int(x^7/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^4+2)*(x^4-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{1}{6}\left(x^4+2\right)\sqrt{x^4-1}$$

input
```
integrate(x^7/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(x^4 + 2)*sqrt(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{x^4\sqrt{x^4-1}}{6} + \frac{\sqrt{x^4-1}}{3}$$

input
```
integrate(x**7/(x**4-1)**(1/2),x)
```

output
```
x**4*sqrt(x**4 - 1)/6 + sqrt(x**4 - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{1}{6}\left(x^4-1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^7/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(x^4 - 1)^(3/2) + 1/2*sqrt(x^4 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{1}{6}\left(x^4-1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^7/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/6*(x^4 - 1)^(3/2) + 1/2*sqrt(x^4 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,(x^4+2)}{6}$$

input
```
int(x^7/(x^4 - 1)^(1/2),x)
```

output
```
((x^4 - 1)^(1/2)*(x^4 + 2))/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int \frac{x^7}{\sqrt{-1+x^4}}\,dx = \frac{4\sqrt{x^4-1}\,x^{10}+5\sqrt{x^4-1}\,x^6-6\sqrt{x^4-1}\,x^2+4x^{12}+3x^8-9x^4+2}{24\sqrt{x^4-1}\,x^4-6\sqrt{x^4-1}+24x^6-18x^2}$$

input
```
int(x^7/(x^4-1)^(1/2),x)
```

output
```
(4*sqrt(x**4 - 1)*x**10 + 5*sqrt(x**4 - 1)*x**6 - 6*sqrt(x**4 - 1)*x**2 +
4*x**12 + 3*x**8 - 9*x**4 + 2)/(6*(4*sqrt(x**4 - 1)*x**4 - sqrt(x**4 - 1)
+ 4*x**6 - 3*x**2))
```

3.290 $\int \frac{x^3}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{-1+x^4}$$

output
```
1/2*(x^4-1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{-1+x^4}$$

input
```
Integrate[x^3/Sqrt[-1 + x^4],x]
```

output
```
Sqrt[-1 + x^4]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{\sqrt{x^4-1}}{2}$$

input
```
Int[x^3/Sqrt[-1 + x^4],x]
```

output
```
Sqrt[-1 + x^4]/2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\sqrt{x^4-1}}{2}$	10
default	$\frac{\sqrt{x^4-1}}{2}$	10
trager	$\frac{\sqrt{x^4-1}}{2}$	10
risch	$\frac{\sqrt{x^4-1}}{2}$	10
pseudoelliptic	$\frac{\sqrt{x^4-1}}{2}$	10
elliptic	$\frac{(x^2+1)(x^2-1)}{2\sqrt{x^4-1}}$	20
gosper	$\frac{(-1+x)(1+x)(x^2+1)}{2\sqrt{x^4-1}}$	21
orering	$\frac{(-1+x)(1+x)(x^2+1)}{2\sqrt{x^4-1}}$	21
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-x^4+1}\right)}{4\sqrt{\pi}\sqrt{\operatorname{signum}(x^4-1)}}$	44

input
```
int(x^3/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^4-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^3/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^4 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{2}$$

input
```
integrate(x**3/(x**4-1)**(1/2),x)
```

output
```
sqrt(x**4 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^3/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\sqrt{x^4-1}$$

input
```
integrate(x^3/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^4 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{2}$$

input
```
int(x^3/(x^4 - 1)^(1/2),x)
```

output
```
(x^4 - 1)^(1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{x^3}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,x^2+x^4-1}{2\sqrt{x^4-1}+2x^2}$$

input
```
int(x^3/(x^4-1)^(1/2),x)
```

output
```
(sqrt(x**4 - 1)*x**2 + x**4 - 1)/(2*(sqrt(x**4 - 1) + x**2))
```

3.291 $\int \frac{1}{x\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)$$

output
```
1/2*arctan((x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{-1+x^4}\right)$$

input
```
Integrate[1/(x*Sqrt[-1 + x^4]),x]
```

output
```
ArcTan[Sqrt[-1 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{ 798}$$
$$\frac{1}{4}\int \frac{1}{x^4\sqrt{x^4-1}}dx^4$$
$$\Big\downarrow \text{ 73}$$
$$\frac{1}{2}\int \frac{1}{x^8+1}d\sqrt{x^4-1}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
Int[1/(x*Sqrt[-1 + x^4]),x]
```

output
```
ArcTan[Sqrt[-1 + x^4]]/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
elliptic	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
pseudoelliptic	$-\frac{\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{2}$	11
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{x^4-1}}{x^2}\right)}{2}$	28
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(\left(-2\ln(2)+4\ln(x)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^4+1}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^4-1)}}$	61

input
```
int(1/x/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(1/(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(sqrt(x^4 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{i\operatorname{acosh}\left(\frac{1}{x^2}\right)}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{1}{x^2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(x**4-1)**(1/2),x)
```

output
```
Piecewise((I*acosh(x**(-2))/2, 1/Abs(x**4) > 1), (-asin(x**(-2))/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arctan(sqrt(x^4 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{1}{2}\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = \frac{\mathrm{atan}\left(\sqrt{x^4-1}\right)}{2}$$

input
```
int(1/(x*(x^4 - 1)^(1/2)),x)
```

output
```
atan((x^4 - 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\sqrt{-1+x^4}}\,dx = atan\left(\sqrt{x^4-1}+x^2\right)$$

input
```
int(1/x/(x^4-1)^(1/2),x)
```

output
```
atan(sqrt(x**4 - 1) + x**2)
```

3.292 $\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{4x^4}+\frac{1}{4}\arctan\left(\sqrt{-1+x^4}\right)$$

output
```
1/4*(x^4-1)^(1/2)/x^4+1/4*arctan((x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx=\frac{1}{4}\left(\frac{\sqrt{-1+x^4}}{x^4}+\arctan\left(\sqrt{-1+x^4}\right)\right)$$

input
```
Integrate[1/(x^5*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^4]/x^4 + ArcTan[Sqrt[-1 + x^4]])/4
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{1}{x^8\sqrt{x^4-1}}dx^4$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{x^4\sqrt{x^4-1}}dx^4+\frac{\sqrt{x^4-1}}{x^4}\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{4}\left(\int \frac{1}{x^8+1}d\sqrt{x^4-1}+\frac{\sqrt{x^4-1}}{x^4}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{4}\left(\arctan\left(\sqrt{x^4-1}\right)+\frac{\sqrt{x^4-1}}{x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^4]/x^4 + ArcTan[Sqrt[-1 + x^4]])/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\sqrt{x^{4}-1}}{4x^{4}}-\frac{\arctan\left(\frac{1}{\sqrt{x^{4}-1}}\right)}{4}$	24
risch	$\frac{\sqrt{x^{4}-1}}{4x^{4}}-\frac{\arctan\left(\frac{1}{\sqrt{x^{4}-1}}\right)}{4}$	24
elliptic	$\frac{\sqrt{x^{4}-1}}{4x^{4}}-\frac{\arctan\left(\frac{1}{\sqrt{x^{4}-1}}\right)}{4}$	24
pseudoelliptic	$\frac{-\arctan\left(\frac{1}{\sqrt{x^{4}-1}}\right)x^{4}+\sqrt{x^{4}-1}}{4x^{4}}$	27
trager	$\frac{\sqrt{x^{4}-1}}{4x^{4}}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{4}-1}}{x^{2}}\right)}{4}$	41
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^{4}-1)}\left(\frac{\sqrt{\pi}}{x^{4}}-\frac{(1-2\ln(2)+4\ln(x)+i\pi)\sqrt{\pi}}{2}-\frac{\sqrt{\pi}\left(-4x^{4}+8\right)}{8x^{4}}+\frac{\sqrt{\pi}\sqrt{-x^{4}+1}}{x^{4}}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{4}+1}}{2}\right)\right)}{4\sqrt{\pi}\sqrt{\operatorname{signum}(x^{4}-1)}}$	100

input
```
int(1/x^5/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^4-1)^(1/2)/x^4-1/4*arctan(1/(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx=\frac{x^4\arctan\left(\sqrt{x^4-1}\right)+\sqrt{x^4-1}}{4\,x^4}$$

input
```
integrate(1/x^5/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(x^4*arctan(sqrt(x^4 - 1)) + sqrt(x^4 - 1))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.29

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{i\,\text{acosh}\left(\frac{1}{x^2}\right)}{4} - \frac{i}{4x^2\sqrt{-1+\frac{1}{x^4}}} + \frac{i}{4x^6\sqrt{-1+\frac{1}{x^4}}} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{\text{asin}\left(\frac{1}{x^2}\right)}{4} + \frac{\sqrt{1-\frac{1}{x^4}}}{4x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(x**4-1)**(1/2),x)
```

output
```
Piecewise((I*acosh(x**(-2))/4 - I/(4*x**2*sqrt(-1 + x**(-4))) + I/(4*x**6*
sqrt(-1 + x**(-4))), 1/Abs(x**4) > 1), (-asin(x**(-2))/4 + sqrt(1 - 1/x**4
)/(4*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{4\,x^4} + \frac{1}{4}\,\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x^5/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(x^4 - 1)/x^4 + 1/4*arctan(sqrt(x^4 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{4\,x^4} + \frac{1}{4}\,\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x^5/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 - 1)/x^4 + 1/4*arctan(sqrt(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx = \frac{\mathrm{atan}\left(\sqrt{x^4-1}\right)}{4} + \frac{\sqrt{x^4-1}}{4\,x^4}$$

input
```
int(1/(x^5*(x^4 - 1)^(1/2)),x)
```

output
```
atan((x^4 - 1)^(1/2))/4 + (x^4 - 1)^(1/2)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.52

$$\int \frac{1}{x^5\sqrt{-1+x^4}}\,dx = \frac{4\sqrt{x^4-1}\,atan\left(\sqrt{x^4-1}+x^2\right)x^6 + 4atan\left(\sqrt{x^4-1}+x^2\right)x^8 - 2atan\left(\sqrt{x^4-1}+x^2\right)x^4 + 2\sqrt{x^4-1}\,x^4}{4x^4\left(2\sqrt{x^4-1}\,x^2+2x^4-1\right)}$$

input
```
int(1/x^5/(x^4-1)^(1/2),x)
```

output
```
(4*sqrt(x**4 - 1)*atan(sqrt(x**4 - 1) + x**2)*x**6 + 4*atan(sqrt(x**4 - 1)
 + x**2)*x**8 - 2*atan(sqrt(x**4 - 1) + x**2)*x**4 + 2*sqrt(x**4 - 1)*x**4
 - sqrt(x**4 - 1) + 2*x**6 - 2*x**2)/(4*x**4*(2*sqrt(x**4 - 1)*x**2 + 2*x*
*4 - 1))
```

3.293 $\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \frac{\sqrt{-1+x^4}}{8x^8} + \frac{3\sqrt{-1+x^4}}{16x^4} + \frac{3}{16}\arctan\left(\sqrt{-1+x^4}\right)$$

output
```
1/8*(x^4-1)^(1/2)/x^8+3/16*(x^4-1)^(1/2)/x^4+3/16*arctan((x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \frac{\sqrt{-1+x^4}(2+3x^4)}{16x^8} + \frac{3}{16}\arctan\left(\sqrt{-1+x^4}\right)$$

input
```
Integrate[1/(x^9*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^4]*(2 + 3*x^4))/(16*x^8) + (3*ArcTan[Sqrt[-1 + x^4]])/16
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^{12}\sqrt{x^4-1}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{1}{x^8\sqrt{x^4-1}}dx^4+\frac{\sqrt{x^4-1}}{2x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{x^4\sqrt{x^4-1}}dx^4+\frac{\sqrt{x^4-1}}{x^4}\right)+\frac{\sqrt{x^4-1}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\int \frac{1}{x^8+1}d\sqrt{x^4-1}+\frac{\sqrt{x^4-1}}{x^4}\right)+\frac{\sqrt{x^4-1}}{2x^8}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\arctan\left(\sqrt{x^4-1}\right)+\frac{\sqrt{x^4-1}}{x^4}\right)+\frac{\sqrt{x^4-1}}{2x^8}\right)$$

input
```
Int[1/(x^9*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^4]/(2*x^8) + (3*(Sqrt[-1 + x^4]/x^4 + ArcTan[Sqrt[-1 + x^4]])
)/4)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result
default	$\frac{\sqrt{x^4-1}}{8x^8}+\frac{3\sqrt{x^4-1}}{16x^4}-\frac{3\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{16}$
risch	$\frac{3x^8-x^4-2}{16x^8\sqrt{x^4-1}}-\frac{3\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{16}$
elliptic	$\frac{\sqrt{x^4-1}}{8x^8}+\frac{3\sqrt{x^4-1}}{16x^4}-\frac{3\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)}{16}$
pseudoelliptic	$\frac{-3\arctan\left(\frac{1}{\sqrt{x^4-1}}\right)x^8+3\sqrt{x^4-1}\,x^4+2\sqrt{x^4-1}}{16x^8}$
trager	$\frac{(3x^4+2)\sqrt{x^4-1}}{16x^8}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)-\sqrt{x^4-1}}{x^2}\right)}{16}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(-\frac{\sqrt{\pi}}{2x^8}-\frac{\sqrt{\pi}}{2x^4}+\frac{3\left(\frac{7}{6}-2\ln(2)+4\ln(x)+i\pi\right)\sqrt{\pi}}{8}+\frac{\sqrt{\pi}\left(-7x^8+8x^4+8\right)}{16x^8}-\frac{\sqrt{\pi}\left(12x^4+8\right)\sqrt{-x^4+1}}{16x^8}-\frac{3\sqrt{\pi}\,\ln\left(\frac{1}{2}+\sqrt{\ldots}\right.}{4}\right.}{4\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^4-1)}}$

input
```
int(1/x^9/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(x^4-1)^(1/2)/x^8+3/16*(x^4-1)^(1/2)/x^4-3/16*arctan(1/(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx=\frac{3\,x^8\arctan\left(\sqrt{x^4-1}\right)+(3\,x^4+2)\sqrt{x^4-1}}{16\,x^8}$$

input
```
integrate(1/x^9/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(3*x^8*arctan(sqrt(x^4 - 1)) + (3*x^4 + 2)*sqrt(x^4 - 1))/x^8
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.74

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{3i\operatorname{acosh}\left(\frac{1}{x^2}\right)}{16} - \frac{3i}{16x^2\sqrt{-1+\frac{1}{x^4}}} + \frac{i}{16x^6\sqrt{-1+\frac{1}{x^4}}} + \frac{i}{8x^{10}\sqrt{-1+\frac{1}{x^4}}} & \text{for } \frac{1}{|x^4|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{x^2}\right)}{16} + \frac{3}{16x^2\sqrt{1-\frac{1}{x^4}}} - \frac{1}{16x^6\sqrt{1-\frac{1}{x^4}}} - \frac{1}{8x^{10}\sqrt{1-\frac{1}{x^4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**9/(x**4-1)**(1/2),x)
```

output
```
Piecewise((3*I*acosh(x**(-2))/16 - 3*I/(16*x**2*sqrt(-1 + x**(-4))) + I/(1
6*x**6*sqrt(-1 + x**(-4))) + I/(8*x**10*sqrt(-1 + x**(-4))), 1/Abs(x**4) >
 1), (-3*asin(x**(-2))/16 + 3/(16*x**2*sqrt(1 - 1/x**4)) - 1/(16*x**6*sqrt
(1 - 1/x**4)) - 1/(8*x**10*sqrt(1 - 1/x**4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \frac{3\,(x^4-1)^{\frac{3}{2}} + 5\,\sqrt{x^4-1}}{16\,\left(2\,x^4 + (x^4-1)^2 - 1\right)} + \frac{3}{16}\,\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x^9/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(3*(x^4 - 1)^(3/2) + 5*sqrt(x^4 - 1))/(2*x^4 + (x^4 - 1)^2 - 1) + 3/1
6*arctan(sqrt(x^4 - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \frac{3\,(x^4-1)^{\frac{3}{2}} + 5\,\sqrt{x^4-1}}{16\,x^8} + \frac{3}{16}\,\arctan\left(\sqrt{x^4-1}\right)$$

input
```
integrate(1/x^9/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/16*(3*(x^4 - 1)^(3/2) + 5*sqrt(x^4 - 1))/x^8 + 3/16*arctan(sqrt(x^4 - 1)
)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \frac{3\,\mathrm{atan}\left(\sqrt{x^4-1}\right)}{16} + \frac{3\,\sqrt{x^4-1}}{16\,x^4} + \frac{\sqrt{x^4-1}}{8\,x^8}$$

input
```
int(1/(x^9*(x^4 - 1)^(1/2)),x)
```

output
```
(3*atan((x^4 - 1)^(1/2)))/16 + (3*(x^4 - 1)^(1/2))/(16*x^4) + (x^4 - 1)^(1
/2)/(8*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 4.15

$$\int \frac{1}{x^9\sqrt{-1+x^4}}\,dx = \frac{48\sqrt{x^4-1}\,atan\left(\sqrt{x^4-1}+x^2\right)x^{14} - 24\sqrt{x^4-1}\,atan\left(\sqrt{x^4-1}+x^2\right)x^{10} + 48atan\left(\sqrt{x^4-1}+x^2\right)x^{16}}{16x}$$

input
```
int(1/x^9/(x^4-1)^(1/2),x)
```

output
```
(48*sqrt(x**4 - 1)*atan(sqrt(x**4 - 1) + x**2)*x**14 - 24*sqrt(x**4 - 1)*a
tan(sqrt(x**4 - 1) + x**2)*x**10 + 48*atan(sqrt(x**4 - 1) + x**2)*x**16 -
48*atan(sqrt(x**4 - 1) + x**2)*x**12 + 6*atan(sqrt(x**4 - 1) + x**2)*x**8
+ 24*sqrt(x**4 - 1)*x**12 - 8*sqrt(x**4 - 1)*x**8 - 13*sqrt(x**4 - 1)*x**4
 + 2*sqrt(x**4 - 1) + 24*x**14 - 20*x**10 - 12*x**6 + 8*x**2)/(16*x**8*(8*
sqrt(x**4 - 1)*x**6 - 4*sqrt(x**4 - 1)*x**2 + 8*x**8 - 8*x**4 + 1))
```

3.294 $\int \frac{x^9}{\sqrt{-1+x^4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^9}{\sqrt{-1+x^4}} \, dx = \frac{3}{16}x^2\sqrt{-1+x^4}+\frac{1}{8}x^6\sqrt{-1+x^4}+\frac{3}{16}\text{arctanh}\left(\frac{x^2}{\sqrt{-1+x^4}}\right)$$

output
```
3/16*x^2*(x^4-1)^(1/2)+1/8*x^6*(x^4-1)^(1/2)+3/16*arctanh(x^2/(x^4-1)^(1/2
))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{\sqrt{-1+x^4}} \, dx = \frac{1}{16}x^2\sqrt{-1+x^4}\left(3+2x^4\right)+\frac{3}{16}\log\left(x^2+\sqrt{-1+x^4}\right)$$

input
```
Integrate[x^9/Sqrt[-1 + x^4],x]
```

output
```
(x^2*Sqrt[-1 + x^4]*(3 + 2*x^4))/16 + (3*Log[x^2 + Sqrt[-1 + x^4]])/16
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {807, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\sqrt{x^4-1}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{\sqrt{x^4-1}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \frac{x^4}{\sqrt{x^4-1}}dx^2+\frac{1}{4}\sqrt{x^4-1}x^6\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4-1}}dx^2+\frac{1}{2}\sqrt{x^4-1}x^2\right)+\frac{1}{4}\sqrt{x^4-1}x^6\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\frac{x^2}{\sqrt{x^4-1}}+\frac{1}{2}\sqrt{x^4-1}x^2\right)+\frac{1}{4}\sqrt{x^4-1}x^6\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{x^4-1}}\right)+\frac{1}{2}\sqrt{x^4-1}x^2\right)+\frac{1}{4}\sqrt{x^4-1}x^6\right)$$

input
```
Int[x^9/Sqrt[-1 + x^4],x]
```

output
```
((x^6*Sqrt[-1 + x^4])/4 + (3*((x^2*Sqrt[-1 + x^4])/2 + ArcTanh[x^2/Sqrt[-1
 + x^4]]/2))/4)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x^{2}\left(2x^{4}+3\right)\sqrt{x^{4}-1}}{16}+\frac{3\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{16}$	35
pseudoelliptic	$\frac{3\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{16}+\frac{\left(2x^{6}+3x^{2}\right)\sqrt{x^{4}-1}}{16}$	36
trager	$\frac{x^{2}\left(2x^{4}+3\right)\sqrt{x^{4}-1}}{16}-\frac{3\ln\left(x^{2}-\sqrt{x^{4}-1}\right)}{16}$	37
default	$\frac{x^{6}\sqrt{x^{4}-1}}{8}+\frac{3x^{2}\sqrt{x^{4}-1}}{16}+\frac{3\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{16}$	40
elliptic	$\frac{x^{6}\sqrt{x^{4}-1}}{8}+\frac{3x^{2}\sqrt{x^{4}-1}}{16}+\frac{3\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{16}$	40
meijerg	$-\frac{i\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\left(-\frac{i\sqrt{\pi}\,x^{2}\left(10x^{4}+15\right)\sqrt{-x^{4}+1}}{20}+\frac{3i\sqrt{\pi}\,\arcsin\left(x^{2}\right)}{4}\right)}{4\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(x^{4}-1\right)}}$	61

input
```
int(x^9/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*x^2*(2*x^4+3)*(x^4-1)^(1/2)+3/16*ln(x^2+(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{\sqrt{-1+x^4}}\,dx = \frac{1}{16}\left(2\,x^6+3\,x^2\right)\sqrt{x^4-1}-\frac{3}{16}\,\log\left(-x^2+\sqrt{x^4-1}\right)$$

input
```
integrate(x^9/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/16*(2*x^6 + 3*x^2)*sqrt(x^4 - 1) - 3/16*log(-x^2 + sqrt(x^4 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.16

$$\int \frac{x^9}{\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{x^{10}}{8\sqrt{x^4-1}}+\frac{x^6}{16\sqrt{x^4-1}}-\frac{3x^2}{16\sqrt{x^4-1}}+\frac{3\operatorname{acosh}\left(x^2\right)}{16} & \text{for}\ \left|x^4\right|>1 \\ -\frac{ix^{10}}{8\sqrt{1-x^4}}-\frac{ix^6}{16\sqrt{1-x^4}}+\frac{3ix^2}{16\sqrt{1-x^4}}-\frac{3i\operatorname{asin}\left(x^2\right)}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**9/(x**4-1)**(1/2),x)
```

output
```
Piecewise((x**10/(8*sqrt(x**4 - 1)) + x**6/(16*sqrt(x**4 - 1)) - 3*x**2/(1
6*sqrt(x**4 - 1)) + 3*acosh(x**2)/16, Abs(x**4) > 1), (-I*x**10/(8*sqrt(1
- x**4)) - I*x**6/(16*sqrt(1 - x**4)) + 3*I*x**2/(16*sqrt(1 - x**4)) - 3*I
*asin(x**2)/16, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(39) = 78.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

$$\int \frac{x^9}{\sqrt{-1+x^4}}\,dx = -\frac{\frac{5\sqrt{x^4-1}}{x^2} - \frac{3\,(x^4-1)^{\frac{3}{2}}}{x^6}}{16\left(\frac{2\,(x^4-1)}{x^4} - \frac{(x^4-1)^2}{x^8} - 1\right)} + \frac{3}{32}\log\left(\frac{\sqrt{x^4-1}}{x^2}+1\right) - \frac{3}{32}\log\left(\frac{\sqrt{x^4-1}}{x^2}-1\right)$$

input
```
integrate(x^9/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(5*sqrt(x^4 - 1)/x^2 - 3*(x^4 - 1)^(3/2)/x^6)/(2*(x^4 - 1)/x^4 - (x^
4 - 1)^2/x^8 - 1) + 3/32*log(sqrt(x^4 - 1)/x^2 + 1) - 3/32*log(sqrt(x^4 -
1)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \frac{x^9}{\sqrt{-1+x^4}}\,dx = \frac{1}{16}\left(2\,x^4+3\right)\sqrt{x^4-1}x^2 - \frac{3}{16}\log\left(x^2-\sqrt{x^4-1}\right)$$

input
```
integrate(x^9/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*x^4 + 3)*sqrt(x^4 - 1)*x^2 - 3/16*log(x^2 - sqrt(x^4 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\sqrt{-1+x^4}}\,dx = \int \frac{x^9}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^9/(x^4 - 1)^(1/2),x)
```

output
```
int(x^9/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.65

$$\int \frac{x^9}{\sqrt{-1+x^4}}\,dx = \frac{24\sqrt{x^4-1}\log\left(\sqrt{x^4-1}+x^2\right)x^6-12\sqrt{x^4-1}\log\left(\sqrt{x^4-1}+x^2\right)x^2+16\sqrt{x^4-1}\,x^{14}+8\sqrt{x^4-1}\,x^{10}-}{128\sqrt{x^4-1}\,x}$$

input
```
int(x^9/(x^4-1)^(1/2),x)
```

output
```
(24*sqrt(x**4 - 1)*log(sqrt(x**4 - 1) + x**2)*x**6 - 12*sqrt(x**4 - 1)*log
(sqrt(x**4 - 1) + x**2)*x**2 + 16*sqrt(x**4 - 1)*x**14 + 8*sqrt(x**4 - 1)*
x**10 - 22*sqrt(x**4 - 1)*x**6 + 3*sqrt(x**4 - 1)*x**2 + 24*log(sqrt(x**4
- 1) + x**2)*x**8 - 24*log(sqrt(x**4 - 1) + x**2)*x**4 + 3*log(sqrt(x**4 -
 1) + x**2) + 16*x**16 - 28*x**8 + 12*x**4)/(16*(8*sqrt(x**4 - 1)*x**6 - 4
*sqrt(x**4 - 1)*x**2 + 8*x**8 - 8*x**4 + 1))
```

3.295 $\int \frac{x^5}{\sqrt{-1+x^4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{x^5}{\sqrt{-1+x^4}} \, dx = \frac{1}{4}x^2\sqrt{-1+x^4}+\frac{1}{4}\text{arctanh}\left(\frac{x^2}{\sqrt{-1+x^4}}\right)$$

output
```
1/4*x^2*(x^4-1)^(1/2)+1/4*arctanh(x^2/(x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{\sqrt{-1+x^4}} \, dx = \frac{1}{4}\left(x^2\sqrt{-1+x^4}+\log\left(x^2+\sqrt{-1+x^4}\right)\right)$$

input
```
Integrate[x^5/Sqrt[-1 + x^4],x]
```

output
```
(x^2*Sqrt[-1 + x^4] + Log[x^2 + Sqrt[-1 + x^4]])/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{x^4-1}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{x^4-1}}dx^2+\frac{1}{2}\sqrt{x^4-1}x^2\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-x^4}d\frac{x^2}{\sqrt{x^4-1}}+\frac{1}{2}\sqrt{x^4-1}x^2\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{x^4-1}}\right)+\frac{1}{2}\sqrt{x^4-1}x^2\right)$$

input
```
Int[x^5/Sqrt[-1 + x^4],x]
```

output
```
((x^2*Sqrt[-1 + x^4])/2 + ArcTanh[x^2/Sqrt[-1 + x^4]]/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^{2}\sqrt{x^{4}-1}}{4}+\frac{\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{4}$	28
risch	$\frac{x^{2}\sqrt{x^{4}-1}}{4}+\frac{\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{4}$	28
elliptic	$\frac{x^{2}\sqrt{x^{4}-1}}{4}+\frac{\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{4}$	28
pseudoelliptic	$\frac{x^{2}\sqrt{x^{4}-1}}{4}+\frac{\ln\left(x^{2}+\sqrt{x^{4}-1}\right)}{4}$	28
trager	$\frac{x^{2}\sqrt{x^{4}-1}}{4}-\frac{\ln\left(x^{2}-\sqrt{x^{4}-1}\right)}{4}$	30
meijerg	$\frac{i\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\,\left(i\sqrt{\pi}\,x^{2}\sqrt{-x^{4}+1}-i\sqrt{\pi}\,\arcsin\left(x^{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(x^{4}-1\right)}}$	54

input
```
int(x^5/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(x^4-1)^(1/2)+1/4*ln(x^2+(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{\sqrt{-1+x^4}}\,dx = \frac{1}{4}\,\sqrt{x^4-1}x^2 - \frac{1}{4}\,\log\left(-x^2+\sqrt{x^4-1}\right)$$

input
```
integrate(x^5/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^4 - 1)*x^2 - 1/4*log(-x^2 + sqrt(x^4 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{x^2\sqrt{x^4-1}}{4} + \frac{\operatorname{acosh}(x^2)}{4} & \text{for } |x^4| > 1 \\ -\frac{ix^6}{4\sqrt{1-x^4}} + \frac{ix^2}{4\sqrt{1-x^4}} - \frac{i\operatorname{asin}(x^2)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(x**4-1)**(1/2),x)
```

output
```
Piecewise((x**2*sqrt(x**4 - 1)/4 + acosh(x**2)/4, Abs(x**4) > 1), (-I*x**6
/(4*sqrt(1 - x**4)) + I*x**2/(4*sqrt(1 - x**4)) - I*asin(x**2)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \frac{x^5}{\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{x^4-1}}{4\,x^2\left(\frac{x^4-1}{x^4}-1\right)} + \frac{1}{8}\log\left(\frac{\sqrt{x^4-1}}{x^2}+1\right) - \frac{1}{8}\log\left(\frac{\sqrt{x^4-1}}{x^2}-1\right)$$

input
```
integrate(x^5/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(x^4 - 1)/(x^2*((x^4 - 1)/x^4 - 1)) + 1/8*log(sqrt(x^4 - 1)/x^2 +
 1) - 1/8*log(sqrt(x^4 - 1)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{\sqrt{-1+x^4}}\,dx = \frac{1}{4}\sqrt{x^4-1}x^2 - \frac{1}{4}\log\left(x^2-\sqrt{x^4-1}\right)$$

input
```
integrate(x^5/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 - 1)*x^2 - 1/4*log(x^2 - sqrt(x^4 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{-1+x^4}}\,dx = \int \frac{x^5}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^5/(x^4 - 1)^(1/2),x)
```

output
```
int(x^5/(x^4 - 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.00

$$\int \frac{x^5}{\sqrt{-1+x^4}}\,dx$$
$$= \frac{2\sqrt{x^4-1}\log\left(\sqrt{x^4-1}+x^2\right)x^2+2\sqrt{x^4-1}\,x^6-\sqrt{x^4-1}\,x^2+2\log\left(\sqrt{x^4-1}+x^2\right)x^4-\log\left(\sqrt{x^4-1}\right.}{8\sqrt{x^4-1}\,x^2+8x^4-4}$$

input
```
int(x^5/(x^4-1)^(1/2),x)
```

output
```
(2*sqrt(x**4 - 1)*log(sqrt(x**4 - 1) + x**2)*x**2 + 2*sqrt(x**4 - 1)*x**6
- sqrt(x**4 - 1)*x**2 + 2*log(sqrt(x**4 - 1) + x**2)*x**4 - log(sqrt(x**4
- 1) + x**2) + 2*x**8 - 2*x**4)/(4*(2*sqrt(x**4 - 1)*x**2 + 2*x**4 - 1))
```

3.296 $\int \frac{x}{\sqrt{-1+x^4}} dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{-1+x^4}}\right)$$

output
```
1/2*arctanh(x^2/(x^4-1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx = \frac{1}{2}\log\left(x^2+\sqrt{-1+x^4}\right)$$

input
```
Integrate[x/Sqrt[-1 + x^4],x]
```

output
```
Log[x^2 + Sqrt[-1 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{ 807}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^4-1}}dx^2$$
$$\Big\downarrow \text{ 224}$$
$$\frac{1}{2}\int \frac{1}{1-x^4}d\frac{x^2}{\sqrt{x^4-1}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{x^4-1}}\right)$$

input
```
Int[x/Sqrt[-1 + x^4],x]
```

output
```
ArcTanh[x^2/Sqrt[-1 + x^4]]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln\left(x^2+\sqrt{x^4-1}\right)}{2}$	15
trager	$\frac{\ln\left(x^2+\sqrt{x^4-1}\right)}{2}$	15
elliptic	$\frac{\ln\left(x^2+\sqrt{x^4-1}\right)}{2}$	15
pseudoelliptic	$\frac{\ln\left(x^2+\sqrt{x^4-1}\right)}{2}$	15
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\,\arcsin(x^2)}{2\sqrt{\operatorname{signum}(x^4-1)}}$	25

input
```
int(x/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+(x^4-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx = -\frac{1}{2}\,\log\left(-x^2+\sqrt{x^4-1}\right)$$

input
```
integrate(x/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{\operatorname{acosh}(x^2)}{2} & \text{for } |x^4| > 1 \\ -\frac{i \operatorname{asin}(x^2)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(x**4-1)**(1/2),x)
```

output
```
Piecewise((acosh(x**2)/2, Abs(x**4) > 1), (-I*asin(x**2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx = \frac{1}{4}\log\left(\frac{\sqrt{x^4-1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4-1}}{x^2}-1\right)$$

input
```
integrate(x/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 - 1)/x^2 + 1) - 1/4*log(sqrt(x^4 - 1)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx = -\frac{1}{2}\log\left(x^2-\sqrt{x^4-1}\right)$$

input
```
integrate(x/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(x^2 - sqrt(x^4 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx=\frac{\ln\left(\sqrt{x^4-1}+x^2\right)}{2}$$

input
```
int(x/(x^4 - 1)^(1/2),x)
```

output
```
log((x^4 - 1)^(1/2) + x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{-1+x^4}}\,dx=\frac{\log\left(\sqrt{x^4-1}+x^2\right)}{2}$$

input
```
int(x/(x^4-1)^(1/2),x)
```

output
```
log(sqrt(x**4 - 1) + x**2)/2
```

3.297 $\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{2x^2}$$

output
```
1/2*(x^4-1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{2x^2}$$

input
```
Integrate[1/(x^3*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(2*x^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{796}$$

$$\frac{\sqrt{x^4-1}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(2*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
trager	$\frac{\sqrt{x^4-1}}{2x^2}$	13
risch	$\frac{\sqrt{x^4-1}}{2x^2}$	13
pseudoelliptic	$\frac{\sqrt{x^4-1}}{2x^2}$	13
default	$\frac{(x^2-1)(x^2+1)}{2x^2\sqrt{x^4-1}}$	23
elliptic	$\frac{(x^2-1)(x^2+1)}{2x^2\sqrt{x^4-1}}$	23
gosper	$\frac{(-1+x)(1+x)(x^2+1)}{2x^2\sqrt{x^4-1}}$	24
orering	$\frac{(-1+x)(1+x)(x^2+1)}{2x^2\sqrt{x^4-1}}$	24
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\,\sqrt{-x^4+1}}{2\sqrt{\operatorname{signum}(x^4-1)}\,x^2}$	33

input
```
int(1/x^3/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^4-1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx = \frac{x^2+\sqrt{x^4-1}}{2\,x^2}$$

input
```
integrate(1/x^3/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^2 + sqrt(x^4 - 1))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{i\sqrt{-1+\frac{1}{x^4}}}{2} & \text{for } \frac{1}{|x^4|} > 1 \\ \frac{\sqrt{1-\frac{1}{x^4}}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(x**4-1)**(1/2),x)
```

output
```
Piecewise((I*sqrt(-1 + x**(-4))/2, 1/Abs(x**4) > 1), (sqrt(1 - 1/x**4)/2,
True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{2\,x^2}$$

input
```
integrate(1/x^3/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 - 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx = \frac{1}{\left(x^2-\sqrt{x^4-1}\right)^2+1}$$

input
```
integrate(1/x^3/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
1/((x^2 - sqrt(x^4 - 1))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{2\,x^2}$$

input
```
int(1/(x^3*(x^4 - 1)^(1/2)),x)
```

output
```
(x^4 - 1)^(1/2)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^3\sqrt{-1+x^4}}\,dx = \frac{2\sqrt{x^4-1}\,x^2+2x^4-1}{2x^2\left(\sqrt{x^4-1}+x^2\right)}$$

input
```
int(1/x^3/(x^4-1)^(1/2),x)
```

output
```
(2*sqrt(x**4 - 1)*x**2 + 2*x**4 - 1)/(2*x**2*(sqrt(x**4 - 1) + x**2))
```

3.298 $\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{6x^6}+\frac{\sqrt{-1+x^4}}{3x^2}$$

output
```
1/6*(x^4-1)^(1/2)/x^6+1/3*(x^4-1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}(1+2x^4)}{6x^6}$$

input
```
Integrate[1/(x^7*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^4]*(1 + 2*x^4))/(6*x^6)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{803}$$
$$\frac{2}{3}\int \frac{1}{x^3\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{6x^6}$$
$$\Big\downarrow \text{796}$$
$$\frac{\sqrt{x^4-1}}{6x^6}+\frac{\sqrt{x^4-1}}{3x^2}$$

input
```
Int[1/(x^7*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(6*x^6) + Sqrt[-1 + x^4]/(3*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
trager	$\frac{(2x^4+1)\sqrt{x^4-1}}{6x^6}$	20
pseudoelliptic	$\frac{(2x^4+1)\sqrt{x^4-1}}{6x^6}$	20
risch	$\frac{2x^8-x^4-1}{6x^6\sqrt{x^4-1}}$	25
default	$\frac{(x^2+1)(x^2-1)(2x^4+1)}{6x^6\sqrt{x^4-1}}$	30
elliptic	$\frac{(x^2+1)(x^2-1)(2x^4+1)}{6x^6\sqrt{x^4-1}}$	30
gosper	$\frac{(-1+x)(1+x)(x^2+1)(2x^4+1)}{6x^6\sqrt{x^4-1}}$	31
orering	$\frac{(-1+x)(1+x)(x^2+1)(2x^4+1)}{6x^6\sqrt{x^4-1}}$	31
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\,(2x^4+1)\sqrt{-x^4+1}}{6\sqrt{\operatorname{signum}(x^4-1)}\,x^6}$	40

input
```
int(1/x^7/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^4+1)/x^6*(x^4-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx = \frac{2\,x^6 + (2\,x^4+1)\sqrt{x^4-1}}{6\,x^6}$$

input
```
integrate(1/x^7/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^6 + (2*x^4 + 1)*sqrt(x^4 - 1))/x^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{\sqrt{x^4-1}}{3x^2} + \frac{\sqrt{x^4-1}}{6x^6} & \text{for } |x^4| > 1 \\ \frac{i\sqrt{1-x^4}}{3x^2} + \frac{i\sqrt{1-x^4}}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**7/(x**4-1)**(1/2),x)
```

output
```
Piecewise((sqrt(x**4 - 1)/(3*x**2) + sqrt(x**4 - 1)/(6*x**6), Abs(x**4) >
1), (I*sqrt(1 - x**4)/(3*x**2) + I*sqrt(1 - x**4)/(6*x**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{2\,x^2} - \frac{(x^4-1)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate(1/x^7/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 - 1)/x^2 - 1/6*(x^4 - 1)^(3/2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx = \frac{2\left(3\left(x^2-\sqrt{x^4-1}\right)^2+1\right)}{3\left(\left(x^2-\sqrt{x^4-1}\right)^2+1\right)^3}$$

input
```
integrate(1/x^7/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(x^2 - sqrt(x^4 - 1))^2 + 1)/((x^2 - sqrt(x^4 - 1))^2 + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}+2\,x^4\,\sqrt{x^4-1}}{6\,x^6}$$

input
```
int(1/(x^7*(x^4 - 1)^(1/2)),x)
```

output
```
((x^4 - 1)^(1/2) + 2*x^4*(x^4 - 1)^(1/2))/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^7\sqrt{-1+x^4}}\,dx = \frac{-3\sqrt{x^4-1}\,x^2-3x^4+1}{6x^6\left(4\sqrt{x^4-1}\,x^4-\sqrt{x^4-1}+4x^6-3x^2\right)}$$

input
```
int(1/x^7/(x^4-1)^(1/2),x)
```

output
```
( - 3*sqrt(x**4 - 1)*x**2 - 3*x**4 + 1)/(6*x**6*(4*sqrt(x**4 - 1)*x**4 - s
qrt(x**4 - 1) + 4*x**6 - 3*x**2))
```

3.299 $\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{10x^{10}}+\frac{2\sqrt{-1+x^4}}{15x^6}+\frac{4\sqrt{-1+x^4}}{15x^2}$$

output
```
1/10*(x^4-1)^(1/2)/x^10+2/15*(x^4-1)^(1/2)/x^6+4/15*(x^4-1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}(3+4x^4+8x^8)}{30x^{10}}$$

input
```
Integrate[1/(x^11*Sqrt[-1 + x^4]),x]
```

output
```
(Sqrt[-1 + x^4]*(3 + 4*x^4 + 8*x^8))/(30*x^10)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{4}{5}\int \frac{1}{x^7\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{10x^{10}}$$

$$\Big\downarrow \text{803}$$

$$\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{x^3\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{6x^6}\right)+\frac{\sqrt{x^4-1}}{10x^{10}}$$

$$\Big\downarrow \text{796}$$

$$\frac{\sqrt{x^4-1}}{10x^{10}}+\frac{4}{5}\left(\frac{\sqrt{x^4-1}}{6x^6}+\frac{\sqrt{x^4-1}}{3x^2}\right)$$

input
```
Int[1/(x^11*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(10*x^10) + (4*(Sqrt[-1 + x^4]/(6*x^6) + Sqrt[-1 + x^4]/(3*
x^2)))/5
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
trager	$\frac{(8x^8+4x^4+3)\sqrt{x^4-1}}{30x^{10}}$	25
pseudoelliptic	$\frac{(8x^8+4x^4+3)\sqrt{x^4-1}}{30x^{10}}$	25
risch	$\frac{8x^{12}-4x^8-x^4-3}{30x^{10}\sqrt{x^4-1}}$	30
default	$\frac{(x^2-1)(x^2+1)(8x^8+4x^4+3)}{30x^{10}\sqrt{x^4-1}}$	35
elliptic	$\frac{(x^2-1)(x^2+1)(8x^8+4x^4+3)}{30x^{10}\sqrt{x^4-1}}$	35
gosper	$\frac{(-1+x)(1+x)(x^2+1)(8x^8+4x^4+3)}{30x^{10}\sqrt{x^4-1}}$	36
orering	$\frac{(-1+x)(1+x)(x^2+1)(8x^8+4x^4+3)}{30x^{10}\sqrt{x^4-1}}$	36
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\left(\frac{8}{3}x^8+\frac{4}{3}x^4+1\right)\sqrt{-x^4+1}}{10\sqrt{\operatorname{signum}(x^4-1)}\,x^{10}}$	45

input
```
int(1/x^11/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(8*x^8+4*x^4+3)/x^10*(x^4-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx = \frac{8\,x^{10} + (8\,x^8 + 4\,x^4 + 3)\sqrt{x^4 - 1}}{30\,x^{10}}$$

input
```
integrate(1/x^11/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(8*x^10 + (8*x^8 + 4*x^4 + 3)*sqrt(x^4 - 1))/x^10
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx = \begin{cases} \frac{4i\sqrt{-1+\frac{1}{x^4}}}{15} + \frac{2i\sqrt{-1+\frac{1}{x^4}}}{15x^4} + \frac{i\sqrt{-1+\frac{1}{x^4}}}{10x^8} & \text{for}\ \frac{1}{|x^4|} > 1 \\ \frac{4\sqrt{1-\frac{1}{x^4}}}{15} + \frac{2\sqrt{1-\frac{1}{x^4}}}{15x^4} + \frac{\sqrt{1-\frac{1}{x^4}}}{10x^8} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**11/(x**4-1)**(1/2),x)
```

output
```
Piecewise((4*I*sqrt(-1 + x**(-4))/15 + 2*I*sqrt(-1 + x**(-4))/(15*x**4) +
I*sqrt(-1 + x**(-4))/(10*x**8), 1/Abs(x**4) > 1), (4*sqrt(1 - 1/x**4)/15 +
 2*sqrt(1 - 1/x**4)/(15*x**4) + sqrt(1 - 1/x**4)/(10*x**8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}}{2\,x^2} - \frac{(x^4-1)^{\frac{3}{2}}}{3\,x^6} + \frac{(x^4-1)^{\frac{5}{2}}}{10\,x^{10}}$$

input
```
integrate(1/x^11/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 - 1)/x^2 - 1/3*(x^4 - 1)^(3/2)/x^6 + 1/10*(x^4 - 1)^(5/2)/x^1
0
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx = \frac{8\left(10\left(x^2-\sqrt{x^4-1}\right)^4+5\left(x^2-\sqrt{x^4-1}\right)^2+1\right)}{15\left(\left(x^2-\sqrt{x^4-1}\right)^2+1\right)^5}$$

input
```
integrate(1/x^11/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
8/15*(10*(x^2 - sqrt(x^4 - 1))^4 + 5*(x^2 - sqrt(x^4 - 1))^2 + 1)/((x^2 -
sqrt(x^4 - 1))^2 + 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,(8\,x^8+4\,x^4+3)}{30\,x^{10}}$$

input
```
int(1/(x^11*(x^4 - 1)^(1/2)),x)
```

output
```
((x^4 - 1)^(1/2)*(4*x^4 + 8*x^8 + 3))/(30*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \frac{1}{x^{11}\sqrt{-1+x^4}}\,dx = \frac{-40\sqrt{x^4-1}\,x^6+15\sqrt{x^4-1}\,x^2-40x^8+35x^4-3}{30x^{10}\left(16\sqrt{x^4-1}\,x^8-12\sqrt{x^4-1}\,x^4+\sqrt{x^4-1}+16x^{10}-20x^6+5x^2\right)}$$

input
```
int(1/x^11/(x^4-1)^(1/2),x)
```

output
```
( - 40*sqrt(x**4 - 1)*x**6 + 15*sqrt(x**4 - 1)*x**2 - 40*x**8 + 35*x**4 -
3)/(30*x**10*(16*sqrt(x**4 - 1)*x**8 - 12*sqrt(x**4 - 1)*x**4 + sqrt(x**4
- 1) + 16*x**10 - 20*x**6 + 5*x**2))
```

3.300 $\int \frac{x^8}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx=\frac{5}{21}x\sqrt{-1+x^4}+\frac{1}{7}x^5\sqrt{-1+x^4}+\frac{5\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{21\sqrt{-1+x^4}}$$

output
```
5/21*x*(x^4-1)^(1/2)+1/7*x^5*(x^4-1)^(1/2)+5/21*(-x^4+1)^(1/2)*EllipticF(x
,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx=\frac{x\left(-5+2x^4+3x^8+5\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)}{21\sqrt{-1+x^4}}$$

input
```
Integrate[x^8/Sqrt[-1 + x^4],x]
```

output
```
(x*(-5 + 2*x^4 + 3*x^8 + 5*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4,
x^4]))/(21*Sqrt[-1 + x^4])
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {843, 843, 763}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{x^4-1}} \, dx$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{7} \int \frac{x^4}{\sqrt{x^4-1}}dx + \frac{1}{7}\sqrt{x^4-1}x^5$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{x^4-1}}dx + \frac{1}{3}\sqrt{x^4-1}x\right) + \frac{1}{7}\sqrt{x^4-1}x^5$$

$$\Big\downarrow \text{763}$$

$$\frac{5}{7}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4-1}} + \frac{1}{3}\sqrt{x^4-1}x\right) + \frac{1}{7}\sqrt{x^4-1}x^5$$

input
```
Int[x^8/Sqrt[-1 + x^4],x]
```

output
```
(x^5*Sqrt[-1 + x^4])/7 + (5*((x*Sqrt[-1 + x^4])/3 + (Sqrt[-1 + x^2]*Sqrt[1
 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(3*Sqrt[2]*Sqr
t[-1 + x^4])))/7
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.95 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\, x^9 \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{9}{4}\right], \left[\frac{13}{4}\right], x^4\right)}{9\sqrt{\operatorname{signum}(x^4-1)}}$	33
risch	$\frac{x(3x^4+5)\sqrt{x^4-1}}{21} - \frac{5i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \operatorname{EllipticF}(ix,i)}{21\sqrt{x^4-1}}$	52
default	$\frac{x^5\sqrt{x^4-1}}{7} + \frac{5x\sqrt{x^4-1}}{21} - \frac{5i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \operatorname{EllipticF}(ix,i)}{21\sqrt{x^4-1}}$	57
elliptic	$\frac{x^5\sqrt{x^4-1}}{7} + \frac{5x\sqrt{x^4-1}}{21} - \frac{5i\sqrt{x^2+1}\,\sqrt{-x^2+1}\ \operatorname{EllipticF}(ix,i)}{21\sqrt{x^4-1}}$	57

input
```
int(x^8/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^9*hypergeom([1/2,9/4],[13
/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx = \frac{1}{21}\left(3\,x^5+5\,x\right)\sqrt{x^4-1}-\frac{5}{21}\,F(\arcsin\left(\frac{1}{x}\right)\,|-1)$$

input
```
integrate(x^8/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(3*x^5 + 5*x)*sqrt(x^4 - 1) - 5/21*elliptic_f(arcsin(1/x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx = -\frac{ix^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|x^4\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(x**4-1)**(1/2),x)
```

output
```
-I*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), x**4)/(4*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx = \int \frac{x^8}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^8/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx = \int \frac{x^8}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^8/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx = \int \frac{x^8}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^8/(x^4 - 1)^(1/2),x)
```

output
```
int(x^8/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,x^5}{7} + \frac{5\sqrt{x^4-1}\,x}{21} + \frac{5\left(\int \frac{\sqrt{x^4-1}}{x^4-1}dx\right)}{21}$$

input
```
int(x^8/(x^4-1)^(1/2),x)
```

output
```
(3*sqrt(x**4 - 1)*x**5 + 5*sqrt(x**4 - 1)*x + 5*int(sqrt(x**4 - 1)/(x**4 -
 1),x))/21
```

3.301 $\int \frac{x^4}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx=\frac{1}{3}x\sqrt{-1+x^4}+\frac{\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{3\sqrt{-1+x^4}}$$

output
```
1/3*x*(x^4-1)^(1/2)+1/3*(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx=\frac{x\left(-1+x^4+\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)\right)}{3\sqrt{-1+x^4}}$$

input
```
Integrate[x^4/Sqrt[-1 + x^4],x]
```

output
```
(x*(-1 + x^4 + Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4]))/(3*Sq
rt[-1 + x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {843, 763}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{x^4-1}} \, dx$$
$$\Big\downarrow \ 843$$
$$\frac{1}{3}\int \frac{1}{\sqrt{x^4-1}}dx+\frac{1}{3}\sqrt{x^4-1}x$$
$$\Big\downarrow \ 763$$
$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4-1}}+\frac{1}{3}\sqrt{x^4-1}x$$

input
```
Int[x^4/Sqrt[-1 + x^4],x]
```

output
```
(x*Sqrt[-1 + x^4])/3 + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqr
t[2]*x)/Sqrt[-1 + x^2]], 1/2])/(3*Sqrt[2]*Sqrt[-1 + x^4])
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\, x^5 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],x^4\right)}{5\sqrt{\operatorname{signum}(x^4-1)}}$	33
default	$\frac{x\sqrt{x^4-1}}{3}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix,i)}{3\sqrt{x^4-1}}$	45
risch	$\frac{x\sqrt{x^4-1}}{3}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix,i)}{3\sqrt{x^4-1}}$	45
elliptic	$\frac{x\sqrt{x^4-1}}{3}-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix,i)}{3\sqrt{x^4-1}}$	45

input
```
int(x^4/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^5*hypergeom([1/2,5/4],[9/
4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.44

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx = \frac{1}{3}\sqrt{x^4-1}x - \frac{1}{3}F\left(\arcsin\left(\frac{1}{x}\right) \mid -1\right)$$

input
```
integrate(x^4/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(x^4 - 1)*x - 1/3*elliptic_f(arcsin(1/x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx = -\frac{ix^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(x**4-1)**(1/2),x)
```

output
```
-I*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4)/(4*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx = \int \frac{x^4}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^4/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx = \int \frac{x^4}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^4/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx = \int \frac{x^4}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^4/(x^4 - 1)^(1/2),x)
```

output
```
int(x^4/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,x}{3} + \frac{\left(\int \frac{\sqrt{x^4-1}}{x^4-1}dx\right)}{3}$$

input
```
int(x^4/(x^4-1)^(1/2),x)
```

output
```
(sqrt(x**4 - 1)*x + int(sqrt(x**4 - 1)/(x**4 - 1),x))/3
```

3.302 $\int \frac{1}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

output
```
(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

input
```
Integrate[1/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[1 - x^4]*EllipticF[ArcSin[x], -1])/Sqrt[-1 + x^4]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {763}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4-1}}\,dx$$

$$\Big\downarrow \text{763}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}$$

input
```
Int[1/Sqrt[-1 + x^4],x]
```

output
```
(Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]]
, 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\, x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^4\right)}{\sqrt{\operatorname{signum}(x^4-1)}}$	30
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix, i)}{\sqrt{x^4-1}}$	34
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\operatorname{EllipticF}(ix, i)}{\sqrt{x^4-1}}$	34

input
```
int(1/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x*hypergeom([1/4,1/2],[5/4],x
^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.24

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = -i\,F(\arcsin(x)\mid -1)$$

input
```
integrate(1/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
-I*elliptic_f(arcsin(x), -1)
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = -\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(x**4-1)**(1/2),x)
```

output
```
-I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4)/(4*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(1/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(1/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^4 - 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \frac{x\sqrt{1-x^4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ x^4\right)}{\sqrt{x^4-1}}$$

input
```
int(1/(x^4 - 1)^(1/2),x)
```

output
```
(x*(1 - x^4)^(1/2)*hypergeom([1/4, 1/2], 5/4, x^4))/(x^4 - 1)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^4-1}dx$$

input
```
int(1/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**4 - 1),x)
```

3.303 $\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{3x^3}+\frac{\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{3\sqrt{-1+x^4}}$$

output
```
1/3*(x^4-1)^(1/2)/x^3+1/3*(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx=-\frac{\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},x^4\right)}{3x^3\sqrt{-1+x^4}}$$

input
```
Integrate[1/(x^4*Sqrt[-1 + x^4]),x]
```

output
```
-1/3*(Sqrt[1 - x^4]*Hypergeometric2F1[-3/4, 1/2, 1/4, x^4])/(x^3*Sqrt[-1 +
 x^4])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {847, 763}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{3x^3}$$

$$\Big\downarrow \text{763}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{3x^3}$$

input
```
Int[1/(x^4*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(3*x^3) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(S
qrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(3*Sqrt[2]*Sqrt[-1 + x^4])
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, \operatorname{hypergeom}\left(\left[-\frac{3}{4},\frac{1}{2}\right],\left[\frac{1}{4}\right],x^{4}\right)}{3 \sqrt{\operatorname{signum}\left(x^{4}-1\right)}\, x^{3}}$	33
default	$\frac{\sqrt{x^{4}-1}}{3 x^{3}}-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x , i\right)}{3 \sqrt{x^{4}-1}}$	47
risch	$\frac{\sqrt{x^{4}-1}}{3 x^{3}}-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x , i\right)}{3 \sqrt{x^{4}-1}}$	47
elliptic	$\frac{\sqrt{x^{4}-1}}{3 x^{3}}-\frac{i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x , i\right)}{3 \sqrt{x^{4}-1}}$	47

input
```
int(1/x^4/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)/x^3*hypergeom([-3/4,1/2],[
1/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx = \frac{-i\,x^3F(\arcsin{(x)}\mid -1)+\sqrt{x^4-1}}{3\,x^3}$$

input
```
integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-I*x^3*elliptic_f(arcsin(x), -1) + sqrt(x^4 - 1))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx = -\frac{i\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| x^4\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(x**4-1)**(1/2),x)
```

output
```
-I*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4)/(4*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^4}\,dx$$

input
```
integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^4}\,dx$$

input
```
integrate(1/x^4/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx = \int \frac{1}{x^4\,\sqrt{x^4-1}}\,dx$$

input
```
int(1/(x^4*(x^4 - 1)^(1/2)),x)
```

output
```
int(1/(x^4*(x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^8-x^4}dx$$

input
```
int(1/x^4/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**8 - x**4),x)
```

3.304 $\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx=\frac{\sqrt{-1+x^4}}{7x^7}+\frac{5\sqrt{-1+x^4}}{21x^3}+\frac{5\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{21\sqrt{-1+x^4}}$$

output
```
1/7*(x^4-1)^(1/2)/x^7+5/21*(x^4-1)^(1/2)/x^3+5/21*(-x^4+1)^(1/2)*EllipticF
(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx=-\frac{\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},x^4\right)}{7x^7\sqrt{-1+x^4}}$$

input
```
Integrate[1/(x^8*Sqrt[-1 + x^4]),x]
```

output
```
-1/7*(Sqrt[1 - x^4]*Hypergeometric2F1[-7/4, 1/2, -3/4, x^4])/(x^7*Sqrt[-1
+ x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {847, 847, 763}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{7}\int \frac{1}{x^4\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{3x^3}\right)+\frac{\sqrt{x^4-1}}{7x^7}$$

$$\Big\downarrow \text{763}$$

$$\frac{5}{7}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{3x^3}\right)+\frac{\sqrt{x^4-1}}{7x^7}$$

input
```
Int[1/(x^8*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(7*x^7) + (5*(Sqrt[-1 + x^4]/(3*x^3) + (Sqrt[-1 + x^2]*Sqrt
[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(3*Sqrt[2]*S
qrt[-1 + x^4])))/7
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, \operatorname{hypergeom}\left(\left[-\frac{7}{4}, \frac{1}{2}\right], \left[-\frac{3}{4}\right], x^{4}\right)}{7 \sqrt{\operatorname{signum}\left(x^{4}-1\right)}\, x^{7}}$	33
default	$\frac{\sqrt{x^{4}-1}}{7 x^{7}}+\frac{5 \sqrt{x^{4}-1}}{21 x^{3}}-\frac{5 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x , i\right)}{21 \sqrt{x^{4}-1}}$	59
risch	$\frac{5 x^{8}-2 x^{4}-3}{21 x^{7} \sqrt{x^{4}-1}}-\frac{5 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x , i\right)}{21 \sqrt{x^{4}-1}}$	59
elliptic	$\frac{\sqrt{x^{4}-1}}{7 x^{7}}+\frac{5 \sqrt{x^{4}-1}}{21 x^{3}}-\frac{5 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \operatorname{EllipticF}\left(i x , i\right)}{21 \sqrt{x^{4}-1}}$	59

input
```
int(1/x^8/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)/x^7*hypergeom([-7/4,1/2],[
-3/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx = \frac{-5i\,x^7F(\arcsin{(x)}\,|-1)+(5\,x^4+3)\sqrt{x^4-1}}{21\,x^7}$$

input
```
integrate(1/x^8/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*x^7*elliptic_f(arcsin(x), -1) + (5*x^4 + 3)*sqrt(x^4 - 1))/x^7
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx = -\frac{i\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|x^4\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(x**4-1)**(1/2),x)
```

output
```
-I*gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), x**4)/(4*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^8}\,dx$$

input
```
integrate(1/x^8/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^8}\,dx$$

input
```
integrate(1/x^8/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx = \int \frac{1}{x^8\,\sqrt{x^4-1}}\,dx$$

input
```
int(1/(x^8*(x^4 - 1)^(1/2)),x)
```

output
```
int(1/(x^8*(x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^{12}-x^8}dx$$

input
```
int(1/x^8/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**12 - x**8),x)
```

3.305 $\int \frac{x^{10}}{\sqrt{-1+x^4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{x^{10}}{\sqrt{-1+x^4}} \, dx = \frac{7}{45}x^3\sqrt{-1+x^4} + \frac{1}{9}x^7\sqrt{-1+x^4} + \frac{7\sqrt{1-x^4}E(\arcsin(x)|-1)}{15\sqrt{-1+x^4}} - \frac{7\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{15\sqrt{-1+x^4}}$$

output
```
7/45*x^3*(x^4-1)^(1/2)+1/9*x^7*(x^4-1)^(1/2)+7/15*(-x^4+1)^(1/2)*EllipticE
(x,I)/(x^4-1)^(1/2)-7/15*(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^{10}}{\sqrt{-1+x^4}} \, dx = \frac{x^3\left(-7+2x^4+5x^8+7\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)\right)}{45\sqrt{-1+x^4}}$$

input
```
Integrate[x^10/Sqrt[-1 + x^4],x]
```

output
```
(x^3*(-7 + 2*x^4 + 5*x^8 + 7*Sqrt[1 - x^4]*Hypergeometric2F1[1/2, 3/4, 7/4
, x^4]))/(45*Sqrt[-1 + x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {843, 843, 835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{7}{9}\int \frac{x^6}{\sqrt{x^4-1}}dx+\frac{1}{9}\sqrt{x^4-1}x^7$$

$$\Big\downarrow \text{843}$$

$$\frac{7}{9}\left(\frac{3}{5}\int \frac{x^2}{\sqrt{x^4-1}}dx+\frac{1}{5}\sqrt{x^4-1}x^3\right)+\frac{1}{9}\sqrt{x^4-1}x^7$$

$$\Big\downarrow \text{835}$$

$$\frac{7}{9}\left(\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)+\frac{1}{5}\sqrt{x^4-1}x^3\right)+\frac{1}{9}\sqrt{x^4-1}x^7$$

$$\Big\downarrow \text{763}$$

$$\frac{7}{9}\left(\frac{3}{5}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)+\frac{1}{5}\sqrt{x^4-1}x^3\right)+\frac{1}{9}\sqrt{x^4-1}x^7$$

$$\Big\downarrow \text{1499}$$

$$\frac{7}{9}\left(\frac{3}{5}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x(x^2+}{\sqrt{x^4-}}\right.\right.$$

$$\frac{1}{9}\sqrt{x^4-1}x^7$$

input
```
Int[x^10/Sqrt[-1 + x^4],x]
```

output
```
(x^7*Sqrt[-1 + x^4])/9 + (7*((x^3*Sqrt[-1 + x^4])/5 + (3*((x*(1 + x^2))/Sq
rt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqr
t[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + (Sqrt[-1 + x^2]*Sqrt[1 + x
^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 +
 x^4])))/5))/9
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, x^{11} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{11}{4}\right],\left[\frac{15}{4}\right], x^{4}\right)}{11 \sqrt{\operatorname{signum}\left(x^{4}-1\right)}}$	33
risch	$\frac{x^{3}\left(5 x^{4}+7\right) \sqrt{x^{4}-1}}{45}-\frac{7 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\,(\operatorname{EllipticF}(i x, i)-\operatorname{EllipticE}(i x, i))}{15 \sqrt{x^{4}-1}}$	64
default	$\frac{x^{7} \sqrt{x^{4}-1}}{9}+\frac{7 x^{3} \sqrt{x^{4}-1}}{45}-\frac{7 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\,(\operatorname{EllipticF}(i x, i)-\operatorname{EllipticE}(i x, i))}{15 \sqrt{x^{4}-1}}$	69
elliptic	$\frac{x^{7} \sqrt{x^{4}-1}}{9}+\frac{7 x^{3} \sqrt{x^{4}-1}}{45}-\frac{7 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\,(\operatorname{EllipticF}(i x, i)-\operatorname{EllipticE}(i x, i))}{15 \sqrt{x^{4}-1}}$	69

input
```
int(x^10/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/11/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^11*hypergeom([1/2,11/4],
[15/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{x^{10}}{\sqrt{-1+x^4}}\, dx = \frac{21\, xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right) - 21\, xF\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right) + \left(5\, x^{8} + 7\, x^{4} + 21\right)\sqrt{x^{4} - 1}}{45\, x}$$

input
```
integrate(x^10/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/45*(21*x*elliptic_e(arcsin(1/x), -1) - 21*x*elliptic_f(arcsin(1/x), -1)
+ (5*x^8 + 7*x^4 + 21)*sqrt(x^4 - 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.30

$$\int \frac{x^{10}}{\sqrt{-1+x^4}}\,dx = -\frac{ix^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle| x^4\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(x**4-1)**(1/2),x)
```

output
```
-I*x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), x**4)/(4*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{-1+x^4}}\,dx = \int \frac{x^{10}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^10/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^10/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{-1+x^4}}\,dx = \int \frac{x^{10}}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^10/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^10/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{-1+x^4}}\,dx = \int \frac{x^{10}}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^10/(x^4 - 1)^(1/2),x)
```

output
```
int(x^10/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,x^7}{9} + \frac{7\sqrt{x^4-1}\,x^3}{45} + \frac{7\left(\int \frac{\sqrt{x^4-1}\,x^2}{x^4-1}dx\right)}{15}$$

input
```
int(x^10/(x^4-1)^(1/2),x)
```

output
```
(5*sqrt(x**4 - 1)*x**7 + 7*sqrt(x**4 - 1)*x**3 + 21*int((sqrt(x**4 - 1)*x*
*2)/(x**4 - 1),x))/45
```

3.306 $\int \frac{x^6}{\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = \frac{1}{5}x^3\sqrt{-1+x^4} + \frac{3\sqrt{1-x^4}E(\arcsin(x)|-1)}{5\sqrt{-1+x^4}} - \frac{3\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{5\sqrt{-1+x^4}}$$

output
```
1/5*x^3*(x^4-1)^(1/2)+3/5*(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1/2)-3/5*
(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = \frac{x^3\left(-1+x^4+\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)\right)}{5\sqrt{-1+x^4}}$$

input
```
Integrate[x^6/Sqrt[-1 + x^4],x]
```

output
```
(x^3*(-1 + x^4 + Sqrt[1 - x^4]*Hypergeometric2F1[1/2, 3/4, 7/4, x^4]))/(5*
Sqrt[-1 + x^4])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(73) = 146.

Time = 0.42 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {843, 835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{x^4-1}}dx$$
$$\Big\downarrow \text{ 843}$$
$$\frac{3}{5}\int \frac{x^2}{\sqrt{x^4-1}}dx+\frac{1}{5}\sqrt{x^4-1}x^3$$
$$\Big\downarrow \text{ 835}$$
$$\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)+\frac{1}{5}\sqrt{x^4-1}x^3$$
$$\Big\downarrow \text{ 763}$$
$$\frac{3}{5}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\int \frac{1-x^2}{\sqrt{x^4-1}}dx\right)+\frac{1}{5}\sqrt{x^4-1}x^3$$
$$\Big\downarrow \text{ 1499}$$
$$\frac{3}{5}\left(\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x(x^2+1)}{\sqrt{x^4-1}}\right)+$$
$$\frac{1}{5}\sqrt{x^4-1}x^3$$

input
```
Int[x^6/Sqrt[-1 + x^4],x]
```

output
```
(x^3*Sqrt[-1 + x^4])/5 + (3*((x*(1 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[
-1 + x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]
)/Sqrt[-1 + x^4] + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]
*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])))/5
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{7}{4}\right],\left[\frac{11}{4}\right],x^{4}\right)}{7 \sqrt{\operatorname{signum}\left(x^{4}-1\right)}}$	33
default	$\frac{x^{3} \sqrt{x^{4}-1}}{5}-\frac{3 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \left(\operatorname{EllipticF}\left(i x , i\right)-\operatorname{EllipticE}\left(i x , i\right)\right)}{5 \sqrt{x^{4}-1}}$	57
risch	$\frac{x^{3} \sqrt{x^{4}-1}}{5}-\frac{3 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \left(\operatorname{EllipticF}\left(i x , i\right)-\operatorname{EllipticE}\left(i x , i\right)\right)}{5 \sqrt{x^{4}-1}}$	57
elliptic	$\frac{x^{3} \sqrt{x^{4}-1}}{5}-\frac{3 i \sqrt{x^{2}+1}\, \sqrt{-x^{2}+1}\, \left(\operatorname{EllipticF}\left(i x , i\right)-\operatorname{EllipticE}\left(i x , i\right)\right)}{5 \sqrt{x^{4}-1}}$	57

input
```
int(x^6/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^7*hypergeom([1/2,7/4],[11
/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx=\frac{3\,xE(\arcsin\left(\frac{1}{x}\right)\mid -1)-3\,xF(\arcsin\left(\frac{1}{x}\right)\mid -1)+(x^4+3)\sqrt{x^4-1}}{5\,x}$$

input
```
integrate(x^6/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(3*x*elliptic_e(arcsin(1/x), -1) - 3*x*elliptic_f(arcsin(1/x), -1) + (
x^4 + 3)*sqrt(x^4 - 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.37

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = -\frac{ix^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|x^4\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(x**4-1)**(1/2),x)
```

output
```
-I*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4)/(4*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = \int \frac{x^6}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^6/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = \int \frac{x^6}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^6/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = \int \frac{x^6}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^6/(x^4 - 1)^(1/2),x)
```

output
```
int(x^6/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{-1+x^4}}\,dx = \frac{\sqrt{x^4-1}\,x^3}{5} + \frac{3\left(\int \frac{\sqrt{x^4-1}\,x^2}{x^4-1}dx\right)}{5}$$

input
```
int(x^6/(x^4-1)^(1/2),x)
```

output
```
(sqrt(x**4 - 1)*x**3 + 3*int((sqrt(x**4 - 1)*x**2)/(x**4 - 1),x))/5
```

3.307 $\int \frac{x^2}{\sqrt{-1+x^4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{x^2}{\sqrt{-1+x^4}} \, dx = \frac{\sqrt{1-x^4} E(\arcsin(x)|-1)}{\sqrt{-1+x^4}} - \frac{\sqrt{1-x^4} \operatorname{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

output
```
(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1/2)-(-x^4+1)^(1/2)*EllipticF(x,I)/
(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{-1+x^4}} \, dx = \frac{x^3 \sqrt{1-x^4} \operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^4\right)}{3\sqrt{-1+x^4}}$$

input
```
Integrate[x^2/Sqrt[-1 + x^4],x]
```

output
```
(x^3*Sqrt[1 - x^4]*Hypergeometric2F1[1/2, 3/4, 7/4, x^4])/(3*Sqrt[-1 + x^4
])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(52) = 104$.

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.42, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{835}$$

$$\int \frac{1}{\sqrt{x^4-1}}dx-\int \frac{1-x^2}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{763}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\int \frac{1-x^2}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{1499}$$

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{x(x^2+1)}{\sqrt{x^4-1}}$$

input
```
Int[x^2/Sqrt[-1 + x^4],x]
```

output
```
(x*(1 + x^2))/Sqrt[-1 + x^4] - (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ellip
ticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] + (Sqrt[-1 +
 x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(S
qrt[2]*Sqrt[-1 + x^4])
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.63

method	result	size
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^4-1)}\, x^3 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^4\right)}{3\sqrt{\operatorname{signum}(x^4-1)}}$	33
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	44
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{\sqrt{x^4-1}}$	44

input
```
int(x^2/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)*x^3*hypergeom([1/2,3/4],[7/
4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = \frac{xE\left(\arcsin\left(\frac{1}{x}\right)|-1\right) - xF\left(\arcsin\left(\frac{1}{x}\right)|-1\right) + \sqrt{x^4-1}}{x}$$

input
```
integrate(x^2/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
(x*elliptic_e(arcsin(1/x), -1) - x*elliptic_f(arcsin(1/x), -1) + sqrt(x^4
- 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = -\frac{ix^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(x**4-1)**(1/2),x)
```

output
```
-I*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4)/(4*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^2/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(x^4 - 1), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^2/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(x^4 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4-1}}\,dx$$

input
```
int(x^2/(x^4 - 1)^(1/2),x)
```

output
```
int(x^2/(x^4 - 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}\,x^2}{x^4-1}dx$$

input
```
int(x^2/(x^4-1)^(1/2),x)
```

output
```
int((sqrt(x**4 - 1)*x**2)/(x**4 - 1),x)
```

3.308 $\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = \frac{\sqrt{-1+x^4}}{x} - \frac{\sqrt{1-x^4}E(\arcsin(x)|-1)}{\sqrt{-1+x^4}} + \frac{\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{\sqrt{-1+x^4}}$$

output
```
(x^4-1)^(1/2)/x-(-x^4+1)^(1/2)*EllipticE(x,I)/(x^4-1)^(1/2)+(-x^4+1)^(1/2)
*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},x^4\right)}{x\sqrt{-1+x^4}}$$

input
```
Integrate[1/(x^2*Sqrt[-1 + x^4]),x]
```

output
```
-((Sqrt[1 - x^4]*Hypergeometric2F1[-1/4, 1/2, 3/4, x^4])/(x*Sqrt[-1 + x^4]
))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time $= 0.41$ (sec) , antiderivative size $= 140$, normalized size of antiderivative $= 2.15$, number of steps used $= 4$, number of rules used $= 4$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used $= \{847, 835, 763, 1499\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^4-1}}dx$$

$$\Big\downarrow \ 847$$

$$\frac{\sqrt{x^4-1}}{x}-\int \frac{x^2}{\sqrt{x^4-1}}dx$$

$$\Big\downarrow \ 835$$

$$-\int \frac{1}{\sqrt{x^4-1}}dx+\int \frac{1-x^2}{\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{x}$$

$$\Big\downarrow \ 763$$

$$\int \frac{1-x^2}{\sqrt{x^4-1}}dx-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{x}$$

$$\Big\downarrow \ 1499$$

$$-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}+\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{x}-\frac{x(x^2+1)}{\sqrt{x^4-1}}$$

input
```
Int[1/(x^2*Sqrt[-1 + x^4]),x]
```

output
```
-((x*(1 + x^2))/Sqrt[-1 + x^4]) + Sqrt[-1 + x^4]/x + (Sqrt[2]*Sqrt[-1 + x^
2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/Sqrt[
-1 + x^4] - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqr
t[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.51

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(x^{4}-1\right)}\, \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{4}\right],x^{4}\right)}{\sqrt{\operatorname{signum}\left(x^{4}-1\right)}\, x}$	33
default	$\frac{\sqrt{x^{4}-1}}{x}+\frac{i\sqrt{x^{2}+1}\,\sqrt{-x^{2}+1}\,\left(\operatorname{EllipticF}\left(ix,i\right)-\operatorname{EllipticE}\left(ix,i\right)\right)}{\sqrt{x^{4}-1}}$	56
risch	$\frac{\sqrt{x^{4}-1}}{x}+\frac{i\sqrt{x^{2}+1}\,\sqrt{-x^{2}+1}\,\left(\operatorname{EllipticF}\left(ix,i\right)-\operatorname{EllipticE}\left(ix,i\right)\right)}{\sqrt{x^{4}-1}}$	56
elliptic	$\frac{\sqrt{x^{4}-1}}{x}+\frac{i\sqrt{x^{2}+1}\,\sqrt{-x^{2}+1}\,\left(\operatorname{EllipticF}\left(ix,i\right)-\operatorname{EllipticE}\left(ix,i\right)\right)}{\sqrt{x^{4}-1}}$	56

input
```
int(1/x^2/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)/x*hypergeom([-1/4,1/2],[3/4]
,x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx=\frac{i\,xE(\arcsin\left(x\right)\,|\,-1)-i\,xF(\arcsin\left(x\right)\,|\,-1)+\sqrt{x^4-1}}{x}$$

input
```
integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
(I*x*elliptic_e(arcsin(x), -1) - I*x*elliptic_f(arcsin(x), -1) + sqrt(x^4
- 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = -\frac{i\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|x^4\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(x**4-1)**(1/2),x)
```

output
```
-I*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4)/(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{\frac{1}{x^4}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ \frac{1}{x^4}\right)}{3\,x}$$

input
```
int(1/(x^2*(x^4 - 1)^(1/2)),x)
```

output
```
-((1/x^4)^(1/2)*hypergeom([1/2, 3/4], 7/4, 1/x^4))/(3*x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^6-x^2}dx$$

input
```
int(1/x^2/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**6 - x**2),x)
```

3.309 $\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = \frac{\sqrt{-1+x^4}}{5x^5}+\frac{3\sqrt{-1+x^4}}{5x}-\frac{3\sqrt{1-x^4}E(\arcsin(x)|-1)}{5\sqrt{-1+x^4}}+\frac{3\sqrt{1-x^4}\,\text{EllipticF}(\arcsin(x),-1)}{5\sqrt{-1+x^4}}$$

output
```
1/5*(x^4-1)^(1/2)/x^5+3/5*(x^4-1)^(1/2)/x-3/5*(-x^4+1)^(1/2)*EllipticE(x,I
)/(x^4-1)^(1/2)+3/5*(-x^4+1)^(1/2)*EllipticF(x,I)/(x^4-1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = -\frac{\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},x^4\right)}{5x^5\sqrt{-1+x^4}}$$

input
```
Integrate[1/(x^6*Sqrt[-1 + x^4]),x]
```

output
```
-1/5*(Sqrt[1 - x^4]*Hypergeometric2F1[-5/4, 1/2, -1/4, x^4])/(x^5*Sqrt[-1
+ x^4])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {847, 847, 835, 763, 1499}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{5}\int \frac{1}{x^2\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{5}\left(\frac{\sqrt{x^4-1}}{x}-\int \frac{x^2}{\sqrt{x^4-1}}dx\right)+\frac{\sqrt{x^4-1}}{5x^5}$$

$$\Big\downarrow \text{835}$$

$$\frac{3}{5}\left(-\int \frac{1}{\sqrt{x^4-1}}dx+\int \frac{1-x^2}{\sqrt{x^4-1}}dx+\frac{\sqrt{x^4-1}}{x}\right)+\frac{\sqrt{x^4-1}}{5x^5}$$

$$\Big\downarrow \text{763}$$

$$\frac{3}{5}\left(\int \frac{1-x^2}{\sqrt{x^4-1}}dx-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{x}\right)+\frac{\sqrt{x^4-1}}{5x^5}$$

$$\Big\downarrow \text{1499}$$

$$\frac{3}{5}\left(-\frac{\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}+\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{x^2+1}E\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{x^4-1}}+\frac{\sqrt{x^4-1}}{x}\right.$$
$$\frac{\sqrt{x^4-1}}{5x^5}$$

input
```
Int[1/(x^6*Sqrt[-1 + x^4]),x]
```

output
```
Sqrt[-1 + x^4]/(5*x^5) + (3*(-((x*(1 + x^2))/Sqrt[-1 + x^4]) + Sqrt[-1 + x
^4]/x + (Sqrt[2]*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticE[ArcSin[(Sqrt[2]*x)
/Sqrt[-1 + x^2]], 1/2])/Sqrt[-1 + x^4] - (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Ell
ipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4]))
)/5
```

Defintions of rubi rules used

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 835
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[
a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1499
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[(-a)*c, 2]}, Simp[e*x*((q + c*x^2)/(c*Sqrt[a + c*x^4])), x] - Simp[Sqrt
[2]*e*q*Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[-a]*c*Sqrt[a + c*x^4]))
*EllipticE[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; EqQ[c*d + e*q, 0]
 && IntegerQ[q]] /; FreeQ[{a, c, d, e}, x] && LtQ[a, 0] && GtQ[c, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.78 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.37

method	result	size
meijerg	$-\frac{\sqrt{-\operatorname{signum}(x^4-1)}\ \operatorname{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{2}\right],\left[-\frac{1}{4}\right],x^4\right)}{5\sqrt{\operatorname{signum}(x^4-1)}\, x^5}$	33
default	$\frac{\sqrt{x^4-1}}{5x^5}+\frac{3\sqrt{x^4-1}}{5x}+\frac{3i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{5\sqrt{x^4-1}}$	69
risch	$\frac{3x^8-2x^4-1}{5x^5\sqrt{x^4-1}}+\frac{3i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{5\sqrt{x^4-1}}$	69
elliptic	$\frac{\sqrt{x^4-1}}{5x^5}+\frac{3\sqrt{x^4-1}}{5x}+\frac{3i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\operatorname{EllipticF}(ix,i)-\operatorname{EllipticE}(ix,i))}{5\sqrt{x^4-1}}$	69

input
```
int(1/x^6/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/signum(x^4-1)^(1/2)*(-signum(x^4-1))^(1/2)/x^5*hypergeom([-5/4,1/2],[
-1/4],x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = \frac{3i\,x^5E(\arcsin{(x)}\mid -1)-3i\,x^5F(\arcsin{(x)}\mid -1)+(3\,x^4+1)\sqrt{x^4-1}}{5\,x^5}$$

input
```
integrate(1/x^6/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(3*I*x^5*elliptic_e(arcsin(x), -1) - 3*I*x^5*elliptic_f(arcsin(x), -1)
 + (3*x^4 + 1)*sqrt(x^4 - 1))/x^5
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = -\frac{i\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| x^4\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(x**4-1)**(1/2),x)
```

output
```
-I*gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), x**4)/(4*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4-1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 - 1)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = \int \frac{1}{x^6\,\sqrt{x^4-1}}\,dx$$

input
```
int(1/(x^6*(x^4 - 1)^(1/2)),x)
```

output
```
int(1/(x^6*(x^4 - 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{-1+x^4}}\,dx = \int \frac{\sqrt{x^4-1}}{x^{10}-x^6}dx$$

input
```
int(1/x^6/(x^4-1)^(1/2),x)
```

output
```
int(sqrt(x**4 - 1)/(x**10 - x**6),x)
```

3.310 $\int \frac{x^2}{\sqrt{3-2x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx=\frac{\sqrt[4]{3}E\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{2^{3/4}}$$

output
```
1/2*3^(1/4)*EllipticE(1/3*2^(1/4)*3^(3/4)*x,I)*2^(1/4)-1/2*3^(1/4)*Ellipti
cF(1/3*2^(1/4)*3^(3/4)*x,I)*2^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx=\frac{x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{2x^4}{3}\right)}{3\sqrt{3}}$$

input
```
Integrate[x^2/Sqrt[3 - 2*x^4],x]
```

output
```
(x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (2*x^4)/3])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {836, 27, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx$$
$$\Big\downarrow \text{836}$$
$$\sqrt{\frac{3}{2}}\int \frac{\sqrt{6}x^2+3}{3\sqrt{3-2x^4}}dx-\sqrt{\frac{3}{2}}\int \frac{1}{\sqrt{3-2x^4}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\sqrt{6}x^2+3}{\sqrt{3-2x^4}}dx}{\sqrt{6}}-\sqrt{\frac{3}{2}}\int \frac{1}{\sqrt{3-2x^4}}dx$$
$$\Big\downarrow \text{762}$$
$$\frac{\int \frac{\sqrt{6}x^2+3}{\sqrt{3-2x^4}}dx}{\sqrt{6}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{2^{3/4}}$$
$$\Big\downarrow \text{1388}$$
$$\frac{\int \frac{\sqrt{\sqrt{6}x^2+3}}{\sqrt{1-\sqrt{\frac{2}{3}}x^2}}dx}{\sqrt{6}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{2^{3/4}}$$
$$\Big\downarrow \text{327}$$
$$\frac{\sqrt[4]{3}E\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\frac{2}{3}}x\right),-1\right)}{2^{3/4}}$$

input
```
Int[x^2/Sqrt[3 - 2*x^4],x]
```

output
```
(3^(1/4)*EllipticE[ArcSin[(2/3)^(1/4)*x], -1])/2^(3/4) - (3^(1/4)*Elliptic
F[ArcSin[(2/3)^(1/4)*x], -1])/2^(3/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.42

method	result	size
meijerg	$\frac{\sqrt{3}\, x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], \frac{2 x^{4}}{3}\right)}{9}$	20
default	$-\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \left(\operatorname{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)-\operatorname{EllipticE}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)\right)}{18 \sqrt{-2 x^{4}+3}}$	69
elliptic	$-\frac{\sqrt{3}\, 6^{\frac{1}{4}} \sqrt{9-3 \sqrt{6}\, x^{2}}\, \sqrt{9+3 \sqrt{6}\, x^{2}}\, \left(\operatorname{EllipticF}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)-\operatorname{EllipticE}\left(\frac{x \sqrt{3}\, 6^{\frac{1}{4}}}{3}, i\right)\right)}{18 \sqrt{-2 x^{4}+3}}$	69

input
```
int(x^2/(-2*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*x^3*hypergeom([1/2,3/4],[7/4],2/3*x^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx = -\frac{6^{\frac{3}{4}}\sqrt{\frac{1}{2}}\sqrt{-2}xE(\arcsin\left(\frac{6^{\frac{1}{4}}\sqrt{\frac{1}{2}}}{x}\right)\mid -1) - 6^{\frac{3}{4}}\sqrt{\frac{1}{2}}\sqrt{-2}xF(\arcsin\left(\frac{6^{\frac{1}{4}}\sqrt{\frac{1}{2}}}{x}\right)\mid -1) + 2\sqrt{-2\,x^4+3}}{4\,x}$$

input
```
integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(6^(3/4)*sqrt(1/2)*sqrt(-2)*x*elliptic_e(arcsin(6^(1/4)*sqrt(1/2)/x),
 -1) - 6^(3/4)*sqrt(1/2)*sqrt(-2)*x*elliptic_f(arcsin(6^(1/4)*sqrt(1/2)/x)
, -1) + 2*sqrt(-2*x^4 + 3))/x
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx = \frac{\sqrt{3}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-2*x**4+3)**(1/2),x)
```

output
```
sqrt(3)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), 2*x**4*exp_polar(2*I*pi)
/3)/(12*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx = \int \frac{x^2}{\sqrt{-2\,x^4+3}}\,dx$$

input
```
integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-2*x^4 + 3), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx = \int \frac{x^2}{\sqrt{-2\,x^4+3}}\,dx$$

input
```
integrate(x^2/(-2*x^4+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-2*x^4 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx=\int \frac{x^2}{\sqrt{3-2\,x^4}}\,dx$$

input
```
int(x^2/(3 - 2*x^4)^(1/2),x)
```

output
```
int(x^2/(3 - 2*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{3-2x^4}}\,dx=-\left(\int \frac{\sqrt{-2x^4+3}\,x^2}{2x^4-3}dx\right)$$

input
```
int(x^2/(-2*x^4+3)^(1/2),x)
```

output
```
- int((sqrt( - 2*x**4 + 3)*x**2)/(2*x**4 - 3),x)
```

3.311 $\int \frac{x^2}{\sqrt{3-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx=\frac{\sqrt[4]{3}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right)\middle|-1\right)}{b^{3/4}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right),-1\right)}{b^{3/4}}$$

output
```
3^(1/4)*EllipticE(1/3*b^(1/4)*x*3^(3/4),I)/b^(3/4)-3^(1/4)*EllipticF(1/3*b
^(1/4)*x*3^(3/4),I)/b^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx=\frac{x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{bx^4}{3}\right)}{3\sqrt{3}}$$

input
```
Integrate[x^2/Sqrt[3 - b*x^4],x]
```

output
```
(x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (b*x^4)/3])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {836, 27, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{3-bx^4}}dx$$

$$\Big\downarrow \text{836}$$

$$\frac{\sqrt{3}\int \frac{\sqrt{b}x^2+\sqrt{3}}{\sqrt{3}\sqrt{3-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{3}\int \frac{1}{\sqrt{3-bx^4}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{b}x^2+\sqrt{3}}{\sqrt{3-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt{3}\int \frac{1}{\sqrt{3-bx^4}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\int \frac{\sqrt{b}x^2+\sqrt{3}}{\sqrt{3-bx^4}}dx}{\sqrt{b}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right),-1\right)}{b^{3/4}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{\int \frac{\sqrt{\sqrt{b}x^2+\sqrt{3}}}{\sqrt{\sqrt{3}-\sqrt{b}x^2}}dx}{\sqrt{b}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right),-1\right)}{b^{3/4}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt[4]{3}E\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right)\middle|-1\right)}{b^{3/4}}-\frac{\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{3}}\right),-1\right)}{b^{3/4}}$$

input
```
Int[x^2/Sqrt[3 - b*x^4],x]
```

output
```
(3^(1/4)*EllipticE[ArcSin[(b^(1/4)*x)/3^(1/4)], -1])/b^(3/4) - (3^(1/4)*El
lipticF[ArcSin[(b^(1/4)*x)/3^(1/4)], -1])/b^(3/4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.39

method	result	size
meijerg	$\frac{\sqrt{3}\,x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{b\,x^{4}}{3}\right)}{9}$	21
default	$-\frac{\sqrt{9-3\sqrt{3}\,\sqrt{b}\,x^{2}}\,\sqrt{9+3\sqrt{3}\,\sqrt{b}\,x^{2}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{b}}}{3},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{b}}}{3},i\right)\right)}{3\sqrt{\sqrt{3}\,\sqrt{b}}\,\sqrt{-b\,x^{4}+3}\,\sqrt{b}}$	94
elliptic	$-\frac{\sqrt{9-3\sqrt{3}\,\sqrt{b}\,x^{2}}\,\sqrt{9+3\sqrt{3}\,\sqrt{b}\,x^{2}}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{b}}}{3},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{3}\,\sqrt{\sqrt{3}\,\sqrt{b}}}{3},i\right)\right)}{3\sqrt{\sqrt{3}\,\sqrt{b}}\,\sqrt{-b\,x^{4}+3}\,\sqrt{b}}$	94

input
```
int(x^2/(-b*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*x^3*hypergeom([1/2,3/4],[7/4],1/3*b*x^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(38) = 76$.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.78

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = -\frac{\frac{\sqrt{3}\sqrt{-b}x\sqrt{\frac{\sqrt{3}}{\sqrt{b}}}E\left(\arcsin\left(\frac{\sqrt{\frac{\sqrt{3}}{\sqrt{b}}}}{x}\right)\mid -1\right)}{\sqrt{b}}-\frac{\sqrt{3}\sqrt{-b}x\sqrt{\frac{\sqrt{3}}{\sqrt{b}}}F\left(\arcsin\left(\frac{\sqrt{\frac{\sqrt{3}}{\sqrt{b}}}}{x}\right)\mid -1\right)}{\sqrt{b}}+\sqrt{-bx^4+3}}{bx}$$

input
```
integrate(x^2/(-b*x^4+3)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(3)*sqrt(-b)*x*sqrt(sqrt(3)/sqrt(b))*elliptic_e(arcsin(sqrt(sqrt(3)/
sqrt(b))/x), -1)/sqrt(b) - sqrt(3)*sqrt(-b)*x*sqrt(sqrt(3)/sqrt(b))*ellipt
ic_f(arcsin(sqrt(sqrt(3)/sqrt(b))/x), -1)/sqrt(b) + sqrt(-b*x^4 + 3))/(b*x
)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = \frac{\sqrt{3}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{3}\right)}{12\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-b*x**4+3)**(1/2),x)
```

output
```
sqrt(3)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)
/3)/(12*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = \int \frac{x^2}{\sqrt{-bx^4+3}}\,dx$$

input
```
integrate(x^2/(-b*x^4+3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-b*x^4 + 3), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = \int \frac{x^2}{\sqrt{-bx^4+3}}\,dx$$

input
```
integrate(x^2/(-b*x^4+3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-b*x^4 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = \int \frac{x^2}{\sqrt{3-b\,x^4}}\,dx$$

input
```
int(x^2/(3 - b*x^4)^(1/2),x)
```

output
```
int(x^2/(3 - b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{3-bx^4}}\,dx = -\left(\int \frac{\sqrt{-b\,x^4+3}\,x^2}{b\,x^4-3}dx\right)$$

input
```
int(x^2/(-b*x^4+3)^(1/2),x)
```

output
```
- int((sqrt( - b*x**4 + 3)*x**2)/(b*x**4 - 3),x)
```

3.312 $\int x^{11}\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^{11}\sqrt{a+cx^4}\,dx=\frac{a^2(a+cx^4)^{3/2}}{6c^3}-\frac{a(a+cx^4)^{5/2}}{5c^3}+\frac{(a+cx^4)^{7/2}}{14c^3}$$

output
```
1/6*a^2*(c*x^4+a)^(3/2)/c^3-1/5*a*(c*x^4+a)^(5/2)/c^3+1/14*(c*x^4+a)^(7/2)
/c^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x^{11}\sqrt{a+cx^4}\,dx=\frac{\sqrt{a+cx^4}(8a^3-4a^2cx^4+3ac^2x^8+15c^3x^{12})}{210c^3}$$

input
```
Integrate[x^11*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(8*a^3 - 4*a^2*c*x^4 + 3*a*c^2*x^8 + 15*c^3*x^12))/(210*c
^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^8\sqrt{cx^4+a}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(cx^4+a\right)^{5/2}}{c^2}-\frac{2a\left(cx^4+a\right)^{3/2}}{c^2}+\frac{a^2\sqrt{cx^4+a}}{c^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2a^2\left(a+cx^4\right)^{3/2}}{3c^3}+\frac{2\left(a+cx^4\right)^{7/2}}{7c^3}-\frac{4a\left(a+cx^4\right)^{5/2}}{5c^3}\right)$$

input
```
Int[x^11*Sqrt[a + c*x^4],x]
```

output
```
((2*a^2*(a + c*x^4)^(3/2))/(3*c^3) - (4*a*(a + c*x^4)^(5/2))/(5*c^3) + (2*
(a + c*x^4)^(7/2))/(7*c^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(c\,x^4+a)^{\frac{3}{2}}(15c^2x^8-12a\,x^4c+8a^2)}{210c^3}$	36
default	$\frac{(c\,x^4+a)^{\frac{3}{2}}(15c^2x^8-12a\,x^4c+8a^2)}{210c^3}$	36
elliptic	$\frac{(c\,x^4+a)^{\frac{3}{2}}(15c^2x^8-12a\,x^4c+8a^2)}{210c^3}$	36
pseudoelliptic	$\frac{(c\,x^4+a)^{\frac{3}{2}}(15c^2x^8-12a\,x^4c+8a^2)}{210c^3}$	36
orering	$\frac{(c\,x^4+a)^{\frac{3}{2}}(15c^2x^8-12a\,x^4c+8a^2)}{210c^3}$	36
trager	$\frac{(15c^3x^{12}+3a\,c^2x^8-4a^2c\,x^4+8a^3)\sqrt{c\,x^4+a}}{210c^3}$	47
risch	$\frac{(15c^3x^{12}+3a\,c^2x^8-4a^2c\,x^4+8a^3)\sqrt{c\,x^4+a}}{210c^3}$	47

input
```
int(x^11*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/210*(c*x^4+a)^(3/2)*(15*c^2*x^8-12*a*c*x^4+8*a^2)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^{11}\sqrt{a+cx^4}\,dx = \frac{(15\,c^3x^{12} + 3\,ac^2x^8 - 4\,a^2cx^4 + 8\,a^3)\sqrt{cx^4+a}}{210\,c^3}$$

input
```
integrate(x^11*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/210*(15*c^3*x^12 + 3*a*c^2*x^8 - 4*a^2*c*x^4 + 8*a^3)*sqrt(c*x^4 + a)/c^
3
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x^{11}\sqrt{a+cx^4}\,dx = \begin{cases} \frac{4a^3\sqrt{a+cx^4}}{105c^3} - \frac{2a^2x^4\sqrt{a+cx^4}}{105c^2} + \frac{ax^8\sqrt{a+cx^4}}{70c} + \frac{x^{12}\sqrt{a+cx^4}}{14} & \text{for } c \neq 0 \\ \frac{\sqrt{a}x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(c*x**4+a)**(1/2),x)
```

output
```
Piecewise((4*a**3*sqrt(a + c*x**4)/(105*c**3) - 2*a**2*x**4*sqrt(a + c*x**
4)/(105*c**2) + a*x**8*sqrt(a + c*x**4)/(70*c) + x**12*sqrt(a + c*x**4)/14
, Ne(c, 0)), (sqrt(a)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^{11}\sqrt{a+cx^4}\,dx = \frac{(cx^4+a)^{\frac{7}{2}}}{14\,c^3} - \frac{(cx^4+a)^{\frac{5}{2}}a}{5\,c^3} + \frac{(cx^4+a)^{\frac{3}{2}}a^2}{6\,c^3}$$

input
```
integrate(x^11*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/14*(c*x^4 + a)^(7/2)/c^3 - 1/5*(c*x^4 + a)^(5/2)*a/c^3 + 1/6*(c*x^4 + a)
^(3/2)*a^2/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^{11}\sqrt{a+cx^4}\,dx=\frac{15\left(cx^4+a\right)^{\frac{7}{2}}-42\left(cx^4+a\right)^{\frac{5}{2}}a+35\left(cx^4+a\right)^{\frac{3}{2}}a^2}{210\,c^3}$$

input
```
integrate(x^11*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/210*(15*(c*x^4 + a)^(7/2) - 42*(c*x^4 + a)^(5/2)*a + 35*(c*x^4 + a)^(3/2
)*a^2)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^{11}\sqrt{a+cx^4}\,dx=\sqrt{c\,x^4+a}\left(\frac{x^{12}}{14}+\frac{4\,a^3}{105\,c^3}+\frac{a\,x^8}{70\,c}-\frac{2\,a^2\,x^4}{105\,c^2}\right)$$

input
```
int(x^11*(a + c*x^4)^(1/2),x)
```

output
```
(a + c*x^4)^(1/2)*(x^12/14 + (4*a^3)/(105*c^3) + (a*x^8)/(70*c) - (2*a^2*x
^4)/(105*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 325, normalized size of antiderivative = 5.51

$$\int x^{11}\sqrt{a+cx^4}\,dx$$
$$= \frac{56\sqrt{c}\,\sqrt{c\,x^4+a}\,a^6x^2+420\sqrt{c}\,\sqrt{c\,x^4+a}\,a^5c\,x^6+693\sqrt{c}\,\sqrt{c\,x^4+a}\,a^4c^2x^{10}+337\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3c^3x^{14}+}{210c^3\left(\sqrt{c\,x^4+a}\,a^3+24\sqrt{c\,x^4+}\right.}$$

input
```
int(x^11*(c*x^4+a)^(1/2),x)
```

output
```
(56*sqrt(c)*sqrt(a + c*x**4)*a**6*x**2 + 420*sqrt(c)*sqrt(a + c*x**4)*a**5
*c*x**6 + 693*sqrt(c)*sqrt(a + c*x**4)*a**4*c**2*x**10 + 337*sqrt(c)*sqrt(
a + c*x**4)*a**3*c**3*x**14 + 920*sqrt(c)*sqrt(a + c*x**4)*a**2*c**4*x**18
 + 1872*sqrt(c)*sqrt(a + c*x**4)*a*c**5*x**22 + 960*sqrt(c)*sqrt(a + c*x**
4)*c**6*x**26 + 8*a**7 + 196*a**6*c*x**4 + 735*a**5*c**2*x**8 + 826*a**4*c
**3*x**12 + 623*a**3*c**4*x**16 + 1736*a**2*c**5*x**20 + 2352*a*c**6*x**24
 + 960*c**7*x**28)/(210*c**3*(sqrt(a + c*x**4)*a**3 + 24*sqrt(a + c*x**4)*
a**2*c*x**4 + 80*sqrt(a + c*x**4)*a*c**2*x**8 + 64*sqrt(a + c*x**4)*c**3*x
**12 + 7*sqrt(c)*a**3*x**2 + 56*sqrt(c)*a**2*c*x**6 + 112*sqrt(c)*a*c**2*x
**10 + 64*sqrt(c)*c**3*x**14))
```

3.313 $\int x^7\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^7\sqrt{a+cx^4}\,dx=-\frac{a(a+cx^4)^{3/2}}{6c^2}+\frac{(a+cx^4)^{5/2}}{10c^2}$$

output
```
-1/6*a*(c*x^4+a)^(3/2)/c^2+1/10*(c*x^4+a)^(5/2)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^7\sqrt{a+cx^4}\,dx=\frac{\sqrt{a+cx^4}(-2a^2+acx^4+3c^2x^8)}{30c^2}$$

input
```
Integrate[x^7*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(-2*a^2 + a*c*x^4 + 3*c^2*x^8))/(30*c^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^4\sqrt{cx^4+a}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{(cx^4+a)^{3/2}}{c}-\frac{a\sqrt{cx^4+a}}{c}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2(a+cx^4)^{5/2}}{5c^2}-\frac{2a(a+cx^4)^{3/2}}{3c^2}\right)$$

input
```
Int[x^7*Sqrt[a + c*x^4],x]
```

output
```
((-2*a*(a + c*x^4)^(3/2))/(3*c^2) + (2*(a + c*x^4)^(5/2))/(5*c^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(-3c\,x^{4}+2a\right)}{30c^{2}}$	25
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(-3c\,x^{4}+2a\right)}{30c^{2}}$	25
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(-3c\,x^{4}+2a\right)}{30c^{2}}$	25
pseudoelliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(-3c\,x^{4}+2a\right)}{30c^{2}}$	25
orering	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(-3c\,x^{4}+2a\right)}{30c^{2}}$	25
trager	$-\frac{\left(-3c^{2}x^{8}-a\,x^{4}c+2a^{2}\right)\sqrt{c\,x^{4}+a}}{30c^{2}}$	36
risch	$-\frac{\left(-3c^{2}x^{8}-a\,x^{4}c+2a^{2}\right)\sqrt{c\,x^{4}+a}}{30c^{2}}$	36

input
```
int(x^7*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(c*x^4+a)^(3/2)*(-3*c*x^4+2*a)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^7\sqrt{a+cx^4}\,dx = \frac{(3\,c^2x^8+acx^4-2\,a^2)\sqrt{cx^4+a}}{30\,c^2}$$

input
```
integrate(x^7*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(3*c^2*x^8 + a*c*x^4 - 2*a^2)*sqrt(c*x^4 + a)/c^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int x^7\sqrt{a+cx^4}\,dx = \begin{cases} -\frac{a^2\sqrt{a+cx^4}}{15c^2}+\frac{ax^4\sqrt{a+cx^4}}{30c}+\frac{x^8\sqrt{a+cx^4}}{10} & \text{for } c\neq 0 \\ \frac{\sqrt{a}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(c*x**4+a)**(1/2),x)
```

output
```
Piecewise((-a**2*sqrt(a + c*x**4)/(15*c**2) + a*x**4*sqrt(a + c*x**4)/(30*
c) + x**8*sqrt(a + c*x**4)/10, Ne(c, 0)), (sqrt(a)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^7\sqrt{a+cx^4}\,dx = \frac{(cx^4+a)^{\frac{5}{2}}}{10\,c^2} - \frac{(cx^4+a)^{\frac{3}{2}}a}{6\,c^2}$$

input
```
integrate(x^7*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/10*(c*x^4 + a)^(5/2)/c^2 - 1/6*(c*x^4 + a)^(3/2)*a/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^7\sqrt{a+cx^4}\,dx = \frac{3\,(cx^4+a)^{\frac{5}{2}}-5\,(cx^4+a)^{\frac{3}{2}}a}{30\,c^2}$$

input
```
integrate(x^7*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/30*(3*(c*x^4 + a)^(5/2) - 5*(c*x^4 + a)^(3/2)*a)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^7\sqrt{a+cx^4}\,dx = \sqrt{c\,x^4+a}\,\left(\frac{x^8}{10}-\frac{a^2}{15\,c^2}+\frac{a\,x^4}{30\,c}\right)$$

input
```
int(x^7*(a + c*x^4)^(1/2),x)
```

output
```
(a + c*x^4)^(1/2)*(x^8/10 - a^2/(15*c^2) + (a*x^4)/(30*c))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 6.03

$$\int x^7\sqrt{a+cx^4}\,dx$$
$$= \frac{-10\sqrt{c}\,\sqrt{c\,x^4+a}\,a^4x^2-35\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3c\,x^6+3\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2c^2x^{10}+76\sqrt{c}\,\sqrt{c\,x^4+a}\,a\,c^3x^{14}+48}{30c^2\left(\sqrt{c\,x^4+a}\,a^2+12\sqrt{c\,x^4+a}\,ac\,x^4+16\sqrt{c\,x^4+a}\,c^2x^8\right.}$$

input
```
int(x^7*(c*x^4+a)^(1/2),x)
```

output
```
( - 10*sqrt(c)*sqrt(a + c*x**4)*a**4*x**2 - 35*sqrt(c)*sqrt(a + c*x**4)*a*
*3*c*x**6 + 3*sqrt(c)*sqrt(a + c*x**4)*a**2*c**2*x**10 + 76*sqrt(c)*sqrt(a
 + c*x**4)*a*c**3*x**14 + 48*sqrt(c)*sqrt(a + c*x**4)*c**4*x**18 - 2*a**5
- 25*a**4*c*x**4 - 40*a**3*c**2*x**8 + 35*a**2*c**3*x**12 + 100*a*c**4*x**
16 + 48*c**5*x**20)/(30*c**2*(sqrt(a + c*x**4)*a**2 + 12*sqrt(a + c*x**4)*
a*c*x**4 + 16*sqrt(a + c*x**4)*c**2*x**8 + 5*sqrt(c)*a**2*x**2 + 20*sqrt(c
)*a*c*x**6 + 16*sqrt(c)*c**2*x**10))
```

3.314 $\int x^3\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^3\sqrt{a+cx^4}\,dx=\frac{(a+cx^4)^{3/2}}{6c}$$

output
```
1/6*(c*x^4+a)^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{a+cx^4}\,dx=\frac{(a+cx^4)^{3/2}}{6c}$$

input
```
Integrate[x^3*Sqrt[a + c*x^4],x]
```

output
```
(a + c*x^4)^(3/2)/(6*c)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{793}$$

$$\frac{\left(a+cx^4\right)^{3/2}}{6c}$$

input
```
Int[x^3*Sqrt[a + c*x^4],x]
```

output
```
(a + c*x^4)^(3/2)/(6*c)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15
derivativedivides	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15
default	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15
trager	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15
risch	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15
pseudoelliptic	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15
orering	$\frac{(c\,x^4+a)^{\frac{3}{2}}}{6c}$	15

input
```
int(x^3*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*x^4+a)^(3/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+cx^4}\,dx=\frac{(cx^4+a)^{\frac{3}{2}}}{6\,c}$$

input
```
integrate(x^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(c*x^4 + a)^(3/2)/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int x^3\sqrt{a+cx^4}\,dx = \begin{cases} \frac{a\sqrt{a+cx^4}}{6c} + \frac{x^4\sqrt{a+cx^4}}{6} & \text{for } c \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(c*x**4+a)**(1/2),x)
```

output
```
Piecewise((a*sqrt(a + c*x**4)/(6*c) + x**4*sqrt(a + c*x**4)/6, Ne(c, 0)),
(sqrt(a)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+cx^4}\,dx = \frac{(cx^4+a)^{\frac{3}{2}}}{6\,c}$$

input
```
integrate(x^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^4 + a)^(3/2)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+cx^4}\,dx = \frac{(cx^4+a)^{\frac{3}{2}}}{6\,c}$$

input
```
integrate(x^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*(c*x^4 + a)^(3/2)/c
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{a+cx^4}\,dx=\frac{(c\,x^4+a)^{3/2}}{6\,c}$$

input
```
int(x^3*(a + c*x^4)^(1/2),x)
```

output
```
(a + c*x^4)^(3/2)/(6*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 7.28

$$\int x^3\sqrt{a+cx^4}\,dx$$
$$=\frac{3\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2x^2+7\sqrt{c}\,\sqrt{c\,x^4+a}\,ac\,x^6+4\sqrt{c}\,\sqrt{c\,x^4+a}\,c^2x^{10}+a^3+6a^2c\,x^4+9a\,c^2x^8+4c^3x^{12}}{6c\left(\sqrt{c\,x^4+a}\,a+4\sqrt{c\,x^4+a}\,c\,x^4+3\sqrt{c}\,a\,x^2+4\sqrt{c}\,c\,x^6\right)}$$

input
```
int(x^3*(c*x^4+a)^(1/2),x)
```

output
```
(3*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 + 7*sqrt(c)*sqrt(a + c*x**4)*a*c*x**
6 + 4*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 + a**3 + 6*a**2*c*x**4 + 9*a*c**
2*x**8 + 4*c**3*x**12)/(6*c*(sqrt(a + c*x**4)*a + 4*sqrt(a + c*x**4)*c*x**
4 + 3*sqrt(c)*a*x**2 + 4*sqrt(c)*c*x**6))
```

3.315 $\int \frac{\sqrt{a+cx^4}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{\sqrt{a+cx^4}}{x} \, dx = \frac{1}{2}\sqrt{a+cx^4} - \frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

output
```
1/2*(c*x^4+a)^(1/2)-1/2*a^(1/2)*arctanh((c*x^4+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+cx^4}}{x} \, dx = \frac{1}{2}\sqrt{a+cx^4} - \frac{1}{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + c*x^4]/x,x]
```

output
```
Sqrt[a + c*x^4]/2 - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{\sqrt{cx^4+a}}{x^4} dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4} \left(a \int \frac{1}{x^4\sqrt{cx^4+a}} dx^4 + 2\sqrt{a+cx^4} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(\frac{2a \int \frac{1}{\frac{x^8}{c}-\frac{a}{c}} d\sqrt{cx^4+a}}{c} + 2\sqrt{a+cx^4} \right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4} \left(2\sqrt{a+cx^4} - 2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}} \right) \right)$$

input
```
Int[Sqrt[a + c*x^4]/x,x]
```

output
```
(2*Sqrt[a + c*x^4] - 2*Sqrt[a]*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
pseudoelliptic	$\frac{\sqrt{c\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right)}{2}$	32
default	$\frac{\sqrt{c\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2}$	41
elliptic	$\frac{\sqrt{c\,x^{4}+a}}{2}-\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2}$	41

input
```
int((c*x^4+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+a)^(1/2)-1/2*a^(1/2)*arctanh((c*x^4+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{a+cx^4}}{x}\,dx = \left[\frac{1}{4}\sqrt{a}\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right) + \frac{1}{2}\sqrt{cx^4+a}, \frac{1}{2}\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^4+a}}\right) + \frac{1}{2}\sqrt{cx^4+a}\right]$$

input
```
integrate((c*x^4+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) + 1/2*sqrt
(c*x^4 + a), 1/2*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^4 + a)) + 1/2*sqrt(c*x^
4 + a)]
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{a+cx^4}}{x}\,dx = -\frac{\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{2} + \frac{a}{2\sqrt{c}x^2\sqrt{\frac{a}{cx^4}+1}} + \frac{\sqrt{c}x^2}{2\sqrt{\frac{a}{cx^4}+1}}$$

input
```
integrate((c*x**4+a)**(1/2)/x,x)
```

output
```
-sqrt(a)*asinh(sqrt(a)/(sqrt(c)*x**2))/2 + a/(2*sqrt(c)*x**2*sqrt(a/(c*x**
4) + 1)) + sqrt(c)*x**2/(2*sqrt(a/(c*x**4) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a+cx^4}}{x}\,dx = \frac{1}{4}\sqrt{a}\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)+\frac{1}{2}\sqrt{cx^4+a}$$

input
```
integrate((c*x^4+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/4*sqrt(a)*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a))) +
 1/2*sqrt(c*x^4 + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+cx^4}}{x}\,dx = \frac{a\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}+\frac{1}{2}\sqrt{cx^4+a}$$

input
```
integrate((c*x^4+a)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*a*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) + 1/2*sqrt(c*x^4 + a)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+cx^4}}{x}\,dx = \frac{\sqrt{c\,x^4+a}}{2}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2}$$

input
```
int((a + c*x^4)^(1/2)/x,x)
```

output
```
(a + c*x^4)^(1/2)/2 - (a^(1/2)*atanh((a + c*x^4)^(1/2)/a^(1/2)))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.14

$$\int \frac{\sqrt{a+cx^4}}{x}\,dx$$
$$= \frac{\sqrt{a}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)-\sqrt{a}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)+\sqrt{c}\,\sqrt{c\,x^4+a}\,x^2+\sqrt{c}\,\sqrt{a}}{2\sqrt{c\,x^4+a}+2\sqrt{c}\,x^2}$$

input
```
int((c*x^4+a)^(1/2)/x,x)
```

output
```
(sqrt(a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/
sqrt(a)) - sqrt(a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt
(c)*x**2)/sqrt(a)) + sqrt(c)*sqrt(a + c*x**4)*x**2 + sqrt(c)*sqrt(a)*log((
sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*x**2 - sqrt(c)*sqrt(a)
*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*x**2 + a + c*x**
4)/(2*(sqrt(a + c*x**4) + sqrt(c)*x**2))
```

3.316 $\int \frac{\sqrt{a+cx^4}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\sqrt{a+cx^4}}{x^5} dx = -\frac{\sqrt{a+cx^4}}{4x^4} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

output
```
-1/4*(c*x^4+a)^(1/2)/x^4-1/4*c*arctanh((c*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+cx^4}}{x^5} dx = -\frac{\sqrt{a+cx^4}}{4x^4} - \frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^5,x]
```

output
```
-1/4*Sqrt[a + c*x^4]/x^4 - (c*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/(4*Sqrt[a]
)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {798, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^5} \, dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4} \int \frac{\sqrt{cx^4+a}}{x^8} dx^4$$

$$\Big\downarrow \ 51$$

$$\frac{1}{4} \left(\frac{1}{2} c \int \frac{1}{x^4 \sqrt{cx^4+a}} dx^4 - \frac{\sqrt{a+cx^4}}{x^4} \right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4} \left(\int \frac{1}{\frac{x^8}{c} - \frac{a}{c}} d\sqrt{cx^4+a} - \frac{\sqrt{a+cx^4}}{x^4} \right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4} \left(-\frac{c \operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{a+cx^4}}{x^4} \right)$$

input
```
Int[Sqrt[a + c*x^4]/x^5,x]
```

output
```
(-(Sqrt[a + c*x^4]/x^4) - (c*ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]])/Sqrt[a])/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{c\left(-\frac{\sqrt{c\,x^{4}+a}}{x^{4} c}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{4}$	41
risch	$-\frac{\sqrt{c\,x^{4}+a}}{4x^{4}}-\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4\sqrt{a}}$	45
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}-\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4\sqrt{a}}+\frac{c\sqrt{c\,x^{4}+a}}{4a}$	63
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{4a\,x^{4}}-\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4\sqrt{a}}+\frac{c\sqrt{c\,x^{4}+a}}{4a}$	63

input
```
int((c*x^4+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*c*(-(c*x^4+a)^(1/2)/x^4/c-1/a^(1/2)*arctanh((c*x^4+a)^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a+cx^4}}{x^5}\,dx = \left[\frac{\sqrt{a}cx^4\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)-2\sqrt{cx^4+a}a}{8ax^4}, \frac{\sqrt{-a}cx^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^4+a}}\right)-\sqrt{cx^4+a}a}{4ax^4}\right]$$

input
```
integrate((c*x^4+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*c*x^4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) - 2
*sqrt(c*x^4 + a)*a)/(a*x^4), 1/4*(sqrt(-a)*c*x^4*arctan(sqrt(-a)/sqrt(c*x^
4 + a)) - sqrt(c*x^4 + a)*a)/(a*x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+cx^4}}{x^5}\,dx = -\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{4x^2}-\frac{c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{4\sqrt{a}}$$

input
```
integrate((c*x**4+a)**(1/2)/x**5,x)
```

output
```
-sqrt(c)*sqrt(a/(c*x**4) + 1)/(4*x**2) - c*asinh(sqrt(a)/(sqrt(c)*x**2))/(
4*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{a+cx^4}}{x^5}\,dx = \frac{c\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{8\sqrt{a}} - \frac{\sqrt{cx^4+a}}{4\,x^4}$$

input
```
integrate((c*x^4+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/8*c*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt(a)
 - 1/4*sqrt(c*x^4 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+cx^4}}{x^5}\,dx = \frac{1}{4}\,c\left(\frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{cx^4+a}}{cx^4}\right)$$

input
```
integrate((c*x^4+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/4*c*(arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) - sqrt(c*x^4 + a)/(c*x^4)
)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+cx^4}}{x^5}\,dx = -\frac{\sqrt{c\,x^4+a}}{4\,x^4} - \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
int((a + c*x^4)^(1/2)/x^5,x)
```

output
```
- (a + c*x^4)^(1/2)/(4*x^4) - (c*atanh((a + c*x^4)^(1/2)/a^(1/2)))/(4*a^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 289, normalized size of antiderivative = 6.15

$$\int \frac{\sqrt{a+cx^4}}{x^5}\,dx$$
$$= \frac{-\sqrt{a}\,\sqrt{c\,x^4+a}\,a-2\sqrt{a}\,\sqrt{c\,x^4+a}\,c\,x^4+2\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)c\,x^6-2\sqrt{c}\,\sqrt{c\,x^4+a}\log}{}$$

input
```
int((c*x^4+a)^(1/2)/x^5,x)
```

output
```
( - sqrt(a)*sqrt(a + c*x**4)*a - 2*sqrt(a)*sqrt(a + c*x**4)*c*x**4 + 2*sqr
t(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt
(a))*c*x**6 - 2*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) +
 sqrt(c)*x**2)/sqrt(a))*c*x**6 - 2*sqrt(c)*sqrt(a)*a*x**2 - 2*sqrt(c)*sqrt
(a)*c*x**6 + log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*c*
x**4 + 2*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c**2*x**
8 - log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*c*x**4 - 2*
log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c**2*x**8)/(4*sqr
t(a)*x**4*(2*sqrt(c)*sqrt(a + c*x**4)*x**2 + a + 2*c*x**4))
```

3.317 $\int \frac{\sqrt{a+cx^4}}{x^9} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\sqrt{a+cx^4}}{x^9} \, dx = -\frac{\sqrt{a+cx^4}}{8x^8} - \frac{c\sqrt{a+cx^4}}{16ax^4} + \frac{c^2 \text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
-1/8*(c*x^4+a)^(1/2)/x^8-1/16*c*(c*x^4+a)^(1/2)/a/x^4+1/16*c^2*arctanh((c*
x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+cx^4}}{x^9} \, dx = \frac{(-2a-cx^4)\sqrt{a+cx^4}}{16ax^8} + \frac{c^2 \text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16a^{3/2}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^9,x]
```

output
```
((-2*a - c*x^4)*Sqrt[a + c*x^4])/(16*a*x^8) + (c^2*ArcTanh[Sqrt[a + c*x^4]
/Sqrt[a]])/(16*a^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{\sqrt{cx^4+a}}{x^{12}}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{1}{4}c\int \frac{1}{x^8\sqrt{cx^4+a}}dx^4-\frac{\sqrt{a+cx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{x^4\sqrt{cx^4+a}}dx^4}{2a}-\frac{\sqrt{a+cx^4}}{ax^4}\right)-\frac{\sqrt{a+cx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{1}{4}c\left(-\frac{\int \frac{1}{\frac{x^8}{c}-\frac{a}{c}}d\sqrt{cx^4+a}}{a}-\frac{\sqrt{a+cx^4}}{ax^4}\right)-\frac{\sqrt{a+cx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{1}{4}c\left(\frac{\text{carctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+cx^4}}{ax^4}\right)-\frac{\sqrt{a+cx^4}}{2x^8}\right)$$

input
```
Int[Sqrt[a + c*x^4]/x^9,x]
```

output
```
(-1/2*Sqrt[a + c*x^4]/x^8 + (c*(-(Sqrt[a + c*x^4]/(a*x^4)) + (c*ArcTanh[Sq
rt[a + c*x^4]/Sqrt[a]])/a^(3/2)))/4)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right) c^{2} x^{8}-\left(c\,x^{4} \sqrt{a}+2 a^{\frac{3}{2}}\right) \sqrt{c\,x^{4}+a}}{16 a^{\frac{3}{2}} x^{8}}$	56
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(c\,x^{4}+2 a\right)}{16 x^{8} a}+\frac{c^{2} \ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{16 a^{\frac{3}{2}}}$	59
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{8 a\,x^{8}}+\frac{c \left(c\,x^{4}+a\right)^{\frac{3}{2}}}{16 a^{2} x^{4}}+\frac{c^{2} \ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{16 a^{\frac{3}{2}}}-\frac{c^{2} \sqrt{c\,x^{4}+a}}{16 a^{2}}$	85
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{8 a\,x^{8}}+\frac{c \left(c\,x^{4}+a\right)^{\frac{3}{2}}}{16 a^{2} x^{4}}+\frac{c^{2} \ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{16 a^{\frac{3}{2}}}-\frac{c^{2} \sqrt{c\,x^{4}+a}}{16 a^{2}}$	85

input
```
int((c*x^4+a)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/16*(arctanh((c*x^4+a)^(1/2)/a^(1/2))*c^2*x^8-(c*x^4*a^(1/2)+2*a^(3/2))*(
c*x^4+a)^(1/2))/a^(3/2)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{a+cx^4}}{x^9}\,dx = \left[\frac{\sqrt{a}c^2x^8\log\left(\frac{cx^4+2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)-2\left(acx^4+2a^2\right)\sqrt{cx^4+a}}{32a^2x^8},\ -\frac{\sqrt{-a}c^2x^8\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^4+a}}\right)+\left(acx^4+2a^2\right)\sqrt{cx^4+a}}{16a^2x^8}\right]$$

input
```
integrate((c*x^4+a)^(1/2)/x^9,x, algorithm="fricas")
```

output
```
[1/32*(sqrt(a)*c^2*x^8*log((c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4)
- 2*(a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/(a^2*x^8), -1/16*(sqrt(-a)*c^2*x^8*
arctan(sqrt(-a)/sqrt(c*x^4 + a)) + (a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/(a^2
*x^8)]
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{a+cx^4}}{x^9}\,dx = -\frac{a}{8\sqrt{c}x^{10}\sqrt{\frac{a}{cx^4}+1}} - \frac{3\sqrt{c}}{16x^6\sqrt{\frac{a}{cx^4}+1}} - \frac{c^{\frac{3}{2}}}{16ax^2\sqrt{\frac{a}{cx^4}+1}} + \frac{c^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{16a^{\frac{3}{2}}}$$

input
```
integrate((c*x**4+a)**(1/2)/x**9,x)
```

output
```
-a/(8*sqrt(c)*x**10*sqrt(a/(c*x**4) + 1)) - 3*sqrt(c)/(16*x**6*sqrt(a/(c*x
**4) + 1)) - c**(3/2)/(16*a*x**2*sqrt(a/(c*x**4) + 1)) + c**2*asinh(sqrt(a
)/(sqrt(c)*x**2))/(16*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+cx^4}}{x^9}\,dx = -\frac{c^2\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{32\,a^{\frac{3}{2}}} - \frac{(cx^4+a)^{\frac{3}{2}}c^2+\sqrt{cx^4+a}\,ac^2}{16\left((cx^4+a)^2a-2\,(cx^4+a)a^2+a^3\right)}$$

input
```
integrate((c*x^4+a)^(1/2)/x^9,x, algorithm="maxima")
```

output
```
-1/32*c^2*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/a^(
3/2) - 1/16*((c*x^4 + a)^(3/2)*c^2 + sqrt(c*x^4 + a)*a*c^2)/((c*x^4 + a)^2
*a - 2*(c*x^4 + a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+cx^4}}{x^9}\,dx = -\frac{\frac{c^3\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{(cx^4+a)^{\frac{3}{2}}c^3+\sqrt{cx^4+a}ac^3}{ac^2x^8}}{16\,c}$$

input
```
integrate((c*x^4+a)^(1/2)/x^9,x, algorithm="giac")
```

output
```
-1/16*(c^3*arctan(sqrt(c*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + ((c*x^4 + a)^(3
/2)*c^3 + sqrt(c*x^4 + a)*a*c^3)/(a*c^2*x^8))/c
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+cx^4}}{x^9}\,dx = \frac{c^2\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{16\,a^{3/2}} - \frac{\sqrt{c\,x^4+a}}{16\,x^8} - \frac{(c\,x^4+a)^{3/2}}{16\,a\,x^8}$$

input
```
int((a + c*x^4)^(1/2)/x^9,x)
```

output
```
(c^2*atanh((a + c*x^4)^(1/2)/a^(1/2)))/(16*a^(3/2)) - (a + c*x^4)^(1/2)/(1
6*x^8) - (a + c*x^4)^(3/2)/(16*a*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 556, normalized size of antiderivative = 7.83

$$\int \frac{\sqrt{a+cx^4}}{x^9}\,dx$$
$$= \frac{-2\sqrt{a}\,\sqrt{c\,x^4+a}\,a^3 - 17\sqrt{a}\,\sqrt{c\,x^4+a}\,a^2c\,x^4 - 24\sqrt{a}\,\sqrt{c\,x^4+a}\,a\,c^2x^8 - 8\sqrt{a}\,\sqrt{c\,x^4+a}\,c^3x^{12} - 4\sqrt{c}\,\sqrt{c}}{}$$

input
```
int((c*x^4+a)^(1/2)/x^9,x)
```

output
```
( - 2*sqrt(a)*sqrt(a + c*x**4)*a**3 - 17*sqrt(a)*sqrt(a + c*x**4)*a**2*c*x
**4 - 24*sqrt(a)*sqrt(a + c*x**4)*a*c**2*x**8 - 8*sqrt(a)*sqrt(a + c*x**4)
*c**3*x**12 - 4*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt(a) +
 sqrt(c)*x**2)/sqrt(a))*a*c**2*x**10 - 8*sqrt(c)*sqrt(a + c*x**4)*log((sqr
t(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c**3*x**14 + 4*sqrt(c)*sq
rt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*
c**2*x**10 + 8*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) +
sqrt(c)*x**2)/sqrt(a))*c**3*x**14 - 8*sqrt(c)*sqrt(a)*a**3*x**2 - 28*sqrt(
c)*sqrt(a)*a**2*c*x**6 - 28*sqrt(c)*sqrt(a)*a*c**2*x**10 - 8*sqrt(c)*sqrt(
a)*c**3*x**14 - log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a
**2*c**2*x**8 - 8*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))
*a*c**3*x**12 - 8*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))
*c**4*x**16 + log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a**
2*c**2*x**8 + 8*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a
*c**3*x**12 + 8*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c
**4*x**16)/(16*sqrt(a)*a*x**8*(4*sqrt(c)*sqrt(a + c*x**4)*a*x**2 + 8*sqrt(
c)*sqrt(a + c*x**4)*c*x**6 + a**2 + 8*a*c*x**4 + 8*c**2*x**8))
```

3.318 $\int x^5\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int x^5\sqrt{a+cx^4}\,dx=\frac{ax^2\sqrt{a+cx^4}}{16c}+\frac{1}{8}x^6\sqrt{a+cx^4}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{16c^{3/2}}$$

output
```
1/16*a*x^2*(c*x^4+a)^(1/2)/c+1/8*x^6*(c*x^4+a)^(1/2)-1/16*a^2*arctanh(c^(1
/2)*x^2/(c*x^4+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^5\sqrt{a+cx^4}\,dx=\frac{x^2\sqrt{a+cx^4}(a+2cx^4)}{16c}-\frac{a^2\log\left(\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{16c^{3/2}}$$

input
```
Integrate[x^5*Sqrt[a + c*x^4],x]
```

output
```
(x^2*Sqrt[a + c*x^4]*(a + 2*c*x^4))/(16*c) - (a^2*Log[Sqrt[c]*x^2 + Sqrt[a
 + c*x^4]])/(16*c^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {807, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^4\sqrt{cx^4+a}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{1}{4}a\int \frac{x^4}{\sqrt{cx^4+a}}dx^2+\frac{1}{4}x^6\sqrt{a+cx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{4}a\left(\frac{x^2\sqrt{a+cx^4}}{2c}-\frac{a\int \frac{1}{\sqrt{cx^4+a}}dx^2}{2c}\right)+\frac{1}{4}x^6\sqrt{a+cx^4}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{4}a\left(\frac{x^2\sqrt{a+cx^4}}{2c}-\frac{a\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{2c}\right)+\frac{1}{4}x^6\sqrt{a+cx^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{1}{4}a\left(\frac{x^2\sqrt{a+cx^4}}{2c}-\frac{a\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2c^{3/2}}\right)+\frac{1}{4}x^6\sqrt{a+cx^4}\right)$$

input
```
Int[x^5*Sqrt[a + c*x^4],x]
```

output
```
((x^6*Sqrt[a + c*x^4])/4 + (a*((x^2*Sqrt[a + c*x^4])/(2*c) - (a*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*c^(3/2))))/4)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{x^{2}\left(2c\,x^{4}+a\right)\sqrt{c\,x^{4}+a}}{16c}-\frac{a^{2}\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{16c^{\frac{3}{2}}}$	53
default	$\frac{x^{2}\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{8c}-\frac{a\,x^{2}\sqrt{c\,x^{4}+a}}{16c}-\frac{a^{2}\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{16c^{\frac{3}{2}}}$	63
elliptic	$\frac{x^{2}\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{8c}-\frac{a\,x^{2}\sqrt{c\,x^{4}+a}}{16c}-\frac{a^{2}\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{16c^{\frac{3}{2}}}$	63
pseudoelliptic	$\frac{2\sqrt{c\,x^{4}+a}\,c^{\frac{3}{2}}x^{6}+a\,x^{2}\sqrt{c\,x^{4}+a}\,\sqrt{c}-\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{x^{2}\sqrt{c}}\right)a^{2}}{16c^{\frac{3}{2}}}$	63

input
```
int(x^5*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*x^2*(2*c*x^4+a)*(c*x^4+a)^(1/2)/c-1/16/c^(3/2)*a^2*ln(c^(1/2)*x^2+(c*
x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.76

$$\int x^{5}\sqrt{a+cx^{4}}\,dx$$
$$=\left[\frac{a^{2}\sqrt{c}\log\left(-2\,cx^{4}+2\,\sqrt{cx^{4}+a}\sqrt{c}x^{2}-a\right)+2\left(2\,c^{2}x^{6}+acx^{2}\right)\sqrt{cx^{4}+a}}{32\,c^{2}},\frac{a^{2}\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+a}\sqrt{-c}}{cx^{2}}\right)+}{16\,c^{2}}\right.$$

input
```
integrate(x^5*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/32*(a^2*sqrt(c)*log(-2*c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*(
2*c^2*x^6 + a*c*x^2)*sqrt(c*x^4 + a))/c^2, 1/16*(a^2*sqrt(-c)*arctan(sqrt(
c*x^4 + a)*sqrt(-c)/(c*x^2)) + (2*c^2*x^6 + a*c*x^2)*sqrt(c*x^4 + a))/c^2]
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int x^5\sqrt{a+cx^4}\,dx=\frac{a^{\frac{3}{2}}x^2}{16c\sqrt{1+\frac{cx^4}{a}}}+\frac{3\sqrt{a}\,x^6}{16\sqrt{1+\frac{cx^4}{a}}}-\frac{a^2\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16c^{\frac{3}{2}}}+\frac{cx^{10}}{8\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate(x**5*(c*x**4+a)**(1/2),x)
```

output
```
a**(3/2)*x**2/(16*c*sqrt(1 + c*x**4/a)) + 3*sqrt(a)*x**6/(16*sqrt(1 + c*x*
*4/a)) - a**2*asinh(sqrt(c)*x**2/sqrt(a))/(16*c**(3/2)) + c*x**10/(8*sqrt(
a)*sqrt(1 + c*x**4/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(58) = 116.

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.62

$$\int x^5\sqrt{a+cx^4}\,dx=\frac{a^2\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{32\,c^{\frac{3}{2}}}+\frac{\frac{\sqrt{cx^4+a}a^2c}{x^2}+\frac{(cx^4+a)^{\frac{3}{2}}a^2}{x^6}}{16\left(c^3-\frac{2\,(cx^4+a)c^2}{x^4}+\frac{(cx^4+a)^2c}{x^8}\right)}$$

input
```
integrate(x^5*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/32*a^2*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x
^2))/c^(3/2) + 1/16*(sqrt(c*x^4 + a)*a^2*c/x^2 + (c*x^4 + a)^(3/2)*a^2/x^6
)/(c^3 - 2*(c*x^4 + a)*c^2/x^4 + (c*x^4 + a)^2*c/x^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int x^5\sqrt{a+cx^4}\,dx = \frac{1}{16}\sqrt{cx^4+a}\left(2\,x^4+\frac{a}{c}\right)x^2+\frac{a^2\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate(x^5*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(c*x^4 + a)*(2*x^4 + a/c)*x^2 + 1/16*a^2*log(abs(-sqrt(c)*x^2 + s
qrt(c*x^4 + a)))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+cx^4}\,dx = \int x^5\,\sqrt{c\,x^4+a}\,dx$$

input
```
int(x^5*(a + c*x^4)^(1/2),x)
```

output
```
int(x^5*(a + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 342, normalized size of antiderivative = 4.62

$$\int x^5\sqrt{a+cx^4}\,dx$$
$$= \frac{-4\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^3x^2-8\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^2c\,x^6+\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3x^2}{}$$

input
```
int(x^5*(c*x^4+a)^(1/2),x)
```

output
```
( - 4*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(
a))*a**3*x**2 - 8*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)
*x**2)/sqrt(a))*a**2*c*x**6 + sqrt(c)*sqrt(a + c*x**4)*a**3*x**2 + 10*sqrt
(c)*sqrt(a + c*x**4)*a**2*c*x**6 + 24*sqrt(c)*sqrt(a + c*x**4)*a*c**2*x**1
0 + 16*sqrt(c)*sqrt(a + c*x**4)*c**3*x**14 - log((sqrt(a + c*x**4) + sqrt(
c)*x**2)/sqrt(a))*a**4 - 8*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*
a**3*c*x**4 - 8*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**2*c**2*x
**8 + 4*a**3*c*x**4 + 20*a**2*c**2*x**8 + 32*a*c**3*x**12 + 16*c**4*x**16)
/(16*c*(4*sqrt(a + c*x**4)*a*c*x**2 + 8*sqrt(a + c*x**4)*c**2*x**6 + sqrt(
c)*a**2 + 8*sqrt(c)*a*c*x**4 + 8*sqrt(c)*c**2*x**8))
```

3.319 $\int x\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int x\sqrt{a+cx^4}\,dx=\frac{1}{4}x^2\sqrt{a+cx^4}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

output
```
1/4*x^2*(c*x^4+a)^(1/2)+1/4*a*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int x\sqrt{a+cx^4}\,dx=\frac{1}{4}x^2\sqrt{a+cx^4}+\frac{a\log\left(\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{4\sqrt{c}}$$

input
```
Integrate[x*Sqrt[a + c*x^4],x]
```

output
```
(x^2*Sqrt[a + c*x^4])/4 + (a*Log[Sqrt[c]*x^2 + Sqrt[a + c*x^4]])/(4*Sqrt[c
])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \sqrt{cx^4+a}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^4+a}}dx^2+\frac{1}{2}x^2\sqrt{a+cx^4}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{1}{2}a\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}+\frac{1}{2}x^2\sqrt{a+cx^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{1}{2}x^2\sqrt{a+cx^4}\right)$$

input
```
Int[x*Sqrt[a + c*x^4],x]
```

output
```
((x^2*Sqrt[a + c*x^4])/2 + (a*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*S
qrt[c]))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^{2} \sqrt{c \,x^{4}+a}}{4}+\frac{a \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{4 \sqrt{c}}$	40
risch	$\frac{x^{2} \sqrt{c \,x^{4}+a}}{4}+\frac{a \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{4 \sqrt{c}}$	40
elliptic	$\frac{x^{2} \sqrt{c \,x^{4}+a}}{4}+\frac{a \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{4 \sqrt{c}}$	40
pseudoelliptic	$-\frac{a \left(-\frac{\sqrt{c \,x^{4}+a}\, x^{2}}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c \,x^{4}+a}}{x^{2} \sqrt{c}}\right)}{\sqrt{c}}\right)}{4}$	44

input
```
int(x*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(c*x^4+a)^(1/2)+1/4*a/c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int x\sqrt{a+cx^4}\,dx = \left[\frac{2\sqrt{cx^4+a}cx^2 + a\sqrt{c}\log\left(-2\,cx^4 - 2\sqrt{cx^4+a}\sqrt{c}x^2 - a\right)}{8\,c}, \frac{\sqrt{cx^4+a}cx^2 - a\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-c}}{cx^2}\right)}{4\,c}\right.$$

input
```
integrate(x*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(c*x^4 + a)*c*x^2 + a*sqrt(c)*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)
*sqrt(c)*x^2 - a))/c, 1/4*(sqrt(c*x^4 + a)*c*x^2 - a*sqrt(-c)*arctan(sqrt(
c*x^4 + a)*sqrt(-c)/(c*x^2)))/c]
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int x\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}x^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{a\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

input
```
integrate(x*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x**2*sqrt(1 + c*x**4/a)/4 + a*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(
c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int x\sqrt{a+cx^4}\,dx = -\frac{a\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{8\sqrt{c}} - \frac{\sqrt{cx^4+a}a}{4\left(c-\frac{cx^4+a}{x^4}\right)x^2}$$

input
```
integrate(x*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*a*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x^2
))/sqrt(c) - 1/4*sqrt(c*x^4 + a)*a/((c - (c*x^4 + a)/x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int x\sqrt{a+cx^4}\,dx = \frac{1}{4}\sqrt{cx^4+a}x^2 - \frac{a\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{4\sqrt{c}}$$

input
```
integrate(x*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^4 + a)*x^2 - 1/4*a*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/s
qrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+cx^4}\,dx = \int x\,\sqrt{c\,x^4+a}\,dx$$

input
```
int(x*(a + c*x^4)^(1/2),x)
```

output
```
int(x*(a + c*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.36

$$\int x\sqrt{a+cx^4}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a\,x^2+\sqrt{c}\,\sqrt{c\,x^4+a}\,a\,x^2+2\sqrt{c}\,\sqrt{c\,x^4+a}\,c\,x^6+\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^2}{8\sqrt{c\,x^4+a}\,c\,x^2+4\sqrt{c}\,a+8\sqrt{c}\,c\,x^4}$$

input
```
int(x*(c*x^4+a)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))
*a*x**2 + sqrt(c)*sqrt(a + c*x**4)*a*x**2 + 2*sqrt(c)*sqrt(a + c*x**4)*c*x
**6 + log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**2 + 2*log((sqrt(a
+ c*x**4) + sqrt(c)*x**2)/sqrt(a))*a*c*x**4 + 2*a*c*x**4 + 2*c**2*x**8)/(4
*(2*sqrt(a + c*x**4)*c*x**2 + sqrt(c)*a + 2*sqrt(c)*c*x**4))
```

3.320 $\int \frac{\sqrt{a+cx^4}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{\sqrt{a+cx^4}}{x^3} dx = -\frac{\sqrt{a+cx^4}}{2x^2} + \frac{1}{2}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)$$

output
```
-1/2*(c*x^4+a)^(1/2)/x^2+1/2*c^(1/2)*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+cx^4}}{x^3} dx = -\frac{\sqrt{a+cx^4}}{2x^2} + \frac{1}{2}\sqrt{c}\log\left(\sqrt{c}x^2 + \sqrt{a+cx^4}\right)$$

input
```
Integrate[Sqrt[a + c*x^4]/x^3,x]
```

output
```
-1/2*Sqrt[a + c*x^4]/x^2 + (Sqrt[c]*Log[Sqrt[c]*x^2 + Sqrt[a + c*x^4]])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {807, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt{cx^4+a}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(c\int \frac{1}{\sqrt{cx^4+a}}dx^2-\frac{\sqrt{a+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(c\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}-\frac{\sqrt{a+cx^4}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)-\frac{\sqrt{a+cx^4}}{x^2}\right)$$

input
```
Int[Sqrt[a + c*x^4]/x^3,x]
```

output
```
(-(Sqrt[a + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/
2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}}{2x^{2}}+\frac{\sqrt{c}\,\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2}$	39
pseudoelliptic	$\frac{\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{x^{2}\sqrt{c}}\right)x^{2}-\sqrt{c\,x^{4}+a}}{2x^{2}}$	42
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{c\,x^{2}\sqrt{c\,x^{4}+a}}{2a}+\frac{\sqrt{c}\,\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2}$	60
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{c\,x^{2}\sqrt{c\,x^{4}+a}}{2a}+\frac{\sqrt{c}\,\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2}$	60

input
```
int((c*x^4+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^4+a)^(1/2)/x^2+1/2*c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a+cx^4}}{x^3}\,dx = \left[\frac{\sqrt{c}x^2\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)-2\,\sqrt{cx^4+a}}{4\,x^2},\ -\frac{\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-c}}{cx^2}\right)+\sqrt{cx^4+a}}{2\,x^2}\right]$$

input
```
integrate((c*x^4+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(c)*x^2*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) - 2*sq
rt(c*x^4 + a))/x^2, -1/2*(sqrt(-c)*x^2*arctan(sqrt(c*x^4 + a)*sqrt(-c)/(c*
x^2)) + sqrt(c*x^4 + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+cx^4}}{x^3}\,dx = -\frac{\sqrt{a}}{2x^2\sqrt{1+\frac{cx^4}{a}}}+\frac{\sqrt{c}\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2}-\frac{cx^2}{2\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((c*x**4+a)**(1/2)/x**3,x)
```

output
```
-sqrt(a)/(2*x**2*sqrt(1 + c*x**4/a)) + sqrt(c)*asinh(sqrt(c)*x**2/sqrt(a))
/2 - c*x**2/(2*sqrt(a)*sqrt(1 + c*x**4/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+cx^4}}{x^3}\,dx = -\frac{1}{4}\sqrt{c}\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)-\frac{\sqrt{cx^4+a}}{2\,x^2}$$

input
```
integrate((c*x^4+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/4*sqrt(c)*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 +
a)/x^2)) - 1/2*sqrt(c*x^4 + a)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+cx^4}}{x^3}\,dx = -\frac{1}{4}\sqrt{c}\log\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2\right)+\frac{a\sqrt{c}}{\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a}$$

input
```
integrate((c*x^4+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/4*sqrt(c)*log((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2) + a*sqrt(c)/((sqrt(c)*
x^2 - sqrt(c*x^4 + a))^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{x^3}\,dx = \int \frac{\sqrt{c\,x^4+a}}{x^3}\,dx$$

input
```
int((a + c*x^4)^(1/2)/x^3,x)
```

output
```
int((a + c*x^4)^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{a+cx^4}}{x^3}\,dx$$
$$=\frac{\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)x^2-2\sqrt{c}\,\sqrt{c\,x^4+a}\,x^2+\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)c\,x^4-a-2c\,x^4}{2x^2\left(\sqrt{c\,x^4+a}+\sqrt{c}\,x^2\right)}$$

input
```
int((c*x^4+a)^(1/2)/x^3,x)
```

output
```
(sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*x
**2 - 2*sqrt(c)*sqrt(a + c*x**4)*x**2 + log((sqrt(a + c*x**4) + sqrt(c)*x*
*2)/sqrt(a))*c*x**4 - a - 2*c*x**4)/(2*x**2*(sqrt(a + c*x**4) + sqrt(c)*x*
*2))
```

3.321 $\int \frac{\sqrt{a+cx^4}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{(a+cx^4)^{3/2}}{6ax^6}$$

output
```
-1/6*(c*x^4+a)^(3/2)/a/x^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{(a+cx^4)^{3/2}}{6ax^6}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^7,x]
```

output
```
-1/6*(a + c*x^4)^(3/2)/(a*x^6)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^7} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+cx^4)^{3/2}}{6ax^6}$$

input
```
Int[Sqrt[a + c*x^4]/x^7,x]
```

output
```
-1/6*(a + c*x^4)^(3/2)/(a*x^6)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18
trager	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18
risch	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18
pseudoelliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18
orering	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}}{6a\,x^{6}}$	18

input
```
int((c*x^4+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*x^4+a)^(3/2)/a/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{(cx^4+a)^{\frac{3}{2}}}{6\,ax^6}$$

input
```
integrate((c*x^4+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-1/6*(c*x^4 + a)^(3/2)/(a*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.45 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{6x^4} - \frac{c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{6a}$$

input
```
integrate((c*x**4+a)**(1/2)/x**7,x)
```

output
```
-sqrt(c)*sqrt(a/(c*x**4) + 1)/(6*x**4) - c**(3/2)*sqrt(a/(c*x**4) + 1)/(6*
a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{(cx^4+a)^{\frac{3}{2}}}{6\,ax^6}$$

input
```
integrate((c*x^4+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/6*(c*x^4 + a)^(3/2)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = \frac{3\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^4c^{\frac{3}{2}}+a^2c^{\frac{3}{2}}}{3\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)^3}$$

input
```
integrate((c*x^4+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*c^(3/2) + a^2*c^(3/2))/((sqrt(c)*
x^2 - sqrt(c*x^4 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = -\frac{\left(c\,x^4+a\right)^{3/2}}{6\,a\,x^6}$$

input
```
int((a + c*x^4)^(1/2)/x^7,x)
```

output
```
-(a + c*x^4)^(3/2)/(6*a*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 6.48

$$\int \frac{\sqrt{a+cx^4}}{x^7}\,dx = \frac{-3\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2x^2-8\sqrt{c}\,\sqrt{c\,x^4+a}\,ac\,x^6-8\sqrt{c}\,\sqrt{c\,x^4+a}\,c^2x^{10}-a^3-6a^2c\,x^4-12a\,c^2x^8-8c^3x^{12}}{6a\,x^6\left(\sqrt{c\,x^4+a}\,a+4\sqrt{c\,x^4+a}\,c\,x^4+3\sqrt{c}\,a\,x^2+4\sqrt{c}\,c\,x^6\right)}$$

input
```
int((c*x^4+a)^(1/2)/x^7,x)
```

output
```
( - 3*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 - 8*sqrt(c)*sqrt(a + c*x**4)*a*c*
x**6 - 8*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 - a**3 - 6*a**2*c*x**4 - 12*a
*c**2*x**8 - 8*c**3*x**12)/(6*a*x**6*(sqrt(a + c*x**4)*a + 4*sqrt(a + c*x*
*4)*c*x**4 + 3*sqrt(c)*a*x**2 + 4*sqrt(c)*c*x**6))
```

3.322 $\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = -\frac{(a+cx^4)^{3/2}}{10ax^{10}}+\frac{c(a+cx^4)^{3/2}}{15a^2x^6}$$

output
```
-1/10*(c*x^4+a)^(3/2)/a/x^10+1/15*c*(c*x^4+a)^(3/2)/a^2/x^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = \frac{\sqrt{a+cx^4}(-3a^2-acx^4+2c^2x^8)}{30a^2x^{10}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^11,x]
```

output
```
(Sqrt[a + c*x^4]*(-3*a^2 - a*c*x^4 + 2*c^2*x^8))/(30*a^2*x^10)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{2c\int \frac{\sqrt{cx^4+a}}{x^7}dx}{5a}-\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{c\left(a+cx^4\right)^{3/2}}{15a^2x^6}-\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}$$

input

```
Int[Sqrt[a + c*x^4]/x^11,x]
```

output

```
-1/10*(a + c*x^4)^(3/2)/(a*x^10) + (c*(a + c*x^4)^(3/2))/(15*a^2*x^6)
```

Defintions of rubi rules used

rule 796

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}} \left(-2c\,x^{4}+3a\right)}{30x^{10}a^{2}}$	28
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}} \left(-2c\,x^{4}+3a\right)}{30x^{10}a^{2}}$	28
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}} \left(-2c\,x^{4}+3a\right)}{30x^{10}a^{2}}$	28
pseudoelliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}} \left(-2c\,x^{4}+3a\right)}{30x^{10}a^{2}}$	28
orering	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}} \left(-2c\,x^{4}+3a\right)}{30x^{10}a^{2}}$	28
trager	$-\frac{\left(-2c^{2}x^{8}+a\,x^{4}c+3a^{2}\right)\sqrt{c\,x^{4}+a}}{30x^{10}a^{2}}$	38
risch	$-\frac{\left(-2c^{2}x^{8}+a\,x^{4}c+3a^{2}\right)\sqrt{c\,x^{4}+a}}{30x^{10}a^{2}}$	38

input
```
int((c*x^4+a)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/30*(c*x^4+a)^(3/2)*(-2*c*x^4+3*a)/x^10/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = \frac{(2\,c^2x^8-acx^4-3\,a^2)\sqrt{cx^4+a}}{30\,a^2x^{10}}$$

input
```
integrate((c*x^4+a)^(1/2)/x^11,x, algorithm="fricas")
```

output
```
1/30*(2*c^2*x^8 - a*c*x^4 - 3*a^2)*sqrt(c*x^4 + a)/(a^2*x^10)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = -\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{10x^8} - \frac{c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{30ax^4} + \frac{c^{\frac{5}{2}}\sqrt{\frac{a}{cx^4}+1}}{15a^2}$$

input
```
integrate((c*x**4+a)**(1/2)/x**11,x)
```

output
```
-sqrt(c)*sqrt(a/(c*x**4) + 1)/(10*x**8) - c**(3/2)*sqrt(a/(c*x**4) + 1)/(3
0*a*x**4) + c**(5/2)*sqrt(a/(c*x**4) + 1)/(15*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = \frac{\frac{5\,(cx^4+a)^{\frac{3}{2}}c}{x^6} - \frac{3\,(cx^4+a)^{\frac{5}{2}}}{x^{10}}}{30\,a^2}$$

input
```
integrate((c*x^4+a)^(1/2)/x^11,x, algorithm="maxima")
```

output
```
1/30*(5*(c*x^4 + a)^(3/2)*c/x^6 - 3*(c*x^4 + a)^(5/2)/x^10)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = \frac{2\left(15\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^6c^{\frac{5}{2}}+5\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^4ac^{\frac{5}{2}}+5\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2a^2c^{\frac{5}{2}}-a^3c^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)^5}$$

input
```
integrate((c*x^4+a)^(1/2)/x^11,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^6*c^(5/2) + 5*(sqrt(c)*x^2 - sqrt
(c*x^4 + a))^4*a*c^(5/2) + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^2*c^(5/2)
 - a^3*c^(5/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = -\frac{\sqrt{c\,x^4+a}\,\left(3\,a^2+a\,c\,x^4-2\,c^2\,x^8\right)}{30\,a^2\,x^{10}}$$

input
```
int((a + c*x^4)^(1/2)/x^11,x)
```

output
```
-((a + c*x^4)^(1/2)*(3*a^2 - 2*c^2*x^8 + a*c*x^4))/(30*a^2*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{a+cx^4}}{x^{11}}\,dx = \frac{-15\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2x^2-65\sqrt{c}\,\sqrt{c\,x^4+a}\,ac\,x^6-60\sqrt{c}\,\sqrt{c\,x^4+a}\,c^2x^{10}-3a^3-40a^2c\,x^4-95a\,c^2x^8-6}{30x^{10}\left(\sqrt{c\,x^4+a}\,a^2+12\sqrt{c\,x^4+a}\,ac\,x^4+16\sqrt{c\,x^4+a}\,c^2x^8+5\sqrt{c}\,a^2x^2+20\sqrt{c}\,ac\,x^6+16\sqrt{c}\,c^2x^{10}\right.}$$

input
```
int((c*x^4+a)^(1/2)/x^11,x)
```

output
```
( - 15*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 - 65*sqrt(c)*sqrt(a + c*x**4)*a*
c*x**6 - 60*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 - 3*a**3 - 40*a**2*c*x**4
- 95*a*c**2*x**8 - 60*c**3*x**12)/(30*x**10*(sqrt(a + c*x**4)*a**2 + 12*sq
rt(a + c*x**4)*a*c*x**4 + 16*sqrt(a + c*x**4)*c**2*x**8 + 5*sqrt(c)*a**2*x
**2 + 20*sqrt(c)*a*c*x**6 + 16*sqrt(c)*c**2*x**10))
```

3.323 $\int \frac{\sqrt{a+cx^4}}{x^{15}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt{a+cx^4}}{x^{15}} \, dx = -\frac{(a+cx^4)^{3/2}}{14ax^{14}} + \frac{2c(a+cx^4)^{3/2}}{35a^2x^{10}} - \frac{4c^2(a+cx^4)^{3/2}}{105a^3x^6}$$

output
```
-1/14*(c*x^4+a)^(3/2)/a/x^14+2/35*c*(c*x^4+a)^(3/2)/a^2/x^10-4/105*c^2*(c*
x^4+a)^(3/2)/a^3/x^6
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+cx^4}}{x^{15}} \, dx = \frac{\sqrt{a+cx^4}(-15a^3-3a^2cx^4+4ac^2x^8-8c^3x^{12})}{210a^3x^{14}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^15,x]
```

output
```
(Sqrt[a + c*x^4]*(-15*a^3 - 3*a^2*c*x^4 + 4*a*c^2*x^8 - 8*c^3*x^12))/(210*
a^3*x^14)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^{15}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4c\int \frac{\sqrt{cx^4+a}}{x^{11}}dx}{7a}-\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^4+a}}{x^7}dx}{5a}-\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}\right)}{7a}-\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{4c\left(\frac{c\left(a+cx^4\right)^{3/2}}{15a^2x^6}-\frac{\left(a+cx^4\right)^{3/2}}{10ax^{10}}\right)}{7a}-\frac{\left(a+cx^4\right)^{3/2}}{14ax^{14}}$$

input
```
Int[Sqrt[a + c*x^4]/x^15,x]
```

output
```
-1/14*(a + c*x^4)^(3/2)/(a*x^14) - (4*c*(-1/10*(a + c*x^4)^(3/2)/(a*x^10)
+ (c*(a + c*x^4)^(3/2))/(15*a^2*x^6)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(8c^{2}x^{8}-12a\,x^{4}c+15a^{2}\right)}{210x^{14}a^{3}}$	39
default	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(8c^{2}x^{8}-12a\,x^{4}c+15a^{2}\right)}{210x^{14}a^{3}}$	39
elliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(8c^{2}x^{8}-12a\,x^{4}c+15a^{2}\right)}{210x^{14}a^{3}}$	39
pseudoelliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(8c^{2}x^{8}-12a\,x^{4}c+15a^{2}\right)}{210x^{14}a^{3}}$	39
orering	$-\frac{\left(c\,x^{4}+a\right)^{\frac{3}{2}}\left(8c^{2}x^{8}-12a\,x^{4}c+15a^{2}\right)}{210x^{14}a^{3}}$	39
trager	$-\frac{\left(8c^{3}x^{12}-4a\,c^{2}x^{8}+3a^{2}c\,x^{4}+15a^{3}\right)\sqrt{c\,x^{4}+a}}{210x^{14}a^{3}}$	50
risch	$-\frac{\left(8c^{3}x^{12}-4a\,c^{2}x^{8}+3a^{2}c\,x^{4}+15a^{3}\right)\sqrt{c\,x^{4}+a}}{210x^{14}a^{3}}$	50

input
```
int((c*x^4+a)^(1/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/210*(c*x^4+a)^(3/2)*(8*c^2*x^8-12*a*c*x^4+15*a^2)/x^14/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx = -\frac{(8\,c^3x^{12}-4\,ac^2x^8+3\,a^2cx^4+15\,a^3)\sqrt{cx^4+a}}{210\,a^3x^{14}}$$

input
```
integrate((c*x^4+a)^(1/2)/x^15,x, algorithm="fricas")
```

output
```
-1/210*(8*c^3*x^12 - 4*a*c^2*x^8 + 3*a^2*c*x^4 + 15*a^3)*sqrt(c*x^4 + a)/(
a^3*x^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(61) = 122.

Time = 1.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 5.28

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx = &-\frac{15a^5c^{\frac{9}{2}}\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}} \\ &-\frac{33a^4c^{\frac{11}{2}}x^4\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}} \\ &-\frac{17a^3c^{\frac{13}{2}}x^8\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}} \\ &-\frac{3a^2c^{\frac{15}{2}}x^{12}\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}} \\ &-\frac{12ac^{\frac{17}{2}}x^{16}\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}} \\ &-\frac{8c^{\frac{19}{2}}x^{20}\sqrt{\frac{a}{cx^4}+1}}{210a^5c^4x^{12}+420a^4c^5x^{16}+210a^3c^6x^{20}}\end{aligned}$$

input
```
integrate((c*x**4+a)**(1/2)/x**15,x)
```

output
```
-15*a**5*c**(9/2)*sqrt(a/(c*x**4) + 1)/(210*a**5*c**4*x**12 + 420*a**4*c**
5*x**16 + 210*a**3*c**6*x**20) - 33*a**4*c**(11/2)*x**4*sqrt(a/(c*x**4) +
1)/(210*a**5*c**4*x**12 + 420*a**4*c**5*x**16 + 210*a**3*c**6*x**20) - 17*
a**3*c**(13/2)*x**8*sqrt(a/(c*x**4) + 1)/(210*a**5*c**4*x**12 + 420*a**4*c
**5*x**16 + 210*a**3*c**6*x**20) - 3*a**2*c**(15/2)*x**12*sqrt(a/(c*x**4)
+ 1)/(210*a**5*c**4*x**12 + 420*a**4*c**5*x**16 + 210*a**3*c**6*x**20) - 1
2*a*c**(17/2)*x**16*sqrt(a/(c*x**4) + 1)/(210*a**5*c**4*x**12 + 420*a**4*c
**5*x**16 + 210*a**3*c**6*x**20) - 8*c**(19/2)*x**20*sqrt(a/(c*x**4) + 1)/
(210*a**5*c**4*x**12 + 420*a**4*c**5*x**16 + 210*a**3*c**6*x**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx = -\frac{\frac{35\,(cx^4+a)^{\frac{3}{2}}c^2}{x^6} - \frac{42\,(cx^4+a)^{\frac{5}{2}}c}{x^{10}} + \frac{15\,(cx^4+a)^{\frac{7}{2}}}{x^{14}}}{210\,a^3}$$

input
```
integrate((c*x^4+a)^(1/2)/x^15,x, algorithm="maxima")
```

output
```
-1/210*(35*(c*x^4 + a)^(3/2)*c^2/x^6 - 42*(c*x^4 + a)^(5/2)*c/x^10 + 15*(c
*x^4 + a)^(7/2)/x^14)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx = \frac{8\left(70\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^8c^{\frac{7}{2}}+35\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^6ac^{\frac{7}{2}}+21\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^4a^2c^{\frac{7}{2}}-7\left(\sqrt{c}x^2-\right.\right.}{105\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)^7}$$

input
```
integrate((c*x^4+a)^(1/2)/x^15,x, algorithm="giac")
```

output
```
8/105*(70*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^8*c^(7/2) + 35*(sqrt(c)*x^2 - sq
rt(c*x^4 + a))^6*a*c^(7/2) + 21*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*a^2*c^(7
/2) - 7*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^3*c^(7/2) + a^4*c^(7/2))/((sqr
t(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx = \frac{2\,c^2\,\sqrt{c\,x^4+a}}{105\,a^2\,x^6} - \frac{c\,\sqrt{c\,x^4+a}}{70\,a\,x^{10}} - \frac{4\,c^3\,\sqrt{c\,x^4+a}}{105\,a^3\,x^2} - \frac{\sqrt{c\,x^4+a}}{14\,x^{14}}$$

input
```
int((a + c*x^4)^(1/2)/x^15,x)
```

output
```
(2*c^2*(a + c*x^4)^(1/2))/(105*a^2*x^6) - (c*(a + c*x^4)^(1/2))/(70*a*x^10
) - (4*c^3*(a + c*x^4)^(1/2))/(105*a^3*x^2) - (a + c*x^4)^(1/2)/(14*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.37

$$\int \frac{\sqrt{a+cx^4}}{x^{15}}\,dx = \frac{-105\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3x^2 - 861\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2c\,x^6 - 1820\sqrt{c}\,\sqrt{c\,x^4+a}\,a\,c^2x^{10} - 1120\sqrt{c}\,\sqrt{c\,x^4+a}\,c^3x^{1}}{210x^{14}\left(\sqrt{c\,x^4+a}\,a^3 + 24\sqrt{c\,x^4+a}\,a^2c\,x^4 + 80\sqrt{c\,x^4+a}\,a\,c^2x^8 + 64\sqrt{c\,x^4+a}\,c^3x^{12} + 7\sqrt{c}\right.}$$

input
```
int((c*x^4+a)^(1/2)/x^15,x)
```

output
```
( - 105*sqrt(c)*sqrt(a + c*x**4)*a**3*x**2 - 861*sqrt(c)*sqrt(a + c*x**4)*
a**2*c*x**6 - 1820*sqrt(c)*sqrt(a + c*x**4)*a*c**2*x**10 - 1120*sqrt(c)*sq
rt(a + c*x**4)*c**3*x**14 - 15*a**4 - 378*a**3*c*x**4 - 1631*a**2*c**2*x**
8 - 2380*a*c**3*x**12 - 1120*c**4*x**16)/(210*x**14*(sqrt(a + c*x**4)*a**3
 + 24*sqrt(a + c*x**4)*a**2*c*x**4 + 80*sqrt(a + c*x**4)*a*c**2*x**8 + 64*
sqrt(a + c*x**4)*c**3*x**12 + 7*sqrt(c)*a**3*x**2 + 56*sqrt(c)*a**2*c*x**6
 + 112*sqrt(c)*a*c**2*x**10 + 64*sqrt(c)*c**3*x**14))
```

3.324 $\int x^4\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int x^4\sqrt{a+cx^4}\,dx=\frac{2ax\sqrt{a+cx^4}}{21c}+\frac{1}{7}x^5\sqrt{a+cx^4}-\frac{a^{7/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21c^{5/4}\sqrt{a+cx^4}}$$

output
```
2/21*a*x*(c*x^4+a)^(1/2)/c+1/7*x^5*(c*x^4+a)^(1/2)-1/21*a^(7/4)*(a^(1/2)+c
^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(5/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.49

$$\int x^4\sqrt{a+cx^4}\,dx=\frac{x\sqrt{a+cx^4}\left(a+cx^4-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{1+\frac{cx^4}{a}}}\right)}{7c}$$

input
```
Integrate[x^4*Sqrt[a + c*x^4],x]
```

output
```
(x*Sqrt[a + c*x^4]*(a + c*x^4 - (a*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)])/Sqrt[1 + (c*x^4)/a]))/(7*c)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {811, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{2}{7}a\int \frac{x^4}{\sqrt{cx^4+a}}dx+\frac{1}{7}x^5\sqrt{a+cx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{7}a\left(\frac{x\sqrt{a+cx^4}}{3c}-\frac{a\int \frac{1}{\sqrt{cx^4+a}}dx}{3c}\right)+\frac{1}{7}x^5\sqrt{a+cx^4}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}a\left(\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}\right)+\frac{1}{7}x^5\sqrt{a+cx^4}$$

input
```
Int[x^4*Sqrt[a + c*x^4],x]
```

output
```
(x^5*Sqrt[a + c*x^4])/7 + (2*a*((x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4])))/7
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x(3c\,x^4+2a)\sqrt{c\,x^4+a}}{21c}-\frac{2a^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{21c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	103
default	$\frac{x^5\sqrt{c\,x^4+a}}{7}+\frac{2ax\sqrt{c\,x^4+a}}{21c}-\frac{2a^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{21c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	108
elliptic	$\frac{x^5\sqrt{c\,x^4+a}}{7}+\frac{2ax\sqrt{c\,x^4+a}}{21c}-\frac{2a^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{21c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	108

input
```
int(x^4*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*x*(3*c*x^4+2*a)/c*(c*x^4+a)^(1/2)-2/21*a^2/c/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int x^4\sqrt{a+cx^4}\,dx=-\frac{2\,a\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)-\left(3\,cx^5+2\,ax\right)\sqrt{cx^4+a}}{21\,c}$$

input
```
integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/21*(2*a*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) - (
3*c*x^5 + 2*a*x)*sqrt(c*x^4 + a))/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int x^4\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+cx^4}\,dx=\int x^4\,\sqrt{c\,x^4+a}\,dx$$

input
```
int(x^4*(a + c*x^4)^(1/2),x)
```

output
```
int(x^4*(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{a+cx^4}\,dx=\frac{2\sqrt{c\,x^4+a}\,ax+3\sqrt{c\,x^4+a}\,c\,x^5-2\left(\int\frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a^2}{21c}$$

input
```
int(x^4*(c*x^4+a)^(1/2),x)
```

output
```
(2*sqrt(a + c*x**4)*a*x + 3*sqrt(a + c*x**4)*c*x**5 - 2*int(sqrt(a + c*x**
4)/(a + c*x**4),x)*a**2)/(21*c)
```

3.325 $\int \sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 105

$$\int \sqrt{a+cx^4}\,dx=\frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2
)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^
(1/2))/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \sqrt{a+cx^4}\,dx=\frac{x(a+cx^4)-\frac{2ia\sqrt{1+\frac{cx^4}{a}}\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

input
```
Integrate[Sqrt[a + c*x^4],x]
```

output
```
(x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{ 748}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{1}{3}x\sqrt{a+cx^4}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

input
```
Int[Sqrt[a + c*x^4],x]
```

output
```
(x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(3*c^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	85
risch	$\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	85
elliptic	$\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	85

input
```
int((c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \sqrt{a+cx^4}\,dx = \frac{2}{3}\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\frac{1}{3}\sqrt{cx^4+a}\,x$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 1/3*sqrt
(c*x^4 + a)*x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx = \frac{\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}\,dx$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}\,dx$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx = \frac{x\,\sqrt{c\,x^4+a}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

input
```
int((a + c*x^4)^(1/2),x)
```

output
```
(x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
 1)^(1/2)
```

Reduce [F]

$$\int \sqrt{a+cx^4}\,dx = \frac{\sqrt{c\,x^4+a}\,x}{3} + \frac{2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a}{3}$$

input
```
int((c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + c*x**4)*x + 2*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a)/3
```

3.326 $\int \frac{\sqrt{a+cx^4}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = -\frac{\sqrt{a+cx^4}}{3x^3} + \frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+cx^4}}$$

output
```
-1/3*(c*x^4+a)^(1/2)/x^3+1/3*c^(3/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(
1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2
*2^(1/2))/a^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = -\frac{\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{cx^4}{a}\right)}{3x^3\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^4,x]
```

output
```
-1/3*(Sqrt[a + c*x^4]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((c*x^4)/a)])/(x
^3*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {809, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^4}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{2}{3}c\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a+cx^4}}{3x^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3x^3}$$

input
```
Int[Sqrt[a + c*x^4]/x^4,x]
```

output
```
-1/3*Sqrt[a + c*x^4]/x^3 + (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(3*a^(1/4)*Sqrt[a + c*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\sqrt{c\,x^{4}+a}}{3x^{3}}+\frac{2c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\;\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	87
risch	$-\frac{\sqrt{c\,x^{4}+a}}{3x^{3}}+\frac{2c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\;\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	87
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{3x^{3}}+\frac{2c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\;\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	87

input
```
int((c*x^4+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+a)^(1/2)/x^3+2/3*c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = -\frac{2\sqrt{a}x^3\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right)+\sqrt{cx^4+a}}{3\,x^3}$$

input
```
integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(a)*x^3*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) +
sqrt(c*x^4 + a))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2)/x**4,x)
```

output
```
sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/
(4*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = \int \frac{\sqrt{cx^4+a}}{x^4}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = \int \frac{\sqrt{c\,x^4+a}}{x^4}\,dx$$

input
```
int((a + c*x^4)^(1/2)/x^4,x)
```

output
```
int((a + c*x^4)^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{x^4}\,dx = \frac{-\sqrt{c\,x^4+a}-2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^8+a\,x^4}dx\right)a\,x^3}{x^3}$$

input
```
int((c*x^4+a)^(1/2)/x^4,x)
```

output
```
( - sqrt(a + c*x**4) - 2*int(sqrt(a + c*x**4)/(a*x**4 + c*x**8),x)*a*x**3)
/x**3
```

3.327 $\int \frac{\sqrt{a+cx^4}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx = -\frac{\sqrt{a+cx^4}}{7x^7}-\frac{2c\sqrt{a+cx^4}}{21ax^3} - \frac{c^{7/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{21a^{5/4}\sqrt{a+cx^4}}$$

output
```
-1/7*(c*x^4+a)^(1/2)/x^7-2/21*c*(c*x^4+a)^(1/2)/a/x^3-1/21*c^(7/4)*(a^(1/2
)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(5/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx = -\frac{\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{cx^4}{a}\right)}{7x^7\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^8,x]
```

output
```
-1/7*(Sqrt[a + c*x^4]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((c*x^4)/a)])/(
x^7*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {809, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{2}{7}c\int \frac{1}{x^4\sqrt{cx^4+a}}dx-\frac{\sqrt{a+cx^4}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{7}c\left(-\frac{c\int \frac{1}{\sqrt{cx^4+a}}dx}{3a}-\frac{\sqrt{a+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+cx^4}}{7x^7}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}c\left(-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}\right)-\frac{\sqrt{a+cx^4}}{7x^7}$$

input
```
Int[Sqrt[a + c*x^4]/x^8,x]
```

output
```
-1/7*Sqrt[a + c*x^4]/x^7 + (2*c*(-1/3*Sqrt[a + c*x^4]/(a*x^3) - (c^(3/4)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + c*x^4])))/7
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,(2c\,x^{4}+3a)}{21x^{7}a}-\frac{2c^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	105
default	$-\frac{\sqrt{c\,x^{4}+a}}{7x^{7}}-\frac{2c\sqrt{c\,x^{4}+a}}{21a\,x^{3}}-\frac{2c^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	110
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{7x^{7}}-\frac{2c\sqrt{c\,x^{4}+a}}{21a\,x^{3}}-\frac{2c^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{21a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	110

input
```
int((c*x^4+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/21*(c*x^4+a)^(1/2)*(2*c*x^4+3*a)/x^7/a-2/21*c^2/a/(I/a^(1/2)*c^(1/2))^(
1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx=\frac{2\,\sqrt{a}cx^7\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\,|-1\right)-(2\,cx^4+3\,a)\sqrt{cx^4+a}}{21\,ax^7}$$

input
```
integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
1/21*(2*sqrt(a)*c*x^7*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1)
- (2*c*x^4 + 3*a)*sqrt(c*x^4 + a))/(a*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2)/x**8,x)
```

output
```
sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), c*x**4*exp_polar(I*pi)/a)
/(4*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+a}}{x^8}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx = \int \frac{\sqrt{cx^4+a}}{x^8}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx=\int \frac{\sqrt{c\,x^4+a}}{x^8}\,dx$$

input
```
int((a + c*x^4)^(1/2)/x^8,x)
```

output
```
int((a + c*x^4)^(1/2)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{x^8}\,dx=\frac{-\sqrt{c\,x^4+a}-2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^{12}+a\,x^8}dx\right)a\,x^7}{5x^7}$$

input
```
int((c*x^4+a)^(1/2)/x^8,x)
```

output
```
( - sqrt(a + c*x**4) - 2*int(sqrt(a + c*x**4)/(a*x**8 + c*x**12),x)*a*x**7
)/(5*x**7)
```

3.328 $\int x^2\sqrt{a+cx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 234

$$\int x^2\sqrt{a+cx^4}\,dx = \frac{1}{5}x^3\sqrt{a+cx^4}+\frac{2ax\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2a^{5/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{a^{5/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}$$

output
```
1/5*x^3*(c*x^4+a)^(1/2)+2/5*a*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(1/2)*x
^2)-2/5*a^(5/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^
(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/c^(3/4)/(c*x
^4+a)^(1/2)+1/5*a^(5/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*
x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(
3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.22

$$\int x^2\sqrt{a+cx^4}\,dx=\frac{x^3\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{3\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[x^2*Sqrt[a + c*x^4],x]
```

output
```
(x^3*Sqrt[a + c*x^4]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)])/(3*S
qrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \; 811$$

$$\frac{2}{5}a\int \frac{x^2}{\sqrt{cx^4+a}}dx+\frac{1}{5}x^3\sqrt{a+cx^4}$$

$$\Big\downarrow \; 834$$

$$\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{5}a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}$$

$\Big\downarrow$ 761

$$\frac{2}{5}a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+$$

$$\frac{1}{5}x^3\sqrt{a+cx^4}$$

$\Big\downarrow$ 1510

$$\frac{2}{5}a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\right.\right.}{\sqrt[4]{\right.}}}{\sqrt[4]{c}\sqrt{a+cx^4}}}{\sqrt{c}}\right.$$

$$\frac{1}{5}x^3\sqrt{a+cx^4}$$

input
```
Int[x^2*Sqrt[a + c*x^4],x]
```

output
```
(x^3*Sqrt[a + c*x^4])/5 + (2*a*(-((-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c
]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[
a + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(2*c^(3/4)*Sqrt[a + c*x^4])))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{x^{3}\sqrt{c\,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}$	112
risch	$\frac{x^{3}\sqrt{c\,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}$	112
elliptic	$\frac{x^{3}\sqrt{c\,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}$	112

input
```
int(x^2*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(c*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1
/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/
2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.38

$$\int x^2\sqrt{a+cx^4}\,dx = \frac{2\,a\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)-2\,a\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+(cx^4+2\,a)\sqrt{cx^4+a}}{5\,cx}$$

input
```
integrate(x^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(2*a*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - 2
*a*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + (c*x^4
+ 2*a)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int x^2\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/
a)/(4*gamma(7/4))
```

Maxima [F]

$$\int x^2\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+cx^4}\,dx=\int x^2\,\sqrt{c\,x^4+a}\,dx$$

input
```
int(x^2*(a + c*x^4)^(1/2),x)
```

output
```
int(x^2*(a + c*x^4)^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{a+cx^4}\,dx = \frac{\sqrt{c\,x^4+a}\,x^3}{5} + \frac{2\left(\int \frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)a}{5}$$

input
```
int(x^2*(c*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + c*x**4)*x**3 + 2*int((sqrt(a + c*x**4)*x**2)/(a + c*x**4),x)*a)/
5
```

3.329 $\int \frac{\sqrt{a+cx^4}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 224

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = -\frac{\sqrt{a+cx^4}}{x}+\frac{2\sqrt{c}x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2} - \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt{a+cx^4}}$$

output

```
-(c*x^4+a)^(1/2)/x+2*c^(1/2)*x*(c*x^4+a)^(1/2)/(a^(1/2)+c^(1/2)*x^2)-2*a^(
1/4)*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/
2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))/(c*x^4+a)^(1/2)
+a^(1/4)*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)
^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(c*x^4+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = -\frac{\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^4}{a}\right)}{x\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^2,x]
```

output
```
-((Sqrt[a + c*x^4]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((c*x^4)/a)])/(x*Sq
rt[1 + (c*x^4)/a]))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {809, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx$$

$$\Big\downarrow \text{809}$$

$$2c\int \frac{x^2}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a+cx^4}}{x}$$

$$\Big\downarrow \text{834}$$

$$2c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)-\frac{\sqrt{a+cx^4}}{x}$$

$$\Big\downarrow \text{27}$$

$$2c\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)-\frac{\sqrt{a+cx^4}}{x}$$

$\Big\downarrow$ 761

$$2c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)-\frac{\sqrt{a+cx^4}}{x}$$

$\Big\downarrow$ 1510

$$2c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}}{\sqrt{c}}\right.$$

$$\frac{\sqrt{a+cx^4}}{x}$$

input
```
Int[Sqrt[a + c*x^4]/x^2,x]
```

output
```
-(Sqrt[a + c*x^4]/x) + 2*c*(-((-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^
2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a +
c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c
^(3/4)*Sqrt[a + c*x^4]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\sqrt{c\,x^{4}+a}}{x}+\frac{2 i \sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	112
risch	$-\frac{\sqrt{c\,x^{4}+a}}{x}+\frac{2 i \sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	112
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{x}+\frac{2 i \sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i \sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	112

input
```
int((c*x^4+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*x^4+a)^(1/2)/x+2*I*c^(1/2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(El
lipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/
2),I))
```

Fricas [F]

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx=\int \frac{\sqrt{cx^4+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2)/x**2,x)
```

output
```
sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), c*x**4*exp_polar(I*pi)/a)/
(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = \int \frac{\sqrt{cx^4+a}}{x^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = \frac{\sqrt{c\,x^4+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a}{c\,x^4}\right)}{x\,\sqrt{\frac{a}{c\,x^4}+1}}$$

input
```
int((a + c*x^4)^(1/2)/x^2,x)
```

output
```
((a + c*x^4)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -a/(c*x^4)))/(x*(a/(c*x^4)
 + 1)^(1/2))
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{x^2}\,dx = \frac{\sqrt{c\,x^4+a}+2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^6+a\,x^2}dx\right)ax}{x}$$

input
```
int((c*x^4+a)^(1/2)/x^2,x)
```

output
```
(sqrt(a + c*x**4) + 2*int(sqrt(a + c*x**4)/(a*x**2 + c*x**6),x)*a*x)/x
```

3.330 $\int \frac{\sqrt{a+cx^4}}{x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 258

$$\int \frac{\sqrt{a+cx^4}}{x^6} \, dx = -\frac{\sqrt{a+cx^4}}{5x^5} - \frac{2c\sqrt{a+cx^4}}{5ax} + \frac{2c^{3/2}x\sqrt{a+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{2c^{5/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+cx^4}} + \frac{c^{5/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5a^{3/4}\sqrt{a+cx^4}}$$

output

```
-1/5*(c*x^4+a)^(1/2)/x^5-2/5*c*(c*x^4+a)^(1/2)/a/x+2/5*c^(3/2)*x*(c*x^4+a)
^(1/2)/a/(a^(1/2)+c^(1/2)*x^2)-2/5*c^(5/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a
)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*2^(1/2))/a^(3/4)/(c*x^4+a)^(1/2)+1/5*c^(5/4)*(a^(1/2)+c^(1/2)*x^2)*(
(c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*
x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx = -\frac{\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{cx^4}{a}\right)}{5x^5\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[Sqrt[a + c*x^4]/x^6,x]
```

output
```
-1/5*(Sqrt[a + c*x^4]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((c*x^4)/a)])/(
x^5*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {809, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{2}{5}c\int \frac{1}{x^2\sqrt{cx^4+a}}dx - \frac{\sqrt{a+cx^4}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$\frac{2}{5}c\left(\frac{c\int \frac{x^2}{\sqrt{cx^4+a}}dx}{a} - \frac{\sqrt{a+cx^4}}{ax}\right) - \frac{\sqrt{a+cx^4}}{5x^5}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{5}c\left(\frac{c\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+cx^4}}{ax}\right)-\frac{\sqrt{a+cx^4}}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}c\left(\frac{c\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+cx^4}}{ax}\right)-\frac{\sqrt{a+cx^4}}{5x^5}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{5}c\left(\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{a}-\frac{\sqrt{a+cx^4}}{ax}\right)-\frac{\sqrt{a+cx^4}}{5x^5}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{5}c\left(\frac{c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+\cdots}}{\sqrt{a}+\cdots}}{\sqrt{c}}\right)}{a}\cdots\right)-\frac{\sqrt{a+cx^4}}{5x^5}$$

input
```
Int[Sqrt[a + c*x^4]/x^6,x]
```

output
```
-1/5*Sqrt[a + c*x^4]/x^5 + (2*c*(-(Sqrt[a + c*x^4]/(a*x)) + (c*(-((-((x*Sq
rt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])))/a))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(2c\,x^{4}+a\right)}{5x^{5}a}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	123
default	$-\frac{\sqrt{c\,x^{4}+a}}{5x^{5}}-\frac{2c\sqrt{c\,x^{4}+a}}{5ax}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	130
elliptic	$-\frac{\sqrt{c\,x^{4}+a}}{5x^{5}}-\frac{2c\sqrt{c\,x^{4}+a}}{5ax}+\frac{2ic^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	130

input
```
int((c*x^4+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(c*x^4+a)^(1/2)*(2*c*x^4+a)/x^5/a+2/5*I*c^(3/2)/a^(1/2)/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(
I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx = -\frac{2\sqrt{a}cx^5\left(-\frac{c}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1) - 2\sqrt{a}cx^5\left(-\frac{c}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1) + (2\,cx^4+a)\sqrt{}}{5\,ax^5}$$

input
```
integrate((c*x^4+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/5*(2*sqrt(a)*c*x^5*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(1/4)), -1)
- 2*sqrt(a)*c*x^5*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (2
*c*x^4 + a)*sqrt(c*x^4 + a))/(a*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2)/x**6,x)
```

output
```
sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), c*x**4*exp_polar(I*pi)/a)
/(4*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx=\int \frac{\sqrt{cx^4+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx=\int \frac{\sqrt{cx^4+a}}{x^6}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx=\int \frac{\sqrt{c\,x^4+a}}{x^6}\,dx$$

input
```
int((a + c*x^4)^(1/2)/x^6,x)
```

output
```
int((a + c*x^4)^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+cx^4}}{x^6}\,dx = \frac{-\sqrt{c\,x^4+a}-2\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^{10}+a\,x^6}dx\right)a\,x^5}{3x^5}$$

input
```
int((c*x^4+a)^(1/2)/x^6,x)
```

output
```
( - sqrt(a + c*x**4) - 2*int(sqrt(a + c*x**4)/(a*x**6 + c*x**10),x)*a*x**5
)/(3*x**5)
```

3.331 $\int x^{11}(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^{11}(a+cx^4)^{3/2}\,dx=\frac{a^2(a+cx^4)^{5/2}}{10c^3}-\frac{a(a+cx^4)^{7/2}}{7c^3}+\frac{(a+cx^4)^{9/2}}{18c^3}$$

output
```
1/10*a^2*(c*x^4+a)^(5/2)/c^3-1/7*a*(c*x^4+a)^(7/2)/c^3+1/18*(c*x^4+a)^(9/2
)/c^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^{11}(a+cx^4)^{3/2}\,dx=\frac{(a+cx^4)^{5/2}(8a^2-20acx^4+35c^2x^8)}{630c^3}$$

input
```
Integrate[x^11*(a + c*x^4)^(3/2),x]
```

output
```
((a + c*x^4)^(5/2)*(8*a^2 - 20*a*c*x^4 + 35*c^2*x^8))/(630*c^3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int x^8\left(cx^4+a\right)^{3/2}dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4}\int \left(\frac{\left(cx^4+a\right)^{7/2}}{c^2}-\frac{2a\left(cx^4+a\right)^{5/2}}{c^2}+\frac{a^2\left(cx^4+a\right)^{3/2}}{c^2}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{2a^2\left(a+cx^4\right)^{5/2}}{5c^3}+\frac{2\left(a+cx^4\right)^{9/2}}{9c^3}-\frac{4a\left(a+cx^4\right)^{7/2}}{7c^3}\right)$$

input
```
Int[x^11*(a + c*x^4)^(3/2),x]
```

output
```
((2*a^2*(a + c*x^4)^(5/2))/(5*c^3) - (4*a*(a + c*x^4)^(7/2))/(7*c^3) + (2*
(a + c*x^4)^(9/2))/(9*c^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}} \left(35 c^{2} x^{8}-20 a\,x^{4} c+8 a^{2}\right)}{630 c^{3}}$	36
pseudoelliptic	$\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}} \left(35 c^{2} x^{8}-20 a\,x^{4} c+8 a^{2}\right)}{630 c^{3}}$	36
orering	$\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}} \left(35 c^{2} x^{8}-20 a\,x^{4} c+8 a^{2}\right)}{630 c^{3}}$	36
default	$\frac{\sqrt{c\,x^{4}+a}\,\left(35 c^{2} x^{8}-20 a\,x^{4} c+8 a^{2}\right) \left(c^{2} x^{8}+2 a\,x^{4} c+a^{2}\right)}{630 c^{3}}$	54
elliptic	$\frac{\sqrt{c\,x^{4}+a}\,\left(35 c^{2} x^{8}-20 a\,x^{4} c+8 a^{2}\right) \left(c^{2} x^{8}+2 a\,x^{4} c+a^{2}\right)}{630 c^{3}}$	54
trager	$\frac{\left(35 c^{4} x^{16}+50 c^{3} a\,x^{12}+3 a^{2} x^{8} c^{2}-4 a^{3} c\,x^{4}+8 a^{4}\right) \sqrt{c\,x^{4}+a}}{630 c^{3}}$	58
risch	$\frac{\left(35 c^{4} x^{16}+50 c^{3} a\,x^{12}+3 a^{2} x^{8} c^{2}-4 a^{3} c\,x^{4}+8 a^{4}\right) \sqrt{c\,x^{4}+a}}{630 c^{3}}$	58

input
```
int(x^11*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/630*(c*x^4+a)^(5/2)*(35*c^2*x^8-20*a*c*x^4+8*a^2)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^{11}\left(a+cx^4\right)^{3/2}\,dx=\frac{\left(35\,c^4x^{16}+50\,ac^3x^{12}+3\,a^2c^2x^8-4\,a^3cx^4+8\,a^4\right)\sqrt{cx^4+a}}{630\,c^3}$$

input
```
integrate(x^11*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/630*(35*c^4*x^16 + 50*a*c^3*x^12 + 3*a^2*c^2*x^8 - 4*a^3*c*x^4 + 8*a^4)*
sqrt(c*x^4 + a)/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(49) = 98.

Time = 0.55 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int x^{11}\left(a+cx^4\right)^{3/2}\,dx=\begin{cases}\frac{4a^4\sqrt{a+cx^4}}{315c^3}-\frac{2a^3x^4\sqrt{a+cx^4}}{315c^2}+\frac{a^2x^8\sqrt{a+cx^4}}{210c}+\frac{5ax^{12}\sqrt{a+cx^4}}{63}+\frac{cx^{16}\sqrt{a+cx^4}}{18} & \text{for } c\neq 0\\ \frac{a^{\frac{3}{2}}x^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(x**11*(c*x**4+a)**(3/2),x)
```

output
```
Piecewise((4*a**4*sqrt(a + c*x**4)/(315*c**3) - 2*a**3*x**4*sqrt(a + c*x**
4)/(315*c**2) + a**2*x**8*sqrt(a + c*x**4)/(210*c) + 5*a*x**12*sqrt(a + c*
x**4)/63 + c*x**16*sqrt(a + c*x**4)/18, Ne(c, 0)), (a**(3/2)*x**12/12, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^{11}(a+cx^4)^{3/2}\,dx=\frac{(cx^4+a)^{\frac{9}{2}}}{18\,c^3}-\frac{(cx^4+a)^{\frac{7}{2}}a}{7\,c^3}+\frac{(cx^4+a)^{\frac{5}{2}}a^2}{10\,c^3}$$

input
```
integrate(x^11*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/18*(c*x^4 + a)^(9/2)/c^3 - 1/7*(c*x^4 + a)^(7/2)*a/c^3 + 1/10*(c*x^4 + a
)^(5/2)*a^2/c^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^{11}(a+cx^4)^{3/2}\,dx=\frac{35\,(cx^4+a)^{\frac{9}{2}}-90\,(cx^4+a)^{\frac{7}{2}}a+63\,(cx^4+a)^{\frac{5}{2}}a^2}{630\,c^3}$$

input
```
integrate(x^11*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/630*(35*(c*x^4 + a)^(9/2) - 90*(c*x^4 + a)^(7/2)*a + 63*(c*x^4 + a)^(5/2
)*a^2)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^{11}(a+cx^4)^{3/2}\,dx=\sqrt{c\,x^4+a}\left(\frac{5\,a\,x^{12}}{63}+\frac{c\,x^{16}}{18}+\frac{4\,a^4}{315\,c^3}-\frac{2\,a^3\,x^4}{315\,c^2}+\frac{a^2\,x^8}{210\,c}\right)$$

input
```
int(x^11*(a + c*x^4)^(3/2),x)
```

output
```
(a + c*x^4)^(1/2)*((5*a*x^12)/63 + (c*x^16)/18 + (4*a^4)/(315*c^3) - (2*a^
3*x^4)/(315*c^2) + (a^2*x^8)/(210*c))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 421, normalized size of antiderivative = 7.14

$$\int x^{11}(a+cx^4)^{3/2}\,dx = \frac{72\sqrt{c}\,\sqrt{c\,x^4+a}\,a^8x^2+924\sqrt{c}\,\sqrt{c\,x^4+a}\,a^7c\,x^6+3003\sqrt{c}\,\sqrt{c\,x^4+a}\,a^6c^2x^{10}+3690\sqrt{c}\,\sqrt{c\,x}}{}$$

input
```
int(x^11*(c*x^4+a)^(3/2),x)
```

output
```
(72*sqrt(c)*sqrt(a + c*x**4)*a**8*x**2 + 924*sqrt(c)*sqrt(a + c*x**4)*a**7
*c*x**6 + 3003*sqrt(c)*sqrt(a + c*x**4)*a**6*c**2*x**10 + 3690*sqrt(c)*sqr
t(a + c*x**4)*a**5*c**3*x**14 + 7355*sqrt(c)*sqrt(a + c*x**4)*a**4*c**4*x*
*18 + 26504*sqrt(c)*sqrt(a + c*x**4)*a**3*c**5*x**22 + 44688*sqrt(c)*sqrt(
a + c*x**4)*a**2*c**6*x**26 + 32960*sqrt(c)*sqrt(a + c*x**4)*a*c**7*x**30
+ 8960*sqrt(c)*sqrt(a + c*x**4)*c**8*x**34 + 8*a**9 + 324*a**8*c*x**4 + 20
79*a**7*c**2*x**8 + 4557*a**6*c**3*x**12 + 5805*a**5*c**4*x**16 + 16731*a*
*4*c**5*x**20 + 45288*a**3*c**6*x**24 + 60048*a**2*c**7*x**28 + 37440*a*c*
*8*x**32 + 8960*c**9*x**36)/(630*c**3*(sqrt(a + c*x**4)*a**4 + 40*sqrt(a +
 c*x**4)*a**3*c*x**4 + 240*sqrt(a + c*x**4)*a**2*c**2*x**8 + 448*sqrt(a +
c*x**4)*a*c**3*x**12 + 256*sqrt(a + c*x**4)*c**4*x**16 + 9*sqrt(c)*a**4*x*
*2 + 120*sqrt(c)*a**3*c*x**6 + 432*sqrt(c)*a**2*c**2*x**10 + 576*sqrt(c)*a
*c**3*x**14 + 256*sqrt(c)*c**4*x**18))
```

3.332 $\int x^7(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^7 (a+cx^4)^{3/2}\,dx=-\frac{a(a+cx^4)^{5/2}}{10c^2}+\frac{(a+cx^4)^{7/2}}{14c^2}$$

output
```
-1/10*a*(c*x^4+a)^(5/2)/c^2+1/14*(c*x^4+a)^(7/2)/c^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^7 (a+cx^4)^{3/2}\,dx=\frac{(a+cx^4)^{5/2}(-2a+5cx^4)}{70c^2}$$

input
```
Integrate[x^7*(a + c*x^4)^(3/2),x]
```

output
```
((a + c*x^4)^(5/2)*(-2*a + 5*c*x^4))/(70*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int x^4 \left(cx^4+a\right)^{3/2}dx^4$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{4}\int \left(\frac{\left(cx^4+a\right)^{5/2}}{c}-\frac{a\left(cx^4+a\right)^{3/2}}{c}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{2\left(a+cx^4\right)^{7/2}}{7c^2}-\frac{2a\left(a+cx^4\right)^{5/2}}{5c^2}\right)$$

input
```
Int[x^7*(a + c*x^4)^(3/2),x]
```

output
```
((-2*a*(a + c*x^4)^(5/2))/(5*c^2) + (2*(a + c*x^4)^(7/2))/(7*c^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(c\,x^4+a)^{\frac{5}{2}}(-5c\,x^4+2a)}{70c^2}$	25
pseudoelliptic	$-\frac{(c\,x^4+a)^{\frac{5}{2}}(-5c\,x^4+2a)}{70c^2}$	25
orering	$-\frac{(c\,x^4+a)^{\frac{5}{2}}(-5c\,x^4+2a)}{70c^2}$	25
default	$-\frac{\sqrt{c\,x^4+a}\,(-5c\,x^4+2a)(c^2x^8+2a\,x^4c+a^2)}{70c^2}$	43
elliptic	$-\frac{\sqrt{c\,x^4+a}\,(-5c\,x^4+2a)(c^2x^8+2a\,x^4c+a^2)}{70c^2}$	43
trager	$-\frac{(-5c^3x^{12}-8a\,c^2x^8-a^2c\,x^4+2a^3)\sqrt{c\,x^4+a}}{70c^2}$	47
risch	$-\frac{(-5c^3x^{12}-8a\,c^2x^8-a^2c\,x^4+2a^3)\sqrt{c\,x^4+a}}{70c^2}$	47

input
```
int(x^7*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/70*(c*x^4+a)^(5/2)*(-5*c*x^4+2*a)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^7 (a + cx^4)^{3/2} \, dx = \frac{(5\, c^3 x^{12} + 8\, ac^2 x^8 + a^2 c x^4 - 2\, a^3) \sqrt{cx^4 + a}}{70\, c^2}$$

input
```
integrate(x^7*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/70*(5*c^3*x^12 + 8*a*c^2*x^8 + a^2*c*x^4 - 2*a^3)*sqrt(c*x^4 + a)/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(31) = 62.

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.18

$$\int x^7 (a + cx^4)^{3/2} \, dx = \begin{cases} -\frac{a^3 \sqrt{a + cx^4}}{35c^2} + \frac{a^2 x^4 \sqrt{a + cx^4}}{70c} + \frac{4ax^8 \sqrt{a + cx^4}}{35} + \frac{cx^{12} \sqrt{a + cx^4}}{14} & \text{for } c \neq 0 \\ \frac{a^{\frac{3}{2}} x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(c*x**4+a)**(3/2),x)
```

output
```
Piecewise((-a**3*sqrt(a + c*x**4)/(35*c**2) + a**2*x**4*sqrt(a + c*x**4)/(
70*c) + 4*a*x**8*sqrt(a + c*x**4)/35 + c*x**12*sqrt(a + c*x**4)/14, Ne(c,
0)), (a**(3/2)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^7 (a + cx^4)^{3/2} \, dx = \frac{(cx^4 + a)^{\frac{7}{2}}}{14\, c^2} - \frac{(cx^4 + a)^{\frac{5}{2}} a}{10\, c^2}$$

input
```
integrate(x^7*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/14*(c*x^4 + a)^(7/2)/c^2 - 1/10*(c*x^4 + a)^(5/2)*a/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^7 (a + cx^4)^{3/2} \, dx = \frac{5 (cx^4 + a)^{\frac{7}{2}} - 7 (cx^4 + a)^{\frac{5}{2}} a}{70 \, c^2}$$

input
```
integrate(x^7*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/70*(5*(c*x^4 + a)^(7/2) - 7*(c*x^4 + a)^(5/2)*a)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^7 (a + cx^4)^{3/2} \, dx = \sqrt{c\,x^4 + a} \left(\frac{4\,a\,x^8}{35} + \frac{c\,x^{12}}{14} - \frac{a^3}{35\,c^2} + \frac{a^2\,x^4}{70\,c} \right)$$

input
```
int(x^7*(a + c*x^4)^(3/2),x)
```

output
```
(a + c*x^4)^(1/2)*((4*a*x^8)/35 + (c*x^12)/14 - a^3/(35*c^2) + (a^2*x^4)/(
70*c))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 325, normalized size of antiderivative = 8.55

$$\int x^7 (a + cx^4)^{3/2} \, dx = \frac{-14\sqrt{c}\,\sqrt{c\,x^4 + a}\,a^6 x^2 - 105\sqrt{c}\,\sqrt{c\,x^4 + a}\,a^5 c\,x^6 - 112\sqrt{c}\,\sqrt{c\,x^4 + a}\,a^4 c^2 x^{10} + 467\sqrt{c}\,\sqrt{c\,x^4}}{70c^2 \left(\sqrt{c\,x^4 + a}\,a^3 -\right.}$$

input
```
int(x^7*(c*x^4+a)^(3/2),x)
```

output
```
( - 14*sqrt(c)*sqrt(a + c*x**4)*a**6*x**2 - 105*sqrt(c)*sqrt(a + c*x**4)*a
**5*c*x**6 - 112*sqrt(c)*sqrt(a + c*x**4)*a**4*c**2*x**10 + 467*sqrt(c)*sq
rt(a + c*x**4)*a**3*c**3*x**14 + 1240*sqrt(c)*sqrt(a + c*x**4)*a**2*c**4*x
**18 + 1072*sqrt(c)*sqrt(a + c*x**4)*a*c**5*x**22 + 320*sqrt(c)*sqrt(a + c
*x**4)*c**6*x**26 - 2*a**7 - 49*a**6*c*x**4 - 175*a**5*c**2*x**8 + 21*a**4
*c**3*x**12 + 973*a**3*c**4*x**16 + 1736*a**2*c**5*x**20 + 1232*a*c**6*x**
24 + 320*c**7*x**28)/(70*c**2*(sqrt(a + c*x**4)*a**3 + 24*sqrt(a + c*x**4)
*a**2*c*x**4 + 80*sqrt(a + c*x**4)*a*c**2*x**8 + 64*sqrt(a + c*x**4)*c**3*
x**12 + 7*sqrt(c)*a**3*x**2 + 56*sqrt(c)*a**2*c*x**6 + 112*sqrt(c)*a*c**2*
x**10 + 64*sqrt(c)*c**3*x**14))
```

3.333 $\int x^3(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^3\left(a+cx^4\right)^{3/2}\,dx=\frac{\left(a+cx^4\right)^{5/2}}{10c}$$

output
```
1/10*(c*x^4+a)^(5/2)/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^3\left(a+cx^4\right)^{3/2}\,dx=\frac{\left(a+cx^4\right)^{5/2}}{10c}$$

input
```
Integrate[x^3*(a + c*x^4)^(3/2),x]
```

output
```
(a + c*x^4)^(5/2)/(10*c)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + cx^4)^{3/2} \, dx$$

$$\Big\downarrow \text{793}$$

$$\frac{(a + cx^4)^{5/2}}{10c}$$

input
```
Int[x^3*(a + c*x^4)^(3/2),x]
```

output
```
(a + c*x^4)^(5/2)/(10*c)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(c\,x^4+a)^{\frac{5}{2}}}{10c}$	15
derivativedivides	$\frac{(c\,x^4+a)^{\frac{5}{2}}}{10c}$	15
default	$\frac{(c\,x^4+a)^{\frac{5}{2}}}{10c}$	15
pseudoelliptic	$\frac{(c\,x^4+a)^{\frac{5}{2}}}{10c}$	15
orering	$\frac{(c\,x^4+a)^{\frac{5}{2}}}{10c}$	15
trager	$\frac{(c^2x^8+2a\,x^4c+a^2)\sqrt{c\,x^4+a}}{10c}$	33
risch	$\frac{(c^2x^8+2a\,x^4c+a^2)\sqrt{c\,x^4+a}}{10c}$	33

input
```
int(x^3*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*(c*x^4+a)^(5/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^3\left(a+cx^4\right)^{3/2}\,dx=\frac{(c^2x^8+2\,acx^4+a^2)\sqrt{cx^4+a}}{10\,c}$$

input
```
integrate(x^3*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(c*x^4 + a)/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(12) = 24$.

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.33

$$\int x^3 \left(a+cx^4\right)^{3/2}\,dx = \begin{cases} \frac{a^2\sqrt{a+cx^4}}{10c} + \frac{ax^4\sqrt{a+cx^4}}{5} + \frac{cx^8\sqrt{a+cx^4}}{10} & \text{for } c \neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(c*x**4+a)**(3/2),x)
```

output
```
Piecewise((a**2*sqrt(a + c*x**4)/(10*c) + a*x**4*sqrt(a + c*x**4)/5 + c*x*
*8*sqrt(a + c*x**4)/10, Ne(c, 0)), (a**(3/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \left(a+cx^4\right)^{3/2}\,dx = \frac{\left(cx^4+a\right)^{\frac{5}{2}}}{10\,c}$$

input
```
integrate(x^3*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/10*(c*x^4 + a)^(5/2)/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \left(a+cx^4\right)^{3/2}\,dx = \frac{\left(cx^4+a\right)^{\frac{5}{2}}}{10\,c}$$

input
```
integrate(x^3*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/10*(c*x^4 + a)^(5/2)/c
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \left(a+cx^4\right)^{3/2} \, dx = \frac{\left(c\,x^4+a\right)^{5/2}}{10\,c}$$

input
```
int(x^3*(a + c*x^4)^(3/2),x)
```

output
```
(a + c*x^4)^(5/2)/(10*c)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 12.61

$$\int x^3 (a + cx^4)^{3/2} \, dx = \frac{5\sqrt{c}\,\sqrt{c\,x^4+a}\,a^4x^2 + 30\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3c\,x^6 + 61\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2c^2x^{10} + 52\sqrt{c}\,\sqrt{c\,x^4+a}\,a}{10c\left(\sqrt{c\,x^4+a}\,a^2 + 12\sqrt{c\,x^4+a}\,ac\,x^4 + 16\sqrt{c\,x^4}\right.}$$

input
```
int(x^3*(c*x^4+a)^(3/2),x)
```

output
```
(5*sqrt(c)*sqrt(a + c*x**4)*a**4*x**2 + 30*sqrt(c)*sqrt(a + c*x**4)*a**3*c
*x**6 + 61*sqrt(c)*sqrt(a + c*x**4)*a**2*c**2*x**10 + 52*sqrt(c)*sqrt(a +
c*x**4)*a*c**3*x**14 + 16*sqrt(c)*sqrt(a + c*x**4)*c**4*x**18 + a**5 + 15*
a**4*c*x**4 + 55*a**3*c**2*x**8 + 85*a**2*c**3*x**12 + 60*a*c**4*x**16 + 1
6*c**5*x**20)/(10*c*(sqrt(a + c*x**4)*a**2 + 12*sqrt(a + c*x**4)*a*c*x**4
+ 16*sqrt(a + c*x**4)*c**2*x**8 + 5*sqrt(c)*a**2*x**2 + 20*sqrt(c)*a*c*x**
6 + 16*sqrt(c)*c**2*x**10))
```

3.334 $\int \frac{(a+cx^4)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{1}{2}a\sqrt{a+cx^4} + \frac{1}{6}(a+cx^4)^{3/2} - \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

output
```
1/2*a*(c*x^4+a)^(1/2)+1/6*(c*x^4+a)^(3/2)-1/2*a^(3/2)*arctanh((c*x^4+a)^(1
/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{1}{6}\sqrt{a+cx^4}(4a+cx^4) - \frac{1}{2}a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + c*x^4)^(3/2)/x,x]
```

output
```
(Sqrt[a + c*x^4]*(4*a + c*x^4))/6 - (a^(3/2)*ArcTanh[Sqrt[a + c*x^4]/Sqrt[
a]])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(cx^4+a)^{3/2}}{x^4}dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\int \frac{\sqrt{cx^4+a}}{x^4}dx^4+\frac{2}{3}(a+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\left(a\int \frac{1}{x^4\sqrt{cx^4+a}}dx^4+2\sqrt{a+cx^4}\right)+\frac{2}{3}(a+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(a\left(\frac{2a\int \frac{1}{\frac{x^8}{c}-\frac{a}{c}}d\sqrt{cx^4+a}}{c}+2\sqrt{a+cx^4}\right)+\frac{2}{3}(a+cx^4)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(a\left(2\sqrt{a+cx^4}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+cx^4)^{3/2}\right)$$

input
```
Int[(a + c*x^4)^(3/2)/x,x]
```

output
```
((2*(a + c*x^4)^(3/2))/3 + a*(2*Sqrt[a + c*x^4] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + c*x^4]/Sqrt[a]]))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

method	result	size
pseudoelliptic	$-\frac{a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right)}{2}+\frac{\sqrt{c\,x^{4}+a}\,\left(c\,x^{4}+4 a \right)}{6}$	41
default	$-\frac{a^{\frac{3}{2}} \ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2}+\frac{\sqrt{c\,x^{4}+a}\,c\,x^{4}}{6}+\frac{2 a \sqrt{c\,x^{4}+a}}{3}$	57
elliptic	$-\frac{a^{\frac{3}{2}} \ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{2}+\frac{\sqrt{c\,x^{4}+a}\,c\,x^{4}}{6}+\frac{2 a \sqrt{c\,x^{4}+a}}{3}$	57

input
```
int((c*x^4+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^(3/2)*arctanh((c*x^4+a)^(1/2)/a^(1/2))+1/6*(c*x^4+a)^(1/2)*(c*x^4+4
*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \left[\frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{cx^4-2\,\sqrt{cx^4+a}\sqrt{a}+2\,a}{x^4}\right)\right.$$
$$\left.+\frac{1}{6}\left(cx^4+4\,a\right)\sqrt{cx^4+a},\,\frac{1}{2}\,\sqrt{-a}a\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^4+a}}\right)+\frac{1}{6}\left(cx^4+4\,a\right)\sqrt{cx^4+a}\right]$$

input
```
integrate((c*x^4+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/4*a^(3/2)*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) + 1/6*(c*x
^4 + 4*a)*sqrt(c*x^4 + a), 1/2*sqrt(-a)*a*arctan(sqrt(-a)/sqrt(c*x^4 + a))
 + 1/6*(c*x^4 + 4*a)*sqrt(c*x^4 + a)]
```

Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}{3}+\frac{a^{\frac{3}{2}}\log\left(\frac{cx^4}{a}\right)}{4}$$
$$-\frac{a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{cx^4}{a}}+1\right)}{2}+\frac{\sqrt{a}cx^4\sqrt{1+\frac{cx^4}{a}}}{6}$$

input
```
integrate((c*x**4+a)**(3/2)/x,x)
```

output
```
2*a**(3/2)*sqrt(1 + c*x**4/a)/3 + a**(3/2)*log(c*x**4/a)/4 - a**(3/2)*log(
sqrt(1 + c*x**4/a) + 1)/2 + sqrt(a)*c*x**4*sqrt(1 + c*x**4/a)/6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{1}{4}\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right) + \frac{1}{6}\,(cx^4+a)^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{cx^4+a}a$$

input
```
integrate((c*x^4+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*a^(3/2)*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a))) +
 1/6*(c*x^4 + a)^(3/2) + 1/2*sqrt(c*x^4 + a)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}} + \frac{1}{6}\,(cx^4+a)^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{cx^4+a}a$$

input
```
integrate((c*x^4+a)^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*a^2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) + 1/6*(c*x^4 + a)^(3/2)
+ 1/2*sqrt(c*x^4 + a)*a
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{a\,\sqrt{c\,x^4+a}}{2} - \frac{a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{2} + \frac{(c\,x^4+a)^{3/2}}{6}$$

input
```
int((a + c*x^4)^(3/2)/x,x)
```

output
```
(a*(a + c*x^4)^(1/2))/2 - (a^(3/2)*atanh((a + c*x^4)^(1/2)/a^(1/2)))/2 + (
a + c*x^4)^(3/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 432, normalized size of antiderivative = 7.32

$$\int \frac{(a+cx^4)^{3/2}}{x}\,dx = \frac{3\sqrt{a}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^2+12\sqrt{a}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)ac\,x}{}$$

input
```
int((c*x^4+a)^(3/2)/x,x)
```

output
```
(3*sqrt(a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2
)/sqrt(a))*a**2 + 12*sqrt(a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt
(a) + sqrt(c)*x**2)/sqrt(a))*a*c*x**4 - 3*sqrt(a)*sqrt(a + c*x**4)*log((sq
rt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a**2 - 12*sqrt(a)*sqrt(a
 + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*c*x*
*4 + 12*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 + 19*sqrt(c)*sqrt(a + c*x**4)*a
*c*x**6 + 4*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 + 9*sqrt(c)*sqrt(a)*log((s
qrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a**2*x**2 + 12*sqrt(c)*
sqrt(a)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*c*x**6
- 9*sqrt(c)*sqrt(a)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a
))*a**2*x**2 - 12*sqrt(c)*sqrt(a)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c
)*x**2)/sqrt(a))*a*c*x**6 + 4*a**3 + 21*a**2*c*x**4 + 21*a*c**2*x**8 + 4*c
**3*x**12)/(6*(sqrt(a + c*x**4)*a + 4*sqrt(a + c*x**4)*c*x**4 + 3*sqrt(c)*
a*x**2 + 4*sqrt(c)*c*x**6))
```

3.335 $\int \frac{(a+cx^4)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{(a+cx^4)^{3/2}}{x^5} dx = \frac{1}{2}c\sqrt{a+cx^4} - \frac{a\sqrt{a+cx^4}}{4x^4} - \frac{3}{4}\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

output
```
1/2*c*(c*x^4+a)^(1/2)-1/4*a*(c*x^4+a)^(1/2)/x^4-3/4*a^(1/2)*c*arctanh((c*x
^4+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{(a+cx^4)^{3/2}}{x^5} dx = \frac{\sqrt{a+cx^4}(-a+2cx^4)}{4x^4} - \frac{3}{4}\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^5,x]
```

output
```
(Sqrt[a + c*x^4]*(-a + 2*c*x^4))/(4*x^4) - (3*Sqrt[a]*c*ArcTanh[Sqrt[a + c
*x^4]/Sqrt[a]])/4
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^5} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(cx^4+a)^{3/2}}{x^8}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{3}{2}c\int \frac{\sqrt{cx^4+a}}{x^4}dx^4-\frac{(a+cx^4)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{3}{2}c\left(a\int \frac{1}{x^4\sqrt{cx^4+a}}dx^4+2\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{3}{2}c\left(\frac{2a\int \frac{1}{\frac{x^8}{c}-\frac{a}{c}}d\sqrt{cx^4+a}}{c}+2\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{3}{2}c\left(2\sqrt{a+cx^4}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)\right)-\frac{(a+cx^4)^{3/2}}{x^4}\right)$$

input
```
Int[(a + c*x^4)^(3/2)/x^5,x]
```

output
```
(-((a + c*x^4)^(3/2)/x^4) + (3*c*(2*Sqrt[a + c*x^4] - 2*Sqrt[a]*ArcTanh[Sq
rt[a + c*x^4]/Sqrt[a]]))/2)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right) a c\,x^{4}+\left(-2 c\,x^{4}+a \right) \sqrt{c\,x^{4}+a}\,\sqrt{a}}{4 \sqrt{a}\,x^{4}}$	52
default	$\frac{c\sqrt{c\,x^{4}+a}}{2}-\frac{a\sqrt{c\,x^{4}+a}}{4 x^{4}}-\frac{3 \sqrt{a}\,c \ln \left(\frac{2 a +2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4}$	58
risch	$\frac{c\sqrt{c\,x^{4}+a}}{2}-\frac{a\sqrt{c\,x^{4}+a}}{4 x^{4}}-\frac{3 \sqrt{a}\,c \ln \left(\frac{2 a +2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4}$	58
elliptic	$\frac{c\sqrt{c\,x^{4}+a}}{2}-\frac{a\sqrt{c\,x^{4}+a}}{4 x^{4}}-\frac{3 \sqrt{a}\,c \ln \left(\frac{2 a +2 \sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{4}$	58

input
```
int((c*x^4+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(3*arctanh((c*x^4+a)^(1/2)/a^(1/2))*a*c*x^4+(-2*c*x^4+a)*(c*x^4+a)^(1
/2)*a^(1/2))/a^(1/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.84

$$\int \frac{(a+cx^4)^{3/2}}{x^5}\,dx = \left[\frac{3\sqrt{a}cx^4\log\left(\frac{cx^4-2\sqrt{cx^4+a}\sqrt{a}+2a}{x^4}\right)+2\,(2\,cx^4-a)\sqrt{cx^4+a}}{8\,x^4}, \frac{3\sqrt{-a}cx^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^4+a}}\right) \ldots}{4 \ldots}\right.$$

input
```
integrate((c*x^4+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*c*x^4*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4) +
 2*(2*c*x^4 - a)*sqrt(c*x^4 + a))/x^4, 1/4*(3*sqrt(-a)*c*x^4*arctan(sqrt(-
a)/sqrt(c*x^4 + a)) + (2*c*x^4 - a)*sqrt(c*x^4 + a))/x^4]
```

Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.48

$$\int \frac{(a+cx^4)^{3/2}}{x^5}\,dx = -\frac{3\sqrt{a}c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{4} - \frac{a^2}{4\sqrt{c}x^6\sqrt{\frac{a}{cx^4}+1}} + \frac{a\sqrt{c}}{4x^2\sqrt{\frac{a}{cx^4}+1}} + \frac{c^{\frac{3}{2}}x^2}{2\sqrt{\frac{a}{cx^4}+1}}$$

input
```
integrate((c*x**4+a)**(3/2)/x**5,x)
```

output
```
-3*sqrt(a)*c*asinh(sqrt(a)/(sqrt(c)*x**2))/4 - a**2/(4*sqrt(c)*x**6*sqrt(a
/(c*x**4) + 1)) + a*sqrt(c)/(4*x**2*sqrt(a/(c*x**4) + 1)) + c**(3/2)*x**2/
(2*sqrt(a/(c*x**4) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{(a+cx^4)^{3/2}}{x^5}\,dx = \frac{3}{8}\sqrt{a}c\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right) + \frac{1}{2}\sqrt{cx^4+a}c - \frac{\sqrt{cx^4+a}a}{4x^4}$$

input
```
integrate((c*x^4+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
3/8*sqrt(a)*c*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))
 + 1/2*sqrt(c*x^4 + a)*c - 1/4*sqrt(c*x^4 + a)*a/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{(a+cx^4)^{3/2}}{x^5}\,dx = \frac{1}{4}\left(\frac{3\,a\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{cx^4+a} - \frac{\sqrt{cx^4+a}a}{cx^4}\right)c$$

input
```
integrate((c*x^4+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/4*(3*a*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(c*x^4 + a) - s
qrt(c*x^4 + a)*a/(c*x^4))*c
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{(a + cx^4)^{3/2}}{x^5}\, dx = \frac{c\,\sqrt{c\,x^4 + a}}{2} - \frac{a\,\sqrt{c\,x^4 + a}}{4\,x^4} - \frac{3\,\sqrt{a}\,c\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{4}$$

input
```
int((a + c*x^4)^(3/2)/x^5,x)
```

output
```
(c*(a + c*x^4)^(1/2))/2 - (a*(a + c*x^4)^(1/2))/(4*x^4) - (3*a^(1/2)*c*ata
nh((a + c*x^4)^(1/2)/a^(1/2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 441, normalized size of antiderivative = 6.89

$$\int \frac{(a + cx^4)^{3/2}}{x^5}\, dx = \frac{3\sqrt{a}\,\sqrt{c\,x^4 + a}\,\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right) ac\,x^4 + 12\sqrt{a}\,\sqrt{c\,x^4 + a}\,\log\left(\frac{\sqrt{c\,x^4+a}-\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{a}}\right) c}{}$$

input
```
int((c*x^4+a)^(3/2)/x^5,x)
```

output
```
(3*sqrt(a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2
)/sqrt(a))*a*c*x**4 + 12*sqrt(a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) -
sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c**2*x**8 - 3*sqrt(a)*sqrt(a + c*x**4)*lo
g((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*c*x**4 - 12*sqrt(
a)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a
))*c**2*x**8 - 3*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 + 2*sqrt(c)*sqrt(a + c
*x**4)*a*c*x**6 + 8*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 + 9*sqrt(c)*sqrt(a
)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*a*c*x**6 + 12*s
qrt(c)*sqrt(a)*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c*
*2*x**10 - 9*sqrt(c)*sqrt(a)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**
2)/sqrt(a))*a*c*x**6 - 12*sqrt(c)*sqrt(a)*log((sqrt(a + c*x**4) + sqrt(a)
+ sqrt(c)*x**2)/sqrt(a))*c**2*x**10 - a**3 - 3*a**2*c*x**4 + 6*a*c**2*x**8
 + 8*c**3*x**12)/(4*x**4*(sqrt(a + c*x**4)*a + 4*sqrt(a + c*x**4)*c*x**4 +
 3*sqrt(c)*a*x**2 + 4*sqrt(c)*c*x**6))
```

3.336 $\int \frac{(a+cx^4)^{3/2}}{x^9} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{(a+cx^4)^{3/2}}{x^9} \, dx = -\frac{a\sqrt{a+cx^4}}{8x^8} - \frac{5c\sqrt{a+cx^4}}{16x^4} - \frac{3c^2 \text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

output
```
-1/8*a*(c*x^4+a)^(1/2)/x^8-5/16*c*(c*x^4+a)^(1/2)/x^4-3/16*c^2*arctanh((c*
x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^4)^{3/2}}{x^9} \, dx = \frac{(-2a-5cx^4)\sqrt{a+cx^4}}{16x^8} - \frac{3c^2 \text{arctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^9,x]
```

output
```
((-2*a - 5*c*x^4)*Sqrt[a + c*x^4])/(16*x^8) - (3*c^2*ArcTanh[Sqrt[a + c*x^
4]/Sqrt[a]])/(16*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^9}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{(cx^4+a)^{3/2}}{x^{12}}dx^4$$

$$\Big\downarrow \; 51$$

$$\frac{1}{4}\left(\frac{3}{4}c\int \frac{\sqrt{cx^4+a}}{x^8}dx^4-\frac{(a+cx^4)^{3/2}}{2x^8}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{x^4\sqrt{cx^4+a}}dx^4-\frac{\sqrt{a+cx^4}}{x^4}\right)-\frac{(a+cx^4)^{3/2}}{2x^8}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(\int \frac{1}{\frac{x^8}{c}-\frac{a}{c}}d\sqrt{cx^4+a}-\frac{\sqrt{a+cx^4}}{x^4}\right)-\frac{(a+cx^4)^{3/2}}{2x^8}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}\left(\frac{3}{4}c\left(-\frac{\text{carctanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+cx^4}}{x^4}\right)-\frac{(a+cx^4)^{3/2}}{2x^8}\right)$$

input
```
Int[(a + c*x^4)^(3/2)/x^9,x]
```

output
```
(-1/2*(a + c*x^4)^(3/2)/x^8 + (3*c*(-(Sqrt[a + c*x^4]/x^4) - (c*ArcTanh[Sq
rt[a + c*x^4]/Sqrt[a]])/Sqrt[a]))/4)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(5c\,x^{4}+2a\right)}{16x^{8}}-\frac{3c^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}$	57
default	$-\frac{a\sqrt{c\,x^{4}+a}}{8x^{8}}-\frac{5c\sqrt{c\,x^{4}+a}}{16x^{4}}-\frac{3c^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}$	63
elliptic	$-\frac{a\sqrt{c\,x^{4}+a}}{8x^{8}}-\frac{5c\sqrt{c\,x^{4}+a}}{16x^{4}}-\frac{3c^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{4}+a}}{x^{2}}\right)}{16\sqrt{a}}$	63
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{\sqrt{a}}\right)c^{2}x^{8}-5c\,x^{4}\sqrt{c\,x^{4}+a}\,\sqrt{a}-2\sqrt{c\,x^{4}+a}\,a^{\frac{3}{2}}}{16x^{8}\sqrt{a}}$	64

input
```
int((c*x^4+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(c*x^4+a)^(1/2)*(5*c*x^4+2*a)/x^8-3/16*c^2/a^(1/2)*ln((2*a+2*a^(1/2)
*(c*x^4+a)^(1/2))/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.96

$$\int \frac{(a+cx^4)^{3/2}}{x^9}\,dx = \left[\frac{3\,\sqrt{a}c^2x^8\log\left(\frac{cx^4-2\,\sqrt{cx^4+a}\sqrt{a}+2\,a}{x^4}\right)-2\,(5\,acx^4+2\,a^2)\sqrt{cx^4+a}}{32\,ax^8}, \frac{3\,\sqrt{-a}c^2x^8\arctan\left(\right.}{}\right.$$

input
```
integrate((c*x^4+a)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/32*(3*sqrt(a)*c^2*x^8*log((c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(a) + 2*a)/x^4
) - 2*(5*a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))/(a*x^8), 1/16*(3*sqrt(-a)*c^2*x
^8*arctan(sqrt(-a)/sqrt(c*x^4 + a)) - (5*a*c*x^4 + 2*a^2)*sqrt(c*x^4 + a))
/(a*x^8)]
```

Sympy [A] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^4)^{3/2}}{x^9}\,dx = -\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{8x^6} - \frac{5c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{16x^2} - \frac{3c^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{16\sqrt{a}}$$

input
```
integrate((c*x**4+a)**(3/2)/x**9,x)
```

output
```
-a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(8*x**6) - 5*c**(3/2)*sqrt(a/(c*x**4) + 1)
/(16*x**2) - 3*c**2*asinh(sqrt(a)/(sqrt(c)*x**2))/(16*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.42

$$\int \frac{(a+cx^4)^{3/2}}{x^9}\,dx = \frac{3\,c^2\log\left(\frac{\sqrt{cx^4+a}-\sqrt{a}}{\sqrt{cx^4+a}+\sqrt{a}}\right)}{32\,\sqrt{a}} - \frac{5\,(cx^4+a)^{\frac{3}{2}}c^2 - 3\,\sqrt{cx^4+a}ac^2}{16\,\left((cx^4+a)^2 - 2\,(cx^4+a)a + a^2\right)}$$

input
```
integrate((c*x^4+a)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
3/32*c^2*log((sqrt(c*x^4 + a) - sqrt(a))/(sqrt(c*x^4 + a) + sqrt(a)))/sqrt
(a) - 1/16*(5*(c*x^4 + a)^(3/2)*c^2 - 3*sqrt(c*x^4 + a)*a*c^2)/((c*x^4 + a
)^2 - 2*(c*x^4 + a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(a+cx^4)^{3/2}}{x^9}\,dx = \frac{\frac{3\,c^3\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5\,(cx^4+a)^{\frac{3}{2}}c^3 - 3\,\sqrt{cx^4+a}ac^3}{c^2x^8}}{16\,c}$$

input
```
integrate((c*x^4+a)^(3/2)/x^9,x, algorithm="giac")
```

output
```
1/16*(3*c^3*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a) - (5*(c*x^4 + a)^(3/
2)*c^3 - 3*sqrt(c*x^4 + a)*a*c^3)/(c^2*x^8))/c
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{(a+cx^4)^{3/2}}{x^9}\,dx = \frac{3\,a\,\sqrt{c\,x^4+a}}{16\,x^8} - \frac{3\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c\,x^4+a}}{\sqrt{a}}\right)}{16\,\sqrt{a}} - \frac{5\,(c\,x^4+a)^{3/2}}{16\,x^8}$$

input
```
int((a + c*x^4)^(3/2)/x^9,x)
```

output
```
(3*a*(a + c*x^4)^(1/2))/(16*x^8) - (3*c^2*atanh((a + c*x^4)^(1/2)/a^(1/2))
)/(16*a^(1/2)) - (5*(a + c*x^4)^(3/2))/(16*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 554, normalized size of antiderivative = 8.03

$$\int \frac{(a + cx^4)^{3/2}}{x^9} \, dx = \frac{-2\sqrt{a}\,\sqrt{c\,x^4+a}\,a^3 - 21\sqrt{a}\,\sqrt{c\,x^4+a}\,a^2c\,x^4 - 56\sqrt{a}\,\sqrt{c\,x^4+a}\,a\,c^2x^8 - 40\sqrt{a}\,\sqrt{c\,x^4}}{}$$

input
```
int((c*x^4+a)^(3/2)/x^9,x)
```

output
```
( - 2*sqrt(a)*sqrt(a + c*x**4)*a**3 - 21*sqrt(a)*sqrt(a + c*x**4)*a**2*c*x
**4 - 56*sqrt(a)*sqrt(a + c*x**4)*a*c**2*x**8 - 40*sqrt(a)*sqrt(a + c*x**4
)*c**3*x**12 + 12*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) - sqrt(a)
 + sqrt(c)*x**2)/sqrt(a))*a*c**2*x**10 + 24*sqrt(c)*sqrt(a + c*x**4)*log((
sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sqrt(a))*c**3*x**14 - 12*sqrt(c
)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/sqrt(a)
)*a*c**2*x**10 - 24*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(
a) + sqrt(c)*x**2)/sqrt(a))*c**3*x**14 - 8*sqrt(c)*sqrt(a)*a**3*x**2 - 44*
sqrt(c)*sqrt(a)*a**2*c*x**6 - 76*sqrt(c)*sqrt(a)*a*c**2*x**10 - 40*sqrt(c)
*sqrt(a)*c**3*x**14 + 3*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)/sq
rt(a))*a**2*c**2*x**8 + 24*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2)
/sqrt(a))*a*c**3*x**12 + 24*log((sqrt(a + c*x**4) - sqrt(a) + sqrt(c)*x**2
)/sqrt(a))*c**4*x**16 - 3*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**2)/
sqrt(a))*a**2*c**2*x**8 - 24*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x**
2)/sqrt(a))*a*c**3*x**12 - 24*log((sqrt(a + c*x**4) + sqrt(a) + sqrt(c)*x*
*2)/sqrt(a))*c**4*x**16)/(16*sqrt(a)*x**8*(4*sqrt(c)*sqrt(a + c*x**4)*a*x*
*2 + 8*sqrt(c)*sqrt(a + c*x**4)*c*x**6 + a**2 + 8*a*c*x**4 + 8*c**2*x**8))
```

3.337 $\int x^5 (a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int x^5 \left(a+cx^4\right)^{3/2}\,dx = \frac{a^2x^2\sqrt{a+cx^4}}{32c}+\frac{1}{16}ax^6\sqrt{a+cx^4}+\frac{1}{12}x^6\left(a+cx^4\right)^{3/2}-\frac{a^3\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{32c^{3/2}}$$

output
```
1/32*a^2*x^2*(c*x^4+a)^(1/2)/c+1/16*a*x^6*(c*x^4+a)^(1/2)+1/12*x^6*(c*x^4+
a)^(3/2)-1/32*a^3*arctanh(c^(1/2)*x^2/(c*x^4+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x^5 \left(a+cx^4\right)^{3/2}\,dx = \frac{x^2\sqrt{a+cx^4}(3a^2+14acx^4+8c^2x^8)}{96c}-\frac{a^3\log\left(\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{32c^{3/2}}$$

input
```
Integrate[x^5*(a + c*x^4)^(3/2),x]
```

output
```
(x^2*Sqrt[a + c*x^4]*(3*a^2 + 14*a*c*x^4 + 8*c^2*x^8))/(96*c) - (a^3*Log[S
qrt[c]*x^2 + Sqrt[a + c*x^4]])/(32*c^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {807, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int x^4 \left(cx^4+a\right)^{3/2} dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} \left(\frac{1}{2} a \int x^4 \sqrt{cx^4+a} dx^2 + \frac{1}{6} x^6 \left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \int \frac{x^4}{\sqrt{cx^4+a}} dx^2 + \frac{1}{4} x^6 \sqrt{a+cx^4}\right) + \frac{1}{6} x^6 \left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x^2 \sqrt{a+cx^4}}{2c} - \frac{a \int \frac{1}{\sqrt{cx^4+a}} dx^2}{2c}\right) + \frac{1}{4} x^6 \sqrt{a+cx^4}\right) + \frac{1}{6} x^6 \left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x^2 \sqrt{a+cx^4}}{2c} - \frac{a \int \frac{1}{1-cx^4} d\frac{x^2}{\sqrt{cx^4+a}}}{2c}\right) + \frac{1}{4} x^6 \sqrt{a+cx^4}\right) + \frac{1}{6} x^6 \left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x^2 \sqrt{a+cx^4}}{2c} - \frac{a \text{arctanh}\left(\frac{\sqrt{c} x^2}{\sqrt{a+cx^4}}\right)}{2c^{3/2}}\right) + \frac{1}{4} x^6 \sqrt{a+cx^4}\right) + \frac{1}{6} x^6 \left(a+cx^4\right)^{3/2}\right)$$

input

```
Int[x^5*(a + c*x^4)^(3/2),x]
```

output
```
((x^6*(a + c*x^4)^(3/2))/6 + (a*((x^6*Sqrt[a + c*x^4])/4 + (a*((x^2*Sqrt[a
 + c*x^4])/(2*c) - (a*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*c^(3/2)))
)/4))/2)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x^{2}\left(8 c^{2} x^{8}+14 a \,x^{4} c +3 a^{2}\right) \sqrt{c \,x^{4}+a}}{96 c}-\frac{a^{3} \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{32 c^{\frac{3}{2}}}$	66
default	$\frac{x^{10} \sqrt{c \,x^{4}+a}\, c}{12}+\frac{7 a \,x^{6} \sqrt{c \,x^{4}+a}}{48}+\frac{a^{2} x^{2} \sqrt{c \,x^{4}+a}}{32 c}-\frac{a^{3} \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{32 c^{\frac{3}{2}}}$	78
elliptic	$\frac{x^{10} \sqrt{c \,x^{4}+a}\, c}{12}+\frac{7 a \,x^{6} \sqrt{c \,x^{4}+a}}{48}+\frac{a^{2} x^{2} \sqrt{c \,x^{4}+a}}{32 c}-\frac{a^{3} \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{32 c^{\frac{3}{2}}}$	78
pseudoelliptic	$\frac{8 \sqrt{c \,x^{4}+a}\, c^{\frac{5}{2}} x^{10}+14 a \,c^{\frac{3}{2}} x^{6} \sqrt{c \,x^{4}+a}+3 a^{2} x^{2} \sqrt{c \,x^{4}+a}\, \sqrt{c}-3 \,\operatorname{arctanh}\left(\frac{\sqrt{c \,x^{4}+a}}{x^{2} \sqrt{c}}\right) a^{3}}{96 c^{\frac{3}{2}}}$	84

input
```
int(x^5*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*x^2*(8*c^2*x^8+14*a*c*x^4+3*a^2)*(c*x^4+a)^(1/2)/c-1/32/c^(3/2)*a^3*l
n(c^(1/2)*x^2+(c*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.64

$$\int x^5(a+cx^4)^{3/2}\,dx = \left[\frac{3\,a^3\sqrt{c}\log\left(-2\,cx^4+2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,(8\,c^3x^{10}+14\,ac^2x^6+3\,a^2cx^2)\sqrt{cx^4+a}}{192\,c^2}, \frac{3}{-}\right.$$

input
```
integrate(x^5*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*a^3*sqrt(c)*log(-2*c*x^4 + 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) +
2*(8*c^3*x^10 + 14*a*c^2*x^6 + 3*a^2*c*x^2)*sqrt(c*x^4 + a))/c^2, 1/96*(3*
a^3*sqrt(-c)*arctan(sqrt(c*x^4 + a)*sqrt(-c)/(c*x^2)) + (8*c^3*x^10 + 14*a
*c^2*x^6 + 3*a^2*c*x^2)*sqrt(c*x^4 + a))/c^2]
```

Sympy [A] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.28

$$\int x^5 \left(a+cx^4\right)^{3/2} \, dx = \frac{a^{\frac{5}{2}}x^2}{32c\sqrt{1+\frac{cx^4}{a}}}+\frac{17a^{\frac{3}{2}}x^6}{96\sqrt{1+\frac{cx^4}{a}}} + \frac{11\sqrt{a}cx^{10}}{48\sqrt{1+\frac{cx^4}{a}}}-\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{32c^{\frac{3}{2}}}+\frac{c^2x^{14}}{12\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate(x**5*(c*x**4+a)**(3/2),x)
```

output
```
a**(5/2)*x**2/(32*c*sqrt(1 + c*x**4/a)) + 17*a**(3/2)*x**6/(96*sqrt(1 + c*
x**4/a)) + 11*sqrt(a)*c*x**10/(48*sqrt(1 + c*x**4/a)) - a**3*asinh(sqrt(c)
*x**2/sqrt(a))/(32*c**(3/2)) + c**2*x**14/(12*sqrt(a)*sqrt(1 + c*x**4/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(75) = 150.

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

$$\int x^5 \left(a+cx^4\right)^{3/2} \, dx = \frac{a^3\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{64\,c^{\frac{3}{2}}} + \frac{\frac{3\sqrt{cx^4+a}a^3c^2}{x^2}-\frac{8\,(cx^4+a)^{\frac{3}{2}}a^3c}{x^6}-\frac{3\,(cx^4+a)^{\frac{5}{2}}a^3}{x^{10}}}{96\left(c^4-\frac{3\,(cx^4+a)c^3}{x^4}+\frac{3\,(cx^4+a)^2c^2}{x^8}-\frac{(cx^4+a)^3c}{x^{12}}\right)}$$

input
```
integrate(x^5*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/64*a^3*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/x
^2))/c^(3/2) + 1/96*(3*sqrt(c*x^4 + a)*a^3*c^2/x^2 - 8*(c*x^4 + a)^(3/2)*a
^3*c/x^6 - 3*(c*x^4 + a)^(5/2)*a^3/x^10)/(c^4 - 3*(c*x^4 + a)*c^3/x^4 + 3*
(c*x^4 + a)^2*c^2/x^8 - (c*x^4 + a)^3*c/x^12)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int x^5(a+cx^4)^{3/2}\,dx = \frac{1}{16}\left(\sqrt{cx^4+a}\left(2x^4+\frac{a}{c}\right)x^2+\frac{a^2\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{c^{\frac{3}{2}}}\right)a + \frac{1}{96}\left(\left(2\left(4x^4+\frac{a}{c}\right)x^4-\frac{3a^2}{c^2}\right)\sqrt{cx^4+a}x^2-\frac{3a^3\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{c^{\frac{5}{2}}}\right)c$$

input
```
integrate(x^5*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/16*(sqrt(c*x^4 + a)*(2*x^4 + a/c)*x^2 + a^2*log(abs(-sqrt(c)*x^2 + sqrt(
c*x^4 + a)))/c^(3/2))*a + 1/96*((2*(4*x^4 + a/c)*x^4 - 3*a^2/c^2)*sqrt(c*x
^4 + a)*x^2 - 3*a^3*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/c^(5/2))*c
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a+cx^4)^{3/2}\,dx = \int x^5\,\left(c\,x^4+a\right)^{3/2}\,dx$$

input
```
int(x^5*(a + c*x^4)^(3/2),x)
```

output
```
int(x^5*(a + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.40

$$\int x^5(a+cx^4)^{3/2}\,dx = \frac{-18\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^5x^2-96\sqrt{c}\,\sqrt{c\,x^4+a}\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^4c\,x^6-96\sqrt{}}{}$$

input
```
int(x^5*(c*x^4+a)^(3/2),x)
```

output
```
( - 18*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt
(a))*a**5*x**2 - 96*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(
c)*x**2)/sqrt(a))*a**4*c*x**6 - 96*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a +
c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**3*c**2*x**10 + 3*sqrt(c)*sqrt(a + c*x*
*4)*a**5*x**2 + 68*sqrt(c)*sqrt(a + c*x**4)*a**4*c*x**6 + 404*sqrt(c)*sqrt
(a + c*x**4)*a**3*c**2*x**10 + 912*sqrt(c)*sqrt(a + c*x**4)*a**2*c**3*x**1
4 + 832*sqrt(c)*sqrt(a + c*x**4)*a*c**4*x**18 + 256*sqrt(c)*sqrt(a + c*x**
4)*c**5*x**22 - 3*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**6 - 54
*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**5*c*x**4 - 144*log((sqr
t(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**4*c**2*x**8 - 96*log((sqrt(a + c
*x**4) + sqrt(c)*x**2)/sqrt(a))*a**3*c**3*x**12 + 18*a**5*c*x**4 + 198*a**
4*c**2*x**8 + 772*a**3*c**3*x**12 + 1296*a**2*c**4*x**16 + 960*a*c**5*x**2
0 + 256*c**6*x**24)/(96*c*(6*sqrt(a + c*x**4)*a**2*c*x**2 + 32*sqrt(a + c*
x**4)*a*c**2*x**6 + 32*sqrt(a + c*x**4)*c**3*x**10 + sqrt(c)*a**3 + 18*sqr
t(c)*a**2*c*x**4 + 48*sqrt(c)*a*c**2*x**8 + 32*sqrt(c)*c**3*x**12))
```

3.338 $\int x(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int x\left(a+cx^4\right)^{3/2}\,dx=\frac{3}{16}ax^2\sqrt{a+cx^4}+\frac{1}{8}x^2\left(a+cx^4\right)^{3/2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{16\sqrt{c}}$$

output
```
3/16*a*x^2*(c*x^4+a)^(1/2)+1/8*x^2*(c*x^4+a)^(3/2)+3/16*a^2*arctanh(c^(1/2
)*x^2/(c*x^4+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int x\left(a+cx^4\right)^{3/2}\,dx=\frac{1}{16}x^2\sqrt{a+cx^4}\left(5a+2cx^4\right)+\frac{3a^2\log\left(\sqrt{c}x^2+\sqrt{a+cx^4}\right)}{16\sqrt{c}}$$

input
```
Integrate[x*(a + c*x^4)^(3/2),x]
```

output
```
(x^2*Sqrt[a + c*x^4]*(5*a + 2*c*x^4))/16 + (3*a^2*Log[Sqrt[c]*x^2 + Sqrt[a
 + c*x^4]])/(16*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {807, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+cx^4\right)^{3/2}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \left(cx^4+a\right)^{3/2}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{4}a\int \sqrt{cx^4+a}dx^2+\frac{1}{4}x^2\left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^4+a}}dx^2+\frac{1}{2}x^2\sqrt{a+cx^4}\right)+\frac{1}{4}x^2\left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}+\frac{1}{2}x^2\sqrt{a+cx^4}\right)+\frac{1}{4}x^2\left(a+cx^4\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{1}{2}x^2\sqrt{a+cx^4}\right)+\frac{1}{4}x^2\left(a+cx^4\right)^{3/2}\right)$$

input
```
Int[x*(a + c*x^4)^(3/2),x]
```

output
```
((x^2*(a + c*x^4)^(3/2))/4 + (3*a*((x^2*Sqrt[a + c*x^4])/2 + (a*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c])))/4)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{x^{2} \left(2 c \,x^{4}+5 a \right) \sqrt{c \,x^{4}+a}}{16}+\frac{3 a^{2} \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{16 \sqrt{c}}$	52
default	$\frac{3 a^{2} \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{16 \sqrt{c}}+\frac{c \,x^{6} \sqrt{c \,x^{4}+a}}{8}+\frac{5 a \,x^{2} \sqrt{c \,x^{4}+a}}{16}$	58
elliptic	$\frac{3 a^{2} \ln \left(\sqrt{c}\, x^{2}+\sqrt{c \,x^{4}+a}\right)}{16 \sqrt{c}}+\frac{c \,x^{6} \sqrt{c \,x^{4}+a}}{8}+\frac{5 a \,x^{2} \sqrt{c \,x^{4}+a}}{16}$	58
pseudoelliptic	$\frac{2 \sqrt{c \,x^{4}+a}\, c^{\frac{3}{2}} x^{6}+5 a \,x^{2} \sqrt{c \,x^{4}+a}\, \sqrt{c}+3 \,\operatorname{arctanh}\left(\frac{\sqrt{c \,x^{4}+a}}{x^{2} \sqrt{c}}\right) a^{2}}{16 \sqrt{c}}$	64

input
```
int(x*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*x^2*(2*c*x^4+5*a)*(c*x^4+a)^(1/2)+3/16*a^2*ln(c^(1/2)*x^2+(c*x^4+a)^(
1/2))/c^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.90

$$\int x(a+cx^4)^{3/2}\,dx = \left[\frac{3\,a^2\sqrt{c}\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,(2\,c^2x^6+5\,acx^2)\sqrt{cx^4+a}}{32\,c}, -\frac{3\,a^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-c}}{cx^2}\right)-(2\,c^2x^6+5\,acx^2)\sqrt{cx^4+a}}{16\,c}\right]$$

input
```
integrate(x*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*a^2*sqrt(c)*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2
*(2*c^2*x^6 + 5*a*c*x^2)*sqrt(c*x^4 + a))/c, -1/16*(3*a^2*sqrt(-c)*arctan(
sqrt(c*x^4 + a)*sqrt(-c)/(c*x^2)) - (2*c^2*x^6 + 5*a*c*x^2)*sqrt(c*x^4 + a
))/c]
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int x(a+cx^4)^{3/2}\,dx = \frac{5a^{\frac{3}{2}}x^2\sqrt{1+\frac{cx^4}{a}}}{16}+\frac{\sqrt{a}cx^6\sqrt{1+\frac{cx^4}{a}}}{8}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{16\sqrt{c}}$$

input
```
integrate(x*(c*x**4+a)**(3/2),x)
```

output
```
5*a**(3/2)*x**2*sqrt(1 + c*x**4/a)/16 + sqrt(a)*c*x**6*sqrt(1 + c*x**4/a)/
8 + 3*a**2*asinh(sqrt(c)*x**2/sqrt(a))/(16*sqrt(c))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int x(a+cx^4)^{3/2}\,dx = -\frac{3\,a^2\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right)}{32\,\sqrt{c}} - \frac{\frac{3\,\sqrt{cx^4+a}a^2c}{x^2} - \frac{5\,(cx^4+a)^{\frac{3}{2}}a^2}{x^6}}{16\left(c^2 - \frac{2\,(cx^4+a)c}{x^4} + \frac{(cx^4+a)^2}{x^8}\right)}$$

input
```
integrate(x*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-3/32*a^2*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 + a)/
x^2))/sqrt(c) - 1/16*(3*sqrt(c*x^4 + a)*a^2*c/x^2 - 5*(c*x^4 + a)^(3/2)*a^
2/x^6)/(c^2 - 2*(c*x^4 + a)*c/x^4 + (c*x^4 + a)^2/x^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\int x(a+cx^4)^{3/2}\,dx = \frac{1}{4}\left(\sqrt{cx^4+a}x^2 - \frac{a\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{\sqrt{c}}\right)a + \frac{1}{16}\left(\sqrt{cx^4+a}\left(2\,x^4+\frac{a}{c}\right)x^2 + \frac{a^2\log\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{c^{\frac{3}{2}}}\right)c$$

input
```
integrate(x*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(c*x^4 + a)*x^2 - a*log(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt
(c))*a + 1/16*(sqrt(c*x^4 + a)*(2*x^4 + a/c)*x^2 + a^2*log(abs(-sqrt(c)*x^
2 + sqrt(c*x^4 + a)))/c^(3/2))*c
```

Mupad [F(-1)]

Timed out.

$$\int x(a+cx^4)^{3/2}\,dx=\int x\,(c\,x^4+a)^{3/2}\,dx$$

input
```
int(x*(a + c*x^4)^(3/2),x)
```

output
```
int(x*(a + c*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.79

$$\int x(a+cx^4)^{3/2}\,dx=\frac{12\sqrt{c}\,\sqrt{c\,x^4+a}\,\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^3x^2+24\sqrt{c}\,\sqrt{c\,x^4+a}\,\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^2c\,x^6+5\sqrt{c}$$

input
```
int(x*(c*x^4+a)^(3/2),x)
```

output
```
(12*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a)
)*a**3*x**2 + 24*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*
x**2)/sqrt(a))*a**2*c*x**6 + 5*sqrt(c)*sqrt(a + c*x**4)*a**3*x**2 + 42*sqr
t(c)*sqrt(a + c*x**4)*a**2*c*x**6 + 56*sqrt(c)*sqrt(a + c*x**4)*a*c**2*x**
10 + 16*sqrt(c)*sqrt(a + c*x**4)*c**3*x**14 + 3*log((sqrt(a + c*x**4) + sq
rt(c)*x**2)/sqrt(a))*a**4 + 24*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(
a))*a**3*c*x**4 + 24*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**2*c
**2*x**8 + 20*a**3*c*x**4 + 68*a**2*c**2*x**8 + 64*a*c**3*x**12 + 16*c**4*
x**16)/(16*(4*sqrt(a + c*x**4)*a*c*x**2 + 8*sqrt(a + c*x**4)*c**2*x**6 + s
qrt(c)*a**2 + 8*sqrt(c)*a*c*x**4 + 8*sqrt(c)*c**2*x**8))
```

3.339 $\int \frac{(a+cx^4)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{(a+cx^4)^{3/2}}{x^3} \, dx = -\frac{a\sqrt{a+cx^4}}{2x^2} + \frac{1}{4}cx^2\sqrt{a+cx^4} + \frac{3}{4}a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)$$

output
```
-1/2*a*(c*x^4+a)^(1/2)/x^2+1/4*c*x^2*(c*x^4+a)^(1/2)+3/4*a*c^(1/2)*arctanh
(c^(1/2)*x^2/(c*x^4+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{(a+cx^4)^{3/2}}{x^3} \, dx = \frac{(-2a+cx^4)\sqrt{a+cx^4}}{4x^2} + \frac{3}{4}a\sqrt{c}\log\left(\sqrt{c}x^2 + \sqrt{a+cx^4}\right)$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^3,x]
```

output
```
((-2*a + c*x^4)*Sqrt[a + c*x^4])/(4*x^2) + (3*a*Sqrt[c]*Log[Sqrt[c]*x^2 +
Sqrt[a + c*x^4]])/4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{(cx^4+a)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(3c\int \sqrt{cx^4+a}dx^2-\frac{(a+cx^4)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(3c\left(\frac{1}{2}a\int \frac{1}{\sqrt{cx^4+a}}dx^2+\frac{1}{2}x^2\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(3c\left(\frac{1}{2}a\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}+\frac{1}{2}x^2\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(3c\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{1}{2}x^2\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{x^2}\right)$$

input
```
Int[(a + c*x^4)^(3/2)/x^3,x]
```

output
```
(-((a + c*x^4)^(3/2)/x^2) + 3*c*((x^2*Sqrt[a + c*x^4])/2 + (a*ArcTanh[(Sqr
t[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c])))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(-c\,x^{4}+2a\right)}{4x^{2}}+\frac{3a\sqrt{c}\,\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{4}$	50
default	$-\frac{a\sqrt{c\,x^{4}+a}}{2x^{2}}+\frac{c\,x^{2}\sqrt{c\,x^{4}+a}}{4}+\frac{3a\sqrt{c}\,\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{4}$	56
elliptic	$-\frac{a\sqrt{c\,x^{4}+a}}{2x^{2}}+\frac{c\,x^{2}\sqrt{c\,x^{4}+a}}{4}+\frac{3a\sqrt{c}\,\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{4}$	56
pseudoelliptic	$\frac{\sqrt{c\,x^{4}+a}\,c\,x^{4}+3\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{x^{2}\sqrt{c}}\right)a\,x^{2}-2a\sqrt{c\,x^{4}+a}}{4x^{2}}$	59

input
```
int((c*x^4+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^4+a)^(1/2)*(-c*x^4+2*a)/x^2+3/4*a*c^(1/2)*ln(c^(1/2)*x^2+(c*x^4+
a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.76

$$\int \frac{(a+cx^4)^{3/2}}{x^3}\,dx = \left[\frac{3\,a\sqrt{c}x^2\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,\sqrt{cx^4+a}(cx^4-2\,a)}{8\,x^2},\right. \\ \left. -\frac{3\,a\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-c}}{cx^2}\right)-\sqrt{cx^4+a}(cx^4-2\,a)}{4\,x^2}\right]$$

input
```
integrate((c*x^4+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*a*sqrt(c)*x^2*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) +
2*sqrt(c*x^4 + a)*(c*x^4 - 2*a))/x^2, -1/4*(3*a*sqrt(-c)*x^2*arctan(sqrt(c
*x^4 + a)*sqrt(-c)/(c*x^2)) - sqrt(c*x^4 + a)*(c*x^4 - 2*a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{(a+cx^4)^{3/2}}{x^3}\,dx = -\frac{a^{\frac{3}{2}}}{2x^2\sqrt{1+\frac{cx^4}{a}}}-\frac{\sqrt{a}cx^2}{4\sqrt{1+\frac{cx^4}{a}}}+\frac{3a\sqrt{c}\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4}+\frac{c^2x^6}{4\sqrt{a}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((c*x**4+a)**(3/2)/x**3,x)
```

output
```
-a**(3/2)/(2*x**2*sqrt(1 + c*x**4/a)) - sqrt(a)*c*x**2/(4*sqrt(1 + c*x**4/
a)) + 3*a*sqrt(c)*asinh(sqrt(c)*x**2/sqrt(a))/4 + c**2*x**6/(4*sqrt(a)*sqr
t(1 + c*x**4/a))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.34

$$\int \frac{(a+cx^4)^{3/2}}{x^3}\,dx = -\frac{3}{8}\,a\sqrt{c}\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right) - \frac{\sqrt{cx^4+a}a}{2\,x^2} - \frac{\sqrt{cx^4+a}ac}{4\left(c-\frac{cx^4+a}{x^4}\right)x^2}$$

input
```
integrate((c*x^4+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-3/8*a*sqrt(c)*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4
+ a)/x^2)) - 1/2*sqrt(c*x^4 + a)*a/x^2 - 1/4*sqrt(c*x^4 + a)*a*c/((c - (c*
x^4 + a)/x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^4)^{3/2}}{x^3}\,dx = \frac{1}{4}\sqrt{cx^4+a}cx^2 - \frac{3}{8}\,a\sqrt{c}\log\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2\right) + \frac{a^2\sqrt{c}}{\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a}$$

input
```
integrate((c*x^4+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^4 + a)*c*x^2 - 3/8*a*sqrt(c)*log((sqrt(c)*x^2 - sqrt(c*x^4 +
a))^2) + a^2*sqrt(c)/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{x^3}\,dx=\int \frac{(c\,x^4+a)^{3/2}}{x^3}\,dx$$

input
```
int((a + c*x^4)^(3/2)/x^3,x)
```

output
```
int((a + c*x^4)^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.86

$$\int \frac{(a+cx^4)^{3/2}}{x^3}\,dx=\frac{12\sqrt{c}\,\sqrt{c\,x^4+a}\,\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a^2x^2+48\sqrt{c}\,\sqrt{c\,x^4+a}\,\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)ac\,x^6-}{}$$

input
```
int((c*x^4+a)^(3/2)/x^3,x)
```

output
```
(12*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a)
)*a**2*x**2 + 48*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*
x**2)/sqrt(a))*a*c*x**6 - 33*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 - 56*sqrt(
c)*sqrt(a + c*x**4)*a*c*x**6 + 16*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 + 36
*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a**2*c*x**4 + 48*log((sqrt
(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a*c**2*x**8 - 8*a**3 - 63*a**2*c*x**
4 - 48*a*c**2*x**8 + 16*c**3*x**12)/(16*x**2*(sqrt(a + c*x**4)*a + 4*sqrt(
a + c*x**4)*c*x**4 + 3*sqrt(c)*a*x**2 + 4*sqrt(c)*c*x**6))
```

3.340 $\int \frac{(a+cx^4)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{(a+cx^4)^{3/2}}{x^7} dx = -\frac{a\sqrt{a+cx^4}}{6x^6} - \frac{2c\sqrt{a+cx^4}}{3x^2} + \frac{1}{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)$$

output
```
-1/6*a*(c*x^4+a)^(1/2)/x^6-2/3*c*(c*x^4+a)^(1/2)/x^2+1/2*c^(3/2)*arctanh(c
^(1/2)*x^2/(c*x^4+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{(a+cx^4)^{3/2}}{x^7} dx = \frac{(-a-4cx^4)\sqrt{a+cx^4}}{6x^6} + \frac{1}{2}c^{3/2}\log\left(\sqrt{c}x^2+\sqrt{a+cx^4}\right)$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^7,x]
```

output
```
((-a - 4*c*x^4)*Sqrt[a + c*x^4])/(6*x^6) + (c^(3/2)*Log[Sqrt[c]*x^2 + Sqrt
[a + c*x^4]])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {807, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{(cx^4+a)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(c\int \frac{\sqrt{cx^4+a}}{x^4}dx^2-\frac{(a+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(c\left(c\int \frac{1}{\sqrt{cx^4+a}}dx^2-\frac{\sqrt{a+cx^4}}{x^2}\right)-\frac{(a+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(c\left(c\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}-\frac{\sqrt{a+cx^4}}{x^2}\right)-\frac{(a+cx^4)^{3/2}}{3x^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(c\left(\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)-\frac{\sqrt{a+cx^4}}{x^2}\right)-\frac{(a+cx^4)^{3/2}}{3x^6}\right)$$

input
```
Int[(a + c*x^4)^(3/2)/x^7,x]
```

output
```
(-1/3*(a + c*x^4)^(3/2)/x^6 + c*(-(Sqrt[a + c*x^4]/x^2) + Sqrt[c]*ArcTanh[
(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(4c\,x^{4}+a\right)}{6x^{6}}+\frac{c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2}$	47
pseudoelliptic	$\frac{3c^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{c\,x^{4}+a}}{x^{2}\sqrt{c}}\right)x^{6}-\sqrt{c\,x^{4}+a}\,\left(4c\,x^{4}+a\right)}{6x^{6}}$	51
default	$\frac{c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2}-\frac{a\sqrt{c\,x^{4}+a}}{6x^{6}}-\frac{2c\sqrt{c\,x^{4}+a}}{3x^{2}}$	55
elliptic	$\frac{c^{\frac{3}{2}}\ln\left(\sqrt{c}\,x^{2}+\sqrt{c\,x^{4}+a}\right)}{2}-\frac{a\sqrt{c\,x^{4}+a}}{6x^{6}}-\frac{2c\sqrt{c\,x^{4}+a}}{3x^{2}}$	55

input
```
int((c*x^4+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*x^4+a)^(1/2)*(4*c*x^4+a)/x^6+1/2*c^(3/2)*ln(c^(1/2)*x^2+(c*x^4+a)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int \frac{(a+cx^4)^{3/2}}{x^7}\,dx = \left[\frac{3\,c^{\frac{3}{2}}x^6\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)-2\,(4\,cx^4+a)\sqrt{cx^4+a}}{12\,x^6},\right.$$
$$\left.-\frac{3\,\sqrt{-c}cx^6\arctan\left(\frac{\sqrt{cx^4+a}\sqrt{-c}}{cx^2}\right)+(4\,cx^4+a)\sqrt{cx^4+a}}{6\,x^6}\right]$$

input
```
integrate((c*x^4+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/12*(3*c^(3/2)*x^6*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) - 2
*(4*c*x^4 + a)*sqrt(c*x^4 + a))/x^6, -1/6*(3*sqrt(-c)*c*x^6*arctan(sqrt(c*
x^4 + a)*sqrt(-c)/(c*x^2)) + (4*c*x^4 + a)*sqrt(c*x^4 + a))/x^6]
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{(a+cx^4)^{3/2}}{x^7}\,dx = -\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{6x^4}-\frac{2c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{3}$$
$$-\frac{c^{\frac{3}{2}}\log\left(\frac{a}{cx^4}\right)}{4}+\frac{c^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{cx^4}+1}+1\right)}{2}$$

input
```
integrate((c*x**4+a)**(3/2)/x**7,x)
```

output
```
-a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(6*x**4) - 2*c**(3/2)*sqrt(a/(c*x**4) + 1)
/3 - c**(3/2)*log(a/(c*x**4))/4 + c**(3/2)*log(sqrt(a/(c*x**4) + 1) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{(a+cx^4)^{3/2}}{x^7}\,dx = -\frac{1}{4}\,c^{\frac{3}{2}}\log\left(-\frac{\sqrt{c}-\frac{\sqrt{cx^4+a}}{x^2}}{\sqrt{c}+\frac{\sqrt{cx^4+a}}{x^2}}\right) - \frac{\sqrt{cx^4+a}c}{2\,x^2} - \frac{(cx^4+a)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate((c*x^4+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/4*c^(3/2)*log(-(sqrt(c) - sqrt(c*x^4 + a)/x^2)/(sqrt(c) + sqrt(c*x^4 +
a)/x^2)) - 1/2*sqrt(c*x^4 + a)*c/x^2 - 1/6*(c*x^4 + a)^(3/2)/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.77

$$\int \frac{(a+cx^4)^{3/2}}{x^7}\,dx = -\frac{1}{4}\,c^{\frac{3}{2}}\log\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2\right) + \frac{2\left(3\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^4ac^{\frac{3}{2}} - 3\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2a^2c^{\frac{3}{2}} + 2\,a^3c^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)^3}$$

input
```
integrate((c*x^4+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/4*c^(3/2)*log((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2) + 2/3*(3*(sqrt(c)*x^2
- sqrt(c*x^4 + a))^4*a*c^(3/2) - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^2*a^2*c
^(3/2) + 2*a^3*c^(3/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{x^7}\,dx = \int \frac{(c\,x^4+a)^{3/2}}{x^7}\,dx$$

input
```
int((a + c*x^4)^(3/2)/x^7,x)
```

output
```
int((a + c*x^4)^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.90

$$\int \frac{(a+cx^4)^{3/2}}{x^7}\,dx = \frac{3\sqrt{c}\,\sqrt{c\,x^4+a}\,\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)a c\,x^6+12\sqrt{c}\,\sqrt{c\,x^4+a}\,\log\left(\frac{\sqrt{c\,x^4+a}+\sqrt{c}\,x^2}{\sqrt{a}}\right)c^2x^{10}-3}{}$$

input
```
int((c*x^4+a)^(3/2)/x^7,x)
```

output
```
(3*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))
*a*c*x**6 + 12*sqrt(c)*sqrt(a + c*x**4)*log((sqrt(a + c*x**4) + sqrt(c)*x*
*2)/sqrt(a))*c**2*x**10 - 3*sqrt(c)*sqrt(a + c*x**4)*a**2*x**2 - 16*sqrt(c
)*sqrt(a + c*x**4)*a*c*x**6 - 16*sqrt(c)*sqrt(a + c*x**4)*c**2*x**10 + 9*l
og((sqrt(a + c*x**4) + sqrt(c)*x**2)/sqrt(a))*a*c**2*x**8 + 12*log((sqrt(a
 + c*x**4) + sqrt(c)*x**2)/sqrt(a))*c**3*x**12 - a**3 - 9*a**2*c*x**4 - 24
*a*c**2*x**8 - 16*c**3*x**12)/(6*x**6*(sqrt(a + c*x**4)*a + 4*sqrt(a + c*x
**4)*c*x**4 + 3*sqrt(c)*a*x**2 + 4*sqrt(c)*c*x**6))
```

3.341 $\int \frac{(a+cx^4)^{3/2}}{x^{11}} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}} dx = -\frac{(a+cx^4)^{5/2}}{10ax^{10}}$$

output
```
-1/10*(c*x^4+a)^(5/2)/a/x^10
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}} dx = -\frac{(a+cx^4)^{5/2}}{10ax^{10}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^11,x]
```

output
```
-1/10*(a + c*x^4)^(5/2)/(a*x^10)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+cx^4)^{5/2}}{10ax^{10}}$$

input
```
Int[(a + c*x^4)^(3/2)/x^11,x]
```

output
```
-1/10*(a + c*x^4)^(5/2)/(a*x^10)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}}}{10a\,x^{10}}$	18
pseudoelliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}}}{10a\,x^{10}}$	18
orering	$-\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}}}{10a\,x^{10}}$	18
default	$-\frac{\sqrt{c\,x^{4}+a}\,\left(c^{2}x^{8}+2a\,x^{4}c+a^{2}\right)}{10x^{10}a}$	36
trager	$-\frac{\sqrt{c\,x^{4}+a}\,\left(c^{2}x^{8}+2a\,x^{4}c+a^{2}\right)}{10x^{10}a}$	36
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(c^{2}x^{8}+2a\,x^{4}c+a^{2}\right)}{10x^{10}a}$	36
elliptic	$-\frac{\sqrt{c\,x^{4}+a}\,\left(c^{2}x^{8}+2a\,x^{4}c+a^{2}\right)}{10x^{10}a}$	36

input
```
int((c*x^4+a)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(c*x^4+a)^(5/2)/a/x^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{(c^2x^8+2\,acx^4+a^2)\sqrt{cx^4+a}}{10\,ax^{10}}$$

input
```
integrate((c*x^4+a)^(3/2)/x^11,x, algorithm="fricas")
```

output
```
-1/10*(c^2*x^8 + 2*a*c*x^4 + a^2)*sqrt(c*x^4 + a)/(a*x^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.

Time = 0.62 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{10x^8} - \frac{c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{5x^4} - \frac{c^{\frac{5}{2}}\sqrt{\frac{a}{cx^4}+1}}{10a}$$

input
```
integrate((c*x**4+a)**(3/2)/x**11,x)
```

output
```
-a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(10*x**8) - c**(3/2)*sqrt(a/(c*x**4) + 1)/
(5*x**4) - c**(5/2)*sqrt(a/(c*x**4) + 1)/(10*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{(cx^4+a)^{\frac{5}{2}}}{10\,ax^{10}}$$

input
```
integrate((c*x^4+a)^(3/2)/x^11,x, algorithm="maxima")
```

output
```
-1/10*(c*x^4 + a)^(5/2)/(a*x^10)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(17) = 34$.

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.38

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx = \frac{5\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^8 c^{\frac{5}{2}} + 10\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^4 a^2 c^{\frac{5}{2}} + a^4 c^{\frac{5}{2}}}{5\left(\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^2-a\right)^5}$$

input
```
integrate((c*x^4+a)^(3/2)/x^11,x, algorithm="giac")
```

output
```
1/5*(5*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^8*c^(5/2) + 10*(sqrt(c)*x^2 - sqrt(
c*x^4 + a))^4*a^2*c^(5/2) + a^4*c^(5/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^
2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx = -\frac{(c\,x^4+a)^{5/2}}{10\,a\,x^{10}}$$

input
```
int((a + c*x^4)^(3/2)/x^11,x)
```

output
```
-(a + c*x^4)^(5/2)/(10*a*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 11.05

$$\int \frac{(a+cx^4)^{3/2}}{x^{11}}\,dx = \frac{-5\sqrt{c}\,\sqrt{c\,x^4+a}\,a^4x^2-30\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3c\,x^6-62\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2c^2x^{10}-64\sqrt{c}\,\sqrt{c}}{10a\,x^{10}\left(\sqrt{c\,x^4+a}\,a^2+12\sqrt{c\,x^4+a}\,ac\,x^4+\right.}$$

input
```
int((c*x^4+a)^(3/2)/x^11,x)
```

output
```
( - 5*sqrt(c)*sqrt(a + c*x**4)*a**4*x**2 - 30*sqrt(c)*sqrt(a + c*x**4)*a**
3*c*x**6 - 62*sqrt(c)*sqrt(a + c*x**4)*a**2*c**2*x**10 - 64*sqrt(c)*sqrt(a
 + c*x**4)*a*c**3*x**14 - 32*sqrt(c)*sqrt(a + c*x**4)*c**4*x**18 - a**5 -
15*a**4*c*x**4 - 55*a**3*c**2*x**8 - 90*a**2*c**3*x**12 - 80*a*c**4*x**16
- 32*c**5*x**20)/(10*a*x**10*(sqrt(a + c*x**4)*a**2 + 12*sqrt(a + c*x**4)*
a*c*x**4 + 16*sqrt(a + c*x**4)*c**2*x**8 + 5*sqrt(c)*a**2*x**2 + 20*sqrt(c
)*a*c*x**6 + 16*sqrt(c)*c**2*x**10))
```

3.342 $\int \frac{(a+cx^4)^{3/2}}{x^{15}} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}} dx = -\frac{(a+cx^4)^{5/2}}{14ax^{14}} + \frac{c(a+cx^4)^{5/2}}{35a^2x^{10}}$$

output
```
-1/14*(c*x^4+a)^(5/2)/a/x^14+1/35*c*(c*x^4+a)^(5/2)/a^2/x^10
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}} dx = \frac{(a+cx^4)^{5/2}(-5a+2cx^4)}{70a^2x^{14}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^15,x]
```

output
```
((a + c*x^4)^(5/2)*(-5*a + 2*c*x^4))/(70*a^2*x^14)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{2c\int \frac{(cx^4+a)^{3/2}}{x^{11}}dx}{7a}-\frac{(a+cx^4)^{5/2}}{14ax^{14}}$$

$$\Big\downarrow \text{796}$$

$$\frac{c(a+cx^4)^{5/2}}{35a^2x^{10}}-\frac{(a+cx^4)^{5/2}}{14ax^{14}}$$

input
```
Int[(a + c*x^4)^(3/2)/x^15,x]
```

output
```
-1/14*(a + c*x^4)^(5/2)/(a*x^14) + (c*(a + c*x^4)^(5/2))/(35*a^2*x^10)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(c\,x^{4}+a)^{\frac{5}{2}}(-2c\,x^{4}+5a)}{70x^{14}a^{2}}$	28
pseudoelliptic	$-\frac{(c\,x^{4}+a)^{\frac{5}{2}}(-2c\,x^{4}+5a)}{70x^{14}a^{2}}$	28
orering	$-\frac{(c\,x^{4}+a)^{\frac{5}{2}}(-2c\,x^{4}+5a)}{70x^{14}a^{2}}$	28
default	$-\frac{\sqrt{c\,x^{4}+a}\,(-2c\,x^{4}+5a)(c^{2}x^{8}+2a\,x^{4}c+a^{2})}{70a^{2}x^{14}}$	46
elliptic	$-\frac{\sqrt{c\,x^{4}+a}\,(-2c\,x^{4}+5a)(c^{2}x^{8}+2a\,x^{4}c+a^{2})}{70a^{2}x^{14}}$	46
trager	$-\frac{(-2c^{3}x^{12}+a\,c^{2}x^{8}+8a^{2}c\,x^{4}+5a^{3})\sqrt{c\,x^{4}+a}}{70x^{14}a^{2}}$	49
risch	$-\frac{(-2c^{3}x^{12}+a\,c^{2}x^{8}+8a^{2}c\,x^{4}+5a^{3})\sqrt{c\,x^{4}+a}}{70x^{14}a^{2}}$	49

input
```
int((c*x^4+a)^(3/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/70*(c*x^4+a)^(5/2)*(-2*c*x^4+5*a)/x^14/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx = \frac{(2\,c^3x^{12}-ac^2x^8-8\,a^2cx^4-5\,a^3)\sqrt{cx^4+a}}{70\,a^2x^{14}}$$

input
```
integrate((c*x^4+a)^(3/2)/x^15,x, algorithm="fricas")
```

output
```
1/70*(2*c^3*x^12 - a*c^2*x^8 - 8*a^2*c*x^4 - 5*a^3)*sqrt(c*x^4 + a)/(a^2*x
^14)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.

Time = 0.89 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.09

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx = -\frac{a\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{14x^{12}} - \frac{4c^{\frac{3}{2}}\sqrt{\frac{a}{cx^4}+1}}{35x^8} - \frac{c^{\frac{5}{2}}\sqrt{\frac{a}{cx^4}+1}}{70ax^4} + \frac{c^{\frac{7}{2}}\sqrt{\frac{a}{cx^4}+1}}{35a^2}$$

input
```
integrate((c*x**4+a)**(3/2)/x**15,x)
```

output
```
-a*sqrt(c)*sqrt(a/(c*x**4) + 1)/(14*x**12) - 4*c**(3/2)*sqrt(a/(c*x**4) +
1)/(35*x**8) - c**(5/2)*sqrt(a/(c*x**4) + 1)/(70*a*x**4) + c**(7/2)*sqrt(a
/(c*x**4) + 1)/(35*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx = \frac{\frac{7\,(cx^4+a)^{\frac{5}{2}}c}{x^{10}} - \frac{5\,(cx^4+a)^{\frac{7}{2}}}{x^{14}}}{70\,a^2}$$

input
```
integrate((c*x^4+a)^(3/2)/x^15,x, algorithm="maxima")
```

output
```
1/70*(7*(c*x^4 + a)^(5/2)*c/x^10 - 5*(c*x^4 + a)^(7/2)/x^14)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(36) = 72$.

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.05

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx = \frac{2\left(35\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^{10}c^{\frac{7}{2}} + 35\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^{8}ac^{\frac{7}{2}} + 70\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right.\right.}{35\left(\left(\sqrt{c}x^2-\sqrt{cx^4-\right.\right.}$$

input
```
integrate((c*x^4+a)^(3/2)/x^15,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^10*c^(7/2) + 35*(sqrt(c)*x^2 - sq
rt(c*x^4 + a))^8*a*c^(7/2) + 70*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^6*a^2*c^(7
/2) + 14*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^4*a^3*c^(7/2) + 7*(sqrt(c)*x^2 -
sqrt(c*x^4 + a))^2*a^4*c^(7/2) - a^5*c^(7/2))/((sqrt(c)*x^2 - sqrt(c*x^4 +
 a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx = \frac{c^3\sqrt{c\,x^4+a}}{35\,a^2\,x^2} - \frac{4\,c\,\sqrt{c\,x^4+a}}{35\,x^{10}} - \frac{a\,\sqrt{c\,x^4+a}}{14\,x^{14}} - \frac{c^2\,\sqrt{c\,x^4+a}}{70\,a\,x^6}$$

input
```
int((a + c*x^4)^(3/2)/x^15,x)
```

output
```
(c^3*(a + c*x^4)^(1/2))/(35*a^2*x^2) - (4*c*(a + c*x^4)^(1/2))/(35*x^10) -
 (a*(a + c*x^4)^(1/2))/(14*x^14) - (c^2*(a + c*x^4)^(1/2))/(70*a*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 261, normalized size of antiderivative = 5.93

$$\int \frac{(a+cx^4)^{3/2}}{x^{15}}\,dx = \frac{-35\sqrt{c}\,\sqrt{c\,x^4+a}\,a^4x^2 - 336\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3c\,x^6 - 1015\sqrt{c}\,\sqrt{c\,x^4+a}\,a^2c^2x^{10} - 1260}{70x^{14}\left(\sqrt{c\,x^4+a}\,a^3 + 24\sqrt{c\,x^4+a}\,a^2c\,x^4 + 80\sqrt{c\,x^4+}\right.}$$

input
```
int((c*x^4+a)^(3/2)/x^15,x)
```

output
```
( - 35*sqrt(c)*sqrt(a + c*x**4)*a**4*x**2 - 336*sqrt(c)*sqrt(a + c*x**4)*a
**3*c*x**6 - 1015*sqrt(c)*sqrt(a + c*x**4)*a**2*c**2*x**10 - 1260*sqrt(c)*
sqrt(a + c*x**4)*a*c**3*x**14 - 560*sqrt(c)*sqrt(a + c*x**4)*c**4*x**18 -
5*a**5 - 133*a**4*c*x**4 - 721*a**3*c**2*x**8 - 1575*a**2*c**3*x**12 - 154
0*a*c**4*x**16 - 560*c**5*x**20)/(70*x**14*(sqrt(a + c*x**4)*a**3 + 24*sqr
t(a + c*x**4)*a**2*c*x**4 + 80*sqrt(a + c*x**4)*a*c**2*x**8 + 64*sqrt(a +
c*x**4)*c**3*x**12 + 7*sqrt(c)*a**3*x**2 + 56*sqrt(c)*a**2*c*x**6 + 112*sq
rt(c)*a*c**2*x**10 + 64*sqrt(c)*c**3*x**14))
```

3.343 $\int \frac{(a+cx^4)^{3/2}}{x^{19}} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}} dx = -\frac{(a+cx^4)^{5/2}}{18ax^{18}} + \frac{2c(a+cx^4)^{5/2}}{63a^2x^{14}} - \frac{4c^2(a+cx^4)^{5/2}}{315a^3x^{10}}$$

output
```
-1/18*(c*x^4+a)^(5/2)/a/x^18+2/63*c*(c*x^4+a)^(5/2)/a^2/x^14-4/315*c^2*(c*
x^4+a)^(5/2)/a^3/x^10
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}} dx = \frac{(a+cx^4)^{5/2}(-35a^2+20acx^4-8c^2x^8)}{630a^3x^{18}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^19,x]
```

output
```
((a + c*x^4)^(5/2)*(-35*a^2 + 20*a*c*x^4 - 8*c^2*x^8))/(630*a^3*x^18)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^4\right)^{3/2}}{x^{19}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4c\int \frac{\left(cx^4+a\right)^{3/2}}{x^{15}}dx}{9a}-\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4c\left(-\frac{2c\int \frac{\left(cx^4+a\right)^{3/2}}{x^{11}}dx}{7a}-\frac{\left(a+cx^4\right)^{5/2}}{14ax^{14}}\right)}{9a}-\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{4c\left(\frac{c\left(a+cx^4\right)^{5/2}}{35a^2x^{10}}-\frac{\left(a+cx^4\right)^{5/2}}{14ax^{14}}\right)}{9a}-\frac{\left(a+cx^4\right)^{5/2}}{18ax^{18}}$$

input
```
Int[(a + c*x^4)^(3/2)/x^19,x]
```

output
```
-1/18*(a + c*x^4)^(5/2)/(a*x^18) - (4*c*(-1/14*(a + c*x^4)^(5/2)/(a*x^14)
+ (c*(a + c*x^4)^(5/2))/(35*a^2*x^10)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}} \left(8 c^{2} x^{8}-20 a\,x^{4} c+35 a^{2}\right)}{630 x^{18} a^{3}}$	39
pseudoelliptic	$-\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}} \left(8 c^{2} x^{8}-20 a\,x^{4} c+35 a^{2}\right)}{630 x^{18} a^{3}}$	39
orering	$-\frac{\left(c\,x^{4}+a\right)^{\frac{5}{2}} \left(8 c^{2} x^{8}-20 a\,x^{4} c+35 a^{2}\right)}{630 x^{18} a^{3}}$	39
default	$-\frac{\sqrt{c\,x^{4}+a}\,\left(8 c^{2} x^{8}-20 a\,x^{4} c+35 a^{2}\right) \left(c^{2} x^{8}+2 a\,x^{4} c+a^{2}\right)}{630 a^{3} x^{18}}$	57
elliptic	$-\frac{\sqrt{c\,x^{4}+a}\,\left(8 c^{2} x^{8}-20 a\,x^{4} c+35 a^{2}\right) \left(c^{2} x^{8}+2 a\,x^{4} c+a^{2}\right)}{630 a^{3} x^{18}}$	57
trager	$-\frac{\left(8 c^{4} x^{16}-4 c^{3} a\,x^{12}+3 a^{2} x^{8} c^{2}+50 a^{3} c\,x^{4}+35 a^{4}\right) \sqrt{c\,x^{4}+a}}{630 x^{18} a^{3}}$	61
risch	$-\frac{\left(8 c^{4} x^{16}-4 c^{3} a\,x^{12}+3 a^{2} x^{8} c^{2}+50 a^{3} c\,x^{4}+35 a^{4}\right) \sqrt{c\,x^{4}+a}}{630 x^{18} a^{3}}$	61

input
```
int((c*x^4+a)^(3/2)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/630*(c*x^4+a)^(5/2)*(8*c^2*x^8-20*a*c*x^4+35*a^2)/x^18/a^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}}\,dx = -\frac{(8\,c^4x^{16}-4\,ac^3x^{12}+3\,a^2c^2x^8+50\,a^3cx^4+35\,a^4)\sqrt{cx^4+a}}{630\,a^3x^{18}}$$

input
```
integrate((c*x^4+a)^(3/2)/x^19,x, algorithm="fricas")
```

output
```
-1/630*(8*c^4*x^16 - 4*a*c^3*x^12 + 3*a^2*c^2*x^8 + 50*a^3*c*x^4 + 35*a^4)
*sqrt(c*x^4 + a)/(a^3*x^18)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(61) = 122.

Time = 1.38 (sec) , antiderivative size = 420, normalized size of antiderivative = 6.18

$$\begin{aligned}\int \frac{(a+cx^4)^{3/2}}{x^{19}}\,dx = &-\frac{35a^6c^{\frac{9}{2}}\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}} \\ &-\frac{120a^5c^{\frac{11}{2}}x^4\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}} \\ &-\frac{138a^4c^{\frac{13}{2}}x^8\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}} \\ &-\frac{52a^3c^{\frac{15}{2}}x^{12}\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}} \\ &-\frac{3a^2c^{\frac{17}{2}}x^{16}\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}} \\ &-\frac{12ac^{\frac{19}{2}}x^{20}\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}} \\ &-\frac{8c^{\frac{21}{2}}x^{24}\sqrt{\frac{a}{cx^4}+1}}{630a^5c^4x^{16}+1260a^4c^5x^{20}+630a^3c^6x^{24}}\end{aligned}$$

input
```
integrate((c*x**4+a)**(3/2)/x**19,x)
```

output
```
-35*a**6*c**(9/2)*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**4*c*
*5*x**20 + 630*a**3*c**6*x**24) - 120*a**5*c**(11/2)*x**4*sqrt(a/(c*x**4)
+ 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5*x**20 + 630*a**3*c**6*x**24) -
138*a**4*c**(13/2)*x**8*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a
**4*c**5*x**20 + 630*a**3*c**6*x**24) - 52*a**3*c**(15/2)*x**12*sqrt(a/(c*
x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5*x**20 + 630*a**3*c**6*x**
24) - 3*a**2*c**(17/2)*x**16*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1
260*a**4*c**5*x**20 + 630*a**3*c**6*x**24) - 12*a*c**(19/2)*x**20*sqrt(a/(
c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260*a**4*c**5*x**20 + 630*a**3*c**6*x
**24) - 8*c**(21/2)*x**24*sqrt(a/(c*x**4) + 1)/(630*a**5*c**4*x**16 + 1260
*a**4*c**5*x**20 + 630*a**3*c**6*x**24)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}}\,dx = -\frac{\frac{63\,(cx^4+a)^{\frac{5}{2}}c^2}{x^{10}} - \frac{90\,(cx^4+a)^{\frac{7}{2}}c}{x^{14}} + \frac{35\,(cx^4+a)^{\frac{9}{2}}}{x^{18}}}{630\,a^3}$$

input
```
integrate((c*x^4+a)^(3/2)/x^19,x, algorithm="maxima")
```

output
```
-1/630*(63*(c*x^4 + a)^(5/2)*c^2/x^10 - 90*(c*x^4 + a)^(7/2)*c/x^14 + 35*(
c*x^4 + a)^(9/2)/x^18)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(56) = 112$.

Time = 0.15 (sec) , antiderivative size = 206, normalized size of antiderivative = 3.03

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}}\,dx = \frac{8\left(210\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^{12}c^{\frac{9}{2}} + 315\left(\sqrt{c}x^2-\sqrt{cx^4+a}\right)^{10}ac^{\frac{9}{2}} + 441\left(\sqrt{c}x^2-\sqrt{cx^4\right.}\right.}{31}$$

input
```
integrate((c*x^4+a)^(3/2)/x^19,x, algorithm="giac")
```

output
```
8/315*(210*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^12*c^(9/2) + 315*(sqrt(c)*x^2 -
 sqrt(c*x^4 + a))^10*a*c^(9/2) + 441*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^8*a^2
*c^(9/2) + 126*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^6*a^3*c^(9/2) + 36*(sqrt(c)
*x^2 - sqrt(c*x^4 + a))^4*a^4*c^(9/2) - 9*(sqrt(c)*x^2 - sqrt(c*x^4 + a))^
2*a^5*c^(9/2) + a^6*c^(9/2))/((sqrt(c)*x^2 - sqrt(c*x^4 + a))^2 - a)^9
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}}\,dx = \frac{2\,c^3\,\sqrt{c\,x^4+a}}{315\,a^2\,x^6} - \frac{5\,c\,\sqrt{c\,x^4+a}}{63\,x^{14}} - \frac{4\,c^4\,\sqrt{c\,x^4+a}}{315\,a^3\,x^2} - \frac{a\,\sqrt{c\,x^4+a}}{18\,x^{18}} - \frac{c^2\,\sqrt{c\,x^4+a}}{210\,a\,x^{10}}$$

input
```
int((a + c*x^4)^(3/2)/x^19,x)
```

output
```
(2*c^3*(a + c*x^4)^(1/2))/(315*a^2*x^6) - (5*c*(a + c*x^4)^(1/2))/(63*x^14
) - (4*c^4*(a + c*x^4)^(1/2))/(315*a^3*x^2) - (a*(a + c*x^4)^(1/2))/(18*x^
18) - (c^2*(a + c*x^4)^(1/2))/(210*a*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 325, normalized size of antiderivative = 4.78

$$\int \frac{(a+cx^4)^{3/2}}{x^{19}}\,dx = \frac{-315\sqrt{c}\,\sqrt{c\,x^4+a}\,a^5x^2 - 4650\sqrt{c}\,\sqrt{c\,x^4+a}\,a^4c\,x^6 - 21147\sqrt{c}\,\sqrt{c\,x^4+a}\,a^3c^2x^{10} - 4}{630x^{18}\left(\sqrt{c\,x^4+a}\,a^4 + 40\sqrt{c\,x^4+a}\,a^3c\,x^4 + 2\right.}$$

input
```
int((c*x^4+a)^(3/2)/x^19,x)
```

output
```
( - 315*sqrt(c)*sqrt(a + c*x**4)*a**5*x**2 - 4650*sqrt(c)*sqrt(a + c*x**4)
*a**4*c*x**6 - 21147*sqrt(c)*sqrt(a + c*x**4)*a**3*c**2*x**10 - 42084*sqrt
(c)*sqrt(a + c*x**4)*a**2*c**3*x**14 - 38640*sqrt(c)*sqrt(a + c*x**4)*a*c*
*4*x**18 - 13440*sqrt(c)*sqrt(a + c*x**4)*c**5*x**22 - 35*a**6 - 1485*a**5
*c*x**4 - 11853*a**4*c**2*x**8 - 38199*a**3*c**3*x**12 - 59724*a**2*c**4*x
**16 - 45360*a*c**5*x**20 - 13440*c**6*x**24)/(630*x**18*(sqrt(a + c*x**4)
*a**4 + 40*sqrt(a + c*x**4)*a**3*c*x**4 + 240*sqrt(a + c*x**4)*a**2*c**2*x
**8 + 448*sqrt(a + c*x**4)*a*c**3*x**12 + 256*sqrt(a + c*x**4)*c**4*x**16
+ 9*sqrt(c)*a**4*x**2 + 120*sqrt(c)*a**3*c*x**6 + 432*sqrt(c)*a**2*c**2*x*
*10 + 576*sqrt(c)*a*c**3*x**14 + 256*sqrt(c)*c**4*x**18))
```

3.344 $\int x^4(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int x^4(a+cx^4)^{3/2}\,dx = \frac{4a^2x\sqrt{a+cx^4}}{77c}+\frac{6}{77}ax^5\sqrt{a+cx^4}+\frac{1}{11}x^5(a+cx^4)^{3/2} - \frac{2a^{11/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{77c^{5/4}\sqrt{a+cx^4}}$$

output
```
4/77*a^2*x*(c*x^4+a)^(1/2)/c+6/77*a*x^5*(c*x^4+a)^(1/2)+1/11*x^5*(c*x^4+a)
^(3/2)-2/77*a^(11/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2
)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(5/4
)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.45

$$\int x^4(a+cx^4)^{3/2}\,dx = \frac{x\sqrt{a+cx^4}\left((a+cx^4)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{1+\frac{cx^4}{a}}}\right)}{11c}$$

input
```
Integrate[x^4*(a + c*x^4)^(3/2),x]
```

output
```
(x*Sqrt[a + c*x^4]*((a + c*x^4)^2 - (a^2*Hypergeometric2F1[-3/2, 1/4, 5/4,
 -((c*x^4)/a)])/Sqrt[1 + (c*x^4)/a]))/(11*c)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {811, 811, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+cx^4\right)^{3/2} dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{6}{11}a\int x^4\sqrt{cx^4+a}dx+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\int \frac{x^4}{\sqrt{cx^4+a}}dx+\frac{1}{7}x^5\sqrt{a+cx^4}\right)+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{x\sqrt{a+cx^4}}{3c}-\frac{a\int \frac{1}{\sqrt{cx^4+a}}dx}{3c}\right)+\frac{1}{7}x^5\sqrt{a+cx^4}\right)+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{x\sqrt{a+cx^4}}{3c}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}\right)+\frac{1}{7}x^5\sqrt{a+cx^4}\right)+\frac{1}{11}x^5\left(a+cx^4\right)^{3/2}$$

input
```
Int[x^4*(a + c*x^4)^(3/2),x]
```

output
```
(x^5*(a + c*x^4)^(3/2))/11 + (6*a*((x^5*Sqrt[a + c*x^4])/7 + (2*a*((x*Sqrt
[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*
c^(5/4)*Sqrt[a + c*x^4])))/7))/11
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x(7c^2x^8+13a\,x^4c+4a^2)\sqrt{c\,x^4+a}}{77c}-\frac{4a^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{77c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	114
default	$\frac{c\,x^9\sqrt{c\,x^4+a}}{11}+\frac{13a\,x^5\sqrt{c\,x^4+a}}{77}+\frac{4a^2x\sqrt{c\,x^4+a}}{77c}-\frac{4a^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{77c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	126
elliptic	$\frac{c\,x^9\sqrt{c\,x^4+a}}{11}+\frac{13a\,x^5\sqrt{c\,x^4+a}}{77}+\frac{4a^2x\sqrt{c\,x^4+a}}{77c}-\frac{4a^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{77c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	126

input
```
int(x^4*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/77*x*(7*c^2*x^8+13*a*c*x^4+4*a^2)/c*(c*x^4+a)^(1/2)-4/77*a^3/c/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)
^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.47

$$\int x^4\left(a+cx^4\right)^{3/2}\,dx = -\frac{4\,a^2\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)-\left(7\,c^2x^9+13\,acx^5+4\,a^2x\right)\sqrt{cx^4+a}}{77\,c}$$

input
```
integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/77*(4*a^2*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) -
 (7*c^2*x^9 + 13*a*c*x^5 + 4*a^2*x)*sqrt(c*x^4 + a))/c
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int x^4(a+cx^4)^{3/2}\,dx=\frac{a^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x**5*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4(a+cx^4)^{3/2}\,dx=\int(cx^4+a)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4(a+cx^4)^{3/2}\,dx=\int(cx^4+a)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + cx^4)^{3/2} \, dx = \int x^4 (c x^4 + a)^{3/2} \, dx$$

input
```
int(x^4*(a + c*x^4)^(3/2),x)
```

output
```
int(x^4*(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^4 (a + cx^4)^{3/2} \, dx = \frac{4\sqrt{c x^4 + a}\, a^2 x + 13\sqrt{c x^4 + a}\, a c x^5 + 7\sqrt{c x^4 + a}\, c^2 x^9 - 4\left(\int \frac{\sqrt{c x^4+a}}{c x^4+a} dx\right) a^3}{77c}$$

input
```
int(x^4*(c*x^4+a)^(3/2),x)
```

output
```
(4*sqrt(a + c*x**4)*a**2*x + 13*sqrt(a + c*x**4)*a*c*x**5 + 7*sqrt(a + c*x
**4)*c**2*x**9 - 4*int(sqrt(a + c*x**4)/(a + c*x**4),x)*a**3)/(77*c)
```

3.345 $\int (a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 122

$$\int (a+cx^4)^{3/2}\,dx = \frac{2}{7}ax\sqrt{a+cx^4} + \frac{1}{7}x(a+cx^4)^{3/2} + \frac{2a^{7/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
2/7*a*x*(c*x^4+a)^(1/2)+1/7*x*(c*x^4+a)^(3/2)+2/7*a^(7/4)*(a^(1/2)+c^(1/2)
*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c
^(1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.39

$$\int (a+cx^4)^{3/2}\,dx = \frac{ax\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(a + c*x^4)^(3/2),x]
```

output
```
(a*x*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((c*x^4)/a)])/Sqrt
[1 + (c*x^4)/a]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {748, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+cx^4\right)^{3/2} \, dx$$

$$\Big\downarrow \text{748}$$

$$\frac{6}{7}a\int \sqrt{cx^4+a}dx+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{748}$$

$$\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{1}{3}x\sqrt{a+cx^4}\right)+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}a\left(\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}\right)+\frac{1}{7}x\left(a+cx^4\right)^{3/2}$$

input
```
Int[(a + c*x^4)^(3/2),x]
```

output
```
(x*(a + c*x^4)^(3/2))/7 + (6*a*((x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])))/7
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x\left(c\,x^{4}+3a\right)\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	96
default	$\frac{x^{5}c\sqrt{c\,x^{4}+a}}{7}+\frac{3ax\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	103
elliptic	$\frac{x^{5}c\sqrt{c\,x^{4}+a}}{7}+\frac{3ax\sqrt{c\,x^{4}+a}}{7}+\frac{4a^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{7\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	103

input
```
int((c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x*(c*x^4+3*a)*(c*x^4+a)^(1/2)+4/7*a^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a
^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*
EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \left(a+cx^4\right)^{3/2} dx = \frac{4}{7} a\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right) | -1) + \frac{1}{7} \left(cx^5 + 3\,ax\right)\sqrt{cx^4+a}$$

input
```
integrate((c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
4/7*a*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 1/7*(c
*x^5 + 3*a*x)*sqrt(c*x^4 + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \left(a+cx^4\right)^{3/2} dx = \frac{a^{\frac{3}{2}} x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(a+cx^4\right)^{3/2} dx = \int \left(cx^4+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \left(a+cx^4\right)^{3/2} dx = \int \left(cx^4+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \left(a+cx^4\right)^{3/2} dx = \frac{x\left(c\,x^4+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\left(\frac{c\,x^4}{a}+1\right)^{3/2}}$$

input
```
int((a + c*x^4)^(3/2),x)
```

output
```
(x*(a + c*x^4)^(3/2)*hypergeom([-3/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
 1)^(3/2)
```

Reduce [F]

$$\int \left(a+cx^4\right)^{3/2} dx = \frac{3\sqrt{c\,x^4+a}\,ax}{7} + \frac{\sqrt{c\,x^4+a}\,c\,x^5}{7} + \frac{4\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^4+a}dx\right)a^2}{7}$$

input
```
int((c*x^4+a)^(3/2),x)
```

output
```
(3*sqrt(a + c*x**4)*a*x + sqrt(a + c*x**4)*c*x**5 + 4*int(sqrt(a + c*x**4)
/(a + c*x**4),x)*a**2)/7
```

3.346 $\int \frac{(a+cx^4)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{(a+cx^4)^{3/2}}{x^4} dx = -\frac{a\sqrt{a+cx^4}}{3x^3} + \frac{1}{3}cx\sqrt{a+cx^4} + \frac{2a^{3/4}c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt{a+cx^4}}$$

output
```
-1/3*a*(c*x^4+a)^(1/2)/x^3+1/3*c*x*(c*x^4+a)^(1/2)+2/3*a^(3/4)*c^(3/4)*(a^
(1/2)+c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.42

$$\int \frac{(a+cx^4)^{3/2}}{x^4} dx = -\frac{a\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{cx^4}{a}\right)}{3x^3\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^4,x]
```

output
```
-1/3*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((c*x^4)/a)])/
(x^3*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {809, 748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{809}$$

$$2c\int \sqrt{cx^4+a}dx-\frac{(a+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{748}$$

$$2c\left(\frac{2}{3}a\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{1}{3}x\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{761}$$

$$2c\left(\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{3x^3}$$

input
```
Int[(a + c*x^4)^(3/2)/x^4,x]
```

output
```
-1/3*(a + c*x^4)^(3/2)/x^3 + 2*c*((x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4]))
```

Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(-c\,x^{4}+a\right)}{3x^{3}}+\frac{4ac\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	96
default	$-\frac{a\sqrt{c\,x^{4}+a}}{3x^{3}}+\frac{cx\sqrt{c\,x^{4}+a}}{3}+\frac{4ac\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	102
elliptic	$-\frac{a\sqrt{c\,x^{4}+a}}{3x^{3}}+\frac{cx\sqrt{c\,x^{4}+a}}{3}+\frac{4ac\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	102

input
```
int((c*x^4+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*x^4+a)^(1/2)*(-c*x^4+a)/x^3+4/3*a*c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^4}\,dx=\int \frac{(cx^4+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
integral((c*x^4 + a)^(3/2)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.34

$$\int \frac{(a+cx^4)^{3/2}}{x^4}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((c*x**4+a)**(3/2)/x**4,x)
```

output
```
a**(3/2)*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), c*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{x^4}\,dx = \int \frac{(c\,x^4+a)^{3/2}}{x^4}\,dx$$

input
```
int((a + c*x^4)^(3/2)/x^4,x)
```

output
```
int((a + c*x^4)^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^4}\,dx = \frac{-5\sqrt{c\,x^4+a}\,a+\sqrt{c\,x^4+a}\,c\,x^4-12\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^8+a\,x^4}dx\right)a^2x^3}{3x^3}$$

input
```
int((c*x^4+a)^(3/2)/x^4,x)
```

output
```
( - 5*sqrt(a + c*x**4)*a + sqrt(a + c*x**4)*c*x**4 - 12*int(sqrt(a + c*x**
4)/(a*x**4 + c*x**8),x)*a**2*x**3)/(3*x**3)
```

3.347 $\int \frac{(a+cx^4)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{(a+cx^4)^{3/2}}{x^8} dx = -\frac{a\sqrt{a+cx^4}}{7x^7} - \frac{3c\sqrt{a+cx^4}}{7x^3} + \frac{2c^{7/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{a+cx^4}}$$

output
```
-1/7*a*(c*x^4+a)^(1/2)/x^7-3/7*c*(c*x^4+a)^(1/2)/x^3+2/7*c^(7/4)*(a^(1/2)+
c^(1/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*a
rctan(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{(a+cx^4)^{3/2}}{x^8} dx = -\frac{a\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{cx^4}{a}\right)}{7x^7\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/7*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((c*x^4)/a)])
/(x^7*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {809, 809, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{6}{7}c\int \frac{\sqrt{cx^4+a}}{x^4}dx-\frac{(a+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{809}$$

$$\frac{6}{7}c\left(\frac{2}{3}c\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a+cx^4}}{3x^3}\right)-\frac{(a+cx^4)^{3/2}}{7x^7}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}c\left(\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3x^3}\right)-\frac{(a+cx^4)^{3/2}}{7x^7}$$

input
```
Int[(a + c*x^4)^(3/2)/x^8,x]
```

output
```
-1/7*(a + c*x^4)^(3/2)/x^7 + (6*c*(-1/3*Sqrt[a + c*x^4]/x^3 + (c^(3/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a + c*x^4])))/7
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(3 c\,x^{4}+a \right)}{7 x^{7}}+\frac{4 c^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$	97
default	$-\frac{a \sqrt{c\,x^{4}+a}}{7 x^{7}}-\frac{3 c \sqrt{c\,x^{4}+a}}{7 x^{3}}+\frac{4 c^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$	105
elliptic	$-\frac{a \sqrt{c\,x^{4}+a}}{7 x^{7}}-\frac{3 c \sqrt{c\,x^{4}+a}}{7 x^{3}}+\frac{4 c^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{7 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$	105

input
```
int((c*x^4+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(c*x^4+a)^(1/2)*(3*c*x^4+a)/x^7+4/7*c^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/
2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx = -\frac{4\sqrt{a}cx^7\left(-\frac{c}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1) + (3\,cx^4+a)\sqrt{cx^4+a}}{7\,x^7}$$

input
```
integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-1/7*(4*sqrt(a)*c*x^7*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1)
+ (3*c*x^4 + a)*sqrt(c*x^4 + a))/x^7
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{3}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((c*x**4+a)**(3/2)/x**8,x)
```

output
```
a**(3/2)*gamma(-7/4)*hyper((-7/4, -3/2), (-3/4,), c*x**4*exp_polar(I*pi)/a
)/(4*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^8, x)
```

Giac [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^8}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx = \int \frac{(c\,x^4+a)^{3/2}}{x^8}\,dx$$

input
```
int((a + c*x^4)^(3/2)/x^8,x)
```

output
```
int((a + c*x^4)^(3/2)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^8}\,dx = \frac{\sqrt{c\,x^4+a}\,a-5\sqrt{c\,x^4+a}\,c\,x^4+12\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^{12}+a\,x^8}dx\right)a^2x^7}{5x^7}$$

input

```
int((c*x^4+a)^(3/2)/x^8,x)
```

output

```
(sqrt(a + c*x**4)*a - 5*sqrt(a + c*x**4)*c*x**4 + 12*int(sqrt(a + c*x**4)/
(a*x**8 + c*x**12),x)*a**2*x**7)/(5*x**7)
```

3.348 $\int x^2(a+cx^4)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 255

$$\int x^2\left(a+cx^4\right)^{3/2}\,dx=\frac{2}{15}ax^3\sqrt{a+cx^4}+\frac{4a^2x\sqrt{a+cx^4}}{15\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}-\frac{4a^{9/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}+\frac{2a^{9/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}$$

output
```
2/15*a*x^3*(c*x^4+a)^(1/2)+4/15*a^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+c^(
1/2)*x^2)+1/9*x^3*(c*x^4+a)^(3/2)-4/15*a^(9/4)*(a^(1/2)+c^(1/2)*x^2)*((c*x
^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)+2/15*a^(9/4)*(a^(1/2)+c^(1/2)*x
^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(c^(
1/4)*x/a^(1/4)),1/2*2^(1/2))/c^(3/4)/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.80 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int x^2\left(a+cx^4\right)^{3/2}\,dx=\frac{ax^3\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{3\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[x^2*(a + c*x^4)^(3/2),x]
```

output
```
(a*x^3*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((c*x^4)/a)])/(3
*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {811, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+cx^4\right)^{3/2}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{2}{3}a\int x^2\sqrt{cx^4+a}dx+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{811}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\int\frac{x^2}{\sqrt{cx^4+a}}dx+\frac{1}{5}x^3\sqrt{a+cx^4}\right)+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}\right)+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}\right)+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}\right)+\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\ldots\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\ldots}{\sqrt{c}}\ldots\right.\right.$$

$$\frac{1}{9}x^3\left(a+cx^4\right)^{3/2}$$

input

```
Int[x^2*(a + c*x^4)^(3/2),x]
```

output

```
(x^3*(a + c*x^4)^(3/2))/9 + (2*a*((x^3*Sqrt[a + c*x^4])/5 + (2*a*(-((-((x*
Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])))/5))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.48

method	result	size
risch	$\frac{x^{3}\left(5 c \,x^{4}+11 a \right) \sqrt{c \,x^{4}+a}}{45}+\frac{4 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{15 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c \,x^{4}+a}\, \sqrt{c}}$	122
default	$\frac{x^{7} \sqrt{c \,x^{4}+a}\, c}{9}+\frac{11 a \,x^{3} \sqrt{c \,x^{4}+a}}{45}+\frac{4 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{15 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c \,x^{4}+a}\, \sqrt{c}}$	128
elliptic	$\frac{x^{7} \sqrt{c \,x^{4}+a}\, c}{9}+\frac{11 a \,x^{3} \sqrt{c \,x^{4}+a}}{45}+\frac{4 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{15 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c \,x^{4}+a}\, \sqrt{c}}$	128

input
```
int(x^2*(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*x^3*(5*c*x^4+11*a)*(c*x^4+a)^(1/2)+4/15*I*a^(5/2)/(I/a^(1/2)*c^(1/2))
^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*
x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x
*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.41

$$\int x^2(a+cx^4)^{3/2}\,dx = \frac{12\,a^2\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-12\,a^2\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+(5\,c}{45\,cx}$$

input
```
integrate(x^2*(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/45*(12*a^2*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
 - 12*a^2*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) +
(5*c^2*x^8 + 11*a*c*x^4 + 12*a^2)*sqrt(c*x^4 + a))/(c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int x^2 (a + cx^4)^{3/2} \, dx = \frac{a^{\frac{3}{2}} x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(c*x**4+a)**(3/2),x)
```

output
```
a**(3/2)*x**3*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int x^2 (a + cx^4)^{3/2} \, dx = \int (cx^4 + a)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2(a+cx^4)^{3/2}\,dx=\int (cx^4+a)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+cx^4)^{3/2}\,dx=\int x^2\,(c\,x^4+a)^{3/2}\,dx$$

input
```
int(x^2*(a + c*x^4)^(3/2),x)
```

output
```
int(x^2*(a + c*x^4)^(3/2), x)
```

Reduce [F]

$$\int x^2(a+cx^4)^{3/2}\,dx=\frac{11\sqrt{c\,x^4+a}\,a\,x^3}{45}+\frac{\sqrt{c\,x^4+a}\,c\,x^7}{9}+\frac{4\left(\int\frac{\sqrt{c\,x^4+a}\,x^2}{c\,x^4+a}dx\right)a^2}{15}$$

input
```
int(x^2*(c*x^4+a)^(3/2),x)
```

output
```
(11*sqrt(a + c*x**4)*a*x**3 + 5*sqrt(a + c*x**4)*c*x**7 + 12*int((sqrt(a +
 c*x**4)*x**2)/(a + c*x**4),x)*a**2)/45
```

3.349 $\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 252

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+cx^4}}{x}+\frac{1}{5}cx^3\sqrt{a+cx^4}+\frac{12a\sqrt{c}x\sqrt{a+cx^4}}{5\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{12a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+cx^4}} + \frac{6a^{5/4}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{a+cx^4}}$$

output
```
-a*(c*x^4+a)^(1/2)/x+1/5*c*x^3*(c*x^4+a)^(1/2)+12*a*c^(1/2)*x*(c*x^4+a)^(1
/2)/(5*a^(1/2)+5*c^(1/2)*x^2)-12/5*a^(5/4)*c^(1/4)*(a^(1/2)+c^(1/2)*x^2)*(
(c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*2^(1/2))/(c*x^4+a)^(1/2)+6/5*a^(5/4)*c^(1/4)*(a^(1/2)+c^(1/2
)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(
c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.20

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{cx^4}{a}\right)}{x\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^2,x]
```

output
```
-((a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((c*x^4)/a)])/(x*
Sqrt[1 + (c*x^4)/a]))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {809, 811, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{809}$$

$$6c\int x^2\sqrt{cx^4+a}dx-\frac{(a+cx^4)^{3/2}}{x}$$

$$\Big\downarrow \text{811}$$

$$6c\left(\frac{2}{5}a\int \frac{x^2}{\sqrt{cx^4+a}}dx+\frac{1}{5}x^3\sqrt{a+cx^4}\right)-\frac{(a+cx^4)^{3/2}}{x}$$

$$\Big\downarrow \text{834}$$

$$6c\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}\right)-\frac{\left(a+cx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{27}$$

$$6c\left(\frac{2}{5}a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}\right)-\frac{\left(a+cx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{761}$$

$$6c\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)+\frac{1}{5}x^3\sqrt{a+cx^4}\right)-\frac{\left(a+cx^4\right)^{3/2}}{x}$$

$$\Big\downarrow \text{1510}$$

$$6c\left(\frac{2}{5}a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}}{\sqrt{c}}\right.\right.$$

$$\frac{\left(a+cx^4\right)^{3/2}}{x}$$

input
```
Int[(a + c*x^4)^(3/2)/x^2,x]
```

output
```
-((a + c*x^4)^(3/2)/x) + 6*c*((x^3*Sqrt[a + c*x^4])/5 + (2*a*(-((-((x*Sqrt
[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])))/5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,\left(-c\,x^{4}+5a\right)}{5x}+\frac{12 i a^{\frac{3}{2}} \sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	122
default	$-\frac{a\sqrt{c\,x^{4}+a}}{x}+\frac{c\,x^{3}\sqrt{c\,x^{4}+a}}{5}+\frac{12 i a^{\frac{3}{2}} \sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	128
elliptic	$-\frac{a\sqrt{c\,x^{4}+a}}{x}+\frac{c\,x^{3}\sqrt{c\,x^{4}+a}}{5}+\frac{12 i a^{\frac{3}{2}} \sqrt{c}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	128

input
```
int((c*x^4+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(c*x^4+a)^(1/2)*(-c*x^4+5*a)/x+12/5*I*a^(3/2)*c^(1/2)/(I/a^(1/2)*c^(1
/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)
/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/
a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx=\int \frac{(cx^4+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral((c*x^4 + a)^(3/2)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.16

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{2},-\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((c*x**4+a)**(3/2)/x**2,x)
```

output
```
a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), c*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.16

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = \frac{(c\,x^4+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ -\frac{a}{c\,x^4}\right)}{5\,x\,\left(\frac{a}{c\,x^4}+1\right)^{3/2}}$$

input
```
int((a + c*x^4)^(3/2)/x^2,x)
```

output
```
((a + c*x^4)^(3/2)*hypergeom([-3/2, -5/4], -1/4, -a/(c*x^4)))/(5*x*(a/(c*x
^4) + 1)^(3/2))
```

Reduce [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^2}\,dx = \frac{7\sqrt{c\,x^4+a}\,a+\sqrt{c\,x^4+a}\,c\,x^4+12\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^6+a\,x^2}dx\right)a^2x}{5x}$$

input
```
int((c*x^4+a)^(3/2)/x^2,x)
```

output
```
(7*sqrt(a + c*x**4)*a + sqrt(a + c*x**4)*c*x**4 + 12*int(sqrt(a + c*x**4)/
(a*x**2 + c*x**6),x)*a**2*x)/(5*x)
```

3.350 $\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 253

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx = -\frac{a\sqrt{a+cx^4}}{5x^5}-\frac{7c\sqrt{a+cx^4}}{5x}+\frac{12c^{3/2}x\sqrt{a+cx^4}}{5\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{12\sqrt[4]{a}c^{5/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{a+cx^4}} + \frac{6\sqrt[4]{a}c^{5/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{5\sqrt{a+cx^4}}$$

output

```
-1/5*a*(c*x^4+a)^(1/2)/x^5-7/5*c*(c*x^4+a)^(1/2)/x+12*c^(3/2)*x*(c*x^4+a)^
(1/2)/(5*a^(1/2)+5*c^(1/2)*x^2)-12/5*a^(1/4)*c^(5/4)*(a^(1/2)+c^(1/2)*x^2)
*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(c^(1/4)*
x/a^(1/4))),1/2*2^(1/2))/(c*x^4+a)^(1/2)+6/5*a^(1/4)*c^(5/4)*(a^(1/2)+c^(1
/2)*x^2)*((c*x^4+a)/(a^(1/2)+c^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arcta
n(c^(1/4)*x/a^(1/4)),1/2*2^(1/2))/(c*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.21

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx=-\frac{a\sqrt{a+cx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{cx^4}{a}\right)}{5x^5\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(a + c*x^4)^(3/2)/x^6,x]
```

output
```
-1/5*(a*Sqrt[a + c*x^4]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((c*x^4)/a)])
/(x^5*Sqrt[1 + (c*x^4)/a])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {809, 809, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^4)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{6}{5}c\int \frac{\sqrt{cx^4+a}}{x^2}dx-\frac{(a+cx^4)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{809}$$

$$\frac{6}{5}c\left(2c\int \frac{x^2}{\sqrt{cx^4+a}}dx-\frac{\sqrt{a+cx^4}}{x}\right)-\frac{(a+cx^4)^{3/2}}{5x^5}$$

$$\Big\downarrow \text{834}$$

$$\frac{6}{5}c\left(2c\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)-\frac{\sqrt{a+cx^4}}{x}\right)-\frac{\left(a+cx^4\right)^{3/2}}{5x^5}$$

$\downarrow$ 27

$$\frac{6}{5}c\left(2c\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)-\frac{\sqrt{a+cx^4}}{x}\right)-\frac{\left(a+cx^4\right)^{3/2}}{5x^5}$$

$\downarrow$ 761

$$\frac{6}{5}c\left(2c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)-\frac{\sqrt{a+cx^4}}{x}\right)-\frac{\left(a+cx^4\right)^{3/2}}{5x^5}$$

$\downarrow$ 1510

$$\frac{6}{5}c\left(2c\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\right.\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}}{\sqrt{c}}\right.\right.$$

$$\frac{\left(a+cx^4\right)^{3/2}}{5x^5}$$

input

```
Int[(a + c*x^4)^(3/2)/x^6,x]
```

output

```
-1/5*(a + c*x^4)^(3/2)/x^5 + (6*c*(-(Sqrt[a + c*x^4]/x) + 2*c*(-((-((x*Sqr
t[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4]))/Sqrt[c]) + (a^(1/4)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4]))))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{\sqrt{c\,x^{4}+a}\,(7c\,x^{4}+a)}{5x^{5}}+\frac{12 i c^{\frac{3}{2}} \sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	120
default	$-\frac{a\sqrt{c\,x^{4}+a}}{5x^{5}}-\frac{7c\sqrt{c\,x^{4}+a}}{5x}+\frac{12 i c^{\frac{3}{2}} \sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	128
elliptic	$-\frac{a\sqrt{c\,x^{4}+a}}{5x^{5}}-\frac{7c\sqrt{c\,x^{4}+a}}{5x}+\frac{12 i c^{\frac{3}{2}} \sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	128

input
```
int((c*x^4+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(c*x^4+a)^(1/2)*(7*c*x^4+a)/x^5+12/5*I*c^(3/2)*a^(1/2)/(I/a^(1/2)*c^(
1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I
/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx=\int \frac{(cx^4+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
integral((c*x^4 + a)^(3/2)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4} \\ -\frac{1}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((c*x**4+a)**(3/2)/x**6,x)
```

output
```
a**(3/2)*gamma(-5/4)*hyper((-3/2, -5/4), (-1/4,), c*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(cx^4+a)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((c*x^4+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx = \int \frac{(c\,x^4+a)^{3/2}}{x^6}\,dx$$

input
```
int((a + c*x^4)^(3/2)/x^6,x)
```

output
```
int((a + c*x^4)^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+cx^4)^{3/2}}{x^6}\,dx = \frac{-\sqrt{c\,x^4+a}\,a+\sqrt{c\,x^4+a}\,c\,x^4-4\left(\int \frac{\sqrt{c\,x^4+a}}{c\,x^{10}+a\,x^6}dx\right)a^2x^5}{x^5}$$

input
```
int((c*x^4+a)^(3/2)/x^6,x)
```

output
```
( - sqrt(a + c*x**4)*a + sqrt(a + c*x**4)*c*x**4 - 4*int(sqrt(a + c*x**4)/
(a*x**6 + c*x**10),x)*a**2*x**5)/x**5
```

3.351 $\int x^7\sqrt{5+3x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^7\sqrt{5+3x^4}\,dx=-\frac{5}{54}\left(5+3x^4\right)^{3/2}+\frac{1}{90}\left(5+3x^4\right)^{5/2}$$

output
```
-5/54*(3*x^4+5)^(3/2)+1/90*(3*x^4+5)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^7\sqrt{5+3x^4}\,dx=\frac{1}{270}\left(5+3x^4\right)^{3/2}\left(-10+9x^4\right)$$

input
```
Integrate[x^7*Sqrt[5 + 3*x^4],x]
```

output
```
((5 + 3*x^4)^(3/2)*(-10 + 9*x^4))/270
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{3x^4+5}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^4\sqrt{3x^4+5}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{1}{3}\left(3x^4+5\right)^{3/2}-\frac{5}{3}\sqrt{3x^4+5}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2}{45}\left(3x^4+5\right)^{5/2}-\frac{10}{27}\left(3x^4+5\right)^{3/2}\right)$$

input
```
Int[x^7*Sqrt[5 + 3*x^4],x]
```

output
```
((-10*(5 + 3*x^4)^(3/2))/27 + (2*(5 + 3*x^4)^(5/2))/45)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(3x^4+5)^{\frac{3}{2}}(9x^4-10)}{270}$	19
default	$\frac{(3x^4+5)^{\frac{3}{2}}(9x^4-10)}{270}$	19
elliptic	$\frac{(3x^4+5)^{\frac{3}{2}}(9x^4-10)}{270}$	19
pseudoelliptic	$\frac{(3x^4+5)^{\frac{3}{2}}(9x^4-10)}{270}$	19
orering	$\frac{(3x^4+5)^{\frac{3}{2}}(9x^4-10)}{270}$	19
trager	$\left(\frac{1}{10}x^8+\frac{1}{18}x^4-\frac{5}{27}\right)\sqrt{3x^4+5}$	23
risch	$\frac{(27x^8+15x^4-50)\sqrt{3x^4+5}}{270}$	24
meijerg	$-\frac{25\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1+\frac{3x^4}{5}\right)^{\frac{3}{2}}\left(-\frac{9x^4}{5}+2\right)}{15}\right)}{72\sqrt{\pi}}$	36

input
```
int(x^7*(3*x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/270*(3*x^4+5)^(3/2)*(9*x^4-10)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^7\sqrt{5+3x^4}\,dx=\frac{1}{270}\left(27\,x^8+15\,x^4-50\right)\sqrt{3\,x^4+5}$$

input
```
integrate(x^7*(3*x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/270*(27*x^8 + 15*x^4 - 50)*sqrt(3*x^4 + 5)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int x^7\sqrt{5+3x^4}\,dx=\frac{x^8\sqrt{3x^4+5}}{10}+\frac{x^4\sqrt{3x^4+5}}{18}-\frac{5\sqrt{3x^4+5}}{27}$$

input
```
integrate(x**7*(3*x**4+5)**(1/2),x)
```

output
```
x**8*sqrt(3*x**4 + 5)/10 + x**4*sqrt(3*x**4 + 5)/18 - 5*sqrt(3*x**4 + 5)/2
7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^7\sqrt{5+3x^4}\,dx=\frac{1}{90}\left(3\,x^4+5\right)^{\frac{5}{2}}-\frac{5}{54}\left(3\,x^4+5\right)^{\frac{3}{2}}$$

input
```
integrate(x^7*(3*x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/90*(3*x^4 + 5)^(5/2) - 5/54*(3*x^4 + 5)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^7\sqrt{5+3x^4}\,dx = \frac{1}{90}\left(3\,x^4+5\right)^{\frac{5}{2}} - \frac{5}{54}\left(3\,x^4+5\right)^{\frac{3}{2}}$$

input
```
integrate(x^7*(3*x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/90*(3*x^4 + 5)^(5/2) - 5/54*(3*x^4 + 5)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x^7\sqrt{5+3x^4}\,dx = \frac{\left(3\,x^4+5\right)^{3/2}\left(9\,x^4-10\right)}{270}$$

input
```
int(x^7*(3*x^4 + 5)^(1/2),x)
```

output
```
((3*x^4 + 5)^(3/2)*(9*x^4 - 10))/270
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 5.26

$$\int x^7\sqrt{5+3x^4}\,dx$$
$$= \frac{3888\sqrt{3x^4+5}\,\sqrt{3}\,x^{18}+10260\sqrt{3x^4+5}\,\sqrt{3}\,x^{14}+675\sqrt{3x^4+5}\,\sqrt{3}\,x^{10}-13125\sqrt{3x^4+5}\,\sqrt{3}\,x^6-6250\sqrt{}}{38880\sqrt{3x^4+5}\,x^8+48600\sqrt{3x^4+5}\,x^4+6750\sqrt{3x^4+5}+388}$$

input
```
int(x^7*(3*x^4+5)^(1/2),x)
```

output
```
(3888*sqrt(3*x**4 + 5)*sqrt(3)*x**18 + 10260*sqrt(3*x**4 + 5)*sqrt(3)*x**1
4 + 675*sqrt(3*x**4 + 5)*sqrt(3)*x**10 - 13125*sqrt(3*x**4 + 5)*sqrt(3)*x*
*6 - 6250*sqrt(3*x**4 + 5)*sqrt(3)*x**2 + 11664*x**20 + 40500*x**16 + 2362
5*x**12 - 45000*x**8 - 46875*x**4 - 6250)/(270*(144*sqrt(3*x**4 + 5)*x**8
+ 180*sqrt(3*x**4 + 5)*x**4 + 25*sqrt(3*x**4 + 5) + 144*sqrt(3)*x**10 + 30
0*sqrt(3)*x**6 + 125*sqrt(3)*x**2))
```

3.352 $\int x^3\sqrt{5+x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int x^3\sqrt{5+x^4}\,dx=\frac{1}{6}\left(5+x^4\right)^{3/2}$$

output
```
1/6*(x^4+5)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{5+x^4}\,dx=\frac{1}{6}\left(5+x^4\right)^{3/2}$$

input
```
Integrate[x^3*Sqrt[5 + x^4],x]
```

output
```
(5 + x^4)^(3/2)/6
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{x^4+5}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{1}{6}\left(x^4+5\right)^{3/2}$$

input
```
Int[x^3*Sqrt[5 + x^4],x]
```

output
```
(5 + x^4)^(3/2)/6
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{(x^4+5)^{\frac{3}{2}}}{6}$	10
derivativedivides	$\frac{(x^4+5)^{\frac{3}{2}}}{6}$	10
default	$\frac{(x^4+5)^{\frac{3}{2}}}{6}$	10
risch	$\frac{(x^4+5)^{\frac{3}{2}}}{6}$	10
pseudoelliptic	$\frac{(x^4+5)^{\frac{3}{2}}}{6}$	10
orering	$\frac{(x^4+5)^{\frac{3}{2}}}{6}$	10
trager	$\left(\frac{x^4}{6}+\frac{5}{6}\right)\sqrt{x^4+5}$	16
meijerg	$-\frac{5\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{8\sqrt{\pi}}$	36

input
```
int(x^3*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^4+5)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{5+x^4}\,dx=\frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^4+5)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(x^4 + 5)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int x^3\sqrt{5+x^4}\,dx = \frac{x^4\sqrt{x^4+5}}{6} + \frac{5\sqrt{x^4+5}}{6}$$

input
```
integrate(x**3*(x**4+5)**(1/2),x)
```

output
```
x**4*sqrt(x**4 + 5)/6 + 5*sqrt(x**4 + 5)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{5+x^4}\,dx = \frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^4+5)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(x^4 + 5)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{5+x^4}\,dx = \frac{1}{6}\left(x^4+5\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(x^4+5)^(1/2),x, algorithm="giac")
```

output
```
1/6*(x^4 + 5)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int x^3\sqrt{5+x^4}\,dx=\frac{(x^4+5)^{3/2}}{6}$$

input
```
int(x^3*(x^4 + 5)^(1/2),x)
```

output
```
(x^4 + 5)^(3/2)/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 6.38

$$\int x^3\sqrt{5+x^4}\,dx = \frac{4\sqrt{x^4+5}\,x^{10}+35\sqrt{x^4+5}\,x^6+75\sqrt{x^4+5}\,x^2+4x^{12}+45x^8+150x^4+125}{24\sqrt{x^4+5}\,x^4+30\sqrt{x^4+5}+24x^6+90x^2}$$

input
```
int(x^3*(x^4+5)^(1/2),x)
```

output
```
(4*sqrt(x**4 + 5)*x**10 + 35*sqrt(x**4 + 5)*x**6 + 75*sqrt(x**4 + 5)*x**2
+ 4*x**12 + 45*x**8 + 150*x**4 + 125)/(6*(4*sqrt(x**4 + 5)*x**4 + 5*sqrt(x
**4 + 5) + 4*x**6 + 15*x**2))
```

3.353 $\int x\sqrt{3+2x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int x\sqrt{3+2x^4}\,dx=\frac{1}{4}x^2\sqrt{3+2x^4}+\frac{3\operatorname{arcsinh}\left(\sqrt{\frac{2}{3}}x^2\right)}{4\sqrt{2}}$$

output
```
1/4*x^2*(2*x^4+3)^(1/2)+3/8*arcsinh(1/3*6^(1/2)*x^2)*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int x\sqrt{3+2x^4}\,dx=\frac{1}{4}x^2\sqrt{3+2x^4}+\frac{3\log\left(\sqrt{2}x^2+\sqrt{3+2x^4}\right)}{4\sqrt{2}}$$

input
```
Integrate[x*Sqrt[3 + 2*x^4],x]
```

output
```
(x^2*Sqrt[3 + 2*x^4])/4 + (3*Log[Sqrt[2]*x^2 + Sqrt[3 + 2*x^4]])/(4*Sqrt[2
])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {807, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{2x^4+3}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \sqrt{2x^4+3}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{\sqrt{2x^4+3}}dx^2+\frac{1}{2}\sqrt{2x^4+3}x^2\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{3\text{arcsinh}\left(\sqrt{\frac{2}{3}}x^2\right)}{2\sqrt{2}}+\frac{1}{2}\sqrt{2x^4+3}x^2\right)$$

input
```
Int[x*Sqrt[3 + 2*x^4],x]
```

output
```
((x^2*Sqrt[3 + 2*x^4])/2 + (3*ArcSinh[Sqrt[2/3]*x^2])/(2*Sqrt[2]))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{x^{2} \sqrt{2 x^{4}+3}}{4}+\frac{3 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x^{2}}{3}\right) \sqrt{2}}{8}$	30
risch	$\frac{x^{2} \sqrt{2 x^{4}+3}}{4}+\frac{3 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x^{2}}{3}\right) \sqrt{2}}{8}$	30
elliptic	$\frac{x^{2} \sqrt{2 x^{4}+3}}{4}+\frac{3 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x^{2}}{3}\right) \sqrt{2}}{8}$	30
pseudoelliptic	$\frac{\sqrt{2}\, \left(\sqrt{2}\, x^{2} \sqrt{2 x^{4}+3}+3 \,\operatorname{arcsinh}\left(\frac{\sqrt{6}\, x^{2}}{3}\right)\right)}{8}$	34
trager	$\frac{x^{2} \sqrt{2 x^{4}+3}}{4}+\frac{3 \operatorname{RootOf}\left(\_Z^{2}-2\right) \ln \left(\operatorname{RootOf}\left(\_Z^{2}-2\right) \sqrt{2 x^{4}+3}+2 x^{2}\right)}{8}$	47
meijerg	$-\frac{3 \sqrt{2}\, \left(-\frac{2 \sqrt{\pi}\, x^{2} \sqrt{3}\, \sqrt{2}\, \sqrt{\frac{2 x^{4}}{3}+1}}{3}-2 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{x^{2} \sqrt{3}\, \sqrt{2}}{3}\right)\right)}{16 \sqrt{\pi}}$	50

input
```
int(x*(2*x^4+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(2*x^4+3)^(1/2)+3/8*arcsinh(1/3*6^(1/2)*x^2)*2^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int x\sqrt{3+2x^4}\,dx=\frac{1}{4}\,\sqrt{2\,x^{4}+3}x^{2}+\frac{3}{16}\,\sqrt{2}\log\left(-4\,x^{4}-2\,\sqrt{2}\sqrt{2\,x^{4}+3}x^{2}-3\right)$$

input
```
integrate(x*(2*x^4+3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2*x^4 + 3)*x^2 + 3/16*sqrt(2)*log(-4*x^4 - 2*sqrt(2)*sqrt(2*x^4 +
 3)*x^2 - 3)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int x\sqrt{3+2x^4}\,dx = \frac{x^6}{2\sqrt{2x^4+3}} + \frac{3x^2}{4\sqrt{2x^4+3}} + \frac{3\sqrt{2}\operatorname{asinh}\left(\frac{\sqrt{6}x^2}{3}\right)}{8}$$

input
```
integrate(x*(2*x**4+3)**(1/2),x)
```

output
```
x**6/(2*sqrt(2*x**4 + 3)) + 3*x**2/(4*sqrt(2*x**4 + 3)) + 3*sqrt(2)*asinh(
sqrt(6)*x**2/3)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(29) = 58.

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.88

$$\int x\sqrt{3+2x^4}\,dx = -\frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2x^4+3}}{x^2}}{\sqrt{2}+\frac{\sqrt{2x^4+3}}{x^2}}\right) + \frac{3\sqrt{2x^4+3}}{4x^2\left(\frac{2x^4+3}{x^4}-2\right)}$$

input
```
integrate(x*(2*x^4+3)^(1/2),x, algorithm="maxima")
```

output
```
-3/16*sqrt(2)*log(-(sqrt(2) - sqrt(2*x^4 + 3)/x^2)/(sqrt(2) + sqrt(2*x^4 +
 3)/x^2)) + 3/4*sqrt(2*x^4 + 3)/(x^2*((2*x^4 + 3)/x^4 - 2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int x\sqrt{3+2x^4}\,dx = \frac{1}{4}\sqrt{2x^4+3}x^2 - \frac{3}{8}\sqrt{2}\log\left(-\sqrt{2}x^2 + \sqrt{2x^4+3}\right)$$

input
```
integrate(x*(2*x^4+3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2*x^4 + 3)*x^2 - 3/8*sqrt(2)*log(-sqrt(2)*x^2 + sqrt(2*x^4 + 3))
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{3+2x^4}\,dx = \int x\sqrt{2x^4+3}\,dx$$

input
```
int(x*(2*x^4 + 3)^(1/2),x)
```

output
```
int(x*(2*x^4 + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.85

$$\int x\sqrt{3+2x^4}\,dx$$
$$= \frac{6\sqrt{2x^4+3}\sqrt{2}\log\left(\frac{\sqrt{2x^4+3}+\sqrt{2}x^2}{\sqrt{3}}\right)x^2 + 4\sqrt{2x^4+3}\sqrt{2}x^6 + 3\sqrt{2x^4+3}\sqrt{2}x^2 + 12\log\left(\frac{\sqrt{2x^4+3}+\sqrt{2}x^2}{\sqrt{3}}\right)x^4}{16\sqrt{2x^4+3}x^2 + 16\sqrt{2}x^4 + 12\sqrt{2}}$$

input
```
int(x*(2*x^4+3)^(1/2),x)
```

output
```
(6*sqrt(2*x**4 + 3)*sqrt(2)*log((sqrt(2*x**4 + 3) + sqrt(2)*x**2)/sqrt(3))
*x**2 + 4*sqrt(2*x**4 + 3)*sqrt(2)*x**6 + 3*sqrt(2*x**4 + 3)*sqrt(2)*x**2
+ 12*log((sqrt(2*x**4 + 3) + sqrt(2)*x**2)/sqrt(3))*x**4 + 9*log((sqrt(2*x
**4 + 3) + sqrt(2)*x**2)/sqrt(3)) + 8*x**8 + 12*x**4)/(4*(4*sqrt(2*x**4 +
3)*x**2 + 4*sqrt(2)*x**4 + 3*sqrt(2)))
```

3.354 $\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx=\frac{a^2\sqrt{a+bx^4}}{2b^3}-\frac{a(a+bx^4)^{3/2}}{3b^3}+\frac{(a+bx^4)^{5/2}}{10b^3}$$

output
```
1/2*a^2*(b*x^4+a)^(1/2)/b^3-1/3*a*(b*x^4+a)^(3/2)/b^3+1/10*(b*x^4+a)^(5/2)
/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx=\frac{\sqrt{a+bx^4}(8a^2-4abx^4+3b^2x^8)}{30b^3}$$

input
```
Integrate[x^11/Sqrt[a + b*x^4],x]
```

output
```
(Sqrt[a + b*x^4]*(8*a^2 - 4*a*b*x^4 + 3*b^2*x^8))/(30*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{\sqrt{bx^4+a}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2\sqrt{bx^4+a}}-\frac{2\sqrt{bx^4+a}a}{b^2}+\frac{\left(bx^4+a\right)^{3/2}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2a^2\sqrt{a+bx^4}}{b^3}+\frac{2\left(a+bx^4\right)^{5/2}}{5b^3}-\frac{4a\left(a+bx^4\right)^{3/2}}{3b^3}\right)$$

input
```
Int[x^11/Sqrt[a + b*x^4],x]
```

output
```
((2*a^2*Sqrt[a + b*x^4])/b^3 - (4*a*(a + b*x^4)^(3/2))/(3*b^3) + (2*(a + b
*x^4)^(5/2))/(5*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36
default	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36
trager	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36
risch	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36
elliptic	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36
pseudoelliptic	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36
orering	$\frac{\sqrt{b\,x^4+a}\,(3b^2x^8-4ab\,x^4+8a^2)}{30b^3}$	36

input
```
int(x^11/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(b*x^4+a)^(1/2)*(3*b^2*x^8-4*a*b*x^4+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx = \frac{(3\,b^2x^8-4\,abx^4+8\,a^2)\sqrt{bx^4+a}}{30\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(3*b^2*x^8 - 4*a*b*x^4 + 8*a^2)*sqrt(b*x^4 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx = \begin{cases} \frac{4a^2\sqrt{a+bx^4}}{15b^3} - \frac{2ax^4\sqrt{a+bx^4}}{15b^2} + \frac{x^8\sqrt{a+bx^4}}{10b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**4+a)**(1/2),x)
```

output
```
Piecewise((4*a**2*sqrt(a + b*x**4)/(15*b**3) - 2*a*x**4*sqrt(a + b*x**4)/(
15*b**2) + x**8*sqrt(a + b*x**4)/(10*b), Ne(b, 0)), (x**12/(12*sqrt(a)), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{5}{2}}}{10\,b^3} - \frac{(bx^4+a)^{\frac{3}{2}}a}{3\,b^3} + \frac{\sqrt{bx^4+a}a^2}{2\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^4 + a)^(5/2)/b^3 - 1/3*(b*x^4 + a)^(3/2)*a/b^3 + 1/2*sqrt(b*x^4
+ a)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}a^2}{2\,b^3} + \frac{3\,(bx^4+a)^{\frac{5}{2}} - 10\,(bx^4+a)^{\frac{3}{2}}a}{30\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^4 + a)*a^2/b^3 + 1/30*(3*(b*x^4 + a)^(5/2) - 10*(b*x^4 + a)^(
3/2)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx = \sqrt{b\,x^4+a}\,\left(\frac{4\,a^2}{15\,b^3} + \frac{x^8}{10\,b} - \frac{2\,a\,x^4}{15\,b^2}\right)$$

input
```
int(x^11/(a + b*x^4)^(1/2),x)
```

output
```
(a + b*x^4)^(1/2)*((4*a^2)/(15*b^3) + x^8/(10*b) - (2*a*x^4)/(15*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.88

$$\int \frac{x^{11}}{\sqrt{a+bx^4}}\,dx = \frac{40\sqrt{b}\,\sqrt{b\,x^4+a}\,a^4x^2 + 140\sqrt{b}\,\sqrt{b\,x^4+a}\,a^3b\,x^6 + 63\sqrt{b}\,\sqrt{b\,x^4+a}\,a^2b^2x^{10} - 4\sqrt{b}\,\sqrt{b\,x^4+a}\,a\,b^3x^{14} + 48}{30b^3\left(\sqrt{b\,x^4+a}\,a^2 + 12\sqrt{b\,x^4+a}\,ab\,x^4 + 16\sqrt{b\,x^4+a}\,b^2x^8\right.}$$

input
```
int(x^11/(b*x^4+a)^(1/2),x)
```

output

```
(40*sqrt(b)*sqrt(a + b*x**4)*a**4*x**2 + 140*sqrt(b)*sqrt(a + b*x**4)*a**3
*b*x**6 + 63*sqrt(b)*sqrt(a + b*x**4)*a**2*b**2*x**10 - 4*sqrt(b)*sqrt(a +
 b*x**4)*a*b**3*x**14 + 48*sqrt(b)*sqrt(a + b*x**4)*b**4*x**18 + 8*a**5 +
100*a**4*b*x**4 + 175*a**3*b**2*x**8 + 55*a**2*b**3*x**12 + 20*a*b**4*x**1
6 + 48*b**5*x**20)/(30*b**3*(sqrt(a + b*x**4)*a**2 + 12*sqrt(a + b*x**4)*a
*b*x**4 + 16*sqrt(a + b*x**4)*b**2*x**8 + 5*sqrt(b)*a**2*x**2 + 20*sqrt(b)
*a*b*x**6 + 16*sqrt(b)*b**2*x**10))
```

3.355 $\int \frac{x^7}{\sqrt{a+bx^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^7}{\sqrt{a+bx^4}} \, dx = -\frac{a\sqrt{a+bx^4}}{2b^2} + \frac{(a+bx^4)^{3/2}}{6b^2}$$

output
```
-1/2*a*(b*x^4+a)^(1/2)/b^2+1/6*(b*x^4+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{\sqrt{a+bx^4}} \, dx = \frac{(-2a+bx^4)\sqrt{a+bx^4}}{6b^2}$$

input
```
Integrate[x^7/Sqrt[a + b*x^4],x]
```

output
```
((-2*a + b*x^4)*Sqrt[a + b*x^4])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt{bx^4+a}}dx^4$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{4}\int \left(\frac{\sqrt{bx^4+a}}{b}-\frac{a}{b\sqrt{bx^4+a}}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{2\left(a+bx^4\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx^4}}{b^2}\right)$$

input
```
Int[x^7/Sqrt[a + b*x^4],x]
```

output
```
((-2*a*Sqrt[a + b*x^4])/b^2 + (2*(a + b*x^4)^(3/2))/(3*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
pseudoelliptic	$\frac{(b\,x^4-2a)\sqrt{b\,x^4+a}}{6b^2}$	24
gosper	$-\frac{\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6b^2}$	25
default	$-\frac{\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6b^2}$	25
trager	$-\frac{\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6b^2}$	25
risch	$-\frac{\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6b^2}$	25
elliptic	$-\frac{\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6b^2}$	25
orering	$-\frac{\sqrt{b\,x^4+a}\,(-b\,x^4+2a)}{6b^2}$	25

input
```
int(x^7/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^4-2*a)*(b*x^4+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}(bx^4-2\,a)}{6\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(b*x^4 + a)*(b*x^4 - 2*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx = \begin{cases} -\frac{a\sqrt{a+bx^4}}{3b^2} + \frac{x^4\sqrt{a+bx^4}}{6b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**4+a)**(1/2),x)
```

output
```
Piecewise((-a*sqrt(a + b*x**4)/(3*b**2) + x**4*sqrt(a + b*x**4)/(6*b), Ne(
b, 0)), (x**8/(8*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{3}{2}}}{6\,b^2} - \frac{\sqrt{bx^4+a}a}{2\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(b*x^4 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{3}{2}}}{6\,b^2} - \frac{\sqrt{bx^4+a}a}{2\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*(b*x^4 + a)^(3/2)/b^2 - 1/2*sqrt(b*x^4 + a)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b\,x^4+a}\,(2\,a-b\,x^4)}{6\,b^2}$$

input
```
int(x^7/(a + b*x^4)^(1/2),x)
```

output
```
-((a + b*x^4)^(1/2)*(2*a - b*x^4))/(6*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.50

$$\int \frac{x^7}{\sqrt{a+bx^4}}\,dx = \frac{-6\sqrt{b}\,\sqrt{b\,x^4+a}\,a^2x^2-5\sqrt{b}\,\sqrt{b\,x^4+a}\,ab\,x^6+4\sqrt{b}\,\sqrt{b\,x^4+a}\,b^2x^{10}-2a^3-9a^2b\,x^4-3a\,b^2x^8+4b^3x^{12}}{6b^2\left(\sqrt{b\,x^4+a}\,a+4\sqrt{b\,x^4+a}\,b\,x^4+3\sqrt{b}\,a\,x^2+4\sqrt{b}\,b\,x^6\right)}$$

input
```
int(x^7/(b*x^4+a)^(1/2),x)
```

output
```
( - 6*sqrt(b)*sqrt(a + b*x**4)*a**2*x**2 - 5*sqrt(b)*sqrt(a + b*x**4)*a*b*
x**6 + 4*sqrt(b)*sqrt(a + b*x**4)*b**2*x**10 - 2*a**3 - 9*a**2*b*x**4 - 3*
a*b**2*x**8 + 4*b**3*x**12)/(6*b**2*(sqrt(a + b*x**4)*a + 4*sqrt(a + b*x**
4)*b*x**4 + 3*sqrt(b)*a*x**2 + 4*sqrt(b)*b*x**6))
```

3.356 $\int \frac{x^3}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx=\frac{\sqrt{a+bx^4}}{2b}$$

output
```
1/2*(b*x^4+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx=\frac{\sqrt{a+bx^4}}{2b}$$

input
```
Integrate[x^3/Sqrt[a + b*x^4],x]
```

output
```
Sqrt[a + b*x^4]/(2*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{\sqrt{a+bx^4}}{2b}$$

input
```
Int[x^3/Sqrt[a + b*x^4],x]
```

output
```
Sqrt[a + b*x^4]/(2*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
derivativedivides	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
default	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
trager	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
risch	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
elliptic	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
pseudoelliptic	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15
orering	$\frac{\sqrt{b\,x^{4}+a}}{2b}$	15

input
```
int(x^3/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^4+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}}{2\,b}$$

input
```
integrate(x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \begin{cases} \frac{\sqrt{a+bx^4}}{2b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**4)/(2*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}}{2\,b}$$

input
```
integrate(x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^4 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}}{2\,b}$$

input
```
integrate(x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^4 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b\,x^4+a}}{2\,b}$$

input
```
int(x^3/(a + b*x^4)^(1/2),x)
```

output
```
(a + b*x^4)^(1/2)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int \frac{x^3}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}{2b\left(\sqrt{b\,x^4+a}+\sqrt{b}\,x^2\right)}$$

input
```
int(x^3/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)/(2*b*(sqrt(a + b*x**4) + sqrt
(b)*x**2))
```

3.357 $\int \frac{1}{x\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^4}}dx$$
$$\Big\downarrow \; 798$$
$$\frac{1}{4}\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4$$
$$\Big\downarrow \; 73$$
$$\frac{\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{2b}$$
$$\Big\downarrow \; 221$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/Sqrt[a]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$	20
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}$	29
elliptic	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2\sqrt{a}}$	29

input
```
int(1/x/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.22

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = \left[\frac{\log\left(\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2\,a}{x^4}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^4+a}}\right)}{2\,a}\right]$$

input
```
integrate(1/x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log((b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4)/sqrt(a), 1/2*sqrt(
-a)*arctan(sqrt(-a)/sqrt(b*x^4 + a))/a]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{2\sqrt{a}}$$

input
```
integrate(1/x/(b*x**4+a)**(1/2),x)
```

output
```
-asinh(sqrt(a)/(sqrt(b)*x**2))/(2*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = \frac{\log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{4\sqrt{a}}$$

input
```
integrate(1/x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

input
```
integrate(1/x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(b*x^4 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^4)^(1/2)),x)
```

output
```
-atanh((a + b*x^4)^(1/2)/a^(1/2))/(2*a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{x\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}\left(\log\left(\frac{\sqrt{b\,x^4+a}-\sqrt{a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)-\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)\right)}{2a}$$

input
```
int(1/x/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a)*(log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a)) - log((
sqrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))))/(2*a)
```

3.358 $\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{4ax^4}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}$$

output
```
-1/4*(b*x^4+a)^(1/2)/a/x^4+1/4*b*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{4ax^4}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[1/(x^5*Sqrt[a + b*x^4]),x]
```

output
```
-1/4*Sqrt[a + b*x^4]/(a*x^4) + (b*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/(4*a^(
3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8\sqrt{bx^4+a}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(-\frac{b\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4}{2a}-\frac{\sqrt{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{a}-\frac{\sqrt{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^4}}{ax^4}\right)$$

input
```
Int[1/(x^5*Sqrt[a + b*x^4]),x]
```

output
```
(-(Sqrt[a + b*x^4]/(a*x^4)) + (b*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/a^(3/2)
)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{a}}\right) b\,x^{4}-\sqrt{a}\,\sqrt{b\,x^{4}+a}}{4 a^{\frac{3}{2}} x^{4}}$	43
default	$-\frac{\sqrt{b\,x^{4}+a}}{4 a\,x^{4}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4 a^{\frac{3}{2}}}$	48
risch	$-\frac{\sqrt{b\,x^{4}+a}}{4 a\,x^{4}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4 a^{\frac{3}{2}}}$	48
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{4 a\,x^{4}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4 a^{\frac{3}{2}}}$	48

input
```
int(1/x^5/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctanh((b*x^4+a)^(1/2)/a^(1/2))*b*x^4-a^(1/2)*(b*x^4+a)^(1/2))/a^(3/
2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = \left[\frac{\sqrt{a}bx^4\log\left(\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-2\sqrt{bx^4+a}a}{8a^2x^4}, -\frac{\sqrt{-a}bx^4\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^4+a}}\right)+\sqrt{bx^4+a}a}{4a^2x^4}\right]$$

input
```
integrate(1/x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*x^4*log((b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^4) - 2
*sqrt(b*x^4 + a)*a)/(a^2*x^4), -1/4*(sqrt(-a)*b*x^4*arctan(sqrt(-a)/sqrt(b
*x^4 + a)) + sqrt(b*x^4 + a)*a)/(a^2*x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}+1}}{4ax^2}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{3}{2}}}$$

input
```
integrate(1/x**5/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**4) + 1)/(4*a*x**2) + b*asinh(sqrt(a)/(sqrt(b)*x**2))
/(4*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{bx^4+a}\,b}{4\,((bx^4+a)a-a^2)} - \frac{b\log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}$$

input
```
integrate(1/x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(b*x^4 + a)*b/((b*x^4 + a)*a - a^2) - 1/8*b*log((sqrt(b*x^4 + a)
- sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = -\frac{1}{4}\,b\left(\frac{\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx^4+a}}{abx^4}\right)$$

input
```
integrate(1/x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*b*(arctan(sqrt(b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^4 + a)/(a
*b*x^4))
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{4\,a^{3/2}} - \frac{\sqrt{b\,x^4+a}}{4\,a\,x^4}$$

input
```
int(1/(x^5*(a + b*x^4)^(1/2)),x)
```

output
```
(b*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x^4)^(1/2)/(4*a*
x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.84

$$\int \frac{1}{x^5\sqrt{a+bx^4}}\,dx = \frac{-\sqrt{a}\,\sqrt{b\,x^4+a}\,a-2\sqrt{a}\,\sqrt{b\,x^4+a}\,b\,x^4-2\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\frac{\sqrt{b\,x^4+a}-\sqrt{a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)b\,x^6+2\sqrt{b}\,\sqrt{b\,x^4+a}\log}{}$$

input
```
int(1/x^5/(b*x^4+a)^(1/2),x)
```

output
```
( - sqrt(a)*sqrt(a + b*x**4)*a - 2*sqrt(a)*sqrt(a + b*x**4)*b*x**4 - 2*sqr
t(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt
(a))*b*x**6 + 2*sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) + sqrt(a) +
 sqrt(b)*x**2)/sqrt(a))*b*x**6 - 2*sqrt(b)*sqrt(a)*a*x**2 - 2*sqrt(b)*sqrt
(a)*b*x**6 - log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a*b*
x**4 - 2*log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b**2*x**
8 + log((sqrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a*b*x**4 + 2*
log((sqrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b**2*x**8)/(4*sqr
t(a)*a*x**4*(2*sqrt(b)*sqrt(a + b*x**4)*x**2 + a + 2*b*x**4))
```

3.359 $\int \frac{x^5}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{3/2}}$$

output
```
1/4*x^2*(b*x^4+a)^(1/2)/b-1/4*a*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/
2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \frac{x^2\sqrt{a+bx^4}}{4b} - \frac{a\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{4b^{3/2}}$$

input
```
Integrate[x^5/Sqrt[a + b*x^4],x]
```

output
```
(x^2*Sqrt[a + b*x^4])/(4*b) - (a*Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]])/(4*b^
(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {807, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{bx^4+a}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^4}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx^2}{2b}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^4}}{2b}-\frac{a\int \frac{1}{1-bx^4}d\frac{x^2}{\sqrt{bx^4+a}}}{2b}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{x^2\sqrt{a+bx^4}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^4],x]
```

output
```
((x^2*Sqrt[a + b*x^4])/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/
(2*b^(3/2)))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b}-\frac{a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{3}{2}}}$	43
risch	$\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b}-\frac{a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{3}{2}}}$	43
elliptic	$\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b}-\frac{a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{3}{2}}}$	43
pseudoelliptic	$-\frac{-\sqrt{b\,x^{4}+a}\,x^{2}\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)a}{4b^{\frac{3}{2}}}$	43

input
```
int(x^5/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(b*x^4+a)^(1/2)/b-1/4*a/b^(3/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.96

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \left[\frac{2\sqrt{bx^4+a}bx^2+a\sqrt{b}\log\left(-2bx^4+2\sqrt{bx^4+a}\sqrt{b}x^2-a\right)}{8b^2}, \frac{\sqrt{bx^4+a}bx^2+a\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+a}\sqrt{-b}}{bx^2}\right)}{4b^2}\right]$$

input
```
integrate(x^5/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(2*sqrt(b*x^4 + a)*b*x^2 + a*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)
*sqrt(b)*x^2 - a))/b^2, 1/4*(sqrt(b*x^4 + a)*b*x^2 + a*sqrt(-b)*arctan(sqr
t(b*x^4 + a)*sqrt(-b)/(b*x^2)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}x^2\sqrt{1+\frac{bx^4}{a}}}{4b} - \frac{a\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}$$

input
```
integrate(x**5/(b*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x**2*sqrt(1 + b*x**4/a)/(4*b) - a*asinh(sqrt(b)*x**2/sqrt(a))/(4*b
**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.53

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{8\,b^{\frac{3}{2}}} - \frac{\sqrt{bx^4+a}a}{4\left(b^2-\frac{(bx^4+a)b}{x^4}\right)x^2}$$

input
```
integrate(x^5/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2)
)/b^(3/2) - 1/4*sqrt(b*x^4 + a)*a/((b^2 - (b*x^4 + a)*b/x^4)*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{bx^4+a}x^2}{4\,b} + \frac{a\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{4\,b^{\frac{3}{2}}}$$

input
```
integrate(x^5/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^4 + a)*x^2/b + 1/4*a*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)))
/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \int \frac{x^5}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x^5/(a + b*x^4)^(1/2),x)
```

output
```
int(x^5/(a + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.25

$$\int \frac{x^5}{\sqrt{a+bx^4}}\,dx = \frac{-2\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)a\,x^2+\sqrt{b}\,\sqrt{b\,x^4+a}\,a\,x^2+2\sqrt{b}\,\sqrt{b\,x^4+a}\,b\,x^6-\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)a}{4b\left(2\sqrt{b\,x^4+a}\,b\,x^2+\sqrt{b}\,a+2\sqrt{b}\,b\,x^4\right)}$$

input
```
int(x^5/(b*x^4+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(
a))*a*x**2 + sqrt(b)*sqrt(a + b*x**4)*a*x**2 + 2*sqrt(b)*sqrt(a + b*x**4)*
b*x**6 - log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))*a**2 - 2*log((sqrt
(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))*a*b*x**4 + 2*a*b*x**4 + 2*b**2*x**8)
/(4*b*(2*sqrt(a + b*x**4)*b*x**2 + sqrt(b)*a + 2*sqrt(b)*b*x**4))
```

3.360 $\int \frac{x}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

output
```
1/2*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx=\frac{\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{2\sqrt{b}}$$

input
```
Integrate[x/Sqrt[a + b*x^4],x]
```

output
```
Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]]/(2*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^4}}dx$$
$$\Big\downarrow \; 807$$
$$\frac{1}{2}\int \frac{1}{\sqrt{bx^4+a}}dx^2$$
$$\Big\downarrow \; 224$$
$$\frac{1}{2}\int \frac{1}{1-bx^4}d\frac{x^2}{\sqrt{bx^4+a}}$$
$$\Big\downarrow \; 219$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}}$$

input
```
Int[x/Sqrt[a + b*x^4],x]
```

output
```
ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/(2*Sqrt[b])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)}{2\sqrt{b}}$	23
default	$\frac{\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}$	24
elliptic	$\frac{\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{2\sqrt{b}}$	24

input
```
int(x/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^(1/2)*arctanh(1/b^(1/2)/x^2*(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.20

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx = \left[\frac{\log\left(-2\,bx^4 - 2\,\sqrt{bx^4+a}\sqrt{b}x^2 - a\right)}{4\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+a}\sqrt{-b}}{bx^2}\right)}{2\,b}\right]$$

input
```
integrate(x/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(-2*b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(b)*x^2 - a)/sqrt(b), -1/2*sqrt(
-b)*arctan(sqrt(b*x^4 + a)*sqrt(-b)/(b*x^2))/b]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
integrate(x/(b*x**4+a)**(1/2),x)
```

output
```
asinh(sqrt(b)*x**2/sqrt(a))/(2*sqrt(b))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx = -\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{4\sqrt{b}}$$

input
```
integrate(x/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt(b*x^4 + a)/x^2))
/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx = -\frac{\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate(x/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)))/sqrt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx=\int \frac{x}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x/(a + b*x^4)^(1/2),x)
```

output
```
int(x/(a + b*x^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{x}{\sqrt{a+bx^4}}\,dx=\frac{\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
int(x/(b*x^4+a)^(1/2),x)
```

output
```
log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))/(2*sqrt(b))
```

3.361 $\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{2ax^2}$$

output
```
-1/2*(b*x^4+a)^(1/2)/a/x^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{2ax^2}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*Sqrt[a + b*x^4]/(a*x^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{796}$$
$$-\frac{\sqrt{a+bx^4}}{2ax^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*x^4]),x]
```

output
```
-1/2*Sqrt[a + b*x^4]/(a*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18
default	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18
trager	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18
risch	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18
pseudoelliptic	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18
orering	$-\frac{\sqrt{b\,x^{4}+a}}{2a\,x^{2}}$	18

input
```
int(1/x^3/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^4+a)^(1/2)/a/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{bx^4+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*x^4 + a)/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}+1}}{2a}$$

input
```
integrate(1/x**3/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**4) + 1)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{bx^4+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^4 + a)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b}}{\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2-a}$$

input
```
integrate(1/x^3/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b)/((sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b\,x^4+a}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + b*x^4)^(1/2)),x)
```

output
```
-(a + b*x^4)^(1/2)/(2*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{1}{x^3\sqrt{a+bx^4}}\,dx = \frac{-2\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2-a-2b\,x^4}{2a\,x^2\left(\sqrt{b\,x^4+a}+\sqrt{b}\,x^2\right)}$$

input
```
int(1/x^3/(b*x^4+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(a + b*x**4)*x**2 - a - 2*b*x**4)/(2*a*x**2*(sqrt(a + b*
x**4) + sqrt(b)*x**2))
```

3.362 $\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{6ax^6}+\frac{b\sqrt{a+bx^4}}{3a^2x^2}$$

output
```
-1/6*(b*x^4+a)^(1/2)/a/x^6+1/3*b*(b*x^4+a)^(1/2)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a+bx^4}(-a+2bx^4)}{6a^2x^6}$$

input
```
Integrate[1/(x^7*Sqrt[a + b*x^4]),x]
```

output
```
(Sqrt[a + b*x^4]*(-a + 2*b*x^4))/(6*a^2*x^6)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \; 803$$

$$-\frac{2b\int \frac{1}{x^3\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{6ax^6}$$

$$\Big\downarrow \; 796$$

$$\frac{b\sqrt{a+bx^4}}{3a^2x^2}-\frac{\sqrt{a+bx^4}}{6ax^6}$$

input
```
Int[1/(x^7*Sqrt[a + b*x^4]),x]
```

output
```
-1/6*Sqrt[a + b*x^4]/(a*x^6) + (b*Sqrt[a + b*x^4])/(3*a^2*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26
default	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26
trager	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26
elliptic	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26
pseudoelliptic	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26
orering	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-2b\,x^{4}+a\right)}{6a^{2}x^{6}}$	26

input
```
int(1/x^7/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^4+a)^(1/2)*(-2*b*x^4+a)/a^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = \frac{(2\,bx^4-a)\sqrt{bx^4+a}}{6\,a^2x^6}$$

input
```
integrate(1/x^7/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*b*x^4 - a)*sqrt(b*x^4 + a)/(a^2*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^4}+1}}{6ax^4} + \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^4}+1}}{3a^2}$$

input
```
integrate(1/x**7/(b*x**4+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**4) + 1)/(6*a*x**4) + b**(3/2)*sqrt(a/(b*x**4) + 1)/(
3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = \frac{\frac{3\sqrt{bx^4+a}b}{x^2} - \frac{(bx^4+a)^{\frac{3}{2}}}{x^6}}{6\,a^2}$$

input
```
integrate(1/x^7/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(3*sqrt(b*x^4 + a)*b/x^2 - (b*x^4 + a)^(3/2)/x^6)/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = \frac{2\left(3\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2-a\right)^3}$$

input
```
integrate(1/x^7/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)*b^(3/2)/((sqrt(b)*x^2 - sqrt
(b*x^4 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b\,x^4+a}\,(a-2\,b\,x^4)}{6\,a^2\,x^6}$$

input
```
int(1/(x^7*(a + b*x^4)^(1/2)),x)
```

output
```
-((a + b*x^4)^(1/2)*(a - 2*b*x^4))/(6*a^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.66

$$\int \frac{1}{x^7\sqrt{a+bx^4}}\,dx = \frac{-3\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2-a-3b\,x^4}{6x^6\left(\sqrt{b\,x^4+a}\,a+4\sqrt{b\,x^4+a}\,b\,x^4+3\sqrt{b}\,a\,x^2+4\sqrt{b}\,b\,x^6\right)}$$

input
```
int(1/x^7/(b*x^4+a)^(1/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(a + b*x**4)*x**2 - a - 3*b*x**4)/(6*x**6*(sqrt(a + b*x*
*4)*a + 4*sqrt(a + b*x**4)*b*x**4 + 3*sqrt(b)*a*x**2 + 4*sqrt(b)*b*x**6))
```

3.363 $\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{10ax^{10}}+\frac{2b\sqrt{a+bx^4}}{15a^2x^6}-\frac{4b^2\sqrt{a+bx^4}}{15a^3x^2}$$

output
```
-1/10*(b*x^4+a)^(1/2)/a/x^10+2/15*b*(b*x^4+a)^(1/2)/a^2/x^6-4/15*b^2*(b*x^
4+a)^(1/2)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a+bx^4}(-3a^2+4abx^4-8b^2x^8)}{30a^3x^{10}}$$

input
```
Integrate[1/(x^11*Sqrt[a + b*x^4]),x]
```

output
```
(Sqrt[a + b*x^4]*(-3*a^2 + 4*a*b*x^4 - 8*b^2*x^8))/(30*a^3*x^10)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\int \frac{1}{x^7\sqrt{bx^4+a}}dx}{5a}-\frac{\sqrt{a+bx^4}}{10ax^{10}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{x^3\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{6ax^6}\right)}{5a}-\frac{\sqrt{a+bx^4}}{10ax^{10}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{4b\left(\frac{b\sqrt{a+bx^4}}{3a^2x^2}-\frac{\sqrt{a+bx^4}}{6ax^6}\right)}{5a}-\frac{\sqrt{a+bx^4}}{10ax^{10}}$$

input
```
Int[1/(x^11*Sqrt[a + b*x^4]),x]
```

output
```
-1/10*Sqrt[a + b*x^4]/(a*x^10) - (4*b*(-1/6*Sqrt[a + b*x^4]/(a*x^6) + (b*S
qrt[a + b*x^4])/(3*a^2*x^2)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39
default	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39
trager	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39
elliptic	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39
pseudoelliptic	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39
orering	$-\frac{\sqrt{b\,x^{4}+a}\,\left(8b^{2}x^{8}-4ab\,x^{4}+3a^{2}\right)}{30a^{3}x^{10}}$	39

input
```
int(1/x^11/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(b*x^4+a)^(1/2)*(8*b^2*x^8-4*a*b*x^4+3*a^2)/a^3/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = -\frac{(8\,b^2x^8-4\,abx^4+3\,a^2)\sqrt{bx^4+a}}{30\,a^3x^{10}}$$

input
```
integrate(1/x^11/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(8*b^2*x^8 - 4*a*b*x^4 + 3*a^2)*sqrt(b*x^4 + a)/(a^3*x^10)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(61) = 122.

Time = 0.77 (sec) , antiderivative size = 298, normalized size of antiderivative = 4.38

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = -\frac{3a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}} - \frac{2a^3b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}} - \frac{3a^2b^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}} - \frac{12ab^{\frac{15}{2}}x^{12}\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}} - \frac{8b^{\frac{17}{2}}x^{16}\sqrt{\frac{a}{bx^4}+1}}{30a^5b^4x^8+60a^4b^5x^{12}+30a^3b^6x^{16}}$$

input
```
integrate(1/x**11/(b*x**4+a)**(1/2),x)
```

output
```
-3*a**4*b**(9/2)*sqrt(a/(b*x**4) + 1)/(30*a**5*b**4*x**8 + 60*a**4*b**5*x*
*12 + 30*a**3*b**6*x**16) - 2*a**3*b**(11/2)*x**4*sqrt(a/(b*x**4) + 1)/(30
*a**5*b**4*x**8 + 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 3*a**2*b**(13
/2)*x**8*sqrt(a/(b*x**4) + 1)/(30*a**5*b**4*x**8 + 60*a**4*b**5*x**12 + 30
*a**3*b**6*x**16) - 12*a*b**(15/2)*x**12*sqrt(a/(b*x**4) + 1)/(30*a**5*b**
4*x**8 + 60*a**4*b**5*x**12 + 30*a**3*b**6*x**16) - 8*b**(17/2)*x**16*sqrt
(a/(b*x**4) + 1)/(30*a**5*b**4*x**8 + 60*a**4*b**5*x**12 + 30*a**3*b**6*x*
*16)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = -\frac{\frac{15\sqrt{bx^4+a}b^2}{x^2} - \frac{10\left(bx^4+a\right)^{\frac{3}{2}}b}{x^6} + \frac{3\left(bx^4+a\right)^{\frac{5}{2}}}{x^{10}}}{30\,a^3}$$

input
```
integrate(1/x^11/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/30*(15*sqrt(b*x^4 + a)*b^2/x^2 - 10*(b*x^4 + a)^(3/2)*b/x^6 + 3*(b*x^4
+ a)^(5/2)/x^10)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = \frac{8\left(10\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^4 - 5\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2 a + a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2 - a\right)^5}$$

input
```
integrate(1/x^11/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
8/15*(10*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^4 - 5*(sqrt(b)*x^2 - sqrt(b*x^4 +
 a))^2*a + a^2)*b^(5/2)/((sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b\,x^4+a}\,\left(3\,a^2-4\,a\,b\,x^4+8\,b^2\,x^8\right)}{30\,a^3\,x^{10}}$$

input
```
int(1/(x^11*(a + b*x^4)^(1/2)),x)
```

output
```
-((a + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^8 - 4*a*b*x^4))/(30*a^3*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.96

$$\int \frac{1}{x^{11}\sqrt{a+bx^4}}\,dx = \frac{-15\sqrt{b}\,\sqrt{b\,x^4+a}\,a\,x^2-40\sqrt{b}\,\sqrt{b\,x^4+a}\,b\,x^6-3a^2-35ab\,x^4-40b^2x^8}{30x^{10}\left(\sqrt{b\,x^4+a}\,a^2+12\sqrt{b\,x^4+a}\,ab\,x^4+16\sqrt{b\,x^4+a}\,b^2x^8+5\sqrt{b}\,a^2x^2+20\sqrt{b}\,ab\,x^6+16\sqrt{b}\,b^2x^{10}\right)}$$

input
```
int(1/x^11/(b*x^4+a)^(1/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a + b*x**4)*a*x**2 - 40*sqrt(b)*sqrt(a + b*x**4)*b*x**
6 - 3*a**2 - 35*a*b*x**4 - 40*b**2*x**8)/(30*x**10*(sqrt(a + b*x**4)*a**2
+ 12*sqrt(a + b*x**4)*a*b*x**4 + 16*sqrt(a + b*x**4)*b**2*x**8 + 5*sqrt(b)
*a**2*x**2 + 20*sqrt(b)*a*b*x**6 + 16*sqrt(b)*b**2*x**10))
```

3.364 $\int \frac{x^8}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 130

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = -\frac{5ax\sqrt{a+bx^4}}{21b^2}+\frac{x^5\sqrt{a+bx^4}}{7b} + \frac{5a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42b^{9/4}\sqrt{a+bx^4}}$$

output
```
-5/21*a*x*(b*x^4+a)^(1/2)/b^2+1/7*x^5*(b*x^4+a)^(1/2)/b+5/42*a^(7/4)*(a^(1
/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM
(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \frac{-5a^2x-2abx^5+3b^2x^9+5a^2x\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{21b^2\sqrt{a+bx^4}}$$

input
```
Integrate[x^8/Sqrt[a + b*x^4],x]
```

output
```
(-5*a^2*x - 2*a*b*x^5 + 3*b^2*x^9 + 5*a^2*x*Sqrt[1 + (b*x^4)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(21*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{843, 843, 761\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\int \frac{x^4}{\sqrt{bx^4+a}}dx}{7b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{761}$$

$$\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{7b}$$

input
```
Int[x^8/Sqrt[a + b*x^4],x]
```

output
```
(x^5*Sqrt[a + b*x^4])/(7*b) - (5*a*((x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])))/(7*b
)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

method	result	size
risch	$-\frac{x\left(-3b\,x^{4}+5a\right)\sqrt{b\,x^{4}+a}}{21b^{2}}+\frac{5a^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	103
default	$\frac{x^{5}\sqrt{b\,x^{4}+a}}{7b}-\frac{5ax\sqrt{b\,x^{4}+a}}{21b^{2}}+\frac{5a^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	111
elliptic	$\frac{x^{5}\sqrt{b\,x^{4}+a}}{7b}-\frac{5ax\sqrt{b\,x^{4}+a}}{21b^{2}}+\frac{5a^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21b^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	111

input
```
int(x^8/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*x*(-3*b*x^4+5*a)/b^2*(b*x^4+a)^(1/2)+5/21*a^2/b^2/(I/a^(1/2)*b^(1/2)
)^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \frac{5\,a\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+(3\,bx^5-5\,ax)\sqrt{bx^4+a}}{21\,b^2}$$

input
```
integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(5*a*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + (3
*b*x^5 - 5*a*x)*sqrt(b*x^4 + a))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.28

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(b*x**4+a)**(1/2),x)
```

output
```
x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \int \frac{x^8}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \int \frac{x^8}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \int \frac{x^8}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x^8/(a + b*x^4)^(1/2),x)
```

output
```
int(x^8/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{a+bx^4}}\,dx = \frac{-5\sqrt{b\,x^4+a}\,ax+3\sqrt{b\,x^4+a}\,b\,x^5+5\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)a^2}{21b^2}$$

input
```
int(x^8/(b*x^4+a)^(1/2),x)
```

output
```
( - 5*sqrt(a + b*x**4)*a*x + 3*sqrt(a + b*x**4)*b*x**5 + 5*int(sqrt(a + b*
x**4)/(a + b*x**4),x)*a**2)/(21*b**2)
```

3.365 $\int \frac{x^4}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \frac{x\sqrt{a+bx^4}}{3b} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

output
```
1/3*x*(b*x^4+a)^(1/2)/b-1/6*a^(3/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1
/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*
2^(1/2))/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \frac{x\left(a+bx^4-a\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{3b\sqrt{a+bx^4}}$$

input
```
Integrate[x^4/Sqrt[a + b*x^4],x]
```

output
```
(x*(a + b*x^4 - a*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((
b*x^4)/a)]))/(3*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}$$

input
```
Int[x^4/Sqrt[a + b*x^4],x]
```

output
```
(x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(6*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{x\sqrt{b\,x^{4}+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	91
risch	$\frac{x\sqrt{b\,x^{4}+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	91
elliptic	$\frac{x\sqrt{b\,x^{4}+a}}{3b}-\frac{a\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	91

input
```
int(x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^4+a)^(1/2)/b-1/3*a/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2
/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}} F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right) \mid -1\right) - \sqrt{bx^4+a}\,x}{3\,b}$$

input
```
integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - sqrt(b
*x^4 + a)*x)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(b*x**4+a)**(1/2),x)
```

output
```
x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \int \frac{x^4}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \int \frac{x^4}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x^4/(a + b*x^4)^(1/2),x)
```

output
```
int(x^4/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b\,x^4+a}\,x-\left(\int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx\right)a}{3b}$$

input
```
int(x^4/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**4)*x - int(sqrt(a + b*x**4)/(a + b*x**4),x)*a)/(3*b)
```

3.366 $\int \frac{1}{\sqrt{a+bx^4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+bx^4}} \, dx = \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
1/2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Invers
eJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(1/4)/b^(1/4)/(b*x^4+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+bx^4}} \, dx = -\frac{i\sqrt{1+\frac{bx^4}{a}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+bx^4}}$$

input
```
Integrate[1/Sqrt[a + b*x^4],x]
```

output
```
((-I)*Sqrt[1 + (b*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{761\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx$$
$$\Big\downarrow \text{761}$$
$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{a+bx^4}}$$

input
```
Int[1/Sqrt[a + b*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[a + b*x^
4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	70

input
```
int(1/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x
^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right) \mid -1\right)}{b}$$

input
```
integrate(1/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(b*x**4+a)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^4 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx = \frac{x\,\sqrt{\frac{b\,x^4}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\sqrt{b\,x^4+a}}$$

input
```
int(1/(a + b*x^4)^(1/2),x)
```

output
```
(x*((b*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^4)/a))/(a + b*x^
4)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+bx^4}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b\,x^4+a}dx$$

input
```
int(1/(b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a + b*x**4),x)
```

3.367 $\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{3ax^3} - \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/3*(b*x^4+a)^(1/2)/a/x^3-1/6*b^(3/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a
^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1
/2*2^(1/2))/a^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^4}{a}\right)}{3x^3\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
-1/3*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^4)/a)])
/(x^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x^4]),x]
```

output
```
-1/3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a +
b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)],
 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	93
risch	$-\frac{\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	93
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{3a\,x^{3}}-\frac{b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{3a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	93

input
```
int(1/x^4/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^4+a)^(1/2)/a/x^3-1/3*b/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*
x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = \frac{\sqrt{a}x^3\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right)-\sqrt{bx^4+a}}{3\,ax^3}$$

input
```
integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(a)*x^3*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) - sqr
t(b*x^4 + a))/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(b*x**4+a)**(1/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{1}{x^4\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int(1/(x^4*(a + b*x^4)^(1/2)),x)
```

output
```
int(1/(x^4*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{a+bx^4}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b\,x^8+a\,x^4}dx$$

input
```
int(1/x^4/(b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a*x**4 + b*x**8),x)
```

3.368 $\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 132

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{7ax^7}+\frac{5b\sqrt{a+bx^4}}{21a^2x^3} + \frac{5b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42a^{9/4}\sqrt{a+bx^4}}$$

output
```
-1/7*(b*x^4+a)^(1/2)/a/x^7+5/21*b*(b*x^4+a)^(1/2)/a^2/x^3+5/42*b^(7/4)*(a^
(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobi
AM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)}{7x^7\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^8*Sqrt[a + b*x^4]),x]
```

output
```
-1/7*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((b*x^4)/a)]
)/(x^7*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{5b\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \text{761}$$

$$-\frac{5b\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}$$

input
```
Int[1/(x^8*Sqrt[a + b*x^4]),x]
```

output
```
-1/7*Sqrt[a + b*x^4]/(a*x^7) - (5*b*(-1/3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/
4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])))/
(7*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{\sqrt{b\,x^{4}+a}\,(-5b\,x^{4}+3a)}{21a^{2}x^{7}}+\frac{5b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	105
default	$-\frac{\sqrt{b\,x^{4}+a}}{7a\,x^{7}}+\frac{5b\sqrt{b\,x^{4}+a}}{21a^{2}x^{3}}+\frac{5b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	113
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{7a\,x^{7}}+\frac{5b\sqrt{b\,x^{4}+a}}{21a^{2}x^{3}}+\frac{5b^{2}\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{21a^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	113

input
```
int(1/x^8/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(b*x^4+a)^(1/2)*(-5*b*x^4+3*a)/a^2/x^7+5/21*b^2/a^2/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/
(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx=-\frac{5\,\sqrt{a}bx^7\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)-(5\,bx^4-3\,a)\sqrt{bx^4+a}}{21\,a^2x^7}$$

input
```
integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/21*(5*sqrt(a)*b*x^7*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1)
 - (5*b*x^4 - 3*a)*sqrt(b*x^4 + a))/(a^2*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(b*x**4+a)**(1/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{1}{x^8\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int(1/(x^8*(a + b*x^4)^(1/2)),x)
```

output
```
int(1/(x^8*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{a+bx^4}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b\,x^{12}+a\,x^8}dx$$

input
```
int(1/x^8/(b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a*x**8 + b*x**12),x)
```

3.369 $\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 261

$$
\begin{aligned}
\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = {} & -\frac{7ax^3\sqrt{a+bx^4}}{45b^2}+\frac{x^7\sqrt{a+bx^4}}{9b}+\frac{7a^2x\sqrt{a+bx^4}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\
& -\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^4}} \\
& +\frac{7a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{30b^{11/4}\sqrt{a+bx^4}}
\end{aligned}
$$

output
```
-7/45*a*x^3*(b*x^4+a)^(1/2)/b^2+1/9*x^7*(b*x^4+a)^(1/2)/b+7/15*a^2*x*(b*x^
4+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x^2)-7/15*a^(9/4)*(a^(1/2)+b^(1/2)*x^2
)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)
*x/a^(1/4))),1/2*2^(1/2))/b^(11/4)/(b*x^4+a)^(1/2)+7/30*a^(9/4)*(a^(1/2)+b
^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(11/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.31

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \frac{x^3\left(-7a^2-2abx^4+5b^2x^8+7a^2\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{45b^2\sqrt{a+bx^4}}$$

input
```
Integrate[x^10/Sqrt[a + b*x^4],x]
```

output
```
(x^3*(-7*a^2 - 2*a*b*x^4 + 5*b^2*x^8 + 7*a^2*Sqrt[1 + (b*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)]))/(45*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {843, 843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 843$$

$$\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\int \frac{x^6}{\sqrt{bx^4+a}}dx}{9b}$$

$$\Big\downarrow \; 843$$

$$\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}}dx}{5b}\right)}{9b}$$

$$\Big\downarrow \text{834}$$

$$\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}$$

$$\Big\downarrow \text{761}$$

$$\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}$$

$$\Big\downarrow \text{1510}$$

$$\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right.}{5b}\right.}{9b}$$

input
```
Int[x^10/Sqrt[a + b*x^4],x]
```

output
```
(x^7*Sqrt[a + b*x^4])/(9*b) - (7*a*((x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*(-(
(-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(
Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b
)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{x^{3}\left(-5 b\,x^{4}+7 a \right) \sqrt{b\,x^{4}+a}}{45 b^{2}}+\frac{7 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 b^{\frac{5}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	125
default	$\frac{x^{7} \sqrt{b\,x^{4}+a}}{9 b}-\frac{7 a\,x^{3} \sqrt{b\,x^{4}+a}}{45 b^{2}}+\frac{7 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 b^{\frac{5}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	133
elliptic	$\frac{x^{7} \sqrt{b\,x^{4}+a}}{9 b}-\frac{7 a\,x^{3} \sqrt{b\,x^{4}+a}}{45 b^{2}}+\frac{7 i a^{\frac{5}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{15 b^{\frac{5}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	133

input
```
int(x^10/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/45*x^3*(-5*b*x^4+7*a)/b^2*(b*x^4+a)^(1/2)+7/15*I*a^(5/2)/b^(5/2)/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/
2))^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-Ellipt
icE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.40

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \frac{21\,a^2\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1) - 21\,a^2\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)|-1) + (5\,b^2x^8 - 7\,abx^4}{45\,b^3x}$$

input
```
integrate(x^10/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/45*(21*a^2*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1)
 - 21*a^2*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) +
(5*b^2*x^8 - 7*a*b*x^4 + 21*a^2)*sqrt(b*x^4 + a))/(b^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(b*x**4+a)**(1/2),x)
```

output
```
x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \int \frac{x^{10}}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^10/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \int \frac{x^{10}}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^10/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \int \frac{x^{10}}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x^10/(a + b*x^4)^(1/2),x)
```

output
```
int(x^10/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{a+bx^4}}\,dx = \frac{-7\sqrt{b\,x^4+a}\,a\,x^3+5\sqrt{b\,x^4+a}\,b\,x^7+21\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)a^2}{45b^2}$$

input

```
int(x^10/(b*x^4+a)^(1/2),x)
```

output

```
( - 7*sqrt(a + b*x**4)*a*x**3 + 5*sqrt(a + b*x**4)*b*x**7 + 21*int((sqrt(a
 + b*x**4)*x**2)/(a + b*x**4),x)*a**2)/(45*b**2)
```

3.370 $\int \frac{x^6}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 237

$$\begin{aligned}\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = {} & \frac{x^3\sqrt{a+bx^4}}{5b} - \frac{3ax\sqrt{a+bx^4}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} \\ & + \frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^4}} \\ & - \frac{3a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10b^{7/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*a*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)
*x^2)+3/5*a^(5/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2
)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b
*x^4+a)^(1/2)-3/10*a^(5/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/
2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/
b^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \frac{x^3\left(a+bx^4-a\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{5b\sqrt{a+bx^4}}$$

input
```
Integrate[x^6/Sqrt[a + b*x^4],x]
```

output
```
(x^3*(a + b*x^4 - a*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -
((b*x^4)/a)]))/(5*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}}dx}{5b}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}$$

$\downarrow$ 761

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}$$

$\downarrow$ 1510

$$\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}\right)}{5b}$$

input
```
Int[x^6/Sqrt[a + b*x^4],x]
```

output
```
(x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sq
rt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
 Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*S
qrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b}-\frac{3 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{5 b^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$	115
risch	$\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b}-\frac{3 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{5 b^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$	115
elliptic	$\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b}-\frac{3 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{5 b^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$	115

input
```
int(x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(b*x^4+a)^(1/2)/b-3/5*I*a^(3/2)/b^(3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*
(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^
(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(
1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.38

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = -\frac{3\,a\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1) - 3\,a\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|-1) - \sqrt{bx^4+a}(bx^4 - 3}{5\,b^2x}$$

input
```
integrate(x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/5*(3*a*sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) -
3*a*sqrt(b)*x*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) - sqrt(b
*x^4 + a)*(b*x^4 - 3*a))/(b^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(b*x**4+a)**(1/2),x)
```

output
```
x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \int \frac{x^6}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \int \frac{x^6}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \int \frac{x^6}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x^6/(a + b*x^4)^(1/2),x)
```

output
```
int(x^6/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b\,x^4+a}\,x^3 - 3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx\right)a}{5b}$$

input
```
int(x^6/(b*x^4+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**4)*x**3 - 3*int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)*a)/
(5*b)
```

3.371 $\int \frac{x^2}{\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 210

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx = \frac{x\sqrt{a+bx^4}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}$$

output
```
x*(b*x^4+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x^2)-a^(1/4)*(a^(1/2)+b^(1/2)*x
^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)+1/2*a^(1/4)*(a^(1/2)+b
^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx = \frac{x^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt{a+bx^4}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^4],x]
```

output
```
(x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^4)/a)])/(
3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{834}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}$$

input
```
Int[x^2/Sqrt[a + b*x^4],x]
```

output
```
-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sq
rt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[
(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)
*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.46

method	result	size
default	$\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$	97
elliptic	$\frac{i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$	97

input
```
int(x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b
^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*
b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx = \frac{\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)-\sqrt{b}x\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)+\sqrt{bx^4+a}}{bx}$$

input
```
integrate(x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b)*x*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x), -1) - sqrt(b)*x
*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x), -1) + sqrt(b*x^4 + a))/(b
*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.18

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(b*x**4+a)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx = \int \frac{x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(b*x^4 + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx=\int \frac{x^2}{\sqrt{bx^4+a}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(b*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx=\int \frac{x^2}{\sqrt{b\,x^4+a}}\,dx$$

input
```
int(x^2/(a + b*x^4)^(1/2),x)
```

output
```
int(x^2/(a + b*x^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+bx^4}}\,dx=\int \frac{\sqrt{b\,x^4+a}\,x^2}{b\,x^4+a}dx$$

input
```
int(x^2/(b*x^4+a)^(1/2),x)
```

output
```
int((sqrt(a + b*x**4)*x**2)/(a + b*x**4),x)
```

3.372 $\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 232

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{ax}+\frac{\sqrt{b}x\sqrt{a+bx^4}}{a\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+bx^4}} + \frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+bx^4}}$$

output
```
-(b*x^4+a)^(1/2)/a/x+b^(1/2)*x*(b*x^4+a)^(1/2)/a/(a^(1/2)+b^(1/2)*x^2)-b^(
1/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*Ellip
ticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/(b*x^4+a)^(1/2)
+1/2*b^(1/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/
2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/(b*x^4
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
-((Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^4)/a)])/(x
*Sqrt[a + b*x^4]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx$$

$$\Big\downarrow \; 847$$

$$\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 834$$

$$\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \text{761}$$

$$\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}$$

$$\Big\downarrow \text{1510}$$

$$\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}\right)}{a} \frac{\sqrt{a+bx^4}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^4]),x]
```

output
```
-(Sqrt[a + b*x^4]/(a*x)) + (b*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]
*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt
[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a
 + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
2*b^(3/4)*Sqrt[a + b*x^4])))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{\sqrt{b\,x^{4}+a}}{a x}+\frac{i \sqrt{b}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{a}\, \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	115
risch	$-\frac{\sqrt{b\,x^{4}+a}}{a x}+\frac{i \sqrt{b}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{a}\, \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	115
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{a x}+\frac{i \sqrt{b}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{\sqrt{a}\, \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	115

input
```
int(1/x^2/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^4+a)^(1/2)/a/x+I*b^(1/2)/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1
/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*(El
lipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/
2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a}x\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)-\sqrt{a}x\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|-1)+\sqrt{bx^4+a}}{ax}$$

input
```
integrate(1/x^2/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*x*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) - sqrt(a)*
x*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + sqrt(b*x^4 + a))/(
a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(b*x**4+a)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+ax^2}}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{\frac{a}{b\,x^4}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^4}\right)}{3\,x\,\sqrt{b\,x^4+a}}$$

input
```
int(1/(x^2*(a + b*x^4)^(1/2)),x)
```

output
```
-((a/(b*x^4) + 1)^(1/2)*hypergeom([1/2, 3/4], 7/4, -a/(b*x^4)))/(3*x*(a +
b*x^4)^(1/2))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{a+bx^4}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b\,x^6+a\,x^2}dx$$

input
```
int(1/x^2/(b*x^4+a)^(1/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a*x**2 + b*x**6),x)
```

3.373 $\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 261

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{a+bx^4}}{5ax^5}+\frac{3b\sqrt{a+bx^4}}{5a^2x}-\frac{3b^{3/2}x\sqrt{a+bx^4}}{5a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{a+bx^4}} - \frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10a^{7/4}\sqrt{a+bx^4}}$$

output
```
-1/5*(b*x^4+a)^(1/2)/a/x^5+3/5*b*(b*x^4+a)^(1/2)/a^2/x-3/5*b^(3/2)*x*(b*x^
4+a)^(1/2)/a^2/(a^(1/2)+b^(1/2)*x^2)+3/5*b^(5/4)*(a^(1/2)+b^(1/2)*x^2)*((b
*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^
(1/4))),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)-3/10*b^(5/4)*(a^(1/2)+b^(1/2)
*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b
^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)}{5x^5\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/5*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^4)/a)]
)/(x^5*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {847, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{a+bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{3b\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{3b\left(\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}$$

$$\Big\downarrow \text{834}$$

$$-\frac{3b\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}$$

$\downarrow$ 27

$$-\frac{3b\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}$$

$\downarrow$ 761

$$-\frac{3b\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}$$

$\downarrow$ 1510

$$-\frac{3b\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}\right)}{a}\right.}{5a}$$

$$\frac{\sqrt{a+bx^4}}{5ax^5}$$

input
```
Int[1/(x^6*Sqrt[a + b*x^4]),x]
```

output
```
-1/5*Sqrt[a + b*x^4]/(a*x^5) - (3*b*(-(Sqrt[a + b*x^4]/(a*x)) + (b*(-((-((
x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/
4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt
[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2
*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/a))/(5*a
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.47

method	result	size
risch	$-\frac{\sqrt{b\,x^4+a}\,(-3b\,x^4+a)}{5a^2x^5}-\frac{3ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	123
default	$-\frac{\sqrt{b\,x^4+a}}{5a\,x^5}+\frac{3b\sqrt{b\,x^4+a}}{5a^2x}-\frac{3ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	133
elliptic	$-\frac{\sqrt{b\,x^4+a}}{5a\,x^5}+\frac{3b\sqrt{b\,x^4+a}}{5a^2x}-\frac{3ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{5a^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	133

input
```
int(1/x^6/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(1/2)*(-3*b*x^4+a)/a^2/x^5-3/5*I*b^(3/2)/a^(3/2)/(I/a^(1/2)
*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^
(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(
x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx = \frac{3\sqrt{a}bx^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right) - 3\sqrt{a}bx^5\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1\right) + (3bx^4-a)\sqrt{bx^4+a}}{5a^2x^5}$$

input
```
integrate(1/x^6/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(3*sqrt(a)*b*x^5*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4)), -1) -
 3*sqrt(a)*b*x^5*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)), -1) + (3*
b*x^4 - a)*sqrt(b*x^4 + a))/(a^2*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(b*x**4+a)**(1/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx=\int \frac{1}{\sqrt{bx^4+ax^6}}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx=\int \frac{1}{x^6\,\sqrt{b\,x^4+a}}\,dx$$

input
```
int(1/(x^6*(a + b*x^4)^(1/2)),x)
```

output
```
int(1/(x^6*(a + b*x^4)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{a+bx^4}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b\,x^{10}+a\,x^6}dx$$

input

```
int(1/x^6/(b*x^4+a)^(1/2),x)
```

output

```
int(sqrt(a + b*x**4)/(a*x**6 + b*x**10),x)
```

3.374 $\int \frac{x^{11}}{(a+bx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}} \, dx = -\frac{a^2}{2b^3\sqrt{a+bx^4}} - \frac{a\sqrt{a+bx^4}}{b^3} + \frac{(a+bx^4)^{3/2}}{6b^3}$$

output
```
-1/2*a^2/b^3/(b*x^4+a)^(1/2)-a*(b*x^4+a)^(1/2)/b^3+1/6*(b*x^4+a)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}} \, dx = \frac{-8a^2 - 4abx^4 + b^2x^8}{6b^3\sqrt{a+bx^4}}$$

input
```
Integrate[x^11/(a + b*x^4)^(3/2),x]
```

output
```
(-8*a^2 - 4*a*b*x^4 + b^2*x^8)/(6*b^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2(bx^4+a)^{3/2}}-\frac{2a}{b^2\sqrt{bx^4+a}}+\frac{\sqrt{bx^4+a}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{2a^2}{b^3\sqrt{a+bx^4}}-\frac{4a\sqrt{a+bx^4}}{b^3}+\frac{2(a+bx^4)^{3/2}}{3b^3}\right)$$

input
```
Int[x^11/(a + b*x^4)^(3/2),x]
```

output
```
((-2*a^2)/(b^3*Sqrt[a + b*x^4]) - (4*a*Sqrt[a + b*x^4])/b^3 + (2*(a + b*x^
4)^(3/2))/(3*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{b^2x^8-4ab\,x^4-8a^2}{6\sqrt{b\,x^4+a}\,b^3}$	35
gosper	$-\frac{-b^2x^8+4ab\,x^4+8a^2}{6\sqrt{b\,x^4+a}\,b^3}$	36
default	$-\frac{-b^2x^8+4ab\,x^4+8a^2}{6\sqrt{b\,x^4+a}\,b^3}$	36
trager	$-\frac{-b^2x^8+4ab\,x^4+8a^2}{6\sqrt{b\,x^4+a}\,b^3}$	36
elliptic	$-\frac{-b^2x^8+4ab\,x^4+8a^2}{6\sqrt{b\,x^4+a}\,b^3}$	36
orering	$-\frac{-b^2x^8+4ab\,x^4+8a^2}{6\sqrt{b\,x^4+a}\,b^3}$	36
risch	$-\frac{(-b\,x^4+5a)\sqrt{b\,x^4+a}}{6b^3}-\frac{a^2}{2b^3\sqrt{b\,x^4+a}}$	43

input
```
int(x^11/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*x^8-4*a*b*x^4-8*a^2)/(b*x^4+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx = \frac{(b^2x^8 - 4\,abx^4 - 8\,a^2)\sqrt{bx^4+a}}{6\,(b^4x^4 + ab^3)}$$

input
```
integrate(x^11/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(b^2*x^8 - 4*a*b*x^4 - 8*a^2)*sqrt(b*x^4 + a)/(b^4*x^4 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx = \begin{cases} -\frac{4a^2}{3b^3\sqrt{a+bx^4}} - \frac{2ax^4}{3b^2\sqrt{a+bx^4}} + \frac{x^8}{6b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-4*a**2/(3*b**3*sqrt(a + b*x**4)) - 2*a*x**4/(3*b**2*sqrt(a + b
*x**4)) + x**8/(6*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**12/(12*a**(3/2)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx = \frac{(bx^4+a)^{\frac{3}{2}}}{6\,b^3} - \frac{\sqrt{bx^4+a}\,a}{b^3} - \frac{a^2}{2\,\sqrt{bx^4+a}\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^4 + a)^(3/2)/b^3 - sqrt(b*x^4 + a)*a/b^3 - 1/2*a^2/(sqrt(b*x^4 +
a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx = -\frac{\frac{3\,a^2}{\sqrt{bx^4+a}b} - \frac{(bx^4+a)^{\frac{3}{2}}b^2-6\sqrt{bx^4+a}ab^2}{b^3}}{6\,b^2}$$

input
```
integrate(x^11/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/6*(3*a^2/(sqrt(b*x^4 + a)*b) - ((b*x^4 + a)^(3/2)*b^2 - 6*sqrt(b*x^4 +
a)*a*b^2)/b^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx = -\frac{6\,a\,(b\,x^4+a)-(b\,x^4+a)^2+3\,a^2}{6\,b^3\,\sqrt{b\,x^4+a}}$$

input
```
int(x^11/(a + b*x^4)^(3/2),x)
```

output
```
-(6*a*(a + b*x^4) - (a + b*x^4)^2 + 3*a^2)/(6*b^3*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{x^{11}}{(a+bx^4)^{3/2}}\,dx = \frac{-8\sqrt{b\,x^4+a}\,a^2-4\sqrt{b\,x^4+a}\,ab\,x^4+\sqrt{b\,x^4+a}\,b^2x^8-8\sqrt{b}\,a^2x^2-4\sqrt{b}\,ab\,x^6+\sqrt{b}\,b^2}{6b^3\left(\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4\right)}$$

input

```
int(x^11/(b*x^4+a)^(3/2),x)
```

output

```
( - 8*sqrt(a + b*x**4)*a**2 - 4*sqrt(a + b*x**4)*a*b*x**4 + sqrt(a + b*x**
4)*b**2*x**8 - 8*sqrt(b)*a**2*x**2 - 4*sqrt(b)*a*b*x**6 + sqrt(b)*b**2*x**
10)/(6*b**3*(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4))
```

3.375 $\int \frac{x^7}{(a+bx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^7}{(a+bx^4)^{3/2}} \, dx = \frac{a}{2b^2\sqrt{a+bx^4}} + \frac{\sqrt{a+bx^4}}{2b^2}$$

output
```
1/2*a/b^2/(b*x^4+a)^(1/2)+1/2*(b*x^4+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x^7}{(a+bx^4)^{3/2}} \, dx = \frac{2a+bx^4}{2b^2\sqrt{a+bx^4}}$$

input
```
Integrate[x^7/(a + b*x^4)^(3/2),x]
```

output
```
(2*a + b*x^4)/(2*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{1}{b\sqrt{bx^4+a}}-\frac{a}{b\,(bx^4+a)^{3/2}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2a}{b^2\sqrt{a+bx^4}}+\frac{2\sqrt{a+bx^4}}{b^2}\right)$$

input
```
Int[x^7/(a + b*x^4)^(3/2),x]
```

output
```
((2*a)/(b^2*Sqrt[a + b*x^4]) + (2*Sqrt[a + b*x^4])/b^2)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{b\,x^{4}+2a}{2\sqrt{b\,x^{4}+a}\,b^{2}}$	24
default	$\frac{b\,x^{4}+2a}{2\sqrt{b\,x^{4}+a}\,b^{2}}$	24
trager	$\frac{b\,x^{4}+2a}{2\sqrt{b\,x^{4}+a}\,b^{2}}$	24
elliptic	$\frac{b\,x^{4}+2a}{2\sqrt{b\,x^{4}+a}\,b^{2}}$	24
pseudoelliptic	$\frac{b\,x^{4}+2a}{2\sqrt{b\,x^{4}+a}\,b^{2}}$	24
orering	$\frac{b\,x^{4}+2a}{2\sqrt{b\,x^{4}+a}\,b^{2}}$	24
risch	$\frac{a}{2b^{2}\sqrt{b\,x^{4}+a}}+\frac{\sqrt{b\,x^{4}+a}}{2b^{2}}$	31

input
```
int(x^7/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^4+2*a)/(b*x^4+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx = \frac{(bx^4+2\,a)\sqrt{bx^4+a}}{2\,(b^3x^4+ab^2)}$$

input
```
integrate(x^7/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(b*x^4 + 2*a)*sqrt(b*x^4 + a)/(b^3*x^4 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx = \begin{cases} \frac{a}{b^2\sqrt{a+bx^4}} + \frac{x^4}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**4+a)**(3/2),x)
```

output
```
Piecewise((a/(b**2*sqrt(a + b*x**4)) + x**4/(2*b*sqrt(a + b*x**4)), Ne(b,
0)), (x**8/(8*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{bx^4+a}}{2\,b^2} + \frac{a}{2\,\sqrt{bx^4+a}b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^4 + a)/b^2 + 1/2*a/(sqrt(b*x^4 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx = \frac{\frac{\sqrt{bx^4+a}}{b}+\frac{a}{\sqrt{bx^4+a}b}}{2\,b}$$

input
```
integrate(x^7/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(b*x^4 + a)/b + a/(sqrt(b*x^4 + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.55

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx = \frac{\frac{b\,x^4}{2}+a}{b^2\,\sqrt{b\,x^4+a}}$$

input
```
int(x^7/(a + b*x^4)^(3/2),x)
```

output
```
(a + (b*x^4)/2)/(b^2*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.79

$$\int \frac{x^7}{(a+bx^4)^{3/2}}\,dx = \frac{2\sqrt{b\,x^4+a}\,a+\sqrt{b\,x^4+a}\,b\,x^4+2\sqrt{b}\,a\,x^2+\sqrt{b}\,b\,x^6}{2b^2\left(\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4\right)}$$

input
```
int(x^7/(b*x^4+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x**4)*a + sqrt(a + b*x**4)*b*x**4 + 2*sqrt(b)*a*x**2 + sqrt(
b)*b*x**6)/(2*b**2*(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4))
```

3.376 $\int \frac{x^3}{(a+bx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x^3}{(a+bx^4)^{3/2}} \, dx = -\frac{1}{2b\sqrt{a+bx^4}}$$

output
```
-1/2/b/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^4)^{3/2}} \, dx = -\frac{1}{2b\sqrt{a+bx^4}}$$

input
```
Integrate[x^3/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*1/(b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{1}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^3/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*1/(b*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15
derivativedivides	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15
default	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15
trager	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15
elliptic	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15
pseudoelliptic	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15
orering	$-\frac{1}{2b\sqrt{b\,x^{4}+a}}$	15

input
```
int(x^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx = -\frac{\sqrt{bx^4+a}}{2\,(b^2x^4+ab)}$$

input
```
integrate(x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(b*x^4 + a)/(b^2*x^4 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx = \begin{cases} -\frac{1}{2b\sqrt{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)**(3/2),x)
```

output
```
Piecewise((-1/(2*b*sqrt(a + b*x**4)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx = -\frac{1}{2\sqrt{bx^4+a}b}$$

input
```
integrate(x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/2/(sqrt(b*x^4 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx = -\frac{1}{2\sqrt{bx^4+a}b}$$

input
```
integrate(x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2/(sqrt(b*x^4 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx = -\frac{1}{2\,b\,\sqrt{b\,x^4+a}}$$

input
```
int(x^3/(a + b*x^4)^(3/2),x)
```

output
```
-1/(2*b*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{x^3}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b\,x^4+a}-\sqrt{b}\,x^2}{2b\left(\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4\right)}$$

input
```
int(x^3/(b*x^4+a)^(3/2),x)
```

output
```
( - (sqrt(a + b*x**4) + sqrt(b)*x**2))/(2*b*(sqrt(b)*sqrt(a + b*x**4)*x**2
 + a + b*x**4))
```

3.377 $\int \frac{1}{x(a+bx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{1}{x(a+bx^4)^{3/2}} \, dx = \frac{1}{2a\sqrt{a+bx^4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/2/a/(b*x^4+a)^(1/2)-1/2*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^4)^{3/2}} \, dx = \frac{1}{2a\sqrt{a+bx^4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^4)^(3/2)),x]
```

output
```
1/(2*a*Sqrt[a + b*x^4]) - ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]]/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4(bx^4+a)^{3/2}}dx^4$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4}{a}+\frac{2}{a\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{2\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{ab}+\frac{2}{a\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(\frac{2}{a\sqrt{a+bx^4}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^4)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*x^4]) - (2*ArcTanh[Sqrt[a + b*x^4]/Sqrt[a]])/a^(3/2))/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$\frac{1}{2a\sqrt{b\,x^{4}+a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$	35
default	$\frac{1}{2a\sqrt{b\,x^{4}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	44
elliptic	$\frac{1}{2a\sqrt{b\,x^{4}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{2a^{\frac{3}{2}}}$	44

input
```
int(1/x/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(b*x^4+a)^(1/2)-1/2*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx = \left[\frac{(bx^4+a)\sqrt{a}\log\left(\frac{bx^4-2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)+2\sqrt{bx^4+a}a}{4\left(a^2bx^4+a^3\right)}, \frac{(bx^4+a)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^4+a}}\right)+\sqrt{bx^4+a}a}{2\left(a^2bx^4+a^3\right)}\right]$$

input
```
integrate(1/x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*x^4 + a)*sqrt(a)*log((b*x^4 - 2*sqrt(b*x^4 + a)*sqrt(a) + 2*a)/x^
4) + 2*sqrt(b*x^4 + a)*a)/(a^2*b*x^4 + a^3), 1/2*((b*x^4 + a)*sqrt(-a)*arc
tan(sqrt(-a)/sqrt(b*x^4 + a)) + sqrt(b*x^4 + a)*a)/(a^2*b*x^4 + a^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(37) = 74$.

Time = 0.85 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{2a^3\sqrt{1+\frac{bx^4}{a}}}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^3\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}+\frac{a^2bx^4\log\left(\frac{bx^4}{a}\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}-\frac{2a^2bx^4\log\left(\sqrt{1+\frac{bx^4}{a}}+1\right)}{4a^{\frac{9}{2}}+4a^{\frac{7}{2}}bx^4}$$

input
```
integrate(1/x/(b*x**4+a)**(3/2),x)
```

output
```
2*a**3*sqrt(1 + b*x**4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**3*log(b*x*
*4/a)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) - 2*a**3*log(sqrt(1 + b*x**4/a) + 1
)/(4*a**(9/2) + 4*a**(7/2)*b*x**4) + a**2*b*x**4*log(b*x**4/a)/(4*a**(9/2)
 + 4*a**(7/2)*b*x**4) - 2*a**2*b*x**4*log(sqrt(1 + b*x**4/a) + 1)/(4*a**(9
/2) + 4*a**(7/2)*b*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{x (a + bx^4)^{3/2}} dx = \frac{\log \left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}} \right)}{4 a^{\frac{3}{2}}} + \frac{1}{2 \sqrt{bx^4 + a}a}$$

input
```
integrate(1/x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^(3/2) +
 1/2/(sqrt(b*x^4 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{1}{x (a + bx^4)^{3/2}} dx = \frac{\arctan \left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}} \right)}{2 \sqrt{-a}a} + \frac{1}{2 \sqrt{bx^4 + a}a}$$

input
```
integrate(1/x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/2/(sqrt(b*x^4 + a)*a
)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{1}{2\,a\,\sqrt{b\,x^4+a}}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}$$

input
```
int(1/(x*(a + b*x^4)^(3/2)),x)
```

output
```
1/(2*a*(a + b*x^4)^(1/2)) - atanh((a + b*x^4)^(1/2)/a^(1/2))/(2*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 5.24

$$\int \frac{1}{x\left(a+bx^4\right)^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{b\,x^4+a}+\sqrt{b}\,\sqrt{b\,x^4+a}\,\log\left(\frac{\sqrt{b\,x^4+a}-\sqrt{a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)x^2-\sqrt{b}\,\sqrt{b\,x^4+a}\,\log\left(\frac{\sqrt{b\,x^4+a}-}{}\right.}{}$$

input
```
int(1/x/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a)*sqrt(a + b*x**4) + sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4)
 - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*x**2 - sqrt(b)*sqrt(a + b*x**4)*log((s
qrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*x**2 + sqrt(b)*sqrt(a)*
x**2 + log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a + log((s
qrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b*x**4 - log((sqrt(a +
b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a - log((sqrt(a + b*x**4) + sqr
t(a) + sqrt(b)*x**2)/sqrt(a))*b*x**4)/(2*sqrt(a)*a*(sqrt(b)*sqrt(a + b*x**
4)*x**2 + a + b*x**4))
```

3.378 $\int \frac{1}{x^5(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{1}{x^5(a+bx^4)^{3/2}}\,dx = -\frac{3b}{4a^2\sqrt{a+bx^4}} - \frac{1}{4ax^4\sqrt{a+bx^4}} + \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{5/2}}$$

output
```
-3/4*b/a^2/(b*x^4+a)^(1/2)-1/4/a/x^4/(b*x^4+a)^(1/2)+3/4*b*arctanh((b*x^4+
a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^5(a+bx^4)^{3/2}}\,dx = \frac{-a-3bx^4}{4a^2x^4\sqrt{a+bx^4}} + \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[1/(x^5*(a + b*x^4)^(3/2)),x]
```

output
```
(-a - 3*b*x^4)/(4*a^2*x^4*Sqrt[a + b*x^4]) + (3*b*ArcTanh[Sqrt[a + b*x^4]/
Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8 (bx^4+a)^{3/2}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(-\frac{3b\int \frac{1}{x^4(bx^4+a)^{3/2}}dx^4}{2a}-\frac{1}{ax^4\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(-\frac{3b\left(\frac{\int \frac{1}{x^4\sqrt{bx^4+a}}dx^4}{a}+\frac{2}{a\sqrt{a+bx^4}}\right)}{2a}-\frac{1}{ax^4\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{3b\left(\frac{2\int \frac{1}{\frac{x^8}{b}-\frac{a}{b}}d\sqrt{bx^4+a}}{ab}+\frac{2}{a\sqrt{a+bx^4}}\right)}{2a}-\frac{1}{ax^4\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{4}\left(-\frac{3b\left(\frac{2}{a\sqrt{a+bx^4}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^4\sqrt{a+bx^4}}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)^(3/2)),x]
```

output
```
(-(1/(a*x^4*Sqrt[a + b*x^4])) - (3*b*(2/(a*Sqrt[a + b*x^4]) - (2*ArcTanh[S
qrt[a + b*x^4]/Sqrt[a]])/a^(3/2)))/(2*a))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{b\,x^{4}+a}}{b\,x^{4}}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2}{\sqrt{b\,x^{4}+a}}\right)}{4a^{2}}$	55
default	$-\frac{1}{4a\,x^{4}\sqrt{b\,x^{4}+a}}-\frac{3b}{4a^{2}\sqrt{b\,x^{4}+a}}+\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{5}{2}}}$	63
risch	$-\frac{\sqrt{b\,x^{4}+a}}{4a^{2}x^{4}}-\frac{b}{2a^{2}\sqrt{b\,x^{4}+a}}+\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{5}{2}}}$	63
elliptic	$-\frac{1}{4a\,x^{4}\sqrt{b\,x^{4}+a}}-\frac{3b}{4a^{2}\sqrt{b\,x^{4}+a}}+\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{4}+a}}{x^{2}}\right)}{4a^{\frac{5}{2}}}$	63

input
```
int(1/x^5/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*b/a^2*(-(b*x^4+a)^(1/2)/b/x^4+3*arctanh((b*x^4+a)^(1/2)/a^(1/2))/a^(1/
2)-2/(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^5\left(a+bx^4\right)^{3/2}}\,dx=\left[\frac{3\left(b^2x^8+abx^4\right)\sqrt{a}\log\left(\frac{bx^4+2\sqrt{bx^4+a}\sqrt{a}+2a}{x^4}\right)-2\left(3\,abx^4+a^2\right)\sqrt{bx^4+a}}{8\left(a^3bx^8+a^4x^4\right)},\right.$$
$$\left.-\frac{3\left(b^2x^8+abx^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^4+a}}\right)+\left(3\,abx^4+a^2\right)\sqrt{bx^4+a}}{4\left(a^3bx^8+a^4x^4\right)}\right]$$

input
```
integrate(1/x^5/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(b^2*x^8 + a*b*x^4)*sqrt(a)*log((b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(a)
 + 2*a)/x^4) - 2*(3*a*b*x^4 + a^2)*sqrt(b*x^4 + a))/(a^3*b*x^8 + a^4*x^4),
 -1/4*(3*(b^2*x^8 + a*b*x^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^4 + a)) + (
3*a*b*x^4 + a^2)*sqrt(b*x^4 + a))/(a^3*b*x^8 + a^4*x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^5 (a+bx^4)^{3/2}} \, dx = -\frac{1}{4a\sqrt{b}x^6\sqrt{\frac{a}{bx^4}+1}} - \frac{3\sqrt{b}}{4a^2x^2\sqrt{\frac{a}{bx^4}+1}} + \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)}{4a^{\frac{5}{2}}}$$

input
```
integrate(1/x**5/(b*x**4+a)**(3/2),x)
```

output
```
-1/(4*a*sqrt(b)*x**6*sqrt(a/(b*x**4) + 1)) - 3*sqrt(b)/(4*a**2*x**2*sqrt(a
/(b*x**4) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x**2))/(4*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^5 (a+bx^4)^{3/2}} \, dx = -\frac{3 (bx^4+a)b - 2 ab}{4 \left((bx^4+a)^{\frac{3}{2}}a^2 - \sqrt{bx^4+a}a^3\right)} - \frac{3 b \log\left(\frac{\sqrt{bx^4+a}-\sqrt{a}}{\sqrt{bx^4+a}+\sqrt{a}}\right)}{8 a^{\frac{5}{2}}}$$

input
```
integrate(1/x^5/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*(b*x^4 + a)*b - 2*a*b)/((b*x^4 + a)^(3/2)*a^2 - sqrt(b*x^4 + a)*a^
3) - 3/8*b*log((sqrt(b*x^4 + a) - sqrt(a))/(sqrt(b*x^4 + a) + sqrt(a)))/a^
(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^5 (a+bx^4)^{3/2}}\,dx = -\frac{3\,b\arctan\left(\frac{\sqrt{bx^4+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2} - \frac{3\,(bx^4+a)b-2\,ab}{4\left((bx^4+a)^{\frac{3}{2}}-\sqrt{bx^4+a}a\right)a^2}$$

input
```
integrate(1/x^5/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-3/4*b*arctan(sqrt(b*x^4 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(b*x^4 + a
)*b - 2*a*b)/(((b*x^4 + a)^(3/2) - sqrt(b*x^4 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5 (a+bx^4)^{3/2}}\,dx = \frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^4+a}}{\sqrt{a}}\right)}{4\,a^{5/2}} - \frac{1}{4\,a\,x^4\,\sqrt{b\,x^4+a}} - \frac{3\,b}{4\,a^2\,\sqrt{b\,x^4+a}}$$

input
```
int(1/(x^5*(a + b*x^4)^(3/2)),x)
```

output
```
(3*b*atanh((a + b*x^4)^(1/2)/a^(1/2)))/(4*a^(5/2)) - 1/(4*a*x^4*(a + b*x^4
)^(1/2)) - (3*b)/(4*a^2*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 513, normalized size of antiderivative = 7.43

$$\int \frac{1}{x^5 (a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{a}\,\sqrt{b\,x^4+a}\,a^2-7\sqrt{a}\,\sqrt{b\,x^4+a}\,ab\,x^4-12\sqrt{a}\,\sqrt{b\,x^4+a}\,b^2x^8-9\sqrt{b}\,\sqrt{b\,x^4+a}}{}$$

input
```
int(1/x^5/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a)*sqrt(a + b*x**4)*a**2 - 7*sqrt(a)*sqrt(a + b*x**4)*a*b*x**4 -
12*sqrt(a)*sqrt(a + b*x**4)*b**2*x**8 - 9*sqrt(b)*sqrt(a + b*x**4)*log((sq
rt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a*b*x**6 - 12*sqrt(b)*sq
rt(a + b*x**4)*log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b*
*2*x**10 + 9*sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) + sqrt(a) + sq
rt(b)*x**2)/sqrt(a))*a*b*x**6 + 12*sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a +
b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b**2*x**10 - 3*sqrt(b)*sqrt(a)*
a**2*x**2 - 13*sqrt(b)*sqrt(a)*a*b*x**6 - 12*sqrt(b)*sqrt(a)*b**2*x**10 -
3*log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a**2*b*x**4 - 1
5*log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a*b**2*x**8 - 1
2*log((sqrt(a + b*x**4) - sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b**3*x**12 + 3*
log((sqrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a**2*b*x**4 + 15*
log((sqrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*a*b**2*x**8 + 12*
log((sqrt(a + b*x**4) + sqrt(a) + sqrt(b)*x**2)/sqrt(a))*b**3*x**12)/(4*sq
rt(a)*a**2*x**4*(3*sqrt(b)*sqrt(a + b*x**4)*a*x**2 + 4*sqrt(b)*sqrt(a + b*
x**4)*b*x**6 + a**2 + 5*a*b*x**4 + 4*b**2*x**8))
```

3.379 $\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^9}{(a+bx^4)^{3/2}}\,dx=-\frac{x^6}{2b\sqrt{a+bx^4}}+\frac{3x^2\sqrt{a+bx^4}}{4b^2}-\frac{3a\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{4b^{5/2}}$$

output
```
-1/2*x^6/b/(b*x^4+a)^(1/2)+3/4*x^2*(b*x^4+a)^(1/2)/b^2-3/4*a*arctanh(b^(1/
2)*x^2/(b*x^4+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{(a+bx^4)^{3/2}}\,dx=\frac{3ax^2+bx^6}{4b^2\sqrt{a+bx^4}}-\frac{3a\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{4b^{5/2}}$$

input
```
Integrate[x^9/(a + b*x^4)^(3/2),x]
```

output
```
(3*a*x^2 + b*x^6)/(4*b^2*Sqrt[a + b*x^4]) - (3*a*Log[Sqrt[b]*x^2 + Sqrt[a
+ b*x^4]])/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 252}$$

$$\frac{1}{2}\left(\frac{3\int \frac{x^4}{\sqrt{bx^4+a}}dx^2}{b}-\frac{x^6}{b\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{x^2\sqrt{a+bx^4}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx^2}{2b}\right)}{b}-\frac{x^6}{b\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{x^2\sqrt{a+bx^4}}{2b}-\frac{a\int \frac{1}{1-bx^4}d\frac{x^2}{\sqrt{bx^4+a}}}{2b}\right)}{b}-\frac{x^6}{b\sqrt{a+bx^4}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(\frac{3\left(\frac{x^2\sqrt{a+bx^4}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^6}{b\sqrt{a+bx^4}}\right)$$

input
```
Int[x^9/(a + b*x^4)^(3/2),x]
```

output
```
(-(x^6/(b*Sqrt[a + b*x^4])) + (3*((x^2*Sqrt[a + b*x^4])/(2*b) - (a*ArcTanh
[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*b^(3/2))))/b)/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x^{6}}{4b\sqrt{b\,x^{4}+a}}+\frac{3a\,x^{2}}{4b^{2}\sqrt{b\,x^{4}+a}}-\frac{3a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{5}{2}}}$	61
risch	$\frac{x^{2}\sqrt{b\,x^{4}+a}}{4b^{2}}+\frac{a\,x^{2}}{2b^{2}\sqrt{b\,x^{4}+a}}-\frac{3a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{5}{2}}}$	61
elliptic	$\frac{x^{6}}{4b\sqrt{b\,x^{4}+a}}+\frac{3a\,x^{2}}{4b^{2}\sqrt{b\,x^{4}+a}}-\frac{3a\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{4b^{\frac{5}{2}}}$	61
pseudoelliptic	$\frac{b^{\frac{3}{2}}x^{6}+3a\,x^{2}\sqrt{b}-3\sqrt{b\,x^{4}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)a}{4b^{\frac{5}{2}}\sqrt{b\,x^{4}+a}}$	61

input
```
int(x^9/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^6/b/(b*x^4+a)^(1/2)+3/4*a/b^2*x^2/(b*x^4+a)^(1/2)-3/4*a/b^(5/2)*ln(b
^(1/2)*x^2+(b*x^4+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.30

$$\int \frac{x^9}{(a+bx^4)^{3/2}}\,dx=\left[\frac{3\,(abx^4+a^2)\sqrt{b}\log\left(-2\,bx^4+2\,\sqrt{bx^4+a}\sqrt{b}x^2-a\right)+2\,(b^2x^6+3\,abx^2)\sqrt{bx^4+a}}{8\,(b^4x^4+ab^3)},\right.$$

input
```
integrate(x^9/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a*b*x^4 + a^2)*sqrt(b)*log(-2*b*x^4 + 2*sqrt(b*x^4 + a)*sqrt(b)*x
^2 - a) + 2*(b^2*x^6 + 3*a*b*x^2)*sqrt(b*x^4 + a))/(b^4*x^4 + a*b^3), 1/4*
(3*(a*b*x^4 + a^2)*sqrt(-b)*arctan(sqrt(b*x^4 + a)*sqrt(-b)/(b*x^2)) + (b^
2*x^6 + 3*a*b*x^2)*sqrt(b*x^4 + a))/(b^4*x^4 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{x^9}{(a+bx^4)^{3/2}}\,dx = \frac{3\sqrt{a}x^2}{4b^2\sqrt{1+\frac{bx^4}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}} + \frac{x^6}{4\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**9/(b*x**4+a)**(3/2),x)
```

output
```
3*sqrt(a)*x**2/(4*b**2*sqrt(1 + b*x**4/a)) - 3*a*asinh(sqrt(b)*x**2/sqrt(a
))/(4*b**(5/2)) + x**6/(4*sqrt(a)*b*sqrt(1 + b*x**4/a))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int \frac{x^9}{(a+bx^4)^{3/2}}\,dx = \frac{2\,ab - \frac{3\,(bx^4+a)a}{x^4}}{4\left(\frac{\sqrt{bx^4+a}b^3}{x^2} - \frac{(bx^4+a)^{\frac{3}{2}}b^2}{x^6}\right)} + \frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^9/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(2*a*b - 3*(b*x^4 + a)*a/x^4)/(sqrt(b*x^4 + a)*b^3/x^2 - (b*x^4 + a)^(
3/2)*b^2/x^6) + 3/8*a*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(sqrt(b) + sqrt
(b*x^4 + a)/x^2))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{\left(\frac{x^4}{b}+\frac{3\,a}{b^2}\right)x^2}{4\,\sqrt{bx^4+a}}+\frac{3\,a\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

input
```
integrate(x^9/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(x^4/b + 3*a/b^2)*x^2/sqrt(b*x^4 + a) + 3/4*a*log(abs(-sqrt(b)*x^2 + s
qrt(b*x^4 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx=\int \frac{x^9}{\left(b\,x^4+a\right)^{3/2}}\,dx$$

input
```
int(x^9/(a + b*x^4)^(3/2),x)
```

output
```
int(x^9/(a + b*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.22

$$\int \frac{x^9}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{-36\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)a^2x^2-48\sqrt{b}\,\sqrt{b\,x^4+a}\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)ab\,x^6}{}$$

input
```
int(x^9/(b*x^4+a)^(3/2),x)
```

output

```
( - 36*sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt
(a))*a**2*x**2 - 48*sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) + sqrt(
b)*x**2)/sqrt(a))*a*b*x**6 + 39*sqrt(b)*sqrt(a + b*x**4)*a**2*x**2 + 88*sq
rt(b)*sqrt(a + b*x**4)*a*b*x**6 + 16*sqrt(b)*sqrt(a + b*x**4)*b**2*x**10 -
 12*log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))*a**3 - 60*log((sqrt(a +
 b*x**4) + sqrt(b)*x**2)/sqrt(a))*a**2*b*x**4 - 48*log((sqrt(a + b*x**4) +
 sqrt(b)*x**2)/sqrt(a))*a*b**2*x**8 + 9*a**3 + 81*a**2*b*x**4 + 96*a*b**2*
x**8 + 16*b**3*x**12)/(16*b**2*(3*sqrt(a + b*x**4)*a*b*x**2 + 4*sqrt(a + b
*x**4)*b**2*x**6 + sqrt(b)*a**2 + 5*sqrt(b)*a*b*x**4 + 4*sqrt(b)*b**2*x**8
))
```

3.380 $\int \frac{x^5}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{x^5}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^2}{2b\sqrt{a+bx^4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2b^{3/2}}$$

output
```
-1/2*x^2/b/(b*x^4+a)^(1/2)+1/2*arctanh(b^(1/2)*x^2/(b*x^4+a)^(1/2))/b^(3/2
)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{x^2}{2b\sqrt{a+bx^4}}+\frac{\log\left(\sqrt{b}x^2+\sqrt{a+bx^4}\right)}{2b^{3/2}}$$

input
```
Integrate[x^5/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x^2/(b*Sqrt[a + b*x^4]) + Log[Sqrt[b]*x^2 + Sqrt[a + b*x^4]]/(2*b^(3/
2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {807, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{(bx^4+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{\sqrt{bx^4+a}}dx^2}{b} - \frac{x^2}{b\sqrt{a+bx^4}} \right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{1-bx^4}d\frac{x^2}{\sqrt{bx^4+a}}}{b} - \frac{x^2}{b\sqrt{a+bx^4}} \right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{b^{3/2}} - \frac{x^2}{b\sqrt{a+bx^4}} \right)$$

input
```
Int[x^5/(a + b*x^4)^(3/2),x]
```

output
```
(-(x^2/(b*Sqrt[a + b*x^4])) + ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]]/b^(3/
2))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$-\frac{x^{2}}{2 b \sqrt{b\,x^{4}+a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{4}+a}}{\sqrt{b}\,x^{2}}\right)}{2 b^{\frac{3}{2}}}$	41
default	$-\frac{x^{2}}{2 b \sqrt{b\,x^{4}+a}}+\frac{\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{2 b^{\frac{3}{2}}}$	42
elliptic	$-\frac{x^{2}}{2 b \sqrt{b\,x^{4}+a}}+\frac{\ln\left(\sqrt{b}\,x^{2}+\sqrt{b\,x^{4}+a}\right)}{2 b^{\frac{3}{2}}}$	42

input
```
int(x^5/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2/b/(b*x^4+a)^(1/2)+1/2/b^(3/2)*arctanh(1/b^(1/2)/x^2*(b*x^4+a)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{(a+bx^4)^{3/2}}\,dx = \left[-\frac{2\sqrt{bx^4+a}bx^2-(bx^4+a)\sqrt{b}\log\left(-2\,bx^4-2\sqrt{bx^4+a}\sqrt{b}x^2-a\right)}{4\,(b^3x^4+ab^2)}, -\frac{\sqrt{bx^4+a}bx^2+(bx^4+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx^4+a}\sqrt{-b}}{bx^2}\right)}{2\,(b^3x^4+ab^2)}\right]$$

input
```
integrate(x^5/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*sqrt(b*x^4 + a)*b*x^2 - (b*x^4 + a)*sqrt(b)*log(-2*b*x^4 - 2*sqrt
(b*x^4 + a)*sqrt(b)*x^2 - a))/(b^3*x^4 + a*b^2), -1/2*(sqrt(b*x^4 + a)*b*x
^2 + (b*x^4 + a)*sqrt(-b)*arctan(sqrt(b*x^4 + a)*sqrt(-b)/(b*x^2)))/(b^3*x
^4 + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{(a+bx^4)^{3/2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} - \frac{x^2}{2\sqrt{a}b\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x**5/(b*x**4+a)**(3/2),x)
```

output
```
asinh(sqrt(b)*x**2/sqrt(a))/(2*b**(3/2)) - x**2/(2*sqrt(a)*b*sqrt(1 + b*x*
*4/a))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{(a+bx^4)^{3/2}}\,dx = -\frac{x^2}{2\sqrt{bx^4+a}\,b} - \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^4+a}}{x^2}}{\sqrt{b}+\frac{\sqrt{bx^4+a}}{x^2}}\right)}{4\,b^{\frac{3}{2}}}$$

input
```
integrate(x^5/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*x^2/(sqrt(b*x^4 + a)*b) - 1/4*log(-(sqrt(b) - sqrt(b*x^4 + a)/x^2)/(s
qrt(b) + sqrt(b*x^4 + a)/x^2))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x^5}{(a+bx^4)^{3/2}}\,dx = -\frac{x^2}{2\sqrt{bx^4+a}\,b} - \frac{\log\left(\left|-\sqrt{b}x^2+\sqrt{bx^4+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^5/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*x^2/(sqrt(b*x^4 + a)*b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a))
)/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^5}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^5/(a + b*x^4)^(3/2),x)
```

output
```
int(x^5/(a + b*x^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{b\,x^4+a}\,\log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)x^2 - 2\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2 + \log\left(\frac{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}{\sqrt{a}}\right)a + \log\left(\right.}{2b\left(\sqrt{b\,x^4+a}\,b\,x^2+\sqrt{b}\,a+\sqrt{b}\,b\,x^4\right)}$$

input
```
int(x^5/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(a + b*x**4)*log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))*x
**2 - 2*sqrt(b)*sqrt(a + b*x**4)*x**2 + log((sqrt(a + b*x**4) + sqrt(b)*x*
*2)/sqrt(a))*a + log((sqrt(a + b*x**4) + sqrt(b)*x**2)/sqrt(a))*b*x**4 - a
 - 2*b*x**4)/(2*b*(sqrt(a + b*x**4)*b*x**2 + sqrt(b)*a + sqrt(b)*b*x**4))
```

3.381 $\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{x^2}{2a\sqrt{a+bx^4}}$$

output
```
1/2*x^2/a/(b*x^4+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^4\right)^{3/2}}\,dx=\frac{x^2}{2a\sqrt{a+bx^4}}$$

input
```
Integrate[x/(a + b*x^4)^(3/2),x]
```

output
```
x^2/(2*a*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx$$
$$\Big\downarrow \text{796}$$
$$\frac{x^2}{2a\sqrt{a+bx^4}}$$

input
```
Int[x/(a + b*x^4)^(3/2),x]
```

output
```
x^2/(2*a*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^{2}}{2 a \sqrt{b \,x^{4}+a}}$	18
default	$\frac{x^{2}}{2 a \sqrt{b \,x^{4}+a}}$	18
trager	$\frac{x^{2}}{2 a \sqrt{b \,x^{4}+a}}$	18
elliptic	$\frac{x^{2}}{2 a \sqrt{b \,x^{4}+a}}$	18
pseudoelliptic	$\frac{x^{2}}{2 a \sqrt{b \,x^{4}+a}}$	18
orering	$\frac{x^{2}}{2 a \sqrt{b \,x^{4}+a}}$	18

input
```
int(x/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a/(b*x^4+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{bx^4+a}x^2}{2\,(abx^4+a^2)}$$

input
```
integrate(x/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(b*x^4 + a)*x^2/(a*b*x^4 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx = \frac{x^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{bx^4}{a}}}$$

input
```
integrate(x/(b*x**4+a)**(3/2),x)
```

output
```
x**2/(2*a**(3/2)*sqrt(1 + b*x**4/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{bx^4+a}a}$$

input
```
integrate(x/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/(sqrt(b*x^4 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{bx^4+a}a}$$

input
```
integrate(x/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*x^2/(sqrt(b*x^4 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx = \frac{x^2}{2\,a\,\sqrt{b\,x^4+a}}$$

input
```
int(x/(a + b*x^4)^(3/2),x)
```

output
```
x^2/(2*a*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{x}{(a+bx^4)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+2b\,x^4}{2a\left(\sqrt{b\,x^4+a}\,b\,x^2+\sqrt{b}\,a+\sqrt{b}\,b\,x^4\right)}$$

input
```
int(x/(b*x^4+a)^(3/2),x)
```

output
```
(2*sqrt(b)*sqrt(a + b*x**4)*x**2 + a + 2*b*x**4)/(2*a*(sqrt(a + b*x**4)*b*
x**2 + sqrt(b)*a + sqrt(b)*b*x**4))
```

3.382 $\int \frac{1}{x^3(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{x^3(a+bx^4)^{3/2}}\,dx = \frac{1}{2ax^2\sqrt{a+bx^4}} - \frac{\sqrt{a+bx^4}}{a^2x^2}$$

output
```
1/2/a/x^2/(b*x^4+a)^(1/2)-(b*x^4+a)^(1/2)/a^2/x^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3(a+bx^4)^{3/2}}\,dx = \frac{-a-2bx^4}{2a^2x^2\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)^(3/2)),x]
```

output
```
(-a - 2*b*x^4)/(2*a^2*x^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \ 803$$

$$-\frac{2b \int \frac{x}{(bx^4+a)^{3/2}}dx}{a}-\frac{1}{2ax^2\sqrt{a+bx^4}}$$

$$\Big\downarrow \ 796$$

$$-\frac{bx^2}{a^2\sqrt{a+bx^4}}-\frac{1}{2ax^2\sqrt{a+bx^4}}$$

input
```
Int[1/(x^3*(a + b*x^4)^(3/2)),x]
```

output
```
-1/2*1/(a*x^2*Sqrt[a + b*x^4]) - (b*x^2)/(a^2*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2b\,x^{4}+a}{2x^{2}\sqrt{b\,x^{4}+a}\,a^{2}}$	26
default	$-\frac{2b\,x^{4}+a}{2x^{2}\sqrt{b\,x^{4}+a}\,a^{2}}$	26
trager	$-\frac{2b\,x^{4}+a}{2x^{2}\sqrt{b\,x^{4}+a}\,a^{2}}$	26
elliptic	$-\frac{2b\,x^{4}+a}{2x^{2}\sqrt{b\,x^{4}+a}\,a^{2}}$	26
pseudoelliptic	$-\frac{2b\,x^{4}+a}{2x^{2}\sqrt{b\,x^{4}+a}\,a^{2}}$	26
orering	$-\frac{2b\,x^{4}+a}{2x^{2}\sqrt{b\,x^{4}+a}\,a^{2}}$	26
risch	$-\frac{\sqrt{b\,x^{4}+a}}{2a^{2}x^{2}}-\frac{x^{2}b}{2\sqrt{b\,x^{4}+a}\,a^{2}}$	37

input
```
int(1/x^3/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x^4+a)/x^2/(b*x^4+a)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3\left(a+bx^4\right)^{3/2}}\,dx=-\frac{\left(2\,bx^4+a\right)\sqrt{bx^4+a}}{2\left(a^2bx^6+a^3x^2\right)}$$

input
```
integrate(1/x^3/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x^4 + a)*sqrt(b*x^4 + a)/(a^2*b*x^6 + a^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3 (a+bx^4)^{3/2}}\,dx = -\frac{1}{2a\sqrt{b}x^4\sqrt{\frac{a}{bx^4}+1}} - \frac{\sqrt{b}}{a^2\sqrt{\frac{a}{bx^4}+1}}$$

input
```
integrate(1/x**3/(b*x**4+a)**(3/2),x)
```

output
```
-1/(2*a*sqrt(b)*x**4*sqrt(a/(b*x**4) + 1)) - sqrt(b)/(a**2*sqrt(a/(b*x**4)
 + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a+bx^4)^{3/2}}\,dx = -\frac{bx^2}{2\sqrt{bx^4+a}a^2} - \frac{\sqrt{bx^4+a}}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*b*x^2/(sqrt(b*x^4 + a)*a^2) - 1/2*sqrt(b*x^4 + a)/(a^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \frac{1}{x^3 (a+bx^4)^{3/2}}\,dx = -\frac{bx^2}{2\sqrt{bx^4+a}a^2} + \frac{\sqrt{b}}{\left(\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2-a\right)a}$$

input
```
integrate(1/x^3/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
-1/2*b*x^2/(sqrt(b*x^4 + a)*a^2) + sqrt(b)/(((sqrt(b)*x^2 - sqrt(b*x^4 + a
))^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^3 (a+bx^4)^{3/2}} \, dx = -\frac{2 b x^4 + a}{2 a^2 x^2 \sqrt{b x^4 + a}}$$

input
```
int(1/(x^3*(a + b*x^4)^(3/2)),x)
```

output
```
-(a + 2*b*x^4)/(2*a^2*x^2*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.54

$$\int \frac{1}{x^3 (a+bx^4)^{3/2}} \, dx = \frac{-4\sqrt{b}\,\sqrt{b\,x^4+a}\,a\,x^2 - 8\sqrt{b}\,\sqrt{b\,x^4+a}\,b\,x^6 - a^2 - 8ab\,x^4 - 8b^2x^8}{2a^2x^2\left(\sqrt{b\,x^4+a}\,a + 2\sqrt{b\,x^4+a}\,b\,x^4 + 2\sqrt{b}\,a\,x^2 + 2\sqrt{b}\,b\,x^6\right)}$$

input
```
int(1/x^3/(b*x^4+a)^(3/2),x)
```

output
```
( - 4*sqrt(b)*sqrt(a + b*x**4)*a*x**2 - 8*sqrt(b)*sqrt(a + b*x**4)*b*x**6
- a**2 - 8*a*b*x**4 - 8*b**2*x**8)/(2*a**2*x**2*(sqrt(a + b*x**4)*a + 2*sq
rt(a + b*x**4)*b*x**4 + 2*sqrt(b)*a*x**2 + 2*sqrt(b)*b*x**6))
```

3.383 $\int \frac{1}{x^7(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{1}{x^7\left(a+bx^4\right)^{3/2}}\,dx=\frac{1}{2ax^6\sqrt{a+bx^4}}-\frac{2\sqrt{a+bx^4}}{3a^2x^6}+\frac{4b\sqrt{a+bx^4}}{3a^3x^2}$$

output
```
1/2/a/x^6/(b*x^4+a)^(1/2)-2/3*(b*x^4+a)^(1/2)/a^2/x^6+4/3*b*(b*x^4+a)^(1/2
)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^7\left(a+bx^4\right)^{3/2}}\,dx=\frac{-a^2+4abx^4+8b^2x^8}{6a^3x^6\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)^(3/2)),x]
```

output
```
(-a^2 + 4*a*b*x^4 + 8*b^2*x^8)/(6*a^3*x^6*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^4)^{3/2}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{4b\int \frac{1}{x^3(bx^4+a)^{3/2}}dx}{3a}-\frac{1}{6ax^6\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{4b\left(-\frac{2b\int \frac{x}{\left(bx^4+a\right)^{3/2}}dx}{a}-\frac{1}{2ax^2\sqrt{a+bx^4}}\right)}{3a}-\frac{1}{6ax^6\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{4b\left(-\frac{bx^2}{a^2\sqrt{a+bx^4}}-\frac{1}{2ax^2\sqrt{a+bx^4}}\right)}{3a}-\frac{1}{6ax^6\sqrt{a+bx^4}}$$

input
```
Int[1/(x^7*(a + b*x^4)^(3/2)),x]
```

output
```
-1/6*1/(a*x^6*Sqrt[a + b*x^4]) - (4*b*(-1/2*1/(a*x^2*Sqrt[a + b*x^4]) - (b
*x^2)/(a^2*Sqrt[a + b*x^4])))/(3*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{-8b^{2}x^{8}-4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{b\,x^{4}+a}\,a^{3}}$	37
default	$-\frac{-8b^{2}x^{8}-4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{b\,x^{4}+a}\,a^{3}}$	37
trager	$-\frac{-8b^{2}x^{8}-4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{b\,x^{4}+a}\,a^{3}}$	37
elliptic	$-\frac{-8b^{2}x^{8}-4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{b\,x^{4}+a}\,a^{3}}$	37
pseudoelliptic	$-\frac{-8b^{2}x^{8}-4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{b\,x^{4}+a}\,a^{3}}$	37
orering	$-\frac{-8b^{2}x^{8}-4ab\,x^{4}+a^{2}}{6x^{6}\sqrt{b\,x^{4}+a}\,a^{3}}$	37
risch	$-\frac{\sqrt{b\,x^{4}+a}\,\left(-5b\,x^{4}+a\right)}{6a^{3}x^{6}}+\frac{x^{2}b^{2}}{2\sqrt{b\,x^{4}+a}\,a^{3}}$	47

input
```
int(1/x^7/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-8*b^2*x^8-4*a*b*x^4+a^2)/x^6/(b*x^4+a)^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^7 (a+bx^4)^{3/2}}\, dx = \frac{(8\, b^2x^8 + 4\, abx^4 - a^2)\sqrt{bx^4 + a}}{6\, (a^3bx^{10} + a^4x^6)}$$

input
```
integrate(1/x^7/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(8*b^2*x^8 + 4*a*b*x^4 - a^2)*sqrt(b*x^4 + a)/(a^3*b*x^10 + a^4*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(60) = 120.

Time = 0.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.58

$$\begin{aligned}\int \frac{1}{x^7 (a+bx^4)^{3/2}}\, dx = &-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4 + 12a^4b^5x^8 + 6a^3b^6x^{12}} \\ &+ \frac{3a^2b^{\frac{11}{2}}x^4\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4 + 12a^4b^5x^8 + 6a^3b^6x^{12}} + \frac{12ab^{\frac{13}{2}}x^8\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4 + 12a^4b^5x^8 + 6a^3b^6x^{12}} \\ &+ \frac{8b^{\frac{15}{2}}x^{12}\sqrt{\frac{a}{bx^4}+1}}{6a^5b^4x^4 + 12a^4b^5x^8 + 6a^3b^6x^{12}}\end{aligned}$$

input
```
integrate(1/x**7/(b*x**4+a)**(3/2),x)
```

output
```
-a**3*b**(9/2)*sqrt(a/(b*x**4) + 1)/(6*a**5*b**4*x**4 + 12*a**4*b**5*x**8
+ 6*a**3*b**6*x**12) + 3*a**2*b**(11/2)*x**4*sqrt(a/(b*x**4) + 1)/(6*a**5*
b**4*x**4 + 12*a**4*b**5*x**8 + 6*a**3*b**6*x**12) + 12*a*b**(13/2)*x**8*s
qrt(a/(b*x**4) + 1)/(6*a**5*b**4*x**4 + 12*a**4*b**5*x**8 + 6*a**3*b**6*x*
*12) + 8*b**(15/2)*x**12*sqrt(a/(b*x**4) + 1)/(6*a**5*b**4*x**4 + 12*a**4*
b**5*x**8 + 6*a**3*b**6*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7 (a+bx^4)^{3/2}}\,dx = \frac{b^2x^2}{2\sqrt{bx^4+a}a^3} + \frac{\frac{6\sqrt{bx^4+a}b}{x^2} - \frac{(bx^4+a)^{\frac{3}{2}}}{x^6}}{6a^3}$$

input
```
integrate(1/x^7/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b^2*x^2/(sqrt(b*x^4 + a)*a^3) + 1/6*(6*sqrt(b*x^4 + a)*b/x^2 - (b*x^4
+ a)^(3/2)/x^6)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(53) = 106.

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{1}{x^7 (a+bx^4)^{3/2}}\,dx = \frac{b^2x^2}{2\sqrt{bx^4+a}a^3} - \frac{3\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^4 b^{\frac{3}{2}} - 12\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2 ab^{\frac{3}{2}} + 5a^2b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x^2-\sqrt{bx^4+a}\right)^2-a\right)^3 a^2}$$

input
```
integrate(1/x^7/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*b^2*x^2/(sqrt(b*x^4 + a)*a^3) - 1/3*(3*(sqrt(b)*x^2 - sqrt(b*x^4 + a))
^4*b^(3/2) - 12*(sqrt(b)*x^2 - sqrt(b*x^4 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2
))/(((sqrt(b)*x^2 - sqrt(b*x^4 + a))^2 - a)^3*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^7 (a+bx^4)^{3/2}} \, dx = -\frac{8 (b x^4+a)^2 - 12 a (b x^4+a) + 3 a^2}{\left(\frac{6 a^4 x^2}{b} - \frac{6 a^3 x^2 (b x^4+a)}{b}\right) \sqrt{b x^4+a}}$$

input
```
int(1/(x^7*(a + b*x^4)^(3/2)),x)
```

output
```
-(8*(a + b*x^4)^2 - 12*a*(a + b*x^4) + 3*a^2)/(((6*a^4*x^2)/b - (6*a^3*x^2
*(a + b*x^4))/b)*(a + b*x^4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^7 (a+bx^4)^{3/2}} \, dx = \frac{-4\sqrt{b}\,\sqrt{b x^4+a}\, x^2 - a - 4b x^4}{6x^6 \left(\sqrt{b x^4+a}\, a^2 + 8\sqrt{b x^4+a}\, ab x^4 + 8\sqrt{b x^4+a}\, b^2 x^8 + 4\sqrt{b}\, a^2 x^2 + 12\sqrt{b}\, ab x^6 + \right.}$$

input
```
int(1/x^7/(b*x^4+a)^(3/2),x)
```

output
```
( - 4*sqrt(b)*sqrt(a + b*x**4)*x**2 - a - 4*b*x**4)/(6*x**6*(sqrt(a + b*x*
*4)*a**2 + 8*sqrt(a + b*x**4)*a*b*x**4 + 8*sqrt(a + b*x**4)*b**2*x**8 + 4*
sqrt(b)*a**2*x**2 + 12*sqrt(b)*a*b*x**6 + 8*sqrt(b)*b**2*x**10))
```

3.384 $\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 151

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = -\frac{x^9}{2b\sqrt{a+bx^4}} - \frac{15ax\sqrt{a+bx^4}}{14b^3} + \frac{9x^5\sqrt{a+bx^4}}{14b^2} + \frac{15a^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{28b^{13/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x^9/b/(b*x^4+a)^(1/2)-15/14*a*x*(b*x^4+a)^(1/2)/b^3+9/14*x^5*(b*x^4+a
)^(1/2)/b^2+15/28*a^(7/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2
)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b
^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \frac{-15a^2x-6abx^5+2b^2x^9+15a^2x\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{14b^3\sqrt{a+bx^4}}$$

input
```
Integrate[x^12/(a + b*x^4)^(3/2),x]
```

output
```
(-15*a^2*x - 6*a*b*x^5 + 2*b^2*x^9 + 15*a^2*x*Sqrt[1 + (b*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])/(14*b^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {817, 843, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{9\int \frac{x^8}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^9}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{9\left(\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\int \frac{x^4}{\sqrt{bx^4+a}}dx}{7b}\right)}{2b}-\frac{x^9}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{9\left(\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int\frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{7b}\right)}{2b}-\frac{x^9}{2b\sqrt{a+bx^4}}$$

$\Big\downarrow$ 761

$$\frac{9\left(\frac{x^5\sqrt{a+bx^4}}{7b}-\frac{5a\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{7b}\right)}{2b}-\frac{x^9}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^12/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x^9/(b*Sqrt[a + b*x^4]) + (9*((x^5*Sqrt[a + b*x^4])/(7*b) - (5*a*((x*
Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)
/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])
/(6*b^(5/4)*Sqrt[a + b*x^4])))/(7*b)))/(2*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^{2} x}{2 b^{3} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{x^{5} \sqrt{b \,x^{4}+a}}{7 b^{2}}-\frac{4 a x \sqrt{b \,x^{4}+a}}{7 b^{3}}+\frac{15 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{14 b^{3} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$
elliptic	$-\frac{a^{2} x}{2 b^{3} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{x^{5} \sqrt{b \,x^{4}+a}}{7 b^{2}}-\frac{4 a x \sqrt{b \,x^{4}+a}}{7 b^{3}}+\frac{15 a^{2} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{14 b^{3} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$
risch	$-\frac{x \left(-b \,x^{4}+4 a \right) \sqrt{b \,x^{4}+a}}{7 b^{3}}+\frac{a^{2} \left(4 a \left(\frac{x}{2 a \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)}{2 a \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}\right)+11 b \left(-\frac{x}{2 b \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{\sqrt{1-\cdots}}{\cdots}\right.\right.}{7 b^{3}}$

input
```
int(x^12/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*a^2*x/((x^4+a/b)*b)^(1/2)+1/7*x^5*(b*x^4+a)^(1/2)/b^2-4/7*a*x*(b*
x^4+a)^(1/2)/b^3+15/14*a^2/b^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/
a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.58

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \frac{15\,(abx^4+a^2)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1) + (2\,b^2x^9-6\,abx^5-15\,a^2x)\sqrt{bx^4+a}}{14\,(b^4x^4+ab^3)}$$

input
```
integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/14*(15*(a*b*x^4 + a^2)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/
4)/x), -1) + (2*b^2*x^9 - 6*a*b*x^5 - 15*a^2*x)*sqrt(b*x^4 + a))/(b^4*x^4
+ a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12/(b*x**4+a)**(3/2),x)
```

output
```
x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(3/2)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^{12}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^12/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^{12}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^12/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^{12}}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^12/(a + b*x^4)^(3/2),x)
```

output
```
int(x^12/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/2}}\,dx = \frac{-15\sqrt{b\,x^4+a}\,a^2x-3\sqrt{b\,x^4+a}\,ab\,x^5+\sqrt{b\,x^4+a}\,b^2x^9+15\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^4+}{7b^3\,(b\,x^4+a)}$$

input
```
int(x^12/(b*x^4+a)^(3/2),x)
```

output
```
( - 15*sqrt(a + b*x**4)*a**2*x - 3*sqrt(a + b*x**4)*a*b*x**5 + sqrt(a + b*
x**4)*b**2*x**9 + 15*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),
x)*a**4 + 15*int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3*
b*x**4)/(7*b**3*(a + b*x**4))
```

3.385 $\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx = -\frac{x^5}{2b\sqrt{a+bx^4}}+\frac{5x\sqrt{a+bx^4}}{6b^2} - \frac{5a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12b^{9/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x^5/b/(b*x^4+a)^(1/2)+5/6*x*(b*x^4+a)^(1/2)/b^2-5/12*a^(3/4)*(a^(1/2)
+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*
arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.51

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx = \frac{5ax+2bx^5-5ax\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{6b^2\sqrt{a+bx^4}}$$

input
```
Integrate[x^8/(a + b*x^4)^(3/2),x]
```

output
```
(5*a*x + 2*b*x^5 - 5*a*x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5
/4, -((b*x^4)/a)])/(6*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {817, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5\int \frac{x^4}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a\int \frac{1}{\sqrt{bx^4+a}}dx}{3b}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(\frac{x\sqrt{a+bx^4}}{3b}-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6b^{5/4}\sqrt{a+bx^4}}\right)}{2b}-\frac{x^5}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^8/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x^5/(b*Sqrt[a + b*x^4]) + (5*((x*Sqrt[a + b*x^4])/(3*b) - (a^(3/4)*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*b^(5/4)*Sqrt[a + b*x^4])))/(2*b)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

method	result
default	$\frac{xa}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$\frac{xa}{2b^2\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{5a\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
risch	$\frac{x\sqrt{b\,x^4+a}}{3b^2}-\frac{a\left(a\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+4b\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\ \mathrm{E}}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b}}\right.\right.}{3b^2}$

input
```
int(x^8/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*x*a/((x^4+a/b)*b)^(1/2)+1/3*x*(b*x^4+a)^(1/2)/b^2-5/6*a/b^2/(I/a^(
1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/
2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{x^8}{\left(a+bx^4\right)^{3/2}}\,dx = -\frac{5\left(bx^4+a\right)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1\right)-\left(2\,bx^5+5\,ax\right)\sqrt{bx^4+a}}{6\left(b^3x^4+ab^2\right)}$$

input
```
integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(5*(b*x^4 + a)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/4)/x)
, -1) - (2*b*x^5 + 5*a*x)*sqrt(b*x^4 + a))/(b^3*x^4 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(b*x**4+a)**(3/2),x)
```

output
```
x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^8}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^8}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^8}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^8/(a + b*x^4)^(3/2),x)
```

output
```
int(x^8/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^4)^{3/2}}\,dx=\frac{5\sqrt{b\,x^4+a}\,ax+\sqrt{b\,x^4+a}\,b\,x^5-5\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^3-5\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^2}{3b^2\left(b\,x^4+a\right)}$$

input
```
int(x^8/(b*x^4+a)^(3/2),x)
```

output
```
(5*sqrt(a + b*x**4)*a*x + sqrt(a + b*x**4)*b*x**5 - 5*int(sqrt(a + b*x**4)
/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3 - 5*int(sqrt(a + b*x**4)/(a**2 +
2*a*b*x**4 + b**2*x**8),x)*a**2*b*x**4)/(3*b**2*(a + b*x**4))
```

3.386 $\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = -\frac{x}{2b\sqrt{a+bx^4}} + \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x/b/(b*x^4+a)^(1/2)+1/4*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(
1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2)
)/a^(1/4)/b^(5/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = \frac{x\left(-1+\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{2b\sqrt{a+bx^4}}$$

input
```
Integrate[x^4/(a + b*x^4)^(3/2),x]
```

output
```
(x*(-1 + Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)
]))/(2*b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx$$
$$\Big\downarrow \ 817$$
$$\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2b}-\frac{x}{2b\sqrt{a+bx^4}}$$
$$\Big\downarrow \ 761$$
$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}b^{5/4}\sqrt{a+bx^4}}-\frac{x}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^4/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x/(b*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*
a^(1/4)*b^(5/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	94
elliptic	$-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	94

input
```
int(x^4/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*x/((x^4+a/b)*b)^(1/2)+1/2/b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*
x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*Ellipti
cF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = -\frac{(bx^4+a)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right) \mid -1\right) + \sqrt{bx^4+a}bx}{2\left(b^3x^4+ab^2\right)}$$

input
```
integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((b*x^4 + a)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
-1) + sqrt(b*x^4 + a)*b*x)/(b^3*x^4 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(b*x**4+a)**(3/2),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^4/(a + b*x^4)^(3/2),x)
```

output
```
int(x^4/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^4)^{3/2}}\,dx = \frac{-\sqrt{b\,x^4+a}\,x+\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)a^2+\left(\int \frac{\sqrt{b\,x^4+a}}{b^2x^8+2ab\,x^4+a^2}dx\right)ab\,x^4}{b\,(b\,x^4+a)}$$

input
```
int(x^4/(b*x^4+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**4)*x + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x*
*8),x)*a**2 + int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a*b*
x**4)/(b*(a + b*x**4))
```

3.387 $\int \frac{1}{(a+bx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 108

$$\int \frac{1}{(a+bx^4)^{3/2}} \, dx = \frac{x}{2a\sqrt{a+bx^4}} + \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}$$

output
```
1/2*x/a/(b*x^4+a)^(1/2)+1/4*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1
/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))
/a^(5/4)/b^(1/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+bx^4)^{3/2}} \, dx = \frac{x+x\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^4}{a}\right)}{2a\sqrt{a+bx^4}}$$

input
```
Integrate[(a + b*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^4)/a)])
/(2*a*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 749}$$

$$\frac{\int \frac{1}{\sqrt{bx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{b}\sqrt{a+bx^4}}+\frac{x}{2a\sqrt{a+bx^4}}$$

input
```
Int[(a + b*x^4)^(-3/2),x]
```

output
```
x/(2*a*Sqrt[a + b*x^4]) + ((Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
5/4)*b^(1/4)*Sqrt[a + b*x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	94
elliptic	$\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$	94

input
```
int(1/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/((x^4+a/b)*b)^(1/2)+1/2/a/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x
^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a+bx^4)^{3/2}}\,dx = -\frac{(bx^4+a)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,|\,-1\right)-\sqrt{bx^4+a}\,bx}{2\,(ab^2x^4+a^2b)}$$

input
```
integrate(1/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((b*x^4 + a)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/4)),
-1) - sqrt(b*x^4 + a)*b*x)/(a*b^2*x^4 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{1}{(a+bx^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(b*x**4+a)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+bx^4)^{3/2}}\,dx = \frac{x\left(\frac{b\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/2}}$$

input
```
int(1/(a + b*x^4)^(3/2),x)
```

output
```
(x*((b*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(b*x^4)/a))/(a + b*x^
4)^(3/2)
```

Reduce [F]

$$\int \frac{1}{(a + bx^4)^{3/2}}\, dx = \int \frac{\sqrt{b\, x^4 + a}}{b^2 x^8 + 2ab\, x^4 + a^2} dx$$

input
```
int(1/(b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)
```

3.388 $\int \frac{1}{x^4(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\frac{1}{2ax^3\sqrt{a+bx^4}}-\frac{5\sqrt{a+bx^4}}{6a^2x^3}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt{a+bx^4}}$$

output
```
1/2/a/x^3/(b*x^4+a)^(1/2)-5/6*(b*x^4+a)^(1/2)/a^2/x^3-5/12*b^(3/4)*(a^(1/2
)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2
*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(9/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^4}{a}\right)}{3ax^3\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^4*(a + b*x^4)^(3/2)),x]
```

output
```
-1/3*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^4)/a)])
/(a*x^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^4)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 819}$$
$$\frac{5\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}$$
$$\Big\downarrow \text{ 847}$$
$$\frac{5\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}$$
$$\Big\downarrow \text{ 761}$$
$$\frac{5\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\sqrt{a+bx^4}}$$

input
```
Int[1/(x^4*(a + b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x^3*Sqrt[a + b*x^4]) + (5*(-1/3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/4)*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipt
icF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + b*x^4])))/(2*
a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.86

method	result
default	$-\frac{\sqrt{b\,x^{4}+a}}{3a^{2}x^{3}}-\frac{bx}{2a^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{5b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{3a^{2}x^{3}}-\frac{bx}{2a^{2}\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{5b\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{6a^{2}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$
risch	$-\frac{\sqrt{b\,x^{4}+a}}{3a^{2}x^{3}}-\frac{b\left(b\left(-\frac{x}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2b\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}\right)+4a\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\cdots}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{\cdots}}\right.\cdots}{3a^{2}}$

input
```
int(1/x^4/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^4+a)^(1/2)/a^2/x^3-1/2*b/a^2*x/((x^4+a/b)*b)^(1/2)-5/6*b/a^2/(I/
a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^
(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\left(a+bx^4\right)^{3/2}}\,dx=\frac{5\left(bx^7+ax^3\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1)-\left(5\,bx^4+2\,a\right)\sqrt{bx^4+a}}{6\left(a^2bx^7+a^3x^3\right)}$$

input
```
integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(5*(b*x^7 + a*x^3)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^(1/
4)), -1) - (5*b*x^4 + 2*a)*sqrt(b*x^4 + a))/(a^2*b*x^7 + a^3*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^4 (a+bx^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}} x^3 \Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(b*x**4+a)**(3/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{x^4 \left(b\,x^4+a\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(a + b*x^4)^(3/2)),x)
```

output
```
int(1/(x^4*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b^2x^{12}+2ab\,x^8+a^2x^4}dx$$

input
```
int(1/x^4/(b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*x**4 + 2*a*b*x**8 + b**2*x**12),x)
```

3.389 $\int \frac{1}{x^8(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx=\frac{1}{2ax^7\sqrt{a+bx^4}}-\frac{9\sqrt{a+bx^4}}{14a^2x^7}+\frac{15b\sqrt{a+bx^4}}{14a^3x^3}+\frac{15b^{7/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{28a^{13/4}\sqrt{a+bx^4}}$$

output
```
1/2/a/x^7/(b*x^4+a)^(1/2)-9/14*(b*x^4+a)^(1/2)/a^2/x^7+15/14*b*(b*x^4+a)^(
1/2)/a^3/x^3+15/28*b^(7/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/
2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/
a^(13/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx=-\frac{\sqrt{1+\frac{bx^4}{a}}\operatorname{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},-\frac{3}{4},-\frac{bx^4}{a}\right)}{7ax^7\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^8*(a + b*x^4)^(3/2)),x]
```

output
```
-1/7*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-7/4, 3/2, -3/4, -((b*x^4)/a)]
)/(a*x^7*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {819, 847, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a+bx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{9\int \frac{1}{x^8\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax^7\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{9\left(-\frac{5b\int \frac{1}{x^4\sqrt{bx^4+a}}dx}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{9\left(-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{bx^4+a}}dx}{3a}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{9\left(-\frac{5b\left(-\frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+bx^4}}-\frac{\sqrt{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt{a+bx^4}}{7ax^7}\right)}{2a}+\frac{1}{2ax^7\sqrt{a+bx^4}}$$

input
```
Int[1/(x^8*(a + b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x^7*Sqrt[a + b*x^4]) + (9*(-1/7*Sqrt[a + b*x^4]/(a*x^7) - (5*b*(-1/
3*Sqrt[a + b*x^4]/(a*x^3) - (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x
^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/
2])/(6*a^(5/4)*Sqrt[a + b*x^4])))/(7*a)))/(2*a)
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{b\,x^4+a}}{7a^2x^7}+\frac{4b\sqrt{b\,x^4+a}}{7a^3x^3}+\frac{b^2x}{2a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{15b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{14a^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$-\frac{\sqrt{b\,x^4+a}}{7a^2x^7}+\frac{4b\sqrt{b\,x^4+a}}{7a^3x^3}+\frac{b^2x}{2a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{15b^2\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{14a^3\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,\left(-4b\,x^4+a\right)}{7a^3x^7}+\frac{b^2\left(11a\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}\right)+4b\left(-\frac{x}{2b\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\dots\right)\right)}{7a^3}$

input
```
int(1/x^8/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^4+a)^(1/2)/a^2/x^7+4/7*b*(b*x^4+a)^(1/2)/a^3/x^3+1/2*b^2/a^3*x/(
(x^4+a/b)*b)^(1/2)+15/14*b^2/a^3/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^
2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*b^(1/2))^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^8\left(a+bx^4\right)^{3/2}}\,dx =$$

$$-\frac{15\left(b^2x^{11}+abx^7\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\,\middle|\,-1\right)-\left(15\,b^2x^8+6\,abx^4-2\,a^2\right)\sqrt{bx^4+a}}{14\left(a^3bx^{11}+a^4x^7\right)}$$

input
```
integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/14*(15*(b^2*x^11 + a*b*x^7)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-
b/a)^(1/4)), -1) - (15*b^2*x^8 + 6*a*b*x^4 - 2*a^2)*sqrt(b*x^4 + a))/(a^3*
b*x^11 + a^4*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^8 (a+bx^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}} x^7 \Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(b*x**4+a)**(3/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{x^8 (b\,x^4+a)^{3/2}}\,dx$$

input
```
int(1/(x^8*(a + b*x^4)^(3/2)),x)
```

output
```
int(1/(x^8*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (a+bx^4)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b^2x^{16}+2ab\,x^{12}+a^2x^8}dx$$

input
```
int(1/x^8/(b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*x**8 + 2*a*b*x**12 + b**2*x**16),x)
```

3.390 $\int \frac{x^{14}}{(a+bx^4)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 282

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}} dx = -\frac{x^{11}}{2b\sqrt{a+bx^4}} - \frac{77ax^3\sqrt{a+bx^4}}{90b^3} + \frac{11x^7\sqrt{a+bx^4}}{18b^2} + \frac{77a^2x\sqrt{a+bx^4}}{30b^{7/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{a+bx^4}} + \frac{77a^{9/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{60b^{15/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x^11/b/(b*x^4+a)^(1/2)-77/90*a*x^3*(b*x^4+a)^(1/2)/b^3+11/18*x^7*(b*x
^4+a)^(1/2)/b^2+77/30*a^2*x*(b*x^4+a)^(1/2)/b^(7/2)/(a^(1/2)+b^(1/2)*x^2)-
77/30*a^(9/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1
/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(15/4)/(b*x^
4+a)^(1/2)+77/60*a^(9/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^
(15/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.28

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\left(77a^2-11abx^4+5b^2x^8-77a^2\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{45b^3\sqrt{a+bx^4}}$$

input
```
Integrate[x^14/(a + b*x^4)^(3/2),x]
```

output
```
(x^3*(77*a^2 - 11*a*b*x^4 + 5*b^2*x^8 - 77*a^2*Sqrt[1 + (b*x^4)/a]*Hyperge
ometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)]))/(45*b^3*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {817, 843, 843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 817}$$

$$\frac{11\int \frac{x^{10}}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{11\left(\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\int \frac{x^6}{\sqrt{bx^4+a}}dx}{9b}\right)}{2b}-\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{11\left(\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int\frac{x^2}{\sqrt{bx^4+a}}dx}{5b}\right)}{9b}\right)}{2b}-\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

$\downarrow$ 834

$$\frac{11\left(\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}\right)}{2b}-\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

$\downarrow$ 27

$$\frac{11\left(\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}\right)}{2b}-\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

$\downarrow$ 761

$$\frac{11\left(\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{9b}\right)}{2b}-\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

$\downarrow$ 1510

$$\frac{11\left(\frac{x^7\sqrt{a+bx^4}}{9b}-\frac{7a\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\,\mathrm{a}\right.}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right.}{5b}\right.}{9b}\right.}{2b}$$

$$\frac{x^{11}}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^14/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x^11/(b*Sqrt[a + b*x^4]) + (11*((x^7*Sqrt[a + b*x^4])/(9*b) - (7*a*((
x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqr
t[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sq
rt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b)))/(9*b)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.56

method	result
default	$-\frac{x^3 a^2}{2b^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x^7\sqrt{b\,x^4+a}}{9b^2}-\frac{16a\,x^3\sqrt{b\,x^4+a}}{45b^3}+\frac{77ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\right.\right.}{30b^{\frac{7}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
elliptic	$-\frac{x^3 a^2}{2b^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}+\frac{x^7\sqrt{b\,x^4+a}}{9b^2}-\frac{16a\,x^3\sqrt{b\,x^4+a}}{45b^3}+\frac{77ia^{\frac{5}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\right.\right.}{30b^{\frac{7}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}}$
risch	$-\frac{x^3\left(-5b\,x^4+16a\right)\sqrt{b\,x^4+a}}{45b^3}+\frac{a^2\left(16a\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^4+a}\,\sqrt{b}}\right)+3\right.}{15}$

input
```
int(x^14/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*x^3*a^2/((x^4+a/b)*b)^(1/2)+1/9*x^7*(b*x^4+a)^(1/2)/b^2-16/45*a*x
^3*(b*x^4+a)^(1/2)/b^3+77/30*I*a^(5/2)/b^(7/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(
1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(
1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1
/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.51

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \frac{231\,(a^2bx^5+a^3x)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)-231\,(a^2bx^5+a^3x)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F}{90\,(b^5x^5+a}$$

input
```
integrate(x^14/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/90*(231*(a^2*b*x^5 + a^3*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b
)^(1/4)/x), -1) - 231*(a^2*b*x^5 + a^3*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(
arcsin((-a/b)^(1/4)/x), -1) + (10*b^3*x^12 - 22*a*b^2*x^8 + 154*a^2*b*x^4
+ 231*a^3)*sqrt(b*x^4 + a))/(b^5*x^5 + a*b^4*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \frac{x^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**14/(b*x**4+a)**(3/2),x)
```

output
```
x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(3/2)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^{14}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^14/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^{14}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^14/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^{14}}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^14/(a + b*x^4)^(3/2),x)
```

output
```
int(x^14/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(a+bx^4)^{3/2}}\,dx = \frac{77\sqrt{b\,x^4+a}\,a^2x^3-11\sqrt{b\,x^4+a}\,ab\,x^7+5\sqrt{b\,x^4+a}\,b^2x^{11}-231\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right) a}{45b^3\,(b\,x^4+a)}$$

input
```
int(x^14/(b*x^4+a)^(3/2),x)
```

output
```
(77*sqrt(a + b*x**4)*a**2*x**3 - 11*sqrt(a + b*x**4)*a*b*x**7 + 5*sqrt(a +
 b*x**4)*b**2*x**11 - 231*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
 b**2*x**8),x)*a**4 - 231*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
 b**2*x**8),x)*a**3*b*x**4)/(45*b**3*(a + b*x**4))
```

3.391 $\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 258

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx = -\frac{x^7}{2b\sqrt{a+bx^4}}+\frac{7x^3\sqrt{a+bx^4}}{10b^2}-\frac{21ax\sqrt{a+bx^4}}{10b^{5/2}\left(\sqrt{a}+\sqrt{b}x^2\right)}+\frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^4}}-\frac{21a^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{20b^{11/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x^7/b/(b*x^4+a)^(1/2)+7/10*x^3*(b*x^4+a)^(1/2)/b^2-21/10*a*x*(b*x^4+a
)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x^2)+21/10*a^(5/4)*(a^(1/2)+b^(1/2)*x^2)*
((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x
/a^(1/4))),1/2*2^(1/2))/b^(11/4)/(b*x^4+a)^(1/2)-21/20*a^(5/4)*(a^(1/2)+b^
(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^(11/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.26

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\left(-7a+bx^4+7a\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{5b^2\sqrt{a+bx^4}}$$

input
```
Integrate[x^10/(a + b*x^4)^(3/2),x]
```

output
```
(x^3*(-7*a + b*x^4 + 7*a*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7
/4, -((b*x^4)/a)]))/(5*b^2*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {817, 843, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7\int \frac{x^6}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}}dx}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{834}$$

$$\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt{a}\int\frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{x^7}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{7\left(\frac{x^3\sqrt{a+bx^4}}{5b}-\frac{3a\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}}{\sqrt{b}}\right)}{5b}\right)}{2b}$$

$$\frac{x^7}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^10/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x^7/(b*Sqrt[a + b*x^4]) + (7*((x^3*Sqrt[a + b*x^4])/(5*b) - (3*a*(-((
-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[
b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(S
qrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Elliptic
F[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(5*b)
))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.53

method	result
default	$\frac{x^{3} a}{2 b^{2} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b^{2}}-\frac{21 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{10 b^{\frac{5}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$
elliptic	$\frac{x^{3} a}{2 b^{2} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b^{2}}-\frac{21 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{10 b^{\frac{5}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}}$
risch	$\frac{x^{3} \sqrt{b \,x^{4}+a}}{5 b^{2}}-\frac{a \left(3 a \left(\frac{x^{3}}{2 a \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}-\frac{i \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 \sqrt{a}\, \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b \,x^{4}+a}\, \sqrt{b}}\right)+8 b \left(-\frac{x^{3}}{2 b \sqrt{\left(x^{4}+\frac{a}{b}\right) b}} \cdots\right)\right)}{5 b^{2}}$

input
```
int(x^10/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*x^3*a/((x^4+a/b)*b)^(1/2)+1/5*x^3*(b*x^4+a)^(1/2)/b^2-21/10*I*a^(3
/2)/b^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I
*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2
))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.51

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx =$$
$$-\frac{21\,(abx^5+a^2x)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)-21\,(abx^5+a^2x)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)}{10\,(b^4x^5+ab^3x)}$$

input
```
integrate(x^10/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(21*(a*b*x^5 + a^2*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^
(1/4)/x), -1) - 21*(a*b*x^5 + a^2*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsi
n((-a/b)^(1/4)/x), -1) - (2*b^2*x^8 - 14*a*b*x^4 - 21*a^2)*sqrt(b*x^4 + a)
)/(b^4*x^5 + a*b^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(b*x**4+a)**(3/2),x)
```

output
```
x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(3/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^{10}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^10/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^{10}}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^10/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^{10}}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^10/(a + b*x^4)^(3/2),x)
```

output
```
int(x^10/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a+bx^4)^{3/2}}\,dx = \frac{-7\sqrt{b\,x^4+a}\,a\,x^3+\sqrt{b\,x^4+a}\,b\,x^7+21\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right)a^3+21\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}d}{5b^2\left(b\,x^4+a\right)}$$

input

```
int(x^10/(b*x^4+a)^(3/2),x)
```

output

```
( - 7*sqrt(a + b*x**4)*a*x**3 + sqrt(a + b*x**4)*b*x**7 + 21*int((sqrt(a +
 b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**3 + 21*int((sqrt(a +
b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)*a**2*b*x**4)/(5*b**2*(a +
 b*x**4))
```

3.392 $\int \frac{x^6}{\left(a+bx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 236

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx = -\frac{x^3}{2b\sqrt{a+bx^4}}+\frac{3x\sqrt{a+bx^4}}{2b^{3/2}\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^4}} + \frac{3\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4b^{7/4}\sqrt{a+bx^4}}$$

output
```
-1/2*x^3/b/(b*x^4+a)^(1/2)+3/2*x*(b*x^4+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*
x^2)-3/2*a^(1/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)
^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/b^(7/4)/(b*
x^4+a)^(1/2)+3/4*a^(1/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)
*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/b^
(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.24

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx=\frac{x^3-x^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)}{b\sqrt{a+bx^4}}$$

input
```
Integrate[x^6/(a + b*x^4)^(3/2),x]
```

output
```
(x^3 - x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/
a)])/(b*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {817, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{3\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{834}$$

$$\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}$$

$\downarrow$ 761

$$\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{2b}-\frac{x^3}{2b\sqrt{a+bx^4}}$$

$\downarrow$ 1510

$$\frac{3\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}\right)}{2b}$$

$$\frac{x^3}{2b\sqrt{a+bx^4}}$$

input
```
Int[x^6/(a + b*x^4)^(3/2),x]
```

output
```
-1/2*x^3/(b*Sqrt[a + b*x^4]) + (3*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqr
t[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] +
Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sq
rt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^
4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2
])/(2*b^(3/4)*Sqrt[a + b*x^4])))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.50

method	result	size
default	$-\frac{x^{3}}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	119
elliptic	$-\frac{x^{3}}{2b\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2b^{\frac{3}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}}$	119

input
```
int(x^6/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*x^3/((x^4+a/b)*b)^(1/2)+3/2*I/b^(3/2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(
1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^
4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx=\frac{3\,(bx^5+ax)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{b}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-3\,(bx^5+ax)\sqrt{b}\left(-\frac{a}{b}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{(-\cdots}{}\right.}{2\,(b^3x^5+ab^2x)}$$

input
```
integrate(x^6/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*(b*x^5 + a*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_e(arcsin((-a/b)^(1/4)/x
), -1) - 3*(b*x^5 + a*x)*sqrt(b)*(-a/b)^(3/4)*elliptic_f(arcsin((-a/b)^(1/
4)/x), -1) + (2*b*x^4 + 3*a)*sqrt(b*x^4 + a))/(b^3*x^5 + a*b^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.16

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(b*x**4+a)**(3/2),x)
```

output
```
x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx=\int \frac{x^6}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^6/(a + b*x^4)^(3/2),x)
```

output
```
int(x^6/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^4)^{3/2}}\,dx=\frac{\sqrt{b\,x^4+a}\,x^3-3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right)a^2-3\left(\int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx\right)ab\,x^4}{b\,(b\,x^4+a)}$$

input
```
int(x^6/(b*x^4+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**4)*x**3 - 3*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4
+ b**2*x**8),x)*a**2 - 3*int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 +
b**2*x**8),x)*a*b*x**4)/(b*(a + b*x**4))
```

3.393 $\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 239

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \frac{x^3}{2a\sqrt{a+bx^4}} - \frac{x\sqrt{a+bx^4}}{2a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x^2\right)} + \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^4}} - \frac{\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{3/4}\sqrt{a+bx^4}}$$

output
```
1/2*x^3/a/(b*x^4+a)^(1/2)-1/2*x*(b*x^4+a)^(1/2)/a/b^(1/2)/(a^(1/2)+b^(1/2)
*x^2)+1/2*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*
EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b
*x^4+a)^(1/2)-1/4*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2
)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(3/4)/b
^(3/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.83 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.23

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a\sqrt{a+bx^4}}$$

input
```
Integrate[x^2/(a + b*x^4)^(3/2),x]
```

output
```
(x^3*Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^4)/a)])/(
3*a*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {819, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 819}$$

$$\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\int \frac{x^2}{\sqrt{bx^4+a}}dx}{2a}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}$$

$$\Big\downarrow \text{761}$$

$$\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}}{2a}$$

$$\Big\downarrow \text{1510}$$

$$\frac{x^3}{2a\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}}{2a}$$

input
```
Int[x^2/(a + b*x^4)^(3/2),x]
```

output
```
x^3/(2*a*Sqrt[a + b*x^4]) - (-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x
^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b
]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a +
 b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sq
rt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*
b^(3/4)*Sqrt[a + b*x^4]))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$	119
elliptic	$\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{b}\,x^{2}}{\sqrt{a}}}\,\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,\sqrt{b\,x^{4}+a}\,\sqrt{b}}$	119

input
```
int(x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3/a/((x^4+a/b)*b)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2
)/b^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)
*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.41

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \frac{\sqrt{bx^4+a}bx^3 + (bx^4+a)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right) \mid -1\right) - (bx^4+a)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}} F}{2\left(ab^2x^4+a^2b\right)}$$

input
```
integrate(x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(b*x^4 + a)*b*x^3 + (b*x^4 + a)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(a
rcsin(x*(-b/a)^(1/4)), -1) - (b*x^4 + a)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(a
rcsin(x*(-b/a)^(1/4)), -1))/(a*b^2*x^4 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(b*x**4+a)**(3/2),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \int \frac{x^2}{(b\,x^4+a)^{3/2}}\,dx$$

input
```
int(x^2/(a + b*x^4)^(3/2),x)
```

output
```
int(x^2/(a + b*x^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^4)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^4+a}\,x^2}{b^2x^8+2ab\,x^4+a^2}dx$$

input
```
int(x^2/(b*x^4+a)^(3/2),x)
```

output
```
int((sqrt(a + b*x**4)*x**2)/(a**2 + 2*a*b*x**4 + b**2*x**8),x)
```

3.394 $\int \frac{1}{x^2(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 260

$$\int \frac{1}{x^2(a+bx^4)^{3/2}}\,dx = \frac{1}{2ax\sqrt{a+bx^4}} - \frac{3\sqrt{a+bx^4}}{2a^2x} + \frac{3\sqrt{b}x\sqrt{a+bx^4}}{2a^2\left(\sqrt{a}+\sqrt{b}x^2\right)} - \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{a+bx^4}} + \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{7/4}\sqrt{a+bx^4}}$$

output
```
1/2/a/x/(b*x^4+a)^(1/2)-3/2*(b*x^4+a)^(1/2)/a^2/x+3/2*b^(1/2)*x*(b*x^4+a)^
(1/2)/a^2/(a^(1/2)+b^(1/2)*x^2)-3/2*b^(1/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+
a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)+3/4*b^(1/4)*(a^(1/2)+b^(1/2)*x^2)*
((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)
*x/a^(1/4)),1/2*2^(1/2))/a^(7/4)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^4}{a}\right)}{ax\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
-((Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^4)/a)])/(a
*x*Sqrt[a + b*x^4]))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {819, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{3\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3\left(\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{834}$$

$$\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{2a}+\frac{1}{2ax\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{3\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}\right)}{a}-\right)}{2a}$$

$$\frac{1}{2ax\sqrt{a+bx^4}}$$

input
```
Int[1/(x^2*(a + b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x*Sqrt[a + b*x^4]) + (3*(-(Sqrt[a + b*x^4]/(a*x)) + (b*(-((-((x*Sqr
t[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*
Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a + b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] +
 Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcT
an[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^4])))/a))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{\sqrt{b\,x^{4}+a}}{a^{2} x}-\frac{b\,x^{3}}{2 a^{2} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{3 i \sqrt{b}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 a^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	137
elliptic	$-\frac{\sqrt{b\,x^{4}+a}}{a^{2} x}-\frac{b\,x^{3}}{2 a^{2} \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{3 i \sqrt{b}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 a^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}$	137
risch	$-\frac{\sqrt{b\,x^{4}+a}}{a^{2} x}+\frac{b^{2} \left(-\frac{x^{3}}{2 b \sqrt{\left(x^{4}+\frac{a}{b}\right) b}}+\frac{3 i \sqrt{a}\, \sqrt{1-\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{b}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}, i\right)\right)}{2 b^{\frac{3}{2}} \sqrt{\frac{i \sqrt{b}}{\sqrt{a}}}\, \sqrt{b\,x^{4}+a}}\right)}{a^{2}}$	144

input
```
int(1/x^2/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^4+a)^(1/2)/a^2/x-1/2*b/a^2*x^3/((x^4+a/b)*b)^(1/2)+3/2*I*b^(1/2)/a^(
3/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2
)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2
),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}}\,dx =$$
$$-\frac{3\,(bx^5+ax)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1) - 3\,(bx^5+ax)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}F(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)|-1) + (}{2\,(a^2bx^5+a^3x)}$$

input
```
integrate(1/x^2/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*(b*x^5 + a*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/a)^(1/4
)), -1) - 3*(b*x^5 + a*x)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(arcsin(x*(-b/a)^
(1/4)), -1) + (3*b*x^4 + 2*a)*sqrt(b*x^4 + a))/(a^2*b*x^5 + a^3*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}} x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(b*x**4+a)**(3/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}} \, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}} \, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}} \, dx = -\frac{\left(\frac{a}{b\,x^4}+1\right)^{3/2} {}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^4}\right)}{7\,x\,(b\,x^4+a)^{3/2}}$$

input
```
int(1/(x^2*(a + b*x^4)^(3/2)),x)
```

output
```
-((a/(b*x^4) + 1)^(3/2)*hypergeom([3/2, 7/4], 11/4, -a/(b*x^4)))/(7*x*(a +
 b*x^4)^(3/2))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^4)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b^2x^{10}+2ab\,x^6+a^2x^2}dx$$

input
```
int(1/x^2/(b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*x**2 + 2*a*b*x**6 + b**2*x**10),x)
```

3.395 $\int \frac{1}{x^6(a+bx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 282

$$\begin{aligned}\int \frac{1}{x^6(a+bx^4)^{3/2}}\,dx &= \frac{1}{2ax^5\sqrt{a+bx^4}} - \frac{7\sqrt{a+bx^4}}{10a^2x^5} \\ &+ \frac{21b\sqrt{a+bx^4}}{10a^3x} - \frac{21b^{3/2}x\sqrt{a+bx^4}}{10a^3\left(\sqrt{a}+\sqrt{b}x^2\right)} \\ &+ \frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{a+bx^4}} \\ &- \frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{20a^{11/4}\sqrt{a+bx^4}}\end{aligned}$$

output
```
1/2/a/x^5/(b*x^4+a)^(1/2)-7/10*(b*x^4+a)^(1/2)/a^2/x^5+21/10*b*(b*x^4+a)^(
1/2)/a^3/x-21/10*b^(3/2)*x*(b*x^4+a)^(1/2)/a^3/(a^(1/2)+b^(1/2)*x^2)+21/10
*b^(5/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))/a^(11/4)/(b*x^4+a)^
(1/2)-21/20*b^(5/4)*(a^(1/2)+b^(1/2)*x^2)*((b*x^4+a)/(a^(1/2)+b^(1/2)*x^2)
^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*x/a^(1/4)),1/2*2^(1/2))/a^(11/4
)/(b*x^4+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.19

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}}\,dx = -\frac{\sqrt{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{bx^4}{a}\right)}{5ax^5\sqrt{a+bx^4}}$$

input
```
Integrate[1/(x^6*(a + b*x^4)^(3/2)),x]
```

output
```
-1/5*(Sqrt[1 + (b*x^4)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^4)/a)]
)/(a*x^5*Sqrt[a + b*x^4])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {819, 847, 847, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7\int \frac{1}{x^6\sqrt{bx^4+a}}dx}{2a}+\frac{1}{2ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7\left(-\frac{3b\int \frac{1}{x^2\sqrt{bx^4+a}}dx}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\int \frac{x^2}{\sqrt{bx^4+a}}dx}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{a}\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\left(\frac{\sqrt{a}\int \frac{1}{\sqrt{bx^4+a}}dx}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x^2}{\sqrt{bx^4+a}}dx}{\sqrt{b}}\right)}{a}-\frac{\sqrt{a+bx^4}}{ax}\right)}{5a}-\frac{\sqrt{a+bx^4}}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\sqrt{a+bx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{7\left(-\frac{3b\left(\frac{b\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^4}}-\frac{\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^4}}-\frac{x\sqrt{a+bx^4}}{\sqrt{a}+\sqrt{b}x^2}}{\sqrt{b}}\right)}{a}}{5a}\right)}{2a}$$

$$\frac{1}{2ax^5\sqrt{a+bx^4}}$$

input
```
Int[1/(x^6*(a + b*x^4)^(3/2)),x]
```

output
```
1/(2*a*x^5*Sqrt[a + b*x^4]) + (7*(-1/5*Sqrt[a + b*x^4]/(a*x^5) - (3*b*(-(S
qrt[a + b*x^4]/(a*x)) + (b*(-((-((x*Sqrt[a + b*x^4])/(Sqrt[a] + Sqrt[b]*x^
2)) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticE[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(b^(1/4)*Sqrt[a +
b*x^4]))/Sqrt[b]) + (a^(1/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqr
t[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*b
^(3/4)*Sqrt[a + b*x^4])))/a))/(5*a)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.56

method	result
default	$-\frac{\sqrt{b\,x^4+a}}{5a^2x^5}+\frac{8b\sqrt{b\,x^4+a}}{5a^3x}+\frac{b^2x^3}{2a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{21ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{10a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
elliptic	$-\frac{\sqrt{b\,x^4+a}}{5a^2x^5}+\frac{8b\sqrt{b\,x^4+a}}{5a^3x}+\frac{b^2x^3}{2a^3\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{21ib^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right.}{10a^{\frac{5}{2}}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}}$
risch	$-\frac{\sqrt{b\,x^4+a}\,(-8b\,x^4+a)}{5a^3x^5}-\frac{b^2\left(3a\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{i\sqrt{1-\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{b\,x^4+a}\sqrt{b}}\right)+8b\left(-\right.\right.}{5a^3}$

input
```
int(1/x^6/(b*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(1/2)/a^2/x^5+8/5*b*(b*x^4+a)^(1/2)/a^3/x+1/2*b^2/a^3*x^3/(
(x^4+a/b)*b)^(1/2)-21/10*I*b^(3/2)/a^(5/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I*
b^(1/2)*x^2/a^(1/2))^(1/2)*(1+I*b^(1/2)*x^2/a^(1/2))^(1/2)/(b*x^4+a)^(1/2)
*(EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*b^(1/2))
^(1/2),I))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^6\left(a+bx^4\right)^{3/2}}\,dx=\frac{21\left(b^2x^9+abx^5\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{b}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-21\left(b^2x^9+abx^5\right)\sqrt{a}\left(-\frac{b}{a}\right)^{\frac{3}{4}}}{10\left(a^3bx^9+a^4x^5\right)}$$

input
```
integrate(1/x^6/(b*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/10*(21*(b^2*x^9 + a*b*x^5)*sqrt(a)*(-b/a)^(3/4)*elliptic_e(arcsin(x*(-b/
a)^(1/4)), -1) - 21*(b^2*x^9 + a*b*x^5)*sqrt(a)*(-b/a)^(3/4)*elliptic_f(ar
csin(x*(-b/a)^(1/4)), -1) + (21*b^2*x^8 + 14*a*b*x^4 - 2*a^2)*sqrt(b*x^4 +
 a))/(a^3*b*x^9 + a^4*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}} \, dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ -\frac{1}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}} x^5 \Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(b*x**4+a)**(3/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}} \, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{2}} x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}}\,dx = \int \frac{1}{x^6 (b\,x^4+a)^{3/2}}\,dx$$

input
```
int(1/(x^6*(a + b*x^4)^(3/2)),x)
```

output
```
int(1/(x^6*(a + b*x^4)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^4)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^4+a}}{b^2x^{14}+2ab\,x^{10}+a^2x^6}\,dx$$

input
```
int(1/x^6/(b*x^4+a)^(3/2),x)
```

output
```
int(sqrt(a + b*x**4)/(a**2*x**6 + 2*a*b*x**10 + b**2*x**14),x)
```

3.396 $\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}}{2} - \frac{1}{3}\left(1+x^4\right)^{3/2} + \frac{1}{10}\left(1+x^4\right)^{5/2}$$

output
```
1/2*(x^4+1)^(1/2)-1/3*(x^4+1)^(3/2)+1/10*(x^4+1)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{1}{30}\sqrt{1+x^4}\left(8-4x^4+3x^8\right)$$

input
```
Integrate[x^11/Sqrt[1 + x^4],x]
```

output
```
(Sqrt[1 + x^4]*(8 - 4*x^4 + 3*x^8))/30
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt{x^4+1}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{x^8}{\sqrt{x^4+1}} dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4} \int \left(\left(x^4+1\right)^{3/2} - 2\sqrt{x^4+1} + \frac{1}{\sqrt{x^4+1}} \right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{2}{5} \left(x^4+1\right)^{5/2} - \frac{4}{3} \left(x^4+1\right)^{3/2} + 2\sqrt{x^4+1} \right)$$

input
```
Int[x^11/Sqrt[1 + x^4],x]
```

output
```
(2*Sqrt[1 + x^4] - (4*(1 + x^4)^(3/2))/3 + (2*(1 + x^4)^(5/2))/5)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
trager	$\sqrt{x^4+1}\,\left(\frac{1}{10}x^8-\frac{2}{15}x^4+\frac{4}{15}\right)$	21
gosper	$\frac{\sqrt{x^4+1}\,(3x^8-4x^4+8)}{30}$	22
default	$\frac{\sqrt{x^4+1}\,(3x^8-4x^4+8)}{30}$	22
risch	$\frac{\sqrt{x^4+1}\,(3x^8-4x^4+8)}{30}$	22
elliptic	$\frac{\sqrt{x^4+1}\,(3x^8-4x^4+8)}{30}$	22
pseudoelliptic	$\frac{\sqrt{x^4+1}\,(3x^8-4x^4+8)}{30}$	22
orering	$\frac{\sqrt{x^4+1}\,(3x^8-4x^4+8)}{30}$	22
meijerg	$\frac{-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,\left(6x^8-8x^4+16\right)\sqrt{x^4+1}}{15}}{4\sqrt{\pi}}$	36

input
```
int(x^11/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^4+1)^(1/2)*(1/10*x^8-2/15*x^4+4/15)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{1}{30}\left(3\,x^8 - 4\,x^4 + 8\right)\sqrt{x^4+1}$$

input
```
integrate(x^11/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(3*x^8 - 4*x^4 + 8)*sqrt(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{x^8\sqrt{x^4+1}}{10} - \frac{2x^4\sqrt{x^4+1}}{15} + \frac{4\sqrt{x^4+1}}{15}$$

input
```
integrate(x**11/(x**4+1)**(1/2),x)
```

output
```
x**8*sqrt(x**4 + 1)/10 - 2*x**4*sqrt(x**4 + 1)/15 + 4*sqrt(x**4 + 1)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{1}{10}\left(x^4+1\right)^{\frac{5}{2}} - \frac{1}{3}\left(x^4+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^11/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/10*(x^4 + 1)^(5/2) - 1/3*(x^4 + 1)^(3/2) + 1/2*sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{1}{10}\left(x^4+1\right)^{\frac{5}{2}} - \frac{1}{3}\left(x^4+1\right)^{\frac{3}{2}} + \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^11/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/10*(x^4 + 1)^(5/2) - 1/3*(x^4 + 1)^(3/2) + 1/2*sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.50

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \sqrt{x^4+1}\left(\frac{x^8}{10} - \frac{2\,x^4}{15} + \frac{4}{15}\right)$$

input
```
int(x^11/(x^4 + 1)^(1/2),x)
```

output
```
(x^4 + 1)^(1/2)*(x^8/10 - (2*x^4)/15 + 4/15)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.28

$$\int \frac{x^{11}}{\sqrt{1+x^4}}\,dx = \frac{48\sqrt{x^4+1}\,x^{18} - 4\sqrt{x^4+1}\,x^{14} + 63\sqrt{x^4+1}\,x^{10} + 140\sqrt{x^4+1}\,x^6 + 40\sqrt{x^4+1}\,x^2 + 48x^{20} + 20x^{16} + 55}{480\sqrt{x^4+1}\,x^8 + 360\sqrt{x^4+1}\,x^4 + 30\sqrt{x^4+1} + 480x^{10} + 600x^6 + 150x^2}$$

input
```
int(x^11/(x^4+1)^(1/2),x)
```

output
```
(48*sqrt(x**4 + 1)*x**18 - 4*sqrt(x**4 + 1)*x**14 + 63*sqrt(x**4 + 1)*x**1
0 + 140*sqrt(x**4 + 1)*x**6 + 40*sqrt(x**4 + 1)*x**2 + 48*x**20 + 20*x**16
 + 55*x**12 + 175*x**8 + 100*x**4 + 8)/(30*(16*sqrt(x**4 + 1)*x**8 + 12*sq
rt(x**4 + 1)*x**4 + sqrt(x**4 + 1) + 16*x**10 + 20*x**6 + 5*x**2))
```

3.397 $\int \frac{x^7}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx=-\frac{1}{2}\sqrt{1+x^4}+\frac{1}{6}\left(1+x^4\right)^{3/2}$$

output
```
-1/2*(x^4+1)^(1/2)+1/6*(x^4+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx=\frac{1}{6}\left(-2+x^4\right)\sqrt{1+x^4}$$

input
```
Integrate[x^7/Sqrt[1 + x^4],x]
```

output
```
((-2 + x^4)*Sqrt[1 + x^4])/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt{x^4+1}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\sqrt{x^4+1}-\frac{1}{\sqrt{x^4+1}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2}{3}\left(x^4+1\right)^{3/2}-2\sqrt{x^4+1}\right)$$

input
```
Int[x^7/Sqrt[1 + x^4],x]
```

output
```
(-2*Sqrt[1 + x^4] + (2*(1 + x^4)^(3/2))/3)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{\sqrt{x^4+1}\,(x^4-2)}{6}$	15
default	$\frac{\sqrt{x^4+1}\,(x^4-2)}{6}$	15
risch	$\frac{\sqrt{x^4+1}\,(x^4-2)}{6}$	15
elliptic	$\frac{\sqrt{x^4+1}\,(x^4-2)}{6}$	15
pseudoelliptic	$\frac{\sqrt{x^4+1}\,(x^4-2)}{6}$	15
orering	$\frac{\sqrt{x^4+1}\,(x^4-2)}{6}$	15
trager	$\sqrt{x^4+1}\left(\frac{x^4}{6}-\frac{1}{3}\right)$	16
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,(-4x^4+8)\sqrt{x^4+1}}{6}}{4\sqrt{\pi}}$	31

input
```
int(x^7/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^4+1)^(1/2)*(x^4-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx = \frac{1}{6}\sqrt{x^4+1}\left(x^4-2\right)$$

input
```
integrate(x^7/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(x^4 + 1)*(x^4 - 2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx = \frac{x^4\sqrt{x^4+1}}{6} - \frac{\sqrt{x^4+1}}{3}$$

input
```
integrate(x**7/(x**4+1)**(1/2),x)
```

output
```
x**4*sqrt(x**4 + 1)/6 - sqrt(x**4 + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx = \frac{1}{6}\left(x^4+1\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^7/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(x^4 + 1)^(3/2) - 1/2*sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx = \frac{1}{6}\left(x^4+1\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^7/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/6*(x^4 + 1)^(3/2) - 1/2*sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}\,(x^4-2)}{6}$$

input
```
int(x^7/(x^4 + 1)^(1/2),x)
```

output
```
((x^4 + 1)^(1/2)*(x^4 - 2))/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.07

$$\int \frac{x^7}{\sqrt{1+x^4}}\,dx = \frac{4\sqrt{x^4+1}\,x^{10} - 5\sqrt{x^4+1}\,x^6 - 6\sqrt{x^4+1}\,x^2 + 4x^{12} - 3x^8 - 9x^4 - 2}{24\sqrt{x^4+1}\,x^4 + 6\sqrt{x^4+1} + 24x^6 + 18x^2}$$

input
```
int(x^7/(x^4+1)^(1/2),x)
```

output
```
(4*sqrt(x**4 + 1)*x**10 - 5*sqrt(x**4 + 1)*x**6 - 6*sqrt(x**4 + 1)*x**2 +
4*x**12 - 3*x**8 - 9*x**4 - 2)/(6*(4*sqrt(x**4 + 1)*x**4 + sqrt(x**4 + 1)
+ 4*x**6 + 3*x**2))
```

3.398 $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}}{2}$$

output
```
1/2*(x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}}{2}$$

input
```
Integrate[x^3/Sqrt[1 + x^4],x]
```

output
```
Sqrt[1 + x^4]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{793\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{x^4+1}} \, dx$$

$$\Big\downarrow \; 793$$

$$\frac{\sqrt{x^4+1}}{2}$$

input
```
Int[x^3/Sqrt[1 + x^4],x]
```

output
```
Sqrt[1 + x^4]/2
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{\sqrt{x^4+1}}{2}$	10
derivativedivides	$\frac{\sqrt{x^4+1}}{2}$	10
default	$\frac{\sqrt{x^4+1}}{2}$	10
trager	$\frac{\sqrt{x^4+1}}{2}$	10
risch	$\frac{\sqrt{x^4+1}}{2}$	10
elliptic	$\frac{\sqrt{x^4+1}}{2}$	10
pseudoelliptic	$\frac{\sqrt{x^4+1}}{2}$	10
orering	$\frac{\sqrt{x^4+1}}{2}$	10
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^4+1}}{4\sqrt{\pi}}$	24

input
```
int(x^3/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^3/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}}{2}$$

input
```
integrate(x**3/(x**4+1)**(1/2),x)
```

output
```
sqrt(x**4 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^3/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{1}{2}\sqrt{x^4+1}$$

input
```
integrate(x^3/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}}{2}$$

input
```
int(x^3/(x^4 + 1)^(1/2),x)
```

output
```
(x^4 + 1)^(1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.46

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}\,x^2+x^4+1}{2\sqrt{x^4+1}+2x^2}$$

input
```
int(x^3/(x^4+1)^(1/2),x)
```

output
```
(sqrt(x**4 + 1)*x**2 + x**4 + 1)/(2*(sqrt(x**4 + 1) + x**2))
```

3.399 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

output
```
-1/2*arctanh((x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{2}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

input
```
Integrate[1/(x*Sqrt[1 + x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[1 + x^4]]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int \frac{1}{x^4\sqrt{x^4+1}}dx^4$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\int \frac{1}{x^8-1}d\sqrt{x^4+1}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{1}{2}\text{arctanh}\left(\sqrt{x^4+1}\right)$$

input
```
Int[1/(x*Sqrt[1 + x^4]),x]
```

output
```
-1/2*ArcTanh[Sqrt[1 + x^4]]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	11
elliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	11
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{4}+1}}\right)}{2}$	11
trager	$\frac{\ln\left(\frac{-1+\sqrt{x^{4}+1}}{x^{2}}\right)}{2}$	17
meijerg	$\frac{\left(-2\ln\left(2\right)+4\ln\left(x\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{4}+1}}{2}\right)}{4\sqrt{\pi}}$	37

input
```
int(1/x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/(x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

input
```
integrate(1/x/(x**4+1)**(1/2),x)
```

output
```
-asinh(x**(-2))/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = -\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int(1/(x*(x^4 + 1)^(1/2)),x)
```

output
```
-atanh((x^4 + 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1}{x\sqrt{1+x^4}}\,dx = \frac{\log\left(\sqrt{x^4+1}+x^2-1\right)}{2}-\frac{\log\left(\sqrt{x^4+1}+x^2+1\right)}{2}$$

input
```
int(1/x/(x^4+1)^(1/2),x)
```

output
```
(log(sqrt(x**4 + 1) + x**2 - 1) - log(sqrt(x**4 + 1) + x**2 + 1))/2
```

3.400 $\int \frac{1}{x^5\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{4x^4}+\frac{1}{4}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

output
```
-1/4*(x^4+1)^(1/2)/x^4+1/4*arctanh((x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{4x^4}+\frac{1}{4}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

input
```
Integrate[1/(x^5*Sqrt[1 + x^4]),x]
```

output
```
-1/4*Sqrt[1 + x^4]/x^4 + ArcTanh[Sqrt[1 + x^4]]/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {798, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{x^4+1}}\,dx$$
$$\Big\downarrow \ 798$$
$$\frac{1}{4}\int \frac{1}{x^8\sqrt{x^4+1}}dx^4$$
$$\Big\downarrow \ 52$$
$$\frac{1}{4}\left(-\frac{1}{2}\int \frac{1}{x^4\sqrt{x^4+1}}dx^4-\frac{\sqrt{x^4+1}}{x^4}\right)$$
$$\Big\downarrow \ 73$$
$$\frac{1}{4}\left(-\int \frac{1}{x^8-1}d\sqrt{x^4+1}-\frac{\sqrt{x^4+1}}{x^4}\right)$$
$$\Big\downarrow \ 220$$
$$\frac{1}{4}\left(\text{arctanh}\left(\sqrt{x^4+1}\right)-\frac{\sqrt{x^4+1}}{x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[1 + x^4]),x]
```

output
```
(-(Sqrt[1 + x^4]/x^4) + ArcTanh[Sqrt[1 + x^4]])/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\sqrt{x^4+1}}{4x^4}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	24
risch	$-\frac{\sqrt{x^4+1}}{4x^4}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	24
elliptic	$-\frac{\sqrt{x^4+1}}{4x^4}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	24
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)x^4-\sqrt{x^4+1}}{4x^4}$	28
trager	$-\frac{\sqrt{x^4+1}}{4x^4}-\frac{\ln\left(\frac{-1+\sqrt{x^4+1}}{x^2}\right)}{4}$	30
meijerg	$\frac{-\frac{\sqrt{\pi}}{x^4}-\frac{(1-2\ln(2)+4\ln(x))\sqrt{\pi}}{2}+\frac{\sqrt{\pi}\left(4x^4+8\right)}{8x^4}-\frac{\sqrt{\pi}\sqrt{x^4+1}}{x^4}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{4\sqrt{\pi}}$	76

input
```
int(1/x^5/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x^4+1)^(1/2)/x^4+1/4*arctanh(1/(x^4+1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx=\frac{x^4\log\left(\sqrt{x^4+1}+1\right)-x^4\log\left(\sqrt{x^4+1}-1\right)-2\sqrt{x^4+1}}{8\,x^4}$$

input
```
integrate(1/x^5/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(x^4*log(sqrt(x^4 + 1) + 1) - x^4*log(sqrt(x^4 + 1) - 1) - 2*sqrt(x^4
+ 1))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{4} - \frac{\sqrt{1+\frac{1}{x^4}}}{4x^2}$$

input
```
integrate(1/x**5/(x**4+1)**(1/2),x)
```

output
```
asinh(x**(-2))/4 - sqrt(1 + x**(-4))/(4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = -\frac{\sqrt{x^4+1}}{4\,x^4} + \frac{1}{8}\,\log\left(\sqrt{x^4+1}+1\right) - \frac{1}{8}\,\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x^5/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(x^4 + 1)/x^4 + 1/8*log(sqrt(x^4 + 1) + 1) - 1/8*log(sqrt(x^4 + 1
) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = -\frac{\sqrt{x^4+1}}{4\,x^4} + \frac{1}{8}\,\log\left(\sqrt{x^4+1}+1\right) - \frac{1}{8}\,\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x^5/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(x^4 + 1)/x^4 + 1/8*log(sqrt(x^4 + 1) + 1) - 1/8*log(sqrt(x^4 + 1
) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = \frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{4} - \frac{\sqrt{x^4+1}}{4\,x^4}$$

input
```
int(1/(x^5*(x^4 + 1)^(1/2)),x)
```

output
```
atanh((x^4 + 1)^(1/2))/4 - (x^4 + 1)^(1/2)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 5.42

$$\int \frac{1}{x^5\sqrt{1+x^4}}\,dx = \frac{-2\sqrt{x^4+1}\,\log\left(\sqrt{x^4+1}+x^2-1\right)x^6+2\sqrt{x^4+1}\,\log\left(\sqrt{x^4+1}+x^2+1\right)x^6-2\sqrt{x^4+1}\,x^4-\sqrt{x^4+1}+}{4}$$

input
```
int(1/x^5/(x^4+1)^(1/2),x)
```

output
```
( - 2*sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2 - 1)*x**6 + 2*sqrt(x**4 + 1
)*log(sqrt(x**4 + 1) + x**2 + 1)*x**6 - 2*sqrt(x**4 + 1)*x**4 - sqrt(x**4
+ 1) - 2*log(sqrt(x**4 + 1) + x**2 - 1)*x**8 - log(sqrt(x**4 + 1) + x**2 -
 1)*x**4 + 2*log(sqrt(x**4 + 1) + x**2 + 1)*x**8 + log(sqrt(x**4 + 1) + x*
*2 + 1)*x**4 - 2*x**6 - 2*x**2)/(4*x**4*(2*sqrt(x**4 + 1)*x**2 + 2*x**4 +
1))
```

3.401 $\int \frac{x^5}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{1}{4}x^2\sqrt{1+x^4} - \frac{\operatorname{arcsinh}(x^2)}{4}$$

output
```
1/4*x^2*(x^4+1)^(1/2)-1/4*arcsinh(x^2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{1}{4}x^2\sqrt{1+x^4} - \frac{1}{4}\log\left(x^2+\sqrt{1+x^4}\right)$$

input
```
Integrate[x^5/Sqrt[1 + x^4],x]
```

output
```
(x^2*Sqrt[1 + x^4])/4 - Log[x^2 + Sqrt[1 + x^4]]/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {807, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{x^4+1}}dx^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\sqrt{x^4+1}-\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx^2\right)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\left(\frac{1}{2}x^2\sqrt{x^4+1}-\frac{\text{arcsinh}(x^2)}{2}\right)$$

input
```
Int[x^5/Sqrt[1 + x^4],x]
```

output
```
((x^2*Sqrt[1 + x^4])/2 - ArcSinh[x^2]/2)/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x^2\sqrt{x^4+1}}{4}-\frac{\operatorname{arcsinh}\left(x^2\right)}{4}$	20
risch	$\frac{x^2\sqrt{x^4+1}}{4}-\frac{\operatorname{arcsinh}\left(x^2\right)}{4}$	20
elliptic	$\frac{x^2\sqrt{x^4+1}}{4}-\frac{\operatorname{arcsinh}\left(x^2\right)}{4}$	20
pseudoelliptic	$\frac{x^2\sqrt{x^4+1}}{4}-\frac{\operatorname{arcsinh}\left(x^2\right)}{4}$	20
trager	$\frac{x^2\sqrt{x^4+1}}{4}+\frac{\ln\left(x^2-\sqrt{x^4+1}\right)}{4}$	30
meijerg	$\frac{\sqrt{\pi}\,x^2\sqrt{x^4+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x^2\right)}{4\sqrt{\pi}}$	30

input
```
int(x^5/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(x^4+1)^(1/2)-1/4*arcsinh(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{x^4+1}x^2 + \frac{1}{4}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate(x^5/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^4 + 1)*x^2 + 1/4*log(-x^2 + sqrt(x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{x^2\sqrt{x^4+1}}{4} - \frac{\operatorname{asinh}(x^2)}{4}$$

input
```
integrate(x**5/(x**4+1)**(1/2),x)
```

output
```
x**2*sqrt(x**4 + 1)/4 - asinh(x**2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(19) = 38.

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}}{4\,x^2\left(\frac{x^4+1}{x^4}-1\right)} - \frac{1}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) + \frac{1}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate(x^5/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(x^4 + 1)/(x^2*((x^4 + 1)/x^4 - 1)) - 1/8*log(sqrt(x^4 + 1)/x^2 +
1) + 1/8*log(sqrt(x^4 + 1)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\sqrt{x^4+1}x^2 + \frac{1}{4}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate(x^5/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^4 + 1)*x^2 + 1/4*log(-x^2 + sqrt(x^4 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \int \frac{x^5}{\sqrt{x^4+1}}\,dx$$

input
```
int(x^5/(x^4 + 1)^(1/2),x)
```

output
```
int(x^5/(x^4 + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.16

$$\int \frac{x^5}{\sqrt{1+x^4}}\,dx = \frac{-2\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2\right)x^2+2\sqrt{x^4+1}\,x^6+\sqrt{x^4+1}\,x^2-2\log\left(\sqrt{x^4+1}+x^2\right)x^4-\log\left(\sqrt{x^4+1}\right.}{8\sqrt{x^4+1}\,x^2+8x^4+4}$$

input
```
int(x^5/(x^4+1)^(1/2),x)
```

output
```
( - 2*sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2)*x**2 + 2*sqrt(x**4 + 1)*x*
*6 + sqrt(x**4 + 1)*x**2 - 2*log(sqrt(x**4 + 1) + x**2)*x**4 - log(sqrt(x*
*4 + 1) + x**2) + 2*x**8 + 2*x**4)/(4*(2*sqrt(x**4 + 1)*x**2 + 2*x**4 + 1)
)
```

3.402 $\int \frac{x}{\sqrt{1+x^4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{x}{\sqrt{1+x^4}} \, dx = \frac{\operatorname{arcsinh}(x^2)}{2}$$

output
```
1/2*arcsinh(x^2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{x}{\sqrt{1+x^4}} \, dx = \frac{1}{2} \log \left(x^2 + \sqrt{1+x^4}\right)$$

input
```
Integrate[x/Sqrt[1 + x^4],x]
```

output
```
Log[x^2 + Sqrt[1 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4+1}}\,dx$$
$$\Big\downarrow \ 807$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx^2$$
$$\Big\downarrow \ 222$$
$$\frac{\operatorname{arcsinh}(x^2)}{2}$$

input
```
Int[x/Sqrt[1 + x^4],x]
```

output
```
ArcSinh[x^2]/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\operatorname{arcsinh}(x^2)}{2}$	7
meijerg	$\frac{\operatorname{arcsinh}(x^2)}{2}$	7
elliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}$	7
pseudoelliptic	$\frac{\operatorname{arcsinh}(x^2)}{2}$	7
trager	$\frac{\ln\left(x^2+\sqrt{x^4+1}\right)}{2}$	15

input
```
int(x/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate(x/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 + 1))
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{x}{\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2}$$

input
```
integrate(x/(x**4+1)**(1/2),x)
```

output
```
asinh(x**2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(6) = 12.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 4.12

$$\int \frac{x}{\sqrt{1+x^4}}\,dx = \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate(x/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{x}{\sqrt{1+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate(x/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x}{\sqrt{1+x^4}}\,dx = \frac{\operatorname{asinh}(x^2)}{2}$$

input
```
int(x/(x^4 + 1)^(1/2),x)
```

output
```
asinh(x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{x}{\sqrt{1+x^4}}\,dx = \frac{\log\left(\sqrt{x^4+1}+x^2\right)}{2}$$

input
```
int(x/(x^4+1)^(1/2),x)
```

output
```
log(sqrt(x**4 + 1) + x**2)/2
```

3.403 $\int \frac{1}{x^3\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{2x^2}$$

output
```
-1/2*(x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{2x^2}$$

input
```
Integrate[1/(x^3*Sqrt[1 + x^4]),x]
```

output
```
-1/2*Sqrt[1 + x^4]/x^2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\sqrt{x^4+1}}{2x^2}$$

input
```
Int[1/(x^3*Sqrt[1 + x^4]),x]
```

output
```
-1/2*Sqrt[1 + x^4]/x^2
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
default	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
trager	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
meijerg	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
risch	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
elliptic	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
pseudoelliptic	$-\frac{\sqrt{x^4+1}}{2x^2}$	13
orering	$-\frac{\sqrt{x^4+1}}{2x^2}$	13

input
```
int(1/(x^4+1)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^4+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{x^2+\sqrt{x^4+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + sqrt(x^4 + 1))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+\frac{1}{x^4}}}{2}$$

input
```
integrate(1/x**3/(x**4+1)**(1/2),x)
```

output
```
-sqrt(1 + x**(-4))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{\sqrt{x^4+1}}{2\,x^2}$$

input
```
integrate(1/x^3/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(x^4 + 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = \frac{1}{\left(x^2-\sqrt{x^4+1}\right)^2-1}$$

input
```
integrate(1/x^3/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
1/((x^2 - sqrt(x^4 + 1))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = -\frac{\sqrt{x^4+1}}{2\,x^2}$$

input
```
int(1/(x^3*(x^4 + 1)^(1/2)),x)
```

output
```
-(x^4 + 1)^(1/2)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{1}{x^3\sqrt{1+x^4}}\,dx = \frac{-2\sqrt{x^4+1}\,x^2-2x^4-1}{2x^2\left(\sqrt{x^4+1}+x^2\right)}$$

input
```
int(1/x^3/(x^4+1)^(1/2),x)
```

output
```
( - 2*sqrt(x**4 + 1)*x**2 - 2*x**4 - 1)/(2*x**2*(sqrt(x**4 + 1) + x**2))
```

3.404 $\int \frac{1}{x^7\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{6x^6}+\frac{\sqrt{1+x^4}}{3x^2}$$

output
```
-1/6*(x^4+1)^(1/2)/x^6+1/3*(x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}(-1+2x^4)}{6x^6}$$

input
```
Integrate[1/(x^7*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[1 + x^4]*(-1 + 2*x^4))/(6*x^6)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7\sqrt{x^4+1}}dx$$
$$\Big\downarrow \text{803}$$
$$-\frac{2}{3}\int \frac{1}{x^3\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{6x^6}$$
$$\Big\downarrow \text{796}$$
$$\frac{\sqrt{x^4+1}}{3x^2}-\frac{\sqrt{x^4+1}}{6x^6}$$

input
```
Int[1/(x^7*Sqrt[1 + x^4]),x]
```

output
```
-1/6*Sqrt[1 + x^4]/x^6 + Sqrt[1 + x^4]/(3*x^2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\sqrt{x^4+1}\,(2x^4-1)}{6x^6}$	20
default	$\frac{\sqrt{x^4+1}\,(2x^4-1)}{6x^6}$	20
trager	$\frac{\sqrt{x^4+1}\,(2x^4-1)}{6x^6}$	20
meijerg	$-\frac{(-2x^4+1)\sqrt{x^4+1}}{6x^6}$	20
elliptic	$\frac{\sqrt{x^4+1}\,(2x^4-1)}{6x^6}$	20
pseudoelliptic	$\frac{\sqrt{x^4+1}\,(2x^4-1)}{6x^6}$	20
orering	$\frac{\sqrt{x^4+1}\,(2x^4-1)}{6x^6}$	20
risch	$\frac{2x^8+x^4-1}{6x^6\sqrt{x^4+1}}$	23

input
```
int(1/x^7/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^4+1)^(1/2)*(2*x^4-1)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{2\,x^6 + (2\,x^4-1)\sqrt{x^4+1}}{6\,x^6}$$

input
```
integrate(1/x^7/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(2*x^6 + (2*x^4 - 1)*sqrt(x^4 + 1))/x^6
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+\frac{1}{x^4}}}{3} - \frac{\sqrt{1+\frac{1}{x^4}}}{6x^4}$$

input
```
integrate(1/x**7/(x**4+1)**(1/2),x)
```

output
```
sqrt(1 + x**(-4))/3 - sqrt(1 + x**(-4))/(6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}}{2\,x^2} - \frac{(x^4+1)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate(1/x^7/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 + 1)/x^2 - 1/6*(x^4 + 1)^(3/2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{2\left(3\left(x^2-\sqrt{x^4+1}\right)^2-1\right)}{3\left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^3}$$

input
```
integrate(1/x^7/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(x^2 - sqrt(x^4 + 1))^2 - 1)/((x^2 - sqrt(x^4 + 1))^2 - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}\,(2\,x^4-1)}{6\,x^6}$$

input
```
int(1/(x^7*(x^4 + 1)^(1/2)),x)
```

output
```
((x^4 + 1)^(1/2)*(2*x^4 - 1))/(6*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^7\sqrt{1+x^4}}\,dx = \frac{-3\sqrt{x^4+1}\,x^2-3x^4-1}{6x^6\left(4\sqrt{x^4+1}\,x^4+\sqrt{x^4+1}+4x^6+3x^2\right)}$$

input
```
int(1/x^7/(x^4+1)^(1/2),x)
```

output
```
( - 3*sqrt(x**4 + 1)*x**2 - 3*x**4 - 1)/(6*x**6*(4*sqrt(x**4 + 1)*x**4 + s
qrt(x**4 + 1) + 4*x**6 + 3*x**2))
```

3.405 $\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{10x^{10}} + \frac{2\sqrt{1+x^4}}{15x^6} - \frac{4\sqrt{1+x^4}}{15x^2}$$

output
```
-1/10*(x^4+1)^(1/2)/x^10+2/15*(x^4+1)^(1/2)/x^6-4/15*(x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = \frac{\sqrt{1+x^4}(-3+4x^4-8x^8)}{30x^{10}}$$

input
```
Integrate[1/(x^11*Sqrt[1 + x^4]),x]
```

output
```
(Sqrt[1 + x^4]*(-3 + 4*x^4 - 8*x^8))/(30*x^10)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{4}{5}\int \frac{1}{x^7\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{10x^{10}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{4}{5}\left(-\frac{2}{3}\int \frac{1}{x^3\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{6x^6}\right)-\frac{\sqrt{x^4+1}}{10x^{10}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{\sqrt{x^4+1}}{10x^{10}}-\frac{4}{5}\left(\frac{\sqrt{x^4+1}}{3x^2}-\frac{\sqrt{x^4+1}}{6x^6}\right)$$

input
```
Int[1/(x^11*Sqrt[1 + x^4]),x]
```

output
```
-1/10*Sqrt[1 + x^4]/x^10 - (4*(-1/6*Sqrt[1 + x^4]/x^6 + Sqrt[1 + x^4]/(3*x
^2)))/5
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\sqrt{x^4+1}\,(8x^8-4x^4+3)}{30x^{10}}$	25
default	$-\frac{\sqrt{x^4+1}\,(8x^8-4x^4+3)}{30x^{10}}$	25
trager	$-\frac{\sqrt{x^4+1}\,(8x^8-4x^4+3)}{30x^{10}}$	25
meijerg	$-\frac{\left(\frac{8}{3}x^8-\frac{4}{3}x^4+1\right)\sqrt{x^4+1}}{10x^{10}}$	25
elliptic	$-\frac{\sqrt{x^4+1}\,(8x^8-4x^4+3)}{30x^{10}}$	25
pseudoelliptic	$-\frac{\sqrt{x^4+1}\,(8x^8-4x^4+3)}{30x^{10}}$	25
orering	$-\frac{\sqrt{x^4+1}\,(8x^8-4x^4+3)}{30x^{10}}$	25
risch	$-\frac{8x^{12}+4x^8-x^4+3}{30x^{10}\sqrt{x^4+1}}$	30

input
```
int(1/x^11/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/30*(x^4+1)^(1/2)*(8*x^8-4*x^4+3)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = -\frac{8\,x^{10} + (8\,x^8 - 4\,x^4 + 3)\sqrt{x^4+1}}{30\,x^{10}}$$

input
```
integrate(1/x^11/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/30*(8*x^10 + (8*x^8 - 4*x^4 + 3)*sqrt(x^4 + 1))/x^10
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = -\frac{4\sqrt{1+\frac{1}{x^4}}}{15} + \frac{2\sqrt{1+\frac{1}{x^4}}}{15x^4} - \frac{\sqrt{1+\frac{1}{x^4}}}{10x^8}$$

input
```
integrate(1/x**11/(x**4+1)**(1/2),x)
```

output
```
-4*sqrt(1 + x**(-4))/15 + 2*sqrt(1 + x**(-4))/(15*x**4) - sqrt(1 + x**(-4)
)/(10*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = -\frac{\sqrt{x^4+1}}{2\,x^2} + \frac{(x^4+1)^{\frac{3}{2}}}{3\,x^6} - \frac{(x^4+1)^{\frac{5}{2}}}{10\,x^{10}}$$

input
```
integrate(1/x^11/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(x^4 + 1)/x^2 + 1/3*(x^4 + 1)^(3/2)/x^6 - 1/10*(x^4 + 1)^(5/2)/x^
10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = \frac{8\left(10\left(x^2-\sqrt{x^4+1}\right)^4-5\left(x^2-\sqrt{x^4+1}\right)^2+1\right)}{15\left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^5}$$

input
```
integrate(1/x^11/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
8/15*(10*(x^2 - sqrt(x^4 + 1))^4 - 5*(x^2 - sqrt(x^4 + 1))^2 + 1)/((x^2 -
sqrt(x^4 + 1))^2 - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = -\frac{\sqrt{x^4+1}\,(8\,x^8-4\,x^4+3)}{30\,x^{10}}$$

input
```
int(1/(x^11*(x^4 + 1)^(1/2)),x)
```

output
```
-((x^4 + 1)^(1/2)*(8*x^8 - 4*x^4 + 3))/(30*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.73

$$\int \frac{1}{x^{11}\sqrt{1+x^4}}\,dx = \frac{-40\sqrt{x^4+1}\,x^6-15\sqrt{x^4+1}\,x^2-40x^8-35x^4-3}{30x^{10}\left(16\sqrt{x^4+1}\,x^8+12\sqrt{x^4+1}\,x^4+\sqrt{x^4+1}+16x^{10}+20x^6+5x^2\right)}$$

input
```
int(1/x^11/(x^4+1)^(1/2),x)
```

output
```
( - 40*sqrt(x**4 + 1)*x**6 - 15*sqrt(x**4 + 1)*x**2 - 40*x**8 - 35*x**4 -
3)/(30*x**10*(16*sqrt(x**4 + 1)*x**8 + 12*sqrt(x**4 + 1)*x**4 + sqrt(x**4
+ 1) + 16*x**10 + 20*x**6 + 5*x**2))
```

3.406 $\int \frac{x^8}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = -\frac{5}{21}x\sqrt{1+x^4}+\frac{1}{7}x^5\sqrt{1+x^4} + \frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{42\sqrt{1+x^4}}$$

output
```
-5/21*x*(x^4+1)^(1/2)+1/7*x^5*(x^4+1)^(1/2)+5/42*(x^2+1)*((x^4+1)/(x^2+1)^
2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \frac{1}{21}x\left(\sqrt{1+x^4}(-5+3x^4)+5\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)\right)$$

input
```
Integrate[x^8/Sqrt[1 + x^4],x]
```

output
```
(x*(Sqrt[1 + x^4]*(-5 + 3*x^4) + 5*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4])
)/21
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {843, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{7}x^5\sqrt{x^4+1}-\frac{5}{7}\int \frac{x^4}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{7}x^5\sqrt{x^4+1}-\frac{5}{7}\left(\frac{1}{3}x\sqrt{x^4+1}-\frac{1}{3}\int \frac{1}{\sqrt{x^4+1}}dx\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{7}x^5\sqrt{x^4+1}-\frac{5}{7}\left(\frac{1}{3}x\sqrt{x^4+1}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{x^4+1}}\right)$$

input
```
Int[x^8/Sqrt[1 + x^4],x]
```

output
```
(x^5*Sqrt[1 + x^4])/7 - (5*((x*Sqrt[1 + x^4])/3 - ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4])))/7
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

method	result	size
meijerg	$\frac{x^{9} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{9}{4}\right], \left[\frac{13}{4}\right], -x^{4}\right)}{9}$	17
risch	$\frac{x \left(3 x^{4}-5\right) \sqrt{x^{4}+1}}{21}+\frac{5 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{21 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	79
default	$\frac{x^{5} \sqrt{x^{4}+1}}{7}-\frac{5 x \sqrt{x^{4}+1}}{21}+\frac{5 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{21 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	84
elliptic	$\frac{x^{5} \sqrt{x^{4}+1}}{7}-\frac{5 x \sqrt{x^{4}+1}}{21}+\frac{5 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{21 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	84

input
```
int(x^8/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*hypergeom([1/2,9/4],[13/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.46

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \frac{1}{21}\left(3\,x^5-5\,x\right)\sqrt{x^4+1}+\frac{5}{21}i\,\sqrt{i}F(\arcsin\left(\frac{\sqrt{i}}{x}\right)\,|-1)$$

input
```
integrate(x^8/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(3*x^5 - 5*x)*sqrt(x^4 + 1) + 5/21*I*sqrt(I)*elliptic_f(arcsin(sqrt(I
)/x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(x**4+1)**(1/2),x)
```

output
```
x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), x**4*exp_polar(I*pi))/(4*gamma(
13/4))
```

Maxima [F]

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \int \frac{x^8}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^8/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \int \frac{x^8}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^8/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^8/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \int \frac{x^8}{\sqrt{x^4+1}}\,dx$$

input
```
int(x^8/(x^4 + 1)^(1/2),x)
```

output
```
int(x^8/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt{1+x^4}}\,dx = \frac{\sqrt{x^4+1}\,x^5}{7} - \frac{5\sqrt{x^4+1}\,x}{21} + \frac{5\left(\int \frac{\sqrt{x^4+1}}{x^4+1}dx\right)}{21}$$

input
```
int(x^8/(x^4+1)^(1/2),x)
```

output
```
(3*sqrt(x**4 + 1)*x**5 - 5*sqrt(x**4 + 1)*x + 5*int(sqrt(x**4 + 1)/(x**4 +
 1),x))/21
```

3.407 $\int \frac{x^4}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx = \frac{1}{3}x\sqrt{1+x^4} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{1+x^4}}$$

output
```
1/3*x*(x^4+1)^(1/2)-1/6*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(
2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx = \frac{1}{3}x\left(\sqrt{1+x^4} - \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)\right)$$

input
```
Integrate[x^4/Sqrt[1 + x^4],x]
```

output
```
(x*(Sqrt[1 + x^4] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{3}x\sqrt{x^4+1}-\frac{1}{3}\int \frac{1}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{3}x\sqrt{x^4+1}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{x^4+1}}$$

input
```
Int[x^4/Sqrt[1 + x^4],x]
```

output
```
(x*Sqrt[1 + x^4])/3 - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(6*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.29

method	result	size
meijerg	$\frac{x^5 \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],-x^4\right)}{5}$	17
default	$\frac{x\sqrt{x^4+1}}{3}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	72
risch	$\frac{x\sqrt{x^4+1}}{3}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	72
elliptic	$\frac{x\sqrt{x^4+1}}{3}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	72

input
```
int(x^4/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([1/2,5/4],[9/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx = \frac{1}{3}\sqrt{x^4+1}x - \frac{1}{3}i\sqrt{i}F(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1)$$

input
```
integrate(x^4/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(x^4 + 1)*x - 1/3*I*sqrt(I)*elliptic_f(arcsin(sqrt(I)/x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(x**4+1)**(1/2),x)
```

output
```
x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9
/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx = \int \frac{x^4}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^4/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx = \int \frac{x^4}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^4/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx=\int \frac{x^4}{\sqrt{x^4+1}}\,dx$$

input
```
int(x^4/(x^4 + 1)^(1/2),x)
```

output
```
int(x^4/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{1+x^4}}\,dx=\frac{\sqrt{x^4+1}\,x}{3}-\frac{\left(\int \frac{\sqrt{x^4+1}}{x^4+1}dx\right)}{3}$$

input
```
int(x^4/(x^4+1)^(1/2),x)
```

output
```
(sqrt(x**4 + 1)*x - int(sqrt(x**4 + 1)/(x**4 + 1),x))/3
```

3.408 $\int \frac{1}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 43

$$\int \frac{1}{\sqrt{1+x^4}}\,dx=\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{1+x^4}}$$

output
```
1/2*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1
/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{1+x^4}}\,dx=-\sqrt[4]{-1}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt[4]{-1}x\right),-1\right)$$

input
```
Integrate[1/Sqrt[1 + x^4],x]
```

output
```
-((-1)^(1/4)*EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^4+1}}\,dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}$$

input
```
Int[1/Sqrt[1 + x^4],x]
```

output
```
((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqr
t[1 + x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.33

method	result	size
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)$	14
default	$\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	60
elliptic	$\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	60

input
```
int(1/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,1/2],[5/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = -i\sqrt{i}F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right)$$

input
```
integrate(1/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-I*sqrt(I)*elliptic_f(arcsin(sqrt(I)*x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,x^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(x**4+1)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(1/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(x^4 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -x^4\right)$$

input
```
int(1/(x^4 + 1)^(1/2),x)
```

output
```
x*hypergeom([1/4, 1/2], 5/4, -x^4)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^4+1}dx$$

input
```
int(1/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**4 + 1),x)
```

3.409 $\int \frac{1}{x^4\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{3x^3} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{1+x^4}}$$

output
```
-1/3*(x^4+1)^(1/2)/x^3-1/6*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobi
AM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-x^4\right)}{3x^3}$$

input
```
Integrate[1/(x^4*Sqrt[1 + x^4]),x]
```

output
```
-1/3*Hypergeometric2F1[-3/4, 1/2, 1/4, -x^4]/x^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{3x^3}$$

$$\Big\downarrow \text{761}$$

$$-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{3x^3}$$

input
```
Int[1/(x^4*Sqrt[1 + x^4]),x]
```

output
```
-1/3*Sqrt[1 + x^4]/x^3 - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[
2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{3}{4},\frac{1}{2}\right],\left[\frac{1}{4}\right],-x^4\right)}{3x^3}$	17
default	$-\frac{\sqrt{x^4+1}}{3x^3}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	74
risch	$-\frac{\sqrt{x^4+1}}{3x^3}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	74
elliptic	$-\frac{\sqrt{x^4+1}}{3x^3}-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{3\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	74

input
```
int(1/x^4/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*hypergeom([-3/4,1/2],[1/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx = \frac{i\sqrt{i}x^3F\left(\arcsin\left(\sqrt{i}x\right)\mid -1\right)-\sqrt{x^4+1}}{3\,x^3}$$

input
```
integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(I*sqrt(I)*x^3*elliptic_f(arcsin(sqrt(I)*x), -1) - sqrt(x^4 + 1))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| x^4 e^{i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(x**4+1)**(1/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_polar(I*pi))/(4*x**3*gamma
(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^4}\,dx$$

input
```
integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^4}\,dx$$

input
```
integrate(1/x^4/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx=\int \frac{1}{x^4\,\sqrt{x^4+1}}\,dx$$

input
```
int(1/(x^4*(x^4 + 1)^(1/2)),x)
```

output
```
int(1/(x^4*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{1+x^4}}\,dx=\int \frac{\sqrt{x^4+1}}{x^8+x^4}dx$$

input
```
int(1/x^4/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**8 + x**4),x)
```

3.410 $\int \frac{1}{x^8\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{7x^7}+\frac{5\sqrt{1+x^4}}{21x^3}+\frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{42\sqrt{1+x^4}}$$

output
```
-1/7*(x^4+1)^(1/2)/x^7+5/21*(x^4+1)^(1/2)/x^3+5/42*(x^2+1)*((x^4+1)/(x^2+1
)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-x^4\right)}{7x^7}$$

input
```
Integrate[1/(x^8*Sqrt[1 + x^4]),x]
```

output
```
-1/7*Hypergeometric2F1[-7/4, 1/2, -3/4, -x^4]/x^7
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {847, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{5}{7}\int \frac{1}{x^4\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{5}{7}\left(-\frac{1}{3}\int \frac{1}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{3x^3}\right)-\frac{\sqrt{x^4+1}}{7x^7}$$

$$\Big\downarrow \text{761}$$

$$-\frac{5}{7}\left(-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{3x^3}\right)-\frac{\sqrt{x^4+1}}{7x^7}$$

input
```
Int[1/(x^8*Sqrt[1 + x^4]),x]
```

output
```
-1/7*Sqrt[1 + x^4]/x^7 - (5*(-1/3*Sqrt[1 + x^4]/x^3 - ((1 + x^2)*Sqrt[(1 +
 x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4])))/7
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.22

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{7}{4},\frac{1}{2}\right],\left[-\frac{3}{4}\right],-x^{4}\right)}{7x^{7}}$	17
default	$-\frac{\sqrt{x^{4}+1}}{7x^{7}}+\frac{5\sqrt{x^{4}+1}}{21x^{3}}+\frac{5\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{21\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	86
risch	$\frac{5x^{8}+2x^{4}-3}{21x^{7}\sqrt{x^{4}+1}}+\frac{5\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{21\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	86
elliptic	$-\frac{\sqrt{x^{4}+1}}{7x^{7}}+\frac{5\sqrt{x^{4}+1}}{21x^{3}}+\frac{5\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{21\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	86

input
```
int(1/x^8/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7*hypergeom([-7/4,1/2],[-3/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = \frac{-5i\sqrt{i}x^7F\left(\arcsin\left(\sqrt{i}x\right)\mid -1\right)+(5x^4-3)\sqrt{x^4+1}}{21x^7}$$

input
```
integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/21*(-5*I*sqrt(I)*x^7*elliptic_f(arcsin(sqrt(I)*x), -1) + (5*x^4 - 3)*sqr
t(x^4 + 1))/x^7
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(x**4+1)**(1/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), x**4*exp_polar(I*pi))/(4*x**7*gamm
a(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^8}\,dx$$

input
```
integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^8}\,dx$$

input
```
integrate(1/x^8/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = \int \frac{1}{x^8\,\sqrt{x^4+1}}\,dx$$

input
```
int(1/(x^8*(x^4 + 1)^(1/2)),x)
```

output
```
int(1/(x^8*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^{12}+x^8}dx$$

input
```
int(1/x^8/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**12 + x**8),x)
```

3.411 $\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx=-\frac{7}{45}x^3\sqrt{1+x^4}+\frac{1}{9}x^7\sqrt{1+x^4}+\frac{7x\sqrt{1+x^4}}{15(1+x^2)}-\frac{7(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{15\sqrt{1+x^4}}+\frac{7(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{30\sqrt{1+x^4}}$$

output
```
-7/45*x^3*(x^4+1)^(1/2)+1/9*x^7*(x^4+1)^(1/2)+7*x*(x^4+1)^(1/2)/(15*x^2+15
)-7/15*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*2^
(1/2))/(x^4+1)^(1/2)+7/30*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiA
M(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx = \frac{1}{45}x^3\left(\sqrt{1+x^4}(-7+5x^4)+7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)\right)$$

input
```
Integrate[x^10/Sqrt[1 + x^4],x]
```

output
```
(x^3*(Sqrt[1 + x^4]*(-7 + 5*x^4) + 7*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4
]))/45
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {843, 843, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\int \frac{x^6}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\int \frac{x^2}{\sqrt{x^4+1}}dx\right)$$

$$\Big\downarrow \text{834}$$

$$\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx\right)\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\int\frac{1-x^2}{\sqrt{x^4+1}}dx\right)\right)$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+1}}+\right.\right.$$

input
```
Int[x^10/Sqrt[1 + x^4],x]
```

output
```
(x^7*Sqrt[1 + x^4])/9 - (7*((x^3*Sqrt[1 + x^4])/5 - (3*((x*Sqrt[1 + x^4])/
(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(2*Sqrt[1 + x^4])))/5))/9
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

method	result	size
meijerg	$\frac{x^{11} \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{11}{4}\right],\left[\frac{15}{4}\right],-x^{4}\right)}{11}$	17
risch	$\frac{x^{3}\left(5 x^{4}-7\right) \sqrt{x^{4}+1}}{45}+\frac{7 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{15 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	102
default	$\frac{x^{7} \sqrt{x^{4}+1}}{9}-\frac{7 x^{3} \sqrt{x^{4}+1}}{45}+\frac{7 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{15 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	107
elliptic	$\frac{x^{7} \sqrt{x^{4}+1}}{9}-\frac{7 x^{3} \sqrt{x^{4}+1}}{45}+\frac{7 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{15 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	107

input
```
int(x^10/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11*hypergeom([1/2,11/4],[15/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx = \frac{21 i \sqrt{i} x E(\arcsin\left(\frac{\sqrt{i}}{x}\right) \mid -1) - 21 i \sqrt{i} x F(\arcsin\left(\frac{\sqrt{i}}{x}\right) \mid -1) + (5 x^8 - 7 x^4 + 21)\sqrt{x^4+1}}{45 x}$$

input
```
integrate(x^10/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/45*(21*I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) - 21*I*sqrt(I)*x*el
liptic_f(arcsin(sqrt(I)/x), -1) + (5*x^8 - 7*x^4 + 21)*sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.21

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(x**4+1)**(1/2),x)
```

output
```
x**11*gamma(11/4)*hyper((1/2, 11/4), (15/4,), x**4*exp_polar(I*pi))/(4*gam
ma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx = \int \frac{x^{10}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^10/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^10/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx=\int \frac{x^{10}}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^10/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^10/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx=\int \frac{x^{10}}{\sqrt{x^4+1}}\,dx$$

input
```
int(x^10/(x^4 + 1)^(1/2),x)
```

output
```
int(x^10/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt{1+x^4}}\,dx=\frac{\sqrt{x^4+1}\,x^7}{9}-\frac{7\sqrt{x^4+1}\,x^3}{45}+\frac{7\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{15}$$

input
```
int(x^10/(x^4+1)^(1/2),x)
```

output
```
(5*sqrt(x**4 + 1)*x**7 - 7*sqrt(x**4 + 1)*x**3 + 21*int((sqrt(x**4 + 1)*x*
*2)/(x**4 + 1),x))/45
```

3.412 $\int \frac{x^6}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx=\frac{1}{5}x^3\sqrt{1+x^4}-\frac{3x\sqrt{1+x^4}}{5\left(1+x^2\right)}+\frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{5\sqrt{1+x^4}}-\frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{10\sqrt{1+x^4}}$$

output
```
1/5*x^3*(x^4+1)^(1/2)-3*x*(x^4+1)^(1/2)/(5*x^2+5)+3/5*(x^2+1)*((x^4+1)/(x^
2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)-3/10*(
x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/
(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx=\frac{1}{5}x^3\left(\sqrt{1+x^4}-\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)\right)$$

input
```
Integrate[x^6/Sqrt[1 + x^4],x]
```

output
```
(x^3*(Sqrt[1 + x^4] - Hypergeometric2F1[1/2, 3/4, 7/4, -x^4]))/5
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {843, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\int \frac{x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{ 834}$$

$$\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\int \frac{1-x^2}{\sqrt{x^4+1}}dx\right)$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{1}{5}x^3\sqrt{x^4+1}-$$
$$\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}x}{x^2+1}\right)$$

input
```
Int[x^6/Sqrt[1 + x^4],x]
```

output
```
(x^3*Sqrt[1 + x^4])/5 - (3*((x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[
(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 +
x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 +
x^4])))/5
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.14

method	result	size
meijerg	$\frac{x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{7}{4}\right],\left[\frac{11}{4}\right],-x^{4}\right)}{7}$	17
default	$\frac{x^{3} \sqrt{x^{4}+1}}{5}-\frac{3 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{5 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	95
risch	$\frac{x^{3} \sqrt{x^{4}+1}}{5}-\frac{3 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{5 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	95
elliptic	$\frac{x^{3} \sqrt{x^{4}+1}}{5}-\frac{3 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{5 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	95

input
```
int(x^6/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*hypergeom([1/2,7/4],[11/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx = \frac{-3 i \sqrt{i} x E(\arcsin\left(\frac{\sqrt{i}}{x}\right) \mid -1) + 3 i \sqrt{i} x F(\arcsin\left(\frac{\sqrt{i}}{x}\right) \mid -1) + \sqrt{x^4+1}(x^4-3)}{5\,x}$$

input
```
integrate(x^6/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(-3*I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) + 3*I*sqrt(I)*x*elli
ptic_f(arcsin(sqrt(I)/x), -1) + sqrt(x^4 + 1)*(x^4 - 3))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(x**4+1)**(1/2),x)
```

output
```
x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi))/(4*gamma(
11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx = \int \frac{x^6}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^6/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^6/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx = \int \frac{x^6}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^6/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^6/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx=\int \frac{x^6}{\sqrt{x^4+1}}\,dx$$

input
```
int(x^6/(x^4 + 1)^(1/2),x)
```

output
```
int(x^6/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt{1+x^4}}\,dx=\frac{\sqrt{x^4+1}\,x^3}{5}-\frac{3\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx\right)}{5}$$

input
```
int(x^6/(x^4+1)^(1/2),x)
```

output
```
(sqrt(x**4 + 1)*x**3 - 3*int((sqrt(x**4 + 1)*x**2)/(x**4 + 1),x))/5
```

3.413 $\int \frac{x^2}{\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 103

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx=\frac{x\sqrt{1+x^4}}{1+x^2}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{1+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{1+x^4}}$$

output
```
x*(x^4+1)^(1/2)/(x^2+1)-(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*
arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)+1/2*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2
)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.21

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx=\frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^4\right)$$

input
```
Integrate[x^2/Sqrt[1 + x^4],x]
```

output
```
(x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4])/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{834}$$

$$\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\int \frac{1-x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{1510}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}x}{x^2+1}$$

input
```
Int[x^2/Sqrt[1 + x^4],x]
```

output
```
(x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*Ellip
ticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.17

method	result	size
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -x^{4}\right)}{3}$	17
default	$\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	82
elliptic	$\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	82

input
```
int(x^2/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([1/2,3/4],[7/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \frac{i\sqrt{i}xE\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1\right) - i\sqrt{i}xF\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1\right) + \sqrt{x^4+1}}{x}$$

input
```
integrate(x^2/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)/x), -1) - I*sqrt(I)*x*elliptic_f(ar
csin(sqrt(I)/x), -1) + sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(x**4+1)**(1/2),x)
```

output
```
x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7
/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^2/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(x^4 + 1), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}}\,dx$$

input
```
integrate(x^2/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(x^4 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \int \frac{x^2}{\sqrt{x^4+1}}\,dx$$

input
```
int(x^2/(x^4 + 1)^(1/2),x)
```

output
```
int(x^2/(x^4 + 1)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}\,x^2}{x^4+1}dx$$

input
```
int(x^2/(x^4+1)^(1/2),x)
```

output
```
int((sqrt(x**4 + 1)*x**2)/(x**4 + 1),x)
```

3.414 $\int \frac{1}{x^2\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 117

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{x}+\frac{x\sqrt{1+x^4}}{1+x^2}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{1+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{1+x^4}}$$

output
```
-(x^4+1)^(1/2)/x+x*(x^4+1)^(1/2)/(x^2+1)-(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)
*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)+1/2*(x^2+1)*((x^4+1
)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-x^4\right)}{x}$$

input
```
Integrate[1/(x^2*Sqrt[1 + x^4]),x]
```

output
```
-(Hypergeometric2F1[-1/4, 1/2, 3/4, -x^4]/x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{847}$$

$$\int \frac{x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}$$

$$\Big\downarrow \text{834}$$

$$\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}$$

$$\Big\downarrow \text{761}$$

$$-\int \frac{1-x^2}{\sqrt{x^4+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}$$

$$\Big\downarrow \text{1510}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}+\frac{\sqrt{x^4+1}x}{x^2+1}$$

input
```
Int[1/(x^2*Sqrt[1 + x^4]),x]
```

output
```
-(Sqrt[1 + x^4]/x) + (x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sq
rt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.15

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{4}\right],-x^4\right)}{x}$	17
default	$-\frac{\sqrt{x^4+1}}{x}+\frac{i\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	95
risch	$-\frac{\sqrt{x^4+1}}{x}+\frac{i\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	95
elliptic	$-\frac{\sqrt{x^4+1}}{x}+\frac{i\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	95

input
```
int(1/x^2/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x*hypergeom([-1/4,1/2],[3/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = \frac{-i\sqrt{i}xE(\arcsin\left(\sqrt{i}x\right)\mid -1)+i\sqrt{i}xF(\arcsin\left(\sqrt{i}x\right)\mid -1)-\sqrt{x^4+1}}{x}$$

input
```
integrate(1/x^2/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(I)*x*elliptic_e(arcsin(sqrt(I)*x), -1) + I*sqrt(I)*x*elliptic_f(a
rcsin(sqrt(I)*x), -1) - sqrt(x^4 + 1))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| x^4 e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(x**4+1)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi))/(4*x*gamma(3/
4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^2}\,dx$$

input
```
integrate(1/x^2/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = -\frac{{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ -x^4\right)}{x}$$

input
```
int(1/(x^2*(x^4 + 1)^(1/2)),x)
```

output
```
-hypergeom([-1/4, 1/2], 3/4, -x^4)/x
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^6+x^2}dx$$

input
```
int(1/x^2/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**6 + x**2),x)
```

3.415 $\int \frac{1}{x^6\sqrt{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = -\frac{\sqrt{1+x^4}}{5x^5}+\frac{3\sqrt{1+x^4}}{5x}-\frac{3x\sqrt{1+x^4}}{5\,(1+x^2)} +\frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{5\sqrt{1+x^4}} -\frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{10\sqrt{1+x^4}}$$

output
```
-1/5*(x^4+1)^(1/2)/x^5+3/5*(x^4+1)^(1/2)/x-3*x*(x^4+1)^(1/2)/(5*x^2+5)+3/5
*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))
/(x^4+1)^(1/2)-3/10*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*ar
ctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-x^4\right)}{5x^5}$$

input
```
Integrate[1/(x^6*Sqrt[1 + x^4]),x]
```

output
```
-1/5*Hypergeometric2F1[-5/4, 1/2, -1/4, -x^4]/x^5
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {847, 847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{3}{5}\int \frac{1}{x^2\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{3}{5}\left(\int \frac{x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}\right)-\frac{\sqrt{x^4+1}}{5x^5}$$

$$\Big\downarrow \text{834}$$

$$-\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}\right)-\frac{\sqrt{x^4+1}}{5x^5}$$

$$\Big\downarrow \text{761}$$

$$-\frac{3}{5}\left(-\int \frac{1-x^2}{\sqrt{x^4+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}\right)-\frac{\sqrt{x^4+1}}{5x^5}$$

$$\Big\downarrow \text{1510}$$

$$-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}+\frac{\sqrt{x^4+}}{x^2+}\right.$$
$$\frac{\sqrt{x^4+1}}{5x^5}$$

input
```
Int[1/(x^6*Sqrt[1 + x^4]),x]
```

output
```
-1/5*Sqrt[1 + x^4]/x^5 - (3*(-(Sqrt[1 + x^4]/x) + (x*Sqrt[1 + x^4])/(1 + x
^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/
Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[
x], 1/2])/(2*Sqrt[1 + x^4])))/5
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{5}{4},\frac{1}{2}\right],\left[-\frac{1}{4}\right],-x^{4}\right)}{5x^{5}}$	17
default	$-\frac{\sqrt{x^{4}+1}}{5x^{5}}+\frac{3\sqrt{x^{4}+1}}{5x}-\frac{3i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{5\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	107
risch	$\frac{3x^{8}+2x^{4}-1}{5x^{5}\sqrt{x^{4}+1}}-\frac{3i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{5\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	107
elliptic	$-\frac{\sqrt{x^{4}+1}}{5x^{5}}+\frac{3\sqrt{x^{4}+1}}{5x}-\frac{3i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{5\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	107

input
```
int(1/x^6/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5*hypergeom([-5/4,1/2],[-1/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = \frac{3i\,\sqrt{i}x^5E(\arcsin\left(\sqrt{i}x\right)\,|-1)-3i\,\sqrt{i}x^5F(\arcsin\left(\sqrt{i}x\right)\,|-1)+(3\,x^4-1)\sqrt{x^4+1}}{5\,x^5}$$

input
```
integrate(1/x^6/(x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
1/5*(3*I*sqrt(I)*x^5*elliptic_e(arcsin(sqrt(I)*x), -1) - 3*I*sqrt(I)*x^5*e
lliptic_f(arcsin(sqrt(I)*x), -1) + (3*x^4 - 1)*sqrt(x^4 + 1))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| x^4 e^{i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(x**4+1)**(1/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), x**4*exp_polar(I*pi))/(4*x**5*gamm
a(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = \int \frac{1}{\sqrt{x^4+1}x^6}\,dx$$

input
```
integrate(1/x^6/(x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^4 + 1)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = \int \frac{1}{x^6\,\sqrt{x^4+1}}\,dx$$

input
```
int(1/(x^6*(x^4 + 1)^(1/2)),x)
```

output
```
int(1/(x^6*(x^4 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt{1+x^4}}\,dx = \int \frac{\sqrt{x^4+1}}{x^{10}+x^6}dx$$

input
```
int(1/x^6/(x^4+1)^(1/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**10 + x**6),x)
```

3.416 $\int \frac{x^{11}}{(1+x^4)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{x^{11}}{(1+x^4)^{3/2}} dx = -\frac{1}{2\sqrt{1+x^4}} - \sqrt{1+x^4} + \frac{1}{6}(1+x^4)^{3/2}$$

output
```
-1/2/(x^4+1)^(1/2)-(x^4+1)^(1/2)+1/6*(x^4+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{x^{11}}{(1+x^4)^{3/2}} dx = \frac{-8-4x^4+x^8}{6\sqrt{1+x^4}}$$

input
```
Integrate[x^11/(1 + x^4)^(3/2),x]
```

output
```
(-8 - 4*x^4 + x^8)/(6*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(x^4+1)^{3/2}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\sqrt{x^4+1}-\frac{2}{\sqrt{x^4+1}}+\frac{1}{(x^4+1)^{3/2}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{2}{3}(x^4+1)^{3/2}-4\sqrt{x^4+1}-\frac{2}{\sqrt{x^4+1}}\right)$$

input
```
Int[x^11/(1 + x^4)^(3/2),x]
```

output
```
(-2/Sqrt[1 + x^4] - 4*Sqrt[1 + x^4] + (2*(1 + x^4)^(3/2))/3)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

method	result	size
gosper	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
default	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
trager	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
risch	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
elliptic	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
pseudoelliptic	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
orering	$\frac{x^{8}-4x^{4}-8}{6\sqrt{x^{4}+1}}$	20
meijerg	$\frac{\frac{8\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-2x^{8}+8x^{4}+16\right)}{6\sqrt{x^{4}+1}}}{2\sqrt{\pi}}$	36

input
```
int(x^11/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(x^8-4*x^4-8)/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.50

$$\int \frac{x^{11}}{(1+x^4)^{3/2}}\,dx = \frac{x^8-4\,x^4-8}{6\,\sqrt{x^4+1}}$$

input
```
integrate(x^11/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(x^8 - 4*x^4 - 8)/sqrt(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{(1+x^4)^{3/2}}\,dx = \frac{x^8}{6\sqrt{x^4+1}} - \frac{2x^4}{3\sqrt{x^4+1}} - \frac{4}{3\sqrt{x^4+1}}$$

input
```
integrate(x**11/(x**4+1)**(3/2),x)
```

output
```
x**8/(6*sqrt(x**4 + 1)) - 2*x**4/(3*sqrt(x**4 + 1)) - 4/(3*sqrt(x**4 + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{(1+x^4)^{3/2}}\,dx = \frac{1}{6}\left(x^4+1\right)^{\frac{3}{2}} - \sqrt{x^4+1} - \frac{1}{2\,\sqrt{x^4+1}}$$

input
```
integrate(x^11/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(x^4 + 1)^(3/2) - sqrt(x^4 + 1) - 1/2/sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^{11}}{(1+x^4)^{3/2}}\,dx = \frac{1}{6}\left(x^4+1\right)^{\frac{3}{2}} - \sqrt{x^4+1} - \frac{1}{2\sqrt{x^4+1}}$$

input
```
integrate(x^11/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/6*(x^4 + 1)^(3/2) - sqrt(x^4 + 1) - 1/2/sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{x^{11}}{(1+x^4)^{3/2}}\,dx = -\frac{6\,x^4 - (x^4+1)^2 + 9}{6\,\sqrt{x^4+1}}$$

input
```
int(x^11/(x^4 + 1)^(3/2),x)
```

output
```
-(6*x^4 - (x^4 + 1)^2 + 9)/(6*(x^4 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \frac{x^{11}}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\,x^8 - 4\sqrt{x^4+1}\,x^4 - 8\sqrt{x^4+1} + x^{10} - 4x^6 - 8x^2}{6\sqrt{x^4+1}\,x^2 + 6x^4 + 6}$$

input
```
int(x^11/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*x**8 - 4*sqrt(x**4 + 1)*x**4 - 8*sqrt(x**4 + 1) + x**10 -
4*x**6 - 8*x**2)/(6*(sqrt(x**4 + 1)*x**2 + x**4 + 1))
```

3.417 $\int \frac{x^7}{(1+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^7}{(1+x^4)^{3/2}} \, dx = \frac{1}{2\sqrt{1+x^4}} + \frac{\sqrt{1+x^4}}{2}$$

output
```
1/2/(x^4+1)^(1/2)+1/2*(x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^7}{(1+x^4)^{3/2}} \, dx = \frac{2+x^4}{2\sqrt{1+x^4}}$$

input
```
Integrate[x^7/(1 + x^4)^(3/2),x]
```

output
```
(2 + x^4)/(2*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(x^4+1)^{3/2}}dx$$
$$\big\downarrow \text{798}$$
$$\frac{1}{4}\int \frac{x^4}{(x^4+1)^{3/2}}dx^4$$
$$\big\downarrow \text{53}$$
$$\frac{1}{4}\int \left(\frac{1}{\sqrt{x^4+1}}-\frac{1}{(x^4+1)^{3/2}}\right)dx^4$$
$$\big\downarrow \text{2009}$$
$$\frac{1}{4}\left(2\sqrt{x^4+1}+\frac{2}{\sqrt{x^4+1}}\right)$$

input
```
Int[x^7/(1 + x^4)^(3/2),x]
```

output
```
(2/Sqrt[1 + x^4] + 2*Sqrt[1 + x^4])/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
gosper	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
default	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
trager	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
risch	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
elliptic	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
pseudoelliptic	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
orering	$\frac{x^4+2}{2\sqrt{x^4+1}}$	15
meijerg	$\frac{-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(4x^4+8\right)}{4\sqrt{x^4+1}}}{2\sqrt{\pi}}$	31

input
```
int(x^7/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^4+2)/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{(1+x^4)^{3/2}}\,dx = \frac{x^4+2}{2\sqrt{x^4+1}}$$

input
```
integrate(x^7/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(x^4 + 2)/sqrt(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^7}{(1+x^4)^{3/2}}\,dx = \frac{x^4}{2\sqrt{x^4+1}} + \frac{1}{\sqrt{x^4+1}}$$

input
```
integrate(x**7/(x**4+1)**(3/2),x)
```

output
```
x**4/(2*sqrt(x**4 + 1)) + 1/sqrt(x**4 + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{(1+x^4)^{3/2}}\,dx = \frac{1}{2}\sqrt{x^4+1} + \frac{1}{2\sqrt{x^4+1}}$$

input
```
integrate(x^7/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^4 + 1) + 1/2/sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^7}{(1+x^4)^{3/2}}\,dx = \frac{1}{2}\sqrt{x^4+1} + \frac{1}{2\sqrt{x^4+1}}$$

input
```
integrate(x^7/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^4 + 1) + 1/2/sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{x^7}{(1+x^4)^{3/2}}\,dx = \frac{x^4+2}{2\sqrt{x^4+1}}$$

input
```
int(x^7/(x^4 + 1)^(3/2),x)
```

output
```
(x^4 + 2)/(2*(x^4 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{x^7}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\,x^4+2\sqrt{x^4+1}+x^6+2x^2}{2\sqrt{x^4+1}\,x^2+2x^4+2}$$

input
```
int(x^7/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*x**4 + 2*sqrt(x**4 + 1) + x**6 + 2*x**2)/(2*(sqrt(x**4 + 1
)*x**2 + x**4 + 1))
```

3.418 $\int \frac{x^3}{(1+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{(1+x^4)^{3/2}} \, dx = -\frac{1}{2\sqrt{1+x^4}}$$

output
```
-1/2/(x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1+x^4)^{3/2}} \, dx = -\frac{1}{2\sqrt{1+x^4}}$$

input
```
Integrate[x^3/(1 + x^4)^(3/2),x]
```

output
```
-1/2*1/Sqrt[1 + x^4]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 793$$

$$-\frac{1}{2\sqrt{x^4+1}}$$

input
```
Int[x^3/(1 + x^4)^(3/2),x]
```

output
```
-1/2*1/Sqrt[1 + x^4]
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{1}{2\sqrt{x^4+1}}$	10
derivativedivides	$-\frac{1}{2\sqrt{x^4+1}}$	10
default	$-\frac{1}{2\sqrt{x^4+1}}$	10
trager	$-\frac{1}{2\sqrt{x^4+1}}$	10
risch	$-\frac{1}{2\sqrt{x^4+1}}$	10
elliptic	$-\frac{1}{2\sqrt{x^4+1}}$	10
pseudoelliptic	$-\frac{1}{2\sqrt{x^4+1}}$	10
orering	$-\frac{1}{2\sqrt{x^4+1}}$	10
meijerg	$\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^4+1}}}{2\sqrt{\pi}}$	22

input
```
int(x^3/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\left(1+x^4\right)^{3/2}}\,dx=-\frac{1}{2\sqrt{x^4+1}}$$

input
```
integrate(x^3/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2/sqrt(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(1+x^4)^{3/2}}\,dx = -\frac{1}{2\sqrt{x^4+1}}$$

input
```
integrate(x**3/(x**4+1)**(3/2),x)
```

output
```
-1/(2*sqrt(x**4 + 1))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{3/2}}\,dx = -\frac{1}{2\,\sqrt{x^4+1}}$$

input
```
integrate(x^3/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/2/sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{3/2}}\,dx = -\frac{1}{2\,\sqrt{x^4+1}}$$

input
```
integrate(x^3/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2/sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{3/2}}\,dx = -\frac{1}{2\sqrt{x^4+1}}$$

input
```
int(x^3/(x^4 + 1)^(3/2),x)
```

output
```
-1/(2*(x^4 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.69

$$\int \frac{x^3}{(1+x^4)^{3/2}}\,dx = \frac{-\sqrt{x^4+1}-x^2}{2\sqrt{x^4+1}\,x^2+2x^4+2}$$

input
```
int(x^3/(x^4+1)^(3/2),x)
```

output
```
( - (sqrt(x**4 + 1) + x**2))/(2*(sqrt(x**4 + 1)*x**2 + x**4 + 1))
```

3.419 $\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2\sqrt{1+x^4}}-\frac{1}{2}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

output
```
1/2/(x^4+1)^(1/2)-1/2*arctanh((x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2\sqrt{1+x^4}}-\frac{1}{2}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

input
```
Integrate[1/(x*(1 + x^4)^(3/2)),x]
```

output
```
1/(2*Sqrt[1 + x^4]) - ArcTanh[Sqrt[1 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {798, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^4+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(x^4+1\right)^{3/2}}dx^4$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\int \frac{1}{x^4\sqrt{x^4+1}}dx^4+\frac{2}{\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(2\int \frac{1}{x^8-1}d\sqrt{x^4+1}+\frac{2}{\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{4}\left(\frac{2}{\sqrt{x^4+1}}-2\text{arctanh}\left(\sqrt{x^4+1}\right)\right)$$

input
```
Int[1/(x*(1 + x^4)^(3/2)),x]
```

output
```
(2/Sqrt[1 + x^4] - 2*ArcTanh[Sqrt[1 + x^4]])/4
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{1}{2\sqrt{x^4+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	21
risch	$\frac{1}{2\sqrt{x^4+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	21
elliptic	$\frac{1}{2\sqrt{x^4+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	21
pseudoelliptic	$\frac{1}{2\sqrt{x^4+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{2}$	21
trager	$\frac{1}{2\sqrt{x^4+1}}+\frac{\ln\left(\frac{-1+\sqrt{x^4+1}}{x^2}\right)}{2}$	27
meijerg	$\frac{\frac{(2-2\ln(2)+4\ln(x))\sqrt{\pi}}{2}-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{x^4+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{2\sqrt{\pi}}$	55

input
```
int(1/x/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(x^4+1)^(1/2)-1/2*arctanh(1/(x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{1}{x(1+x^4)^{3/2}}\,dx = -\frac{(x^4+1)\log\left(\sqrt{x^4+1}+1\right)-(x^4+1)\log\left(\sqrt{x^4+1}-1\right)-2\sqrt{x^4+1}}{4(x^4+1)}$$

input
```
integrate(1/x/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*((x^4 + 1)*log(sqrt(x^4 + 1) + 1) - (x^4 + 1)*log(sqrt(x^4 + 1) - 1)
- 2*sqrt(x^4 + 1))/(x^4 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(22) = 44$.

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.11

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx = \frac{x^4\log\left(x^4\right)}{4x^4+4} - \frac{2x^4\log\left(\sqrt{x^4+1}+1\right)}{4x^4+4} + \frac{2\sqrt{x^4+1}}{4x^4+4} + \frac{\log\left(x^4\right)}{4x^4+4} - \frac{2\log\left(\sqrt{x^4+1}+1\right)}{4x^4+4}$$

input
```
integrate(1/x/(x**4+1)**(3/2),x)
```

output
```
x**4*log(x**4)/(4*x**4 + 4) - 2*x**4*log(sqrt(x**4 + 1) + 1)/(4*x**4 + 4)
+ 2*sqrt(x**4 + 1)/(4*x**4 + 4) + log(x**4)/(4*x**4 + 4) - 2*log(sqrt(x**4
 + 1) + 1)/(4*x**4 + 4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx = \frac{1}{2\sqrt{x^4+1}} - \frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) + \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2/sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2\sqrt{x^4+1}}-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2/sqrt(x^4 + 1) - 1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1
)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2\sqrt{x^4+1}}-\frac{\operatorname{atanh}\left(\sqrt{x^4+1}\right)}{2}$$

input
```
int(1/(x*(x^4 + 1)^(3/2)),x)
```

output
```
1/(2*(x^4 + 1)^(1/2)) - atanh((x^4 + 1)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 135, normalized size of antiderivative = 4.82

$$\int \frac{1}{x\left(1+x^4\right)^{3/2}}\,dx=\frac{\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2-1\right)x^2-\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2+1\right)x^2+\sqrt{x^4+1}\cdots}{}$$

input
```
int(1/x/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2 - 1)*x**2 - sqrt(x**4 + 1)*log(s
qrt(x**4 + 1) + x**2 + 1)*x**2 + sqrt(x**4 + 1) + log(sqrt(x**4 + 1) + x**
2 - 1)*x**4 + log(sqrt(x**4 + 1) + x**2 - 1) - log(sqrt(x**4 + 1) + x**2 +
 1)*x**4 - log(sqrt(x**4 + 1) + x**2 + 1) + x**2)/(2*(sqrt(x**4 + 1)*x**2
+ x**4 + 1))
```

3.420 $\int \frac{1}{x^5(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{1}{x^5\left(1+x^4\right)^{3/2}}\,dx=-\frac{3}{4\sqrt{1+x^4}}-\frac{1}{4x^4\sqrt{1+x^4}}+\frac{3}{4}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

output
```
-3/4/(x^4+1)^(1/2)-1/4/x^4/(x^4+1)^(1/2)+3/4*arctanh((x^4+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\left(1+x^4\right)^{3/2}}\,dx=\frac{-1-3x^4}{4x^4\sqrt{1+x^4}}+\frac{3}{4}\text{arctanh}\left(\sqrt{1+x^4}\right)$$

input
```
Integrate[1/(x^5*(1 + x^4)^(3/2)),x]
```

output
```
(-1 - 3*x^4)/(4*x^4*Sqrt[1 + x^4]) + (3*ArcTanh[Sqrt[1 + x^4]])/4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {798, 52, 61, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^8 (x^4+1)^{3/2}} dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{3}{2} \int \frac{1}{x^4 (x^4+1)^{3/2}} dx^4 - \frac{1}{x^4 \sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(-\frac{3}{2} \left(\int \frac{1}{x^4 \sqrt{x^4+1}} dx^4 + \frac{2}{\sqrt{x^4+1}}\right) - \frac{1}{x^4 \sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(-\frac{3}{2} \left(2 \int \frac{1}{x^8-1} d\sqrt{x^4+1} + \frac{2}{\sqrt{x^4+1}}\right) - \frac{1}{x^4 \sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{4} \left(-\frac{3}{2} \left(\frac{2}{\sqrt{x^4+1}} - 2\text{arctanh}\left(\sqrt{x^4+1}\right)\right) - \frac{1}{x^4 \sqrt{x^4+1}}\right)$$

input
```
Int[1/(x^5*(1 + x^4)^(3/2)),x]
```

output
```
(-(1/(x^4*Sqrt[1 + x^4])) - (3*(2/Sqrt[1 + x^4] - 2*ArcTanh[Sqrt[1 + x^4]]
))/2)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{3x^4+1}{4x^4\sqrt{x^4+1}}+\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	31
default	$-\frac{1}{4x^4\sqrt{x^4+1}}-\frac{3}{4\sqrt{x^4+1}}+\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	33
elliptic	$-\frac{1}{4x^4\sqrt{x^4+1}}-\frac{3}{4\sqrt{x^4+1}}+\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	33
pseudoelliptic	$-\frac{1}{4x^4\sqrt{x^4+1}}-\frac{3}{4\sqrt{x^4+1}}+\frac{3\,\text{arctanh}\left(\frac{1}{\sqrt{x^4+1}}\right)}{4}$	33
trager	$-\frac{3x^4+1}{4x^4\sqrt{x^4+1}}+\frac{3\ln\left(\frac{1+\sqrt{x^4+1}}{x^2}\right)}{4}$	37
meijerg	$\frac{-\frac{\sqrt{\pi}}{2x^4}-\frac{3\left(\frac{5}{3}-2\ln(2)+4\ln(x)\right)\sqrt{\pi}}{4}+\frac{\sqrt{\pi}\left(20x^4+8\right)}{16x^4}-\frac{\sqrt{\pi}\left(24x^4+8\right)}{16x^4\sqrt{x^4+1}}+\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^4+1}}{2}\right)}{2}}{2\sqrt{\pi}}$	84

input
```
int(1/x^5/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(3*x^4+1)/x^4/(x^4+1)^(1/2)+3/4*arctanh(1/(x^4+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{1}{x^5\left(1+x^4\right)^{3/2}}\,dx=\frac{3\left(x^8+x^4\right)\log\left(\sqrt{x^4+1}+1\right)-3\left(x^8+x^4\right)\log\left(\sqrt{x^4+1}-1\right)-2\left(3\,x^4+1\right)\sqrt{x^4+}}{8\left(x^8+x^4\right)}$$

input
```
integrate(1/x^5/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(3*(x^8 + x^4)*log(sqrt(x^4 + 1) + 1) - 3*(x^8 + x^4)*log(sqrt(x^4 + 1
) - 1) - 2*(3*x^4 + 1)*sqrt(x^4 + 1))/(x^8 + x^4)
```

Sympy [A] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5 (1+x^4)^{3/2}}\,dx = \frac{3\operatorname{asinh}\left(\frac{1}{x^2}\right)}{4} - \frac{3}{4x^2\sqrt{1+\frac{1}{x^4}}} - \frac{1}{4x^6\sqrt{1+\frac{1}{x^4}}}$$

input
```
integrate(1/x**5/(x**4+1)**(3/2),x)
```

output
```
3*asinh(x**(-2))/4 - 3/(4*x**2*sqrt(1 + x**(-4))) - 1/(4*x**6*sqrt(1 + x**
(-4)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^5 (1+x^4)^{3/2}}\,dx = -\frac{3x^4+1}{4\left((x^4+1)^{\frac{3}{2}} - \sqrt{x^4+1}\right)} + \frac{3}{8}\log\left(\sqrt{x^4+1}+1\right) - \frac{3}{8}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x^5/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*x^4 + 1)/((x^4 + 1)^(3/2) - sqrt(x^4 + 1)) + 3/8*log(sqrt(x^4 + 1)
 + 1) - 3/8*log(sqrt(x^4 + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^5 (1+x^4)^{3/2}}\,dx = -\frac{3x^4+1}{4\left((x^4+1)^{\frac{3}{2}} - \sqrt{x^4+1}\right)} + \frac{3}{8}\log\left(\sqrt{x^4+1}+1\right) - \frac{3}{8}\log\left(\sqrt{x^4+1}-1\right)$$

input
```
integrate(1/x^5/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(3*x^4 + 1)/((x^4 + 1)^(3/2) - sqrt(x^4 + 1)) + 3/8*log(sqrt(x^4 + 1)
 + 1) - 3/8*log(sqrt(x^4 + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^5 \left(1+x^4\right)^{3/2}}\,dx = \frac{3\,\mathrm{atanh}\left(\sqrt{x^4+1}\right)}{4} - \frac{3}{4\,\sqrt{x^4+1}} - \frac{1}{4\,x^4\,\sqrt{x^4+1}}$$

input
```
int(1/(x^5*(x^4 + 1)^(3/2)),x)
```

output
```
(3*atanh((x^4 + 1)^(1/2)))/4 - 3/(4*(x^4 + 1)^(1/2)) - 1/(4*x^4*(x^4 + 1)^
(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 6.39

$$\int \frac{1}{x^5 \left(1+x^4\right)^{3/2}}\,dx = \frac{-12\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2-1\right)x^{10} - 9\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2-1\right)x^6 + 12}{}$$

input
```
int(1/x^5/(x^4+1)^(3/2),x)
```

output
```
( - 12*sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2 - 1)*x**10 - 9*sqrt(x**4 +
 1)*log(sqrt(x**4 + 1) + x**2 - 1)*x**6 + 12*sqrt(x**4 + 1)*log(sqrt(x**4
+ 1) + x**2 + 1)*x**10 + 9*sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2 + 1)*x
**6 - 12*sqrt(x**4 + 1)*x**8 - 7*sqrt(x**4 + 1)*x**4 - sqrt(x**4 + 1) - 12
*log(sqrt(x**4 + 1) + x**2 - 1)*x**12 - 15*log(sqrt(x**4 + 1) + x**2 - 1)*
x**8 - 3*log(sqrt(x**4 + 1) + x**2 - 1)*x**4 + 12*log(sqrt(x**4 + 1) + x**
2 + 1)*x**12 + 15*log(sqrt(x**4 + 1) + x**2 + 1)*x**8 + 3*log(sqrt(x**4 +
1) + x**2 + 1)*x**4 - 12*x**10 - 13*x**6 - 3*x**2)/(4*x**4*(4*sqrt(x**4 +
1)*x**6 + 3*sqrt(x**4 + 1)*x**2 + 4*x**8 + 5*x**4 + 1))
```

3.421 $\int \frac{x^9}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = -\frac{x^6}{2\sqrt{1+x^4}}+\frac{3}{4}x^2\sqrt{1+x^4}-\frac{3\text{arcsinh}(x^2)}{4}$$

output
```
-1/2*x^6/(x^4+1)^(1/2)+3/4*x^2*(x^4+1)^(1/2)-3/4*arcsinh(x^2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = \frac{3x^2+x^6}{4\sqrt{1+x^4}}-\frac{3}{4}\log\left(x^2+\sqrt{1+x^4}\right)$$

input
```
Integrate[x^9/(1 + x^4)^(3/2),x]
```

output
```
(3*x^2 + x^6)/(4*Sqrt[1 + x^4]) - (3*Log[x^2 + Sqrt[1 + x^4]])/4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 252, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^8}{(x^4+1)^{3/2}}dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2} \left(3 \int \frac{x^4}{\sqrt{x^4+1}}dx^2-\frac{x^6}{\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(3 \left(\frac{1}{2} x^2 \sqrt{x^4+1}-\frac{1}{2} \int \frac{1}{\sqrt{x^4+1}}dx^2\right)-\frac{x^6}{\sqrt{x^4+1}}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2} \left(3 \left(\frac{1}{2} x^2 \sqrt{x^4+1}-\frac{\text{arcsinh}\left(x^2\right)}{2}\right)-\frac{x^6}{\sqrt{x^4+1}}\right)$$

input
```
Int[x^9/(1 + x^4)^(3/2),x]
```

output
```
(-(x^6/Sqrt[1 + x^4]) + 3*((x^2*Sqrt[1 + x^4])/2 - ArcSinh[x^2]/2))/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{x^2(x^4+3)}{4\sqrt{x^4+1}}-\frac{3\,\mathrm{arcsinh}(x^2)}{4}$	25
default	$\frac{x^6}{4\sqrt{x^4+1}}+\frac{3x^2}{4\sqrt{x^4+1}}-\frac{3\,\mathrm{arcsinh}(x^2)}{4}$	32
elliptic	$\frac{x^6}{4\sqrt{x^4+1}}+\frac{3x^2}{4\sqrt{x^4+1}}-\frac{3\,\mathrm{arcsinh}(x^2)}{4}$	32
pseudoelliptic	$\frac{x^6}{4\sqrt{x^4+1}}+\frac{3x^2}{4\sqrt{x^4+1}}-\frac{3\,\mathrm{arcsinh}(x^2)}{4}$	32
trager	$\frac{x^2(x^4+3)}{4\sqrt{x^4+1}}+\frac{3\ln\left(x^2-\sqrt{x^4+1}\right)}{4}$	35
meijerg	$\frac{\frac{\sqrt{\pi}\,x^2\left(5x^4+15\right)}{10\sqrt{x^4+1}}-\frac{3\sqrt{\pi}\,\mathrm{arcsinh}\left(x^2\right)}{2}}{2\sqrt{\pi}}$	38

input
```
int(x^9/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(x^4+3)/(x^4+1)^(1/2)-3/4*arcsinh(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = \frac{2\,x^4+3\,(x^4+1)\log\left(-x^2+\sqrt{x^4+1}\right)+(x^6+3\,x^2)\sqrt{x^4+1}+2}{4\,(x^4+1)}$$

input
```
integrate(x^9/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*x^4 + 3*(x^4 + 1)*log(-x^2 + sqrt(x^4 + 1)) + (x^6 + 3*x^2)*sqrt(x^
4 + 1) + 2)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = \frac{x^6}{4\sqrt{x^4+1}} + \frac{3x^2}{4\sqrt{x^4+1}} - \frac{3\operatorname{asinh}(x^2)}{4}$$

input
```
integrate(x**9/(x**4+1)**(3/2),x)
```

output
```
x**6/(4*sqrt(x**4 + 1)) + 3*x**2/(4*sqrt(x**4 + 1)) - 3*asinh(x**2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = -\frac{\frac{3(x^4+1)}{x^4} - 2}{4\left(\frac{\sqrt{x^4+1}}{x^2} - \frac{(x^4+1)^{\frac{3}{2}}}{x^6}\right)} - \frac{3}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) + \frac{3}{8}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate(x^9/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(3*(x^4 + 1)/x^4 - 2)/(sqrt(x^4 + 1)/x^2 - (x^4 + 1)^(3/2)/x^6) - 3/8
*log(sqrt(x^4 + 1)/x^2 + 1) + 3/8*log(sqrt(x^4 + 1)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = \frac{(x^4+3)x^2}{4\sqrt{x^4+1}} + \frac{3}{4}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate(x^9/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/4*(x^4 + 3)*x^2/sqrt(x^4 + 1) + 3/4*log(-x^2 + sqrt(x^4 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = \int \frac{x^9}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^9/(x^4 + 1)^(3/2),x)
```

output
```
int(x^9/(x^4 + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 4.29

$$\int \frac{x^9}{(1+x^4)^{3/2}}\,dx = \frac{-48\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2\right)x^6 - 36\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2\right)x^2 + 16\sqrt{x^4+1}\,x}{}$$

input
```
int(x^9/(x^4+1)^(3/2),x)
```

output
```
( - 48*sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2)*x**6 - 36*sqrt(x**4 + 1)*
log(sqrt(x**4 + 1) + x**2)*x**2 + 16*sqrt(x**4 + 1)*x**10 + 88*sqrt(x**4 +
 1)*x**6 + 39*sqrt(x**4 + 1)*x**2 - 48*log(sqrt(x**4 + 1) + x**2)*x**8 - 6
0*log(sqrt(x**4 + 1) + x**2)*x**4 - 12*log(sqrt(x**4 + 1) + x**2) + 16*x**
12 + 96*x**8 + 81*x**4 + 9)/(16*(4*sqrt(x**4 + 1)*x**6 + 3*sqrt(x**4 + 1)*
x**2 + 4*x**8 + 5*x**4 + 1))
```

3.422 $\int \frac{x^5}{(1+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{x^5}{(1+x^4)^{3/2}} \, dx = -\frac{x^2}{2\sqrt{1+x^4}} + \frac{\operatorname{arcsinh}(x^2)}{2}$$

output
```
-1/2*x^2/(x^4+1)^(1/2)+1/2*arcsinh(x^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{x^5}{(1+x^4)^{3/2}} \, dx = -\frac{x^2}{2\sqrt{1+x^4}} + \frac{1}{2}\log\left(x^2 + \sqrt{1+x^4}\right)$$

input
```
Integrate[x^5/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x^2/Sqrt[1 + x^4] + Log[x^2 + Sqrt[1 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {807, 252, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{(x^4+1)^{3/2}} dx^2$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{2} \left(\int \frac{1}{\sqrt{x^4+1}} dx^2 - \frac{x^2}{\sqrt{x^4+1}} \right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2} \left(\operatorname{arcsinh}(x^2) - \frac{x^2}{\sqrt{x^4+1}} \right)$$

input
```
Int[x^5/(1 + x^4)^(3/2),x]
```

output
```
(-(x^2/Sqrt[1 + x^4]) + ArcSinh[x^2])/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x^{2}}{2\sqrt{x^{4}+1}}+\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}$	20
risch	$-\frac{x^{2}}{2\sqrt{x^{4}+1}}+\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}$	20
elliptic	$-\frac{x^{2}}{2\sqrt{x^{4}+1}}+\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}$	20
pseudoelliptic	$-\frac{x^{2}}{2\sqrt{x^{4}+1}}+\frac{\operatorname{arcsinh}\left(x^{2}\right)}{2}$	20
trager	$-\frac{x^{2}}{2\sqrt{x^{4}+1}}+\frac{\ln\left(x^{2}+\sqrt{x^{4}+1}\right)}{2}$	28
meijerg	$\frac{-\frac{\sqrt{\pi}\, x^{2}}{\sqrt{x^{4}+1}}+\sqrt{\pi}\, \operatorname{arcsinh}\left(x^{2}\right)}{2\sqrt{\pi}}$	30

input
```
int(x^5/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2/(x^4+1)^(1/2)+1/2*arcsinh(x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{x^5}{(1+x^4)^{3/2}}\,dx = -\frac{x^4+\sqrt{x^4+1}x^2+(x^4+1)\log\left(-x^2+\sqrt{x^4+1}\right)+1}{2\,(x^4+1)}$$

input
```
integrate(x^5/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(x^4 + sqrt(x^4 + 1)*x^2 + (x^4 + 1)*log(-x^2 + sqrt(x^4 + 1)) + 1)/(
x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{x^5}{(1+x^4)^{3/2}}\,dx = -\frac{x^2}{2\sqrt{x^4+1}}+\frac{\operatorname{asinh}(x^2)}{2}$$

input
```
integrate(x**5/(x**4+1)**(3/2),x)
```

output
```
-x**2/(2*sqrt(x**4 + 1)) + asinh(x**2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{x^5}{(1+x^4)^{3/2}}\,dx = -\frac{x^2}{2\,\sqrt{x^4+1}}+\frac{1}{4}\,\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\,\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

input
```
integrate(x^5/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*x^2/sqrt(x^4 + 1) + 1/4*log(sqrt(x^4 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4
 + 1)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{(1+x^4)^{3/2}}\,dx = -\frac{x^2}{2\sqrt{x^4+1}} - \frac{1}{2}\log\left(-x^2+\sqrt{x^4+1}\right)$$

input
```
integrate(x^5/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*x^2/sqrt(x^4 + 1) - 1/2*log(-x^2 + sqrt(x^4 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(1+x^4)^{3/2}}\,dx = \int \frac{x^5}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^5/(x^4 + 1)^(3/2),x)
```

output
```
int(x^5/(x^4 + 1)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.44

$$\int \frac{x^5}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\log\left(\sqrt{x^4+1}+x^2\right)x^2 - 2\sqrt{x^4+1}\,x^2 + \log\left(\sqrt{x^4+1}+x^2\right)x^4 + \log\left(\sqrt{x^4+1}\right.}{2\sqrt{x^4+1}\,x^2+2x^4+2}$$

input
```
int(x^5/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*log(sqrt(x**4 + 1) + x**2)*x**2 - 2*sqrt(x**4 + 1)*x**2 +
log(sqrt(x**4 + 1) + x**2)*x**4 + log(sqrt(x**4 + 1) + x**2) - 2*x**4 - 1)
/(2*(sqrt(x**4 + 1)*x**2 + x**4 + 1))
```

3.423 $\int \frac{x}{(1+x^4)^{3/2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{(1+x^4)^{3/2}} dx = \frac{x^2}{2\sqrt{1+x^4}}$$

output
```
1/2*x^2/(x^4+1)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(1+x^4)^{3/2}} dx = \frac{x^2}{2\sqrt{1+x^4}}$$

input
```
Integrate[x/(1 + x^4)^(3/2),x]
```

output
```
x^2/(2*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 796$$

$$\frac{x^2}{2\sqrt{x^4+1}}$$

input
```
Int[x/(1 + x^4)^(3/2),x]
```

output
```
x^2/(2*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{x^2}{2\sqrt{x^4+1}}$	13
default	$\frac{x^2}{2\sqrt{x^4+1}}$	13
trager	$\frac{x^2}{2\sqrt{x^4+1}}$	13
meijerg	$\frac{x^2}{2\sqrt{x^4+1}}$	13
risch	$\frac{x^2}{2\sqrt{x^4+1}}$	13
elliptic	$\frac{x^2}{2\sqrt{x^4+1}}$	13
pseudoelliptic	$\frac{x^2}{2\sqrt{x^4+1}}$	13
orering	$\frac{x^2}{2\sqrt{x^4+1}}$	13

input
```
int(x/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(x^4+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x}{(1+x^4)^{3/2}}\,dx = \frac{x^4+\sqrt{x^4+1}x^2+1}{2\,(x^4+1)}$$

input
```
integrate(x/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(x^4 + sqrt(x^4 + 1)*x^2 + 1)/(x^4 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{x^4+1}}$$

input
```
integrate(x/(x**4+1)**(3/2),x)
```

output
```
x**2/(2*sqrt(x**4 + 1))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x^4)^{3/2}}\,dx = \frac{x^2}{2\,\sqrt{x^4+1}}$$

input
```
integrate(x/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(x^4 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x^4)^{3/2}}\,dx = \frac{x^2}{2\,\sqrt{x^4+1}}$$

input
```
integrate(x/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*x^2/sqrt(x^4 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{(1+x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{x^4+1}}$$

input
```
int(x/(x^4 + 1)^(3/2),x)
```

output
```
x^2/(2*(x^4 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{x}{(1+x^4)^{3/2}}\,dx = \frac{2\sqrt{x^4+1}\,x^2+2x^4+1}{2\sqrt{x^4+1}\,x^2+2x^4+2}$$

input
```
int(x/(x^4+1)^(3/2),x)
```

output
```
(2*sqrt(x**4 + 1)*x**2 + 2*x**4 + 1)/(2*(sqrt(x**4 + 1)*x**2 + x**4 + 1))
```

3.424 $\int \frac{1}{x^3(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x^3\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2x^2\sqrt{1+x^4}}-\frac{\sqrt{1+x^4}}{x^2}$$

output
```
1/2/x^2/(x^4+1)^(1/2)-(x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\left(1+x^4\right)^{3/2}}\,dx=\frac{-1-2x^4}{2x^2\sqrt{1+x^4}}$$

input
```
Integrate[1/(x^3*(1 + x^4)^(3/2)),x]
```

output
```
(-1 - 2*x^4)/(2*x^2*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \ 803$$

$$-2 \int \frac{x}{(x^4+1)^{3/2}}dx - \frac{1}{2x^2\sqrt{x^4+1}}$$

$$\Big\downarrow \ 796$$

$$-\frac{x^2}{\sqrt{x^4+1}} - \frac{1}{2\sqrt{x^4+1}x^2}$$

input
```
Int[1/(x^3*(1 + x^4)^(3/2)),x]
```

output
```
-1/2*1/(x^2*Sqrt[1 + x^4]) - x^2/Sqrt[1 + x^4]
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20
default	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20
trager	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20
meijerg	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20
risch	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20
elliptic	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20
pseudoelliptic	$\frac{-2x^4-1}{2x^2\sqrt{x^4+1}}$	20
orering	$-\frac{2x^4+1}{2x^2\sqrt{x^4+1}}$	20

input
```
int(1/x^3/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*x^4+1)/x^2/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3 (1+x^4)^{3/2}}\,dx = -\frac{2\,x^6 + 2\,x^2 + (2\,x^4 + 1)\sqrt{x^4 + 1}}{2\,(x^6 + x^2)}$$

input
```
integrate(1/x^3/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(2*x^6 + 2*x^2 + (2*x^4 + 1)*sqrt(x^4 + 1))/(x^6 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{x^3(1+x^4)^{3/2}}\,dx = -\frac{2x^4\sqrt{x^4+1}}{2x^6+2x^2} - \frac{\sqrt{x^4+1}}{2x^6+2x^2}$$

input
```
integrate(1/x**3/(x**4+1)**(3/2),x)
```

output
```
-2*x**4*sqrt(x**4 + 1)/(2*x**6 + 2*x**2) - sqrt(x**4 + 1)/(2*x**6 + 2*x**2
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(1+x^4)^{3/2}}\,dx = -\frac{x^2}{2\sqrt{x^4+1}} - \frac{\sqrt{x^4+1}}{2x^2}$$

input
```
integrate(1/x^3/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*x^2/sqrt(x^4 + 1) - 1/2*sqrt(x^4 + 1)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3(1+x^4)^{3/2}}\,dx = -\frac{x^2}{2\sqrt{x^4+1}} + \frac{1}{\left(x^2-\sqrt{x^4+1}\right)^2-1}$$

input
```
integrate(1/x^3/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
-1/2*x^2/sqrt(x^4 + 1) + 1/((x^2 - sqrt(x^4 + 1))^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^3 (1+x^4)^{3/2}} \, dx = -\frac{x^4+\frac{1}{2}}{x^2 \sqrt{x^4+1}}$$

input
```
int(1/(x^3*(x^4 + 1)^(3/2)),x)
```

output
```
-(x^4 + 1/2)/(x^2*(x^4 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^3 (1+x^4)^{3/2}} \, dx = \frac{-8\sqrt{x^4+1}\, x^6-4\sqrt{x^4+1}\, x^2-8x^8-8x^4-1}{2x^2 \left(2\sqrt{x^4+1}\, x^4+\sqrt{x^4+1}+2x^6+2x^2\right)}$$

input
```
int(1/x^3/(x^4+1)^(3/2),x)
```

output
```
( - 8*sqrt(x**4 + 1)*x**6 - 4*sqrt(x**4 + 1)*x**2 - 8*x**8 - 8*x**4 - 1)/(
2*x**2*(2*sqrt(x**4 + 1)*x**4 + sqrt(x**4 + 1) + 2*x**6 + 2*x**2))
```

3.425 $\int \frac{1}{x^7(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^7\,(1+x^4)^{3/2}}\,dx = \frac{1}{2x^6\sqrt{1+x^4}} - \frac{2\sqrt{1+x^4}}{3x^6} + \frac{4\sqrt{1+x^4}}{3x^2}$$

output
```
1/2/x^6/(x^4+1)^(1/2)-2/3*(x^4+1)^(1/2)/x^6+4/3*(x^4+1)^(1/2)/x^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7\,(1+x^4)^{3/2}}\,dx = \frac{-1+4x^4+8x^8}{6x^6\sqrt{1+x^4}}$$

input
```
Integrate[1/(x^7*(1 + x^4)^(3/2)),x]
```

output
```
(-1 + 4*x^4 + 8*x^8)/(6*x^6*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{4}{3} \int \frac{1}{x^3 (x^4+1)^{3/2}}dx - \frac{1}{6x^6\sqrt{x^4+1}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{4}{3} \left(-2 \int \frac{x}{(x^4+1)^{3/2}}dx - \frac{1}{2x^2\sqrt{x^4+1}}\right) - \frac{1}{6x^6\sqrt{x^4+1}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{1}{6x^6\sqrt{x^4+1}} - \frac{4}{3}\left(-\frac{x^2}{\sqrt{x^4+1}} - \frac{1}{2\sqrt{x^4+1}x^2}\right)$$

input
```
Int[1/(x^7*(1 + x^4)^(3/2)),x]
```

output
```
-1/6*1/(x^6*Sqrt[1 + x^4]) - (4*(-1/2*1/(x^2*Sqrt[1 + x^4]) - x^2/Sqrt[1 +
 x^4]))/3
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
gosper	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25
default	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25
trager	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25
meijerg	$-\frac{-8x^8-4x^4+1}{6x^6\sqrt{x^4+1}}$	25
risch	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25
elliptic	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25
pseudoelliptic	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25
orering	$\frac{8x^8+4x^4-1}{6x^6\sqrt{x^4+1}}$	25

input
```
int(1/x^7/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(8*x^8+4*x^4-1)/x^6/(x^4+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^7 (1+x^4)^{3/2}} \, dx = \frac{8 \, x^{10} + 8 \, x^6 + (8 \, x^8 + 4 \, x^4 - 1)\sqrt{x^4+1}}{6 \, (x^{10} + x^6)}$$

input
```
integrate(1/x^7/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/6*(8*x^10 + 8*x^6 + (8*x^8 + 4*x^4 - 1)*sqrt(x^4 + 1))/(x^10 + x^6)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^7 (1+x^4)^{3/2}}\,dx = \frac{8x^8\sqrt{1+\frac{1}{x^4}}}{6x^8+6x^4} + \frac{4x^4\sqrt{1+\frac{1}{x^4}}}{6x^8+6x^4} - \frac{\sqrt{1+\frac{1}{x^4}}}{6x^8+6x^4}$$

input
```
integrate(1/x**7/(x**4+1)**(3/2),x)
```

output
```
8*x**8*sqrt(1 + x**(-4))/(6*x**8 + 6*x**4) + 4*x**4*sqrt(1 + x**(-4))/(6*x
**8 + 6*x**4) - sqrt(1 + x**(-4))/(6*x**8 + 6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^7 (1+x^4)^{3/2}}\,dx = \frac{x^2}{2\sqrt{x^4+1}} + \frac{\sqrt{x^4+1}}{x^2} - \frac{(x^4+1)^{\frac{3}{2}}}{6\,x^6}$$

input
```
integrate(1/x^7/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^2/sqrt(x^4 + 1) + sqrt(x^4 + 1)/x^2 - 1/6*(x^4 + 1)^(3/2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^7 (1+x^4)^{3/2}} \, dx = \frac{x^2}{2 \sqrt{x^4+1}} - \frac{3 \left(x^2-\sqrt{x^4+1}\right)^4 - 12 \left(x^2-\sqrt{x^4+1}\right)^2 + 5}{3 \left(\left(x^2-\sqrt{x^4+1}\right)^2-1\right)^3}$$

input
```
integrate(1/x^7/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
1/2*x^2/sqrt(x^4 + 1) - 1/3*(3*(x^2 - sqrt(x^4 + 1))^4 - 12*(x^2 - sqrt(x^
4 + 1))^2 + 5)/((x^2 - sqrt(x^4 + 1))^2 - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^7 (1+x^4)^{3/2}} \, dx = -\frac{12 x^4 - 8 \left(x^4+1\right)^2 + 9}{6 x^6 \sqrt{x^4+1}}$$

input
```
int(1/(x^7*(x^4 + 1)^(3/2)),x)
```

output
```
-(12*x^4 - 8*(x^4 + 1)^2 + 9)/(6*x^6*(x^4 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^7 (1+x^4)^{3/2}} \, dx = \frac{-4\sqrt{x^4+1}\, x^2 - 4x^4 - 1}{6x^6 \left(8\sqrt{x^4+1}\, x^8 + 8\sqrt{x^4+1}\, x^4 + \sqrt{x^4+1} + 8x^{10} + 12x^6 + 4x^2\right)}$$

input
```
int(1/x^7/(x^4+1)^(3/2),x)
```

output
```
( - 4*sqrt(x**4 + 1)*x**2 - 4*x**4 - 1)/(6*x**6*(8*sqrt(x**4 + 1)*x**8 + 8
*sqrt(x**4 + 1)*x**4 + sqrt(x**4 + 1) + 8*x**10 + 12*x**6 + 4*x**2))
```

3.426 $\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = -\frac{x^9}{2\sqrt{1+x^4}} - \frac{15}{14}x\sqrt{1+x^4} + \frac{9}{14}x^5\sqrt{1+x^4} + \frac{15(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{28\sqrt{1+x^4}}$$

output
```
-1/2*x^9/(x^4+1)^(1/2)-15/14*x*(x^4+1)^(1/2)+9/14*x^5*(x^4+1)^(1/2)+15/28*
(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))
/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = \frac{x\left(-15-6x^4+2x^8+15\sqrt{1+x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)\right)}{14\sqrt{1+x^4}}$$

input
```
Integrate[x^12/(1 + x^4)^(3/2),x]
```

output
```
(x*(-15 - 6*x^4 + 2*x^8 + 15*Sqrt[1 + x^4]*Hypergeometric2F1[1/4, 1/2, 5/4
, -x^4]))/(14*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {817, 843, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(x^4+1)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{9}{2}\int \frac{x^8}{\sqrt{x^4+1}}dx-\frac{x^9}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{9}{2}\left(\frac{1}{7}x^5\sqrt{x^4+1}-\frac{5}{7}\int \frac{x^4}{\sqrt{x^4+1}}dx\right)-\frac{x^9}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{9}{2}\left(\frac{1}{7}x^5\sqrt{x^4+1}-\frac{5}{7}\left(\frac{1}{3}x\sqrt{x^4+1}-\frac{1}{3}\int \frac{1}{\sqrt{x^4+1}}dx\right)\right)-\frac{x^9}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{9}{2}\left(\frac{1}{7}x^5\sqrt{x^4+1}-\frac{5}{7}\left(\frac{1}{3}x\sqrt{x^4+1}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{x^4+1}}\right)\right)-\frac{x^9}{2\sqrt{x^4+1}}$$

input
```
Int[x^12/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x^9/Sqrt[1 + x^4] + (9*((x^5*Sqrt[1 + x^4])/7 - (5*((x*Sqrt[1 + x^4])
/3 - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(
6*Sqrt[1 + x^4])))/7))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.19

method	result	size
meijerg	$\frac{x^{13} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{13}{4}\right],\left[\frac{17}{4}\right],-x^{4}\right)}{13}$	17
risch	$\frac{x\left(2 x^{8}-6 x^{4}-15\right)}{14 \sqrt{x^{4}+1}}+\frac{15 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	84
default	$-\frac{x}{2 \sqrt{x^{4}+1}}+\frac{x^{5} \sqrt{x^{4}+1}}{7}-\frac{4 x \sqrt{x^{4}+1}}{7}+\frac{15 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	94
elliptic	$-\frac{x}{2 \sqrt{x^{4}+1}}+\frac{x^{5} \sqrt{x^{4}+1}}{7}-\frac{4 x \sqrt{x^{4}+1}}{7}+\frac{15 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	94

input
```
int(x^12/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/13*x^13*hypergeom([3/2,13/4],[17/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = -\frac{15\sqrt{i}(-i\,x^4-i)F\left(\arcsin\left(\frac{\sqrt{i}}{x}\right)\,|-1\right)-(2\,x^9-6\,x^5-15\,x)\sqrt{x^4+1}}{14\,(x^4+1)}$$

input
```
integrate(x^12/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/14*(15*sqrt(I)*(-I*x^4 - I)*elliptic_f(arcsin(sqrt(I)/x), -1) - (2*x^9
- 6*x^5 - 15*x)*sqrt(x^4 + 1))/(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = \frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{13}{4}\\\frac{17}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12/(x**4+1)**(3/2),x)
```

output
```
x**13*gamma(13/4)*hyper((3/2, 13/4), (17/4,), x**4*exp_polar(I*pi))/(4*gam
ma(17/4))
```

Maxima [F]

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{12}}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^12/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^12/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{12}}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^12/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^12/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{12}}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^12/(x^4 + 1)^(3/2),x)
```

output
```
int(x^12/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{12}}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\,x^9 - 3\sqrt{x^4+1}\,x^5 - 15\sqrt{x^4+1}\,x + 15\left(\int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx\right)x^4 + 15\left(\int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx\right)}{7x^4+7}$$

input
```
int(x^12/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*x**9 - 3*sqrt(x**4 + 1)*x**5 - 15*sqrt(x**4 + 1)*x + 15*in
t(sqrt(x**4 + 1)/(x**8 + 2*x**4 + 1),x)*x**4 + 15*int(sqrt(x**4 + 1)/(x**8
 + 2*x**4 + 1),x))/(7*(x**4 + 1))
```

3.427 $\int \frac{x^8}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{x^8}{(1+x^4)^{3/2}}\,dx = -\frac{x^5}{2\sqrt{1+x^4}}+\frac{5}{6}x\sqrt{1+x^4} - \frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{12\sqrt{1+x^4}}$$

output
```
-1/2*x^5/(x^4+1)^(1/2)+5/6*x*(x^4+1)^(1/2)-5/12*(x^2+1)*((x^4+1)/(x^2+1)^2
)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.91 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

$$\int \frac{x^8}{(1+x^4)^{3/2}}\,dx = \frac{x\left(5+2x^4-5\sqrt{1+x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)\right)}{6\sqrt{1+x^4}}$$

input
```
Integrate[x^8/(1 + x^4)^(3/2),x]
```

output
```
(x*(5 + 2*x^4 - 5*Sqrt[1 + x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/(
6*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {817, 843, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5}{2}\int \frac{x^4}{\sqrt{x^4+1}}dx-\frac{x^5}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{2}\left(\frac{1}{3}x\sqrt{x^4+1}-\frac{1}{3}\int \frac{1}{\sqrt{x^4+1}}dx\right)-\frac{x^5}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5}{2}\left(\frac{1}{3}x\sqrt{x^4+1}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{6\sqrt{x^4+1}}\right)-\frac{x^5}{2\sqrt{x^4+1}}$$

input
```
Int[x^8/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x^5/Sqrt[1 + x^4] + (5*((x*Sqrt[1 + x^4])/3 - ((1 + x^2)*Sqrt[(1 + x^
4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4])))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.23

method	result	size
meijerg	$\frac{x^{9} \operatorname{hypergeom}\left(\left[\frac{3}{2}, \frac{9}{4}\right], \left[\frac{13}{4}\right], -x^{4}\right)}{9}$	17
risch	$\frac{x \left(2 x^{4}+5\right)}{6 \sqrt{x^{4}+1}}-\frac{5 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{6 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	79
default	$\frac{x}{2 \sqrt{x^{4}+1}}+\frac{x \sqrt{x^{4}+1}}{3}-\frac{5 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{6 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	82
elliptic	$\frac{x}{2 \sqrt{x^{4}+1}}+\frac{x \sqrt{x^{4}+1}}{3}-\frac{5 \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)}{6 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	82

input
```
int(x^8/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*x^9*hypergeom([3/2,9/4],[13/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{(1+x^4)^{3/2}}\,dx = -\frac{5\sqrt{i}(i\,x^4+i)F(\arcsin\left(\frac{\sqrt{i}}{x}\right)\mid -1) - (2\,x^5+5\,x)\sqrt{x^4+1}}{6\,(x^4+1)}$$

input
```
integrate(x^8/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(5*sqrt(I)*(I*x^4 + I)*elliptic_f(arcsin(sqrt(I)/x), -1) - (2*x^5 + 5
*x)*sqrt(x^4 + 1))/(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{x^8}{(1+x^4)^{3/2}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(x**4+1)**(3/2),x)
```

output
```
x**9*gamma(9/4)*hyper((3/2, 9/4), (13/4,), x**4*exp_polar(I*pi))/(4*gamma(
13/4))
```

Maxima [F]

$$\int \frac{x^8}{(1+x^4)^{3/2}} \, dx = \int \frac{x^8}{(x^4+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^8/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^8/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^8}{(1+x^4)^{3/2}} \, dx = \int \frac{x^8}{(x^4+1)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^8/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^8/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(1+x^4)^{3/2}} \, dx = \int \frac{x^8}{(x^4+1)^{3/2}} \, dx$$

input
```
int(x^8/(x^4 + 1)^(3/2),x)
```

output
```
int(x^8/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^8}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\,x^5 + 5\sqrt{x^4+1}\,x - 5\left(\int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx\right)x^4 - 5\left(\int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx\right)}{3x^4+3}$$

input
```
int(x^8/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*x**5 + 5*sqrt(x**4 + 1)*x - 5*int(sqrt(x**4 + 1)/(x**8 + 2
*x**4 + 1),x)*x**4 - 5*int(sqrt(x**4 + 1)/(x**8 + 2*x**4 + 1),x))/(3*(x**4
 + 1))
```

3.428 $\int \frac{x^4}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = -\frac{x}{2\sqrt{1+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output

```
-1/2*x/(x^4+1)^(1/2)+1/4*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM
(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = \frac{1}{2}x\left(-\frac{1}{\sqrt{1+x^4}} + \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^4\right)\right)$$

input

```
Integrate[x^4/(1 + x^4)^(3/2),x]
```

output

```
(x*(-(1/Sqrt[1 + x^4]) + Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {817, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx-\frac{x}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{x}{2\sqrt{x^4+1}}$$

input
```
Int[x^4/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.29

method	result	size
meijerg	$\frac{x^{5} \operatorname{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^{4}\right)}{5}$	17
default	$-\frac{x}{2\sqrt{x^{4}+1}}+\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	72
risch	$-\frac{x}{2\sqrt{x^{4}+1}}+\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	72
elliptic	$-\frac{x}{2\sqrt{x^{4}+1}}+\frac{\sqrt{-i x^{2}+1}\,\sqrt{i x^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	72

input
```
int(x^4/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*hypergeom([5/4,3/2],[9/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\left(1+x^4\right)^{3/2}}\,dx=\frac{\sqrt{i}(-i\,x^4-i)F(\arcsin\left(\sqrt{i}x\right)\mid-1)-\sqrt{x^4+1}x}{2\left(x^4+1\right)}$$

input
```
integrate(x^4/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(I)*(-I*x^4 - I)*elliptic_f(arcsin(sqrt(I)*x), -1) - sqrt(x^4 + 1
)*x)/(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{9}{4} \end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(x**4+1)**(3/2),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi))/(4*gamma(9
/4))
```

Maxima [F]

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = \int \frac{x^4}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = \int \frac{x^4}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = \int \frac{x^4}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^4/(x^4 + 1)^(3/2),x)
```

output
```
int(x^4/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{(1+x^4)^{3/2}}\,dx = \frac{-\sqrt{x^4+1}\,x + \left(\int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx\right)x^4 + \int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx}{x^4+1}$$

input
```
int(x^4/(x^4+1)^(3/2),x)
```

output
```
( - sqrt(x**4 + 1)*x + int(sqrt(x**4 + 1)/(x**8 + 2*x**4 + 1),x)*x**4 + in
t(sqrt(x**4 + 1)/(x**8 + 2*x**4 + 1),x))/(x**4 + 1)
```

3.429 $\int \frac{1}{(1+x^4)^{3/2}} dx$

Optimal result

Integrand size = 9, antiderivative size = 58

$$\int \frac{1}{(1+x^4)^{3/2}} dx = \frac{x}{2\sqrt{1+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x), \frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
1/2*x/(x^4+1)^(1/2)+1/4*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(
2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.52

$$\int \frac{1}{(1+x^4)^{3/2}} dx = \frac{1}{2}x\left(\frac{1}{\sqrt{1+x^4}} + \text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -x^4\right)\right)$$

input
```
Integrate[(1 + x^4)^(-3/2),x]
```

output
```
(x*(1/Sqrt[1 + x^4] + Hypergeometric2F1[1/4, 1/2, 5/4, -x^4]))/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 749$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+1}}dx+\frac{x}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \; 761$$

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}+\frac{x}{2\sqrt{x^4+1}}$$

input
```
Int[(1 + x^4)^(-3/2),x]
```

output
```
x/(2*Sqrt[1 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*A
rcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.24

method	result	size
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-x^{4}\right)$	14
default	$\frac{x}{2\sqrt{x^{4}+1}}+\frac{\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	72
risch	$\frac{x}{2\sqrt{x^{4}+1}}+\frac{\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	72
elliptic	$\frac{x}{2\sqrt{x^{4}+1}}+\frac{\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	72

input
```
int(1/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/4,3/2],[5/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1+x^4)^{3/2}}\,dx=\frac{\sqrt{i}(-i\,x^4-i)F\left(\arcsin\left(\sqrt{i}x\right)\,|-1\right)+\sqrt{x^4+1}x}{2\,(x^4+1)}$$

input
```
integrate(1/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(I)*(-I*x^4 - I)*elliptic_f(arcsin(sqrt(I)*x), -1) + sqrt(x^4 + 1
)*x)/(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1+x^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{4}\end{matrix}\right| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(x**4+1)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

Maxima [F]

$$\int \frac{1}{(1+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x^4 + 1)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(1+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((x^4 + 1)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.21

$$\int \frac{1}{(1+x^4)^{3/2}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -x^4\right)$$

input
```
int(1/(x^4 + 1)^(3/2),x)
```

output
```
x*hypergeom([1/4, 3/2], 5/4, -x^4)
```

Reduce [F]

$$\int \frac{1}{(1+x^4)^{3/2}}\,dx = \int \frac{\sqrt{x^4+1}}{x^8+2x^4+1}dx$$

input
```
int(1/(x^4+1)^(3/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**8 + 2*x**4 + 1),x)
```

3.430 $\int \frac{1}{x^4(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{1}{x^4\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2x^3\sqrt{1+x^4}}-\frac{5\sqrt{1+x^4}}{6x^3}-\frac{5(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{12\sqrt{1+x^4}}$$

output
```
1/2/x^3/(x^4+1)^(1/2)-5/6*(x^4+1)^(1/2)/x^3-5/12*(x^2+1)*((x^4+1)/(x^2+1)^
2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^4\left(1+x^4\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-x^4\right)}{3x^3}$$

input
```
Integrate[1/(x^4*(1 + x^4)^(3/2)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/4, 3/2, 1/4, -x^4]/x^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {819, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{5}{2} \int \frac{1}{x^4 \sqrt{x^4+1}} dx + \frac{1}{2x^3 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{2} \left(-\frac{1}{3} \int \frac{1}{\sqrt{x^4+1}} dx - \frac{\sqrt{x^4+1}}{3x^3} \right) + \frac{1}{2x^3 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5}{2} \left(-\frac{(x^2+1) \sqrt{\frac{x^4+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \arctan(x), \frac{1}{2}\right)}{6\sqrt{x^4+1}} - \frac{\sqrt{x^4+1}}{3x^3} \right) + \frac{1}{2x^3 \sqrt{x^4+1}}$$

input
```
Int[1/(x^4*(1 + x^4)^(3/2)),x]
```

output
```
1/(2*x^3*Sqrt[1 + x^4]) + (5*(-1/3*Sqrt[1 + x^4]/x^3 - ((1 + x^2)*Sqrt[(1
+ x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(6*Sqrt[1 + x^4])))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.22

method	result	size
meijerg	$-\frac{\mathrm{hypergeom}\left(\left[-\frac{3}{4},\frac{3}{2}\right],\left[\frac{1}{4}\right],-x^{4}\right)}{3x^{3}}$	17
risch	$-\frac{5x^{4}+2}{6x^{3}\sqrt{x^{4}+1}}-\frac{5\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{6\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	81
default	$-\frac{\sqrt{x^{4}+1}}{3x^{3}}-\frac{x}{2\sqrt{x^{4}+1}}-\frac{5\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{6\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	84
elliptic	$-\frac{\sqrt{x^{4}+1}}{3x^{3}}-\frac{x}{2\sqrt{x^{4}+1}}-\frac{5\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{6\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	84

input
```
int(1/x^4/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/x^3*hypergeom([-3/4,3/2],[1/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^4 (1+x^4)^{3/2}}\,dx = -\frac{5\sqrt{i}(-i\,x^7 - i\,x^3)F\left(\arcsin\left(\sqrt{i}x\right)\mid -1\right) + (5\,x^4+2)\sqrt{x^4+1}}{6\,(x^7+x^3)}$$

input
```
integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(5*sqrt(I)*(-I*x^7 - I*x^3)*elliptic_f(arcsin(sqrt(I)*x), -1) + (5*x^
4 + 2)*sqrt(x^4 + 1))/(x^7 + x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^4 (1+x^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle| x^4 e^{i\pi}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(x**4+1)**(3/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_polar(I*pi))/(4*x**3*gamma
(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4 \left(1+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \left(1+x^4\right)^{3/2}}\,dx = \int \frac{1}{\left(x^4+1\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(1+x^4\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(x^4+1\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*(x^4 + 1)^(3/2)),x)
```

output
```
int(1/(x^4*(x^4 + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (1 + x^4)^{3/2}} \, dx = \int \frac{\sqrt{x^4 + 1}}{x^{12} + 2x^8 + x^4} dx$$

input
```
int(1/x^4/(x^4+1)^(3/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**12 + 2*x**8 + x**4),x)
```

3.431 $\int \frac{1}{x^8(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{1}{x^8\left(1+x^4\right)^{3/2}}\,dx=\frac{1}{2x^7\sqrt{1+x^4}}-\frac{9\sqrt{1+x^4}}{14x^7}+\frac{15\sqrt{1+x^4}}{14x^3}+\frac{15(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{28\sqrt{1+x^4}}$$

output
```
1/2/x^7/(x^4+1)^(1/2)-9/14*(x^4+1)^(1/2)/x^7+15/14*(x^4+1)^(1/2)/x^3+15/28
*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2)
)/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.24

$$\int \frac{1}{x^8\left(1+x^4\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{2},-\frac{3}{4},-x^4\right)}{7x^7}$$

input
```
Integrate[1/(x^8*(1 + x^4)^(3/2)),x]
```

output
```
-1/7*Hypergeometric2F1[-7/4, 3/2, -3/4, -x^4]/x^7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {819, 847, 847, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (x^4+1)^{3/2}} \, dx$$

$$\Big\downarrow \text{819}$$

$$\frac{9}{2} \int \frac{1}{x^8 \sqrt{x^4+1}}dx + \frac{1}{2x^7 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{9}{2} \left(-\frac{5}{7} \int \frac{1}{x^4 \sqrt{x^4+1}}dx - \frac{\sqrt{x^4+1}}{7x^7}\right) + \frac{1}{2x^7 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{9}{2} \left(-\frac{5}{7} \left(-\frac{1}{3} \int \frac{1}{\sqrt{x^4+1}}dx - \frac{\sqrt{x^4+1}}{3x^3}\right) - \frac{\sqrt{x^4+1}}{7x^7}\right) + \frac{1}{2x^7 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{9}{2} \left(-\frac{5}{7} \left(-\frac{(x^2+1) \sqrt{\frac{x^4+1}{(x^2+1)^2}} \text{EllipticF}\left(2 \arctan(x), \frac{1}{2}\right)}{6\sqrt{x^4+1}} - \frac{\sqrt{x^4+1}}{3x^3}\right) - \frac{\sqrt{x^4+1}}{7x^7}\right) + \frac{1}{2x^7 \sqrt{x^4+1}}$$

input
```
Int[1/(x^8*(1 + x^4)^(3/2)),x]
```

output
```
1/(2*x^7*Sqrt[1 + x^4]) + (9*(-1/7*Sqrt[1 + x^4]/x^7 - (5*(-1/3*Sqrt[1 + x
^4]/x^3 - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/
2])/(6*Sqrt[1 + x^4])))/7))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.18

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{7}{4},\frac{3}{2}\right],\left[-\frac{3}{4}\right],-x^{4}\right)}{7x^{7}}$	17
risch	$\frac{15x^{8}+6x^{4}-2}{14x^{7}\sqrt{x^{4}+1}}+\frac{15\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	86
default	$-\frac{\sqrt{x^{4}+1}}{7x^{7}}+\frac{4\sqrt{x^{4}+1}}{7x^{3}}+\frac{x}{2\sqrt{x^{4}+1}}+\frac{15\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	96
elliptic	$-\frac{\sqrt{x^{4}+1}}{7x^{7}}+\frac{4\sqrt{x^{4}+1}}{7x^{3}}+\frac{x}{2\sqrt{x^{4}+1}}+\frac{15\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{14\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	96

input
```
int(1/x^8/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/x^7*hypergeom([-7/4,3/2],[-3/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^8 (1+x^4)^{3/2}}\,dx = -\frac{15\sqrt{i}(i\,x^{11}+i\,x^7)F(\arcsin\left(\sqrt{i}x\right)\mid -1)-(15\,x^8+6\,x^4-2)\sqrt{x^4+1}}{14\,(x^{11}+x^7)}$$

input
```
integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/14*(15*sqrt(I)*(I*x^11 + I*x^7)*elliptic_f(arcsin(sqrt(I)*x), -1) - (15
*x^8 + 6*x^4 - 2)*sqrt(x^4 + 1))/(x^11 + x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^8 (1+x^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{7}{4},\frac{3}{2}\\-\frac{3}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(x**4+1)**(3/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 3/2), (-3/4,), x**4*exp_polar(I*pi))/(4*x**7*gamm
a(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8 (1+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}} x^8} \, dx$$

input
```
integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (1+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}} x^8} \, dx$$

input
```
integrate(1/x^8/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (1+x^4)^{3/2}} \, dx = \int \frac{1}{x^8 (x^4+1)^{3/2}} \, dx$$

input
```
int(1/(x^8*(x^4 + 1)^(3/2)),x)
```

output
```
int(1/(x^8*(x^4 + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (1+x^4)^{3/2}}\,dx = \int \frac{\sqrt{x^4+1}}{x^{16}+2x^{12}+x^8}dx$$

input
```
int(1/x^8/(x^4+1)^(3/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**16 + 2*x**12 + x**8),x)
```

3.432 $\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 156

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = -\frac{x^{11}}{2\sqrt{1+x^4}}-\frac{77}{90}x^3\sqrt{1+x^4}+\frac{11}{18}x^7\sqrt{1+x^4} + \frac{77x\sqrt{1+x^4}}{30\,(1+x^2)}-\frac{77(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{30\sqrt{1+x^4}} + \frac{77(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{60\sqrt{1+x^4}}$$

output
```
-1/2*x^11/(x^4+1)^(1/2)-77/90*x^3*(x^4+1)^(1/2)+11/18*x^7*(x^4+1)^(1/2)+77
*x*(x^4+1)^(1/2)/(30*x^2+30)-77/30*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*Ellip
ticE(sin(2*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)+77/60*(x^2+1)*((x^4+1)/(x
^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.35

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = \frac{x^3\left(77-11x^4+5x^8-77\sqrt{1+x^4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-x^4\right)\right)}{45\sqrt{1+x^4}}$$

input
```
Integrate[x^14/(1 + x^4)^(3/2),x]
```

output
```
(x^3*(77 - 11*x^4 + 5*x^8 - 77*Sqrt[1 + x^4]*Hypergeometric2F1[3/4, 3/2, 7
/4, -x^4]))/(45*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {817, 843, 843, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{11}{2}\int \frac{x^{10}}{\sqrt{x^4+1}}dx-\frac{x^{11}}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{11}{2}\left(\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\int \frac{x^6}{\sqrt{x^4+1}}dx\right)-\frac{x^{11}}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{11}{2}\left(\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\int \frac{x^2}{\sqrt{x^4+1}}dx\right)\right)-\frac{x^{11}}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \; 834$$

$$\frac{11}{2}\left(\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\int\frac{1}{\sqrt{x^4+1}}dx-\int\frac{1-x^2}{\sqrt{x^4+1}}dx\right)\right)\right)-\frac{x^{11}}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \; 761$$

$$\frac{11}{2}\left(\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\int\frac{1-x^2}{\sqrt{x^4+1}}dx\right)\right)\right)$$

$$\frac{x^{11}}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \; 1510$$

$$\frac{11}{2}\left(\frac{1}{9}x^7\sqrt{x^4+1}-\frac{7}{9}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E}{\sqrt{x^4+}}\right.\right.\right.$$

$$\frac{x^{11}}{2\sqrt{x^4+1}}$$

input
```
Int[x^14/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x^11/Sqrt[1 + x^4] + (11*((x^7*Sqrt[1 + x^4])/9 - (7*((x^3*Sqrt[1 + x
^4])/5 - (3*((x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 +
x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 +
x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])))/5))/9))
/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.11

method	result
meijerg	$\frac{x^{15} \operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{15}{4}\right],\left[\frac{19}{4}\right],-x^{4}\right)}{15}$
risch	$\frac{x^{3}\left(10x^{8}-22x^{4}-77\right)}{90\sqrt{x^{4}+1}}+\frac{77i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{30\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$
default	$-\frac{x^{3}}{2\sqrt{x^{4}+1}}+\frac{x^{7}\sqrt{x^{4}+1}}{9}-\frac{16x^{3}\sqrt{x^{4}+1}}{45}+\frac{77i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{30\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$
elliptic	$-\frac{x^{3}}{2\sqrt{x^{4}+1}}+\frac{x^{7}\sqrt{x^{4}+1}}{9}-\frac{16x^{3}\sqrt{x^{4}+1}}{45}+\frac{77i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{30\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$

input
```
int(x^14/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x^15*hypergeom([3/2,15/4],[19/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.54

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx =$$
$$-\frac{231\,\sqrt{i}(-i\,x^5-i\,x)E(\arcsin\left(\frac{\sqrt{i}}{x}\right)\,|-1)+231\,\sqrt{i}(i\,x^5+i\,x)F(\arcsin\left(\frac{\sqrt{i}}{x}\right)\,|-1)-(10\,x^{12}-22\,x^8+15}{90\,(x^5+x)}$$

input
```
integrate(x^14/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/90*(231*sqrt(I)*(-I*x^5 - I*x)*elliptic_e(arcsin(sqrt(I)/x), -1) + 231*
sqrt(I)*(I*x^5 + I*x)*elliptic_f(arcsin(sqrt(I)/x), -1) - (10*x^12 - 22*x^
8 + 154*x^4 + 231)*sqrt(x^4 + 1))/(x^5 + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.19

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = \frac{x^{15}\Gamma\left(\frac{15}{4}\right) {}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle| x^4 e^{i\pi}\right)}{4\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**14/(x**4+1)**(3/2),x)
```

output
```
x**15*gamma(15/4)*hyper((3/2, 15/4), (19/4,), x**4*exp_polar(I*pi))/(4*gam
ma(19/4))
```

Maxima [F]

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{14}}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^14/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{14}}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^14/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^14/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{14}}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^14/(x^4 + 1)^(3/2),x)
```

output
```
int(x^14/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(1+x^4)^{3/2}}\,dx = \frac{5\sqrt{x^4+1}\,x^{11}-11\sqrt{x^4+1}\,x^7+77\sqrt{x^4+1}\,x^3-231\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^8+2x^4+1}dx\right)x^4-231\left(\int \frac{\sqrt{x^4+}}{x^8+2x}\right.}{45x^4+45}$$

input
```
int(x^14/(x^4+1)^(3/2),x)
```

output
```
(5*sqrt(x**4 + 1)*x**11 - 11*sqrt(x**4 + 1)*x**7 + 77*sqrt(x**4 + 1)*x**3
- 231*int((sqrt(x**4 + 1)*x**2)/(x**8 + 2*x**4 + 1),x)*x**4 - 231*int((sqr
t(x**4 + 1)*x**2)/(x**8 + 2*x**4 + 1),x))/(45*(x**4 + 1))
```

3.433 $\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = -\frac{x^7}{2\sqrt{1+x^4}}+\frac{7}{10}x^3\sqrt{1+x^4} - \frac{21x\sqrt{1+x^4}}{10\,(1+x^2)}+\frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{10\sqrt{1+x^4}} - \frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{20\sqrt{1+x^4}}$$

output
```
-1/2*x^7/(x^4+1)^(1/2)+7/10*x^3*(x^4+1)^(1/2)-21*x*(x^4+1)^(1/2)/(10*x^2+1
0)+21/10*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*
2^(1/2))/(x^4+1)^(1/2)-21/20*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJaco
biAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.93 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.34

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = \frac{x^3\left(-7+x^4+7\sqrt{1+x^4}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-x^4\right)\right)}{5\sqrt{1+x^4}}$$

input
```
Integrate[x^10/(1 + x^4)^(3/2),x]
```

output
```
(x^3*(-7 + x^4 + 7*Sqrt[1 + x^4]*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4]))/
(5*Sqrt[1 + x^4])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {817, 843, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(x^4+1)^{3/2}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{7}{2}\int \frac{x^6}{\sqrt{x^4+1}}dx-\frac{x^7}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{843}$$

$$\frac{7}{2}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\int \frac{x^2}{\sqrt{x^4+1}}dx\right)-\frac{x^7}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{834}$$

$$\frac{7}{2}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx\right)\right)-\frac{x^7}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \ 761$$

$$\frac{7}{2}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\int\frac{1-x^2}{\sqrt{x^4+1}}dx\right)\right)-\frac{x^7}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \ 1510$$

$$\frac{7}{2}\left(\frac{1}{5}x^3\sqrt{x^4+1}-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\operatorname{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}+\right.\right.-\frac{x^7}{2\sqrt{x^4+1}}$$

input
```
Int[x^10/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x^7/Sqrt[1 + x^4] + (7*((x^3*Sqrt[1 + x^4])/5 - (3*((x*Sqrt[1 + x^4])
/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])))/5))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

method	result	size
meijerg	$\frac{x^{11} \operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{11}{4}\right],\left[\frac{15}{4}\right],-x^{4}\right)}{11}$	17
risch	$\frac{x^{3}\left(2 x^{4}+7\right)}{10 \sqrt{x^{4}+1}}-\frac{21 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{10 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	102
default	$\frac{x^{3}}{2 \sqrt{x^{4}+1}}+\frac{x^{3} \sqrt{x^{4}+1}}{5}-\frac{21 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{10 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	107
elliptic	$\frac{x^{3}}{2 \sqrt{x^{4}+1}}+\frac{x^{3} \sqrt{x^{4}+1}}{5}-\frac{21 i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{10 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	107

input
```
int(x^10/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*x^11*hypergeom([3/2,11/4],[15/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = -\frac{21\sqrt{i}(i\,x^5+i\,x)E(\arcsin\left(\frac{\sqrt{i}}{x}\right)|-1)+21\sqrt{i}(-i\,x^5-i\,x)F(\arcsin\left(\frac{\sqrt{i}}{x}\right)|-1)-(2\,x^8-14\,x^4-21)\sqrt{x}}{10\,(x^5+x)}$$

input
```
integrate(x^10/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(21*sqrt(I)*(I*x^5 + I*x)*elliptic_e(arcsin(sqrt(I)/x), -1) + 21*sqr
t(I)*(-I*x^5 - I*x)*elliptic_f(arcsin(sqrt(I)/x), -1) - (2*x^8 - 14*x^4 -
21)*sqrt(x^4 + 1))/(x^5 + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.21

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\right|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(x**4+1)**(3/2),x)
```

output
```
x**11*gamma(11/4)*hyper((3/2, 11/4), (15/4,), x**4*exp_polar(I*pi))/(4*gam
ma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{10}}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^10/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{10}}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^10/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^10/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = \int \frac{x^{10}}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^10/(x^4 + 1)^(3/2),x)
```

output
```
int(x^10/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\,x^7 - 7\sqrt{x^4+1}\,x^3 + 21\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^8+2x^4+1}dx\right)x^4 + 21\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^8+2x^4+1}dx\right)}{5x^4+5}$$

input
```
int(x^10/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*x**7 - 7*sqrt(x**4 + 1)*x**3 + 21*int((sqrt(x**4 + 1)*x**2
)/(x**8 + 2*x**4 + 1),x)*x**4 + 21*int((sqrt(x**4 + 1)*x**2)/(x**8 + 2*x**
4 + 1),x))/(5*(x**4 + 1))
```

3.434 $\int \frac{x^6}{(1+x^4)^{3/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{x^6}{(1+x^4)^{3/2}} \, dx = -\frac{x^3}{2\sqrt{1+x^4}} + \frac{3x\sqrt{1+x^4}}{2(1+x^2)} - \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
-1/2*x^3/(x^4+1)^(1/2)+3*x*(x^4+1)^(1/2)/(2*x^2+2)-3/2*(x^2+1)*((x^4+1)/(x
^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)+3/4*(
x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/
(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = x^3\left(\frac{1}{\sqrt{1+x^4}} - \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-x^4\right)\right)$$

input
```
Integrate[x^6/(1 + x^4)^(3/2),x]
```

output
```
x^3*(1/Sqrt[1 + x^4] - Hypergeometric2F1[3/4, 3/2, 7/4, -x^4])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {817, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(x^4+1)^{3/2}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{3}{2}\int \frac{x^2}{\sqrt{x^4+1}}dx - \frac{x^3}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{834}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{x^4+1}}dx - \int \frac{1-x^2}{\sqrt{x^4+1}}dx\right) - \frac{x^3}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{3}{2}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}} - \int \frac{1-x^2}{\sqrt{x^4+1}}dx\right) - \frac{x^3}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{3}{2}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\mathrm{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+1}}+\frac{\sqrt{x^4+1}x}{x^2+1}\right)-\frac{x^3}{2\sqrt{x^4+1}}$$

input
```
Int[x^6/(1 + x^4)^(3/2),x]
```

output
```
-1/2*x^3/Sqrt[1 + x^4] + (3*((x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt
[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 +
 x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 +
 x^4])))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.14

method	result	size
meijerg	$\frac{x^7 \operatorname{hypergeom}\left(\left[\frac{3}{2},\frac{7}{4}\right],\left[\frac{11}{4}\right],-x^4\right)}{7}$	17
default	$-\frac{x^3}{2\sqrt{x^4+1}}+\frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	95
risch	$-\frac{x^3}{2\sqrt{x^4+1}}+\frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	95
elliptic	$-\frac{x^3}{2\sqrt{x^4+1}}+\frac{3i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	95

input
```
int(x^6/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*hypergeom([3/2,7/4],[11/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = -\frac{3\sqrt{i}(-i\,x^5-i\,x)E(\arcsin\left(\frac{\sqrt{i}}{x}\right)|-1)+3\sqrt{i}(i\,x^5+i\,x)F(\arcsin\left(\frac{\sqrt{i}}{x}\right)|-1)-(2\,x^4+3)\sqrt{x^4+1}}{2\,(x^5+x)}$$

input
```
integrate(x^6/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*sqrt(I)*(-I*x^5 - I*x)*elliptic_e(arcsin(sqrt(I)/x), -1) + 3*sqrt(
I)*(I*x^5 + I*x)*elliptic_f(arcsin(sqrt(I)/x), -1) - (2*x^4 + 3)*sqrt(x^4
+ 1))/(x^5 + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|x^4e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(x**4+1)**(3/2),x)
```

output
```
x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi))/(4*gamma(
11/4))
```

Maxima [F]

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = \int \frac{x^6}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^6/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = \int \frac{x^6}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^6/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^6/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = \int \frac{x^6}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^6/(x^4 + 1)^(3/2),x)
```

output
```
int(x^6/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^6}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}\,x^3 - 3\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^8+2x^4+1}dx\right)x^4 - 3\left(\int \frac{\sqrt{x^4+1}\,x^2}{x^8+2x^4+1}dx\right)}{x^4+1}$$

input
```
int(x^6/(x^4+1)^(3/2),x)
```

output
```
(sqrt(x**4 + 1)*x**3 - 3*int((sqrt(x**4 + 1)*x**2)/(x**8 + 2*x**4 + 1),x)*
x**4 - 3*int((sqrt(x**4 + 1)*x**2)/(x**8 + 2*x**4 + 1),x))/(x**4 + 1)
```

3.435 $\int \frac{x^2}{\left(1+x^4\right)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{x^2}{\left(1+x^4\right)^{3/2}} dx = \frac{x^3}{2\sqrt{1+x^4}} - \frac{x\sqrt{1+x^4}}{2\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{2\sqrt{1+x^4}} - \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
1/2*x^3/(x^4+1)^(1/2)-x*(x^4+1)^(1/2)/(2*x^2+2)+1/2*(x^2+1)*((x^4+1)/(x^2+
1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)-1/4*(x^2
+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^
4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.18

$$\int \frac{x^2}{\left(1+x^4\right)^{3/2}} dx = \frac{1}{3}x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-x^4\right)$$

input
```
Integrate[x^2/(1 + x^4)^(3/2),x]
```

output
```
(x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4])/3
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {819, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(x^4+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{819}$$

$$\frac{x^3}{2\sqrt{x^4+1}}-\frac{1}{2}\int \frac{x^2}{\sqrt{x^4+1}}dx$$

$$\Big\downarrow \text{834}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{x^4+1}}dx-\int \frac{1}{\sqrt{x^4+1}}dx\right)+\frac{x^3}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{x^4+1}}dx-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}\right)+\frac{x^3}{2\sqrt{x^4+1}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{1}{2}\left(-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}+\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{\left(x^2+1\right)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}x}{x^2+1}\right)+\frac{x^3}{2\sqrt{x^4+1}}$$

input
```
Int[x^2/(1 + x^4)^(3/2),x]
```

output
```
x^3/(2*Sqrt[1 + x^4]) + (-((x*Sqrt[1 + x^4])/(1 + x^2)) + ((1 + x^2)*Sqrt[
(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] - ((1 +
x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 +
x^4]))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.14

method	result	size
meijerg	$\frac{x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],-x^{4}\right)}{3}$	17
default	$\frac{x^{3}}{2 \sqrt{x^{4}+1}}-\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{2 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	95
risch	$\frac{x^{3}}{2 \sqrt{x^{4}+1}}-\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{2 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	95
elliptic	$\frac{x^{3}}{2 \sqrt{x^{4}+1}}-\frac{i \sqrt{-i x^{2}+1}\, \sqrt{i x^{2}+1}\, \left(\operatorname{EllipticF}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)-\operatorname{EllipticE}\left(x \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right), i\right)\right)}{2 \left(\frac{\sqrt{2}}{2}+\frac{i \sqrt{2}}{2}\right) \sqrt{x^{4}+1}}$	95

input
```
int(x^2/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*hypergeom([3/4,3/2],[7/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{(1+x^4)^{3/2}}\,dx = \frac{\sqrt{x^4+1}x^3 + \sqrt{i}(i\,x^4+i)E(\arcsin\left(\sqrt{i}x\right)\,|-1) + \sqrt{i}(-i\,x^4-i)F(\arcsin\left(\sqrt{i}x\right)\,|-1)}{2\,(x^4+1)}$$

input
```
integrate(x^2/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(x^4 + 1)*x^3 + sqrt(I)*(I*x^4 + I)*elliptic_e(arcsin(sqrt(I)*x),
 -1) + sqrt(I)*(-I*x^4 - I)*elliptic_f(arcsin(sqrt(I)*x), -1))/(x^4 + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^2}{(1+x^4)^{3/2}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle| x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(x**4+1)**(3/2),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi))/(4*gamma(7
/4))
```

Maxima [F]

$$\int \frac{x^2}{(1+x^4)^{3/2}}\,dx = \int \frac{x^2}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^4 + 1)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(1+x^4)^{3/2}}\,dx = \int \frac{x^2}{(x^4+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(x^4 + 1)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(1+x^4)^{3/2}}\,dx = \int \frac{x^2}{(x^4+1)^{3/2}}\,dx$$

input
```
int(x^2/(x^4 + 1)^(3/2),x)
```

output
```
int(x^2/(x^4 + 1)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(1+x^4)^{3/2}}\,dx = \int \frac{\sqrt{x^4+1}\,x^2}{x^8+2x^4+1}dx$$

input
```
int(x^2/(x^4+1)^(3/2),x)
```

output
```
int((sqrt(x**4 + 1)*x**2)/(x**8 + 2*x**4 + 1),x)
```

3.436 $\int \frac{1}{x^2(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{x^2(1+x^4)^{3/2}}\,dx = \frac{1}{2x\sqrt{1+x^4}} - \frac{3\sqrt{1+x^4}}{2x} + \frac{3x\sqrt{1+x^4}}{2(1+x^2)} - \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{2\sqrt{1+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

output
```
1/2/x/(x^4+1)^(1/2)-3/2*(x^4+1)^(1/2)/x+3*x*(x^4+1)^(1/2)/(2*x^2+2)-3/2*(x
^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(sin(2*arctan(x)),1/2*2^(1/2))/(x
^4+1)^(1/2)+3/4*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*InverseJacobiAM(2*arctan
(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 (1+x^4)^{3/2}} dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-x^4\right)}{x}$$

input
```
Integrate[1/(x^2*(1 + x^4)^(3/2)),x]
```

output
```
-(Hypergeometric2F1[-1/4, 3/2, 3/4, -x^4]/x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {819, 847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (x^4+1)^{3/2}} dx$$

$$\Big\downarrow \text{819}$$

$$\frac{3}{2}\int \frac{1}{x^2\sqrt{x^4+1}}dx+\frac{1}{2x\sqrt{x^4+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{2}\left(\int \frac{x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}\right)+\frac{1}{2x\sqrt{x^4+1}}$$

$$\Big\downarrow \text{834}$$

$$\frac{3}{2}\left(\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}\right)+\frac{1}{2x\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{3}{2}\left(-\int \frac{1-x^2}{\sqrt{x^4+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}\right)+\frac{1}{2x\sqrt{x^4+1}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{3}{2}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}+\frac{\sqrt{x^4+1}x}{x^2+1}\right)+\frac{1}{2x\sqrt{x^4+1}}$$

input
```
Int[1/(x^2*(1 + x^4)^(3/2)),x]
```

output
```
1/(2*x*Sqrt[1 + x^4]) + (3*(-(Sqrt[1 + x^4]/x) + (x*Sqrt[1 + x^4])/(1 + x^
2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/2])/S
qrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x
], 1/2])/(2*Sqrt[1 + x^4])))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.12

method	result	size
meijerg	$-\frac{\operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{3}{2}\right],\left[\frac{3}{4}\right],-x^{4}\right)}{x}$	17
risch	$-\frac{3x^{4}+2}{2x\sqrt{x^{4}+1}}+\frac{3i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	102
default	$-\frac{\sqrt{x^{4}+1}}{x}-\frac{x^{3}}{2\sqrt{x^{4}+1}}+\frac{3i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	107
elliptic	$-\frac{\sqrt{x^{4}+1}}{x}-\frac{x^{3}}{2\sqrt{x^{4}+1}}+\frac{3i\sqrt{-ix^{2}+1}\,\sqrt{ix^{2}+1}\,\left(\operatorname{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\operatorname{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{2\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^{4}+1}}$	107

input
```
int(1/x^2/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/x*hypergeom([-1/4,3/2],[3/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^2 (1+x^4)^{3/2}} \, dx = -\frac{3 \sqrt{i}(i x^5 + i x) E(\arcsin\left(\sqrt{i}x\right) | -1) + 3 \sqrt{i}(-i x^5 - i x) F(\arcsin\left(\sqrt{i}x\right) | -1) + (3 x^4 + 2)\sqrt{x^4 + 1}}{2 (x^5 + x)}$$

input
```
integrate(1/x^2/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(3*sqrt(I)*(I*x^5 + I*x)*elliptic_e(arcsin(sqrt(I)*x), -1) + 3*sqrt(I
)*(-I*x^5 - I*x)*elliptic_f(arcsin(sqrt(I)*x), -1) + (3*x^4 + 2)*sqrt(x^4
+ 1))/(x^5 + x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^2 (1+x^4)^{3/2}} \, dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{3}{4} \end{matrix}\middle| x^4 e^{i\pi}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(x**4+1)**(3/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi))/(4*x*gamma(3/
4))
```

Maxima [F]

$$\int \frac{1}{x^2 (1+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (1+x^4)^{3/2}}\,dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}} x^2}\,dx$$

input
```
integrate(1/x^2/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2 (1+x^4)^{3/2}}\,dx = -\frac{{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ -x^4\right)}{x}$$

input
```
int(1/(x^2*(x^4 + 1)^(3/2)),x)
```

output
```
-hypergeom([-1/4, 3/2], 3/4, -x^4)/x
```

Reduce [F]

$$\int \frac{1}{x^2\left(1+x^4\right)^{3/2}}\,dx = \int \frac{\sqrt{x^4+1}}{x^{10}+2x^6+x^2}dx$$

input
```
int(1/x^2/(x^4+1)^(3/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**10 + 2*x**6 + x**2),x)
```

3.437 $\int \frac{1}{x^6(1+x^4)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 156

$$\begin{aligned}\int \frac{1}{x^6\left(1+x^4\right)^{3/2}}\,dx &= \frac{1}{2x^5\sqrt{1+x^4}}-\frac{7\sqrt{1+x^4}}{10x^5}+\frac{21\sqrt{1+x^4}}{10x}\\ &\quad-\frac{21x\sqrt{1+x^4}}{10\left(1+x^2\right)}+\frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}E\left(2\arctan(x)\,\middle|\,\frac{1}{2}\right)}{10\sqrt{1+x^4}}\\ &\quad-\frac{21(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{20\sqrt{1+x^4}}\end{aligned}$$

output

```
1/2/x^5/(x^4+1)^(1/2)-7/10*(x^4+1)^(1/2)/x^5+21/10*(x^4+1)^(1/2)/x-21*x*(x
^4+1)^(1/2)/(10*x^2+10)+21/10*(x^2+1)*((x^4+1)/(x^2+1)^2)^(1/2)*EllipticE(
sin(2*arctan(x)),1/2*2^(1/2))/(x^4+1)^(1/2)-21/20*(x^2+1)*((x^4+1)/(x^2+1)
^2)^(1/2)*InverseJacobiAM(2*arctan(x),1/2*2^(1/2))/(x^4+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^6 (1+x^4)^{3/2}} dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-x^4\right)}{5x^5}$$

input
```
Integrate[1/(x^6*(1 + x^4)^(3/2)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/4, 3/2, -1/4, -x^4]/x^5
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {819, 847, 847, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (x^4+1)^{3/2}} dx$$

$$\Big\downarrow \text{819}$$

$$\frac{7}{2} \int \frac{1}{x^6 \sqrt{x^4+1}} dx + \frac{1}{2x^5 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{2} \left(-\frac{3}{5} \int \frac{1}{x^2 \sqrt{x^4+1}} dx - \frac{\sqrt{x^4+1}}{5x^5}\right) + \frac{1}{2x^5 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{847}$$

$$\frac{7}{2} \left(-\frac{3}{5} \left(\int \frac{x^2}{\sqrt{x^4+1}} dx - \frac{\sqrt{x^4+1}}{x}\right) - \frac{\sqrt{x^4+1}}{5x^5}\right) + \frac{1}{2x^5 \sqrt{x^4+1}}$$

$$\Big\downarrow \text{834}$$

$$\frac{7}{2}\left(-\frac{3}{5}\left(\int \frac{1}{\sqrt{x^4+1}}dx-\int \frac{1-x^2}{\sqrt{x^4+1}}dx-\frac{\sqrt{x^4+1}}{x}\right)-\frac{\sqrt{x^4+1}}{5x^5}\right)+\frac{1}{2x^5\sqrt{x^4+1}}$$

$$\Big\downarrow \text{761}$$

$$\frac{7}{2}\left(-\frac{3}{5}\left(-\int \frac{1-x^2}{\sqrt{x^4+1}}dx+\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}\right)-\frac{\sqrt{x^4+1}}{5x^5}\right)+\frac{1}{2x^5\sqrt{x^4+1}}$$

$$\Big\downarrow \text{1510}$$

$$\frac{7}{2}\left(-\frac{3}{5}\left(\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\arctan(x),\frac{1}{2}\right)}{2\sqrt{x^4+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}E\left(2\arctan(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+1}}-\frac{\sqrt{x^4+1}}{x}+\frac{\sqrt{x}}{x}\right.\right.$$

$$\frac{1}{2x^5\sqrt{x^4+1}}$$

input
```
Int[1/(x^6*(1 + x^4)^(3/2)),x]
```

output
```
1/(2*x^5*Sqrt[1 + x^4]) + (7*(-1/5*Sqrt[1 + x^4]/x^5 - (3*(-(Sqrt[1 + x^4]
/x) + (x*Sqrt[1 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^4)/(1 + x^2)^2]
*EllipticE[2*ArcTan[x], 1/2])/Sqrt[1 + x^4] + ((1 + x^2)*Sqrt[(1 + x^4)/(1
 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(2*Sqrt[1 + x^4])))/5))/2
```

Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 819
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Simp[(m + n*(p +
1) + 1)/(a*n*(p + 1))   Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a
, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.11

method	result	size
meijerg	$-\frac{\text{hypergeom}\left(\left[-\frac{5}{4},\frac{3}{2}\right],\left[-\frac{1}{4}\right],-x^4\right)}{5x^5}$	17
risch	$\frac{21x^8+14x^4-2}{10x^5\sqrt{x^4+1}}-\frac{21i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{10\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	107
default	$-\frac{\sqrt{x^4+1}}{5x^5}+\frac{8\sqrt{x^4+1}}{5x}+\frac{x^3}{2\sqrt{x^4+1}}-\frac{21i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{10\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	119
elliptic	$-\frac{\sqrt{x^4+1}}{5x^5}+\frac{8\sqrt{x^4+1}}{5x}+\frac{x^3}{2\sqrt{x^4+1}}-\frac{21i\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\left(\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)-\text{EllipticE}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)\right)}{10\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}$	119

input
```
int(1/x^6/(x^4+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^5*hypergeom([-5/4,3/2],[-1/4],-x^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^6 (1+x^4)^{3/2}}\,dx =$$
$$-\frac{21\,\sqrt{i}(-i\,x^9-i\,x^5)E(\arcsin\left(\sqrt{i}x\right)\,|-1)+21\,\sqrt{i}(i\,x^9+i\,x^5)F(\arcsin\left(\sqrt{i}x\right)\,|-1)-(21\,x^8+14\,x^4-}{10\,(x^9+x^5)}$$

input
```
integrate(1/x^6/(x^4+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/10*(21*sqrt(I)*(-I*x^9 - I*x^5)*elliptic_e(arcsin(sqrt(I)*x), -1) + 21*
sqrt(I)*(I*x^9 + I*x^5)*elliptic_f(arcsin(sqrt(I)*x), -1) - (21*x^8 + 14*x
^4 - 2)*sqrt(x^4 + 1))/(x^9 + x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^6 (1+x^4)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ -\frac{1}{4} \end{matrix}\middle| x^4 e^{i\pi}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(x**4+1)**(3/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), x**4*exp_polar(I*pi))/(4*x**5*gamm
a(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6 (1+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate(1/x^6/(x^4+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (1+x^4)^{3/2}} \, dx = \int \frac{1}{(x^4+1)^{\frac{3}{2}} x^6} \, dx$$

input
```
integrate(1/x^6/(x^4+1)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)^(3/2)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (1+x^4)^{3/2}} \, dx = \int \frac{1}{x^6 (x^4+1)^{3/2}} \, dx$$

input
```
int(1/(x^6*(x^4 + 1)^(3/2)),x)
```

output
```
int(1/(x^6*(x^4 + 1)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (1+x^4)^{3/2}} \, dx = \int \frac{\sqrt{x^4+1}}{x^{14}+2x^{10}+x^6} dx$$

input
```
int(1/x^6/(x^4+1)^(3/2),x)
```

output
```
int(sqrt(x**4 + 1)/(x**14 + 2*x**10 + x**6),x)
```

3.438 $\int \frac{x}{\sqrt{-4+x^4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{x}{\sqrt{-4+x^4}} \, dx = \frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{-4+x^4}}\right)$$

output
```
1/2*arctanh(x^2/(x^4-4)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-4+x^4}} \, dx = \frac{1}{2} \log \left(x^2 + \sqrt{-4+x^4}\right)$$

input
```
Integrate[x/Sqrt[-4 + x^4],x]
```

output
```
Log[x^2 + Sqrt[-4 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {807, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4-4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4-4}}dx^2$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\int \frac{1}{1-x^4}d\frac{x^2}{\sqrt{x^4-4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{x^2}{\sqrt{x^4-4}}\right)$$

input
```
Int[x/Sqrt[-4 + x^4],x]
```

output
```
ArcTanh[x^2/Sqrt[-4 + x^4]]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\ln\left(x^2+\sqrt{x^4-4}\right)}{2}$	15
elliptic	$\frac{\ln\left(x^2+\sqrt{x^4-4}\right)}{2}$	15
pseudoelliptic	$\frac{\ln\left(x^2+\sqrt{x^4-4}\right)}{2}$	15
trager	$-\frac{\ln\left(x^2-\sqrt{x^4-4}\right)}{2}$	17
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{x^4}{4}\right)}\,\arcsin\left(\frac{x^2}{2}\right)}{2\sqrt{\operatorname{signum}\left(-1+\frac{x^4}{4}\right)}}$	31

input
```
int(x/(x^4-4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2+(x^4-4)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{-4+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4-4}\right)$$

input
```
integrate(x/(x^4-4)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 - 4))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{-4+x^4}}\,dx = \begin{cases} \frac{\operatorname{acosh}\left(\frac{x^2}{2}\right)}{2} & \text{for } |x^4| > 4 \\ -\frac{i\operatorname{asin}\left(\frac{x^2}{2}\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(x**4-4)**(1/2),x)
```

output
```
Piecewise((acosh(x**2/2)/2, Abs(x**4) > 4), (-I*asin(x**2/2)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{x}{\sqrt{-4+x^4}}\,dx = \frac{1}{4}\log\left(\frac{\sqrt{x^4-4}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4-4}}{x^2}-1\right)$$

input
```
integrate(x/(x^4-4)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 - 4)/x^2 + 1) - 1/4*log(sqrt(x^4 - 4)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{-4+x^4}}\,dx = -\frac{1}{2}\log\left(x^2-\sqrt{x^4-4}\right)$$

input
```
integrate(x/(x^4-4)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(x^2 - sqrt(x^4 - 4))
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{-4+x^4}}\,dx = \frac{\ln\left(\sqrt{x^4-4}+x^2\right)}{2}$$

input
```
int(x/(x^4 - 4)^(1/2),x)
```

output
```
log((x^4 - 4)^(1/2) + x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{-4+x^4}}\,dx = \frac{\log\left(\frac{\sqrt{x^4-4}}{2}+\frac{x^2}{2}\right)}{2}$$

input
```
int(x/(x^4-4)^(1/2),x)
```

output
```
log((sqrt(x**4 - 4) + x**2)/2)/2
```

3.439 $\int \frac{x}{\sqrt{4+x^4}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{x}{\sqrt{4+x^4}} \, dx = \frac{1}{2}\text{arcsinh}\left(\frac{x^2}{2}\right)$$

output
```
1/2*arcsinh(1/2*x^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{4+x^4}} \, dx = \frac{1}{2} \log \left(x^2+\sqrt{4+x^4}\right)$$

input
```
Integrate[x/Sqrt[4 + x^4],x]
```

output
```
Log[x^2 + Sqrt[4 + x^4]]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^4+4}}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^4+4}}dx^2$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\text{arcsinh}\left(\frac{x^2}{2}\right)$$

input
```
Int[x/Sqrt[4 + x^4],x]
```

output
```
ArcSinh[x^2/2]/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{2}\right)}{2}$	9
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{2}\right)}{2}$	9
elliptic	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{2}\right)}{2}$	9
pseudoelliptic	$\frac{\operatorname{arcsinh}\left(\frac{x^2}{2}\right)}{2}$	9
trager	$-\frac{\ln\left(x^2-\sqrt{x^4+4}\right)}{2}$	17

input
```
int(x/(x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(1/2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{4+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+4}\right)$$

input
```
integrate(x/(x^4+4)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 + 4))
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{\sqrt{4+x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{x^2}{2}\right)}{2}$$

input
```
integrate(x/(x**4+4)**(1/2),x)
```

output
```
asinh(x**2/2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{x}{\sqrt{4+x^4}}\,dx = \frac{1}{4}\log\left(\frac{\sqrt{x^4+4}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+4}}{x^2}-1\right)$$

input
```
integrate(x/(x^4+4)^(1/2),x, algorithm="maxima")
```

output
```
1/4*log(sqrt(x^4 + 4)/x^2 + 1) - 1/4*log(sqrt(x^4 + 4)/x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{4+x^4}}\,dx = -\frac{1}{2}\log\left(-x^2+\sqrt{x^4+4}\right)$$

input
```
integrate(x/(x^4+4)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(-x^2 + sqrt(x^4 + 4))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{4+x^4}}\,dx = \frac{\operatorname{asinh}\left(\frac{x^2}{2}\right)}{2}$$

input
```
int(x/(x^4 + 4)^(1/2),x)
```

output
```
asinh(x^2/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{x}{\sqrt{4+x^4}}\,dx = \frac{\log\left(\frac{\sqrt{x^4+4}}{2}+\frac{x^2}{2}\right)}{2}$$

input
```
int(x/(x^4+4)^(1/2),x)
```

output
```
log((sqrt(x**4 + 4) + x**2)/2)/2
```

3.440 $\int x^7 \sqrt[3]{1+x^4}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int x^7 \sqrt[3]{1+x^4}\, dx = -\frac{3}{16}\left(1+x^4\right)^{4/3} + \frac{3}{28}\left(1+x^4\right)^{7/3}$$

output
```
-3/16*(x^4+1)^(4/3)+3/28*(x^4+1)^(7/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int x^7 \sqrt[3]{1+x^4}\, dx = \frac{3}{112}\left(1+x^4\right)^{4/3}\left(-3+4x^4\right)$$

input
```
Integrate[x^7*(1 + x^4)^(1/3),x]
```

output
```
(3*(1 + x^4)^(4/3)*(-3 + 4*x^4))/112
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \sqrt[3]{x^4+1} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int x^4 \sqrt[3]{x^4+1} dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4} \int \left(\left(x^4+1\right)^{4/3}-\sqrt[3]{x^4+1}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{3}{7} \left(x^4+1\right)^{7/3}-\frac{3}{4} \left(x^4+1\right)^{4/3}\right)$$

input
```
Int[x^7*(1 + x^4)^(1/3),x]
```

output
```
((-3*(1 + x^4)^(4/3))/4 + (3*(1 + x^4)^(7/3))/7)/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{3(x^4+1)^{\frac{4}{3}}(4x^4-3)}{112}$	17
meijerg	$\frac{x^8 \operatorname{hypergeom}([-\frac{1}{3},2],[3],-x^4)}{8}$	17
pseudoelliptic	$\frac{3(x^4+1)^{\frac{4}{3}}(4x^4-3)}{112}$	17
orering	$\frac{3(x^4+1)^{\frac{4}{3}}(4x^4-3)}{112}$	17
risch	$\frac{3(x^4+1)^{\frac{1}{3}}(4x^8+x^4-3)}{112}$	20
trager	$\left(\frac{3}{28}x^8+\frac{3}{112}x^4-\frac{9}{112}\right)(x^4+1)^{\frac{1}{3}}$	21

input
```
int(x^7*(x^4+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/112*(x^4+1)^(4/3)*(4*x^4-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^7 \sqrt[3]{1+x^4}\,dx = \frac{3}{112}\left(4\,x^8+x^4-3\right)\left(x^4+1\right)^{\frac{1}{3}}$$

input
```
integrate(x^7*(x^4+1)^(1/3),x, algorithm="fricas")
```

output
```
3/112*(4*x^8 + x^4 - 3)*(x^4 + 1)^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^7 \sqrt[3]{1+x^4}\,dx = \frac{3x^8\sqrt[3]{x^4+1}}{28} + \frac{3x^4\sqrt[3]{x^4+1}}{112} - \frac{9\sqrt[3]{x^4+1}}{112}$$

input
```
integrate(x**7*(x**4+1)**(1/3),x)
```

output
```
3*x**8*(x**4 + 1)**(1/3)/28 + 3*x**4*(x**4 + 1)**(1/3)/112 - 9*(x**4 + 1)*
*(1/3)/112
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^7 \sqrt[3]{1+x^4}\,dx = \frac{3}{28}\left(x^4+1\right)^{\frac{7}{3}} - \frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^7*(x^4+1)^(1/3),x, algorithm="maxima")
```

output
```
3/28*(x^4 + 1)^(7/3) - 3/16*(x^4 + 1)^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int x^7 \sqrt[3]{1+x^4}\,dx = \frac{3}{28}\left(x^4+1\right)^{\frac{7}{3}} - \frac{3}{16}\left(x^4+1\right)^{\frac{4}{3}}$$

input
```
integrate(x^7*(x^4+1)^(1/3),x, algorithm="giac")
```

output
```
3/28*(x^4 + 1)^(7/3) - 3/16*(x^4 + 1)^(4/3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int x^7 \sqrt[3]{1+x^4}\,dx = \frac{3\left(x^4+1\right)^{4/3}\left(4\,x^4-3\right)}{112}$$

input
```
int(x^7*(x^4 + 1)^(1/3),x)
```

output
```
(3*(x^4 + 1)^(4/3)*(4*x^4 - 3))/112
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int x^7 \sqrt[3]{1+x^4}\,dx = \frac{3\left(\sqrt{x^4+1}\,x^2+x^4+1\right)^{\frac{2}{3}}\left(4x^8+x^4-3\right)}{112\left(\sqrt{x^4+1}+x^2\right)^{\frac{2}{3}}}$$

input
```
int(x^7*(x^4+1)^(1/3),x)
```

output
```
(3*(sqrt(x**4 + 1)*x**2 + x**4 + 1)**(2/3)*(4*x**8 + x**4 - 3))/(112*(sqrt
(x**4 + 1) + x**2)**(2/3))
```

3.441 $\int \frac{x^3}{(1+x^4)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\sqrt[3]{1+x^4}}$$

output
```
-3/4/(x^4+1)^(1/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\sqrt[3]{1+x^4}}$$

input
```
Integrate[x^3/(1 + x^4)^(4/3),x]
```

output
```
-3/(4*(1 + x^4)^(1/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(x^4+1)^{4/3}} \, dx$$

$$\Big\downarrow \text{ 793}$$

$$-\frac{3}{4\sqrt[3]{x^4+1}}$$

input
```
Int[x^3/(1 + x^4)^(4/3),x]
```

output
```
-3/(4*(1 + x^4)^(1/3))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
derivativedivides	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
default	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
trager	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
risch	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
pseudoelliptic	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
orering	$-\frac{3}{4(x^4+1)^{\frac{1}{3}}}$	10
meijerg	$\frac{x^4 \operatorname{hypergeom}\left(\left[1,\frac{4}{3}\right],[2],-x^4\right)}{4}$	17

input
```
int(x^3/(x^4+1)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/4/(x^4+1)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\,(x^4+1)^{\frac{1}{3}}}$$

input
```
integrate(x^3/(x^4+1)^(4/3),x, algorithm="fricas")
```

output
```
-3/4/(x^4 + 1)^(1/3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\sqrt[3]{x^4+1}}$$

input
```
integrate(x**3/(x**4+1)**(4/3),x)
```

output
```
-3/(4*(x**4 + 1)**(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\,(x^4+1)^{\frac{1}{3}}}$$

input
```
integrate(x^3/(x^4+1)^(4/3),x, algorithm="maxima")
```

output
```
-3/4/(x^4 + 1)^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\,(x^4+1)^{\frac{1}{3}}}$$

input
```
integrate(x^3/(x^4+1)^(4/3),x, algorithm="giac")
```

output
```
-3/4/(x^4 + 1)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = -\frac{3}{4\,(x^4+1)^{1/3}}$$

input
```
int(x^3/(x^4 + 1)^(4/3),x)
```

output
```
-3/(4*(x^4 + 1)^(1/3))
```

Reduce [F]

$$\int \frac{x^3}{(1+x^4)^{4/3}}\,dx = \int \frac{x^3}{(x^4+1)^{\frac{1}{3}}\,x^4+(x^4+1)^{\frac{1}{3}}}dx$$

input
```
int(x^3/(x^4+1)^(4/3),x)
```

output
```
int(x**3/((x**4 + 1)**(1/3)*x***4 + (x**4 + 1)**(1/3)),x)
```

3.442 $\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx=\frac{3}{8}\left(1+x^4\right)^{2/3}$$

output
```
3/8*(x^4+1)^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx=\frac{3}{8}\left(1+x^4\right)^{2/3}$$

input
```
Integrate[x^3/(1 + x^4)^(1/3),x]
```

output
```
(3*(1 + x^4)^(2/3))/8
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{x^4+1}}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{3}{8}\left(x^4+1\right)^{2/3}$$

input
```
Int[x^3/(1 + x^4)^(1/3),x]
```

output
```
(3*(1 + x^4)^(2/3))/8
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
derivativedivides	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
default	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
trager	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
risch	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
pseudoelliptic	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
orering	$\frac{3(x^4+1)^{\frac{2}{3}}}{8}$	10
meijerg	$\frac{x^4 \operatorname{hypergeom}\left(\left[\frac{1}{3},1\right],[2],-x^4\right)}{4}$	17

input
```
int(x^3/(x^4+1)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/8*(x^4+1)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx = \frac{3}{8}\left(x^4+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^3/(x^4+1)^(1/3),x, algorithm="fricas")
```

output
```
3/8*(x^4 + 1)^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx = \frac{3(x^4+1)^{\frac{2}{3}}}{8}$$

input
```
integrate(x**3/(x**4+1)**(1/3),x)
```

output
```
3*(x**4 + 1)**(2/3)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx = \frac{3}{8}\,\left(x^4+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^3/(x^4+1)^(1/3),x, algorithm="maxima")
```

output
```
3/8*(x^4 + 1)^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx = \frac{3}{8}\,\left(x^4+1\right)^{\frac{2}{3}}$$

input
```
integrate(x^3/(x^4+1)^(1/3),x, algorithm="giac")
```

output
```
3/8*(x^4 + 1)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx=\frac{3\,{\left(x^4+1\right)}^{2/3}}{8}$$

input
```
int(x^3/(x^4 + 1)^(1/3),x)
```

output
```
(3*(x^4 + 1)^(2/3))/8
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[3]{1+x^4}}\,dx=\int \frac{x^3}{\left(x^4+1\right)^{\frac{1}{3}}}dx$$

input
```
int(x^3/(x^4+1)^(1/3),x)
```

output
```
int(x**3/(x**4 + 1)**(1/3),x)
```

3.443 $\int x^{19} \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^{19} \sqrt[4]{a+bx^4}\,dx = \frac{a^4(a+bx^4)^{5/4}}{5b^5} - \frac{4a^3(a+bx^4)^{9/4}}{9b^5} + \frac{6a^2(a+bx^4)^{13/4}}{13b^5} - \frac{4a(a+bx^4)^{17/4}}{17b^5} + \frac{(a+bx^4)^{21/4}}{21b^5}$$

output
```
1/5*a^4*(b*x^4+a)^(5/4)/b^5-4/9*a^3*(b*x^4+a)^(9/4)/b^5+6/13*a^2*(b*x^4+a)
^(13/4)/b^5-4/17*a*(b*x^4+a)^(17/4)/b^5+1/21*(b*x^4+a)^(21/4)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int x^{19} \sqrt[4]{a+bx^4}\,dx = \frac{(a+bx^4)^{5/4}(2048a^4 - 2560a^3bx^4 + 2880a^2b^2x^8 - 3120ab^3x^{12} + 3315b^4x^{16})}{69615b^5}$$

input
```
Integrate[x^19*(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(5/4)*(2048*a^4 - 2560*a^3*b*x^4 + 2880*a^2*b^2*x^8 - 3120*a*
b^3*x^12 + 3315*b^4*x^16))/(69615*b^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^{16}\sqrt[4]{bx^4+a}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{(bx^4+a)^{17/4}}{b^4}-\frac{4a(bx^4+a)^{13/4}}{b^4}+\frac{6a^2(bx^4+a)^{9/4}}{b^4}-\frac{4a^3(bx^4+a)^{5/4}}{b^4}+\frac{a^4\sqrt[4]{bx^4+a}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^4(a+bx^4)^{5/4}}{5b^5}-\frac{16a^3(a+bx^4)^{9/4}}{9b^5}+\frac{24a^2(a+bx^4)^{13/4}}{13b^5}+\frac{4(a+bx^4)^{21/4}}{21b^5}-\frac{16a(a+bx^4)^{17/4}}{17b^5}\right)$$

input
```
Int[x^19*(a + b*x^4)^(1/4),x]
```

output
```
((4*a^4*(a + b*x^4)^(5/4))/(5*b^5) - (16*a^3*(a + b*x^4)^(9/4))/(9*b^5) +
(24*a^2*(a + b*x^4)^(13/4))/(13*b^5) - (16*a*(a + b*x^4)^(17/4))/(17*b^5)
+ (4*(a + b*x^4)^(21/4))/(21*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(3315 x^{16} b^{4}-3120 a\,b^{3} x^{12}+2880 a^{2} b^{2} x^{8}-2560 a^{3} b\,x^{4}+2048 a^{4}\right)}{69615 b^{5}}$	58
pseudoelliptic	$\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(3315 x^{16} b^{4}-3120 a\,b^{3} x^{12}+2880 a^{2} b^{2} x^{8}-2560 a^{3} b\,x^{4}+2048 a^{4}\right)}{69615 b^{5}}$	58
orering	$\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(3315 x^{16} b^{4}-3120 a\,b^{3} x^{12}+2880 a^{2} b^{2} x^{8}-2560 a^{3} b\,x^{4}+2048 a^{4}\right)}{69615 b^{5}}$	58
trager	$\frac{\left(3315 b^{5} x^{20}+195 a\,b^{4} x^{16}-240 a^{2} b^{3} x^{12}+320 a^{3} b^{2} x^{8}-512 a^{4} b\,x^{4}+2048 a^{5}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{69615 b^{5}}$	69
risch	$\frac{\left(3315 b^{5} x^{20}+195 a\,b^{4} x^{16}-240 a^{2} b^{3} x^{12}+320 a^{3} b^{2} x^{8}-512 a^{4} b\,x^{4}+2048 a^{5}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{69615 b^{5}}$	69

input
```
int(x^19*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/69615*(b*x^4+a)^(5/4)*(3315*b^4*x^16-3120*a*b^3*x^12+2880*a^2*b^2*x^8-25
60*a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx$$
$$=\frac{(3315\,b^5x^{20}+195\,ab^4x^{16}-240\,a^2b^3x^{12}+320\,a^3b^2x^8-512\,a^4bx^4+2048\,a^5)(bx^4+a)^{\frac{1}{4}}}{69615\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/69615*(3315*b^5*x^20 + 195*a*b^4*x^16 - 240*a^2*b^3*x^12 + 320*a^3*b^2*x
^8 - 512*a^4*b*x^4 + 2048*a^5)*(b*x^4 + a)^(1/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.33

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx$$
$$=\begin{cases}\frac{2048a^5\sqrt[4]{a+bx^4}}{69615b^5}-\frac{512a^4x^4\sqrt[4]{a+bx^4}}{69615b^4}+\frac{64a^3x^8\sqrt[4]{a+bx^4}}{13923b^3}-\frac{16a^2x^{12}\sqrt[4]{a+bx^4}}{4641b^2}+\frac{ax^{16}\sqrt[4]{a+bx^4}}{357b}+\frac{x^{20}\sqrt[4]{a+b}}{21} \\ \frac{\sqrt[4]{a}x^{20}}{20}\end{cases}$$

input
```
integrate(x**19*(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((2048*a**5*(a + b*x**4)**(1/4)/(69615*b**5) - 512*a**4*x**4*(a +
 b*x**4)**(1/4)/(69615*b**4) + 64*a**3*x**8*(a + b*x**4)**(1/4)/(13923*b**
3) - 16*a**2*x**12*(a + b*x**4)**(1/4)/(4641*b**2) + a*x**16*(a + b*x**4)*
*(1/4)/(357*b) + x**20*(a + b*x**4)**(1/4)/21, Ne(b, 0)), (a**(1/4)*x**20/
20, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{21}{4}}}{21\,b^5} - \frac{4\,(bx^4+a)^{\frac{17}{4}}a}{17\,b^5} + \frac{6\,(bx^4+a)^{\frac{13}{4}}a^2}{13\,b^5} - \frac{4\,(bx^4+a)^{\frac{9}{4}}a^3}{9\,b^5} + \frac{(bx^4+a)^{\frac{5}{4}}a^4}{5\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/21*(b*x^4 + a)^(21/4)/b^5 - 4/17*(b*x^4 + a)^(17/4)*a/b^5 + 6/13*(b*x^4
+ a)^(13/4)*a^2/b^5 - 4/9*(b*x^4 + a)^(9/4)*a^3/b^5 + 1/5*(b*x^4 + a)^(5/4
)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx$$
$$= \frac{3315\,(bx^4+a)^{\frac{21}{4}} - 16380\,(bx^4+a)^{\frac{17}{4}}a + 32130\,(bx^4+a)^{\frac{13}{4}}a^2 - 30940\,(bx^4+a)^{\frac{9}{4}}a^3 + 13923\,(bx^4+a)^{\frac{5}{4}}a}{69615\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/69615*(3315*(b*x^4 + a)^(21/4) - 16380*(b*x^4 + a)^(17/4)*a + 32130*(b*x
^4 + a)^(13/4)*a^2 - 30940*(b*x^4 + a)^(9/4)*a^3 + 13923*(b*x^4 + a)^(5/4)
*a^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{x^{20}}{21}+\frac{2048\,a^5}{69615\,b^5}+\frac{a\,x^{16}}{357\,b}-\frac{512\,a^4\,x^4}{69615\,b^4}+\frac{64\,a^3\,x^8}{13923\,b^3}-\frac{16\,a^2\,x^{12}}{4641\,b^2}\right)$$

input
```
int(x^19*(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(1/4)*(x^20/21 + (2048*a^5)/(69615*b^5) + (a*x^16)/(357*b) - (
512*a^4*x^4)/(69615*b^4) + (64*a^3*x^8)/(13923*b^3) - (16*a^2*x^12)/(4641*
b^2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int x^{19}\sqrt[4]{a+bx^4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(3315b^5x^{20}+195a\,b^4x^{16}-240a^2b^3x^{12}+320a^3b^2x^8-512a^4b\,x^4+2048a^5)}{69615\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^5}$$

input
```
int(x^19*(b*x^4+a)^(1/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(2048*a**5 - 512*a**4*b*
x**4 + 320*a**3*b**2*x**8 - 240*a**2*b**3*x**12 + 195*a*b**4*x**16 + 3315*
b**5*x**20))/(69615*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**5)
```

3.444 $\int x^{15}\sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx=-\frac{a^3(a+bx^4)^{5/4}}{5b^4}+\frac{a^2(a+bx^4)^{9/4}}{3b^4}-\frac{3a(a+bx^4)^{13/4}}{13b^4}+\frac{(a+bx^4)^{17/4}}{17b^4}$$

output
```
-1/5*a^3*(b*x^4+a)^(5/4)/b^4+1/3*a^2*(b*x^4+a)^(9/4)/b^4-3/13*a*(b*x^4+a)^
(13/4)/b^4+1/17*(b*x^4+a)^(17/4)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx=\frac{\sqrt[4]{a+bx^4}(-128a^4+32a^3bx^4-20a^2b^2x^8+15ab^3x^{12}+195b^4x^{16})}{3315b^4}$$

input
```
Integrate[x^15*(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(1/4)*(-128*a^4 + 32*a^3*b*x^4 - 20*a^2*b^2*x^8 + 15*a*b^3*x^
12 + 195*b^4*x^16))/(3315*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15} \sqrt[4]{a+bx^4} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int x^{12} \sqrt[4]{bx^4+a} dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4} \int \left(\frac{(bx^4+a)^{13/4}}{b^3} - \frac{3a(bx^4+a)^{9/4}}{b^3} + \frac{3a^2(bx^4+a)^{5/4}}{b^3} - \frac{a^3\sqrt[4]{bx^4+a}}{b^3}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^3(a+bx^4)^{5/4}}{5b^4} + \frac{4a^2(a+bx^4)^{9/4}}{3b^4} + \frac{4(a+bx^4)^{17/4}}{17b^4} - \frac{12a(a+bx^4)^{13/4}}{13b^4}\right)$$

input
```
Int[x^15*(a + b*x^4)^(1/4),x]
```

output
```
((-4*a^3*(a + b*x^4)^(5/4))/(5*b^4) + (4*a^2*(a + b*x^4)^(9/4))/(3*b^4) -
(12*a*(a + b*x^4)^(13/4))/(13*b^4) + (4*(a + b*x^4)^(17/4))/(17*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(b x^4+a)^{\frac{5}{4}} (-195 b^3 x^{12}+180 a b^2 x^8-160 a^2 b x^4+128 a^3)}{3315 b^4}$	47
pseudoelliptic	$-\frac{(b x^4+a)^{\frac{5}{4}} (-195 b^3 x^{12}+180 a b^2 x^8-160 a^2 b x^4+128 a^3)}{3315 b^4}$	47
orering	$-\frac{(b x^4+a)^{\frac{5}{4}} (-195 b^3 x^{12}+180 a b^2 x^8-160 a^2 b x^4+128 a^3)}{3315 b^4}$	47
trager	$-\frac{(-195 x^{16} b^4-15 a b^3 x^{12}+20 a^2 b^2 x^8-32 a^3 b x^4+128 a^4) (b x^4+a)^{\frac{1}{4}}}{3315 b^4}$	58
risch	$-\frac{(-195 x^{16} b^4-15 a b^3 x^{12}+20 a^2 b^2 x^8-32 a^3 b x^4+128 a^4) (b x^4+a)^{\frac{1}{4}}}{3315 b^4}$	58

input
```
int(x^15*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3315*(b*x^4+a)^(5/4)*(-195*b^3*x^12+180*a*b^2*x^8-160*a^2*b*x^4+128*a^3
)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{15} \sqrt[4]{a+bx^4}\,dx = \frac{(195\,b^4x^{16}+15\,ab^3x^{12}-20\,a^2b^2x^8+32\,a^3bx^4-128\,a^4)(bx^4+a)^{\frac{1}{4}}}{3315\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/3315*(195*b^4*x^16 + 15*a*b^3*x^12 - 20*a^2*b^2*x^8 + 32*a^3*b*x^4 - 128
*a^4)*(b*x^4 + a)^(1/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx$$

$$= \begin{cases} -\frac{128a^4\sqrt[4]{a+bx^4}}{3315b^4} + \frac{32a^3x^4\sqrt[4]{a+bx^4}}{3315b^3} - \frac{4a^2x^8\sqrt[4]{a+bx^4}}{663b^2} + \frac{ax^{12}\sqrt[4]{a+bx^4}}{221b} + \frac{x^{16}\sqrt[4]{a+bx^4}}{17} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^{16}}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15*(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-128*a**4*(a + b*x**4)**(1/4)/(3315*b**4) + 32*a**3*x**4*(a + b
*x**4)**(1/4)/(3315*b**3) - 4*a**2*x**8*(a + b*x**4)**(1/4)/(663*b**2) + a
*x**12*(a + b*x**4)**(1/4)/(221*b) + x**16*(a + b*x**4)**(1/4)/17, Ne(b, 0
)), (a**(1/4)*x**16/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{17}{4}}}{17\,b^4} - \frac{3\,(bx^4+a)^{\frac{13}{4}}a}{13\,b^4} + \frac{(bx^4+a)^{\frac{9}{4}}a^2}{3\,b^4} - \frac{(bx^4+a)^{\frac{5}{4}}a^3}{5\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/17*(b*x^4 + a)^(17/4)/b^4 - 3/13*(b*x^4 + a)^(13/4)*a/b^4 + 1/3*(b*x^4 +
 a)^(9/4)*a^2/b^4 - 1/5*(b*x^4 + a)^(5/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx = \frac{195\,(bx^4+a)^{\frac{17}{4}} - 765\,(bx^4+a)^{\frac{13}{4}}a + 1105\,(bx^4+a)^{\frac{9}{4}}a^2 - 663\,(bx^4+a)^{\frac{5}{4}}a^3}{3315\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/3315*(195*(b*x^4 + a)^(17/4) - 765*(b*x^4 + a)^(13/4)*a + 1105*(b*x^4 +
a)^(9/4)*a^2 - 663*(b*x^4 + a)^(5/4)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{x^{16}}{17} - \frac{128\,a^4}{3315\,b^4} + \frac{a\,x^{12}}{221\,b} + \frac{32\,a^3\,x^4}{3315\,b^3} - \frac{4\,a^2\,x^8}{663\,b^2}\right)$$

input
```
int(x^15*(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(1/4)*(x^16/17 - (128*a^4)/(3315*b^4) + (a*x^12)/(221*b) + (32
*a^3*x^4)/(3315*b^3) - (4*a^2*x^8)/(663*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int x^{15}\sqrt[4]{a+bx^4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(195b^4x^{16}+15a\,b^3x^{12}-20a^2b^2x^8+32a^3b\,x^4-128a^4)}{3315\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^4}$$

input
```
int(x^15*(b*x^4+a)^(1/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*( - 128*a**4 + 32*a**3*b
*x**4 - 20*a**2*b**2*x**8 + 15*a*b**3*x**12 + 195*b**4*x**16))/(3315*sqrt(
sqrt(a + b*x**4) + sqrt(b)*x**2)*b**4)
```

3.445 $\int x^{11}\sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx=\frac{a^2(a+bx^4)^{5/4}}{5b^3}-\frac{2a(a+bx^4)^{9/4}}{9b^3}+\frac{(a+bx^4)^{13/4}}{13b^3}$$

output
```
1/5*a^2*(b*x^4+a)^(5/4)/b^3-2/9*a*(b*x^4+a)^(9/4)/b^3+1/13*(b*x^4+a)^(13/4
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx=\frac{(a+bx^4)^{5/4}(32a^2-40abx^4+45b^2x^8)}{585b^3}$$

input
```
Integrate[x^11*(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(5/4)*(32*a^2 - 40*a*b*x^4 + 45*b^2*x^8))/(585*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \sqrt[4]{a+bx^4} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int x^8 \sqrt[4]{bx^4+a} dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4} \int \left(\frac{\left(bx^4+a\right)^{9/4}}{b^2} - \frac{2a\left(bx^4+a\right)^{5/4}}{b^2} + \frac{a^2 \sqrt[4]{bx^4+a}}{b^2}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{4a^2\left(a+bx^4\right)^{5/4}}{5b^3} + \frac{4\left(a+bx^4\right)^{13/4}}{13b^3} - \frac{8a\left(a+bx^4\right)^{9/4}}{9b^3}\right)$$

input
```
Int[x^11*(a + b*x^4)^(1/4),x]
```

output
```
((4*a^2*(a + b*x^4)^(5/4))/(5*b^3) - (8*a*(a + b*x^4)^(9/4))/(9*b^3) + (4*
(a + b*x^4)^(13/4))/(13*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{5}{4}}(45b^2x^8-40ab\,x^4+32a^2)}{585b^3}$	36
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{5}{4}}(45b^2x^8-40ab\,x^4+32a^2)}{585b^3}$	36
orering	$\frac{(b\,x^4+a)^{\frac{5}{4}}(45b^2x^8-40ab\,x^4+32a^2)}{585b^3}$	36
trager	$\frac{(45b^3x^{12}+5a\,b^2x^8-8a^2b\,x^4+32a^3)(b\,x^4+a)^{\frac{1}{4}}}{585b^3}$	47
risch	$\frac{(45b^3x^{12}+5a\,b^2x^8-8a^2b\,x^4+32a^3)(b\,x^4+a)^{\frac{1}{4}}}{585b^3}$	47

input
```
int(x^11*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/585*(b*x^4+a)^(5/4)*(45*b^2*x^8-40*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx = \frac{(45\,b^3x^{12}+5\,ab^2x^8-8\,a^2bx^4+32\,a^3)(bx^4+a)^{\frac{1}{4}}}{585\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/585*(45*b^3*x^12 + 5*a*b^2*x^8 - 8*a^2*b*x^4 + 32*a^3)*(b*x^4 + a)^(1/4)
/b^3
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx = \begin{cases} \frac{32a^3\sqrt[4]{a+bx^4}}{585b^3} - \frac{8a^2x^4\sqrt[4]{a+bx^4}}{585b^2} + \frac{ax^8\sqrt[4]{a+bx^4}}{117b} + \frac{x^{12}\sqrt[4]{a+bx^4}}{13} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((32*a**3*(a + b*x**4)**(1/4)/(585*b**3) - 8*a**2*x**4*(a + b*x**
4)**(1/4)/(585*b**2) + a*x**8*(a + b*x**4)**(1/4)/(117*b) + x**12*(a + b*x
**4)**(1/4)/13, Ne(b, 0)), (a**(1/4)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{13}{4}}}{13\,b^3} - \frac{2\,(bx^4+a)^{\frac{9}{4}}a}{9\,b^3} + \frac{(bx^4+a)^{\frac{5}{4}}a^2}{5\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/13*(b*x^4 + a)^(13/4)/b^3 - 2/9*(b*x^4 + a)^(9/4)*a/b^3 + 1/5*(b*x^4 + a
)^(5/4)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx = \frac{45\,(bx^4+a)^{\frac{13}{4}} - 130\,(bx^4+a)^{\frac{9}{4}}a + 117\,(bx^4+a)^{\frac{5}{4}}a^2}{585\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/585*(45*(b*x^4 + a)^(13/4) - 130*(b*x^4 + a)^(9/4)*a + 117*(b*x^4 + a)^(
5/4)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{x^{12}}{13}+\frac{32\,a^3}{585\,b^3}+\frac{a\,x^8}{117\,b}-\frac{8\,a^2\,x^4}{585\,b^2}\right)$$

input
```
int(x^11*(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(1/4)*(x^12/13 + (32*a^3)/(585*b^3) + (a*x^8)/(117*b) - (8*a^2
*x^4)/(585*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int x^{11}\sqrt[4]{a+bx^4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(45b^3x^{12}+5a\,b^2x^8-8a^2b\,x^4+32a^3)}{585\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^3}$$

input
```
int(x^11*(b*x^4+a)^(1/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(32*a**3 - 8*a**2*b*x**4
 + 5*a*b**2*x**8 + 45*b**3*x**12))/(585*sqrt(sqrt(a + b*x**4) + sqrt(b)*x*
*2)*b**3)
```

3.446 $\int x^7 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = -\frac{a(a+bx^4)^{5/4}}{5b^2} + \frac{(a+bx^4)^{9/4}}{9b^2}$$

output
```
-1/5*a*(b*x^4+a)^(5/4)/b^2+1/9*(b*x^4+a)^(9/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a+bx^4}(-4a^2+abx^4+5b^2x^8)}{45b^2}$$

input
```
Integrate[x^7*(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(1/4)*(-4*a^2 + a*b*x^4 + 5*b^2*x^8))/(45*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int x^4 \sqrt[4]{bx^4+a}dx^4$$

$$\Big\downarrow \; 53$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{5/4}}{b}-\frac{a\sqrt[4]{bx^4+a}}{b}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{4\left(a+bx^4\right)^{9/4}}{9b^2}-\frac{4a\left(a+bx^4\right)^{5/4}}{5b^2}\right)$$

input
```
Int[x^7*(a + b*x^4)^(1/4),x]
```

output
```
((-4*a*(a + b*x^4)^(5/4))/(5*b^2) + (4*(a + b*x^4)^(9/4))/(9*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}\left(-5b\,x^{4}+4a\right)}{45b^{2}}$	25
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}\left(-5b\,x^{4}+4a\right)}{45b^{2}}$	25
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}}\left(-5b\,x^{4}+4a\right)}{45b^{2}}$	25
trager	$-\frac{\left(-5b^{2}x^{8}-ab\,x^{4}+4a^{2}\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{45b^{2}}$	36
risch	$-\frac{\left(-5b^{2}x^{8}-ab\,x^{4}+4a^{2}\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{45b^{2}}$	36

input
```
int(x^7*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(b*x^4+a)^(5/4)*(-5*b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^7\sqrt[4]{a+bx^4}\,dx=\frac{(5\,b^2x^8+abx^4-4\,a^2)(bx^4+a)^{\frac{1}{4}}}{45\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/45*(5*b^2*x^8 + a*b*x^4 - 4*a^2)*(b*x^4 + a)^(1/4)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = \begin{cases} -\frac{4a^2\sqrt[4]{a+bx^4}}{45b^2} + \frac{ax^4\sqrt[4]{a+bx^4}}{45b} + \frac{x^8\sqrt[4]{a+bx^4}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-4*a**2*(a + b*x**4)**(1/4)/(45*b**2) + a*x**4*(a + b*x**4)**(1
/4)/(45*b) + x**8*(a + b*x**4)**(1/4)/9, Ne(b, 0)), (a**(1/4)*x**8/8, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{9}{4}}}{9\,b^2} - \frac{(bx^4+a)^{\frac{5}{4}}a}{5\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/9*(b*x^4 + a)^(9/4)/b^2 - 1/5*(b*x^4 + a)^(5/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = \frac{5\,(bx^4+a)^{\frac{9}{4}} - 9\,(bx^4+a)^{\frac{5}{4}}a}{45\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/45*(5*(b*x^4 + a)^(9/4) - 9*(b*x^4 + a)^(5/4)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{x^8}{9} - \frac{4\,a^2}{45\,b^2} + \frac{a\,x^4}{45\,b}\right)$$

input
```
int(x^7*(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(1/4)*(x^8/9 - (4*a^2)/(45*b^2) + (a*x^4)/(45*b))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int x^7 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(5b^2x^8+ab\,x^4-4a^2)}{45\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^2}$$

input
```
int(x^7*(b*x^4+a)^(1/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*( - 4*a**2 + a*b*x**4 +
5*b**2*x**8))/(45*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**2)
```

3.447 $\int x^3 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{(a+bx^4)^{5/4}}{5b}$$

output
```
1/5*(b*x^4+a)^(5/4)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{(a+bx^4)^{5/4}}{5b}$$

input
```
Integrate[x^3*(a + b*x^4)^(1/4),x]
```

output
```
(a + b*x^4)^(5/4)/(5*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^4\right)^{5/4}}{5b}$$

input
```
Int[x^3*(a + b*x^4)^(1/4),x]
```

output
```
(a + b*x^4)^(5/4)/(5*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15
derivativedivides	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15
default	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15
trager	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15
risch	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15
pseudoelliptic	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15
orering	$\frac{(b\,x^{4}+a)^{\frac{5}{4}}}{5b}$	15

input
```
int(x^3*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/5*(b*x^4+a)^(5/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{5}{4}}}{5\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/5*(b*x^4 + a)^(5/4)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \begin{cases} \frac{a\sqrt[4]{a+bx^4}}{5b} + \frac{x^4\sqrt[4]{a+bx^4}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((a*(a + b*x**4)**(1/4)/(5*b) + x**4*(a + b*x**4)**(1/4)/5, Ne(b,
 0)), (a**(1/4)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{5}{4}}}{5\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/5*(b*x^4 + a)^(5/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{(bx^4+a)^{\frac{5}{4}}}{5\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/5*(b*x^4 + a)^(5/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{(b\,x^4+a)^{5/4}}{5\,b}$$

input
```
int(x^3*(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(5/4)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.89

$$\int x^3 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(b\,x^4+a)}{5\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b}$$

input
```
int(x^3*(b*x^4+a)^(1/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(a + b*x**4))/(5*sqrt(sq
rt(a + b*x**4) + sqrt(b)*x**2)*b)
```

3.448 $\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx=\sqrt[4]{a+bx^4}-\frac{1}{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

output
```
(b*x^4+a)^(1/4)-1/2*a^(1/4)*arctan((b*x^4+a)^(1/4)/a^(1/4))-1/2*a^(1/4)*ar
ctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx=\sqrt[4]{a+bx^4}-\frac{1}{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

input
```
Integrate[(a + b*x^4)^(1/4)/x,x]
```

output
```
(a + b*x^4)^(1/4) - (a^(1/4)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/2 - (a^(1/
4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {798, 60, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{\sqrt[4]{bx^4+a}}{x^4}dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4} \left(a \int \frac{1}{x^4 (bx^4+a)^{3/4}}dx^4 + 4\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(\frac{4a \int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{b} + 4\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4} \left(\frac{4a \left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}} - \frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{b} + 4\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{4a\left(-\frac{b\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)$$

$\downarrow$ 219

$$\frac{1}{4}\left(\frac{4a\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)$$

input

```
Int[(a + b*x^4)^(1/4)/x,x]
```

output

```
(4*(a + b*x^4)^(1/4) + (4*a*(-1/2*(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/a^
(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/b)/4
```

Defintions of rubi rules used

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\frac{\ln\left(\frac{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)a^{\frac{1}{4}}}{4}-\frac{a^{\frac{1}{4}}\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2}$	71

input
```
int((b*x^4+a)^(1/4)/x,x,method=_RETURNVERBOSE)
```

output
```
(b*x^4+a)^(1/4)-1/4*ln((-(b*x^4+a)^(1/4)-a^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4
)))*a^(1/4)-1/2*a^(1/4)*arctan((b*x^4+a)^(1/4)/a^(1/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx = -\frac{1}{4}\,a^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}\right)-\frac{1}{4}i\,a^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}+i\,a^{\frac{1}{4}}\right)+\frac{1}{4}i\,a^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}-i\,a^{\frac{1}{4}}\right)+\frac{1}{4}\,a^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}\right)+\left(bx^4+a\right)^{\frac{1}{4}}$$

input
```
integrate((b*x^4+a)^(1/4)/x,x, algorithm="fricas")
```

output
```
-1/4*a^(1/4)*log((b*x^4 + a)^(1/4) + a^(1/4)) - 1/4*I*a^(1/4)*log((b*x^4 +
 a)^(1/4) + I*a^(1/4)) + 1/4*I*a^(1/4)*log((b*x^4 + a)^(1/4) - I*a^(1/4))
+ 1/4*a^(1/4)*log((b*x^4 + a)^(1/4) - a^(1/4)) + (b*x^4 + a)^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx = -\frac{\sqrt[4]{b}x\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x,x)
```

output
```
-b**(1/4)*x*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), a*exp_polar(I*pi)/(b*x
**4))/(4*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx = -\frac{1}{2}\,a^{\frac{1}{4}}\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) + \frac{1}{4}\,a^{\frac{1}{4}}\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) + \left(bx^4+a\right)^{\frac{1}{4}}$$

input
```
integrate((b*x^4+a)^(1/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(1/4)*arctan((b*x^4 + a)^(1/4)/a^(1/4)) + 1/4*a^(1/4)*log(((b*x^4 +
 a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4))) + (b*x^4 + a)^(1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(48) = 96$.

Time = 0.12 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{x}\,dx = &-\frac{1}{4}\,\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)\\ &-\frac{1}{4}\,\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)\\ &-\frac{1}{8}\,\sqrt{2}(-a)^{\frac{1}{4}}\log\left(\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)\\ &+\frac{1}{8}\,\sqrt{2}(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)\\ &+\left(bx^4+a\right)^{\frac{1}{4}}\end{aligned}$$

input
```
integrate((b*x^4+a)^(1/4)/x,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
+ a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) - 1/8*sqrt(2)*(-a)^(1/4)
*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)) +
1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*
x^4 + a) + sqrt(-a)) + (b*x^4 + a)^(1/4)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt[4]{a+bx^4}}{x} \, dx = \left(b\,x^4+a\right)^{1/4} - \frac{a^{1/4}\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2} - \frac{a^{1/4}\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2}$$

input
```
int((a + b*x^4)^(1/4)/x,x)
```

output
```
(a + b*x^4)^(1/4) - (a^(1/4)*atanh((a + b*x^4)^(1/4)/a^(1/4)))/2 - (a^(1/4
)*atan((a + b*x^4)^(1/4)/a^(1/4)))/2
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x} \, dx = \left(b\,x^4+a\right)^{\frac{1}{4}} + \left(\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b\,x^5+ax}dx\right) a$$

input
```
int((b*x^4+a)^(1/4)/x,x)
```

output
```
(a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x + b*x**5),x)*a
```

3.449 $\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx=-\frac{\sqrt[4]{a+bx^4}}{4x^4}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}$$

output
```
-1/4*(b*x^4+a)^(1/4)/x^4-1/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/8
*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx=-\frac{\sqrt[4]{a+bx^4}}{4x^4}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^5,x]
```

output
```
-1/4*(a + b*x^4)^(1/4)/x^4 - (b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(3
/4)) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(3/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 51, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{\sqrt[4]{bx^4+a}}{x^8}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{1}{4}b\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^4-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)$$

input
```
Int[(a + b*x^4)^(1/4)/x^5,x]
```

output
```
(-((a + b*x^4)^(1/4)/x^4) - (b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/
4)) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{b\left(-\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{x^{4} b}-\frac{\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)}{4a^{\frac{3}{4}}}-\frac{\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2a^{\frac{3}{4}}}\right)}{4}$	78

input
```
int((b*x^4+a)^(1/4)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*(-(b*x^4+a)^(1/4)/x^4/b-1/4*ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^
(1/4)-a^(1/4)))/a^(3/4)-1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx = -\frac{\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}} x^4 \log\left((bx^4+a)^{\frac{1}{4}} b + a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\right) + i\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}} x^4 \log\left((bx^4+a)^{\frac{1}{4}} b + i\,a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\right) - i\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}} x^4 \log\left(\right.}{16\,x^4}$$

input
```
integrate((b*x^4+a)^(1/4)/x^5,x, algorithm="fricas")
```

output
```
-1/16*((b^4/a^3)^(1/4)*x^4*log((b*x^4 + a)^(1/4)*b + a*(b^4/a^3)^(1/4)) +
I*(b^4/a^3)^(1/4)*x^4*log((b*x^4 + a)^(1/4)*b + I*a*(b^4/a^3)^(1/4)) - I*(
b^4/a^3)^(1/4)*x^4*log((b*x^4 + a)^(1/4)*b - I*a*(b^4/a^3)^(1/4)) - (b^4/a
^3)^(1/4)*x^4*log((b*x^4 + a)^(1/4)*b - a*(b^4/a^3)^(1/4)) + 4*(b*x^4 + a)
^(1/4))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx = -\frac{\sqrt[4]{b}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**5,x)
```

output
```
-b**(1/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4)
)/(4*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx = -\frac{b\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{8\,a^{\frac{3}{4}}}+\frac{b\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{16\,a^{\frac{3}{4}}}-\frac{(bx^4+a)^{\frac{1}{4}}}{4\,x^4}$$

input
```
integrate((b*x^4+a)^(1/4)/x^5,x, algorithm="maxima")
```

output
```
-1/8*b*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 1/16*b*log(((b*x^4 + a)
^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4) - 1/4*(b*x^4 + a)
^(1/4)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(55) = 110$.

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.73

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx = -\frac{1}{32}\,b\left(\frac{2\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a}+\frac{2\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a}\right.$$

input
```
integrate((b*x^4+a)^(1/4)/x^5,x, algorithm="giac")
```

output
```
-1/32*b*(2*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(
b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 2*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a + sqrt(2)*(-a)^(
1/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)
)/a - sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(
b*x^4 + a) + sqrt(-a))/a + 8*(b*x^4 + a)^(1/4)/(b*x^4))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx = -\frac{(b\,x^4+a)^{1/4}}{4\,x^4}-\frac{b\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}-\frac{b\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}$$

input
```
int((a + b*x^4)^(1/4)/x^5,x)
```

output
```
- (a + b*x^4)^(1/4)/(4*x^4) - (b*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/
4)) - (b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4))
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^5}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}}+\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^5+ax}dx\right)b\,x^4}{4x^4}$$

input
```
int((b*x^4+a)^(1/4)/x^5,x)
```

output
```
( - (a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x + b*x**5),x)*b*x**4
)/(4*x**4)
```

3.450 $\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = -\frac{\sqrt[4]{a+bx^4}}{8x^8} - \frac{b\sqrt[4]{a+bx^4}}{32ax^4} + \frac{3b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}} + \frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}$$

output
```
-1/8*(b*x^4+a)^(1/4)/x^8-1/32*b*(b*x^4+a)^(1/4)/a/x^4+3/64*b^2*arctan((b*x
^4+a)^(1/4)/a^(1/4))/a^(7/4)+3/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(
7/4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = \frac{(-4a-bx^4)\sqrt[4]{a+bx^4}}{32ax^8} + \frac{3b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}} + \frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^9,x]
```

output
```
((-4*a - b*x^4)*(a + b*x^4)^(1/4))/(32*a*x^8) + (3*b^2*ArcTan[(a + b*x^4)^
(1/4)/a^(1/4)])/(64*a^(7/4)) + (3*b^2*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/
(64*a^(7/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 51, 52, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{\sqrt[4]{bx^4+a}}{x^{12}}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{1}{8}b\int \frac{1}{x^8\left(bx^4+a\right)^{3/4}}dx^4-\frac{\sqrt[4]{a+bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{1}{8}b\left(-\frac{3b\int \frac{1}{x^4(bx^4+a)^{3/4}}dx^4}{4a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a+bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{1}{8}b\left(-\frac{3\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a+bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(\frac{1}{8}b\left(-\frac{3\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a+bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{1}{8}b\left(-\frac{3\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a+bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{1}{8}b\left(-\frac{3\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a+bx^4}}{2x^8}\right)$$

input
```
Int[(a + b*x^4)^(1/4)/x^9,x]
```

output
```
(-1/2*(a + b*x^4)^(1/4)/x^8 + (b*(-((a + b*x^4)^(1/4)/(a*x^4)) - (3*(-1/2*
(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/
4)/a^(1/4)])/(2*a^(3/4))))/a))/8)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

method	result	size
pseudoelliptic	$\frac{3\ln\left(\frac{(b\,x^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}{(b\,x^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)b^2x^8+6\arctan\left(\frac{(b\,x^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b^2x^8-4b\,x^4(b\,x^4+a)^{\frac{1}{4}}a^{\frac{3}{4}}-16a^{\frac{7}{4}}(b\,x^4+a)^{\frac{1}{4}}}{128a^{\frac{7}{4}}x^8}$	104

input
```
int((b*x^4+a)^(1/4)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/128*(3*ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*b^2*x^8+6
*arctan((b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8-4*b*x^4*(b*x^4+a)^(1/4)*a^(3/4)-1
6*a^(7/4)*(b*x^4+a)^(1/4))/a^(7/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = \frac{3\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\log\left(3\,(bx^4+a)^{\frac{1}{4}}b^2+3\,a^2\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\right)+3i\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\log\left(3\,(bx^4+a)^{\frac{1}{4}}b^2+3i\,a^2\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\right)-3i\,a}{}$$

input
```
integrate((b*x^4+a)^(1/4)/x^9,x, algorithm="fricas")
```

output
```
1/128*(3*a*(b^8/a^7)^(1/4)*x^8*log(3*(b*x^4 + a)^(1/4)*b^2 + 3*a^2*(b^8/a^
7)^(1/4)) + 3*I*a*(b^8/a^7)^(1/4)*x^8*log(3*(b*x^4 + a)^(1/4)*b^2 + 3*I*a^
2*(b^8/a^7)^(1/4)) - 3*I*a*(b^8/a^7)^(1/4)*x^8*log(3*(b*x^4 + a)^(1/4)*b^2
 - 3*I*a^2*(b^8/a^7)^(1/4)) - 3*a*(b^8/a^7)^(1/4)*x^8*log(3*(b*x^4 + a)^(1
/4)*b^2 - 3*a^2*(b^8/a^7)^(1/4)) - 4*(b*x^4 + 4*a)*(b*x^4 + a)^(1/4))/(a*x
^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = -\frac{\sqrt[4]{b}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4x^7\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**9,x)
```

output
```
-b**(1/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), a*exp_polar(I*pi)/(b*x**4
))/(4*x**7*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = -\frac{(bx^4+a)^{\frac{5}{4}}b^2+3\,(bx^4+a)^{\frac{1}{4}}ab^2}{32\left((bx^4+a)^2a-2\,(bx^4+a)a^2+a^3\right)} + \frac{3\left(\frac{2\,b^2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b^2\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a}$$

input
```
integrate((b*x^4+a)^(1/4)/x^9,x, algorithm="maxima")
```

output
```
-1/32*((b*x^4 + a)^(5/4)*b^2 + 3*(b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 + a)^2*a
 - 2*(b*x^4 + a)*a^2 + a^3) + 3/128*(2*b^2*arctan((b*x^4 + a)^(1/4)/a^(1/4
))/a^(3/4) - b^2*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^
(1/4)))/a^(3/4))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = \frac{\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^3\log\left(\sqrt{2}(bx\right.}{}}{256\,b}$$

input
```
integrate((b*x^4+a)^(1/4)/x^9,x, algorithm="giac")
```

output
```
1/256*(6*sqrt(2)*(-a)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^3*arctan(-1/2
*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sq
rt(2)*(-a)^(1/4)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4
 + a) + sqrt(-a))/a^2 + 3*sqrt(2)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^
(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) - 8*((b*x^4 + a)^(5/4)*
b^3 + 3*(b*x^4 + a)^(1/4)*a*b^3)/(a*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = \frac{3\,b^2\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{64\,a^{7/4}} - \frac{3\,{\left(b\,x^4+a\right)}^{1/4}}{32\,x^8} - \frac{{\left(b\,x^4+a\right)}^{5/4}}{32\,a\,x^8} - \frac{b^2\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}\,1{}\mathrm{i}}{a^{1/4}}\right)\,3{}\mathrm{i}}{64\,a^{7/4}}$$

input
```
int((a + b*x^4)^(1/4)/x^9,x)
```

output
```
(3*b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(7/4)) - (3*(a + b*x^4)^(1/4
))/(32*x^8) - (b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*3i)/(64*a^(7/4)) -
 (a + b*x^4)^(5/4)/(32*a*x^8)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^9}\,dx = \frac{-4(b\,x^4+a)^{\frac{1}{4}}\,a-(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4-3\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^5+ax}dx\right)b^2x^8}{32a\,x^8}$$

input
```
int((b*x^4+a)^(1/4)/x^9,x)
```

output
```
( - 4*(a + b*x**4)**(1/4)*a - (a + b*x**4)**(1/4)*b*x**4 - 3*int((a + b*x*
*4)**(1/4)/(a*x + b*x**5),x)*b**2*x**8)/(32*a*x**8)
```

3.451 $\int x^9 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = -\frac{2a^2x^2\sqrt[4]{a+bx^4}}{77b^2}+\frac{ax^6\sqrt[4]{a+bx^4}}{77b}+\frac{1}{11}x^{10}\sqrt[4]{a+bx^4}+\frac{4a^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a+bx^4\right)^{3/4}}$$

output

```
-2/77*a^2*x^2*(b*x^4+a)^(1/4)/b^2+1/77*a*x^6*(b*x^4+a)^(1/4)/b+1/11*x^10*(
b*x^4+a)^(1/4)+4/77*a^(7/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b
^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = \frac{x^2\sqrt[4]{a+bx^4}\left(\sqrt[4]{1+\frac{bx^4}{a}}\left(-6a^2+abx^4+7b^2x^8\right)+6a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{77b^2\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x^9*(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(a + b*x^4)^(1/4)*((1 + (b*x^4)/a)^(1/4)*(-6*a^2 + a*b*x^4 + 7*b^2*x^
8) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^4)/a)]))/(77*b^2*(1 +
(b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 248, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \sqrt[4]{a+bx^4} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^8 \sqrt[4]{bx^4+a}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{1}{11}a\int \frac{x^8}{(bx^4+a)^{3/4}}dx^2+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3b}\right)}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b(a+bx^4)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)$$

input
```
Int[x^9*(a + b*x^4)^(1/4),x]
```

output
```
((2*x^10*(a + b*x^4)^(1/4))/11 + (a*((2*x^6*(a + b*x^4)^(1/4))/(7*b) - (6*
a*((2*x^2*(a + b*x^4)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*Elli
pticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))))
/(7*b)))/11)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^{9} \left(b\,x^{4}+a\right)^{\frac{1}{4}}d x$$

input
```
int(x^9*(b*x^4+a)^(1/4),x)
```

output
```
int(x^9*(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^9 \sqrt[4]{a+bx^4}\,dx=\int \left(bx^{4}+a\right)^{\frac{1}{4}}x^{9}\,dx$$

input
```
integrate(x^9*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*x^9, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^{10}{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{10}$$

input
```
integrate(x**9*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**10*hyper((-1/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/10
```

Maxima [F]

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}}x^9\,dx$$

input
```
integrate(x^9*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^9, x)
```

Giac [F]

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}}x^9\,dx$$

input
```
integrate(x^9*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \sqrt[4]{a+bx^4}\,dx = \int x^9 \left(b\,x^4+a\right)^{1/4}\,dx$$

input
```
int(x^9*(a + b*x^4)^(1/4),x)
```

output
```
int(x^9*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^9 \sqrt[4]{a+bx^4}\,dx$$
$$= \frac{-2(b\,x^4+a)^{\frac{1}{4}}\,a^2x^2+(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^6+7(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{10}+4\left(\int \frac{x}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^3}{77b^2}$$

input
```
int(x^9*(b*x^4+a)^(1/4),x)
```

output
```
( - 2*(a + b*x**4)**(1/4)*a**2*x**2 + (a + b*x**4)**(1/4)*a*b*x**6 + 7*(a
+ b*x**4)**(1/4)*b**2*x**10 + 4*int(((a + b*x**4)**(1/4)*x)/(a + b*x**4),x
)*a**3)/(77*b**2)
```

3.452 $\int x^5 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^5 \sqrt[4]{a+bx^4}\,dx = \frac{ax^2\sqrt[4]{a+bx^4}}{21b}+\frac{1}{7}x^6\sqrt[4]{a+bx^4} - \frac{2a^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
1/21*a*x^2*(b*x^4+a)^(1/4)/b+1/7*x^6*(b*x^4+a)^(1/4)-2/21*a^(5/2)*(1+b*x^4
/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)
/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.45 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^5 \sqrt[4]{a+bx^4}\,dx = \frac{x^2\sqrt[4]{a+bx^4}\left(a+bx^4-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{7b}$$

input
```
Integrate[x^5*(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(a + b*x^4)^(1/4)*(a + b*x^4 - (a*Hypergeometric2F1[-1/4, 1/2, 3/2, -
((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(7*b)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 248, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt[4]{a+bx^4} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^4 \sqrt[4]{bx^4+a}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{1}{7}a\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{7}a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3b}\right)+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{1}{7}a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b\left(a+bx^4\right)^{3/4}}\right)+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{1}{7}a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}\right)+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)$$

input
```
Int[x^5*(a + b*x^4)^(1/4),x]
```

output
```
((2*x^6*(a + b*x^4)^(1/4))/7 + (a*((2*x^2*(a + b*x^4)^(1/4))/(3*b) - (4*a^
(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/(3*b^(3/2)*(a + b*x^4)^(3/4))))/7)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^{5} \left(b\,x^{4}+a\right)^{\frac{1}{4}}d x$$

input
```
int(x^5*(b*x^4+a)^(1/4),x)
```

output
```
int(x^5*(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^5 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^5\,dx$$

input
```
integrate(x^5*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

$$\int x^5 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^6{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6}$$

input
```
integrate(x**5*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**6*hyper((-1/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/6
```

Maxima [F]

$$\int x^5 \sqrt[4]{a+bx^4}\, dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^5\, dx$$

input
```
integrate(x^5*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^5, x)
```

Giac [F]

$$\int x^5 \sqrt[4]{a+bx^4}\, dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^5\, dx$$

input
```
integrate(x^5*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \sqrt[4]{a+bx^4}\, dx = \int x^5 \left(b\,x^4+a\right)^{1/4}\, dx$$

input
```
int(x^5*(a + b*x^4)^(1/4),x)
```

output
```
int(x^5*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^5 \sqrt[4]{a+bx^4}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^2+3(b\,x^4+a)^{\frac{1}{4}}\,b\,x^6-2\left(\int \frac{x}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{21b}$$

input
```
int(x^5*(b*x^4+a)^(1/4),x)
```

output
```
((a + b*x**4)**(1/4)*a*x**2 + 3*(a + b*x**4)**(1/4)*b*x**6 - 2*int(((a + b
*x**4)**(1/4)*x)/(a + b*x**4),x)*a**2)/(21*b)
```

3.453 $\int x\sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int x\sqrt[4]{a+bx^4}\,dx=\frac{1}{3}x^2\sqrt[4]{a+bx^4}+\frac{a^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
1/3*x^2*(b*x^4+a)^(1/4)+1/3*a^(3/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*
arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int x\sqrt[4]{a+bx^4}\,dx=\frac{x^2\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{2\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x*(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(a + b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^4)/a)])/(2
*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {807, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \sqrt[4]{bx^4+a}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{1}{3}a\int \frac{1}{(bx^4+a)^{3/4}}dx^2+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3\left(a+bx^4\right)^{3/4}}+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{2a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)$$

input
```
Int[x*(a + b*x^4)^(1/4),x]
```

output
```
((2*x^2*(a + b*x^4)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^4)^(3/4)))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x\left(b\,x^{4}+a\right)^{\frac{1}{4}}d x$$

input
```
int(x*(b*x^4+a)^(1/4),x)
```

output
```
int(x*(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x\sqrt[4]{a+bx^4}\,dx = \int (bx^4+a)^{\frac{1}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int x\sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^2{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2}$$

input
```
integrate(x*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**2*hyper((-1/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/2
```

Maxima [F]

$$\int x\sqrt[4]{a+bx^4}\,dx = \int (bx^4+a)^{\frac{1}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*x, x)
```

Giac [F]

$$\int x\sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x\,dx$$

input
```
integrate(x*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[4]{a+bx^4}\,dx = \int x\,\left(b\,x^4+a\right)^{1/4}\,dx$$

input
```
int(x*(a + b*x^4)^(1/4),x)
```

output
```
int(x*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x\sqrt[4]{a+bx^4}\,dx = \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}x^2}{3} + \frac{\left(\int \frac{x}{\left(b\,x^4+a\right)^{\frac{3}{4}}}dx\right)a}{3}$$

input
```
int(x*(b*x^4+a)^(1/4),x)
```

output
```
((a + b*x**4)**(1/4)*x**2 + int(((a + b*x**4)**(1/4)*x)/(a + b*x**4),x)*a)
/3
```

3.454 $\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx = -\frac{\sqrt[4]{a+bx^4}}{2x^2}+\frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\,(a+bx^4)^{3/4}}$$

output
```
-1/2*(b*x^4+a)^(1/4)/x^2+1/2*a^(1/2)*b^(1/2)*(1+b*x^4/a)^(3/4)*InverseJaco
biAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx = -\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^3,x]
```

output
```
-1/2*((a + b*x^4)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x^4)/a)])/
(x^2*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {807, 247, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt[4]{bx^4+a}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \frac{1}{(bx^4+a)^{3/4}}dx^2-\frac{\sqrt[4]{a+bx^4}}{x^2}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^2}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^2}\right)$$

input
```
Int[(a + b*x^4)^(1/4)/x^3,x]
```

output
```
(-((a + b*x^4)^(1/4)/x^2) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*Ellipti
cF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(a + b*x^4)^(3/4))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{x^3}dx$$

input
```
int((b*x^4+a)^(1/4)/x^3,x)
```

output
```
int((b*x^4+a)^(1/4)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2x^2}$$

input
```
integrate((b*x**4+a)**(1/4)/x**3,x)
```

output
```
-a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*x**2)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx=\int \frac{(bx^4+a)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx=\int \frac{(b\,x^4+a)^{1/4}}{x^3}\,dx$$

input
```
int((a + b*x^4)^(1/4)/x^3,x)
```

output
```
int((a + b*x^4)^(1/4)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^3}\,dx=\frac{-(b\,x^4+a)^{\frac{1}{4}}-\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^7+a\,x^3}dx\right)a\,x^2}{x^2}$$

input
```
int((b*x^4+a)^(1/4)/x^3,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**3 + b*x**7),x)*a*
x**2))/x**2
```

3.455 $\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx = -\frac{\sqrt[4]{a+bx^4}}{6x^6}-\frac{b\sqrt[4]{a+bx^4}}{12ax^2} -\frac{b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\sqrt{a}\,(a+bx^4)^{3/4}}$$

output
```
-1/6*(b*x^4+a)^(1/4)/x^6-1/12*b*(b*x^4+a)^(1/4)/a/x^2-1/12*b^(3/2)*(1+b*x^
4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2
)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx = -\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},-\frac{1}{2},-\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^7,x]
```

output
```
-1/6*((a + b*x^4)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x^4)/a)])
/(x^6*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 247, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt[4]{bx^4+a}}{x^8}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{1}{6}b\int \frac{1}{x^4 (bx^4+a)^{3/4}}dx^2-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{b\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)$$

input
```
Int[(a + b*x^4)^(1/4)/x^7,x]
```

output
```
(-1/3*(a + b*x^4)^(1/4)/x^6 + (b*(-((a + b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b]*
(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt
[a]*(a + b*x^4)^(3/4))))/6)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{1}{4}}}{x^{7}}dx$$

input
```
int((b*x^4+a)^(1/4)/x^7,x)
```

output
```
int((b*x^4+a)^(1/4)/x^7,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^7,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^7, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6x^6}$$

input
```
integrate((b*x**4+a)**(1/4)/x**7,x)
```

output
```
-a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*x**6)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^7, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx=\int \frac{\left(b\,x^4+a\right)^{1/4}}{x^7}\,dx$$

input
```
int((a + b*x^4)^(1/4)/x^7,x)
```

output
```
int((a + b*x^4)^(1/4)/x^7, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^7}\,dx=\frac{-\left(b\,x^4+a\right)^{\frac{1}{4}}-\left(\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b\,x^{11}+a\,x^7}dx\right)a\,x^6}{5x^6}$$

input
```
int((b*x^4+a)^(1/4)/x^7,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**7 + b*x**11),x)*a
*x**6))/(5*x**6)
```

3.456 $\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{10x^{10}} - \frac{b\sqrt[4]{a+bx^4}}{60ax^6} + \frac{b^2\sqrt[4]{a+bx^4}}{24a^2x^2} + \frac{b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24a^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/10*(b*x^4+a)^(1/4)/x^10-1/60*b*(b*x^4+a)^(1/4)/a/x^6+1/24*b^2*(b*x^4+a)
^(1/4)/a^2/x^2+1/24*b^(5/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b
^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = -\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{4},-\frac{3}{2},-\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^11,x]
```

output
```
-1/10*((a + b*x^4)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x^4)/a)]
)/(x^10*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 247, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2} \int \frac{\sqrt[4]{bx^4+a}}{x^{12}}dx^2$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{2} \left(\frac{1}{10} b \int \frac{1}{x^8 \left(bx^4+a\right)^{3/4}}dx^2-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2} \left(\frac{1}{10} b \left(-\frac{5b \int \frac{1}{x^4 \left(bx^4+a\right)^{3/4}}dx^2}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2} \left(\frac{1}{10} b \left(-\frac{5b \left(-\frac{b \int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{ 231}$$

$$\frac{1}{2}\left(\frac{1}{10}b\left(-\frac{5b\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)$$

$\downarrow$ 229

$$\frac{1}{2}\left(\frac{1}{10}b\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)$$

input
```
Int[(a + b*x^4)^(1/4)/x^11,x]
```

output
```
(-1/5*(a + b*x^4)^(1/4)/x^10 + (b*(-1/3*(a + b*x^4)^(1/4)/(a*x^6) - (5*b*(
-((a + b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[Ar
cTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4))))/(6*a)))/1
0)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{x^{11}}dx$$

input
```
int((b*x^4+a)^(1/4)/x^11,x)
```

output
```
int((b*x^4+a)^(1/4)/x^11,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx=\int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^11,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^11, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{10x^{10}}$$

input
```
integrate((b*x**4+a)**(1/4)/x**11,x)
```

output
```
-a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*x**10
)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^11,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^11, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^11,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = \int \frac{\left(b\,x^4+a\right)^{1/4}}{x^{11}}\,dx$$

input
```
int((a + b*x^4)^(1/4)/x^11,x)
```

output
```
int((a + b*x^4)^(1/4)/x^11, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{11}}\,dx = \frac{-\left(b\,x^4+a\right)^{\frac{1}{4}}-\left(\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b\,x^{15}+a\,x^{11}}dx\right)a\,x^{10}}{9x^{10}}$$

input
```
int((b*x^4+a)^(1/4)/x^11,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**11 + b*x**15),x)*
a*x**10))/(9*x**10)
```

3.457 $\int x^6 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \frac{ax^3\sqrt[4]{a+bx^4}}{32b} + \frac{1}{8}x^7\sqrt[4]{a+bx^4} + \frac{3a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}} - \frac{3a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}}$$

output
```
1/32*a*x^3*(b*x^4+a)^(1/4)/b+1/8*x^7*(b*x^4+a)^(1/4)+3/64*a^2*arctan(b^(1/
4)*x/(b*x^4+a)^(1/4))/b^(7/4)-3/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/
b^(7/4)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \frac{2b^{3/4}x^3\sqrt[4]{a+bx^4}(a+4bx^4) + 3a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) - 3a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{7/4}}$$

input
```
Integrate[x^6*(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(3/4)*x^3*(a + b*x^4)^(1/4)*(a + 4*b*x^4) + 3*a^2*ArcTan[(b^(1/4)*x)/
(a + b*x^4)^(1/4)] - 3*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(
7/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {811, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[4]{a+bx^4}\, dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{1}{8}a\int \frac{x^6}{(bx^4+a)^{3/4}}dx+\frac{1}{8}x^7\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}$$

input

```
Int[x^6*(a + b*x^4)^(1/4),x]
```

output

```
(x^7*(a + b*x^4)^(1/4))/8 + (a*((x^3*(a + b*x^4)^(1/4))/(4*b) - (3*a*(-1/2
*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a +
b*x^4)^(1/4)]/(2*b^(3/4))))/(4*b)))/8
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$\frac{16(b\,x^4+a)^{\frac{1}{4}}b^{\frac{7}{4}}x^7+4a\,x^3(b\,x^4+a)^{\frac{1}{4}}b^{\frac{3}{4}}-3\ln\left(\frac{b^{\frac{1}{4}}x+(b\,x^4+a)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+(b\,x^4+a)^{\frac{1}{4}}}\right)a^2-6\arctan\left(\frac{(b\,x^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)a^2}{128b^{\frac{7}{4}}}$	104

input
```
int(x^6*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(16*(b*x^4+a)^(1/4)*b^(7/4)*x^7+4*a*x^3*(b*x^4+a)^(1/4)*b^(3/4)-3*ln
((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^2-6*arctan(1/
b^(1/4)/x*(b*x^4+a)^(1/4))*a^2)/b^(7/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.12

$$\int x^6\sqrt[4]{a+bx^4}\,dx = -\frac{3\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b\log\left(\frac{3\left(\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x+(bx^4+a)^{\frac{1}{4}}a^2\right)}{x}\right)-3\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b\log\left(-\frac{3\left(\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x-(bx^4+a)^{\frac{1}{4}}a^2\right)}{x}\right)-3i\left(\frac{a^8}{b^7}\right)^{\frac{1}{4}}b\log$$

input
```
integrate(x^6*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/128*(3*(a^8/b^7)^(1/4)*b*log(3*((a^8/b^7)^(1/4)*b^2*x + (b*x^4 + a)^(1/
4)*a^2)/x) - 3*(a^8/b^7)^(1/4)*b*log(-3*((a^8/b^7)^(1/4)*b^2*x - (b*x^4 +
a)^(1/4)*a^2)/x) - 3*I*(a^8/b^7)^(1/4)*b*log(-3*(I*(a^8/b^7)^(1/4)*b^2*x -
 (b*x^4 + a)^(1/4)*a^2)/x) + 3*I*(a^8/b^7)^(1/4)*b*log(-3*(-I*(a^8/b^7)^(1
/4)*b^2*x - (b*x^4 + a)^(1/4)*a^2)/x) - 4*(4*b*x^7 + a*x^3)*(b*x^4 + a)^(1
/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.48

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \frac{\frac{3(bx^4+a)^{\frac{1}{4}}a^2b}{x} + \frac{(bx^4+a)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^3 - \frac{2(bx^4+a)b^2}{x^4} + \frac{(bx^4+a)^2b}{x^8}\right)} - \frac{3\left(\frac{2a^2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{128b}$$

input
```
integrate(x^6*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/32*(3*(b*x^4 + a)^(1/4)*a^2*b/x + (b*x^4 + a)^(5/4)*a^2/x^5)/(b^3 - 2*(b
*x^4 + a)*b^2/x^4 + (b*x^4 + a)^2*b/x^8) - 3/128*(2*a^2*arctan((b*x^4 + a)
^(1/4)/(b^(1/4)*x))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^
(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b
```

Giac [F]

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^6\,dx$$

input
```
integrate(x^6*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \int x^6 \left(b\,x^4+a\right)^{1/4}\,dx$$

input
```
int(x^6*(a + b*x^4)^(1/4),x)
```

output
```
int(x^6*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^6 \sqrt[4]{a+bx^4}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^3+4(b\,x^4+a)^{\frac{1}{4}}\,b\,x^7-3\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{32b}$$

input
```
int(x^6*(b*x^4+a)^(1/4),x)
```

output
```
((a + b*x**4)**(1/4)*a*x**3 + 4*(a + b*x**4)**(1/4)*b*x**7 - 3*int(((a + b
*x**4)**(1/4)*x**2)/(a + b*x**4),x)*a**2)/(32*b)
```

3.458 $\int x^2\sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int x^2\sqrt[4]{a+bx^4}\,dx=\frac{1}{4}x^3\sqrt[4]{a+bx^4}-\frac{a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}}+\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}}$$

output
```
1/4*x^3*(b*x^4+a)^(1/4)-1/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)+1/
8*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96

$$\int x^2\sqrt[4]{a+bx^4}\,dx=\frac{2b^{3/4}x^3\sqrt[4]{a+bx^4}-a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{3/4}}$$

input
```
Integrate[x^2*(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(3/4)*x^3*(a + b*x^4)^(1/4) - a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]
 + a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {811, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{1}{4}a\int \frac{x^2}{(bx^4+a)^{3/4}}dx+\frac{1}{4}x^3\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{1}{4}a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{4}x^3\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{1}{4}a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}$$

input
```
Int[x^2*(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(a + b*x^4)^(1/4))/4 + (a*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]
/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4))))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{1}{4}} x^{3} b^{\frac{3}{4}}+\ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a}{16 b^{\frac{3}{4}}}$	81

input
```
int(x^2*(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*(b*x^4+a)^(1/4)*x^3*b^(3/4)+ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/
4)*x+(b*x^4+a)^(1/4)))*a+2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a)/b^(3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.32

$$\int x^2 \sqrt[4]{a+bx^4}\,dx = \frac{1}{4}\left(bx^4+a\right)^{\frac{1}{4}}x^3 + \frac{1}{16}\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+\left(bx^4+a\right)^{\frac{1}{4}}a}{x}\right) - \frac{1}{16}\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx-\left(bx^4+a\right)^{\frac{1}{4}}a}{x}\right) + \frac{1}{16}i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+\left(bx^4+a\right)^{\frac{1}{4}}a}{x}\right) - \frac{1}{16}i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+\left(bx^4+a\right)^{\frac{1}{4}}a}{x}\right)$$

input
```
integrate(x^2*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/4*(b*x^4 + a)^(1/4)*x^3 + 1/16*(a^4/b^3)^(1/4)*log(((a^4/b^3)^(1/4)*b*x
+ (b*x^4 + a)^(1/4)*a)/x) - 1/16*(a^4/b^3)^(1/4)*log(-((a^4/b^3)^(1/4)*b*x
 - (b*x^4 + a)^(1/4)*a)/x) + 1/16*I*(a^4/b^3)^(1/4)*log((I*(a^4/b^3)^(1/4)
*b*x + (b*x^4 + a)^(1/4)*a)/x) - 1/16*I*(a^4/b^3)^(1/4)*log((-I*(a^4/b^3)^
(1/4)*b*x + (b*x^4 + a)^(1/4)*a)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int x^2 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.31

$$\int x^2 \sqrt[4]{a+bx^4}\,dx = \frac{a\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{8\,b^{\frac{3}{4}}} - \frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{16\,b^{\frac{3}{4}}} - \frac{(bx^4+a)^{\frac{1}{4}}a}{4\left(b-\frac{bx^4+a}{x^4}\right)x}$$

input
```
integrate(x^2*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/8*a*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - 1/16*a*log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4) - 1/4*(b*
x^4 + a)^(1/4)*a/((b - (b*x^4 + a)/x^4)*x)
```

Giac [F]

$$\int x^2 \sqrt[4]{a+bx^4}\, dx = \int (bx^4+a)^{\frac{1}{4}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[4]{a+bx^4}\, dx = \int x^2 \left(b\,x^4+a\right)^{1/4} dx$$

input
```
int(x^2*(a + b*x^4)^(1/4),x)
```

output
```
int(x^2*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^2 \sqrt[4]{a+bx^4}\, dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\, x^3}{4} + \frac{\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}} dx\right) a}{4}$$

input
```
int(x^2*(b*x^4+a)^(1/4),x)
```

output
```
((a + b*x**4)**(1/4)*x**3 + int(((a + b*x**4)**(1/4)*x**2)/(a + b*x**4),x)
*a)/4
```

3.459 $\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx=-\frac{\sqrt[4]{a+bx^4}}{x}-\frac{1}{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)$$

output
```
-(b*x^4+a)^(1/4)/x-1/2*b^(1/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))+1/2*b^(1/
4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx=-\frac{\sqrt[4]{a+bx^4}}{x}-\frac{1}{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^2,x]
```

output
```
-((a + b*x^4)^(1/4)/x) - (b^(1/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/2
 + (b^(1/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {809, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2} \, dx$$

$$\Big\downarrow \text{809}$$

$$b\int \frac{x^2}{(bx^4+a)^{3/4}}dx - \frac{\sqrt[4]{a+bx^4}}{x}$$

$$\Big\downarrow \text{854}$$

$$b\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}} - \frac{\sqrt[4]{a+bx^4}}{x}$$

$$\Big\downarrow \text{827}$$

$$b\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}} - \frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right) - \frac{\sqrt[4]{a+bx^4}}{x}$$

$$\Big\downarrow \text{216}$$

$$b\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right) - \frac{\sqrt[4]{a+bx^4}}{x}$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}} - \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right) - \frac{\sqrt[4]{a+bx^4}}{x}$$

input
```
Int[(a + b*x^4)^(1/4)/x^2,x]
```

output
```
-((a + b*x^4)^(1/4)/x) + b*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(
3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

method	result	size
pseudoelliptic	$\frac{\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)b^{\frac{1}{4}}x+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)b^{\frac{1}{4}}x-4\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{4x}$	81

input
```
int((b*x^4+a)^(1/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*b^(1/4)*
x+2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*b^(1/4)*x-4*(b*x^4+a)^(1/4))/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^4+a)^(1/4)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx=\frac{\sqrt[4]{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**2,x)
```

output
```
a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx = \frac{1}{2}\,b^{\frac{1}{4}}\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right) - \frac{1}{4}\,b^{\frac{1}{4}}\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right) - \frac{(bx^4+a)^{\frac{1}{4}}}{x}$$

input
```
integrate((b*x^4+a)^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/2*b^(1/4)*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)) - 1/4*b^(1/4)*log(-(b^(1
/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x)) - (b*x^4 + a)^
(1/4)/x
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx = -\frac{\left(b\,x^4+a\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{b\,x^4}{a}\right)}{x\,\left(\frac{b\,x^4}{a}+1\right)^{1/4}}$$

input
```
int((a + b*x^4)^(1/4)/x^2,x)
```

output
```
-((a + b*x^4)^(1/4)*hypergeom([-1/4, -1/4], 3/4, -(b*x^4)/a))/(x*((b*x^4)/
a + 1)^(1/4))
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^2}\,dx = \int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{x^2}dx$$

input
```
int((b*x^4+a)^(1/4)/x^2,x)
```

output
```
int((a + b*x**4)**(1/4)/x**2,x)
```

3.460 $\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx=-\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

output
```
-1/5*(b*x^4+a)^(5/4)/a/x^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx=-\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^6,x]
```

output
```
-1/5*(a + b*x^4)^(5/4)/(a*x^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\left(a+bx^4\right)^{5/4}}{5ax^5}$$

input
```
Int[(a + b*x^4)^(1/4)/x^6,x]
```

output
```
-1/5*(a + b*x^4)^(5/4)/(a*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{5}{4}}}{5a\,x^5}$	18
trager	$-\frac{(b\,x^4+a)^{\frac{5}{4}}}{5a\,x^5}$	18
risch	$-\frac{(b\,x^4+a)^{\frac{5}{4}}}{5a\,x^5}$	18
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{5}{4}}}{5a\,x^5}$	18
orering	$-\frac{(b\,x^4+a)^{\frac{5}{4}}}{5a\,x^5}$	18

input
```
int((b*x^4+a)^(1/4)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(5/4)/a/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = -\frac{(bx^4+a)^{\frac{5}{4}}}{5\,ax^5}$$

input
```
integrate((b*x^4+a)^(1/4)/x^6,x, algorithm="fricas")
```

output
```
-1/5*(b*x^4 + a)^(5/4)/(a*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{4x^4\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{4a\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**6,x)
```

output
```
b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*x**4*gamma(-1/4)) + b**(5/
4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*a*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = -\frac{(bx^4+a)^{\frac{5}{4}}}{5\,ax^5}$$

input
```
integrate((b*x^4+a)^(1/4)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(b*x^4 + a)^(5/4)/(a*x^5)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = -\frac{\left(b\,x^4+a\right)^{5/4}}{5\,a\,x^5}$$

input
```
int((a + b*x^4)^(1/4)/x^6,x)
```

output
```
-(a + b*x^4)^(5/4)/(5*a*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[4]{a+bx^4}}{x^6}\,dx = -\frac{\left(b\,x^4+a\right)^{\frac{5}{4}}}{5a\,x^5}$$

input
```
int((b*x^4+a)^(1/4)/x^6,x)
```

output
```
( - (a + b*x**4)**(1/4)*(a + b*x**4))/(5*a*x**5)
```

3.461 $\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = -\frac{(a+bx^4)^{5/4}}{9ax^9} + \frac{4b(a+bx^4)^{5/4}}{45a^2x^5}$$

output
```
-1/9*(b*x^4+a)^(5/4)/a/x^9+4/45*b*(b*x^4+a)^(5/4)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = \frac{\sqrt[4]{a+bx^4}(-5a^2-abx^4+4b^2x^8)}{45a^2x^9}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^10,x]
```

output
```
((a + b*x^4)^(1/4)*(-5*a^2 - a*b*x^4 + 4*b^2*x^8))/(45*a^2*x^9)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{4b\int \frac{\sqrt[4]{bx^4+a}}{x^6}dx}{9a}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}$$

$$\Big\downarrow \text{796}$$

$$\frac{4b\left(a+bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}$$

input
```
Int[(a + b*x^4)^(1/4)/x^10,x]
```

output
```
-1/9*(a + b*x^4)^(5/4)/(a*x^9) + (4*b*(a + b*x^4)^(5/4))/(45*a^2*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b x^4+a)^{\frac{5}{4}} (-4b x^4+5a)}{45x^9 a^2}$	28
pseudoelliptic	$-\frac{(b x^4+a)^{\frac{5}{4}} (-4b x^4+5a)}{45x^9 a^2}$	28
orering	$-\frac{(b x^4+a)^{\frac{5}{4}} (-4b x^4+5a)}{45x^9 a^2}$	28
trager	$-\frac{(-4b^2x^8+ab x^4+5a^2) (b x^4+a)^{\frac{1}{4}}}{45x^9 a^2}$	38
risch	$-\frac{(-4b^2x^8+ab x^4+5a^2) (b x^4+a)^{\frac{1}{4}}}{45x^9 a^2}$	38

input
```
int((b*x^4+a)^(1/4)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/45*(b*x^4+a)^(5/4)*(-4*b*x^4+5*a)/x^9/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx=\frac{(4\,b^2x^8-abx^4-5\,a^2)(bx^4+a)^{\frac{1}{4}}}{45\,a^2x^9}$$

input
```
integrate((b*x^4+a)^(1/4)/x^10,x, algorithm="fricas")
```

output
```
1/45*(4*b^2*x^8 - a*b*x^4 - 5*a^2)*(b*x^4 + a)^(1/4)/(a^2*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(37) = 74$.

Time = 0.63 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{16x^8\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{16ax^4\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{4a^2\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**10,x)
```

output
```
-5*b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(16*x**8*gamma(-1/4)) - b*
*(5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(16*a*x**4*gamma(-1/4)) + b**(9
/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(4*a**2*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = \frac{\frac{9\,(bx^4+a)^{\frac{5}{4}}b}{x^5} - \frac{5\,(bx^4+a)^{\frac{9}{4}}}{x^9}}{45\,a^2}$$

input
```
integrate((b*x^4+a)^(1/4)/x^10,x, algorithm="maxima")
```

output
```
1/45*(9*(b*x^4 + a)^(5/4)*b/x^5 - 5*(b*x^4 + a)^(9/4)/x^9)/a^2
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{10}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^10,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = -\frac{\left(b\,x^4+a\right)^{1/4}\,\left(5\,a^2+a\,b\,x^4-4\,b^2\,x^8\right)}{45\,a^2\,x^9}$$

input
```
int((a + b*x^4)^(1/4)/x^10,x)
```

output
```
-((a + b*x^4)^(1/4)*(5*a^2 - 4*b^2*x^8 + a*b*x^4))/(45*a^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{10}}\,dx = \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(4b^2x^8-ab\,x^4-5a^2\right)}{45a^2x^9}$$

input
```
int((b*x^4+a)^(1/4)/x^10,x)
```

output
```
((a + b*x**4)**(1/4)*( - 5*a**2 - a*b*x**4 + 4*b**2*x**8))/(45*a**2*x**9)
```

3.462 $\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx=-\frac{(a+bx^4)^{5/4}}{13ax^{13}}+\frac{8b(a+bx^4)^{5/4}}{117a^2x^9}-\frac{32b^2(a+bx^4)^{5/4}}{585a^3x^5}$$

output
```
-1/13*(b*x^4+a)^(5/4)/a/x^13+8/117*b*(b*x^4+a)^(5/4)/a^2/x^9-32/585*b^2*(b
*x^4+a)^(5/4)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx=\frac{\sqrt[4]{a+bx^4}(-45a^3-5a^2bx^4+8ab^2x^8-32b^3x^{12})}{585a^3x^{13}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^14,x]
```

output
```
((a + b*x^4)^(1/4)*(-45*a^3 - 5*a^2*b*x^4 + 8*a*b^2*x^8 - 32*b^3*x^12))/(5
85*a^3*x^13)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{8b\int \frac{\sqrt[4]{bx^4+a}}{x^{10}}dx}{13a}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{8b\left(-\frac{4b\int \frac{\sqrt[4]{bx^4+a}}{x^6}dx}{9a}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{8b\left(\frac{4b\left(a+bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}$$

input
```
Int[(a + b*x^4)^(1/4)/x^14,x]
```

output
```
-1/13*(a + b*x^4)^(5/4)/(a*x^13) - (8*b*(-1/9*(a + b*x^4)^(5/4)/(a*x^9) +
(4*b*(a + b*x^4)^(5/4))/(45*a^2*x^5)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(32 b^{2} x^{8}-40 a b\,x^{4}+45 a^{2}\right)}{585 x^{13} a^{3}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(32 b^{2} x^{8}-40 a b\,x^{4}+45 a^{2}\right)}{585 x^{13} a^{3}}$	39
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(32 b^{2} x^{8}-40 a b\,x^{4}+45 a^{2}\right)}{585 x^{13} a^{3}}$	39
trager	$-\frac{\left(32 b^{3} x^{12}-8 a\,b^{2} x^{8}+5 a^{2} b\,x^{4}+45 a^{3}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{585 x^{13} a^{3}}$	50
risch	$-\frac{\left(32 b^{3} x^{12}-8 a\,b^{2} x^{8}+5 a^{2} b\,x^{4}+45 a^{3}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{585 x^{13} a^{3}}$	50

input
```
int((b*x^4+a)^(1/4)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/585*(b*x^4+a)^(5/4)*(32*b^2*x^8-40*a*b*x^4+45*a^2)/x^13/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx = -\frac{(32\,b^3x^{12} - 8\,ab^2x^8 + 5\,a^2bx^4 + 45\,a^3)(bx^4+a)^{\frac{1}{4}}}{585\,a^3x^{13}}$$

input
```
integrate((b*x^4+a)^(1/4)/x^14,x, algorithm="fricas")
```

output
```
-1/585*(32*b^3*x^12 - 8*a*b^2*x^8 + 5*a^2*b*x^4 + 45*a^3)*(b*x^4 + a)^(1/4
)/(a^3*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(61) = 122.

Time = 0.98 (sec) , antiderivative size = 520, normalized size of antiderivative = 7.65

$$\begin{aligned}\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx &= \frac{45a^5b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)} \\ &+ \frac{95a^4b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)} \\ &+ \frac{47a^3b^{\frac{25}{4}}x^8\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)} \\ &+ \frac{21a^2b^{\frac{29}{4}}x^{12}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)} \\ &+ \frac{56ab^{\frac{33}{4}}x^{16}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)} \\ &+ \frac{32b^{\frac{37}{4}}x^{20}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{13}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{1}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{1}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{1}{4}\right)}\end{aligned}$$

input
```
integrate((b*x**4+a)**(1/4)/x**14,x)
```

output
```
45*a**5*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12
*gamma(-1/4) + 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(
-1/4)) + 95*a**4*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a
**5*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**
6*x**20*gamma(-1/4)) + 47*a**3*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamm
a(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*x**16*gamma(-1/4)
 + 64*a**3*b**6*x**20*gamma(-1/4)) + 21*a**2*b**(29/4)*x**12*(a/(b*x**4) +
 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*a**4*b**5*x*
*16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 56*a*b**(33/4)*x**16*(
a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) + 128*
a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 32*b**(37/
4)*x**20*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1
/4) + 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx = -\frac{\frac{117\,(bx^4+a)^{\frac{5}{4}}b^2}{x^5} - \frac{130\,(bx^4+a)^{\frac{9}{4}}b}{x^9} + \frac{45\,(bx^4+a)^{\frac{13}{4}}}{x^{13}}}{585\,a^3}$$

input
```
integrate((b*x^4+a)^(1/4)/x^14,x, algorithm="maxima")
```

output
```
-1/585*(117*(b*x^4 + a)^(5/4)*b^2/x^5 - 130*(b*x^4 + a)^(9/4)*b/x^9 + 45*(
b*x^4 + a)^(13/4)/x^13)/a^3
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{14}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^14,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx = \frac{8\,b^2\,(b\,x^4+a)^{1/4}}{585\,a^2\,x^5} - \frac{b\,(b\,x^4+a)^{1/4}}{117\,a\,x^9} - \frac{32\,b^3\,(b\,x^4+a)^{1/4}}{585\,a^3\,x} - \frac{(b\,x^4+a)^{1/4}}{13\,x^{13}}$$

input
```
int((a + b*x^4)^(1/4)/x^14,x)
```

output
```
(8*b^2*(a + b*x^4)^(1/4))/(585*a^2*x^5) - (b*(a + b*x^4)^(1/4))/(117*a*x^9
) - (32*b^3*(a + b*x^4)^(1/4))/(585*a^3*x) - (a + b*x^4)^(1/4)/(13*x^13)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{14}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,(-32b^3x^{12}+8a\,b^2x^8-5a^2b\,x^4-45a^3)}{585a^3x^{13}}$$

input
```
int((b*x^4+a)^(1/4)/x^14,x)
```

output
```
((a + b*x**4)**(1/4)*( - 45*a**3 - 5*a**2*b*x**4 + 8*a*b**2*x**8 - 32*b**3
*x**12))/(585*a**3*x**13)
```

3.463 $\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = -\frac{(a+bx^4)^{5/4}}{17ax^{17}}+\frac{12b(a+bx^4)^{5/4}}{221a^2x^{13}}-\frac{32b^2(a+bx^4)^{5/4}}{663a^3x^9}+\frac{128b^3(a+bx^4)^{5/4}}{3315a^4x^5}$$

output
```
-1/17*(b*x^4+a)^(5/4)/a/x^17+12/221*b*(b*x^4+a)^(5/4)/a^2/x^13-32/663*b^2*
(b*x^4+a)^(5/4)/a^3/x^9+128/3315*b^3*(b*x^4+a)^(5/4)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \frac{(a+bx^4)^{5/4}\left(-195a^3+180a^2bx^4-160ab^2x^8+128b^3x^{12}\right)}{3315a^4x^{17}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^18,x]
```

output
```
((a + b*x^4)^(5/4)*(-195*a^3 + 180*a^2*b*x^4 - 160*a*b^2*x^8 + 128*b^3*x^1
2))/(3315*a^4*x^17)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{12b\int \frac{\sqrt[4]{bx^4+a}}{x^{14}}dx}{17a}-\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{12b\left(-\frac{8b\int \frac{\sqrt[4]{bx^4+a}}{x^{10}}dx}{13a}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{\sqrt[4]{bx^4+a}}{x^6}dx}{9a}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{12b\left(-\frac{8b\left(\frac{4b\left(a+bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a+bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{5/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a+bx^4\right)^{5/4}}{17ax^{17}}$$

input
```
Int[(a + b*x^4)^(1/4)/x^18,x]
```

output
```
-1/17*(a + b*x^4)^(5/4)/(a*x^17) - (12*b*(-1/13*(a + b*x^4)^(5/4)/(a*x^13)
 - (8*b*(-1/9*(a + b*x^4)^(5/4)/(a*x^9) + (4*b*(a + b*x^4)^(5/4))/(45*a^2*
x^5)))/(13*a)))/(17*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(-128 b^{3} x^{12}+160 a\,b^{2} x^{8}-180 a^{2} b\,x^{4}+195 a^{3}\right)}{3315 x^{17} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(-128 b^{3} x^{12}+160 a\,b^{2} x^{8}-180 a^{2} b\,x^{4}+195 a^{3}\right)}{3315 x^{17} a^{4}}$	50
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{5}{4}} \left(-128 b^{3} x^{12}+160 a\,b^{2} x^{8}-180 a^{2} b\,x^{4}+195 a^{3}\right)}{3315 x^{17} a^{4}}$	50
trager	$-\frac{\left(-128 x^{16} b^{4}+32 a\,b^{3} x^{12}-20 a^{2} b^{2} x^{8}+15 a^{3} b\,x^{4}+195 a^{4}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{3315 x^{17} a^{4}}$	61
risch	$-\frac{\left(-128 x^{16} b^{4}+32 a\,b^{3} x^{12}-20 a^{2} b^{2} x^{8}+15 a^{3} b\,x^{4}+195 a^{4}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{3315 x^{17} a^{4}}$	61

input
```
int((b*x^4+a)^(1/4)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/3315*(b*x^4+a)^(5/4)*(-128*b^3*x^12+160*a*b^2*x^8-180*a^2*b*x^4+195*a^3
)/x^17/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \frac{(128\,b^4x^{16} - 32\,ab^3x^{12} + 20\,a^2b^2x^8 - 15\,a^3bx^4 - 195\,a^4)(bx^4+a)^{\frac{1}{4}}}{3315\,a^4x^{17}}$$

input
```
integrate((b*x^4+a)^(1/4)/x^18,x, algorithm="fricas")
```

output
```
1/3315*(128*b^4*x^16 - 32*a*b^3*x^12 + 20*a^2*b^2*x^8 - 15*a^3*b*x^4 - 195
*a^4)*(b*x^4 + a)^(1/4)/(a^4*x^17)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(85) = 170.

Time = 1.38 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.21

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**4+a)**(1/4)/x**18,x)
```

output
```
-585*a**7*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x*
*16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*
gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 1800*a**6*b**(41/4)*x**4
*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 7
68*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a
**4*b**12*x**28*gamma(-1/4)) - 1830*a**5*b**(45/4)*x**8*(a/(b*x**4) + 1)**
(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20
*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gam
ma(-1/4)) - 636*a**4*b**(49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/
(256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*
a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 231*a**
3*b**(53/4)*x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**1
6*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*ga
mma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 924*a**2*b**(57/4)*x**20*(
a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 768
*a**6*b**10*x**20*gamma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**
4*b**12*x**28*gamma(-1/4)) + 1056*a*b**(61/4)*x**24*(a/(b*x**4) + 1)**(1/4
)*gamma(-17/4)/(256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gam
ma(-1/4) + 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-
1/4)) + 384*b**(65/4)*x**28*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(256*a...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \frac{\frac{663\,(bx^4+a)^{\frac{5}{4}}b^3}{x^5} - \frac{1105\,(bx^4+a)^{\frac{9}{4}}b^2}{x^9} + \frac{765\,(bx^4+a)^{\frac{13}{4}}b}{x^{13}} - \frac{195\,(bx^4+a)^{\frac{17}{4}}}{x^{17}}}{3315\,a^4}$$

input
```
integrate((b*x^4+a)^(1/4)/x^18,x, algorithm="maxima")
```

output
```
1/3315*(663*(b*x^4 + a)^(5/4)*b^3/x^5 - 1105*(b*x^4 + a)^(9/4)*b^2/x^9 + 7
65*(b*x^4 + a)^(13/4)*b/x^13 - 195*(b*x^4 + a)^(17/4)/x^17)/a^4
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{18}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^18,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^18, x)
```

Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \frac{128\,b^4\,(b\,x^4+a)^{1/4}}{3315\,a^4\,x} - \frac{b\,(b\,x^4+a)^{1/4}}{221\,a\,x^{13}} - \frac{(b\,x^4+a)^{1/4}}{17\,x^{17}} - \frac{32\,b^3\,(b\,x^4+a)^{1/4}}{3315\,a^3\,x^5} + \frac{4\,b^2\,(b\,x^4+a)^{1/4}}{663\,a^2\,x^9}$$

input
```
int((a + b*x^4)^(1/4)/x^18,x)
```

output
```
(128*b^4*(a + b*x^4)^(1/4))/(3315*a^4*x) - (b*(a + b*x^4)^(1/4))/(221*a*x^
13) - (a + b*x^4)^(1/4)/(17*x^17) - (32*b^3*(a + b*x^4)^(1/4))/(3315*a^3*x
^5) + (4*b^2*(a + b*x^4)^(1/4))/(663*a^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{18}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,(128b^4x^{16} - 32a\,b^3x^{12} + 20a^2b^2x^8 - 15a^3b\,x^4 - 195a^4)}{3315a^4x^{17}}$$

input
```
int((b*x^4+a)^(1/4)/x^18,x)
```

output
```
((a + b*x**4)**(1/4)*( - 195*a**4 - 15*a**3*b*x**4 + 20*a**2*b**2*x**8 - 3
2*a*b**3*x**12 + 128*b**4*x**16))/(3315*a**4*x**17)
```

3.464 $\int x^{12}\sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx=\frac{3a^3x\sqrt[4]{a+bx^4}}{112b^3}-\frac{3a^2x^5\sqrt[4]{a+bx^4}}{280b^2}+\frac{ax^9\sqrt[4]{a+bx^4}}{140b}+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}+\frac{3a^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{112b^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
3/112*a^3*x*(b*x^4+a)^(1/4)/b^3-3/280*a^2*x^5*(b*x^4+a)^(1/4)/b^2+1/140*a*
x^9*(b*x^4+a)^(1/4)/b+1/14*x^13*(b*x^4+a)^(1/4)+3/112*a^(7/2)*(1+a/b/x^4)^
(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)
/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.54 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.70

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx$$
$$=\frac{x\sqrt[4]{a+bx^4}\left(\sqrt[4]{1+\frac{bx^4}{a}}(15a^3-3a^2bx^4+2ab^2x^8+20b^3x^{12})-15a^3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{280b^3\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x^12*(a + b*x^4)^(1/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*((1 + (b*x^4)/a)^(1/4)*(15*a^3 - 3*a^2*b*x^4 + 2*a*b^
2*x^8 + 20*b^3*x^12) - 15*a^3*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^4)/
a)]))/(280*b^3*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {811, 843, 843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx$$
$$\Big\downarrow \text{811}$$
$$\frac{1}{14}a\int \frac{x^{12}}{(bx^4+a)^{3/4}}dx+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$
$$\Big\downarrow \text{843}$$
$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\int \frac{x^8}{(bx^4+a)^{3/4}}dx}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$
$$\Big\downarrow \text{843}$$

$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int\frac{x^4}{\left(bx^4+a\right)^{3/4}}dx}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$

$\Big\downarrow$ 843

$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int\frac{1}{\left(bx^4+a\right)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$

$\Big\downarrow$ 768

$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$

$\Big\downarrow$ 858

$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}\right)+$$

$$\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$

$\downarrow$ 807

$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}\right)+$$

$$\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$

$\downarrow$ 229

$$\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}$$

input
```
Int[x^12*(a + b*x^4)^(1/4),x]
```

output
```
(x^13*(a + b*x^4)^(1/4))/14 + (a*((x^9*(a + b*x^4)^(1/4))/(10*b) - (9*a*((
x^5*(a + b*x^4)^(1/4))/(6*b) - (5*a*((x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a
]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])
/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(6*b)))/(10*b)))/14
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{12} \left(b\,x^{4}+a\right)^{\frac{1}{4}}d x$$

input
```
int(x^12*(b*x^4+a)^(1/4),x)
```

output
```
int(x^12*(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^{12} \sqrt[4]{a+bx^4}\, dx = \int \left(bx^{4}+a\right)^{\frac{1}{4}} x^{12}\, dx$$

input
```
integrate(x^12*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*x^12, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**13*gamma(13/4)*hyper((-1/4, 13/4), (17/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(17/4))
```

Maxima [F]

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx = \int (bx^4+a)^{\frac{1}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^12, x)
```

Giac [F]

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx = \int (bx^4+a)^{\frac{1}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx=\int x^{12}\,\left(b\,x^4+a\right)^{1/4}\,dx$$

input
```
int(x^12*(a + b*x^4)^(1/4),x)
```

output
```
int(x^12*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^{12}\sqrt[4]{a+bx^4}\,dx$$
$$=\frac{15(b\,x^4+a)^{\frac{1}{4}}\,a^3x-6(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^5+4(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^9+40(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{13}-15\left(\int\frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)}{560b^3}$$

input
```
int(x^12*(b*x^4+a)^(1/4),x)
```

output
```
(15*(a + b*x**4)**(1/4)*a**3*x - 6*(a + b*x**4)**(1/4)*a**2*b*x**5 + 4*(a
+ b*x**4)**(1/4)*a*b**2*x**9 + 40*(a + b*x**4)**(1/4)*b**3*x**13 - 15*int(
(a + b*x**4)**(1/4)/(a + b*x**4),x)*a**4)/(560*b**3)
```

3.465 $\int x^8 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = -\frac{a^2x\sqrt[4]{a+bx^4}}{24b^2}+\frac{ax^5\sqrt[4]{a+bx^4}}{60b}+\frac{1}{10}x^9\sqrt[4]{a+bx^4} -\frac{a^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/24*a^2*x*(b*x^4+a)^(1/4)/b^2+1/60*a*x^5*(b*x^4+a)^(1/4)/b+1/10*x^9*(b*x
^4+a)^(1/4)-1/24*a^(5/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(
b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \frac{x\sqrt[4]{a+bx^4}\left(\sqrt[4]{1+\frac{bx^4}{a}}\left(-5a^2+abx^4+6b^2x^8\right)+5a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{60b^2\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x^8*(a + b*x^4)^(1/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*((1 + (b*x^4)/a)^(1/4)*(-5*a^2 + a*b*x^4 + 6*b^2*x^8)
 + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^4)/a)]))/(60*b^2*(1 + (b
*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {811, 843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{10}a\int \frac{x^8}{(bx^4+a)^{3/4}}dx+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int \frac{x^4}{(bx^4+a)^{3/4}}dx}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

$\downarrow$ 858

$$\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

$\downarrow$ 807

$$\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

$\downarrow$ 229

$$\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}$$

input
```
Int[x^8*(a + b*x^4)^(1/4),x]
```

output
```
(x^9*(a + b*x^4)^(1/4))/10 + (a*((x^5*(a + b*x^4)^(1/4))/(6*b) - (5*a*((x*
(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[Ar
cTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(6*b)))
/10
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^8 \left(b\,x^4+a\right)^{\frac{1}{4}}dx$$

input
```
int(x^8*(b*x^4+a)^(1/4),x)
```

output
```
int(x^8*(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^8\,dx$$

input
```
integrate(x^8*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**9*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^8\,dx$$

input
```
integrate(x^8*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^8, x)
```

Giac [F]

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}} x^8\,dx$$

input
```
integrate(x^8*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \sqrt[4]{a+bx^4}\,dx = \int x^8 \left(b\,x^4+a\right)^{1/4}\,dx$$

input
```
int(x^8*(a + b*x^4)^(1/4),x)
```

output
```
int(x^8*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^8 \sqrt[4]{a+bx^4}\,dx$$
$$= \frac{-5(b\,x^4+a)^{\frac{1}{4}}\,a^2x+2(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^5+12(b\,x^4+a)^{\frac{1}{4}}\,b^2x^9+5\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^3}{120b^2}$$

input
```
int(x^8*(b*x^4+a)^(1/4),x)
```

output
```
( - 5*(a + b*x**4)**(1/4)*a**2*x + 2*(a + b*x**4)**(1/4)*a*b*x**5 + 12*(a
+ b*x**4)**(1/4)*b**2*x**9 + 5*int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**
3)/(120*b**2)
```

3.466 $\int x^4 \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int x^4 \sqrt[4]{a+bx^4}\,dx=\frac{ax\sqrt[4]{a+bx^4}}{12b}+\frac{1}{6}x^5\sqrt[4]{a+bx^4}+\frac{a^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
1/12*a*x*(b*x^4+a)^(1/4)/b+1/6*x^5*(b*x^4+a)^(1/4)+1/12*a^(3/2)*(1+a/b/x^4
)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/
2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int x^4 \sqrt[4]{a+bx^4}\,dx=\frac{x\sqrt[4]{a+bx^4}\left(a+bx^4-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{6b}$$

input
```
Integrate[x^4*(a + b*x^4)^(1/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*(a + b*x^4 - (a*Hypergeometric2F1[-1/4, 1/4, 5/4, -((
b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(6*b)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {811, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[4]{a+bx^4} \, dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{1}{6}a\int \frac{x^4}{(bx^4+a)^{3/4}}dx+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{6}a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{1}{6}a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{1}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{6}a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}$$

input
```
Int[x^4*(a + b*x^4)^(1/4),x]
```

output
```
(x^5*(a + b*x^4)^(1/4))/6 + (a*((x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1
+ a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*S
qrt[b]*(a + b*x^4)^(3/4))))/6
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{4} \left(b \,x^{4}+a \right)^{\frac{1}{4}}d x$$

input
```
int(x^4*(b*x^4+a)^(1/4),x)
```

output
```
int(x^4*(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^4 \sqrt[4]{a+bx^4} \, dx = \int { {\left(b x^{4} + a\right)}^{\frac{1}{4}} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int x^4 \sqrt[4]{a+bx^4}\,dx = \frac{\sqrt[4]{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4 \sqrt[4]{a+bx^4}\,dx = \int (bx^4+a)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^4, x)
```

Giac [F]

$$\int x^4 \sqrt[4]{a+bx^4}\,dx = \int (bx^4+a)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[4]{a+bx^4}\,dx = \int x^4 \left(b\,x^4+a\right)^{1/4}\,dx$$

input
```
int(x^4*(a + b*x^4)^(1/4),x)
```

output
```
int(x^4*(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^4 \sqrt[4]{a+bx^4}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,ax+2(b\,x^4+a)^{\frac{1}{4}}\,b\,x^5-\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{12b}$$

input
```
int(x^4*(b*x^4+a)^(1/4),x)
```

output
```
((a + b*x**4)**(1/4)*a*x + 2*(a + b*x**4)**(1/4)*b*x**5 - int((a + b*x**4)
**(1/4)/(a + b*x**4),x)*a**2)/(12*b)
```

3.467 $\int \sqrt[4]{a+bx^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 80

$$\int \sqrt[4]{a+bx^4}\,dx = \frac{1}{2}x\sqrt[4]{a+bx^4} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\left(a+bx^4\right)^{3/4}}$$

output
```
1/2*x*(b*x^4+a)^(1/4)-1/2*a^(1/2)*b^(1/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJac
obiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \sqrt[4]{a+bx^4}\,dx = \frac{x\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^4)/a)])/(1 +
 (b*x^4)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a+bx^4}\,dx$$

$$\Big\downarrow \text{748}$$

$$\frac{1}{2}a\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{768}$$

$$\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2\left(a+bx^4\right)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\left(a+bx^4\right)^{3/4}}$$

input
```
Int[(a + b*x^4)^(1/4),x]
```

output
```
(x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{1}{4}}dx$$

input
```
int((b*x^4+a)^(1/4),x)
```

output
```
int((b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{a+bx^4}\,dx=\int \left(bx^4+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \sqrt[4]{a+bx^4}\,dx=\frac{\sqrt[4]{a}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a+bx^4}\,dx = \int \left(bx^4+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \sqrt[4]{a+bx^4}\,dx = \frac{x\left(b\,x^4+a\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\,\frac{1}{4};\;\frac{5}{4};\;-\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^{1/4}}$$

input
```
int((a + b*x^4)^(1/4),x)
```

output
```
(x*(a + b*x^4)^(1/4)*hypergeom([-1/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
 1)^(1/4)
```

Reduce [F]

$$\int \sqrt[4]{a+bx^4}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,x}{2} + \frac{\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a}{2}$$

input
```
int((b*x^4+a)^(1/4),x)
```

output
```
((a + b*x**4)**(1/4)*x + int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a)/2
```

3.468 $\int \frac{\sqrt[4]{a+bx^4}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4} dx = -\frac{\sqrt[4]{a+bx^4}}{3x^3} - \frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/3*(b*x^4+a)^(1/4)/x^3-1/3*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM
(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4} dx = -\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{1}{4},-\frac{bx^4}{a}\right)}{3x^3\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^4,x]
```

output
```
-1/3*((a + b*x^4)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x^4)/a)])/
(x^3*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {809, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{3}b\int \frac{1}{(bx^4+a)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

$$\Big\downarrow \text{768}$$

$$\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

$$\Big\downarrow \text{858}$$

$$-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

$$\Big\downarrow \text{807}$$

$$-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{6\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

$$\Big\downarrow \text{229}$$

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}$$

input
```
Int[(a + b*x^4)^(1/4)/x^4,x]
```

output
```
-1/3*(a + b*x^4)^(1/4)/x^3 - (b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[
ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*Sqrt[a]*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{x^4}dx$$

input
```
int((b*x^4+a)^(1/4)/x^4,x)
```

output
```
int((b*x^4+a)^(1/4)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx = -\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{2x^2}$$

input
```
integrate((b*x**4+a)**(1/4)/x**4,x)
```

output
```
-b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**4))/(2*x**2)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx = \int \frac{(b\,x^4+a)^{1/4}}{x^4}\,dx$$

input
```
int((a + b*x^4)^(1/4)/x^4,x)
```

output
```
int((a + b*x^4)^(1/4)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^4}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}} - \left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^8+a\,x^4}dx\right)a\,x^3}{2x^3}$$

input
```
int((b*x^4+a)^(1/4)/x^4,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**4 + b*x**8),x)*a*
x**3))/(2*x**3)
```

3.469 $\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx=-\frac{\sqrt[4]{a+bx^4}}{7x^7}-\frac{b\sqrt[4]{a+bx^4}}{21ax^3}+\frac{2b^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/7*(b*x^4+a)^(1/4)/x^7-1/21*b*(b*x^4+a)^(1/4)/a/x^3+2/21*b^(5/2)*(1+a/b/
x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^
(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx=-\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},-\frac{3}{4},-\frac{bx^4}{a}\right)}{7x^7\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^8,x]
```

output
```
-1/7*((a + b*x^4)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x^4)/a)])
/(x^7*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {809, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{7}b\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{7}b\left(-\frac{2b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{7}b\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{7}b\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{7}b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{7}b\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}$$

input
```
Int[(a + b*x^4)^(1/4)/x^8,x]
```

output
```
-1/7*(a + b*x^4)^(1/4)/x^7 + (b*(-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(3/
2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(3*a^(3/2)*(a + b*x^4)^(3/4))))/7
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{1}{4}}}{x^{8}}dx$$

input
```
int((b*x^4+a)^(1/4)/x^8,x)
```

output
```
int((b*x^4+a)^(1/4)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx=\int \frac{(bx^4+a)^{\frac{1}{4}}}{x^8}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx = -\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{6x^6}$$

input
```
integrate((b*x**4+a)**(1/4)/x**8,x)
```

output
```
-b**(1/4)*hyper((-1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*x**6)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^8}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8} dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^8} dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8} dx = \int \frac{(b\,x^4+a)^{1/4}}{x^8} dx$$

input
```
int((a + b*x^4)^(1/4)/x^8,x)
```

output
```
int((a + b*x^4)^(1/4)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^8} dx = \frac{-(b\,x^4+a)^{\frac{1}{4}} - \left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{12}+a\,x^8}dx\right) a\,x^7}{6x^7}$$

input
```
int((b*x^4+a)^(1/4)/x^8,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**8 + b*x**12),x)*a
*x**7))/(6*x**7)
```

3.470 $\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{11x^{11}} - \frac{b\sqrt[4]{a+bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a+bx^4}}{77a^2x^3} - \frac{4b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77a^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/11*(b*x^4+a)^(1/4)/x^11-1/77*b*(b*x^4+a)^(1/4)/a/x^7+2/77*b^2*(b*x^4+a)
^(1/4)/a^2/x^3-4/77*b^(7/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arcc
ot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = -\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},-\frac{bx^4}{a}\right)}{11x^{11}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^12,x]
```

output
```
-1/11*((a + b*x^4)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x^4)/a)
])/(x^11*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {809, 847, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{11}b\int \frac{1}{x^8\left(bx^4+a\right)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{11}b\left(-\frac{6b\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{11}b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{11} b \left(-\frac{6b \left(-\frac{2bx^3 \left(\frac{a}{bx^4}+1\right)^{3/4} \int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4} x^3} dx}{3a(a+bx^4)^{3/4}} - \frac{\sqrt[4]{a+bx^4}}{3ax^3} \right)}{7a} - \frac{\sqrt[4]{a+bx^4}}{7ax^7} \right) - \frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

$\downarrow$ 858

$$\frac{1}{11} b \left(-\frac{6b \left(\frac{2bx^3 \left(\frac{a}{bx^4}+1\right)^{3/4} \int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4} x} d\frac{1}{x}}{3a(a+bx^4)^{3/4}} - \frac{\sqrt[4]{a+bx^4}}{3ax^3} \right)}{7a} - \frac{\sqrt[4]{a+bx^4}}{7ax^7} \right) - \frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

$\downarrow$ 807

$$\frac{1}{11} b \left(-\frac{6b \left(\frac{bx^3 \left(\frac{a}{bx^4}+1\right)^{3/4} \int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}} d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}} - \frac{\sqrt[4]{a+bx^4}}{3ax^3} \right)}{7a} - \frac{\sqrt[4]{a+bx^4}}{7ax^7} \right) - \frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

$\downarrow$ 229

$$\frac{1}{11} b \left(-\frac{6b \left(\frac{2b^{3/2}x^3 \left(\frac{a}{bx^4}+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}} - \frac{\sqrt[4]{a+bx^4}}{3ax^3} \right)}{7a} - \frac{\sqrt[4]{a+bx^4}}{7ax^7} \right) - \frac{\sqrt[4]{a+bx^4}}{11x^{11}}$$

input
```
Int[(a + b*x^4)^(1/4)/x^12,x]
```

output
```
-1/11*(a + b*x^4)^(1/4)/x^11 + (b*(-1/7*(a + b*x^4)^(1/4)/(a*x^7) - (6*b*(
-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*Elli
pticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))
/(7*a)))/11
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{x^{12}}dx$$

input
```
int((b*x^4+a)^(1/4)/x^12,x)
```

output
```
int((b*x^4+a)^(1/4)/x^12,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^12,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^12, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = \frac{\sqrt[4]{a}\Gamma\left(-\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{11}{4},-\frac{1}{4}\\-\frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{11}\Gamma\left(-\frac{7}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**12,x)
```

output
```
a**(1/4)*gamma(-11/4)*hyper((-11/4, -1/4), (-7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**11*gamma(-7/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^12,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^12, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^12,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx=\int \frac{{\left(b\,x^4+a\right)}^{1/4}}{x^{12}} \,d x$$

input
```
int((a + b*x^4)^(1/4)/x^12,x)
```

output
```
int((a + b*x^4)^(1/4)/x^12, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{12}}\,dx=\frac{-\left(b\,x^{4}+a \right)^{\frac{1}{4}}-\left(\int \frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}}}{b\,x^{16}+a\,x^{12}}d x \right) a\,x^{11}}{10 x^{11}}$$

input
```
int((b*x^4+a)^(1/4)/x^12,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**12 + b*x**16),x)*
a*x**11))/(10*x**11)
```

3.471 $\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{15x^{15}} - \frac{b\sqrt[4]{a+bx^4}}{165ax^{11}} + \frac{2b^2\sqrt[4]{a+bx^4}}{231a^2x^7} - \frac{4b^3\sqrt[4]{a+bx^4}}{231a^3x^3} + \frac{8b^{9/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231a^{7/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/15*(b*x^4+a)^(1/4)/x^15-1/165*b*(b*x^4+a)^(1/4)/a/x^11+2/231*b^2*(b*x^4
+a)^(1/4)/a^2/x^7-4/231*b^3*(b*x^4+a)^(1/4)/a^3/x^3+8/231*b^(9/2)*(1+a/b/x
^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(
7/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = -\frac{\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{1}{4},-\frac{11}{4},-\frac{bx^4}{a}\right)}{15x^{15}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(1/4)/x^16,x]
```

output
```
-1/15*((a + b*x^4)^(1/4)*Hypergeometric2F1[-15/4, -1/4, -11/4, -((b*x^4)/a
)])/(x^15*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {809, 847, 847, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{15}b\int \frac{1}{x^{12}\left(bx^4+a\right)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{15}b\left(-\frac{10b\int \frac{1}{x^8\left(bx^4+a\right)^{3/4}}dx}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{2b\int\frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$\downarrow$ 768

$$\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$\downarrow$ 858

$$\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$$\Big\downarrow \; 807$$

$$\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

$$\Big\downarrow \; 229$$

$$\frac{1}{15}b\left(-\frac{10b\left(-\frac{6b\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a+bx^4}}{15x^{15}}$$

input
```
Int[(a + b*x^4)^(1/4)/x^16,x]
```

output
```
-1/15*(a + b*x^4)^(1/4)/x^15 + (b*(-1/11*(a + b*x^4)^(1/4)/(a*x^11) - (10*
b*(-1/7*(a + b*x^4)^(1/4)/(a*x^7) - (6*b*(-1/3*(a + b*x^4)^(1/4)/(a*x^3) +
 (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^
2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/(7*a)))/(11*a)))/15
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{1}{4}}}{x^{16}}dx$$

input
```
int((b*x^4+a)^(1/4)/x^16,x)
```

output
```
int((b*x^4+a)^(1/4)/x^16,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^16,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/x^16, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = \frac{\sqrt[4]{a}\Gamma\left(-\frac{15}{4}\right){}_2F_1\left(\begin{matrix}-\frac{15}{4},-\frac{1}{4}\\-\frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{15}\Gamma\left(-\frac{11}{4}\right)}$$

input
```
integrate((b*x**4+a)**(1/4)/x**16,x)
```

output
```
a**(1/4)*gamma(-15/4)*hyper((-15/4, -1/4), (-11/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**15*gamma(-11/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^16,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^16, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{1}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(1/4)/x^16,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(1/4)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = \int \frac{(b\,x^4+a)^{1/4}}{x^{16}}\,dx$$

input
```
int((a + b*x^4)^(1/4)/x^16,x)
```

output
```
int((a + b*x^4)^(1/4)/x^16, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^4}}{x^{16}}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}} - \left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{20}+a\,x^{16}}dx\right) a\,x^{15}}{14x^{15}}$$

input
```
int((b*x^4+a)^(1/4)/x^16,x)
```

output
```
( - ((a + b*x**4)**(1/4) + int((a + b*x**4)**(1/4)/(a*x**16 + b*x**20),x)*
a*x**15))/(14*x**15)
```

3.472 $\int x^{19}(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^{19}(a+bx^4)^{3/4}\,dx=\frac{a^4(a+bx^4)^{7/4}}{7b^5}-\frac{4a^3(a+bx^4)^{11/4}}{11b^5}+\frac{2a^2(a+bx^4)^{15/4}}{5b^5}-\frac{4a(a+bx^4)^{19/4}}{19b^5}+\frac{(a+bx^4)^{23/4}}{23b^5}$$

output
```
1/7*a^4*(b*x^4+a)^(7/4)/b^5-4/11*a^3*(b*x^4+a)^(11/4)/b^5+2/5*a^2*(b*x^4+a
)^(15/4)/b^5-4/19*a*(b*x^4+a)^(19/4)/b^5+1/23*(b*x^4+a)^(23/4)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int x^{19}(a+bx^4)^{3/4}\,dx=\frac{(a+bx^4)^{7/4}(2048a^4-3584a^3bx^4+4928a^2b^2x^8-6160ab^3x^{12}+7315b^4x^{16})}{168245b^5}$$

input
```
Integrate[x^19*(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(7/4)*(2048*a^4 - 3584*a^3*b*x^4 + 4928*a^2*b^2*x^8 - 6160*a*
b^3*x^12 + 7315*b^4*x^16))/(168245*b^5)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{19}\left(a+bx^4\right)^{3/4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^{16}\left(bx^4+a\right)^{3/4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{19/4}}{b^4}-\frac{4a\left(bx^4+a\right)^{15/4}}{b^4}+\frac{6a^2\left(bx^4+a\right)^{11/4}}{b^4}-\frac{4a^3\left(bx^4+a\right)^{7/4}}{b^4}+\frac{a^4\left(bx^4+a\right)^{3/4}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^4\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{16a^3\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{8a^2\left(a+bx^4\right)^{15/4}}{5b^5}+\frac{4\left(a+bx^4\right)^{23/4}}{23b^5}-\frac{16a\left(a+bx^4\right)^{19/4}}{19b^5}\right)$$

input
```
Int[x^19*(a + b*x^4)^(3/4),x]
```

output
```
((4*a^4*(a + b*x^4)^(7/4))/(7*b^5) - (16*a^3*(a + b*x^4)^(11/4))/(11*b^5)
+ (8*a^2*(a + b*x^4)^(15/4))/(5*b^5) - (16*a*(a + b*x^4)^(19/4))/(19*b^5)
+ (4*(a + b*x^4)^(23/4))/(23*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{7}{4}}(7315x^{16}b^4-6160a\,b^3x^{12}+4928a^2b^2x^8-3584a^3b\,x^4+2048a^4)}{168245b^5}$	58
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{7}{4}}(7315x^{16}b^4-6160a\,b^3x^{12}+4928a^2b^2x^8-3584a^3b\,x^4+2048a^4)}{168245b^5}$	58
orering	$\frac{(b\,x^4+a)^{\frac{7}{4}}(7315x^{16}b^4-6160a\,b^3x^{12}+4928a^2b^2x^8-3584a^3b\,x^4+2048a^4)}{168245b^5}$	58
trager	$\frac{(7315b^5x^{20}+1155a\,b^4x^{16}-1232a^2b^3x^{12}+1344a^3b^2x^8-1536a^4b\,x^4+2048a^5)(b\,x^4+a)^{\frac{3}{4}}}{168245b^5}$	69
risch	$\frac{(7315b^5x^{20}+1155a\,b^4x^{16}-1232a^2b^3x^{12}+1344a^3b^2x^8-1536a^4b\,x^4+2048a^5)(b\,x^4+a)^{\frac{3}{4}}}{168245b^5}$	69

input
```
int(x^19*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/168245*(b*x^4+a)^(7/4)*(7315*b^4*x^16-6160*a*b^3*x^12+4928*a^2*b^2*x^8-3
584*a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int x^{19}\left(a+bx^4\right)^{3/4}\,dx=\frac{(7315\,b^5x^{20}+1155\,ab^4x^{16}-1232\,a^2b^3x^{12}+1344\,a^3b^2x^8-1536\,a^4bx^4+2048\,a^5)(bx^4+a)^{\frac{3}{4}}}{168245\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/168245*(7315*b^5*x^20 + 1155*a*b^4*x^16 - 1232*a^2*b^3*x^12 + 1344*a^3*b
^2*x^8 - 1536*a^4*b*x^4 + 2048*a^5)*(b*x^4 + a)^(3/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int x^{19}\left(a+bx^4\right)^{3/4}\,dx=\begin{cases}\frac{2048a^5(a+bx^4)^{\frac{3}{4}}}{168245b^5}-\frac{1536a^4x^4(a+bx^4)^{\frac{3}{4}}}{168245b^4}+\frac{192a^3x^8(a+bx^4)^{\frac{3}{4}}}{24035b^3}-\frac{16a^2x^{12}(a+bx^4)^{\frac{3}{4}}}{2185b^2}+\frac{3ax^{16}(a+bx^4)^{\frac{3}{4}}}{437b}+\frac{x^{20}(a+\dots}{23\dots} \\ \frac{a^{\frac{3}{4}}x^{20}}{20}\end{cases}$$

input
```
integrate(x**19*(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((2048*a**5*(a + b*x**4)**(3/4)/(168245*b**5) - 1536*a**4*x**4*(a
 + b*x**4)**(3/4)/(168245*b**4) + 192*a**3*x**8*(a + b*x**4)**(3/4)/(24035
*b**3) - 16*a**2*x**12*(a + b*x**4)**(3/4)/(2185*b**2) + 3*a*x**16*(a + b*
x**4)**(3/4)/(437*b) + x**20*(a + b*x**4)**(3/4)/23, Ne(b, 0)), (a**(3/4)*
x**20/20, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int x^{19}\left(a+bx^4\right)^{3/4}\,dx=\frac{\left(bx^4+a\right)^{\frac{23}{4}}}{23\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{19}{4}}a}{19\,b^5}+\frac{2\left(bx^4+a\right)^{\frac{15}{4}}a^2}{5\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{11}{4}}a^3}{11\,b^5}+\frac{\left(bx^4+a\right)^{\frac{7}{4}}a^4}{7\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/23*(b*x^4 + a)^(23/4)/b^5 - 4/19*(b*x^4 + a)^(19/4)*a/b^5 + 2/5*(b*x^4 +
 a)^(15/4)*a^2/b^5 - 4/11*(b*x^4 + a)^(11/4)*a^3/b^5 + 1/7*(b*x^4 + a)^(7/
4)*a^4/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.61

$$\int x^{19}\left(a+bx^4\right)^{3/4}\,dx=\frac{\frac{23\left(1155\left(bx^4+a\right)^{\frac{19}{4}}-5852\left(bx^4+a\right)^{\frac{15}{4}}a+11970\left(bx^4+a\right)^{\frac{11}{4}}a^2-12540\left(bx^4+a\right)^{\frac{7}{4}}a^3+7315\left(bx^4+a\right)^{\frac{3}{4}}a^4\right)a}{b^4}+\frac{5\left(4389\left(bx^4+\right.\right.}{}}{504735\,b}$$

input
```
integrate(x^19*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/504735*(23*(1155*(b*x^4 + a)^(19/4) - 5852*(b*x^4 + a)^(15/4)*a + 11970*
(b*x^4 + a)^(11/4)*a^2 - 12540*(b*x^4 + a)^(7/4)*a^3 + 7315*(b*x^4 + a)^(3
/4)*a^4)*a/b^4 + 5*(4389*(b*x^4 + a)^(23/4) - 26565*(b*x^4 + a)^(19/4)*a +
 67298*(b*x^4 + a)^(15/4)*a^2 - 91770*(b*x^4 + a)^(11/4)*a^3 + 72105*(b*x^
4 + a)^(7/4)*a^4 - 33649*(b*x^4 + a)^(3/4)*a^5)/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int x^{19}(a+bx^4)^{3/4}\,dx=(b\,x^4+a)^{3/4}\left(\frac{x^{20}}{23}+\frac{2048\,a^5}{168245\,b^5}+\frac{3\,a\,x^{16}}{437\,b}-\frac{1536\,a^4\,x^4}{168245\,b^4}+\frac{192\,a^3\,x^8}{24035\,b^3}-\frac{16\,a^2\,x^{12}}{2185\,b^2}\right)$$

input
```
int(x^19*(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(3/4)*(x^20/23 + (2048*a^5)/(168245*b^5) + (3*a*x^16)/(437*b)
- (1536*a^4*x^4)/(168245*b^4) + (192*a^3*x^8)/(24035*b^3) - (16*a^2*x^12)/
(2185*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 557, normalized size of antiderivative = 5.51

$$\int x^{19}(a+bx^4)^{3/4}\,dx=\frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\left(22528\sqrt{b}\,\sqrt{b\,x^4+a}\,a^{10}x^2+433664\sqrt{b}\,\sqrt{b\,x^4+a}\,a^9b\,x^6+220\right.}{}$$

input
```
int(x^19*(b*x^4+a)^(3/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(22528*sqrt(b)*sqrt(a +
b*x**4)*a**10*x**2 + 433664*sqrt(b)*sqrt(a + b*x**4)*a**9*b*x**6 + 2200000
*sqrt(b)*sqrt(a + b*x**4)*a**8*b**2*x**10 + 4156944*sqrt(b)*sqrt(a + b*x**
4)*a**7*b**3*x**14 + 2839265*sqrt(b)*sqrt(a + b*x**4)*a**6*b**4*x**18 + 37
3221*sqrt(b)*sqrt(a + b*x**4)*a**5*b**5*x**22 + 1774788*sqrt(b)*sqrt(a + b
*x**4)*a**4*b**6*x**26 + 10171504*sqrt(b)*sqrt(a + b*x**4)*a**3*b**7*x**30
 + 22589952*sqrt(b)*sqrt(a + b*x**4)*a**2*b**8*x**34 + 21781760*sqrt(b)*sq
rt(a + b*x**4)*a*b**9*x**38 + 7490560*sqrt(b)*sqrt(a + b*x**4)*b**10*x**42
 + 2048*a**11 + 123392*a**10*b*x**4 + 1177408*a**9*b**2*x**8 + 3945328*a**
8*b**3*x**12 + 5535219*a**7*b**4*x**16 + 2999306*a**6*b**5*x**20 + 755019*
a**5*b**6*x**24 + 5105556*a**4*b**7*x**28 + 19211920*a**3*b**8*x**32 + 325
44512*a**2*b**9*x**36 + 25527040*a*b**10*x**40 + 7490560*b**11*x**44))/(16
8245*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**5*(sqrt(a + b*x**4)*a**5 + 6
0*sqrt(a + b*x**4)*a**4*b*x**4 + 560*sqrt(a + b*x**4)*a**3*b**2*x**8 + 179
2*sqrt(a + b*x**4)*a**2*b**3*x**12 + 2304*sqrt(a + b*x**4)*a*b**4*x**16 +
1024*sqrt(a + b*x**4)*b**5*x**20 + 11*sqrt(b)*a**5*x**2 + 220*sqrt(b)*a**4
*b*x**6 + 1232*sqrt(b)*a**3*b**2*x**10 + 2816*sqrt(b)*a**2*b**3*x**14 + 28
16*sqrt(b)*a*b**4*x**18 + 1024*sqrt(b)*b**5*x**22))
```

3.473 $\int x^{15}(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{15}(a+bx^4)^{3/4}\,dx = -\frac{a^3(a+bx^4)^{7/4}}{7b^4} + \frac{3a^2(a+bx^4)^{11/4}}{11b^4} - \frac{a(a+bx^4)^{15/4}}{5b^4} + \frac{(a+bx^4)^{19/4}}{19b^4}$$

output
```
-1/7*a^3*(b*x^4+a)^(7/4)/b^4+3/11*a^2*(b*x^4+a)^(11/4)/b^4-1/5*a*(b*x^4+a)
^(15/4)/b^4+1/19*(b*x^4+a)^(19/4)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^{15}(a+bx^4)^{3/4}\,dx = \frac{(a+bx^4)^{7/4}(-128a^3+224a^2bx^4-308ab^2x^8+385b^3x^{12})}{7315b^4}$$

input
```
Integrate[x^15*(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(7/4)*(-128*a^3 + 224*a^2*b*x^4 - 308*a*b^2*x^8 + 385*b^3*x^1
2))/(7315*b^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15}\left(a+bx^4\right)^{3/4}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int x^{12}\left(bx^4+a\right)^{3/4}dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4}\int\left(\frac{\left(bx^4+a\right)^{15/4}}{b^3}-\frac{3a\left(bx^4+a\right)^{11/4}}{b^3}+\frac{3a^2\left(bx^4+a\right)^{7/4}}{b^3}-\frac{a^3\left(bx^4+a\right)^{3/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{4a^3\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{12a^2\left(a+bx^4\right)^{11/4}}{11b^4}+\frac{4\left(a+bx^4\right)^{19/4}}{19b^4}-\frac{4a\left(a+bx^4\right)^{15/4}}{5b^4}\right)$$

input
```
Int[x^15*(a + b*x^4)^(3/4),x]
```

output
```
((-4*a^3*(a + b*x^4)^(7/4))/(7*b^4) + (12*a^2*(a + b*x^4)^(11/4))/(11*b^4)
 - (4*a*(a + b*x^4)^(15/4))/(5*b^4) + (4*(a + b*x^4)^(19/4))/(19*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}}\left(-385b^{3}x^{12}+308a\,b^{2}x^{8}-224a^{2}b\,x^{4}+128a^{3}\right)}{7315b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}}\left(-385b^{3}x^{12}+308a\,b^{2}x^{8}-224a^{2}b\,x^{4}+128a^{3}\right)}{7315b^{4}}$	47
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}}\left(-385b^{3}x^{12}+308a\,b^{2}x^{8}-224a^{2}b\,x^{4}+128a^{3}\right)}{7315b^{4}}$	47
trager	$-\frac{\left(-385x^{16}b^{4}-77a\,b^{3}x^{12}+84a^{2}b^{2}x^{8}-96a^{3}b\,x^{4}+128a^{4}\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{7315b^{4}}$	58
risch	$-\frac{\left(-385x^{16}b^{4}-77a\,b^{3}x^{12}+84a^{2}b^{2}x^{8}-96a^{3}b\,x^{4}+128a^{4}\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{7315b^{4}}$	58

input
```
int(x^15*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/7315*(b*x^4+a)^(7/4)*(-385*b^3*x^12+308*a*b^2*x^8-224*a^2*b*x^4+128*a^3
)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{15}\left(a+bx^4\right)^{3/4}\,dx=\frac{\left(385\,b^4x^{16}+77\,ab^3x^{12}-84\,a^2b^2x^8+96\,a^3bx^4-128\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{7315\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/7315*(385*b^4*x^16 + 77*a*b^3*x^12 - 84*a^2*b^2*x^8 + 96*a^3*b*x^4 - 128
*a^4)*(b*x^4 + a)^(3/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int x^{15}(a+bx^4)^{3/4}\,dx=\begin{cases}-\frac{128a^4(a+bx^4)^{\frac{3}{4}}}{7315b^4}+\frac{96a^3x^4(a+bx^4)^{\frac{3}{4}}}{7315b^3}-\frac{12a^2x^8(a+bx^4)^{\frac{3}{4}}}{1045b^2}+\frac{ax^{12}(a+bx^4)^{\frac{3}{4}}}{95b}+\frac{x^{16}(a+bx^4)^{\frac{3}{4}}}{19} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{4}}x^{16}}{16} & \text{otherwise}\end{cases}$$

input
```
integrate(x**15*(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-128*a**4*(a + b*x**4)**(3/4)/(7315*b**4) + 96*a**3*x**4*(a + b
*x**4)**(3/4)/(7315*b**3) - 12*a**2*x**8*(a + b*x**4)**(3/4)/(1045*b**2) +
 a*x**12*(a + b*x**4)**(3/4)/(95*b) + x**16*(a + b*x**4)**(3/4)/19, Ne(b,
0)), (a**(3/4)*x**16/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{15}(a+bx^4)^{3/4}\,dx=\frac{(bx^4+a)^{\frac{19}{4}}}{19\,b^4}-\frac{(bx^4+a)^{\frac{15}{4}}a}{5\,b^4}+\frac{3\,(bx^4+a)^{\frac{11}{4}}a^2}{11\,b^4}-\frac{(bx^4+a)^{\frac{7}{4}}a^3}{7\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/19*(b*x^4 + a)^(19/4)/b^4 - 1/5*(b*x^4 + a)^(15/4)*a/b^4 + 3/11*(b*x^4 +
 a)^(11/4)*a^2/b^4 - 1/7*(b*x^4 + a)^(7/4)*a^3/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(64) = 128$.

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int x^{15}\left(a+bx^4\right)^{3/4}\,dx = \frac{\frac{19\left(77\,(bx^4+a)^{\frac{15}{4}}-315\,(bx^4+a)^{\frac{11}{4}}a+495\,(bx^4+a)^{\frac{7}{4}}a^2-385\,(bx^4+a)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\,(bx^4+a)^{\frac{19}{4}}-5852\,(bx^4+a)^{\frac{15}{4}}a+1197}{}}{21945\,b}$$

input
```
integrate(x^15*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/21945*(19*(77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 495*(b*x^4
 + a)^(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^4 + a)^(19
/4) - 5852*(b*x^4 + a)^(15/4)*a + 11970*(b*x^4 + a)^(11/4)*a^2 - 12540*(b*
x^4 + a)^(7/4)*a^3 + 7315*(b*x^4 + a)^(3/4)*a^4)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^{15}\left(a+bx^4\right)^{3/4}\,dx = \left(b\,x^4+a\right)^{3/4}\left(\frac{x^{16}}{19}-\frac{128\,a^4}{7315\,b^4}+\frac{a\,x^{12}}{95\,b}+\frac{96\,a^3\,x^4}{7315\,b^3}-\frac{12\,a^2\,x^8}{1045\,b^2}\right)$$

input
```
int(x^15*(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(3/4)*(x^16/19 - (128*a^4)/(7315*b^4) + (a*x^12)/(95*b) + (96*
a^3*x^4)/(7315*b^3) - (12*a^2*x^8)/(1045*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 461, normalized size of antiderivative = 5.76

$$\int x^{15}(a+bx^4)^{3/4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,\left(-1152\sqrt{b}\,\sqrt{b\,x^4+a}\,a^8x^2-14496\sqrt{b}\,\sqrt{b\,x^4+a}\,a^7b\,x^6-4453\right.}{}$$

input

```
int(x^15*(b*x^4+a)^(3/4),x)
```

output

```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*( - 1152*sqrt(b)*sqrt(a
+ b*x**4)*a**8*x**2 - 14496*sqrt(b)*sqrt(a + b*x**4)*a**7*b*x**6 - 44532*s
qrt(b)*sqrt(a + b*x**4)*a**6*b**2*x**10 - 41643*sqrt(b)*sqrt(a + b*x**4)*a
**5*b**3*x**14 - 1055*sqrt(b)*sqrt(a + b*x**4)*a**4*b**4*x**18 + 55656*sqr
t(b)*sqrt(a + b*x**4)*a**3*b**5*x**22 + 189168*sqrt(b)*sqrt(a + b*x**4)*a*
*2*b**6*x**26 + 241472*sqrt(b)*sqrt(a + b*x**4)*a*b**7*x**30 + 98560*sqrt(
b)*sqrt(a + b*x**4)*b**8*x**34 - 128*a**9 - 5152*a**8*b*x**4 - 31988*a**7*
b**2*x**8 - 64551*a**6*b**3*x**12 - 44042*a**5*b**4*x**16 + 14369*a**4*b**
5*x**20 + 126216*a**3*b**6*x**24 + 297584*a**2*b**7*x**28 + 290752*a*b**8*
x**32 + 98560*b**9*x**36))/(7315*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**
4*(sqrt(a + b*x**4)*a**4 + 40*sqrt(a + b*x**4)*a**3*b*x**4 + 240*sqrt(a +
b*x**4)*a**2*b**2*x**8 + 448*sqrt(a + b*x**4)*a*b**3*x**12 + 256*sqrt(a +
b*x**4)*b**4*x**16 + 9*sqrt(b)*a**4*x**2 + 120*sqrt(b)*a**3*b*x**6 + 432*s
qrt(b)*a**2*b**2*x**10 + 576*sqrt(b)*a*b**3*x**14 + 256*sqrt(b)*b**4*x**18
))
```

3.474 $\int x^{11}(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^{11}\left(a+bx^4\right)^{3/4}\,dx=\frac{a^2(a+bx^4)^{7/4}}{7b^3}-\frac{2a(a+bx^4)^{11/4}}{11b^3}+\frac{(a+bx^4)^{15/4}}{15b^3}$$

output
```
1/7*a^2*(b*x^4+a)^(7/4)/b^3-2/11*a*(b*x^4+a)^(11/4)/b^3+1/15*(b*x^4+a)^(15
/4)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^{11}\left(a+bx^4\right)^{3/4}\,dx=\frac{(a+bx^4)^{7/4}\,(32a^2-56abx^4+77b^2x^8)}{1155b^3}$$

input
```
Integrate[x^11*(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(7/4)*(32*a^2 - 56*a*b*x^4 + 77*b^2*x^8))/(1155*b^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} \left(a+b x^4\right)^{3/4} \, dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4} \int x^8 \left(b x^4+a\right)^{3/4}dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4} \int \left(\frac{\left(b x^4+a\right)^{11/4}}{b^2}-\frac{2 a \left(b x^4+a\right)^{7/4}}{b^2}+\frac{a^2 \left(b x^4+a\right)^{3/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4} \left(\frac{4 a^2 \left(a+b x^4\right)^{7/4}}{7 b^3}+\frac{4 \left(a+b x^4\right)^{15/4}}{15 b^3}-\frac{8 a \left(a+b x^4\right)^{11/4}}{11 b^3}\right)$$

input
```
Int[x^11*(a + b*x^4)^(3/4),x]
```

output
```
((4*a^2*(a + b*x^4)^(7/4))/(7*b^3) - (8*a*(a + b*x^4)^(11/4))/(11*b^3) + (
4*(a + b*x^4)^(15/4))/(15*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{7}{4}}(77b^2x^8-56ab\,x^4+32a^2)}{1155b^3}$	36
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{7}{4}}(77b^2x^8-56ab\,x^4+32a^2)}{1155b^3}$	36
orering	$\frac{(b\,x^4+a)^{\frac{7}{4}}(77b^2x^8-56ab\,x^4+32a^2)}{1155b^3}$	36
trager	$\frac{(77b^3x^{12}+21a\,b^2x^8-24a^2b\,x^4+32a^3)(b\,x^4+a)^{\frac{3}{4}}}{1155b^3}$	47
risch	$\frac{(77b^3x^{12}+21a\,b^2x^8-24a^2b\,x^4+32a^3)(b\,x^4+a)^{\frac{3}{4}}}{1155b^3}$	47

input
```
int(x^11*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/1155*(b*x^4+a)^(7/4)*(77*b^2*x^8-56*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^{11}\left(a+bx^4\right)^{3/4}\,dx=\frac{(77\,b^3x^{12}+21\,ab^2x^8-24\,a^2bx^4+32\,a^3)(bx^4+a)^{\frac{3}{4}}}{1155\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/1155*(77*b^3*x^12 + 21*a*b^2*x^8 - 24*a^2*b*x^4 + 32*a^3)*(b*x^4 + a)^(3
/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x^{11}(a+bx^4)^{3/4}\,dx = \begin{cases} \frac{32a^3(a+bx^4)^{\frac{3}{4}}}{1155b^3} - \frac{8a^2x^4(a+bx^4)^{\frac{3}{4}}}{385b^2} + \frac{ax^8(a+bx^4)^{\frac{3}{4}}}{55b} + \frac{x^{12}(a+bx^4)^{\frac{3}{4}}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{4}}x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((32*a**3*(a + b*x**4)**(3/4)/(1155*b**3) - 8*a**2*x**4*(a + b*x*
*4)**(3/4)/(385*b**2) + a*x**8*(a + b*x**4)**(3/4)/(55*b) + x**12*(a + b*x
**4)**(3/4)/15, Ne(b, 0)), (a**(3/4)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^{11}(a+bx^4)^{3/4}\,dx = \frac{(bx^4+a)^{\frac{15}{4}}}{15\,b^3} - \frac{2\,(bx^4+a)^{\frac{11}{4}}a}{11\,b^3} + \frac{(bx^4+a)^{\frac{7}{4}}a^2}{7\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/15*(b*x^4 + a)^(15/4)/b^3 - 2/11*(b*x^4 + a)^(11/4)*a/b^3 + 1/7*(b*x^4 +
 a)^(7/4)*a^2/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(47) = 94$.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.80

$$\int x^{11}\left(a+bx^4\right)^{3/4}\,dx = \frac{\frac{5\left(21\,(bx^4+a)^{\frac{11}{4}} - 66\,(bx^4+a)^{\frac{7}{4}}a + 77\,(bx^4+a)^{\frac{3}{4}}a^2\right)a}{b^2} + \frac{77\,(bx^4+a)^{\frac{15}{4}} - 315\,(bx^4+a)^{\frac{11}{4}}a + 495\,(bx^4+a)^{\frac{7}{4}}a^2 - 385\,(bx^4+a)}{b^2}}{1155\,b}$$

input
```
integrate(x^11*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/1155*(5*(21*(b*x^4 + a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)
^(3/4)*a^2)*a/b^2 + (77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 49
5*(b*x^4 + a)^(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)/b^2)/b
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^{11}\left(a+bx^4\right)^{3/4}\,dx = \left(b\,x^4+a\right)^{3/4}\left(\frac{x^{12}}{15} + \frac{32\,a^3}{1155\,b^3} + \frac{a\,x^8}{55\,b} - \frac{8\,a^2\,x^4}{385\,b^2}\right)$$

input
```
int(x^11*(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(3/4)*(x^12/15 + (32*a^3)/(1155*b^3) + (a*x^8)/(55*b) - (8*a^2
*x^4)/(385*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 365, normalized size of antiderivative = 6.19

$$\int x^{11}(a+bx^4)^{3/4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,\left(224\sqrt{b}\,\sqrt{b\,x^4+a}\,a^6x^2+1624\sqrt{b}\,\sqrt{b\,x^4+a}\,a^5b\,x^6+2387\sqrt{b}\,\cdots\right.}{1155\sqrt{\sqrt{\cdots}}}$$

input
```
int(x^11*(b*x^4+a)^(3/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(224*sqrt(b)*sqrt(a + b*
x**4)*a**6*x**2 + 1624*sqrt(b)*sqrt(a + b*x**4)*a**5*b*x**6 + 2387*sqrt(b)
*sqrt(a + b*x**4)*a**4*b**2*x**10 + 1075*sqrt(b)*sqrt(a + b*x**4)*a**3*b**
3*x**14 + 5128*sqrt(b)*sqrt(a + b*x**4)*a**2*b**4*x**18 + 9968*sqrt(b)*sqr
t(a + b*x**4)*a*b**5*x**22 + 4928*sqrt(b)*sqrt(a + b*x**4)*b**6*x**26 + 32
*a**7 + 776*a**6*b*x**4 + 2749*a**5*b**2*x**8 + 2714*a**4*b**3*x**12 + 270
1*a**3*b**4*x**16 + 9496*a**2*b**5*x**20 + 12432*a*b**6*x**24 + 4928*b**7*
x**28))/(1155*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**3*(sqrt(a + b*x**4)
*a**3 + 24*sqrt(a + b*x**4)*a**2*b*x**4 + 80*sqrt(a + b*x**4)*a*b**2*x**8
+ 64*sqrt(a + b*x**4)*b**3*x**12 + 7*sqrt(b)*a**3*x**2 + 56*sqrt(b)*a**2*b
*x**6 + 112*sqrt(b)*a*b**2*x**10 + 64*sqrt(b)*b**3*x**14))
```

3.475 $\int x^7(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^7\left(a+bx^4\right)^{3/4}\,dx=-\frac{a(a+bx^4)^{7/4}}{7b^2}+\frac{(a+bx^4)^{11/4}}{11b^2}$$

output
```
-1/7*a*(b*x^4+a)^(7/4)/b^2+1/11*(b*x^4+a)^(11/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int x^7\left(a+bx^4\right)^{3/4}\,dx=\frac{(a+bx^4)^{3/4}\left(-4a^2+3abx^4+7b^2x^8\right)}{77b^2}$$

input
```
Integrate[x^7*(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(3/4)*(-4*a^2 + 3*a*b*x^4 + 7*b^2*x^8))/(77*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^4 \left(bx^4+a\right)^{3/4} dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{7/4}}{b}-\frac{a\left(bx^4+a\right)^{3/4}}{b}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4\left(a+bx^4\right)^{11/4}}{11b^2}-\frac{4a\left(a+bx^4\right)^{7/4}}{7b^2}\right)$$

input
```
Int[x^7*(a + b*x^4)^(3/4),x]
```

output
```
((-4*a*(a + b*x^4)^(7/4))/(7*b^2) + (4*(a + b*x^4)^(11/4))/(11*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{7}{4}}(-7b\,x^4+4a)}{77b^2}$	25
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{7}{4}}(-7b\,x^4+4a)}{77b^2}$	25
orering	$-\frac{(b\,x^4+a)^{\frac{7}{4}}(-7b\,x^4+4a)}{77b^2}$	25
trager	$-\frac{(-7b^2x^8-3ab\,x^4+4a^2)(b\,x^4+a)^{\frac{3}{4}}}{77b^2}$	36
risch	$-\frac{(-7b^2x^8-3ab\,x^4+4a^2)(b\,x^4+a)^{\frac{3}{4}}}{77b^2}$	36

input
```
int(x^7*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/77*(b*x^4+a)^(7/4)*(-7*b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x^7(a+bx^4)^{3/4}\,dx=\frac{(7\,b^2x^8+3\,abx^4-4\,a^2)(bx^4+a)^{\frac{3}{4}}}{77\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/77*(7*b^2*x^8 + 3*a*b*x^4 - 4*a^2)*(b*x^4 + a)^(3/4)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.

Time = 0.44 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int x^7 (a+bx^4)^{3/4}\,dx = \begin{cases} -\frac{4a^2(a+bx^4)^{\frac{3}{4}}}{77b^2} + \frac{3ax^4(a+bx^4)^{\frac{3}{4}}}{77b} + \frac{x^8(a+bx^4)^{\frac{3}{4}}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{4}}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-4*a**2*(a + b*x**4)**(3/4)/(77*b**2) + 3*a*x**4*(a + b*x**4)**
(3/4)/(77*b) + x**8*(a + b*x**4)**(3/4)/11, Ne(b, 0)), (a**(3/4)*x**8/8, T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^7 (a+bx^4)^{3/4}\,dx = \frac{(bx^4+a)^{\frac{11}{4}}}{11\,b^2} - \frac{(bx^4+a)^{\frac{7}{4}}a}{7\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/11*(b*x^4 + a)^(11/4)/b^2 - 1/7*(b*x^4 + a)^(7/4)*a/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int x^7 (a + bx^4)^{3/4} \, dx = \frac{\frac{11 \left(3 (bx^4+a)^{\frac{7}{4}} - 7 (bx^4+a)^{\frac{3}{4}} a\right) a}{b} + \frac{21 (bx^4+a)^{\frac{11}{4}} - 66 (bx^4+a)^{\frac{7}{4}} a + 77 (bx^4+a)^{\frac{3}{4}} a^2}{b}}{231 b}$$

input
```
integrate(x^7*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/231*(11*(3*(b*x^4 + a)^(7/4) - 7*(b*x^4 + a)^(3/4)*a)*a/b + (21*(b*x^4 +
 a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)^(3/4)*a^2)/b)/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^7 (a + bx^4)^{3/4} \, dx = (b\,x^4 + a)^{3/4} \left(\frac{x^8}{11} - \frac{4\,a^2}{77\,b^2} + \frac{3\,a\,x^4}{77\,b}\right)$$

input
```
int(x^7*(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(3/4)*(x^8/11 - (4*a^2)/(77*b^2) + (3*a*x^4)/(77*b))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 269, normalized size of antiderivative = 7.08

$$\int x^7 (a + bx^4)^{3/4} \, dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2 + a + b\,x^4}\,\left(-20\sqrt{b}\,\sqrt{b\,x^4+a}\,a^4x^2 - 65\sqrt{b}\,\sqrt{b\,x^4+a}\,a^3b\,x^6 + 31\sqrt{b}\,\sqrt{b\,x\right.}{77\sqrt{\sqrt{b\,x^4+a} + \sqrt{b}\,x^2}\,b^2\left(\sqrt{b\,x^4+a}\,a\right.}$$

input
```
int(x^7*(b*x^4+a)^(3/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*( - 20*sqrt(b)*sqrt(a +
b*x**4)*a**4*x**2 - 65*sqrt(b)*sqrt(a + b*x**4)*a**3*b*x**6 + 31*sqrt(b)*s
qrt(a + b*x**4)*a**2*b**2*x**10 + 188*sqrt(b)*sqrt(a + b*x**4)*a*b**3*x**1
4 + 112*sqrt(b)*sqrt(a + b*x**4)*b**4*x**18 - 4*a**5 - 49*a**4*b*x**4 - 66
*a**3*b**2*x**8 + 111*a**2*b**3*x**12 + 244*a*b**4*x**16 + 112*b**5*x**20)
)/(77*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**2*(sqrt(a + b*x**4)*a**2 +
12*sqrt(a + b*x**4)*a*b*x**4 + 16*sqrt(a + b*x**4)*b**2*x**8 + 5*sqrt(b)*a
**2*x**2 + 20*sqrt(b)*a*b*x**6 + 16*sqrt(b)*b**2*x**10))
```

3.476 $\int x^3(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^3(a+bx^4)^{3/4}\,dx=\frac{(a+bx^4)^{7/4}}{7b}$$

output
```
1/7*(b*x^4+a)^(7/4)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^4)^{3/4}\,dx=\frac{(a+bx^4)^{7/4}}{7b}$$

input
```
Integrate[x^3*(a + b*x^4)^(3/4),x]
```

output
```
(a + b*x^4)^(7/4)/(7*b)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx^4)^{3/4} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$\frac{(a + bx^4)^{7/4}}{7b}$$

input
```
Int[x^3*(a + b*x^4)^(3/4),x]
```

output
```
(a + b*x^4)^(7/4)/(7*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15
derivativedivides	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15
default	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15
trager	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15
risch	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15
orering	$\frac{(b\,x^4+a)^{\frac{7}{4}}}{7b}$	15

input
```
int(x^3*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/7*(b*x^4+a)^(7/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\left(a+bx^4\right)^{3/4}\,dx=\frac{(bx^4+a)^{\frac{7}{4}}}{7\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/7*(b*x^4 + a)^(7/4)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int x^3\left(a+bx^4\right)^{3/4}\,dx=\begin{cases}\frac{a\left(a+bx^4\right)^{\frac{3}{4}}}{7b}+\frac{x^4\left(a+bx^4\right)^{\frac{3}{4}}}{7} & \text{for } b\neq 0\\ \frac{a^{\frac{3}{4}}x^4}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((a*(a + b*x**4)**(3/4)/(7*b) + x**4*(a + b*x**4)**(3/4)/7, Ne(b,
 0)), (a**(3/4)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\left(a+bx^4\right)^{3/4}\,dx=\frac{\left(bx^4+a\right)^{\frac{7}{4}}}{7\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/7*(b*x^4 + a)^(7/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3\left(a+bx^4\right)^{3/4}\,dx=\frac{\left(bx^4+a\right)^{\frac{7}{4}}}{7\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/7*(b*x^4 + a)^(7/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 (a + bx^4)^{3/4}\,dx = \frac{(b\,x^4+a)^{7/4}}{7\,b}$$

input
```
int(x^3*(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(7/4)/(7*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 9.50

$$\int x^3 (a + bx^4)^{3/4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,\left(3\sqrt{b}\,\sqrt{b\,x^4+a}\,a^2x^2+7\sqrt{b}\,\sqrt{b\,x^4+a}\,ab\,x^6+4\sqrt{b}\,\sqrt{b\,x^4+a}\,b\right.}{7\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b\left(\sqrt{b\,x^4+a}\,a+4\sqrt{b\,x^4+a}\,b\,x^4+3\sqrt{b}\,a\,x\right.}$$

input
```
int(x^3*(b*x^4+a)^(3/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(3*sqrt(b)*sqrt(a + b*x*
*4)*a**2*x**2 + 7*sqrt(b)*sqrt(a + b*x**4)*a*b*x**6 + 4*sqrt(b)*sqrt(a + b
*x**4)*b**2*x**10 + a**3 + 6*a**2*b*x**4 + 9*a*b**2*x**8 + 4*b**3*x**12))/
(7*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b*(sqrt(a + b*x**4)*a + 4*sqrt(a
+ b*x**4)*b*x**4 + 3*sqrt(b)*a*x**2 + 4*sqrt(b)*b*x**6))
```

3.477 $\int \frac{(a+bx^4)^{3/4}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{(a+bx^4)^{3/4}}{x} dx = \frac{1}{3}(a+bx^4)^{3/4} + \frac{1}{2}a^{3/4}\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

output
```
1/3*(b*x^4+a)^(3/4)+1/2*a^(3/4)*arctan((b*x^4+a)^(1/4)/a^(1/4))-1/2*a^(3/4
)*arctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^{3/4}}{x} dx = \frac{1}{3}(a+bx^4)^{3/4} + \frac{1}{2}a^{3/4}\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

input
```
Integrate[(a + b*x^4)^(3/4)/x,x]
```

output
```
(a + b*x^4)^(3/4)/3 + (a^(3/4)*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/2 - (a^(
3/4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {798, 60, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^{3/4}}{x^4}dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4+\frac{4}{3}(a+bx^4)^{3/4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{4a\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{b}+\frac{4}{3}(a+bx^4)^{3/4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{4}{3}(a+bx^4)^{3/4}-\frac{4a\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{b}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{4}{3}(a+bx^4)^{3/4}-4a\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{4}{3}(a+bx^4)^{3/4}-4a\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{4}{3}\left(a+bx^4\right)^{3/4}-4a\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{4}{3}\left(a+bx^4\right)^{3/4}-4a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)\right)$$

input
```
Int[(a + b*x^4)^(3/4)/x,x]
```

output
```
((4*(a + b*x^4)^(3/4))/3 - 4*a*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(
1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3}+\frac{a^{\frac{3}{4}}\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2}-\frac{\ln\left(\frac{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)a^{\frac{3}{4}}}{4}$	73

input
```
int((b*x^4+a)^(3/4)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^4+a)^(3/4)+1/2*a^(3/4)*arctan((b*x^4+a)^(1/4)/a^(1/4))-1/4*ln((-(
b*x^4+a)^(1/4)-a^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4)))*a^(3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.80

$$\int \frac{(a+bx^4)^{3/4}}{x}\,dx = -\frac{1}{4}\left(a^3\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2+\left(a^3\right)^{\frac{3}{4}}\right) + \frac{1}{4}i\left(a^3\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2+i\left(a^3\right)^{\frac{3}{4}}\right) - \frac{1}{4}i\left(a^3\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2-i\left(a^3\right)^{\frac{3}{4}}\right) + \frac{1}{4}\left(a^3\right)^{\frac{1}{4}}\log\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2-\left(a^3\right)^{\frac{3}{4}}\right) + \frac{1}{3}\left(bx^4+a\right)^{\frac{3}{4}}$$

input
```
integrate((b*x^4+a)^(3/4)/x,x, algorithm="fricas")
```

output
```
-1/4*(a^3)^(1/4)*log((b*x^4 + a)^(1/4)*a^2 + (a^3)^(3/4)) + 1/4*I*(a^3)^(1
/4)*log((b*x^4 + a)^(1/4)*a^2 + I*(a^3)^(3/4)) - 1/4*I*(a^3)^(1/4)*log((b*
x^4 + a)^(1/4)*a^2 - I*(a^3)^(3/4)) + 1/4*(a^3)^(1/4)*log((b*x^4 + a)^(1/4
)*a^2 - (a^3)^(3/4)) + 1/3*(b*x^4 + a)^(3/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{(a+bx^4)^{3/4}}{x}\,dx = -\frac{b^{\frac{3}{4}}x^3\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x,x)
```

output
```
-b**(3/4)*x**3*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), a*exp_polar(I*pi)/(
b*x**4))/(4*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^4)^{3/4}}{x}\,dx = \frac{1}{4}\,a\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)+\frac{1}{3}\,(bx^4+a)^{\frac{3}{4}}$$

input
```
integrate((b*x^4+a)^(3/4)/x,x, algorithm="maxima")
```

output
```
1/4*a*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4
) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)) + 1/3*(b*x^4 + a)^(3/
4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(50) = 100.

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.64

$$\begin{aligned}\int \frac{(a+bx^4)^{3/4}}{x}\,dx =& -\frac{1}{4}\,\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)\\&-\frac{1}{4}\,\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)\\&+\frac{1}{8}\,\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)\\&-\frac{1}{8}\,\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)+\frac{1}{3}\,(bx^4+a)^{\frac{3}{4}}\end{aligned}$$

input
```
integrate((b*x^4+a)^(3/4)/x,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
+ a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) + 1/8*sqrt(2)*(-a)^(3/4)
*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)) -
1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*
x^4 + a) + sqrt(-a)) + 1/3*(b*x^4 + a)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^4)^{3/4}}{x} dx = \frac{a^{3/4} \operatorname{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2} - \frac{a^{3/4} \operatorname{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2} + \frac{(b\,x^4+a)^{3/4}}{3}$$

input
```
int((a + b*x^4)^(3/4)/x,x)
```

output
```
(a^(3/4)*atan((a + b*x^4)^(1/4)/a^(1/4)))/2 - (a^(3/4)*atanh((a + b*x^4)^(
1/4)/a^(1/4)))/2 + (a + b*x^4)^(3/4)/3
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x} dx = \frac{(b\,x^4+a)^{\frac{3}{4}}}{3} + \left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^5+ax} dx\right) a$$

input
```
int((b*x^4+a)^(3/4)/x,x)
```

output
```
((a + b*x**4)**(3/4) + 3*int((a + b*x**4)**(3/4)/(a*x + b*x**5),x)*a)/3
```

3.478 $\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx = -\frac{(a+bx^4)^{3/4}}{4x^4} + \frac{3b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}} - \frac{3b\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}$$

output
```
-1/4*(b*x^4+a)^(3/4)/x^4+3/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-3/8
*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx = -\frac{(a+bx^4)^{3/4}}{4x^4} + \frac{3b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}} - \frac{3b\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^5,x]
```

output
```
-1/4*(a + b*x^4)^(3/4)/x^4 + (3*b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^
(1/4)) - (3*b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {798, 51, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^{3/4}}{x^8}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{3}{4}b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4-\frac{(a+bx^4)^{3/4}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(3\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}-\frac{(a+bx^4)^{3/4}}{x^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(-3\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}-\frac{(a+bx^4)^{3/4}}{x^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-3b\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}-\frac{(a+bx^4)^{3/4}}{x^4}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(-3b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)-\frac{(a+bx^4)^{3/4}}{x^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-3b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(-3b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{x^4}\right)$$

input
```
Int[(a + b*x^4)^(3/4)/x^5,x]
```

output
```
(-((a + b*x^4)^(3/4)/x^4) - 3*b*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^
(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{6 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b\,x^{4}-3 \ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right) b\,x^{4}-4\left(b\,x^{4}+a\right)^{\frac{3}{4}} a^{\frac{1}{4}}}{16 x^{4} a^{\frac{1}{4}}}$	82

input
```
int((b*x^4+a)^(3/4)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/16*(6*arctan((b*x^4+a)^(1/4)/a^(1/4))*b*x^4-3*ln(((b*x^4+a)^(1/4)+a^(1/4
))/((b*x^4+a)^(1/4)-a^(1/4)))*b*x^4-4*(b*x^4+a)^(3/4)*a^(1/4))/x^4/a^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.47

$$\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx = -\frac{3\left(\frac{b^4}{a}\right)^{\frac{1}{4}}x^4\log\left(27\,(bx^4+a)^{\frac{1}{4}}b^3+27\left(\frac{b^4}{a}\right)^{\frac{3}{4}}a\right)-3i\left(\frac{b^4}{a}\right)^{\frac{1}{4}}x^4\log\left(27\,(bx^4+a)^{\frac{1}{4}}b^3+27i\left(\frac{b^4}{a}\right)^{\frac{3}{4}}a\right)+3i}{}$$

input
```
integrate((b*x^4+a)^(3/4)/x^5,x, algorithm="fricas")
```

output
```
-1/16*(3*(b^4/a)^(1/4)*x^4*log(27*(b*x^4 + a)^(1/4)*b^3 + 27*(b^4/a)^(3/4)
*a) - 3*I*(b^4/a)^(1/4)*x^4*log(27*(b*x^4 + a)^(1/4)*b^3 + 27*I*(b^4/a)^(3
/4)*a) + 3*I*(b^4/a)^(1/4)*x^4*log(27*(b*x^4 + a)^(1/4)*b^3 - 27*I*(b^4/a)
^(3/4)*a) - 3*(b^4/a)^(1/4)*x^4*log(27*(b*x^4 + a)^(1/4)*b^3 - 27*(b^4/a)^
(3/4)*a) + 4*(b*x^4 + a)^(3/4))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx = -\frac{b^{\frac{3}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**5,x)
```

output
```
-b**(3/4)*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), a*exp_polar(I*pi)/(b*x**4)
)/(4*x*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^5}\,dx = \frac{3}{16}\,b\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)-\frac{(bx^4+a)^{\frac{3}{4}}}{4\,x^4}$$

input
```
integrate((b*x^4+a)^(3/4)/x^5,x, algorithm="maxima")
```

output
```
3/16*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/
4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)) - 1/4*(b*x^4 + a)^(3
/4)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(55) = 110$.

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.75

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^5}\,dx =$$

$$-\frac{1}{32}\left(\frac{6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a}+\frac{6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a}-\right.$$

input
```
integrate((b*x^4+a)^(3/4)/x^5,x, algorithm="giac")
```

output
```
-1/32*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*
x^4 + a)^(1/4))/(-a)^(1/4))/a + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a - 3*sqrt(2)*(-a)^(
3/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)
)/a + 3*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqr
t(b*x^4 + a) + sqrt(-a))/a + 8*(b*x^4 + a)^(3/4)/(b*x^4))*b
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx = \frac{3\,b\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{1/4}} - \frac{(b\,x^4+a)^{3/4}}{4\,x^4} - \frac{3\,b\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{1/4}}$$

input
```
int((a + b*x^4)^(3/4)/x^5,x)
```

output
```
(3*b*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(1/4)) - (a + b*x^4)^(3/4)/(4*x
^4) - (3*b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(1/4))
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^5}\,dx = \frac{-(b\,x^4+a)^{\frac{3}{4}}+3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^5+ax}dx\right)b\,x^4}{4x^4}$$

input
```
int((b*x^4+a)^(3/4)/x^5,x)
```

output
```
( - (a + b*x**4)**(3/4) + 3*int((a + b*x**4)**(3/4)/(a*x + b*x**5),x)*b*x*
*4)/(4*x**4)
```

3.479 $\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = -\frac{(a+bx^4)^{3/4}}{8x^8} - \frac{3b(a+bx^4)^{3/4}}{32ax^4} - \frac{3b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}} + \frac{3b^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}$$

output
```
-1/8*(b*x^4+a)^(3/4)/x^8-3/32*b*(b*x^4+a)^(3/4)/a/x^4-3/64*b^2*arctan((b*x
^4+a)^(1/4)/a^(1/4))/a^(5/4)+3/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(
5/4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = \frac{(-4a-3bx^4)(a+bx^4)^{3/4}}{32ax^8} - \frac{3b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}} + \frac{3b^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{5/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^9,x]
```

output
```
((-4*a - 3*b*x^4)*(a + b*x^4)^(3/4))/(32*a*x^8) - (3*b^2*ArcTan[(a + b*x^4
)^(1/4)/a^(1/4)])/(64*a^(5/4)) + (3*b^2*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]
)/(64*a^(5/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {798, 51, 52, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^{3/4}}{x^{12}}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{3}{8}b\int \frac{1}{x^8\sqrt[4]{bx^4+a}}dx^4-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(-\frac{b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{4a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(-\frac{\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(\frac{\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(\frac{b\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{3}{8}b\left(\frac{b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)-\frac{(a+bx^4)^{3/4}}{2x^8}\right)$$

input
```
Int[(a + b*x^4)^(3/4)/x^9,x]
```

output
```
(-1/2*(a + b*x^4)^(3/4)/x^8 + (3*b*(-((a + b*x^4)^(3/4)/(a*x^4)) + (b*(-1/
2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^
(1/4)]/(2*a^(1/4))))/a))/8)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{3 \ln \left(\frac{-\left(b \,x^{4}+a \right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b \,x^{4}+a \right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) b^{2} x^{8}-6 \arctan \left(\frac{\left(b \,x^{4}+a \right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b^{2} x^{8}-12 b \,x^{4} a^{\frac{1}{4}} \left(b \,x^{4}+a \right)^{\frac{3}{4}}-16 a^{\frac{5}{4}} \left(b \,x^{4}+a \right)^{\frac{3}{4}}}{128 a^{\frac{5}{4}} x^{8}}$	108

input
```
int((b*x^4+a)^(3/4)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/128*(3*ln((-(b*x^4+a)^(1/4)-a^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4)))*b^2*x^8
-6*arctan((b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8-12*b*x^4*a^(1/4)*(b*x^4+a)^(3/4
)-16*a^(5/4)*(b*x^4+a)^(3/4))/a^(5/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = \frac{3\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}} ax^8 \log\left(27\,(bx^4+a)^{\frac{1}{4}}b^6 + 27\left(\frac{b^8}{a^5}\right)^{\frac{3}{4}}a^4\right) - 3i\left(\frac{b^8}{a^5}\right)^{\frac{1}{4}} ax^8 \log\left(27\,(bx^4+a)^{\frac{1}{4}}b\right.}{}$$

input
```
integrate((b*x^4+a)^(3/4)/x^9,x, algorithm="fricas")
```

output
```
1/128*(3*(b^8/a^5)^(1/4)*a*x^8*log(27*(b*x^4 + a)^(1/4)*b^6 + 27*(b^8/a^5)
^(3/4)*a^4) - 3*I*(b^8/a^5)^(1/4)*a*x^8*log(27*(b*x^4 + a)^(1/4)*b^6 + 27*
I*(b^8/a^5)^(3/4)*a^4) + 3*I*(b^8/a^5)^(1/4)*a*x^8*log(27*(b*x^4 + a)^(1/4
)*b^6 - 27*I*(b^8/a^5)^(3/4)*a^4) - 3*(b^8/a^5)^(1/4)*a*x^8*log(27*(b*x^4
+ a)^(1/4)*b^6 - 27*(b^8/a^5)^(3/4)*a^4) - 4*(3*b*x^4 + 4*a)*(b*x^4 + a)^(
3/4))/(a*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = -\frac{b^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**9,x)
```

output
```
-b**(3/4)*gamma(5/4)*hyper((-3/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4)
)/(4*x**5*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = -\frac{3\,b^2\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a} - \frac{3\,(bx^4+a)^{\frac{7}{4}}b^2+(bx^4+a)^{\frac{3}{4}}ab^2}{32\,\left((bx^4+a)^2a-2\,(bx^4+a)a^2+a^3\right)}$$

input
```
integrate((b*x^4+a)^(3/4)/x^9,x, algorithm="maxima")
```

output
```
-3/128*b^2*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)
^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a - 1/32*(3*(b*x
^4 + a)^(7/4)*b^2 + (b*x^4 + a)^(3/4)*a*b^2)/((b*x^4 + a)^2*a - 2*(b*x^4 +
 a)*a^2 + a^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = \frac{\frac{6\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{6\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2}+}{}$$

input
```
integrate((b*x^4+a)^(3/4)/x^9,x, algorithm="giac")
```

output
```
1/256*(6*sqrt(2)*(-a)^(3/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(3/4)*b^3*arctan(-1/2
*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sq
rt(2)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqr
t(-a))/((-a)^(1/4)*a) + 3*sqrt(2)*(-a)^(3/4)*b^3*log(-sqrt(2)*(b*x^4 + a)^
(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^2 - 8*(3*(b*x^4 + a)^(7/4
)*b^3 + (b*x^4 + a)^(3/4)*a*b^3)/(a*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = -\frac{(b\,x^4+a)^{3/4}}{32\,x^8} - \frac{3\,b^2\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{64\,a^{5/4}} - \frac{3\,(b\,x^4+a)^{7/4}}{32\,a\,x^8} - \frac{b^2\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,3\mathrm{i}}{64\,a^{5/4}}$$

input
```
int((a + b*x^4)^(3/4)/x^9,x)
```

output
```
- (a + b*x^4)^(3/4)/(32*x^8) - (3*b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64
*a^(5/4)) - (b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*3i)/(64*a^(5/4)) - (
3*(a + b*x^4)^(7/4))/(32*a*x^8)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^9}\,dx = \frac{-4(b\,x^4+a)^{\frac{3}{4}}\,a-3(b\,x^4+a)^{\frac{3}{4}}\,b\,x^4-3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^5+ax}dx\right)b^2x^8}{32a\,x^8}$$

input
```
int((b*x^4+a)^(3/4)/x^9,x)
```

output
```
( - 4*(a + b*x**4)**(3/4)*a - 3*(a + b*x**4)**(3/4)*b*x**4 - 3*int((a + b*
x**4)**(3/4)/(a*x + b*x**5),x)*b**2*x**8)/(32*a*x**8)
```

3.480 $\int x^9(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int x^9\left(a+bx^4\right)^{3/4}\,dx=\frac{4a^3x^2}{65b^2\sqrt[4]{a+bx^4}}-\frac{2a^2x^2(a+bx^4)^{3/4}}{65b^2}+\frac{ax^6(a+bx^4)^{3/4}}{39b}+\frac{1}{13}x^{10}\left(a+bx^4\right)^{3/4}-\frac{4a^{7/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^4}}$$

output

```
4/65*a^3*x^2/b^2/(b*x^4+a)^(1/4)-2/65*a^2*x^2*(b*x^4+a)^(3/4)/b^2+1/39*a*x
^6*(b*x^4+a)^(3/4)/b+1/13*x^10*(b*x^4+a)^(3/4)-4/65*a^(7/2)*(1+b*x^4/a)^(1
/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4
+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int x^9(a+bx^4)^{3/4}\,dx = \frac{x^2(a+bx^4)^{3/4}\left(\left(1+\frac{bx^4}{a}\right)^{3/4}(-2a^2+abx^4+3b^2x^8)+2a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\right.\right.}{39b^2\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[x^9*(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(a + b*x^4)^(3/4)*((1 + (b*x^4)/a)^(3/4)*(-2*a^2 + a*b*x^4 + 3*b^2*x^
8) + 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^4)/a)]))/(39*b^2*(1 +
(b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {807, 248, 262, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9\left(a+bx^4\right)^{3/4}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^8\left(bx^4+a\right)^{3/4}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{3}{13}a\int \frac{x^8}{\sqrt[4]{bx^4+a}}dx^2+\frac{2}{13}x^{10}\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3}{13}a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\int\frac{x^4}{\sqrt[4]{bx^4+a}}dx^2}{3b}\right)+\frac{2}{13}x^{10}\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{3}{13}a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\int\frac{1}{\sqrt[4]{bx^4+a}}dx^2}{5b}\right)}{3b}\right)+\frac{2}{13}x^{10}\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{3}{13}a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)+\frac{2}{13}x^{10}\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{3}{13}a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)+\frac{2}{13}x^{10}\left(a+b\right.\right.$$

$\downarrow$ 212

$$\frac{1}{2}\left(\frac{3}{13}a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)+\frac{2}{13}x^{10}\right.$$

input

```
Int[x^9*(a + b*x^4)^(3/4),x]
```

output
```
((2*x^10*(a + b*x^4)^(3/4))/13 + (3*a*((2*x^6*(a + b*x^4)^(3/4))/(9*b) - (
2*a*((2*x^2*(a + b*x^4)^(3/4))/(5*b) - (2*a*(1 + (b*x^4)/a)^(1/4)*((2*x^2)
/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^4)^(1/4))))/(3*b)))/13)/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^9 \left(b\,x^4+a\right)^{\frac{3}{4}}dx$$

input
```
int(x^9*(b*x^4+a)^(3/4),x)
```

output
```
int(x^9*(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int x^9 \left(a+bx^4\right)^{3/4}\,dx=\int \left(bx^4+a\right)^{\frac{3}{4}}x^9\,dx$$

input
```
integrate(x^9*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^9, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.19

$$\int x^9 \left(a+bx^4\right)^{3/4}\,dx=\frac{a^{\frac{3}{4}}x^{10}{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10}$$

input
```
integrate(x**9*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**10*hyper((-3/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/10
```

Maxima [F]

$$\int x^9 \left(a+bx^4\right)^{3/4} dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^9 \, dx$$

input
```
integrate(x^9*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^9, x)
```

Giac [F]

$$\int x^9 \left(a+bx^4\right)^{3/4} dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^9 \, dx$$

input
```
integrate(x^9*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \left(a+bx^4\right)^{3/4} dx = \int x^9 \left(b\,x^4+a\right)^{3/4} dx$$

input
```
int(x^9*(a + b*x^4)^(3/4),x)
```

output
```
int(x^9*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^9 (a+bx^4)^{3/4}\,dx = \frac{-6(b\,x^4+a)^{\frac{3}{4}}\,a^2x^2+5(b\,x^4+a)^{\frac{3}{4}}\,ab\,x^6+15(b\,x^4+a)^{\frac{3}{4}}\,b^2x^{10}+12\left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)a^3}{195b^2}$$

input
```
int(x^9*(b*x^4+a)^(3/4),x)
```

output
```
( - 6*(a + b*x**4)**(3/4)*a**2*x**2 + 5*(a + b*x**4)**(3/4)*a*b*x**6 + 15*
(a + b*x**4)**(3/4)*b**2*x**10 + 12*int(((a + b*x**4)**(3/4)*x)/(a + b*x**
4),x)*a**3)/(195*b**2)
```

3.481 $\int x^5(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int x^5\left(a+bx^4\right)^{3/4}\,dx=-\frac{2a^2x^2}{15b\sqrt[4]{a+bx^4}}+\frac{ax^2\left(a+bx^4\right)^{3/4}}{15b}+\frac{1}{9}x^6\left(a+bx^4\right)^{3/4}+\frac{2a^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-2/15*a^2*x^2/b/(b*x^4+a)^(1/4)+1/15*a*x^2*(b*x^4+a)^(3/4)/b+1/9*x^6*(b*x^
4+a)^(3/4)+2/15*a^(5/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)
*x^2/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.54 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int x^5\left(a+bx^4\right)^{3/4}\,dx=\frac{x^2\left(a+bx^4\right)^{3/4}\left(a+bx^4-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{\left(1+\frac{bx^4}{a}\right)^{3/4}}\right)}{9b}$$

input
```
Integrate[x^5*(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(a + b*x^4)^(3/4)*(a + b*x^4 - (a*Hypergeometric2F1[-3/4, 1/2, 3/2, -
((b*x^4)/a)])/(1 + (b*x^4)/a)^(3/4)))/(9*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {807, 248, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^4 \left(bx^4+a\right)^{3/4} dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{1}{3}a\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx^2+\frac{2}{9}x^6\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{5b}\right)+\frac{2}{9}x^6\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5b\sqrt[4]{a+bx^4}}\right)+\frac{2}{9}x^6\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5b\sqrt[4]{a+bx^4}}\right)+\frac{2}{9}x^6\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^4}}\right)+\frac{2}{9}x^6\left(a+bx^4\right)^{3/4}\right)$$

input
```
Int[x^5*(a + b*x^4)^(3/4),x]
```

output
```
((2*x^6*(a + b*x^4)^(3/4))/9 + (a*((2*x^2*(a + b*x^4)^(3/4))/(5*b) - (2*a*
(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*Elliptic
E[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^4)^(1/4))))
/3)/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^{5} \left(b \,x^{4}+a \right)^{\frac{3}{4}}d x$$

input
```
int(x^5*(b*x^4+a)^(3/4),x)
```

output
```
int(x^5*(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int x^5 \left(a+bx^4\right)^{3/4} dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^5 \, dx$$

input
```
integrate(x^5*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int x^5 \left(a+bx^4\right)^{3/4} dx = \frac{a^{\frac{3}{4}} x^6 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6}$$

input
```
integrate(x**5*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**6*hyper((-3/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/6
```

Maxima [F]

$$\int x^5 \left(a+bx^4\right)^{3/4} dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^5 \, dx$$

input
```
integrate(x^5*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^5, x)
```

Giac [F]

$$\int x^5 (a+bx^4)^{3/4} \, dx = \int (bx^4+a)^{\frac{3}{4}} x^5 \, dx$$

input
```
integrate(x^5*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (a+bx^4)^{3/4} \, dx = \int x^5 \left(b\,x^4+a\right)^{3/4} \, dx$$

input
```
int(x^5*(a + b*x^4)^(3/4),x)
```

output
```
int(x^5*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^5 (a+bx^4)^{3/4} \, dx = \frac{3(b\,x^4+a)^{\frac{3}{4}}\,a\,x^2+5(b\,x^4+a)^{\frac{3}{4}}\,b\,x^6-6\left(\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)a^2}{45b}$$

input
```
int(x^5*(b*x^4+a)^(3/4),x)
```

output
```
(3*(a + b*x**4)**(3/4)*a*x**2 + 5*(a + b*x**4)**(3/4)*b*x**6 - 6*int(((a +
b*x**4)**(3/4)*x)/(a + b*x**4),x)*a**2)/(45*b)
```

3.482 $\int x(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\frac{3ax^2}{5\sqrt[4]{a+bx^4}}+\frac{1}{5}x^2\left(a+bx^4\right)^{3/4}-\frac{3a^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
3/5*a*x^2/(b*x^4+a)^(1/4)+1/5*x^2*(b*x^4+a)^(3/4)-3/5*a^(3/2)*(1+b*x^4/a)^
(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(1/2)/(b*x
^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\frac{x^2(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{2\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[x*(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^4)/a)])/(2
*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {807, 211, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^4\right)^{3/4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \left(bx^4+a\right)^{3/4}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{3}{5}a\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2+\frac{2}{5}x^2\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{3a\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5\sqrt[4]{a+bx^4}}+\frac{2}{5}x^2\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{3a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5\sqrt[4]{a+bx^4}}+\frac{2}{5}x^2\left(a+bx^4\right)^{3/4}\right)$$

$$\Big\downarrow \; 212$$

$$\frac{1}{2}\left(\frac{3a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^4}}+\frac{2}{5}x^2\left(a+bx^4\right)^{3/4}\right)$$

input
```
Int[x*(a + b*x^4)^(3/4),x]
```

output
```
((2*x^2*(a + b*x^4)^(3/4))/5 + (3*a*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b
*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/Sqrt[b]))/(5*(a + b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x\left(b\,x^{4}+a\right)^{\frac{3}{4}}d x$$

input
```
int(x*(b*x^4+a)^(3/4),x)
```

output
```
int(x*(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\int\left(bx^4+a\right)^{\frac{3}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.30

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\frac{a^{\frac{3}{4}}x^2{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2}$$

input
```
integrate(x*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**2*hyper((-3/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/2
```

Maxima [F]

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\int\left(bx^4+a\right)^{\frac{3}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)*x, x)
```

Giac [F]

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\int\left(bx^4+a\right)^{\frac{3}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\int x\,\left(b\,x^4+a\right)^{3/4}\,dx$$

input
```
int(x*(a + b*x^4)^(3/4),x)
```

output
```
int(x*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x\left(a+bx^4\right)^{3/4}\,dx=\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}x^2}{5}+\frac{3\left(\int \frac{x}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right)a}{5}$$

input
```
int(x*(b*x^4+a)^(3/4),x)
```

output
```
((a + b*x**4)**(3/4)*x**2 + 3*int(((a + b*x**4)**(3/4)*x)/(a + b*x**4),x)*
a)/5
```

3.483 $\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx=\frac{3bx^2}{2\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{2x^2}-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}$$

output
```
3/2*b*x^2/(b*x^4+a)^(1/4)-1/2*(b*x^4+a)^(3/4)/x^2-3/2*a^(1/2)*b^(1/2)*(1+b
*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/(b*x
^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx=-\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx^4}{a}\right)}{2x^2\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^3,x]
```

output
```
-1/2*((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^4)/a)])/
(x^2*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 247, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{(bx^4+a)^{3/4}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{3}{2}b\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2-\frac{(a+bx^4)^{3/4}}{x^2}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{x^2}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{x^2}\right)$$

$\downarrow$ 212

$$\frac{1}{2}\left(\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{x^2}\right)$$

input
```
Int[(a + b*x^4)^(3/4)/x^3,x]
```

output
```
(-((a + b*x^4)^(3/4)/x^2) + (3*b*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^
4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sq
rt[b]))/(2*(a + b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{3}{4}}}{x^{3}}dx$$

input
```
int((b*x^4+a)^(3/4)/x^3,x)
```

output
```
int((b*x^4+a)^(3/4)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2x^2}$$

input
```
integrate((b*x**4+a)**(3/4)/x**3,x)
```

output
```
-a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*x**2)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{x^3}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^3,x)
```

output
```
int((a + b*x^4)^(3/4)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^3}\,dx = \frac{(b\,x^4+a)^{\frac{3}{4}} + 3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^7+a\,x^3}dx\right)a\,x^2}{x^2}$$

input
```
int((b*x^4+a)^(3/4)/x^3,x)
```

output
```
((a + b*x**4)**(3/4) + 3*int((a + b*x**4)**(3/4)/(a*x**3 + b*x**7),x)*a*x*
*2)/x**2
```

3.484 $\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx = \frac{b^2x^2}{4a\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{6x^6} - \frac{b(a+bx^4)^{3/4}}{4ax^2} - \frac{b^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}\sqrt[4]{a+bx^4}}$$

output
```
1/4*b^2*x^2/a/(b*x^4+a)^(1/4)-1/6*(b*x^4+a)^(3/4)/x^6-1/4*b*(b*x^4+a)^(3/4
)/a/x^2-1/4*b^(3/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2
/a^(1/2))),2^(1/2))/a^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx = -\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},-\frac{1}{2},-\frac{bx^4}{a}\right)}{6x^6\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^7,x]
```

output
```
-1/6*((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^4)/a)])
/(x^6*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 247, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^7}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\left(bx^4+a\right)^{3/4}}{x^8}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^2-\frac{\left(a+bx^4\right)^{3/4}}{3x^6}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)-\frac{\left(a+bx^4\right)^{3/4}}{3x^6}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)-\frac{\left(a+bx^4\right)^{3/4}}{3x^6}\right)$$

$$\Big\downarrow \ 225$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)-\frac{\left(a+bx^4\right)^{3/4}}{3x^6}\right)$$

$$\Big\downarrow \ 212$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)-\frac{\left(a+bx^4\right)^{3/4}}{3x^6}\right)$$

input
```
Int[(a + b*x^4)^(3/4)/x^7,x]
```

output
```
(-1/3*(a + b*x^4)^(3/4)/x^6 + (b*(-((a + b*x^4)^(3/4)/(a*x^2)) + (b*(1 + (
b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcT
an[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*a*(a + b*x^4)^(1/4))))/2)/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{x^7}dx$$

input
```
int((b*x^4+a)^(3/4)/x^7,x)
```

output
```
int((b*x^4+a)^(3/4)/x^7,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^7,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^7, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6x^6}$$

input
```
integrate((b*x**4+a)**(3/4)/x**7,x)
```

output
```
-a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*x**6)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^7, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx=\int \frac{(b\,x^4+a)^{3/4}}{x^7}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^7,x)
```

output
```
int((a + b*x^4)^(3/4)/x^7, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^7}\,dx=\frac{-(b\,x^4+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^{11}+a\,x^7}dx\right)a\,x^6}{3x^6}$$

input
```
int((b*x^4+a)^(3/4)/x^7,x)
```

output
```
( - (a + b*x**4)**(3/4) - 3*int((a + b*x**4)**(3/4)/(a*x**7 + b*x**11),x)*
a*x**6)/(3*x**6)
```

3.485 $\int \frac{(a+bx^4)^{3/4}}{x^{11}} dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}} dx = -\frac{3b^3x^2}{40a^2\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{10x^{10}} - \frac{b(a+bx^4)^{3/4}}{20ax^6} + \frac{3b^2(a+bx^4)^{3/4}}{40a^2x^2} + \frac{3b^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-3/40*b^3*x^2/a^2/(b*x^4+a)^(1/4)-1/10*(b*x^4+a)^(3/4)/x^10-1/20*b*(b*x^4+
a)^(3/4)/a/x^6+3/40*b^2*(b*x^4+a)^(3/4)/a^2/x^2+3/40*b^(5/2)*(1+b*x^4/a)^(
1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^
4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}} dx = -\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{4},-\frac{3}{2},-\frac{bx^4}{a}\right)}{10x^{10}\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^11,x]
```

output
```
-1/10*((a + b*x^4)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x^4)/a)]
)/(x^10*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {807, 247, 264, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^{11}}\,dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2}\int \frac{\left(bx^4+a\right)^{3/4}}{x^{12}}dx^2$$

$$\Big\downarrow \; 247$$

$$\frac{1}{2}\left(\frac{3}{10}b\int \frac{1}{x^8\sqrt[4]{bx^4+a}}dx^2-\frac{\left(a+bx^4\right)^{3/4}}{5x^{10}}\right)$$

$$\Big\downarrow \; 264$$

$$\frac{1}{2}\left(\frac{3}{10}b\left(-\frac{b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^2}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^{10}}\right)$$

$$\Big\downarrow \; 264$$

$$\frac{1}{2}\left(\frac{3}{10}b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{3}{10}b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{3}{10}b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{3}{10}b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^{10}}\right)$$

input
```
Int[(a + b*x^4)^(3/4)/x^11,x]
```

output
```
(-1/5*(a + b*x^4)^(3/4)/x^10 + (3*b*(-1/3*(a + b*x^4)^(3/4)/(a*x^6) - (b*(
-((a + b*x^4)^(3/4)/(a*x^2)) + (b*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x
^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/S
qrt[b]))/(2*a*(a + b*x^4)^(1/4))))/(2*a)))/10)/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{x^{11}}dx$$

input
```
int((b*x^4+a)^(3/4)/x^11,x)
```

output
```
int((b*x^4+a)^(3/4)/x^11,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^11,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^11, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{3}{4}\\ -\frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10x^{10}}$$

input
```
integrate((b*x**4+a)**(3/4)/x**11,x)
```

output
```
-a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*x**10
)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^11,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^11, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^11,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{x^{11}}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^11,x)
```

output
```
int((a + b*x^4)^(3/4)/x^11, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{11}}\,dx = \frac{-(b\,x^4+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^{15}+a\,x^{11}}dx\right)a\,x^{10}}{7x^{10}}$$

input
```
int((b*x^4+a)^(3/4)/x^11,x)
```

output
```
( - (a + b*x**4)**(3/4) - 3*int((a + b*x**4)**(3/4)/(a*x**11 + b*x**15),x)
*a*x**10)/(7*x**10)
```

3.486 $\int x^{12}(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\frac{45a^3x\left(a+bx^4\right)^{3/4}}{2048b^3}-\frac{9a^2x^5\left(a+bx^4\right)^{3/4}}{512b^2}+\frac{ax^9\left(a+bx^4\right)^{3/4}}{64b}+\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}-\frac{45a^4\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{13/4}}-\frac{45a^4\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{13/4}}$$

output
```
45/2048*a^3*x*(b*x^4+a)^(3/4)/b^3-9/512*a^2*x^5*(b*x^4+a)^(3/4)/b^2+1/64*a
*x^9*(b*x^4+a)^(3/4)/b+1/16*x^13*(b*x^4+a)^(3/4)-45/4096*a^4*arctan(b^(1/4
)*x/(b*x^4+a)^(1/4))/b^(13/4)-45/4096*a^4*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4
))/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\frac{2\sqrt[4]{b}x\left(a+bx^4\right)^{3/4}\left(45a^3-36a^2bx^4+32ab^2x^8+128b^3x^{12}\right)-45a^4\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-45}{4096b^{13/4}}$$

input
```
Integrate[x^12*(a + b*x^4)^(3/4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(45*a^3 - 36*a^2*b*x^4 + 32*a*b^2*x^8 + 128
*b^3*x^12) - 45*a^4*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - 45*a^4*ArcTanh
[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4096*b^(13/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {811, 843, 843, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12} \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{3}{16} a \int \frac{x^{12}}{\sqrt[4]{bx^4+a}}dx+\frac{1}{16} x^{13} \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{16} a \left(\frac{x^9 \left(a+bx^4\right)^{3/4}}{12 b}-\frac{3 a \int \frac{x^8}{\sqrt[4]{bx^4+a}}dx}{4 b}\right)+\frac{1}{16} x^{13} \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{16} a \left(\frac{x^9 \left(a+bx^4\right)^{3/4}}{12 b}-\frac{3 a \left(\frac{x^5 \left(a+bx^4\right)^{3/4}}{8 b}-\frac{5 a \int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{8 b}\right)}{4 b}\right)+\frac{1}{16} x^{13} \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{16}a\left(\frac{x^9\left(a+bx^4\right)^{3/4}}{12b}-\frac{3a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{8b}\right)}{4b}\right)+$$

$$\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 770

$$\frac{3}{16}a\left(\frac{x^9\left(a+bx^4\right)^{3/4}}{12b}-\frac{3a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}\right)}{4b}\right)+$$

$$\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 756

$$\frac{3}{16}a\left(\frac{x^9\left(a+bx^4\right)^{3/4}}{12b}-\frac{3a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{8b}\right)}{4b}\right)+\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \; 216$$

$$\frac{3}{16}a\left(\frac{x^9(a+bx^4)^{3/4}}{12b}-\frac{3a\left(\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}\right)}{4b}\right)$$

$$\frac{1}{16}x^{13}(a+bx^4)^{3/4}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{16}a\left(\frac{x^9\left(a+bx^4\right)^{3/4}}{12b}-\frac{3a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}\right)}{4b}\right)+$$

$$\frac{1}{16}x^{13}\left(a+bx^4\right)^{3/4}$$

input
```
Int[x^12*(a + b*x^4)^(3/4),x]
```

output
```
(x^13*(a + b*x^4)^(3/4))/16 + (3*a*((x^9*(a + b*x^4)^(3/4))/(12*b) - (3*a*
((x^5*(a + b*x^4)^(3/4))/(8*b) - (5*a*((x*(a + b*x^4)^(3/4))/(4*b) - (a*(A
rcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/(8*b)))/(4*b)))/16
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{512\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{13}{4}} x^{13}+128 a\,b^{\frac{9}{4}} x^{9}\left(b\,x^{4}+a\right)^{\frac{3}{4}}-144 a^{2} b^{\frac{5}{4}} x^{5}\left(b\,x^{4}+a\right)^{\frac{3}{4}}+180 a^{3} x\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{1}{4}}+90 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a^{4}-}{8192 b^{\frac{13}{4}}}$

input
```
int(x^12*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/8192/b^(13/4)*(512*(b*x^4+a)^(3/4)*b^(13/4)*x^13+128*a*b^(9/4)*x^9*(b*x^
4+a)^(3/4)-144*a^2*b^(5/4)*x^5*(b*x^4+a)^(3/4)+180*a^3*x*(b*x^4+a)^(3/4)*b
^(1/4)+90*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^4-45*ln((b^(1/4)*x+(b*x^4+
a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.64

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx =$$

$$-\frac{45\left(\frac{a^{16}}{b^{13}}\right)^{\frac{1}{4}} b^{3}\log\left(\frac{91125\left(\left(bx^{4}+a\right)^{\frac{1}{4}} a^{12}+\left(\frac{a^{16}}{b^{13}}\right)^{\frac{3}{4}} b^{10} x\right)}{x}\right)-45i\left(\frac{a^{16}}{b^{13}}\right)^{\frac{1}{4}} b^{3}\log\left(\frac{91125\left(\left(bx^{4}+a\right)^{\frac{1}{4}} a^{12}+i\left(\frac{a^{16}}{b^{13}}\right)^{\frac{3}{4}} b^{10} x\right)}{x}\right)-}{}$$

input
```
integrate(x^12*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/8192*(45*(a^16/b^13)^(1/4)*b^3*log(91125*((b*x^4 + a)^(1/4)*a^12 + (a^1
6/b^13)^(3/4)*b^10*x)/x) - 45*I*(a^16/b^13)^(1/4)*b^3*log(91125*((b*x^4 +
a)^(1/4)*a^12 + I*(a^16/b^13)^(3/4)*b^10*x)/x) + 45*I*(a^16/b^13)^(1/4)*b^
3*log(91125*((b*x^4 + a)^(1/4)*a^12 - I*(a^16/b^13)^(3/4)*b^10*x)/x) - 45*
(a^16/b^13)^(1/4)*b^3*log(91125*((b*x^4 + a)^(1/4)*a^12 - (a^16/b^13)^(3/4
)*b^10*x)/x) - 4*(128*b^3*x^13 + 32*a*b^2*x^9 - 36*a^2*b*x^5 + 45*a^3*x)*(
b*x^4 + a)^(3/4))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.99 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\frac{a^{\frac{3}{4}}x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**13*gamma(13/4)*hyper((-3/4, 13/4), (17/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(17/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.51

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\frac{45\,a^4\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{8192\,b^3}+\frac{\frac{15\left(bx^4+a\right)^{\frac{3}{4}}a^4b^3}{x^3}+\frac{239\left(bx^4+a\right)^{\frac{7}{4}}a^4b^2}{x^7}-\frac{171\left(bx^4+a\right)^{\frac{11}{4}}a^4b}{x^{11}}+\frac{45\left(bx^4+a\right)^{\frac{15}{4}}a^4}{x^{15}}}{2048\left(b^7-\frac{4\left(bx^4+a\right)b^6}{x^4}+\frac{6\left(bx^4+a\right)^2b^5}{x^8}-\frac{4\left(bx^4+a\right)^3b^4}{x^{12}}+\frac{\left(bx^4+a\right)^4b^3}{x^{16}}\right)}$$

input
```
integrate(x^12*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
45/8192*a^4*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/
4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^3 +
1/2048*(15*(b*x^4 + a)^(3/4)*a^4*b^3/x^3 + 239*(b*x^4 + a)^(7/4)*a^4*b^2/x
^7 - 171*(b*x^4 + a)^(11/4)*a^4*b/x^11 + 45*(b*x^4 + a)^(15/4)*a^4/x^15)/(
b^7 - 4*(b*x^4 + a)*b^6/x^4 + 6*(b*x^4 + a)^2*b^5/x^8 - 4*(b*x^4 + a)^3*b^
4/x^12 + (b*x^4 + a)^4*b^3/x^16)
```

Giac [F]

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\int \left(bx^4+a\right)^{\frac{3}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\int x^{12}\,{\left(b\,x^4+a\right)}^{3/4}\,dx$$

input
```
int(x^12*(a + b*x^4)^(3/4),x)
```

output
```
int(x^12*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^{12}\left(a+bx^4\right)^{3/4}\,dx=\frac{45(b\,x^4+a)^{\frac{3}{4}}\,a^3x-36(b\,x^4+a)^{\frac{3}{4}}\,a^2b\,x^5+32(b\,x^4+a)^{\frac{3}{4}}\,a\,b^2x^9+128(b\,x^4+a)^{\frac{3}{4}}\,b^3x^{13}-45}{2048b^3}$$

input
```
int(x^12*(b*x^4+a)^(3/4),x)
```

output
```
(45*(a + b*x**4)**(3/4)*a**3*x - 36*(a + b*x**4)**(3/4)*a**2*b*x**5 + 32*(
a + b*x**4)**(3/4)*a*b**2*x**9 + 128*(a + b*x**4)**(3/4)*b**3*x**13 - 45*i
nt((a + b*x**4)**(3/4)/(a + b*x**4),x)*a**4)/(2048*b**3)
```

3.487 $\int x^8(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int x^8(a+bx^4)^{3/4}\,dx=-\frac{5a^2x(a+bx^4)^{3/4}}{128b^2}+\frac{ax^5(a+bx^4)^{3/4}}{32b}+\frac{1}{12}x^9(a+bx^4)^{3/4}+\frac{5a^3\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}+\frac{5a^3\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{256b^{9/4}}$$

output
```
-5/128*a^2*x*(b*x^4+a)^(3/4)/b^2+1/32*a*x^5*(b*x^4+a)^(3/4)/b+1/12*x^9*(b*
x^4+a)^(3/4)+5/256*a^3*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+5/256*a^3
*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int x^8(a+bx^4)^{3/4}\,dx=\frac{2\sqrt[4]{b}x(a+bx^4)^{3/4}(-15a^2+12abx^4+32b^2x^8)+15a^3\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+15a^3\text{arctanh}\Big(}{768b^{9/4}}$$

input
```
Integrate[x^8*(a + b*x^4)^(3/4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(-15*a^2 + 12*a*b*x^4 + 32*b^2*x^8) + 15*a^
3*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 15*a^3*ArcTanh[(b^(1/4)*x)/(a +
b*x^4)^(1/4)])/(768*b^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {811, 843, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{4}a\int \frac{x^8}{\sqrt[4]{bx^4+a}}dx+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{4}a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{8b}\right)+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{4}a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{8b}\right)+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{770}$$

$$\frac{1}{4}a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int\frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}\right)+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 756

$$\frac{1}{4}a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{8b}\right)+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 216

$$\frac{1}{4}a\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}\right)+\frac{1}{12}x^9\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 219

$$\frac{1}{4}a\left(\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}\right)+\frac{1}{12}x^9(a+bx^4)^{3/4}$$

input
```
Int[x^8*(a + b*x^4)^(3/4),x]
```

output
```
(x^9*(a + b*x^4)^(3/4))/12 + (a*((x^5*(a + b*x^4)^(3/4))/(8*b) - (5*a*((x*
(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^
(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/(8*b
)))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{128\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{9}{4}} x^{9}+48 a\,b^{\frac{5}{4}} x^{5} \left(b\,x^{4}+a\right)^{\frac{3}{4}}-60 a^{2} x \left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{1}{4}}-30 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a^{3}+15 \ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{1536 b^{\frac{9}{4}}}$

input
```
int(x^8*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(128*(b*x^4+a)^(3/4)*b^(9/4)*x^9+48*a*b^(5/4)*x^5*(b*x^4+a)^(3/4)-6
0*a^2*x*(b*x^4+a)^(3/4)*b^(1/4)-30*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^3
+15*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^3)/b^(9
/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.87

$$\int x^8(a+bx^4)^{3/4}\,dx = \frac{15\left(\frac{a^{12}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\frac{125\left((bx^4+a)^{\frac{1}{4}}a^9+\left(\frac{a^{12}}{b^9}\right)^{\frac{3}{4}}b^7x\right)}{x}\right)-15i\left(\frac{a^{12}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\frac{125\left((bx^4+a)^{\frac{1}{4}}a^9+i\left(\frac{a^{12}}{b^9}\right)^{\frac{3}{4}}b^7 \right.}{x}\right.}{}$$

input
```
integrate(x^8*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/1536*(15*(a^12/b^9)^(1/4)*b^2*log(125*((b*x^4 + a)^(1/4)*a^9 + (a^12/b^9
)^(3/4)*b^7*x)/x) - 15*I*(a^12/b^9)^(1/4)*b^2*log(125*((b*x^4 + a)^(1/4)*a
^9 + I*(a^12/b^9)^(3/4)*b^7*x)/x) + 15*I*(a^12/b^9)^(1/4)*b^2*log(125*((b*
x^4 + a)^(1/4)*a^9 - I*(a^12/b^9)^(3/4)*b^7*x)/x) - 15*(a^12/b^9)^(1/4)*b^
2*log(125*((b*x^4 + a)^(1/4)*a^9 - (a^12/b^9)^(3/4)*b^7*x)/x) + 4*(32*b^2*
x^9 + 12*a*b*x^5 - 15*a^2*x)*(b*x^4 + a)^(3/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int x^8(a+bx^4)^{3/4}\,dx = \frac{a^{\frac{3}{4}}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**9*gamma(9/4)*hyper((-3/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.51

$$\int x^8 \left(a+bx^4\right)^{3/4}\,dx = -\frac{5\,a^3\left(\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{512\,b^2} - \frac{\frac{5\left(bx^4+a\right)^{\frac{3}{4}}a^3b^2}{x^3}+\frac{42\left(bx^4+a\right)^{\frac{7}{4}}a^3b}{x^7}-\frac{15\left(bx^4+a\right)^{\frac{11}{4}}a^3}{x^{11}}}{384\left(b^5-\frac{3\left(bx^4+a\right)b^4}{x^4}+\frac{3\left(bx^4+a\right)^2b^3}{x^8}-\frac{\left(bx^4+a\right)^3b^2}{x^{12}}\right)}$$

input
```
integrate(x^8*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-5/512*a^3*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4
) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^2 - 1
/384*(5*(b*x^4 + a)^(3/4)*a^3*b^2/x^3 + 42*(b*x^4 + a)^(7/4)*a^3*b/x^7 - 1
5*(b*x^4 + a)^(11/4)*a^3/x^11)/(b^5 - 3*(b*x^4 + a)*b^4/x^4 + 3*(b*x^4 + a
)^2*b^3/x^8 - (b*x^4 + a)^3*b^2/x^12)
```

Giac [F]

$$\int x^8 \left(a+bx^4\right)^{3/4} \, dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^8 \, dx$$

input
```
integrate(x^8*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a+bx^4\right)^{3/4} \, dx = \int x^8 \left(b\,x^4+a\right)^{3/4} \,dx$$

input
```
int(x^8*(a + b*x^4)^(3/4),x)
```

output
```
int(x^8*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^8 \left(a+bx^4\right)^{3/4} \, dx = \frac{-15(b\,x^4+a)^{\frac{3}{4}} a^2 x+12(b\,x^4+a)^{\frac{3}{4}} ab\,x^5+32(b\,x^4+a)^{\frac{3}{4}} b^2 x^9+15\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right) a^3}{384b^2}$$

input
```
int(x^8*(b*x^4+a)^(3/4),x)
```

output
```
( - 15*(a + b*x**4)**(3/4)*a**2*x + 12*(a + b*x**4)**(3/4)*a*b*x**5 + 32*(
a + b*x**4)**(3/4)*b**2*x**9 + 15*int((a + b*x**4)**(3/4)/(a + b*x**4),x)*
a**3)/(384*b**2)
```

3.488 $\int x^4(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^4(a+bx^4)^{3/4}\,dx=\frac{3ax(a+bx^4)^{3/4}}{32b}+\frac{1}{8}x^5(a+bx^4)^{3/4}-\frac{3a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}-\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}$$

output
```
3/32*a*x*(b*x^4+a)^(3/4)/b+1/8*x^5*(b*x^4+a)^(3/4)-3/64*a^2*arctan(b^(1/4)
*x/(b*x^4+a)^(1/4))/b^(5/4)-3/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^
(5/4)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int x^4(a+bx^4)^{3/4}\,dx=\frac{2\sqrt[4]{b}x(a+bx^4)^{3/4}(3a+4bx^4)-3a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-3a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{5/4}}$$

input
```
Integrate[x^4*(a + b*x^4)^(3/4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(3*a + 4*b*x^4) - 3*a^2*ArcTan[(b^(1/4)*x)/
(a + b*x^4)^(1/4)] - 3*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^(
5/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {811, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{3}{8}a\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{3}{8}a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{ 770}$$

$$\frac{3}{8}a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)+\frac{1}{8}x^5\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{3}{8}a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)+\frac{1}{8}x^5(a+bx^4)^{3/4}$$

$\downarrow$ 216

$$\frac{3}{8}a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)+\frac{1}{8}x^5(a+bx^4)^{3/4}$$

$\downarrow$ 219

$$\frac{3}{8}a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)+\frac{1}{8}x^5(a+bx^4)^{3/4}$$

input
```
Int[x^4*(a + b*x^4)^(3/4),x]
```

output
```
(x^5*(a + b*x^4)^(3/4))/8 + (3*a*((x*(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTan
[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x
^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/8
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$\frac{16(b\,x^{4}+a)^{\frac{3}{4}} b^{\frac{5}{4}} x^{5}+12 a x (b\,x^{4}+a)^{\frac{3}{4}} b^{\frac{1}{4}}-3\ln\left(\frac{b^{\frac{1}{4}} x+(b\,x^{4}+a)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+(b\,x^{4}+a)^{\frac{1}{4}}}\right) a^{2}+6\arctan\left(\frac{(b\,x^{4}+a)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a^{2}}{128 b^{\frac{5}{4}}}$	102

input
```
int(x^4*(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(16*(b*x^4+a)^(3/4)*b^(5/4)*x^5+12*a*x*(b*x^4+a)^(3/4)*b^(1/4)-3*ln(
(b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^2+6*arctan(1/b
^(1/4)/x*(b*x^4+a)^(1/4))*a^2)/b^(5/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.13

$$\int x^4 \left(a+b x^4\right)^{3/4}\,dx = -\frac{3\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}} b\log\left(\frac{27\left((bx^4+a)^{\frac{1}{4}}a^6+\left(\frac{a^8}{b^5}\right)^{\frac{3}{4}}b^4x\right)}{x}\right)-3i\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}} b\log\left(\frac{27\left((bx^4+a)^{\frac{1}{4}}a^6+i\left(\frac{a^8}{b^5}\right)^{\frac{3}{4}}b^4x\right)}{x}\right)+3i\left(\frac{a^8}{b^5}\right)^{\frac{1}{4}} b\log}{128}$$

input
```
integrate(x^4*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/128*(3*(a^8/b^5)^(1/4)*b*log(27*((b*x^4 + a)^(1/4)*a^6 + (a^8/b^5)^(3/4
)*b^4*x)/x) - 3*I*(a^8/b^5)^(1/4)*b*log(27*((b*x^4 + a)^(1/4)*a^6 + I*(a^8
/b^5)^(3/4)*b^4*x)/x) + 3*I*(a^8/b^5)^(1/4)*b*log(27*((b*x^4 + a)^(1/4)*a^
6 - I*(a^8/b^5)^(3/4)*b^4*x)/x) - 3*(a^8/b^5)^(1/4)*b*log(27*((b*x^4 + a)^
(1/4)*a^6 - (a^8/b^5)^(3/4)*b^4*x)/x) - 4*(4*b*x^5 + 3*a*x)*(b*x^4 + a)^(3
/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int x^4 \left(a+bx^4\right)^{3/4} dx = \frac{a^{\frac{3}{4}} x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**5*gamma(5/4)*hyper((-3/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.47

$$\int x^4 \left(a+bx^4\right)^{3/4} dx = \frac{3\,a^2 \left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b} + \frac{\frac{\left(bx^4+a\right)^{\frac{3}{4}} a^2 b}{x^3} + \frac{3\left(bx^4+a\right)^{\frac{7}{4}} a^2}{x^7}}{32\left(b^3 - \frac{2\left(bx^4+a\right)b^2}{x^4} + \frac{\left(bx^4+a\right)^2 b}{x^8}\right)}$$

input
```
integrate(x^4*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
3/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b + 1/32
*((b*x^4 + a)^(3/4)*a^2*b/x^3 + 3*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^3 - 2*(b*x
^4 + a)*b^2/x^4 + (b*x^4 + a)^2*b/x^8)
```

Giac [F]

$$\int x^4 \left(a+bx^4\right)^{3/4} dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^4\right)^{3/4} dx = \int x^4 \left(b\,x^4+a\right)^{3/4} dx$$

input
```
int(x^4*(a + b*x^4)^(3/4),x)
```

output
```
int(x^4*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^4 \left(a+bx^4\right)^{3/4} dx = \frac{3(b\,x^4+a)^{\frac{3}{4}}\,ax+4(b\,x^4+a)^{\frac{3}{4}}\,b\,x^5-3\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)a^2}{32b}$$

input
```
int(x^4*(b*x^4+a)^(3/4),x)
```

output
```
(3*(a + b*x**4)**(3/4)*a*x + 4*(a + b*x**4)**(3/4)*b*x**5 - 3*int((a + b*x
**4)**(3/4)/(a + b*x**4),x)*a**2)/(32*b)
```

3.489 $\int (a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 75

$$\int \left(a+bx^4\right)^{3/4}\,dx=\frac{1}{4}x\left(a+bx^4\right)^{3/4}+\frac{3a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}+\frac{3a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}$$

output
```
1/4*x*(b*x^4+a)^(3/4)+3/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)+3/8*
a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \left(a+bx^4\right)^{3/4}\,dx=\frac{1}{4}x\left(a+bx^4\right)^{3/4}+\frac{3a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}+\frac{3a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8\sqrt[4]{b}}$$

input
```
Integrate[(a + b*x^4)^(3/4),x]
```

output
```
(x*(a + b*x^4)^(3/4))/4 + (3*a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b
^(1/4)) + (3*a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {748, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{748}$$

$$\frac{3}{4}a\int \frac{1}{\sqrt[4]{bx^4+a}}dx+\frac{1}{4}x\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{770}$$

$$\frac{3}{4}a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{4}x\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{756}$$

$$\frac{3}{4}a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{4}a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)+\frac{1}{4}x\left(a+bx^4\right)^{3/4}$$

input
```
Int[(a + b*x^4)^(3/4),x]
```

output
```
(x*(a + b*x^4)^(3/4))/4 + (3*a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b
^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{4\left(b\,x^{4}+a\right)^{\frac{3}{4}} x\,b^{\frac{1}{4}}-6\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a+3\ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a}{16 b^{\frac{1}{4}}}$	80

input
```
int((b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*(b*x^4+a)^(3/4)*x*b^(1/4)-6*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a+
3*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a)/b^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.51

$$\begin{aligned}\int \left(a+bx^4\right)^{3/4}\,dx &= \frac{1}{4}\left(bx^4+a\right)^{\frac{3}{4}}x \\ &+ \frac{3}{16}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3+\left(\frac{a^4}{b}\right)^{\frac{3}{4}}bx\right)}{x}\right) \\ &- \frac{3}{16}i\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3+i\left(\frac{a^4}{b}\right)^{\frac{3}{4}}bx\right)}{x}\right) \\ &+ \frac{3}{16}i\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3-i\left(\frac{a^4}{b}\right)^{\frac{3}{4}}bx\right)}{x}\right) \\ &- \frac{3}{16}\left(\frac{a^4}{b}\right)^{\frac{1}{4}}\log\left(\frac{27\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3-\left(\frac{a^4}{b}\right)^{\frac{3}{4}}bx\right)}{x}\right)\end{aligned}$$

input
```
integrate((b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/4*(b*x^4 + a)^(3/4)*x + 3/16*(a^4/b)^(1/4)*log(27*((b*x^4 + a)^(1/4)*a^3
 + (a^4/b)^(3/4)*b*x)/x) - 3/16*I*(a^4/b)^(1/4)*log(27*((b*x^4 + a)^(1/4)*
a^3 + I*(a^4/b)^(3/4)*b*x)/x) + 3/16*I*(a^4/b)^(1/4)*log(27*((b*x^4 + a)^(
1/4)*a^3 - I*(a^4/b)^(3/4)*b*x)/x) - 3/16*(a^4/b)^(1/4)*log(27*((b*x^4 + a
)^(1/4)*a^3 - (a^4/b)^(3/4)*b*x)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \left(a+bx^4\right)^{3/4}\,dx=\frac{a^{\frac{3}{4}}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x*gamma(1/4)*hyper((-3/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \left(a+bx^4\right)^{3/4}\,dx=-\frac{3}{16}\,a\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)-\frac{\left(bx^4+a\right)^{\frac{3}{4}}a}{4\left(b-\frac{bx^4+a}{x^4}\right)x^3}$$

input
```
integrate((b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-3/16*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
 (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) - 1/4*(b*x
^4 + a)^(3/4)*a/((b - (b*x^4 + a)/x^4)*x^3)
```

Giac [F]

$$\int \left(a+bx^4\right)^{3/4} \, dx = \int \left(bx^4+a\right)^{\frac{3}{4}} \, dx$$

input
```
integrate((b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \left(a+bx^4\right)^{3/4} \, dx = \frac{x\,{\left(b\,x^4+a\right)}^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{{\left(\frac{b\,x^4}{a}+1\right)}^{3/4}}$$

input
```
int((a + b*x^4)^(3/4),x)
```

output
```
(x*(a + b*x^4)^(3/4)*hypergeom([-3/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
 1)^(3/4)
```

Reduce [F]

$$\int \left(a+bx^4\right)^{3/4} dx = \frac{(b\,x^4+a)^{\frac{3}{4}}\,x}{4} + \frac{3\left(\int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)a}{4}$$

input
```
int((b*x^4+a)^(3/4),x)
```

output
```
((a + b*x**4)**(3/4)*x + 3*int((a + b*x**4)**(3/4)/(a + b*x**4),x)*a)/4
```

3.490 $\int \frac{(a+bx^4)^{3/4}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{(a+bx^4)^{3/4}}{x^4} dx = -\frac{(a+bx^4)^{3/4}}{3x^3} + \frac{1}{2}b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)$$

output
```
-1/3*(b*x^4+a)^(3/4)/x^3+1/2*b^(3/4)*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))+1/2
*b^(3/4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^{3/4}}{x^4} dx = -\frac{(a+bx^4)^{3/4}}{3x^3} + \frac{1}{2}b^{3/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{1}{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^4,x]
```

output
```
-1/3*(a + b*x^4)^(3/4)/x^3 + (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)
])/2 + (b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {809, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^4}dx$$

$$\Big\downarrow \text{ 809}$$

$$b\int \frac{1}{\sqrt[4]{bx^4+a}}dx-\frac{(a+bx^4)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 770}$$

$$b\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}-\frac{(a+bx^4)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 756}$$

$$b\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)-\frac{(a+bx^4)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 216}$$

$$b\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)-\frac{(a+bx^4)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 219}$$

$$b\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)-\frac{(a+bx^4)^{3/4}}{3x^3}$$

input
```
Int[(a + b*x^4)^(3/4)/x^4,x]
```

output
```
-1/3*(a + b*x^4)^(3/4)/x^3 + b*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b
^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

method	result	size
pseudoelliptic	$\frac{-6 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) b^{\frac{3}{4}} x^{3}+3 \ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) b^{\frac{3}{4}} x^{3}-4\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{12 x^{3}}$	86

input
```
int((b*x^4+a)^(3/4)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*(-6*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*b^(3/4)*x^3+3*ln((b^(1/4)*x+(
b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*b^(3/4)*x^3-4*(b*x^4+a)^(3/4
))/x^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(3/4)/x^4,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{(a+bx^4)^{3/4}}{x^4}\,dx = \frac{a^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**4,x)
```

output
```
a**(3/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^4)^{3/4}}{x^4}\,dx = -\frac{1}{4}\,b\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right) - \frac{(bx^4+a)^{\frac{3}{4}}}{3\,x^3}$$

input
```
integrate((b*x^4+a)^(3/4)/x^4,x, algorithm="maxima")
```

output
```
-1/4*b*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
(b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)) - 1/3*(b*x^
4 + a)^(3/4)/x^3
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^4}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{x^4}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^4,x)
```

output
```
int((a + b*x^4)^(3/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^4}\,dx = \int \frac{(b\,x^4+a)^{\frac{3}{4}}}{x^4}dx$$

input
```
int((b*x^4+a)^(3/4)/x^4,x)
```

output
```
int((a + b*x**4)**(3/4)/x**4,x)
```

3.491 $\int \frac{(a+bx^4)^{3/4}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx^4)^{3/4}}{x^8} dx = -\frac{(a+bx^4)^{7/4}}{7ax^7}$$

output
```
-1/7*(b*x^4+a)^(7/4)/a/x^7
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^{3/4}}{x^8} dx = -\frac{(a+bx^4)^{7/4}}{7ax^7}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^8,x]
```

output
```
-1/7*(a + b*x^4)^(7/4)/(a*x^7)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^8} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+bx^4)^{7/4}}{7ax^7}$$

input
```
Int[(a + b*x^4)^(3/4)/x^8,x]
```

output
```
-1/7*(a + b*x^4)^(7/4)/(a*x^7)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{7}{4}}}{7a\,x^7}$	18
trager	$-\frac{(b\,x^4+a)^{\frac{7}{4}}}{7a\,x^7}$	18
risch	$-\frac{(b\,x^4+a)^{\frac{7}{4}}}{7a\,x^7}$	18
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{7}{4}}}{7a\,x^7}$	18
orering	$-\frac{(b\,x^4+a)^{\frac{7}{4}}}{7a\,x^7}$	18

input
```
int((b*x^4+a)^(3/4)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^4+a)^(7/4)/a/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^{3/4}}{x^8}\,dx = -\frac{(bx^4+a)^{\frac{7}{4}}}{7\,ax^7}$$

input
```
integrate((b*x^4+a)^(3/4)/x^8,x, algorithm="fricas")
```

output
```
-1/7*(b*x^4 + a)^(7/4)/(a*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.56 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx^4)^{3/4}}{x^8}\,dx = \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4x^4\Gamma\left(-\frac{3}{4}\right)} + \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**8,x)
```

output
```
b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*x**4*gamma(-3/4)) + b**(7/
4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*a*gamma(-3/4))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^{3/4}}{x^8}\,dx = -\frac{(bx^4+a)^{\frac{7}{4}}}{7\,ax^7}$$

input
```
integrate((b*x^4+a)^(3/4)/x^8,x, algorithm="maxima")
```

output
```
-1/7*(b*x^4 + a)^(7/4)/(a*x^7)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^8}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^{3/4}}{x^8}\,dx = -\frac{(b\,x^4+a)^{7/4}}{7\,a\,x^7}$$

input
```
int((a + b*x^4)^(3/4)/x^8,x)
```

output
```
-(a + b*x^4)^(7/4)/(7*a*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^{3/4}}{x^8}\,dx = -\frac{(b\,x^4+a)^{\frac{7}{4}}}{7a\,x^7}$$

input
```
int((b*x^4+a)^(3/4)/x^8,x)
```

output
```
( - (a + b*x**4)**(3/4)*(a + b*x**4))/(7*a*x**7)
```

3.492 $\int \frac{(a+bx^4)^{3/4}}{x^{12}} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}} dx = -\frac{(a+bx^4)^{7/4}}{11ax^{11}} + \frac{4b(a+bx^4)^{7/4}}{77a^2x^7}$$

output
```
-1/11*(b*x^4+a)^(7/4)/a/x^11+4/77*b*(b*x^4+a)^(7/4)/a^2/x^7
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}} dx = \frac{(a+bx^4)^{7/4}(-7a+4bx^4)}{77a^2x^{11}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^12,x]
```

output
```
((a + b*x^4)^(7/4)*(-7*a + 4*b*x^4))/(77*a^2*x^11)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\int \frac{(bx^4+a)^{3/4}}{x^8}dx}{11a}-\frac{(a+bx^4)^{7/4}}{11ax^{11}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{4b(a+bx^4)^{7/4}}{77a^2x^7}-\frac{(a+bx^4)^{7/4}}{11ax^{11}}$$

input
```
Int[(a + b*x^4)^(3/4)/x^12,x]
```

output
```
-1/11*(a + b*x^4)^(7/4)/(a*x^11) + (4*b*(a + b*x^4)^(7/4))/(77*a^2*x^7)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{7}{4}}(-4b\,x^4+7a)}{77x^{11}a^2}$	28
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{7}{4}}(-4b\,x^4+7a)}{77x^{11}a^2}$	28
orering	$-\frac{(b\,x^4+a)^{\frac{7}{4}}(-4b\,x^4+7a)}{77x^{11}a^2}$	28
trager	$-\frac{(-4b^2x^8+3ab\,x^4+7a^2)(b\,x^4+a)^{\frac{3}{4}}}{77x^{11}a^2}$	39
risch	$-\frac{(-4b^2x^8+3ab\,x^4+7a^2)(b\,x^4+a)^{\frac{3}{4}}}{77x^{11}a^2}$	39

input
```
int((b*x^4+a)^(3/4)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/77*(b*x^4+a)^(7/4)*(-4*b*x^4+7*a)/x^11/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx = \frac{(4\,b^2x^8-3\,abx^4-7\,a^2)(bx^4+a)^{\frac{3}{4}}}{77\,a^2x^{11}}$$

input
```
integrate((b*x^4+a)^(3/4)/x^12,x, algorithm="fricas")
```

output
```
1/77*(4*b^2*x^8 - 3*a*b*x^4 - 7*a^2)*(b*x^4 + a)^(3/4)/(a^2*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(37) = 74$.

Time = 0.85 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx = -\frac{7b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16x^8\Gamma\left(-\frac{3}{4}\right)} - \frac{3b^{\frac{7}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{16ax^4\Gamma\left(-\frac{3}{4}\right)} + \frac{b^{\frac{11}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{4a^2\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**12,x)
```

output
```
-7*b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(16*x**8*gamma(-3/4)) - 3
*b**(7/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(16*a*x**4*gamma(-3/4)) + b
**(11/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(4*a**2*gamma(-3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx = \frac{\frac{11\,(bx^4+a)^{\frac{7}{4}}b}{x^7} - \frac{7\,(bx^4+a)^{\frac{11}{4}}}{x^{11}}}{77\,a^2}$$

input
```
integrate((b*x^4+a)^(3/4)/x^12,x, algorithm="maxima")
```

output
```
1/77*(11*(b*x^4 + a)^(7/4)*b/x^7 - 7*(b*x^4 + a)^(11/4)/x^11)/a^2
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^12,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^12, x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx=-\frac{7\,a^2\,(b\,x^4+a)^{3/4}-4\,b^2\,x^8\,(b\,x^4+a)^{3/4}+3\,a\,b\,x^4\,(b\,x^4+a)^{3/4}}{77\,a^2\,x^{11}}$$

input
```
int((a + b*x^4)^(3/4)/x^12,x)
```

output
```
-(7*a^2*(a + b*x^4)^(3/4) - 4*b^2*x^8*(a + b*x^4)^(3/4) + 3*a*b*x^4*(a + b
*x^4)^(3/4))/(77*a^2*x^11)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^4)^{3/4}}{x^{12}}\,dx=\frac{(b\,x^4+a)^{\frac{3}{4}}\,(4b^2x^8-3ab\,x^4-7a^2)}{77a^2x^{11}}$$

input
```
int((b*x^4+a)^(3/4)/x^12,x)
```

output
```
((a + b*x**4)**(3/4)*( - 7*a**2 - 3*a*b*x**4 + 4*b**2*x**8))/(77*a**2*x**1
1)
```

3.493 $\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = -\frac{(a+bx^4)^{7/4}}{15ax^{15}} + \frac{8b(a+bx^4)^{7/4}}{165a^2x^{11}} - \frac{32b^2(a+bx^4)^{7/4}}{1155a^3x^7}$$

output
```
-1/15*(b*x^4+a)^(7/4)/a/x^15+8/165*b*(b*x^4+a)^(7/4)/a^2/x^11-32/1155*b^2*
(b*x^4+a)^(7/4)/a^3/x^7
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = \frac{(a+bx^4)^{7/4}(-77a^2+56abx^4-32b^2x^8)}{1155a^3x^{15}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^16,x]
```

output
```
((a + b*x^4)^(7/4)*(-77*a^2 + 56*a*b*x^4 - 32*b^2*x^8))/(1155*a^3*x^15)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{8b\int \frac{(bx^4+a)^{3/4}}{x^{12}}dx}{15a}-\frac{(a+bx^4)^{7/4}}{15ax^{15}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{8b\left(-\frac{4b\int \frac{(bx^4+a)^{3/4}}{x^8}dx}{11a}-\frac{(a+bx^4)^{7/4}}{11ax^{11}}\right)}{15a}-\frac{(a+bx^4)^{7/4}}{15ax^{15}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{8b\left(\frac{4b(a+bx^4)^{7/4}}{77a^2x^7}-\frac{(a+bx^4)^{7/4}}{11ax^{11}}\right)}{15a}-\frac{(a+bx^4)^{7/4}}{15ax^{15}}$$

input

```
Int[(a + b*x^4)^(3/4)/x^16,x]
```

output

```
-1/15*(a + b*x^4)^(7/4)/(a*x^15) - (8*b*(-1/11*(a + b*x^4)^(7/4)/(a*x^11)
+ (4*b*(a + b*x^4)^(7/4))/(77*a^2*x^7)))/(15*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(32 b^{2} x^{8}-56 a b\,x^{4}+77 a^{2}\right)}{1155 a^{3} x^{15}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(32 b^{2} x^{8}-56 a b\,x^{4}+77 a^{2}\right)}{1155 a^{3} x^{15}}$	39
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(32 b^{2} x^{8}-56 a b\,x^{4}+77 a^{2}\right)}{1155 a^{3} x^{15}}$	39
trager	$-\frac{\left(32 b^{3} x^{12}-24 a\,b^{2} x^{8}+21 a^{2} b\,x^{4}+77 a^{3}\right) \left(b\,x^{4}+a\right)^{\frac{3}{4}}}{1155 a^{3} x^{15}}$	50
risch	$-\frac{\left(32 b^{3} x^{12}-24 a\,b^{2} x^{8}+21 a^{2} b\,x^{4}+77 a^{3}\right) \left(b\,x^{4}+a\right)^{\frac{3}{4}}}{1155 a^{3} x^{15}}$	50

input
```
int((b*x^4+a)^(3/4)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(b*x^4+a)^(7/4)*(32*b^2*x^8-56*a*b*x^4+77*a^2)/a^3/x^15
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = -\frac{(32\,b^3x^{12} - 24\,ab^2x^8 + 21\,a^2bx^4 + 77\,a^3)(bx^4+a)^{\frac{3}{4}}}{1155\,a^3x^{15}}$$

input
```
integrate((b*x^4+a)^(3/4)/x^16,x, algorithm="fricas")
```

output
```
-1/1155*(32*b^3*x^12 - 24*a*b^2*x^8 + 21*a^2*b*x^4 + 77*a^3)*(b*x^4 + a)^(
3/4)/(a^3*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(61) = 122.

Time = 1.23 (sec) , antiderivative size = 520, normalized size of antiderivative = 7.65

$$\begin{aligned}\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx &= \frac{77a^5b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)} \\ &+ \frac{175a^4b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)} \\ &+ \frac{95a^3b^{\frac{27}{4}}x^8\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)} \\ &+ \frac{5a^2b^{\frac{31}{4}}x^{12}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)} \\ &+ \frac{40ab^{\frac{35}{4}}x^{16}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)} \\ &+ \frac{32b^{\frac{39}{4}}x^{20}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{15}{4}\right)}{64a^5b^4x^{12}\Gamma\left(-\frac{3}{4}\right)+128a^4b^5x^{16}\Gamma\left(-\frac{3}{4}\right)+64a^3b^6x^{20}\Gamma\left(-\frac{3}{4}\right)}\end{aligned}$$

input
```
integrate((b*x**4+a)**(3/4)/x**16,x)
```

output
```
77*a**5*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12
*gamma(-3/4) + 128*a**4*b**5*x**16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(
-3/4)) + 175*a**4*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*
a**5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*x**16*gamma(-3/4) + 64*a**3*b*
*6*x**20*gamma(-3/4)) + 95*a**3*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gam
ma(-15/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*x**16*gamma(-3/4
) + 64*a**3*b**6*x**20*gamma(-3/4)) + 5*a**2*b**(31/4)*x**12*(a/(b*x**4) +
 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*a**4*b**5*x*
*16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(-3/4)) + 40*a*b**(35/4)*x**16*(
a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12*gamma(-3/4) + 128*
a**4*b**5*x**16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(-3/4)) + 32*b**(39/
4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(64*a**5*b**4*x**12*gamma(-3
/4) + 128*a**4*b**5*x**16*gamma(-3/4) + 64*a**3*b**6*x**20*gamma(-3/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = -\frac{\frac{165\,(bx^4+a)^{\frac{7}{4}}b^2}{x^7} - \frac{210\,(bx^4+a)^{\frac{11}{4}}b}{x^{11}} + \frac{77\,(bx^4+a)^{\frac{15}{4}}}{x^{15}}}{1155\,a^3}$$

input
```
integrate((b*x^4+a)^(3/4)/x^16,x, algorithm="maxima")
```

output
```
-1/1155*(165*(b*x^4 + a)^(7/4)*b^2/x^7 - 210*(b*x^4 + a)^(11/4)*b/x^11 + 7
7*(b*x^4 + a)^(15/4)/x^15)/a^3
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^16,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^16, x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = \frac{8\,b^2\,(b\,x^4+a)^{3/4}}{385\,a^2\,x^7} - \frac{b\,(b\,x^4+a)^{3/4}}{55\,a\,x^{11}} - \frac{32\,b^3\,(b\,x^4+a)^{3/4}}{1155\,a^3\,x^3} - \frac{(b\,x^4+a)^{3/4}}{15\,x^{15}}$$

input
```
int((a + b*x^4)^(3/4)/x^16,x)
```

output
```
(8*b^2*(a + b*x^4)^(3/4))/(385*a^2*x^7) - (b*(a + b*x^4)^(3/4))/(55*a*x^11
) - (32*b^3*(a + b*x^4)^(3/4))/(1155*a^3*x^3) - (a + b*x^4)^(3/4)/(15*x^15
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^4)^{3/4}}{x^{16}}\,dx = \frac{(b\,x^4+a)^{\frac{3}{4}}\,(-32b^3x^{12}+24a\,b^2x^8-21a^2b\,x^4-77a^3)}{1155a^3x^{15}}$$

input
```
int((b*x^4+a)^(3/4)/x^16,x)
```

output
```
((a + b*x**4)**(3/4)*( - 77*a**3 - 21*a**2*b*x**4 + 24*a*b**2*x**8 - 32*b*
*3*x**12))/(1155*a**3*x**15)
```

3.494 $\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = -\frac{(a+bx^4)^{7/4}}{19ax^{19}}+\frac{4b(a+bx^4)^{7/4}}{95a^2x^{15}} -\frac{32b^2(a+bx^4)^{7/4}}{1045a^3x^{11}}+\frac{128b^3(a+bx^4)^{7/4}}{7315a^4x^7}$$

output
```
-1/19*(b*x^4+a)^(7/4)/a/x^19+4/95*b*(b*x^4+a)^(7/4)/a^2/x^15-32/1045*b^2*(
b*x^4+a)^(7/4)/a^3/x^11+128/7315*b^3*(b*x^4+a)^(7/4)/a^4/x^7
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \frac{(a+bx^4)^{7/4}(-385a^3+308a^2bx^4-224ab^2x^8+128b^3x^{12})}{7315a^4x^{19}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^20,x]
```

output
```
((a + b*x^4)^(7/4)*(-385*a^3 + 308*a^2*b*x^4 - 224*a*b^2*x^8 + 128*b^3*x^1
2))/(7315*a^4*x^19)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{12b\int \frac{(bx^4+a)^{3/4}}{x^{16}}dx}{19a}-\frac{(a+bx^4)^{7/4}}{19ax^{19}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{12b\left(-\frac{8b\int \frac{(bx^4+a)^{3/4}}{x^{12}}dx}{15a}-\frac{(a+bx^4)^{7/4}}{15ax^{15}}\right)}{19a}-\frac{(a+bx^4)^{7/4}}{19ax^{19}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{(bx^4+a)^{3/4}}{x^8}dx}{11a}-\frac{(a+bx^4)^{7/4}}{11ax^{11}}\right)}{15a}-\frac{(a+bx^4)^{7/4}}{15ax^{15}}\right)}{19a}-\frac{(a+bx^4)^{7/4}}{19ax^{19}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{12b\left(-\frac{8b\left(\frac{4b(a+bx^4)^{7/4}}{77a^2x^7}-\frac{(a+bx^4)^{7/4}}{11ax^{11}}\right)}{15a}-\frac{(a+bx^4)^{7/4}}{15ax^{15}}\right)}{19a}-\frac{(a+bx^4)^{7/4}}{19ax^{19}}$$

input
```
Int[(a + b*x^4)^(3/4)/x^20,x]
```

output
```
-1/19*(a + b*x^4)^(7/4)/(a*x^19) - (12*b*(-1/15*(a + b*x^4)^(7/4)/(a*x^15)
 - (8*b*(-1/11*(a + b*x^4)^(7/4)/(a*x^11) + (4*b*(a + b*x^4)^(7/4))/(77*a^
2*x^7)))/(15*a)))/(19*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(-128 b^{3} x^{12}+224 a\,b^{2} x^{8}-308 a^{2} b\,x^{4}+385 a^{3}\right)}{7315 x^{19} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(-128 b^{3} x^{12}+224 a\,b^{2} x^{8}-308 a^{2} b\,x^{4}+385 a^{3}\right)}{7315 x^{19} a^{4}}$	50
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{7}{4}} \left(-128 b^{3} x^{12}+224 a\,b^{2} x^{8}-308 a^{2} b\,x^{4}+385 a^{3}\right)}{7315 x^{19} a^{4}}$	50
trager	$-\frac{\left(-128 x^{16} b^{4}+96 a\,b^{3} x^{12}-84 a^{2} b^{2} x^{8}+77 a^{3} b\,x^{4}+385 a^{4}\right) \left(b\,x^{4}+a\right)^{\frac{3}{4}}}{7315 x^{19} a^{4}}$	61
risch	$-\frac{\left(-128 x^{16} b^{4}+96 a\,b^{3} x^{12}-84 a^{2} b^{2} x^{8}+77 a^{3} b\,x^{4}+385 a^{4}\right) \left(b\,x^{4}+a\right)^{\frac{3}{4}}}{7315 x^{19} a^{4}}$	61

input
```
int((b*x^4+a)^(3/4)/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/7315*(b*x^4+a)^(7/4)*(-128*b^3*x^12+224*a*b^2*x^8-308*a^2*b*x^4+385*a^3
)/x^19/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \frac{(128\,b^4x^{16}-96\,ab^3x^{12}+84\,a^2b^2x^8-77\,a^3bx^4-385\,a^4)(bx^4+a)^{\frac{3}{4}}}{7315\,a^4x^{19}}$$

input
```
integrate((b*x^4+a)^(3/4)/x^20,x, algorithm="fricas")
```

output
```
1/7315*(128*b^4*x^16 - 96*a*b^3*x^12 + 84*a^2*b^2*x^8 - 77*a^3*b*x^4 - 385
*a^4)*(b*x^4 + a)^(3/4)/(a^4*x^19)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(85) = 170.

Time = 1.83 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.21

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**4+a)**(3/4)/x**20,x)
```

output
```
-1155*a**7*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x
**16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24
*gamma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4)) - 3696*a**6*b**(43/4)*x**
4*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) +
768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*
a**4*b**12*x**28*gamma(-3/4)) - 3906*a**5*b**(47/4)*x**8*(a/(b*x**4) + 1)*
*(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x**2
0*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*ga
mma(-3/4)) - 1380*a**4*b**(51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4
)/(256*a**7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 76
8*a**5*b**11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4)) + 45*a*
*3*b**(55/4)*x**16*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**
16*gamma(-3/4) + 768*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*g
amma(-3/4) + 256*a**4*b**12*x**28*gamma(-3/4)) + 540*a**2*b**(59/4)*x**20*
(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) + 76
8*a**6*b**10*x**20*gamma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*a*
*4*b**12*x**28*gamma(-3/4)) + 864*a*b**(63/4)*x**24*(a/(b*x**4) + 1)**(3/4
)*gamma(-19/4)/(256*a**7*b**9*x**16*gamma(-3/4) + 768*a**6*b**10*x**20*gam
ma(-3/4) + 768*a**5*b**11*x**24*gamma(-3/4) + 256*a**4*b**12*x**28*gamma(-
3/4)) + 384*b**(67/4)*x**28*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(256*a...
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \frac{\frac{1045\,(bx^4+a)^{\frac{7}{4}}b^3}{x^7} - \frac{1995\,(bx^4+a)^{\frac{11}{4}}b^2}{x^{11}} + \frac{1463\,(bx^4+a)^{\frac{15}{4}}b}{x^{15}} - \frac{385\,(bx^4+a)^{\frac{19}{4}}}{x^{19}}}{7315\,a^4}$$

input
```
integrate((b*x^4+a)^(3/4)/x^20,x, algorithm="maxima")
```

output
```
1/7315*(1045*(b*x^4 + a)^(7/4)*b^3/x^7 - 1995*(b*x^4 + a)^(11/4)*b^2/x^11
+ 1463*(b*x^4 + a)^(15/4)*b/x^15 - 385*(b*x^4 + a)^(19/4)/x^19)/a^4
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{20}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^20,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^20, x)
```

Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \frac{128\,b^4\,(b\,x^4+a)^{3/4}}{7315\,a^4\,x^3} - \frac{b\,(b\,x^4+a)^{3/4}}{95\,a\,x^{15}} - \frac{(b\,x^4+a)^{3/4}}{19\,x^{19}} - \frac{96\,b^3\,(b\,x^4+a)^{3/4}}{7315\,a^3\,x^7} + \frac{12\,b^2\,(b\,x^4+a)^{3/4}}{1045\,a^2\,x^{11}}$$

input
```
int((a + b*x^4)^(3/4)/x^20,x)
```

output
```
(128*b^4*(a + b*x^4)^(3/4))/(7315*a^4*x^3) - (b*(a + b*x^4)^(3/4))/(95*a*x
^15) - (a + b*x^4)^(3/4)/(19*x^19) - (96*b^3*(a + b*x^4)^(3/4))/(7315*a^3*
x^7) + (12*b^2*(a + b*x^4)^(3/4))/(1045*a^2*x^11)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^4)^{3/4}}{x^{20}}\,dx = \frac{(b\,x^4+a)^{\frac{3}{4}}\,(128b^4x^{16} - 96a\,b^3x^{12} + 84a^2b^2x^8 - 77a^3b\,x^4 - 385a^4)}{7315a^4x^{19}}$$

input
```
int((b*x^4+a)^(3/4)/x^20,x)
```

output
```
((a + b*x**4)**(3/4)*( - 385*a**4 - 77*a**3*b*x**4 + 84*a**2*b**2*x**8 - 9
6*a*b**3*x**12 + 128*b**4*x**16))/(7315*a**4*x**19)
```

3.495 $\int x^{10}(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx=\frac{3a^3x^3}{80b^2\sqrt[4]{a+bx^4}}-\frac{a^2x^3(a+bx^4)^{3/4}}{40b^2}+\frac{3ax^7(a+bx^4)^{3/4}}{140b}+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}+\frac{3a^{7/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{80b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
3/80*a^3*x^3/b^2/(b*x^4+a)^(1/4)-1/40*a^2*x^3*(b*x^4+a)^(3/4)/b^2+3/140*a*
x^7*(b*x^4+a)^(3/4)/b+1/14*x^11*(b*x^4+a)^(3/4)+3/80*a^(7/2)*(1+a/b/x^4)^(
1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(b*
x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.63 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.64

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx=\frac{x^3(a+bx^4)^{3/4}\left(\left(1+\frac{bx^4}{a}\right)^{3/4}(-7a^2+3abx^4+10b^2x^8)+7a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4}\right.\right.}{140b^2\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[x^10*(a + b*x^4)^(3/4),x]
```

output
```
(x^3*(a + b*x^4)^(3/4)*((1 + (b*x^4)/a)^(3/4)*(-7*a^2 + 3*a*b*x^4 + 10*b^2
*x^8) + 7*a^2*Hypergeometric2F1[-3/4, 3/4, 7/4, -((b*x^4)/a)]))/(140*b^2*(
1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {811, 843, 843, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{14}a\int \frac{x^{10}}{\sqrt[4]{bx^4+a}}dx+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{14}a\left(\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}-\frac{7a\int \frac{x^6}{\sqrt[4]{bx^4+a}}dx}{10b}\right)+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{14}a\left(\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}-\frac{7a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\int\frac{x^2}{\sqrt[4]{bx^4+a}}dx}{2b}\right)}{10b}\right)+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 839

$$\frac{3}{14}a\left(\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}-\frac{7a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int\frac{x^2}{\left(bx^4+a\right)^{5/4}}dx\right)}{2b}\right)}{10b}\right)+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 813

$$\frac{3}{14}a\left(\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}-\frac{7a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}\right)+\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 858

$$\frac{3}{14}a\left(\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}-\frac{7a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}\right)+$$

$$\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$\downarrow$ 807

$$\frac{3}{14}a\left(\frac{x^7\left(a+bx^4\right)^{3/4}}{10b}-\frac{7a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}\right)+$$

$$\frac{1}{14}x^{11}\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{212}$$

$$\frac{3}{14}a\left(\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}\right)+\frac{1}{14}x^{11}(a+bx^4)^{3/4}$$

input
```
Int[x^10*(a + b*x^4)^(3/4),x]
```

output
```
(x^11*(a + b*x^4)^(3/4))/14 + (3*a*((x^7*(a + b*x^4)^(3/4))/(10*b) - (7*a*
((x^3*(a + b*x^4)^(3/4))/(6*b) - (a*(x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*
(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*
Sqrt[b]*(a + b*x^4)^(1/4))))/(2*b)))/(10*b)))/14
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{10}\left(b\,x^{4}+a\right)^{\frac{3}{4}}d x$$

input
```
int(x^10*(b*x^4+a)^(3/4),x)
```

output
```
int(x^10*(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx=\int \left(bx^4+a\right)^{\frac{3}{4}}x^{10}\,dx$$

input
```
integrate(x^10*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^10, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx=\frac{a^{\frac{3}{4}}x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**11*gamma(11/4)*hyper((-3/4, 11/4), (15/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(15/4))
```

Maxima [F]

$$\int x^{10}\left(a+bx^4\right)^{3/4}\,dx=\int \left(bx^4+a\right)^{\frac{3}{4}}x^{10}\,dx$$

input
```
integrate(x^10*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^10, x)
```

Giac [F]

$$\int x^{10}(a+bx^4)^{3/4}\,dx=\int (bx^4+a)^{\frac{3}{4}}x^{10}\,dx$$

input
```
integrate(x^10*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{10}(a+bx^4)^{3/4}\,dx=\int x^{10}\,\left(b\,x^4+a\right)^{3/4}\,dx$$

input
```
int(x^10*(a + b*x^4)^(3/4),x)
```

output
```
int(x^10*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^{10}(a+bx^4)^{3/4}\,dx=\frac{-7(b\,x^4+a)^{\frac{3}{4}}a^2x^3+6(b\,x^4+a)^{\frac{3}{4}}ab\,x^7+20(b\,x^4+a)^{\frac{3}{4}}b^2x^{11}+21\left(\int\frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}}dx\right)a^3}{280b^2}$$

input
```
int(x^10*(b*x^4+a)^(3/4),x)
```

output
```
( - 7*(a + b*x**4)**(3/4)*a**2*x**3 + 6*(a + b*x**4)**(3/4)*a*b*x**7 + 20*
(a + b*x**4)**(3/4)*b**2*x**11 + 21*int(((a + b*x**4)**(3/4)*x**2)/(a + b*
x**4),x)*a**3)/(280*b**2)
```

3.496 $\int x^6(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int x^6\left(a+bx^4\right)^{3/4}\,dx=-\frac{3a^2x^3}{40b\sqrt[4]{a+bx^4}}+\frac{ax^3\left(a+bx^4\right)^{3/4}}{20b}+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}-\frac{3a^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-3/40*a^2*x^3/b/(b*x^4+a)^(1/4)+1/20*a*x^3*(b*x^4+a)^(3/4)/b+1/10*x^7*(b*x
^4+a)^(3/4)-3/40*a^(5/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1
/2)*x^2/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.51

$$\int x^6\left(a+bx^4\right)^{3/4}\,dx=\frac{x^3\left(a+bx^4\right)^{3/4}\left(a+bx^4-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{\left(1+\frac{bx^4}{a}\right)^{3/4}}\right)}{10b}$$

input
```
Integrate[x^6*(a + b*x^4)^(3/4),x]
```

output
```
(x^3*(a + b*x^4)^(3/4)*(a + b*x^4 - (a*Hypergeometric2F1[-3/4, 3/4, 7/4, -
((b*x^4)/a)])/(1 + (b*x^4)/a)^(3/4)))/(10*b)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {811, 843, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^4\right)^{3/4} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{3}{10}a\int \frac{x^6}{\sqrt[4]{bx^4+a}}dx+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{3}{10}a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx}{2b}\right)+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{839}$$

$$\frac{3}{10}a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx\right)}{2b}\right)+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{813}$$

$$\frac{3}{10}a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{2b}\right)+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 858

$$\frac{3}{10}a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 807

$$\frac{3}{10}a\left(\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)+\frac{1}{10}x^7\left(a+bx^4\right)^{3/4}$$

$\Big\downarrow$ 212

$$\frac{3}{10}a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)+\frac{1}{10}x^7(a+bx^4)^{3/4}$$

input
```
Int[x^6*(a + b*x^4)^(3/4),x]
```

output
```
(x^7*(a + b*x^4)^(3/4))/10 + (3*a*((x^3*(a + b*x^4)^(3/4))/(6*b) - (a*(x^3
/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan
[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(1/4))))/(2*b)))/10
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^6 \left(b\,x^4+a\right)^{\frac{3}{4}}dx$$

input
```
int(x^6*(b*x^4+a)^(3/4),x)
```

output
```
int(x^6*(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int x^6 (a+bx^4)^{3/4}\,dx = \int (bx^4+a)^{\frac{3}{4}} x^6\,dx$$

input
```
integrate(x^6*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int x^6 (a+bx^4)^{3/4}\,dx = \frac{a^{\frac{3}{4}} x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**7*gamma(7/4)*hyper((-3/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(11/4))
```

Maxima [F]

$$\int x^6 (a+bx^4)^{3/4}\,dx = \int (bx^4+a)^{\frac{3}{4}} x^6\,dx$$

input
```
integrate(x^6*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^6, x)
```

Giac [F]

$$\int x^6 \left(a+bx^4\right)^{3/4} \, dx = \int \left(bx^4+a\right)^{\frac{3}{4}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a+bx^4\right)^{3/4} \, dx = \int x^6 \left(b\,x^4+a\right)^{3/4} \, dx$$

input
```
int(x^6*(a + b*x^4)^(3/4),x)
```

output
```
int(x^6*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^6 \left(a+bx^4\right)^{3/4} \, dx = \frac{\left(b\,x^4+a\right)^{\frac{3}{4}} a\,x^3+2\left(b\,x^4+a\right)^{\frac{3}{4}} b\,x^7-3\left(\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right) a^2}{20b}$$

input
```
int(x^6*(b*x^4+a)^(3/4),x)
```

output
```
((a + b*x**4)**(3/4)*a*x**3 + 2*(a + b*x**4)**(3/4)*b*x**7 - 3*int(((a + b
*x**4)**(3/4)*x**2)/(a + b*x**4),x)*a**2)/(20*b)
```

3.497 $\int x^2(a+bx^4)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int x^2\left(a+bx^4\right)^{3/4}\,dx=\frac{ax^3}{4\sqrt[4]{a+bx^4}}+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}+\frac{a^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
1/4*a*x^3/(b*x^4+a)^(1/4)+1/6*x^3*(b*x^4+a)^(3/4)+1/4*a^(3/2)*(1+a/b/x^4)^
(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(1/2)/(b
*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int x^2\left(a+bx^4\right)^{3/4}\,dx=\frac{x^3(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[x^2*(a + b*x^4)^(3/4),x]
```

output
```
(x^3*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 7/4, -((b*x^4)/a)])/(3
*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {811, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^4\right)^{3/4} \, dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{2}a\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx+\frac{1}{6}x^3 \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{839}$$

$$\frac{1}{2}a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx\right)+\frac{1}{6}x^3 \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{813}$$

$$\frac{1}{2}a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)+\frac{1}{6}x^3 \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{2}a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)+\frac{1}{6}x^3 \left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}a\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)+\frac{1}{6}x^3\left(a+bx^4\right)^{3/4}$$

input
```
Int[x^2*(a + b*x^4)^(3/4),x]
```

output
```
(x^3*(a + b*x^4)^(3/4))/6 + (a*(x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 +
a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[
b]*(a + b*x^4)^(1/4))))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{2} \left(b\,x^{4}+a\right)^{\frac{3}{4}}d x$$

input
```
int(x^2*(b*x^4+a)^(3/4),x)
```

output
```
int(x^2*(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^4\right)^{3/4} \, dx = \int \left(bx^{4}+a\right)^{\frac{3}{4}}x^{2}\, dx$$

input
```
integrate(x^2*(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int x^2(a+bx^4)^{3/4}\,dx = \frac{a^{\frac{3}{4}}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(b*x**4+a)**(3/4),x)
```

output
```
a**(3/4)*x**3*gamma(3/4)*hyper((-3/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4))
```

Maxima [F]

$$\int x^2(a+bx^4)^{3/4}\,dx = \int (bx^4+a)^{\frac{3}{4}}x^2\,dx$$

input
```
integrate(x^2*(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^4)^{3/4}\,dx = \int (bx^4+a)^{\frac{3}{4}}x^2\,dx$$

input
```
integrate(x^2*(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^4\right)^{3/4} \, dx = \int x^2 \left(b\,x^4+a\right)^{3/4} \,dx$$

input
```
int(x^2*(a + b*x^4)^(3/4),x)
```

output
```
int(x^2*(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int x^2 \left(a+bx^4\right)^{3/4} \, dx = \frac{\left(b\,x^4+a\right)^{\frac{3}{4}} x^3}{6} + \frac{\left(\int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx\right) a}{2}$$

input
```
int(x^2*(b*x^4+a)^(3/4),x)
```

output
```
((a + b*x**4)**(3/4)*x**3 + 3*int(((a + b*x**4)**(3/4)*x**2)/(a + b*x**4),
x)*a)/6
```

3.498 $\int \frac{(a+bx^4)^{3/4}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{(a+bx^4)^{3/4}}{x^2} dx = \frac{3bx^3}{2\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{x} + \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt[4]{a+bx^4}}$$

output
```
3/2*b*x^3/(b*x^4+a)^(1/4)-(b*x^4+a)^(3/4)/x+3/2*a^(1/2)*b^(1/2)*(1+a/b/x^4
)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/(b*x^4+a
)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^4)^{3/4}}{x^2} dx = -\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^2,x]
```

output
```
-(((a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, -1/4, 3/4, -((b*x^4)/a)])/(x*
(1 + (b*x^4)/a)^(3/4)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {809, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^2}dx$$

$$\Big\downarrow \text{809}$$

$$3b\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx-\frac{\left(a+bx^4\right)^{3/4}}{x}$$

$$\Big\downarrow \text{839}$$

$$3b\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx\right)-\frac{\left(a+bx^4\right)^{3/4}}{x}$$

$$\Big\downarrow \text{813}$$

$$3b\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{x}$$

$$\Big\downarrow \text{858}$$

$$3b\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{x}$$

$$\Big\downarrow \text{807}$$

$$3b\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{x}$$

$$\Big\downarrow 212$$

$$3b\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{x}$$

input
```
Int[(a + b*x^4)^(3/4)/x^2,x]
```

output
```
-((a + b*x^4)^(3/4)/x) + 3*b*(x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 + a/
(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]
*(a + b*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{3}{4}}}{x^{2}}dx$$

input
```
int((b*x^4+a)^(3/4)/x^2,x)
```

output
```
int((b*x^4+a)^(3/4)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^4)^{3/4}}{x^2}\,dx = \frac{a^{\frac{3}{4}}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**2,x)
```

output
```
a**(3/4)*gamma(-1/4)*hyper((-3/4, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^4)^{3/4}}{x^2}\,dx = \frac{(b\,x^4+a)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a}{b\,x^4}\right)}{2\,x\,\left(\frac{a}{b\,x^4}+1\right)^{3/4}}$$

input
```
int((a + b*x^4)^(3/4)/x^2,x)
```

output
```
((a + b*x^4)^(3/4)*hypergeom([-3/4, -1/2], 1/2, -a/(b*x^4)))/(2*x*(a/(b*x^
4) + 1)^(3/4))
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^2}\,dx = \frac{(b\,x^4+a)^{\frac{3}{4}}+3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^6+a\,x^2}dx\right)ax}{2x}$$

input
```
int((b*x^4+a)^(3/4)/x^2,x)
```

output
```
((a + b*x**4)**(3/4) + 3*int((a + b*x**4)**(3/4)/(a*x**2 + b*x**6),x)*a*x)
/(2*x)
```

3.499 $\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx = -\frac{3b}{5x\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{5x^5}+\frac{3b^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{a}\sqrt[4]{a+bx^4}}$$

output
```
-3/5*b/x/(b*x^4+a)^(1/4)-1/5*(b*x^4+a)^(3/4)/x^5+3/5*b^(3/2)*(1+a/b/x^4)^(
1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/(b*
x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx = -\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{3}{4},-\frac{1}{4},-\frac{bx^4}{a}\right)}{5x^5\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^6,x]
```

output
```
-1/5*((a + b*x^4)^(3/4)*Hypergeometric2F1[-5/4, -3/4, -1/4, -((b*x^4)/a)])
/(x^5*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {809, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^6}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{3}{5}b\int \frac{1}{x^2\sqrt[4]{bx^4+a}}dx-\frac{(a+bx^4)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{841}$$

$$\frac{3}{5}b\left(-b\int \frac{x^2}{(bx^4+a)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)-\frac{(a+bx^4)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{813}$$

$$\frac{3}{5}b\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)-\frac{(a+bx^4)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{858}$$

$$\frac{3}{5}b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)-\frac{(a+bx^4)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{5}b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{212}$$

$$\frac{3}{5}b\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)-\frac{\left(a+bx^4\right)^{3/4}}{5x^5}$$

input
```
Int[(a + b*x^4)^(3/4)/x^6,x]
```

output
```
-1/5*(a + b*x^4)^(3/4)/x^5 + (3*b*(-(1/(x*(a + b*x^4)^(1/4))) + (Sqrt[b]*(
1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(Sqr
t[a]*(a + b*x^4)^(1/4))))/5
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{x^6}dx$$

input
```
int((b*x^4+a)^(3/4)/x^6,x)
```

output
```
int((b*x^4+a)^(3/4)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx = -\frac{b^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{2x^2}$$

input
```
integrate((b*x**4+a)**(3/4)/x**6,x)
```

output
```
-b**(3/4)*hyper((-3/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**4))/(2*x**2)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx=\int \frac{(b\,x^4+a)^{3/4}}{x^6}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^6,x)
```

output
```
int((a + b*x^4)^(3/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^6}\,dx=\frac{-(b\,x^4+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^{10}+a\,x^6}dx\right)a\,x^5}{2x^5}$$

input
```
int((b*x^4+a)^(3/4)/x^6,x)
```

output
```
( - (a + b*x**4)**(3/4) - 3*int((a + b*x**4)**(3/4)/(a*x**6 + b*x**10),x)*
a*x**5)/(2*x**5)
```

3.500 $\int \frac{(a+bx^4)^{3/4}}{x^{10}} dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}} dx = \frac{2b^2}{15ax\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{9x^9} - \frac{b(a+bx^4)^{3/4}}{15ax^5} - \frac{2b^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
2/15*b^2/a/x/(b*x^4+a)^(1/4)-1/9*(b*x^4+a)^(3/4)/x^9-1/15*b*(b*x^4+a)^(3/4
)/a/x^5-2/15*b^(5/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*
x^2/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}} dx = -\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{4},-\frac{5}{4},-\frac{bx^4}{a}\right)}{9x^9\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^10,x]
```

output
```
-1/9*((a + b*x^4)^(3/4)*Hypergeometric2F1[-9/4, -3/4, -5/4, -((b*x^4)/a)])
/(x^9*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {809, 847, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{1}{3}b\int \frac{1}{x^6\sqrt[4]{bx^4+a}}dx-\frac{(a+bx^4)^{3/4}}{9x^9}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{3}b\left(-\frac{2b\int \frac{1}{x^2\sqrt[4]{bx^4+a}}dx}{5a}-\frac{(a+bx^4)^{3/4}}{5ax^5}\right)-\frac{(a+bx^4)^{3/4}}{9x^9}$$

$$\Big\downarrow \text{841}$$

$$\frac{1}{3}b\left(-\frac{2b\left(-b\int \frac{x^2}{(bx^4+a)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{(a+bx^4)^{3/4}}{5ax^5}\right)-\frac{(a+bx^4)^{3/4}}{9x^9}$$

$$\Big\downarrow \text{813}$$

$$\frac{1}{3}b\left(-\frac{2b\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)-\frac{\left(a+bx^4\right)^{3/4}}{9x^9}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{3}b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)-\frac{\left(a+bx^4\right)^{3/4}}{9x^9}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{3}b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)-\frac{\left(a+bx^4\right)^{3/4}}{9x^9}$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{3}b\left(-\frac{2b\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)-\frac{\left(a+bx^4\right)^{3/4}}{9x^9}$$

input
```
Int[(a + b*x^4)^(3/4)/x^10,x]
```

output
```
-1/9*(a + b*x^4)^(3/4)/x^9 + (b*(-1/5*(a + b*x^4)^(3/4)/(a*x^5) - (2*b*(-(
1/(x*(a + b*x^4)^(1/4))) + (Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcT
an[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(Sqrt[a]*(a + b*x^4)^(1/4))))/(5*a)))/3
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{x^{10}}d x$$

input
```
int((b*x^4+a)^(3/4)/x^10,x)
```

output
```
int((b*x^4+a)^(3/4)/x^10,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}\,dx=\int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{10}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^10,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^10, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}\,dx = -\frac{b^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{6x^6}$$

input
```
integrate((b*x**4+a)**(3/4)/x**10,x)
```

output
```
-b**(3/4)*hyper((-3/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*x**6)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{10}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^10,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^10, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{10}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^10,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{x^{10}}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^10,x)
```

output
```
int((a + b*x^4)^(3/4)/x^10, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{10}}\,dx = \frac{-(b\,x^4+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^{14}+a\,x^{10}}dx\right)a\,x^9}{6x^9}$$

input
```
int((b*x^4+a)^(3/4)/x^10,x)
```

output
```
( - (a + b*x**4)**(3/4) - 3*int((a + b*x**4)**(3/4)/(a*x**10 + b*x**14),x)
*a*x**9)/(6*x**9)
```

3.501 $\int \frac{(a+bx^4)^{3/4}}{x^{14}} dx$

Optimal result

Integrand size = 15, antiderivative size = 150

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}} dx = -\frac{4b^3}{65a^2x\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{13x^{13}} - \frac{b(a+bx^4)^{3/4}}{39ax^9} + \frac{2b^2(a+bx^4)^{3/4}}{65a^2x^5} + \frac{4b^{7/2}\sqrt[4]{1+\frac{a}{bx^4}}xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{65a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-4/65*b^3/a^2/x/(b*x^4+a)^(1/4)-1/13*(b*x^4+a)^(3/4)/x^13-1/39*b*(b*x^4+a)
^(3/4)/a/x^9+2/65*b^2*(b*x^4+a)^(3/4)/a^2/x^5+4/65*b^(7/2)*(1+a/b/x^4)^(1/
4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^
4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}} dx = -\frac{(a+bx^4)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},-\frac{3}{4},-\frac{9}{4},-\frac{bx^4}{a}\right)}{13x^{13}\left(1+\frac{bx^4}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(3/4)/x^14,x]
```

output
```
-1/13*((a + b*x^4)^(3/4)*Hypergeometric2F1[-13/4, -3/4, -9/4, -((b*x^4)/a)
])/(x^13*(1 + (b*x^4)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {809, 847, 847, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{3/4}}{x^{14}}dx$$

$$\Big\downarrow \text{ 809}$$

$$\frac{3}{13}b\int \frac{1}{x^{10}\sqrt[4]{bx^4+a}}dx-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{3}{13}b\left(-\frac{2b\int \frac{1}{x^6\sqrt[4]{bx^4+a}}dx}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\int \frac{1}{x^2\sqrt[4]{bx^4+a}}dx}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

$$\Big\downarrow \text{ 841}$$

$$\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(-b\int\frac{x^2}{(bx^4+a)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{(a+bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a+bx^4)^{3/4}}{9ax^9}\right)-\frac{(a+bx^4)^{3/4}}{13x^{13}}$$

$\big\downarrow$ 813

$$\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{(a+bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a+bx^4)^{3/4}}{9ax^9}\right)-\frac{(a+bx^4)^{3/4}}{13x^{13}}$$

$\big\downarrow$ 858

$$\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

$$\Big\downarrow \; 807$$

$$\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

$$\Big\downarrow \text{212}$$

$$\frac{3}{13}b\left(-\frac{2b\left(-\frac{2b\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)-\frac{\left(a+bx^4\right)^{3/4}}{13x^{13}}$$

input
```
Int[(a + b*x^4)^(3/4)/x^14,x]
```

output
```
-1/13*(a + b*x^4)^(3/4)/x^13 + (3*b*(-1/9*(a + b*x^4)^(3/4)/(a*x^9) - (2*b
*(-1/5*(a + b*x^4)^(3/4)/(a*x^5) - (2*b*(-(1/(x*(a + b*x^4)^(1/4))) + (Sqr
t[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(Sqrt[a]*(a + b*x^4)^(1/4))))/(5*a)))/(3*a)))/13
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{x^{14}}dx$$

input
```
int((b*x^4+a)^(3/4)/x^14,x)
```

output
```
int((b*x^4+a)^(3/4)/x^14,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{14}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^14,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/x^14, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}}\,dx = \frac{a^{\frac{3}{4}}\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{13}{4},-\frac{3}{4}\\ -\frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4x^{13}\Gamma\left(-\frac{9}{4}\right)}$$

input
```
integrate((b*x**4+a)**(3/4)/x**14,x)
```

output
```
a**(3/4)*gamma(-13/4)*hyper((-13/4, -3/4), (-9/4,), b*x**4*exp_polar(I*pi)
/a)/(4*x**13*gamma(-9/4))
```

Maxima [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{14}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^14,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^14, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}}\,dx = \int \frac{(bx^4+a)^{\frac{3}{4}}}{x^{14}}\,dx$$

input
```
integrate((b*x^4+a)^(3/4)/x^14,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(3/4)/x^14, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}}\,dx = \int \frac{(b\,x^4+a)^{3/4}}{x^{14}}\,dx$$

input
```
int((a + b*x^4)^(3/4)/x^14,x)
```

output
```
int((a + b*x^4)^(3/4)/x^14, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{3/4}}{x^{14}}\,dx = \frac{-(b\,x^4+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^4+a)^{\frac{3}{4}}}{b\,x^{18}+a\,x^{14}}dx\right)a\,x^{13}}{10x^{13}}$$

input
```
int((b*x^4+a)^(3/4)/x^14,x)
```

output
```
( - (a + b*x**4)**(3/4) - 3*int((a + b*x**4)**(3/4)/(a*x**14 + b*x**18),x)
*a*x**13)/(10*x**13)
```

3.502 $\int x^{19}(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^{19}\left(a+bx^4\right)^{5/4}\,dx=\frac{a^4\left(a+bx^4\right)^{9/4}}{9b^5}-\frac{4a^3\left(a+bx^4\right)^{13/4}}{13b^5}+\frac{6a^2\left(a+bx^4\right)^{17/4}}{17b^5}-\frac{4a\left(a+bx^4\right)^{21/4}}{21b^5}+\frac{\left(a+bx^4\right)^{25/4}}{25b^5}$$

output
```
1/9*a^4*(b*x^4+a)^(9/4)/b^5-4/13*a^3*(b*x^4+a)^(13/4)/b^5+6/17*a^2*(b*x^4+
a)^(17/4)/b^5-4/21*a*(b*x^4+a)^(21/4)/b^5+1/25*(b*x^4+a)^(25/4)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int x^{19}\left(a+bx^4\right)^{5/4}\,dx=\frac{\left(a+bx^4\right)^{9/4}\left(2048a^4-4608a^3bx^4+7488a^2b^2x^8-10608ab^3x^{12}+13923b^4x^{16}\right)}{348075b^5}$$

input
```
Integrate[x^19*(a + b*x^4)^(5/4),x]
```

output
```
((a + b*x^4)^(9/4)*(2048*a^4 - 4608*a^3*b*x^4 + 7488*a^2*b^2*x^8 - 10608*a
*b^3*x^12 + 13923*b^4*x^16))/(348075*b^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{19} \left(a + bx^4\right)^{5/4} dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int x^{16} \left(bx^4 + a\right)^{5/4} dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4} \int \left(\frac{\left(bx^4 + a\right)^{21/4}}{b^4} - \frac{4a \left(bx^4 + a\right)^{17/4}}{b^4} + \frac{6a^2 \left(bx^4 + a\right)^{13/4}}{b^4} - \frac{4a^3 \left(bx^4 + a\right)^{9/4}}{b^4} + \frac{a^4 \left(bx^4 + a\right)^{5/4}}{b^4}\right) dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{4a^4 \left(a + bx^4\right)^{9/4}}{9b^5} - \frac{16a^3 \left(a + bx^4\right)^{13/4}}{13b^5} + \frac{24a^2 \left(a + bx^4\right)^{17/4}}{17b^5} + \frac{4 \left(a + bx^4\right)^{25/4}}{25b^5} - \frac{16a \left(a + bx^4\right)^{21/4}}{21b^5}\right)$$

input
```
Int[x^19*(a + b*x^4)^(5/4),x]
```

output
```
((4*a^4*(a + b*x^4)^(9/4))/(9*b^5) - (16*a^3*(a + b*x^4)^(13/4))/(13*b^5)
+ (24*a^2*(a + b*x^4)^(17/4))/(17*b^5) - (16*a*(a + b*x^4)^(21/4))/(21*b^5
) + (4*(a + b*x^4)^(25/4))/(25*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{9}{4}}\left(13923x^{16}b^4-10608a\,b^3x^{12}+7488a^2b^2x^8-4608a^3b\,x^4+2048a^4\right)}{348075b^5}$	58
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{9}{4}}\left(13923x^{16}b^4-10608a\,b^3x^{12}+7488a^2b^2x^8-4608a^3b\,x^4+2048a^4\right)}{348075b^5}$	58
orering	$\frac{(b\,x^4+a)^{\frac{9}{4}}\left(13923x^{16}b^4-10608a\,b^3x^{12}+7488a^2b^2x^8-4608a^3b\,x^4+2048a^4\right)}{348075b^5}$	58
trager	$\frac{\left(13923b^6x^{24}+17238a\,b^5x^{20}+195a^2b^4x^{16}-240a^3b^3x^{12}+320a^4b^2x^8-512a^5b\,x^4+2048a^6\right)(b\,x^4+a)^{\frac{1}{4}}}{348075b^5}$	80
risch	$\frac{\left(13923b^6x^{24}+17238a\,b^5x^{20}+195a^2b^4x^{16}-240a^3b^3x^{12}+320a^4b^2x^8-512a^5b\,x^4+2048a^6\right)(b\,x^4+a)^{\frac{1}{4}}}{348075b^5}$	80

input
```
int(x^19*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/348075*(b*x^4+a)^(9/4)*(13923*b^4*x^16-10608*a*b^3*x^12+7488*a^2*b^2*x^8
-4608*a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int x^{19}(a+bx^4)^{5/4}\,dx = \frac{(13923\,b^6x^{24}+17238\,ab^5x^{20}+195\,a^2b^4x^{16}-240\,a^3b^3x^{12}+320\,a^4b^2x^8-512\,a^5bx^4+2048\,a$$

$$348075\,b^5$$

input
```
integrate(x^19*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/348075*(13923*b^6*x^24 + 17238*a*b^5*x^20 + 195*a^2*b^4*x^16 - 240*a^3*b
^3*x^12 + 320*a^4*b^2*x^8 - 512*a^5*b*x^4 + 2048*a^6)*(b*x^4 + a)^(1/4)/b^
5
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.54

$$\int x^{19}(a+bx^4)^{5/4}\,dx = \begin{cases} \frac{2048a^6\sqrt[4]{a+bx^4}}{348075b^5} - \frac{512a^5x^4\sqrt[4]{a+bx^4}}{348075b^4} + \frac{64a^4x^8\sqrt[4]{a+bx^4}}{69615b^3} - \frac{16a^3x^{12}\sqrt[4]{a+bx^4}}{23205b^2} + \frac{a^2x^{16}\sqrt[4]{a+bx^4}}{1785b} \\ \frac{a^{\frac{5}{4}}x^{20}}{20} \end{cases}$$

input
```
integrate(x**19*(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((2048*a**6*(a + b*x**4)**(1/4)/(348075*b**5) - 512*a**5*x**4*(a
+ b*x**4)**(1/4)/(348075*b**4) + 64*a**4*x**8*(a + b*x**4)**(1/4)/(69615*b
**3) - 16*a**3*x**12*(a + b*x**4)**(1/4)/(23205*b**2) + a**2*x**16*(a + b*
x**4)**(1/4)/(1785*b) + 26*a*x**20*(a + b*x**4)**(1/4)/525 + b*x**24*(a +
b*x**4)**(1/4)/25, Ne(b, 0)), (a**(5/4)*x**20/20, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int x^{19}\left(a+bx^4\right)^{5/4}\,dx=\frac{\left(bx^4+a\right)^{\frac{25}{4}}}{25\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{21}{4}}a}{21\,b^5}+\frac{6\left(bx^4+a\right)^{\frac{17}{4}}a^2}{17\,b^5}-\frac{4\left(bx^4+a\right)^{\frac{13}{4}}a^3}{13\,b^5}+\frac{\left(bx^4+a\right)^{\frac{9}{4}}a^4}{9\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/25*(b*x^4 + a)^(25/4)/b^5 - 4/21*(b*x^4 + a)^(21/4)*a/b^5 + 6/17*(b*x^4
+ a)^(17/4)*a^2/b^5 - 4/13*(b*x^4 + a)^(13/4)*a^3/b^5 + 1/9*(b*x^4 + a)^(9
/4)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int x^{19}\left(a+bx^4\right)^{5/4}\,dx=\frac{13923\left(bx^4+a\right)^{\frac{25}{4}}-66300\left(bx^4+a\right)^{\frac{21}{4}}a+122850\left(bx^4+a\right)^{\frac{17}{4}}a^2-107100\left(bx^4+a\right)^{\frac{13}{4}}a^3+3}{348075\,b^5}$$

input
```
integrate(x^19*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/348075*(13923*(b*x^4 + a)^(25/4) - 66300*(b*x^4 + a)^(21/4)*a + 122850*(
b*x^4 + a)^(17/4)*a^2 - 107100*(b*x^4 + a)^(13/4)*a^3 + 38675*(b*x^4 + a)^
(9/4)*a^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int x^{19}(a+bx^4)^{5/4}\,dx=(b\,x^4+a)^{1/4}\left(\frac{26\,a\,x^{20}}{525}+\frac{b\,x^{24}}{25}+\frac{2048\,a^6}{348075\,b^5}-\frac{512\,a^5\,x^4}{348075\,b^4}+\frac{64\,a^4\,x^8}{69615\,b^3}-\frac{16\,a^3\,x^{12}}{23205\,b^2}+\frac{a^2\,x^{16}}{1785\,b}\right)$$

input
```
int(x^19*(a + b*x^4)^(5/4),x)
```

output
```
(a + b*x^4)^(1/4)*((26*a*x^20)/525 + (b*x^24)/25 + (2048*a^6)/(348075*b^5)
 - (512*a^5*x^4)/(348075*b^4) + (64*a^4*x^8)/(69615*b^3) - (16*a^3*x^12)/(
23205*b^2) + (a^2*x^16)/(1785*b))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int x^{19}(a+bx^4)^{5/4}\,dx=\frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(13923b^6x^{24}+17238a\,b^5x^{20}+195a^2b^4x^{16}-240a^3b^3x^{12}+320}{348075\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^5}$$

input
```
int(x^19*(b*x^4+a)^(5/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(2048*a**6 - 512*a**5*b*
x**4 + 320*a**4*b**2*x**8 - 240*a**3*b**3*x**12 + 195*a**2*b**4*x**16 + 17
238*a*b**5*x**20 + 13923*b**6*x**24))/(348075*sqrt(sqrt(a + b*x**4) + sqrt
(b)*x**2)*b**5)
```

3.503 $\int x^{15}(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^{15}\left(a+bx^4\right)^{5/4}\,dx=-\frac{a^3\left(a+bx^4\right)^{9/4}}{9b^4}+\frac{3a^2\left(a+bx^4\right)^{13/4}}{13b^4}-\frac{3a\left(a+bx^4\right)^{17/4}}{17b^4}+\frac{\left(a+bx^4\right)^{21/4}}{21b^4}$$

output
```
-1/9*a^3*(b*x^4+a)^(9/4)/b^4+3/13*a^2*(b*x^4+a)^(13/4)/b^4-3/17*a*(b*x^4+a
)^(17/4)/b^4+1/21*(b*x^4+a)^(21/4)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^{15}\left(a+bx^4\right)^{5/4}\,dx=\frac{\left(a+bx^4\right)^{9/4}\left(-128a^3+288a^2bx^4-468ab^2x^8+663b^3x^{12}\right)}{13923b^4}$$

input
```
Integrate[x^15*(a + b*x^4)^(5/4),x]
```

output
```
((a + b*x^4)^(9/4)*(-128*a^3 + 288*a^2*b*x^4 - 468*a*b^2*x^8 + 663*b^3*x^1
2))/(13923*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15} \left(a+b x^4\right)^{5/4} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int x^{12} \left(b x^4+a\right)^{5/4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4} \int \left(\frac{\left(b x^4+a\right)^{17/4}}{b^3}-\frac{3 a \left(b x^4+a\right)^{13/4}}{b^3}+\frac{3 a^2 \left(b x^4+a\right)^{9/4}}{b^3}-\frac{a^3 \left(b x^4+a\right)^{5/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{4 a^3 \left(a+b x^4\right)^{9/4}}{9 b^4}+\frac{12 a^2 \left(a+b x^4\right)^{13/4}}{13 b^4}+\frac{4 \left(a+b x^4\right)^{21/4}}{21 b^4}-\frac{12 a \left(a+b x^4\right)^{17/4}}{17 b^4}\right)$$

input
```
Int[x^15*(a + b*x^4)^(5/4),x]
```

output
```
((-4*a^3*(a + b*x^4)^(9/4))/(9*b^4) + (12*a^2*(a + b*x^4)^(13/4))/(13*b^4)
 - (12*a*(a + b*x^4)^(17/4))/(17*b^4) + (4*(a + b*x^4)^(21/4))/(21*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{9}{4}}\,(-663b^3x^{12}+468a\,b^2x^8-288a^2b\,x^4+128a^3)}{13923b^4}$	47
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{9}{4}}\,(-663b^3x^{12}+468a\,b^2x^8-288a^2b\,x^4+128a^3)}{13923b^4}$	47
orering	$-\frac{(b\,x^4+a)^{\frac{9}{4}}\,(-663b^3x^{12}+468a\,b^2x^8-288a^2b\,x^4+128a^3)}{13923b^4}$	47
trager	$-\frac{(-663b^5x^{20}-858a\,b^4x^{16}-15a^2b^3x^{12}+20a^3b^2x^8-32a^4b\,x^4+128a^5)\,(b\,x^4+a)^{\frac{1}{4}}}{13923b^4}$	69
risch	$-\frac{(-663b^5x^{20}-858a\,b^4x^{16}-15a^2b^3x^{12}+20a^3b^2x^8-32a^4b\,x^4+128a^5)\,(b\,x^4+a)^{\frac{1}{4}}}{13923b^4}$	69

input
```
int(x^15*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/13923*(b*x^4+a)^(9/4)*(-663*b^3*x^12+468*a*b^2*x^8-288*a^2*b*x^4+128*a^
3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x^{15}(a+bx^4)^{5/4}\,dx=\frac{(663\,b^5x^{20}+858\,ab^4x^{16}+15\,a^2b^3x^{12}-20\,a^3b^2x^8+32\,a^4bx^4-128\,a^5)(bx^4+a)^{\frac{1}{4}}}{13923\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/13923*(663*b^5*x^20 + 858*a*b^4*x^16 + 15*a^2*b^3*x^12 - 20*a^3*b^2*x^8
+ 32*a^4*b*x^4 - 128*a^5)*(b*x^4 + a)^(1/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int x^{15}(a+bx^4)^{5/4}\,dx=\begin{cases}-\frac{128a^5\sqrt[4]{a+bx^4}}{13923b^4}+\frac{32a^4x^4\sqrt[4]{a+bx^4}}{13923b^3}-\frac{20a^3x^8\sqrt[4]{a+bx^4}}{13923b^2}+\frac{5a^2x^{12}\sqrt[4]{a+bx^4}}{4641b}+\frac{22ax^{16}\sqrt[4]{a+bx^4}}{357} \\ \frac{a^{\frac{5}{4}}x^{16}}{16}\end{cases}$$

input
```
integrate(x**15*(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((-128*a**5*(a + b*x**4)**(1/4)/(13923*b**4) + 32*a**4*x**4*(a +
b*x**4)**(1/4)/(13923*b**3) - 20*a**3*x**8*(a + b*x**4)**(1/4)/(13923*b**2
) + 5*a**2*x**12*(a + b*x**4)**(1/4)/(4641*b) + 22*a*x**16*(a + b*x**4)**(
1/4)/357 + b*x**20*(a + b*x**4)**(1/4)/21, Ne(b, 0)), (a**(5/4)*x**16/16,
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{15}(a+bx^4)^{5/4}\,dx=\frac{(bx^4+a)^{\frac{21}{4}}}{21\,b^4}-\frac{3\,(bx^4+a)^{\frac{17}{4}}a}{17\,b^4}+\frac{3\,(bx^4+a)^{\frac{13}{4}}a^2}{13\,b^4}-\frac{(bx^4+a)^{\frac{9}{4}}a^3}{9\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/21*(b*x^4 + a)^(21/4)/b^4 - 3/17*(b*x^4 + a)^(17/4)*a/b^4 + 3/13*(b*x^4
+ a)^(13/4)*a^2/b^4 - 1/9*(b*x^4 + a)^(9/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^{15}(a+bx^4)^{5/4}\,dx = \frac{663\,(bx^4+a)^{\frac{21}{4}} - 2457\,(bx^4+a)^{\frac{17}{4}}a + 3213\,(bx^4+a)^{\frac{13}{4}}a^2 - 1547\,(bx^4+a)^{\frac{9}{4}}a^3}{13923\,b^4}$$

input
```
integrate(x^15*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/13923*(663*(b*x^4 + a)^(21/4) - 2457*(b*x^4 + a)^(17/4)*a + 3213*(b*x^4
+ a)^(13/4)*a^2 - 1547*(b*x^4 + a)^(9/4)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^{15}(a+bx^4)^{5/4}\,dx = (b\,x^4+a)^{1/4}\left(\frac{22\,a\,x^{16}}{357}+\frac{b\,x^{20}}{21}-\frac{128\,a^5}{13923\,b^4}+\frac{32\,a^4\,x^4}{13923\,b^3}-\frac{20\,a^3\,x^8}{13923\,b^2}+\frac{5\,a^2\,x^{12}}{4641\,b}\right)$$

input
```
int(x^15*(a + b*x^4)^(5/4),x)
```

output
```
(a + b*x^4)^(1/4)*((22*a*x^16)/357 + (b*x^20)/21 - (128*a^5)/(13923*b^4) +
 (32*a^4*x^4)/(13923*b^3) - (20*a^3*x^8)/(13923*b^2) + (5*a^2*x^12)/(4641*
b))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int x^{15}(a+bx^4)^{5/4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(663b^5x^{20}+858a\,b^4x^{16}+15a^2b^3x^{12}-20a^3b^2x^8+32a^4b\,x^4-1}{13923\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^4}$$

input
```
int(x^15*(b*x^4+a)^(5/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*( - 128*a**5 + 32*a**4*b
*x**4 - 20*a**3*b**2*x**8 + 15*a**2*b**3*x**12 + 858*a*b**4*x**16 + 663*b*
*5*x**20))/(13923*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b**4)
```

3.504 $\int x^{11}(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^{11}\left(a+bx^4\right)^{5/4}\,dx=\frac{a^2(a+bx^4)^{9/4}}{9b^3}-\frac{2a(a+bx^4)^{13/4}}{13b^3}+\frac{(a+bx^4)^{17/4}}{17b^3}$$

output
```
1/9*a^2*(b*x^4+a)^(9/4)/b^3-2/13*a*(b*x^4+a)^(13/4)/b^3+1/17*(b*x^4+a)^(17
/4)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^{11}\left(a+bx^4\right)^{5/4}\,dx=\frac{(a+bx^4)^{9/4}\,(32a^2-72abx^4+117b^2x^8)}{1989b^3}$$

input
```
Integrate[x^11*(a + b*x^4)^(5/4),x]
```

output
```
((a + b*x^4)^(9/4)*(32*a^2 - 72*a*b*x^4 + 117*b^2*x^8))/(1989*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^4\right)^{5/4}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^8\left(bx^4+a\right)^{5/4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{13/4}}{b^2}-\frac{2a\left(bx^4+a\right)^{9/4}}{b^2}+\frac{a^2\left(bx^4+a\right)^{5/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^2\left(a+bx^4\right)^{9/4}}{9b^3}+\frac{4\left(a+bx^4\right)^{17/4}}{17b^3}-\frac{8a\left(a+bx^4\right)^{13/4}}{13b^3}\right)$$

input
```
Int[x^11*(a + b*x^4)^(5/4),x]
```

output
```
((4*a^2*(a + b*x^4)^(9/4))/(9*b^3) - (8*a*(a + b*x^4)^(13/4))/(13*b^3) + (
4*(a + b*x^4)^(17/4))/(17*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}}\left(117 b^{2} x^{8}-72 a b\,x^{4}+32 a^{2}\right)}{1989 b^{3}}$	36
pseudoelliptic	$\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}}\left(117 b^{2} x^{8}-72 a b\,x^{4}+32 a^{2}\right)}{1989 b^{3}}$	36
orering	$\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}}\left(117 b^{2} x^{8}-72 a b\,x^{4}+32 a^{2}\right)}{1989 b^{3}}$	36
trager	$\frac{\left(117 x^{16} b^{4}+162 a\,b^{3} x^{12}+5 a^{2} b^{2} x^{8}-8 a^{3} b\,x^{4}+32 a^{4}\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{1989 b^{3}}$	58
risch	$\frac{\left(117 x^{16} b^{4}+162 a\,b^{3} x^{12}+5 a^{2} b^{2} x^{8}-8 a^{3} b\,x^{4}+32 a^{4}\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{1989 b^{3}}$	58

input
```
int(x^11*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/1989*(b*x^4+a)^(9/4)*(117*b^2*x^8-72*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^{11}\left(a+bx^4\right)^{5/4}\,dx = \frac{(117\,b^4x^{16}+162\,ab^3x^{12}+5\,a^2b^2x^8-8\,a^3bx^4+32\,a^4)(bx^4+a)^{\frac{1}{4}}}{1989\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/1989*(117*b^4*x^16 + 162*a*b^3*x^12 + 5*a^2*b^2*x^8 - 8*a^3*b*x^4 + 32*a
^4)*(b*x^4 + a)^(1/4)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(51) = 102$.

Time = 0.89 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.86

$$\int x^{11}(a+bx^4)^{5/4}\,dx = \begin{cases} \frac{32a^4\sqrt[4]{a+bx^4}}{1989b^3} - \frac{8a^3x^4\sqrt[4]{a+bx^4}}{1989b^2} + \frac{5a^2x^8\sqrt[4]{a+bx^4}}{1989b} + \frac{18ax^{12}\sqrt[4]{a+bx^4}}{221} + \frac{bx^{16}\sqrt[4]{a+bx^4}}{17} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{4}}x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((32*a**4*(a + b*x**4)**(1/4)/(1989*b**3) - 8*a**3*x**4*(a + b*x*
*4)**(1/4)/(1989*b**2) + 5*a**2*x**8*(a + b*x**4)**(1/4)/(1989*b) + 18*a*x
**12*(a + b*x**4)**(1/4)/221 + b*x**16*(a + b*x**4)**(1/4)/17, Ne(b, 0)),
(a**(5/4)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^{11}(a+bx^4)^{5/4}\,dx = \frac{(bx^4+a)^{\frac{17}{4}}}{17\,b^3} - \frac{2\,(bx^4+a)^{\frac{13}{4}}a}{13\,b^3} + \frac{(bx^4+a)^{\frac{9}{4}}a^2}{9\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/17*(b*x^4 + a)^(17/4)/b^3 - 2/13*(b*x^4 + a)^(13/4)*a/b^3 + 1/9*(b*x^4 +
 a)^(9/4)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^{11}(a+bx^4)^{5/4}\,dx = \frac{117\,(bx^4+a)^{\frac{17}{4}} - 306\,(bx^4+a)^{\frac{13}{4}}a + 221\,(bx^4+a)^{\frac{9}{4}}a^2}{1989\,b^3}$$

input
```
integrate(x^11*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/1989*(117*(b*x^4 + a)^(17/4) - 306*(b*x^4 + a)^(13/4)*a + 221*(b*x^4 + a
)^(9/4)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^{11}(a+bx^4)^{5/4}\,dx = (b\,x^4+a)^{1/4}\left(\frac{18\,a\,x^{12}}{221}+\frac{b\,x^{16}}{17}+\frac{32\,a^4}{1989\,b^3}-\frac{8\,a^3\,x^4}{1989\,b^2}+\frac{5\,a^2\,x^8}{1989\,b}\right)$$

input
```
int(x^11*(a + b*x^4)^(5/4),x)
```

output
```
(a + b*x^4)^(1/4)*((18*a*x^12)/221 + (b*x^16)/17 + (32*a^4)/(1989*b^3) - (
8*a^3*x^4)/(1989*b^2) + (5*a^2*x^8)/(1989*b))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.49

$$\int x^{11}(a+bx^4)^{5/4}\,dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4+a}\,x^2+a+b\,x^4}\,(117b^4x^{16}+162a\,b^3x^{12}+5a^2b^2x^8-8a^3b\,x^4+32a^4)}{1989\sqrt{\sqrt{b\,x^4+a}+\sqrt{b}\,x^2}\,b^3}$$

input
```
int(x^11*(b*x^4+a)^(5/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(32*a**4 - 8*a**3*b*x**4
 + 5*a**2*b**2*x**8 + 162*a*b**3*x**12 + 117*b**4*x**16))/(1989*sqrt(sqrt(
a + b*x**4) + sqrt(b)*x**2)*b**3)
```

3.505 $\int x^7(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^7\left(a+bx^4\right)^{5/4}\,dx=-\frac{a(a+bx^4)^{9/4}}{9b^2}+\frac{(a+bx^4)^{13/4}}{13b^2}$$

output
```
-1/9*a*(b*x^4+a)^(9/4)/b^2+1/13*(b*x^4+a)^(13/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^7\left(a+bx^4\right)^{5/4}\,dx=\frac{(a+bx^4)^{9/4}\left(-4a+9bx^4\right)}{117b^2}$$

input
```
Integrate[x^7*(a + b*x^4)^(5/4),x]
```

output
```
((a + b*x^4)^(9/4)*(-4*a + 9*b*x^4))/(117*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^4\right)^{5/4} dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int x^4 \left(bx^4+a\right)^{5/4} dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{9/4}}{b}-\frac{a\left(bx^4+a\right)^{5/4}}{b}\right) dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{4\left(a+bx^4\right)^{13/4}}{13b^2}-\frac{4a\left(a+bx^4\right)^{9/4}}{9b^2}\right)$$

input
```
Int[x^7*(a + b*x^4)^(5/4),x]
```

output
```
((-4*a*(a + b*x^4)^(9/4))/(9*b^2) + (4*(a + b*x^4)^(13/4))/(13*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(b x^4+a)^{\frac{9}{4}} (-9b x^4+4a)}{117b^2}$	25
pseudoelliptic	$-\frac{(b x^4+a)^{\frac{9}{4}} (-9b x^4+4a)}{117b^2}$	25
orering	$-\frac{(b x^4+a)^{\frac{9}{4}} (-9b x^4+4a)}{117b^2}$	25
trager	$-\frac{(-9b^3x^{12}-14a b^2x^8-a^2b x^4+4a^3)(b x^4+a)^{\frac{1}{4}}}{117b^2}$	47
risch	$-\frac{(-9b^3x^{12}-14a b^2x^8-a^2b x^4+4a^3)(b x^4+a)^{\frac{1}{4}}}{117b^2}$	47

input
```
int(x^7*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/117*(b*x^4+a)^(9/4)*(-9*b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^7 (a+bx^4)^{5/4} \, dx = \frac{(9\,b^3x^{12}+14\,ab^2x^8+a^2bx^4-4\,a^3)(bx^4+a)^{\frac{1}{4}}}{117\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/117*(9*b^3*x^12 + 14*a*b^2*x^8 + a^2*b*x^4 - 4*a^3)*(b*x^4 + a)^(1/4)/b^
2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int x^7(a+bx^4)^{5/4}\,dx = \begin{cases} -\frac{4a^3\sqrt[4]{a+bx^4}}{117b^2} + \frac{a^2x^4\sqrt[4]{a+bx^4}}{117b} + \frac{14ax^8\sqrt[4]{a+bx^4}}{117} + \frac{bx^{12}\sqrt[4]{a+bx^4}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{4}}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((-4*a**3*(a + b*x**4)**(1/4)/(117*b**2) + a**2*x**4*(a + b*x**4)
**(1/4)/(117*b) + 14*a*x**8*(a + b*x**4)**(1/4)/117 + b*x**12*(a + b*x**4)
**(1/4)/13, Ne(b, 0)), (a**(5/4)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^7(a+bx^4)^{5/4}\,dx = \frac{(bx^4+a)^{\frac{13}{4}}}{13\,b^2} - \frac{(bx^4+a)^{\frac{9}{4}}a}{9\,b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/13*(b*x^4 + a)^(13/4)/b^2 - 1/9*(b*x^4 + a)^(9/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^7 (a + bx^4)^{5/4} \, dx = \frac{9 (bx^4 + a)^{\frac{13}{4}} - 13 (bx^4 + a)^{\frac{9}{4}} a}{117 \, b^2}$$

input
```
integrate(x^7*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/117*(9*(b*x^4 + a)^(13/4) - 13*(b*x^4 + a)^(9/4)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^7 (a + bx^4)^{5/4} \, dx = (b\,x^4 + a)^{1/4} \left(\frac{14\,a\,x^8}{117} + \frac{b\,x^{12}}{13} - \frac{4\,a^3}{117\,b^2} + \frac{a^2\,x^4}{117\,b} \right)$$

input
```
int(x^7*(a + b*x^4)^(5/4),x)
```

output
```
(a + b*x^4)^(1/4)*((14*a*x^8)/117 + (b*x^12)/13 - (4*a^3)/(117*b^2) + (a^2
*x^4)/(117*b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int x^7 (a + bx^4)^{5/4} \, dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4 + a}\,x^2 + a + b\,x^4}\,(9b^3x^{12} + 14a\,b^2x^8 + a^2b\,x^4 - 4a^3)}{117\sqrt{\sqrt{b\,x^4 + a} + \sqrt{b}\,x^2}\,b^2}$$

input
```
int(x^7*(b*x^4+a)^(5/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*( - 4*a**3 + a**2*b*x**4
 + 14*a*b**2*x**8 + 9*b**3*x**12))/(117*sqrt(sqrt(a + b*x**4) + sqrt(b)*x*
*2)*b**2)
```

3.506 $\int x^3(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int x^3\left(a+bx^4\right)^{5/4}\,dx=\frac{\left(a+bx^4\right)^{9/4}}{9b}$$

output
```
1/9*(b*x^4+a)^(9/4)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^4\right)^{5/4}\,dx=\frac{\left(a+bx^4\right)^{9/4}}{9b}$$

input
```
Integrate[x^3*(a + b*x^4)^(5/4),x]
```

output
```
(a + b*x^4)^(9/4)/(9*b)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx^4)^{5/4} \, dx$$

$$\Big\downarrow \ 793$$

$$\frac{(a + bx^4)^{9/4}}{9b}$$

input
```
Int[x^3*(a + b*x^4)^(5/4),x]
```

output
```
(a + b*x^4)^(9/4)/(9*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{9}{4}}}{9b}$	15
derivativedivides	$\frac{(b\,x^4+a)^{\frac{9}{4}}}{9b}$	15
default	$\frac{(b\,x^4+a)^{\frac{9}{4}}}{9b}$	15
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{9}{4}}}{9b}$	15
orering	$\frac{(b\,x^4+a)^{\frac{9}{4}}}{9b}$	15
trager	$\frac{(b^2x^8+2ab\,x^4+a^2)\,(b\,x^4+a)^{\frac{1}{4}}}{9b}$	33
risch	$\frac{(b^2x^8+2ab\,x^4+a^2)\,(b\,x^4+a)^{\frac{1}{4}}}{9b}$	33

input
```
int(x^3*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/9*(b*x^4+a)^(9/4)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x^3(a+bx^4)^{5/4}\,dx = \frac{(b^2x^8+2\,abx^4+a^2)(bx^4+a)^{\frac{1}{4}}}{9\,b}$$

input
```
integrate(x^3*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/9*(b^2*x^8 + 2*a*b*x^4 + a^2)*(b*x^4 + a)^(1/4)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int x^3 (a + bx^4)^{5/4}\, dx = \begin{cases} \frac{a^2 \sqrt[4]{a + bx^4}}{9b} + \frac{2ax^4 \sqrt[4]{a + bx^4}}{9} + \frac{bx^8 \sqrt[4]{a + bx^4}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{4}} x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((a**2*(a + b*x**4)**(1/4)/(9*b) + 2*a*x**4*(a + b*x**4)**(1/4)/9
 + b*x**8*(a + b*x**4)**(1/4)/9, Ne(b, 0)), (a**(5/4)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 (a + bx^4)^{5/4}\, dx = \frac{(bx^4 + a)^{\frac{9}{4}}}{9\, b}$$

input
```
integrate(x^3*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/9*(b*x^4 + a)^(9/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 (a + bx^4)^{5/4}\, dx = \frac{(bx^4 + a)^{\frac{9}{4}}}{9\, b}$$

input
```
integrate(x^3*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/9*(b*x^4 + a)^(9/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x^3 (a + bx^4)^{5/4} \, dx = \frac{(b\,x^4 + a)^{9/4}}{9\,b}$$

input
```
int(x^3*(a + b*x^4)^(5/4),x)
```

output
```
(a + b*x^4)^(9/4)/(9*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.50

$$\int x^3 (a + bx^4)^{5/4} \, dx = \frac{\sqrt{\sqrt{b}\,\sqrt{b\,x^4 + a}\,x^2 + a + b\,x^4}\,(b^2x^8 + 2ab\,x^4 + a^2)}{9\sqrt{\sqrt{b\,x^4 + a} + \sqrt{b}\,x^2}\,b}$$

input
```
int(x^3*(b*x^4+a)^(5/4),x)
```

output
```
(sqrt(sqrt(b)*sqrt(a + b*x**4)*x**2 + a + b*x**4)*(a**2 + 2*a*b*x**4 + b**
2*x**8))/(9*sqrt(sqrt(a + b*x**4) + sqrt(b)*x**2)*b)
```

3.507 $\int \frac{(a+bx^4)^{5/4}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{(a+bx^4)^{5/4}}{x} dx = a\sqrt[4]{a+bx^4} + \frac{1}{5}(a+bx^4)^{5/4} - \frac{1}{2}a^{5/4}\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

output
```
a*(b*x^4+a)^(1/4)+1/5*(b*x^4+a)^(5/4)-1/2*a^(5/4)*arctan((b*x^4+a)^(1/4)/a
^(1/4))-1/2*a^(5/4)*arctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^{5/4}}{x} dx = \frac{1}{5}\sqrt[4]{a+bx^4}(6a+bx^4) - \frac{1}{2}a^{5/4}\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{1}{2}a^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

input
```
Integrate[(a + b*x^4)^(5/4)/x,x]
```

output
```
((a + b*x^4)^(1/4)*(6*a + b*x^4))/5 - (a^(5/4)*ArcTan[(a + b*x^4)^(1/4)/a^
(1/4)])/2 - (a^(5/4)*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 60, 60, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^{5/4}}{x^4}dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\int \frac{\sqrt[4]{bx^4+a}}{x^4}dx^4+\frac{4}{5}(a+bx^4)^{5/4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(a\left(a\int \frac{1}{x^4(bx^4+a)^{3/4}}dx^4+4\sqrt[4]{a+bx^4}\right)+\frac{4}{5}(a+bx^4)^{5/4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(a\left(\frac{4a\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{b}+4\sqrt[4]{a+bx^4}\right)+\frac{4}{5}(a+bx^4)^{5/4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(a\left(\frac{4a\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)+\frac{4}{5}(a+bx^4)^{5/4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(a\left(\frac{4a\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)+\frac{4}{5}\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(a\left(\frac{4a\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)+\frac{4}{5}\left(a+bx^4\right)^{5/4}\right)$$

input
```
Int[(a + b*x^4)^(5/4)/x,x]
```

output
```
((4*(a + b*x^4)^(5/4))/5 + a*(4*(a + b*x^4)^(1/4) + (4*a*(-1/2*(b*ArcTan[(
a + b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]
)/(2*a^(3/4))))/b))/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$\frac{\left(-\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)\right)a^{\frac{5}{4}}}{4}+\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(b\,x^{4}+6a\right)}{5}$	78

input
```
int((b*x^4+a)^(5/4)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))-2*arctan((b*
x^4+a)^(1/4)/a^(1/4)))*a^(5/4)+1/5*(b*x^4+a)^(1/4)*(b*x^4+6*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx^4)^{5/4}}{x} dx = \frac{1}{5}(bx^4+6a)(bx^4+a)^{\frac{1}{4}} - \frac{1}{4}(a^5)^{\frac{1}{4}}\log\left((bx^4+a)^{\frac{1}{4}}a+(a^5)^{\frac{1}{4}}\right) - \frac{1}{4}i(a^5)^{\frac{1}{4}}\log\left((bx^4+a)^{\frac{1}{4}}a+i(a^5)^{\frac{1}{4}}\right) + \frac{1}{4}i(a^5)^{\frac{1}{4}}\log\left((bx^4+a)^{\frac{1}{4}}a-i(a^5)^{\frac{1}{4}}\right) + \frac{1}{4}(a^5)^{\frac{1}{4}}\log\left((bx^4+a)^{\frac{1}{4}}a-(a^5)^{\frac{1}{4}}\right)$$

input
```
integrate((b*x^4+a)^(5/4)/x,x, algorithm="fricas")
```

output
```
1/5*(b*x^4 + 6*a)*(b*x^4 + a)^(1/4) - 1/4*(a^5)^(1/4)*log((b*x^4 + a)^(1/4
)*a + (a^5)^(1/4)) - 1/4*I*(a^5)^(1/4)*log((b*x^4 + a)^(1/4)*a + I*(a^5)^(
1/4)) + 1/4*I*(a^5)^(1/4)*log((b*x^4 + a)^(1/4)*a - I*(a^5)^(1/4)) + 1/4*(
a^5)^(1/4)*log((b*x^4 + a)^(1/4)*a - (a^5)^(1/4))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^4)^{5/4}}{x} dx = -\frac{b^{\frac{5}{4}}x^5\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{5}{4} \\ -\frac{1}{4} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x,x)
```

output
```
-b**(5/4)*x**5*gamma(-5/4)*hyper((-5/4, -5/4), (-1/4,), a*exp_polar(I*pi)/
(b*x**4))/(4*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^{5/4}}{x}\,dx = -\frac{1}{2}\,a^{\frac{5}{4}}\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) + \frac{1}{4}\,a^{\frac{5}{4}}\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) + \frac{1}{5}\,(bx^4+a)^{\frac{5}{4}} + (bx^4+a)^{\frac{1}{4}}a$$

input
```
integrate((b*x^4+a)^(5/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(5/4)*arctan((b*x^4 + a)^(1/4)/a^(1/4)) + 1/4*a^(5/4)*log(((b*x^4 +
 a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4))) + 1/5*(b*x^4 + a)^(5/4
) + (b*x^4 + a)^(1/4)*a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(61) = 122.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx^4)^{5/4}}{x}\,dx = -\frac{1}{4}\,\sqrt{2}(-a)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right) - \frac{1}{4}\,\sqrt{2}(-a)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right) - \frac{1}{8}\,\sqrt{2}(-a)^{\frac{1}{4}}\,a\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right) + \frac{1}{8}\,\sqrt{2}(-a)^{\frac{1}{4}}\,a\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right) + \frac{1}{5}\,(bx^4+a)^{\frac{5}{4}} + (bx^4+a)^{\frac{1}{4}}a$$

input
```
integrate((b*x^4+a)^(5/4)/x,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^
4 + a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(1/4)*a*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) - 1/8*sqrt(2)*(-a)^(
1/4)*a*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-
a)) + 1/8*sqrt(2)*(-a)^(1/4)*a*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) +
 sqrt(b*x^4 + a) + sqrt(-a)) + 1/5*(b*x^4 + a)^(5/4) + (b*x^4 + a)^(1/4)*a
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^4)^{5/4}}{x}\,dx = a\left(b\,x^4+a\right)^{1/4} - \frac{a^{5/4}\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{2} + \frac{\left(b\,x^4+a\right)^{5/4}}{5} + \frac{a^{5/4}\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,1\mathrm{i}}{2}$$

input
```
int((a + b*x^4)^(5/4)/x,x)
```

output
```
(a^(5/4)*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*1i)/2 - (a^(5/4)*atan((a + b
*x^4)^(1/4)/a^(1/4)))/2 + a*(a + b*x^4)^(1/4) + (a + b*x^4)^(5/4)/5
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x}\,dx = \frac{6\left(b\,x^4+a\right)^{\frac{1}{4}}a}{5} + \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}b\,x^4}{5} + \left(\int \frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b\,x^5+ax}dx\right)a^2$$

input
```
int((b*x^4+a)^(5/4)/x,x)
```

output
```
(6*(a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b*x**4)
**(1/4)/(a*x + b*x**5),x)*a**2)/5
```

3.508 $\int \frac{(a+bx^4)^{5/4}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{(a+bx^4)^{5/4}}{x^5} \, dx = b\sqrt[4]{a+bx^4} - \frac{a\sqrt[4]{a+bx^4}}{4x^4} - \frac{5}{8}\sqrt[4]{a}b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - \frac{5}{8}\sqrt[4]{a}b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)$$

output
```
b*(b*x^4+a)^(1/4)-1/4*a*(b*x^4+a)^(1/4)/x^4-5/8*a^(1/4)*b*arctan((b*x^4+a)
^(1/4)/a^(1/4))-5/8*a^(1/4)*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^4)^{5/4}}{x^5} \, dx = \frac{1}{8}\left(-\frac{2(a-4bx^4)\sqrt[4]{a+bx^4}}{x^4} - 5\sqrt[4]{a}b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - 5\sqrt[4]{a}b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)\right)$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^5,x]
```

output
```
((-2*(a - 4*b*x^4)*(a + b*x^4)^(1/4))/x^4 - 5*a^(1/4)*b*ArcTan[(a + b*x^4)
^(1/4)/a^(1/4)] - 5*a^(1/4)*b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 51, 60, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{(bx^4+a)^{5/4}}{x^8}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{5}{4}b\int \frac{\sqrt[4]{bx^4+a}}{x^4}dx^4-\frac{(a+bx^4)^{5/4}}{x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{5}{4}b\left(a\int \frac{1}{x^4(bx^4+a)^{3/4}}dx^4+4\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{5}{4}b\left(\frac{4a\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{b}+4\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{x^4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(\frac{5}{4}b\left(\frac{4a\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{x^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{5}{4}b\left(\frac{4a\left(-\frac{b\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{5}{4}b\left(\frac{4a\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}+4\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x^4}\right)$$

input
```
Int[(a + b*x^4)^(5/4)/x^5,x]
```

output
```
(-((a + b*x^4)^(5/4)/x^4) + (5*b*(4*(a + b*x^4)^(1/4) + (4*a*(-1/2*(b*ArcT
an[(a + b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1
/4)])/(2*a^(3/4))))/b))/4)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

method	result	size
pseudoelliptic	$\frac{-5 b\,x^{4} \left(\ln \left(\frac{-\left(b\,x^{4}+a \right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a \right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)+2 \arctan \left(\frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)\right) a^{\frac{1}{4}}-4 \left(b\,x^{4}+a \right)^{\frac{1}{4}} \left(-4 b\,x^{4}+a \right)}{16 x^{4}}$	88

input
```
int((b*x^4+a)^(5/4)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/16*(-5*b*x^4*(ln((-(b*x^4+a)^(1/4)-a^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4)))+
2*arctan((b*x^4+a)^(1/4)/a^(1/4)))*a^(1/4)-4*(b*x^4+a)^(1/4)*(-4*b*x^4+a))
/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^4)^{5/4}}{x^5}\,dx =$$
$$-\frac{5\,(ab^4)^{\frac{1}{4}}\,x^4\log\left(5\,(bx^4+a)^{\frac{1}{4}}b+5\,(ab^4)^{\frac{1}{4}}\right)+5i\,(ab^4)^{\frac{1}{4}}\,x^4\log\left(5\,(bx^4+a)^{\frac{1}{4}}b+5i\,(ab^4)^{\frac{1}{4}}\right)-5i\,(ab^4)^{\frac{1}{4}}\,x^4\log}{16\,x^4}$$

input
```
integrate((b*x^4+a)^(5/4)/x^5,x, algorithm="fricas")
```

output
```
-1/16*(5*(a*b^4)^(1/4)*x^4*log(5*(b*x^4 + a)^(1/4)*b + 5*(a*b^4)^(1/4)) +
5*I*(a*b^4)^(1/4)*x^4*log(5*(b*x^4 + a)^(1/4)*b + 5*I*(a*b^4)^(1/4)) - 5*I
*(a*b^4)^(1/4)*x^4*log(5*(b*x^4 + a)^(1/4)*b - 5*I*(a*b^4)^(1/4)) - 5*(a*b
^4)^(1/4)*x^4*log(5*(b*x^4 + a)^(1/4)*b - 5*(a*b^4)^(1/4)) - 4*(4*b*x^4 -
a)*(b*x^4 + a)^(1/4))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

$$\int \frac{(a+bx^4)^{5/4}}{x^5}\,dx = -\frac{b^{\frac{5}{4}} x \Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**5,x)
```

output
```
-b**(5/4)*x*gamma(-1/4)*hyper((-5/4, -1/4), (3/4,), a*exp_polar(I*pi)/(b*x
**4))/(4*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^{5/4}}{x^5}\,dx = -\frac{5}{16}\left(\frac{2\,b\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}} - \frac{b\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)a + (bx^4+a)^{\frac{1}{4}}b - \frac{(bx^4+a)^{\frac{1}{4}}a}{4\,x^4}$$

input
```
integrate((b*x^4+a)^(5/4)/x^5,x, algorithm="maxima")
```

output
```
-5/16*(2*b*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((b*x^4 + a)^
(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4))*a + (b*x^4 + a)^(
1/4)*b - 1/4*(b*x^4 + a)^(1/4)*a/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(67) = 134$.

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31

$$\int \frac{(a+bx^4)^{5/4}}{x^5}\,dx =$$
$$-\frac{1}{32}\left(10\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)+10\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-\right.}{2\,(-a\right.}\right.$$

input
```
integrate((b*x^4+a)^(5/4)/x^5,x, algorithm="giac")
```

output
```
-1/32*(10*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b
*x^4 + a)^(1/4))/(-a)^(1/4)) + 10*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4)) + 5*sqrt(2)*(-a)^(1/
4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))
- 5*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*
x^4 + a) + sqrt(-a)) - 32*(b*x^4 + a)^(1/4) + 8*(b*x^4 + a)^(1/4)*a/(b*x^4
))*b
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^4)^{5/4}}{x^5}\,dx = b\,(b\,x^4+a)^{1/4}-\frac{a\,(b\,x^4+a)^{1/4}}{4\,x^4} -\frac{5\,a^{1/4}\,b\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8}+\frac{a^{1/4}\,b\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,5\mathrm{i}}{8}$$

input
```
int((a + b*x^4)^(5/4)/x^5,x)
```

output
```
b*(a + b*x^4)^(1/4) - (a*(a + b*x^4)^(1/4))/(4*x^4) - (5*a^(1/4)*b*atan((a
 + b*x^4)^(1/4)/a^(1/4)))/8 + (a^(1/4)*b*atan(((a + b*x^4)^(1/4)*1i)/a^(1/
4))*5i)/8
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^5}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}}\,a+4(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^5+ax}dx\right)ab\,x^4}{4x^4}$$

input
```
int((b*x^4+a)^(5/4)/x^5,x)
```

output
```
( - (a + b*x**4)**(1/4)*a + 4*(a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b*x*
*4)**(1/4)/(a*x + b*x**5),x)*a*b*x**4)/(4*x**4)
```

3.509 $\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = -\frac{a\sqrt[4]{a+bx^4}}{8x^8}-\frac{9b\sqrt[4]{a+bx^4}}{32x^4} - \frac{5b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}-\frac{5b^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}$$

output
```
-1/8*a*(b*x^4+a)^(1/4)/x^8-9/32*b*(b*x^4+a)^(1/4)/x^4-5/64*b^2*arctan((b*x
^4+a)^(1/4)/a^(1/4))/a^(3/4)-5/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(
3/4)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = \frac{(-4a-9bx^4)\sqrt[4]{a+bx^4}}{32x^8} - \frac{5b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}-\frac{5b^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^9,x]
```

output
```
((-4*a - 9*b*x^4)*(a + b*x^4)^(1/4))/(32*x^8) - (5*b^2*ArcTan[(a + b*x^4)^
(1/4)/a^(1/4)])/(64*a^(3/4)) - (5*b^2*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/
(64*a^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {798, 51, 51, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{\left(bx^4+a\right)^{5/4}}{x^{12}}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{5}{8}b\int \frac{\sqrt[4]{bx^4+a}}{x^8}dx^4-\frac{\left(a+bx^4\right)^{5/4}}{2x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(\frac{5}{8}b\left(\frac{1}{4}b\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^4-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{5}{8}b\left(\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{2x^8}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(\frac{5}{8}b\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{2x^8}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}\left(\frac{5}{8}b\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{2x^8}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}\left(\frac{5}{8}b\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{2x^8}\right)$$

input
```
Int[(a + b*x^4)^(5/4)/x^9,x]
```

output
```
(-1/2*(a + b*x^4)^(5/4)/x^8 + (5*b*(-((a + b*x^4)^(1/4)/x^4) - (b*ArcTan[(
a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1
/4)])/(2*a^(3/4))))/8)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{-5\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)b^{2}x^{8}-10\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b^{2}x^{8}-36b\,x^{4}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{\frac{3}{4}}-16a^{\frac{7}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{128x^{8}a^{\frac{3}{4}}}$	104

input
```
int((b*x^4+a)^(5/4)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/128*(-5*ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*b^2*x^8-
10*arctan((b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8-36*b*x^4*(b*x^4+a)^(1/4)*a^(3/4
)-16*a^(7/4)*(b*x^4+a)^(1/4))/x^8/a^(3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx =$$
$$-\frac{5\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}x^8\log\left(5\,(bx^4+a)^{\frac{1}{4}}b^2+5\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}a\right)+5i\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}x^8\log\left(5\,(bx^4+a)^{\frac{1}{4}}b^2+5i\left(\frac{b^8}{a^3}\right)^{\frac{1}{4}}a\right)-5i\left(\frac{b^8}{a^3}\right.}{}$$

input
```
integrate((b*x^4+a)^(5/4)/x^9,x, algorithm="fricas")
```

output
```
-1/128*(5*(b^8/a^3)^(1/4)*x^8*log(5*(b*x^4 + a)^(1/4)*b^2 + 5*(b^8/a^3)^(1
/4)*a) + 5*I*(b^8/a^3)^(1/4)*x^8*log(5*(b*x^4 + a)^(1/4)*b^2 + 5*I*(b^8/a^
3)^(1/4)*a) - 5*I*(b^8/a^3)^(1/4)*x^8*log(5*(b*x^4 + a)^(1/4)*b^2 - 5*I*(b
^8/a^3)^(1/4)*a) - 5*(b^8/a^3)^(1/4)*x^8*log(5*(b*x^4 + a)^(1/4)*b^2 - 5*(
b^8/a^3)^(1/4)*a) + 4*(9*b*x^4 + 4*a)*(b*x^4 + a)^(1/4))/x^8
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = -\frac{b^{\frac{5}{4}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix} -\frac{5}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\right| \frac{ae^{i\pi}}{bx^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**9,x)
```

output
```
-b**(5/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4)
)/(4*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = -\frac{5\,b^2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{64\,a^{\frac{3}{4}}} + \frac{5\,b^2\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{128\,a^{\frac{3}{4}}} - \frac{9\,(bx^4+a)^{\frac{5}{4}}b^2 - 5\,(bx^4+a)^{\frac{1}{4}}ab^2}{32\,\left((bx^4+a)^2 - 2\,(bx^4+a)a + a^2\right)}$$

input
```
integrate((b*x^4+a)^(5/4)/x^9,x, algorithm="maxima")
```

output
```
-5/64*b^2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 5/128*b^2*log(((b*x^
4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4) - 1/32*(9*(
b*x^4 + a)^(5/4)*b^2 - 5*(b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 + a)^2 - 2*(b*x^
4 + a)*a + a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.34

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = \frac{\frac{10\,\sqrt{2}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}} + \frac{10\,\sqrt{2}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}} + \frac{5\,\sqrt{2}b^3\log(}{}}{}$$

input
```
integrate((b*x^4+a)^(5/4)/x^9,x, algorithm="giac")
```

output
```
1/256*(10*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
a)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 10*sqrt(2)*b^3*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/(-a)^(3/4) + 5*sqrt(2)*
b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))
/(-a)^(3/4) + 5*sqrt(2)*(-a)^(1/4)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)
^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a - 8*(9*(b*x^4 + a)^(5/4)*b^3 - 5*(b
*x^4 + a)^(1/4)*a*b^3)/(b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = \frac{5\,a\,(b\,x^4+a)^{1/4}}{32\,x^8} - \frac{5\,b^2\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{64\,a^{3/4}} - \frac{9\,(b\,x^4+a)^{5/4}}{32\,x^8} + \frac{b^2\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,5\mathrm{i}}{64\,a^{3/4}}$$

input
```
int((a + b*x^4)^(5/4)/x^9,x)
```

output
```
(b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*5i)/(64*a^(3/4)) - (5*b^2*atan((
a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(3/4)) - (9*(a + b*x^4)^(5/4))/(32*x^8) +
 (5*a*(a + b*x^4)^(1/4))/(32*x^8)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^9}\,dx = \frac{-4(b\,x^4+a)^{\frac{1}{4}}\,a - 9(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4 + 5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^5+ax}dx\right)b^2x^8}{32x^8}$$

input
```
int((b*x^4+a)^(5/4)/x^9,x)
```

output
```
( - 4*(a + b*x**4)**(1/4)*a - 9*(a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b*
x**4)**(1/4)/(a*x + b*x**5),x)*b**2*x**8)/(32*x**8)
```

3.510 $\int x^9(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int x^9\left(a+bx^4\right)^{5/4}\,dx=-\frac{2a^3x^2\sqrt[4]{a+bx^4}}{231b^2}+\frac{a^2x^6\sqrt[4]{a+bx^4}}{231b}+\frac{1}{33}ax^{10}\sqrt[4]{a+bx^4}+\frac{1}{15}x^{10}\left(a+bx^4\right)^{5/4}+\frac{4a^{9/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231b^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-2/231*a^3*x^2*(b*x^4+a)^(1/4)/b^2+1/231*a^2*x^6*(b*x^4+a)^(1/4)/b+1/33*a*
x^10*(b*x^4+a)^(1/4)+1/15*x^10*(b*x^4+a)^(5/4)+4/231*a^(9/2)*(1+b*x^4/a)^(
3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(b*x
^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.85 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int x^9 \left(a+bx^4\right)^{5/4} dx = \frac{x^2\sqrt[4]{a+bx^4}\left(-\left(\left(6a-11bx^4\right)\left(a+bx^4\right)^2\right)+\frac{6a^3 \text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{165b^2}$$

input
```
Integrate[x^9*(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(a + b*x^4)^(1/4)*(-((6*a - 11*b*x^4)*(a + b*x^4)^2) + (6*a^3*Hyperge
ometric2F1[-5/4, 1/2, 3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(165*b^2
)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {807, 248, 248, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^4\right)^{5/4} dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int x^8 \left(bx^4+a\right)^{5/4} dx^2$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{2}\left(\frac{1}{3}a\int x^8 \sqrt[4]{bx^4+a}dx^2+\frac{2}{15}x^{10}\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{1}{11}a\int \frac{x^8}{(bx^4+a)^{3/4}}dx^2+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)+\frac{2}{15}x^{10}\left(a+bx^4\right)^{5/4}\right)$$

$\big\downarrow$ 262

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)+\frac{2}{15}x^{10}\left(a+bx^4\right)^{5/4}\right)$$

$\big\downarrow$ 262

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{3b}\right)}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)+\frac{2}{15}x^{10}\left(a+bx^4\right.$$

$\big\downarrow$ 231

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b(a+bx^4)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+bx^4}\right)+\frac{2}{15}x\right.$$

$\big\downarrow$ 229

$$\frac{1}{2}\left(\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a+b\right.\right.$$

input
```
Int[x^9*(a + b*x^4)^(5/4),x]
```

output
```
((2*x^10*(a + b*x^4)^(5/4))/15 + (a*((2*x^10*(a + b*x^4)^(1/4))/11 + (a*((
2*x^6*(a + b*x^4)^(1/4))/(7*b) - (6*a*((2*x^2*(a + b*x^4)^(1/4))/(3*b) - (
4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2,
 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))))/(7*b)))/11))/3)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^9 \left(b\,x^4+a\right)^{\frac{5}{4}}dx$$

input
```
int(x^9*(b*x^4+a)^(5/4),x)
```

output
```
int(x^9*(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int x^9 \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^9\, dx$$

input
```
integrate(x^9*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^13 + a*x^9)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.20

$$\int x^9 \left(a+bx^4\right)^{5/4} dx = \frac{a^{\frac{5}{4}} x^{10} {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{10}$$

input
```
integrate(x**9*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**10*hyper((-5/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/10
```

Maxima [F]

$$\int x^9 (a+bx^4)^{5/4}\,dx = \int (bx^4+a)^{\frac{5}{4}} x^9\,dx$$

input
```
integrate(x^9*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^9, x)
```

Giac [F]

$$\int x^9 (a+bx^4)^{5/4}\,dx = \int (bx^4+a)^{\frac{5}{4}} x^9\,dx$$

input
```
integrate(x^9*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 (a+bx^4)^{5/4}\,dx = \int x^9\,(b\,x^4+a)^{5/4}\,dx$$

input
```
int(x^9*(a + b*x^4)^(5/4),x)
```

output
```
int(x^9*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^9 (a+bx^4)^{5/4}\,dx = \frac{-10(b\,x^4+a)^{\frac{1}{4}}\,a^3x^2+5(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^6+112(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^{10}+77(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{14}+2}{1155b^2}$$

input
```
int(x^9*(b*x^4+a)^(5/4),x)
```

output
```
( - 10*(a + b*x**4)**(1/4)*a**3*x**2 + 5*(a + b*x**4)**(1/4)*a**2*b*x**6 +
 112*(a + b*x**4)**(1/4)*a*b**2*x**10 + 77*(a + b*x**4)**(1/4)*b**3*x**14
+ 20*int(((a + b*x**4)**(1/4)*x)/(a + b*x**4),x)*a**4)/(1155*b**2)
```

3.511 $\int x^5(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int x^5\left(a+bx^4\right)^{5/4}\,dx=\frac{5a^2x^2\sqrt[4]{a+bx^4}}{231b}+\frac{5}{77}ax^6\sqrt[4]{a+bx^4}+\frac{1}{11}x^6\left(a+bx^4\right)^{5/4}-\frac{10a^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
5/231*a^2*x^2*(b*x^4+a)^(1/4)/b+5/77*a*x^6*(b*x^4+a)^(1/4)+1/11*x^6*(b*x^4
+a)^(5/4)-10/231*a^(7/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1
/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.83 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int x^5\left(a+bx^4\right)^{5/4}\,dx=\frac{x^2\sqrt[4]{a+bx^4}\left(\left(a+bx^4\right)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{11b}$$

input
```
Integrate[x^5*(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(a + b*x^4)^(1/4)*((a + b*x^4)^2 - (a^2*Hypergeometric2F1[-5/4, 1/2,
3/2, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(11*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {807, 248, 248, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^4\right)^{5/4} dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int x^4 \left(bx^4+a\right)^{5/4} dx^2$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{2}\left(\frac{5}{11}a\int x^4 \sqrt[4]{bx^4+a}dx^2+\frac{2}{11}x^6\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{2}\left(\frac{5}{11}a\left(\frac{1}{7}a\int \frac{x^4}{\left(bx^4+a\right)^{3/4}}dx^2+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)+\frac{2}{11}x^6\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{3b}\right)+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)+\frac{2}{11}x^6\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{ 231}$$

$$\frac{1}{2}\left(\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b\left(a+bx^4\right)^{3/4}}\right)+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)+\frac{2}{11}x^6\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}\right)+\frac{2}{7}x^6\sqrt[4]{a+bx^4}\right)+\frac{2}{11}x^6(\right.$$

input
```
Int[x^5*(a + b*x^4)^(5/4),x]
```

output
```
((2*x^6*(a + b*x^4)^(5/4))/11 + (5*a*((2*x^6*(a + b*x^4)^(1/4))/7 + (a*((2
*x^2*(a + b*x^4)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))))/7))/
11)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^{5} \left(b\,x^{4}+a \right)^{\frac{5}{4}}d x$$

input
```
int(x^5*(b*x^4+a)^(5/4),x)
```

output
```
int(x^5*(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int x^5 \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^5 \, dx$$

input
```
integrate(x^5*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^9 + a*x^5)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

$$\int x^5 (a+bx^4)^{5/4}\,dx = \frac{a^{\frac{5}{4}} x^6 {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6}$$

input
```
integrate(x**5*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**6*hyper((-5/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/6
```

Maxima [F]

$$\int x^5 (a+bx^4)^{5/4}\,dx = \int (bx^4+a)^{\frac{5}{4}} x^5\,dx$$

input
```
integrate(x^5*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^5, x)
```

Giac [F]

$$\int x^5 (a+bx^4)^{5/4}\,dx = \int (bx^4+a)^{\frac{5}{4}} x^5\,dx$$

input
```
integrate(x^5*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 (a+bx^4)^{5/4} \, dx = \int x^5 \left(b x^4 + a\right)^{5/4} dx$$

input
```
int(x^5*(a + b*x^4)^(5/4),x)
```

output
```
int(x^5*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^5 (a+bx^4)^{5/4} \, dx = \frac{5(b x^4+a)^{\frac{1}{4}} a^2 x^2 + 36(b x^4+a)^{\frac{1}{4}} ab x^6 + 21(b x^4+a)^{\frac{1}{4}} b^2 x^{10} - 10\left(\int \frac{x}{(b x^4+a)^{\frac{3}{4}}} dx\right) a^3}{231b}$$

input
```
int(x^5*(b*x^4+a)^(5/4),x)
```

output
```
(5*(a + b*x**4)**(1/4)*a**2*x**2 + 36*(a + b*x**4)**(1/4)*a*b*x**6 + 21*(a
 + b*x**4)**(1/4)*b**2*x**10 - 10*int(((a + b*x**4)**(1/4)*x)/(a + b*x**4)
,x)*a**3)/(231*b)
```

3.512 $\int x(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int x\left(a+bx^4\right)^{5/4}\,dx=\frac{5}{21}ax^2\sqrt[4]{a+bx^4}+\frac{1}{7}x^2\left(a+bx^4\right)^{5/4}+\frac{5a^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\left(a+bx^4\right)^{3/4}}$$

output
```
5/21*a*x^2*(b*x^4+a)^(1/4)+1/7*x^2*(b*x^4+a)^(5/4)+5/21*a^(5/2)*(1+b*x^4/a
)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(
b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int x\left(a+bx^4\right)^{5/4}\,dx=\frac{ax^2\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{2\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[x*(a + b*x^4)^(5/4),x]
```

output
```
(a*x^2*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^4)/a)])/
(2*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {807, 211, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^4\right)^{5/4}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \left(bx^4+a\right)^{5/4}dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{5}{7}a\int \sqrt[4]{bx^4+a}dx^2+\frac{2}{7}x^2\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{5}{7}a\left(\frac{1}{3}a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)+\frac{2}{7}x^2\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{5}{7}a\left(\frac{a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3\left(a+bx^4\right)^{3/4}}+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)+\frac{2}{7}x^2\left(a+bx^4\right)^{5/4}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{5}{7}a\left(\frac{2a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)+\frac{2}{7}x^2\left(a+bx^4\right)^{5/4}\right)$$

input
```
Int[x*(a + b*x^4)^(5/4),x]
```

output
```
((2*x^2*(a + b*x^4)^(5/4))/7 + (5*a*((2*x^2*(a + b*x^4)^(1/4))/3 + (2*a^(3
/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
3*Sqrt[b]*(a + b*x^4)^(3/4))))/7)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x\left(b\,x^{4}+a\right)^{\frac{5}{4}}dx$$

input
```
int(x*(b*x^4+a)^(5/4),x)
```

output
```
int(x*(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int x\left(a+bx^4\right)^{5/4}\,dx=\int \left(bx^4+a\right)^{\frac{5}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^5 + a*x)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.30

$$\int x\left(a+bx^4\right)^{5/4}\,dx=\frac{a^{\frac{5}{4}}x^2{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2}$$

input
```
integrate(x*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**2*hyper((-5/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/2
```

Maxima [F]

$$\int x(a+bx^4)^{5/4}\,dx=\int (bx^4+a)^{\frac{5}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*x, x)
```

Giac [F]

$$\int x(a+bx^4)^{5/4}\,dx=\int (bx^4+a)^{\frac{5}{4}}x\,dx$$

input
```
integrate(x*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+bx^4)^{5/4}\,dx=\int x\,(b\,x^4+a)^{5/4}\,dx$$

input
```
int(x*(a + b*x^4)^(5/4),x)
```

output
```
int(x*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x\left(a+bx^4\right)^{5/4}\,dx=\frac{8(b\,x^4+a)^{\frac{1}{4}}\,a\,x^2}{21}+\frac{(b\,x^4+a)^{\frac{1}{4}}\,b\,x^6}{7}+\frac{5\left(\int\frac{x}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{21}$$

input
```
int(x*(b*x^4+a)^(5/4),x)
```

output
```
(8*(a + b*x**4)**(1/4)*a*x**2 + 3*(a + b*x**4)**(1/4)*b*x**6 + 5*int(((a +
 b*x**4)**(1/4)*x)/(a + b*x**4),x)*a**2)/21
```

3.513 $\int \frac{(a+bx^4)^{5/4}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{(a+bx^4)^{5/4}}{x^3} dx = \frac{5}{6}bx^2\sqrt[4]{a+bx^4} - \frac{(a+bx^4)^{5/4}}{2x^2} + \frac{5a^{3/2}\sqrt{b}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{6(a+bx^4)^{3/4}}$$

output
```
5/6*b*x^2*(b*x^4+a)^(1/4)-1/2*(b*x^4+a)^(5/4)/x^2+5/6*a^(3/2)*b^(1/2)*(1+b
*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(b*
x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx^4)^{5/4}}{x^3} dx = -\frac{a\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^3,x]
```

output
```
-1/2*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, -((b*x^4)/a)]
)/(x^2*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 247, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2}\int \frac{(bx^4+a)^{5/4}}{x^4}dx^2$$

$$\Big\downarrow \ 247$$

$$\frac{1}{2}\left(\frac{5}{2}b\int \sqrt[4]{bx^4+a}dx^2-\frac{(a+bx^4)^{5/4}}{x^2}\right)$$

$$\Big\downarrow \ 211$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(\frac{1}{3}a\int \frac{1}{(bx^4+a)^{3/4}}dx^2+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{x^2}\right)$$

$$\Big\downarrow \ 231$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(\frac{a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3(a+bx^4)^{3/4}}+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{x^2}\right)$$

$$\Big\downarrow \ 229$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(\frac{2a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{2}{3}x^2\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x^2}\right)$$

input
```
Int[(a + b*x^4)^(5/4)/x^3,x]
```

output
```
(-((a + b*x^4)^(5/4)/x^2) + (5*b*((2*x^2*(a + b*x^4)^(1/4))/3 + (2*a^(3/2)
*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*S
qrt[b]*(a + b*x^4)^(3/4))))/2)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{x^3}dx$$

input
```
int((b*x^4+a)^(5/4)/x^3,x)
```

output
```
int((b*x^4+a)^(5/4)/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx = -\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{4},-\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2x^2}$$

input
```
integrate((b*x**4+a)**(5/4)/x**3,x)
```

output
```
-a**(5/4)*hyper((-5/4, -1/2), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*x**2)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^3}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx=\int \frac{(b\,x^4+a)^{5/4}}{x^3}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^3,x)
```

output
```
int((a + b*x^4)^(5/4)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^3}\,dx=\frac{-4(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4-5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^7+a\,x^3}dx\right)a^2x^2}{3x^2}$$

input
```
int((b*x^4+a)^(5/4)/x^3,x)
```

output
```
( - 4*(a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4 - 5*int((a + b*x*
*4)**(1/4)/(a*x**3 + b*x**7),x)*a**2*x**2)/(3*x**2)
```

3.514 $\int \frac{(a+bx^4)^{5/4}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{(a+bx^4)^{5/4}}{x^7} dx = -\frac{5b\sqrt[4]{a+bx^4}}{12x^2} - \frac{(a+bx^4)^{5/4}}{6x^6} + \frac{5\sqrt{a}b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12(a+bx^4)^{3/4}}$$

output
```
-5/12*b*(b*x^4+a)^(1/4)/x^2-1/6*(b*x^4+a)^(5/4)/x^6+5/12*a^(1/2)*b^(3/2)*(
1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/
(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx^4)^{5/4}}{x^7} dx = -\frac{a\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{2},-\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^7,x]
```

output
```
-1/6*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, -((b*x^4)/a)
])/(x^6*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {807, 247, 247, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^7} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{\left(bx^4+a\right)^{5/4}}{x^8}dx^2$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{2}\left(\frac{5}{6}b\int \frac{\sqrt[4]{bx^4+a}}{x^4}dx^2-\frac{\left(a+bx^4\right)^{5/4}}{3x^6}\right)$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{1}{2}b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2-\frac{\sqrt[4]{a+bx^4}}{x^2}\right)-\frac{\left(a+bx^4\right)^{5/4}}{3x^6}\right)$$

$$\Big\downarrow \text{ 231}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^2}\right)-\frac{\left(a+bx^4\right)^{5/4}}{3x^6}\right)$$

$$\Big\downarrow \text{ 229}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{x^2}\right)-\frac{(a+bx^4)^{5/4}}{3x^6}\right)$$

input
```
Int[(a + b*x^4)^(5/4)/x^7,x]
```

output
```
(-1/3*(a + b*x^4)^(5/4)/x^6 + (5*b*(-((a + b*x^4)^(1/4)/x^2) + (Sqrt[a]*Sq
rt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/(a + b*x^4)^(3/4)))/6)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{x^7}dx$$

input
```
int((b*x^4+a)^(5/4)/x^7,x)
```

output
```
int((b*x^4+a)^(5/4)/x^7,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^7,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^7, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^4)^{5/4}}{x^7}\,dx = -\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6x^6}$$

input
```
integrate((b*x**4+a)**(5/4)/x**7,x)
```

output
```
-a**(5/4)*hyper((-3/2, -5/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*x**6)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^7, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^7}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^7}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^7,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^7}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^7}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^7,x)
```

output
```
int((a + b*x^4)^(5/4)/x^7, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^7}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}}\,a-6(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4-5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^7+a\,x^3}dx\right)ab\,x^6}{6x^6}$$

input
```
int((b*x^4+a)^(5/4)/x^7,x)
```

output
```
( - (a + b*x**4)**(1/4)*a - 6*(a + b*x**4)**(1/4)*b*x**4 - 5*int((a + b*x*
*4)**(1/4)/(a*x**3 + b*x**7),x)*a*b*x**6)/(6*x**6)
```

3.515 $\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = -\frac{b\sqrt[4]{a+bx^4}}{12x^6} - \frac{b^2\sqrt[4]{a+bx^4}}{24ax^2} - \frac{(a+bx^4)^{5/4}}{10x^{10}} - \frac{b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24\sqrt{a}\,(a+bx^4)^{3/4}}$$

output
```
-1/12*b*(b*x^4+a)^(1/4)/x^6-1/24*b^2*(b*x^4+a)^(1/4)/a/x^2-1/10*(b*x^4+a)^
(5/4)/x^10-1/24*b^(5/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/
2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = -\frac{a\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{4},-\frac{3}{2},-\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^11,x]
```

output
```
-1/10*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, -((b*x^4)/a
)])/(x^10*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 247, 247, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{(bx^4+a)^{5/4}}{x^{12}}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \frac{\sqrt[4]{bx^4+a}}{x^8}dx^2-\frac{(a+bx^4)^{5/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{1}{6}b\int \frac{1}{x^4\,(bx^4+a)^{3/4}}dx^2-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)-\frac{(a+bx^4)^{5/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{1}{6}b\left(-\frac{b\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)-\frac{(a+bx^4)^{5/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{1}{6}b\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)-\frac{\left(a+bx^4\right)^{5/4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{1}{6}b\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a+bx^4}}{3x^6}\right)-\frac{\left(a+bx^4\right)^{5/4}}{5x^{10}}\right)$$

input
```
Int[(a + b*x^4)^(5/4)/x^11,x]
```

output
```
(-1/5*(a + b*x^4)^(5/4)/x^10 + (b*(-1/3*(a + b*x^4)^(1/4)/x^6 + (b*(-((a +
 b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(
Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4))))/6))/2)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{x^{11}}dx$$

input
```
int((b*x^4+a)^(5/4)/x^11,x)
```

output
```
int((b*x^4+a)^(5/4)/x^11,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^11,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^11, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = -\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{10x^{10}}$$

input
```
integrate((b*x**4+a)**(5/4)/x**11,x)
```

output
```
-a**(5/4)*hyper((-5/2, -5/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*x**10
)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^11,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^11, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{11}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^11,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^{11}}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^11,x)
```

output
```
int((a + b*x^4)^(5/4)/x^11, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{11}}\,dx = \frac{-4(b\,x^4+a)^{\frac{1}{4}}\,a-9(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{15}+a\,x^{11}}dx\right)a^2x^{10}}{45x^{10}}$$

input
```
int((b*x^4+a)^(5/4)/x^11,x)
```

output
```
( - 4*(a + b*x**4)**(1/4)*a - 9*(a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b*
x**4)**(1/4)/(a*x**11 + b*x**15),x)*a**2*x**10)/(45*x**10)
```

3.516 $\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = -\frac{b\sqrt[4]{a+bx^4}}{28x^{10}} - \frac{b^2\sqrt[4]{a+bx^4}}{168ax^6} + \frac{5b^3\sqrt[4]{a+bx^4}}{336a^2x^2} - \frac{(a+bx^4)^{5/4}}{14x^{14}} + \frac{5b^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{336a^{3/2}(a+bx^4)^{3/4}}$$

output
```
-1/28*b*(b*x^4+a)^(1/4)/x^10-1/168*b^2*(b*x^4+a)^(1/4)/a/x^6+5/336*b^3*(b*
x^4+a)^(1/4)/a^2/x^2-1/14*(b*x^4+a)^(5/4)/x^14+5/336*b^(7/2)*(1+b*x^4/a)^(
3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(b*x
^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.36

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = -\frac{a\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{4},-\frac{5}{2},-\frac{bx^4}{a}\right)}{14x^{14}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^15,x]
```

output
```
-1/14*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-7/2, -5/4, -5/2, -((b*x^4)/a
)])/(x^14*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {807, 247, 247, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{(bx^4+a)^{5/4}}{x^{16}}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{5}{14}b\int \frac{\sqrt[4]{bx^4+a}}{x^{12}}dx^2-\frac{(a+bx^4)^{5/4}}{7x^{14}}\right)$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(\frac{5}{14}b\left(\frac{1}{10}b\int \frac{1}{x^8(bx^4+a)^{3/4}}dx^2-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)-\frac{(a+bx^4)^{5/4}}{7x^{14}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{5}{14}b\left(\frac{1}{10}b\left(-\frac{5b\int \frac{1}{x^4(bx^4+a)^{3/4}}dx^2}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)-\frac{(a+bx^4)^{5/4}}{7x^{14}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{5}{14}b\left(\frac{1}{10}b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{7x^{14}}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{5}{14}b\left(\frac{1}{10}b\left(-\frac{5b\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{7x^{14}}\right.$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{5}{14}b\left(\frac{1}{10}b\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a+bx^4}}{5x^{10}}\right)-\frac{(a\right.$$

input
```
Int[(a + b*x^4)^(5/4)/x^15,x]
```

output
```
(-1/7*(a + b*x^4)^(5/4)/x^14 + (5*b*(-1/5*(a + b*x^4)^(1/4)/x^10 + (b*(-1/
3*(a + b*x^4)^(1/4)/(a*x^6) - (5*b*(-((a + b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b
]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sq
rt[a]*(a + b*x^4)^(3/4))))/(6*a)))/10))/14)/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{x^{15}}dx$$

input
```
int((b*x^4+a)^(5/4)/x^15,x)
```

output
```
int((b*x^4+a)^(5/4)/x^15,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{15}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^15,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^15, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = -\frac{a^{\frac{5}{4}}{}_2F_1\left(\left.\begin{matrix} -\frac{7}{2}, -\frac{5}{4} \\ -\frac{5}{2} \end{matrix}\right| \frac{bx^4e^{i\pi}}{a}\right)}{14x^{14}}$$

input
```
integrate((b*x**4+a)**(5/4)/x**15,x)
```

output
```
-a**(5/4)*hyper((-7/2, -5/4), (-5/2,), b*x**4*exp_polar(I*pi)/a)/(14*x**14
)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{15}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^15,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^15, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{15}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^15,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^15, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^{15}}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^15,x)
```

output
```
int((a + b*x^4)^(5/4)/x^15, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{15}}\,dx = \frac{-8(b\,x^4+a)^{\frac{1}{4}}\,a-13(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{19}+a\,x^{15}}dx\right)a^2x^{14}}{117x^{14}}$$

input
```
int((b*x^4+a)^(5/4)/x^15,x)
```

output
```
( - 8*(a + b*x**4)**(1/4)*a - 13*(a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b
*x**4)**(1/4)/(a*x**15 + b*x**19),x)*a**2*x**14)/(117*x**14)
```

3.517 $\int x^{10}(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int x^{10}(a+bx^4)^{5/4}\,dx=-\frac{35a^3x^3\sqrt[4]{a+bx^4}}{6144b^2}+\frac{5a^2x^7\sqrt[4]{a+bx^4}}{1536b}+\frac{5}{192}ax^{11}\sqrt[4]{a+bx^4}+\frac{1}{16}x^{11}(a+bx^4)^{5/4}-\frac{35a^4\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{11/4}}+\frac{35a^4\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4096b^{11/4}}$$

output
```
-35/6144*a^3*x^3*(b*x^4+a)^(1/4)/b^2+5/1536*a^2*x^7*(b*x^4+a)^(1/4)/b+5/19
2*a*x^11*(b*x^4+a)^(1/4)+1/16*x^11*(b*x^4+a)^(5/4)-35/4096*a^4*arctan(b^(1
/4)*x/(b*x^4+a)^(1/4))/b^(11/4)+35/4096*a^4*arctanh(b^(1/4)*x/(b*x^4+a)^(1
/4))/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

$$\int x^{10}(a+bx^4)^{5/4}\,dx=\frac{2b^{3/4}x^3\sqrt[4]{a+bx^4}(-35a^3+20a^2bx^4+544ab^2x^8+384b^3x^{12})-105a^4\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+}{12288b^{11/4}}$$

input
```
Integrate[x^10*(a + b*x^4)^(5/4),x]
```

output
```
(2*b^(3/4)*x^3*(a + b*x^4)^(1/4)*(-35*a^3 + 20*a^2*b*x^4 + 544*a*b^2*x^8 +
 384*b^3*x^12) - 105*a^4*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 105*a^4*A
rcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(12288*b^(11/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {811, 811, 843, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10} \left(a+b x^4\right)^{5/4} \, dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{5}{16} a \int x^{10} \sqrt[4]{b x^4+a} dx+\frac{1}{16} x^{11} \left(a+b x^4\right)^{5/4}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{5}{16} a \left(\frac{1}{12} a \int \frac{x^{10}}{\left(b x^4+a\right)^{3/4}}dx+\frac{1}{12} x^{11} \sqrt[4]{a+b x^4}\right)+\frac{1}{16} x^{11} \left(a+b x^4\right)^{5/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5}{16} a \left(\frac{1}{12} a \left(\frac{x^7 \sqrt[4]{a+b x^4}}{8 b}-\frac{7 a \int \frac{x^6}{\left(b x^4+a\right)^{3/4}}dx}{8 b}\right)+\frac{1}{12} x^{11} \sqrt[4]{a+b x^4}\right)+\frac{1}{16} x^{11} \left(a+b x^4\right)^{5/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5}{16}a\left(\frac{1}{12}a\left(\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int\frac{x^2}{\left(bx^4+a\right)^{3/4}}dx}{4b}\right)}{8b}\right)+\frac{1}{12}x^{11}\sqrt[4]{a+bx^4}\right)+$$
$$\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}$$

$$\downarrow \text{854}$$

$$\frac{5}{16}a\left(\frac{1}{12}a\left(\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int\frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}\right)+\frac{1}{12}x^{11}\sqrt[4]{a+bx^4}\right)+$$
$$\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}$$

$$\downarrow \text{827}$$

$$\frac{5}{16}a\left(\frac{1}{12}a\left(\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}\right)}{8b}\right)+\frac{1}{12}x^{1}\right.$$
$$\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}$$

$$\downarrow \text{216}$$

$$\frac{5}{16}a\left(\frac{1}{12}a\left(\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)}{8b}\right)+\frac{1}{12}x^{11}\sqrt[4]{a}\cdots\right)$$

$$\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}$$

$\Big\downarrow$ 219

$$\frac{5}{16}a\left(\frac{1}{12}a\left(\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)}{8b}\right)+\frac{1}{12}x^{11}\sqrt[4]{a}\cdots\right)$$

$$\frac{1}{16}x^{11}\left(a+bx^4\right)^{5/4}$$

input
```
Int[x^10*(a + b*x^4)^(5/4),x]
```

output
```
(x^11*(a + b*x^4)^(5/4))/16 + (5*a*((x^11*(a + b*x^4)^(1/4))/12 + (a*((x^7
*(a + b*x^4)^(1/4))/(8*b) - (7*a*((x^3*(a + b*x^4)^(1/4))/(4*b) - (3*a*(-1
/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*b^(3/4))))/(4*b)))/(8*b)))/12))/16
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\frac{35\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)a^{4}}{8192}+\frac{35\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)a^{4}}{4096}-\frac{35x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(-\frac{384b^{\frac{15}{4}}x^{12}}{35}-\frac{544a\,b^{\frac{11}{4}}x^{8}}{35}-\frac{4a^{2}b^{\frac{7}{4}}x^{4}}{7}+a^{3}b^{\frac{3}{4}}\right)}{6144}}{b^{\frac{11}{4}}}$

input
```
int(x^10*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
35/4096*(1/2*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*
a^4+arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^4-2/3*x^3*(b*x^4+a)^(1/4)*(-384/
35*b^(15/4)*x^12-544/35*a*b^(11/4)*x^8-4/7*a^2*b^(7/4)*x^4+a^3*b^(3/4)))/b
^(11/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.67

$$\int x^{10}(a+bx^4)^{5/4}\,dx = \frac{105\left(\frac{a^{16}}{b^{11}}\right)^{\frac{1}{4}}b^{2}\log\left(\frac{35\left((bx^{4}+a)^{\frac{1}{4}}a^{4}+\left(\frac{a^{16}}{b^{11}}\right)^{\frac{1}{4}}b^{3}x\right)}{x}\right)+105i\left(\frac{a^{16}}{b^{11}}\right)^{\frac{1}{4}}b^{2}\log\left(\frac{35\left((bx^{4}+a)^{\frac{1}{4}}a^{4}+i\left(\frac{a^{16}}{b^{11}}\right)^{\frac{1}{4}}b\right.}{x}\right.}{}$$

input
```
integrate(x^10*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/24576*(105*(a^16/b^11)^(1/4)*b^2*log(35*((b*x^4 + a)^(1/4)*a^4 + (a^16/b
^11)^(1/4)*b^3*x)/x) + 105*I*(a^16/b^11)^(1/4)*b^2*log(35*((b*x^4 + a)^(1/
4)*a^4 + I*(a^16/b^11)^(1/4)*b^3*x)/x) - 105*I*(a^16/b^11)^(1/4)*b^2*log(3
5*((b*x^4 + a)^(1/4)*a^4 - I*(a^16/b^11)^(1/4)*b^3*x)/x) - 105*(a^16/b^11)
^(1/4)*b^2*log(35*((b*x^4 + a)^(1/4)*a^4 - (a^16/b^11)^(1/4)*b^3*x)/x) + 4
*(384*b^3*x^15 + 544*a*b^2*x^11 + 20*a^2*b*x^7 - 35*a^3*x^3)*(b*x^4 + a)^(
1/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int x^{10}\left(a+bx^4\right)^{5/4}\,dx=\frac{a^{\frac{5}{4}}x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**11*gamma(11/4)*hyper((-5/4, 11/4), (15/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.55

$$\int x^{10}\left(a+bx^4\right)^{5/4}\,dx = -\frac{\frac{105\,(bx^4+a)^{\frac{1}{4}}a^4b^3}{x}-\frac{399\,(bx^4+a)^{\frac{5}{4}}a^4b^2}{x^5}-\frac{125\,(bx^4+a)^{\frac{9}{4}}a^4b}{x^9}+\frac{35\,(bx^4+a)^{\frac{13}{4}}a^4}{x^{13}}}{6144\left(b^6-\frac{4\,(bx^4+a)b^5}{x^4}+\frac{6\,(bx^4+a)^2b^4}{x^8}-\frac{4\,(bx^4+a)^3b^3}{x^{12}}+\frac{(bx^4+a)^4b^2}{x^{16}}\right)} + \frac{35\left(\frac{2\,a^4\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}}-\frac{a^4\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{8192\,b^2}$$

input

```
integrate(x^10*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output

```
-1/6144*(105*(b*x^4 + a)^(1/4)*a^4*b^3/x - 399*(b*x^4 + a)^(5/4)*a^4*b^2/x
^5 - 125*(b*x^4 + a)^(9/4)*a^4*b/x^9 + 35*(b*x^4 + a)^(13/4)*a^4/x^13)/(b^
6 - 4*(b*x^4 + a)*b^5/x^4 + 6*(b*x^4 + a)^2*b^4/x^8 - 4*(b*x^4 + a)^3*b^3/
x^12 + (b*x^4 + a)^4*b^2/x^16) + 35/8192*(2*a^4*arctan((b*x^4 + a)^(1/4)/(
b^(1/4)*x))/b^(3/4) - a^4*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) +
(b*x^4 + a)^(1/4)/x))/b^(3/4))/b^2
```

Giac [F]

$$\int x^{10}\left(a+bx^4\right)^{5/4}\,dx = \int \left(bx^4+a\right)^{\frac{5}{4}}x^{10}\,dx$$

input

```
integrate(x^10*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output

```
integrate((b*x^4 + a)^(5/4)*x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{10}(a+bx^4)^{5/4}\,dx=\int x^{10}\left(b\,x^4+a\right)^{5/4}dx$$

input
```
int(x^10*(a + b*x^4)^(5/4),x)
```

output
```
int(x^10*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^{10}(a+bx^4)^{5/4}\,dx=\frac{-35(b\,x^4+a)^{\frac{1}{4}}\,a^3x^3+20(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^7+544(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^{11}+384(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{15}+}{6144b^2}$$

input
```
int(x^10*(b*x^4+a)^(5/4),x)
```

output
```
( - 35*(a + b*x**4)**(1/4)*a**3*x**3 + 20*(a + b*x**4)**(1/4)*a**2*b*x**7
+ 544*(a + b*x**4)**(1/4)*a*b**2*x**11 + 384*(a + b*x**4)**(1/4)*b**3*x**1
5 + 105*int(((a + b*x**4)**(1/4)*x**2)/(a + b*x**4),x)*a**4)/(6144*b**2)
```

3.518 $\int x^6(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int x^6\left(a+bx^4\right)^{5/4}\,dx=\frac{5a^2x^3\sqrt[4]{a+bx^4}}{384b}+\frac{5}{96}ax^7\sqrt[4]{a+bx^4}+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}+\frac{5a^3\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}-\frac{5a^3\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{256b^{7/4}}$$

output
```
5/384*a^2*x^3*(b*x^4+a)^(1/4)/b+5/96*a*x^7*(b*x^4+a)^(1/4)+1/12*x^7*(b*x^4
+a)^(5/4)+5/256*a^3*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)-5/256*a^3*ar
ctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int x^6\left(a+bx^4\right)^{5/4}\,dx=\frac{2b^{3/4}x^3\sqrt[4]{a+bx^4}(5a^2+52abx^4+32b^2x^8)+15a^3\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-15a^3\text{arctanh}\left(\frac{\sqrt[4]{}}{\sqrt[4]{a}}\right.}{768b^{7/4}}$$

input
```
Integrate[x^6*(a + b*x^4)^(5/4),x]
```

output
```
(2*b^(3/4)*x^3*(a + b*x^4)^(1/4)*(5*a^2 + 52*a*b*x^4 + 32*b^2*x^8) + 15*a^
3*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - 15*a^3*ArcTanh[(b^(1/4)*x)/(a +
b*x^4)^(1/4)])/(768*b^(7/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {811, 811, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^4\right)^{5/4} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{12}a\int x^6\sqrt[4]{bx^4+a}dx+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{12}a\left(\frac{1}{8}a\int \frac{x^6}{(bx^4+a)^{3/4}}dx+\frac{1}{8}x^7\sqrt[4]{a+bx^4}\right)+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{12}a\left(\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}\right)+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{854}$$

$$\frac{5}{12}a\left(\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}\right)+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{827}$$

$$\frac{5}{12}a\left(\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}\right)+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

$\Big\downarrow$ 216

$$\frac{5}{12}a\left(\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}\right)+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

$\Big\downarrow$ 219

$$\frac{5}{12}a\left(\frac{1}{8}a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a+bx^4}\right)+\frac{1}{12}x^7\left(a+bx^4\right)^{5/4}$$

input
```
Int[x^6*(a + b*x^4)^(5/4),x]
```

output
```
(x^7*(a + b*x^4)^(5/4))/12 + (5*a*((x^7*(a + b*x^4)^(1/4))/8 + (a*((x^3*(a
 + b*x^4)^(1/4))/(4*b) - (3*a*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/
b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4))))/(4*b)))/8))
/12
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{128\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{\frac{11}{4}} x^{11}+208 a\,b^{\frac{7}{4}} x^{7}\left(b\,x^{4}+a\right)^{\frac{1}{4}}+20 a^{2} x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{\frac{3}{4}}-15\ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a^{3}-30\arctan\left(\frac{\left(b\,x^{4}+a\right)}{b^{\frac{1}{4}} x}\right.}{1536 b^{\frac{7}{4}}}$

input
```
int(x^6*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(128*(b*x^4+a)^(1/4)*b^(11/4)*x^11+208*a*b^(7/4)*x^7*(b*x^4+a)^(1/4
)+20*a^2*x^3*(b*x^4+a)^(1/4)*b^(3/4)-15*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b
^(1/4)*x+(b*x^4+a)^(1/4)))*a^3-30*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^3)
/b^(7/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.84

$$\int x^6\left(a+bx^4\right)^{5/4}\,dx =$$

$$-\frac{15\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}} b\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3+\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}}b^2x\right)}{x}\right)+15i\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}} b\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^3+i\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}}b^2x\right)}{x}\right)-15i\left(\frac{a^{12}}{b^7}\right)^{\frac{1}{4}}\cdots}{\cdots}$$

input
```
integrate(x^6*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/1536*(15*(a^12/b^7)^(1/4)*b*log(5*((b*x^4 + a)^(1/4)*a^3 + (a^12/b^7)^(
1/4)*b^2*x)/x) + 15*I*(a^12/b^7)^(1/4)*b*log(5*((b*x^4 + a)^(1/4)*a^3 + I*
(a^12/b^7)^(1/4)*b^2*x)/x) - 15*I*(a^12/b^7)^(1/4)*b*log(5*((b*x^4 + a)^(1
/4)*a^3 - I*(a^12/b^7)^(1/4)*b^2*x)/x) - 15*(a^12/b^7)^(1/4)*b*log(5*((b*x
^4 + a)^(1/4)*a^3 - (a^12/b^7)^(1/4)*b^2*x)/x) - 4*(32*b^2*x^11 + 52*a*b*x
^7 + 5*a^2*x^3)*(b*x^4 + a)^(1/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int x^6 (a+bx^4)^{5/4}\,dx = \frac{a^{\frac{5}{4}} x^7 \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**7*gamma(7/4)*hyper((-5/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.54

$$\int x^6 (a+bx^4)^{5/4}\,dx = \frac{\frac{15\,(bx^4+a)^{\frac{1}{4}}a^3b^2}{x} - \frac{42\,(bx^4+a)^{\frac{5}{4}}a^3b}{x^5} - \frac{5\,(bx^4+a)^{\frac{9}{4}}a^3}{x^9}}{384\left(b^4 - \frac{3\,(bx^4+a)b^3}{x^4} + \frac{3\,(bx^4+a)^2b^2}{x^8} - \frac{(bx^4+a)^3b}{x^{12}}\right)} - \frac{5\left(\frac{2\,a^3\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a^3\log\left(-\frac{b^{\frac{1}{4}} - \frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{512\,b}$$

input
```
integrate(x^6*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/384*(15*(b*x^4 + a)^(1/4)*a^3*b^2/x - 42*(b*x^4 + a)^(5/4)*a^3*b/x^5 - 5
*(b*x^4 + a)^(9/4)*a^3/x^9)/(b^4 - 3*(b*x^4 + a)*b^3/x^4 + 3*(b*x^4 + a)^2
*b^2/x^8 - (b*x^4 + a)^3*b/x^12) - 5/512*(2*a^3*arctan((b*x^4 + a)^(1/4)/(
b^(1/4)*x))/b^(3/4) - a^3*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) +
(b*x^4 + a)^(1/4)/x))/b^(3/4))/b
```

Giac [F]

$$\int x^6 \left(a+bx^4\right)^{5/4} \, dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a+bx^4\right)^{5/4} \, dx = \int x^6 \, \left(b\,x^4+a\right)^{5/4} \,dx$$

input
```
int(x^6*(a + b*x^4)^(5/4),x)
```

output
```
int(x^6*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^6 (a+bx^4)^{5/4}\,dx = \frac{5(b\,x^4+a)^{\frac{1}{4}}\,a^2x^3+52(b\,x^4+a)^{\frac{1}{4}}\,ab\,x^7+32(b\,x^4+a)^{\frac{1}{4}}\,b^2x^{11}-15\left(\int\frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^3}{384b}$$

input
```
int(x^6*(b*x^4+a)^(5/4),x)
```

output
```
(5*(a + b*x**4)**(1/4)*a**2*x**3 + 52*(a + b*x**4)**(1/4)*a*b*x**7 + 32*(a
 + b*x**4)**(1/4)*b**2*x**11 - 15*int(((a + b*x**4)**(1/4)*x**2)/(a + b*x*
*4),x)*a**3)/(384*b)
```

3.519 $\int x^2(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int x^2\left(a+bx^4\right)^{5/4}\,dx=\frac{5}{32}ax^3\sqrt[4]{a+bx^4}+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}-\frac{5a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{5a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}$$

output
```
5/32*a*x^3*(b*x^4+a)^(1/4)+1/8*x^3*(b*x^4+a)^(5/4)-5/64*a^2*arctan(b^(1/4)
*x/(b*x^4+a)^(1/4))/b^(3/4)+5/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^
(3/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int x^2\left(a+bx^4\right)^{5/4}\,dx=\frac{1}{32}x^3\sqrt[4]{a+bx^4}\left(9a+4bx^4\right)-\frac{5a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}+\frac{5a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{3/4}}$$

input
```
Integrate[x^2*(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(a + b*x^4)^(1/4)*(9*a + 4*b*x^4))/32 - (5*a^2*ArcTan[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)])/(64*b^(3/4)) + (5*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/
4)])/(64*b^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {811, 811, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^4\right)^{5/4} dx$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{8}a\int x^2\sqrt[4]{bx^4+a}dx+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{8}a\left(\frac{1}{4}a\int \frac{x^2}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{854}$$

$$\frac{5}{8}a\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{827}$$

$$\frac{5}{8}a\left(\frac{1}{4}a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)+$$
$$\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{216}$$

$$\frac{5}{8}a\left(\frac{1}{4}a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}$$

$\downarrow$ 219

$$\frac{5}{8}a\left(\frac{1}{4}a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)+\frac{1}{8}x^3\left(a+bx^4\right)^{5/4}$$

input
```
Int[x^2*(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(a + b*x^4)^(5/4))/8 + (5*a*((x^3*(a + b*x^4)^(1/4))/4 + (a*(-1/2*Arc
Tan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^
4)^(1/4)]/(2*b^(3/4))))/4))/8
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{16(b\,x^{4}+a)^{\frac{1}{4}} b^{\frac{7}{4}} x^{7}+36 a\,x^{3} (b\,x^{4}+a)^{\frac{1}{4}} b^{\frac{3}{4}}+5\ln\left(\frac{b^{\frac{1}{4}} x+(b\,x^{4}+a)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+(b\,x^{4}+a)^{\frac{1}{4}}}\right) a^{2}+10\arctan\left(\frac{(b\,x^{4}+a)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a^{2}}{128 b^{\frac{3}{4}}}$	104

input
```
int(x^2*(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(16*(b*x^4+a)^(1/4)*b^(7/4)*x^7+36*a*x^3*(b*x^4+a)^(1/4)*b^(3/4)+5*l
n((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^2+10*arctan(
1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^2)/b^(3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.00

$$\int x^2 (a+bx^4)^{5/4} \, dx = \frac{1}{32}\left(4\,bx^7+9\,ax^3\right)\left(bx^4+a\right)^{\frac{1}{4}} + \frac{5}{128}\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2+\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}bx\right)}{x}\right) + \frac{5}{128}i\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2+i\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}bx\right)}{x}\right) - \frac{5}{128}i\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2-i\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}bx\right)}{x}\right) - \frac{5}{128}\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{5\left(\left(bx^4+a\right)^{\frac{1}{4}}a^2-\left(\frac{a^8}{b^3}\right)^{\frac{1}{4}}bx\right)}{x}\right)$$

input
```
integrate(x^2*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/32*(4*b*x^7 + 9*a*x^3)*(b*x^4 + a)^(1/4) + 5/128*(a^8/b^3)^(1/4)*log(5*(
(b*x^4 + a)^(1/4)*a^2 + (a^8/b^3)^(1/4)*b*x)/x) + 5/128*I*(a^8/b^3)^(1/4)*
log(5*((b*x^4 + a)^(1/4)*a^2 + I*(a^8/b^3)^(1/4)*b*x)/x) - 5/128*I*(a^8/b^
3)^(1/4)*log(5*((b*x^4 + a)^(1/4)*a^2 - I*(a^8/b^3)^(1/4)*b*x)/x) - 5/128*
(a^8/b^3)^(1/4)*log(5*((b*x^4 + a)^(1/4)*a^2 - (a^8/b^3)^(1/4)*b*x)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int x^2 \left(a+bx^4\right)^{5/4} dx = \frac{a^{\frac{5}{4}} x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.44

$$\int x^2 \left(a+bx^4\right)^{5/4} dx = \frac{5\,a^2 \arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{64\,b^{\frac{3}{4}}} - \frac{5\,a^2 \log\left(-\frac{b^{\frac{1}{4}} - \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{128\,b^{\frac{3}{4}}} - \frac{\frac{5\left(bx^4+a\right)^{\frac{1}{4}} a^2 b}{x} - \frac{9\left(bx^4+a\right)^{\frac{5}{4}} a^2}{x^5}}{32\left(b^2 - \frac{2\left(bx^4+a\right)b}{x^4} + \frac{\left(bx^4+a\right)^2}{x^8}\right)}$$

input
```
integrate(x^2*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
5/64*a^2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - 5/128*a^2*log(-(b
^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4) - 1
/32*(5*(b*x^4 + a)^(1/4)*a^2*b/x - 9*(b*x^4 + a)^(5/4)*a^2/x^5)/(b^2 - 2*(
b*x^4 + a)*b/x^4 + (b*x^4 + a)^2/x^8)
```

Giac [F]

$$\int x^2 (a + bx^4)^{5/4} \, dx = \int (bx^4 + a)^{\frac{5}{4}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^4)^{5/4} \, dx = \int x^2 \left(b x^4 + a\right)^{5/4} dx$$

input
```
int(x^2*(a + b*x^4)^(5/4),x)
```

output
```
int(x^2*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^2 (a + bx^4)^{5/4} \, dx = \frac{9(b x^4 + a)^{\frac{1}{4}} a x^3}{32} + \frac{(b x^4 + a)^{\frac{1}{4}} b x^7}{8} + \frac{5\left(\int \frac{x^2}{(b x^4+a)^{\frac{3}{4}}} dx\right) a^2}{32}$$

input
```
int(x^2*(b*x^4+a)^(5/4),x)
```

output
```
(9*(a + b*x**4)**(1/4)*a*x**3 + 4*(a + b*x**4)**(1/4)*b*x**7 + 5*int(((a +
 b*x**4)**(1/4)*x**2)/(a + b*x**4),x)*a**2)/32
```

3.520 $\int \frac{(a+bx^4)^{5/4}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\begin{aligned}\int \frac{(a+bx^4)^{5/4}}{x^2} dx =& \frac{5}{4}bx^3\sqrt[4]{a+bx^4} - \frac{(a+bx^4)^{5/4}}{x} \\ &- \frac{5}{8}a\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{5}{8}a\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\end{aligned}$$

output
```
5/4*b*x^3*(b*x^4+a)^(1/4)-(b*x^4+a)^(5/4)/x-5/8*a*b^(1/4)*arctan(b^(1/4)*x
/(b*x^4+a)^(1/4))+5/8*a*b^(1/4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{(a+bx^4)^{5/4}}{x^2} dx =& \frac{(-4a+bx^4)\sqrt[4]{a+bx^4}}{4x} \\ &- \frac{5}{8}a\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \frac{5}{8}a\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\end{aligned}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^2,x]
```

output
```
((-4*a + b*x^4)*(a + b*x^4)^(1/4))/(4*x) - (5*a*b^(1/4)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/8 + (5*a*b^(1/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4
)])/8
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {809, 811, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^2}dx$$

$$\Big\downarrow \text{809}$$

$$5b\int x^2\sqrt[4]{bx^4+a}dx-\frac{\left(a+bx^4\right)^{5/4}}{x}$$

$$\Big\downarrow \text{811}$$

$$5b\left(\frac{1}{4}a\int \frac{x^2}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x}$$

$$\Big\downarrow \text{854}$$

$$5b\left(\frac{1}{4}a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x}$$

$$\Big\downarrow \text{827}$$

$$5b\left(\frac{1}{4}a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x}$$

$$\Big\downarrow \text{216}$$

$$5b\left(\frac{1}{4}a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x}$$

$$\Big\downarrow \; 219$$

$$5b\left(\frac{1}{4}a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)+\frac{1}{4}x^3\sqrt[4]{a+bx^4}\right)-\frac{\left(a+bx^4\right)^{5/4}}{x}$$

input
```
Int[(a + b*x^4)^(5/4)/x^2,x]
```

output
```
-((a + b*x^4)^(5/4)/x) + 5*b*((x^3*(a + b*x^4)^(1/4))/4 + (a*(-1/2*ArcTan[
(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(
1/4)]/(2*b^(3/4))))/4)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$\frac{5ax\left(\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)\right)b^{\frac{1}{4}}-4\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(-b\,x^{4}+4a\right)}{16x}$	90

input
```
int((b*x^4+a)^(5/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(5*a*x*(ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))+
2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*b^(1/4)-4*(b*x^4+a)^(1/4)*(-b*x^4+4
*a))/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(5/4)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx^4)^{5/4}}{x^2}\,dx = \frac{a^{\frac{5}{4}}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**2,x)
```

output
```
a**(5/4)*gamma(-1/4)*hyper((-5/4, -1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^4)^{5/4}}{x^2}\,dx = \frac{5}{16}\left(2\,b^{\frac{1}{4}}\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right) - b^{\frac{1}{4}}\log\left(-\frac{b^{\frac{1}{4}} - \frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)\right)a - \frac{(bx^4+a)^{\frac{1}{4}}a}{x} - \frac{(bx^4+a)^{\frac{1}{4}}ab}{4\left(b - \frac{bx^4+a}{x^4}\right)x}$$

input
```
integrate((b*x^4+a)^(5/4)/x^2,x, algorithm="maxima")
```

output
```
5/16*(2*b^(1/4)*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)) - b^(1/4)*log(-(b^(1
/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x)))*a - (b*x^4 +
a)^(1/4)*a/x - 1/4*(b*x^4 + a)^(1/4)*a*b/((b - (b*x^4 + a)/x^4)*x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^2}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{(a+bx^4)^{5/4}}{x^2}\,dx = -\frac{(b\,x^4+a)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{b\,x^4}{a}\right)}{x\,\left(\frac{b\,x^4}{a}+1\right)^{5/4}}$$

input
```
int((a + b*x^4)^(5/4)/x^2,x)
```

output
```
-((a + b*x^4)^(5/4)*hypergeom([-5/4, -1/4], 3/4, -(b*x^4)/a))/(x*((b*x^4)/
a + 1)^(5/4))
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^2}\,dx = \frac{-4(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)abx}{4x}$$

input
```
int((b*x^4+a)^(5/4)/x^2,x)
```

output
```
( - 4*(a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4 + 5*int(((a + b*x
**4)**(1/4)*x**2)/(a + b*x**4),x)*a*b*x)/(4*x)
```

3.521 $\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = -\frac{b\sqrt[4]{a+bx^4}}{x}-\frac{(a+bx^4)^{5/4}}{5x^5} - \frac{1}{2}b^{5/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\frac{1}{2}b^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)$$

output
```
-b*(b*x^4+a)^(1/4)/x-1/5*(b*x^4+a)^(5/4)/x^5-1/2*b^(5/4)*arctan(b^(1/4)*x/
(b*x^4+a)^(1/4))+1/2*b^(5/4)*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \frac{1}{10}\left(-\frac{2\sqrt[4]{a+bx^4}(a+6bx^4)}{x^5} - 5b^{5/4}\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+5b^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right)$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^6,x]
```

output
```
((-2*(a + b*x^4)^(1/4)*(a + 6*b*x^4))/x^5 - 5*b^(5/4)*ArcTan[(b^(1/4)*x)/(
a + b*x^4)^(1/4)] + 5*b^(5/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/10
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {809, 809, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx$$

$$\Big\downarrow \ 809$$

$$b\int \frac{\sqrt[4]{bx^4+a}}{x^2}dx-\frac{(a+bx^4)^{5/4}}{5x^5}$$

$$\Big\downarrow \ 809$$

$$b\left(b\int \frac{x^2}{(bx^4+a)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{x}\right)-\frac{(a+bx^4)^{5/4}}{5x^5}$$

$$\Big\downarrow \ 854$$

$$b\left(b\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}-\frac{\sqrt[4]{a+bx^4}}{x}\right)-\frac{(a+bx^4)^{5/4}}{5x^5}$$

$$\Big\downarrow \ 827$$

$$b\left(b\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)-\frac{\sqrt[4]{a+bx^4}}{x}\right)-\frac{(a+bx^4)^{5/4}}{5x^5}$$

$$\Big\downarrow \ 216$$

$$b\left(b\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)-\frac{\sqrt[4]{a+bx^4}}{x}\right)-\frac{(a+bx^4)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$b\left(b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)-\frac{\sqrt[4]{a+bx^4}}{x}\right)-\frac{\left(a+bx^4\right)^{5/4}}{5x^5}$$

input
```
Int[(a + b*x^4)^(5/4)/x^6,x]
```

output
```
-1/5*(a + b*x^4)^(5/4)/x^5 + b*(-((a + b*x^4)^(1/4)/x) + b*(-1/2*ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/
4)]/(2*b^(3/4))))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{5x^5\left(\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^4+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^4+a\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\left(b\,x^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)\right)b^{\frac{5}{4}}-4\left(b\,x^4+a\right)^{\frac{1}{4}}\left(6b\,x^4+a\right)}{20x^5}$	89

input
```
int((b*x^4+a)^(5/4)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/20*(5*x^5*(ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))+
2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))*b^(5/4)-4*(b*x^4+a)^(1/4)*(6*b*x^4+
a))/x^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((b*x^4+a)^(5/4)/x^6,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \frac{a^{\frac{5}{4}}\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{5}{4} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^5\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**6,x)
```

output
```
a**(5/4)*gamma(-5/4)*hyper((-5/4, -5/4), (-1/4,), b*x**4*exp_polar(I*pi)/a
)/(4*x**5*gamma(-1/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \frac{1}{2}\,b^{\frac{5}{4}}\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right) - \frac{1}{4}\,b^{\frac{5}{4}}\log\left(-\frac{b^{\frac{1}{4}} - \frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}} + \frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right) - \frac{(bx^4+a)^{\frac{1}{4}}b}{x} - \frac{(bx^4+a)^{\frac{5}{4}}}{5\,x^5}$$

input
```
integrate((b*x^4+a)^(5/4)/x^6,x, algorithm="maxima")
```

output
```
1/2*b^(5/4)*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x)) - 1/4*b^(5/4)*log(-(b^(1
/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x)) - (b*x^4 + a)^
(1/4)*b/x - 1/5*(b*x^4 + a)^(5/4)/x^5
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^6}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^6,x)
```

output
```
int((a + b*x^4)^(5/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^6}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}}\,a-(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{x^2}dx\right)b\,x^5}{5x^5}$$

input
```
int((b*x^4+a)^(5/4)/x^6,x)
```

output
```
( - (a + b*x**4)**(1/4)*a - (a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b*x**4
)**(1/4)/x**2,x)*b*x**5)/(5*x**5)
```

3.522 $\int \frac{(a+bx^4)^{5/4}}{x^{10}} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}} dx = -\frac{(a+bx^4)^{9/4}}{9ax^9}$$

output
```
-1/9*(b*x^4+a)^(9/4)/a/x^9
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}} dx = -\frac{(a+bx^4)^{9/4}}{9ax^9}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^10,x]
```

output
```
-1/9*(a + b*x^4)^(9/4)/(a*x^9)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = $\{796\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{(a+bx^4)^{9/4}}{9ax^9}$$

input
```
Int[(a + b*x^4)^(5/4)/x^10,x]
```

output
```
-1/9*(a + b*x^4)^(9/4)/(a*x^9)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b x^4+a)^{\frac{9}{4}}}{9 a x^9}$	18
pseudoelliptic	$-\frac{(b x^4+a)^{\frac{9}{4}}}{9 a x^9}$	18
orering	$-\frac{(b x^4+a)^{\frac{9}{4}}}{9 a x^9}$	18
trager	$-\frac{(b^2 x^8+2 a b x^4+a^2)(b x^4+a)^{\frac{1}{4}}}{9 a x^9}$	36
risch	$-\frac{(b^2 x^8+2 a b x^4+a^2)(b x^4+a)^{\frac{1}{4}}}{9 a x^9}$	36

input
```
int((b*x^4+a)^(5/4)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*(b*x^4+a)^(9/4)/a/x^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx = -\frac{(b^2x^8+2\,abx^4+a^2)(bx^4+a)^{\frac{1}{4}}}{9\,ax^9}$$

input
```
integrate((b*x^4+a)^(5/4)/x^10,x, algorithm="fricas")
```

output
```
-1/9*(b^2*x^8 + 2*a*b*x^4 + a^2)*(b*x^4 + a)^(1/4)/(a*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(17) = 34$.

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx = \frac{a\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{4x^8\Gamma\left(-\frac{5}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{2x^4\Gamma\left(-\frac{5}{4}\right)} + \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{4a\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**10,x)
```

output
```
a*b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(4*x**8*gamma(-5/4)) + b**(
5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(2*x**4*gamma(-5/4)) + b**(9/4)*(
a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(4*a*gamma(-5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx = -\frac{(bx^4+a)^{\frac{9}{4}}}{9\,ax^9}$$

input
```
integrate((b*x^4+a)^(5/4)/x^10,x, algorithm="maxima")
```

output
```
-1/9*(b*x^4 + a)^(9/4)/(a*x^9)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{10}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^10,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx = -\frac{(b\,x^4+a)^{9/4}}{9\,a\,x^9}$$

input
```
int((a + b*x^4)^(5/4)/x^10,x)
```

output
```
-(a + b*x^4)^(9/4)/(9*a*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^4)^{5/4}}{x^{10}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,(-b^2x^8-2ab\,x^4-a^2)}{9a\,x^9}$$

input
```
int((b*x^4+a)^(5/4)/x^10,x)
```

output
```
((a + b*x**4)**(1/4)*( - a**2 - 2*a*b*x**4 - b**2*x**8))/(9*a*x**9)
```

3.523 $\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx = -\frac{(a+bx^4)^{9/4}}{13ax^{13}} + \frac{4b(a+bx^4)^{9/4}}{117a^2x^9}$$

output
```
-1/13*(b*x^4+a)^(9/4)/a/x^13+4/117*b*(b*x^4+a)^(9/4)/a^2/x^9
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx = \frac{(a+bx^4)^{9/4}(-9a+4bx^4)}{117a^2x^{13}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^14,x]
```

output
```
((a + b*x^4)^(9/4)*(-9*a + 4*b*x^4))/(117*a^2*x^13)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{4b\int \frac{(bx^4+a)^{5/4}}{x^{10}}dx}{13a}-\frac{(a+bx^4)^{9/4}}{13ax^{13}}$$

$$\Big\downarrow \text{796}$$

$$\frac{4b(a+bx^4)^{9/4}}{117a^2x^9}-\frac{(a+bx^4)^{9/4}}{13ax^{13}}$$

input
```
Int[(a + b*x^4)^(5/4)/x^14,x]
```

output
```
-1/13*(a + b*x^4)^(9/4)/(a*x^13) + (4*b*(a + b*x^4)^(9/4))/(117*a^2*x^9)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{9}{4}}(-4b\,x^4+9a)}{117x^{13}a^2}$	28
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{9}{4}}(-4b\,x^4+9a)}{117x^{13}a^2}$	28
orering	$-\frac{(b\,x^4+a)^{\frac{9}{4}}(-4b\,x^4+9a)}{117x^{13}a^2}$	28
trager	$-\frac{(-4b^3x^{12}+a\,b^2x^8+14a^2b\,x^4+9a^3)(b\,x^4+a)^{\frac{1}{4}}}{117x^{13}a^2}$	49
risch	$-\frac{(-4b^3x^{12}+a\,b^2x^8+14a^2b\,x^4+9a^3)(b\,x^4+a)^{\frac{1}{4}}}{117x^{13}a^2}$	49

input
```
int((b*x^4+a)^(5/4)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/117*(b*x^4+a)^(9/4)*(-4*b*x^4+9*a)/x^13/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx = \frac{(4\,b^3x^{12}-ab^2x^8-14\,a^2bx^4-9\,a^3)(bx^4+a)^{\frac{1}{4}}}{117\,a^2x^{13}}$$

input
```
integrate((b*x^4+a)^(5/4)/x^14,x, algorithm="fricas")
```

output
```
1/117*(4*b^3*x^12 - a*b^2*x^8 - 14*a^2*b*x^4 - 9*a^3)*(b*x^4 + a)^(1/4)/(a
^2*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(37) = 74$.

Time = 1.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.36

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx = -\frac{9a\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{13}{4}\right)}{16x^{12}\Gamma\left(-\frac{5}{4}\right)} - \frac{7b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{13}{4}\right)}{8x^8\Gamma\left(-\frac{5}{4}\right)} - \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{13}{4}\right)}{16ax^4\Gamma\left(-\frac{5}{4}\right)} + \frac{b^{\frac{13}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{13}{4}\right)}{4a^2\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**14,x)
```

output
```
-9*a*b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(16*x**12*gamma(-5/4))
- 7*b**(5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(8*x**8*gamma(-5/4)) - b
**(9/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(16*a*x**4*gamma(-5/4)) + b**
(13/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(4*a**2*gamma(-5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx = \frac{\frac{13\,(bx^4+a)^{\frac{9}{4}}b}{x^9} - \frac{9\,(bx^4+a)^{\frac{13}{4}}}{x^{13}}}{117\,a^2}$$

input
```
integrate((b*x^4+a)^(5/4)/x^14,x, algorithm="maxima")
```

output
```
1/117*(13*(b*x^4 + a)^(9/4)*b/x^9 - 9*(b*x^4 + a)^(13/4)/x^13)/a^2
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx=\int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{14}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^14,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx=\frac{4\,b^3\,(b\,x^4+a)^{1/4}}{117\,a^2\,x}-\frac{14\,b\,(b\,x^4+a)^{1/4}}{117\,x^9}-\frac{a\,(b\,x^4+a)^{1/4}}{13\,x^{13}}-\frac{b^2\,(b\,x^4+a)^{1/4}}{117\,a\,x^5}$$

input
```
int((a + b*x^4)^(5/4)/x^14,x)
```

output
```
(4*b^3*(a + b*x^4)^(1/4))/(117*a^2*x) - (14*b*(a + b*x^4)^(1/4))/(117*x^9)
 - (a*(a + b*x^4)^(1/4))/(13*x^13) - (b^2*(a + b*x^4)^(1/4))/(117*a*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^4)^{5/4}}{x^{14}}\,dx=\frac{(b\,x^4+a)^{\frac{1}{4}}\,(4b^3x^{12}-a\,b^2x^8-14a^2b\,x^4-9a^3)}{117a^2x^{13}}$$

input
```
int((b*x^4+a)^(5/4)/x^14,x)
```

output
```
((a + b*x**4)**(1/4)*( - 9*a**3 - 14*a**2*b*x**4 - a*b**2*x**8 + 4*b**3*x**
*12))/(117*a**2*x**13)
```

3.524 $\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = -\frac{(a+bx^4)^{9/4}}{17ax^{17}} + \frac{8b(a+bx^4)^{9/4}}{221a^2x^{13}} - \frac{32b^2(a+bx^4)^{9/4}}{1989a^3x^9}$$

output
```
-1/17*(b*x^4+a)^(9/4)/a/x^17+8/221*b*(b*x^4+a)^(9/4)/a^2/x^13-32/1989*b^2*
(b*x^4+a)^(9/4)/a^3/x^9
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = \frac{(a+bx^4)^{9/4}(-117a^2+72abx^4-32b^2x^8)}{1989a^3x^{17}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^18,x]
```

output
```
((a + b*x^4)^(9/4)*(-117*a^2 + 72*a*b*x^4 - 32*b^2*x^8))/(1989*a^3*x^17)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^4\right)^{5/4}}{x^{18}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{8b\int \frac{\left(bx^4+a\right)^{5/4}}{x^{14}}dx}{17a}-\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{8b\left(-\frac{4b\int \frac{\left(bx^4+a\right)^{5/4}}{x^{10}}dx}{13a}-\frac{\left(a+bx^4\right)^{9/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{8b\left(\frac{4b\left(a+bx^4\right)^{9/4}}{117a^2x^9}-\frac{\left(a+bx^4\right)^{9/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a+bx^4\right)^{9/4}}{17ax^{17}}$$

input
```
Int[(a + b*x^4)^(5/4)/x^18,x]
```

output
```
-1/17*(a + b*x^4)^(9/4)/(a*x^17) - (8*b*(-1/13*(a + b*x^4)^(9/4)/(a*x^13)
+ (4*b*(a + b*x^4)^(9/4))/(117*a^2*x^9)))/(17*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(32 b^{2} x^{8}-72 a b\,x^{4}+117 a^{2}\right)}{1989 x^{17} a^{3}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(32 b^{2} x^{8}-72 a b\,x^{4}+117 a^{2}\right)}{1989 x^{17} a^{3}}$	39
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(32 b^{2} x^{8}-72 a b\,x^{4}+117 a^{2}\right)}{1989 x^{17} a^{3}}$	39
trager	$-\frac{\left(32 x^{16} b^{4}-8 a\,b^{3} x^{12}+5 a^{2} b^{2} x^{8}+162 a^{3} b\,x^{4}+117 a^{4}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{1989 x^{17} a^{3}}$	61
risch	$-\frac{\left(32 x^{16} b^{4}-8 a\,b^{3} x^{12}+5 a^{2} b^{2} x^{8}+162 a^{3} b\,x^{4}+117 a^{4}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{1989 x^{17} a^{3}}$	61

input
```
int((b*x^4+a)^(5/4)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/1989*(b*x^4+a)^(9/4)*(32*b^2*x^8-72*a*b*x^4+117*a^2)/x^17/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = -\frac{(32\,b^4x^{16}-8\,ab^3x^{12}+5\,a^2b^2x^8+162\,a^3bx^4+117\,a^4)(bx^4+a)^{\frac{1}{4}}}{1989\,a^3x^{17}}$$

input
```
integrate((b*x^4+a)^(5/4)/x^18,x, algorithm="fricas")
```

output
```
-1/1989*(32*b^4*x^16 - 8*a*b^3*x^12 + 5*a^2*b^2*x^8 + 162*a^3*b*x^4 + 117*
a^4)*(b*x^4 + a)^(1/4)/(a^3*x^17)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(61) = 122$.

Time = 1.65 (sec) , antiderivative size = 609, normalized size of antiderivative = 8.96

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = \frac{117a^6b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$
$$+\frac{396a^5b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$
$$+\frac{446a^4b^{\frac{25}{4}}x^8\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$
$$+\frac{164a^3b^{\frac{29}{4}}x^{12}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$
$$+\frac{21a^2b^{\frac{33}{4}}x^{16}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$
$$+\frac{56ab^{\frac{37}{4}}x^{20}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$
$$+\frac{32b^{\frac{41}{4}}x^{24}\sqrt[4]{\frac{a}{bx^4}+1}\,\Gamma\left(-\frac{17}{4}\right)}{64a^5b^4x^{16}\Gamma\left(-\frac{5}{4}\right)+128a^4b^5x^{20}\Gamma\left(-\frac{5}{4}\right)+64a^3b^6x^{24}\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**18,x)
```

output
```
117*a**6*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**1
6*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma
(-5/4)) + 396*a**5*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64
*a**5*b**4*x**16*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b
**6*x**24*gamma(-5/4)) + 446*a**4*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*g
amma(-17/4)/(64*a**5*b**4*x**16*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5
/4) + 64*a**3*b**6*x**24*gamma(-5/4)) + 164*a**3*b**(29/4)*x**12*(a/(b*x**
4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16*gamma(-5/4) + 128*a**4*b**
5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-5/4)) + 21*a**2*b**(33/4)*
x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16*gamma(-5/4)
 + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(-5/4)) + 56*
a*b**(37/4)*x**20*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*b**4*x**16
*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x**24*gamma(
-5/4)) + 32*b**(41/4)*x**24*(a/(b*x**4) + 1)**(1/4)*gamma(-17/4)/(64*a**5*
b**4*x**16*gamma(-5/4) + 128*a**4*b**5*x**20*gamma(-5/4) + 64*a**3*b**6*x*
*24*gamma(-5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = -\frac{\frac{221\,(bx^4+a)^{\frac{9}{4}}b^2}{x^9} - \frac{306\,(bx^4+a)^{\frac{13}{4}}b}{x^{13}} + \frac{117\,(bx^4+a)^{\frac{17}{4}}}{x^{17}}}{1989\,a^3}$$

input
```
integrate((b*x^4+a)^(5/4)/x^18,x, algorithm="maxima")
```

output
```
-1/1989*(221*(b*x^4 + a)^(9/4)*b^2/x^9 - 306*(b*x^4 + a)^(13/4)*b/x^13 + 1
17*(b*x^4 + a)^(17/4)/x^17)/a^3
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{18}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^18,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^18, x)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = \frac{8\,b^3\,(b\,x^4+a)^{1/4}}{1989\,a^2\,x^5} - \frac{18\,b\,(b\,x^4+a)^{1/4}}{221\,x^{13}} - \frac{32\,b^4\,(b\,x^4+a)^{1/4}}{1989\,a^3\,x} - \frac{a\,(b\,x^4+a)^{1/4}}{17\,x^{17}} - \frac{5\,b^2\,(b\,x^4+a)^{1/4}}{1989\,a\,x^9}$$

input
```
int((a + b*x^4)^(5/4)/x^18,x)
```

output
```
(8*b^3*(a + b*x^4)^(1/4))/(1989*a^2*x^5) - (18*b*(a + b*x^4)^(1/4))/(221*x
^13) - (32*b^4*(a + b*x^4)^(1/4))/(1989*a^3*x) - (a*(a + b*x^4)^(1/4))/(17
*x^17) - (5*b^2*(a + b*x^4)^(1/4))/(1989*a*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^4)^{5/4}}{x^{18}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,(-32b^4x^{16}+8a\,b^3x^{12}-5a^2b^2x^8-162a^3b\,x^4-117a^4)}{1989a^3x^{17}}$$

input
```
int((b*x^4+a)^(5/4)/x^18,x)
```

output
```
((a + b*x**4)**(1/4)*( - 117*a**4 - 162*a**3*b*x**4 - 5*a**2*b**2*x**8 + 8
*a*b**3*x**12 - 32*b**4*x**16))/(1989*a**3*x**17)
```

3.525 $\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = -\frac{(a+bx^4)^{9/4}}{21ax^{21}}+\frac{4b(a+bx^4)^{9/4}}{119a^2x^{17}} - \frac{32b^2(a+bx^4)^{9/4}}{1547a^3x^{13}}+\frac{128b^3(a+bx^4)^{9/4}}{13923a^4x^9}$$

output
```
-1/21*(b*x^4+a)^(9/4)/a/x^21+4/119*b*(b*x^4+a)^(9/4)/a^2/x^17-32/1547*b^2*
(b*x^4+a)^(9/4)/a^3/x^13+128/13923*b^3*(b*x^4+a)^(9/4)/a^4/x^9
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \frac{(a+bx^4)^{9/4}(-663a^3+468a^2bx^4-288ab^2x^8+128b^3x^{12})}{13923a^4x^{21}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^22,x]
```

output
```
((a + b*x^4)^(9/4)*(-663*a^3 + 468*a^2*b*x^4 - 288*a*b^2*x^8 + 128*b^3*x^1
2))/(13923*a^4*x^21)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\int \frac{(bx^4+a)^{5/4}}{x^{18}}dx}{7a}-\frac{(a+bx^4)^{9/4}}{21ax^{21}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\left(-\frac{8b\int \frac{(bx^4+a)^{5/4}}{x^{14}}dx}{17a}-\frac{(a+bx^4)^{9/4}}{17ax^{17}}\right)}{7a}-\frac{(a+bx^4)^{9/4}}{21ax^{21}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\left(-\frac{8b\left(-\frac{4b\int \frac{(bx^4+a)^{5/4}}{x^{10}}dx}{13a}-\frac{(a+bx^4)^{9/4}}{13ax^{13}}\right)}{17a}-\frac{(a+bx^4)^{9/4}}{17ax^{17}}\right)}{7a}-\frac{(a+bx^4)^{9/4}}{21ax^{21}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{4b\left(-\frac{8b\left(\frac{4b(a+bx^4)^{9/4}}{117a^2x^9}-\frac{(a+bx^4)^{9/4}}{13ax^{13}}\right)}{17a}-\frac{(a+bx^4)^{9/4}}{17ax^{17}}\right)}{7a}-\frac{(a+bx^4)^{9/4}}{21ax^{21}}$$

input
```
Int[(a + b*x^4)^(5/4)/x^22,x]
```

output
```
-1/21*(a + b*x^4)^(9/4)/(a*x^21) - (4*b*(-1/17*(a + b*x^4)^(9/4)/(a*x^17)
- (8*b*(-1/13*(a + b*x^4)^(9/4)/(a*x^13) + (4*b*(a + b*x^4)^(9/4))/(117*a^
2*x^9)))/(17*a)))/(7*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(-128 b^{3} x^{12}+288 a\,b^{2} x^{8}-468 a^{2} b\,x^{4}+663 a^{3}\right)}{13923 x^{21} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(-128 b^{3} x^{12}+288 a\,b^{2} x^{8}-468 a^{2} b\,x^{4}+663 a^{3}\right)}{13923 x^{21} a^{4}}$	50
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(-128 b^{3} x^{12}+288 a\,b^{2} x^{8}-468 a^{2} b\,x^{4}+663 a^{3}\right)}{13923 x^{21} a^{4}}$	50
trager	$-\frac{\left(-128 b^{5} x^{20}+32 a\,b^{4} x^{16}-20 a^{2} b^{3} x^{12}+15 a^{3} b^{2} x^{8}+858 a^{4} b\,x^{4}+663 a^{5}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{13923 x^{21} a^{4}}$	72
risch	$-\frac{\left(-128 b^{5} x^{20}+32 a\,b^{4} x^{16}-20 a^{2} b^{3} x^{12}+15 a^{3} b^{2} x^{8}+858 a^{4} b\,x^{4}+663 a^{5}\right) \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{13923 x^{21} a^{4}}$	72

input
```
int((b*x^4+a)^(5/4)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/13923*(b*x^4+a)^(9/4)*(-128*b^3*x^12+288*a*b^2*x^8-468*a^2*b*x^4+663*a^
3)/x^21/a^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \frac{(128\,b^5x^{20}-32\,ab^4x^{16}+20\,a^2b^3x^{12}-15\,a^3b^2x^8-858\,a^4bx^4-663\,a^5)(bx^4+a)^{\frac{1}{4}}}{13923\,a^4x^{21}}$$

input
```
integrate((b*x^4+a)^(5/4)/x^22,x, algorithm="fricas")
```

output
```
1/13923*(128*b^5*x^20 - 32*a*b^4*x^16 + 20*a^2*b^3*x^12 - 15*a^3*b^2*x^8 -
 858*a^4*b*x^4 - 663*a^5)*(b*x^4 + a)^(1/4)/(a^4*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(85) = 170.

Time = 2.49 (sec) , antiderivative size = 954, normalized size of antiderivative = 10.37

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**4+a)**(5/4)/x**22,x)
```

output
```
-1989*a**8*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x
**20*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28
*gamma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) - 8541*a**7*b**(41/4)*x**
4*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) +
768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*
a**4*b**12*x**32*gamma(-5/4)) - 13734*a**6*b**(45/4)*x**8*(a/(b*x**4) + 1)
**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10*x**
24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x**32*g
amma(-5/4)) - 9786*a**5*b**(49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-21/
4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 7
68*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) - 2625
*a**4*b**(53/4)*x**16*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*
x**20*gamma(-5/4) + 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**2
8*gamma(-5/4) + 256*a**4*b**12*x**32*gamma(-5/4)) + 231*a**3*b**(57/4)*x**
20*(a/(b*x**4) + 1)**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) +
 768*a**6*b**10*x**24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256
*a**4*b**12*x**32*gamma(-5/4)) + 924*a**2*b**(61/4)*x**24*(a/(b*x**4) + 1)
**(1/4)*gamma(-21/4)/(256*a**7*b**9*x**20*gamma(-5/4) + 768*a**6*b**10*x**
24*gamma(-5/4) + 768*a**5*b**11*x**28*gamma(-5/4) + 256*a**4*b**12*x**32*g
amma(-5/4)) + 1056*a*b**(65/4)*x**28*(a/(b*x**4) + 1)**(1/4)*gamma(-21/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \frac{\frac{1547\,(bx^4+a)^{\frac{9}{4}}b^3}{x^9} - \frac{3213\,(bx^4+a)^{\frac{13}{4}}b^2}{x^{13}} + \frac{2457\,(bx^4+a)^{\frac{17}{4}}b}{x^{17}} - \frac{663\,(bx^4+a)^{\frac{21}{4}}}{x^{21}}}{13923\,a^4}$$

input
```
integrate((b*x^4+a)^(5/4)/x^22,x, algorithm="maxima")
```

output
```
1/13923*(1547*(b*x^4 + a)^(9/4)*b^3/x^9 - 3213*(b*x^4 + a)^(13/4)*b^2/x^13
 + 2457*(b*x^4 + a)^(17/4)*b/x^17 - 663*(b*x^4 + a)^(21/4)/x^21)/a^4
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{22}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^22,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^22, x)
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \frac{128\,b^5\,(b\,x^4+a)^{1/4}}{13923\,a^4\,x} - \frac{22\,b\,(b\,x^4+a)^{1/4}}{357\,x^{17}} - \frac{a\,(b\,x^4+a)^{1/4}}{21\,x^{21}} - \frac{32\,b^4\,(b\,x^4+a)^{1/4}}{13923\,a^3\,x^5} + \frac{20\,b^3\,(b\,x^4+a)^{1/4}}{13923\,a^2\,x^9} - \frac{5\,b^2\,(b\,x^4+a)^{1/4}}{4641\,a\,x^{13}}$$

input
```
int((a + b*x^4)^(5/4)/x^22,x)
```

output
```
(128*b^5*(a + b*x^4)^(1/4))/(13923*a^4*x) - (22*b*(a + b*x^4)^(1/4))/(357*
x^17) - (a*(a + b*x^4)^(1/4))/(21*x^21) - (32*b^4*(a + b*x^4)^(1/4))/(1392
3*a^3*x^5) + (20*b^3*(a + b*x^4)^(1/4))/(13923*a^2*x^9) - (5*b^2*(a + b*x^
4)^(1/4))/(4641*a*x^13)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^4)^{5/4}}{x^{22}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}\,(128b^5x^{20}-32a\,b^4x^{16}+20a^2b^3x^{12}-15a^3b^2x^8-858a^4b\,x^4-663a^5)}{13923a^4x^{21}}$$

input
```
int((b*x^4+a)^(5/4)/x^22,x)
```

output
```
((a + b*x**4)**(1/4)*( - 663*a**5 - 858*a**4*b*x**4 - 15*a**3*b**2*x**8 +
20*a**2*b**3*x**12 - 32*a*b**4*x**16 + 128*b**5*x**20))/(13923*a**4*x**21)
```

3.526 $\int x^{12}(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 171

$$\int x^{12}\left(a+bx^4\right)^{5/4}\,dx=\frac{5a^4x\sqrt[4]{a+bx^4}}{672b^3}-\frac{a^3x^5\sqrt[4]{a+bx^4}}{336b^2}+\frac{a^2x^9\sqrt[4]{a+bx^4}}{504b}+\frac{5}{252}ax^{13}\sqrt[4]{a+bx^4}+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}+\frac{5a^{9/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{672b^{5/2}\left(a+bx^4\right)^{3/4}}$$

output

```
5/672*a^4*x*(b*x^4+a)^(1/4)/b^3-1/336*a^3*x^5*(b*x^4+a)^(1/4)/b^2+1/504*a^
2*x^9*(b*x^4+a)^(1/4)/b+5/252*a*x^13*(b*x^4+a)^(1/4)+1/18*x^13*(b*x^4+a)^(
5/4)+5/672*a^(9/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2
)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.52

$$\int x^{12}\left(a+bx^4\right)^{5/4}\,dx=\frac{x\sqrt[4]{a+bx^4}\left(\left(a+bx^4\right)^2\left(9a^2-18abx^4+28b^2x^8\right)-\frac{9a^4\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{504b^3}$$

input
```
Integrate[x^12*(a + b*x^4)^(5/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*((a + b*x^4)^2*(9*a^2 - 18*a*b*x^4 + 28*b^2*x^8) - (9
*a^4*Hypergeometric2F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4
)))/(504*b^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {811, 811, 843, 843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\left(a+bx^4\right)^{5/4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{18}a\int x^{12}\sqrt[4]{bx^4+a}dx+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{18}a\left(\frac{1}{14}a\int \frac{x^{12}}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}\right)+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \ 843$$

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\int\frac{x^8}{(bx^4+a)^{3/4}}dx}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}\right)+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \ 843$$

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int\frac{x^4}{(bx^4+a)^{3/4}}dx}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}\right)+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \ 843$$

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int\frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a+bx^4}\right)+\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \ 768$$

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a\right)$$

$$\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$\downarrow$ 858

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a\right)$$

$$\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$\downarrow$ 807

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a\right)$$

$$\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

$\downarrow$ 229

$$\frac{5}{18}a\left(\frac{1}{14}a\left(\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}\right)+\frac{1}{1}\right)$$

$$\frac{1}{18}x^{13}\left(a+bx^4\right)^{5/4}$$

input
```
Int[x^12*(a + b*x^4)^(5/4),x]
```

output
```
(x^13*(a + b*x^4)^(5/4))/18 + (5*a*((x^13*(a + b*x^4)^(1/4))/14 + (a*((x^9
*(a + b*x^4)^(1/4))/(10*b) - (9*a*((x^5*(a + b*x^4)^(1/4))/(6*b) - (5*a*((
x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[
ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(6*b)
))/(10*b)))/14))/18
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{12}\left(b\,x^{4}+a\right)^{\frac{5}{4}}dx$$

input
```
int(x^12*(b*x^4+a)^(5/4),x)
```

output
```
int(x^12*(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int x^{12}\left(a+bx^4\right)^{5/4}\,dx=\int\left(bx^4+a\right)^{\frac{5}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^16 + a*x^12)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.23

$$\int x^{12}\left(a+bx^4\right)^{5/4}\,dx=\frac{a^{\frac{5}{4}}x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**13*gamma(13/4)*hyper((-5/4, 13/4), (17/4,), b*x**4*exp_polar(I
*pi)/a)/(4*gamma(17/4))
```

Maxima [F]

$$\int x^{12}(a+bx^4)^{5/4}\,dx=\int (bx^4+a)^{\frac{5}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^12, x)
```

Giac [F]

$$\int x^{12}(a+bx^4)^{5/4}\,dx=\int (bx^4+a)^{\frac{5}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{12}(a+bx^4)^{5/4}\,dx=\int x^{12}\,\left(b\,x^4+a\right)^{5/4}\,dx$$

input
```
int(x^12*(a + b*x^4)^(5/4),x)
```

output
```
int(x^12*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^{12}(a+bx^4)^{5/4}\,dx = \frac{15(b\,x^4+a)^{\frac{1}{4}}\,a^4x-6(b\,x^4+a)^{\frac{1}{4}}\,a^3b\,x^5+4(b\,x^4+a)^{\frac{1}{4}}\,a^2b^2x^9+152(b\,x^4+a)^{\frac{1}{4}}\,a\,b^3x^{13}+112}{2016b^3}$$

input
```
int(x^12*(b*x^4+a)^(5/4),x)
```

output
```
(15*(a + b*x**4)**(1/4)*a**4*x - 6*(a + b*x**4)**(1/4)*a**3*b*x**5 + 4*(a
+ b*x**4)**(1/4)*a**2*b**2*x**9 + 152*(a + b*x**4)**(1/4)*a*b**3*x**13 + 1
12*(a + b*x**4)**(1/4)*b**4*x**17 - 15*int((a + b*x**4)**(1/4)/(a + b*x**4
),x)*a**5)/(2016*b**3)
```

3.527 $\int x^8(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int x^8\left(a+bx^4\right)^{5/4}\,dx=-\frac{5a^3x\sqrt[4]{a+bx^4}}{336b^2}+\frac{a^2x^5\sqrt[4]{a+bx^4}}{168b}+\frac{1}{28}ax^9\sqrt[4]{a+bx^4}+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}-\frac{5a^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{336b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-5/336*a^3*x*(b*x^4+a)^(1/4)/b^2+1/168*a^2*x^5*(b*x^4+a)^(1/4)/b+1/28*a*x^
9*(b*x^4+a)^(1/4)+1/14*x^9*(b*x^4+a)^(5/4)-5/336*a^(7/2)*(1+a/b/x^4)^(3/4)
*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x
^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.94 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int x^8\left(a+bx^4\right)^{5/4}\,dx=\frac{x\sqrt[4]{a+bx^4}\left(-\left(\left(a-2bx^4\right)\left(a+bx^4\right)^2\right)+\frac{a^3\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{28b^2}$$

input
```
Integrate[x^8*(a + b*x^4)^(5/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*(-((a - 2*b*x^4)*(a + b*x^4)^2) + (a^3*Hypergeometric
2F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(28*b^2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {811, 811, 843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\left(a+bx^4\right)^{5/4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{14}a\int x^8\sqrt[4]{bx^4+a}dx+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{811}$$

$$\frac{5}{14}a\left(\frac{1}{10}a\int \frac{x^8}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{14}a\left(\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int\frac{x^4}{(bx^4+a)^{3/4}}dx}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{843}$$

$$\frac{5}{14}a\left(\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int\frac{1}{\left(bx^4+a\right)^{3/4}}dx}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{768}$$

$$\frac{5}{14}a\left(\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{858}$$

$$\frac{5}{14}a\left(\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{807}$$

$$\frac{5}{14}a\left(\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

$\downarrow$ 229

$$\frac{5}{14}a\left(\frac{1}{10}a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a+bx^4}\right)+\frac{1}{14}x^9\left(a+bx^4\right)^{5/4}$$

input
```
Int[x^8*(a + b*x^4)^(5/4),x]
```

output
```
(x^9*(a + b*x^4)^(5/4))/14 + (5*a*((x^9*(a + b*x^4)^(1/4))/10 + (a*((x^5*(
a + b*x^4)^(1/4))/(6*b) - (5*a*((x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1
+ a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*S
qrt[b]*(a + b*x^4)^(3/4))))/(6*b)))/10))/14
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^8 (b x^4 + a)^{\frac{5}{4}} dx$$

input
```
int(x^8*(b*x^4+a)^(5/4),x)
```

output
```
int(x^8*(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int x^8 \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^8 \, dx$$

input
```
integrate(x^8*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^12 + a*x^8)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.27

$$\int x^8 \left(a+bx^4\right)^{5/4} dx = \frac{a^{\frac{5}{4}} x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**9*gamma(9/4)*hyper((-5/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi
)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int x^8 \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^8 \, dx$$

input
```
integrate(x^8*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^8, x)
```

Giac [F]

$$\int x^8 (a+bx^4)^{5/4}\,dx = \int (bx^4+a)^{\frac{5}{4}} x^8\,dx$$

input
```
integrate(x^8*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 (a+bx^4)^{5/4}\,dx = \int x^8\,(b\,x^4+a)^{5/4}\,dx$$

input
```
int(x^8*(a + b*x^4)^(5/4),x)
```

output
```
int(x^8*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^8 (a+bx^4)^{5/4}\,dx = \frac{-5(b\,x^4+a)^{\frac{1}{4}}\,a^3x+2(b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^5+36(b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^9+24(b\,x^4+a)^{\frac{1}{4}}\,b^3x^{13}+5\Big(\int}{336b^2}$$

input
```
int(x^8*(b*x^4+a)^(5/4),x)
```

output
```
( - 5*(a + b*x**4)**(1/4)*a**3*x + 2*(a + b*x**4)**(1/4)*a**2*b*x**5 + 36*
(a + b*x**4)**(1/4)*a*b**2*x**9 + 24*(a + b*x**4)**(1/4)*b**3*x**13 + 5*in
t((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**4)/(336*b**2)
```

3.528 $\int x^4(a+bx^4)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int x^4(a+bx^4)^{5/4}\,dx=\frac{a^2x\sqrt[4]{a+bx^4}}{24b}+\frac{1}{12}ax^5\sqrt[4]{a+bx^4}+\frac{1}{10}x^5(a+bx^4)^{5/4}+\frac{a^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24\sqrt{b}(a+bx^4)^{3/4}}$$

output
```
1/24*a^2*x*(b*x^4+a)^(1/4)/b+1/12*a*x^5*(b*x^4+a)^(1/4)+1/10*x^5*(b*x^4+a)
^(5/4)+1/24*a^(5/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/
2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.54 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.54

$$\int x^4(a+bx^4)^{5/4}\,dx=\frac{x\sqrt[4]{a+bx^4}\left((a+bx^4)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}\right)}{10b}$$

input
```
Integrate[x^4*(a + b*x^4)^(5/4),x]
```

output
```
(x*(a + b*x^4)^(1/4)*((a + b*x^4)^2 - (a^2*Hypergeometric2F1[-5/4, 1/4, 5/
4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^(1/4)))/(10*b)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {811, 811, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^4)^{5/4} \, dx$$

$$\Big\downarrow \text{ 811}$$

$$\frac{1}{2}a\int x^4\sqrt[4]{bx^4+a}dx+\frac{1}{10}x^5(a+bx^4)^{5/4}$$

$$\Big\downarrow \text{ 811}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\int \frac{x^4}{(bx^4+a)^{3/4}}dx+\frac{1}{6}x^5\sqrt[4]{a+bx^4}\right)+\frac{1}{10}x^5(a+bx^4)^{5/4}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}\right)+\frac{1}{10}x^5(a+bx^4)^{5/4}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}\right)+\frac{1}{10}x^5(a+bx^4)^{5/4}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}\right)+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}$$

$\Big\downarrow$ 807

$$\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}\right)+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}$$

$\Big\downarrow$ 229

$$\frac{1}{2}a\left(\frac{1}{6}a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a+bx^4}\right)+\frac{1}{10}x^5\left(a+bx^4\right)^{5/4}$$

input
```
Int[x^4*(a + b*x^4)^(5/4),x]
```

output
```
(x^5*(a + b*x^4)^(5/4))/10 + (a*((x^5*(a + b*x^4)^(1/4))/6 + (a*((x*(a + b
*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[S
qrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/6))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{4} \left(b \,x^{4}+a \right)^{\frac{5}{4}}d x$$

input
```
int(x^4*(b*x^4+a)^(5/4),x)
```

output
```
int(x^4*(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a*x^4)*(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.32

$$\int x^4 \left(a+bx^4\right)^{5/4} dx = \frac{a^{\frac{5}{4}} x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x**5*gamma(5/4)*hyper((-5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)
/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4 \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a+bx^4\right)^{5/4} \, dx = \int \left(bx^4+a\right)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^4\right)^{5/4} \, dx = \int x^4 \left(b\,x^4+a\right)^{5/4} \, dx$$

input
```
int(x^4*(a + b*x^4)^(5/4),x)
```

output
```
int(x^4*(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int x^4 \left(a+bx^4\right)^{5/4} \, dx = \frac{5\left(b\,x^4+a\right)^{\frac{1}{4}} a^2 x + 22\left(b\,x^4+a\right)^{\frac{1}{4}} a b\,x^5 + 12\left(b\,x^4+a\right)^{\frac{1}{4}} b^2 x^9 - 5\left(\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}} dx\right) a^3}{120b}$$

input
```
int(x^4*(b*x^4+a)^(5/4),x)
```

output
```
(5*(a + b*x**4)**(1/4)*a**2*x + 22*(a + b*x**4)**(1/4)*a*b*x**5 + 12*(a +
b*x**4)**(1/4)*b**2*x**9 - 5*int((a + b*x**4)**(1/4)/(a + b*x**4),x)*a**3)
/(120*b)
```

3.529 $\int (a+bx^4)^{5/4} dx$

Optimal result

Integrand size = 11, antiderivative size = 97

$$\int \left(a+bx^4\right)^{5/4} dx = \frac{5}{12}ax\sqrt[4]{a+bx^4} + \frac{1}{6}x\left(a+bx^4\right)^{5/4} - \frac{5a^{3/2}\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\left(a+bx^4\right)^{3/4}}$$

output

```
5/12*a*x*(b*x^4+a)^(1/4)+1/6*x*(b*x^4+a)^(5/4)-5/12*a^(3/2)*b^(1/2)*(1+a/b
/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(
b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

$$\int \left(a+bx^4\right)^{5/4} dx = \frac{ax\sqrt[4]{a+bx^4} \operatorname{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4),x]
```

output
```
(a*x*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, 1/4, 5/4, -((b*x^4)/a)])/(1
 + (b*x^4)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {748, 748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^4\right)^{5/4} dx$$

$$\Big\downarrow \text{748}$$

$$\frac{5}{6}a\int \sqrt[4]{bx^4+a}dx+\frac{1}{6}x\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{748}$$

$$\frac{5}{6}a\left(\frac{1}{2}a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{768}$$

$$\frac{5}{6}a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2\left(a+bx^4\right)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{858}$$

$$\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2\left(a+bx^4\right)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{807}$$

$$\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4\left(a+bx^4\right)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{6}a\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\left(a+bx^4\right)^{3/4}}\right)+\frac{1}{6}x\left(a+bx^4\right)^{5/4}$$

input
```
Int[(a + b*x^4)^(5/4),x]
```

output
```
(x*(a + b*x^4)^(5/4))/6 + (5*a*((x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]
*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/
(2*(a + b*x^4)^(3/4))))/6
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \left(b\,x^{4}+a\right)^{\frac{5}{4}}d x$$

input
```
int((b*x^4+a)^(5/4),x)
```

output
```
int((b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \left(a+bx^4\right)^{5/4}\,dx=\int \left(bx^{4}+a\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

$$\int \left(a+bx^4\right)^{5/4}\,dx=\frac{a^{\frac{5}{4}}x\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4),x)
```

output
```
a**(5/4)*x*gamma(1/4)*hyper((-5/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4))
```

Maxima [F]

$$\int \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} dx$$

input
```
integrate((b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \left(a+bx^4\right)^{5/4} dx = \int \left(bx^4+a\right)^{\frac{5}{4}} dx$$

input
```
integrate((b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

$$\int \left(a+bx^4\right)^{5/4} dx = \frac{x\,\left(b\,x^4+a\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(\frac{b\,x^4}{a}+1\right)^{5/4}}$$

input
```
int((a + b*x^4)^(5/4),x)
```

output
```
(x*(a + b*x^4)^(5/4)*hypergeom([-5/4, 1/4], 5/4, -(b*x^4)/a))/((b*x^4)/a +
 1)^(5/4)
```

Reduce [F]

$$\int \left(a+bx^4\right)^{5/4} dx = \frac{7(b\,x^4+a)^{\frac{1}{4}}\,ax}{12}+\frac{(b\,x^4+a)^{\frac{1}{4}}\,b\,x^5}{6}+\frac{5\left(\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{12}$$

input
```
int((b*x^4+a)^(5/4),x)
```

output
```
(7*(a + b*x**4)**(1/4)*a*x + 2*(a + b*x**4)**(1/4)*b*x**5 + 5*int((a + b*x
**4)**(1/4)/(a + b*x**4),x)*a**2)/12
```

3.530 $\int \frac{(a+bx^4)^{5/4}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{(a+bx^4)^{5/4}}{x^4} dx = \frac{5}{6}bx\sqrt[4]{a+bx^4} - \frac{(a+bx^4)^{5/4}}{3x^3} - \frac{5\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{6(a+bx^4)^{3/4}}$$

output
```
5/6*b*x*(b*x^4+a)^(1/4)-1/3*(b*x^4+a)^(5/4)/x^3-5/6*a^(1/2)*b^(3/2)*(1+a/b
/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(
b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx^4)^{5/4}}{x^4} dx = -\frac{a\sqrt[4]{a+bx^4} \operatorname{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)}{3x^3\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^4,x]
```

output
```
-1/3*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-5/4, -3/4, 1/4, -((b*x^4)/a)]
)/(x^3*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {809, 748, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^4}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{5}{3}b\int \sqrt[4]{bx^4+a}dx-\frac{(a+bx^4)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{748}$$

$$\frac{5}{3}b\left(\frac{1}{2}a\int \frac{1}{(bx^4+a)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{768}$$

$$\frac{5}{3}b\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2(a+bx^4)^{3/4}}+\frac{1}{2}x\sqrt[4]{a+bx^4}\right)-\frac{(a+bx^4)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{5}{3}b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2(a+bx^4)^{3/4}}\right)-\frac{(a+bx^4)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{807}$$

$$\frac{5}{3}b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4\left(a+bx^4\right)^{3/4}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{3}b\left(\frac{1}{2}x\sqrt[4]{a+bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\left(a+bx^4\right)^{3/4}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{3x^3}$$

input
```
Int[(a + b*x^4)^(5/4)/x^4,x]
```

output
```
-1/3*(a + b*x^4)^(5/4)/x^3 + (5*b*((x*(a + b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt
[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2
])/(2*(a + b*x^4)^(3/4))))/3
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^{\frac{5}{4}}}{x^4}dx$$

input
```
int((b*x^4+a)^(5/4)/x^4,x)
```

output
```
int((b*x^4+a)^(5/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^4}\,dx=\int \frac{(bx^4+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^4)^{5/4}}{x^4}\,dx = \frac{a^{\frac{5}{4}}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^3\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**4,x)
```

output
```
a**(5/4)*gamma(-3/4)*hyper((-5/4, -3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)
/(4*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^4}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^4}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^4}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^4,x)
```

output
```
int((a + b*x^4)^(5/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^4}\,dx = \frac{-3(b\,x^4+a)^{\frac{1}{4}}\,a+2(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4-5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^8+a\,x^4}dx\right)a^2x^3}{4x^3}$$

input
```
int((b*x^4+a)^(5/4)/x^4,x)
```

output
```
( - 3*(a + b*x**4)**(1/4)*a + 2*(a + b*x**4)**(1/4)*b*x**4 - 5*int((a + b*
x**4)**(1/4)/(a*x**4 + b*x**8),x)*a**2*x**3)/(4*x**3)
```

3.531 $\int \frac{(a+bx^4)^{5/4}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx = -\frac{5b\sqrt[4]{a+bx^4}}{21x^3}-\frac{(a+bx^4)^{5/4}}{7x^7} - \frac{5b^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21\sqrt{a}\,(a+bx^4)^{3/4}}$$

output
```
-5/21*b*(b*x^4+a)^(1/4)/x^3-1/7*(b*x^4+a)^(5/4)/x^7-5/21*b^(5/2)*(1+a/b/x^
4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1
/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx = -\frac{a\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{5}{4},-\frac{3}{4},-\frac{bx^4}{a}\right)}{7x^7\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^8,x]
```

output
```
-1/7*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-7/4, -5/4, -3/4, -((b*x^4)/a)
])/(x^7*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {809, 809, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^8}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{5}{7}b\int \frac{\sqrt[4]{bx^4+a}}{x^4}dx-\frac{(a+bx^4)^{5/4}}{7x^7}$$

$$\Big\downarrow \text{809}$$

$$\frac{5}{7}b\left(\frac{1}{3}b\int \frac{1}{(bx^4+a)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{3x^3}\right)-\frac{(a+bx^4)^{5/4}}{7x^7}$$

$$\Big\downarrow \text{768}$$

$$\frac{5}{7}b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}\right)-\frac{(a+bx^4)^{5/4}}{7x^7}$$

$$\Big\downarrow \text{858}$$

$$\frac{5}{7}b\left(-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}\right)-\frac{(a+bx^4)^{5/4}}{7x^7}$$

$$\Big\downarrow \text{807}$$

$$\frac{5}{7}b\left(-\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{6\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}\right)-\frac{\left(a+bx^4\right)^{5/4}}{7x^7}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{5}{7}b\left(-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3x^3}\right)-\frac{\left(a+bx^4\right)^{5/4}}{7x^7}$$

input
```
Int[(a + b*x^4)^(5/4)/x^8,x]
```

output
```
-1/7*(a + b*x^4)^(5/4)/x^7 + (5*b*(-1/3*(a + b*x^4)^(1/4)/x^3 - (b^(3/2)*(
1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3
*Sqrt[a]*(a + b*x^4)^(3/4))))/7
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{5}{4}}}{x^{8}}dx$$

input
```
int((b*x^4+a)^(5/4)/x^8,x)
```

output
```
int((b*x^4+a)^(5/4)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^8}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx = -\frac{b^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{2x^2}$$

input
```
integrate((b*x**4+a)**(5/4)/x**8,x)
```

output
```
-b**(5/4)*hyper((-5/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**4))/(2*x**2)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^8}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^8}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx=\int \frac{(b\,x^4+a)^{5/4}}{x^8}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^8,x)
```

output
```
int((a + b*x^4)^(5/4)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^8}\,dx=\frac{-(b\,x^4+a)^{\frac{1}{4}}\,a-6(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{12}+a\,x^8}dx\right)a^2x^7}{12x^7}$$

input
```
int((b*x^4+a)^(5/4)/x^8,x)
```

output
```
( - (a + b*x**4)**(1/4)*a - 6*(a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b*x*
*4)**(1/4)/(a*x**8 + b*x**12),x)*a**2*x**7)/(12*x**7)
```

3.532 $\int \frac{(a+bx^4)^{5/4}}{x^{12}} dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}} dx = -\frac{5b\sqrt[4]{a+bx^4}}{77x^7} - \frac{5b^2\sqrt[4]{a+bx^4}}{231ax^3} - \frac{(a+bx^4)^{5/4}}{11x^{11}} + \frac{10b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231a^{3/2}(a+bx^4)^{3/4}}$$

output
```
-5/77*b*(b*x^4+a)^(1/4)/x^7-5/231*b^2*(b*x^4+a)^(1/4)/a/x^3-1/11*(b*x^4+a)
^(5/4)/x^11+10/231*b^(7/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arcco
t(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}} dx = -\frac{a\sqrt[4]{a+bx^4} \operatorname{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{5}{4},-\frac{7}{4},-\frac{bx^4}{a}\right)}{11x^{11}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^12,x]
```

output
```
-1/11*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-11/4, -5/4, -7/4, -((b*x^4)/
a)])/(x^11*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {809, 809, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}dx$$

$$\Big\downarrow \text{809}$$

$$\frac{5}{11}b\int \frac{\sqrt[4]{bx^4+a}}{x^8}dx-\frac{(a+bx^4)^{5/4}}{11x^{11}}$$

$$\Big\downarrow \text{809}$$

$$\frac{5}{11}b\left(\frac{1}{7}b\int \frac{1}{x^4(bx^4+a)^{3/4}}dx-\frac{\sqrt[4]{a+bx^4}}{7x^7}\right)-\frac{(a+bx^4)^{5/4}}{11x^{11}}$$

$$\Big\downarrow \text{847}$$

$$\frac{5}{11}b\left(\frac{1}{7}b\left(-\frac{2b\int \frac{1}{(bx^4+a)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}\right)-\frac{(a+bx^4)^{5/4}}{11x^{11}}$$

$$\Big\downarrow \text{768}$$

$$\frac{5}{11}b\left(\frac{1}{7}b\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}\right)-\frac{(a+bx^4)^{5/4}}{11x^{11}}$$

$$\Big\downarrow \text{858}$$

$$\frac{5}{11}b\left(\frac{1}{7}b\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}\right)-\frac{\left(a+bx^4\right)^{5/4}}{11x^{11}}$$

$$\Big\downarrow \text{807}$$

$$\frac{5}{11}b\left(\frac{1}{7}b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}\right)-\frac{\left(a+bx^4\right)^{5/4}}{11x^{11}}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{11}b\left(\frac{1}{7}b\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^4}}{7x^7}\right)-\frac{\left(a+bx^4\right)^{5/4}}{11x^{11}}$$

input
```
Int[(a + b*x^4)^(5/4)/x^12,x]
```

output
```
-1/11*(a + b*x^4)^(5/4)/x^11 + (5*b*(-1/7*(a + b*x^4)^(1/4)/x^7 + (b*(-1/3
*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*Elliptic
F[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/7))
/11
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{5}{4}}}{x^{12}}dx$$

input
```
int((b*x^4+a)^(5/4)/x^12,x)
```

output
```
int((b*x^4+a)^(5/4)/x^12,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^12,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^12, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}\,dx = -\frac{b^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{6x^6}$$

input
```
integrate((b*x**4+a)**(5/4)/x**12,x)
```

output
```
-b**(5/4)*hyper((-5/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*x**6)
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^12,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^12, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{12}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^12,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^{12}}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^12,x)
```

output
```
int((a + b*x^4)^(5/4)/x^12, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{12}}\,dx = \frac{-(b\,x^4+a)^{\frac{1}{4}}\,a-2(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{16}+a\,x^{12}}dx\right)a^2x^{11}}{12x^{11}}$$

input
```
int((b*x^4+a)^(5/4)/x^12,x)
```

output
```
( - (a + b*x**4)**(1/4)*a - 2*(a + b*x**4)**(1/4)*b*x**4 + int((a + b*x**4
)**(1/4)/(a*x**12 + b*x**16),x)*a**2*x**11)/(12*x**11)
```

3.533 $\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = -\frac{b\sqrt[4]{a+bx^4}}{33x^{11}} - \frac{b^2\sqrt[4]{a+bx^4}}{231ax^7} + \frac{2b^3\sqrt[4]{a+bx^4}}{231a^2x^3} - \frac{(a+bx^4)^{5/4}}{15x^{15}} - \frac{4b^{9/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231a^{5/2}(a+bx^4)^{3/4}}$$

output
```
-1/33*b*(b*x^4+a)^(1/4)/x^11-1/231*b^2*(b*x^4+a)^(1/4)/a/x^7+2/231*b^3*(b*
x^4+a)^(1/4)/a^2/x^3-1/15*(b*x^4+a)^(5/4)/x^15-4/231*b^(9/2)*(1+a/b/x^4)^(
3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/
(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = -\frac{a\sqrt[4]{a+bx^4}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{5}{4},-\frac{11}{4},-\frac{bx^4}{a}\right)}{15x^{15}\sqrt[4]{1+\frac{bx^4}{a}}}$$

input
```
Integrate[(a + b*x^4)^(5/4)/x^16,x]
```

output
```
-1/15*(a*(a + b*x^4)^(1/4)*Hypergeometric2F1[-15/4, -5/4, -11/4, -((b*x^4)
/a)])/(x^15*(1 + (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {809, 809, 847, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}} \, dx$$

$$\Big\downarrow \text{ 809}$$

$$\frac{1}{3}b \int \frac{\sqrt[4]{bx^4+a}}{x^{12}}dx - \frac{(a+bx^4)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{ 809}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\int \frac{1}{x^8(bx^4+a)^{3/4}}dx - \frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right) - \frac{(a+bx^4)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\int \frac{1}{x^4(bx^4+a)^{3/4}}dx}{7a} - \frac{\sqrt[4]{a+bx^4}}{7ax^7}\right) - \frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right) - \frac{(a+bx^4)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{2b\int\frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{3}b\left(\frac{1}{11}b\left(-\frac{6b\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a+bx^4}}{11x^{11}}\right)-\frac{\left(a+bx^4\right)^{5/4}}{15x^{15}}$$

input
```
Int[(a + b*x^4)^(5/4)/x^16,x]
```

output
```
-1/15*(a + b*x^4)^(5/4)/x^15 + (b*(-1/11*(a + b*x^4)^(1/4)/x^11 + (b*(-1/7
*(a + b*x^4)^(1/4)/(a*x^7) - (6*b*(-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(
3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2,
2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/(7*a)))/11))/3
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{4}+a)^{\frac{5}{4}}}{x^{16}}dx$$

input
```
int((b*x^4+a)^(5/4)/x^16,x)
```

output
```
int((b*x^4+a)^(5/4)/x^16,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^16,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(5/4)/x^16, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = \frac{a^{\frac{5}{4}}\Gamma\left(-\frac{15}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{15}{4}, -\frac{5}{4} \\ -\frac{11}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4x^{15}\Gamma\left(-\frac{11}{4}\right)}$$

input
```
integrate((b*x**4+a)**(5/4)/x**16,x)
```

output
```
a**(5/4)*gamma(-15/4)*hyper((-15/4, -5/4), (-11/4,), b*x**4*exp_polar(I*pi
)/a)/(4*x**15*gamma(-11/4))
```

Maxima [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^16,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^16, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = \int \frac{(bx^4+a)^{\frac{5}{4}}}{x^{16}}\,dx$$

input
```
integrate((b*x^4+a)^(5/4)/x^16,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(5/4)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = \int \frac{(b\,x^4+a)^{5/4}}{x^{16}}\,dx$$

input
```
int((a + b*x^4)^(5/4)/x^16,x)
```

output
```
int((a + b*x^4)^(5/4)/x^16, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^{5/4}}{x^{16}}\,dx = \frac{-9(b\,x^4+a)^{\frac{1}{4}}\,a-14(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4+5\left(\int \frac{(b\,x^4+a)^{\frac{1}{4}}}{b\,x^{20}+a\,x^{16}}dx\right)a^2x^{15}}{140x^{15}}$$

input
```
int((b*x^4+a)^(5/4)/x^16,x)
```

output
```
( - 9*(a + b*x**4)**(1/4)*a - 14*(a + b*x**4)**(1/4)*b*x**4 + 5*int((a + b
*x**4)**(1/4)/(a*x**16 + b*x**20),x)*a**2*x**15)/(140*x**15)
```

3.534 $\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \frac{a^4(a+bx^4)^{3/4}}{3b^5} - \frac{4a^3(a+bx^4)^{7/4}}{7b^5} + \frac{6a^2(a+bx^4)^{11/4}}{11b^5} - \frac{4a(a+bx^4)^{15/4}}{15b^5} + \frac{(a+bx^4)^{19/4}}{19b^5}$$

output
```
1/3*a^4*(b*x^4+a)^(3/4)/b^5-4/7*a^3*(b*x^4+a)^(7/4)/b^5+6/11*a^2*(b*x^4+a)
^(11/4)/b^5-4/15*a*(b*x^4+a)^(15/4)/b^5+1/19*(b*x^4+a)^(19/4)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}(2048a^4 - 1536a^3bx^4 + 1344a^2b^2x^8 - 1232ab^3x^{12} + 1155b^4x^{16})}{21945b^5}$$

input
```
Integrate[x^19/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(2048*a^4 - 1536*a^3*b*x^4 + 1344*a^2*b^2*x^8 - 1232*a*
b^3*x^12 + 1155*b^4*x^16))/(21945*b^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{16}}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^4}{b^4\sqrt[4]{bx^4+a}}-\frac{4(bx^4+a)^{3/4}a^3}{b^4}+\frac{6(bx^4+a)^{7/4}a^2}{b^4}-\frac{4(bx^4+a)^{11/4}a}{b^4}+\frac{(bx^4+a)^{15/4}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^4(a+bx^4)^{3/4}}{3b^5}-\frac{16a^3(a+bx^4)^{7/4}}{7b^5}+\frac{24a^2(a+bx^4)^{11/4}}{11b^5}+\frac{4(a+bx^4)^{19/4}}{19b^5}-\frac{16a(a+bx^4)^{15/4}}{15b^5}\right)$$

input
```
Int[x^19/(a + b*x^4)^(1/4),x]
```

output
```
((4*a^4*(a + b*x^4)^(3/4))/(3*b^5) - (16*a^3*(a + b*x^4)^(7/4))/(7*b^5) +
(24*a^2*(a + b*x^4)^(11/4))/(11*b^5) - (16*a*(a + b*x^4)^(15/4))/(15*b^5)
+ (4*(a + b*x^4)^(19/4))/(19*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result	size
gosper	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(1155x^{16}b^{4}-1232a\,b^{3}x^{12}+1344a^{2}b^{2}x^{8}-1536a^{3}b\,x^{4}+2048a^{4}\right)}{21945b^{5}}$	58
trager	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(1155x^{16}b^{4}-1232a\,b^{3}x^{12}+1344a^{2}b^{2}x^{8}-1536a^{3}b\,x^{4}+2048a^{4}\right)}{21945b^{5}}$	58
risch	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(1155x^{16}b^{4}-1232a\,b^{3}x^{12}+1344a^{2}b^{2}x^{8}-1536a^{3}b\,x^{4}+2048a^{4}\right)}{21945b^{5}}$	58
pseudoelliptic	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(1155x^{16}b^{4}-1232a\,b^{3}x^{12}+1344a^{2}b^{2}x^{8}-1536a^{3}b\,x^{4}+2048a^{4}\right)}{21945b^{5}}$	58
orering	$\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(1155x^{16}b^{4}-1232a\,b^{3}x^{12}+1344a^{2}b^{2}x^{8}-1536a^{3}b\,x^{4}+2048a^{4}\right)}{21945b^{5}}$	58

input
```
int(x^19/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/21945*(b*x^4+a)^(3/4)*(1155*b^4*x^16-1232*a*b^3*x^12+1344*a^2*b^2*x^8-15
36*a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.56

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(1155\,b^4x^{16} - 1232\,ab^3x^{12} + 1344\,a^2b^2x^8 - 1536\,a^3bx^4 + 2048\,a^4)(bx^4+a)^{\frac{3}{4}}}{21945\,b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/21945*(1155*b^4*x^16 - 1232*a*b^3*x^12 + 1344*a^2*b^2*x^8 - 1536*a^3*b*x
^4 + 2048*a^4)*(b*x^4 + a)^(3/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} \frac{2048a^4(a+bx^4)^{\frac{3}{4}}}{21945b^5} - \frac{512a^3x^4(a+bx^4)^{\frac{3}{4}}}{7315b^4} + \frac{64a^2x^8(a+bx^4)^{\frac{3}{4}}}{1045b^3} - \frac{16ax^{12}(a+bx^4)^{\frac{3}{4}}}{285b^2} + \frac{x^{16}(a+bx^4)^{\frac{3}{4}}}{19b} & \text{for } b \neq 0 \\ \frac{x^{20}}{20\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**19/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((2048*a**4*(a + b*x**4)**(3/4)/(21945*b**5) - 512*a**3*x**4*(a +
 b*x**4)**(3/4)/(7315*b**4) + 64*a**2*x**8*(a + b*x**4)**(3/4)/(1045*b**3)
 - 16*a*x**12*(a + b*x**4)**(3/4)/(285*b**2) + x**16*(a + b*x**4)**(3/4)/(
19*b), Ne(b, 0)), (x**20/(20*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{19}{4}}}{19\,b^5} - \frac{4\,(bx^4+a)^{\frac{15}{4}}a}{15\,b^5} + \frac{6\,(bx^4+a)^{\frac{11}{4}}a^2}{11\,b^5} - \frac{4\,(bx^4+a)^{\frac{7}{4}}a^3}{7\,b^5} + \frac{(bx^4+a)^{\frac{3}{4}}a^4}{3\,b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/19*(b*x^4 + a)^(19/4)/b^5 - 4/15*(b*x^4 + a)^(15/4)*a/b^5 + 6/11*(b*x^4
+ a)^(11/4)*a^2/b^5 - 4/7*(b*x^4 + a)^(7/4)*a^3/b^5 + 1/3*(b*x^4 + a)^(3/4
)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \frac{1155\,(bx^4+a)^{\frac{19}{4}} - 5852\,(bx^4+a)^{\frac{15}{4}}a + 11970\,(bx^4+a)^{\frac{11}{4}}a^2 - 12540\,(bx^4+a)^{\frac{7}{4}}a^3 + 7315\,(bx^4+a)^{\frac{3}{4}}a^4}{21945\,b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/21945*(1155*(b*x^4 + a)^(19/4) - 5852*(b*x^4 + a)^(15/4)*a + 11970*(b*x^
4 + a)^(11/4)*a^2 - 12540*(b*x^4 + a)^(7/4)*a^3 + 7315*(b*x^4 + a)^(3/4)*a
^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \left(b\,x^4+a\right)^{3/4}\left(\frac{2048\,a^4}{21945\,b^5}+\frac{x^{16}}{19\,b}-\frac{16\,a\,x^{12}}{285\,b^2}-\frac{512\,a^3\,x^4}{7315\,b^4}+\frac{64\,a^2\,x^8}{1045\,b^3}\right)$$

input
```
int(x^19/(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(3/4)*((2048*a^4)/(21945*b^5) + x^16/(19*b) - (16*a*x^12)/(285
*b^2) - (512*a^3*x^4)/(7315*b^4) + (64*a^2*x^8)/(1045*b^3))
```

Reduce [F]

$$\int \frac{x^{19}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{19}}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^19/(b*x^4+a)^(1/4),x)
```

output
```
int(x**19/(a + b*x**4)**(1/4),x)
```

3.535 $\int \frac{x^{15}}{\sqrt[4]{a+bx^4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}} dx = -\frac{a^3(a+bx^4)^{3/4}}{3b^4}+\frac{3a^2(a+bx^4)^{7/4}}{7b^4}-\frac{3a(a+bx^4)^{11/4}}{11b^4}+\frac{(a+bx^4)^{15/4}}{15b^4}$$

output
```
-1/3*a^3*(b*x^4+a)^(3/4)/b^4+3/7*a^2*(b*x^4+a)^(7/4)/b^4-3/11*a*(b*x^4+a)^
(11/4)/b^4+1/15*(b*x^4+a)^(15/4)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}} dx = \frac{(a+bx^4)^{3/4}(-128a^3+96a^2bx^4-84ab^2x^8+77b^3x^{12})}{1155b^4}$$

input
```
Integrate[x^15/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(-128*a^3 + 96*a^2*b*x^4 - 84*a*b^2*x^8 + 77*b^3*x^12))
/(1155*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{12}}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(-\frac{a^3}{b^3\sqrt[4]{bx^4+a}}+\frac{3(bx^4+a)^{3/4}a^2}{b^3}-\frac{3(bx^4+a)^{7/4}a}{b^3}+\frac{(bx^4+a)^{11/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^3(a+bx^4)^{3/4}}{3b^4}+\frac{12a^2(a+bx^4)^{7/4}}{7b^4}+\frac{4(a+bx^4)^{15/4}}{15b^4}-\frac{12a(a+bx^4)^{11/4}}{11b^4}\right)$$

input
```
Int[x^15/(a + b*x^4)^(1/4),x]
```

output
```
((-4*a^3*(a + b*x^4)^(3/4))/(3*b^4) + (12*a^2*(a + b*x^4)^(7/4))/(7*b^4) -
 (12*a*(a + b*x^4)^(11/4))/(11*b^4) + (4*(a + b*x^4)^(15/4))/(15*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-77 b^{3} x^{12}+84 a\,b^{2} x^{8}-96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	47
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-77 b^{3} x^{12}+84 a\,b^{2} x^{8}-96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	47
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-77 b^{3} x^{12}+84 a\,b^{2} x^{8}-96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-77 b^{3} x^{12}+84 a\,b^{2} x^{8}-96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	47
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-77 b^{3} x^{12}+84 a\,b^{2} x^{8}-96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	47

input
```
int(x^15/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(b*x^4+a)^(3/4)*(-77*b^3*x^12+84*a*b^2*x^8-96*a^2*b*x^4+128*a^3)/b
^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(77\,b^3x^{12}-84\,ab^2x^8+96\,a^2bx^4-128\,a^3)(bx^4+a)^{\frac{3}{4}}}{1155\,b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/1155*(77*b^3*x^12 - 84*a*b^2*x^8 + 96*a^2*b*x^4 - 128*a^3)*(b*x^4 + a)^(
3/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} -\frac{128a^3(a+bx^4)^{\frac{3}{4}}}{1155b^4}+\frac{32a^2x^4(a+bx^4)^{\frac{3}{4}}}{385b^3}-\frac{4ax^8(a+bx^4)^{\frac{3}{4}}}{55b^2}+\frac{x^{12}(a+bx^4)^{\frac{3}{4}}}{15b} & \text{for } b\neq 0 \\ \frac{x^{16}}{16\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-128*a**3*(a + b*x**4)**(3/4)/(1155*b**4) + 32*a**2*x**4*(a + b
*x**4)**(3/4)/(385*b**3) - 4*a*x**8*(a + b*x**4)**(3/4)/(55*b**2) + x**12*
(a + b*x**4)**(3/4)/(15*b), Ne(b, 0)), (x**16/(16*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{15}{4}}}{15\,b^4} - \frac{3\,(bx^4+a)^{\frac{11}{4}}a}{11\,b^4} + \frac{3\,(bx^4+a)^{\frac{7}{4}}a^2}{7\,b^4} - \frac{(bx^4+a)^{\frac{3}{4}}a^3}{3\,b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/15*(b*x^4 + a)^(15/4)/b^4 - 3/11*(b*x^4 + a)^(11/4)*a/b^4 + 3/7*(b*x^4 +
 a)^(7/4)*a^2/b^4 - 1/3*(b*x^4 + a)^(3/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx = \frac{77\,(bx^4+a)^{\frac{15}{4}} - 315\,(bx^4+a)^{\frac{11}{4}}a + 495\,(bx^4+a)^{\frac{7}{4}}a^2 - 385\,(bx^4+a)^{\frac{3}{4}}a^3}{1155\,b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/1155*(77*(b*x^4 + a)^(15/4) - 315*(b*x^4 + a)^(11/4)*a + 495*(b*x^4 + a)
^(7/4)*a^2 - 385*(b*x^4 + a)^(3/4)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx = -\left(b\,x^4+a\right)^{3/4}\left(\frac{128\,a^3}{1155\,b^4} - \frac{x^{12}}{15\,b} + \frac{4\,a\,x^8}{55\,b^2} - \frac{32\,a^2\,x^4}{385\,b^3}\right)$$

input
```
int(x^15/(a + b*x^4)^(1/4),x)
```

output
```
-(a + b*x^4)^(3/4)*((128*a^3)/(1155*b^4) - x^12/(15*b) + (4*a*x^8)/(55*b^2
) - (32*a^2*x^4)/(385*b^3))
```

Reduce [F]

$$\int \frac{x^{15}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{15}}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^15/(b*x^4+a)^(1/4),x)
```

output
```
int(x**15/(a + b*x**4)**(1/4),x)
```

3.536 $\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx=\frac{a^2(a+bx^4)^{3/4}}{3b^3}-\frac{2a(a+bx^4)^{7/4}}{7b^3}+\frac{(a+bx^4)^{11/4}}{11b^3}$$

output
```
1/3*a^2*(b*x^4+a)^(3/4)/b^3-2/7*a*(b*x^4+a)^(7/4)/b^3+1/11*(b*x^4+a)^(11/4
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx=\frac{(a+bx^4)^{3/4}(32a^2-24abx^4+21b^2x^8)}{231b^3}$$

input
```
Integrate[x^11/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(32*a^2 - 24*a*b*x^4 + 21*b^2*x^8))/(231*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2\sqrt[4]{bx^4+a}}-\frac{2\left(bx^4+a\right)^{3/4}a}{b^2}+\frac{\left(bx^4+a\right)^{7/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^2\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{4\left(a+bx^4\right)^{11/4}}{11b^3}-\frac{8a\left(a+bx^4\right)^{7/4}}{7b^3}\right)$$

input
```
Int[x^11/(a + b*x^4)^(1/4),x]
```

output
```
((4*a^2*(a + b*x^4)^(3/4))/(3*b^3) - (8*a*(a + b*x^4)^(7/4))/(7*b^3) + (4*
(a + b*x^4)^(11/4))/(11*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{3}{4}}\,(21b^2x^8-24ab\,x^4+32a^2)}{231b^3}$	36
trager	$\frac{(b\,x^4+a)^{\frac{3}{4}}\,(21b^2x^8-24ab\,x^4+32a^2)}{231b^3}$	36
risch	$\frac{(b\,x^4+a)^{\frac{3}{4}}\,(21b^2x^8-24ab\,x^4+32a^2)}{231b^3}$	36
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{3}{4}}\,(21b^2x^8-24ab\,x^4+32a^2)}{231b^3}$	36
orering	$\frac{(b\,x^4+a)^{\frac{3}{4}}\,(21b^2x^8-24ab\,x^4+32a^2)}{231b^3}$	36

input
```
int(x^11/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/231*(b*x^4+a)^(3/4)*(21*b^2*x^8-24*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(21\,b^2x^8 - 24\,abx^4 + 32\,a^2)(bx^4+a)^{\frac{3}{4}}}{231\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/231*(21*b^2*x^8 - 24*a*b*x^4 + 32*a^2)*(b*x^4 + a)^(3/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} \frac{32a^2(a+bx^4)^{\frac{3}{4}}}{231b^3} - \frac{8ax^4(a+bx^4)^{\frac{3}{4}}}{77b^2} + \frac{x^8(a+bx^4)^{\frac{3}{4}}}{11b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((32*a**2*(a + b*x**4)**(3/4)/(231*b**3) - 8*a*x**4*(a + b*x**4)*
*(3/4)/(77*b**2) + x**8*(a + b*x**4)**(3/4)/(11*b), Ne(b, 0)), (x**12/(12*
a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{11}{4}}}{11\,b^3} - \frac{2\,(bx^4+a)^{\frac{7}{4}}a}{7\,b^3} + \frac{(bx^4+a)^{\frac{3}{4}}a^2}{3\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/11*(b*x^4 + a)^(11/4)/b^3 - 2/7*(b*x^4 + a)^(7/4)*a/b^3 + 1/3*(b*x^4 + a
)^(3/4)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx = \frac{21\,(bx^4+a)^{\frac{11}{4}} - 66\,(bx^4+a)^{\frac{7}{4}}a + 77\,(bx^4+a)^{\frac{3}{4}}a^2}{231\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/231*(21*(b*x^4 + a)^(11/4) - 66*(b*x^4 + a)^(7/4)*a + 77*(b*x^4 + a)^(3/
4)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx = \left(b\,x^4+a\right)^{3/4}\left(\frac{32\,a^2}{231\,b^3}+\frac{x^8}{11\,b}-\frac{8\,a\,x^4}{77\,b^2}\right)$$

input
```
int(x^11/(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(3/4)*((32*a^2)/(231*b^3) + x^8/(11*b) - (8*a*x^4)/(77*b^2))
```

Reduce [F]

$$\int \frac{x^{11}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{11}}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^11/(b*x^4+a)^(1/4),x)
```

output
```
int(x**11/(a + b*x**4)**(1/4),x)
```

3.537 $\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = -\frac{a(a+bx^4)^{3/4}}{3b^2}+\frac{(a+bx^4)^{7/4}}{7b^2}$$

output
```
-1/3*a*(b*x^4+a)^(3/4)/b^2+1/7*(b*x^4+a)^(7/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}(-4a+3bx^4)}{21b^2}$$

input
```
Integrate[x^7/(a + b*x^4)^(1/4),x]
```

output
```
((a + b*x^4)^(3/4)*(-4*a + 3*b*x^4))/(21*b^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{3/4}}{b}-\frac{a}{b\sqrt[4]{bx^4+a}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4\left(a+bx^4\right)^{7/4}}{7b^2}-\frac{4a\left(a+bx^4\right)^{3/4}}{3b^2}\right)$$

input
```
Int[x^7/(a + b*x^4)^(1/4),x]
```

output
```
((-4*a*(a + b*x^4)^(3/4))/(3*b^2) + (4*(a + b*x^4)^(7/4))/(7*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-3b\,x^{4}+4a\right)}{21b^{2}}$	25
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-3b\,x^{4}+4a\right)}{21b^{2}}$	25
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-3b\,x^{4}+4a\right)}{21b^{2}}$	25
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-3b\,x^{4}+4a\right)}{21b^{2}}$	25
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-3b\,x^{4}+4a\right)}{21b^{2}}$	25

input
```
int(x^7/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(b*x^4+a)^(3/4)*(-3*b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx=\frac{(3\,bx^4-4\,a)(bx^4+a)^{\frac{3}{4}}}{21\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/21*(3*b*x^4 - 4*a)*(b*x^4 + a)^(3/4)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} -\frac{4a(a+bx^4)^{\frac{3}{4}}}{21b^2} + \frac{x^4(a+bx^4)^{\frac{3}{4}}}{7b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-4*a*(a + b*x**4)**(3/4)/(21*b**2) + x**4*(a + b*x**4)**(3/4)/(
7*b), Ne(b, 0)), (x**8/(8*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{7}{4}}}{7\,b^2} - \frac{(bx^4+a)^{\frac{3}{4}}a}{3\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/7*(b*x^4 + a)^(7/4)/b^2 - 1/3*(b*x^4 + a)^(3/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = \frac{3\,(bx^4+a)^{\frac{7}{4}} - 7\,(bx^4+a)^{\frac{3}{4}}a}{21\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/21*(3*(b*x^4 + a)^(7/4) - 7*(b*x^4 + a)^(3/4)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = -\left(b\,x^4+a\right)^{3/4}\left(\frac{4\,a}{21\,b^2}-\frac{x^4}{7\,b}\right)$$

input
```
int(x^7/(a + b*x^4)^(1/4),x)
```

output
```
-(a + b*x^4)^(3/4)*((4*a)/(21*b^2) - x^4/(7*b))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^7}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^7/(b*x^4+a)^(1/4),x)
```

output
```
int(x**7/(a + b*x**4)**(1/4),x)
```

3.538 $\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx=\frac{\left(a+bx^4\right)^{3/4}}{3b}$$

output
```
1/3*(b*x^4+a)^(3/4)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx=\frac{\left(a+bx^4\right)^{3/4}}{3b}$$

input
```
Integrate[x^3/(a + b*x^4)^(1/4),x]
```

output
```
(a + b*x^4)^(3/4)/(3*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \ 793$$

$$\frac{\left(a+bx^4\right)^{3/4}}{3b}$$

input
```
Int[x^3/(a + b*x^4)^(1/4),x]
```

output
```
(a + b*x^4)^(3/4)/(3*b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15
derivativedivides	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15
default	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15
trager	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15
risch	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15
orering	$\frac{(b\,x^4+a)^{\frac{3}{4}}}{3b}$	15

input
```
int(x^3/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^4+a)^(3/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}}{3\,b}$$

input
```
integrate(x^3/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/3*(b*x^4 + a)^(3/4)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx = \begin{cases} \frac{(a+bx^4)^{\frac{3}{4}}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)**(1/4),x)
```

output
```
Piecewise(((a + b*x**4)**(3/4)/(3*b), Ne(b, 0)), (x**4/(4*a**(1/4)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}}{3\,b}$$

input
```
integrate(x^3/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/3*(b*x^4 + a)^(3/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}}{3\,b}$$

input
```
integrate(x^3/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/3*(b*x^4 + a)^(3/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx=\frac{{\left(b\,x^4+a\right)}^{3/4}}{3\,b}$$

input
```
int(x^3/(a + b*x^4)^(1/4),x)
```

output
```
(a + b*x^4)^(3/4)/(3*b)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^3}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^3/(b*x^4+a)^(1/4),x)
```

output
```
int(x**3/(a + b*x**4)**(1/4),x)
```

3.539 $\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

output
```
1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-1/2*arctanh((b*x^4+a)^(1/4)/a^
(1/4))/a^(1/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

input
```
Integrate[1/(x*(a + b*x^4)^(1/4)),x]
```

output
```
(ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] - ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(
2*a^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4$$

$$\Big\downarrow \text{73}$$

$$\frac{\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

input
```
Int[1/(x*(a + b*x^4)^(1/4)),x]
```

output
```
ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4)) - ArcTanh[(a + b*x^4)^(1/4)/
a^(1/4)]/(2*a^(1/4))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)-\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)}{4a^{\frac{1}{4}}}$	57

input
```
int(1/x/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^(1/4)*(2*arctan((b*x^4+a)^(1/4)/a^(1/4))-ln(((b*x^4+a)^(1/4)+a^(1/4)
)/((b*x^4+a)^(1/4)-a^(1/4))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx = -\frac{\log\left((bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}+\frac{i\,\log\left((bx^4+a)^{\frac{1}{4}}+i\,a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}} - \frac{i\,\log\left((bx^4+a)^{\frac{1}{4}}-i\,a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left((bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}$$

input
```
integrate(1/x/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/4*log((b*x^4 + a)^(1/4) + a^(1/4))/a^(1/4) + 1/4*I*log((b*x^4 + a)^(1/4
) + I*a^(1/4))/a^(1/4) - 1/4*I*log((b*x^4 + a)^(1/4) - I*a^(1/4))/a^(1/4)
+ 1/4*log((b*x^4 + a)^(1/4) - a^(1/4))/a^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx = -\frac{\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x/(b*x**4+a)**(1/4),x)
```

output
```
-gamma(1/4)*hyper((1/4, 1/4), (5/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(1
/4)*x*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx = \frac{\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{1}{4}}} + \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{1}{4}}}$$

input
```
integrate(1/x/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + 1/4*log(((b*x^4 + a)^(1/4)
 - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(39) = 78$.

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.38

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx = -\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} + \frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a} - \frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a}$$

input
```
integrate(1/x/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
+ a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 1/8*sqrt(2)*(-a)^(
3/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)
)/a - 1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + s
qrt(b*x^4 + a) + sqrt(-a))/a
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx = \frac{\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)-\mathrm{atanh}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}$$

input
```
int(1/(x*(a + b*x^4)^(1/4)),x)
```

output
```
(atan((a + b*x^4)^(1/4)/a^(1/4)) - atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a^
(1/4))
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x}dx$$

input
```
int(1/x/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x),x)
```

3.540 $\int \frac{1}{x^5 \sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{1}{x^5 \sqrt[4]{a+bx^4}}\,dx = -\frac{(a+bx^4)^{3/4}}{4ax^4} - \frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}$$

output
```
-1/4*(b*x^4+a)^(3/4)/a/x^4-1/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)+1
/8*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 \sqrt[4]{a+bx^4}}\,dx = -\frac{(a+bx^4)^{3/4}}{4ax^4} - \frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}$$

input
```
Integrate[1/(x^5*(a + b*x^4)^(1/4)),x]
```

output
```
-1/4*(a + b*x^4)^(3/4)/(a*x^4) - (b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(8*
a^(5/4)) + (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(5/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {798, 52, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^8 \sqrt[4]{bx^4+a}} dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{b \int \frac{1}{x^4 \sqrt[4]{bx^4+a}} dx^4}{4a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(-\frac{\int -\frac{bx^8}{a-x^{16}} d\sqrt[4]{bx^4+a}}{a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} \left(\frac{\int \frac{bx^8}{a-x^{16}} d\sqrt[4]{bx^4+a}}{a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \left(\frac{b \int \frac{x^8}{a-x^{16}} d\sqrt[4]{bx^4+a}}{a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4} \left(\frac{b \left(\frac{1}{2} \int \frac{1}{\sqrt{a}-x^8} d\sqrt[4]{bx^4+a} - \frac{1}{2} \int \frac{1}{x^8+\sqrt{a}} d\sqrt[4]{bx^4+a}\right)}{a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}\left(\frac{b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{(a+bx^4)^{3/4}}{ax^4}\right)$$

input

```
Int[1/(x^5*(a + b*x^4)^(1/4)),x]
```

output

```
(-((a + b*x^4)^(3/4)/(a*x^4)) + (b*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]
/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a)/4
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b\,x^{4}+\ln\left(\frac{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)b\,x^{4}-4\left(b\,x^{4}+a\right)^{\frac{3}{4}}a^{\frac{1}{4}}}{16a^{\frac{5}{4}}x^{4}}$	85

input
```
int(1/x^5/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(-2*arctan((b*x^4+a)^(1/4)/a^(1/4))*b*x^4+ln((-(b*x^4+a)^(1/4)-a^(1/4
))/(-(b*x^4+a)^(1/4)+a^(1/4)))*b*x^4-4*(b*x^4+a)^(3/4)*a^(1/4))/a^(5/4)/x^
4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx = \frac{ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(\frac{b^4}{a^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^3\right)-i\,ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(\frac{b^4}{a^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}b^3\right)+i\,ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\mathrm{lo}}{16\,ax^4}$$

input
```
integrate(1/x^5/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/16*(a*x^4*(b^4/a^5)^(1/4)*log(a^4*(b^4/a^5)^(3/4) + (b*x^4 + a)^(1/4)*b^
3) - I*a*x^4*(b^4/a^5)^(1/4)*log(I*a^4*(b^4/a^5)^(3/4) + (b*x^4 + a)^(1/4)
*b^3) + I*a*x^4*(b^4/a^5)^(1/4)*log(-I*a^4*(b^4/a^5)^(3/4) + (b*x^4 + a)^(
1/4)*b^3) - a*x^4*(b^4/a^5)^(1/4)*log(-a^4*(b^4/a^5)^(3/4) + (b*x^4 + a)^(
1/4)*b^3) - 4*(b*x^4 + a)^(3/4))/(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx = -\frac{\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(1/x**5/(b*x**4+a)**(1/4),x)
```

output
```
-gamma(5/4)*hyper((1/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(1
/4)*x**5*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx = -\frac{b\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{16\,a}-\frac{(bx^4+a)^{\frac{3}{4}}b}{4\,((bx^4+a)a-a^2)}$$

input
```
integrate(1/x^5/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/16*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1
/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a - 1/4*(b*x^4 + a)
^(3/4)*b/((b*x^4 + a)*a - a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(58) = 116$.

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.67

$$\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx = \frac{1}{32}\,b\left(\frac{2\,\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{2\,\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2}-\right.$$

input
```
integrate(1/x^5/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/32*b*(2*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b
*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 2*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 - sqrt(2)*(-a
)^(3/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(
-a))/a^2 + sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) +
sqrt(b*x^4 + a) + sqrt(-a))/a^2 - 8*(b*x^4 + a)^(3/4)/(a*b*x^4))
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx = \frac{b\,\mathrm{atanh}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}} - \frac{b\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}} - \frac{\left(b\,x^4+a\right)^{3/4}}{4\,a\,x^4}$$

input
```
int(1/(x^5*(a + b*x^4)^(1/4)),x)
```

output
```
(b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4)) - (b*atan((a + b*x^4)^(1/
4)/a^(1/4)))/(8*a^(5/4)) - (a + b*x^4)^(3/4)/(4*a*x^4)
```

Reduce [F]

$$\int \frac{1}{x^5\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^5}dx$$

input
```
int(1/x^5/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**5),x)
```

3.541 $\int \frac{1}{x^9 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^9 \sqrt[4]{a+bx^4}} \, dx = -\frac{(a+bx^4)^{3/4}}{8ax^8} + \frac{5b(a+bx^4)^{3/4}}{32a^2x^4} + \frac{5b^2 \arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2 \operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}}$$

output

```
-1/8*(b*x^4+a)^(3/4)/a/x^8+5/32*b*(b*x^4+a)^(3/4)/a^2/x^4+5/64*b^2*arctan(
(b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)-5/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/4))
/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9 \sqrt[4]{a+bx^4}} \, dx = \frac{(a+bx^4)^{3/4}(-4a+5bx^4)}{32a^2x^8} + \frac{5b^2 \arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2 \operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}}$$

input
```
Integrate[1/(x^9*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-4*a + 5*b*x^4))/(32*a^2*x^8) + (5*b^2*ArcTan[(a + b*x
^4)^(1/4)/a^(1/4)])/(64*a^(9/4)) - (5*b^2*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4
)])/(64*a^(9/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {798, 52, 52, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^{12} \sqrt[4]{bx^4+a}} dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{5b \int \frac{1}{x^8 \sqrt[4]{bx^4+a}} dx^4}{8a} - \frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{5b \left(-\frac{b \int \frac{1}{x^4 \sqrt[4]{bx^4+a}} dx^4}{4a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a} - \frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{5b\left(-\frac{\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a}-\frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(-\frac{5b\left(\frac{\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a}-\frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{5b\left(\frac{b\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a}-\frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(-\frac{5b\left(\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a}-\frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-\frac{5b\left(\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a}-\frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(-\frac{5b\left(\frac{b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^4}\right)}{8a}-\frac{\left(a+bx^4\right)^{3/4}}{2ax^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^4)^(1/4)),x]
```

output
```
(-1/2*(a + b*x^4)^(3/4)/(a*x^8) - (5*b*(-((a + b*x^4)^(3/4)/(a*x^4)) + (b*
(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4
)/a^(1/4)]/(2*a^(1/4))))/a))/(8*a))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{10\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b^{2}x^{8}-5\ln\left(\frac{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)b^{2}x^{8}+20b\,x^{4}a^{\frac{1}{4}}\left(b\,x^{4}+a\right)^{\frac{3}{4}}-16a^{\frac{5}{4}}\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{128a^{\frac{9}{4}}x^{8}}$	108

input
```
int(1/x^9/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(10*arctan((b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8-5*ln((-(b*x^4+a)^(1/4)-a
^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4)))*b^2*x^8+20*b*x^4*a^(1/4)*(b*x^4+a)^(3/
4)-16*a^(5/4)*(b*x^4+a)^(3/4))/a^(9/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^9\sqrt[4]{a+bx^4}}\,dx =$$

$$-\frac{5\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(\frac{b^8}{a^9}\right)^{\frac{3}{4}}+125\,(bx^4+a)^{\frac{1}{4}}b^6\right)-5i\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\log\left(125i\,a^7\left(\frac{b^8}{a^9}\right)^{\frac{3}{4}}+125\,(bx^4+\right.}{}$$

input
```
integrate(1/x^9/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/128*(5*a^2*x^8*(b^8/a^9)^(1/4)*log(125*a^7*(b^8/a^9)^(3/4) + 125*(b*x^4
 + a)^(1/4)*b^6) - 5*I*a^2*x^8*(b^8/a^9)^(1/4)*log(125*I*a^7*(b^8/a^9)^(3/
4) + 125*(b*x^4 + a)^(1/4)*b^6) + 5*I*a^2*x^8*(b^8/a^9)^(1/4)*log(-125*I*a
^7*(b^8/a^9)^(3/4) + 125*(b*x^4 + a)^(1/4)*b^6) - 5*a^2*x^8*(b^8/a^9)^(1/4
)*log(-125*a^7*(b^8/a^9)^(3/4) + 125*(b*x^4 + a)^(1/4)*b^6) - 4*(5*b*x^4 -
 4*a)*(b*x^4 + a)^(3/4))/(a^2*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^9 \sqrt[4]{a+bx^4}}\,dx = -\frac{\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4\sqrt[4]{b}x^9\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(1/x**9/(b*x**4+a)**(1/4),x)
```

output
```
-gamma(9/4)*hyper((1/4, 9/4), (13/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(
1/4)*x**9*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^9 \sqrt[4]{a+bx^4}}\,dx = \frac{5\,b^2\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a^2} + \frac{5\,(bx^4+a)^{\frac{7}{4}}b^2 - 9\,(bx^4+a)^{\frac{3}{4}}ab^2}{32\,\left((bx^4+a)^2a^2 - 2\,(bx^4+a)a^3 + a^4\right)}$$

input
```
integrate(1/x^9/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
5/128*b^2*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^
(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2 + 1/32*(5*(b*
x^4 + a)^(7/4)*b^2 - 9*(b*x^4 + a)^(3/4)*a*b^2)/((b*x^4 + a)^2*a^2 - 2*(b*
x^4 + a)*a^3 + a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.35

$$\int \frac{1}{x^9\sqrt[4]{a+bx^4}}\,dx = \frac{\frac{10\sqrt{2}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}a^2}+\frac{10\sqrt{2}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}a^2}+\frac{5\sqrt{2}(-a)^{\frac{3}{4}}b^3\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a\right.}{a^3}}{256\,b}$$

input
```
integrate(1/x^9/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/256*(10*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^2) + 10*sqrt(2)*b^3*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^2)
+ 5*sqrt(2)*(-a)^(3/4)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt
(b*x^4 + a) + sqrt(-a))/a^3 + 5*sqrt(2)*b^3*log(-sqrt(2)*(b*x^4 + a)^(1/4)
*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^2) + 8*(5*(b*x^4 +
 a)^(7/4)*b^3 - 9*(b*x^4 + a)^(3/4)*a*b^3)/(a^2*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^9\sqrt[4]{a+bx^4}}\,dx = \frac{5\,b^2\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{64\,a^{9/4}}-\frac{9\,{\left(b\,x^4+a\right)}^{3/4}}{32\,a\,x^8}+\frac{5\,{\left(b\,x^4+a\right)}^{7/4}}{32\,a^2\,x^8}+\frac{b^2\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,5\mathrm{i}}{64\,a^{9/4}}$$

input
```
int(1/(x^9*(a + b*x^4)^(1/4)),x)
```

output
```
(5*b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(9/4)) + (b^2*atan(((a + b*x
^4)^(1/4)*1i)/a^(1/4))*5i)/(64*a^(9/4)) - (9*(a + b*x^4)^(3/4))/(32*a*x^8)
 + (5*(a + b*x^4)^(7/4))/(32*a^2*x^8)
```

Reduce [F]

$$\int \frac{1}{x^9\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^9}dx$$

input
```
int(1/x^9/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**9),x)
```

3.542 $\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx = -\frac{8a^3x^2}{39b^3\sqrt[4]{a+bx^4}} + \frac{4a^2x^2(a+bx^4)^{3/4}}{39b^3} - \frac{10ax^6(a+bx^4)^{3/4}}{117b^2} + \frac{x^{10}(a+bx^4)^{3/4}}{13b} + \frac{8a^{7/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^4}}$$

output

```
-8/39*a^3*x^2/b^3/(b*x^4+a)^(1/4)+4/39*a^2*x^2*(b*x^4+a)^(3/4)/b^3-10/117*
a*x^6*(b*x^4+a)^(3/4)/b^2+1/13*x^10*(b*x^4+a)^(3/4)/b+8/39*a^(7/2)*(1+b*x^
4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(7/2)
/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^2\left(12a^3+2a^2bx^4-ab^2x^8+9b^3x^{12}-12a^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{117b^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^13/(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(12*a^3 + 2*a^2*b*x^4 - a*b^2*x^8 + 9*b^3*x^12 - 12*a^3*(1 + (b*x^4)/
a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(117*b^3*(a + b*
x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {807, 262, 262, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2}\int \frac{x^{12}}{\sqrt[4]{bx^4+a}}dx^2$$

$$\Big\downarrow \ 262$$

$$\frac{1}{2}\left(\frac{2x^{10}\left(a+bx^4\right)^{3/4}}{13b}-\frac{10a\int \frac{x^8}{\sqrt[4]{bx^4+a}}dx^2}{13b}\right)$$

$$\Big\downarrow \ 262$$

$$\frac{1}{2}\left(\frac{2x^{10}\left(a+bx^4\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\int\frac{x^4}{\sqrt[4]{bx^4+a}}dx^2}{3b}\right)}{13b}\right)$$

$\downarrow$ 262

$$\frac{1}{2}\left(\frac{2x^{10}\left(a+bx^4\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\int\frac{1}{\sqrt[4]{bx^4+a}}dx^2}{5b}\right)}{3b}\right)}{13b}\right)$$

$\downarrow$ 227

$$\frac{1}{2}\left(\frac{2x^{10}\left(a+bx^4\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)}{13b}\right)$$

$\Big\downarrow$ 225

$$\frac{1}{2}\left(\frac{2x^{10}\left(a+bx^4\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)}{13b}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{2x^{10}\left(a+bx^4\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)}{13b}\right)$$

input
```
Int[x^13/(a + b*x^4)^(1/4),x]
```

output
```
((2*x^10*(a + b*x^4)^(3/4))/(13*b) - (10*a*((2*x^6*(a + b*x^4)^(3/4))/(9*b
) - (2*a*((2*x^2*(a + b*x^4)^(3/4))/(5*b) - (2*a*(1 + (b*x^4)/a)^(1/4)*((2
*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sq
rt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^4)^(1/4))))/(3*b)))/(13*b))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{13}}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^13/(b*x^4+a)^(1/4),x)
```

output
```
int(x^13/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^{13}}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^13/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^{14}{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{14\sqrt[4]{a}}$$

input
```
integrate(x**13/(b*x**4+a)**(1/4),x)
```

output
```
x**14*hyper((1/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(1/4))
```

Maxima [F]

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^{13}}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^13/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^13/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{13}}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(x^13/(a + b*x^4)^(1/4),x)
```

output
```
int(x^13/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{13}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{13}}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^13/(b*x^4+a)^(1/4),x)
```

output
```
int(x**13/(a + b*x**4)**(1/4),x)
```

3.543 $\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx=\frac{4a^2x^2}{15b^2\sqrt[4]{a+bx^4}}-\frac{2ax^2(a+bx^4)^{3/4}}{15b^2}+\frac{x^6(a+bx^4)^{3/4}}{9b}-\frac{4a^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
4/15*a^2*x^2/b^2/(b*x^4+a)^(1/4)-2/15*a*x^2*(b*x^4+a)^(3/4)/b^2+1/9*x^6*(b
*x^4+a)^(3/4)/b-4/15*a^(5/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^
(1/2)*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^2\left(-6a^2-abx^4+5b^2x^8+6a^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{45b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^9/(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(-6*a^2 - a*b*x^4 + 5*b^2*x^8 + 6*a^2*(1 + (b*x^4)/a)^(1/4)*Hypergeom
etric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(45*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 262, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{x^8}{\sqrt[4]{bx^4+a}}dx^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx^2}{3b}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{5b}\right)}{3b}\right)$$

$$\Big\downarrow \text{ 227}$$

$$\frac{1}{2}\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)$$

$\downarrow$ 225

$$\frac{1}{2}\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)$$

$\downarrow$ 212

$$\frac{1}{2}\left(\frac{2x^6\left(a+bx^4\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^4}}\right)}{3b}\right)$$

input
```
Int[x^9/(a + b*x^4)^(1/4),x]
```

output
```
((2*x^6*(a + b*x^4)^(3/4))/(9*b) - (2*a*((2*x^2*(a + b*x^4)^(3/4))/(5*b) -
 (2*a*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*El
lipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^4)^(1
/4))))/(3*b))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^9}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^9/(b*x^4+a)^(1/4),x)
```

output
```
int(x^9/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^9}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^9/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^{10}{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10\sqrt[4]{a}}$$

input
```
integrate(x**9/(b*x**4+a)**(1/4),x)
```

output
```
x**10*hyper((1/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(1/4))
```

Maxima [F]

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^9/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^9/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^9}{\left(b\,x^4+a\right)^{1/4}}\,dx$$

input
```
int(x^9/(a + b*x^4)^(1/4),x)
```

output
```
int(x^9/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^9}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^9}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^9/(b*x^4+a)^(1/4),x)
```

output
```
int(x**9/(a + b*x**4)**(1/4),x)
```

3.544 $\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx = -\frac{2ax^2}{5b\sqrt[4]{a+bx^4}}+\frac{x^2(a+bx^4)^{3/4}}{5b}+\frac{2a^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-2/5*a*x^2/b/(b*x^4+a)^(1/4)+1/5*x^2*(b*x^4+a)^(3/4)/b+2/5*a^(3/2)*(1+b*x^
4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(3/2)
/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^2\left(a+bx^4-a\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{5b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^5/(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(a + b*x^4 - a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 -((b*x^4)/a)]))/(5*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{5b}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{5b\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{5b\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{2x^2\left(a+bx^4\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[x^5/(a + b*x^4)^(1/4),x]
```

output
```
((2*x^2*(a + b*x^4)^(3/4))/(5*b) - (2*a*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1
+ (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2,
 2])/Sqrt[b]))/(5*b*(a + b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^5/(b*x^4+a)^(1/4),x)
```

output
```
int(x^5/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^5}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^5/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^6\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6\sqrt[4]{a}}$$

input
```
integrate(x**5/(b*x**4+a)**(1/4),x)
```

output
```
x**6*hyper((1/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4))
```

Maxima [F]

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^5}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^5}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^5/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^5}{{\left(b\,x^4+a\right)}^{1/4}}\,dx$$

input

```
int(x^5/(a + b*x^4)^(1/4),x)
```

output

```
int(x^5/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^5}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input

```
int(x^5/(b*x^4+a)^(1/4),x)
```

output

```
int(x**5/(a + b*x**4)**(1/4),x)
```

3.545 $\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^2}{\sqrt[4]{a+bx^4}}-\frac{\sqrt{a}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
x^2/(b*x^4+a)^(1/4)-a^(1/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(
1/2)*x^2/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.69

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)}{2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x/(a + b*x^4)^(1/4),x]
```

output
```
(x^2*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)])
/(2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {807, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \ 807$$

$$\frac{1}{2}\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2$$

$$\Big\downarrow \ 227$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 225$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 212$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2\sqrt[4]{a+bx^4}}$$

input
```
Int[x/(a + b*x^4)^(1/4),x]
```

output
```
((1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*Ellipti
cE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x/(b*x^4+a)^(1/4),x)
```

output
```
int(x/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^2{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{2\sqrt[4]{a}}$$

input
```
integrate(x/(b*x**4+a)**(1/4),x)
```

output
```
x**2*hyper((1/4, 1/2), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4))
```

Maxima [F]

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(x/(a + b*x^4)^(1/4),x)
```

output
```
int(x/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}}\,dx$$

input
```
int(x/(b*x^4+a)^(1/4),x)
```

output
```
int(x/(a + b*x**4)**(1/4),x)
```

3.546 $\int \frac{1}{x^3 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}} \, dx = \frac{bx^2}{2a\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{2ax^2} - \frac{\sqrt{b}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^4}}$$

output
```
1/2*b*x^2/a/(b*x^4+a)^(1/4)-1/2*(b*x^4+a)^(3/4)/a/x^2-1/2*b^(1/2)*(1+b*x^4
/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/
(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}} \, dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)^(1/4)),x]
```

output
```
-1/2*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x^4)/a)
])/(x^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^4 \sqrt[4]{bx^4+a}} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{b \int \frac{1}{\sqrt[4]{bx^4+a}} dx^2}{2a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^2} \right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2} \left(\frac{b \sqrt[4]{\frac{bx^4}{a}+1} \int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}} dx^2}{2a \sqrt[4]{a+bx^4}} - \frac{\left(a+bx^4\right)^{3/4}}{ax^2} \right)$$

$$\Big\downarrow \text{225}$$

$$\frac{1}{2}\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)^(1/4)),x]
```

output
```
(-((a + b*x^4)^(3/4)/(a*x^2)) + (b*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*
x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/
Sqrt[b]))/(2*a*(a + b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b \,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^3/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^3/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}}\, dx = \int \frac{1}{{\left(b x^{4} + a\right)}^{\frac{1}{4}} x^{3}}\, dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^7 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2\sqrt[4]{a}x^2}$$

input
```
integrate(1/x**3/(b*x**4+a)**(1/4),x)
```

output
```
-hyper((-1/2, 1/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(1/4)*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{x^3\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(1/(x^3*(a + b*x^4)^(1/4)),x)
```

output
```
int(1/(x^3*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^3}dx$$

input
```
int(1/x^3/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**3),x)
```

3.547 $\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = -\frac{b^2x^2}{4a^2\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{6ax^6} + \frac{b(a+bx^4)^{3/4}}{4a^2x^2} + \frac{b^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/4*b^2*x^2/a^2/(b*x^4+a)^(1/4)-1/6*(b*x^4+a)^(3/4)/a/x^6+1/4*b*(b*x^4+a)
^(3/4)/a^2/x^2+1/4*b^(3/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1
/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},-\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)^(1/4)),x]
```

output
```
-1/6*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x^4)/a
)])/(x^6*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 264, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^8 \sqrt[4]{bx^4+a}} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{b \int \frac{1}{x^4 \sqrt[4]{bx^4+a}} dx^2}{2a} - \frac{\left(a+bx^4\right)^{3/4}}{3ax^6} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{b \left(\frac{b \int \frac{1}{\sqrt[4]{bx^4+a}} dx^2}{2a} - \frac{\left(a+bx^4\right)^{3/4}}{ax^2} \right)}{2a} - \frac{\left(a+bx^4\right)^{3/4}}{3ax^6} \right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)$$

$\Big\downarrow$ 225

$$\frac{1}{2}\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)$$

$\Big\downarrow$ 212

$$\frac{1}{2}\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^4}}-\frac{(a+bx^4)^{3/4}}{ax^2}\right)}{2a}-\frac{(a+bx^4)^{3/4}}{3ax^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)^(1/4)),x]
```

output
```
(-1/3*(a + b*x^4)^(3/4)/(a*x^6) - (b*(-((a + b*x^4)^(3/4)/(a*x^2)) + (b*(1
 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 + (b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[
ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*a*(a + b*x^4)^(1/4))))/(
2*a))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^7/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^7/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^7\sqrt[4]{a+bx^4}}\,dx=\int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^11 + a*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{6\sqrt[4]{a}x^6}$$

input
```
integrate(1/x**7/(b*x**4+a)**(1/4),x)
```

output
```
-hyper((-3/2, 1/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(1/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{x^7\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(1/(x^7*(a + b*x^4)^(1/4)),x)
```

output
```
int(1/(x^7*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^7}dx$$

input
```
int(1/x^7/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**7),x)
```

3.548 $\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 152

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = \frac{7b^3x^2}{40a^3\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{10ax^{10}} + \frac{7b(a+bx^4)^{3/4}}{60a^2x^6} - \frac{7b^2(a+bx^4)^{3/4}}{40a^3x^2} - \frac{7b^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
7/40*b^3*x^2/a^3/(b*x^4+a)^(1/4)-1/10*(b*x^4+a)^(3/4)/a/x^10+7/60*b*(b*x^4
+a)^(3/4)/a^2/x^6-7/40*b^2*(b*x^4+a)^(3/4)/a^3/x^2-7/40*b^(5/2)*(1+b*x^4/a
)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/(b
*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},-\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^11*(a + b*x^4)^(1/4)),x]
```

output
```
-1/10*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x^4)/
a)])/(x^10*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {807, 264, 264, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^{12}\sqrt[4]{bx^4+a}}dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{7b\int \frac{1}{x^8\sqrt[4]{bx^4+a}}dx^2}{10a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^2}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^4+a}}dx^2}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 227

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^4}{a}+1}}dx^2}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 225

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 212

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^4}{a}+1}\left(\frac{2x^2}{\sqrt[4]{\frac{bx^4}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^4}}-\frac{\left(a+bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a+bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^{10}}\right)$$

input

```
Int[1/(x^11*(a + b*x^4)^(1/4)),x]
```

output

```
(-1/5*(a + b*x^4)^(3/4)/(a*x^10) - (7*b*(-1/3*(a + b*x^4)^(3/4)/(a*x^6) -
(b*(-((a + b*x^4)^(3/4)/(a*x^2)) + (b*(1 + (b*x^4)/a)^(1/4)*((2*x^2)/(1 +
(b*x^4)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/Sqrt[b]))/(2*a*(a + b*x^4)^(1/4))))/(2*a)))/(10*a))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{11} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^11/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^11/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^15 + a*x^11), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10\sqrt[4]{a}x^{10}}$$

input
```
integrate(1/x**11/(b*x**4+a)**(1/4),x)
```

output
```
-hyper((-5/2, 1/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(1/4)*x**10)
```

Maxima [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^11), x)
```

Giac [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{x^{11}\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(1/(x^11*(a + b*x^4)^(1/4)),x)
```

output
```
int(1/(x^11*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{11}}dx$$

input
```
int(1/x^11/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**11),x)
```

3.549 $\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = -\frac{5ax(a+bx^4)^{3/4}}{32b^2}+\frac{x^5(a+bx^4)^{3/4}}{8b}+\frac{5a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}+\frac{5a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}$$

output
```
-5/32*a*x*(b*x^4+a)^(3/4)/b^2+1/8*x^5*(b*x^4+a)^(3/4)/b+5/64*a^2*arctan(b^
(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)+5/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4
))/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = \frac{2\sqrt[4]{b}x(a+bx^4)^{3/4}(-5a+4bx^4)+5a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+5a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{9/4}}$$

input
```
Integrate[x^8/(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4)*(-5*a + 4*b*x^4) + 5*a^2*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)] + 5*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(64*b^
(9/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {843, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{8b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{8b}$$

$$\Big\downarrow \text{770}$$

$$\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}$$

$$\Big\downarrow \text{756}$$

$$\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int\frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{8b}$$

$$\Big\downarrow \ 216$$

$$\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}$$

$$\Big\downarrow \ 219$$

$$\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}$$

input
```
Int[x^8/(a + b*x^4)^(1/4),x]
```

output
```
(x^5*(a + b*x^4)^(3/4))/(8*b) - (5*a*((x*(a + b*x^4)^(3/4))/(4*b) - (a*(Ar
cTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$\frac{16\left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{5}{4}} x^{5}-20 a x \left(b\,x^{4}+a\right)^{\frac{3}{4}} b^{\frac{1}{4}}-10 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a^{2}+5 \ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a^{2}}{128 b^{\frac{9}{4}}}$	102

input
```
int(x^8/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(16*(b*x^4+a)^(3/4)*b^(5/4)*x^5-20*a*x*(b*x^4+a)^(3/4)*b^(1/4)-10*ar
ctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^2+5*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b
^(1/4)*x+(b*x^4+a)^(1/4)))*a^2)/b^(9/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.16

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = \frac{5\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{125\left(b^7x\left(\frac{a^8}{b^9}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}a^6\right)}{x}\right)-5\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{125\left(b^7x\left(\frac{a^8}{b^9}\right)^{\frac{3}{4}}-(bx^4+a)^{\frac{1}{4}}a^6\right)}{x}\right)+5i\,b^2\left(\frac{a^8}{b^9}\right)^{\frac{1}{4}}}{}$$

input
```
integrate(x^8/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/128*(5*b^2*(a^8/b^9)^(1/4)*log(125*(b^7*x*(a^8/b^9)^(3/4) + (b*x^4 + a)^
(1/4)*a^6)/x) - 5*b^2*(a^8/b^9)^(1/4)*log(-125*(b^7*x*(a^8/b^9)^(3/4) - (b
*x^4 + a)^(1/4)*a^6)/x) + 5*I*b^2*(a^8/b^9)^(1/4)*log(-125*(I*b^7*x*(a^8/b
^9)^(3/4) - (b*x^4 + a)^(1/4)*a^6)/x) - 5*I*b^2*(a^8/b^9)^(1/4)*log(-125*(
-I*b^7*x*(a^8/b^9)^(3/4) - (b*x^4 + a)^(1/4)*a^6)/x) + 4*(4*b*x^5 - 5*a*x)
*(b*x^4 + a)^(3/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.36

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(b*x**4+a)**(1/4),x)
```

output
```
x**9*gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(1/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.45

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = -\frac{5\,a^2\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b^2} + \frac{\frac{9\,(bx^4+a)^{\frac{3}{4}}a^2b}{x^3}-\frac{5\,(bx^4+a)^{\frac{7}{4}}a^2}{x^7}}{32\left(b^4-\frac{2\,(bx^4+a)b^3}{x^4}+\frac{(bx^4+a)^2b^2}{x^8}\right)}$$

input
```
integrate(x^8/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-5/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4
) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b^2 + 1
/32*(9*(b*x^4 + a)^(3/4)*a^2*b/x^3 - 5*(b*x^4 + a)^(7/4)*a^2/x^7)/(b^4 - 2
*(b*x^4 + a)*b^3/x^4 + (b*x^4 + a)^2*b^2/x^8)
```

Giac [F]

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^8}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^8/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(x^8/(a + b*x^4)^(1/4),x)
```

output
```
int(x^8/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^8}{(b\,x^4+a)^{\frac{1}{4}}}\,dx$$

input
```
int(x^8/(b*x^4+a)^(1/4),x)
```

output
```
int(x**8/(a + b*x**4)**(1/4),x)
```

3.550 $\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx=\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}-\frac{a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

output
```
1/4*x*(b*x^4+a)^(3/4)/b-1/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)-1/
8*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx=\frac{2\sqrt[4]{b}x(a+bx^4)^{3/4}-a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{5/4}}$$

input
```
Integrate[x^4/(a + b*x^4)^(1/4),x]
```

output
```
(2*b^(1/4)*x*(a + b*x^4)^(3/4) - a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] -
 a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(5/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \; 843$$

$$\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}$$

$$\Big\downarrow \; 770$$

$$\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}$$

$$\Big\downarrow \; 756$$

$$\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}$$

$$\Big\downarrow \; 216$$

$$\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}$$

$$\Big\downarrow \; 219$$

$$\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}$$

input
```
Int[x^4/(a + b*x^4)^(1/4),x]
```

output
```
(x*(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2
*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$-\frac{-4\left(b\,x^{4}+a\right)^{\frac{3}{4}} x\,b^{\frac{1}{4}}-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a+\ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a}{16 b^{\frac{5}{4}}}$	79

input
```
int(x^4/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/16/b^(5/4)*(-4*(b*x^4+a)^(3/4)*x*b^(1/4)-2*arctan(1/b^(1/4)/x*(b*x^4+a)
^(1/4))*a+ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx = -\frac{b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{b^4x\left(\frac{a^4}{b^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}a^3}{x}\right)-b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(-\frac{b^4x\left(\frac{a^4}{b^5}\right)^{\frac{3}{4}}-(bx^4+a)^{\frac{1}{4}}a^3}{x}\right)-i\,b\left(\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{i\,b^4x\left(\frac{a^4}{b^5}\right)^{\frac{3}{4}}+\ldots}{x}\right)\ldots}{16\,b}$$

input
```
integrate(x^4/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/16*(b*(a^4/b^5)^(1/4)*log((b^4*x*(a^4/b^5)^(3/4) + (b*x^4 + a)^(1/4)*a^
3)/x) - b*(a^4/b^5)^(1/4)*log(-(b^4*x*(a^4/b^5)^(3/4) - (b*x^4 + a)^(1/4)*
a^3)/x) - I*b*(a^4/b^5)^(1/4)*log((I*b^4*x*(a^4/b^5)^(3/4) + (b*x^4 + a)^(
1/4)*a^3)/x) + I*b*(a^4/b^5)^(1/4)*log((-I*b^4*x*(a^4/b^5)^(3/4) + (b*x^4
+ a)^(1/4)*a^3)/x) - 4*(b*x^4 + a)^(3/4)*x)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(b*x**4+a)**(1/4),x)
```

output
```
x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(1/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.38

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx = \frac{a\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{16\,b} - \frac{(bx^4+a)^{\frac{3}{4}}a}{4\left(b^2-\frac{(bx^4+a)b}{x^4}\right)x^3}$$

input
```
integrate(x^4/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/16*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) -
(b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - 1/4*(b*
x^4 + a)^(3/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x^3)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(x^4/(a + b*x^4)^(1/4),x)
```

output
```
int(x^4/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(b*x^4+a)^(1/4),x)
```

output
```
int(x**4/(a + b*x**4)**(1/4),x)
```

3.551 $\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 57

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

output
```
1/2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(1/4)+1/2*arctanh(b^(1/4)*x/(b*x^4
+a)^(1/4))/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \frac{2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-\log\left(1-\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\log\left(1+\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{4\sqrt[4]{b}}$$

input
```
Integrate[(a + b*x^4)^(-1/4),x]
```

output
```
(2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] - Log[1 - (b^(1/4)*x)/(a + b*x^4)
^(1/4)] + Log[1 + (b^(1/4)*x)/(a + b*x^4)^(1/4)])/(4*b^(1/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 770}$$

$$\int \frac{1}{1-\frac{bx^4}{a+bx^4}}d\frac{x}{\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}$$

input
```
Int[(a + b*x^4)^(-1/4),x]
```

output
```
ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)]/(2*b^(1/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)+\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)}{4b^{\frac{1}{4}}}$	61

input
```
int(1/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))+ln((b^(1/4)*x+(b*x^4+a)^(1/4))
/(-b^(1/4)*x+(b*x^4+a)^(1/4))))/b^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \frac{\log\left(\frac{b^{\frac{1}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right)}{4\,b^{\frac{1}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}x-(bx^4+a)^{\frac{1}{4}}}{x}\right)}{4\,b^{\frac{1}{4}}} - \frac{i\,\log\left(\frac{i\,b^{\frac{1}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right)}{4\,b^{\frac{1}{4}}} + \frac{i\,\log\left(\frac{-i\,b^{\frac{1}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right)}{4\,b^{\frac{1}{4}}}$$

input
```
integrate(1/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/4*log((b^(1/4)*x + (b*x^4 + a)^(1/4))/x)/b^(1/4) - 1/4*log(-(b^(1/4)*x -
 (b*x^4 + a)^(1/4))/x)/b^(1/4) - 1/4*I*log((I*b^(1/4)*x + (b*x^4 + a)^(1/4
))/x)/b^(1/4) + 1/4*I*log((-I*b^(1/4)*x + (b*x^4 + a)^(1/4))/x)/b^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{4}\\\frac{5}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(b*x**4+a)**(1/4),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/
4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = -\frac{\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{2\,b^{\frac{1}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{4\,b^{\frac{1}{4}}}$$

input
```
integrate(1/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) - 1/4*log(-(b^(1/4) - (
b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \frac{x\left(\frac{b\,x^4}{a}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{\left(b\,x^4+a\right)^{1/4}}$$

input
```
int(1/(a + b*x^4)^(1/4),x)
```

output
```
(x*((b*x^4)/a + 1)^(1/4)*hypergeom([1/4, 1/4], 5/4, -(b*x^4)/a))/(a + b*x^
4)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^4+a)^(1/4),x)
```

output
```
int(1/(a + b*x**4)**(1/4),x)
```

3.552 $\int \frac{1}{x^4 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}} \, dx = -\frac{(a+bx^4)^{3/4}}{3ax^3}$$

output
```
-1/3*(b*x^4+a)^(3/4)/a/x^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}} \, dx = -\frac{(a+bx^4)^{3/4}}{3ax^3}$$

input
```
Integrate[1/(x^4*(a + b*x^4)^(1/4)),x]
```

output
```
-1/3*(a + b*x^4)^(3/4)/(a*x^3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \ 796$$

$$-\frac{\left(a+bx^4\right)^{3/4}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^4)^(1/4)),x]
```

output
```
-1/3*(a + b*x^4)^(3/4)/(a*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a\,x^{3}}$	18
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a\,x^{3}}$	18
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a\,x^{3}}$	18
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a\,x^{3}}$	18
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a\,x^{3}}$	18

input
```
int(1/x^4/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^4+a)^(3/4)/a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt[4]{a+bx^4}}\,dx = -\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,ax^3}$$

input
```
integrate(1/x^4/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/3*(b*x^4 + a)^(3/4)/(a*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}}\,dx = \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(b*x**4+a)**(1/4),x)
```

output
```
b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-3/4)/(4*a*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}}\,dx = -\frac{(bx^4+a)^{\frac{3}{4}}}{3\,ax^3}$$

input
```
integrate(1/x^4/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/3*(b*x^4 + a)^(3/4)/(a*x^3)
```

Giac [F]

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4\sqrt[4]{a+bx^4}}\,dx = -\frac{\left(b\,x^4+a\right)^{3/4}}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a + b*x^4)^(1/4)),x)
```

output
```
-(a + b*x^4)^(3/4)/(3*a*x^3)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/x^4/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**4),x)
```

3.553 $\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx = -\frac{(a+bx^4)^{3/4}}{7ax^7}+\frac{4b(a+bx^4)^{3/4}}{21a^2x^3}$$

output
```
-1/7*(b*x^4+a)^(3/4)/a/x^7+4/21*b*(b*x^4+a)^(3/4)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}(-3a+4bx^4)}{21a^2x^7}$$

input
```
Integrate[1/(x^8*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-3*a + 4*b*x^4))/(21*a^2*x^7)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx}{7a}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}$$

input
```
Int[1/(x^8*(a + b*x^4)^(1/4)),x]
```

output
```
-1/7*(a + b*x^4)^(3/4)/(a*x^7) + (4*b*(a + b*x^4)^(3/4))/(21*a^2*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-4b\,x^4+3a)}{21a^2x^7}$	28
trager	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-4b\,x^4+3a)}{21a^2x^7}$	28
risch	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-4b\,x^4+3a)}{21a^2x^7}$	28
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-4b\,x^4+3a)}{21a^2x^7}$	28
orering	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-4b\,x^4+3a)}{21a^2x^7}$	28

input
```
int(1/x^8/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(b*x^4+a)^(3/4)*(-4*b*x^4+3*a)/a^2/x^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx = \frac{(4\,bx^4-3\,a)(bx^4+a)^{\frac{3}{4}}}{21\,a^2x^7}$$

input
```
integrate(1/x^8/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/21*(4*b*x^4 - 3*a)*(b*x^4 + a)^(3/4)/(a^2*x^7)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx = -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16ax^4\Gamma\left(\frac{1}{4}\right)} + \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a^2\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**8/(b*x**4+a)**(1/4),x)
```

output
```
-3*b**(3/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(16*a*x**4*gamma(1/4)) + b
**(7/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(4*a**2*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx = \frac{\frac{7\,(bx^4+a)^{\frac{3}{4}}b}{x^3} - \frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}}{21\,a^2}$$

input
```
integrate(1/x^8/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/21*(7*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^4 + a)^(7/4)/x^7)/a^2
```

Giac [F]

$$\int \frac{1}{x^8 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx = -\frac{3\,a\,{\left(b\,x^4+a\right)}^{3/4}-4\,b\,x^4\,{\left(b\,x^4+a\right)}^{3/4}}{21\,a^2\,x^7}$$

input
```
int(1/(x^8*(a + b*x^4)^(1/4)),x)
```

output
```
-(3*a*(a + b*x^4)^(3/4) - 4*b*x^4*(a + b*x^4)^(3/4))/(21*a^2*x^7)
```

Reduce [F]

$$\int \frac{1}{x^8\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^8}dx$$

input
```
int(1/x^8/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**8),x)
```

3.554 $\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{(a+bx^4)^{3/4}}{11ax^{11}}+\frac{8b(a+bx^4)^{3/4}}{77a^2x^7}-\frac{32b^2(a+bx^4)^{3/4}}{231a^3x^3}$$

output
```
-1/11*(b*x^4+a)^(3/4)/a/x^11+8/77*b*(b*x^4+a)^(3/4)/a^2/x^7-32/231*b^2*(b*
x^4+a)^(3/4)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}\left(-21a^2+24abx^4-32b^2x^8\right)}{231a^3x^{11}}$$

input
```
Integrate[1/(x^12*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-21*a^2 + 24*a*b*x^4 - 32*b^2*x^8))/(231*a^3*x^11)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12} \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{803}$$

$$-\frac{8b \int \frac{1}{x^8 \sqrt[4]{bx^4+a}}dx}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

$$\Big\downarrow \text{803}$$

$$-\frac{8b \left(-\frac{4b \int \frac{1}{x^4 \sqrt[4]{bx^4+a}}dx}{7a}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{8b \left(\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}$$

input
```
Int[1/(x^12*(a + b*x^4)^(1/4)),x]
```

output
```
-1/11*(a + b*x^4)^(3/4)/(a*x^11) - (8*b*(-1/7*(a + b*x^4)^(3/4)/(a*x^7) +
(4*b*(a + b*x^4)^(3/4))/(21*a^2*x^3)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}-24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	39
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}-24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	39
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}-24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}-24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	39
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}-24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	39

input
```
int(1/x^12/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/231*(b*x^4+a)^(3/4)*(32*b^2*x^8-24*a*b*x^4+21*a^2)/a^3/x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{(32\,b^2x^8-24\,abx^4+21\,a^2)(bx^4+a)^{\frac{3}{4}}}{231\,a^3x^{11}}$$

input
```
integrate(1/x^12/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/231*(32*b^2*x^8 - 24*a*b*x^4 + 21*a^2)*(b*x^4 + a)^(3/4)/(a^3*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(61) = 122.

Time = 0.91 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.97

$$\begin{aligned}\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx &= \frac{21a^4b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}\\&+\frac{18a^3b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}\\&+\frac{5a^2b^{\frac{27}{4}}x^8\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}\\&+\frac{40ab^{\frac{31}{4}}x^{12}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}\\&+\frac{32b^{\frac{35}{4}}x^{16}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{1}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{1}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{1}{4}\right)}\end{aligned}$$

input
```
integrate(1/x**12/(b*x**4+a)**(1/4),x)
```

output
```
21*a**4*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4*x**8*
gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16*gamma(1/4
)) + 18*a**3*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*
b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b**6*x**16
*gamma(1/4)) + 5*a**2*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/
(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 64*a**3*b
**6*x**16*gamma(1/4)) + 40*a*b**(31/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma
(-11/4)/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/4) + 6
4*a**3*b**6*x**16*gamma(1/4)) + 32*b**(35/4)*x**16*(a/(b*x**4) + 1)**(3/4)
*gamma(-11/4)/(64*a**5*b**4*x**8*gamma(1/4) + 128*a**4*b**5*x**12*gamma(1/
4) + 64*a**3*b**6*x**16*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{\frac{77\,(bx^4+a)^{\frac{3}{4}}b^2}{x^3} - \frac{66\,(bx^4+a)^{\frac{7}{4}}b}{x^7} + \frac{21\,(bx^4+a)^{\frac{11}{4}}}{x^{11}}}{231\,a^3}$$

input
```
integrate(1/x^12/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/231*(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 66*(b*x^4 + a)^(7/4)*b/x^7 + 21*(b*
x^4 + a)^(11/4)/x^11)/a^3
```

Giac [F]

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{12}}\,dx$$

input
```
integrate(1/x^12/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = -\frac{21\,a^2\,{\left(b\,x^4+a\right)}^{3/4}+32\,b^2\,x^8\,{\left(b\,x^4+a\right)}^{3/4}-24\,a\,b\,x^4\,{\left(b\,x^4+a\right)}^{3/4}}{231\,a^3\,x^{11}}$$

input
```
int(1/(x^12*(a + b*x^4)^(1/4)),x)
```

output
```
-(21*a^2*(a + b*x^4)^(3/4) + 32*b^2*x^8*(a + b*x^4)^(3/4) - 24*a*b*x^4*(a
+ b*x^4)^(3/4))/(231*a^3*x^11)
```

Reduce [F]

$$\int \frac{1}{x^{12}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}x^{12}}dx$$

input
```
int(1/x^12/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**12),x)
```

3.555 $\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = -\frac{(a+bx^4)^{3/4}}{15ax^{15}} + \frac{4b(a+bx^4)^{3/4}}{55a^2x^{11}} - \frac{32b^2(a+bx^4)^{3/4}}{385a^3x^7} + \frac{128b^3(a+bx^4)^{3/4}}{1155a^4x^3}$$

output
```
-1/15*(b*x^4+a)^(3/4)/a/x^15+4/55*b*(b*x^4+a)^(3/4)/a^2/x^11-32/385*b^2*(b
*x^4+a)^(3/4)/a^3/x^7+128/1155*b^3*(b*x^4+a)^(3/4)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}\left(-77a^3+84a^2bx^4-96ab^2x^8+128b^3x^{12}\right)}{1155a^4x^{15}}$$

input
```
Integrate[1/(x^16*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-77*a^3 + 84*a^2*b*x^4 - 96*a*b^2*x^8 + 128*b^3*x^12))
/(1155*a^4*x^15)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\int \frac{1}{x^{12}\sqrt[4]{bx^4+a}}dx}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\left(-\frac{8b\int \frac{1}{x^8\sqrt[4]{bx^4+a}}dx}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx}{7a}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{4b\left(-\frac{8b\left(\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}$$

input
```
Int[1/(x^16*(a + b*x^4)^(1/4)),x]
```

output
```
-1/15*(a + b*x^4)^(3/4)/(a*x^15) - (4*b*(-1/11*(a + b*x^4)^(3/4)/(a*x^11)
- (8*b*(-1/7*(a + b*x^4)^(3/4)/(a*x^7) + (4*b*(a + b*x^4)^(3/4))/(21*a^2*x
^3)))/(11*a)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-128 b^{3} x^{12}+96 a\,b^{2} x^{8}-84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	50
trager	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-128 b^{3} x^{12}+96 a\,b^{2} x^{8}-84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	50
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-128 b^{3} x^{12}+96 a\,b^{2} x^{8}-84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-128 b^{3} x^{12}+96 a\,b^{2} x^{8}-84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	50
orering	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}} \left(-128 b^{3} x^{12}+96 a\,b^{2} x^{8}-84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	50

input
```
int(1/x^16/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(b*x^4+a)^(3/4)*(-128*b^3*x^12+96*a*b^2*x^8-84*a^2*b*x^4+77*a^3)/x
^15/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{(128\,b^3x^{12} - 96\,ab^2x^8 + 84\,a^2bx^4 - 77\,a^3)(bx^4+a)^{\frac{3}{4}}}{1155\,a^4x^{15}}$$

input
```
integrate(1/x^16/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/1155*(128*b^3*x^12 - 96*a*b^2*x^8 + 84*a^2*b*x^4 - 77*a^3)*(b*x^4 + a)^(
3/4)/(a^4*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(85) = 170.

Time = 1.26 (sec) , antiderivative size = 692, normalized size of antiderivative = 7.52

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**16/(b*x**4+a)**(1/4),x)
```

output
```
-231*a**6*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x*
*12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*ga
mma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) - 441*a**5*b**(43/4)*x**4*(a/(
b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**
6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**1
2*x**24*gamma(1/4)) - 225*a**4*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamm
a(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4)
 + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 45
*a**3*b**(51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*
x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*
gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 540*a**2*b**(55/4)*x**16*(
a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*
a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b
**12*x**24*gamma(1/4)) + 864*a*b**(59/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gam
ma(-15/4)/(256*a**7*b**9*x**12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4
) + 768*a**5*b**11*x**20*gamma(1/4) + 256*a**4*b**12*x**24*gamma(1/4)) + 3
84*b**(63/4)*x**24*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(256*a**7*b**9*x**
12*gamma(1/4) + 768*a**6*b**10*x**16*gamma(1/4) + 768*a**5*b**11*x**20*gam
ma(1/4) + 256*a**4*b**12*x**24*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{\frac{385\,(bx^4+a)^{\frac{3}{4}}b^3}{x^3} - \frac{495\,(bx^4+a)^{\frac{7}{4}}b^2}{x^7} + \frac{315\,(bx^4+a)^{\frac{11}{4}}b}{x^{11}} - \frac{77\,(bx^4+a)^{\frac{15}{4}}}{x^{15}}}{1155\,a^4}$$

input
```
integrate(1/x^16/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/1155*(385*(b*x^4 + a)^(3/4)*b^3/x^3 - 495*(b*x^4 + a)^(7/4)*b^2/x^7 + 31
5*(b*x^4 + a)^(11/4)*b/x^11 - 77*(b*x^4 + a)^(15/4)/x^15)/a^4
```

Giac [F]

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{16}}\,dx$$

input
```
integrate(1/x^16/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^16), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \frac{4\,b\,{\left(b\,x^4+a\right)}^{3/4}}{55\,a^2\,x^{11}} - \frac{{\left(b\,x^4+a\right)}^{3/4}}{15\,a\,x^{15}} + \frac{128\,b^3\,{\left(b\,x^4+a\right)}^{3/4}}{1155\,a^4\,x^3} - \frac{32\,b^2\,{\left(b\,x^4+a\right)}^{3/4}}{385\,a^3\,x^7}$$

input
```
int(1/(x^16*(a + b*x^4)^(1/4)),x)
```

output
```
(4*b*(a + b*x^4)^(3/4))/(55*a^2*x^11) - (a + b*x^4)^(3/4)/(15*a*x^15) + (1
28*b^3*(a + b*x^4)^(3/4))/(1155*a^4*x^3) - (32*b^2*(a + b*x^4)^(3/4))/(385
*a^3*x^7)
```

Reduce [F]

$$\int \frac{1}{x^{16}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{16}}dx$$

input
```
int(1/x^16/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**16),x)
```

3.556 $\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = -\frac{(a+bx^4)^{3/4}}{19ax^{19}}+\frac{16b(a+bx^4)^{3/4}}{285a^2x^{15}}-\frac{64b^2(a+bx^4)^{3/4}}{1045a^3x^{11}}+\frac{512b^3(a+bx^4)^{3/4}}{7315a^4x^7}-\frac{2048b^4(a+bx^4)^{3/4}}{21945a^5x^3}$$

output
```
-1/19*(b*x^4+a)^(3/4)/a/x^19+16/285*b*(b*x^4+a)^(3/4)/a^2/x^15-64/1045*b^2
*(b*x^4+a)^(3/4)/a^3/x^11+512/7315*b^3*(b*x^4+a)^(3/4)/a^4/x^7-2048/21945*
b^4*(b*x^4+a)^(3/4)/a^5/x^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = \frac{(a+bx^4)^{3/4}(-1155a^4+1232a^3bx^4-1344a^2b^2x^8+1536ab^3x^{12}-2048b^4x^{16})}{21945a^5x^{19}}$$

input
```
Integrate[1/(x^20*(a + b*x^4)^(1/4)),x]
```

output
```
((a + b*x^4)^(3/4)*(-1155*a^4 + 1232*a^3*b*x^4 - 1344*a^2*b^2*x^8 + 1536*a
*b^3*x^12 - 2048*b^4*x^16))/(21945*a^5*x^19)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\int \frac{1}{x^{16}\sqrt[4]{bx^4+a}}dx}{19a}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\left(-\frac{4b\int \frac{1}{x^{12}\sqrt[4]{bx^4+a}}dx}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\left(-\frac{4b\left(-\frac{8b\int \frac{1}{x^{8}\sqrt[4]{bx^4+a}}dx}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\left(-\frac{4b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx}{7a}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{796}$$

$$-\frac{16b\left(-\frac{4b\left(-\frac{8b\left(\frac{4b\left(a+bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a+bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a+bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a+bx^4\right)^{3/4}}{19ax^{19}}$$

input
```
Int[1/(x^20*(a + b*x^4)^(1/4)),x]
```

output
```
-1/19*(a + b*x^4)^(3/4)/(a*x^19) - (16*b*(-1/15*(a + b*x^4)^(3/4)/(a*x^15)
 - (4*b*(-1/11*(a + b*x^4)^(3/4)/(a*x^11) - (8*b*(-1/7*(a + b*x^4)^(3/4)/(
a*x^7) + (4*b*(a + b*x^4)^(3/4))/(21*a^2*x^3)))/(11*a)))/(5*a)))/(19*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(2048x^{16}b^4-1536a\,b^3x^{12}+1344a^2b^2x^8-1232a^3b\,x^4+1155a^4)}{21945x^{19}a^5}$	61
trager	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(2048x^{16}b^4-1536a\,b^3x^{12}+1344a^2b^2x^8-1232a^3b\,x^4+1155a^4)}{21945x^{19}a^5}$	61
risch	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(2048x^{16}b^4-1536a\,b^3x^{12}+1344a^2b^2x^8-1232a^3b\,x^4+1155a^4)}{21945x^{19}a^5}$	61
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(2048x^{16}b^4-1536a\,b^3x^{12}+1344a^2b^2x^8-1232a^3b\,x^4+1155a^4)}{21945x^{19}a^5}$	61
orering	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(2048x^{16}b^4-1536a\,b^3x^{12}+1344a^2b^2x^8-1232a^3b\,x^4+1155a^4)}{21945x^{19}a^5}$	61

input
```
int(1/x^20/(b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21945*(b*x^4+a)^(3/4)*(2048*b^4*x^16-1536*a*b^3*x^12+1344*a^2*b^2*x^8-1
232*a^3*b*x^4+1155*a^4)/x^19/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = -\frac{\left(2048\,b^4x^{16}-1536\,ab^3x^{12}+1344\,a^2b^2x^8-1232\,a^3bx^4+1155\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{21945\,a^5x^{19}}$$

input
```
integrate(1/x^20/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/21945*(2048*b^4*x^16 - 1536*a*b^3*x^12 + 1344*a^2*b^2*x^8 - 1232*a^3*b*
x^4 + 1155*a^4)*(b*x^4 + a)^(3/4)/(a^5*x^19)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(109) = 218.

Time = 1.86 (sec) , antiderivative size = 1046, normalized size of antiderivative = 9.02

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**20/(b*x**4+a)**(1/4),x)
```

output
```
3465*a**8*b**(67/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*
x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a**7*b**18*x**2
4*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b**20*x**32*ga
mma(1/4)) + 10164*a**7*b**(71/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)
/(1024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 61
44*a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a
**5*b**20*x**32*gamma(1/4)) + 10038*a**6*b**(75/4)*x**8*(a/(b*x**4) + 1)**
(3/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**
20*gamma(1/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*g
amma(1/4) + 1024*a**5*b**20*x**32*gamma(1/4)) + 3204*a**5*b**(79/4)*x**12*
(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*gamma(1/4) + 4
096*a**8*b**17*x**20*gamma(1/4) + 6144*a**7*b**18*x**24*gamma(1/4) + 4096*
a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b**20*x**32*gamma(1/4)) + 585*a**4
*b**(83/4)*x**16*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1024*a**9*b**16*x**
16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*a**7*b**18*x**24*g
amma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5*b**20*x**32*gamma
(1/4)) + 9360*a**3*b**(87/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-19/4)/(1
024*a**9*b**16*x**16*gamma(1/4) + 4096*a**8*b**17*x**20*gamma(1/4) + 6144*
a**7*b**18*x**24*gamma(1/4) + 4096*a**6*b**19*x**28*gamma(1/4) + 1024*a**5
*b**20*x**32*gamma(1/4)) + 22464*a**2*b**(91/4)*x**24*(a/(b*x**4) + 1)*...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = -\frac{\frac{7315\,(bx^4+a)^{\frac{3}{4}}b^4}{x^3} - \frac{12540\,(bx^4+a)^{\frac{7}{4}}b^3}{x^7} + \frac{11970\,(bx^4+a)^{\frac{11}{4}}b^2}{x^{11}} - \frac{5852\,(bx^4+a)^{\frac{15}{4}}b}{x^{15}} + \frac{1155\,(bx^4+a)^{\frac{19}{4}}}{x^{19}}}{21945\,a^5}$$

input
```
integrate(1/x^20/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/21945*(7315*(b*x^4 + a)^(3/4)*b^4/x^3 - 12540*(b*x^4 + a)^(7/4)*b^3/x^7
 + 11970*(b*x^4 + a)^(11/4)*b^2/x^11 - 5852*(b*x^4 + a)^(15/4)*b/x^15 + 11
55*(b*x^4 + a)^(19/4)/x^19)/a^5
```

Giac [F]

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{20}}\,dx$$

input
```
integrate(1/x^20/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^20), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = \frac{16\,b\,(b\,x^4+a)^{3/4}}{285\,a^2\,x^{15}} - \frac{(b\,x^4+a)^{3/4}}{19\,a\,x^{19}} - \frac{2048\,b^4\,(b\,x^4+a)^{3/4}}{21945\,a^5\,x^3} + \frac{512\,b^3\,(b\,x^4+a)^{3/4}}{7315\,a^4\,x^7} - \frac{64\,b^2\,(b\,x^4+a)^{3/4}}{1045\,a^3\,x^{11}}$$

input
```
int(1/(x^20*(a + b*x^4)^(1/4)),x)
```

output
```
(16*b*(a + b*x^4)^(3/4))/(285*a^2*x^15) - (a + b*x^4)^(3/4)/(19*a*x^19) -
(2048*b^4*(a + b*x^4)^(3/4))/(21945*a^5*x^3) + (512*b^3*(a + b*x^4)^(3/4))
/(7315*a^4*x^7) - (64*b^2*(a + b*x^4)^(3/4))/(1045*a^3*x^11)
```

Reduce [F]

$$\int \frac{1}{x^{20}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{20}}dx$$

input
```
int(1/x^20/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**20),x)
```

3.557 $\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \frac{7a^2x^3}{40b^2\sqrt[4]{a+bx^4}} - \frac{7ax^3(a+bx^4)^{3/4}}{60b^2} + \frac{x^7(a+bx^4)^{3/4}}{10b} + \frac{7a^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
7/40*a^2*x^3/b^2/(b*x^4+a)^(1/4)-7/60*a*x^3*(b*x^4+a)^(3/4)/b^2+1/10*x^7*(
b*x^4+a)^(3/4)/b+7/40*a^(5/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot
(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^3\left(-7a^2-abx^4+6b^2x^8+7a^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{60b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^10/(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(-7*a^2 - a*b*x^4 + 6*b^2*x^8 + 7*a^2*(1 + (b*x^4)/a)^(1/4)*Hypergeom
etric2F1[1/4, 3/4, 7/4, -((b*x^4)/a)]))/(60*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {843, 843, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\int \frac{x^6}{\sqrt[4]{bx^4+a}}dx}{10b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx}{2b}\right)}{10b}$$

$$\Big\downarrow \text{ 839}$$

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{(bx^4+a)^{5/4}}dx\right)}{2b}\right)}{10b}$$

$$\Big\downarrow \text{ 813}$$

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}$$

$\downarrow$ 858

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}$$

$\downarrow$ 807

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}$$

$\downarrow$ 212

$$\frac{x^7(a+bx^4)^{3/4}}{10b}-\frac{7a\left(\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}\right)}{10b}$$

input
```
Int[x^10/(a + b*x^4)^(1/4),x]
```

output
```
(x^7*(a + b*x^4)^(3/4))/(10*b) - (7*a*((x^3*(a + b*x^4)^(3/4))/(6*b) - (a*
(x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[Ar
cTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(1/4))))/(2*b)))
/(10*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{10}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^10/(b*x^4+a)^(1/4),x)
```

output
```
int(x^10/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^10/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(b*x**4+a)**(1/4),x)
```

output
```
x**11*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(1/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^10/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^10/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{10}}{{\left(b\,x^4+a\right)}^{1/4}} \,dx$$

input
```
int(x^10/(a + b*x^4)^(1/4),x)
```

output
```
int(x^10/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^{10}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}d x$$

input
```
int(x^10/(b*x^4+a)^(1/4),x)
```

output
```
int(x**10/(a + b*x**4)**(1/4),x)
```

3.558 $\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx = -\frac{ax^3}{4b\sqrt[4]{a+bx^4}}+\frac{x^3(a+bx^4)^{3/4}}{6b}-\frac{a^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/4*a*x^3/b/(b*x^4+a)^(1/4)+1/6*x^3*(b*x^4+a)^(3/4)/b-1/4*a^(3/2)*(1+a/b/
x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(3/
2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.61 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^3\left(a+bx^4-a\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{6b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^6/(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(a + b*x^4 - a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4,
 -((b*x^4)/a)]))/(6*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {843, 839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\int \frac{x^2}{\sqrt[4]{bx^4+a}}dx}{2b}$$

$$\Big\downarrow \text{839}$$

$$\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx\right)}{2b}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^3\left(a+bx^4\right)^{3/4}}{6b}-\frac{a\left(\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}\right)}{2b}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^3 (a+bx^4)^{3/4}}{6b} - \frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}} + \frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^3 (a+bx^4)^{3/4}}{6b} - \frac{a\left(\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}} + \frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}$$

$$\Big\downarrow \text{212}$$

$$\frac{x^3 (a+bx^4)^{3/4}}{6b} - \frac{a\left(\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}} + \frac{x^3}{2\sqrt[4]{a+bx^4}}\right)}{2b}$$

input
```
Int[x^6/(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(a + b*x^4)^(3/4))/(6*b) - (a*(x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(
1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*S
qrt[b]*(a + b*x^4)^(1/4))))/(2*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(b*x^4+a)^(1/4),x)
```

output
```
int(x^6/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^6}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(b*x**4+a)**(1/4),x)
```

output
```
x**7*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(1/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx=\int \frac{x^6}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(x^6/(a + b*x^4)^(1/4),x)
```

output
```
int(x^6/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(b*x^4+a)^(1/4),x)
```

output
```
int(x**6/(a + b*x**4)**(1/4),x)
```

3.559 $\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^3}{2\sqrt[4]{a+bx^4}}+\frac{\sqrt{a}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
1/2*x^3/(b*x^4+a)^(1/4)+1/2*a^(1/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*
arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx=\frac{x^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^2/(a + b*x^4)^(1/4),x]
```

output
```
(x^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^4)/a)])
/(3*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {839, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{839}$$

$$\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{1}{2}a\int \frac{x^2}{(bx^4+a)^{5/4}}dx$$

$$\Big\downarrow \text{813}$$

$$\frac{x^3}{2\sqrt[4]{a+bx^4}}-\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a+bx^4}}+\frac{x^3}{2\sqrt[4]{a+bx^4}}$$

input
```
Int[x^2/(a + b*x^4)^(1/4),x]
```

output
```
x^3/(2*(a + b*x^4)^(1/4)) + (Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[Arc
Tan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 839
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[x^3/(2*(a + b*x^4
)^(1/4)), x] - Simp[a/2   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}
, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(b*x^4+a)^(1/4),x)
```

output
```
int(x^2/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(b*x**4+a)**(1/4),x)
```

output
```
x**3*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(1/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^2}{(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(x^2/(a + b*x^4)^(1/4),x)
```

output
```
int(x^2/(a + b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^4}}\,dx = \int \frac{x^2}{\left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(b*x^4+a)^(1/4),x)
```

output
```
int(x**2/(a + b*x**4)**(1/4),x)
```

3.560 $\int \frac{1}{x^2 \sqrt[4]{a+bx^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^4}} \, dx = -\frac{1}{x\sqrt[4]{a+bx^4}} + \frac{\sqrt{b}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}$$

output
```
-1/x/(b*x^4+a)^(1/4)+b^(1/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(
b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^4}} \, dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{x\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^2*(a + b*x^4)^(1/4)),x]
```

output
```
-(((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x^4)/a)])/
(x*(a + b*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{ 841}$$

$$-b\int \frac{x^2}{(bx^4+a)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^2*(a + b*x^4)^(1/4)),x]
```

output
```
-(1/(x*(a + b*x^4)^(1/4))) + (Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[Ar
cTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(Sqrt[a]*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^2 (b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^2/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^2/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(b*x**4+a)**(1/4),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1/
4)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx = -\frac{\left(\frac{a}{b\,x^4}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{a}{b\,x^4}\right)}{2\,x\,\left(b\,x^4+a\right)^{1/4}}$$

input
```
int(1/(x^2*(a + b*x^4)^(1/4)),x)
```

output
```
-((a/(b*x^4) + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -a/(b*x^4)))/(2*x*(a +
b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} x^2}dx$$

input
```
int(1/x^2/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**2),x)
```

3.561 $\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx = \frac{2b}{5ax\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{5ax^5} - \frac{2b^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
2/5*b/a/x/(b*x^4+a)^(1/4)-1/5*(b*x^4+a)^(3/4)/a/x^5-2/5*b^(3/2)*(1+a/b/x^4
)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/
(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},-\frac{bx^4}{a}\right)}{5x^5\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^6*(a + b*x^4)^(1/4)),x]
```

output
```
-1/5*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x^4)/a
)])/(x^5*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {847, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^4}} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{2b \int \frac{1}{x^2 \sqrt[4]{bx^4+a}}dx}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{841}$$

$$-\frac{2b\left(-b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{813}$$

$$-\frac{2b\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \textbf{807}$$

$$-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \textbf{212}$$

$$-\frac{2b\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^4)^(1/4)),x]
```

output
```
-1/5*(a + b*x^4)^(3/4)/(a*x^5) - (2*b*(-(1/(x*(a + b*x^4)^(1/4))) + (Sqrt[
b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/
(Sqrt[a]*(a + b*x^4)^(1/4))))/(5*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^6 \left(b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^6/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^6/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^10 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{6\sqrt[4]{b}x^6}$$

input
```
integrate(1/x**6/(b*x**4+a)**(1/4),x)
```

output
```
-hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**4))/(6*b**(1/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{x^6\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(1/(x^6*(a + b*x^4)^(1/4)),x)
```

output
```
int(1/(x^6*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^6}dx$$

input
```
int(1/x^6/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**6),x)
```

3.562 $\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = -\frac{4b^2}{15a^2x\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{9ax^9} + \frac{2b(a+bx^4)^{3/4}}{15a^2x^5} + \frac{4b^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-4/15*b^2/a^2/x/(b*x^4+a)^(1/4)-1/9*(b*x^4+a)^(3/4)/a/x^9+2/15*b*(b*x^4+a)
^(3/4)/a^2/x^5+4/15*b^(5/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b
^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},-\frac{bx^4}{a}\right)}{9x^9\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^10*(a + b*x^4)^(1/4)),x]
```

output
```
-1/9*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x^4)/a
)])/(x^9*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {847, 847, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{2b\int \frac{1}{x^6\sqrt[4]{bx^4+a}}dx}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2b\left(-\frac{2b\int \frac{1}{x^2\sqrt[4]{bx^4+a}}dx}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{841}$$

$$-\frac{2b\left(-\frac{2b\left(-b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{813}$$

$$-\frac{2b\left(-\frac{2b\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{212}$$

$$-\frac{2b\left(-\frac{2b\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{(a+bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a+bx^4)^{3/4}}{9ax^9}$$

input
```
Int[1/(x^10*(a + b*x^4)^(1/4)),x]
```

output
```
-1/9*(a + b*x^4)^(3/4)/(a*x^9) - (2*b*(-1/5*(a + b*x^4)^(3/4)/(a*x^5) - (2
*b*(-(1/(x*(a + b*x^4)^(1/4))) + (Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*Elliptic
E[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(Sqrt[a]*(a + b*x^4)^(1/4))))/(5*a)
))/(3*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{10} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^10/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^10/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{10} \sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^14 + a*x^10), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{9}{4}, \frac{1}{4} \\ -\frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}x^9\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate(1/x**10/(b*x**4+a)**(1/4),x)
```

output
```
gamma(-9/4)*hyper((-9/4, 1/4), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(1
/4)*x**9*gamma(-5/4))
```

Maxima [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{x^{10}\,{\left(b\,x^4+a\right)}^{1/4}}\,dx$$

input
```
int(1/(x^10*(a + b*x^4)^(1/4)),x)
```

output
```
int(1/(x^10*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\,x^{10}}dx$$

input
```
int(1/x^10/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**10),x)
```

3.563 $\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \frac{8b^3}{39a^3x\sqrt[4]{a+bx^4}} - \frac{(a+bx^4)^{3/4}}{13ax^{13}} + \frac{10b(a+bx^4)^{3/4}}{117a^2x^9} - \frac{4b^2(a+bx^4)^{3/4}}{39a^3x^5} - \frac{8b^{7/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
8/39*b^3/a^3/x/(b*x^4+a)^(1/4)-1/13*(b*x^4+a)^(3/4)/a/x^13+10/117*b*(b*x^4
+a)^(3/4)/a^2/x^9-4/39*b^2*(b*x^4+a)^(3/4)/a^3/x^5-8/39*b^(7/2)*(1+a/b/x^4
)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(7/2)/
(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{1}{4},-\frac{9}{4},-\frac{bx^4}{a}\right)}{13x^{13}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^14*(a + b*x^4)^(1/4)),x]
```

output
```
-1/13*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, -((b*x^4)
/a)])/(x^13*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {847, 847, 847, 841, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{10b\int \frac{1}{x^{10}\sqrt[4]{bx^4+a}}dx}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{10b\left(-\frac{2b\int \frac{1}{x^6\sqrt[4]{bx^4+a}}dx}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{10b\left(-\frac{2b\left(-\frac{2b\int \frac{1}{x^2\sqrt[4]{bx^4+a}}dx}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{841}$$

$$-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(-b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \; 813$$

$$-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \; 858$$

$$-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{10b\left(-\frac{2b\left(-\frac{2b\left(\frac{\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^4}}-\frac{1}{x\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{\left(a+bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a+bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a+bx^4\right)^{3/4}}{13ax^{13}}$$

input
```
Int[1/(x^14*(a + b*x^4)^(1/4)),x]
```

output
```
-1/13*(a + b*x^4)^(3/4)/(a*x^13) - (10*b*(-1/9*(a + b*x^4)^(3/4)/(a*x^9) -
 (2*b*(-1/5*(a + b*x^4)^(3/4)/(a*x^5) - (2*b*(-(1/(x*(a + b*x^4)^(1/4))) +
 (Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/
2, 2])/(Sqrt[a]*(a + b*x^4)^(1/4))))/(5*a)))/(3*a)))/(13*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{14} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^14/(b*x^4+a)^(1/4),x)
```

output
```
int(1/x^14/(b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b*x^18 + a*x^14), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \frac{\Gamma\left(-\frac{13}{4}\right){}_2F_1\left(\begin{matrix} -\frac{13}{4},\frac{1}{4} \\ -\frac{9}{4} \end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}x^{13}\Gamma\left(-\frac{9}{4}\right)}$$

input
```
integrate(1/x**14/(b*x**4+a)**(1/4),x)
```

output
```
gamma(-13/4)*hyper((-13/4, 1/4), (-9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(1/4)*x**13*gamma(-9/4))
```

Maxima [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^14), x)
```

Giac [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{1}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(1/4)*x^14), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{x^{14}\,(b\,x^4+a)^{1/4}}\,dx$$

input
```
int(1/(x^14*(a + b*x^4)^(1/4)),x)
```

output
```
int(1/(x^14*(a + b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a+bx^4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,x^{14}}dx$$

input
```
int(1/x^14/(b*x^4+a)^(1/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*x**14),x)
```

3.564 $\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \frac{a^4\sqrt[4]{a+bx^4}}{b^5} - \frac{4a^3(a+bx^4)^{5/4}}{5b^5} + \frac{2a^2(a+bx^4)^{9/4}}{3b^5} - \frac{4a(a+bx^4)^{13/4}}{13b^5} + \frac{(a+bx^4)^{17/4}}{17b^5}$$

output
```
a^4*(b*x^4+a)^(1/4)/b^5-4/5*a^3*(b*x^4+a)^(5/4)/b^5+2/3*a^2*(b*x^4+a)^(9/4
)/b^5-4/13*a*(b*x^4+a)^(13/4)/b^5+1/17*(b*x^4+a)^(17/4)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}(2048a^4-512a^3bx^4+320a^2b^2x^8-240ab^3x^{12}+195b^4x^{16})}{3315b^5}$$

input
```
Integrate[x^19/(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(1/4)*(2048*a^4 - 512*a^3*b*x^4 + 320*a^2*b^2*x^8 - 240*a*b^3
*x^12 + 195*b^4*x^16))/(3315*b^5)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{x^{16}}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \; 53$$

$$\frac{1}{4}\int \left(\frac{a^4}{b^4(bx^4+a)^{3/4}}-\frac{4\sqrt[4]{bx^4+a}a^3}{b^4}+\frac{6(bx^4+a)^{5/4}a^2}{b^4}-\frac{4(bx^4+a)^{9/4}a}{b^4}+\frac{(bx^4+a)^{13/4}}{b^4}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{4a^4\sqrt[4]{a+bx^4}}{b^5}-\frac{16a^3(a+bx^4)^{5/4}}{5b^5}+\frac{8a^2(a+bx^4)^{9/4}}{3b^5}+\frac{4(a+bx^4)^{17/4}}{17b^5}-\frac{16a(a+bx^4)^{13/4}}{13b^5}\right)$$

input
```
Int[x^19/(a + b*x^4)^(3/4),x]
```

output
```
((4*a^4*(a + b*x^4)^(1/4))/b^5 - (16*a^3*(a + b*x^4)^(5/4))/(5*b^5) + (8*a
^2*(a + b*x^4)^(9/4))/(3*b^5) - (16*a*(a + b*x^4)^(13/4))/(13*b^5) + (4*(a
 + b*x^4)^(17/4))/(17*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{1}{4}}\left(195x^{16}b^4-240a\,b^3x^{12}+320a^2b^2x^8-512a^3b\,x^4+2048a^4\right)}{3315b^5}$	58
trager	$\frac{(b\,x^4+a)^{\frac{1}{4}}\left(195x^{16}b^4-240a\,b^3x^{12}+320a^2b^2x^8-512a^3b\,x^4+2048a^4\right)}{3315b^5}$	58
risch	$\frac{(b\,x^4+a)^{\frac{1}{4}}\left(195x^{16}b^4-240a\,b^3x^{12}+320a^2b^2x^8-512a^3b\,x^4+2048a^4\right)}{3315b^5}$	58
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{1}{4}}\left(195x^{16}b^4-240a\,b^3x^{12}+320a^2b^2x^8-512a^3b\,x^4+2048a^4\right)}{3315b^5}$	58
orering	$\frac{(b\,x^4+a)^{\frac{1}{4}}\left(195x^{16}b^4-240a\,b^3x^{12}+320a^2b^2x^8-512a^3b\,x^4+2048a^4\right)}{3315b^5}$	58

input
```
int(x^19/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/3315*(b*x^4+a)^(1/4)*(195*b^4*x^16-240*a*b^3*x^12+320*a^2*b^2*x^8-512*a^
3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.58

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \frac{(195\,b^4x^{16} - 240\,ab^3x^{12} + 320\,a^2b^2x^8 - 512\,a^3bx^4 + 2048\,a^4)(bx^4+a)^{\frac{1}{4}}}{3315\,b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/3315*(195*b^4*x^16 - 240*a*b^3*x^12 + 320*a^2*b^2*x^8 - 512*a^3*b*x^4 +
2048*a^4)*(b*x^4 + a)^(1/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \begin{cases} \frac{2048a^4\sqrt[4]{a+bx^4}}{3315b^5} - \frac{512a^3x^4\sqrt[4]{a+bx^4}}{3315b^4} + \frac{64a^2x^8\sqrt[4]{a+bx^4}}{663b^3} - \frac{16ax^{12}\sqrt[4]{a+bx^4}}{221b^2} + \frac{x^{16}\sqrt[4]{a+}}{17b} \\ \frac{x^{20}}{20a^{\frac{3}{4}}} \end{cases}$$

input
```
integrate(x**19/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((2048*a**4*(a + b*x**4)**(1/4)/(3315*b**5) - 512*a**3*x**4*(a +
b*x**4)**(1/4)/(3315*b**4) + 64*a**2*x**8*(a + b*x**4)**(1/4)/(663*b**3) -
 16*a*x**12*(a + b*x**4)**(1/4)/(221*b**2) + x**16*(a + b*x**4)**(1/4)/(17
*b), Ne(b, 0)), (x**20/(20*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{17}{4}}}{17\,b^5} - \frac{4\,(bx^4+a)^{\frac{13}{4}}a}{13\,b^5} + \frac{2\,(bx^4+a)^{\frac{9}{4}}a^2}{3\,b^5} - \frac{4\,(bx^4+a)^{\frac{5}{4}}a^3}{5\,b^5} + \frac{(bx^4+a)^{\frac{1}{4}}a^4}{b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/17*(b*x^4 + a)^(17/4)/b^5 - 4/13*(b*x^4 + a)^(13/4)*a/b^5 + 2/3*(b*x^4 +
 a)^(9/4)*a^2/b^5 - 4/5*(b*x^4 + a)^(5/4)*a^3/b^5 + (b*x^4 + a)^(1/4)*a^4/
b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}a^4}{b^5} + \frac{195\,(bx^4+a)^{\frac{17}{4}} - 1020\,(bx^4+a)^{\frac{13}{4}}a + 2210\,(bx^4+a)^{\frac{9}{4}}a^2 - 2652\,(bx^4+a)^{\frac{5}{4}}a^3}{3315\,b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
(b*x^4 + a)^(1/4)*a^4/b^5 + 1/3315*(195*(b*x^4 + a)^(17/4) - 1020*(b*x^4 +
 a)^(13/4)*a + 2210*(b*x^4 + a)^(9/4)*a^2 - 2652*(b*x^4 + a)^(5/4)*a^3)/b^
5
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{2048\,a^4}{3315\,b^5}+\frac{x^{16}}{17\,b}-\frac{16\,a\,x^{12}}{221\,b^2}-\frac{512\,a^3\,x^4}{3315\,b^4}+\frac{64\,a^2\,x^8}{663\,b^3}\right)$$

input
```
int(x^19/(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(1/4)*((2048*a^4)/(3315*b^5) + x^16/(17*b) - (16*a*x^12)/(221*
b^2) - (512*a^3*x^4)/(3315*b^4) + (64*a^2*x^8)/(663*b^3))
```

Reduce [F]

$$\int \frac{x^{19}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{19}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^19/(b*x^4+a)^(3/4),x)
```

output
```
int(x**19/(a + b*x**4)**(3/4),x)
```

3.565 $\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx=-\frac{a^3\sqrt[4]{a+bx^4}}{b^4}+\frac{3a^2(a+bx^4)^{5/4}}{5b^4}-\frac{a(a+bx^4)^{9/4}}{3b^4}+\frac{(a+bx^4)^{13/4}}{13b^4}$$

output
```
-a^3*(b*x^4+a)^(1/4)/b^4+3/5*a^2*(b*x^4+a)^(5/4)/b^4-1/3*a*(b*x^4+a)^(9/4)
/b^4+1/13*(b*x^4+a)^(13/4)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx=\frac{\sqrt[4]{a+bx^4}(-128a^3+32a^2bx^4-20ab^2x^8+15b^3x^{12})}{195b^4}$$

input
```
Integrate[x^15/(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(1/4)*(-128*a^3 + 32*a^2*b*x^4 - 20*a*b^2*x^8 + 15*b^3*x^12))
/(195*b^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{x^{12}}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \; 53$$

$$\frac{1}{4}\int \left(-\frac{a^3}{b^3\,(bx^4+a)^{3/4}}+\frac{3\sqrt[4]{bx^4+a}a^2}{b^3}-\frac{3(bx^4+a)^{5/4}a}{b^3}+\frac{(bx^4+a)^{9/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(-\frac{4a^3\sqrt[4]{a+bx^4}}{b^4}+\frac{12a^2(a+bx^4)^{5/4}}{5b^4}+\frac{4(a+bx^4)^{13/4}}{13b^4}-\frac{4a(a+bx^4)^{9/4}}{3b^4}\right)$$

input
```
Int[x^15/(a + b*x^4)^(3/4),x]
```

output
```
((-4*a^3*(a + b*x^4)^(1/4))/b^4 + (12*a^2*(a + b*x^4)^(5/4))/(5*b^4) - (4*
a*(a + b*x^4)^(9/4))/(3*b^4) + (4*(a + b*x^4)^(13/4))/(13*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(-15b^3x^{12}+20a\,b^2x^8-32a^2b\,x^4+128a^3)}{195b^4}$	47
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(-15b^3x^{12}+20a\,b^2x^8-32a^2b\,x^4+128a^3)}{195b^4}$	47
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(-15b^3x^{12}+20a\,b^2x^8-32a^2b\,x^4+128a^3)}{195b^4}$	47
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(-15b^3x^{12}+20a\,b^2x^8-32a^2b\,x^4+128a^3)}{195b^4}$	47
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}(-15b^3x^{12}+20a\,b^2x^8-32a^2b\,x^4+128a^3)}{195b^4}$	47

input
```
int(x^15/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/195*(b*x^4+a)^(1/4)*(-15*b^3*x^12+20*a*b^2*x^8-32*a^2*b*x^4+128*a^3)/b^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx = \frac{(15\,b^3x^{12}-20\,ab^2x^8+32\,a^2bx^4-128\,a^3)(bx^4+a)^{\frac{1}{4}}}{195\,b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/195*(15*b^3*x^12 - 20*a*b^2*x^8 + 32*a^2*b*x^4 - 128*a^3)*(b*x^4 + a)^(1
/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx = \begin{cases} -\frac{128a^3\sqrt[4]{a+bx^4}}{195b^4}+\frac{32a^2x^4\sqrt[4]{a+bx^4}}{195b^3}-\frac{4ax^8\sqrt[4]{a+bx^4}}{39b^2}+\frac{x^{12}\sqrt[4]{a+bx^4}}{13b} & \text{for } b\neq 0 \\ \frac{x^{16}}{16a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-128*a**3*(a + b*x**4)**(1/4)/(195*b**4) + 32*a**2*x**4*(a + b*
x**4)**(1/4)/(195*b**3) - 4*a*x**8*(a + b*x**4)**(1/4)/(39*b**2) + x**12*(
a + b*x**4)**(1/4)/(13*b), Ne(b, 0)), (x**16/(16*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{13}{4}}}{13\,b^4}-\frac{(bx^4+a)^{\frac{9}{4}}a}{3\,b^4}+\frac{3\,(bx^4+a)^{\frac{5}{4}}a^2}{5\,b^4}-\frac{(bx^4+a)^{\frac{1}{4}}a^3}{b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/13*(b*x^4 + a)^(13/4)/b^4 - 1/3*(b*x^4 + a)^(9/4)*a/b^4 + 3/5*(b*x^4 + a
)^(5/4)*a^2/b^4 - (b*x^4 + a)^(1/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx = -\frac{(bx^4+a)^{\frac{1}{4}}a^3}{b^4} + \frac{15\,(bx^4+a)^{\frac{13}{4}} - 65\,(bx^4+a)^{\frac{9}{4}}a + 117\,(bx^4+a)^{\frac{5}{4}}a^2}{195\,b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-(b*x^4 + a)^(1/4)*a^3/b^4 + 1/195*(15*(b*x^4 + a)^(13/4) - 65*(b*x^4 + a)
^(9/4)*a + 117*(b*x^4 + a)^(5/4)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx = -\left(b\,x^4+a\right)^{1/4}\left(\frac{128\,a^3}{195\,b^4} - \frac{x^{12}}{13\,b} + \frac{4\,a\,x^8}{39\,b^2} - \frac{32\,a^2\,x^4}{195\,b^3}\right)$$

input
```
int(x^15/(a + b*x^4)^(3/4),x)
```

output
```
-(a + b*x^4)^(1/4)*((128*a^3)/(195*b^4) - x^12/(13*b) + (4*a*x^8)/(39*b^2)
 - (32*a^2*x^4)/(195*b^3))
```

Reduce [F]

$$\int \frac{x^{15}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{15}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^15/(b*x^4+a)^(3/4),x)
```

output
```
int(x**15/(a + b*x**4)**(3/4),x)
```

3.566 $\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx=\frac{a^2\sqrt[4]{a+bx^4}}{b^3}-\frac{2a(a+bx^4)^{5/4}}{5b^3}+\frac{(a+bx^4)^{9/4}}{9b^3}$$

output
```
a^2*(b*x^4+a)^(1/4)/b^3-2/5*a*(b*x^4+a)^(5/4)/b^3+1/9*(b*x^4+a)^(9/4)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx=\frac{\sqrt[4]{a+bx^4}(32a^2-8abx^4+5b^2x^8)}{45b^3}$$

input
```
Integrate[x^11/(a + b*x^4)^(3/4),x]
```

output
```
((a + b*x^4)^(1/4)*(32*a^2 - 8*a*b*x^4 + 5*b^2*x^8))/(45*b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2(bx^4+a)^{3/4}}-\frac{2\sqrt[4]{bx^4+a}a}{b^2}+\frac{(bx^4+a)^{5/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a^2\sqrt[4]{a+bx^4}}{b^3}+\frac{4(a+bx^4)^{9/4}}{9b^3}-\frac{8a(a+bx^4)^{5/4}}{5b^3}\right)$$

input
```
Int[x^11/(a + b*x^4)^(3/4),x]
```

output
```
((4*a^2*(a + b*x^4)^(1/4))/b^3 - (8*a*(a + b*x^4)^(5/4))/(5*b^3) + (4*(a +
 b*x^4)^(9/4))/(9*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5b^2x^8-8ab\,x^4+32a^2)}{45b^3}$	36
trager	$\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5b^2x^8-8ab\,x^4+32a^2)}{45b^3}$	36
risch	$\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5b^2x^8-8ab\,x^4+32a^2)}{45b^3}$	36
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5b^2x^8-8ab\,x^4+32a^2)}{45b^3}$	36
orering	$\frac{(b\,x^4+a)^{\frac{1}{4}}\,(5b^2x^8-8ab\,x^4+32a^2)}{45b^3}$	36

input
```
int(x^11/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/45*(b*x^4+a)^(1/4)*(5*b^2*x^8-8*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx = \frac{(5\,b^2x^8 - 8\,abx^4 + 32\,a^2)(bx^4+a)^{\frac{1}{4}}}{45\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/45*(5*b^2*x^8 - 8*a*b*x^4 + 32*a^2)*(b*x^4 + a)^(1/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx = \begin{cases} \frac{32a^2\sqrt[4]{a+bx^4}}{45b^3} - \frac{8ax^4\sqrt[4]{a+bx^4}}{45b^2} + \frac{x^8\sqrt[4]{a+bx^4}}{9b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((32*a**2*(a + b*x**4)**(1/4)/(45*b**3) - 8*a*x**4*(a + b*x**4)**
(1/4)/(45*b**2) + x**8*(a + b*x**4)**(1/4)/(9*b), Ne(b, 0)), (x**12/(12*a*
*(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{9}{4}}}{9\,b^3} - \frac{2\,(bx^4+a)^{\frac{5}{4}}a}{5\,b^3} + \frac{(bx^4+a)^{\frac{1}{4}}a^2}{b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/9*(b*x^4 + a)^(9/4)/b^3 - 2/5*(b*x^4 + a)^(5/4)*a/b^3 + (b*x^4 + a)^(1/4
)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}a^2}{b^3} + \frac{5\,(bx^4+a)^{\frac{9}{4}} - 18\,(bx^4+a)^{\frac{5}{4}}a}{45\,b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
(b*x^4 + a)^(1/4)*a^2/b^3 + 1/45*(5*(b*x^4 + a)^(9/4) - 18*(b*x^4 + a)^(5/
4)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx = \left(b\,x^4+a\right)^{1/4}\left(\frac{32\,a^2}{45\,b^3} + \frac{x^8}{9\,b} - \frac{8\,a\,x^4}{45\,b^2}\right)$$

input
```
int(x^11/(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(1/4)*((32*a^2)/(45*b^3) + x^8/(9*b) - (8*a*x^4)/(45*b^2))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{11}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^11/(b*x^4+a)^(3/4),x)
```

output
```
int(x**11/(a + b*x**4)**(3/4),x)
```

3.567 $\int \frac{x^7}{(a+bx^4)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x^7}{(a+bx^4)^{3/4}} \, dx = -\frac{a\sqrt[4]{a+bx^4}}{b^2} + \frac{(a+bx^4)^{5/4}}{5b^2}$$

output
```
-a*(b*x^4+a)^(1/4)/b^2+1/5*(b*x^4+a)^(5/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{x^7}{(a+bx^4)^{3/4}} \, dx = \frac{(-4a+bx^4)\sqrt[4]{a+bx^4}}{5b^2}$$

input
```
Integrate[x^7/(a + b*x^4)^(3/4),x]
```

output
```
((-4*a + b*x^4)*(a + b*x^4)^(1/4))/(5*b^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)^{3/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\sqrt[4]{bx^4+a}}{b}-\frac{a}{b\,(bx^4+a)^{3/4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4(a+bx^4)^{5/4}}{5b^2}-\frac{4a\sqrt[4]{a+bx^4}}{b^2}\right)$$

input
```
Int[x^7/(a + b*x^4)^(3/4),x]
```

output
```
((-4*a*(a + b*x^4)^(1/4))/b^2 + (4*(a + b*x^4)^(5/4))/(5*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{(b\,x^4-4a)\,(b\,x^4+a)^{\frac{1}{4}}}{5b^2}$	24
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-b\,x^4+4a)}{5b^2}$	25
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-b\,x^4+4a)}{5b^2}$	25
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-b\,x^4+4a)}{5b^2}$	25
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-b\,x^4+4a)}{5b^2}$	25

input
```
int(x^7/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/5*(b*x^4-4*a)*(b*x^4+a)^(1/4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}(bx^4-4\,a)}{5\,b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/5*(b*x^4 + a)^(1/4)*(b*x^4 - 4*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx = \begin{cases} -\frac{4a\sqrt[4]{a+bx^4}}{5b^2} + \frac{x^4\sqrt[4]{a+bx^4}}{5b} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-4*a*(a + b*x**4)**(1/4)/(5*b**2) + x**4*(a + b*x**4)**(1/4)/(5
*b), Ne(b, 0)), (x**8/(8*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{5}{4}}}{5\,b^2} - \frac{(bx^4+a)^{\frac{1}{4}}a}{b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/5*(b*x^4 + a)^(5/4)/b^2 - (b*x^4 + a)^(1/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{5}{4}}}{5\,b^2} - \frac{(bx^4+a)^{\frac{1}{4}}a}{b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/5*(b*x^4 + a)^(5/4)/b^2 - (b*x^4 + a)^(1/4)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx = -\frac{(b\,x^4+a)^{1/4}\,(4\,a-b\,x^4)}{5\,b^2}$$

input
```
int(x^7/(a + b*x^4)^(3/4),x)
```

output
```
-((a + b*x^4)^(1/4)*(4*a - b*x^4))/(5*b^2)
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^7}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^7/(b*x^4+a)^(3/4),x)
```

output
```
int(x**7/(a + b*x**4)**(3/4),x)
```

3.568 $\int \frac{x^3}{\left(a+bx^4\right)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{x^3}{\left(a+bx^4\right)^{3/4}} \, dx = \frac{\sqrt[4]{a+bx^4}}{b}$$

output
```
(b*x^4+a)^(1/4)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a+bx^4\right)^{3/4}} \, dx = \frac{\sqrt[4]{a+bx^4}}{b}$$

input
```
Integrate[x^3/(a + b*x^4)^(3/4),x]
```

output
```
(a + b*x^4)^(1/4)/b
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \; 793$$

$$\frac{\sqrt[4]{a+bx^4}}{b}$$

input
```
Int[x^3/(a + b*x^4)^(3/4),x]
```

output
```
(a + b*x^4)^(1/4)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14
derivativedivides	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14
default	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14
trager	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14
risch	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14
pseudoelliptic	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14
orering	$\frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$	14

input
```
int(x^3/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
(b*x^4+a)^(1/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}}{b}$$

input
```
integrate(x^3/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
(b*x^4 + a)^(1/4)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx = \begin{cases} \frac{\sqrt[4]{a+bx^4}}{b} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)**(3/4),x)
```

output
```
Piecewise(((a + b*x**4)**(1/4)/b, Ne(b, 0)), (x**4/(4*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}}{b}$$

input
```
integrate(x^3/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
(b*x^4 + a)^(1/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx = \frac{(bx^4+a)^{\frac{1}{4}}}{b}$$

input
```
integrate(x^3/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
(b*x^4 + a)^(1/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx = \frac{(b\,x^4+a)^{1/4}}{b}$$

input
```
int(x^3/(a + b*x^4)^(3/4),x)
```

output
```
(a + b*x^4)^(1/4)/b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx^4)^{3/4}}\,dx = \frac{(b\,x^4+a)^{\frac{1}{4}}}{b}$$

input
```
int(x^3/(b*x^4+a)^(3/4),x)
```

output
```
(a + b*x**4)**(1/4)/b
```

3.569 $\int \frac{1}{x(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

output
```
-1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/2*arctanh((b*x^4+a)^(1/4)/a
^(1/4))/a^(3/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) + \operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

input
```
Integrate[1/(x*(a + b*x^4)^(3/4)),x]
```

output
```
-1/2*(ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4
)])/a^(3/4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {798, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx^4$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{b}$$

$$\Big\downarrow \text{756}$$

$$\frac{-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}}{b}$$

input
```
Int[1/(x*(a + b*x^4)^(3/4)),x]
```

output
```
(-1/2*(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^
4)^(1/4)/a^(1/4)])/(2*a^(3/4)))/b
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result	size
pseudoelliptic	$-\frac{\ln\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{4a^{\frac{3}{4}}}$	55

input
```
int(1/x/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(3/4)*(ln(((b*x^4+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))+2*ar
ctan((b*x^4+a)^(1/4)/a^(1/4)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.89

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}}\log\left(a\frac{1}{a^3}^{\frac{1}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}\right)-\frac{1}{4}i\frac{1}{a^3}^{\frac{1}{4}}\log\left(i\,a\frac{1}{a^3}^{\frac{1}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}\right)+\frac{1}{4}i\frac{1}{a^3}^{\frac{1}{4}}\log\left(-i\,a\frac{1}{a^3}^{\frac{1}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}\right)+\frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-a\frac{1}{a^3}^{\frac{1}{4}}+\left(bx^4+a\right)^{\frac{1}{4}}\right)$$

input
```
integrate(1/x/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/4*(a^(-3))^(1/4)*log(a*(a^(-3))^(1/4) + (b*x^4 + a)^(1/4)) - 1/4*I*(a^(
-3))^(1/4)*log(I*a*(a^(-3))^(1/4) + (b*x^4 + a)^(1/4)) + 1/4*I*(a^(-3))^(1
/4)*log(-I*a*(a^(-3))^(1/4) + (b*x^4 + a)^(1/4)) + 1/4*(a^(-3))^(1/4)*log(
-a*(a^(-3))^(1/4) + (b*x^4 + a)^(1/4))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x/(b*x**4+a)**(3/4),x)
```

output
```
-gamma(3/4)*hyper((3/4, 3/4), (7/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(3
/4)*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{3}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{3}{4}}}$$

input
```
integrate(1/x/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 1/4*log(((b*x^4 + a)^(1/4
) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(39) = 78$.

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.38

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^4+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}(-a)^{\frac{1}{4}}\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a} + \frac{\sqrt{2}(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a}$$

input
```
integrate(1/x/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
+ a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a - 1/8*sqrt(2)*(-a)^(
1/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a)
)/a + 1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + s
qrt(b*x^4 + a) + sqrt(-a))/a
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)+\mathrm{atanh}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}}$$

input
```
int(1/(x*(a + b*x^4)^(3/4)),x)
```

output
```
-(atan((a + b*x^4)^(1/4)/a^(1/4)) + atanh((a + b*x^4)^(1/4)/a^(1/4)))/(2*a
^(3/4))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}x}dx$$

input
```
int(1/x/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x),x)
```

3.570 $\int \frac{1}{x^5 (a+bx^4)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}} \, dx = -\frac{\sqrt[4]{a+bx^4}}{4ax^4} + \frac{3b \arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}} + \frac{3b \text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

output
```
-1/4*(b*x^4+a)^(1/4)/a/x^4+3/8*b*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)+3
/8*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}} \, dx = -\frac{\sqrt[4]{a+bx^4}}{4ax^4} + \frac{3b \arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}} + \frac{3b \text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

input
```
Integrate[1/(x^5*(a + b*x^4)^(3/4)),x]
```

output
```
-1/4*(a + b*x^4)^(1/4)/(a*x^4) + (3*b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/(
8*a^(7/4)) + (3*b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(7/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 52, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8 \left(bx^4+a\right)^{3/4}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(-\frac{3b\int \frac{1}{x^4 \left(bx^4+a\right)^{3/4}}dx^4}{4a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{3\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(-\frac{3\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-\frac{3\left(-\frac{b\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(-\frac{3\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)^(3/4)),x]
```

output
```
(-((a + b*x^4)^(1/4)/(a*x^4)) - (3*(-1/2*(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/
4)])/a^(3/4) - (b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/a)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

method	result	size
pseudoelliptic	$\frac{6 \arctan \left(\frac{\left(b \,x^{4}+a \right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b \,x^{4}+3 \ln \left(\frac{-\left(b \,x^{4}+a \right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b \,x^{4}+a \right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) b \,x^{4}-4 \left(b \,x^{4}+a \right)^{\frac{1}{4}} a^{\frac{3}{4}}}{16 a^{\frac{7}{4}} x^{4}}$	86

input
```
int(1/x^5/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(6*arctan((b*x^4+a)^(1/4)/a^(1/4))*b*x^4+3*ln((-(b*x^4+a)^(1/4)-a^(1/
4))/(-(b*x^4+a)^(1/4)+a^(1/4)))*b*x^4-4*(b*x^4+a)^(1/4)*a^(3/4))/a^(7/4)/x
^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}}\,dx = \frac{3\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(3\,a^2\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+3\,(bx^4+a)^{\frac{1}{4}}b\right)+3i\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(3i\,a^2\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+3\,(\right.}{}$$

input
```
integrate(1/x^5/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/16*(3*a*x^4*(b^4/a^7)^(1/4)*log(3*a^2*(b^4/a^7)^(1/4) + 3*(b*x^4 + a)^(1
/4)*b) + 3*I*a*x^4*(b^4/a^7)^(1/4)*log(3*I*a^2*(b^4/a^7)^(1/4) + 3*(b*x^4
+ a)^(1/4)*b) - 3*I*a*x^4*(b^4/a^7)^(1/4)*log(-3*I*a^2*(b^4/a^7)^(1/4) + 3
*(b*x^4 + a)^(1/4)*b) - 3*a*x^4*(b^4/a^7)^(1/4)*log(-3*a^2*(b^4/a^7)^(1/4)
 + 3*(b*x^4 + a)^(1/4)*b) - 4*(b*x^4 + a)^(1/4))/(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}}\,dx = -\frac{\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^7\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(1/x**5/(b*x**4+a)**(3/4),x)
```

output
```
-gamma(7/4)*hyper((3/4, 7/4), (11/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(
3/4)*x**7*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}}\,dx = -\frac{(bx^4+a)^{\frac{1}{4}}b}{4\left((bx^4+a)a-a^2\right)} + \frac{3\left(\frac{2\,b\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}} - \frac{b\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{16\,a}$$

input
```
integrate(1/x^5/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/4*(b*x^4 + a)^(1/4)*b/((b*x^4 + a)*a - a^2) + 3/16*(2*b*arctan((b*x^4 +
 a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 +
 a)^(1/4) + a^(1/4)))/a^(3/4))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(58) = 116.

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.68

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}}\,dx = \frac{1}{32}\,b\left(\frac{6\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{6\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{\right.}\right.}{}\right)}{a^2}\right.$$

input
```
integrate(1/x^5/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/32*b*(6*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b
*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*sqrt(2)*(
-a)^(1/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqr
t(-a))/a^2 - 3*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4
) + sqrt(b*x^4 + a) + sqrt(-a))/a^2 - 8*(b*x^4 + a)^(1/4)/(a*b*x^4))
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}}\,dx = \frac{3\,b\,\mathrm{atan}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}} - \frac{{\left(b\,x^4+a\right)}^{1/4}}{4\,a\,x^4} + \frac{3\,b\,\mathrm{atanh}\left(\frac{{\left(b\,x^4+a\right)}^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}$$

input
```
int(1/(x^5*(a + b*x^4)^(3/4)),x)
```

output
```
(3*b*atan((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4)) - (a + b*x^4)^(1/4)/(4*a
*x^4) + (3*b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4))
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}} x^5}dx$$

input
```
int(1/x^5/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**5),x)
```

3.571 $\int \frac{1}{x^9(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^9(a+bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{8ax^8}+\frac{7b\sqrt[4]{a+bx^4}}{32a^2x^4} - \frac{21b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}} - \frac{21b^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}$$

output
```
-1/8*(b*x^4+a)^(1/4)/a/x^8+7/32*b*(b*x^4+a)^(1/4)/a^2/x^4-21/64*b^2*arctan
((b*x^4+a)^(1/4)/a^(1/4))/a^(11/4)-21/64*b^2*arctanh((b*x^4+a)^(1/4)/a^(1/
4))/a^(11/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9(a+bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}(-4a+7bx^4)}{32a^2x^8} - \frac{21b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}} - \frac{21b^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}$$

input
```
Integrate[1/(x^9*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-4*a + 7*b*x^4))/(32*a^2*x^8) - (21*b^2*ArcTan[(a + b*
x^4)^(1/4)/a^(1/4)])/(64*a^(11/4)) - (21*b^2*ArcTanh[(a + b*x^4)^(1/4)/a^(
1/4)])/(64*a^(11/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {798, 52, 52, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^{12} (bx^4+a)^{3/4}} dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{7b \int \frac{1}{x^8 (bx^4+a)^{3/4}} dx^4}{8a} - \frac{\sqrt[4]{a+bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{7b \left(-\frac{3b \int \frac{1}{x^4 (bx^4+a)^{3/4}} dx^4}{4a} - \frac{\sqrt[4]{a+bx^4}}{ax^4}\right)}{8a} - \frac{\sqrt[4]{a+bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{7b\left(-\frac{3\int \frac{1}{\frac{x^{16}}{b}-\frac{a}{b}}d\sqrt[4]{bx^4+a}}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a+bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{ 756}$$

$$\frac{1}{4}\left(-\frac{7b\left(-\frac{3\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a+bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\left(-\frac{7b\left(-\frac{3\left(-\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}}{2\sqrt{a}}-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a+bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}\left(-\frac{7b\left(-\frac{3\left(-\frac{b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a+bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a+bx^4}}{2ax^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^4)^(3/4)),x]
```

output
```
(-1/2*(a + b*x^4)^(1/4)/(a*x^8) - (7*b*(-((a + b*x^4)^(1/4)/(a*x^4)) - (3*
(-1/2*(b*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)])/a^(3/4) - (b*ArcTanh[(a + b*x^
4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/a))/(8*a))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$\frac{-21 \ln \left(\frac{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) b^{2} x^{8}-42 \arctan \left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b^{2} x^{8}+28 b\,x^{4} \left(b\,x^{4}+a\right)^{\frac{1}{4}} a^{\frac{3}{4}}-16 a^{\frac{7}{4}} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{128 a^{\frac{11}{4}} x^{8}}$	108

input
```
int(1/x^9/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-21*ln((-(b*x^4+a)^(1/4)-a^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4)))*b^2*x
^8-42*arctan((b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8+28*b*x^4*(b*x^4+a)^(1/4)*a^(
3/4)-16*a^(7/4)*(b*x^4+a)^(1/4))/a^(11/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}}\,dx = -\frac{21\,a^2x^8\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}\log\left(21\,a^3\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}+21\,(bx^4+a)^{\frac{1}{4}}b^2\right)+21i\,a^2x^8\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}\log\left(21i\,a^3\left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}}+21\,(bx^4+a)\right.}{}$$

input
```
integrate(1/x^9/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/128*(21*a^2*x^8*(b^8/a^11)^(1/4)*log(21*a^3*(b^8/a^11)^(1/4) + 21*(b*x^
4 + a)^(1/4)*b^2) + 21*I*a^2*x^8*(b^8/a^11)^(1/4)*log(21*I*a^3*(b^8/a^11)^
(1/4) + 21*(b*x^4 + a)^(1/4)*b^2) - 21*I*a^2*x^8*(b^8/a^11)^(1/4)*log(-21*
I*a^3*(b^8/a^11)^(1/4) + 21*(b*x^4 + a)^(1/4)*b^2) - 21*a^2*x^8*(b^8/a^11)
^(1/4)*log(-21*a^3*(b^8/a^11)^(1/4) + 21*(b*x^4 + a)^(1/4)*b^2) - 4*(7*b*x
^4 - 4*a)*(b*x^4 + a)^(1/4))/(a^2*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}}\,dx = -\frac{\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\,\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{3}{4}}x^{11}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(1/x**9/(b*x**4+a)**(3/4),x)
```

output
```
-gamma(11/4)*hyper((3/4, 11/4), (15/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b*
*(3/4)*x**11*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}}\,dx = \frac{7\,(bx^4+a)^{\frac{5}{4}}b^2 - 11\,(bx^4+a)^{\frac{1}{4}}ab^2}{32\,\left((bx^4+a)^2a^2 - 2\,(bx^4+a)a^3 + a^4\right)} - \frac{21\left(\frac{2\,b^2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}} - \frac{b^2\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^2}$$

input
```
integrate(1/x^9/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/32*(7*(b*x^4 + a)^(5/4)*b^2 - 11*(b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 + a)^2
*a^2 - 2*(b*x^4 + a)*a^3 + a^4) - 21/128*(2*b^2*arctan((b*x^4 + a)^(1/4)/a
^(1/4))/a^(3/4) - b^2*log(((b*x^4 + a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4)
 + a^(1/4)))/a^(3/4))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(80) = 160.

Time = 0.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.35

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}}\,dx = \frac{\frac{42\sqrt{2}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^2} + \frac{42\sqrt{2}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{3}{4}}a^2} + \frac{21\sqrt{2}b^3}{}}{}$$

input
```
integrate(1/x^9/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/256*(42*sqrt(2)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 +
a)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^2) + 42*sqrt(2)*b^3*arctan(-1/2*sqrt(2
)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(3/4)*a^2)
+ 21*sqrt(2)*b^3*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a
) + sqrt(-a))/((-a)^(3/4)*a^2) + 21*sqrt(2)*(-a)^(1/4)*b^3*log(-sqrt(2)*(b
*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^3 + 8*(7*(b*x^4
 + a)^(5/4)*b^3 - 11*(b*x^4 + a)^(1/4)*a*b^3)/(a^2*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}} \, dx = \frac{7 \left(b\,x^4+a\right)^{5/4}}{32\,a^2\,x^8}-\frac{11 \left(b\,x^4+a\right)^{1/4}}{32\,a\,x^8} - \frac{21\,b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{11/4}}+\frac{b^2\,\mathrm{atan}\left(\frac{\left(b\,x^4+a\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,21\mathrm{i}}{64\,a^{11/4}}$$

input
```
int(1/(x^9*(a + b*x^4)^(3/4)),x)
```

output
```
(b^2*atan(((a + b*x^4)^(1/4)*1i)/a^(1/4))*21i)/(64*a^(11/4)) - (21*b^2*ata
n((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(11/4)) - (11*(a + b*x^4)^(1/4))/(32*a
*x^8) + (7*(a + b*x^4)^(5/4))/(32*a^2*x^8)
```

Reduce [F]

$$\int \frac{1}{x^9 (a+bx^4)^{3/4}} \, dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}} x^9}dx$$

input
```
int(1/x^9/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**9),x)
```

3.572 $\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \frac{20a^2x^2\sqrt[4]{a+bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a+bx^4}}{77b^2} + \frac{x^{10}\sqrt[4]{a+bx^4}}{11b} - \frac{40a^{7/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77b^{7/2}(a+bx^4)^{3/4}}$$

output
```
20/77*a^2*x^2*(b*x^4+a)^(1/4)/b^3-10/77*a*x^6*(b*x^4+a)^(1/4)/b^2+1/11*x^1
0*(b*x^4+a)^(1/4)/b-40/77*a^(7/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*ar
ctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \frac{x^2\left(20a^3+10a^2bx^4-3ab^2x^8+7b^3x^{12}-20a^3\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{77b^3(a+bx^4)^{3/4}}$$

input
```
Integrate[x^13/(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(20*a^3 + 10*a^2*b*x^4 - 3*a*b^2*x^8 + 7*b^3*x^12 - 20*a^3*(1 + (b*x^
4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(77*b^3*(a +
b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {807, 262, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^4+a)^{3/4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^{10}\sqrt[4]{a+bx^4}}{11b}-\frac{10a\int \frac{x^8}{(bx^4+a)^{3/4}}dx^2}{11b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^{10}\sqrt[4]{a+bx^4}}{11b}-\frac{10a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{7b}\right)}{11b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^{10}\sqrt[4]{a+bx^4}}{11b}-\frac{10a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int\frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{3b}\right)}{7b}\right)}{11b}\right)$$

$$\Big\downarrow \textbf{231}$$

$$\frac{1}{2}\left(\frac{2x^{10}\sqrt[4]{a+bx^4}}{11b}-\frac{10a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b\left(a+bx^4\right)^{3/4}}\right)}{7b}\right)}{11b}\right)$$

$$\Big\downarrow \textbf{229}$$

$$\frac{1}{2}\left(\frac{2x^{10}\sqrt[4]{a+bx^4}}{11b}-\frac{10a\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^4\right)^{3/4}}\right)}{7b}\right)}{11b}\right)$$

input
```
Int[x^13/(a + b*x^4)^(3/4),x]
```

output
```
((2*x^10*(a + b*x^4)^(1/4))/(11*b) - (10*a*((2*x^6*(a + b*x^4)^(1/4))/(7*b
) - (6*a*((2*x^2*(a + b*x^4)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^4)/a)^(3/
4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(
3/4))))/(7*b)))/(11*b))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{13}}{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(x^13/(b*x^4+a)^(3/4),x)
```

output
```
int(x^13/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^13/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \frac{x^{14}{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{14a^{\frac{3}{4}}}$$

input
```
integrate(x**13/(b*x**4+a)**(3/4),x)
```

output
```
x**14*hyper((3/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(3/4))
```

Maxima [F]

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^13/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^13/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^13/(a + b*x^4)^(3/4),x)
```

output
```
int(x^13/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{13}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^13/(b*x^4+a)^(3/4),x)
```

output
```
int(x**13/(a + b*x**4)**(3/4),x)
```

3.573 $\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx=-\frac{2ax^2\sqrt[4]{a+bx^4}}{7b^2}+\frac{x^6\sqrt[4]{a+bx^4}}{7b}+\frac{4a^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{7b^{5/2}(a+bx^4)^{3/4}}$$

output
```
-2/7*a*x^2*(b*x^4+a)^(1/4)/b^2+1/7*x^6*(b*x^4+a)^(1/4)/b+4/7*a^(5/2)*(1+b*
x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5
/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx=\frac{x^2\left(-2a^2-abx^4+b^2x^8+2a^2\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{7b^2(a+bx^4)^{3/4}}$$

input
```
Integrate[x^9/(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(-2*a^2 - a*b*x^4 + b^2*x^8 + 2*a^2*(1 + (b*x^4)/a)^(3/4)*Hypergeomet
ric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)]))/(7*b^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)^{3/4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2}{7b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3b}\right)}{7b}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b(a+bx^4)^{3/4}}\right)}{7b}\right)$$

$\downarrow$ 229

$$\frac{1}{2}\left(\frac{2x^6\sqrt[4]{a+bx^4}}{7b}-\frac{6a\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}\right)}{7b}\right)$$

input
```
Int[x^9/(a + b*x^4)^(3/4),x]
```

output
```
((2*x^6*(a + b*x^4)^(1/4))/(7*b) - (6*a*((2*x^2*(a + b*x^4)^(1/4))/(3*b) -
 (4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/
2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4))))/(7*b))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^9}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^9/(b*x^4+a)^(3/4),x)
```

output
```
int(x^9/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^9/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx = \frac{x^{10}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{3}{4}}}$$

input
```
integrate(x**9/(b*x**4+a)**(3/4),x)
```

output
```
x**10*hyper((3/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(3/4))
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^9/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^9/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^9}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^9/(a + b*x^4)^(3/4),x)
```

output
```
int(x^9/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^9}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^9}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^9/(b*x^4+a)^(3/4),x)
```

output
```
int(x**9/(a + b*x**4)**(3/4),x)
```

3.574 $\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx = \frac{x^2\sqrt[4]{a+bx^4}}{3b} - \frac{2a^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}$$

output
```
1/3*x^2*(b*x^4+a)^(1/4)/b-2/3*a^(3/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/
2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.59 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx = \frac{x^2\left(a+bx^4-a\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{3b(a+bx^4)^{3/4}}$$

input
```
Integrate[x^5/(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(a + b*x^4 - a*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 -((b*x^4)/a)]))/(3*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {807, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)^{3/4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{3b}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{2a\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{3b(a+bx^4)^{3/4}}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(\frac{2x^2\sqrt[4]{a+bx^4}}{3b}-\frac{4a^{3/2}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^4)^{3/4}}\right)$$

input
```
Int[x^5/(a + b*x^4)^(3/4),x]
```

output
```
((2*x^2*(a + b*x^4)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^4)/a)^(3/4)*Ellipt
icF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^4)^(3/4)))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^5}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^5/(b*x^4+a)^(3/4),x)
```

output
```
int(x^5/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx=\int \frac{x^5}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^5/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.33

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx=\frac{x^6{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}}$$

input
```
integrate(x**5/(b*x**4+a)**(3/4),x)
```

output
```
x**6*hyper((3/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4))
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx=\int \frac{x^5}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^5}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^5/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^5}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^5/(a + b*x^4)^(3/4),x)
```

output
```
int(x^5/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^5}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^5/(b*x^4+a)^(3/4),x)
```

output
```
int(x**5/(a + b*x**4)**(3/4),x)
```

3.575 $\int \frac{x}{(a+bx^4)^{3/4}} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x}{(a+bx^4)^{3/4}} dx = \frac{\sqrt{a}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^4)^{3/4}}$$

output
```
a^(1/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),
2^(1/2))/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x}{(a+bx^4)^{3/4}} dx = \frac{x^2\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^4}{a}\right)}{2\,(a+bx^4)^{3/4}}$$

input
```
Integrate[x/(a + b*x^4)^(3/4),x]
```

output
```
(x^2*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^4)/a)])
/(2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {807, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)^{3/4}} dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)^{3/4}}dx^2$$

$$\Big\downarrow \text{ 231}$$

$$\frac{\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{\sqrt{a}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^4)^{3/4}}$$

input
```
Int[x/(a + b*x^4)^(3/4),x]
```

output
```
(Sqrt[a]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(x/(b*x^4+a)^(3/4),x)
```

output
```
int(x/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^4)^{3/4}}\,dx=\int \frac{x}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int \frac{x}{(a+bx^4)^{3/4}}\,dx = \frac{x^2 {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}}$$

input
```
integrate(x/(b*x**4+a)**(3/4),x)
```

output
```
x**2*hyper((1/2, 3/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4))
```

Maxima [F]

$$\int \frac{x}{(a+bx^4)^{3/4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^4)^{3/4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)^{3/4}}\,dx = \int \frac{x}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x/(a + b*x^4)^(3/4),x)
```

output
```
int(x/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^4)^{3/4}}\,dx = \int \frac{x}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x/(b*x^4+a)^(3/4),x)
```

output
```
int(x/(a + b*x**4)**(3/4),x)
```

3.576 $\int \frac{1}{x^3(a+bx^4)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{1}{x^3(a+bx^4)^{3/4}} dx = -\frac{\sqrt[4]{a+bx^4}}{2ax^2} - \frac{\sqrt{b}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{a}(a+bx^4)^{3/4}}$$

output
```
-1/2*(b*x^4+a)^(1/4)/a/x^2-1/2*b^(1/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1
/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3(a+bx^4)^{3/4}} dx = -\frac{\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{2x^2(a+bx^4)^{3/4}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)^(3/4)),x]
```

output
```
-1/2*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x^4)/a)
])/(x^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {807, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4)^{3/4}} \, dx$$
$$\Big\downarrow \ 807$$
$$\frac{1}{2}\int \frac{1}{x^4 (bx^4+a)^{3/4}}dx^2$$
$$\Big\downarrow \ 264$$
$$\frac{1}{2}\left(-\frac{b\int \frac{1}{(bx^4+a)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)$$
$$\Big\downarrow \ 231$$
$$\frac{1}{2}\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a\,(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)$$
$$\Big\downarrow \ 229$$
$$\frac{1}{2}\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^4)^(3/4)),x]
```

output
```
(-((a + b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[A
rcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4)))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b \,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^3/(b*x^4+a)^(3/4),x)
```

output
```
int(1/x^3/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3 \left(a+bx^4\right)^{3/4}}\, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^3}\, dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/(b*x^7 + a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^3 \left(a+bx^4\right)^{3/4}}\, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}x^2}$$

input
```
integrate(1/x**3/(b*x**4+a)**(3/4),x)
```

output
```
-hyper((-1/2, 3/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(3/4)*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 \left(a+bx^4\right)^{3/4}}\, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^3}\, dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^4)^{3/4}} dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^3} dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4)^{3/4}} dx = \int \frac{1}{x^3 (b\,x^4+a)^{3/4}} dx$$

input
```
int(1/(x^3*(a + b*x^4)^(3/4)),x)
```

output
```
int(1/(x^3*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^4)^{3/4}} dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}} x^3} dx$$

input
```
int(1/x^3/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**3),x)
```

3.577 $\int \frac{1}{x^7 (a+bx^4)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}} dx = -\frac{\sqrt[4]{a+bx^4}}{6ax^6} + \frac{5b\sqrt[4]{a+bx^4}}{12a^2x^2} + \frac{5b^{3/2}\left(1+\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}(a+bx^4)^{3/4}}$$

output
```
-1/6*(b*x^4+a)^(1/4)/a/x^6+5/12*b*(b*x^4+a)^(1/4)/a^2/x^2+5/12*b^(3/2)*(1+
b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^
(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}} dx = -\frac{\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},-\frac{bx^4}{a}\right)}{6x^6(a+bx^4)^{3/4}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)^(3/4)),x]
```

output
```
-1/6*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x^4)/a
)])/(x^6*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^8 (bx^4+a)^{3/4}} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{5b \int \frac{1}{x^4 (bx^4+a)^{3/4}} dx^2}{6a} - \frac{\sqrt[4]{a+bx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{5b \left(-\frac{b \int \frac{1}{(bx^4+a)^{3/4}} dx^2}{2a} - \frac{\sqrt[4]{a+bx^4}}{ax^2} \right)}{6a} - \frac{\sqrt[4]{a+bx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(-\frac{5b\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)^(3/4)),x]
```

output
```
(-1/3*(a + b*x^4)^(1/4)/(a*x^6) - (5*b*(-((a + b*x^4)^(1/4)/(a*x^2)) - (Sq
rt[b]*(1 + (b*x^4)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/(Sqrt[a]*(a + b*x^4)^(3/4))))/(6*a))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^7/(b*x^4+a)^(3/4),x)
```

output
```
int(1/x^7/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/(b*x^11 + a*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{3}{4}}x^6}$$

input
```
integrate(1/x**7/(b*x**4+a)**(3/4),x)
```

output
```
-hyper((-3/2, 3/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(3/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}} \, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^7} \, dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}} \, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^7} \, dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{x^7\,(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(1/(x^7*(a + b*x^4)^(3/4)),x)
```

output
```
int(1/(x^7*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^7}dx$$

input
```
int(1/x^7/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**7),x)
```

3.578 $\int \frac{1}{x^{11}(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{10ax^{10}}+\frac{3b\sqrt[4]{a+bx^4}}{20a^2x^6}-\frac{3b^2\sqrt[4]{a+bx^4}}{8a^3x^2} - \frac{3b^{5/2}\left(1+\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{8a^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/10*(b*x^4+a)^(1/4)/a/x^10+3/20*b*(b*x^4+a)^(1/4)/a^2/x^6-3/8*b^2*(b*x^4
+a)^(1/4)/a^3/x^2-3/8*b^(5/2)*(1+b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arctan
(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},-\frac{bx^4}{a}\right)}{10x^{10}\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^11*(a + b*x^4)^(3/4)),x]
```

output
```
-1/10*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x^4)/
a)])/(x^10*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 264, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11} (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^{12} (bx^4+a)^{3/4}} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{9b \int \frac{1}{x^8 (bx^4+a)^{3/4}} dx^2}{10a} - \frac{\sqrt[4]{a+bx^4}}{5ax^{10}} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(-\frac{9b \left(-\frac{5b \int \frac{1}{x^4 \left(bx^4+a\right)^{3/4}} dx^2}{6a} - \frac{\sqrt[4]{a+bx^4}}{3ax^6} \right)}{10a} - \frac{\sqrt[4]{a+bx^4}}{5ax^{10}} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{9b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)}{10a}-\frac{\sqrt[4]{a+bx^4}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 231$$

$$\frac{1}{2}\left(-\frac{9b\left(-\frac{5b\left(-\frac{b\left(\frac{bx^4}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{3/4}}dx^2}{2a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)}{10a}-\frac{\sqrt[4]{a+bx^4}}{5ax^{10}}\right)$$

$$\Big\downarrow \ 229$$

$$\frac{1}{2}\left(-\frac{9b\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^4}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a+bx^4}}{3ax^6}\right)}{10a}-\frac{\sqrt[4]{a+bx^4}}{5ax^{10}}\right)$$

input
```
Int[1/(x^11*(a + b*x^4)^(3/4)),x]
```

output
```
(-1/5*(a + b*x^4)^(1/4)/(a*x^10) - (9*b*(-1/3*(a + b*x^4)^(1/4)/(a*x^6) -
(5*b*(-((a + b*x^4)^(1/4)/(a*x^2)) - (Sqrt[b]*(1 + (b*x^4)/a)^(3/4)*Ellipt
icF[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4))))/(6*
a)))/(10*a))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{11} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^11/(b*x^4+a)^(3/4),x)
```

output
```
int(1/x^11/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11} (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/(b*x^15 + a*x^11), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^{11} (a+bx^4)^{3/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{3}{4}}x^{10}}$$

input
```
integrate(1/x**11/(b*x**4+a)**(3/4),x)
```

output
```
-hyper((-5/2, 3/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(3/4)*x**10)
```

Maxima [F]

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^11), x)
```

Giac [F]

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{3/4}}\,dx = \int \frac{1}{x^{11}\,\left(b\,x^4+a\right)^{3/4}}\,dx$$

input
```
int(1/(x^11*(a + b*x^4)^(3/4)),x)
```

output
```
int(1/(x^11*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11} \left(a+b x^4\right)^{3/4}} \, dx=\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{3}{4}} x^{11}}dx$$

input
```
int(1/x^11/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**11),x)
```

3.579 $\int \frac{x^{10}}{(a+bx^4)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}} dx = -\frac{7ax^3\sqrt[4]{a+bx^4}}{32b^2}+\frac{x^7\sqrt[4]{a+bx^4}}{8b} - \frac{21a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}+\frac{21a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{11/4}}$$

output
```
-7/32*a*x^3*(b*x^4+a)^(1/4)/b^2+1/8*x^7*(b*x^4+a)^(1/4)/b-21/64*a^2*arctan
(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(11/4)+21/64*a^2*arctanh(b^(1/4)*x/(b*x^4+a)
^(1/4))/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}} dx = \frac{2b^{3/4}x^3\sqrt[4]{a+bx^4}(-7a+4bx^4)-21a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+21a^2\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx}}\right)}{64b^{11/4}}$$

input
```
Integrate[x^10/(a + b*x^4)^(3/4),x]
```

output
```
(2*b^(3/4)*x^3*(a + b*x^4)^(1/4)*(-7*a + 4*b*x^4) - 21*a^2*ArcTan[(b^(1/4)
*x)/(a + b*x^4)^(1/4)] + 21*a^2*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(6
4*b^(11/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {843, 843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\int \frac{x^6}{(bx^4+a)^{3/4}}dx}{8b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{4b}\right)}{8b}$$

$$\Big\downarrow \text{854}$$

$$\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}$$

$$\Big\downarrow \text{827}$$

$$\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}\right)}{8b}$$

$$\downarrow 216$$

$$\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int\frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)}{8b}$$

$$\downarrow 219$$

$$\frac{x^7\sqrt[4]{a+bx^4}}{8b}-\frac{7a\left(\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}\right)}{8b}$$

input
```
Int[x^10/(a + b*x^4)^(3/4),x]
```

output
```
(x^7*(a + b*x^4)^(1/4))/(8*b) - (7*a*((x^3*(a + b*x^4)^(1/4))/(4*b) - (3*a
*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)
/(a + b*x^4)^(1/4)]/(2*b^(3/4))))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$\frac{16\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{\frac{7}{4}} x^{7}-28 a\,x^{3} \left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{\frac{3}{4}}+21 \ln \left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a^{2}+42 \arctan \left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a^{2}}{128 b^{\frac{11}{4}}}$	104

input
```
int(x^10/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/128/b^(11/4)*(16*(b*x^4+a)^(1/4)*b^(7/4)*x^7-28*a*x^3*(b*x^4+a)^(1/4)*b^
(3/4)+21*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^2+
42*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.14

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}\,dx = \frac{21\,b^2\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(\frac{21\left(b^3x\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+(bx^4+a)^{\frac{1}{4}}a^2\right)}{x}\right) - 21\,b^2\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-\frac{21\left(b^3x\left(\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}-(bx^4+\right.}{x}\right.}{}$$

input
```
integrate(x^10/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/128*(21*b^2*(a^8/b^11)^(1/4)*log(21*(b^3*x*(a^8/b^11)^(1/4) + (b*x^4 + a
)^(1/4)*a^2)/x) - 21*b^2*(a^8/b^11)^(1/4)*log(-21*(b^3*x*(a^8/b^11)^(1/4)
- (b*x^4 + a)^(1/4)*a^2)/x) - 21*I*b^2*(a^8/b^11)^(1/4)*log(-21*(I*b^3*x*(
a^8/b^11)^(1/4) - (b*x^4 + a)^(1/4)*a^2)/x) + 21*I*b^2*(a^8/b^11)^(1/4)*lo
g(-21*(-I*b^3*x*(a^8/b^11)^(1/4) - (b*x^4 + a)^(1/4)*a^2)/x) + 4*(4*b*x^7
- 7*a*x^3)*(b*x^4 + a)^(1/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(b*x**4+a)**(3/4),x)
```

output
```
x**11*gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(3/4)*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}\,dx = \frac{\frac{11\,(bx^4+a)^{\frac{1}{4}}a^2b}{x} - \frac{7\,(bx^4+a)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^4 - \frac{2\,(bx^4+a)b^3}{x^4} + \frac{(bx^4+a)^2b^2}{x^8}\right)} + \frac{21\left(\frac{2\,a^2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}} - \frac{a^2\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{128\,b^2}$$

input
```
integrate(x^10/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/32*(11*(b*x^4 + a)^(1/4)*a^2*b/x - 7*(b*x^4 + a)^(5/4)*a^2/x^5)/(b^4 - 2
*(b*x^4 + a)*b^3/x^4 + (b*x^4 + a)^2*b^2/x^8) + 21/128*(2*a^2*arctan((b*x^
4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - a^2*log(-(b^(1/4) - (b*x^4 + a)^(1/4)/
x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b^2
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^10/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{10}}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^10/(a + b*x^4)^(3/4),x)
```

output
```
int(x^10/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{10}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^10/(b*x^4+a)^(3/4),x)
```

output
```
int(x**10/(a + b*x**4)**(3/4),x)
```

3.580 $\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \frac{x^3\sqrt[4]{a+bx^4}}{4b}+\frac{3a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}-\frac{3a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}$$

output
```
1/4*x^3*(b*x^4+a)^(1/4)/b+3/8*a*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)-
3/8*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \frac{2b^{3/4}x^3\sqrt[4]{a+bx^4}+3a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)-3a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{7/4}}$$

input
```
Integrate[x^6/(a + b*x^4)^(3/4),x]
```

output
```
(2*b^(3/4)*x^3*(a + b*x^4)^(1/4) + 3*a*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4
)] - 3*a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(7/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {843, 854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{(bx^4+a)^{3/4}}dx}{4b}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^4+a}\left(1-\frac{bx^4}{bx^4+a}\right)}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}\right)}{4b}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{x^3\sqrt[4]{a+bx^4}}{4b}-\frac{3a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}\right)}{4b}$$

input
```
Int[x^6/(a + b*x^4)^(3/4),x]
```

output
```
(x^3*(a + b*x^4)^(1/4))/(4*b) - (3*a*(-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^
(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(3/4))))/(4*b
)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

method	result	size
pseudoelliptic	$-\frac{3\left(-\frac{4\left(b\,x^{4}+a\right)^{\frac{1}{4}} x^{3} b^{\frac{3}{4}}}{3}+\ln\left(\frac{b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right) a+2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right) a\right)}{16 b^{\frac{7}{4}}}$	81

input
```
int(x^6/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-3/16/b^(7/4)*(-4/3*(b*x^4+a)^(1/4)*x^3*b^(3/4)+ln((b^(1/4)*x+(b*x^4+a)^(1
/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a+2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))
*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.51

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \frac{4\,(bx^4+a)^{\frac{1}{4}}x^3 - 3\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(b^2x\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}+(bx^4+a)^{\frac{1}{4}}a\right)}{x}\right) + 3\,b\left(\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(b^2x\left(\frac{a^4}{b^7}\right.\right.}{}\right.}{}$$

input
```
integrate(x^6/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/16*(4*(b*x^4 + a)^(1/4)*x^3 - 3*b*(a^4/b^7)^(1/4)*log(3*(b^2*x*(a^4/b^7)
^(1/4) + (b*x^4 + a)^(1/4)*a)/x) + 3*b*(a^4/b^7)^(1/4)*log(-3*(b^2*x*(a^4/
b^7)^(1/4) - (b*x^4 + a)^(1/4)*a)/x) + 3*I*b*(a^4/b^7)^(1/4)*log(-3*(I*b^2
*x*(a^4/b^7)^(1/4) - (b*x^4 + a)^(1/4)*a)/x) - 3*I*b*(a^4/b^7)^(1/4)*log(-
3*(-I*b^2*x*(a^4/b^7)^(1/4) - (b*x^4 + a)^(1/4)*a)/x))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.46

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(b*x**4+a)**(3/4),x)
```

output
```
x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(3/4)*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = -\frac{3\left(\frac{2\,a\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{3}{4}}}-\frac{a\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{3}{4}}}\right)}{16\,b}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}a}{4\left(b^2-\frac{(bx^4+a)b}{x^4}\right)x}$$

input
```
integrate(x^6/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-3/16*(2*a*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - a*log(-(b^(1/4)
 - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4))/b - 1/4*
(b*x^4 + a)^(1/4)*a/((b^2 - (b*x^4 + a)*b/x^4)*x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^6/(a + b*x^4)^(3/4),x)
```

output
```
int(x^6/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(b*x^4+a)^(3/4),x)
```

output
```
int(x**6/(a + b*x**4)**(3/4),x)
```

3.581 $\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

output
```
-1/2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(3/4)+1/2*arctanh(b^(1/4)*x/(b*x^
4+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\left(a+bx^4\right)^{3/4}}\,dx=\frac{-\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

input
```
Integrate[x^2/(a + b*x^4)^(3/4),x]
```

output
```
(-ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^
(1/4)])/(2*b^(3/4))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {854, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{854}$$

$$\int \frac{x^2}{\sqrt{a+bx^4}\left(1-\frac{bx^4}{a+bx^4}\right)}d\frac{x}{\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{3/4}}$$

input
```
Int[x^2/(a + b*x^4)^(3/4),x]
```

output
```
-1/2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/b^(3/4) + ArcTanh[(b^(1/4)*x)/(
a + b*x^4)^(1/4)]/(2*b^(3/4))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{\ln\left(\frac{b^{\frac{1}{4}} x+\left(b \,x^{4}+a \right)^{\frac{1}{4}}}{-b^{\frac{1}{4}} x+\left(b \,x^{4}+a \right)^{\frac{1}{4}}}\right)+2 \arctan\left(\frac{\left(b \,x^{4}+a \right)^{\frac{1}{4}}}{b^{\frac{1}{4}} x}\right)}{4 b^{\frac{3}{4}}}$	61

input
```
int(x^2/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^(3/4)*(ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))+
2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.21

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx = \frac{1}{4}\frac{1}{b^3}^{\frac{1}{4}}\log\left(\frac{b\frac{1}{b^3}^{\frac{1}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right) - \frac{1}{4}\frac{1}{b^3}^{\frac{1}{4}}\log\left(-\frac{b\frac{1}{b^3}^{\frac{1}{4}}x-(bx^4+a)^{\frac{1}{4}}}{x}\right) + \frac{1}{4}i\frac{1}{b^3}^{\frac{1}{4}}\log\left(\frac{i\,b\frac{1}{b^3}^{\frac{1}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right) - \frac{1}{4}i\frac{1}{b^3}^{\frac{1}{4}}\log\left(\frac{-i\,b\frac{1}{b^3}^{\frac{1}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/4*(b^(-3))^(1/4)*log((b*(b^(-3))^(1/4)*x + (b*x^4 + a)^(1/4))/x) - 1/4*(
b^(-3))^(1/4)*log(-(b*(b^(-3))^(1/4)*x - (b*x^4 + a)^(1/4))/x) + 1/4*I*(b^
(-3))^(1/4)*log((I*b*(b^(-3))^(1/4)*x + (b*x^4 + a)^(1/4))/x) - 1/4*I*(b^(
-3))^(1/4)*log((-I*b*(b^(-3))^(1/4)*x + (b*x^4 + a)^(1/4))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(b*x**4+a)**(3/4),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/4)*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx = \frac{\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{2\,b^{\frac{3}{4}}} - \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{4\,b^{\frac{3}{4}}}$$

input
```
integrate(x^2/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(3/4) - 1/4*log(-(b^(1/4) - (b
*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(3/4)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^2/(a + b*x^4)^(3/4),x)
```

output
```
int(x^2/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(b*x^4+a)^(3/4),x)
```

output
```
int(x**2/(a + b*x**4)**(3/4),x)
```

3.582 $\int \frac{1}{x^2(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{x^2(a+bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{ax}$$

output
```
-(b*x^4+a)^(1/4)/a/x
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{ax}$$

input
```
Integrate[1/(x^2*(a + b*x^4)^(3/4)),x]
```

output
```
-((a + b*x^4)^(1/4)/(a*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \; 796$$

$$-\frac{\sqrt[4]{a+bx^4}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^4)^(3/4)),x]
```

output
```
-((a + b*x^4)^(1/4)/(a*x))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}}{ax}$	18
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}}{ax}$	18
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}}{ax}$	18
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{1}{4}}}{ax}$	18
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}}{ax}$	18

input
```
int(1/x^2/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^4+a)^(1/4)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\left(a+bx^4\right)^{3/4}}\,dx = -\frac{(bx^4+a)^{\frac{1}{4}}}{ax}$$

input
```
integrate(1/x^2/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-(b*x^4 + a)^(1/4)/(a*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{1}{x^2 (a + bx^4)^{3/4}} \, dx = \frac{\sqrt[4]{b} \sqrt[4]{\frac{a}{bx^4} + 1} \Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(b*x**4+a)**(3/4),x)
```

output
```
b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-1/4)/(4*a*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a + bx^4)^{3/4}} \, dx = -\frac{(bx^4 + a)^{\frac{1}{4}}}{ax}$$

input
```
integrate(1/x^2/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-(b*x^4 + a)^(1/4)/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4)^{3/4}} \, dx = \int \frac{1}{(bx^4 + a)^{\frac{3}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a+bx^4)^{3/4}} \, dx = -\frac{(b\,x^4+a)^{1/4}}{a\,x}$$

input
```
int(1/(x^2*(a + b*x^4)^(3/4)),x)
```

output
```
-(a + b*x^4)^(1/4)/(a*x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^4)^{3/4}} \, dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(1/x^2/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**2),x)
```

3.583 $\int \frac{1}{x^6 \left(a+bx^4\right)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^6 \left(a+bx^4\right)^{3/4}} \, dx = -\frac{\sqrt[4]{a+bx^4}}{5ax^5}+\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}$$

output
```
-1/5*(b*x^4+a)^(1/4)/a/x^5+4/5*b*(b*x^4+a)^(1/4)/a^2/x
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^6 \left(a+bx^4\right)^{3/4}} \, dx = \frac{\sqrt[4]{a+bx^4}(-a+4bx^4)}{5a^2x^5}$$

input
```
Integrate[1/(x^6*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-a + 4*b*x^4))/(5*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b \int \frac{1}{x^2 (bx^4+a)^{3/4}}dx}{5a}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^4)^(3/4)),x]
```

output
```
-1/5*(a + b*x^4)^(1/4)/(a*x^5) + (4*b*(a + b*x^4)^(1/4))/(5*a^2*x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-4b\,x^4+a)}{5a^2x^5}$	26
trager	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-4b\,x^4+a)}{5a^2x^5}$	26
risch	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-4b\,x^4+a)}{5a^2x^5}$	26
pseudoelliptic	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-4b\,x^4+a)}{5a^2x^5}$	26
orering	$-\frac{(b\,x^4+a)^{\frac{1}{4}}\,(-4b\,x^4+a)}{5a^2x^5}$	26

input
```
int(1/x^6/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^4+a)^(1/4)*(-4*b*x^4+a)/a^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^6\,(a+bx^4)^{3/4}}\,dx = \frac{(4\,bx^4-a)(bx^4+a)^{\frac{1}{4}}}{5\,a^2x^5}$$

input
```
integrate(1/x^6/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/5*(4*b*x^4 - a)*(b*x^4 + a)^(1/4)/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^6 (a+bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{16ax^4\Gamma\left(\frac{3}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{4a^2\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**6/(b*x**4+a)**(3/4),x)
```

output
```
-b**(1/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(16*a*x**4*gamma(3/4)) + b**
(5/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(4*a**2*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^6 (a+bx^4)^{3/4}}\,dx = \frac{\frac{5\,(bx^4+a)^{\frac{1}{4}}b}{x} - \frac{(bx^4+a)^{\frac{5}{4}}}{x^5}}{5\,a^2}$$

input
```
integrate(1/x^6/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/5*(5*(b*x^4 + a)^(1/4)*b/x - (b*x^4 + a)^(5/4)/x^5)/a^2
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6 (a+bx^4)^{3/4}} dx = -\frac{(b\,x^4+a)^{1/4}\,(a-4\,b\,x^4)}{5\,a^2\,x^5}$$

input
```
int(1/(x^6*(a + b*x^4)^(3/4)),x)
```

output
```
-((a + b*x^4)^(1/4)*(a - 4*b*x^4))/(5*a^2*x^5)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^4)^{3/4}} dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^6}dx$$

input
```
int(1/x^6/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**6),x)
```

3.584 $\int \frac{1}{x^{10}(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a+bx^4}}{9ax^9}+\frac{8b\sqrt[4]{a+bx^4}}{45a^2x^5}-\frac{32b^2\sqrt[4]{a+bx^4}}{45a^3x}$$

output
```
-1/9*(b*x^4+a)^(1/4)/a/x^9+8/45*b*(b*x^4+a)^(1/4)/a^2/x^5-32/45*b^2*(b*x^4
+a)^(1/4)/a^3/x
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx=\frac{\sqrt[4]{a+bx^4}(-5a^2+8abx^4-32b^2x^8)}{45a^3x^9}$$

input
```
Integrate[1/(x^10*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-5*a^2 + 8*a*b*x^4 - 32*b^2*x^8))/(45*a^3*x^9)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} (a+bx^4)^{3/4}} dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{8b\int \frac{1}{x^6(bx^4+a)^{3/4}}dx}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{8b\left(-\frac{4b\int \frac{1}{x^2\left(bx^4+a\right)^{3/4}}dx}{5a}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{8b\left(\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}$$

input
```
Int[1/(x^10*(a + b*x^4)^(3/4)),x]
```

output
```
-1/9*(a + b*x^4)^(1/4)/(a*x^9) - (8*b*(-1/5*(a + b*x^4)^(1/4)/(a*x^5) + (4
*b*(a + b*x^4)^(1/4))/(5*a^2*x)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{(b\,x^{4}+a)^{\frac{1}{4}}\left(32 b^{2} x^{8}-8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	39
trager	$-\frac{(b\,x^{4}+a)^{\frac{1}{4}}\left(32 b^{2} x^{8}-8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	39
risch	$-\frac{(b\,x^{4}+a)^{\frac{1}{4}}\left(32 b^{2} x^{8}-8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	39
pseudoelliptic	$-\frac{(b\,x^{4}+a)^{\frac{1}{4}}\left(32 b^{2} x^{8}-8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	39
orering	$-\frac{(b\,x^{4}+a)^{\frac{1}{4}}\left(32 b^{2} x^{8}-8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	39

input
```
int(1/x^10/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(b*x^4+a)^(1/4)*(32*b^2*x^8-8*a*b*x^4+5*a^2)/a^3/x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{10} (a+bx^4)^{3/4}} \, dx = -\frac{(32 \, b^2 x^8 - 8 \, abx^4 + 5 \, a^2)(bx^4 + a)^{\frac{1}{4}}}{45 \, a^3 x^9}$$

input
```
integrate(1/x^10/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/45*(32*b^2*x^8 - 8*a*b*x^4 + 5*a^2)*(b*x^4 + a)^(1/4)/(a^3*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(60) = 120$.

Time = 0.85 (sec) , antiderivative size = 406, normalized size of antiderivative = 5.97

$$\begin{aligned}\int \frac{1}{x^{10} (a+bx^4)^{3/4}} \, dx &= \frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{2a^3b^{\frac{21}{4}}x^4\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{21a^2b^{\frac{25}{4}}x^8\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{56ab^{\frac{29}{4}}x^{12}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)} \\ &+ \frac{32b^{\frac{33}{4}}x^{16}\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(-\frac{9}{4}\right)}{64a^5b^4x^8\Gamma\left(\frac{3}{4}\right)+128a^4b^5x^{12}\Gamma\left(\frac{3}{4}\right)+64a^3b^6x^{16}\Gamma\left(\frac{3}{4}\right)}\end{aligned}$$

input
```
integrate(1/x**10/(b*x**4+a)**(3/4),x)
```

output
```
5*a**4*b**(17/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*ga
mma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gamma(3/4))
 + 2*a**3*b**(21/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4
*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*x**16*gam
ma(3/4)) + 21*a**2*b**(25/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-9/4)/(64*
a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**3*b**6*
x**16*gamma(3/4)) + 56*a*b**(29/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-9/
4)/(64*a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 64*a**
3*b**6*x**16*gamma(3/4)) + 32*b**(33/4)*x**16*(a/(b*x**4) + 1)**(1/4)*gamm
a(-9/4)/(64*a**5*b**4*x**8*gamma(3/4) + 128*a**4*b**5*x**12*gamma(3/4) + 6
4*a**3*b**6*x**16*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{10} (a+bx^4)^{3/4}}\,dx = -\frac{\frac{45\,(bx^4+a)^{\frac{1}{4}}b^2}{x} - \frac{18\,(bx^4+a)^{\frac{5}{4}}b}{x^5} + \frac{5\,(bx^4+a)^{\frac{9}{4}}}{x^9}}{45\,a^3}$$

input
```
integrate(1/x^10/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/45*(45*(b*x^4 + a)^(1/4)*b^2/x - 18*(b*x^4 + a)^(5/4)*b/x^5 + 5*(b*x^4
+ a)^(9/4)/x^9)/a^3
```

Giac [F]

$$\int \frac{1}{x^{10} (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^10), x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx=-\frac{{\left(b\,x^4+a\right)}^{1/4}\,\left(5\,a^2-8\,a\,b\,x^4+32\,b^2\,x^8\right)}{45\,a^3\,x^9}$$

input
```
int(1/(x^10*(a + b*x^4)^(3/4)),x)
```

output
```
-((a + b*x^4)^(1/4)*(5*a^2 + 32*b^2*x^8 - 8*a*b*x^4))/(45*a^3*x^9)
```

Reduce [F]

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{10}}dx$$

input
```
int(1/x^10/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**10),x)
```

3.585 $\int \frac{1}{x^{14}(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}+\frac{4b\sqrt[4]{a+bx^4}}{39a^2x^9}-\frac{32b^2\sqrt[4]{a+bx^4}}{195a^3x^5}+\frac{128b^3\sqrt[4]{a+bx^4}}{195a^4x}$$

output
```
-1/13*(b*x^4+a)^(1/4)/a/x^13+4/39*b*(b*x^4+a)^(1/4)/a^2/x^9-32/195*b^2*(b*
x^4+a)^(1/4)/a^3/x^5+128/195*b^3*(b*x^4+a)^(1/4)/a^4/x
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \frac{\sqrt[4]{a+bx^4}\left(-15a^3+20a^2bx^4-32ab^2x^8+128b^3x^{12}\right)}{195a^4x^{13}}$$

input
```
Integrate[1/(x^14*(a + b*x^4)^(3/4)),x]
```

output
```
((a + b*x^4)^(1/4)*(-15*a^3 + 20*a^2*b*x^4 - 32*a*b^2*x^8 + 128*b^3*x^12))
/(195*a^4*x^13)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14} (a+bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \; 803$$

$$-\frac{12b\int \frac{1}{x^{10}(bx^4+a)^{3/4}}dx}{13a}-\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}$$

$$\Big\downarrow \; 803$$

$$-\frac{12b\left(-\frac{8b\int \frac{1}{x^6\left(bx^4+a\right)^{3/4}}dx}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}\right)}{13a}-\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}$$

$$\Big\downarrow \; 803$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{x^2\left(bx^4+a\right)^{3/4}}dx}{5a}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}\right)}{13a}-\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}$$

$$\Big\downarrow \; 796$$

$$-\frac{12b\left(-\frac{8b\left(\frac{4b\sqrt[4]{a+bx^4}}{5a^2x}-\frac{\sqrt[4]{a+bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a+bx^4}}{9ax^9}\right)}{13a}-\frac{\sqrt[4]{a+bx^4}}{13ax^{13}}$$

input
```
Int[1/(x^14*(a + b*x^4)^(3/4)),x]
```

output
```
-1/13*(a + b*x^4)^(1/4)/(a*x^13) - (12*b*(-1/9*(a + b*x^4)^(1/4)/(a*x^9) -
 (8*b*(-1/5*(a + b*x^4)^(1/4)/(a*x^5) + (4*b*(a + b*x^4)^(1/4))/(5*a^2*x))
)/(9*a)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}} \left(-128 b^{3} x^{12}+32 a\,b^{2} x^{8}-20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	50
trager	$-\frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}} \left(-128 b^{3} x^{12}+32 a\,b^{2} x^{8}-20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	50
risch	$-\frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}} \left(-128 b^{3} x^{12}+32 a\,b^{2} x^{8}-20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}} \left(-128 b^{3} x^{12}+32 a\,b^{2} x^{8}-20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	50
orering	$-\frac{\left(b\,x^{4}+a \right)^{\frac{1}{4}} \left(-128 b^{3} x^{12}+32 a\,b^{2} x^{8}-20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	50

input
```
int(1/x^14/(b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/195*(b*x^4+a)^(1/4)*(-128*b^3*x^12+32*a*b^2*x^8-20*a^2*b*x^4+15*a^3)/x^
13/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^{14} (a+bx^4)^{3/4}}\,dx = \frac{(128\,b^3x^{12} - 32\,ab^2x^8 + 20\,a^2bx^4 - 15\,a^3)(bx^4+a)^{\frac{1}{4}}}{195\,a^4x^{13}}$$

input
```
integrate(1/x^14/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
1/195*(128*b^3*x^12 - 32*a*b^2*x^8 + 20*a^2*b*x^4 - 15*a^3)*(b*x^4 + a)^(1
/4)/(a^4*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. 2(83) = 166.

Time = 1.24 (sec) , antiderivative size = 692, normalized size of antiderivative = 7.52

$$\int \frac{1}{x^{14} (a+bx^4)^{3/4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**14/(b*x**4+a)**(3/4),x)
```

output
```
-45*a**6*b**(37/4)*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**
12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gam
ma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) - 75*a**5*b**(41/4)*x**4*(a/(b*
x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*
b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*
x**24*gamma(3/4)) - 51*a**4*b**(45/4)*x**8*(a/(b*x**4) + 1)**(1/4)*gamma(-
13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) +
768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 231*a
**3*b**(49/4)*x**12*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x*
*12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*ga
mma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 924*a**2*b**(53/4)*x**16*(a/
(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a*
*6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**
12*x**24*gamma(3/4)) + 1056*a*b**(57/4)*x**20*(a/(b*x**4) + 1)**(1/4)*gamm
a(-13/4)/(256*a**7*b**9*x**12*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4)
 + 768*a**5*b**11*x**20*gamma(3/4) + 256*a**4*b**12*x**24*gamma(3/4)) + 38
4*b**(61/4)*x**24*(a/(b*x**4) + 1)**(1/4)*gamma(-13/4)/(256*a**7*b**9*x**1
2*gamma(3/4) + 768*a**6*b**10*x**16*gamma(3/4) + 768*a**5*b**11*x**20*gamm
a(3/4) + 256*a**4*b**12*x**24*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx = \frac{\frac{195\left(bx^4+a\right)^{\frac{1}{4}}b^3}{x} - \frac{117\left(bx^4+a\right)^{\frac{5}{4}}b^2}{x^5} + \frac{65\left(bx^4+a\right)^{\frac{9}{4}}b}{x^9} - \frac{15\left(bx^4+a\right)^{\frac{13}{4}}}{x^{13}}}{195\,a^4}$$

input
```
integrate(1/x^14/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/195*(195*(b*x^4 + a)^(1/4)*b^3/x - 117*(b*x^4 + a)^(5/4)*b^2/x^5 + 65*(b
*x^4 + a)^(9/4)*b/x^9 - 15*(b*x^4 + a)^(13/4)/x^13)/a^4
```

Giac [F]

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^14), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx=\frac{4\,b\,\left(b\,x^4+a\right)^{1/4}}{39\,a^2\,x^9}-\frac{\left(b\,x^4+a\right)^{1/4}}{13\,a\,x^{13}}+\frac{128\,b^3\,\left(b\,x^4+a\right)^{1/4}}{195\,a^4\,x}-\frac{32\,b^2\,\left(b\,x^4+a\right)^{1/4}}{195\,a^3\,x^5}$$

input
```
int(1/(x^14*(a + b*x^4)^(3/4)),x)
```

output
```
(4*b*(a + b*x^4)^(1/4))/(39*a^2*x^9) - (a + b*x^4)^(1/4)/(13*a*x^13) + (12
8*b^3*(a + b*x^4)^(1/4))/(195*a^4*x) - (32*b^2*(a + b*x^4)^(1/4))/(195*a^3
*x^5)
```

Reduce [F]

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{3}{4}}\,x^{14}}dx$$

input
```
int(1/x^14/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**14),x)
```

3.586 $\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \frac{3a^2x\sqrt[4]{a+bx^4}}{8b^3}-\frac{3ax^5\sqrt[4]{a+bx^4}}{20b^2}+\frac{x^9\sqrt[4]{a+bx^4}}{10b}+\frac{3a^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{8b^{5/2}(a+bx^4)^{3/4}}$$

output
```
3/8*a^2*x*(b*x^4+a)^(1/4)/b^3-3/20*a*x^5*(b*x^4+a)^(1/4)/b^2+1/10*x^9*(b*x
^4+a)^(1/4)/b+3/8*a^(5/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot
(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.00 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \frac{15a^3x+9a^2bx^5-2ab^2x^9+4b^3x^{13}-15a^3x\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\right.}{40b^3(a+bx^4)^{3/4}}$$

input
```
Integrate[x^12/(a + b*x^4)^(3/4),x]
```

output
```
(15*a^3*x + 9*a^2*b*x^5 - 2*a*b^2*x^9 + 4*b^3*x^13 - 15*a^3*x*(1 + (b*x^4)
/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(40*b^3*(a + b*x
^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {843, 843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\int \frac{x^8}{(bx^4+a)^{3/4}}dx}{10b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int \frac{x^4}{\left(bx^4+a\right)^{3/4}}dx}{6b}\right)}{10b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{2b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}\right)}{6b}\right)}{10b}$$

$\Big\downarrow$ 858

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}$$

$\Big\downarrow$ 807

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}$$

$\Big\downarrow$ 229

$$\frac{x^9\sqrt[4]{a+bx^4}}{10b}-\frac{9a\left(\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}\right)}{10b}$$

input
```
Int[x^12/(a + b*x^4)^(3/4),x]
```

output
```
(x^9*(a + b*x^4)^(1/4))/(10*b) - (9*a*((x^5*(a + b*x^4)^(1/4))/(6*b) - (5*
a*((x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*Ellipt
icF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(
6*b)))/(10*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{12}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^12/(b*x^4+a)^(3/4),x)
```

output
```
int(x^12/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^12/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12/(b*x**4+a)**(3/4),x)
```

output
```
x**13*gamma(13/4)*hyper((3/4, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(3/4)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^12/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^12/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^12/(a + b*x^4)^(3/4),x)
```

output
```
int(x^12/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{12}}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^12/(b*x^4+a)^(3/4),x)
```

output
```
int(x**12/(a + b*x**4)**(3/4),x)
```

3.587 $\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = -\frac{5ax\sqrt[4]{a+bx^4}}{12b^2}+\frac{x^5\sqrt[4]{a+bx^4}}{6b} - \frac{5a^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\mathrm{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12b^{3/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-5/12*a*x*(b*x^4+a)^(1/4)/b^2+1/6*x^5*(b*x^4+a)^(1/4)/b-5/12*a^(3/2)*(1+a/
b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/
b^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = \frac{-5a^2x-3abx^5+2b^2x^9+5a^2x\left(1+\frac{bx^4}{a}\right)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{12b^2\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[x^8/(a + b*x^4)^(3/4),x]
```

output
```
(-5*a^2*x - 3*a*b*x^5 + 2*b^2*x^9 + 5*a^2*x*(1 + (b*x^4)/a)^(3/4)*Hypergeo
metric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(12*b^2*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {843, 843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\int \frac{x^4}{(bx^4+a)^{3/4}}dx}{6b}$$

$$\Big\downarrow \text{843}$$

$$\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}\right)}{6b}$$

$$\Big\downarrow \text{768}$$

$$\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b(a+bx^4)^{3/4}}\right)}{6b}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}$$

$$\Big\downarrow \text{229}$$

$$\frac{x^5\sqrt[4]{a+bx^4}}{6b}-\frac{5a\left(\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a+bx^4)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}\right)}{6b}$$

input
```
Int[x^8/(a + b*x^4)^(3/4),x]
```

output
```
(x^5*(a + b*x^4)^(1/4))/(6*b) - (5*a*((x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[
a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(2*Sqrt[b]*(a + b*x^4)^(3/4))))/(6*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^8/(b*x^4+a)^(3/4),x)
```

output
```
int(x^8/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx=\int \frac{x^8}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^8/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(b*x**4+a)**(3/4),x)
```

output
```
x**9*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(3/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^8}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^8/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^8}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^8/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^8}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^8/(a + b*x^4)^(3/4),x)
```

output
```
int(x^8/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^8}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^8/(b*x^4+a)^(3/4),x)
```

output
```
int(x**8/(a + b*x**4)**(3/4),x)
```

3.588 $\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \frac{x\sqrt[4]{a+bx^4}}{2b} + \frac{\sqrt{a}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{b}\,(a+bx^4)^{3/4}}$$

output
```
1/2*x*(b*x^4+a)^(1/4)/b+1/2*a^(1/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(
1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \frac{x\left(a+bx^4-a\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)\right)}{2b\,(a+bx^4)^{3/4}}$$

input
```
Integrate[x^4/(a + b*x^4)^(3/4),x]
```

output
```
(x*(a + b*x^4 - a*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -
((b*x^4)/a)]))/(2*b*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {843, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{a\int \frac{1}{(bx^4+a)^{3/4}}dx}{2b}$$

$$\Big\downarrow \text{768}$$

$$\frac{x\sqrt[4]{a+bx^4}}{2b}-\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{2b\left(a+bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{2b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}$$

$$\Big\downarrow \text{807}$$

$$\frac{ax^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt{a}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a+bx^4\right)^{3/4}}+\frac{x\sqrt[4]{a+bx^4}}{2b}$$

input
```
Int[x^4/(a + b*x^4)^(3/4),x]
```

output
```
(x*(a + b*x^4)^(1/4))/(2*b) + (Sqrt[a]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF
[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(b*x^4+a)^(3/4),x)
```

output
```
int(x^4/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^4 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\,\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(b*x**4+a)**(3/4),x)
```

output
```
x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(x^4/(a + b*x^4)^(3/4),x)
```

output
```
int(x^4/(a + b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^4)^{3/4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(b*x^4+a)^(3/4),x)
```

output
```
int(x**4/(a + b*x**4)**(3/4),x)
```

3.589 $\int \frac{1}{(a+bx^4)^{3/4}} dx$

Optimal result

Integrand size = 11, antiderivative size = 61

$$\int \frac{1}{(a+bx^4)^{3/4}} dx = -\frac{\sqrt{b}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^4)^{3/4}}$$

output
```
-b^(1/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1
/2)),2^(1/2))/a^(1/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a+bx^4)^{3/4}} dx = \frac{x\left(1+\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^4}{a}\right)}{(a+bx^4)^{3/4}}$$

input
```
Integrate[(a + b*x^4)^(-3/4),x]
```

output
```
(x*(1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^4)/a)])/(
a + b*x^4)^(3/4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{768}$$

$$\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{2(a+bx^4)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{\sqrt{b}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}\,(a+bx^4)^{3/4}}$$

input
```
Int[(a + b*x^4)^(-3/4),x]
```

output
```
-((Sqrt[b]*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(Sqrt[a]*(a + b*x^4)^(3/4)))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^4+a)^(3/4),x)
```

output
```
int(1/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(-3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a+bx^4)^{3/4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(b*x**4+a)**(3/4),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{1}{(a+bx^4)^{3/4}}\,dx = \frac{x\left(\frac{b\,x^4}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{(b\,x^4+a)^{3/4}}$$

input
```
int(1/(a + b*x^4)^(3/4),x)
```

output
```
(x*((b*x^4)/a + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(b*x^4)/a))/(a + b*x^
4)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{3/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^4+a)^(3/4),x)
```

output
```
int(1/(a + b*x**4)**(3/4),x)
```

3.590 $\int \frac{1}{x^4(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{1}{x^4(a+bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{3ax^3} + \frac{2b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4} x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}$$

output
```
-1/3*(b*x^4+a)^(1/4)/a/x^3+2/3*b^(3/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobi
AM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4(a+bx^4)^{3/4}}\,dx = -\frac{\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},-\frac{bx^4}{a}\right)}{3x^3(a+bx^4)^{3/4}}$$

input
```
Integrate[1/(x^4*(a + b*x^4)^(3/4)),x]
```

output
```
-1/3*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^4)/a)
])/(x^3*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{2b\int \frac{1}{(bx^4+a)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{768}$$

$$-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{807}$$

$$\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

$$\Big\downarrow \text{229}$$

$$\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^4)^(3/4)),x]
```

output
```
-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*Elli
pticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{4}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^4/(b*x^4+a)^(3/4),x)
```

output
```
int(1/x^4/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/(b*x^8 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4 (a+bx^4)^{3/4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| {\frac{bx^{4}e^{i\pi}}{a}} \right)}{4a^{\frac{3}{4}}x^{3}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/x**4/(b*x**4+a)**(3/4),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3/
4)*x**3*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{x^4\,(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(1/(x^4*(a + b*x^4)^(3/4)),x)
```

output
```
int(1/(x^4*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/x^4/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**4),x)
```

3.591 $\int \frac{1}{x^8(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a+bx^4}}{7ax^7}+\frac{2b\sqrt[4]{a+bx^4}}{7a^2x^3}-\frac{4b^{5/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{7a^{5/2}\left(a+bx^4\right)^{3/4}}$$

output
```
-1/7*(b*x^4+a)^(1/4)/a/x^7+2/7*b*(b*x^4+a)^(1/4)/a^2/x^3-4/7*b^(5/2)*(1+a/
b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/
a^(5/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^8\left(a+bx^4\right)^{3/4}}\,dx=-\frac{\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},-\frac{bx^4}{a}\right)}{7x^7\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^8*(a + b*x^4)^(3/4)),x]
```

output
```
-1/7*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^4)/a
)])/(x^7*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {847, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a+b x^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{6 b \int \frac{1}{x^4 \left(b x^4+a\right)^{3/4}}dx}{7 a}-\frac{\sqrt[4]{a+b x^4}}{7 a x^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{6 b \left(-\frac{2 b \int \frac{1}{\left(b x^4+a\right)^{3/4}}dx}{3 a}-\frac{\sqrt[4]{a+b x^4}}{3 a x^3}\right)}{7 a}-\frac{\sqrt[4]{a+b x^4}}{7 a x^7}$$

$$\Big\downarrow \text{768}$$

$$-\frac{6 b \left(-\frac{2 b x^3 \left(\frac{a}{b x^4}+1\right)^{3/4} \int \frac{1}{\left(\frac{a}{b x^4}+1\right)^{3/4} x^3}dx}{3 a \left(a+b x^4\right)^{3/4}}-\frac{\sqrt[4]{a+b x^4}}{3 a x^3}\right)}{7 a}-\frac{\sqrt[4]{a+b x^4}}{7 a x^7}$$

$$\Big\downarrow \text{858}$$

$$-\frac{6 b \left(\frac{2 b x^3 \left(\frac{a}{b x^4}+1\right)^{3/4} \int \frac{1}{\left(\frac{a}{b x^4}+1\right)^{3/4} x}d\frac{1}{x}}{3 a \left(a+b x^4\right)^{3/4}}-\frac{\sqrt[4]{a+b x^4}}{3 a x^3}\right)}{7 a}-\frac{\sqrt[4]{a+b x^4}}{7 a x^7}$$

$$\Big\downarrow \; 807$$

$$-\frac{6b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}$$

$$\Big\downarrow \; 229$$

$$-\frac{6b\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a+bx^4)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}$$

input
```
Int[1/(x^8*(a + b*x^4)^(3/4)),x]
```

output
```
-1/7*(a + b*x^4)^(1/4)/(a*x^7) - (6*b*(-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2
*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]
/2, 2])/(3*a^(3/2)*(a + b*x^4)^(3/4))))/(7*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{8} \left(b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^8/(b*x^4+a)^(3/4),x)
```

output
```
int(1/x^8/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^8 \left(a+bx^4\right)^{3/4}}\,dx=\int \frac{1}{{\left(bx^{4}+a\right)}^{\frac{3}{4}}x^{8}}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/(b*x^12 + a*x^8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^8 (a+bx^4)^{3/4}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{3}{4} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}}x^7\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/x**8/(b*x**4+a)**(3/4),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 3/4), (-3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(3
/4)*x**7*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{x^8 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{x^8\,(b\,x^4+a)^{3/4}}\,dx$$

input
```
int(1/(x^8*(a + b*x^4)^(3/4)),x)
```

output
```
int(1/(x^8*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 (a+bx^4)^{3/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{3}{4}}\,x^8}dx$$

input
```
int(1/x^8/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**8),x)
```

3.592 $\int \frac{1}{x^{12}(a+bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}+\frac{10b\sqrt[4]{a+bx^4}}{77a^2x^7}-\frac{20b^2\sqrt[4]{a+bx^4}}{77a^3x^3} + \frac{40b^{7/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77a^{7/2}\left(a+bx^4\right)^{3/4}}$$

output

```
-1/11*(b*x^4+a)^(1/4)/a/x^11+10/77*b*(b*x^4+a)^(1/4)/a^2/x^7-20/77*b^2*(b*
x^4+a)^(1/4)/a^3/x^3+40/77*b^(7/2)*(1+a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1
/2*arccot(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(7/2)/(b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{3/4}}\,dx = -\frac{\left(1+\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},-\frac{7}{4},-\frac{bx^4}{a}\right)}{11x^{11}\left(a+bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^12*(a + b*x^4)^(3/4)),x]
```

output
```
-1/11*((1 + (b*x^4)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x^4)
/a)])/(x^11*(a + b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {847, 847, 847, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12} (a+bx^4)^{3/4}} dx$$

$$\Big\downarrow \text{847}$$

$$-\frac{10b\int \frac{1}{x^8(bx^4+a)^{3/4}}dx}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{10b\left(-\frac{6b\int \frac{1}{x^4\left(bx^4+a\right)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{\left(bx^4+a\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x^3}dx}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \textbf{858}$$

$$-\frac{10b\left(-\frac{6b\left(\frac{2bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{3/4}x}d\frac{1}{x}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \textbf{807}$$

$$-\frac{10b\left(-\frac{6b\left(\frac{bx^3\left(\frac{a}{bx^4}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

$$\Big\downarrow \textbf{229}$$

$$-\frac{10b\left(-\frac{6b\left(\frac{2b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}}-\frac{\sqrt[4]{a+bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a+bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a+bx^4}}{11ax^{11}}$$

input
```
Int[1/(x^12*(a + b*x^4)^(3/4)),x]
```

output
```
-1/11*(a + b*x^4)^(1/4)/(a*x^11) - (10*b*(-1/7*(a + b*x^4)^(1/4)/(a*x^7) -
 (6*b*(-1/3*(a + b*x^4)^(1/4)/(a*x^3) + (2*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x
^3*EllipticF[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a + b*x^4)^(
3/4))))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{12} \left(b\,x^{4}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^12/(b*x^4+a)^(3/4),x)
```

output
```
int(1/x^12/(b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{12} (a+bx^4)^{3/4}}\, dx = \int \frac{1}{(bx^4+a)^{\frac{3}{4}} x^{12}}\, dx$$

input
```
integrate(1/x^12/(b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(1/4)/(b*x^16 + a*x^12), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{12} (a+bx^4)^{3/4}}\, dx = \frac{\Gamma\left(-\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{11}{4}, \frac{3}{4} \\ -\frac{7}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{3}{4}} x^{11} \Gamma\left(-\frac{7}{4}\right)}$$

input
```
integrate(1/x**12/(b*x**4+a)**(3/4),x)
```

output
```
gamma(-11/4)*hyper((-11/4, 3/4), (-7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(3/4)*x**11*gamma(-7/4))
```

Maxima [F]

$$\int \frac{1}{x^{12} (a + bx^4)^{3/4}} \, dx = \int \frac{1}{(bx^4 + a)^{\frac{3}{4}} x^{12}} \, dx$$

input
```
integrate(1/x^12/(b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^12), x)
```

Giac [F]

$$\int \frac{1}{x^{12} (a + bx^4)^{3/4}} \, dx = \int \frac{1}{(bx^4 + a)^{\frac{3}{4}} x^{12}} \, dx$$

input
```
integrate(1/x^12/(b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(3/4)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{12} (a + bx^4)^{3/4}} \, dx = \int \frac{1}{x^{12} (b\,x^4 + a)^{3/4}} \, dx$$

input
```
int(1/(x^12*(a + b*x^4)^(3/4)),x)
```

output
```
int(1/(x^12*(a + b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{12} (a + bx^4)^{3/4}} \, dx = \int \frac{1}{(b\, x^4 + a)^{\frac{3}{4}}\, x^{12}} dx$$

input
```
int(1/x^12/(b*x^4+a)^(3/4),x)
```

output
```
int(1/((a + b*x**4)**(3/4)*x**12),x)
```

3.593 $\int \frac{x^{19}}{\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{x^{19}}{\left(a+bx^4\right)^{5/4}}\,dx=-\frac{a^4}{b^5\sqrt[4]{a+bx^4}}-\frac{4a^3\left(a+bx^4\right)^{3/4}}{3b^5}+\frac{6a^2\left(a+bx^4\right)^{7/4}}{7b^5}-\frac{4a\left(a+bx^4\right)^{11/4}}{11b^5}+\frac{\left(a+bx^4\right)^{15/4}}{15b^5}$$

output
```
-a^4/b^5/(b*x^4+a)^(1/4)-4/3*a^3*(b*x^4+a)^(3/4)/b^5+6/7*a^2*(b*x^4+a)^(7/
4)/b^5-4/11*a*(b*x^4+a)^(11/4)/b^5+1/15*(b*x^4+a)^(15/4)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int \frac{x^{19}}{\left(a+bx^4\right)^{5/4}}\,dx=\frac{-2048a^4-512a^3bx^4+192a^2b^2x^8-112ab^3x^{12}+77b^4x^{16}}{1155b^5\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^19/(a + b*x^4)^(5/4),x]
```

output
```
(-2048*a^4 - 512*a^3*b*x^4 + 192*a^2*b^2*x^8 - 112*a*b^3*x^12 + 77*b^4*x^1
6)/(1155*b^5*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{16}}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^4}{b^4(bx^4+a)^{5/4}}-\frac{4a^3}{b^4\sqrt[4]{bx^4+a}}+\frac{6(bx^4+a)^{3/4}a^2}{b^4}-\frac{4(bx^4+a)^{7/4}a}{b^4}+\frac{(bx^4+a)^{11/4}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^4}{b^5\sqrt[4]{a+bx^4}}-\frac{16a^3(a+bx^4)^{3/4}}{3b^5}+\frac{24a^2(a+bx^4)^{7/4}}{7b^5}-\frac{16a(a+bx^4)^{11/4}}{11b^5}+\frac{4(a+bx^4)^{15/4}}{15b^5}\right)$$

input
```
Int[x^19/(a + b*x^4)^(5/4),x]
```

output
```
((-4*a^4)/(b^5*(a + b*x^4)^(1/4)) - (16*a^3*(a + b*x^4)^(3/4))/(3*b^5) + (
24*a^2*(a + b*x^4)^(7/4))/(7*b^5) - (16*a*(a + b*x^4)^(11/4))/(11*b^5) + (
4*(a + b*x^4)^(15/4))/(15*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{-77x^{16}b^{4}+112a\,b^{3}x^{12}-192a^{2}b^{2}x^{8}+512a^{3}b\,x^{4}+2048a^{4}}{1155\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{5}}$	58
trager	$-\frac{-77x^{16}b^{4}+112a\,b^{3}x^{12}-192a^{2}b^{2}x^{8}+512a^{3}b\,x^{4}+2048a^{4}}{1155\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{5}}$	58
pseudoelliptic	$\frac{77x^{16}b^{4}-112a\,b^{3}x^{12}+192a^{2}b^{2}x^{8}-512a^{3}b\,x^{4}-2048a^{4}}{1155\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{5}}$	58
orering	$-\frac{-77x^{16}b^{4}+112a\,b^{3}x^{12}-192a^{2}b^{2}x^{8}+512a^{3}b\,x^{4}+2048a^{4}}{1155\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{5}}$	58
risch	$-\frac{\left(-77b^{3}x^{12}+189a\,b^{2}x^{8}-381a^{2}b\,x^{4}+893a^{3}\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{1155b^{5}}-\frac{a^{4}}{b^{5}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	65

input
```
int(x^19/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(-77*b^4*x^16+112*a*b^3*x^12-192*a^2*b^2*x^8+512*a^3*b*x^4+2048*a^
4)/(b*x^4+a)^(1/4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}\,dx = \frac{(77\,b^4x^{16}-112\,ab^3x^{12}+192\,a^2b^2x^8-512\,a^3bx^4-2048\,a^4)(bx^4+a)^{\frac{3}{4}}}{1155\,(b^6x^4+ab^5)}$$

input
```
integrate(x^19/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/1155*(77*b^4*x^16 - 112*a*b^3*x^12 + 192*a^2*b^2*x^8 - 512*a^3*b*x^4 - 2
048*a^4)*(b*x^4 + a)^(3/4)/(b^6*x^4 + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}\,dx = \begin{cases} -\frac{2048a^4}{1155b^5\sqrt[4]{a+bx^4}} - \frac{512a^3x^4}{1155b^4\sqrt[4]{a+bx^4}} + \frac{64a^2x^8}{385b^3\sqrt[4]{a+bx^4}} - \frac{16ax^{12}}{165b^2\sqrt[4]{a+bx^4}} + \frac{x^{16}}{15b\sqrt[4]{a+bx^4}} \\ \frac{x^{20}}{20a^{\frac{5}{4}}} \end{cases}$$

input
```
integrate(x**19/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((-2048*a**4/(1155*b**5*(a + b*x**4)**(1/4)) - 512*a**3*x**4/(115
5*b**4*(a + b*x**4)**(1/4)) + 64*a**2*x**8/(385*b**3*(a + b*x**4)**(1/4))
 - 16*a*x**12/(165*b**2*(a + b*x**4)**(1/4)) + x**16/(15*b*(a + b*x**4)**(1
/4)), Ne(b, 0)), (x**20/(20*a**(5/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.82

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+a)^{\frac{15}{4}}}{15\,b^5} - \frac{4\,(bx^4+a)^{\frac{11}{4}}a}{11\,b^5} + \frac{6\,(bx^4+a)^{\frac{7}{4}}a^2}{7\,b^5} - \frac{4\,(bx^4+a)^{\frac{3}{4}}a^3}{3\,b^5} - \frac{a^4}{(bx^4+a)^{\frac{1}{4}}b^5}$$

input
```
integrate(x^19/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/15*(b*x^4 + a)^(15/4)/b^5 - 4/11*(b*x^4 + a)^(11/4)*a/b^5 + 6/7*(b*x^4 +
 a)^(7/4)*a^2/b^5 - 4/3*(b*x^4 + a)^(3/4)*a^3/b^5 - a^4/((b*x^4 + a)^(1/4)
*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}\,dx = -\frac{\frac{1155\,a^4}{(bx^4+a)^{\frac{1}{4}}b} - \frac{77\,(bx^4+a)^{\frac{15}{4}}b^{14} - 420\,(bx^4+a)^{\frac{11}{4}}ab^{14} + 990\,(bx^4+a)^{\frac{7}{4}}a^2b^{14} - 1540\,(bx^4+a)^{\frac{3}{4}}a^3b^{14}}{b^{15}}}{1155\,b^4}$$

input
```
integrate(x^19/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
-1/1155*(1155*a^4/((b*x^4 + a)^(1/4)*b) - (77*(b*x^4 + a)^(15/4)*b^14 - 42
0*(b*x^4 + a)^(11/4)*a*b^14 + 990*(b*x^4 + a)^(7/4)*a^2*b^14 - 1540*(b*x^4
 + a)^(3/4)*a^3*b^14)/b^15)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}\,dx = -\frac{420\,a\,(b\,x^4+a)^3+1540\,a^3\,(b\,x^4+a)-77\,(b\,x^4+a)^4+1155\,a^4-990\,a^2\,(b\,x^4+a)^2}{1155\,b^5\,(b\,x^4+a)^{1/4}}$$

input
```
int(x^19/(a + b*x^4)^(5/4),x)
```

output
```
-(420*a*(a + b*x^4)^3 + 1540*a^3*(a + b*x^4) - 77*(a + b*x^4)^4 + 1155*a^4
 - 990*a^2*(a + b*x^4)^2)/(1155*b^5*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^{19}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{19}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^19/(b*x^4+a)^(5/4),x)
```

output
```
int(x**19/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.594 $\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx=\frac{a^3}{b^4\sqrt[4]{a+bx^4}}+\frac{a^2(a+bx^4)^{3/4}}{b^4}-\frac{3a(a+bx^4)^{7/4}}{7b^4}+\frac{(a+bx^4)^{11/4}}{11b^4}$$

output
```
a^3/b^4/(b*x^4+a)^(1/4)+a^2*(b*x^4+a)^(3/4)/b^4-3/7*a*(b*x^4+a)^(7/4)/b^4+
1/11*(b*x^4+a)^(11/4)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx=\frac{128a^3+32a^2bx^4-12ab^2x^8+7b^3x^{12}}{77b^4\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^15/(a + b*x^4)^(5/4),x]
```

output
```
(128*a^3 + 32*a^2*b*x^4 - 12*a*b^2*x^8 + 7*b^3*x^12)/(77*b^4*(a + b*x^4)^(
1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{x^{12}}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4}\int \left(-\frac{a^3}{b^3 (bx^4+a)^{5/4}}+\frac{3a^2}{b^3 \sqrt[4]{bx^4+a}}-\frac{3\left(bx^4+a\right)^{3/4}a}{b^3}+\frac{\left(bx^4+a\right)^{7/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{4a^3}{b^4\sqrt[4]{a+bx^4}}+\frac{4a^2\left(a+bx^4\right)^{3/4}}{b^4}-\frac{12a\left(a+bx^4\right)^{7/4}}{7b^4}+\frac{4\left(a+bx^4\right)^{11/4}}{11b^4}\right)$$

input
```
Int[x^15/(a + b*x^4)^(5/4),x]
```

output
```
((4*a^3)/(b^4*(a + b*x^4)^(1/4)) + (4*a^2*(a + b*x^4)^(3/4))/b^4 - (12*a*(
a + b*x^4)^(7/4))/(7*b^4) + (4*(a + b*x^4)^(11/4))/(11*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{7b^3x^{12}-12a\,b^2x^8+32a^2b\,x^4+128a^3}{77(b\,x^4+a)^{\frac{1}{4}}b^4}$	47
trager	$\frac{7b^3x^{12}-12a\,b^2x^8+32a^2b\,x^4+128a^3}{77(b\,x^4+a)^{\frac{1}{4}}b^4}$	47
pseudoelliptic	$\frac{7b^3x^{12}-12a\,b^2x^8+32a^2b\,x^4+128a^3}{77(b\,x^4+a)^{\frac{1}{4}}b^4}$	47
orering	$\frac{7b^3x^{12}-12a\,b^2x^8+32a^2b\,x^4+128a^3}{77(b\,x^4+a)^{\frac{1}{4}}b^4}$	47
risch	$\frac{(7b^2x^8-19ab\,x^4+51a^2)(b\,x^4+a)^{\frac{3}{4}}}{77b^4}+\frac{a^3}{b^4(b\,x^4+a)^{\frac{1}{4}}}$	53

input
```
int(x^15/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/77*(7*b^3*x^12-12*a*b^2*x^8+32*a^2*b*x^4+128*a^3)/(b*x^4+a)^(1/4)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx = \frac{(7\,b^3x^{12}-12\,ab^2x^8+32\,a^2bx^4+128\,a^3)(bx^4+a)^{\frac{3}{4}}}{77\,(b^5x^4+ab^4)}$$

input
```
integrate(x^15/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/77*(7*b^3*x^12 - 12*a*b^2*x^8 + 32*a^2*b*x^4 + 128*a^3)*(b*x^4 + a)^(3/4
)/(b^5*x^4 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx = \begin{cases} \frac{128a^3}{77b^4\sqrt[4]{a+bx^4}} + \frac{32a^2x^4}{77b^3\sqrt[4]{a+bx^4}} - \frac{12ax^8}{77b^2\sqrt[4]{a+bx^4}} + \frac{x^{12}}{11b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{16}}{16a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((128*a**3/(77*b**4*(a + b*x**4)**(1/4)) + 32*a**2*x**4/(77*b**3*
(a + b*x**4)**(1/4)) - 12*a*x**8/(77*b**2*(a + b*x**4)**(1/4)) + x**12/(11
*b*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**16/(16*a**(5/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+a)^{\frac{11}{4}}}{11\,b^4} - \frac{3\,(bx^4+a)^{\frac{7}{4}}a}{7\,b^4} + \frac{(bx^4+a)^{\frac{3}{4}}a^2}{b^4} + \frac{a^3}{(bx^4+a)^{\frac{1}{4}}b^4}$$

input
```
integrate(x^15/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/11*(b*x^4 + a)^(11/4)/b^4 - 3/7*(b*x^4 + a)^(7/4)*a/b^4 + (b*x^4 + a)^(3
/4)*a^2/b^4 + a^3/((b*x^4 + a)^(1/4)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx = \frac{a^3}{(bx^4+a)^{\frac{1}{4}}b^4} + \frac{7\,(bx^4+a)^{\frac{11}{4}}b^{40} - 33\,(bx^4+a)^{\frac{7}{4}}ab^{40} + 77\,(bx^4+a)^{\frac{3}{4}}a^2b^{40}}{77\,b^{44}}$$

input
```
integrate(x^15/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
a^3/((b*x^4 + a)^(1/4)*b^4) + 1/77*(7*(b*x^4 + a)^(11/4)*b^40 - 33*(b*x^4
+ a)^(7/4)*a*b^40 + 77*(b*x^4 + a)^(3/4)*a^2*b^40)/b^44
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx = \frac{77\,a^2\,(b\,x^4+a) - 33\,a\,(b\,x^4+a)^2 + 7\,(b\,x^4+a)^3 + 77\,a^3}{77\,b^4\,(b\,x^4+a)^{1/4}}$$

input
```
int(x^15/(a + b*x^4)^(5/4),x)
```

output
```
(77*a^2*(a + b*x^4) - 33*a*(a + b*x^4)^2 + 7*(a + b*x^4)^3 + 77*a^3)/(77*b
^4*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^{15}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{15}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^15/(b*x^4+a)^(5/4),x)
```

output
```
int(x**15/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.595 $\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = -\frac{a^2}{b^3\sqrt[4]{a+bx^4}} - \frac{2a(a+bx^4)^{3/4}}{3b^3} + \frac{(a+bx^4)^{7/4}}{7b^3}$$

output
```
-a^2/b^3/(b*x^4+a)^(1/4)-2/3*a*(b*x^4+a)^(3/4)/b^3+1/7*(b*x^4+a)^(7/4)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = \frac{-32a^2-8abx^4+3b^2x^8}{21b^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^11/(a + b*x^4)^(5/4),x]
```

output
```
(-32*a^2 - 8*a*b*x^4 + 3*b^2*x^8)/(21*b^3*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4}\int \frac{x^8}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \ 53$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2\left(bx^4+a\right)^{5/4}}-\frac{2a}{b^2\sqrt[4]{bx^4+a}}+\frac{\left(bx^4+a\right)^{3/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(-\frac{4a^2}{b^3\sqrt[4]{a+bx^4}}-\frac{8a\left(a+bx^4\right)^{3/4}}{3b^3}+\frac{4\left(a+bx^4\right)^{7/4}}{7b^3}\right)$$

input
```
Int[x^11/(a + b*x^4)^(5/4),x]
```

output
```
((-4*a^2)/(b^3*(a + b*x^4)^(1/4)) - (8*a*(a + b*x^4)^(3/4))/(3*b^3) + (4*(
a + b*x^4)^(7/4))/(7*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{-3b^{2}x^{8}+8ab\,x^{4}+32a^{2}}{21\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{3}}$	36
trager	$-\frac{-3b^{2}x^{8}+8ab\,x^{4}+32a^{2}}{21\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{3}}$	36
pseudoelliptic	$\frac{3b^{2}x^{8}-8ab\,x^{4}-32a^{2}}{21\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{3}}$	36
orering	$-\frac{-3b^{2}x^{8}+8ab\,x^{4}+32a^{2}}{21\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{3}}$	36
risch	$-\frac{\left(-3b\,x^{4}+11a\right)\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{21b^{3}}-\frac{a^{2}}{b^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	43

input
```
int(x^11/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-3*b^2*x^8+8*a*b*x^4+32*a^2)/(b*x^4+a)^(1/4)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = \frac{(3\,b^2x^8 - 8\,abx^4 - 32\,a^2)(bx^4+a)^{\frac{3}{4}}}{21\,(b^4x^4 + ab^3)}$$

input
```
integrate(x^11/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/21*(3*b^2*x^8 - 8*a*b*x^4 - 32*a^2)*(b*x^4 + a)^(3/4)/(b^4*x^4 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = \begin{cases} -\frac{32a^2}{21b^3\sqrt[4]{a+bx^4}} - \frac{8ax^4}{21b^2\sqrt[4]{a+bx^4}} + \frac{x^8}{7b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((-32*a**2/(21*b**3*(a + b*x**4)**(1/4)) - 8*a*x**4/(21*b**2*(a +
 b*x**4)**(1/4)) + x**8/(7*b*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**12/(12*a
**(5/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+a)^{\frac{7}{4}}}{7\,b^3} - \frac{2\,(bx^4+a)^{\frac{3}{4}}a}{3\,b^3} - \frac{a^2}{(bx^4+a)^{\frac{1}{4}}b^3}$$

input
```
integrate(x^11/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/7*(b*x^4 + a)^(7/4)/b^3 - 2/3*(b*x^4 + a)^(3/4)*a/b^3 - a^2/((b*x^4 + a)
^(1/4)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = -\frac{\frac{21\,a^2}{(bx^4+a)^{\frac{1}{4}}\,b} - \frac{3\,(bx^4+a)^{\frac{7}{4}}\,b^6 - 14\,(bx^4+a)^{\frac{3}{4}}\,ab^6}{b^7}}{21\,b^2}$$

input
```
integrate(x^11/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
-1/21*(21*a^2/((b*x^4 + a)^(1/4)*b) - (3*(b*x^4 + a)^(7/4)*b^6 - 14*(b*x^4
 + a)^(3/4)*a*b^6)/b^7)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = -\frac{\frac{2\,a\,(b\,x^4+a)}{3} - \frac{(b\,x^4+a)^2}{7} + a^2}{b^3\,(b\,x^4+a)^{1/4}}$$

input
```
int(x^11/(a + b*x^4)^(5/4),x)
```

output
```
-((2*a*(a + b*x^4))/3 - (a + b*x^4)^2/7 + a^2)/(b^3*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^{11}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{11}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^11/(b*x^4+a)^(5/4),x)
```

output
```
int(x**11/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.596 $\int \frac{x^7}{(a+bx^4)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \frac{x^7}{(a+bx^4)^{5/4}} \, dx = \frac{a}{b^2 \sqrt[4]{a+bx^4}} + \frac{(a+bx^4)^{3/4}}{3b^2}$$

output
```
a/b^2/(b*x^4+a)^(1/4)+1/3*(b*x^4+a)^(3/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^7}{(a+bx^4)^{5/4}} \, dx = \frac{4a+bx^4}{3b^2 \sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^7/(a + b*x^4)^(5/4),x]
```

output
```
(4*a + b*x^4)/(3*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{1}{b\sqrt[4]{bx^4+a}}-\frac{a}{b\,(bx^4+a)^{5/4}}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4a}{b^2\sqrt[4]{a+bx^4}}+\frac{4(a+bx^4)^{3/4}}{3b^2}\right)$$

input
```
Int[x^7/(a + b*x^4)^(5/4),x]
```

output
```
((4*a)/(b^2*(a + b*x^4)^(1/4)) + (4*(a + b*x^4)^(3/4))/(3*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

method	result	size
gosper	$\frac{b\,x^{4}+4a}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	24
trager	$\frac{b\,x^{4}+4a}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	24
pseudoelliptic	$\frac{b\,x^{4}+4a}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	24
orering	$\frac{b\,x^{4}+4a}{3\left(b\,x^{4}+a\right)^{\frac{1}{4}} b^{2}}$	24
risch	$\frac{a}{b^{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}+\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3 b^{2}}$	30

input
```
int(x^7/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^4+4*a)/(b*x^4+a)^(1/4)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+4\,a)(bx^4+a)^{\frac{3}{4}}}{3\,(b^3x^4+ab^2)}$$

input
```
integrate(x^7/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/3*(b*x^4 + 4*a)*(b*x^4 + a)^(3/4)/(b^3*x^4 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx = \begin{cases} \frac{4a}{3b^2\sqrt[4]{a+bx^4}} + \frac{x^4}{3b\sqrt[4]{a+bx^4}} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((4*a/(3*b**2*(a + b*x**4)**(1/4)) + x**4/(3*b*(a + b*x**4)**(1/4
)), Ne(b, 0)), (x**8/(8*a**(5/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}}{3\,b^2} + \frac{a}{(bx^4+a)^{\frac{1}{4}}b^2}$$

input
```
integrate(x^7/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/3*(b*x^4 + a)^(3/4)/b^2 + a/((b*x^4 + a)^(1/4)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx = \frac{\frac{(bx^4+a)^{\frac{3}{4}}}{b} + \frac{3\,a}{(bx^4+a)^{\frac{1}{4}}\,b}}{3\,b}$$

input
```
integrate(x^7/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
1/3*((b*x^4 + a)^(3/4)/b + 3*a/((b*x^4 + a)^(1/4)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx = \frac{b\,x^4+4\,a}{3\,b^2\,(b\,x^4+a)^{1/4}}$$

input
```
int(x^7/(a + b*x^4)^(5/4),x)
```

output
```
(4*a + b*x^4)/(3*b^2*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^7}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^7/(b*x^4+a)^(5/4),x)
```

output
```
int(x**7/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.597 $\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{b\sqrt[4]{a+bx^4}}$$

output
```
-1/b/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^3/(a + b*x^4)^(5/4),x]
```

output
```
-(1/(b*(a + b*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{1}{b\sqrt[4]{a+bx^4}}$$

input
```
Int[x^3/(a + b*x^4)^(5/4),x]
```

output
```
-(1/(b*(a + b*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{b(b\,x^4+a)^{\frac{1}{4}}}$	15
derivativedivides	$-\frac{1}{b(b\,x^4+a)^{\frac{1}{4}}}$	15
default	$-\frac{1}{b(b\,x^4+a)^{\frac{1}{4}}}$	15
trager	$-\frac{1}{b(b\,x^4+a)^{\frac{1}{4}}}$	15
pseudoelliptic	$-\frac{1}{b(b\,x^4+a)^{\frac{1}{4}}}$	15
orering	$-\frac{1}{b(b\,x^4+a)^{\frac{1}{4}}}$	15

input
```
int(x^3/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x^4+a)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = -\frac{(bx^4+a)^{\frac{3}{4}}}{b^2x^4+ab}$$

input
```
integrate(x^3/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-(b*x^4 + a)^(3/4)/(b^2*x^4 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = \begin{cases} -\frac{1}{b\sqrt[4]{a+bx^4}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**4+a)**(5/4),x)
```

output
```
Piecewise((-1/(b*(a + b*x**4)**(1/4)), Ne(b, 0)), (x**4/(4*a**(5/4)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{(bx^4+a)^{\frac{1}{4}}b}$$

input
```
integrate(x^3/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/((b*x^4 + a)^(1/4)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{(bx^4+a)^{\frac{1}{4}}b}$$

input
```
integrate(x^3/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
-1/((b*x^4 + a)^(1/4)*b)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = -\frac{1}{b\,(b\,x^4+a)^{1/4}}$$

input
```
int(x^3/(a + b*x^4)^(5/4),x)
```

output
```
-1/(b*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^3}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^3/(b*x^4+a)^(5/4),x)
```

output
```
int(x**3/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.598 $\int \frac{1}{x(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx=\frac{1}{a\sqrt[4]{a+bx^4}}+\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}$$

output
```
1/a/(b*x^4+a)^(1/4)+1/2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)-1/2*arctan
h((b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx=\frac{\frac{2\sqrt[4]{a}}{\sqrt[4]{a+bx^4}}+\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2a^{5/4}}$$

input
```
Integrate[1/(x*(a + b*x^4)^(5/4)),x]
```

output
```
((2*a^(1/4))/(a + b*x^4)^(1/4) + ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] - ArcTa
nh[(a + b*x^4)^(1/4)/a^(1/4)])/(2*a^(5/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {798, 61, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4(bx^4+a)^{5/4}}dx^4$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}\left(\frac{\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{a}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{4\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int \frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}\right)$$

$\Big\downarrow$ 216

$$\frac{1}{4}\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{4}\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)$$

input
```
Int[1/(x*(a + b*x^4)^(5/4)),x]
```

output
```
(4/(a*(a + b*x^4)^(1/4)) - (4*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(1
/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.26

method	result	size
pseudoelliptic	$-\frac{-2\arctan\left(\frac{(b x^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)(b x^4+a)^{\frac{1}{4}}+\ln\left(\frac{(b x^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}{(b x^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)(b x^4+a)^{\frac{1}{4}}-4a^{\frac{1}{4}}}{4a^{\frac{5}{4}}(b x^4+a)^{\frac{1}{4}}}$	88

input
```
int(1/x/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*arctan((b*x^4+a)^(1/4)/a^(1/4))*(b*x^4+a)^(1/4)+ln(((b*x^4+a)^(1/
4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*(b*x^4+a)^(1/4)-4*a^(1/4))/a^(5/4)/
(b*x^4+a)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.60

$$\int \frac{1}{x(a+bx^4)^{5/4}}\,dx = -\frac{(abx^4+a^2)\left(\frac{1}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(\frac{1}{a^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}\right)-(i\,abx^4+i\,a^2)\left(\frac{1}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(\frac{1}{a^5}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}\right)-(-i\,abx^4-i\cdots}{4\,(abx^4+\cdots}$$

input
```
integrate(1/x/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/4*((a*b*x^4 + a^2)*(a^(-5))^(1/4)*log(a^4*(a^(-5))^(3/4) + (b*x^4 + a)^
(1/4)) - (I*a*b*x^4 + I*a^2)*(a^(-5))^(1/4)*log(I*a^4*(a^(-5))^(3/4) + (b*
x^4 + a)^(1/4)) - (-I*a*b*x^4 - I*a^2)*(a^(-5))^(1/4)*log(-I*a^4*(a^(-5))^
(3/4) + (b*x^4 + a)^(1/4)) - (a*b*x^4 + a^2)*(a^(-5))^(1/4)*log(-a^4*(a^(-
5))^(3/4) + (b*x^4 + a)^(1/4)) - 4*(b*x^4 + a)^(3/4))/(a*b*x^4 + a^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.56

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx = -\frac{\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(1/x/(b*x**4+a)**(5/4),x)
```

output
```
-gamma(5/4)*hyper((5/4, 5/4), (9/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(5
/4)*x**5*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx = \frac{\frac{2\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{4\,a}+\frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}a}$$

input
```
integrate(1/x/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/4*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4)
- a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a + 1/((b*x^4 + a)^(1/4
)*a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.84

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx = -\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a^2} - \frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a^2} + \frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a^2} - \frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{8\,a^2} + \frac{1}{(bx^4+a)^{\frac{1}{4}}a}$$

input
```
integrate(1/x/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4
+ a)^(1/4))/(-a)^(1/4))/a^2 - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 1/8*sqrt(2)*(-
a)^(3/4)*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt
(-a))/a^2 - 1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/
4) + sqrt(b*x^4 + a) + sqrt(-a))/a^2 + 1/((b*x^4 + a)^(1/4)*a)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx = \frac{\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}} - \frac{\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{2\,a^{5/4}} + \frac{1}{a\,(b\,x^4+a)^{1/4}}$$

input
```
int(1/(x*(a + b*x^4)^(5/4)),x)
```

output
```
atan((a + b*x^4)^(1/4)/a^(1/4))/(2*a^(5/4)) - atanh((a + b*x^4)^(1/4)/a^(1
/4))/(2*a^(5/4)) + 1/(a*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}ax+\left(b\,x^4+a\right)^{\frac{1}{4}}b\,x^5}dx$$

input
```
int(1/x/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x + (a + b*x**4)**(1/4)*b*x**5),x)
```

3.599 $\int \frac{1}{x^5 (a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}} dx = -\frac{5b}{4a^2\sqrt[4]{a+bx^4}} - \frac{1}{4ax^4\sqrt[4]{a+bx^4}} - \frac{5b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}} + \frac{5b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}$$

output
```
-5/4*b/a^2/(b*x^4+a)^(1/4)-1/4/a/x^4/(b*x^4+a)^(1/4)-5/8*b*arctan((b*x^4+a
)^(1/4)/a^(1/4))/a^(9/4)+5/8*b*arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}} dx = \frac{-\frac{2\sqrt[4]{a}(a+5bx^4)}{x^4\sqrt[4]{a+bx^4}} - 5b\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) + 5b\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{8a^{9/4}}$$

input
```
Integrate[1/(x^5*(a + b*x^4)^(5/4)),x]
```

output
```
((-2*a^(1/4)*(a + 5*b*x^4))/(x^4*(a + b*x^4)^(1/4)) - 5*b*ArcTan[(a + b*x^
4)^(1/4)/a^(1/4)] + 5*b*ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)])/(8*a^(9/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {798, 52, 61, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{1}{x^8 (bx^4+a)^{5/4}} dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4} \left(-\frac{5b \int \frac{1}{x^4 (bx^4+a)^{5/4}} dx^4}{4a} - \frac{1}{ax^4 \sqrt[4]{a+bx^4}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4} \left(-\frac{5b \left(\frac{\int \frac{1}{x^4 \sqrt[4]{bx^4+a}} dx^4}{a} + \frac{4}{a \sqrt[4]{a+bx^4}} \right)}{4a} - \frac{1}{ax^4 \sqrt[4]{a+bx^4}} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(-\frac{5b \left(\frac{4 \int -\frac{bx^8}{a-x^{16}} d\sqrt[4]{bx^4+a}}{ab} + \frac{4}{a \sqrt[4]{a+bx^4}} \right)}{4a} - \frac{1}{ax^4 \sqrt[4]{a+bx^4}} \right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int\frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 827

$$\frac{1}{4}\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 216

$$\frac{1}{4}\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 219

$$\frac{1}{4}\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[1/(x^5*(a + b*x^4)^(5/4)),x]
```

output
```
(-(1/(a*x^4*(a + b*x^4)^(1/4))) - (5*b*(4/(a*(a + b*x^4)^(1/4)) - (4*(-1/2
*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(
1/4)]/(2*a^(1/4))))/a))/(4*a))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11

method	result	size
pseudoelliptic	$-\frac{5\left(\arctan\left(\frac{(b\,x^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b\,x^4 (b\,x^4+a)^{\frac{1}{4}}-\frac{\ln\left(\frac{(b\,x^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}{(b\,x^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right) b\,x^4 (b\,x^4+a)^{\frac{1}{4}}}{2}+2b\,x^4 a^{\frac{1}{4}}+\frac{2a^{\frac{5}{4}}}{5}\right)}{8a^{\frac{9}{4}} (b\,x^4+a)^{\frac{1}{4}} x^4}$	108

input
```
int(1/x^5/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-5/8*(arctan((b*x^4+a)^(1/4)/a^(1/4))*b*x^4*(b*x^4+a)^(1/4)-1/2*ln(((b*x^4
+a)^(1/4)+a^(1/4))/((b*x^4+a)^(1/4)-a^(1/4)))*b*x^4*(b*x^4+a)^(1/4)+2*b*x^
4*a^(1/4)+2/5*a^(5/4))/a^(9/4)/(b*x^4+a)^(1/4)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.80

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}}\,dx = \frac{5\,(a^2bx^8+a^3x^4)\left(\frac{b^4}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(\frac{b^4}{a^9}\right)^{\frac{3}{4}}+125\,(bx^4+a)^{\frac{1}{4}}b^3\right)-5\,(i\,a^2bx^8+i\,a^3x^4}{}$$

input
```
integrate(1/x^5/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/16*(5*(a^2*b*x^8 + a^3*x^4)*(b^4/a^9)^(1/4)*log(125*a^7*(b^4/a^9)^(3/4)
+ 125*(b*x^4 + a)^(1/4)*b^3) - 5*(I*a^2*b*x^8 + I*a^3*x^4)*(b^4/a^9)^(1/4)
*log(125*I*a^7*(b^4/a^9)^(3/4) + 125*(b*x^4 + a)^(1/4)*b^3) - 5*(-I*a^2*b*
x^8 - I*a^3*x^4)*(b^4/a^9)^(1/4)*log(-125*I*a^7*(b^4/a^9)^(3/4) + 125*(b*x
^4 + a)^(1/4)*b^3) - 5*(a^2*b*x^8 + a^3*x^4)*(b^4/a^9)^(1/4)*log(-125*a^7*
(b^4/a^9)^(3/4) + 125*(b*x^4 + a)^(1/4)*b^3) - 4*(5*b*x^4 + a)*(b*x^4 + a)
^(3/4))/(a^2*b*x^8 + a^3*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}}\,dx = -\frac{\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}}x^9\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(1/x**5/(b*x**4+a)**(5/4),x)
```

output
```
-gamma(9/4)*hyper((5/4, 9/4), (13/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b**(
5/4)*x**9*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}}\,dx = -\frac{5\,(bx^4+a)b-4\,ab}{4\left((bx^4+a)^{\frac{5}{4}}a^2-(bx^4+a)^{\frac{1}{4}}a^3\right)} - \frac{5\,b\left(\frac{2\,\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{16\,a^2}$$

input
```
integrate(1/x^5/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/4*(5*(b*x^4 + a)*b - 4*a*b)/((b*x^4 + a)^(5/4)*a^2 - (b*x^4 + a)^(1/4)*
a^3) - 5/16*b*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 +
 a)^(1/4) - a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(73) = 146$.

Time = 0.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}}\,dx = \frac{5\sqrt{2}(-a)^{\frac{3}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,a^3} + \frac{5\sqrt{2}(-a)^{\frac{3}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{16\,a^3} + \frac{5\sqrt{2}b\log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,(-a)^{\frac{1}{4}}\,a^2} + \frac{5\sqrt{2}(-a)^{\frac{3}{4}}\,b\log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{32\,a^3} - \frac{5\,(bx^4+a)b-4\,ab}{4\left((bx^4+a)^{\frac{5}{4}}-(bx^4+a)^{\frac{1}{4}}a\right)a^2}$$

input
```
integrate(1/x^5/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
5/16*sqrt(2)*(-a)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^
4 + a)^(1/4))/(-a)^(1/4))/a^3 + 5/16*sqrt(2)*(-a)^(3/4)*b*arctan(-1/2*sqrt
(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 5/32*sqrt
(2)*b*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a
))/((-a)^(1/4)*a^2) + 5/32*sqrt(2)*(-a)^(3/4)*b*log(-sqrt(2)*(b*x^4 + a)^(
1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/a^3 - 1/4*(5*(b*x^4 + a)*b -
 4*a*b)/(((b*x^4 + a)^(5/4) - (b*x^4 + a)^(1/4)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}}\,dx = \frac{5\,b\,\mathrm{atanh}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{9/4}} - \frac{5\,b\,\mathrm{atan}\left(\frac{(b\,x^4+a)^{1/4}}{a^{1/4}}\right)}{8\,a^{9/4}} - \frac{\frac{b}{a}-\frac{5\,b\,(b\,x^4+a)}{4\,a^2}}{a\,(b\,x^4+a)^{1/4}-(b\,x^4+a)^{5/4}}$$

input

```
int(1/(x^5*(a + b*x^4)^(5/4)),x)
```

output

```
(5*b*atanh((a + b*x^4)^(1/4)/a^(1/4)))/(8*a^(9/4)) - (5*b*atan((a + b*x^4)
^(1/4)/a^(1/4)))/(8*a^(9/4)) - (b/a - (5*b*(a + b*x^4))/(4*a^2))/(a*(a + b
*x^4)^(1/4) - (a + b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^5+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^9}dx$$

input

```
int(1/x^5/(b*x^4+a)^(5/4),x)
```

output

```
int(1/((a + b*x**4)**(1/4)*a*x**5 + (a + b*x**4)**(1/4)*b*x**9),x)
```

3.600 $\int \frac{1}{x^9(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{1}{x^9(a+bx^4)^{5/4}}\,dx = \frac{45b^2}{32a^3\sqrt[4]{a+bx^4}} - \frac{1}{8ax^8\sqrt[4]{a+bx^4}} + \frac{9b}{32a^2x^4\sqrt[4]{a+bx^4}} + \frac{45b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}} - \frac{45b^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}}$$

output
```
45/32*b^2/a^3/(b*x^4+a)^(1/4)-1/8/a/x^8/(b*x^4+a)^(1/4)+9/32*b/a^2/x^4/(b*
x^4+a)^(1/4)+45/64*b^2*arctan((b*x^4+a)^(1/4)/a^(1/4))/a^(13/4)-45/64*b^2*
arctanh((b*x^4+a)^(1/4)/a^(1/4))/a^(13/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^9(a+bx^4)^{5/4}}\,dx = \frac{\frac{2\sqrt[4]{a}(-4a^2+9abx^4+45b^2x^8)}{x^8\sqrt[4]{a+bx^4}} + 45b^2\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right) - 45b^2\text{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{64a^{13/4}}$$

input
```
Integrate[1/(x^9*(a + b*x^4)^(5/4)),x]
```

output
```
((2*a^(1/4)*(-4*a^2 + 9*a*b*x^4 + 45*b^2*x^8))/(x^8*(a + b*x^4)^(1/4)) + 4
5*b^2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)] - 45*b^2*ArcTanh[(a + b*x^4)^(1/4)
/a^(1/4)])/(64*a^(13/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {798, 52, 52, 61, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \left(a+bx^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \ 798$$

$$\frac{1}{4} \int \frac{1}{x^{12} \left(bx^4+a\right)^{5/4}} dx^4$$

$$\Big\downarrow \ 52$$

$$\frac{1}{4} \left(-\frac{9b \int \frac{1}{x^8 (bx^4+a)^{5/4}} dx^4}{8a} - \frac{1}{2ax^8 \sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{4} \left(-\frac{9b \left(-\frac{5b \int \frac{1}{x^4 \left(bx^4+a\right)^{5/4}} dx^4}{4a} - \frac{1}{ax^4 \sqrt[4]{a+bx^4}}\right)}{8a} - \frac{1}{2ax^8 \sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \ 61$$

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{\int \frac{1}{x^4\sqrt[4]{bx^4+a}}dx^4}{a}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 73

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{4\int -\frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}+\frac{4}{a\sqrt[4]{a+bx^4}}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 25

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{ab}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 27

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\int\frac{x^8}{a-x^{16}}d\sqrt[4]{bx^4+a}}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 827

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{1}{2}\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{bx^4+a}\right)}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 216

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{1}{2}\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{bx^4+a}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

$\downarrow$ 219

$$\frac{1}{4}\left(-\frac{9b\left(-\frac{5b\left(\frac{4}{a\sqrt[4]{a+bx^4}}-\frac{4\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a+bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}\right)}{4a}-\frac{1}{ax^4\sqrt[4]{a+bx^4}}\right)}{8a}-\frac{1}{2ax^8\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[1/(x^9*(a + b*x^4)^(5/4)),x]
```

output
```
(-1/2*1/(a*x^8*(a + b*x^4)^(1/4)) - (9*b*(-(1/(a*x^4*(a + b*x^4)^(1/4))) -
 (5*b*(4/(a*(a + b*x^4)^(1/4)) - (4*(-1/2*ArcTan[(a + b*x^4)^(1/4)/a^(1/4)
]/a^(1/4) + ArcTanh[(a + b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a))/(4*a)))/(
8*a))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$\frac{\frac{45 \arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b^{2} x^{8} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{64}-\frac{45 \ln\left(\frac{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) b^{2} x^{8} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}{128}+\frac{45 b^{2} x^{8} a^{\frac{1}{4}}}{32}+\frac{9 a^{\frac{5}{4}} b\,x^{4}}{32}-\frac{a^{\frac{9}{4}}}{8}}{a^{\frac{13}{4}} x^{8} \left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	127

input
```
int(1/x^9/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
45/64/(b*x^4+a)^(1/4)*(arctan((b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8*(b*x^4+a)^(
1/4)-1/2*ln((-(b*x^4+a)^(1/4)-a^(1/4))/(-(b*x^4+a)^(1/4)+a^(1/4)))*b^2*x^8
*(b*x^4+a)^(1/4)+2*b^2*x^8*a^(1/4)+2/5*a^(5/4)*b*x^4-8/45*a^(9/4))/a^(13/4
)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^9 \left(a+bx^4\right)^{5/4}}\,dx =$$

$$-\frac{45\left(a^3bx^{12}+a^4x^8\right)\left(\frac{b^8}{a^{13}}\right)^{\frac{1}{4}}\log\left(91125\,a^{10}\left(\frac{b^8}{a^{13}}\right)^{\frac{3}{4}}+91125\left(bx^4+a\right)^{\frac{1}{4}}b^6\right)+45\left(-i\,a^3bx^{12}-i\,a^4x^8\right)\left(\frac{b^8}{a^{13}}\right)^{\frac{1}{4}}}{}$$

input
```
integrate(1/x^9/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/128*(45*(a^3*b*x^12 + a^4*x^8)*(b^8/a^13)^(1/4)*log(91125*a^10*(b^8/a^1
3)^(3/4) + 91125*(b*x^4 + a)^(1/4)*b^6) + 45*(-I*a^3*b*x^12 - I*a^4*x^8)*(
b^8/a^13)^(1/4)*log(91125*I*a^10*(b^8/a^13)^(3/4) + 91125*(b*x^4 + a)^(1/4
)*b^6) + 45*(I*a^3*b*x^12 + I*a^4*x^8)*(b^8/a^13)^(1/4)*log(-91125*I*a^10*
(b^8/a^13)^(3/4) + 91125*(b*x^4 + a)^(1/4)*b^6) - 45*(a^3*b*x^12 + a^4*x^8
)*(b^8/a^13)^(1/4)*log(-91125*a^10*(b^8/a^13)^(3/4) + 91125*(b*x^4 + a)^(1
/4)*b^6) - 4*(45*b^2*x^8 + 9*a*b*x^4 - 4*a^2)*(b*x^4 + a)^(3/4))/(a^3*b*x^
12 + a^4*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^9 (a + bx^4)^{5/4}} \, dx = -\frac{\Gamma\left(\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{13}{4} \\ \frac{17}{4} \end{matrix} \middle| \frac{ae^{i\pi}}{bx^4}\right)}{4b^{\frac{5}{4}} x^{13} \Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(1/x**9/(b*x**4+a)**(5/4),x)
```

output
```
-gamma(13/4)*hyper((5/4, 13/4), (17/4,), a*exp_polar(I*pi)/(b*x**4))/(4*b*
*(5/4)*x**13*gamma(17/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^9 (a+bx^4)^{5/4}}\,dx = \frac{45\,(bx^4+a)^2b^2 - 81\,(bx^4+a)ab^2 + 32\,a^2b^2}{32\left((bx^4+a)^{\frac{9}{4}}a^3 - 2\,(bx^4+a)^{\frac{5}{4}}a^4 + (bx^4+a)^{\frac{1}{4}}a^5\right)} + \frac{45\,b^2\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}} + \frac{\log\left(\frac{(bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a^3}$$

input
```
integrate(1/x^9/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/32*(45*(b*x^4 + a)^2*b^2 - 81*(b*x^4 + a)*a*b^2 + 32*a^2*b^2)/((b*x^4 +
a)^(9/4)*a^3 - 2*(b*x^4 + a)^(5/4)*a^4 + (b*x^4 + a)^(1/4)*a^5) + 45/128*b
^2*(2*arctan((b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((b*x^4 + a)^(1/4) -
 a^(1/4))/((b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(97) = 194$.

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^9 (a+bx^4)^{5/4}} dx = \frac{45 \sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2 (bx^4+a)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right)}{128 (-a)^{\frac{1}{4}} a^3} + \frac{45 \sqrt{2}b^2 \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2 (bx^4+a)^{\frac{1}{4}}\right)}{2 (-a)^{\frac{1}{4}}}\right)}{128 (-a)^{\frac{1}{4}} a^3} + \frac{45 \sqrt{2}(-a)^{\frac{3}{4}} b^2 \log\left(\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{256 a^4} + \frac{45 \sqrt{2}b^2 \log\left(-\sqrt{2}(bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^4+a}+\sqrt{-a}\right)}{256 (-a)^{\frac{1}{4}} a^3} + \frac{b^2}{(bx^4+a)^{\frac{1}{4}} a^3} + \frac{13 (bx^4+a)^{\frac{7}{4}} b^2 - 17 (bx^4+a)^{\frac{3}{4}} a b^2}{32 a^3 b^2 x^8}$$

input

```
integrate(1/x^9/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output

```
45/128*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^4 + a)^
(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^3) + 45/128*sqrt(2)*b^2*arctan(-1/2*sqrt(
2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^4 + a)^(1/4))/(-a)^(1/4))/((-a)^(1/4)*a^3)
 + 45/256*sqrt(2)*(-a)^(3/4)*b^2*log(sqrt(2)*(b*x^4 + a)^(1/4)*(-a)^(1/4)
+ sqrt(b*x^4 + a) + sqrt(-a))/a^4 + 45/256*sqrt(2)*b^2*log(-sqrt(2)*(b*x^4
 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^4 + a) + sqrt(-a))/((-a)^(1/4)*a^3) + b^
2/((b*x^4 + a)^(1/4)*a^3) + 1/32*(13*(b*x^4 + a)^(7/4)*b^2 - 17*(b*x^4 + a
)^(3/4)*a*b^2)/(a^3*b^2*x^8)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^9 (a+bx^4)^{5/4}} dx = \frac{\frac{b^2}{a} - \frac{81 b^2 (b x^4+a)}{32 a^2} + \frac{45 b^2 (b x^4+a)^2}{32 a^3}}{(b x^4+a)^{9/4} - 2 a (b x^4+a)^{5/4} + a^2 (b x^4+a)^{1/4}} + \frac{45 b^2 \operatorname{atan}\left(\frac{(b x^4+a)^{1/4}}{a^{1/4}}\right)}{64 a^{13/4}} - \frac{45 b^2 \operatorname{atanh}\left(\frac{(b x^4+a)^{1/4}}{a^{1/4}}\right)}{64 a^{13/4}}$$

input
```
int(1/(x^9*(a + b*x^4)^(5/4)),x)
```

output
```
(b^2/a - (81*b^2*(a + b*x^4))/(32*a^2) + (45*b^2*(a + b*x^4)^2)/(32*a^3))/
((a + b*x^4)^(9/4) - 2*a*(a + b*x^4)^(5/4) + a^2*(a + b*x^4)^(1/4)) + (45*
b^2*atan((a + b*x^4)^(1/4)/a^(1/4)))/(64*a^(13/4)) - (45*b^2*atanh((a + b*
x^4)^(1/4)/a^(1/4)))/(64*a^(13/4))
```

Reduce [F]

$$\int \frac{1}{x^9 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^9+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{13}}dx$$

input
```
int(1/x^9/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**9 + (a + b*x**4)**(1/4)*b*x**13),x)
```

3.601 $\int \frac{x^{13}}{(a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}} dx = \frac{4a^2x^2}{3b^3\sqrt[4]{a+bx^4}} - \frac{2ax^6}{9b^2\sqrt[4]{a+bx^4}} + \frac{x^{10}}{9b\sqrt[4]{a+bx^4}} - \frac{8a^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
4/3*a^2*x^2/b^3/(b*x^4+a)^(1/4)-2/9*a*x^6/b^2/(b*x^4+a)^(1/4)+1/9*x^10/b/(
b*x^4+a)^(1/4)-8/3*a^(5/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1
/2)*x^2/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}} dx = \frac{x^2\left(-12a^2-2abx^4+b^2x^8+12a^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{9b^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^13/(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(-12*a^2 - 2*a*b*x^4 + b^2*x^8 + 12*a^2*(1 + (b*x^4)/a)^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, -((b*x^4)/a)]))/(9*b^3*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {807, 250, 250, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^4+a)^{5/4}}dx^2$$

$$\Big\downarrow \text{250}$$

$$\frac{1}{2}\left(\frac{2x^{10}}{9b\sqrt[4]{a+bx^4}}-\frac{10a\int \frac{x^8}{(bx^4+a)^{5/4}}dx^2}{9b}\right)$$

$$\Big\downarrow \text{250}$$

$$\frac{1}{2}\left(\frac{2x^{10}}{9b\sqrt[4]{a+bx^4}}-\frac{10a\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\int \frac{x^4}{(bx^4+a)^{5/4}}dx^2}{5b}\right)}{9b}\right)$$

$$\Big\downarrow \text{250}$$

$$\frac{1}{2}\left(\frac{2x^{10}}{9b\sqrt[4]{a+bx^4}}-\frac{10a\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2a\int\frac{1}{\left(bx^4+a\right)^{5/4}}dx^2}{b}\right)}{5b}\right)}{9b}\right)$$

$\Big\downarrow$ 213

$$\frac{1}{2}\left(\frac{2x^{10}}{9b\sqrt[4]{a+bx^4}}-\frac{10a\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{b\sqrt[4]{a+bx^4}}\right)}{5b}\right)}{9b}\right)$$

$\Big\downarrow$ 212

$$\frac{1}{2}\left(\frac{2x^{10}}{9b\sqrt[4]{a+bx^4}}-\frac{10a\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}\right)}{5b}\right)}{9b}\right)$$

input
```
Int[x^13/(a + b*x^4)^(5/4),x]
```

output
```
((2*x^10)/(9*b*(a + b*x^4)^(1/4)) - (10*a*((2*x^6)/(5*b*(a + b*x^4)^(1/4))
 - (6*a*((2*x^2)/(b*(a + b*x^4)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*
EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^4)^(1/4))
))/(5*b)))/(9*b))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{13}}{\left(b\,x^{4}+a\right)^{\frac{5}{4}}}dx$$

input
```
int(x^13/(b*x^4+a)^(5/4),x)
```

output
```
int(x^13/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^13/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}\,dx = \frac{x^{14}{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{14a^{\frac{5}{4}}}$$

input
```
integrate(x**13/(b*x**4+a)**(5/4),x)
```

output
```
x**14*hyper((5/4, 7/2), (9/2,), b*x**4*exp_polar(I*pi)/a)/(14*a**(5/4))
```

Maxima [F]

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^13/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{13}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^13/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^13/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}\,dx=\int \frac{x^{13}}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^13/(a + b*x^4)^(5/4),x)
```

output
```
int(x^13/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{13}}{(a+bx^4)^{5/4}}\,dx=\int \frac{x^{13}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^13/(b*x^4+a)^(5/4),x)
```

output
```
int(x**13/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.602 $\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = -\frac{6ax^2}{5b^2\sqrt[4]{a+bx^4}}+\frac{x^6}{5b\sqrt[4]{a+bx^4}} + \frac{12a^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-6/5*a*x^2/b^2/(b*x^4+a)^(1/4)+1/5*x^6/b/(b*x^4+a)^(1/4)+12/5*a^(3/2)*(1+b
*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(5
/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.80 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \frac{x^2\left(6a+bx^4-6a\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{5b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^9/(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(6*a + b*x^4 - 6*a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2,
3/2, -((b*x^4)/a)]))/(5*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 250, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^4+a)^{5/4}}dx^2$$

$$\Big\downarrow \text{250}$$

$$\frac{1}{2}\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\int \frac{x^4}{(bx^4+a)^{5/4}}dx^2}{5b}\right)$$

$$\Big\downarrow \text{250}$$

$$\frac{1}{2}\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2a\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{b}\right)}{5b}\right)$$

$$\Big\downarrow \text{213}$$

$$\frac{1}{2}\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{b\sqrt[4]{a+bx^4}}\right)}{5b}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{2x^6}{5b\sqrt[4]{a+bx^4}}-\frac{6a\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}\right)}{5b}\right)$$

input
```
Int[x^9/(a + b*x^4)^(5/4),x]
```

output
```
((2*x^6)/(5*b*(a + b*x^4)^(1/4)) - (6*a*((2*x^2)/(b*(a + b*x^4)^(1/4)) - (
4*Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2,
 2])/(b^(3/2)*(a + b*x^4)^(1/4))))/(5*b))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^9}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^9/(b*x^4+a)^(5/4),x)
```

output
```
int(x^9/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^9/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.26

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \frac{x^{10}{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{5}{4}}}$$

input
```
integrate(x**9/(b*x**4+a)**(5/4),x)
```

output
```
x**10*hyper((5/4, 5/2), (7/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(5/4))
```

Maxima [F]

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^9/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^9}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^9/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^9/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^9}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^9/(a + b*x^4)^(5/4),x)
```

output
```
int(x^9/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^9}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^9}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^9/(b*x^4+a)^(5/4),x)
```

output
```
int(x**9/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.603 $\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 77

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \frac{x^2}{b\sqrt[4]{a+bx^4}} - \frac{2\sqrt{a}\sqrt[4]{1+\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
x^2/b/(b*x^4+a)^(1/4)-2*a^(1/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan
(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \frac{x^2\left(-1+\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^5/(a + b*x^4)^(5/4),x]
```

output
```
(x^2*(-1 + (1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^4
)/a)]))/(b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {807, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^4+a)^{5/4}}dx^2$$

$$\Big\downarrow \text{250}$$

$$\frac{1}{2}\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2a\int \frac{1}{(bx^4+a)^{5/4}}dx^2}{b}\right)$$

$$\Big\downarrow \text{213}$$

$$\frac{1}{2}\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{2\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{b\sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}\left(\frac{2x^2}{b\sqrt[4]{a+bx^4}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[x^5/(a + b*x^4)^(5/4),x]
```

output
```
((2*x^2)/(b*(a + b*x^4)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^4)/a)^(1/4)*Elliptic
E[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^5}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^5/(b*x^4+a)^(5/4),x)
```

output
```
int(x^5/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^5}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^5/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \frac{x^6 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}}$$

input
```
integrate(x**5/(b*x**4+a)**(5/4),x)
```

output
```
x**6*hyper((5/4, 3/2), (5/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4))
```

Maxima [F]

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^5}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^5/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^5}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^5/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^5/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^5}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^5/(a + b*x^4)^(5/4),x)
```

output
```
int(x^5/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^5}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^5/(b*x^4+a)^(5/4),x)
```

output
```
int(x**5/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.604 $\int \frac{x}{(a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x}{(a+bx^4)^{5/4}} dx = \frac{\sqrt[4]{1+\frac{bx^4}{a}} E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/
a^(1/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^4)^{5/4}} dx = -\frac{x^2\left(-2+\sqrt[4]{1+\frac{bx^4}{a}}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^4}{a}\right)\right)}{2a\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x/(a + b*x^4)^(5/4),x]
```

output
```
-1/2*(x^2*(-2 + (1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((
b*x^4)/a)]))/(a*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {807, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{(bx^4+a)^{5/4}}dx^2$$

$$\Big\downarrow \text{213}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}\int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{2a\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^4}}$$

input
```
Int[x/(a + b*x^4)^(5/4),x]
```

output
```
((1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqr
t[a]*Sqrt[b]*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x/(b*x^4+a)^(5/4),x)
```

output
```
int(x/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx=\int \frac{x}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx = \frac{x^2 {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^4 e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}}$$

input
```
integrate(x/(b*x**4+a)**(5/4),x)
```

output
```
x**2*hyper((1/2, 5/4), (3/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4))
```

Maxima [F]

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx = \int \frac{x}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx = \int \frac{x}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x/(a + b*x^4)^(5/4),x)
```

output
```
int(x/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x}{(a+bx^4)^{5/4}}\,dx = \int \frac{x}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x/(b*x^4+a)^(5/4),x)
```

output
```
int(x/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.605 $\int \frac{1}{x^3(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{1}{x^3\left(a+bx^4\right)^{5/4}}\,dx=-\frac{1}{2ax^2\sqrt[4]{a+bx^4}}-\frac{3\sqrt{b}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/2/a/x^2/(b*x^4+a)^(1/4)-3/2*b^(1/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2
*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^3\left(a+bx^4\right)^{5/4}}\,dx=-\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{4},\frac{1}{2},-\frac{bx^4}{a}\right)}{2ax^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^3*(a + b*x^4)^(5/4)),x]
```

output
```
-1/2*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x^4)/a)
])/(a*x^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {807, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^4+a)^{5/4}} dx^2$$

$$\Big\downarrow \text{251}$$

$$\frac{1}{2} \left(-\frac{3b \int \frac{1}{(bx^4+a)^{5/4}} dx^2}{2a} - \frac{1}{ax^2 \sqrt[4]{a+bx^4}} \right)$$

$$\Big\downarrow \text{213}$$

$$\frac{1}{2} \left(-\frac{3b \sqrt[4]{\frac{bx^4}{a}+1} \int \frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}} dx^2}{2a^2 \sqrt[4]{a+bx^4}} - \frac{1}{ax^2 \sqrt[4]{a+bx^4}} \right)$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2} \left(-\frac{3\sqrt{b} \sqrt[4]{\frac{bx^4}{a}+1} E\left(\frac{1}{2} \arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right)}{a^{3/2} \sqrt[4]{a+bx^4}} - \frac{1}{ax^2 \sqrt[4]{a+bx^4}} \right)$$

input
```
Int[1/(x^3*(a + b*x^4)^(5/4)),x]
```

output
```
(-(1/(a*x^2*(a + b*x^4)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^4)/a)^(1/4)*Ellipti
cE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(b\,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^3/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^3/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^11 + 2*a*b*x^7 + a^2*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}x^2}$$

input
```
integrate(1/x**3/(b*x**4+a)**(5/4),x)
```

output
```
-hyper((-1/2, 5/4), (1/2,), b*x**4*exp_polar(I*pi)/a)/(2*a**(5/4)*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{x^3\,(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(1/(x^3*(a + b*x^4)^(5/4)),x)
```

output
```
int(1/(x^3*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^3+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^7}dx$$

input
```
int(1/x^3/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**3 + (a + b*x**4)**(1/4)*b*x**7),x)
```

3.606 $\int \frac{1}{x^7 (a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}} dx = -\frac{1}{6ax^6\sqrt[4]{a+bx^4}} + \frac{7b}{12a^2x^2\sqrt[4]{a+bx^4}} + \frac{7b^{3/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/6/a/x^6/(b*x^4+a)^(1/4)+7/12*b/a^2/x^2/(b*x^4+a)^(1/4)+7/4*b^(3/2)*(1+b
*x^4/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(5
/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}} dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{4},-\frac{1}{2},-\frac{bx^4}{a}\right)}{6ax^6\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^7*(a + b*x^4)^(5/4)),x]
```

output
```
-1/6*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x^4)/a
)])/(a*x^6*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {807, 251, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \left(a+bx^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^8 \left(bx^4+a\right)^{5/4}}dx^2$$

$$\Big\downarrow \text{251}$$

$$\frac{1}{2} \left(-\frac{7b \int \frac{1}{x^4 \left(bx^4+a\right)^{5/4}}dx^2}{6a}-\frac{1}{3ax^6 \sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{251}$$

$$\frac{1}{2} \left(-\frac{7b \left(-\frac{3b \int \frac{1}{\left(bx^4+a\right)^{5/4}}dx^2}{2a}-\frac{1}{ax^2 \sqrt[4]{a+bx^4}}\right)}{6a}-\frac{1}{3ax^6 \sqrt[4]{a+bx^4}}\right)$$

$$\Big\downarrow \text{213}$$

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{2a^2\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{6a}-\frac{1}{3ax^6\sqrt[4]{a+bx^4}}\right)$$

$\Big\downarrow$ 212

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{6a}-\frac{1}{3ax^6\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[1/(x^7*(a + b*x^4)^(5/4)),x]
```

output
```
(-1/3*1/(a*x^6*(a + b*x^4)^(1/4)) - (7*b*(-(1/(a*x^2*(a + b*x^4)^(1/4))) -
 (3*Sqrt[b]*(1 + (b*x^4)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/
2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))))/(6*a))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(b\,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^7/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^7/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^7 \left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{{\left(bx^{4}+a\right)}^{\frac{5}{4}}x^{7}}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^15 + 2*a*b*x^11 + a^2*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{6a^{\frac{5}{4}}x^6}$$

input
```
integrate(1/x**7/(b*x**4+a)**(5/4),x)
```

output
```
-hyper((-3/2, 5/4), (-1/2,), b*x**4*exp_polar(I*pi)/a)/(6*a**(5/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}} \, dx = \int \frac{1}{x^7 \left(b\,x^4+a\right)^{5/4}} \, dx$$

input
```
int(1/(x^7*(a + b*x^4)^(5/4)),x)
```

output
```
int(1/(x^7*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 (a+bx^4)^{5/4}} \, dx = \int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a\,x^7+\left(b\,x^4+a\right)^{\frac{1}{4}} b\,x^{11}} dx$$

input
```
int(1/x^7/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**7 + (a + b*x**4)**(1/4)*b*x**11),x)
```

3.607 $\int \frac{1}{x^{11}(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{1}{x^{11}(a+bx^4)^{5/4}}\,dx = -\frac{1}{10ax^{10}\sqrt[4]{a+bx^4}}+\frac{11b}{60a^2x^6\sqrt[4]{a+bx^4}} - \frac{77b^2}{120a^3x^2\sqrt[4]{a+bx^4}}-\frac{77b^{5/2}\sqrt[4]{1+\frac{bx^4}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/10/a/x^10/(b*x^4+a)^(1/4)+11/60*b/a^2/x^6/(b*x^4+a)^(1/4)-77/120*b^2/a^
3/x^2/(b*x^4+a)^(1/4)-77/40*b^(5/2)*(1+b*x^4/a)^(1/4)*EllipticE(sin(1/2*ar
ctan(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{11}(a+bx^4)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{4},-\frac{3}{2},-\frac{bx^4}{a}\right)}{10ax^{10}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^11*(a + b*x^4)^(5/4)),x]
```

output
```
-1/10*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x^4)/
a)])/(a*x^10*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {807, 251, 251, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11} \left(a+b x^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2} \int \frac{1}{x^{12} \left(b x^4+a\right)^{5/4}}dx^2$$

$$\Big\downarrow \text{ 251}$$

$$\frac{1}{2} \left(-\frac{11 b \int \frac{1}{x^8 \left(b x^4+a\right)^{5/4}}dx^2}{10 a}-\frac{1}{5 a x^{10} \sqrt[4]{a+b x^4}}\right)$$

$$\Big\downarrow \text{ 251}$$

$$\frac{1}{2} \left(-\frac{11 b \left(-\frac{7 b \int \frac{1}{x^4 \left(b x^4+a\right)^{5/4}}dx^2}{6 a}-\frac{1}{3 a x^6 \sqrt[4]{a+b x^4}}\right)}{10 a}-\frac{1}{5 a x^{10} \sqrt[4]{a+b x^4}}\right)$$

$$\Big\downarrow \text{ 251}$$

$$\frac{1}{2}\left(-\frac{11b\left(-\frac{7b\left(-\frac{3b\int\frac{1}{\left(bx^4+a\right)^{5/4}}dx^2}{2a}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{6a}-\frac{1}{3ax^6\sqrt[4]{a+bx^4}}\right)}{10a}-\frac{1}{5ax^{10}\sqrt[4]{a+bx^4}}\right)$$

$\Big\downarrow$ 213

$$\frac{1}{2}\left(-\frac{11b\left(-\frac{7b\left(-\frac{3b\sqrt[4]{\frac{bx^4}{a}+1}\int\frac{1}{\left(\frac{bx^4}{a}+1\right)^{5/4}}dx^2}{2a^2\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{6a}-\frac{1}{3ax^6\sqrt[4]{a+bx^4}}\right)}{10a}-\frac{1}{5ax^{10}\sqrt[4]{a+bx^4}}\right)$$

$\Big\downarrow$ 212

$$\frac{1}{2}\left(-\frac{11b\left(-\frac{7b\left(-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^4}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax^2\sqrt[4]{a+bx^4}}\right)}{6a}-\frac{1}{3ax^6\sqrt[4]{a+bx^4}}\right)}{10a}-\frac{1}{5ax^{10}\sqrt[4]{a+bx^4}}\right)$$

input
```
Int[1/(x^11*(a + b*x^4)^(5/4)),x]
```

output
```
(-1/5*1/(a*x^10*(a + b*x^4)^(1/4)) - (11*b*(-1/3*1/(a*x^6*(a + b*x^4)^(1/4
)) - (7*b*(-(1/(a*x^2*(a + b*x^4)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^4)/a)^(1/
4)*EllipticE[ArcTan[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/
4))))/(6*a)))/(10*a))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{11} \left(b\,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^11/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^11/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11} \left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{{\left(b x^{4} + a\right)}^{\frac{5}{4}} x^{11}}\,{d x}$$

input
```
integrate(1/x^11/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^19 + 2*a*b*x^15 + a^2*x^11), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{10a^{\frac{5}{4}}x^{10}}$$

input
```
integrate(1/x**11/(b*x**4+a)**(5/4),x)
```

output
```
-hyper((-5/2, 5/4), (-3/2,), b*x**4*exp_polar(I*pi)/a)/(10*a**(5/4)*x**10)
```

Maxima [F]

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx = \int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^11), x)
```

Giac [F]

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx = \int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{x^{11}\,{\left(b\,x^4+a\right)}^{5/4}} \,dx$$

input
```
int(1/(x^11*(a + b*x^4)^(5/4)),x)
```

output
```
int(1/(x^11*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{1}{4}} a\,x^{11}+\left(b\,x^{4}+a\right)^{\frac{1}{4}} b\,x^{15}}dx$$

input
```
int(1/x^11/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**11 + (a + b*x**4)**(1/4)*b*x**15),x)
```

3.608 $\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = -\frac{x^9}{b\sqrt[4]{a+bx^4}}-\frac{45ax(a+bx^4)^{3/4}}{32b^3}+\frac{9x^5(a+bx^4)^{3/4}}{8b^2}+\frac{45a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}+\frac{45a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}$$

output
```
-x^9/b/(b*x^4+a)^(1/4)-45/32*a*x*(b*x^4+a)^(3/4)/b^3+9/8*x^5*(b*x^4+a)^(3/
4)/b^2+45/64*a^2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(13/4)+45/64*a^2*arct
anh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(13/4)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = \frac{\frac{2\sqrt[4]{b}x(-45a^2-9abx^4+4b^2x^8)}{\sqrt[4]{a+bx^4}}+45a^2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)+45a^2\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{64b^{13/4}}$$

input
```
Integrate[x^12/(a + b*x^4)^(5/4),x]
```

output
```
((2*b^(1/4)*x*(-45*a^2 - 9*a*b*x^4 + 4*b^2*x^8))/(a + b*x^4)^(1/4) + 45*a^
2*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)] + 45*a^2*ArcTanh[(b^(1/4)*x)/(a +
b*x^4)^(1/4)])/(64*b^(13/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {817, 843, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{9\int \frac{x^8}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{9\left(\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{8b}\right)}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{9\left(\frac{x^5(a+bx^4)^{3/4}}{8b}-\frac{5a\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{8b}\right)}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{9\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{8b}\right)}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

$\downarrow$ 756

$$\frac{9\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{8b}\right)}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

$\downarrow$ 216

$$\frac{9\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}\right)}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

$\downarrow$ 219

$$\frac{9\left(\frac{x^5\left(a+bx^4\right)^{3/4}}{8b}-\frac{5a\left(\frac{x\left(a+bx^4\right)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{8b}\right)}{b}-\frac{x^9}{b\sqrt[4]{a+bx^4}}$$

input
```
Int[x^12/(a + b*x^4)^(5/4),x]
```

output
```
-(x^9/(b*(a + b*x^4)^(1/4))) + (9*((x^5*(a + b*x^4)^(3/4))/(8*b) - (5*a*((
x*(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*
b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/(8
*b)))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

method	result	size
pseudoelliptic	$-\frac{45\left(-\frac{8b^{\frac{9}{4}}x^{9}}{45}+\frac{2a\,b^{\frac{5}{4}}x^{5}}{5}+\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)a^{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}-\frac{\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)a^{2}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{2}+2x\,a^{2}b^{\frac{1}{4}}\right)}{64\left(b\,x^{4}+a\right)^{\frac{1}{4}}b^{\frac{13}{4}}}$	121

input
```
int(x^12/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-45/64/(b*x^4+a)^(1/4)*(-8/45*b^(9/4)*x^9+2/5*a*b^(5/4)*x^5+arctan(1/b^(1/
4)/x*(b*x^4+a)^(1/4))*a^2*(b*x^4+a)^(1/4)-1/2*ln((b^(1/4)*x+(b*x^4+a)^(1/4
))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a^2*(b*x^4+a)^(1/4)+2*x*a^2*b^(1/4))/b^(1
3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.37

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = \frac{45\,(b^4x^4+ab^3)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\log\left(\frac{91125\left(b^{10}x\left(\frac{a^8}{b^{13}}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}a^6\right)}{x}\right)-45\,(b^4x^4+ab^3)\left(\frac{a^8}{b^{13}}\right)^{\frac{1}{4}}\mathrm{lo}}{}$$

input
```
integrate(x^12/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/128*(45*(b^4*x^4 + a*b^3)*(a^8/b^13)^(1/4)*log(91125*(b^10*x*(a^8/b^13)^
(3/4) + (b*x^4 + a)^(1/4)*a^6)/x) - 45*(b^4*x^4 + a*b^3)*(a^8/b^13)^(1/4)*
log(-91125*(b^10*x*(a^8/b^13)^(3/4) - (b*x^4 + a)^(1/4)*a^6)/x) - 45*(-I*b
^4*x^4 - I*a*b^3)*(a^8/b^13)^(1/4)*log(-91125*(I*b^10*x*(a^8/b^13)^(3/4) -
 (b*x^4 + a)^(1/4)*a^6)/x) - 45*(I*b^4*x^4 + I*a*b^3)*(a^8/b^13)^(1/4)*log
(-91125*(-I*b^10*x*(a^8/b^13)^(3/4) - (b*x^4 + a)^(1/4)*a^6)/x) + 4*(4*b^2
*x^9 - 9*a*b*x^5 - 45*a^2*x)*(b*x^4 + a)^(3/4))/(b^4*x^4 + a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = \frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},\frac{13}{4}\\\frac{17}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12/(b*x**4+a)**(5/4),x)
```

output
```
x**13*gamma(13/4)*hyper((5/4, 13/4), (17/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(5/4)*gamma(17/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.40

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = -\frac{32\,a^2b^2 - \frac{81\,(bx^4+a)a^2b}{x^4} + \frac{45\,(bx^4+a)^2a^2}{x^8}}{32\left(\frac{(bx^4+a)^{\frac{1}{4}}b^5}{x} - \frac{2\,(bx^4+a)^{\frac{5}{4}}b^4}{x^5} + \frac{(bx^4+a)^{\frac{9}{4}}b^3}{x^9}\right)} - \frac{45\,a^2\left(\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{128\,b^3}$$

input
```
integrate(x^12/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/32*(32*a^2*b^2 - 81*(b*x^4 + a)*a^2*b/x^4 + 45*(b*x^4 + a)^2*a^2/x^8)/(
(b*x^4 + a)^(1/4)*b^5/x - 2*(b*x^4 + a)^(5/4)*b^4/x^5 + (b*x^4 + a)^(9/4)*
b^3/x^9) - 45/128*a^2*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + l
og(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/
4))/b^3
```

Giac [F]

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{12}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^12/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^12/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{12}}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^12/(a + b*x^4)^(5/4),x)
```

output
```
int(x^12/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{12}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{12}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^12/(b*x^4+a)^(5/4),x)
```

output
```
int(x**12/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.609 $\int \frac{x^8}{(a+bx^4)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int \frac{x^8}{(a+bx^4)^{5/4}} \, dx = -\frac{x^5}{b\sqrt[4]{a+bx^4}} + \frac{5x(a+bx^4)^{3/4}}{4b^2} - \frac{5a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}} - \frac{5a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}$$

output
```
-x^5/b/(b*x^4+a)^(1/4)+5/4*x*(b*x^4+a)^(3/4)/b^2-5/8*a*arctan(b^(1/4)*x/(b
*x^4+a)^(1/4))/b^(9/4)-5/8*a*arctanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{(a+bx^4)^{5/4}} \, dx = \frac{\frac{2\sqrt[4]{b}x(5a+bx^4)}{\sqrt[4]{a+bx^4}} - 5a\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) - 5a\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8b^{9/4}}$$

input
```
Integrate[x^8/(a + b*x^4)^(5/4),x]
```

output
```
((2*b^(1/4)*x*(5*a + b*x^4))/(a + b*x^4)^(1/4) - 5*a*ArcTan[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)] - 5*a*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*b^(9/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {817, 843, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{817}$$

$$\frac{5\int \frac{x^4}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{843}$$

$$\frac{5\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{5\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{5\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{5\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

$\Big\downarrow$ 219

$$\frac{5\left(\frac{x(a+bx^4)^{3/4}}{4b}-\frac{a\left(\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt[4]{a+bx^4}}$$

input
```
Int[x^8/(a + b*x^4)^(5/4),x]
```

output
```
-(x^5/(b*(a + b*x^4)^(1/4))) + (5*((x*(a + b*x^4)^(3/4))/(4*b) - (a*(ArcTa
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*
x^4)^(1/4)]/(2*b^(1/4))))/(4*b)))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

method	result	size
pseudoelliptic	$\frac{4b^{\frac{5}{4}}x^{5}+10\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)a\left(b\,x^{4}+a\right)^{\frac{1}{4}}-5\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)a\left(b\,x^{4}+a\right)^{\frac{1}{4}}+20xa\,b^{\frac{1}{4}}}{16b^{\frac{9}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	107

input
```
int(x^8/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*b^(5/4)*x^5+10*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*a*(b*x^4+a)^(1/
4)-5*ln((b^(1/4)*x+(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*a*(b*x^4
+a)^(1/4)+20*x*a*b^(1/4))/b^(9/4)/(b*x^4+a)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.89

$$\int \frac{x^8}{(a+bx^4)^{5/4}}\,dx =$$

$$-\frac{5\left(b^3x^4+ab^2\right)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{125\left(b^7x\left(\frac{a^4}{b^9}\right)^{\frac{3}{4}}+(bx^4+a)^{\frac{1}{4}}a^3\right)}{x}\right)-5\left(b^3x^4+ab^2\right)\left(\frac{a^4}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{125\left(b^7x\left(\frac{a^4}{b^9}\right)^{\frac{3}{4}}-(bx^4+a)\right.}{x}\right.}{}$$

input
```
integrate(x^8/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/16*(5*(b^3*x^4 + a*b^2)*(a^4/b^9)^(1/4)*log(125*(b^7*x*(a^4/b^9)^(3/4)
+ (b*x^4 + a)^(1/4)*a^3)/x) - 5*(b^3*x^4 + a*b^2)*(a^4/b^9)^(1/4)*log(-125
*(b^7*x*(a^4/b^9)^(3/4) - (b*x^4 + a)^(1/4)*a^3)/x) + 5*(I*b^3*x^4 + I*a*b
^2)*(a^4/b^9)^(1/4)*log(-125*(I*b^7*x*(a^4/b^9)^(3/4) - (b*x^4 + a)^(1/4)*
a^3)/x) + 5*(-I*b^3*x^4 - I*a*b^2)*(a^4/b^9)^(1/4)*log(-125*(-I*b^7*x*(a^4
/b^9)^(3/4) - (b*x^4 + a)^(1/4)*a^3)/x) - 4*(b*x^5 + 5*a*x)*(b*x^4 + a)^(3
/4))/(b^3*x^4 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.38

$$\int \frac{x^8}{(a+bx^4)^{5/4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(b*x**4+a)**(5/4),x)
```

output
```
x**9*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(5/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.34

$$\int \frac{x^8}{(a+bx^4)^{5/4}}\,dx = \frac{4\,ab - \frac{5\,(bx^4+a)a}{x^4}}{4\left(\frac{(bx^4+a)^{\frac{1}{4}}b^3}{x} - \frac{(bx^4+a)^{\frac{5}{4}}b^2}{x^5}\right)} + \frac{5\,a\left(\frac{2\,\arctan\left(\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}} + \frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{\left(bx^4+a\right)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}\right)}{16\,b^2}$$

input
```
integrate(x^8/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
1/4*(4*a*b - 5*(b*x^4 + a)*a/x^4)/((b*x^4 + a)^(1/4)*b^3/x - (b*x^4 + a)^(
5/4)*b^2/x^5) + 5/16*a*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) +
log(-(b^(1/4) - (b*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1
/4))/b^2
```

Giac [F]

$$\int \frac{x^8}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^8}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^8/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^8/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^4)^{5/4}}\,dx=\int \frac{x^8}{{\left(b\,x^4+a\right)}^{5/4}}\,dx$$

input
```
int(x^8/(a + b*x^4)^(5/4),x)
```

output
```
int(x^8/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^8}{(a+bx^4)^{5/4}}\,dx=\int \frac{x^8}{\left(b\,x^4+a\right)^{\frac{1}{4}} a+\left(b\,x^4+a\right)^{\frac{1}{4}} b\,x^4}dx$$

input
```
int(x^8/(b*x^4+a)^(5/4),x)
```

output
```
int(x**8/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.610 $\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = -\frac{x}{b\sqrt[4]{a+bx^4}} + \frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}$$

output
```
-x/b/(b*x^4+a)^(1/4)+1/2*arctan(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)+1/2*arc
tanh(b^(1/4)*x/(b*x^4+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = \frac{-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}} + \arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right) + \operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2b^{5/4}}$$

input
```
Integrate[x^4/(a + b*x^4)^(5/4),x]
```

output
```
((-2*b^(1/4)*x)/(a + b*x^4)^(1/4) + ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]
+ ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*b^(5/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {817, 770, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{817}$$

$$\frac{\int \frac{1}{\sqrt[4]{bx^4+a}}dx}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{770}$$

$$\frac{\int \frac{1}{1-\frac{bx^4}{bx^4+a}}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{756}$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{1}{2}\int \frac{1}{\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}+1}d\frac{x}{\sqrt[4]{bx^4+a}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\frac{\sqrt{b}x^2}{\sqrt{bx^4+a}}}d\frac{x}{\sqrt[4]{bx^4+a}}+\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt[4]{b}}}{b}-\frac{x}{b\sqrt[4]{a+bx^4}}$$

input
```
Int[x^4/(a + b*x^4)^(5/4),x]
```

output
```
-(x/(b*(a + b*x^4)^(1/4))) + (ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(
1/4)) + ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*b^(1/4)))/b
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

method	result	size
pseudoelliptic	$\frac{-2\arctan\left(\frac{\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}+\ln\left(\frac{b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}{-b^{\frac{1}{4}}x+\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}-4b^{\frac{1}{4}}x}{4b^{\frac{5}{4}}\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	95

input
```
int(x^4/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*arctan(1/b^(1/4)/x*(b*x^4+a)^(1/4))*(b*x^4+a)^(1/4)+ln((b^(1/4)*x+
(b*x^4+a)^(1/4))/(-b^(1/4)*x+(b*x^4+a)^(1/4)))*(b*x^4+a)^(1/4)-4*b^(1/4)*x
)/b^(5/4)/(b*x^4+a)^(1/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.78

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = \frac{(b^2x^4+ab)\frac{1}{b^5}^{\frac{1}{4}}\log\left(\frac{b^4\frac{1}{b^5}^{\frac{3}{4}}x+(bx^4+a)^{\frac{1}{4}}}{x}\right)-(b^2x^4+ab)\frac{1}{b^5}^{\frac{1}{4}}\log\left(-\frac{b^4\frac{1}{b^5}^{\frac{3}{4}}x-(bx^4+a)^{\frac{1}{4}}}{x}\right)+(-}{}$$

input
```
integrate(x^4/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/4*((b^2*x^4 + a*b)*(b^(-5))^(1/4)*log((b^4*(b^(-5))^(3/4)*x + (b*x^4 + a
)^(1/4))/x) - (b^2*x^4 + a*b)*(b^(-5))^(1/4)*log(-(b^4*(b^(-5))^(3/4)*x -
(b*x^4 + a)^(1/4))/x) + (-I*b^2*x^4 - I*a*b)*(b^(-5))^(1/4)*log((I*b^4*(b^
(-5))^(3/4)*x + (b*x^4 + a)^(1/4))/x) + (I*b^2*x^4 + I*a*b)*(b^(-5))^(1/4)
*log((-I*b^4*(b^(-5))^(3/4)*x + (b*x^4 + a)^(1/4))/x) - 4*(b*x^4 + a)^(3/4
)*x)/(b^2*x^4 + a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(b*x**4+a)**(5/4),x)
```

output
```
x**5*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = -\frac{\frac{2\arctan\left(\frac{(bx^4+a)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}\right)}{b^{\frac{1}{4}}}+\frac{\log\left(-\frac{b^{\frac{1}{4}}-\frac{(bx^4+a)^{\frac{1}{4}}}{x}}{b^{\frac{1}{4}}+\frac{(bx^4+a)^{\frac{1}{4}}}{x}}\right)}{b^{\frac{1}{4}}}}{4\,b}-\frac{x}{(bx^4+a)^{\frac{1}{4}}b}$$

input
```
integrate(x^4/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-1/4*(2*arctan((b*x^4 + a)^(1/4)/(b^(1/4)*x))/b^(1/4) + log(-(b^(1/4) - (b
*x^4 + a)^(1/4)/x)/(b^(1/4) + (b*x^4 + a)^(1/4)/x))/b^(1/4))/b - x/((b*x^4
 + a)^(1/4)*b)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^4}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^4/(a + b*x^4)^(5/4),x)
```

output
```
int(x^4/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^4}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^4/(b*x^4+a)^(5/4),x)
```

output
```
int(x**4/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.611 $\int \frac{1}{(a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{(a+bx^4)^{5/4}} dx = \frac{x}{a\sqrt[4]{a+bx^4}}$$

output
```
x/a/(b*x^4+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^4)^{5/4}} dx = \frac{x}{a\sqrt[4]{a+bx^4}}$$

input
```
Integrate[(a + b*x^4)^(-5/4),x]
```

output
```
x/(a*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx$$
$$\Big\downarrow \text{746}$$
$$\frac{x}{a\sqrt[4]{a+bx^4}}$$

input
```
Int[(a + b*x^4)^(-5/4),x]
```

output
```
x/(a*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x}{a\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	15
trager	$\frac{x}{a\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	15
pseudoelliptic	$\frac{x}{a\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	15
orering	$\frac{x}{a\left(b\,x^{4}+a\right)^{\frac{1}{4}}}$	15

input
```
int(1/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
x/a/(b*x^4+a)^(1/4)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx = \frac{(bx^4+a)^{\frac{3}{4}}x}{abx^4+a^2}$$

input
```
integrate(1/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
(b*x^4 + a)^(3/4)*x/(a*b*x^4 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right)}{4a^{\frac{5}{4}}\sqrt[4]{1+\frac{bx^4}{a}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(b*x**4+a)**(5/4),x)
```

output
```
x*gamma(1/4)/(4*a**(5/4)*(1 + b*x**4/a)**(1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx = \frac{x}{(bx^4+a)^{\frac{1}{4}}a}$$

input
```
integrate(1/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
x/((b*x^4 + a)^(1/4)*a)
```

Giac [F]

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx = \frac{x}{a\,(b\,x^4+a)^{1/4}}$$

input
```
int(1/(a + b*x^4)^(5/4),x)
```

output
```
x/(a*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{(a+bx^4)^{5/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(1/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.612 $\int \frac{1}{x^4(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{ax^3\sqrt[4]{a+bx^4}} - \frac{4\left(a+bx^4\right)^{3/4}}{3a^2x^3}$$

output
```
1/a/x^3/(b*x^4+a)^(1/4)-4/3*(b*x^4+a)^(3/4)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx = \frac{-a-4bx^4}{3a^2x^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^4*(a + b*x^4)^(5/4)),x]
```

output
```
(-a - 4*b*x^4)/(3*a^2*x^3*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{4b\int \frac{1}{(bx^4+a)^{5/4}}dx}{3a}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 746}$$

$$-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^4*(a + b*x^4)^(5/4)),x]
```

output
```
-1/3*1/(a*x^3*(a + b*x^4)^(1/4)) - (4*b*x)/(3*a^2*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{4b\,x^{4}+a}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	26
trager	$-\frac{4b\,x^{4}+a}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	26
pseudoelliptic	$-\frac{4b\,x^{4}+a}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	26
orering	$-\frac{4b\,x^{4}+a}{3x^{3}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	26
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}}{3a^{2}x^{3}}-\frac{xb}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{2}}$	35

input
```
int(1/x^4/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(4*b*x^4+a)/x^3/(b*x^4+a)^(1/4)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx=-\frac{(4\,bx^4+a)(bx^4+a)^{\frac{3}{4}}}{3\,(a^2bx^7+a^3x^3)}$$

input
```
integrate(1/x^4/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/3*(4*b*x^4 + a)*(b*x^4 + a)^(3/4)/(a^2*b*x^7 + a^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx=\frac{\Gamma\left(-\frac{3}{4}\right)}{16a\sqrt[4]{b}x^4\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(\frac{5}{4}\right)}+\frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a^2\sqrt[4]{\frac{a}{bx^4}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**4/(b*x**4+a)**(5/4),x)
```

output
```
gamma(-3/4)/(16*a*b**(1/4)*x**4*(a/(b*x**4) + 1)**(1/4)*gamma(5/4)) + b**(
3/4)*gamma(-3/4)/(4*a**2*(a/(b*x**4) + 1)**(1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx=-\frac{bx}{\left(bx^4+a\right)^{\frac{1}{4}}a^2}-\frac{\left(bx^4+a\right)^{\frac{3}{4}}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-b*x/((b*x^4 + a)^(1/4)*a^2) - 1/3*(b*x^4 + a)^(3/4)/(a^2*x^3)
```

Giac [F]

$$\int \frac{1}{x^4\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4 \left(a+bx^4\right)^{5/4}}\,dx=-\frac{4\,b\,x^4+a}{3\,a^2\,x^3\,{\left(b\,x^4+a\right)}^{1/4}}$$

input
```
int(1/(x^4*(a + b*x^4)^(5/4)),x)
```

output
```
-(a + 4*b*x^4)/(3*a^2*x^3*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^4 \left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}} a\,x^4+\left(b\,x^4+a\right)^{\frac{1}{4}} b\,x^8}dx$$

input
```
int(1/x^4/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**4 + (a + b*x**4)**(1/4)*b*x**8),x)
```

3.613 $\int \frac{1}{x^8(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}}\,dx = \frac{1}{ax^7\sqrt[4]{a+bx^4}} - \frac{8(a+bx^4)^{3/4}}{7a^2x^7} + \frac{32b(a+bx^4)^{3/4}}{21a^3x^3}$$

output
```
1/a/x^7/(b*x^4+a)^(1/4)-8/7*(b*x^4+a)^(3/4)/a^2/x^7+32/21*b*(b*x^4+a)^(3/4
)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}}\,dx = \frac{-3a^2+8abx^4+32b^2x^8}{21a^3x^7\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^8*(a + b*x^4)^(5/4)),x]
```

output
```
(-3*a^2 + 8*a*b*x^4 + 32*b^2*x^8)/(21*a^3*x^7*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{8b\int \frac{1}{x^4(bx^4+a)^{5/4}}dx}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{8b\left(-\frac{4b\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{3a}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 746}$$

$$-\frac{8b\left(-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^8*(a + b*x^4)^(5/4)),x]
```

output
```
-1/7*1/(a*x^7*(a + b*x^4)^(1/4)) - (8*b*(-1/3*1/(a*x^3*(a + b*x^4)^(1/4))
- (4*b*x)/(3*a^2*(a + b*x^4)^(1/4))))/(7*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{-32b^2x^8-8ab\,x^4+3a^2}{21x^7(b\,x^4+a)^{\frac{1}{4}}a^3}$	39
trager	$-\frac{-32b^2x^8-8ab\,x^4+3a^2}{21x^7(b\,x^4+a)^{\frac{1}{4}}a^3}$	39
pseudoelliptic	$\frac{32b^2x^8+8ab\,x^4-3a^2}{21x^7(b\,x^4+a)^{\frac{1}{4}}a^3}$	39
orering	$-\frac{-32b^2x^8-8ab\,x^4+3a^2}{21x^7(b\,x^4+a)^{\frac{1}{4}}a^3}$	39
risch	$-\frac{(b\,x^4+a)^{\frac{3}{4}}(-11b\,x^4+3a)}{21a^3x^7}+\frac{x\,b^2}{(b\,x^4+a)^{\frac{1}{4}}a^3}$	46

input
```
int(1/x^8/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-32*b^2*x^8-8*a*b*x^4+3*a^2)/x^7/(b*x^4+a)^(1/4)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^8 \left(a+bx^4\right)^{5/4}}\,dx = \frac{(32\,b^2x^8+8\,abx^4-3\,a^2)(bx^4+a)^{\frac{3}{4}}}{21\,(a^3bx^{11}+a^4x^7)}$$

input
```
integrate(1/x^8/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/21*(32*b^2*x^8 + 8*a*b*x^4 - 3*a^2)*(b*x^4 + a)^(3/4)/(a^3*b*x^11 + a^4*
x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(58) = 116.

Time = 0.85 (sec) , antiderivative size = 323, normalized size of antiderivative = 5.21

$$\begin{aligned}\int \frac{1}{x^8 \left(a+bx^4\right)^{5/4}}\,dx = &-\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}\\
&+\frac{5a^2b^{\frac{23}{4}}x^4\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}\\
&+\frac{40ab^{\frac{27}{4}}x^8\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}\\
&+\frac{32b^{\frac{31}{4}}x^{12}\left(\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{64a^5b^4x^4\Gamma\left(\frac{5}{4}\right)+128a^4b^5x^8\Gamma\left(\frac{5}{4}\right)+64a^3b^6x^{12}\Gamma\left(\frac{5}{4}\right)}\end{aligned}$$

input
```
integrate(1/x**8/(b*x**4+a)**(5/4),x)
```

output
```
-3*a**3*b**(19/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4*x**4*g
amma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamma(5/4))
 + 5*a**2*b**(23/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5*b**4
*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12*gamm
a(5/4)) + 40*a*b**(27/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*a**5
*b**4*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x**12
*gamma(5/4)) + 32*b**(31/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(64*
a**5*b**4*x**4*gamma(5/4) + 128*a**4*b**5*x**8*gamma(5/4) + 64*a**3*b**6*x
**12*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}}\,dx = \frac{b^2x}{(bx^4+a)^{\frac{1}{4}}a^3} + \frac{\frac{14\,(bx^4+a)^{\frac{3}{4}}b}{x^3} - \frac{3\,(bx^4+a)^{\frac{7}{4}}}{x^7}}{21\,a^3}$$

input
```
integrate(1/x^8/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
b^2*x/((b*x^4 + a)^(1/4)*a^3) + 1/21*(14*(b*x^4 + a)^(3/4)*b/x^3 - 3*(b*x^
4 + a)^(7/4)/x^7)/a^3
```

Giac [F]

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}}\,dx = -\frac{32\,(b\,x^4+a)^2-56\,a\,(b\,x^4+a)+21\,a^2}{\left(\frac{21\,a^4\,x^3}{b}-\frac{21\,a^3\,x^3\,(b\,x^4+a)}{b}\right)\,(b\,x^4+a)^{1/4}}$$

input
```
int(1/(x^8*(a + b*x^4)^(5/4)),x)
```

output
```
-(32*(a + b*x^4)^2 - 56*a*(a + b*x^4) + 21*a^2)/(((21*a^4*x^3)/b - (21*a^3
*x^3*(a + b*x^4))/b)*(a + b*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^8 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^8+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{12}}dx$$

input
```
int(1/x^8/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**8 + (a + b*x**4)**(1/4)*b*x**12),x)
```

3.614 $\int \frac{1}{x^{12}(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx=\frac{1}{ax^{11}\sqrt[4]{a+bx^4}}-\frac{12\left(a+bx^4\right)^{3/4}}{11a^2x^{11}}+\frac{96b\left(a+bx^4\right)^{3/4}}{77a^3x^7}-\frac{128b^2\left(a+bx^4\right)^{3/4}}{77a^4x^3}$$

output
```
1/a/x^11/(b*x^4+a)^(1/4)-12/11*(b*x^4+a)^(3/4)/a^2/x^11+96/77*b*(b*x^4+a)^
(3/4)/a^3/x^7-128/77*b^2*(b*x^4+a)^(3/4)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx=\frac{-7a^3+12a^2bx^4-32ab^2x^8-128b^3x^{12}}{77a^4x^{11}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^12*(a + b*x^4)^(5/4)),x]
```

output
```
(-7*a^3 + 12*a^2*b*x^4 - 32*a*b^2*x^8 - 128*b^3*x^12)/(77*a^4*x^11*(a + b*
x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {803, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{12b\int \frac{1}{x^8(bx^4+a)^{5/4}}dx}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{12b\left(-\frac{8b\int \frac{1}{x^4\left(bx^4+a\right)^{5/4}}dx}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4b\int \frac{1}{\left(bx^4+a\right)^{5/4}}dx}{3a}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 746}$$

$$-\frac{12b\left(-\frac{8b\left(-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^12*(a + b*x^4)^(5/4)),x]
```

output
```
-1/11*1/(a*x^11*(a + b*x^4)^(1/4)) - (12*b*(-1/7*1/(a*x^7*(a + b*x^4)^(1/4
)) - (8*b*(-1/3*1/(a*x^3*(a + b*x^4)^(1/4)) - (4*b*x)/(3*a^2*(a + b*x^4)^(
1/4))))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{128b^3x^{12}+32a\,b^2x^8-12a^2b\,x^4+7a^3}{77x^{11}\left(b\,x^4+a\right)^{\frac{1}{4}}a^4}$	50
trager	$-\frac{128b^3x^{12}+32a\,b^2x^8-12a^2b\,x^4+7a^3}{77x^{11}\left(b\,x^4+a\right)^{\frac{1}{4}}a^4}$	50
pseudoelliptic	$\frac{-128b^3x^{12}-32a\,b^2x^8+12a^2b\,x^4-7a^3}{77x^{11}\left(b\,x^4+a\right)^{\frac{1}{4}}a^4}$	50
orering	$-\frac{128b^3x^{12}+32a\,b^2x^8-12a^2b\,x^4+7a^3}{77x^{11}\left(b\,x^4+a\right)^{\frac{1}{4}}a^4}$	50
risch	$-\frac{\left(b\,x^4+a\right)^{\frac{3}{4}}\left(51b^2x^8-19ab\,x^4+7a^2\right)}{77a^4x^{11}}-\frac{x\,b^3}{\left(b\,x^4+a\right)^{\frac{1}{4}}a^4}$	58

input
```
int(1/x^12/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/77*(128*b^3*x^12+32*a*b^2*x^8-12*a^2*b*x^4+7*a^3)/x^11/(b*x^4+a)^(1/4)/
a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx = -\frac{(128\,b^3x^{12}+32\,ab^2x^8-12\,a^2bx^4+7\,a^3)(bx^4+a)^{\frac{3}{4}}}{77\,(a^4bx^{15}+a^5x^{11})}$$

input
```
integrate(1/x^12/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/77*(128*b^3*x^12 + 32*a*b^2*x^8 - 12*a^2*b*x^4 + 7*a^3)*(b*x^4 + a)^(3/
4)/(a^4*b*x^15 + a^5*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(82) = 164.

Time = 1.26 (sec) , antiderivative size = 592, normalized size of antiderivative = 6.88

$$\int \frac{1}{x^{12}\left(a+bx^4\right)^{5/4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**12/(b*x**4+a)**(5/4),x)
```

output
```
21*a**5*b**(39/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8
*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma
(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 6*a**4*b**(43/4)*x**4*(a/(b*x**
4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**1
0*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**2
0*gamma(5/4)) + 45*a**3*b**(47/4)*x**8*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4
)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a
**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gamma(5/4)) + 540*a**2*b
**(51/4)*x**12*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*ga
mma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/
4) + 256*a**4*b**12*x**20*gamma(5/4)) + 864*a*b**(55/4)*x**16*(a/(b*x**4)
+ 1)**(3/4)*gamma(-11/4)/(256*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x
**12*gamma(5/4) + 768*a**5*b**11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*g
amma(5/4)) + 384*b**(59/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(256
*a**7*b**9*x**8*gamma(5/4) + 768*a**6*b**10*x**12*gamma(5/4) + 768*a**5*b*
*11*x**16*gamma(5/4) + 256*a**4*b**12*x**20*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{12} (a + bx^4)^{5/4}} \, dx = -\frac{b^3 x}{(bx^4 + a)^{\frac{1}{4}} a^4} - \frac{\frac{77 (bx^4+a)^{\frac{3}{4}} b^2}{x^3} - \frac{33 (bx^4+a)^{\frac{7}{4}} b}{x^7} + \frac{7 (bx^4+a)^{\frac{11}{4}}}{x^{11}}}{77 a^4}$$

input
```
integrate(1/x^12/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
-b^3*x/((b*x^4 + a)^(1/4)*a^4) - 1/77*(77*(b*x^4 + a)^(3/4)*b^2/x^3 - 33*(
b*x^4 + a)^(7/4)*b/x^7 + 7*(b*x^4 + a)^(11/4)/x^11)/a^4
```

Giac [F]

$$\int \frac{1}{x^{12} (a+bx^4)^{5/4}} \, dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}} x^{12}} \, dx$$

input
```
integrate(1/x^12/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{12} (a+bx^4)^{5/4}} \, dx = \frac{19 b (b x^4+a)^{3/4}}{77 a^3 x^7} - \frac{b^3 x}{a^4 (b x^4+a)^{1/4}} - \frac{(b x^4+a)^{3/4}}{11 a^2 x^{11}} - \frac{51 b^2 (b x^4+a)^{3/4}}{77 a^4 x^3}$$

input
```
int(1/(x^12*(a + b*x^4)^(5/4)),x)
```

output
```
(19*b*(a + b*x^4)^(3/4))/(77*a^3*x^7) - (b^3*x)/(a^4*(a + b*x^4)^(1/4)) -
(a + b*x^4)^(3/4)/(11*a^2*x^11) - (51*b^2*(a + b*x^4)^(3/4))/(77*a^4*x^3)
```

Reduce [F]

$$\int \frac{1}{x^{12} (a+bx^4)^{5/4}} \, dx = \int \frac{1}{(b x^4+a)^{\frac{1}{4}} a x^{12} + (b x^4+a)^{\frac{1}{4}} b x^{16}} dx$$

input
```
int(1/x^12/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**12 + (a + b*x**4)**(1/4)*b*x**16),x)
```

3.615 $\int \frac{1}{x^{16}(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx = \frac{1}{ax^{15}\sqrt[4]{a+bx^4}} - \frac{16(a+bx^4)^{3/4}}{15a^2x^{15}} + \frac{64b(a+bx^4)^{3/4}}{55a^3x^{11}} - \frac{512b^2(a+bx^4)^{3/4}}{385a^4x^7} + \frac{2048b^3(a+bx^4)^{3/4}}{1155a^5x^3}$$

output
```
1/a/x^15/(b*x^4+a)^(1/4)-16/15*(b*x^4+a)^(3/4)/a^2/x^15+64/55*b*(b*x^4+a)^
(3/4)/a^3/x^11-512/385*b^2*(b*x^4+a)^(3/4)/a^4/x^7+2048/1155*b^3*(b*x^4+a)
^(3/4)/a^5/x^3
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx = \frac{-77a^4+112a^3bx^4-192a^2b^2x^8+512ab^3x^{12}+2048b^4x^{16}}{1155a^5x^{15}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^16*(a + b*x^4)^(5/4)),x]
```

output
```
(-77*a^4 + 112*a^3*b*x^4 - 192*a^2*b^2*x^8 + 512*a*b^3*x^12 + 2048*b^4*x^1
6)/(1155*a^5*x^15*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {803, 803, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\int \frac{1}{x^{12}\left(bx^4+a\right)^{5/4}}dx}{15a}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\left(-\frac{12b\int \frac{1}{x^8\left(bx^4+a\right)^{5/4}}dx}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}\right)}{15a}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\left(-\frac{12b\left(-\frac{8b\int \frac{1}{x^4\left(bx^4+a\right)^{5/4}}dx}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}\right)}{15a}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 803}$$

$$-\frac{16b\left(-\frac{12b\left(-\frac{8b\left(-\frac{4b\int\frac{1}{(bx^4+a)^{5/4}}dx}{3a}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}\right)}{15a}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

$\Big\downarrow$ 746

$$-\frac{16b\left(-\frac{12b\left(-\frac{8b\left(-\frac{4bx}{3a^2\sqrt[4]{a+bx^4}}-\frac{1}{3ax^3\sqrt[4]{a+bx^4}}\right)}{7a}-\frac{1}{7ax^7\sqrt[4]{a+bx^4}}\right)}{11a}-\frac{1}{11ax^{11}\sqrt[4]{a+bx^4}}\right)}{15a}-\frac{1}{15ax^{15}\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^16*(a + b*x^4)^(5/4)),x]
```

output
```
-1/15*1/(a*x^15*(a + b*x^4)^(1/4)) - (16*b*(-1/11*1/(a*x^11*(a + b*x^4)^(1
/4)) - (12*b*(-1/7*1/(a*x^7*(a + b*x^4)^(1/4)) - (8*b*(-1/3*1/(a*x^3*(a +
b*x^4)^(1/4)) - (4*b*x)/(3*a^2*(a + b*x^4)^(1/4))))/(7*a)))/(11*a)))/(15*a
)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{-2048x^{16}b^{4}-512a\,b^{3}x^{12}+192a^{2}b^{2}x^{8}-112a^{3}b\,x^{4}+77a^{4}}{1155x^{15}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{5}}$	61
trager	$-\frac{-2048x^{16}b^{4}-512a\,b^{3}x^{12}+192a^{2}b^{2}x^{8}-112a^{3}b\,x^{4}+77a^{4}}{1155x^{15}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{5}}$	61
pseudoelliptic	$\frac{2048x^{16}b^{4}+512a\,b^{3}x^{12}-192a^{2}b^{2}x^{8}+112a^{3}b\,x^{4}-77a^{4}}{1155x^{15}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{5}}$	61
orering	$-\frac{-2048x^{16}b^{4}-512a\,b^{3}x^{12}+192a^{2}b^{2}x^{8}-112a^{3}b\,x^{4}+77a^{4}}{1155x^{15}\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{5}}$	61
risch	$-\frac{\left(b\,x^{4}+a\right)^{\frac{3}{4}}\left(-893b^{3}x^{12}+381a\,b^{2}x^{8}-189a^{2}b\,x^{4}+77a^{3}\right)}{1155a^{5}x^{15}}+\frac{x\,b^{4}}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}a^{5}}$	68

input
```
int(1/x^16/(b*x^4+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(-2048*b^4*x^16-512*a*b^3*x^12+192*a^2*b^2*x^8-112*a^3*b*x^4+77*a^
4)/x^15/(b*x^4+a)^(1/4)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx = \frac{\left(2048\,b^4x^{16}+512\,ab^3x^{12}-192\,a^2b^2x^8+112\,a^3bx^4-77\,a^4\right)\left(bx^4+a\right)^{\frac{3}{4}}}{1155\left(a^5bx^{19}+a^6x^{15}\right)}$$

input
```
integrate(1/x^16/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
1/1155*(2048*b^4*x^16 + 512*a*b^3*x^12 - 192*a^2*b^2*x^8 + 112*a^3*b*x^4 -
 77*a^4)*(b*x^4 + a)^(3/4)/(a^5*b*x^19 + a^6*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(105) = 210.

Time = 1.79 (sec) , antiderivative size = 928, normalized size of antiderivative = 8.44

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**16/(b*x**4+a)**(5/4),x)
```

output
```
-231*a**7*b**(67/4)*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*
x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**18*x**2
0*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x**28*ga
mma(5/4)) - 357*a**6*b**(71/4)*x**4*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(
1024*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144
*a**7*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**
5*b**20*x**28*gamma(5/4)) - 261*a**5*b**(75/4)*x**8*(a/(b*x**4) + 1)**(3/4
)*gamma(-15/4)/(1024*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*g
amma(5/4) + 6144*a**7*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma
(5/4) + 1024*a**5*b**20*x**28*gamma(5/4)) + 585*a**4*b**(79/4)*x**12*(a/(b
*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*x**12*gamma(5/4) + 4096*a
**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**18*x**20*gamma(5/4) + 4096*a**6*
b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x**28*gamma(5/4)) + 9360*a**3*b**
(83/4)*x**16*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024*a**9*b**16*x**12*g
amma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**7*b**18*x**20*gamma
(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b**20*x**28*gamma(5/4
)) + 22464*a**2*b**(87/4)*x**20*(a/(b*x**4) + 1)**(3/4)*gamma(-15/4)/(1024
*a**9*b**16*x**12*gamma(5/4) + 4096*a**8*b**17*x**16*gamma(5/4) + 6144*a**
7*b**18*x**20*gamma(5/4) + 4096*a**6*b**19*x**24*gamma(5/4) + 1024*a**5*b*
*20*x**28*gamma(5/4)) + 19968*a*b**(91/4)*x**24*(a/(b*x**4) + 1)**(3/4)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx = \frac{b^4x}{\left(bx^4+a\right)^{\frac{1}{4}}a^5} + \frac{\frac{1540\left(bx^4+a\right)^{\frac{3}{4}}b^3}{x^3} - \frac{990\left(bx^4+a\right)^{\frac{7}{4}}b^2}{x^7} + \frac{420\left(bx^4+a\right)^{\frac{11}{4}}b}{x^{11}} - \frac{77\left(bx^4+a\right)^{\frac{15}{4}}}{x^{15}}}{1155\,a^5}$$

input
```
integrate(1/x^16/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
b^4*x/((b*x^4 + a)^(1/4)*a^5) + 1/1155*(1540*(b*x^4 + a)^(3/4)*b^3/x^3 - 9
90*(b*x^4 + a)^(7/4)*b^2/x^7 + 420*(b*x^4 + a)^(11/4)*b/x^11 - 77*(b*x^4 +
 a)^(15/4)/x^15)/a^5
```

Giac [F]

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{16}}\,dx$$

input
```
integrate(1/x^16/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^16), x)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx=\frac{b^4\,x}{a^5\,\left(b\,x^4+a\right)^{1/4}}-\frac{\left(b\,x^4+a\right)^{3/4}}{15\,a^2\,x^{15}}+\frac{9\,b\,\left(b\,x^4+a\right)^{3/4}}{55\,a^3\,x^{11}}+\frac{893\,b^3\,\left(b\,x^4+a\right)^{3/4}}{1155\,a^5\,x^3}-\frac{127\,b^2\,\left(b\,x^4+a\right)^{3/4}}{385\,a^4\,x^7}$$

input
```
int(1/(x^16*(a + b*x^4)^(5/4)),x)
```

output
```
(b^4*x)/(a^5*(a + b*x^4)^(1/4)) - (a + b*x^4)^(3/4)/(15*a^2*x^15) + (9*b*(
a + b*x^4)^(3/4))/(55*a^3*x^11) + (893*b^3*(a + b*x^4)^(3/4))/(1155*a^5*x^
3) - (127*b^2*(a + b*x^4)^(3/4))/(385*a^4*x^7)
```

Reduce [F]

$$\int \frac{1}{x^{16}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}a\,x^{16}+\left(b\,x^4+a\right)^{\frac{1}{4}}b\,x^{20}}dx$$

input
```
int(1/x^16/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**16 + (a + b*x**4)**(1/4)*b*x**20),x)
```

3.616 $\int \frac{x^{14}}{(a+bx^4)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}} dx = \frac{77a^2x^3}{120b^3\sqrt[4]{a+bx^4}} - \frac{11ax^7}{60b^2\sqrt[4]{a+bx^4}} + \frac{x^{11}}{10b\sqrt[4]{a+bx^4}} + \frac{77a^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{7/2}\sqrt[4]{a+bx^4}}$$

output

```
77/120*a^2*x^3/b^3/(b*x^4+a)^(1/4)-11/60*a*x^7/b^2/(b*x^4+a)^(1/4)+1/10*x^
11/b/(b*x^4+a)^(1/4)+77/40*a^(5/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*a
rccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.62

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}} dx = \frac{x^3\left(77a^2-22abx^4+12b^2x^8-77a^2\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{120b^3\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^14/(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(77*a^2 - 22*a*b*x^4 + 12*b^2*x^8 - 77*a^2*(1 + (b*x^4)/a)^(1/4)*Hype
rgeometric2F1[3/4, 5/4, 7/4, -((b*x^4)/a)]))/(120*b^3*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {815, 815, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\int \frac{x^{10}}{(bx^4+a)^{5/4}}dx}{10b}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\int \frac{x^6}{\left(bx^4+a\right)^{5/4}}dx}{6b}\right)}{10b}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{10b}$$

$\Big\downarrow$ 858

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{10b}$$

$\Big\downarrow$ 807

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{10b}$$

$\Big\downarrow$ 212

$$\frac{x^{11}}{10b\sqrt[4]{a+bx^4}}-\frac{11a\left(\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}\right)}{10b}$$

input
```
Int[x^14/(a + b*x^4)^(5/4),x]
```

output
```
x^11/(10*b*(a + b*x^4)^(1/4)) - (11*a*(x^7/(6*b*(a + b*x^4)^(1/4)) - (7*a*
(x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*Elliptic
E[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))))/(6*
b)))/(10*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{14}}{\left(b\,x^{4}+a\right)^{\frac{5}{4}}}dx$$

input
```
int(x^14/(b*x^4+a)^(5/4),x)
```

output
```
int(x^14/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{14}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^14/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^14/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.29

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}\,dx = \frac{x^{15}\Gamma\left(\frac{15}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{15}{4}\\ \frac{19}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{19}{4}\right)}$$

input
```
integrate(x**14/(b*x**4+a)**(5/4),x)
```

output
```
x**15*gamma(15/4)*hyper((5/4, 15/4), (19/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(5/4)*gamma(19/4))
```

Maxima [F]

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{14}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^14/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^14/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{14}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^14/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^14/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{14}}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^14/(a + b*x^4)^(5/4),x)
```

output
```
int(x^14/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{14}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{14}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^14/(b*x^4+a)^(5/4),x)
```

output
```
int(x**14/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.617 $\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx = -\frac{7ax^3}{12b^2\sqrt[4]{a+bx^4}}+\frac{x^7}{6b\sqrt[4]{a+bx^4}} - \frac{7a^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-7/12*a*x^3/b^2/(b*x^4+a)^(1/4)+1/6*x^7/b/(b*x^4+a)^(1/4)-7/4*a^(3/2)*(1+a
/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^
(5/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.64

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx = \frac{x^3\left(-7a+2bx^4+7a\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)\right)}{12b^2\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^10/(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(-7*a + 2*b*x^4 + 7*a*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/
4, 7/4, -((b*x^4)/a)]))/(12*b^2*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {815, 815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{815}$$

$$\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\int \frac{x^6}{(bx^4+a)^{5/4}}dx}{6b}$$

$$\Big\downarrow \text{815}$$

$$\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{2b}\right)}{6b}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}\right)}{6b}$$

$$\Big\downarrow \text{858}$$

$$\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}$$

$$\Big\downarrow \text{212}$$

$$\frac{x^7}{6b\sqrt[4]{a+bx^4}}-\frac{7a\left(\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}\right)}{6b}$$

input
```
Int[x^10/(a + b*x^4)^(5/4),x]
```

output
```
x^7/(6*b*(a + b*x^4)^(1/4)) - (7*a*(x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[
a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/
(2*b^(3/2)*(a + b*x^4)^(1/4))))/(6*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{10}}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^10/(b*x^4+a)^(5/4),x)
```

output
```
int(x^10/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^10/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(b*x**4+a)**(5/4),x)
```

output
```
x**11*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*x**4*exp_polar(I*pi)/a)/(4
*a**(5/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^10/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^{10}}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^10/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^10/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx=\int \frac{x^{10}}{{\left(b\,x^4+a\right)}^{5/4}}\,dx$$

input
```
int(x^10/(a + b*x^4)^(5/4),x)
```

output
```
int(x^10/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a+bx^4)^{5/4}}\,dx=\int \frac{x^{10}}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^10/(b*x^4+a)^(5/4),x)
```

output
```
int(x**10/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.618 $\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx=\frac{x^3}{2b\sqrt[4]{a+bx^4}}+\frac{3\sqrt{a}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
1/2*x^3/b/(b*x^4+a)^(1/4)+3/2*a^(1/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/
2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx=\frac{x^3-x^3\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{2b\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^6/(a + b*x^4)^(5/4),x]
```

output
```
(x^3 - x^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^4
)/a)])/(2*b*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {815, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{815}$$

$$\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3a\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{2b}$$

$$\Big\downarrow \text{813}$$

$$\frac{x^3}{2b\sqrt[4]{a+bx^4}}-\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{2b^2\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{3ax\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{4b^2\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{3\sqrt{a}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^4}}+\frac{x^3}{2b\sqrt[4]{a+bx^4}}$$

input
```
Int[x^6/(a + b*x^4)^(5/4),x]
```

output
```
x^3/(2*b*(a + b*x^4)^(1/4)) + (3*Sqrt[a]*(1 + a/(b*x^4))^(1/4)*x*EllipticE
[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*b^(3/2)*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 815
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m - 3)/(b*(
m - 4)*(a + b*x^4)^(1/4)), x] - Simp[a*((m - 3)/(b*(m - 4)))   Int[x^(m - 4
)/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && IGtQ[(m - 2
)/4, 0]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^6/(b*x^4+a)^(5/4),x)
```

output
```
int(x^6/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^6/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.45

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(b*x**4+a)**(5/4),x)
```

output
```
x**7*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*a*
*(5/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^6}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^6/(a + b*x^4)^(5/4),x)
```

output
```
int(x^6/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^6}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^6/(b*x^4+a)^(5/4),x)
```

output
```
int(x**6/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.619 $\int \frac{x^2}{(a+bx^4)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^2}{(a+bx^4)^{5/4}} \, dx = -\frac{\sqrt[4]{1+\frac{a}{bx^4}}\, xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^4}}$$

output
```
-(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2
))/a^(1/2)/b^(1/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(a+bx^4)^{5/4}} \, dx = \frac{x^3 \sqrt[4]{1+\frac{bx^4}{a}} \text{ Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^4}{a}\right)}{3a\sqrt[4]{a+bx^4}}$$

input
```
Integrate[x^2/(a + b*x^4)^(5/4),x]
```

output
```
(x^3*(1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^4)/a)])
/(3*a*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{813}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{2b\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^4}}$$

input
```
Int[x^2/(a + b*x^4)^(5/4),x]
```

output
```
-(((1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/
(Sqrt[a]*Sqrt[b]*(a + b*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^2/(b*x^4+a)^(5/4),x)
```

output
```
int(x^2/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)*x^2/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(a+bx^4)^{5/4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(b*x**4+a)**(5/4),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^2}{(bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^2}{(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^2/(a + b*x^4)^(5/4),x)
```

output
```
int(x^2/(a + b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^4)^{5/4}}\,dx = \int \frac{x^2}{(b\,x^4+a)^{\frac{1}{4}}\,a+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^2/(b*x^4+a)^(5/4),x)
```

output
```
int(x**2/((a + b*x**4)**(1/4)*a + (a + b*x**4)**(1/4)*b*x**4),x)
```

3.620 $\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx = -\frac{1}{ax\sqrt[4]{a+bx^4}}+\frac{2\sqrt{b}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/a/x/(b*x^4+a)^(1/4)+2*b^(1/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arc
cot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\left(a+bx^4\right)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{bx^4}{a}\right)}{ax\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^2*(a + b*x^4)^(5/4)),x]
```

output
```
-(((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x^4)/a)])/
(a*x*(a + b*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^4)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 816}$$

$$-\frac{2b\int \frac{x^2}{(bx^4+a)^{5/4}}dx}{a}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^2*(a + b*x^4)^(5/4)),x]
```

output
```
-(1/(a*x*(a + b*x^4)^(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*Elliptic
E[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b \,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^2/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^2/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{{\left(bx^{4}+a\right)}^{\frac{5}{4}}x^{2}}\,dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^10 + 2*a*b*x^6 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^2 \left(a+bx^4\right)^{5/4}}\,dx=\frac{\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(b*x**4+a)**(5/4),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5/
4)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^4)^{5/4}} \, dx = \int \frac{1}{(bx^4 + a)^{\frac{5}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^4)^{5/4}} \, dx = \int \frac{1}{(bx^4 + a)^{\frac{5}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2 (a + bx^4)^{5/4}} \, dx = -\frac{\left(\frac{a}{b\,x^4}+1\right)^{5/4} {}_2\mathrm{F}_1\left(\frac{5}{4},\frac{3}{2};\ \frac{5}{2};\ -\frac{a}{b\,x^4}\right)}{6\,x\,\left(b\,x^4+a\right)^{5/4}}$$

input
```
int(1/(x^2*(a + b*x^4)^(5/4)),x)
```

output
```
-((a/(b*x^4) + 1)^(5/4)*hypergeom([5/4, 3/2], 5/2, -a/(b*x^4)))/(6*x*(a +
b*x^4)^(5/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^4)^{5/4}} \, dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^2+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^6} dx$$

input
```
int(1/x^2/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**2 + (a + b*x**4)**(1/4)*b*x**6),x)
```

3.621 $\int \frac{1}{x^6(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{1}{x^6(a+bx^4)^{5/4}}\,dx = -\frac{1}{5ax^5\sqrt[4]{a+bx^4}}+\frac{6b}{5a^2x\sqrt[4]{a+bx^4}} - \frac{12b^{3/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/5/a/x^5/(b*x^4+a)^(1/4)+6/5*b/a^2/x/(b*x^4+a)^(1/4)-12/5*b^(3/2)*(1+a/b
/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(5
/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^6(a+bx^4)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},-\frac{1}{4},-\frac{bx^4}{a}\right)}{5ax^5\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^6*(a + b*x^4)^(5/4)),x]
```

output
```
-1/5*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x^4)/a
)])/(a*x^5*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {816, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+bx^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{816}$$

$$-\frac{6b \int \frac{1}{x^2 (bx^4+a)^{5/4}}dx}{5a}-\frac{1}{5ax^5 \sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{816}$$

$$-\frac{6b \left(-\frac{2b \int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{a}-\frac{1}{ax \sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5 \sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{813}$$

$$-\frac{6b \left(-\frac{2x \sqrt[4]{\frac{a}{bx^4}+1} \int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4} x^3}dx}{a \sqrt[4]{a+bx^4}}-\frac{1}{ax \sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5 \sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{6b\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{6b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{6b\left(\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^6*(a + b*x^4)^(5/4)),x]
```

output
```
-1/5*1/(a*x^5*(a + b*x^4)^(1/4)) - (6*b*(-(1/(a*x*(a + b*x^4)^(1/4))) + (2
*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2
, 2])/(a^(3/2)*(a + b*x^4)^(1/4))))/(5*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{4}+a\right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^6/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^6/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(a+bx^4\right)^{5/4}}\,dx = \int \frac{1}{\left(bx^{4}+a\right)^{\frac{5}{4}}x^{6}}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^14 + 2*a*b*x^10 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^6 (a+bx^4)^{5/4}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{4} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^4 e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}} x^5 \Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/x**6/(b*x**4+a)**(5/4),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 5/4), (-1/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5
/4)*x**5*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}} x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}} x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a + bx^4)^{5/4}} \, dx = \int \frac{1}{x^6 \left(b\, x^4 + a\right)^{5/4}} \, dx$$

input
```
int(1/(x^6*(a + b*x^4)^(5/4)),x)
```

output
```
int(1/(x^6*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a + bx^4)^{5/4}} \, dx = \int \frac{1}{\left(b\, x^4 + a\right)^{\frac{1}{4}} a\, x^6 + \left(b\, x^4 + a\right)^{\frac{1}{4}} b\, x^{10}} dx$$

input
```
int(1/x^6/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**6 + (a + b*x**4)**(1/4)*b*x**10),x)
```

3.622 $\int \frac{1}{x^{10}(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 129

$$\int \frac{1}{x^{10}(a+bx^4)^{5/4}}\,dx = -\frac{1}{9ax^9\sqrt[4]{a+bx^4}}+\frac{2b}{9a^2x^5\sqrt[4]{a+bx^4}} -\frac{4b^2}{3a^3x\sqrt[4]{a+bx^4}}+\frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}\sqrt[4]{a+bx^4}}$$

output
```
-1/9/a/x^9/(b*x^4+a)^(1/4)+2/9*b/a^2/x^5/(b*x^4+a)^(1/4)-4/3*b^2/a^3/x/(b*
x^4+a)^(1/4)+8/3*b^(5/2)*(1+a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1
/2)*x^2/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^{10}(a+bx^4)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{5}{4},-\frac{5}{4},-\frac{bx^4}{a}\right)}{9ax^9\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^10*(a + b*x^4)^(5/4)),x]
```

output
```
-1/9*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x^4)/a
)])/(a*x^9*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {816, 816, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} \left(a+b x^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 816}$$

$$-\frac{10 b \int \frac{1}{x^6 \left(b x^4+a\right)^{5/4}}dx}{9 a}-\frac{1}{9 a x^9 \sqrt[4]{a+b x^4}}$$

$$\Big\downarrow \text{ 816}$$

$$-\frac{10 b \left(-\frac{6 b \int \frac{1}{x^2 \left(b x^4+a\right)^{5/4}}dx}{5 a}-\frac{1}{5 a x^5 \sqrt[4]{a+b x^4}}\right)}{9 a}-\frac{1}{9 a x^9 \sqrt[4]{a+b x^4}}$$

$$\Big\downarrow \text{ 816}$$

$$-\frac{10 b \left(-\frac{6 b \left(-\frac{2 b \int \frac{x^2}{\left(b x^4+a\right)^{5/4}}dx}{a}-\frac{1}{a x \sqrt[4]{a+b x^4}}\right)}{5 a}-\frac{1}{5 a x^5 \sqrt[4]{a+b x^4}}\right)}{9 a}-\frac{1}{9 a x^9 \sqrt[4]{a+b x^4}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}$$

$\big\downarrow$ 858

$$-\frac{10b\left(-\frac{6b\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}$$

$\big\downarrow$ 807

$$-\frac{10b\left(-\frac{6b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}$$

$\big\downarrow$ 212

$$-\frac{10b\left(-\frac{6b\left(\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^10*(a + b*x^4)^(5/4)),x]
```

output
```
-1/9*1/(a*x^9*(a + b*x^4)^(1/4)) - (10*b*(-1/5*1/(a*x^5*(a + b*x^4)^(1/4))
 - (6*b*(-(1/(a*x*(a + b*x^4)^(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x
*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4)
)))/(5*a)))/(9*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{10} \left(b\,x^{4}+a \right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^10/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^10/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{10} \left(a+bx^4\right)^{5/4}}\, dx = \int \frac{1}{{\left(b x^{4} + a\right)}^{\frac{5}{4}} x^{10}}\, dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^18 + 2*a*b*x^14 + a^2*x^10), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx=\frac{\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},\frac{5}{4}\\ -\frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x^9\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate(1/x**10/(b*x**4+a)**(5/4),x)
```

output
```
gamma(-9/4)*hyper((-9/4, 5/4), (-5/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**(5
/4)*x**9*gamma(-5/4))
```

Maxima [F]

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10} (a+bx^4)^{5/4}} \, dx = \int \frac{1}{x^{10} \, (b\,x^4+a)^{5/4}} \, dx$$

input
```
int(1/(x^10*(a + b*x^4)^(5/4)),x)
```

output
```
int(1/(x^10*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10} (a+bx^4)^{5/4}} \, dx = \int \frac{1}{(b\,x^4+a)^{\frac{1}{4}}\,a\,x^{10}+(b\,x^4+a)^{\frac{1}{4}}\,b\,x^{14}} dx$$

input
```
int(1/x^10/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**10 + (a + b*x**4)**(1/4)*b*x**14),x)
```

3.623 $\int \frac{1}{x^{14}(a+bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 153

$$\int \frac{1}{x^{14}(a+bx^4)^{5/4}}\,dx = -\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}+\frac{14b}{117a^2x^9\sqrt[4]{a+bx^4}} -\frac{28b^2}{117a^3x^5\sqrt[4]{a+bx^4}}+\frac{56b^3}{39a^4x\sqrt[4]{a+bx^4}}-\frac{112b^{7/2}\sqrt[4]{1+\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{9/2}\sqrt[4]{a+bx^4}}$$

output

```
-1/13/a/x^13/(b*x^4+a)^(1/4)+14/117*b/a^2/x^9/(b*x^4+a)^(1/4)-28/117*b^2/a
^3/x^5/(b*x^4+a)^(1/4)+56/39*b^3/a^4/x/(b*x^4+a)^(1/4)-112/39*b^(7/2)*(1+a
/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^
(9/2)/(b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^{14}(a+bx^4)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{5}{4},-\frac{9}{4},-\frac{bx^4}{a}\right)}{13ax^{13}\sqrt[4]{a+bx^4}}$$

input
```
Integrate[1/(x^14*(a + b*x^4)^(5/4)),x]
```

output
```
-1/13*((1 + (b*x^4)/a)^(1/4)*Hypergeometric2F1[-13/4, 5/4, -9/4, -((b*x^4)
/a)])/(a*x^13*(a + b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {816, 816, 816, 816, 813, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14} \left(a+bx^4\right)^{5/4}} \, dx$$

$$\Big\downarrow \ 816$$

$$-\frac{14b \int \frac{1}{x^{10}\left(bx^4+a\right)^{5/4}}dx}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 816$$

$$-\frac{14b\left(-\frac{10b\int \frac{1}{x^6\left(bx^4+a\right)^{5/4}}dx}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 816$$

$$-\frac{14b\left(-\frac{10b\left(-\frac{6b\int \frac{1}{x^2\left(bx^4+a\right)^{5/4}}dx}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \ 816$$

$$-\frac{14b\left(-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{x^2}{\left(bx^4+a\right)^{5/4}}dx}{a}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$\downarrow$ 813

$$-\frac{14b\left(-\frac{10b\left(-\frac{6b\left(-\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x^3}dx}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$\downarrow$ 858

$$-\frac{14b\left(-\frac{10b\left(-\frac{6b\left(\frac{2x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^4}+1\right)^{5/4}x}d\frac{1}{x}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 807$$

$$-\frac{14b\left(-\frac{10b\left(-\frac{6b\left(\frac{x\sqrt[4]{\frac{a}{bx^4}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x^2}}{a\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

$$\Big\downarrow \; 212$$

$$-\frac{14b\left(-\frac{10b\left(-\frac{6b\left(\frac{2\sqrt{b}x\sqrt[4]{\frac{a}{bx^4}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^4}}-\frac{1}{ax\sqrt[4]{a+bx^4}}\right)}{5a}-\frac{1}{5ax^5\sqrt[4]{a+bx^4}}\right)}{9a}-\frac{1}{9ax^9\sqrt[4]{a+bx^4}}\right)}{13a}-\frac{1}{13ax^{13}\sqrt[4]{a+bx^4}}$$

input
```
Int[1/(x^14*(a + b*x^4)^(5/4)),x]
```

output
```
-1/13*1/(a*x^13*(a + b*x^4)^(1/4)) - (14*b*(-1/9*1/(a*x^9*(a + b*x^4)^(1/4
)) - (10*b*(-1/5*1/(a*x^5*(a + b*x^4)^(1/4)) - (6*b*(-(1/(a*x*(a + b*x^4)^
(1/4))) + (2*Sqrt[b]*(1 + a/(b*x^4))^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqr
t[b]*x^2)]/2, 2])/(a^(3/2)*(a + b*x^4)^(1/4))))/(5*a)))/(9*a)))/(13*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 816
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x^(m + 1)/(a*(
m + 1)*(a + b*x^4)^(1/4)), x] - Simp[b*(m/(a*(m + 1)))   Int[x^(m + 4)/(a +
 b*x^4)^(5/4), x], x] /; FreeQ[{a, b}, x] && PosQ[b/a] && ILtQ[(m - 2)/4, 0
]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{14} \left(b\,x^{4}+a\right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^14/(b*x^4+a)^(5/4),x)
```

output
```
int(1/x^14/(b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{14} (a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^(3/4)/(b^2*x^22 + 2*a*b*x^18 + a^2*x^14), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^{14} (a+bx^4)^{5/4}}\,dx = \frac{\Gamma\left(-\frac{13}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{13}{4}, \frac{5}{4} \\ -\frac{9}{4} \end{matrix}\middle| \frac{bx^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}x^{13}\Gamma\left(-\frac{9}{4}\right)}$$

input
```
integrate(1/x**14/(b*x**4+a)**(5/4),x)
```

output
```
gamma(-13/4)*hyper((-13/4, 5/4), (-9/4,), b*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*x**13*gamma(-9/4))
```

Maxima [F]

$$\int \frac{1}{x^{14}(a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^14), x)
```

Giac [F]

$$\int \frac{1}{x^{14}(a+bx^4)^{5/4}}\,dx = \int \frac{1}{(bx^4+a)^{\frac{5}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)^(5/4)*x^14), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{14}(a+bx^4)^{5/4}}\,dx = \int \frac{1}{x^{14}\,(b\,x^4+a)^{5/4}}\,dx$$

input
```
int(1/(x^14*(a + b*x^4)^(5/4)),x)
```

output
```
int(1/(x^14*(a + b*x^4)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{14}\left(a+bx^4\right)^{5/4}}\,dx=\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}a\,x^{14}+\left(b\,x^4+a\right)^{\frac{1}{4}}b\,x^{18}}dx$$

input
```
int(1/x^14/(b*x^4+a)^(5/4),x)
```

output
```
int(1/((a + b*x**4)**(1/4)*a*x**14 + (a + b*x**4)**(1/4)*b*x**18),x)
```

3.624 $\int \frac{1}{x^2 \sqrt[4]{2+3x^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^4}} \, dx = -\frac{1}{x \sqrt[4]{2+3x^4}} + \frac{\sqrt[4]{3} \sqrt[4]{3+\frac{2}{x^4}} \, x E\left(\frac{1}{2}\cot^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{2+3x^4}}$$

output
```
-1/x/(3*x^4+2)^(1/4)+1/2*3^(1/4)*(3+2/x^4)^(1/4)*x*EllipticE(sin(1/2*arcco
t(1/2*6^(1/2)*x^2)),2^(1/2))*2^(1/2)/(3*x^4+2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^4}} \, dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, -\frac{3x^4}{2}\right)}{\sqrt[4]{2}x}$$

input
```
Integrate[1/(x^2*(2 + 3*x^4)^(1/4)),x]
```

output
```
-(Hypergeometric2F1[-1/4, 1/4, 3/4, (-3*x^4)/2]/(2^(1/4)*x))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {841, 813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{3x^4+2}}dx$$

$$\Big\downarrow \text{ 841}$$

$$-3\int \frac{x^2}{(3x^4+2)^{5/4}}dx-\frac{1}{x\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{3\sqrt[4]{3}}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{\sqrt[4]{3}\sqrt[4]{3x^4+2}}-\frac{1}{x\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{\sqrt[4]{3x^4+2}}-\frac{1}{x\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{3\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{3x^4+2}}-\frac{1}{x\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^2}\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{3x^4+2}}-\frac{1}{x\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{\frac{2}{x^4}+3}xE\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{3x^4+2}}-\frac{1}{x\sqrt[4]{3x^4+2}}$$

input
```
Int[1/(x^2*(2 + 3*x^4)^(1/4)),x]
```

output
```
-(1/(x*(2 + 3*x^4)^(1/4))) + (3^(1/4)*(3 + 2/x^4)^(1/4)*x*EllipticE[ArcTan
[Sqrt[2/3]/x^2]/2, 2])/(Sqrt[2]*(2 + 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.29

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{3x^4}{2}\right)}{2x}$	20
risch	$-\frac{\left(3x^4+2\right)^{\frac{3}{4}}}{2x}+\frac{2^{\frac{3}{4}} x^3 \text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3x^4}{2}\right)}{2}$	35

input
```
int(1/x^2/(3*x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(3/4)/x*hypergeom([-1/4,1/4],[3/4],-3/2*x^4)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{2+3x^4}}\,dx=\int \frac{1}{\left(3\,x^4+2\right)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4+2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^4 + 2)^(3/4)/(3*x^6 + 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.42

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^4}}\,dx = -\frac{3^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{2e^{i\pi}}{3x^4}\right)}{6x^2}$$

input
```
integrate(1/x**2/(3*x**4+2)**(1/4),x)
```

output
```
-3**(3/4)*hyper((1/4, 1/2), (3/2,), 2*exp_polar(I*pi)/(3*x**4))/(6*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^4}}\,dx = \int \frac{1}{(3\,x^4+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^4 + 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^4}}\,dx = \int \frac{1}{(3\,x^4+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^4 + 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2\sqrt[4]{2+3x^4}}\,dx = -\frac{3^{3/4}\left(\frac{2}{x^4}+3\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{2}{3x^4}\right)}{6x\left(3x^4+2\right)^{1/4}}$$

input
```
int(1/(x^2*(3*x^4 + 2)^(1/4)),x)
```

output
```
-(3^(3/4)*(2/x^4 + 3)^(1/4)*hypergeom([1/4, 1/2], 3/2, -2/(3*x^4)))/(6*x*(
3*x^4 + 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{2+3x^4}}\,dx = \int \frac{1}{\left(3x^4+2\right)^{\frac{1}{4}}x^2}dx$$

input
```
int(1/x^2/(3*x^4+2)^(1/4),x)
```

output
```
int(1/((3*x**4 + 2)**(1/4)*x**2),x)
```

3.625 $\int \frac{1}{x^2 \sqrt[4]{-2+3x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^4}}\,dx = -\frac{\sqrt[4]{3}xE\left(\frac{1}{2}\csc^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{x^4}}$$

output
```
-1/2*3^(1/4)*x*EllipticE(sin(1/2*arccsc(1/2*6^(1/2)*x^2)),2^(1/2))*2^(1/2)
/(x^4)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^4}}\,dx = -\frac{\sqrt[4]{1-\frac{3x^4}{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3x^4}{2}\right)}{x\sqrt[4]{-2+3x^4}}$$

input
```
Integrate[1/(x^2*(-2 + 3*x^4)^(1/4)),x]
```

output
```
-(((1 - (3*x^4)/2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (3*x^4)/2])/(x*
(-2 + 3*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{3x^4-2}} \, dx$$

$$\Big\downarrow \text{842}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x \int \frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{\sqrt[4]{3}\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x \int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\sqrt[4]{3-\frac{2}{x^4}}x \int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x}d\frac{1}{x}}{\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{\sqrt[4]{3-\frac{2}{x^4}}x \int \frac{1}{\sqrt[4]{3-\frac{2}{x^2}}}d\frac{1}{x^2}}{2\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt[4]{3}\sqrt[4]{3-\frac{2}{x^4}}xE\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{3x^4-2}}$$

input
```
Int[1/(x^2*(-2 + 3*x^4)^(1/4)),x]
```

output
```
-((3^(1/4)*(3 - 2/x^4)^(1/4)*x*EllipticE[ArcSin[Sqrt[2/3]/x^2]/2, 2])/(Sqr
t[2]*(-2 + 3*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)\right)^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4}, \frac{1}{4}\right], \left[\frac{3}{4}\right], \frac{3 x^{4}}{2}\right)}{2 \operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)^{\frac{1}{4}} x}$	42
risch	$\frac{\left(3 x^{4}-2\right)^{\frac{3}{4}}}{2 x}-\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)\right)^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], \frac{3 x^{4}}{2}\right)}{2 \operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)^{\frac{1}{4}}}$	57

input
```
int(1/x^2/(3*x^4-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(3/4)/signum(-1+3/2*x^4)^(1/4)*(-signum(-1+3/2*x^4))^(1/4)/x*hyperg
eom([-1/4,1/4],[3/4],3/2*x^4)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^4}}\,dx = \int \frac{1}{\left(3\,x^4-2\right)^{\frac{1}{4}} x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4-2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^4 - 2)^(3/4)/(3*x^6 - 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt[4]{-2+3x^4}}\,dx = -\frac{3^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} \frac{1}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{2e^{2i\pi}}{3x^4}\right)}{6x^2}$$

input
```
integrate(1/x**2/(3*x**4-2)**(1/4),x)
```

output
```
-3**(3/4)*hyper((1/4, 1/2), (3/2,), 2*exp_polar(2*I*pi)/(3*x**4))/(6*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{-2+3x^4}}\,dx = \int \frac{1}{(3\,x^4-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^4 - 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{-2+3x^4}}\,dx = \int \frac{1}{(3\,x^4-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^4 - 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^4}} \, dx = -\frac{3^{3/4} \left(3 - \frac{2}{x^4}\right)^{1/4} {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{3}{2}; \frac{2}{3x^4}\right)}{6x \left(3x^4-2\right)^{1/4}}$$

input
```
int(1/(x^2*(3*x^4 - 2)^(1/4)),x)
```

output
```
-(3^(3/4)*(3 - 2/x^4)^(1/4)*hypergeom([1/4, 1/2], 3/2, 2/(3*x^4)))/(6*x*(3
*x^4 - 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^4}} \, dx = \int \frac{1}{\left(3x^4-2\right)^{\frac{1}{4}} x^2} dx$$

input
```
int(1/x^2/(3*x^4-2)^(1/4),x)
```

output
```
int(1/((3*x**4 - 2)**(1/4)*x**2),x)
```

3.626 $\int \frac{1}{x^2 \sqrt[4]{a+3x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{1}{x^2 \sqrt[4]{a+3x^4}}\,dx = -\frac{1}{x\sqrt[4]{a+3x^4}} + \frac{\sqrt[4]{3}\sqrt[4]{3+\frac{a}{x^4}}\,x E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{3}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+3x^4}}$$

output
```
-1/x/(3*x^4+a)^(1/4)+3^(1/4)*(3+a/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(3^
(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/(3*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2 \sqrt[4]{a+3x^4}}\,dx = -\frac{\sqrt[4]{1+\frac{3x^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{3x^4}{a}\right)}{x\sqrt[4]{a+3x^4}}$$

input
```
Integrate[1/(x^2*(a + 3*x^4)^(1/4)),x]
```

output
```
-(((1 + (3*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (-3*x^4)/a])/(x
*(a + 3*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {841, 813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{a+3x^4}}dx$$

$$\Big\downarrow \text{ 841}$$

$$-3\int \frac{x^2}{(3x^4+a)^{5/4}}dx-\frac{1}{x\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{3\sqrt[4]{3}}{\left(\frac{a}{x^4}+3\right)^{5/4}x^3}dx}{\sqrt[4]{3}\sqrt[4]{a+3x^4}}-\frac{1}{x\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{1}{\left(\frac{a}{x^4}+3\right)^{5/4}x^3}dx}{\sqrt[4]{a+3x^4}}-\frac{1}{x\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{3x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{1}{\left(\frac{a}{x^4}+3\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+3x^4}}-\frac{1}{x\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{1}{\left(\frac{a}{x^2}+3\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+3x^4}}-\frac{1}{x\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow 212$$

$$\frac{\sqrt[4]{3}x\sqrt[4]{\frac{a}{x^4}+3}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{3}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+3x^4}}-\frac{1}{x\sqrt[4]{a+3x^4}}$$

input
```
Int[1/(x^2*(a + 3*x^4)^(1/4)),x]
```

output
```
-(1/(x*(a + 3*x^4)^(1/4))) + (3^(1/4)*(3 + a/x^4)^(1/4)*x*EllipticE[ArcTan
[Sqrt[a]/(Sqrt[3]*x^2)]/2, 2])/(Sqrt[a]*(a + 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^2\,(3x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(1/x^2/(3*x^4+a)^(1/4),x)
```

output
```
int(1/x^2/(3*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{a+3x^4}}\,dx = \int \frac{1}{(3\,x^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^4 + a)^(3/4)/(3*x^6 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^2\sqrt[4]{a+3x^4}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{3x^4e^{i\pi}}{a}\right)}{4\sqrt[4]{a}x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/x**2/(3*x**4+a)**(1/4),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/4), (3/4,), 3*x**4*exp_polar(I*pi)/a)/(4*a**(1/
4)*x*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{a+3x^4}}\,dx = \int \frac{1}{\left(3\,x^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^4 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{a+3x^4}}\,dx = \int \frac{1}{\left(3\,x^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^4 + a)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2 \sqrt[4]{a+3x^4}} \, dx = -\frac{3^{3/4} \left(\frac{a}{x^4}+3\right)^{1/4} {}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{a}{3\,x^4}\right)}{6\,x\,\left(3\,x^4+a\right)^{1/4}}$$

input
```
int(1/(x^2*(a + 3*x^4)^(1/4)),x)
```

output
```
-(3^(3/4)*(a/x^4 + 3)^(1/4)*hypergeom([1/4, 1/2], 3/2, -a/(3*x^4)))/(6*x*(
a + 3*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[4]{a+3x^4}} \, dx = \int \frac{1}{\left(3x^4+a\right)^{\frac{1}{4}} x^2} dx$$

input
```
int(1/x^2/(3*x^4+a)^(1/4),x)
```

output
```
int(1/((a + 3*x**4)**(1/4)*x**2),x)
```

3.627 $\int \frac{1}{x^2 \sqrt[4]{2-3x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^4}}\,dx = -\frac{\sqrt[4]{3}\sqrt[4]{3-\frac{2}{x^4}}\,x E\left(\frac{1}{2}\csc^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{2-3x^4}}$$

output
```
-1/2*3^(1/4)*(3-2/x^4)^(1/4)*x*EllipticE(sin(1/2*arccsc(1/2*6^(1/2)*x^2)),
2^(1/2))*2^(1/2)/(-3*x^4+2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3x^4}{2}\right)}{\sqrt[4]{2}x}$$

input
```
Integrate[1/(x^2*(2 - 3*x^4)^(1/4)),x]
```

output
```
-(Hypergeometric2F1[-1/4, 1/4, 3/4, (3*x^4)/2]/(2^(1/4)*x))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[4]{2-3x^4}}dx$$

$$\Big\downarrow \text{ 842}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{\sqrt[4]{3}\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x}d\frac{1}{x}}{\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^2}}}d\frac{1}{x^2}}{2\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{ 226}$$

$$-\frac{\sqrt[4]{3}\sqrt[4]{3-\frac{2}{x^4}}xE\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{2-3x^4}}$$

input
```
Int[1/(x^2*(2 - 3*x^4)^(1/4)),x]
```

output
```
-((3^(1/4)*(3 - 2/x^4)^(1/4)*x*EllipticE[ArcSin[Sqrt[2/3]/x^2]/2, 2])/(Sqr
t[2]*(2 - 3*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.38

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4}\right],\left[\frac{3}{4}\right],\frac{3x^4}{2}\right)}{2x}$	20
risch	$\frac{3x^4-2}{2x(-3x^4+2)^{\frac{1}{4}}}-\frac{2^{\frac{3}{4}} x^3 \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{3x^4}{2}\right)}{2}$	42

input
```
int(1/x^2/(-3*x^4+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(3/4)/x*hypergeom([-1/4,1/4],[3/4],3/2*x^4)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^4}}\,dx=\int \frac{1}{\left(-3\,x^4+2\right)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^4 + 2)^(3/4)/(3*x^6 - 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^2\sqrt[4]{2-3x^4}}\,dx=\frac{3^{\frac{3}{4}}ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{2}{3x^4}\right)}{6x^2}$$

input
```
integrate(1/x**2/(-3*x**4+2)**(1/4),x)
```

output
```
3**(3/4)*I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), 2/(3*x**4))/(6*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^4}}\,dx=\int \frac{1}{(-3\,x^4+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^4 + 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^4}}\,dx=\int \frac{1}{(-3\,x^4+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^4 + 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^4}} \, dx = -\frac{3^{3/4} \left(3-\frac{2}{x^4}\right)^{1/4} {}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{2}{3\,x^4}\right)}{6\,x\,\left(2-3\,x^4\right)^{1/4}}$$

input
```
int(1/(x^2*(2 - 3*x^4)^(1/4)),x)
```

output
```
-(3^(3/4)*(3 - 2/x^4)^(1/4)*hypergeom([1/4, 1/2], 3/2, 2/(3*x^4)))/(6*x*(2
 - 3*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^4}} \, dx = \int \frac{1}{\left(-3x^4+2\right)^{\frac{1}{4}} x^2} dx$$

input
```
int(1/x^2/(-3*x^4+2)^(1/4),x)
```

output
```
int(1/(( - 3*x**4 + 2)**(1/4)*x**2),x)
```

3.628 $\int \frac{1}{x^2 \sqrt[4]{-2-3x^4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{1}{x^2 \sqrt[4]{-2-3x^4}}\,dx = -\frac{1}{x\sqrt[4]{-2-3x^4}} + \frac{\sqrt[4]{3}xE\left(\frac{1}{2}\cot^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{-x^4}}$$

output
```
-1/x/(-3*x^4-2)^(1/4)+1/2*3^(1/4)*x*EllipticE(sin(1/2*arccot(1/2*6^(1/2)*x
^2)),2^(1/2))*2^(1/2)/(-x^4)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \sqrt[4]{-2-3x^4}}\,dx = -\frac{\sqrt[4]{1+\frac{3x^4}{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{3x^4}{2}\right)}{x\sqrt[4]{-2-3x^4}}$$

input
```
Integrate[1/(x^2*(-2 - 3*x^4)^(1/4)),x]
```

output
```
-(((1 + (3*x^4)/2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (-3*x^4)/2])/(x
*(-2 - 3*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {841, 813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{-3x^4-2}}dx$$

$$\Big\downarrow \text{ 841}$$

$$3\int \frac{x^2}{(-3x^4-2)^{5/4}}dx-\frac{1}{x\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{ 813}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{3\sqrt[4]{3}}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{\sqrt[4]{3}\sqrt[4]{-3x^4-2}}-\frac{1}{x\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{\sqrt[4]{-3x^4-2}}-\frac{1}{x\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{3\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{-3x^4-2}}-\frac{1}{x\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^2}\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{-3x^4-2}}-\frac{1}{x\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{\frac{2}{x^4}+3}xE\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}\sqrt[4]{-3x^4-2}}-\frac{1}{x\sqrt[4]{-3x^4-2}}$$

input
```
Int[1/(x^2*(-2 - 3*x^4)^(1/4)),x]
```

output
```
-(1/(x*(-2 - 3*x^4)^(1/4))) + (3^(1/4)*(3 + 2/x^4)^(1/4)*x*EllipticE[ArcTa
n[Sqrt[2/3]/x^2]/2, 2])/(Sqrt[2]*(-2 - 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 841
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> -Simp[(x*(a + b*x^
4)^(1/4))^(-1), x] - Simp[b   Int[x^2/(a + b*x^4)^(5/4), x], x] /; FreeQ[{a
, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.41

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left([-\frac{1}{4},\frac{1}{4}],[\frac{3}{4}],-\frac{3x^4}{2}\right)}{2x}$	23
risch	$-\frac{3x^4+2}{2x(-3x^4-2)^{\frac{1}{4}}}-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^3 \operatorname{hypergeom}\left([\frac{1}{4},\frac{3}{4}],[\frac{7}{4}],-\frac{3x^4}{2}\right)}{2}$	45

input
```
int(1/x^2/(-3*x^4-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1)^(3/4)*2^(3/4)/x*hypergeom([-1/4,1/4],[3/4],-3/2*x^4)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^4}}\,dx=\int \frac{1}{(-3\,x^4-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4-2)^(1/4),x, algorithm="fricas")
```

output
```
1/2*(2*x*integral(-3*(-3*x^4 - 2)^(3/4)*x^2/(3*x^4 + 2), x) + (-3*x^4 - 2)
^(3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^4}}\,dx = -\frac{3^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{2e^{i\pi}}{3x^4}\right)}{6x^2}$$

input
```
integrate(1/x**2/(-3*x**4-2)**(1/4),x)
```

output
```
-3**(3/4)*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 2*exp_polar(I*pi)/(3*x**4
))/(6*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^4}}\,dx = \int \frac{1}{(-3\,x^4-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^4 - 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^4}}\,dx = \int \frac{1}{(-3\,x^4-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^4 - 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^4}}\,dx = -\frac{3^{3/4}\left(\frac{2}{x^4}+3\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{2}{3x^4}\right)}{6x\left(-3x^4-2\right)^{1/4}}$$

input
```
int(1/(x^2*(- 3*x^4 - 2)^(1/4)),x)
```

output
```
-(3^(3/4)*(2/x^4 + 3)^(1/4)*hypergeom([1/4, 1/2], 3/2, -2/(3*x^4)))/(6*x*(
- 3*x^4 - 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^4}}\,dx = \int \frac{1}{\left(-3x^4-2\right)^{\frac{1}{4}}x^2}dx$$

input
```
int(1/x^2/(-3*x^4-2)^(1/4),x)
```

output
```
int(1/(( - 3*x**4 - 2)**(1/4)*x**2),x)
```

3.629 $\int \frac{1}{x^2 \sqrt[4]{a-3x^4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^2 \sqrt[4]{a-3x^4}} \, dx = -\frac{\sqrt[4]{3}\sqrt[4]{3-\frac{a}{x^4}}\, x E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{3}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-3x^4}}$$

output
```
-3^(1/4)*(3-a/x^4)^(1/4)*x*EllipticE(sin(1/2*arccsc(3^(1/2)*x^2/a^(1/2))),
2^(1/2))/a^(1/2)/(-3*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^2 \sqrt[4]{a-3x^4}} \, dx = -\frac{\sqrt[4]{1-\frac{3x^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3x^4}{a}\right)}{x\sqrt[4]{a-3x^4}}$$

input
```
Integrate[1/(x^2*(a - 3*x^4)^(1/4)),x]
```

output
```
-(((1 - (3*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (3*x^4)/a])/(x*
(a - 3*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{a-3x^4}}dx$$

$$\Big\downarrow \text{842}$$

$$\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{a}{x^4}}x^3}dx}{\sqrt[4]{3}\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{1}{\sqrt[4]{3-\frac{a}{x^4}}x^3}dx}{\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{1}{\sqrt[4]{3-\frac{a}{x^4}}x}d\frac{1}{x}}{\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{1}{\sqrt[4]{3-\frac{a}{x^2}}}d\frac{1}{x^2}}{2\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt[4]{3}x\sqrt[4]{3-\frac{a}{x^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{3}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-3x^4}}$$

input
```
Int[1/(x^2*(a - 3*x^4)^(1/4)),x]
```

output
```
-((3^(1/4)*(3 - a/x^4)^(1/4)*x*EllipticE[ArcSin[Sqrt[a]/(Sqrt[3]*x^2)]/2,
2])/(Sqrt[a]*(a - 3*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^2 \left(-3x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^2/(-3*x^4+a)^(1/4),x)
```

output
```
int(1/x^2/(-3*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{a-3x^4}}\,dx = \int \frac{1}{\left(-3\,x^4+a\right)^{\frac{1}{4}} x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^4 + a)^(3/4)/(3*x^6 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^2 \sqrt[4]{a-3x^4}}\,dx = \frac{3^{\frac{3}{4}} i e^{\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{a}{3x^4}\right)}{6x^2}$$

input
```
integrate(1/x**2/(-3*x**4+a)**(1/4),x)
```

output
```
3**(3/4)*I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(3*x**4))/(6*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{a-3x^4}}\,dx=\int \frac{1}{\left(-3\,x^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^4 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{a-3x^4}}\,dx=\int \frac{1}{\left(-3\,x^4+a\right)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^4 + a)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt[4]{a-3x^4}}\,dx=-\frac{3^{3/4}\,\left(3-\frac{a}{x^4}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{a}{3\,x^4}\right)}{6\,x\,\left(a-3\,x^4\right)^{1/4}}$$

input
```
int(1/(x^2*(a - 3*x^4)^(1/4)),x)
```

output
```
-(3^(3/4)*(3 - a/x^4)^(1/4)*hypergeom([1/4, 1/2], 3/2, a/(3*x^4)))/(6*x*(a
 - 3*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[4]{a-3x^4}}\,dx = \int \frac{1}{\left(-3x^4+a\right)^{\frac{1}{4}} x^2}dx$$

input
```
int(1/x^2/(-3*x^4+a)^(1/4),x)
```

output
```
int(1/((a - 3*x**4)**(1/4)*x**2),x)
```

3.630 $\int \frac{x^2}{(2+3x^4)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \frac{x^2}{(2+3x^4)^{5/4}} \, dx = -\frac{\sqrt[4]{3+\frac{2}{x^4}}\, x E\left(\frac{1}{2}\cot^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{2+3x^4}}$$

output
```
-1/6*3^(1/4)*(3+2/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(1/2*6^(1/2)*x^2)),
2^(1/2))*2^(1/2)/(3*x^4+2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{(2+3x^4)^{5/4}} \, dx = \frac{x^3 \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3x^4}{2}\right)}{6\sqrt[4]{2}}$$

input
```
Integrate[x^2/(2 + 3*x^4)^(5/4),x]
```

output
```
(x^3*Hypergeometric2F1[3/4, 5/4, 7/4, (-3*x^4)/2])/(6*2^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^4+2)^{5/4}} \, dx$$

$$\Big\downarrow \text{813}$$

$$\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{3\sqrt[4]{3}}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{3\sqrt[4]{3}\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^2}\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{3x^4+2}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}xE\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{3x^4+2}}$$

input
```
Int[x^2/(2 + 3*x^4)^(5/4),x]
```

output
```
-(((3 + 2/x^4)^(1/4)*x*EllipticE[ArcTan[Sqrt[2/3]/x^2]/2, 2])/(Sqrt[2]*3^(
3/4)*(2 + 3*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.38

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right],\left[\frac{7}{4}\right],-\frac{3 x^{4}}{2}\right)}{12}$	20
risch	$\frac{x^{3}}{2\left(3 x^{4}+2\right)^{\frac{1}{4}}}-\frac{2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3 x^{4}}{2}\right)}{6}$	35

input
```
int(x^2/(3*x^4+2)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(3/4)*x^3*hypergeom([3/4,5/4],[7/4],-3/2*x^4)
```

Fricas [F]

$$\int \frac{x^2}{(2+3x^4)^{5/4}}\,dx=\int \frac{x^{2}}{\left(3\,x^{4}+2\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4+2)^(5/4),x, algorithm="fricas")
```

output
```
integral((3*x^4 + 2)^(3/4)*x^2/(9*x^8 + 12*x^4 + 4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(2+3x^4)^{5/4}}\,dx=\frac{2^{\frac{3}{4}}x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{3x^{4}e^{i\pi}}{2}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(3*x**4+2)**(5/4),x)
```

output
```
2**(3/4)*x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), 3*x**4*exp_polar(I*pi)/
2)/(16*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4+2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4+2)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^4 + 2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4+2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4+2)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^4 + 2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{{\left(3\,x^4+2\right)}^{5/4}} \,dx$$

input
```
int(x^2/(3*x^4 + 2)^(5/4),x)
```

output
```
int(x^2/(3*x^4 + 2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{3\,(3x^4+2)^{\frac{1}{4}}\,x^4+2\,(3x^4+2)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(3*x^4+2)^(5/4),x)
```

output
```
int(x**2/(3*(3*x**4 + 2)**(1/4)*x**4 + 2*(3*x**4 + 2)**(1/4)),x)
```

3.631 $\int \frac{x^2}{(-2+3x^4)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{x^2}{(-2+3x^4)^{5/4}} dx = -\frac{1}{3x\sqrt[4]{-2+3x^4}} + \frac{xE\left(\frac{1}{2}\csc^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{x^4}}$$

output
```
-1/3/x/(3*x^4-2)^(1/4)+1/6*3^(1/4)*x*EllipticE(sin(1/2*arccsc(1/2*6^(1/2)*
x^2)),2^(1/2))*2^(1/2)/(x^4)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(-2+3x^4)^{5/4}} dx = -\frac{x^3\sqrt[4]{1-\frac{3x^4}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{3x^4}{2}\right)}{6\sqrt[4]{-2+3x^4}}$$

input
```
Integrate[x^2/(-2 + 3*x^4)^(5/4),x]
```

output
```
-1/6*(x^3*(1 - (3*x^4)/2)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (3*x^4)/2
])/(-2 + 3*x^4)^(1/4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {814, 842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^4-2)^{5/4}} \, dx$$

$$\Big\downarrow \text{814}$$

$$-\frac{1}{3}\int \frac{1}{x^2\sqrt[4]{3x^4-2}}dx-\frac{1}{3x\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{842}$$

$$-\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{3\sqrt[4]{3}\sqrt[4]{3x^4-2}}-\frac{1}{3x\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{3\sqrt[4]{3x^4-2}}-\frac{1}{3x\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x}d\frac{1}{x}}{3\sqrt[4]{3x^4-2}}-\frac{1}{3x\sqrt[4]{3x^4-2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int\frac{1}{\sqrt[4]{3-\frac{2}{x^2}}}d\frac{1}{x^2}}{6\sqrt[4]{3x^4-2}}-\frac{1}{3x\sqrt[4]{3x^4-2}}$$

$\big\downarrow$ 226

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}xE\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{3x^4-2}}-\frac{1}{3x\sqrt[4]{3x^4-2}}$$

input
```
Int[x^2/(-2 + 3*x^4)^(5/4),x]
```

output
```
-1/3*1/(x*(-2 + 3*x^4)^(1/4)) + ((3 - 2/x^4)^(1/4)*x*EllipticE[ArcSin[Sqrt
[2/3]/x^2]/2, 2])/(Sqrt[2]*3^(3/4)*(-2 + 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 814
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> -Simp[(b*x*(a + b*x^4)
^(1/4))^(-1), x] - Simp[1/b   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)\right)^{\frac{5}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{5}{4}\right],\left[\frac{7}{4}\right],\frac{3 x^{4}}{2}\right)}{12 \operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)^{\frac{5}{4}}}$	42
risch	$-\frac{x^{3}}{2 \left(3 x^{4}-2\right)^{\frac{1}{4}}}+\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)\right)^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{3 x^{4}}{2}\right)}{6 \operatorname{signum}\left(-1+\frac{3 x^{4}}{2}\right)^{\frac{1}{4}}}$	57

input
```
int(x^2/(3*x^4-2)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(3/4)/signum(-1+3/2*x^4)^(5/4)*(-signum(-1+3/2*x^4))^(5/4)*x^3*hype
rgeom([3/4,5/4],[7/4],3/2*x^4)
```

Fricas [F]

$$\int \frac{x^2}{(-2+3x^4)^{5/4}}\,dx = \int \frac{x^{2}}{\left(3\,x^{4}-2\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4-2)^(5/4),x, algorithm="fricas")
```

output
```
integral((3*x^4 - 2)^(3/4)*x^2/(9*x^8 - 12*x^4 + 4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(-2+3x^4)^{5/4}}\,dx = -\frac{2^{\frac{3}{4}} i x^3 e^{-\frac{3i\pi}{4}} \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{3x^4}{2}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(3*x**4-2)**(5/4),x)
```

output
```
-2**(3/4)*I*x**3*exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), 3*x**
4/2)/(16*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(-2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4-2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4-2)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^4 - 2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(-2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4-2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4-2)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^4 - 2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4-2)^{5/4}}\,dx$$

input
```
int(x^2/(3*x^4 - 2)^(5/4),x)
```

output
```
int(x^2/(3*x^4 - 2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(-2+3x^4)^{5/4}}\,dx = \int \frac{x^2}{3\,(3x^4-2)^{\frac{1}{4}}\,x^4-2\,(3x^4-2)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(3*x^4-2)^(5/4),x)
```

output
```
int(x**2/(3*(3*x**4 - 2)**(1/4)*x**4 - 2*(3*x**4 - 2)**(1/4)),x)
```

3.632 $\int \frac{x^2}{(a+3x^4)^{5/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{x^2}{(a+3x^4)^{5/4}} \, dx = -\frac{\sqrt[4]{3+\frac{a}{x^4}}\, x E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{3}x^2}{\sqrt{a}}\right)\middle|2\right)}{3^{3/4}\sqrt{a}\sqrt[4]{a+3x^4}}$$

output
```
-1/3*3^(1/4)*(3+a/x^4)^(1/4)*x*EllipticE(sin(1/2*arccot(3^(1/2)*x^2/a^(1/2
))),2^(1/2))/a^(1/2)/(3*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+3x^4)^{5/4}} \, dx = \frac{x^3 \sqrt[4]{1+\frac{3x^4}{a}}\ \text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3x^4}{a}\right)}{3a\sqrt[4]{a+3x^4}}$$

input
```
Integrate[x^2/(a + 3*x^4)^(5/4),x]
```

output
```
(x^3*(1 + (3*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (-3*x^4)/a])/(
3*a*(a + 3*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx$$

$$\Big\downarrow \text{813}$$

$$\frac{x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{3\sqrt[4]{3}}{\left(\frac{a}{x^4}+3\right)^{5/4}x^3}dx}{3\sqrt[4]{3}\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{1}{\left(\frac{a}{x^4}+3\right)^{5/4}x^3}dx}{\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{1}{\left(\frac{a}{x^4}+3\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{x\sqrt[4]{\frac{a}{x^4}+3}\int \frac{1}{\left(\frac{a}{x^2}+3\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{a+3x^4}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{x\sqrt[4]{\frac{a}{x^4}+3}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{3}x^2}\right)\Big|2\right)}{3^{3/4}\sqrt{a}\sqrt[4]{a+3x^4}}$$

input
```
Int[x^2/(a + 3*x^4)^(5/4),x]
```

output
```
-(((3 + a/x^4)^(1/4)*x*EllipticE[ArcTan[Sqrt[a]/(Sqrt[3]*x^2)]/2, 2])/(3^(
3/4)*Sqrt[a]*(a + 3*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^2}{(3x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^2/(3*x^4+a)^(5/4),x)
```

output
```
int(x^2/(3*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((3*x^4 + a)^(3/4)*x^2/(9*x^8 + 6*a*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(3*x**4+a)**(5/4),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), 3*x**4*exp_polar(I*pi)/a)/(4*a**
(5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3\,x^4+a)^{5/4}}\,dx$$

input
```
int(x^2/(a + 3*x^4)^(5/4),x)
```

output
```
int(x^2/(a + 3*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+3x^4)^{5/4}}\,dx = \int \frac{x^2}{(3x^4+a)^{\frac{1}{4}}\,a+3\,(3x^4+a)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(x^2/(3*x^4+a)^(5/4),x)
```

output
```
int(x**2/((a + 3*x**4)**(1/4)*a + 3*(a + 3*x**4)**(1/4)*x**4),x)
```

3.633 $\int \frac{x^2}{\left(2-3x^4\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{x^2}{\left(2-3x^4\right)^{5/4}}\,dx=\frac{1}{3x\sqrt[4]{2-3x^4}}-\frac{\sqrt[4]{3-\frac{2}{x^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{2-3x^4}}$$

output
```
1/3/x/(-3*x^4+2)^(1/4)-1/6*3^(1/4)*(3-2/x^4)^(1/4)*x*EllipticE(sin(1/2*arc
csc(1/2*6^(1/2)*x^2)),2^(1/2))*2^(1/2)/(-3*x^4+2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \frac{x^2}{\left(2-3x^4\right)^{5/4}}\,dx=\frac{x^3\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{3x^4}{2}\right)}{6\sqrt[4]{2}}$$

input
```
Integrate[x^2/(2 - 3*x^4)^(5/4),x]
```

output
```
(x^3*Hypergeometric2F1[3/4, 5/4, 7/4, (3*x^4)/2])/(6*2^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {814, 842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2-3x^4)^{5/4}}dx$$

$$\Big\downarrow \text{814}$$

$$\frac{1}{3}\int \frac{1}{x^2\sqrt[4]{2-3x^4}}dx+\frac{1}{3x\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{842}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{3\sqrt[4]{3}\sqrt[4]{2-3x^4}}+\frac{1}{3x\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x^3}dx}{3\sqrt[4]{2-3x^4}}+\frac{1}{3x\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{3x\sqrt[4]{2-3x^4}}-\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int \frac{1}{\sqrt[4]{3-\frac{2}{x^4}}x}d\frac{1}{x}}{3\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{3x\sqrt[4]{2-3x^4}}-\frac{\sqrt[4]{3-\frac{2}{x^4}}x\int\frac{1}{\sqrt[4]{3-\frac{2}{x^2}}}d\frac{1}{x^2}}{6\sqrt[4]{2-3x^4}}$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{3x\sqrt[4]{2-3x^4}}-\frac{\sqrt[4]{3-\frac{2}{x^4}}xE\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{2-3x^4}}$$

input
```
Int[x^2/(2 - 3*x^4)^(5/4),x]
```

output
```
1/(3*x*(2 - 3*x^4)^(1/4)) - ((3 - 2/x^4)^(1/4)*x*EllipticE[ArcSin[Sqrt[2/3
]/x^2]/2, 2])/(Sqrt[2]*3^(3/4)*(2 - 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 814
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> -Simp[(b*x*(a + b*x^4)
^(1/4))^(-1), x] - Simp[1/b   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.28

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right], \left[\frac{7}{4}\right], \frac{3 x^{4}}{2}\right)}{12}$	20
risch	$\frac{x^{3}}{2\left(-3 x^{4}+2\right)^{\frac{1}{4}}}-\frac{2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{7}{4}\right], \frac{3 x^{4}}{2}\right)}{6}$	35

input
```
int(x^2/(-3*x^4+2)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/12*2^(3/4)*x^3*hypergeom([3/4,5/4],[7/4],3/2*x^4)
```

Fricas [F]

$$\int \frac{x^2}{(2-3x^4)^{5/4}}\,dx=\int \frac{x^{2}}{\left(-3\,x^{4}+2\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4+2)^(5/4),x, algorithm="fricas")
```

output
```
integral((-3*x^4 + 2)^(3/4)*x^2/(9*x^8 - 12*x^4 + 4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{(2-3x^4)^{5/4}}\,dx = \frac{2^{\frac{3}{4}}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{3x^4e^{2i\pi}}{2}\right)}{16\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-3*x**4+2)**(5/4),x)
```

output
```
2**(3/4)*x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), 3*x**4*exp_polar(2*I*pi
)/2)/(16*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(2-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4+2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4+2)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^4 + 2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(2-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4+2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4+2)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^4 + 2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(2-3\,x^4)^{5/4}}\,dx$$

input
```
int(x^2/(2 - 3*x^4)^(5/4),x)
```

output
```
int(x^2/(2 - 3*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(2-3x^4)^{5/4}}\,dx = -\left(\int \frac{x^2}{3\,(-3x^4+2)^{\frac{1}{4}}\,x^4-2\,(-3x^4+2)^{\frac{1}{4}}}dx\right)$$

input
```
int(x^2/(-3*x^4+2)^(5/4),x)
```

output
```
- int(x**2/(3*( - 3*x**4 + 2)**(1/4)*x**4 - 2*( - 3*x**4 + 2)**(1/4)),x)
```

3.634 $\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx = \frac{xE\left(\frac{1}{2}\cot^{-1}\left(\sqrt{\frac{3}{2}}x^2\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{-x^4}}$$

output
```
1/6*3^(1/4)*x*EllipticE(sin(1/2*arccot(1/2*6^(1/2)*x^2)),2^(1/2))*2^(1/2)/
(-x^4)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx = -\frac{x^3\sqrt[4]{1+\frac{3x^4}{2}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{3x^4}{2}\right)}{6\sqrt[4]{-2-3x^4}}$$

input
```
Integrate[x^2/(-2 - 3*x^4)^(5/4),x]
```

output
```
-1/6*(x^3*(1 + (3*x^4)/2)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (-3*x^4)/
2])/(-2 - 3*x^4)^(1/4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {813, 27, 858, 807, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(-3x^4-2)^{5/4}}dx$$

$$\Big\downarrow \text{813}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{3\sqrt[4]{3}}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{3\sqrt[4]{3}\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x^3}dx}{\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^4}\right)^{5/4}x}d\frac{1}{x}}{\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt[4]{\frac{2}{x^4}+3}x\int \frac{1}{\left(3+\frac{2}{x^2}\right)^{5/4}}d\frac{1}{x^2}}{2\sqrt[4]{-3x^4-2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{2}{x^4}+3}xE\left(\frac{1}{2}\arctan\left(\frac{\sqrt{\frac{2}{3}}}{x^2}\right)\middle|2\right)}{\sqrt{2}3^{3/4}\sqrt[4]{-3x^4-2}}$$

input
```
Int[x^2/(-2 - 3*x^4)^(5/4),x]
```

output
```
((3 + 2/x^4)^(1/4)*x*EllipticE[ArcTan[Sqrt[2/3]/x^2]/2, 2])/(Sqrt[2]*3^(3/
4)*(-2 - 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 813
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> Simp[x*((1 + a/(b*x^4)
)^(1/4)/(b*(a + b*x^4)^(1/4)))   Int[1/(x^3*(1 + a/(b*x^4))^(5/4)), x], x]
/; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.59

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{4}\right],\left[\frac{7}{4}\right],-\frac{3 x^{4}}{2}\right)}{12}$	23
risch	$-\frac{x^{3}}{2\left(-3 x^{4}-2\right)^{\frac{1}{4}}}-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3 x^{4}}{2}\right)}{6}$	38

input
```
int(x^2/(-3*x^4-2)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-1)^(3/4)*2^(3/4)*x^3*hypergeom([3/4,5/4],[7/4],-3/2*x^4)
```

Fricas [F]

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx=\int \frac{x^{2}}{\left(-3\,x^{4}-2\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4-2)^(5/4),x, algorithm="fricas")
```

output
```
1/2*((-3*x^4 - 2)^(3/4)*x^3 + 2*(3*x^4 + 2)*integral(-(-3*x^4 - 2)^(3/4)*x
^2/(3*x^4 + 2), x))/(3*x^4 + 2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx=\frac{2^{\frac{3}{4}} x^{3} e^{\frac{3 i \pi}{4}} \Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{4}\\ \frac{7}{4}\end{matrix}\right| \frac{3 x^{4} e^{i \pi}}{2}\right)}{16 \Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-3*x**4-2)**(5/4),x)
```

output
```
2**(3/4)*x**3*exp(3*I*pi/4)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), 3*x**4*ex
p_polar(I*pi)/2)/(16*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4-2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4-2)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^4 - 2)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4-2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4-2)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^4 - 2)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4-2)^{5/4}}\,dx$$

input
```
int(x^2/(- 3*x^4 - 2)^(5/4),x)
```

output
```
int(x^2/(- 3*x^4 - 2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(-2-3x^4)^{5/4}}\,dx = -\left(\int \frac{x^2}{3\,(-3x^4-2)^{\frac{1}{4}}\,x^4+2\,(-3x^4-2)^{\frac{1}{4}}}dx\right)$$

input
```
int(x^2/(-3*x^4-2)^(5/4),x)
```

output
```
- int(x**2/(3*( - 3*x**4 - 2)**(1/4)*x**4 + 2*( - 3*x**4 - 2)**(1/4)),x)
```

3.635 $\int \frac{x^2}{(a-3x^4)^{5/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{x^2}{(a-3x^4)^{5/4}} dx = \frac{1}{3x\sqrt[4]{a-3x^4}} - \frac{\sqrt[4]{3-\frac{a}{x^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{3}x^2}{\sqrt{a}}\right)\middle|2\right)}{3^{3/4}\sqrt{a}\sqrt[4]{a-3x^4}}$$

output
```
1/3/x/(-3*x^4+a)^(1/4)-1/3*3^(1/4)*(3-a/x^4)^(1/4)*x*EllipticE(sin(1/2*arc
csc(3^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/(-3*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.57 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(a-3x^4)^{5/4}} dx = \frac{x^3\sqrt[4]{1-\frac{3x^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{3x^4}{a}\right)}{3a\sqrt[4]{a-3x^4}}$$

input
```
Integrate[x^2/(a - 3*x^4)^(5/4),x]
```

output
```
(x^3*(1 - (3*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (3*x^4)/a])/(3
*a*(a - 3*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {814, 842, 27, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-3x^4)^{5/4}}dx$$

$$\Big\downarrow \text{814}$$

$$\frac{1}{3}\int \frac{1}{x^2\sqrt[4]{a-3x^4}}dx+\frac{1}{3x\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{842}$$

$$\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{\sqrt[4]{3}}{\sqrt[4]{3-\frac{a}{x^4}}x^3}dx}{3\sqrt[4]{3}\sqrt[4]{a-3x^4}}+\frac{1}{3x\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{1}{\sqrt[4]{3-\frac{a}{x^4}}x^3}dx}{3\sqrt[4]{a-3x^4}}+\frac{1}{3x\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{3x\sqrt[4]{a-3x^4}}-\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{1}{\sqrt[4]{3-\frac{a}{x^4}}x}d\frac{1}{x}}{3\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{3x\sqrt[4]{a-3x^4}}-\frac{x\sqrt[4]{3-\frac{a}{x^4}}\int \frac{1}{\sqrt[4]{3-\frac{a}{x^2}}}d\frac{1}{x^2}}{6\sqrt[4]{a-3x^4}}$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{3x\sqrt[4]{a-3x^4}}-\frac{x\sqrt[4]{3-\frac{a}{x^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{3}x^2}\right)\middle|2\right)}{3^{3/4}\sqrt{a}\sqrt[4]{a-3x^4}}$$

input
```
Int[x^2/(a - 3*x^4)^(5/4),x]
```

output
```
1/(3*x*(a - 3*x^4)^(1/4)) - ((3 - a/x^4)^(1/4)*x*EllipticE[ArcSin[Sqrt[a]/
(Sqrt[3]*x^2)]/2, 2])/(3^(3/4)*Sqrt[a]*(a - 3*x^4)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 814
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> -Simp[(b*x*(a + b*x^4)
^(1/4))^(-1), x] - Simp[1/b   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^2}{(-3x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^2/(-3*x^4+a)^(5/4),x)
```

output
```
int(x^2/(-3*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-3*x^4 + a)^(3/4)*x^2/(9*x^8 - 6*a*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.51

$$\int \frac{x^2}{(a-3x^4)^{5/4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{3x^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-3*x**4+a)**(5/4),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), 3*x**4*exp_polar(2*I*pi)/a)/(4*a
**(5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a-3x^4)^{5/4}}\,dx = \int \frac{x^2}{(-3\,x^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-3x^4)^{5/4}}\,dx = \int \frac{x^2}{{\left(a-3\,x^4\right)}^{5/4}} \,d x$$

input
```
int(x^2/(a - 3*x^4)^(5/4),x)
```

output
```
int(x^2/(a - 3*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-3x^4)^{5/4}}\,dx = \int \frac{x^{2}}{\left(-3 x^{4}+a \right)^{\frac{1}{4}} a -3 \left(-3 x^{4}+a \right)^{\frac{1}{4}} x^{4}}d x$$

input
```
int(x^2/(-3*x^4+a)^(5/4),x)
```

output
```
int(x**2/((a - 3*x**4)**(1/4)*a - 3*(a - 3*x**4)**(1/4)*x**4),x)
```

3.636 $\int x^{19} \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int x^{19} \sqrt[4]{a-bx^4}\,dx = -\frac{a^4(a-bx^4)^{5/4}}{5b^5} + \frac{4a^3(a-bx^4)^{9/4}}{9b^5} - \frac{6a^2(a-bx^4)^{13/4}}{13b^5} + \frac{4a(a-bx^4)^{17/4}}{17b^5} - \frac{(a-bx^4)^{21/4}}{21b^5}$$

output
```
-1/5*a^4*(-b*x^4+a)^(5/4)/b^5+4/9*a^3*(-b*x^4+a)^(9/4)/b^5-6/13*a^2*(-b*x^
4+a)^(13/4)/b^5+4/17*a*(-b*x^4+a)^(17/4)/b^5-1/21*(-b*x^4+a)^(21/4)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69

$$\int x^{19} \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a-bx^4}(-2048a^5-512a^4bx^4-320a^3b^2x^8-240a^2b^3x^{12}-195ab^4x^{16}+3315b^5x^{20})}{69615b^5}$$

input
```
Integrate[x^19*(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-2048*a^5 - 512*a^4*b*x^4 - 320*a^3*b^2*x^8 - 240*a^2*
b^3*x^12 - 195*a*b^4*x^16 + 3315*b^5*x^20))/(69615*b^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{19}\sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^{16}\sqrt[4]{a-bx^4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{(a-bx^4)^{17/4}}{b^4}-\frac{4a(a-bx^4)^{13/4}}{b^4}+\frac{6a^2(a-bx^4)^{9/4}}{b^4}-\frac{4a^3(a-bx^4)^{5/4}}{b^4}+\frac{a^4\sqrt[4]{a-bx^4}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^4(a-bx^4)^{5/4}}{5b^5}+\frac{16a^3(a-bx^4)^{9/4}}{9b^5}-\frac{24a^2(a-bx^4)^{13/4}}{13b^5}-\frac{4(a-bx^4)^{21/4}}{21b^5}+\frac{16a(a-bx^4)^{17/4}}{17b^5}\right)$$

input
```
Int[x^19*(a - b*x^4)^(1/4),x]
```

output
```
((-4*a^4*(a - b*x^4)^(5/4))/(5*b^5) + (16*a^3*(a - b*x^4)^(9/4))/(9*b^5) -
 (24*a^2*(a - b*x^4)^(13/4))/(13*b^5) + (16*a*(a - b*x^4)^(17/4))/(17*b^5)
 - (4*(a - b*x^4)^(21/4))/(21*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(3315x^{16}b^{4}+3120a\,b^{3}x^{12}+2880a^{2}b^{2}x^{8}+2560a^{3}b\,x^{4}+2048a^{4}\right)}{69615b^{5}}$	59
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(3315x^{16}b^{4}+3120a\,b^{3}x^{12}+2880a^{2}b^{2}x^{8}+2560a^{3}b\,x^{4}+2048a^{4}\right)}{69615b^{5}}$	59
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(3315x^{16}b^{4}+3120a\,b^{3}x^{12}+2880a^{2}b^{2}x^{8}+2560a^{3}b\,x^{4}+2048a^{4}\right)}{69615b^{5}}$	59
trager	$-\frac{\left(-3315b^{5}x^{20}+195a\,b^{4}x^{16}+240a^{2}b^{3}x^{12}+320a^{3}b^{2}x^{8}+512a^{4}b\,x^{4}+2048a^{5}\right)\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{69615b^{5}}$	70
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}}\left(-3315b^{5}x^{20}+195a\,b^{4}x^{16}+240a^{2}b^{3}x^{12}+320a^{3}b^{2}x^{8}+512a^{4}b\,x^{4}+2048a^{5}\right)}{69615b^{5}\left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	97

input
```
int(x^19*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/69615*(-b*x^4+a)^(5/4)*(3315*b^4*x^16+3120*a*b^3*x^12+2880*a^2*b^2*x^8+
2560*a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.65

$$\int x^{19}\sqrt[4]{a - bx^4}\,dx$$
$$= \frac{(3315\,b^5x^{20} - 195\,ab^4x^{16} - 240\,a^2b^3x^{12} - 320\,a^3b^2x^8 - 512\,a^4bx^4 - 2048\,a^5)(-bx^4 + a)^{\frac{1}{4}}}{69615\,b^5}$$

input
```
integrate(x^19*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/69615*(3315*b^5*x^20 - 195*a*b^4*x^16 - 240*a^2*b^3*x^12 - 320*a^3*b^2*x
^8 - 512*a^4*b*x^4 - 2048*a^5)*(-b*x^4 + a)^(1/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int x^{19}\sqrt[4]{a - bx^4}\,dx$$
$$= \begin{cases} -\frac{2048a^5\sqrt[4]{a - bx^4}}{69615b^5} - \frac{512a^4x^4\sqrt[4]{a - bx^4}}{69615b^4} - \frac{64a^3x^8\sqrt[4]{a - bx^4}}{13923b^3} - \frac{16a^2x^{12}\sqrt[4]{a - bx^4}}{4641b^2} - \frac{ax^{16}\sqrt[4]{a - bx^4}}{357b} + \frac{x^{20}\sqrt[4]{a - bx^4}}{21} \\ \frac{\sqrt[4]{a}x^{20}}{20} \end{cases}$$

input
```
integrate(x**19*(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-2048*a**5*(a - b*x**4)**(1/4)/(69615*b**5) - 512*a**4*x**4*(a
- b*x**4)**(1/4)/(69615*b**4) - 64*a**3*x**8*(a - b*x**4)**(1/4)/(13923*b*
*3) - 16*a**2*x**12*(a - b*x**4)**(1/4)/(4641*b**2) - a*x**16*(a - b*x**4)
**(1/4)/(357*b) + x**20*(a - b*x**4)**(1/4)/21, Ne(b, 0)), (a**(1/4)*x**20
/20, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int x^{19}\sqrt[4]{a-bx^4}\,dx = -\frac{(-bx^4+a)^{\frac{21}{4}}}{21\,b^5} + \frac{4\,(-bx^4+a)^{\frac{17}{4}}a}{17\,b^5} - \frac{6\,(-bx^4+a)^{\frac{13}{4}}a^2}{13\,b^5} + \frac{4\,(-bx^4+a)^{\frac{9}{4}}a^3}{9\,b^5} - \frac{(-bx^4+a)^{\frac{5}{4}}a^4}{5\,b^5}$$

input
```
integrate(x^19*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/21*(-b*x^4 + a)^(21/4)/b^5 + 4/17*(-b*x^4 + a)^(17/4)*a/b^5 - 6/13*(-b*
x^4 + a)^(13/4)*a^2/b^5 + 4/9*(-b*x^4 + a)^(9/4)*a^3/b^5 - 1/5*(-b*x^4 + a
)^(5/4)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int x^{19}\sqrt[4]{a-bx^4}\,dx$$
$$= \frac{3315\,(bx^4-a)^5(-bx^4+a)^{\frac{1}{4}} + 16380\,(bx^4-a)^4(-bx^4+a)^{\frac{1}{4}}a + 32130\,(bx^4-a)^3(-bx^4+a)^{\frac{1}{4}}a^2 + 3094}{69615\,b^5}$$

input
```
integrate(x^19*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/69615*(3315*(b*x^4 - a)^5*(-b*x^4 + a)^(1/4) + 16380*(b*x^4 - a)^4*(-b*x
^4 + a)^(1/4)*a + 32130*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4)*a^2 + 30940*(b*x^
4 - a)^2*(-b*x^4 + a)^(1/4)*a^3 - 13923*(-b*x^4 + a)^(5/4)*a^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int x^{19}\sqrt[4]{a-bx^4}\,dx = -\left(a-b\,x^4\right)^{1/4}\left(\frac{2048\,a^5}{69615\,b^5}-\frac{x^{20}}{21}+\frac{a\,x^{16}}{357\,b}+\frac{512\,a^4\,x^4}{69615\,b^4}+\frac{64\,a^3\,x^8}{13923\,b^3}+\frac{16\,a^2\,x^{12}}{4641\,b^2}\right)$$

input
```
int(x^19*(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((2048*a^5)/(69615*b^5) - x^20/21 + (a*x^16)/(357*b) +
(512*a^4*x^4)/(69615*b^4) + (64*a^3*x^8)/(13923*b^3) + (16*a^2*x^12)/(4641
*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.65

$$\int x^{19}\sqrt[4]{a-bx^4}\,dx = \frac{(-b\,x^4+a)^{\frac{1}{4}}\,(3315b^5x^{20}-195a\,b^4x^{16}-240a^2b^3x^{12}-320a^3b^2x^8-512a^4b\,x^4-2048a^5)}{69615b^5}$$

input
```
int(x^19*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*( - 2048*a**5 - 512*a**4*b*x**4 - 320*a**3*b**2*x**8
- 240*a**2*b**3*x**12 - 195*a*b**4*x**16 + 3315*b**5*x**20))/(69615*b**5)
```

3.637 $\int x^{15} \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int x^{15} \sqrt[4]{a-bx^4}\,dx = -\frac{a^3(a-bx^4)^{5/4}}{5b^4}+\frac{a^2(a-bx^4)^{9/4}}{3b^4}-\frac{3a(a-bx^4)^{13/4}}{13b^4}+\frac{(a-bx^4)^{17/4}}{17b^4}$$

output
```
-1/5*a^3*(-b*x^4+a)^(5/4)/b^4+1/3*a^2*(-b*x^4+a)^(9/4)/b^4-3/13*a*(-b*x^4+
a)^(13/4)/b^4+1/17*(-b*x^4+a)^(17/4)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int x^{15} \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a-bx^4}(-128a^4-32a^3bx^4-20a^2b^2x^8-15ab^3x^{12}+195b^4x^{16})}{3315b^4}$$

input
```
Integrate[x^15*(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-128*a^4 - 32*a^3*b*x^4 - 20*a^2*b^2*x^8 - 15*a*b^3*x^
12 + 195*b^4*x^16))/(3315*b^4)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^{12}\sqrt[4]{a-bx^4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(-\frac{(a-bx^4)^{13/4}}{b^3}+\frac{3a(a-bx^4)^{9/4}}{b^3}-\frac{3a^2(a-bx^4)^{5/4}}{b^3}+\frac{a^3\sqrt[4]{a-bx^4}}{b^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^3(a-bx^4)^{5/4}}{5b^4}+\frac{4a^2(a-bx^4)^{9/4}}{3b^4}+\frac{4(a-bx^4)^{17/4}}{17b^4}-\frac{12a(a-bx^4)^{13/4}}{13b^4}\right)$$

input
```
Int[x^15*(a - b*x^4)^(1/4),x]
```

output
```
((-4*a^3*(a - b*x^4)^(5/4))/(5*b^4) + (4*a^2*(a - b*x^4)^(9/4))/(3*b^4) -
(12*a*(a - b*x^4)^(13/4))/(13*b^4) + (4*(a - b*x^4)^(17/4))/(17*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(195 b^{3} x^{12}+180 a\,b^{2} x^{8}+160 a^{2} b\,x^{4}+128 a^{3}\right)}{3315 b^{4}}$	48
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(195 b^{3} x^{12}+180 a\,b^{2} x^{8}+160 a^{2} b\,x^{4}+128 a^{3}\right)}{3315 b^{4}}$	48
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(195 b^{3} x^{12}+180 a\,b^{2} x^{8}+160 a^{2} b\,x^{4}+128 a^{3}\right)}{3315 b^{4}}$	48
trager	$-\frac{\left(-195 x^{16} b^{4}+15 a\,b^{3} x^{12}+20 a^{2} b^{2} x^{8}+32 a^{3} b\,x^{4}+128 a^{4}\right) \left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{3315 b^{4}}$	59
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(-195 x^{16} b^{4}+15 a\,b^{3} x^{12}+20 a^{2} b^{2} x^{8}+32 a^{3} b\,x^{4}+128 a^{4}\right)}{3315 b^{4} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	86

input
```
int(x^15*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3315*(-b*x^4+a)^(5/4)*(195*b^3*x^12+180*a*b^2*x^8+160*a^2*b*x^4+128*a^3
)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx = \frac{(195\,b^4x^{16}-15\,ab^3x^{12}-20\,a^2b^2x^8-32\,a^3bx^4-128\,a^4)(-bx^4+a)^{\frac{1}{4}}}{3315\,b^4}$$

input
```
integrate(x^15*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/3315*(195*b^4*x^16 - 15*a*b^3*x^12 - 20*a^2*b^2*x^8 - 32*a^3*b*x^4 - 128
*a^4)*(-b*x^4 + a)^(1/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx = \begin{cases} -\frac{128a^4\sqrt[4]{a-bx^4}}{3315b^4} - \frac{32a^3x^4\sqrt[4]{a-bx^4}}{3315b^3} - \frac{4a^2x^8\sqrt[4]{a-bx^4}}{663b^2} - \frac{ax^{12}\sqrt[4]{a-bx^4}}{221b} + \frac{x^{16}\sqrt[4]{a-bx^4}}{17} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^{16}}{16} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15*(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-128*a**4*(a - b*x**4)**(1/4)/(3315*b**4) - 32*a**3*x**4*(a - b
*x**4)**(1/4)/(3315*b**3) - 4*a**2*x**8*(a - b*x**4)**(1/4)/(663*b**2) - a
*x**12*(a - b*x**4)**(1/4)/(221*b) + x**16*(a - b*x**4)**(1/4)/17, Ne(b, 0
)), (a**(1/4)*x**16/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx = \frac{(-bx^4+a)^{\frac{17}{4}}}{17\,b^4} - \frac{3\,(-bx^4+a)^{\frac{13}{4}}a}{13\,b^4} + \frac{(-bx^4+a)^{\frac{9}{4}}a^2}{3\,b^4} - \frac{(-bx^4+a)^{\frac{5}{4}}a^3}{5\,b^4}$$

input
```
integrate(x^15*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/17*(-b*x^4 + a)^(17/4)/b^4 - 3/13*(-b*x^4 + a)^(13/4)*a/b^4 + 1/3*(-b*x^
4 + a)^(9/4)*a^2/b^4 - 1/5*(-b*x^4 + a)^(5/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx$$
$$= \frac{195\,(bx^4-a)^4(-bx^4+a)^{\frac{1}{4}} + 765\,(bx^4-a)^3(-bx^4+a)^{\frac{1}{4}}a + 1105\,(bx^4-a)^2(-bx^4+a)^{\frac{1}{4}}a^2 - 663\,(-bx^4}{3315\,b^4}$$

input
```
integrate(x^15*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/3315*(195*(b*x^4 - a)^4*(-b*x^4 + a)^(1/4) + 765*(b*x^4 - a)^3*(-b*x^4 +
 a)^(1/4)*a + 1105*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4)*a^2 - 663*(-b*x^4 + a)
^(5/4)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx = -\left(a-b\,x^4\right)^{1/4}\left(\frac{128\,a^4}{3315\,b^4} - \frac{x^{16}}{17} + \frac{a\,x^{12}}{221\,b} + \frac{32\,a^3\,x^4}{3315\,b^3} + \frac{4\,a^2\,x^8}{663\,b^2}\right)$$

input
```
int(x^15*(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((128*a^4)/(3315*b^4) - x^16/17 + (a*x^12)/(221*b) + (3
2*a^3*x^4)/(3315*b^3) + (4*a^2*x^8)/(663*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int x^{15}\sqrt[4]{a-bx^4}\,dx = \frac{(-b\,x^4+a)^{\frac{1}{4}}\left(195b^4x^{16} - 15a\,b^3x^{12} - 20a^2b^2x^8 - 32a^3b\,x^4 - 128a^4\right)}{3315b^4}$$

input
```
int(x^15*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*( - 128*a**4 - 32*a**3*b*x**4 - 20*a**2*b**2*x**8 - 1
5*a*b**3*x**12 + 195*b**4*x**16))/(3315*b**4)
```

3.638 $\int x^{11} \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int x^{11} \sqrt[4]{a-bx^4}\,dx = -\frac{a^2(a-bx^4)^{5/4}}{5b^3} + \frac{2a(a-bx^4)^{9/4}}{9b^3} - \frac{(a-bx^4)^{13/4}}{13b^3}$$

output
```
-1/5*a^2*(-b*x^4+a)^(5/4)/b^3+2/9*a*(-b*x^4+a)^(9/4)/b^3-1/13*(-b*x^4+a)^(
13/4)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int x^{11} \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a-bx^4}(-32a^3-8a^2bx^4-5ab^2x^8+45b^3x^{12})}{585b^3}$$

input
```
Integrate[x^11*(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-32*a^3 - 8*a^2*b*x^4 - 5*a*b^2*x^8 + 45*b^3*x^12))/(5
85*b^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^8\sqrt[4]{a-bx^4}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{(a-bx^4)^{9/4}}{b^2}-\frac{2a(a-bx^4)^{5/4}}{b^2}+\frac{a^2\sqrt[4]{a-bx^4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^2(a-bx^4)^{5/4}}{5b^3}-\frac{4(a-bx^4)^{13/4}}{13b^3}+\frac{8a(a-bx^4)^{9/4}}{9b^3}\right)$$

input
```
Int[x^11*(a - b*x^4)^(1/4),x]
```

output
```
((-4*a^2*(a - b*x^4)^(5/4))/(5*b^3) + (8*a*(a - b*x^4)^(9/4))/(9*b^3) - (4
*(a - b*x^4)^(13/4))/(13*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(45b^{2}x^{8}+40ab\,x^{4}+32a^{2}\right)}{585b^{3}}$	37
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(45b^{2}x^{8}+40ab\,x^{4}+32a^{2}\right)}{585b^{3}}$	37
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(45b^{2}x^{8}+40ab\,x^{4}+32a^{2}\right)}{585b^{3}}$	37
trager	$-\frac{\left(-45b^{3}x^{12}+5a\,b^{2}x^{8}+8a^{2}b\,x^{4}+32a^{3}\right)\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{585b^{3}}$	48
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}}\left(-45b^{3}x^{12}+5a\,b^{2}x^{8}+8a^{2}b\,x^{4}+32a^{3}\right)}{585b^{3}\left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	75

input
```
int(x^11*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/585*(-b*x^4+a)^(5/4)*(45*b^2*x^8+40*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx=\frac{(45\,b^3x^{12}-5\,ab^2x^8-8\,a^2bx^4-32\,a^3)(-bx^4+a)^{\frac{1}{4}}}{585\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/585*(45*b^3*x^12 - 5*a*b^2*x^8 - 8*a^2*b*x^4 - 32*a^3)*(-b*x^4 + a)^(1/4
)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx = \begin{cases} -\frac{32a^3\sqrt[4]{a-bx^4}}{585b^3} - \frac{8a^2x^4\sqrt[4]{a-bx^4}}{585b^2} - \frac{ax^8\sqrt[4]{a-bx^4}}{117b} + \frac{x^{12}\sqrt[4]{a-bx^4}}{13} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11*(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-32*a**3*(a - b*x**4)**(1/4)/(585*b**3) - 8*a**2*x**4*(a - b*x*
*4)**(1/4)/(585*b**2) - a*x**8*(a - b*x**4)**(1/4)/(117*b) + x**12*(a - b*
x**4)**(1/4)/13, Ne(b, 0)), (a**(1/4)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx = -\frac{(-bx^4+a)^{\frac{13}{4}}}{13\,b^3} + \frac{2\,(-bx^4+a)^{\frac{9}{4}}a}{9\,b^3} - \frac{(-bx^4+a)^{\frac{5}{4}}a^2}{5\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/13*(-b*x^4 + a)^(13/4)/b^3 + 2/9*(-b*x^4 + a)^(9/4)*a/b^3 - 1/5*(-b*x^4
 + a)^(5/4)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx = \frac{45\,(bx^4-a)^3(-bx^4+a)^{\frac{1}{4}}+130\,(bx^4-a)^2(-bx^4+a)^{\frac{1}{4}}a-117\,(-bx^4+a)^{\frac{5}{4}}a^2}{585\,b^3}$$

input
```
integrate(x^11*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/585*(45*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4) + 130*(b*x^4 - a)^2*(-b*x^4 + a
)^(1/4)*a - 117*(-b*x^4 + a)^(5/4)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx = -\left(a-b\,x^4\right)^{1/4}\left(\frac{32\,a^3}{585\,b^3}-\frac{x^{12}}{13}+\frac{a\,x^8}{117\,b}+\frac{8\,a^2\,x^4}{585\,b^2}\right)$$

input
```
int(x^11*(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((32*a^3)/(585*b^3) - x^12/13 + (a*x^8)/(117*b) + (8*a^
2*x^4)/(585*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int x^{11}\sqrt[4]{a-bx^4}\,dx = \frac{(-b\,x^4+a)^{\frac{1}{4}}\,(45b^3x^{12}-5a\,b^2x^8-8a^2b\,x^4-32a^3)}{585b^3}$$

input
```
int(x^11*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*( - 32*a**3 - 8*a**2*b*x**4 - 5*a*b**2*x**8 + 45*b**3
*x**12))/(585*b**3)
```

3.639 $\int x^7 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int x^7 \sqrt[4]{a-bx^4}\,dx = -\frac{a(a-bx^4)^{5/4}}{5b^2} + \frac{(a-bx^4)^{9/4}}{9b^2}$$

output
```
-1/5*a*(-b*x^4+a)^(5/4)/b^2+1/9*(-b*x^4+a)^(9/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x^7 \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a-bx^4}(-4a^2-abx^4+5b^2x^8)}{45b^2}$$

input
```
Integrate[x^7*(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-4*a^2 - a*b*x^4 + 5*b^2*x^8))/(45*b^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4}\int x^4 \sqrt[4]{a-bx^4}dx^4$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{4}\int \left(\frac{a\sqrt[4]{a-bx^4}}{b}-\frac{\left(a-bx^4\right)^{5/4}}{b}\right)dx^4$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(\frac{4\left(a-bx^4\right)^{9/4}}{9b^2}-\frac{4a\left(a-bx^4\right)^{5/4}}{5b^2}\right)$$

input
```
Int[x^7*(a - b*x^4)^(1/4),x]
```

output
```
((-4*a*(a - b*x^4)^(5/4))/(5*b^2) + (4*(a - b*x^4)^(9/4))/(9*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(5b\,x^{4}+4a\right)}{45b^{2}}$	26
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(5b\,x^{4}+4a\right)}{45b^{2}}$	26
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(5b\,x^{4}+4a\right)}{45b^{2}}$	26
trager	$-\frac{\left(-5b^{2}x^{8}+ab\,x^{4}+4a^{2}\right) \left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{45b^{2}}$	36
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(-5b^{2}x^{8}+ab\,x^{4}+4a^{2}\right)}{45b^{2} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	63

input
```
int(x^7*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(-b*x^4+a)^(5/4)*(5*b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int x^7 \sqrt[4]{a-bx^4}\,dx = \frac{(5\,b^2x^8 - abx^4 - 4\,a^2)(-bx^4 + a)^{\frac{1}{4}}}{45\,b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/45*(5*b^2*x^8 - a*b*x^4 - 4*a^2)*(-b*x^4 + a)^(1/4)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int x^7 \sqrt[4]{a - bx^4}\, dx = \begin{cases} -\frac{4a^2 \sqrt[4]{a-bx^4}}{45b^2} - \frac{ax^4 \sqrt[4]{a-bx^4}}{45b} + \frac{x^8 \sqrt[4]{a-bx^4}}{9} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-4*a**2*(a - b*x**4)**(1/4)/(45*b**2) - a*x**4*(a - b*x**4)**(1
/4)/(45*b) + x**8*(a - b*x**4)**(1/4)/9, Ne(b, 0)), (a**(1/4)*x**8/8, True
))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int x^7 \sqrt[4]{a - bx^4}\, dx = \frac{(-bx^4 + a)^{\frac{9}{4}}}{9\, b^2} - \frac{(-bx^4 + a)^{\frac{5}{4}} a}{5\, b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/9*(-b*x^4 + a)^(9/4)/b^2 - 1/5*(-b*x^4 + a)^(5/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int x^7 \sqrt[4]{a - bx^4}\, dx = \frac{5 \left(bx^4 - a\right)^2 \left(-bx^4 + a\right)^{\frac{1}{4}} - 9 \left(-bx^4 + a\right)^{\frac{5}{4}} a}{45\, b^2}$$

input
```
integrate(x^7*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/45*(5*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4) - 9*(-b*x^4 + a)^(5/4)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x^7 \sqrt[4]{a - bx^4}\, dx = -\left(a - b\, x^4\right)^{1/4} \left(\frac{4\, a^2}{45\, b^2} - \frac{x^8}{9} + \frac{a\, x^4}{45\, b}\right)$$

input
```
int(x^7*(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((4*a^2)/(45*b^2) - x^8/9 + (a*x^4)/(45*b))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int x^7 \sqrt[4]{a - bx^4}\, dx = \frac{\left(-b\, x^4 + a\right)^{\frac{1}{4}} \left(5b^2 x^8 - ab\, x^4 - 4a^2\right)}{45 b^2}$$

input
```
int(x^7*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*( - 4*a**2 - a*b*x**4 + 5*b**2*x**8))/(45*b**2)
```

3.640 $\int x^3 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int x^3 \sqrt[4]{a-bx^4}\,dx = -\frac{(a-bx^4)^{5/4}}{5b}$$

output
```
-1/5*(-b*x^4+a)^(5/4)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^3 \sqrt[4]{a-bx^4}\,dx = -\frac{(a-bx^4)^{5/4}}{5b}$$

input
```
Integrate[x^3*(a - b*x^4)^(1/4),x]
```

output
```
-1/5*(a - b*x^4)^(5/4)/b
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \ 793$$

$$-\frac{\left(a-bx^4\right)^{5/4}}{5b}$$

input
```
Int[x^3*(a - b*x^4)^(1/4),x]
```

output
```
-1/5*(a - b*x^4)^(5/4)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}}{5b}$	16
derivativedivides	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}}{5b}$	16
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}}{5b}$	16
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}}{5b}$	16
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}}{5b}$	16
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}}{5b}$	16
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}}\left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}}}{5b\left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	43

input
```
int(x^3*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(5/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int x^3\sqrt[4]{a-bx^4}\,dx=\frac{(bx^4-a)(-bx^4+a)^{\frac{1}{4}}}{5\,b}$$

input
```
integrate(x^3*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/5*(b*x^4 - a)*(-b*x^4 + a)^(1/4)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int x^3 \sqrt[4]{a - bx^4}\, dx = \begin{cases} -\frac{a \sqrt[4]{a-bx^4}}{5b} + \frac{x^4 \sqrt[4]{a-bx^4}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt[4]{a}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-a*(a - b*x**4)**(1/4)/(5*b) + x**4*(a - b*x**4)**(1/4)/5, Ne(b
, 0)), (a**(1/4)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \sqrt[4]{a - bx^4}\, dx = -\frac{(-bx^4 + a)^{\frac{5}{4}}}{5\, b}$$

input
```
integrate(x^3*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/5*(-b*x^4 + a)^(5/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \sqrt[4]{a - bx^4}\, dx = -\frac{(-bx^4 + a)^{\frac{5}{4}}}{5\, b}$$

input
```
integrate(x^3*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/5*(-b*x^4 + a)^(5/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \sqrt[4]{a-bx^4}\,dx = -\frac{\left(a-b\,x^4\right)^{5/4}}{5\,b}$$

input
```
int(x^3*(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(5/4)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int x^3 \sqrt[4]{a-bx^4}\,dx = \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}\left(b\,x^4-a\right)}{5b}$$

input
```
int(x^3*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*( - a + b*x**4))/(5*b)
```

3.641 $\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx=\sqrt[4]{a-bx^4}-\frac{1}{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)$$

output
```
(-b*x^4+a)^(1/4)-1/2*a^(1/4)*arctan((-b*x^4+a)^(1/4)/a^(1/4))-1/2*a^(1/4)*
arctanh((-b*x^4+a)^(1/4)/a^(1/4))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx=\sqrt[4]{a-bx^4}-\frac{1}{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)-\frac{1}{2}\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)$$

input
```
Integrate[(a - b*x^4)^(1/4)/x,x]
```

output
```
(a - b*x^4)^(1/4) - (a^(1/4)*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/2 - (a^(1/
4)*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {798, 60, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4} \int \frac{\sqrt[4]{a-bx^4}}{x^4} dx^4$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4} \left(a \int \frac{1}{x^4 (a-bx^4)^{3/4}} dx^4 + 4\sqrt[4]{a-bx^4} \right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4} \left(4\sqrt[4]{a-bx^4} - \frac{4a \int \frac{1}{\frac{a}{b}-\frac{x^{16}}{b}} d\sqrt[4]{a-bx^4}}{b} \right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4} \left(4\sqrt[4]{a-bx^4} - \frac{4a \left(\frac{b \int \frac{1}{\sqrt{a}-x^8} d\sqrt[4]{a-bx^4}}{2\sqrt{a}} + \frac{b \int \frac{1}{x^8+\sqrt{a}} d\sqrt[4]{a-bx^4}}{2\sqrt{a}} \right)}{b} \right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(4\sqrt[4]{a-bx^4}-\frac{4a\left(\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}\right)$$

$\downarrow$ 219

$$\frac{1}{4}\left(4\sqrt[4]{a-bx^4}-\frac{4a\left(\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{b}\right)$$

input
```
Int[(a - b*x^4)^(1/4)/x,x]
```

output
```
(4*(a - b*x^4)^(1/4) - (4*a*((b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3
/4)) + (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/b)/4
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

method	result	size
pseudoelliptic	$\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-\frac{\ln\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)a^{\frac{1}{4}}}{4}-\frac{a^{\frac{1}{4}}\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2}$	71

input
```
int((-b*x^4+a)^(1/4)/x,x,method=_RETURNVERBOSE)
```

output
```
(-b*x^4+a)^(1/4)-1/4*ln(((-b*x^4+a)^(1/4)+a^(1/4))/((-b*x^4+a)^(1/4)-a^(1/
4)))*a^(1/4)-1/2*a^(1/4)*arctan((-b*x^4+a)^(1/4)/a^(1/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx = -\frac{1}{4}\,a^{\frac{1}{4}}\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}\right) - \frac{1}{4}i\,a^{\frac{1}{4}}\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}+i\,a^{\frac{1}{4}}\right) + \frac{1}{4}i\,a^{\frac{1}{4}}\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}-i\,a^{\frac{1}{4}}\right) + \frac{1}{4}\,a^{\frac{1}{4}}\log\left(\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}\right) + \left(-bx^4+a\right)^{\frac{1}{4}}$$

input
```
integrate((-b*x^4+a)^(1/4)/x,x, algorithm="fricas")
```

output
```
-1/4*a^(1/4)*log((-b*x^4 + a)^(1/4) + a^(1/4)) - 1/4*I*a^(1/4)*log((-b*x^4
 + a)^(1/4) + I*a^(1/4)) + 1/4*I*a^(1/4)*log((-b*x^4 + a)^(1/4) - I*a^(1/4
)) + 1/4*a^(1/4)*log((-b*x^4 + a)^(1/4) - a^(1/4)) + (-b*x^4 + a)^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx = -\frac{\sqrt[4]{b}xe^{\frac{i\pi}{4}}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/4)/x,x)
```

output
```
-b**(1/4)*x*exp(I*pi/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), a/(b*x**4)
)/(4*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx = -\frac{1}{2}\,a^{\frac{1}{4}}\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) + \frac{1}{4}\,a^{\frac{1}{4}}\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) + \left(-bx^4+a\right)^{\frac{1}{4}}$$

input
```
integrate((-b*x^4+a)^(1/4)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(1/4)*arctan((-b*x^4 + a)^(1/4)/a^(1/4)) + 1/4*a^(1/4)*log(((-b*x^4
 + a)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4))) + (-b*x^4 + a)^(1/4
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(51) = 102.

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx = -\frac{1}{4}\,\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right) - \frac{1}{4}\,\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right) - \frac{1}{8}\,\sqrt{2}(-a)^{\frac{1}{4}}\log\left(\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4+a}+\sqrt{-a}\right) + \frac{1}{8}\,\sqrt{2}(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4+a}+\sqrt{-a}\right) + \left(-bx^4+a\right)^{\frac{1}{4}}$$

input
```
integrate((-b*x^4+a)^(1/4)/x,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(-b*x^4
 + a)^(1/4))/(-a)^(1/4)) - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(sqr
t(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4)) - 1/8*sqrt(2)*(-a)^(1/
4)*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a)
) + 1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sq
rt(-b*x^4 + a) + sqrt(-a)) + (-b*x^4 + a)^(1/4)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx = \left(a-b\,x^4\right)^{1/4} - \frac{a^{1/4}\,\mathrm{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{2} - \frac{a^{1/4}\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{2}$$

input
```
int((a - b*x^4)^(1/4)/x,x)
```

output
```
(a - b*x^4)^(1/4) - (a^(1/4)*atanh((a - b*x^4)^(1/4)/a^(1/4)))/2 - (a^(1/4
)*atan((a - b*x^4)^(1/4)/a^(1/4)))/2
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x}\,dx = \left(-b\,x^4+a\right)^{\frac{1}{4}} + \left(\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{-b\,x^5+ax}dx\right)a$$

input
```
int((-b*x^4+a)^(1/4)/x,x)
```

output
```
(a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x - b*x**5),x)*a
```

3.642 $\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = -\frac{\sqrt[4]{a-bx^4}}{4x^4} + \frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}} + \frac{b\,\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{3/4}}$$

output
```
-1/4*(-b*x^4+a)^(1/4)/x^4+1/8*b*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)+1
/8*b*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = \frac{1}{8}\left(-\frac{2\sqrt[4]{a-bx^4}}{x^4} + \frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{b\,\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{a^{3/4}}\right)$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^5,x]
```

output
```
((-2*(a - b*x^4)^(1/4))/x^4 + (b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/a^(3/4
) + (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/a^(3/4))/8
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {798, 51, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{\sqrt[4]{a-bx^4}}{x^8}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(-\frac{1}{4}b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx^4-\frac{\sqrt[4]{a-bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\int \frac{1}{\frac{a}{b}-\frac{x^{16}}{b}}d\sqrt[4]{a-bx^4}-\frac{\sqrt[4]{a-bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}-\frac{\sqrt[4]{a-bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{x^4}\right)$$

input
```
Int[(a - b*x^4)^(1/4)/x^5,x]
```

output
```
(-((a - b*x^4)^(1/4)/x^4) + (b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/
4)) + (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)))/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.04

method	result	size
pseudoelliptic	$-\frac{b\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{x^{4} b}-\frac{\ln\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}\right)}{4 a^{\frac{3}{4}}}-\frac{\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2 a^{\frac{3}{4}}}\right)}{4}$	81

input
```
int((-b*x^4+a)^(1/4)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*((-b*x^4+a)^(1/4)/x^4/b-1/4*ln(((-b*x^4+a)^(1/4)+a^(1/4))/((-b*x^4+
a)^(1/4)-a^(1/4)))/a^(3/4)-1/2*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = \frac{\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}} x^4 \log\left((-bx^4+a)^{\frac{1}{4}} b + a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\right) + i\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}} x^4 \log\left((-bx^4+a)^{\frac{1}{4}} b + i\,a\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}}\right) - i\left(\frac{b^4}{a^3}\right)^{\frac{1}{4}} x^4 \log}{16\,x^4}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^5,x, algorithm="fricas")
```

output
```
1/16*((b^4/a^3)^(1/4)*x^4*log((-b*x^4 + a)^(1/4)*b + a*(b^4/a^3)^(1/4)) +
I*(b^4/a^3)^(1/4)*x^4*log((-b*x^4 + a)^(1/4)*b + I*a*(b^4/a^3)^(1/4)) - I*
(b^4/a^3)^(1/4)*x^4*log((-b*x^4 + a)^(1/4)*b - I*a*(b^4/a^3)^(1/4)) - (b^4
/a^3)^(1/4)*x^4*log((-b*x^4 + a)^(1/4)*b - a*(b^4/a^3)^(1/4)) - 4*(-b*x^4
+ a)^(1/4))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = \frac{\sqrt[4]{b}e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**5,x)
```

output
```
b**(1/4)*exp(-3*I*pi/4)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), a/(b*x**4))/
(4*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = \frac{b\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{8\,a^{\frac{3}{4}}} - \frac{b\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{16\,a^{\frac{3}{4}}} - \frac{(-bx^4+a)^{\frac{1}{4}}}{4\,x^4}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^5,x, algorithm="maxima")
```

output
```
1/8*b*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) - 1/16*b*log(((-b*x^4 + a
)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4) - 1/4*(-b*x^4 +
 a)^(1/4)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(58) = 116.

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.82

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx$$

$$= \frac{\frac{2\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a} + \frac{2\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a} + \frac{\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}(-\cdots\right.}{\cdots}}{32b}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^5,x, algorithm="giac")
```

output
```
1/32*(2*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*
(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 2*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/2*s
qrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a + sqrt(2)
*(-a)^(1/4)*b^2*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 +
a) + sqrt(-a))/a + sqrt(2)*b^2*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4)
+ sqrt(-b*x^4 + a) + sqrt(-a))/(-a)^(3/4) - 8*(-b*x^4 + a)^(1/4)*b/x^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = \frac{b\,\mathrm{atan}\left(\frac{{\left(a-b\,x^4\right)}^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}} - \frac{{\left(a-b\,x^4\right)}^{1/4}}{4\,x^4} + \frac{b\,\mathrm{atanh}\left(\frac{{\left(a-b\,x^4\right)}^{1/4}}{a^{1/4}}\right)}{8\,a^{3/4}}$$

input
```
int((a - b*x^4)^(1/4)/x^5,x)
```

output
```
(b*atan((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4)) - (a - b*x^4)^(1/4)/(4*x^4
) + (b*atanh((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(3/4))
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^5}\,dx = \frac{-(-b\,x^4+a)^{\frac{1}{4}}-\left(\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{-b\,x^5+ax}dx\right)b\,x^4}{4x^4}$$

input
```
int((-b*x^4+a)^(1/4)/x^5,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x - b*x**5),x)*b*x**
4))/(4*x**4)
```

3.643 $\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = -\frac{\sqrt[4]{a-bx^4}}{8x^8} + \frac{b\sqrt[4]{a-bx^4}}{32ax^4} + \frac{3b^2\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}} + \frac{3b^2\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}$$

output
```
-1/8*(-b*x^4+a)^(1/4)/x^8+1/32*b*(-b*x^4+a)^(1/4)/a/x^4+3/64*b^2*arctan((-
b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)+3/64*b^2*arctanh((-b*x^4+a)^(1/4)/a^(1/4))
/a^(7/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = \frac{\sqrt[4]{a-bx^4}(-4a+bx^4)}{32ax^8} + \frac{3b^2\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}} + \frac{3b^2\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{7/4}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^9,x]
```

output
```
((a - b*x^4)^(1/4)*(-4*a + b*x^4))/(32*a*x^8) + (3*b^2*ArcTan[(a - b*x^4)^
(1/4)/a^(1/4)])/(64*a^(7/4)) + (3*b^2*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/
(64*a^(7/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {798, 51, 52, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}dx^4$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{4}\left(-\frac{1}{8}b\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}dx^4-\frac{\sqrt[4]{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(-\frac{1}{8}b\left(\frac{3b\int \frac{1}{x^4(a-bx^4)^{3/4}}dx^4}{4a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{1}{8}b\left(-\frac{3\int \frac{1}{\frac{a}{b}-\frac{x^{16}}{b}}d\sqrt[4]{a-bx^4}}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{4}\left(-\frac{1}{8}b\left(-\frac{3\left(\frac{b\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\int\frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{4}\left(-\frac{1}{8}b\left(-\frac{3\left(\frac{b\int\frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a-bx^4}}{2x^8}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}\left(-\frac{1}{8}b\left(-\frac{3\left(\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)-\frac{\sqrt[4]{a-bx^4}}{2x^8}\right)$$

input
```
Int[(a - b*x^4)^(1/4)/x^9,x]
```

output
```
(-1/2*(a - b*x^4)^(1/4)/x^8 - (b*(-((a - b*x^4)^(1/4)/(a*x^4)) - (3*((b*Ar
cTan[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)) + (b*ArcTanh[(a - b*x^4)^(1/4
)/a^(1/4)])/(2*a^(3/4))))/a))/8)/4
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

method	result	size
pseudoelliptic	$\frac{3 \ln \left(\frac{-\left(-b \,x^{4}+a \right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b \,x^{4}+a \right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) b^{2} x^{8}+6 \arctan \left(\frac{\left(-b \,x^{4}+a \right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b^{2} x^{8}+4 b \,x^{4} a^{\frac{3}{4}} \left(-b \,x^{4}+a \right)^{\frac{1}{4}}-16 a^{\frac{7}{4}} \left(-b \,x^{4}+a \right)^{\frac{1}{4}}}{128 a^{\frac{7}{4}} x^{8}}$	113

input
```
int((-b*x^4+a)^(1/4)/x^9,x,method=_RETURNVERBOSE)
```

output
```
1/128*(3*ln((-(-b*x^4+a)^(1/4)-a^(1/4))/(-(-b*x^4+a)^(1/4)+a^(1/4)))*b^2*x
^8+6*arctan((-b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8+4*b*x^4*a^(3/4)*(-b*x^4+a)^(
1/4)-16*a^(7/4)*(-b*x^4+a)^(1/4))/a^(7/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = \frac{3\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\log\left(3\left(-bx^4+a\right)^{\frac{1}{4}}b^2+3\,a^2\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\right)+3i\,a\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}x^8\log\left(3\left(-bx^4+a\right)^{\frac{1}{4}}b^2+3i\,a^2\left(\frac{b^8}{a^7}\right)^{\frac{1}{4}}\right)-$$

input
```
integrate((-b*x^4+a)^(1/4)/x^9,x, algorithm="fricas")
```

output
```
1/128*(3*a*(b^8/a^7)^(1/4)*x^8*log(3*(-b*x^4 + a)^(1/4)*b^2 + 3*a^2*(b^8/a
^7)^(1/4)) + 3*I*a*(b^8/a^7)^(1/4)*x^8*log(3*(-b*x^4 + a)^(1/4)*b^2 + 3*I*
a^2*(b^8/a^7)^(1/4)) - 3*I*a*(b^8/a^7)^(1/4)*x^8*log(3*(-b*x^4 + a)^(1/4)*
b^2 - 3*I*a^2*(b^8/a^7)^(1/4)) - 3*a*(b^8/a^7)^(1/4)*x^8*log(3*(-b*x^4 + a
)^(1/4)*b^2 - 3*a^2*(b^8/a^7)^(1/4)) + 4*(b*x^4 - 4*a)*(-b*x^4 + a)^(1/4))
/(a*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = -\frac{\sqrt[4]{b}e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4x^7\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**9,x)
```

output
```
-b**(1/4)*exp(I*pi/4)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), a/(b*x**4))/(
4*x**7*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = -\frac{(-bx^4+a)^{\frac{5}{4}}b^2+3(-bx^4+a)^{\frac{1}{4}}ab^2}{32\left((bx^4-a)^2a+2(bx^4-a)a^2+a^3\right)} + \frac{3\left(\frac{2b^2\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}-\frac{b^2\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^9,x, algorithm="maxima")
```

output
```
-1/32*((-b*x^4 + a)^(5/4)*b^2 + 3*(-b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 - a)^2
*a + 2*(b*x^4 - a)*a^2 + a^3) + 3/128*(2*b^2*arctan((-b*x^4 + a)^(1/4)/a^(
1/4))/a^(3/4) - b^2*log(((-b*x^4 + a)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4)
 + a^(1/4)))/a^(3/4))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx$$
$$= \frac{\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^3\log\left(\sqrt{2}\right.}{}}{25}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^9,x, algorithm="giac")
```

output
```
1/256*(6*sqrt(2)*(-a)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^3*arctan(-1/
2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*
sqrt(2)*(-a)^(1/4)*b^3*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b
*x^4 + a) + sqrt(-a))/a^2 + 3*sqrt(2)*b^3*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*
(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) - 8*((-b*x^4 + a)
^(5/4)*b^3 + 3*(-b*x^4 + a)^(1/4)*a*b^3)/(a*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = \frac{3\,b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{64\,a^{7/4}} - \frac{3\,\left(a-b\,x^4\right)^{1/4}}{32\,x^8} - \frac{\left(a-b\,x^4\right)^{5/4}}{32\,a\,x^8} - \frac{b^2\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,3\mathrm{i}}{64\,a^{7/4}}$$

input
```
int((a - b*x^4)^(1/4)/x^9,x)
```

output
```
(3*b^2*atan((a - b*x^4)^(1/4)/a^(1/4)))/(64*a^(7/4)) - (3*(a - b*x^4)^(1/4
))/(32*x^8) - (b^2*atan(((a - b*x^4)^(1/4)*1i)/a^(1/4))*3i)/(64*a^(7/4)) -
 (a - b*x^4)^(5/4)/(32*a*x^8)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^9}\,dx = \frac{-4(-b\,x^4+a)^{\frac{1}{4}}\,a+(-b\,x^4+a)^{\frac{1}{4}}\,b\,x^4-3\left(\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{-b\,x^5+ax}dx\right)b^2x^8}{32a\,x^8}$$

input
```
int((-b*x^4+a)^(1/4)/x^9,x)
```

output
```
( - 4*(a - b*x**4)**(1/4)*a + (a - b*x**4)**(1/4)*b*x**4 - 3*int((a - b*x*
*4)**(1/4)/(a*x - b*x**5),x)*b**2*x**8)/(32*a*x**8)
```

3.644 $\int x^9 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int x^9 \sqrt[4]{a-bx^4}\,dx = -\frac{2a^2x^2\sqrt[4]{a-bx^4}}{77b^2} - \frac{ax^6\sqrt[4]{a-bx^4}}{77b} + \frac{1}{11}x^{10}\sqrt[4]{a-bx^4} + \frac{4a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-2/77*a^2*x^2*(-b*x^4+a)^(1/4)/b^2-1/77*a*x^6*(-b*x^4+a)^(1/4)/b+1/11*x^10
*(-b*x^4+a)^(1/4)+4/77*a^(7/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsi
n(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int x^9 \sqrt[4]{a-bx^4}\,dx = \frac{x^2\sqrt[4]{a-bx^4}\left(-\sqrt[4]{1-\frac{bx^4}{a}}(6a^2+abx^4-7b^2x^8)+6a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)\right)}{77b^2\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^9*(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(a - b*x^4)^(1/4)*(-((1 - (b*x^4)/a)^(1/4)*(6*a^2 + a*b*x^4 - 7*b^2*x
^8)) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^4)/a]))/(77*b^2*(1 - (
b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {807, 248, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int x^8 \sqrt[4]{a-bx^4}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}\left(\frac{1}{11}a\int \frac{x^8}{\left(a-bx^4\right)^{3/4}}dx^2+\frac{2}{11}x^{10}\sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{6a\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}dx^2}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{6a\left(\frac{2a\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx^2}{3b}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \; 231$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^4}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{3b(a-bx^4)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \; 230$$

$$\frac{1}{2}\left(\frac{1}{11}a\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^4)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)+\frac{2}{11}x^{10}\sqrt[4]{a-bx^4}\right)$$

input
```
Int[x^9*(a - b*x^4)^(1/4),x]
```

output
```
((2*x^10*(a - b*x^4)^(1/4))/11 + (a*((-2*x^6*(a - b*x^4)^(1/4))/(7*b) + (6
*a*((-2*x^2*(a - b*x^4)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^4)/a)^(3/4)*El
lipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^4)^(3/4))
))/(7*b)))/11)/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^{9} \left(-b\,x^{4}+a \right)^{\frac{1}{4}}d x$$

input
```
int(x^9*(-b*x^4+a)^(1/4),x)
```

output
```
int(x^9*(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^9 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}}x^9\,dx$$

input
```
integrate(x^9*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)*x^9, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.24

$$\int x^9 \sqrt[4]{a - bx^4}\, dx = \frac{\sqrt[4]{a}x^{10}{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{10}$$

input
```
integrate(x**9*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**10*hyper((-1/4, 5/2), (7/2,), b*x**4*exp_polar(2*I*pi)/a)/10
```

Maxima [F]

$$\int x^9 \sqrt[4]{a - bx^4}\, dx = \int \left(-bx^4 + a\right)^{\frac{1}{4}} x^9\, dx$$

input
```
integrate(x^9*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^9, x)
```

Giac [F]

$$\int x^9 \sqrt[4]{a - bx^4}\, dx = \int \left(-bx^4 + a\right)^{\frac{1}{4}} x^9\, dx$$

input
```
integrate(x^9*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^9, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^9 \sqrt[4]{a-bx^4}\,dx = \int x^9\,\left(a-b\,x^4\right)^{1/4}\,dx$$

input
```
int(x^9*(a - b*x^4)^(1/4),x)
```

output
```
int(x^9*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^9 \sqrt[4]{a-bx^4}\,dx = \frac{-2(-b\,x^4+a)^{\frac{1}{4}}\,a^2x^2-(-b\,x^4+a)^{\frac{1}{4}}\,ab\,x^6+7(-b\,x^4+a)^{\frac{1}{4}}\,b^2x^{10}+4\left(\int \frac{x}{(-b\,x^4+a)^{\frac{3}{4}}}dx\right)a^3}{77b^2}$$

input
```
int(x^9*(-b*x^4+a)^(1/4),x)
```

output
```
( - 2*(a - b*x**4)**(1/4)*a**2*x**2 - (a - b*x**4)**(1/4)*a*b*x**6 + 7*(a
- b*x**4)**(1/4)*b**2*x**10 + 4*int(((a - b*x**4)**(1/4)*x)/(a - b*x**4),x
)*a**3)/(77*b**2)
```

3.645 $\int x^5 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = -\frac{ax^2\sqrt[4]{a-bx^4}}{21b}+\frac{1}{7}x^6\sqrt[4]{a-bx^4}+\frac{2a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/21*a*x^2*(-b*x^4+a)^(1/4)/b+1/7*x^6*(-b*x^4+a)^(1/4)+2/21*a^(5/2)*(1-b*
x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3
/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = \frac{x^2\sqrt[4]{a-bx^4}\left(-a+bx^4+\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}}\right)}{7b}$$

input
```
Integrate[x^5*(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(a - b*x^4)^(1/4)*(-a + b*x^4 + (a*Hypergeometric2F1[-1/4, 1/2, 3/2,
(b*x^4)/a])/(1 - (b*x^4)/a)^(1/4)))/(7*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {807, 248, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt[4]{a-bx^4} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int x^4 \sqrt[4]{a-bx^4}dx^2$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} \left(\frac{1}{7} a \int \frac{x^4}{\left(a-bx^4\right)^{3/4}}dx^2+\frac{2}{7} x^6 \sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{1}{7} a \left(\frac{2a \int \frac{1}{\left(a-bx^4\right)^{3/4}}dx^2}{3b}-\frac{2x^2 \sqrt[4]{a-bx^4}}{3b}\right)+\frac{2}{7} x^6 \sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2} \left(\frac{1}{7} a \left(\frac{2a \left(1-\frac{bx^4}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{3b \left(a-bx^4\right)^{3/4}}-\frac{2x^2 \sqrt[4]{a-bx^4}}{3b}\right)+\frac{2}{7} x^6 \sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}\left(\frac{1}{7}a\left(\frac{4a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)+\frac{2}{7}x^6\sqrt[4]{a-bx^4}\right)$$

input
```
Int[x^5*(a - b*x^4)^(1/4),x]
```

output
```
((2*x^6*(a - b*x^4)^(1/4))/7 + (a*((-2*x^2*(a - b*x^4)^(1/4))/(3*b) + (4*a
^(3/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2]
)/(3*b^(3/2)*(a - b*x^4)^(3/4))))/7)/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x^{5}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}dx$$

input
```
int(x^5*(-b*x^4+a)^(1/4),x)
```

output
```
int(x^5*(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}} x^5\,dx$$

input
```
integrate(x^5*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)*x^5, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a}x^6\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{6}$$

input
```
integrate(x**5*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**6*hyper((-1/4, 3/2), (5/2,), b*x**4*exp_polar(2*I*pi)/a)/6
```

Maxima [F]

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}}x^5\,dx$$

input
```
integrate(x^5*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^5, x)
```

Giac [F]

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}}x^5\,dx$$

input
```
integrate(x^5*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \sqrt[4]{a-bx^4}\,dx = \int x^5\,\left(a-b\,x^4\right)^{1/4}\,dx$$

input
```
int(x^5*(a - b*x^4)^(1/4),x)
```

output
```
int(x^5*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^5 \sqrt[4]{a - bx^4}\, dx = \frac{-(-b\,x^4+a)^{\frac{1}{4}}\,a\,x^2 + 3(-b\,x^4+a)^{\frac{1}{4}}\,b\,x^6 + 2\left(\int \frac{x}{(-b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{21b}$$

input
```
int(x^5*(-b*x^4+a)^(1/4),x)
```

output
```
( - (a - b*x**4)**(1/4)*a*x**2 + 3*(a - b*x**4)**(1/4)*b*x**6 + 2*int(((a
- b*x**4)**(1/4)*x)/(a - b*x**4),x)*a**2)/(21*b)
```

3.646 $\int x\sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int x\sqrt[4]{a-bx^4}\,dx=\frac{1}{3}x^2\sqrt[4]{a-bx^4}+\frac{a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\,(a-bx^4)^{3/4}}$$

output
```
1/3*x^2*(-b*x^4+a)^(1/4)+1/3*a^(3/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(1/2
*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int x\sqrt[4]{a-bx^4}\,dx=\frac{x^2\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)}{2\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x*(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(a - b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^4)/a])/(2*(1
 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {807, 211, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sqrt[4]{a-bx^4} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \sqrt[4]{a-bx^4} dx^2$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2} \left(\frac{1}{3} a \int \frac{1}{\left(a-bx^4\right)^{3/4}} dx^2 + \frac{2}{3} x^2 \sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2} \left(\frac{a \left(1-\frac{bx^4}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}} dx^2}{3 \left(a-bx^4\right)^{3/4}} + \frac{2}{3} x^2 \sqrt[4]{a-bx^4}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2} \left(\frac{2a^{3/2} \left(1-\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2} \arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right), 2\right)}{3\sqrt{b} \left(a-bx^4\right)^{3/4}} + \frac{2}{3} x^2 \sqrt[4]{a-bx^4}\right)$$

input
```
Int[x*(a - b*x^4)^(1/4),x]
```

output
```
((2*x^2*(a - b*x^4)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[
ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^4)^(3/4)))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int x\left(-b\,x^{4}+a\right)^{\frac{1}{4}}d x$$

input
```
int(x*(-b*x^4+a)^(1/4),x)
```

output
```
int(x*(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x\sqrt[4]{a-bx^4}\,dx=\int \left(-bx^4+a\right)^{\frac{1}{4}}x\,dx$$

input
```
integrate(x*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.38

$$\int x\sqrt[4]{a-bx^4}\,dx=\frac{\sqrt[4]{a}x^2{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{2}$$

input
```
integrate(x*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**2*hyper((-1/4, 1/2), (3/2,), b*x**4*exp_polar(2*I*pi)/a)/2
```

Maxima [F]

$$\int x\sqrt[4]{a-bx^4}\,dx=\int \left(-bx^4+a\right)^{\frac{1}{4}}x\,dx$$

input
```
integrate(x*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x, x)
```

Giac [F]

$$\int x\sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}}x\,dx$$

input
```
integrate(x*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt[4]{a-bx^4}\,dx = \int x\,\left(a-b\,x^4\right)^{1/4}\,dx$$

input
```
int(x*(a - b*x^4)^(1/4),x)
```

output
```
int(x*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x\sqrt[4]{a-bx^4}\,dx = \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}x^2}{3}+\frac{\left(\int \frac{x}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}dx\right)a}{3}$$

input
```
int(x*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*x**2 + int(((a - b*x**4)**(1/4)*x)/(a - b*x**4),x)*a)
/3
```

3.647 $\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx=-\frac{\sqrt[4]{a-bx^4}}{2x^2}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\left(a-bx^4\right)^{3/4}}$$

output

```
-1/2*(-b*x^4+a)^(1/4)/x^2-1/2*a^(1/2)*b^(1/2)*(1-b*x^4/a)^(3/4)*InverseJac
obiAM(1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx=-\frac{\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{1-\frac{bx^4}{a}}}$$

input

```
Integrate[(a - b*x^4)^(1/4)/x^3,x]
```

output
```
-1/2*((a - b*x^4)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (b*x^4)/a])/(x^
2*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {807, 247, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt[4]{a-bx^4}}{x^4}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(-\frac{1}{2}b\int \frac{1}{(a-bx^4)^{3/4}}dx^2-\frac{\sqrt[4]{a-bx^4}}{x^2}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(-\frac{b\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{2(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{x^2}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}\left(-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{x^2}\right)$$

input
```
Int[(a - b*x^4)^(1/4)/x^3,x]
```

output
```
(-((a - b*x^4)^(1/4)/x^2) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^4)/a)^(3/4)*Ellipti
cF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(a - b*x^4)^(3/4))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{x^3}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^3,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^3,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^3,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{2x^2}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**3,x)
```

output
```
-a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*x**2)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^3,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^3}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^3,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = \int \frac{(a-b\,x^4)^{1/4}}{x^3}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^3,x)
```

output
```
int((a - b*x^4)^(1/4)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^3}\,dx = \frac{-(-b\,x^4+a)^{\frac{1}{4}} - \left(\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{-b\,x^7+a\,x^3}dx\right)a\,x^2}{x^2}$$

input
```
int((-b*x^4+a)^(1/4)/x^3,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**3 - b*x**7),x)*a*
x**2))/x**2
```

3.648 $\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx = -\frac{\sqrt[4]{a-bx^4}}{6x^6}+\frac{b\sqrt[4]{a-bx^4}}{12ax^2} - \frac{b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\sqrt{a}\,(a-bx^4)^{3/4}}$$

output
```
-1/6*(-b*x^4+a)^(1/4)/x^6+1/12*b*(-b*x^4+a)^(1/4)/a/x^2-1/12*b^(3/2)*(1-b*
x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1
/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx = -\frac{\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},-\frac{1}{2},\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^7,x]
```

output
```
-1/6*((a - b*x^4)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (b*x^4)/a])/(x
^6*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 247, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt[4]{a-bx^4}}{x^8}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(-\frac{1}{6}b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx^2-\frac{\sqrt[4]{a-bx^4}}{3x^6}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{1}{6}b\left(\frac{b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a-bx^4}}{3x^6}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(-\frac{1}{6}b\left(\frac{b\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{2a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a-bx^4}}{3x^6}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}\left(-\frac{1}{6}b\left(\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)-\frac{\sqrt[4]{a-bx^4}}{3x^6}\right)$$

input
```
Int[(a - b*x^4)^(1/4)/x^7,x]
```

output
```
(-1/3*(a - b*x^4)^(1/4)/x^6 - (b*(-((a - b*x^4)^(1/4)/(a*x^2)) + (Sqrt[b]*
(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt
[a]*(a - b*x^4)^(3/4))))/6)/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{x^7}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^7,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^7,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^7}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^7,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^7, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{6x^6}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**7,x)
```

output
```
-a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*x**6
)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a - bx^4}}{x^7}\, dx = \int \frac{(-bx^4 + a)^{\frac{1}{4}}}{x^7}\, dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^7,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^7, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a - bx^4}}{x^7}\, dx = \int \frac{(-bx^4 + a)^{\frac{1}{4}}}{x^7}\, dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^7,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx=\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^7}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^7,x)
```

output
```
int((a - b*x^4)^(1/4)/x^7, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^7}\,dx=\frac{-\left(-b\,x^4+a\right)^{\frac{1}{4}}-\left(\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{-b\,x^{11}+a\,x^7}dx\right)a\,x^6}{5x^6}$$

input
```
int((-b*x^4+a)^(1/4)/x^7,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**7 - b*x**11),x)*a
*x**6))/(5*x**6)
```

3.649 $\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 130

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{10x^{10}}+\frac{b\sqrt[4]{a-bx^4}}{60ax^6}+\frac{b^2\sqrt[4]{a-bx^4}}{24a^2x^2} - \frac{b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24a^{3/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/10*(-b*x^4+a)^(1/4)/x^10+1/60*b*(-b*x^4+a)^(1/4)/a/x^6+1/24*b^2*(-b*x^4
+a)^(1/4)/a^2/x^2-1/24*b^(5/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsi
n(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx = -\frac{\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{4},-\frac{3}{2},\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^11,x]
```

output
```
-1/10*((a - b*x^4)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (b*x^4)/a])/(
x^10*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {807, 247, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}dx^2$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}\left(-\frac{1}{10}b\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}dx^2-\frac{\sqrt[4]{a-bx^4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{1}{10}b\left(\frac{5b\int \frac{1}{x^4(a-bx^4)^{3/4}}dx^2}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a-bx^4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{1}{10}b\left(\frac{5b\left(\frac{b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a-bx^4}}{5x^{10}}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(-\frac{1}{10}b\left(\frac{5b\left(\frac{b\left(1-\frac{bx^4}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{2a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a-bx^4}}{5x^{10}}\right)$$

$\Big\downarrow$ 230

$$\frac{1}{2}\left(-\frac{1}{10}b\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)-\frac{\sqrt[4]{a-bx^4}}{5x^{10}}\right)$$

input
```
Int[(a - b*x^4)^(1/4)/x^11,x]
```

output
```
(-1/5*(a - b*x^4)^(1/4)/x^10 - (b*(-1/3*(a - b*x^4)^(1/4)/(a*x^6) + (5*b*(
-((a - b*x^4)^(1/4)/(a*x^2)) + (Sqrt[b]*(1 - (b*x^4)/a)^(3/4)*EllipticF[Ar
cSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^4)^(3/4))))/(6*a)))/1
0)/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^{4}+a)^{\frac{1}{4}}}{x^{11}}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^11,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^11,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{11}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^11,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^11, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{10x^{10}}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**11,x)
```

output
```
-a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*x**
10)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{11}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^11,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^11, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{11}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^11,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^11, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx=\int \frac{{\left(a-b\,x^4\right)}^{1/4}}{x^{11}}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^11,x)
```

output
```
int((a - b*x^4)^(1/4)/x^11, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{11}}\,dx=\frac{-\left(-b\,x^4+a\right)^{\frac{1}{4}}-\left(\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{-b\,x^{15}+a\,x^{11}}dx\right)a\,x^{10}}{9x^{10}}$$

input
```
int((-b*x^4+a)^(1/4)/x^11,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**11 - b*x**15),x)*
a*x**10))/(9*x**10)
```

3.650 $\int x^6 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 199

$$\int x^6 \sqrt[4]{a-bx^4}\,dx = -\frac{ax^3\sqrt[4]{a-bx^4}}{32b}+\frac{1}{8}x^7\sqrt[4]{a-bx^4} - \frac{3a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}b^{7/4}}+\frac{3a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}b^{7/4}} - \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{64\sqrt{2}b^{7/4}}$$

output
```
-1/32*a*x^3*(-b*x^4+a)^(1/4)/b+1/8*x^7*(-b*x^4+a)^(1/4)+3/128*a^2*arctan(-
1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(7/4)+3/128*a^2*arctan(1+2
^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(7/4)-3/128*a^2*arctanh(2^(1/
2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/b^
(7/4)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.81

$$\int x^6 \sqrt[4]{a-bx^4}\,dx = \frac{4b^{3/4}x^3\sqrt[4]{a-bx^4}(-a+4bx^4)+3\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{-\sqrt{b}x^2+\sqrt{a-bx^4}}\right)-3\sqrt{2}a^2\text{arctanh}\left(\frac{\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)}{128b^{7/4}}$$

input
```
Integrate[x^6*(a - b*x^4)^(1/4),x]
```

output
```
(4*b^(3/4)*x^3*(a - b*x^4)^(1/4)*(-a + 4*b*x^4) + 3*Sqrt[2]*a^2*ArcTan[(Sq
rt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))/(-(Sqrt[b]*x^2) + Sqrt[a - b*x^4])] - 3
*Sqrt[2]*a^2*ArcTanh[(Sqrt[b]*x^2 + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a
 - b*x^4)^(1/4))])/(128*b^(7/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {811, 843, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{8}a\int \frac{x^6}{(a-bx^4)^{3/4}}dx+\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{8}a\left(\frac{3a\int \frac{x^2}{(a-bx^4)^{3/4}}dx}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{8}a\left(\frac{3a\int \frac{x^2}{\sqrt{a-bx^4}\left(\frac{bx^4}{a-bx^4}+1\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 826

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)+\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 1476

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-}}}{2\sqrt{b}}\right.}{4b}\right.$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 1082

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right.$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)+$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\cdots}{\sqrt[4]{b}\left(\cdots\right)}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{4b}\cdots\right)$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$\downarrow$ 25

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{\cdots}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-\cdots}}\cdots\right)}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{4b}\cdots\right)$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 27

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a-b\cdots}}}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{\cdots}}{\sqrt[4]{b}\sqrt[4]{\cdots}}}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{4b}\cdots\right)$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{8}a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{4b}\right.$$

$$\frac{1}{8}x^7\sqrt[4]{a-bx^4}$$

input
```
Int[x^6*(a - b*x^4)^(1/4),x]
```

output
```
(x^7*(a - b*x^4)^(1/4))/8 + (a*(-1/4*(x^3*(a - b*x^4)^(1/4))/b + (3*a*((-(
ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))) + Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/(2*Sqrt[
b]) - (-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a
 - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4]
 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)))/(2*Sqrt[b])
))/(4*b)))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{-3\ln\left(\frac{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)\sqrt{2}\,a^{2}b+6\arctan\left(-\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)\sqrt{2}\,a^{2}b-6\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right.}{b^{\frac{1}{4}}x}\right.}{256b^{\frac{11}{4}}}$

input
```
int(x^6*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/256*(-3*ln((b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1
/2))/(-b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2)))*2
^(1/2)*a^2*b+6*arctan(-2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1)*2^(1/2)*a^2*b
-6*arctan(2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1)*2^(1/2)*a^2*b+32*(b*x^4-1/
4*a)*(-b*x^4+a)^(1/4)*b^(7/4)*x^3)/b^(11/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.17

$$\int x^6 \sqrt[4]{a-bx^4}\,dx =$$

$$-\frac{3\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}} b\log\left(\frac{3\left(\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x+\left(-bx^4+a\right)^{\frac{1}{4}}a^2\right)}{x}\right)-3\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}} b\log\left(-\frac{3\left(\left(-\frac{a^8}{b^7}\right)^{\frac{1}{4}}b^2x-\left(-bx^4+a\right)^{\frac{1}{4}}a^2\right)}{x}\right)-3i\left(\right.}{}$$

input
```
integrate(x^6*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/128*(3*(-a^8/b^7)^(1/4)*b*log(3*((-a^8/b^7)^(1/4)*b^2*x + (-b*x^4 + a)^
(1/4)*a^2)/x) - 3*(-a^8/b^7)^(1/4)*b*log(-3*((-a^8/b^7)^(1/4)*b^2*x - (-b*
x^4 + a)^(1/4)*a^2)/x) - 3*I*(-a^8/b^7)^(1/4)*b*log(-3*(I*(-a^8/b^7)^(1/4)
*b^2*x - (-b*x^4 + a)^(1/4)*a^2)/x) + 3*I*(-a^8/b^7)^(1/4)*b*log(-3*(-I*(-
a^8/b^7)^(1/4)*b^2*x - (-b*x^4 + a)^(1/4)*a^2)/x) - 4*(4*b*x^7 - a*x^3)*(-
b*x^4 + a)^(1/4))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.21

$$\int x^6 \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a}x^7\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**4*exp_polar(2*I*
pi)/a)/(4*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.37

$$\int x^6\sqrt[4]{a-bx^4}\,dx=\frac{\frac{3\left(-bx^4+a\right)^{\frac14}a^2b}{x}-\frac{\left(-bx^4+a\right)^{\frac54}a^2}{x^5}}{32\left(b^3-\frac{2\left(bx^4-a\right)b^2}{x^4}+\frac{\left(bx^4-a\right)^2b}{x^8}\right)}-\frac{3\left(\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}+\frac{2\left(-bx^4+a\right)^{\frac14}}{x}\right)}{2b^{\frac14}}\right)}{b^{\frac34}}+\frac{2\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac14}-\frac{2\left(-bx^4+a\right)^{\frac14}}{x}\right)}{2b^{\frac14}}\right)}{b^{\frac34}}+\frac{\sqrt{2}a^2\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac14}b^{\frac14}}{x}+\cdots\right)}{b^{\frac34}}\cdots\right)}{256\,b}$$

input
```
integrate(x^6*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/32*(3*(-b*x^4 + a)^(1/4)*a^2*b/x - (-b*x^4 + a)^(5/4)*a^2/x^5)/(b^3 - 2*
(b*x^4 - a)*b^2/x^4 + (b*x^4 - a)^2*b/x^8) - 3/256*(2*sqrt(2)*a^2*arctan(1
/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) + 2
*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-b*x^4 + a)^(1/4)/x
)/b^(1/4))/b^(3/4) + sqrt(2)*a^2*log(sqrt(b) + sqrt(2)*(-b*x^4 + a)^(1/4)*
b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) - sqrt(2)*a^2*log(sqrt(b) - sqrt
(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4))/b
```

Giac [F]

$$\int x^6\sqrt[4]{a-bx^4}\,dx=\int\left(-bx^4+a\right)^{\frac14}x^6\,dx$$

input
```
integrate(x^6*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[4]{a-bx^4}\,dx = \int x^6\,{\left(a-b\,x^4\right)}^{1/4}\,dx$$

input
```
int(x^6*(a - b*x^4)^(1/4),x)
```

output
```
int(x^6*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^6 \sqrt[4]{a-bx^4}\,dx = \frac{-(-b\,x^4+a)^{\frac{1}{4}}\,a\,x^3+4(-b\,x^4+a)^{\frac{1}{4}}\,b\,x^7+3\left(\int \frac{x^2}{(-b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{32b}$$

input
```
int(x^6*(-b*x^4+a)^(1/4),x)
```

output
```
( - (a - b*x**4)**(1/4)*a*x**3 + 4*(a - b*x**4)**(1/4)*b*x**7 + 3*int(((a
- b*x**4)**(1/4)*x**2)/(a - b*x**4),x)*a**2)/(32*b)
```

3.651 $\int x^2 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 170

$$\int x^2 \sqrt[4]{a-bx^4}\,dx = \frac{1}{4}x^3\sqrt[4]{a-bx^4} - \frac{a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{3/4}} + \frac{a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{3/4}} - \frac{a\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{8\sqrt{2}b^{3/4}}$$

output

```
1/4*x^3*(-b*x^4+a)^(1/4)+1/16*a*arctan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/
4))*2^(1/2)/b^(3/4)+1/16*a*arctan(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^
(1/2)/b^(3/4)-1/16*a*arctanh(2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)
*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.86

$$\int x^2\sqrt[4]{a-bx^4}\,dx = \frac{4b^{3/4}x^3\sqrt[4]{a-bx^4}+\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{-\sqrt{b}x^2+\sqrt{a-bx^4}}\right)-\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)}{16b^{3/4}}$$

input
```
Integrate[x^2*(a - b*x^4)^(1/4),x]
```

output
```
(4*b^(3/4)*x^3*(a - b*x^4)^(1/4) + Sqrt[2]*a*ArcTan[(Sqrt[2]*b^(1/4)*x*(a
- b*x^4)^(1/4))/(-(Sqrt[b]*x^2) + Sqrt[a - b*x^4])] - Sqrt[2]*a*ArcTanh[(S
qrt[b]*x^2 + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))])/(16*
b^(3/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {811, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{4}a\int \frac{x^2}{(a-bx^4)^{3/4}}dx+\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{4}a\int \frac{x^2}{\sqrt{a-bx^4}\left(\frac{bx^4}{a-bx^4}+1\right)}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{4}a\left(\frac{\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\Big\downarrow$ 1476

$$\frac{1}{4}a\left(\frac{\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\Big\downarrow$ 1082

$$\frac{1}{4}a\left(\frac{\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\Big\downarrow$ 217

$$\frac{1}{4}a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\Big\downarrow$ 1479

$$\frac{1}{4}a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\cdots}{2\sqrt{b}}\right.$$

$$\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 25

$$\frac{1}{4}a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\cdots}{2\sqrt{b}}\right.$$

$$\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 27

$$\frac{1}{4}a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\cdots}{2\sqrt{b}}\right.$$

$$\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 1103

$$\frac{1}{4}a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}\right.$$

$$\frac{1}{4}x^3\sqrt[4]{a-bx^4}$$

input
```
Int[x^2*(a - b*x^4)^(1/4),x]
```

output
```
(x^3*(a - b*x^4)^(1/4))/4 + (a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x
^4)^(1/4)]/(Sqrt[2]*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)
^(1/4)]/(Sqrt[2]*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[
a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Lo
g[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4
)]/(2*Sqrt[2]*b^(1/4)))/(2*Sqrt[b])))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{a\left(\ln\left(\frac{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)-2\arctan\left(-\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)+2\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)\right)}{32b^{\frac{3}{4}}}$

input
```
int(x^2*(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(a*(ln((b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1
/2))/(-b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2)))-2
*arctan(-2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1)+2*arctan(2^(1/2)/b^(1/4)*(-
b*x^4+a)^(1/4)/x+1))*2^(1/2)-8*(-b*x^4+a)^(1/4)*x^3*b^(3/4))/b^(3/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\begin{aligned}\int x^2\sqrt[4]{a-bx^4}\,dx =& \frac{1}{4}\left(-bx^4+a\right)^{\frac{1}{4}}x^3-\frac{1}{16}\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+\left(-bx^4+a\right)^{\frac{1}{4}}a}{x}\right)\\&+\frac{1}{16}\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(-\frac{\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx-\left(-bx^4+a\right)^{\frac{1}{4}}a}{x}\right)\\&-\frac{1}{16}i\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{i\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+\left(-bx^4+a\right)^{\frac{1}{4}}a}{x}\right)\\&+\frac{1}{16}i\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(-\frac{a^4}{b^3}\right)^{\frac{1}{4}}bx+\left(-bx^4+a\right)^{\frac{1}{4}}a}{x}\right)\end{aligned}$$

input
```
integrate(x^2*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
1/4*(-b*x^4 + a)^(1/4)*x^3 - 1/16*(-a^4/b^3)^(1/4)*log(((-a^4/b^3)^(1/4)*b
*x + (-b*x^4 + a)^(1/4)*a)/x) + 1/16*(-a^4/b^3)^(1/4)*log(-((-a^4/b^3)^(1/
4)*b*x - (-b*x^4 + a)^(1/4)*a)/x) - 1/16*I*(-a^4/b^3)^(1/4)*log((I*(-a^4/b
^3)^(1/4)*b*x + (-b*x^4 + a)^(1/4)*a)/x) + 1/16*I*(-a^4/b^3)^(1/4)*log((-I
*(-a^4/b^3)^(1/4)*b*x + (-b*x^4 + a)^(1/4)*a)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.24

$$\int x^2 \sqrt[4]{a-bx^4}\,dx = \frac{\sqrt[4]{a}x^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**3*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.26

$$\int x^2\sqrt[4]{a-bx^4}\,dx = -\frac{\sqrt{2}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{16\,b^{\frac{3}{4}}} - \frac{\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{16\,b^{\frac{3}{4}}} - \frac{\sqrt{2}a\log\left(\sqrt{b}+\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{32\,b^{\frac{3}{4}}} + \frac{\sqrt{2}a\log\left(\sqrt{b}-\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{32\,b^{\frac{3}{4}}} + \frac{\left(-bx^4+a\right)^{\frac{1}{4}}a}{4\left(b-\frac{bx^4-a}{x^4}\right)x}$$

input
```
integrate(x^2*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)
/x)/b^(1/4))/b^(3/4) - 1/16*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
 - 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) - 1/32*sqrt(2)*a*log(sqrt(b) +
 sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) + 1/
32*sqrt(2)*a*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*
x^4 + a)/x^2)/b^(3/4) + 1/4*(-b*x^4 + a)^(1/4)*a/((b - (b*x^4 - a)/x^4)*x)
```

Giac [F]

$$\int x^2 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[4]{a-bx^4}\,dx = \int x^2 \left(a-b\,x^4\right)^{1/4}\,dx$$

input
```
int(x^2*(a - b*x^4)^(1/4),x)
```

output
```
int(x^2*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^2 \sqrt[4]{a-bx^4}\,dx = \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^3}{4} + \frac{\left(\int \frac{x^2}{\left(-b\,x^4+a\right)^{\frac{3}{4}}}dx\right) a}{4}$$

input
```
int(x^2*(-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*x**3 + int(((a - b*x**4)**(1/4)*x**2)/(a - b*x**4),x)
*a)/4
```

3.652 $\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 165

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx=-\frac{\sqrt[4]{a-bx^4}}{x}+\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}}-\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}}+\frac{\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{2\sqrt{2}}$$

output
```
-(-b*x^4+a)^(1/4)/x-1/4*b^(1/4)*arctan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/
4))*2^(1/2)-1/4*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/
2)+1/4*b^(1/4)*arctanh(2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)*x^2/(
-b*x^4+a)^(1/2)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2} dx$$
$$= \frac{-4\sqrt[4]{a-bx^4}+\sqrt{2}\sqrt[4]{b}x\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{\sqrt{b}x^2-\sqrt{a-bx^4}}\right)+\sqrt{2}\sqrt[4]{b}x\text{arctanh}\left(\frac{\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)}{4x}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^2,x]
```

output
```
(-4*(a - b*x^4)^(1/4) + Sqrt[2]*b^(1/4)*x*ArcTan[(Sqrt[2]*b^(1/4)*x*(a - b
*x^4)^(1/4))/(Sqrt[b]*x^2 - Sqrt[a - b*x^4])] + Sqrt[2]*b^(1/4)*x*ArcTanh[
(Sqrt[b]*x^2 + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))])/(4
*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {809, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2} dx$$
$$\Big\downarrow \text{809}$$
$$-b\int \frac{x^2}{(a-bx^4)^{3/4}}dx-\frac{\sqrt[4]{a-bx^4}}{x}$$
$$\Big\downarrow \text{854}$$
$$-b\int \frac{x^2}{\sqrt{a-bx^4}\left(\frac{bx^4}{a-bx^4}+1\right)}d\frac{x}{\sqrt[4]{a-bx^4}}-\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \text{826}$$

$$-b\left(\frac{\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)-\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \text{1476}$$

$$-b\left(\frac{\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)-\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \text{1082}$$

$$-b\left(\frac{\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)-\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \text{217}$$

$$-b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)-\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \text{1479}$$

$$-b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\cdots}{2\sqrt{b}}\right.\cdots$$

$$\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \; 25$$

$$-b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\cdots}{2\sqrt{b}}\right.\cdots$$

$$\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \; 27$$

$$-b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\cdots}{2\sqrt{b}}\right.\cdots$$

$$\frac{\sqrt[4]{a-bx^4}}{x}$$

$$\Big\downarrow \; 1103$$

$$-b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}\right.$$

$$\frac{\sqrt[4]{a-bx^4}}{x}$$

input
```
Int[(a - b*x^4)^(1/4)/x^2,x]
```

output
```
-((a - b*x^4)^(1/4)/x) - b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^
(1/4)]/(Sqrt[2]*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/
4)]/(Sqrt[2]*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a -
b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1
+ (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(
2*Sqrt[2]*b^(1/4)))/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{x}+\frac{\ln\left(\frac{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)b^{\frac{1}{4}}\sqrt{2}}{8}+\frac{\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)b^{\frac{1}{4}}\sqrt{2}}{4}-$ arcta

input
```
int((-b*x^4+a)^(1/4)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(-b*x^4+a)^(1/4)/x+1/8*ln((b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2
+(-b*x^4+a)^(1/2))/(-b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^
4+a)^(1/2)))*b^(1/4)*2^(1/2)+1/4*arctan(2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x
+1)*b^(1/4)*2^(1/2)-1/4*arctan(-2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1)*b^(1
/4)*2^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = \frac{\sqrt[4]{a}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**2,x)
```

output
```
a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**4*exp_polar(2*I*pi)/
a)/(4*x*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = \frac{1}{4}\sqrt{2}b^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\,(-bx^4+a)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right) + \frac{1}{4}\sqrt{2}b^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\,(-bx^4+a)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right) + \frac{1}{8}\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}+\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right) - \frac{1}{8}\sqrt{2}b^{\frac{1}{4}}\log\left(\sqrt{b}-\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right) - \frac{(-bx^4+a)^{\frac{1}{4}}}{x}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*b^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(
1/4)/x)/b^(1/4)) + 1/4*sqrt(2)*b^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4
) - 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4)) + 1/8*sqrt(2)*b^(1/4)*log(sqrt(b) + s
qrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2) - 1/8*sqrt(2)*
b^(1/4)*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 +
 a)/x^2) - (-b*x^4 + a)^(1/4)/x
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^2,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = -\frac{(a-b\,x^4)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{b\,x^4}{a}\right)}{x\,\left(1-\frac{b\,x^4}{a}\right)^{1/4}}$$

input
```
int((a - b*x^4)^(1/4)/x^2,x)
```

output
```
-((a - b*x^4)^(1/4)*hypergeom([-1/4, -1/4], 3/4, (b*x^4)/a))/(x*(1 - (b*x^
4)/a)^(1/4))
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^2}\,dx = \int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{x^2}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^2,x)
```

output
```
int((a - b*x**4)**(1/4)/x**2,x)
```

3.653 $\int \frac{\sqrt[4]{a-bx^4}}{x^6} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6} \, dx = -\frac{(a-bx^4)^{5/4}}{5ax^5}$$

output
```
-1/5*(-b*x^4+a)^(5/4)/a/x^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6} \, dx = -\frac{(a-bx^4)^{5/4}}{5ax^5}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^6,x]
```

output
```
-1/5*(a - b*x^4)^(5/4)/(a*x^5)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6} \, dx$$
$$\Big\downarrow \; 796$$
$$-\frac{\left(a-bx^4\right)^{5/4}}{5ax^5}$$

input
```
Int[(a - b*x^4)^(1/4)/x^6,x]
```

output
```
-1/5*(a - b*x^4)^(5/4)/(a*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(-b x^4+a)^{\frac{5}{4}}}{5 a x^5}$	19
trager	$-\frac{(-b x^4+a)^{\frac{5}{4}}}{5 a x^5}$	19
pseudoelliptic	$-\frac{(-b x^4+a)^{\frac{5}{4}}}{5 a x^5}$	19
orering	$-\frac{(-b x^4+a)^{\frac{5}{4}}}{5 a x^5}$	19
risch	$-\frac{(-b x^4+a)^{\frac{5}{4}} \left((-b x^4+a)^3\right)^{\frac{1}{4}}}{5 x^5 \left(-(b x^4-a)^3\right)^{\frac{1}{4}} a}$	46

input
```
int((-b*x^4+a)^(1/4)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(5/4)/a/x^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx = \frac{(bx^4-a)(-bx^4+a)^{\frac{1}{4}}}{5\,ax^5}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^6,x, algorithm="fricas")
```

output
```
1/5*(b*x^4 - a)*(-b*x^4 + a)^(1/4)/(a*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 158, normalized size of antiderivative = 7.18

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx = \begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{5}{4}\right)}{4x^4\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{5}{4}\right)}{4a\Gamma\left(-\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right| > 1 \\ \frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^4}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{4x^4\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{4a\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**6,x)
```

output
```
Piecewise((b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(4*x**4*gamma(-1/4
)) - b**(5/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(4*a*gamma(-1/4)), Abs(a
/(b*x**4)) > 1), (b**(1/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-5/4
)/(4*x**4*gamma(-1/4)) - b**(5/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gam
ma(-5/4)/(4*a*gamma(-1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx = -\frac{(-bx^4+a)^{\frac{5}{4}}}{5\,ax^5}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(-b*x^4 + a)^(5/4)/(a*x^5)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^6,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx=-\frac{(a-b\,x^4)^{5/4}}{5\,a\,x^5}$$

input
```
int((a - b*x^4)^(1/4)/x^6,x)
```

output
```
-(a - b*x^4)^(5/4)/(5*a*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt[4]{a-bx^4}}{x^6}\,dx=\frac{(-b\,x^4+a)^{\frac{1}{4}}\,(b\,x^4-a)}{5a\,x^5}$$

input
```
int((-b*x^4+a)^(1/4)/x^6,x)
```

output
```
((a - b*x**4)**(1/4)*( - a + b*x**4))/(5*a*x**5)
```

3.654 $\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx=-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}$$

output
```
-1/9*(-b*x^4+a)^(5/4)/a/x^9-4/45*b*(-b*x^4+a)^(5/4)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx=\frac{\sqrt[4]{a-bx^4}\left(-5a^2+abx^4+4b^2x^8\right)}{45a^2x^9}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^10,x]
```

output
```
((a - b*x^4)^(1/4)*(-5*a^2 + a*b*x^4 + 4*b^2*x^8))/(45*a^2*x^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}} \, dx$$

$$\Big\downarrow \; 803$$

$$\frac{4b\int \frac{\sqrt[4]{a-bx^4}}{x^6}dx}{9a}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}$$

$$\Big\downarrow \; 796$$

$$-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}$$

input
```
Int[(a - b*x^4)^(1/4)/x^10,x]
```

output
```
-1/9*(a - b*x^4)^(5/4)/(a*x^9) - (4*b*(a - b*x^4)^(5/4))/(45*a^2*x^5)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(4b\,x^{4}+5a\right)}{45x^{9}a^{2}}$	29
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(4b\,x^{4}+5a\right)}{45x^{9}a^{2}}$	29
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(4b\,x^{4}+5a\right)}{45x^{9}a^{2}}$	29
trager	$-\frac{\left(-4b^{2}x^{8}-ab\,x^{4}+5a^{2}\right) \left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{45x^{9}a^{2}}$	40
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(-4b^{2}x^{8}-ab\,x^{4}+5a^{2}\right)}{45x^{9} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}} a^{2}}$	67

input
```
int((-b*x^4+a)^(1/4)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/45*(-b*x^4+a)^(5/4)*(4*b*x^4+5*a)/x^9/a^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx = \frac{(4\,b^2x^8+abx^4-5\,a^2)(-bx^4+a)^{\frac{1}{4}}}{45\,a^2x^9}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^10,x, algorithm="fricas")
```

output
```
1/45*(4*b^2*x^8 + a*b*x^4 - 5*a^2)*(-b*x^4 + a)^(1/4)/(a^2*x^9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 406, normalized size of antiderivative = 8.83

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx$$
$$= \begin{cases} -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{9}{4}\right)}{16x^8\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{9}{4}\right)}{16ax^4\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{9}{4}\right)}{4a^2\Gamma\left(-\frac{1}{4}\right)} \\ \frac{5a^3b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{x^4\left(-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8\Gamma\left(-\frac{1}{4}\right)\right)} - \frac{6a^2b^{\frac{9}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8\Gamma\left(-\frac{1}{4}\right)} - \frac{3ab^{\frac{13}{4}}x^4\sqrt[4]{-\frac{a}{bx^4}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-16a^3bx^4\Gamma\left(-\frac{1}{4}\right)+16a^2b^2x^8\Gamma\left(-\frac{1}{4}\right)} + \frac{4b^{\frac{17}{4}}x^8\sqrt[4]{-}}{-16a^3bx^4\Gamma(-} \end{cases}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**10,x)
```

output
```
Piecewise((-5*b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-9/4)/(16*x**8*gamma(
-1/4)) + b**(5/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-9/4)/(16*a*x**4*gamma(-1/
4)) + b**(9/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-9/4)/(4*a**2*gamma(-1/4)), A
bs(a/(b*x**4)) > 1), (5*a**3*b**(5/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)
*gamma(-9/4)/(x**4*(-16*a**3*b*x**4*gamma(-1/4) + 16*a**2*b**2*x**8*gamma(
-1/4))) - 6*a**2*b**(9/4)*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)
/(-16*a**3*b*x**4*gamma(-1/4) + 16*a**2*b**2*x**8*gamma(-1/4)) - 3*a*b**(1
3/4)*x**4*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-16*a**3*b*x**
4*gamma(-1/4) + 16*a**2*b**2*x**8*gamma(-1/4)) + 4*b**(17/4)*x**8*(-a/(b*x
**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-16*a**3*b*x**4*gamma(-1/4) + 16
*a**2*b**2*x**8*gamma(-1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx = -\frac{\frac{9\left(-bx^4+a\right)^{\frac{5}{4}}b}{x^5} + \frac{5\left(-bx^4+a\right)^{\frac{9}{4}}}{x^9}}{45\,a^2}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^10,x, algorithm="maxima")
```

output
```
-1/45*(9*(-b*x^4 + a)^(5/4)*b/x^5 + 5*(-b*x^4 + a)^(9/4)/x^9)/a^2
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{10}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^10,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx = \frac{(a-b\,x^4)^{1/4}\,(-5\,a^2+a\,b\,x^4+4\,b^2\,x^8)}{45\,a^2\,x^9}$$

input
```
int((a - b*x^4)^(1/4)/x^10,x)
```

output
```
((a - b*x^4)^(1/4)*(4*b^2*x^8 - 5*a^2 + a*b*x^4))/(45*a^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}\,dx = \frac{(-b\,x^4+a)^{\frac{1}{4}}\,(4b^2x^8+ab\,x^4-5a^2)}{45a^2x^9}$$

input
```
int((-b*x^4+a)^(1/4)/x^10,x)
```

output
```
((a - b*x**4)**(1/4)*( - 5*a**2 + a*b*x**4 + 4*b**2*x**8))/(45*a**2*x**9)
```

3.655 $\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx=-\frac{(a-bx^4)^{5/4}}{13ax^{13}}-\frac{8b(a-bx^4)^{5/4}}{117a^2x^9}-\frac{32b^2(a-bx^4)^{5/4}}{585a^3x^5}$$

output
```
-1/13*(-b*x^4+a)^(5/4)/a/x^13-8/117*b*(-b*x^4+a)^(5/4)/a^2/x^9-32/585*b^2*
(-b*x^4+a)^(5/4)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx=\frac{(a-bx^4)^{5/4}(-45a^2-40abx^4-32b^2x^8)}{585a^3x^{13}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^14,x]
```

output
```
((a - b*x^4)^(5/4)*(-45*a^2 - 40*a*b*x^4 - 32*b^2*x^8))/(585*a^3*x^13)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{8b\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}dx}{13a}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{8b\left(\frac{4b\int \frac{\sqrt[4]{a-bx^4}}{x^6}dx}{9a}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{8b\left(-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}$$

input
```
Int[(a - b*x^4)^(1/4)/x^14,x]
```

output
```
-1/13*(a - b*x^4)^(5/4)/(a*x^13) + (8*b*(-1/9*(a - b*x^4)^(5/4)/(a*x^9) -
(4*b*(a - b*x^4)^(5/4))/(45*a^2*x^5)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(32 b^{2} x^{8}+40 a b\,x^{4}+45 a^{2}\right)}{585 x^{13} a^{3}}$	40
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(32 b^{2} x^{8}+40 a b\,x^{4}+45 a^{2}\right)}{585 x^{13} a^{3}}$	40
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(32 b^{2} x^{8}+40 a b\,x^{4}+45 a^{2}\right)}{585 x^{13} a^{3}}$	40
trager	$-\frac{\left(-32 b^{3} x^{12}-8 a\,b^{2} x^{8}-5 a^{2} b\,x^{4}+45 a^{3}\right) \left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{585 x^{13} a^{3}}$	51
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(-32 b^{3} x^{12}-8 a\,b^{2} x^{8}-5 a^{2} b\,x^{4}+45 a^{3}\right)}{585 x^{13} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}} a^{3}}$	78

input
```
int((-b*x^4+a)^(1/4)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/585*(-b*x^4+a)^(5/4)*(32*b^2*x^8+40*a*b*x^4+45*a^2)/x^13/a^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx = \frac{(32\,b^3x^{12}+8\,ab^2x^8+5\,a^2bx^4-45\,a^3)(-bx^4+a)^{\frac{1}{4}}}{585\,a^3x^{13}}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^14,x, algorithm="fricas")
```

output
```
1/585*(32*b^3*x^12 + 8*a*b^2*x^8 + 5*a^2*b*x^4 - 45*a^3)*(-b*x^4 + a)^(1/4
)/(a^3*x^13)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 1090, normalized size of antiderivative = 15.35

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**14,x)
```

output
```
Piecewise((45*a**5*b**(17/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5
*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x
**20*gamma(-1/4)) - 95*a**4*b**(21/4)*x**4*(a/(b*x**4) - 1)**(1/4)*gamma(-
13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) +
64*a**3*b**6*x**20*gamma(-1/4)) + 47*a**3*b**(25/4)*x**8*(a/(b*x**4) - 1)*
*(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*
gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) - 21*a**2*b**(29/4)*x**12*(a
/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a
**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) + 56*a*b**(33
/4)*x**16*(a/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-
1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) -
 32*b**(37/4)*x**20*(a/(b*x**4) - 1)**(1/4)*gamma(-13/4)/(64*a**5*b**4*x**
12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamm
a(-1/4)), Abs(a/(b*x**4)) > 1), (45*a**5*b**(17/4)*(-a/(b*x**4) + 1)**(1/4
)*exp(I*pi/4)*gamma(-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5
*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-1/4)) - 95*a**4*b**(21/4)*x
**4*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(-13/4)/(64*a**5*b**4*x**12*
gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4) + 64*a**3*b**6*x**20*gamma(-
1/4)) + 47*a**3*b**(25/4)*x**8*(-a/(b*x**4) + 1)**(1/4)*exp(I*pi/4)*gamma(
-13/4)/(64*a**5*b**4*x**12*gamma(-1/4) - 128*a**4*b**5*x**16*gamma(-1/4...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx = -\frac{\frac{117\,(-bx^4+a)^{\frac{5}{4}}b^2}{x^5}+\frac{130\,(-bx^4+a)^{\frac{9}{4}}b}{x^9}+\frac{45\,(-bx^4+a)^{\frac{13}{4}}}{x^{13}}}{585\,a^3}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^14,x, algorithm="maxima")
```

output
```
-1/585*(117*(-b*x^4 + a)^(5/4)*b^2/x^5 + 130*(-b*x^4 + a)^(9/4)*b/x^9 + 45
*(-b*x^4 + a)^(13/4)/x^13)/a^3
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{14}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^14,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx=\frac{b\,(a-b\,x^4)^{1/4}}{117\,a\,x^9}-\frac{(a-b\,x^4)^{1/4}}{13\,x^{13}}+\frac{32\,b^3\,(a-b\,x^4)^{1/4}}{585\,a^3\,x}+\frac{8\,b^2\,(a-b\,x^4)^{1/4}}{585\,a^2\,x^5}$$

input
```
int((a - b*x^4)^(1/4)/x^14,x)
```

output
```
(b*(a - b*x^4)^(1/4))/(117*a*x^9) - (a - b*x^4)^(1/4)/(13*x^13) + (32*b^3*
(a - b*x^4)^(1/4))/(585*a^3*x) + (8*b^2*(a - b*x^4)^(1/4))/(585*a^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}\,dx=\frac{(-b\,x^4+a)^{\frac{1}{4}}\,(32b^3x^{12}+8a\,b^2x^8+5a^2b\,x^4-45a^3)}{585a^3x^{13}}$$

input
```
int((-b*x^4+a)^(1/4)/x^14,x)
```

output
```
((a - b*x**4)**(1/4)*( - 45*a**3 + 5*a**2*b*x**4 + 8*a*b**2*x**8 + 32*b**3
*x**12))/(585*a**3*x**13)
```

3.656 $\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = -\frac{(a-bx^4)^{5/4}}{17ax^{17}} - \frac{12b(a-bx^4)^{5/4}}{221a^2x^{13}} - \frac{32b^2(a-bx^4)^{5/4}}{663a^3x^9} - \frac{128b^3(a-bx^4)^{5/4}}{3315a^4x^5}$$

output
```
-1/17*(-b*x^4+a)^(5/4)/a/x^17-12/221*b*(-b*x^4+a)^(5/4)/a^2/x^13-32/663*b^
2*(-b*x^4+a)^(5/4)/a^3/x^9-128/3315*b^3*(-b*x^4+a)^(5/4)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = \frac{(a-bx^4)^{5/4}(-195a^3-180a^2bx^4-160ab^2x^8-128b^3x^{12})}{3315a^4x^{17}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^18,x]
```

output
```
((a - b*x^4)^(5/4)*(-195*a^3 - 180*a^2*b*x^4 - 160*a*b^2*x^8 - 128*b^3*x^1
2))/(3315*a^4*x^17)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}dx$$

$$\Big\downarrow \text{803}$$

$$\frac{12b\int \frac{\sqrt[4]{a-bx^4}}{x^{14}}dx}{17a}-\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}$$

$$\Big\downarrow \text{803}$$

$$\frac{12b\left(\frac{8b\int \frac{\sqrt[4]{a-bx^4}}{x^{10}}dx}{13a}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}$$

$$\Big\downarrow \text{803}$$

$$\frac{12b\left(\frac{8b\left(\frac{4b\int \frac{\sqrt[4]{a-bx^4}}{x^6}dx}{9a}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}$$

$$\Big\downarrow \text{796}$$

$$\frac{12b\left(\frac{8b\left(-\frac{4b\left(a-bx^4\right)^{5/4}}{45a^2x^5}-\frac{\left(a-bx^4\right)^{5/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{5/4}}{13ax^{13}}\right)}{17a}-\frac{\left(a-bx^4\right)^{5/4}}{17ax^{17}}$$

input

```
Int[(a - b*x^4)^(1/4)/x^18,x]
```

output
```
-1/17*(a - b*x^4)^(5/4)/(a*x^17) + (12*b*(-1/13*(a - b*x^4)^(5/4)/(a*x^13)
 + (8*b*(-1/9*(a - b*x^4)^(5/4)/(a*x^9) - (4*b*(a - b*x^4)^(5/4))/(45*a^2*
x^5)))/(13*a)))/(17*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(128 b^{3} x^{12}+160 a\,b^{2} x^{8}+180 a^{2} b\,x^{4}+195 a^{3}\right)}{3315 x^{17} a^{4}}$	51
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(128 b^{3} x^{12}+160 a\,b^{2} x^{8}+180 a^{2} b\,x^{4}+195 a^{3}\right)}{3315 x^{17} a^{4}}$	51
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{5}{4}} \left(128 b^{3} x^{12}+160 a\,b^{2} x^{8}+180 a^{2} b\,x^{4}+195 a^{3}\right)}{3315 x^{17} a^{4}}$	51
trager	$-\frac{\left(-128 x^{16} b^{4}-32 a\,b^{3} x^{12}-20 a^{2} b^{2} x^{8}-15 a^{3} b\,x^{4}+195 a^{4}\right) \left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{3315 x^{17} a^{4}}$	62
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(-128 x^{16} b^{4}-32 a\,b^{3} x^{12}-20 a^{2} b^{2} x^{8}-15 a^{3} b\,x^{4}+195 a^{4}\right)}{3315 x^{17} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}} a^{4}}$	89

input
```
int((-b*x^4+a)^(1/4)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/3315*(-b*x^4+a)^(5/4)*(128*b^3*x^12+160*a*b^2*x^8+180*a^2*b*x^4+195*a^3
)/x^17/a^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = \frac{(128\,b^4x^{16}+32\,ab^3x^{12}+20\,a^2b^2x^8+15\,a^3bx^4-195\,a^4)(-bx^4+a)^{\frac{1}{4}}}{3315\,a^4x^{17}}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^18,x, algorithm="fricas")
```

output
```
1/3315*(128*b^4*x^16 + 32*a*b^3*x^12 + 20*a^2*b^2*x^8 + 15*a^3*b*x^4 - 195
*a^4)*(-b*x^4 + a)^(1/4)/(a^4*x^17)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 1756, normalized size of antiderivative = 18.29

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = \text{Too large to display}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**18,x)
```

output
```
Piecewise((585*a**7*b**(37/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a
**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b
**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 1800*a**6*b**
(41/4)*x**4*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gam
ma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-
1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 1830*a**5*b**(45/4)*x**8*(a/(b*
x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**
6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b*
*12*x**28*gamma(-1/4)) - 636*a**4*b**(49/4)*x**12*(a/(b*x**4) - 1)**(1/4)*
gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamm
a(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1
/4)) - 231*a**3*b**(53/4)*x**16*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256
*a**7*b**9*x**16*gamma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5
*b**11*x**24*gamma(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) + 924*a**2*b*
*(57/4)*x**20*(a/(b*x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*g
amma(-1/4) + 768*a**6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma
(-1/4) + 256*a**4*b**12*x**28*gamma(-1/4)) - 1056*a*b**(61/4)*x**24*(a/(b*
x**4) - 1)**(1/4)*gamma(-17/4)/(-256*a**7*b**9*x**16*gamma(-1/4) + 768*a**
6*b**10*x**20*gamma(-1/4) - 768*a**5*b**11*x**24*gamma(-1/4) + 256*a**4*b*
*12*x**28*gamma(-1/4)) + 384*b**(65/4)*x**28*(a/(b*x**4) - 1)**(1/4)*ga...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = -\frac{\frac{663\,(-bx^4+a)^{\frac{5}{4}}b^3}{x^5} + \frac{1105\,(-bx^4+a)^{\frac{9}{4}}b^2}{x^9} + \frac{765\,(-bx^4+a)^{\frac{13}{4}}b}{x^{13}} + \frac{195\,(-bx^4+a)^{\frac{17}{4}}}{x^{17}}}{3315\,a^4}$$

input
```
integrate((-b*x^4+a)^(1/4)/x^18,x, algorithm="maxima")
```

output
```
-1/3315*(663*(-b*x^4 + a)^(5/4)*b^3/x^5 + 1105*(-b*x^4 + a)^(9/4)*b^2/x^9
+ 765*(-b*x^4 + a)^(13/4)*b/x^13 + 195*(-b*x^4 + a)^(17/4)/x^17)/a^4
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{18}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^18,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^18, x)
```

Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = \frac{b\,{\left(a-b\,x^4\right)}^{1/4}}{221\,a\,x^{13}} - \frac{{\left(a-b\,x^4\right)}^{1/4}}{17\,x^{17}} + \frac{128\,b^4\,{\left(a-b\,x^4\right)}^{1/4}}{3315\,a^4\,x} + \frac{32\,b^3\,{\left(a-b\,x^4\right)}^{1/4}}{3315\,a^3\,x^5} + \frac{4\,b^2\,{\left(a-b\,x^4\right)}^{1/4}}{663\,a^2\,x^9}$$

input
```
int((a - b*x^4)^(1/4)/x^18,x)
```

output
```
(b*(a - b*x^4)^(1/4))/(221*a*x^13) - (a - b*x^4)^(1/4)/(17*x^17) + (128*b^
4*(a - b*x^4)^(1/4))/(3315*a^4*x) + (32*b^3*(a - b*x^4)^(1/4))/(3315*a^3*x
^5) + (4*b^2*(a - b*x^4)^(1/4))/(663*a^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{18}}\,dx = \frac{(-b\,x^4+a)^{\frac{1}{4}}\,(128b^4x^{16}+32a\,b^3x^{12}+20a^2b^2x^8+15a^3b\,x^4-195a^4)}{3315a^4x^{17}}$$

input
```
int((-b*x^4+a)^(1/4)/x^18,x)
```

output
```
((a - b*x**4)**(1/4)*( - 195*a**4 + 15*a**3*b*x**4 + 20*a**2*b**2*x**8 + 3
2*a*b**3*x**12 + 128*b**4*x**16))/(3315*a**4*x**17)
```

3.657 $\int x^{12}\sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 156

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx=-\frac{3a^3x\sqrt[4]{a-bx^4}}{112b^3}-\frac{3a^2x^5\sqrt[4]{a-bx^4}}{280b^2}-\frac{ax^9\sqrt[4]{a-bx^4}}{140b}+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}-\frac{3a^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{112b^{5/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-3/112*a^3*x*(-b*x^4+a)^(1/4)/b^3-3/280*a^2*x^5*(-b*x^4+a)^(1/4)/b^2-1/140
*a*x^9*(-b*x^4+a)^(1/4)/b+1/14*x^13*(-b*x^4+a)^(1/4)-3/112*a^(7/2)*(1-a/b/
x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^
(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.51 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.69

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx = \frac{x\sqrt[4]{a-bx^4}\left(-\sqrt[4]{1-\frac{bx^4}{a}}(15a^3+3a^2bx^4+2ab^2x^8-20b^3x^{12})+15a^3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)\right)}{280b^3\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^12*(a - b*x^4)^(1/4),x]
```

output
```
(x*(a - b*x^4)^(1/4)*(-((1 - (b*x^4)/a)^(1/4)*(15*a^3 + 3*a^2*b*x^4 + 2*a*
b^2*x^8 - 20*b^3*x^12)) + 15*a^3*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^4)
/a]))/(280*b^3*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {811, 843, 843, 843, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{14}a\int \frac{x^{12}}{(a-bx^4)^{3/4}}dx+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{14}a\left(\frac{9a\int \frac{x^8}{(a-bx^4)^{3/4}}dx}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$\downarrow$ 843

$$\frac{1}{14}a\left(\frac{9a\left(\frac{5a\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}dx}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$\downarrow$ 843

$$\frac{1}{14}a\left(\frac{9a\left(\frac{5a\left(\frac{a\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{2b}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+$$

$$\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$\downarrow$ 768

$$\frac{1}{14}a\left(\frac{9a\left(\frac{5a\left(\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+$$

$$\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{14}a\left(\frac{9a\left(\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+$$

$$\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{14}a\left(\frac{9a\left(\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 230

$$\frac{1}{14}a\left(\frac{9a\left(\frac{5a\left(-\frac{\sqrt{a}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}\right)+\frac{1}{14}x^{13}\sqrt[4]{a-bx^4}$$

input
```
Int[x^12*(a - b*x^4)^(1/4),x]
```

output
```
(x^13*(a - b*x^4)^(1/4))/14 + (a*(-1/10*(x^9*(a - b*x^4)^(1/4))/b + (9*a*(
-1/6*(x^5*(a - b*x^4)^(1/4))/b + (5*a*(-1/2*(x*(a - b*x^4)^(1/4))/b - (Sqr
t[a]*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2,
2])/(2*Sqrt[b]*(a - b*x^4)^(3/4))))/(6*b)))/(10*b)))/14
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{12}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}dx$$

input
```
int(x^12*(-b*x^4+a)^(1/4),x)
```

output
```
int(x^12*(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx=\int\left(-bx^4+a\right)^{\frac{1}{4}}x^{12}\,dx$$

input
```
integrate(x^12*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)*x^12, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx=\frac{\sqrt[4]{a}x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**13*gamma(13/4)*hyper((-1/4, 13/4), (17/4,), b*x**4*exp_polar(2
*I*pi)/a)/(4*gamma(17/4))
```

Maxima [F]

$$\int x^{12} \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}} x^{12}\,dx$$

input
```
integrate(x^12*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^12, x)
```

Giac [F]

$$\int x^{12} \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}} x^{12}\,dx$$

input
```
integrate(x^12*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{12} \sqrt[4]{a-bx^4}\,dx = \int x^{12}\,\left(a-b\,x^4\right)^{1/4}\,dx$$

input
```
int(x^12*(a - b*x^4)^(1/4),x)
```

output
```
int(x^12*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^{12}\sqrt[4]{a-bx^4}\,dx$$
$$= \frac{-15(-b\,x^4+a)^{\frac{1}{4}}\,a^3x-6(-b\,x^4+a)^{\frac{1}{4}}\,a^2b\,x^5-4(-b\,x^4+a)^{\frac{1}{4}}\,a\,b^2x^9+40(-b\,x^4+a)^{\frac{1}{4}}\,b^3x^{13}+15\left(\int\frac{}{(-b}\right.}{560b^3}$$

input
```
int(x^12*(-b*x^4+a)^(1/4),x)
```

output
```
( - 15*(a - b*x**4)**(1/4)*a**3*x - 6*(a - b*x**4)**(1/4)*a**2*b*x**5 - 4*
(a - b*x**4)**(1/4)*a*b**2*x**9 + 40*(a - b*x**4)**(1/4)*b**3*x**13 + 15*i
nt((a - b*x**4)**(1/4)/(a - b*x**4),x)*a**4)/(560*b**3)
```

3.658 $\int x^8 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int x^8 \sqrt[4]{a-bx^4}\,dx = -\frac{a^2x\sqrt[4]{a-bx^4}}{24b^2}-\frac{ax^5\sqrt[4]{a-bx^4}}{60b}+\frac{1}{10}x^9\sqrt[4]{a-bx^4} - \frac{a^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{24b^{3/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/24*a^2*x*(-b*x^4+a)^(1/4)/b^2-1/60*a*x^5*(-b*x^4+a)^(1/4)/b+1/10*x^9*(-
b*x^4+a)^(1/4)-1/24*a^(5/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arcc
sc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

$$\int x^8 \sqrt[4]{a-bx^4}\,dx = \frac{x\sqrt[4]{a-bx^4}\left(-\sqrt[4]{1-\frac{bx^4}{a}}\left(5a^2+abx^4-6b^2x^8\right)+5a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)\right)}{60b^2\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[x^8*(a - b*x^4)^(1/4),x]
```

output
```
(x*(a - b*x^4)^(1/4)*(-((1 - (b*x^4)/a)^(1/4)*(5*a^2 + a*b*x^4 - 6*b^2*x^8
)) + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^4)/a]))/(60*b^2*(1 - (b*
x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {811, 843, 843, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{10} a \int \frac{x^8}{\left(a-bx^4\right)^{3/4}}dx+\frac{1}{10} x^9 \sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{10} a \left(\frac{5a \int \frac{x^4}{\left(a-bx^4\right)^{3/4}}dx}{6b}-\frac{x^5 \sqrt[4]{a-bx^4}}{6b}\right)+\frac{1}{10} x^9 \sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{10} a \left(\frac{5a \left(\frac{a \int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{2b}-\frac{x \sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5 \sqrt[4]{a-bx^4}}{6b}\right)+\frac{1}{10} x^9 \sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{10}a\left(\frac{5a\left(\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 858

$$\frac{1}{10}a\left(\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 807

$$\frac{1}{10}a\left(\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a-bx^4}$$

$\big\downarrow$ 230

$$\frac{1}{10}a\left(\frac{5a\left(-\frac{\sqrt{a}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)+\frac{1}{10}x^9\sqrt[4]{a-bx^4}$$

input
```
Int[x^8*(a - b*x^4)^(1/4),x]
```

output
```
(x^9*(a - b*x^4)^(1/4))/10 + (a*(-1/6*(x^5*(a - b*x^4)^(1/4))/b + (5*a*(-1
/2*(x*(a - b*x^4)^(1/4))/b - (Sqrt[a]*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[
ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(3/4))))/(6*b)
))/10
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{8} \left(-b\,x^{4}+a\right)^{\frac{1}{4}}dx$$

input
```
int(x^8*(-b*x^4+a)^(1/4),x)
```

output
```
int(x^8*(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^8 \sqrt[4]{a-bx^4}\, dx = \int \left(-bx^{4}+a\right)^{\frac{1}{4}}x^{8}\, dx$$

input
```
integrate(x^8*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)*x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.31

$$\int x^8 \sqrt[4]{a-bx^4}\, dx = \frac{\sqrt[4]{a}x^{9}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| {\frac{bx^{4}e^{2i\pi}}{a}} \right)}{4\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**9*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**4*exp_polar(2*I*
pi)/a)/(4*gamma(13/4))
```

Maxima [F]

$$\int x^8 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}}x^8\,dx$$

input
```
integrate(x^8*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^8, x)
```

Giac [F]

$$\int x^8 \sqrt[4]{a-bx^4}\,dx = \int \left(-bx^4+a\right)^{\frac{1}{4}}x^8\,dx$$

input
```
integrate(x^8*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8 \sqrt[4]{a-bx^4}\,dx = \int x^8\,\left(a-b\,x^4\right)^{1/4}\,dx$$

input
```
int(x^8*(a - b*x^4)^(1/4),x)
```

output
```
int(x^8*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^8 \sqrt[4]{a-bx^4}\,dx$$
$$= \frac{-5(-b\,x^4+a)^{\frac{1}{4}}\,a^2x-2(-b\,x^4+a)^{\frac{1}{4}}\,ab\,x^5+12(-b\,x^4+a)^{\frac{1}{4}}\,b^2x^9+5\left(\int\frac{1}{(-b\,x^4+a)^{\frac{3}{4}}}dx\right)a^3}{120b^2}$$

input
```
int(x^8*(-b*x^4+a)^(1/4),x)
```

output
```
( - 5*(a - b*x**4)**(1/4)*a**2*x - 2*(a - b*x**4)**(1/4)*a*b*x**5 + 12*(a
- b*x**4)**(1/4)*b**2*x**9 + 5*int((a - b*x**4)**(1/4)/(a - b*x**4),x)*a**
3)/(120*b**2)
```

3.659 $\int x^4 \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int x^4 \sqrt[4]{a-bx^4}\,dx = -\frac{ax\sqrt[4]{a-bx^4}}{12b}+\frac{1}{6}x^5\sqrt[4]{a-bx^4} - \frac{a^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12\sqrt{b}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/12*a*x*(-b*x^4+a)^(1/4)/b+1/6*x^5*(-b*x^4+a)^(1/4)-1/12*a^(3/2)*(1-a/b/
x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^
(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int x^4 \sqrt[4]{a-bx^4}\,dx = \frac{x\sqrt[4]{a-bx^4}\left(-a+bx^4+\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}}\right)}{6b}$$

input
```
Integrate[x^4*(a - b*x^4)^(1/4),x]
```

output
```
(x*(a - b*x^4)^(1/4)*(-a + b*x^4 + (a*Hypergeometric2F1[-1/4, 1/4, 5/4, (b
*x^4)/a])/(1 - (b*x^4)/a)^(1/4)))/(6*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {811, 843, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{811}$$

$$\frac{1}{6}a\int \frac{x^4}{(a-bx^4)^{3/4}}dx+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{6}a\left(\frac{a\int \frac{1}{(a-bx^4)^{3/4}}dx}{2b}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{6}a\left(\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2b\,(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{6}a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2b\,(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{6}a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{6}a\left(-\frac{\sqrt{a}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)+\frac{1}{6}x^5\sqrt[4]{a-bx^4}$$

input
```
Int[x^4*(a - b*x^4)^(1/4),x]
```

output
```
(x^5*(a - b*x^4)^(1/4))/6 + (a*(-1/2*(x*(a - b*x^4)^(1/4))/b - (Sqrt[a]*(1
 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*
Sqrt[b]*(a - b*x^4)^(3/4))))/6
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 811
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && I
GtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m
, p, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int x^{4} \left(-b \,x^{4}+a \right)^{\frac{1}{4}}d x$$

input
```
int(x^4*(-b*x^4+a)^(1/4),x)
```

output
```
int(x^4*(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int x^4 \sqrt[4]{a-b x^4} \, dx=\int { {\left(-b x^{4} + a\right)}^{\frac{1}{4}} x^{4} \,d x }$$

input
```
integrate(x^4*(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int x^4\sqrt[4]{a-bx^4}\,dx=\frac{\sqrt[4]{a}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4*(-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x**5*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**4*exp_polar(2*I*p
i)/a)/(4*gamma(9/4))
```

Maxima [F]

$$\int x^4\sqrt[4]{a-bx^4}\,dx=\int(-bx^4+a)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt[4]{a-bx^4}\,dx=\int(-bx^4+a)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[4]{a-bx^4}\,dx=\int x^4\,{\left(a-b\,x^4\right)}^{1/4}\,dx$$

input
```
int(x^4*(a - b*x^4)^(1/4),x)
```

output
```
int(x^4*(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int x^4 \sqrt[4]{a-bx^4}\,dx=\frac{-(-b\,x^4+a)^{\frac{1}{4}}\,ax+2(-b\,x^4+a)^{\frac{1}{4}}\,b\,x^5+\left(\int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}}dx\right)a^2}{12b}$$

input
```
int(x^4*(-b*x^4+a)^(1/4),x)
```

output
```
( - (a - b*x**4)**(1/4)*a*x + 2*(a - b*x**4)**(1/4)*b*x**5 + int((a - b*x*
*4)**(1/4)/(a - b*x**4),x)*a**2)/(12*b)
```

3.660 $\int \sqrt[4]{a-bx^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \sqrt[4]{a-bx^4}\,dx=\frac{1}{2}x\sqrt[4]{a-bx^4}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\left(a-bx^4\right)^{3/4}}$$

output
```
1/2*x*(-b*x^4+a)^(1/4)-1/2*a^(1/2)*b^(1/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJa
cobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \sqrt[4]{a-bx^4}\,dx=\frac{x\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4),x]
```

output
```
(x*(a - b*x^4)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^4)/a])/(1 - (b
*x^4)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {748, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a-bx^4}\,dx$$

$$\Big\downarrow \text{748}$$

$$\frac{1}{2}a\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx+\frac{1}{2}x\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{768}$$

$$\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2\left(a-bx^4\right)^{3/4}}+\frac{1}{2}x\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{2}x\sqrt[4]{a-bx^4}-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2\left(a-bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}x\sqrt[4]{a-bx^4}-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4\left(a-bx^4\right)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}x\sqrt[4]{a-bx^4}-\frac{\sqrt{a}\sqrt{b}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\left(a-bx^4\right)^{3/4}}$$

input
```
Int[(a - b*x^4)^(1/4),x]
```

output
```
(x*(a - b*x^4)^(1/4))/2 - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*(a - b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \left(-b\,x^{4}+a\right)^{\frac{1}{4}}d x$$

input
```
int((-b*x^4+a)^(1/4),x)
```

output
```
int((-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{a-bx^4}\,dx=\int \left(-bx^{4}+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

$$\int \sqrt[4]{a-bx^4}\,dx=\frac{\sqrt[4]{a}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\right|\frac{bx^{4}e^{2i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-b*x**4+a)**(1/4),x)
```

output
```
a**(1/4)*x*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)/
a)/(4*gamma(5/4))
```

Maxima [F]

$$\int \sqrt[4]{a-bx^4}\,dx=\int \left(-bx^4+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a-bx^4}\,dx=\int \left(-bx^4+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.46

$$\int \sqrt[4]{a-bx^4}\,dx=\frac{x\,{\left(a-b\,x^4\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{{\left(1-\frac{b\,x^4}{a}\right)}^{1/4}}$$

input
```
int((a - b*x^4)^(1/4),x)
```

output
```
(x*(a - b*x^4)^(1/4)*hypergeom([-1/4, 1/4], 5/4, (b*x^4)/a))/(1 - (b*x^4)/
a)^(1/4)
```

Reduce [F]

$$\int \sqrt[4]{a-bx^4}\,dx = \frac{(-b\,x^4+a)^{\frac{1}{4}}\,x}{2} + \frac{\left(\int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}}dx\right)a}{2}$$

input
```
int((-b*x^4+a)^(1/4),x)
```

output
```
((a - b*x**4)**(1/4)*x + int((a - b*x**4)**(1/4)/(a - b*x**4),x)*a)/2
```

3.661 $\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx=-\frac{\sqrt[4]{a-bx^4}}{3x^3}+\frac{b^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\,(a-bx^4)^{3/4}}$$

output
```
-1/3*(-b*x^4+a)^(1/4)/x^3+1/3*b^(3/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiA
M(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx=-\frac{\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{1}{4},\frac{bx^4}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^4,x]
```

output
```
-1/3*((a - b*x^4)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, (b*x^4)/a])/(x^
3*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {809, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{1}{3}b\int \frac{1}{(a-bx^4)^{3/4}}dx-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

$$\Big\downarrow \text{768}$$

$$-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

$$\Big\downarrow \text{858}$$

$$\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

$$\Big\downarrow \text{807}$$

$$\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{6\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

$$\Big\downarrow \text{230}$$

$$\frac{b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{bx^2}}\right),2\right)}{3\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3x^3}$$

input
```
Int[(a - b*x^4)^(1/4)/x^4,x]
```

output
```
-1/3*(a - b*x^4)^(1/4)/x^3 + (b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[
ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*Sqrt[a]*(a - b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{x^4}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^4,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^4,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx = -\frac{i\sqrt[4]{b}e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{2x^2}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**4,x)
```

output
```
-I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 1/2), (3/2,), a/(b*x**4))/(2*x**2)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^4,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx=\int \frac{(a-b\,x^4)^{1/4}}{x^4}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^4,x)
```

output
```
int((a - b*x^4)^(1/4)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^4}\,dx = \frac{-(-b\,x^4+a)^{\frac{1}{4}} - \left(\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{-b\,x^8+a\,x^4}dx\right) a\,x^3}{2x^3}$$

input
```
int((-b*x^4+a)^(1/4)/x^4,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**4 - b*x**8),x)*a*
x**3))/(2*x**3)
```

3.662 $\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx = -\frac{\sqrt[4]{a-bx^4}}{7x^7}+\frac{b\sqrt[4]{a-bx^4}}{21ax^3} + \frac{2b^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{3/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/7*(-b*x^4+a)^(1/4)/x^7+1/21*b*(-b*x^4+a)^(1/4)/a/x^3+2/21*b^(5/2)*(1-a/
b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/
a^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx = -\frac{\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},-\frac{3}{4},\frac{bx^4}{a}\right)}{7x^7\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^8,x]
```

output
```
-1/7*((a - b*x^4)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, (b*x^4)/a])/(x
^7*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {809, 847, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{1}{7}b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{7}b\left(\frac{2b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{768}$$

$$-\frac{1}{7}b\left(\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{858}$$

$$-\frac{1}{7}b\left(-\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{7}b\left(-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

$$\Big\downarrow \text{230}$$

$$-\frac{1}{7}b\left(-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^4}}{7x^7}$$

input
```
Int[(a - b*x^4)^(1/4)/x^8,x]
```

output
```
-1/7*(a - b*x^4)^(1/4)/x^7 - (b*(-1/3*(a - b*x^4)^(1/4)/(a*x^3) - (2*b^(3/
2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2]
)/(3*a^(3/2)*(a - b*x^4)^(3/4))))/7
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{x^{8}}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^8,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx = \int \frac{\left(-bx^{4}+a\right)^{\frac{1}{4}}}{x^{8}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^8,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx = \frac{i\sqrt[4]{b}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{6x^6}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**8,x)
```

output
```
I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 3/2), (5/2,), a/(b*x**4))/(6*x**6)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^8,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^8}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^8,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx=\int \frac{(a-b\,x^4)^{1/4}}{x^8}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^8,x)
```

output
```
int((a - b*x^4)^(1/4)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^8}\,dx=\frac{-(-b\,x^4+a)^{\frac{1}{4}}-\left(\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{-b\,x^{12}+a\,x^8}dx\right)a\,x^7}{6x^7}$$

input
```
int((-b*x^4+a)^(1/4)/x^8,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**8 - b*x**12),x)*a
*x**7))/(6*x**7)
```

3.663 $\int \frac{\sqrt[4]{a-bx^4}}{x^{12}} dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}} dx = -\frac{\sqrt[4]{a-bx^4}}{11x^{11}} + \frac{b\sqrt[4]{a-bx^4}}{77ax^7} + \frac{2b^2\sqrt[4]{a-bx^4}}{77a^2x^3} + \frac{4b^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77a^{5/2}\left(a-bx^4\right)^{3/4}}$$

output

```
-1/11*(-b*x^4+a)^(1/4)/x^11+1/77*b*(-b*x^4+a)^(1/4)/a/x^7+2/77*b^2*(-b*x^4
+a)^(1/4)/a^2/x^3+4/77*b^(7/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*a
rccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}} dx = -\frac{\sqrt[4]{a-bx^4} \text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},\frac{bx^4}{a}\right)}{11x^{11}\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^12,x]
```

output
```
-1/11*((a - b*x^4)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (b*x^4)/a])/
(x^11*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {809, 847, 847, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{1}{11}b\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}dx-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{11}b\left(\frac{6b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{11}b\left(\frac{6b\left(\frac{2b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{1}{11}b\left(\frac{6b\left(\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{1}{11}b\left(\frac{6b\left(-\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{11}b\left(\frac{6b\left(-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{1}{11}b\left(\frac{6b\left(-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)-\frac{\sqrt[4]{a-bx^4}}{11x^{11}}$$

input
```
Int[(a - b*x^4)^(1/4)/x^12,x]
```

output
```
-1/11*(a - b*x^4)^(1/4)/x^11 - (b*(-1/7*(a - b*x^4)^(1/4)/(a*x^7) + (6*b*(
-1/3*(a - b*x^4)^(1/4)/(a*x^3) - (2*b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*Elli
pticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a - b*x^4)^(3/4))))
/(7*a)))/11
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^{4}+a)^{\frac{1}{4}}}{x^{12}}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^12,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^12,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{12}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^12,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^12, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx = -\frac{i\sqrt[4]{b}e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{10x^{10}}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**12,x)
```

output
```
-I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 5/2), (7/2,), a/(b*x**4))/(10*x**10)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{12}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^12,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^12, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{12}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^12,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^12, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx=\int \frac{\left(a-b\,x^4\right)^{1/4}}{x^{12}}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^12,x)
```

output
```
int((a - b*x^4)^(1/4)/x^12, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{12}}\,dx=\frac{-\left(-b\,x^4+a\right)^{\frac{1}{4}}-\left(\int \frac{\left(-b\,x^4+a\right)^{\frac{1}{4}}}{-b\,x^{16}+a\,x^{12}}dx\right)a\,x^{11}}{10x^{11}}$$

input
```
int((-b*x^4+a)^(1/4)/x^12,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**12 - b*x**16),x)*
a*x**11))/(10*x**11)
```

3.664 $\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 158

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{15x^{15}}+\frac{b\sqrt[4]{a-bx^4}}{165ax^{11}}+\frac{2b^2\sqrt[4]{a-bx^4}}{231a^2x^7}+\frac{4b^3\sqrt[4]{a-bx^4}}{231a^3x^3}+\frac{8b^{9/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{231a^{7/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/15*(-b*x^4+a)^(1/4)/x^15+1/165*b*(-b*x^4+a)^(1/4)/a/x^11+2/231*b^2*(-b*
x^4+a)^(1/4)/a^2/x^7+4/231*b^3*(-b*x^4+a)^(1/4)/a^3/x^3+8/231*b^(9/2)*(1-a
/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))
/a^(7/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx = -\frac{\sqrt[4]{a-bx^4}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{1}{4},-\frac{11}{4},\frac{bx^4}{a}\right)}{15x^{15}\sqrt[4]{1-\frac{bx^4}{a}}}$$

input
```
Integrate[(a - b*x^4)^(1/4)/x^16,x]
```

output
```
-1/15*((a - b*x^4)^(1/4)*Hypergeometric2F1[-15/4, -1/4, -11/4, (b*x^4)/a])
/(x^15*(1 - (b*x^4)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {809, 847, 847, 847, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx$$

$$\Big\downarrow \text{809}$$

$$-\frac{1}{15}b\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}dx-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{15}b\left(\frac{10b\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}dx}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{15}b\left(\frac{10b\left(\frac{6b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{847}$$

$$-\frac{1}{15}b\left(\frac{10b\left(\frac{6b\left(\frac{2b\int\frac{1}{\left(a-bx^4\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{1}{15}b\left(\frac{10b\left(\frac{6b\left(\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{1}{15}b\left(\frac{10b\left(\frac{6b\left(-\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{15}b\left(\frac{10b\left(\frac{6b\left(-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{1}{15}b\left(\frac{10b\left(\frac{6b\left(-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}\right)-\frac{\sqrt[4]{a-bx^4}}{15x^{15}}$$

input
```
Int[(a - b*x^4)^(1/4)/x^16,x]
```

output
```
-1/15*(a - b*x^4)^(1/4)/x^15 - (b*(-1/11*(a - b*x^4)^(1/4)/(a*x^11) + (10*
b*(-1/7*(a - b*x^4)^(1/4)/(a*x^7) + (6*b*(-1/3*(a - b*x^4)^(1/4)/(a*x^3) -
 (2*b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^
2)]/2, 2])/(3*a^(3/2)*(a - b*x^4)^(3/4))))/(7*a)))/(11*a)))/15
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 809
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Simp[b*n*(p/(c^n*(m + 1)))   I
nt[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ
[n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntB
inomialQ[a, b, c, n, m, p, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{x^{16}}dx$$

input
```
int((-b*x^4+a)^(1/4)/x^16,x)
```

output
```
int((-b*x^4+a)^(1/4)/x^16,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx=\int \frac{\left(-bx^{4}+a\right)^{\frac{1}{4}}}{x^{16}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^16,x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(1/4)/x^16, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx = \frac{i\sqrt[4]{b}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{a}{bx^4}\right)}{14x^{14}}$$

input
```
integrate((-b*x**4+a)**(1/4)/x**16,x)
```

output
```
I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 7/2), (9/2,), a/(b*x**4))/(14*x**14)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx = \int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{16}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^16,x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^16, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx=\int \frac{(-bx^4+a)^{\frac{1}{4}}}{x^{16}}\,dx$$

input
```
integrate((-b*x^4+a)^(1/4)/x^16,x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(1/4)/x^16, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx=\int \frac{(a-b\,x^4)^{1/4}}{x^{16}}\,dx$$

input
```
int((a - b*x^4)^(1/4)/x^16,x)
```

output
```
int((a - b*x^4)^(1/4)/x^16, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^4}}{x^{16}}\,dx=\frac{-(-b\,x^4+a)^{\frac{1}{4}}-\left(\int \frac{(-b\,x^4+a)^{\frac{1}{4}}}{-b\,x^{20}+a\,x^{16}}dx\right)a\,x^{15}}{14x^{15}}$$

input
```
int((-b*x^4+a)^(1/4)/x^16,x)
```

output
```
( - ((a - b*x**4)**(1/4) + int((a - b*x**4)**(1/4)/(a*x**16 - b*x**20),x)*
a*x**15))/(14*x**15)
```

3.665 $\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{a^4(a-bx^4)^{3/4}}{3b^5}+\frac{4a^3(a-bx^4)^{7/4}}{7b^5}-\frac{6a^2(a-bx^4)^{11/4}}{11b^5}+\frac{4a(a-bx^4)^{15/4}}{15b^5}-\frac{(a-bx^4)^{19/4}}{19b^5}$$

output
```
-1/3*a^4*(-b*x^4+a)^(3/4)/b^5+4/7*a^3*(-b*x^4+a)^(7/4)/b^5-6/11*a^2*(-b*x^
4+a)^(11/4)/b^5+4/15*a*(-b*x^4+a)^(15/4)/b^5-1/19*(-b*x^4+a)^(19/4)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = \frac{(a-bx^4)^{3/4}(-2048a^4-1536a^3bx^4-1344a^2b^2x^8-1232ab^3x^{12}-1155b^4x^{16})}{21945b^5}$$

input
```
Integrate[x^19/(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(3/4)*(-2048*a^4 - 1536*a^3*b*x^4 - 1344*a^2*b^2*x^8 - 1232*a
*b^3*x^12 - 1155*b^4*x^16))/(21945*b^5)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{16}}{\sqrt[4]{a-bx^4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^4}{b^4\sqrt[4]{a-bx^4}}-\frac{4\left(a-bx^4\right)^{3/4}a^3}{b^4}+\frac{6\left(a-bx^4\right)^{7/4}a^2}{b^4}-\frac{4\left(a-bx^4\right)^{11/4}a}{b^4}+\frac{\left(a-bx^4\right)^{15/4}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^4\left(a-bx^4\right)^{3/4}}{3b^5}+\frac{16a^3\left(a-bx^4\right)^{7/4}}{7b^5}-\frac{24a^2\left(a-bx^4\right)^{11/4}}{11b^5}-\frac{4\left(a-bx^4\right)^{19/4}}{19b^5}+\frac{16a\left(a-bx^4\right)^{15/4}}{15b^5}\right)$$

input
```
Int[x^19/(a - b*x^4)^(1/4),x]
```

output
```
((-4*a^4*(a - b*x^4)^(3/4))/(3*b^5) + (16*a^3*(a - b*x^4)^(7/4))/(7*b^5) -
 (24*a^2*(a - b*x^4)^(11/4))/(11*b^5) + (16*a*(a - b*x^4)^(15/4))/(15*b^5)
 - (4*(a - b*x^4)^(19/4))/(19*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(1155 x^{16} b^{4}+1232 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1536 a^{3} b\,x^{4}+2048 a^{4}\right)}{21945 b^{5}}$	59
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(1155 x^{16} b^{4}+1232 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1536 a^{3} b\,x^{4}+2048 a^{4}\right)}{21945 b^{5}}$	59
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(1155 x^{16} b^{4}+1232 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1536 a^{3} b\,x^{4}+2048 a^{4}\right)}{21945 b^{5}}$	59
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(1155 x^{16} b^{4}+1232 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1536 a^{3} b\,x^{4}+2048 a^{4}\right)}{21945 b^{5}}$	59
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(1155 x^{16} b^{4}+1232 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1536 a^{3} b\,x^{4}+2048 a^{4}\right)}{21945 b^{5}}$	59

input
```
int(x^19/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21945*(-b*x^4+a)^(3/4)*(1155*b^4*x^16+1232*a*b^3*x^12+1344*a^2*b^2*x^8+
1536*a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(1155\,b^4x^{16}+1232\,ab^3x^{12}+1344\,a^2b^2x^8+1536\,a^3bx^4+2048\,a^4)(-bx^4+a)^{\frac{3}{4}}}{21945\,b^5}$$

input
```
integrate(x^19/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/21945*(1155*b^4*x^16 + 1232*a*b^3*x^12 + 1344*a^2*b^2*x^8 + 1536*a^3*b*
x^4 + 2048*a^4)*(-b*x^4 + a)^(3/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = \begin{cases} -\frac{2048a^4(a-bx^4)^{\frac{3}{4}}}{21945b^5} - \frac{512a^3x^4(a-bx^4)^{\frac{3}{4}}}{7315b^4} - \frac{64a^2x^8(a-bx^4)^{\frac{3}{4}}}{1045b^3} - \frac{16ax^{12}(a-bx^4)^{\frac{3}{4}}}{285b^2} - \frac{x^{16}(a-bx^4)^{\frac{3}{4}}}{19b} & \text{for } b \neq 0 \\ \frac{x^{20}}{20\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**19/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-2048*a**4*(a - b*x**4)**(3/4)/(21945*b**5) - 512*a**3*x**4*(a
- b*x**4)**(3/4)/(7315*b**4) - 64*a**2*x**8*(a - b*x**4)**(3/4)/(1045*b**3
) - 16*a*x**12*(a - b*x**4)**(3/4)/(285*b**2) - x**16*(a - b*x**4)**(3/4)/
(19*b), Ne(b, 0)), (x**20/(20*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{19}{4}}}{19\,b^5} + \frac{4\,(-bx^4+a)^{\frac{15}{4}}a}{15\,b^5} - \frac{6\,(-bx^4+a)^{\frac{11}{4}}a^2}{11\,b^5} + \frac{4\,(-bx^4+a)^{\frac{7}{4}}a^3}{7\,b^5} - \frac{(-bx^4+a)^{\frac{3}{4}}a^4}{3\,b^5}$$

input
```
integrate(x^19/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/19*(-b*x^4 + a)^(19/4)/b^5 + 4/15*(-b*x^4 + a)^(15/4)*a/b^5 - 6/11*(-b*
x^4 + a)^(11/4)*a^2/b^5 + 4/7*(-b*x^4 + a)^(7/4)*a^3/b^5 - 1/3*(-b*x^4 + a
)^(3/4)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{1155\,(bx^4-a)^4(-bx^4+a)^{\frac{3}{4}} + 5852\,(bx^4-a)^3(-bx^4+a)^{\frac{3}{4}}a + 11970\,(bx^4-a)^2(-bx^4+a)^{\frac{3}{4}}a^2 - 1254}{21945\,b^5}$$

input
```
integrate(x^19/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/21945*(1155*(b*x^4 - a)^4*(-b*x^4 + a)^(3/4) + 5852*(b*x^4 - a)^3*(-b*x
^4 + a)^(3/4)*a + 11970*(b*x^4 - a)^2*(-b*x^4 + a)^(3/4)*a^2 - 12540*(-b*x
^4 + a)^(7/4)*a^3 + 7315*(-b*x^4 + a)^(3/4)*a^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.57

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = -\left(a-b\,x^4\right)^{3/4}\left(\frac{2048\,a^4}{21945\,b^5}+\frac{x^{16}}{19\,b}+\frac{16\,a\,x^{12}}{285\,b^2}+\frac{512\,a^3\,x^4}{7315\,b^4}+\frac{64\,a^2\,x^8}{1045\,b^3}\right)$$

input
```
int(x^19/(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(3/4)*((2048*a^4)/(21945*b^5) + x^16/(19*b) + (16*a*x^12)/(28
5*b^2) + (512*a^3*x^4)/(7315*b^4) + (64*a^2*x^8)/(1045*b^3))
```

Reduce [F]

$$\int \frac{x^{19}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{19}}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^19/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**19/(a - b*x**4)**(1/4),x)
```

3.666 $\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx=-\frac{a^3(a-bx^4)^{3/4}}{3b^4}+\frac{3a^2(a-bx^4)^{7/4}}{7b^4}-\frac{3a(a-bx^4)^{11/4}}{11b^4}+\frac{(a-bx^4)^{15/4}}{15b^4}$$

output
```
-1/3*a^3*(-b*x^4+a)^(3/4)/b^4+3/7*a^2*(-b*x^4+a)^(7/4)/b^4-3/11*a*(-b*x^4+
a)^(11/4)/b^4+1/15*(-b*x^4+a)^(15/4)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx=\frac{(a-bx^4)^{3/4}(-128a^3-96a^2bx^4-84ab^2x^8-77b^3x^{12})}{1155b^4}$$

input
```
Integrate[x^15/(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(3/4)*(-128*a^3 - 96*a^2*b*x^4 - 84*a*b^2*x^8 - 77*b^3*x^12))
/(1155*b^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{12}}{\sqrt[4]{a-bx^4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^3}{b^3\sqrt[4]{a-bx^4}}-\frac{3\left(a-bx^4\right)^{3/4}a^2}{b^3}+\frac{3\left(a-bx^4\right)^{7/4}a}{b^3}-\frac{\left(a-bx^4\right)^{11/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^3\left(a-bx^4\right)^{3/4}}{3b^4}+\frac{12a^2\left(a-bx^4\right)^{7/4}}{7b^4}+\frac{4\left(a-bx^4\right)^{15/4}}{15b^4}-\frac{12a\left(a-bx^4\right)^{11/4}}{11b^4}\right)$$

input
```
Int[x^15/(a - b*x^4)^(1/4),x]
```

output
```
((-4*a^3*(a - b*x^4)^(3/4))/(3*b^4) + (12*a^2*(a - b*x^4)^(7/4))/(7*b^4) -
 (12*a*(a - b*x^4)^(11/4))/(11*b^4) + (4*(a - b*x^4)^(15/4))/(15*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(77 b^{3} x^{12}+84 a\,b^{2} x^{8}+96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	48
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(77 b^{3} x^{12}+84 a\,b^{2} x^{8}+96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	48
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(77 b^{3} x^{12}+84 a\,b^{2} x^{8}+96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	48
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(77 b^{3} x^{12}+84 a\,b^{2} x^{8}+96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	48
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(77 b^{3} x^{12}+84 a\,b^{2} x^{8}+96 a^{2} b\,x^{4}+128 a^{3}\right)}{1155 b^{4}}$	48

input
```
int(x^15/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(-b*x^4+a)^(3/4)*(77*b^3*x^12+84*a*b^2*x^8+96*a^2*b*x^4+128*a^3)/b
^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.56

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(77\,b^3x^{12}+84\,ab^2x^8+96\,a^2bx^4+128\,a^3)(-bx^4+a)^{\frac{3}{4}}}{1155\,b^4}$$

input
```
integrate(x^15/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/1155*(77*b^3*x^12 + 84*a*b^2*x^8 + 96*a^2*b*x^4 + 128*a^3)*(-b*x^4 + a)
^(3/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx = \begin{cases} -\frac{128a^3(a-bx^4)^{\frac{3}{4}}}{1155b^4} - \frac{32a^2x^4(a-bx^4)^{\frac{3}{4}}}{385b^3} - \frac{4ax^8(a-bx^4)^{\frac{3}{4}}}{55b^2} - \frac{x^{12}(a-bx^4)^{\frac{3}{4}}}{15b} & \text{for } b \neq 0 \\ \frac{x^{16}}{16\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-128*a**3*(a - b*x**4)**(3/4)/(1155*b**4) - 32*a**2*x**4*(a - b
*x**4)**(3/4)/(385*b**3) - 4*a*x**8*(a - b*x**4)**(3/4)/(55*b**2) - x**12*
(a - b*x**4)**(3/4)/(15*b), Ne(b, 0)), (x**16/(16*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx = \frac{(-bx^4+a)^{\frac{15}{4}}}{15\,b^4} - \frac{3\,(-bx^4+a)^{\frac{11}{4}}a}{11\,b^4} + \frac{3\,(-bx^4+a)^{\frac{7}{4}}a^2}{7\,b^4} - \frac{(-bx^4+a)^{\frac{3}{4}}a^3}{3\,b^4}$$

input
```
integrate(x^15/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/15*(-b*x^4 + a)^(15/4)/b^4 - 3/11*(-b*x^4 + a)^(11/4)*a/b^4 + 3/7*(-b*x^
4 + a)^(7/4)*a^2/b^4 - 1/3*(-b*x^4 + a)^(3/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{77\,(bx^4-a)^3(-bx^4+a)^{\frac{3}{4}} + 315\,(bx^4-a)^2(-bx^4+a)^{\frac{3}{4}}a - 495\,(-bx^4+a)^{\frac{7}{4}}a^2 + 385\,(-bx^4+a)^{\frac{3}{4}}a^3}{1155\,b^4}$$

input
```
integrate(x^15/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/1155*(77*(b*x^4 - a)^3*(-b*x^4 + a)^(3/4) + 315*(b*x^4 - a)^2*(-b*x^4 +
 a)^(3/4)*a - 495*(-b*x^4 + a)^(7/4)*a^2 + 385*(-b*x^4 + a)^(3/4)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.58

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx = -\left(a-b\,x^4\right)^{3/4}\left(\frac{128\,a^3}{1155\,b^4} + \frac{x^{12}}{15\,b} + \frac{4\,a\,x^8}{55\,b^2} + \frac{32\,a^2\,x^4}{385\,b^3}\right)$$

input
```
int(x^15/(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(3/4)*((128*a^3)/(1155*b^4) + x^12/(15*b) + (4*a*x^8)/(55*b^2
) + (32*a^2*x^4)/(385*b^3))
```

Reduce [F]

$$\int \frac{x^{15}}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^{15}}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^15/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**15/(a - b*x**4)**(1/4),x)
```

3.667 $\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 62

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx=-\frac{a^2(a-bx^4)^{3/4}}{3b^3}+\frac{2a(a-bx^4)^{7/4}}{7b^3}-\frac{(a-bx^4)^{11/4}}{11b^3}$$

output
```
-1/3*a^2*(-b*x^4+a)^(3/4)/b^3+2/7*a*(-b*x^4+a)^(7/4)/b^3-1/11*(-b*x^4+a)^(
11/4)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx=\frac{(a-bx^4)^{3/4}(-32a^2-24abx^4-21b^2x^8)}{231b^3}$$

input
```
Integrate[x^11/(a - b*x^4)^(1/4),x]
```

output
```
((a - b*x^4)^(3/4)*(-32*a^2 - 24*a*b*x^4 - 21*b^2*x^8))/(231*b^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{\sqrt[4]{a-bx^4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2\sqrt[4]{a-bx^4}}-\frac{2\left(a-bx^4\right)^{3/4}a}{b^2}+\frac{\left(a-bx^4\right)^{7/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^2\left(a-bx^4\right)^{3/4}}{3b^3}-\frac{4\left(a-bx^4\right)^{11/4}}{11b^3}+\frac{8a\left(a-bx^4\right)^{7/4}}{7b^3}\right)$$

input
```
Int[x^11/(a - b*x^4)^(1/4),x]
```

output
```
((-4*a^2*(a - b*x^4)^(3/4))/(3*b^3) + (8*a*(a - b*x^4)^(7/4))/(7*b^3) - (4
*(a - b*x^4)^(11/4))/(11*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(21 b^{2} x^{8}+24 a b\,x^{4}+32 a^{2}\right)}{231 b^{3}}$	37
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(21 b^{2} x^{8}+24 a b\,x^{4}+32 a^{2}\right)}{231 b^{3}}$	37
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(21 b^{2} x^{8}+24 a b\,x^{4}+32 a^{2}\right)}{231 b^{3}}$	37
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(21 b^{2} x^{8}+24 a b\,x^{4}+32 a^{2}\right)}{231 b^{3}}$	37
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(21 b^{2} x^{8}+24 a b\,x^{4}+32 a^{2}\right)}{231 b^{3}}$	37

input
```
int(x^11/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/231*(-b*x^4+a)^(3/4)*(21*b^2*x^8+24*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(21\,b^2x^8+24\,abx^4+32\,a^2)(-bx^4+a)^{\frac{3}{4}}}{231\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/231*(21*b^2*x^8 + 24*a*b*x^4 + 32*a^2)*(-b*x^4 + a)^(3/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx = \begin{cases} -\frac{32a^2(a-bx^4)^{\frac{3}{4}}}{231b^3} - \frac{8ax^4(a-bx^4)^{\frac{3}{4}}}{77b^2} - \frac{x^8(a-bx^4)^{\frac{3}{4}}}{11b} & \text{for } b\neq 0 \\ \frac{x^{12}}{12\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-32*a**2*(a - b*x**4)**(3/4)/(231*b**3) - 8*a*x**4*(a - b*x**4)
**(3/4)/(77*b**2) - x**8*(a - b*x**4)**(3/4)/(11*b), Ne(b, 0)), (x**12/(12
*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{11}{4}}}{11\,b^3} + \frac{2\,(-bx^4+a)^{\frac{7}{4}}a}{7\,b^3} - \frac{(-bx^4+a)^{\frac{3}{4}}a^2}{3\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/11*(-b*x^4 + a)^(11/4)/b^3 + 2/7*(-b*x^4 + a)^(7/4)*a/b^3 - 1/3*(-b*x^4
 + a)^(3/4)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{21\,(bx^4-a)^2(-bx^4+a)^{\frac{3}{4}} - 66\,(-bx^4+a)^{\frac{7}{4}}a + 77\,(-bx^4+a)^{\frac{3}{4}}a^2}{231\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/231*(21*(b*x^4 - a)^2*(-b*x^4 + a)^(3/4) - 66*(-b*x^4 + a)^(7/4)*a + 77
*(-b*x^4 + a)^(3/4)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx = -\left(a-b\,x^4\right)^{3/4}\left(\frac{32\,a^2}{231\,b^3}+\frac{x^8}{11\,b}+\frac{8\,a\,x^4}{77\,b^2}\right)$$

input
```
int(x^11/(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(3/4)*((32*a^2)/(231*b^3) + x^8/(11*b) + (8*a*x^4)/(77*b^2))
```

Reduce [F]

$$\int \frac{x^{11}}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^{11}}{(-b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^11/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**11/(a - b*x**4)**(1/4),x)
```

3.668 $\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx=-\frac{a(a-bx^4)^{3/4}}{3b^2}+\frac{(a-bx^4)^{7/4}}{7b^2}$$

output
```
-1/3*a*(-b*x^4+a)^(3/4)/b^2+1/7*(-b*x^4+a)^(7/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx=\frac{(-4a-3bx^4)\,(a-bx^4)^{3/4}}{21b^2}$$

input
```
Integrate[x^7/(a - b*x^4)^(1/4),x]
```

output
```
((-4*a - 3*b*x^4)*(a - b*x^4)^(3/4))/(21*b^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^4}{\sqrt[4]{a-bx^4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a}{b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{b}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{4\left(a-bx^4\right)^{7/4}}{7b^2}-\frac{4a\left(a-bx^4\right)^{3/4}}{3b^2}\right)$$

input
```
Int[x^7/(a - b*x^4)^(1/4),x]
```

output
```
((-4*a*(a - b*x^4)^(3/4))/(3*b^2) + (4*(a - b*x^4)^(7/4))/(7*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}\left(3b\,x^{4}+4a\right)}{21b^{2}}$	26
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}\left(3b\,x^{4}+4a\right)}{21b^{2}}$	26
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}\left(3b\,x^{4}+4a\right)}{21b^{2}}$	26
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}\left(3b\,x^{4}+4a\right)}{21b^{2}}$	26
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}\left(3b\,x^{4}+4a\right)}{21b^{2}}$	26

input
```
int(x^7/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-b*x^4+a)^(3/4)*(3*b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx=-\frac{(3\,bx^4+4\,a)(-bx^4+a)^{\frac{3}{4}}}{21\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/21*(3*b*x^4 + 4*a)*(-b*x^4 + a)^(3/4)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx = \begin{cases} -\frac{4a(a-bx^4)^{\frac{3}{4}}}{21b^2} - \frac{x^4(a-bx^4)^{\frac{3}{4}}}{7b} & \text{for } b \neq 0 \\ \frac{x^8}{8\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-4*a*(a - b*x**4)**(3/4)/(21*b**2) - x**4*(a - b*x**4)**(3/4)/(
7*b), Ne(b, 0)), (x**8/(8*a**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx = \frac{(-bx^4+a)^{\frac{7}{4}}}{7\,b^2} - \frac{(-bx^4+a)^{\frac{3}{4}}a}{3\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/7*(-b*x^4 + a)^(7/4)/b^2 - 1/3*(-b*x^4 + a)^(3/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx = \frac{3\left(-bx^4+a\right)^{\frac{7}{4}} - 7\left(-bx^4+a\right)^{\frac{3}{4}}a}{21\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
1/21*(3*(-b*x^4 + a)^(7/4) - 7*(-b*x^4 + a)^(3/4)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx = -\left(a-b\,x^4\right)^{3/4}\left(\frac{4\,a}{21\,b^2}+\frac{x^4}{7\,b}\right)$$

input
```
int(x^7/(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(3/4)*((4*a)/(21*b^2) + x^4/(7*b))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^7}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^7/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**7/(a - b*x**4)**(1/4),x)
```

3.669 $\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{3b}$$

output
```
-1/3*(-b*x^4+a)^(3/4)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{3b}$$

input
```
Integrate[x^3/(a - b*x^4)^(1/4),x]
```

output
```
-1/3*(a - b*x^4)^(3/4)/b
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}} \, dx$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{\left(a-bx^4\right)^{3/4}}{3b}$$

input
```
Int[x^3/(a - b*x^4)^(1/4),x]
```

output
```
-1/3*(a - b*x^4)^(3/4)/b
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16
derivativedivides	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16
default	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16
trager	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16
risch	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16
pseudoelliptic	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16
orering	$-\frac{(-b\,x^4+a)^{\frac{3}{4}}}{3b}$	16

input
```
int(x^3/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^4+a)^(3/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{3}{4}}}{3\,b}$$

input
```
integrate(x^3/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/3*(-b*x^4 + a)^(3/4)/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = \begin{cases} -\frac{(a-bx^4)^{\frac{3}{4}}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt[4]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-(a - b*x**4)**(3/4)/(3*b), Ne(b, 0)), (x**4/(4*a**(1/4)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{3}{4}}}{3\,b}$$

input
```
integrate(x^3/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/3*(-b*x^4 + a)^(3/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{3}{4}}}{3\,b}$$

input
```
integrate(x^3/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/3*(-b*x^4 + a)^(3/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx=-\frac{{\left(a-b\,x^4\right)}^{3/4}}{3\,b}$$

input
```
int(x^3/(a - b*x^4)^(1/4),x)
```

output
```
-(a - b*x^4)^(3/4)/(3*b)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^3}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^3/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**3/(a - b*x**4)**(1/4),x)
```

3.670 $\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

output
```
1/2*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(1/4)-1/2*arctanh((-b*x^4+a)^(1/4)/
a^(1/4))/a^(1/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

input
```
Integrate[1/(x*(a - b*x^4)^(1/4)),x]
```

output
```
(ArcTan[(a - b*x^4)^(1/4)/a^(1/4)] - ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(
2*a^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {798, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^4\sqrt[4]{a-bx^4}}dx^4$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{a-bx^4}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^8}{a-x^{16}}d\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}-\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}$$

input
```
Int[1/(x*(a - b*x^4)^(1/4)),x]
```

output
```
ArcTan[(a - b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4)) - ArcTanh[(a - b*x^4)^(1/4)/
a^(1/4)]/(2*a^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

method	result	size
pseudoelliptic	$\frac{2\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)-\ln\left(\frac{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4a^{\frac{1}{4}}}$	64

input

```
int(1/x/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output

```
1/4/a^(1/4)*(2*arctan((-b*x^4+a)^(1/4)/a^(1/4))-ln((-(-b*x^4+a)^(1/4)-a^(1
/4))/(-(-b*x^4+a)^(1/4)+a^(1/4))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx = -\frac{\log\left((-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}+\frac{i\,\log\left((-bx^4+a)^{\frac{1}{4}}+i\,a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}} - \frac{i\,\log\left((-bx^4+a)^{\frac{1}{4}}-i\,a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}+\frac{\log\left((-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}\right)}{4\,a^{\frac{1}{4}}}$$

input

```
integrate(1/x/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output

```
-1/4*log((-b*x^4 + a)^(1/4) + a^(1/4))/a^(1/4) + 1/4*I*log((-b*x^4 + a)^(1
/4) + I*a^(1/4))/a^(1/4) - 1/4*I*log((-b*x^4 + a)^(1/4) - I*a^(1/4))/a^(1/
4) + 1/4*log((-b*x^4 + a)^(1/4) - a^(1/4))/a^(1/4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx = -\frac{e^{-\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\sqrt[4]{b}x\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x/(-b*x**4+a)**(1/4),x)
```

output
```
-exp(-I*pi/4)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), a/(b*x**4))/(4*b**(1/4)
*x*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx = \frac{\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{1}{4}}} + \frac{\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{1}{4}}}$$

input
```
integrate(1/x/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + 1/4*log(((-b*x^4 + a)^(1/
4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(41) = 82$.

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.37

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx = -\frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} + \frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(\sqrt{2}(-bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4+a}+\sqrt{-a}\right)}{8\,a} - \frac{\sqrt{2}(-a)^{\frac{3}{4}}\log\left(-\sqrt{2}(-bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4+a}+\sqrt{-a}\right)}{8\,a}$$

input
```
integrate(1/x/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(-b*x^4
 + a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a + 1/8*sqrt(2)*(-a)
^(3/4)*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt
(-a))/a - 1/8*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4
) + sqrt(-b*x^4 + a) + sqrt(-a))/a
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx = \frac{\mathrm{atan}\left(\frac{{\left(a-b\,x^4\right)}^{1/4}}{a^{1/4}}\right)-\mathrm{atanh}\left(\frac{{\left(a-b\,x^4\right)}^{1/4}}{a^{1/4}}\right)}{2\,a^{1/4}}$$

input
```
int(1/(x*(a - b*x^4)^(1/4)),x)
```

output
```
(atan((a - b*x^4)^(1/4)/a^(1/4)) - atanh((a - b*x^4)^(1/4)/a^(1/4)))/(2*a^
(1/4))
```

Reduce [F]

$$\int \frac{1}{x\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} x}dx$$

input
```
int(1/x/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x),x)
```

3.671 $\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{4ax^4} + \frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}$$

output
```
-1/4*(-b*x^4+a)^(3/4)/a/x^4+1/8*b*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
-1/8*b*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{4ax^4} + \frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}} - \frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{5/4}}$$

input
```
Integrate[1/(x^5*(a - b*x^4)^(1/4)),x]
```

output
```
-1/4*(a - b*x^4)^(3/4)/(a*x^4) + (b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(8*
a^(5/4)) - (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(5/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {798, 52, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{b\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}dx^4}{4a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(-\frac{\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{a-bx^4}}{a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(-\frac{b\int \frac{x^8}{a-x^{16}}d\sqrt[4]{a-bx^4}}{a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}\right)}{a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{4}\left(-\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}-\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^4}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(-\frac{b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^4}\right)$$

input
```
Int[1/(x^5*(a - b*x^4)^(1/4)),x]
```

output
```
(-((a - b*x^4)^(3/4)/(a*x^4)) - (b*(-1/2*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)]
/a^(1/4) + ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)]/(2*a^(1/4))))/a)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

method	result	size
pseudoelliptic	$-\frac{-2\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b\,x^{4}+\ln\left(\frac{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)b\,x^{4}+4\left(-b\,x^{4}+a\right)^{\frac{3}{4}}a^{\frac{1}{4}}}{16a^{\frac{5}{4}}x^{4}}$	89

input
```
int(1/x^5/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-2*arctan((-b*x^4+a)^(1/4)/a^(1/4))*b*x^4+ln((-(-b*x^4+a)^(1/4)-a^(
1/4))/(-(-b*x^4+a)^(1/4)+a^(1/4)))*b*x^4+4*(-b*x^4+a)^(3/4)*a^(1/4))/a^(5/
4)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^5\sqrt[4]{a-bx^4}}\,dx = -\frac{ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(\frac{b^4}{a^5}\right)^{\frac{3}{4}}+(-bx^4+a)^{\frac{1}{4}}b^3\right)-i\,ax^4\left(\frac{b^4}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(\frac{b^4}{a^5}\right)^{\frac{3}{4}}+(-bx^4+a)^{\frac{1}{4}}b^3\right)+i\,ax^4\left(\right.}{}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/16*(a*x^4*(b^4/a^5)^(1/4)*log(a^4*(b^4/a^5)^(3/4) + (-b*x^4 + a)^(1/4)*
b^3) - I*a*x^4*(b^4/a^5)^(1/4)*log(I*a^4*(b^4/a^5)^(3/4) + (-b*x^4 + a)^(1
/4)*b^3) + I*a*x^4*(b^4/a^5)^(1/4)*log(-I*a^4*(b^4/a^5)^(3/4) + (-b*x^4 +
a)^(1/4)*b^3) - a*x^4*(b^4/a^5)^(1/4)*log(-a^4*(b^4/a^5)^(3/4) + (-b*x^4 +
 a)^(1/4)*b^3) + 4*(-b*x^4 + a)^(3/4))/(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^5\sqrt[4]{a-bx^4}}\,dx = \frac{e^{\frac{3i\pi}{4}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\sqrt[4]{b}x^5\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(1/x**5/(-b*x**4+a)**(1/4),x)
```

output
```
exp(3*I*pi/4)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), a/(b*x**4))/(4*b**(1/4)
*x**5*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^5\sqrt[4]{a-bx^4}}\,dx = \frac{b\left(\frac{2\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{16\,a}-\frac{(-bx^4+a)^{\frac{3}{4}}b}{4\left((bx^4-a)a+a^2\right)}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
1/16*b*(2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((-b*x^4 + a)^(
1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a - 1/4*(-b*x^4 +
 a)^(3/4)*b/((b*x^4 - a)*a + a^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(61) = 122.

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.79

$$\int \frac{1}{x^5\sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{2\sqrt{2}(-a)^{\frac{3}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{2\sqrt{2}(-a)^{\frac{3}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2}+\frac{\sqrt{2}b^2\log\left(\sqrt{2}(-bx\cdots\right.}{\cdots}}{32\,b}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/32*(2*sqrt(2)*(-a)^(3/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 2*sqrt(2)*(-a)^(3/4)*b^2*arctan(-1/
2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + sq
rt(2)*b^2*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + s
qrt(-a))/((-a)^(1/4)*a) + sqrt(2)*(-a)^(3/4)*b^2*log(-sqrt(2)*(-b*x^4 + a)
^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/a^2 + 8*(-b*x^4 + a)^(3/4
)*b/(a*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx = \frac{b\,\mathrm{atan}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}} - \frac{\left(a-b\,x^4\right)^{3/4}}{4\,a\,x^4} - \frac{b\,\mathrm{atanh}\left(\frac{\left(a-b\,x^4\right)^{1/4}}{a^{1/4}}\right)}{8\,a^{5/4}}$$

input
```
int(1/(x^5*(a - b*x^4)^(1/4)),x)
```

output
```
(b*atan((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4)) - (a - b*x^4)^(3/4)/(4*a*x
^4) - (b*atanh((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(5/4))
```

Reduce [F]

$$\int \frac{1}{x^5 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^5}dx$$

input
```
int(1/x^5/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**5),x)
```

3.672 $\int \frac{1}{x^9 \sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{1}{x^9 \sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{8ax^8} - \frac{5b(a-bx^4)^{3/4}}{32a^2x^4} + \frac{5b^2\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}}$$

output
```
-1/8*(-b*x^4+a)^(3/4)/a/x^8-5/32*b*(-b*x^4+a)^(3/4)/a^2/x^4+5/64*b^2*arcta
n((-b*x^4+a)^(1/4)/a^(1/4))/a^(9/4)-5/64*b^2*arctanh((-b*x^4+a)^(1/4)/a^(1
/4))/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9 \sqrt[4]{a-bx^4}}\,dx = \frac{(-4a-5bx^4)(a-bx^4)^{3/4}}{32a^2x^8} + \frac{5b^2\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}} - \frac{5b^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{9/4}}$$

input
```
Integrate[1/(x^9*(a - b*x^4)^(1/4)),x]
```

output
```
((-4*a - 5*b*x^4)*(a - b*x^4)^(3/4))/(32*a^2*x^8) + (5*b^2*ArcTan[(a - b*x
^4)^(1/4)/a^(1/4)])/(64*a^(9/4)) - (5*b^2*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4
)])/(64*a^(9/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {798, 52, 52, 73, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 \sqrt[4]{a-bx^4}} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4} \int \frac{1}{x^{12} \sqrt[4]{a-bx^4}} dx^4$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{4} \left(\frac{5b \int \frac{1}{x^8 \sqrt[4]{a-bx^4}} dx^4}{8a} - \frac{\left(a-bx^4\right)^{3/4}}{2ax^8} \right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{4} \left(\frac{5b \left(\frac{b \int \frac{1}{x^4 \sqrt[4]{a-bx^4}} dx^4}{4a} - \frac{\left(a-bx^4\right)^{3/4}}{ax^4} \right)}{8a} - \frac{\left(a-bx^4\right)^{3/4}}{2ax^8} \right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\left(\frac{5b\left(-\frac{\int \frac{bx^8}{a-x^{16}}d\sqrt[4]{a-bx^4}}{a}-\frac{(a-bx^4)^{3/4}}{ax^4}\right)}{8a}-\frac{(a-bx^4)^{3/4}}{2ax^8}\right)$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\frac{5b\left(-\frac{b\int \frac{x^8}{a-x^{16}}d\sqrt[4]{a-bx^4}}{a}-\frac{(a-bx^4)^{3/4}}{ax^4}\right)}{8a}-\frac{(a-bx^4)^{3/4}}{2ax^8}\right)$$

$\Big\downarrow$ 827

$$\frac{1}{4}\left(\frac{5b\left(-\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}-\frac{1}{2}\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}\right)}{a}-\frac{(a-bx^4)^{3/4}}{ax^4}\right)}{8a}-\frac{(a-bx^4)^{3/4}}{2ax^8}\right)$$

$\Big\downarrow$ 216

$$\frac{1}{4}\left(\frac{5b\left(-\frac{b\left(\frac{1}{2}\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}-\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{(a-bx^4)^{3/4}}{ax^4}\right)}{8a}-\frac{(a-bx^4)^{3/4}}{2ax^8}\right)$$

$\Big\downarrow$ 219

$$\frac{1}{4}\left(\frac{5b\left(-\frac{b\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}\right)}{a}-\frac{(a-bx^4)^{3/4}}{ax^4}\right)}{8a}-\frac{(a-bx^4)^{3/4}}{2ax^8}\right)$$

input
```
Int[1/(x^9*(a - b*x^4)^(1/4)),x]
```

output
```
(-1/2*(a - b*x^4)^(3/4)/(a*x^8) + (5*b*(-((a - b*x^4)^(3/4)/(a*x^4)) - (b*
(-1/2*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)]/a^(1/4) + ArcTanh[(a - b*x^4)^(1/4
)/a^(1/4)]/(2*a^(1/4))))/a))/(8*a))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{10 \arctan \left(\frac{\left(-b \,x^{4}+a \right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right) b^{2} x^{8}-5 \ln \left(\frac{-\left(-b \,x^{4}+a \right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b \,x^{4}+a \right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right) b^{2} x^{8}-20 b \left(-b \,x^{4}+a \right)^{\frac{3}{4}} x^{4} a^{\frac{1}{4}}-16 \left(-b \,x^{4}+a \right)^{\frac{3}{4}} a^{\frac{5}{4}}}{128 a^{\frac{9}{4}} x^{8}}$	113

input
```
int(1/x^9/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(10*arctan((-b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8-5*ln((-(-b*x^4+a)^(1/4)
-a^(1/4))/(-(-b*x^4+a)^(1/4)+a^(1/4)))*b^2*x^8-20*b*(-b*x^4+a)^(3/4)*x^4*a
^(1/4)-16*(-b*x^4+a)^(3/4)*a^(5/4))/a^(9/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx = -\frac{5\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(\frac{b^8}{a^9}\right)^{\frac{3}{4}}+125\,(-bx^4+a)^{\frac{1}{4}}b^6\right)-5i\,a^2x^8\left(\frac{b^8}{a^9}\right)^{\frac{1}{4}}\log\left(125i\,a^7\left(\frac{b^8}{a^9}\right)^{\frac{3}{4}}+125\,(-bx$$

input
```
integrate(1/x^9/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/128*(5*a^2*x^8*(b^8/a^9)^(1/4)*log(125*a^7*(b^8/a^9)^(3/4) + 125*(-b*x^
4 + a)^(1/4)*b^6) - 5*I*a^2*x^8*(b^8/a^9)^(1/4)*log(125*I*a^7*(b^8/a^9)^(3
/4) + 125*(-b*x^4 + a)^(1/4)*b^6) + 5*I*a^2*x^8*(b^8/a^9)^(1/4)*log(-125*I
*a^7*(b^8/a^9)^(3/4) + 125*(-b*x^4 + a)^(1/4)*b^6) - 5*a^2*x^8*(b^8/a^9)^(
1/4)*log(-125*a^7*(b^8/a^9)^(3/4) + 125*(-b*x^4 + a)^(1/4)*b^6) + 4*(5*b*x
^4 + 4*a)*(-b*x^4 + a)^(3/4))/(a^2*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx = -\frac{e^{-\frac{i\pi}{4}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4\sqrt[4]{b}x^9\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(1/x**9/(-b*x**4+a)**(1/4),x)
```

output
```
-exp(-I*pi/4)*gamma(9/4)*hyper((1/4, 9/4), (13/4,), a/(b*x**4))/(4*b**(1/4
)*x**9*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx = \frac{5\,b^2\left(\frac{2\,\arctan\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}+\frac{\log\left(\frac{\left(-bx^4+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{\left(-bx^4+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}\right)}{128\,a^2} + \frac{5\left(-bx^4+a\right)^{\frac{7}{4}}b^2-9\left(-bx^4+a\right)^{\frac{3}{4}}ab^2}{32\left(\left(bx^4-a\right)^2a^2+2\left(bx^4-a\right)a^3+a^4\right)}$$

input
```
integrate(1/x^9/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
5/128*b^2*(2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(1/4) + log(((-b*x^4 + a
)^(1/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(1/4))/a^2 + 1/32*(5*
(-b*x^4 + a)^(7/4)*b^2 - 9*(-b*x^4 + a)^(3/4)*a*b^2)/((b*x^4 - a)^2*a^2 +
2*(b*x^4 - a)*a^3 + a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(84) = 168$.

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{10\,\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{10\,\sqrt{2}(-a)^{\frac{3}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(-bx^4+a\right)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^3}+\frac{5\,\sqrt{2}b^3\log\left(\sqrt{2}(\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(1/x^9/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
-1/256*(10*sqrt(2)*(-a)^(3/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) +
 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 10*sqrt(2)*(-a)^(3/4)*b^3*arctan(
-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 +
 5*sqrt(2)*b^3*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a
) + sqrt(-a))/((-a)^(1/4)*a^2) + 5*sqrt(2)*(-a)^(3/4)*b^3*log(-sqrt(2)*(-b
*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/a^3 - 8*(5*(-b*x
^4 + a)^(7/4)*b^3 - 9*(-b*x^4 + a)^(3/4)*a*b^3)/(a^2*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx = \frac{5\,b^2\,\mathrm{atan}\left(\frac{{\left(a-b\,x^4\right)}^{1/4}}{a^{1/4}}\right)}{64\,a^{9/4}} - \frac{9\,{\left(a-b\,x^4\right)}^{3/4}}{32\,a\,x^8} + \frac{5\,{\left(a-b\,x^4\right)}^{7/4}}{32\,a^2\,x^8} + \frac{b^2\,\mathrm{atan}\left(\frac{{\left(a-b\,x^4\right)}^{1/4}\,1{}\mathrm{i}}{a^{1/4}}\right)\,5{}\mathrm{i}}{64\,a^{9/4}}$$

input
```
int(1/(x^9*(a - b*x^4)^(1/4)),x)
```

output
```
(5*b^2*atan((a - b*x^4)^(1/4)/a^(1/4)))/(64*a^(9/4)) + (b^2*atan(((a - b*x
^4)^(1/4)*1i)/a^(1/4))*5i)/(64*a^(9/4)) - (9*(a - b*x^4)^(3/4))/(32*a*x^8)
 + (5*(a - b*x^4)^(7/4))/(32*a^2*x^8)
```

Reduce [F]

$$\int \frac{1}{x^9\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^9}dx$$

input
```
int(1/x^9/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x***9),x)
```

3.673 $\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = -\frac{4a^2x^2(a-bx^4)^{3/4}}{39b^3} - \frac{10ax^6(a-bx^4)^{3/4}}{117b^2} - \frac{x^{10}(a-bx^4)^{3/4}}{13b} + \frac{8a^{7/2}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^4}}$$

output
```
-4/39*a^2*x^2*(-b*x^4+a)^(3/4)/b^3-10/117*a*x^6*(-b*x^4+a)^(3/4)/b^2-1/13*
x^10*(-b*x^4+a)^(3/4)/b+8/39*a^(7/2)*(1-b*x^4/a)^(1/4)*EllipticE(sin(1/2*a
rcsin(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.61 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^2\left(-12a^3+2a^2bx^4+ab^2x^8+9b^3x^{12}+12a^3\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)\right)}{117b^3\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^13/(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(-12*a^3 + 2*a^2*b*x^4 + a*b^2*x^8 + 9*b^3*x^12 + 12*a^3*(1 - (b*x^4)
/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^4)/a]))/(117*b^3*(a - b*x^
4)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {807, 262, 262, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{\sqrt[4]{a-bx^4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{10a\int \frac{x^8}{\sqrt[4]{a-bx^4}}dx^2}{13b}-\frac{2x^{10}\left(a-bx^4\right)^{3/4}}{13b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{2a\int \frac{x^4}{\sqrt[4]{a-bx^4}}dx^2}{3b}-\frac{2x^6\left(a-bx^4\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^{10}\left(a-bx^4\right)^{3/4}}{13b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{2a\left(\frac{2a\int\frac{1}{\sqrt[4]{a-bx^4}}dx^2}{5b}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^6\left(a-bx^4\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^{10}\left(a-bx^4\right)^{3/4}}{13b}\right)$$

$$\Big\downarrow \ 227$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{2a\left(\frac{2a\sqrt[4]{1-\frac{bx^4}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{5b\sqrt[4]{a-bx^4}}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^6\left(a-bx^4\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^{10}\left(a-bx^4\right)^{3/4}}{13b}\right)$$

$$\Big\downarrow \ 226$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{2a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^6\left(a-bx^4\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^{10}\left(a-bx^4\right)^{3/4}}{13b}\right)$$

input
```
Int[x^13/(a - b*x^4)^(1/4),x]
```

output
```
((-2*x^10*(a - b*x^4)^(3/4))/(13*b) + (10*a*((-2*x^6*(a - b*x^4)^(3/4))/(9
*b) + (2*a*((-2*x^2*(a - b*x^4)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^4)/a)^
(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^4
)^(1/4))))/(3*b)))/(13*b))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{13}}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^13/(-b*x^4+a)^(1/4),x)
```

output
```
int(x^13/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^{13}}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^13/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x^13/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.22

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^{14}{}_2F_1\left(\begin{matrix} \frac{1}{4},\frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{14\sqrt[4]{a}}$$

input
```
integrate(x**13/(-b*x**4+a)**(1/4),x)
```

output
```
x**14*hyper((1/4, 7/2), (9/2,), b*x**4*exp_polar(2*I*pi)/a)/(14*a**(1/4))
```

Maxima [F]

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{13}}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^13/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^13/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{13}}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^13/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^13/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{13}}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

input
```
int(x^13/(a - b*x^4)^(1/4),x)
```

output
```
int(x^13/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{13}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{13}}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^13/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**13/(a - b*x**4)**(1/4),x)
```

3.674 $\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = -\frac{2ax^2(a-bx^4)^{3/4}}{15b^2} - \frac{x^6(a-bx^4)^{3/4}}{9b} + \frac{4a^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^4}}$$

output
```
-2/15*a*x^2*(-b*x^4+a)^(3/4)/b^2-1/9*x^6*(-b*x^4+a)^(3/4)/b+4/15*a^(5/2)*(
1-b*x^4/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b
^(5/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^2\left(-6a^2+abx^4+5b^2x^8+6a^2\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)\right)}{45b^2\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^9/(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(-6*a^2 + a*b*x^4 + 5*b^2*x^8 + 6*a^2*(1 - (b*x^4)/a)^(1/4)*Hypergeom
etric2F1[1/4, 1/2, 3/2, (b*x^4)/a]))/(45*b^2*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 262, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{\sqrt[4]{a-bx^4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2a\int \frac{x^4}{\sqrt[4]{a-bx^4}}dx^2}{3b}-\frac{2x^6\left(a-bx^4\right)^{3/4}}{9b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2a\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^4}}dx^2}{5b}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^6\left(a-bx^4\right)^{3/4}}{9b}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{2a\left(\frac{2a\sqrt[4]{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{5b\sqrt[4]{a-bx^4}}-\frac{2x^2(a-bx^4)^{3/4}}{5b}\right)}{3b}-\frac{2x^6(a-bx^4)^{3/4}}{9b}\right)$$

$\downarrow$ 226

$$\frac{1}{2}\left(\frac{2a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}-\frac{2x^2(a-bx^4)^{3/4}}{5b}\right)}{3b}-\frac{2x^6(a-bx^4)^{3/4}}{9b}\right)$$

input
```
Int[x^9/(a - b*x^4)^(1/4),x]
```

output
```
((-2*x^6*(a - b*x^4)^(3/4))/(9*b) + (2*a*((-2*x^2*(a - b*x^4)^(3/4))/(5*b)
 + (4*a^(3/2)*(1 - (b*x^4)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(5*b^(3/2)*(a - b*x^4)^(1/4))))/(3*b))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^9}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^9/(-b*x^4+a)^(1/4),x)
```

output
```
int(x^9/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^9}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^9/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x^9/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^{10}{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{10\sqrt[4]{a}}$$

input
```
integrate(x**9/(-b*x**4+a)**(1/4),x)
```

output
```
x**10*hyper((1/4, 5/2), (7/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(1/4))
```

Maxima [F]

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^9}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^9/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^9/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^9}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^9/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^9/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^9}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

input
```
int(x^9/(a - b*x^4)^(1/4),x)
```

output
```
int(x^9/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^9}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^9}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^9/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**9/(a - b*x**4)**(1/4),x)
```

3.675 $\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx=-\frac{x^2(a-bx^4)^{3/4}}{5b}+\frac{2a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/5*x^2*(-b*x^4+a)^(3/4)/b+2/5*a^(3/2)*(1-b*x^4/a)^(1/4)*EllipticE(sin(1/
2*arcsin(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx=\frac{x^2\left(-a+bx^4+a\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)\right)}{5b\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^5/(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2
, (b*x^4)/a]))/(5*b*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {807, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt[4]{a-bx^4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^4}}dx^2}{5b}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(\frac{2a\sqrt[4]{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{5b\sqrt[4]{a-bx^4}}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{2}\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^4}}-\frac{2x^2\left(a-bx^4\right)^{3/4}}{5b}\right)$$

input
```
Int[x^5/(a - b*x^4)^(1/4),x]
```

output
```
((-2*x^2*(a - b*x^4)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^4)/a)^(1/4)*Ellip
ticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{5}}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}d x$$

input
```
int(x^5/(-b*x^4+a)^(1/4),x)
```

output
```
int(x^5/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^5}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x^5/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.34

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^6 {}_2F_1\left(\left.\begin{matrix}\frac{1}{4}, \frac{3}{2}\\ \frac{5}{2}\end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{6\sqrt[4]{a}}$$

input
```
integrate(x**5/(-b*x**4+a)**(1/4),x)
```

output
```
x**6*hyper((1/4, 3/2), (5/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(1/4))
```

Maxima [F]

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^5}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^5/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^5}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^5/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^5/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^5}{(a-b\,x^4)^{1/4}}\,dx$$

input
```
int(x^5/(a - b*x^4)^(1/4),x)
```

output
```
int(x^5/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^5}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^5/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**5/(a - b*x**4)**(1/4),x)
```

3.676 $\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx=\frac{\sqrt{a}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^4}}$$

output
```
a^(1/2)*(1-b*x^4/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x^2/a^(1/2))),2
^(1/2))/b^(1/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.97 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx=\frac{x^2\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^4}{a}\right)}{2\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x/(a - b*x^4)^(1/4),x]
```

output
```
(x^2*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^4)/a])/(2
*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {807, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \; 807$$

$$\frac{1}{2}\int \frac{1}{\sqrt[4]{a-bx^4}}dx^2$$

$$\Big\downarrow \; 227$$

$$\frac{\sqrt[4]{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{2\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \; 226$$

$$\frac{\sqrt{a}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^4}}$$

input
```
Int[x/(a - b*x^4)^(1/4),x]
```

output
```
(Sqrt[a]*(1 - (b*x^4)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a - b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}d x$$

input
```
int(x/(-b*x^4+a)^(1/4),x)
```

output
```
int(x/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^2 {}_2F_1\left(\left.\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\right| \frac{bx^4e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}}$$

input
```
integrate(x/(-b*x**4+a)**(1/4),x)
```

output
```
x**2*hyper((1/4, 1/2), (3/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(1/4))
```

Maxima [F]

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

input
```
int(x/(a - b*x^4)^(1/4),x)
```

output
```
int(x/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x/(-b*x^4+a)^(1/4),x)
```

output
```
int(x/(a - b*x**4)**(1/4),x)
```

3.677 $\int \frac{1}{x^3 \sqrt[4]{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{1}{x^3 \sqrt[4]{a-bx^4}} \, dx = -\frac{(a-bx^4)^{3/4}}{2ax^2} - \frac{\sqrt{b}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a-bx^4}}$$

output
```
-1/2*(-b*x^4+a)^(3/4)/a/x^2-1/2*b^(1/2)*(1-b*x^4/a)^(1/4)*EllipticE(sin(1/
2*arcsin(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^3 \sqrt[4]{a-bx^4}} \, dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{bx^4}{a}\right)}{2x^2\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^3*(a - b*x^4)^(1/4)),x]
```

output
```
-1/2*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (b*x^4)/a])/
(x^2*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {807, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^4}}dx^2}{2a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^2}\right)$$

$$\Big\downarrow \text{227}$$

$$\frac{1}{2}\left(-\frac{b\sqrt[4]{1-\frac{bx^4}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{2a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{ax^2}\right)$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{2}\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{ax^2}\right)$$

input
```
Int[1/(x^3*(a - b*x^4)^(1/4)),x]
```

output
```
(-((a - b*x^4)^(3/4)/(a*x^2)) - (Sqrt[b]*(1 - (b*x^4)/a)^(1/4)*EllipticE[A
rcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^4)^(1/4)))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(-b\,x^{4}+a \right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^3/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^3/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^7 - a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^3\sqrt[4]{a-bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}x^2}$$

input
```
integrate(1/x**3/(-b*x**4+a)**(1/4),x)
```

output
```
-hyper((-1/2, 1/4), (1/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(1/4)*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{x^3\,{\left(a-b\,x^4\right)}^{1/4}}\,dx$$

input
```
int(1/(x^3*(a - b*x^4)^(1/4)),x)
```

output
```
int(1/(x^3*(a - b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^3}dx$$

input
```
int(1/x^3/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**3),x)
```

3.678 $\int \frac{1}{x^7 \sqrt[4]{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{1}{x^7 \sqrt[4]{a-bx^4}} \, dx = -\frac{(a-bx^4)^{3/4}}{6ax^6} - \frac{b(a-bx^4)^{3/4}}{4a^2x^2} - \frac{b^{3/2}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4a^{3/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/6*(-b*x^4+a)^(3/4)/a/x^6-1/4*b*(-b*x^4+a)^(3/4)/a^2/x^2-1/4*b^(3/2)*(1-
b*x^4/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(
3/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^7 \sqrt[4]{a-bx^4}} \, dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},\frac{bx^4}{a}\right)}{6x^6\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^7*(a - b*x^4)^(1/4)),x]
```

output
```
-1/6*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (b*x^4)/a])
/(x^6*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 \sqrt[4]{a-bx^4}} \, dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{2} \int \frac{1}{x^8 \sqrt[4]{a-bx^4}} dx^2$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2} \left(\frac{b \int \frac{1}{x^4 \sqrt[4]{a-bx^4}} dx^2}{2a} - \frac{\left(a-bx^4\right)^{3/4}}{3ax^6} \right)$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2} \left(\frac{b \left(-\frac{b \int \frac{1}{\sqrt[4]{a-bx^4}} dx^2}{2a} - \frac{\left(a-bx^4\right)^{3/4}}{ax^2} \right)}{2a} - \frac{\left(a-bx^4\right)^{3/4}}{3ax^6} \right)$$

$$\Big\downarrow \text{ 227}$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^4}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{2a\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{ax^2}\right)}{2a}-\frac{(a-bx^4)^{3/4}}{3ax^6}\right)$$

$\downarrow$ 226

$$\frac{1}{2}\left(\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{ax^2}\right)}{2a}-\frac{(a-bx^4)^{3/4}}{3ax^6}\right)$$

input
```
Int[1/(x^7*(a - b*x^4)^(1/4)),x]
```

output
```
(-1/3*(a - b*x^4)^(3/4)/(a*x^6) + (b*(-((a - b*x^4)^(3/4)/(a*x^2)) - (Sqrt
[b]*(1 - (b*x^4)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(
Sqrt[a]*(a - b*x^4)^(1/4))))/(2*a))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(-b\,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^7/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^7/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^7 \sqrt[4]{a-bx^4}}\, dx = \int \frac{1}{{\left(-b x^{4} + a\right)}^{\frac{1}{4}} x^{7}}\, dx$$

input
```
integrate(1/x^7/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^11 - a*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^7\sqrt[4]{a-bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{6\sqrt[4]{a}x^6}$$

input
```
integrate(1/x**7/(-b*x**4+a)**(1/4),x)
```

output
```
-hyper((-3/2, 1/4), (-1/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(1/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{x^7\,{\left(a-b\,x^4\right)}^{1/4}}\,dx$$

input
```
int(1/(x^7*(a - b*x^4)^(1/4)),x)
```

output
```
int(1/(x^7*(a - b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^7\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^7}dx$$

input
```
int(1/x^7/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**7),x)
```

3.679 $\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{10ax^{10}} - \frac{7b(a-bx^4)^{3/4}}{60a^2x^6} - \frac{7b^2(a-bx^4)^{3/4}}{40a^3x^2} - \frac{7b^{5/2}\sqrt[4]{1-\frac{bx^4}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40a^{5/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/10*(-b*x^4+a)^(3/4)/a/x^10-7/60*b*(-b*x^4+a)^(3/4)/a^2/x^6-7/40*b^2*(-b
*x^4+a)^(3/4)/a^3/x^2-7/40*b^(5/2)*(1-b*x^4/a)^(1/4)*EllipticE(sin(1/2*arc
sin(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},\frac{bx^4}{a}\right)}{10x^{10}\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^11*(a - b*x^4)^(1/4)),x]
```

output
```
-1/10*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (b*x^4)/a]
)/(x^10*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {807, 264, 264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{7b\int \frac{1}{x^8\sqrt[4]{a-bx^4}}dx^2}{10a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{7b\left(\frac{b\int \frac{1}{x^4\sqrt[4]{a-bx^4}}dx^2}{2a}-\frac{\left(a-bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{7b\left(\frac{b\left(-\frac{b\int\frac{1}{\sqrt[4]{a-bx^4}}dx^2}{2a}-\frac{\left(a-bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a-bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 227

$$\frac{1}{2}\left(\frac{7b\left(\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^4}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^4}{a}}}dx^2}{2a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{ax^2}\right)}{2a}-\frac{\left(a-bx^4\right)^{3/4}}{3ax^6}\right)}{10a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^{10}}\right)$$

$\Big\downarrow$ 226

$$\frac{1}{2}\left(\frac{7b\left(\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^4}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{ax^2}\right)}{2a}-\frac{(a-bx^4)^{3/4}}{3ax^6}\right)}{10a}-\frac{(a-bx^4)^{3/4}}{5ax^{10}}\right)$$

input
```
Int[1/(x^11*(a - b*x^4)^(1/4)),x]
```

output
```
(-1/5*(a - b*x^4)^(3/4)/(a*x^10) + (7*b*(-1/3*(a - b*x^4)^(3/4)/(a*x^6) +
(b*(-((a - b*x^4)^(3/4)/(a*x^2)) - (Sqrt[b]*(1 - (b*x^4)/a)^(1/4)*Elliptic
E[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^4)^(1/4))))/(2*a)
))/(10*a))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{11} \left(-b\,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^11/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^11/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11} \sqrt[4]{a-b x^4}}\, dx = \int \frac{1}{{\left(-b x^{4} + a\right)}^{\frac{1}{4}} x^{11}}\,{d x}$$

input
```
integrate(1/x^11/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^15 - a*x^11), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{10\sqrt[4]{a}x^{10}}$$

input
```
integrate(1/x**11/(-b*x**4+a)**(1/4),x)
```

output
```
-hyper((-5/2, 1/4), (-3/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(1/4)*x**1
0)
```

Maxima [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^11), x)
```

Giac [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{x^{11}\,{\left(a-b\,x^4\right)}^{1/4}}\,dx$$

input
```
int(1/(x^11*(a - b*x^4)^(1/4)),x)
```

output
```
int(1/(x^11*(a - b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^{11}}dx$$

input
```
int(1/x^11/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**11),x)
```

3.680 $\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx = -\frac{5ax(a-bx^4)^{3/4}}{32b^2} - \frac{x^5(a-bx^4)^{3/4}}{8b} - \frac{5a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}b^{9/4}} + \frac{5a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}b^{9/4}} + \frac{5a^2\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{64\sqrt{2}b^{9/4}}$$

output
```
-5/32*a*x*(-b*x^4+a)^(3/4)/b^2-1/8*x^5*(-b*x^4+a)^(3/4)/b+5/128*a^2*arctan
(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(9/4)+5/128*a^2*arctan(1
+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(9/4)+5/128*a^2*arctanh(2^(
1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/
b^(9/4)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx = \frac{-4\sqrt[4]{b}x(a-bx^4)^{3/4}(5a+4bx^4)-5\sqrt{2}a^2\arctan\left(\frac{-\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)+5\sqrt{2}a^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{\sqrt{b}x^2+\sqrt{a-bx^4}}\right)}{128b^{9/4}}$$

input
```
Integrate[x^8/(a - b*x^4)^(1/4),x]
```

output
```
(-4*b^(1/4)*x*(a - b*x^4)^(3/4)*(5*a + 4*b*x^4) - 5*Sqrt[2]*a^2*ArcTan[(-(
Sqrt[b]*x^2) + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))] + 5
*Sqrt[2]*a^2*ArcTanh[(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))/(Sqrt[b]*x^2 +
Sqrt[a - b*x^4])])/(128*b^(9/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {843, 843, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{5a\int \frac{x^4}{\sqrt[4]{a-bx^4}}dx}{8b}-\frac{x^5(a-bx^4)^{3/4}}{8b}$$

$$\Big\downarrow \text{843}$$

$$\frac{5a\left(\frac{a\int \frac{1}{\sqrt[4]{a-bx^4}}dx}{4b}-\frac{x(a-bx^4)^{3/4}}{4b}\right)}{8b}-\frac{x^5(a-bx^4)^{3/4}}{8b}$$

$\downarrow$ 770

$$\frac{5a\left(\frac{a\int \frac{1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{4b}-\frac{x(a-bx^4)^{3/4}}{4b}\right)}{8b}-\frac{x^5(a-bx^4)^{3/4}}{8b}$$

$\downarrow$ 755

$$\frac{5a\left(\frac{a\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b}-\frac{x(a-bx^4)^{3/4}}{4b}\right)}{8b}-\frac{x^5(a-bx^4)^{3/4}}{8b}$$

$\downarrow$ 1476

$$\frac{5a\left(\frac{a\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\left(\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-\cdots}}}{2\sqrt{b}}\cdots\right.\right.}{4b}\cdots}{8b}$$

$$\frac{x^5(a-bx^4)^{3/4}}{8b}$$

$\downarrow$ 1082

$$\frac{5a\left(\frac{a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b}-\frac{x(a-bx^4)^{3/4}}{4b}\right)}{8b}$$

$$\frac{x^5(a-bx^4)^{3/4}}{8b}$$

$\downarrow$ 217

$$\frac{5a\left(\frac{a\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)\right)}{4b}-\frac{x\left(a-bx^4\right)^{3/4}}{4b}\right)}{8b}-\frac{x^5\left(a-bx^4\right)^{3/4}}{8b}$$

$\downarrow$ 1479

$$\frac{5a\left(\frac{a\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\left(\cdots\right.}{4b}\cdots}{8b}$$

$$\frac{x^5\left(a-bx^4\right)^{3/4}}{8b}$$

$\downarrow$ 25

$$5a\left(\frac{a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\left(\frac{\text{arcta}}{\quad}\right.\right.}{4b}\right.}{8b}$$

$$\frac{x^5\left(a-bx^4\right)^{3/4}}{8b}$$

$\downarrow$ 27

$$5a\left(\frac{a\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^{}}}\right.}{\sqrt{2}\sqrt[4]{b}}\right.\right.}{4b}\right.}{8b}$$

$$\frac{x^5\left(a-bx^4\right)^{3/4}}{8b}$$

$\downarrow$ 1103

$$\frac{5a\left(\frac{a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}\right.\right.}{4b}\right.}{8b}$$

$$\frac{x^5\left(a-bx^4\right)^{3/4}}{8b}$$

input
```
Int[x^8/(a - b*x^4)^(1/4),x]
```

output
```
-1/8*(x^5*(a - b*x^4)^(3/4))/b + (5*a*(-1/4*(x*(a - b*x^4)^(3/4))/b + (a*(
(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))) +
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/2 + (
-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^
4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqr
t[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)))/2))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-32\left(-b\,x^4+a\right)^{\frac{3}{4}}x^5 b^{\frac{5}{4}}-40\left(-b\,x^4+a\right)^{\frac{3}{4}}x a\,b^{\frac{1}{4}}-10\arctan\left(\frac{\sqrt{2}\left(-b\,x^4+a\right)^{\frac{1}{4}}+b^{\frac{1}{4}}x}{b^{\frac{1}{4}}x}\right)\sqrt{2}\,a^2-10\arctan\left(\frac{\sqrt{2}\left(-b\,x^4+a\right)^{\frac{1}{4}}-b^{\frac{1}{4}}x}{b^{\frac{1}{4}}x}\right.}{256b^{\frac{9}{4}}}$

input
```
int(x^8/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/256*(-32*(-b*x^4+a)^(3/4)*x^5*b^(5/4)-40*(-b*x^4+a)^(3/4)*x*a*b^(1/4)-10
*arctan((2^(1/2)*(-b*x^4+a)^(1/4)+b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a^2-10*arc
tan((2^(1/2)*(-b*x^4+a)^(1/4)-b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a^2-5*ln((-b^(
1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2))/(b^(1/4)*(-b
*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2)))*2^(1/2)*a^2)/b^(9/4
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.19

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx =$$

$$-\frac{5\,b^2\left(-\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(\frac{125\left(b^7x\left(-\frac{a^8}{b^9}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}a^6\right)}{x}\right)-5\,b^2\left(-\frac{a^8}{b^9}\right)^{\frac{1}{4}}\log\left(-\frac{125\left(b^7x\left(-\frac{a^8}{b^9}\right)^{\frac{3}{4}}-\left(-bx^4+a\right)^{\frac{1}{4}}a^6\right)}{x}\right)+}{}$$

input
```
integrate(x^8/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/128*(5*b^2*(-a^8/b^9)^(1/4)*log(125*(b^7*x*(-a^8/b^9)^(3/4) + (-b*x^4 +
 a)^(1/4)*a^6)/x) - 5*b^2*(-a^8/b^9)^(1/4)*log(-125*(b^7*x*(-a^8/b^9)^(3/4
) - (-b*x^4 + a)^(1/4)*a^6)/x) + 5*I*b^2*(-a^8/b^9)^(1/4)*log(-125*(I*b^7*
x*(-a^8/b^9)^(3/4) - (-b*x^4 + a)^(1/4)*a^6)/x) - 5*I*b^2*(-a^8/b^9)^(1/4)
*log(-125*(-I*b^7*x*(-a^8/b^9)^(3/4) - (-b*x^4 + a)^(1/4)*a^6)/x) + 4*(4*b
*x^5 + 5*a*x)*(-b*x^4 + a)^(3/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.20

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(-b*x**4+a)**(1/4),x)
```

output
```
x**9*gamma(9/4)*hyper((1/4, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
a**(1/4)*gamma(13/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.32

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx = -\frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\sqrt{-\cdots}\right.}{b^{\frac{1}{4}}}\cdots\right.}{256\,b^2}$$

$$-\frac{\frac{9\left(-bx^4+a\right)^{\frac{3}{4}}a^2b}{x^3}+\frac{5\left(-bx^4+a\right)^{\frac{7}{4}}a^2}{x^7}}{32\left(b^4-\frac{2\left(bx^4-a\right)b^3}{x^4}+\frac{\left(bx^4-a\right)^2b^2}{x^8}\right)}$$

input
```
integrate(x^8/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-5/256*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/
4)/x)/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) -
2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(b) + sqrt(2)*(
-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4) + sqrt(2)*log(
sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(
1/4))*a^2/b^2 - 1/32*(9*(-b*x^4 + a)^(3/4)*a^2*b/x^3 + 5*(-b*x^4 + a)^(7/4
)*a^2/x^7)/(b^4 - 2*(b*x^4 - a)*b^3/x^4 + (b*x^4 - a)^2*b^2/x^8)
```

Giac [F]

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^8}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^8/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^8}{\left(a-b\,x^4\right)^{1/4}} \,dx$$

input
```
int(x^8/(a - b*x^4)^(1/4),x)
```

output
```
int(x^8/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^8}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^8}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^8/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**8/(a - b*x**4)**(1/4),x)
```

3.681 $\int \frac{x^4}{\sqrt[4]{a-bx^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}} dx = -\frac{x(a-bx^4)^{3/4}}{4b} - \frac{a \arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{5/4}} + \frac{a \arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{5/4}} + \frac{a \text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{8\sqrt{2}b^{5/4}}$$

output
```
-1/4*x*(-b*x^4+a)^(3/4)/b+1/16*a*arctan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1
/4))*2^(1/2)/b^(5/4)+1/16*a*arctan(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2
^(1/2)/b^(5/4)+1/16*a*arctanh(2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2
)*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = -\frac{4\sqrt[4]{b}x(a-bx^4)^{3/4}+\sqrt{2}a\arctan\left(\frac{-\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)-\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{\sqrt{b}x^2+\sqrt{a-bx^4}}\right)}{16b^{5/4}}$$

input
```
Integrate[x^4/(a - b*x^4)^(1/4),x]
```

output
```
-1/16*(4*b^(1/4)*x*(a - b*x^4)^(3/4) + Sqrt[2]*a*ArcTan[(-(Sqrt[b]*x^2) +
Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))] - Sqrt[2]*a*ArcTan
h[(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))/(Sqrt[b]*x^2 + Sqrt[a - b*x^4])])/
b^(5/4)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.42, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {843, 770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{a\int \frac{1}{\sqrt[4]{a-bx^4}}dx}{4b}-\frac{x(a-bx^4)^{3/4}}{4b}$$

$$\Big\downarrow \text{ 770}$$

$$\frac{a\int \frac{1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{4b}-\frac{x(a-bx^4)^{3/4}}{4b}$$

$\downarrow$ 755

$$\frac{a\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b}-\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$\downarrow$ 1476

$$\frac{a\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\left(\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-}}}{2\sqrt{b}}\right.\right.}{4b}$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$\downarrow$ 1082

$$\frac{a\left(\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}\right)}{4b}-$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$\downarrow$ 217

$$\frac{a\left(\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)\right)}{4b}-$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$\downarrow$ 1479

$$a\left(\frac{\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\cdots}{4b}\right.$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \; 25$$

$$a\left(\frac{\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\left(\frac{\text{arc}\cdots}{\cdots}\right.}{4b}\right.$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \; 27$$

$$a\left(\frac{\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a-\cdots}}\right.}{\sqrt{2}\sqrt[4]{b}\cdots}\right.}{4b}\right.$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

$$\Big\downarrow \; 1103$$

$$\frac{a\left(\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{}}{\sqrt{a}}\right.}{2\sqrt{2}\sqrt[4]{b}}\right.\right.}{4b}$$

$$\frac{x\left(a-bx^4\right)^{3/4}}{4b}$$

input
```
Int[x^4/(a - b*x^4)^(1/4),x]
```

output
```
-1/4*(x*(a - b*x^4)^(3/4))/b + (a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a -
b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x
^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/2 + (-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x
^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1 + (
Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*S
qrt[2]*b^(1/4)))/2))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-b^{\frac{1}{4}}x}{b^{\frac{1}{4}}x}\right)\sqrt{2}\,a+\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+b^{\frac{1}{4}}x}{b^{\frac{1}{4}}x}\right)\sqrt{2}\,a+\frac{\ln\left(\frac{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)}{2}}{16b^{\frac{5}{4}}}$

input
```
int(x^4/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(arctan((2^(1/2)*(-b*x^4+a)^(1/4)-b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a+ar
ctan((2^(1/2)*(-b*x^4+a)^(1/4)+b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a+1/2*ln((-b^
(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2))/(b^(1/4)*(-
b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2)))*2^(1/2)*a+4*(-b*x^
4+a)^(3/4)*x*b^(1/4))/b^(5/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.25

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = -\frac{b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{b^4x\left(-\frac{a^4}{b^5}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}a^3}{x}\right)-b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\left(-\frac{b^4x\left(-\frac{a^4}{b^5}\right)^{\frac{3}{4}}-\left(-bx^4+a\right)^{\frac{1}{4}}a^3}{x}\right)-i\,b\left(-\frac{a^4}{b^5}\right)^{\frac{1}{4}}\log\Bigg(}{16\,b}$$

input
```
integrate(x^4/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/16*(b*(-a^4/b^5)^(1/4)*log((b^4*x*(-a^4/b^5)^(3/4) + (-b*x^4 + a)^(1/4)
*a^3)/x) - b*(-a^4/b^5)^(1/4)*log(-(b^4*x*(-a^4/b^5)^(3/4) - (-b*x^4 + a)^
(1/4)*a^3)/x) - I*b*(-a^4/b^5)^(1/4)*log((I*b^4*x*(-a^4/b^5)^(3/4) + (-b*x
^4 + a)^(1/4)*a^3)/x) + I*b*(-a^4/b^5)^(1/4)*log((-I*b^4*x*(-a^4/b^5)^(3/4
) + (-b*x^4 + a)^(1/4)*a^3)/x) + 4*(-b*x^4 + a)^(3/4)*x)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.23

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(-b*x**4+a)**(1/4),x)
```

output
```
x**5*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(1/4)*gamma(9/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.28

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}\left(-bx^4+a\right)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx\cdots}}{x^2}\cdots\right.}{b^{\frac{1}{4}}}\cdots}{32\,b} - \frac{\left(-bx^4+a\right)^{\frac{3}{4}}a}{4\left(b^2-\frac{\left(bx^4-a\right)b}{x^4}\right)x^3}$$

input
```
integrate(x^4/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4
)/x)/b^(1/4))/b^(1/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2
*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(1/4) - sqrt(2)*log(sqrt(b) + sqrt(2)*(-
b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4) + sqrt(2)*log(s
qrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1
/4))*a/b - 1/4*(-b*x^4 + a)^(3/4)*a/((b^2 - (b*x^4 - a)*b/x^4)*x^3)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^4}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^4}{(a-b\,x^4)^{1/4}}\,dx$$

input
```
int(x^4/(a - b*x^4)^(1/4),x)
```

output
```
int(x^4/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^4}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**4/(a - b*x**4)**(1/4),x)
```

3.682 $\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 148

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\sqrt[4]{b}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{2\sqrt{2}\sqrt[4]{b}}$$

output

```
1/4*arctan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(1/4)+1/4*arct
an(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(1/4)+1/4*arctanh(2^(1/
2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/b^
(1/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx = \frac{-2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)+2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)-\log\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)+\log\left(1+\right.}{4\sqrt{2}\sqrt[4]{b}}$$

input
```
Integrate[(a - b*x^4)^(-1/4),x]
```

output
```
(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] + 2*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] - Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4]
- (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)] + Log[1 + (Sqrt[b]*x^2)/Sqrt[a -
b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)])/(4*Sqrt[2]*b^(1/4))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {770, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx$$

$$\Big\downarrow \text{770}$$

$$\int \frac{1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\left(\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}\right)+$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)+$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2}\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}\right)+$$
$$\frac{1}{2}\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}\right)$$

input

```
Int[(a - b*x^4)^(-1/4),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))) +
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/2 + (
-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^
4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqr
t[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)+2\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)-2\arctan\left(-\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)\right)}{8b^{\frac{1}{4}}}$

input
```
int(1/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^(1/4)*2^(1/2)*(ln((-b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+
(-b*x^4+a)^(1/2))/(b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+
a)^(1/2)))+2*arctan(2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1)-2*arctan(-2^(1/2
)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx = -\frac{1}{4}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(\frac{bx\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{4}\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(-\frac{bx\left(-\frac{1}{b}\right)^{\frac{3}{4}}-\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right) + \frac{1}{4}i\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(\frac{i\,bx\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{4}i\left(-\frac{1}{b}\right)^{\frac{1}{4}}\log\left(\frac{-i\,bx\left(-\frac{1}{b}\right)^{\frac{3}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(1/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/4*(-1/b)^(1/4)*log((b*x*(-1/b)^(3/4) + (-b*x^4 + a)^(1/4))/x) + 1/4*(-1
/b)^(1/4)*log(-(b*x*(-1/b)^(3/4) - (-b*x^4 + a)^(1/4))/x) + 1/4*I*(-1/b)^(
1/4)*log((I*b*x*(-1/b)^(3/4) + (-b*x^4 + a)^(1/4))/x) - 1/4*I*(-1/b)^(1/4)
*log((-I*b*x*(-1/b)^(3/4) + (-b*x^4 + a)^(1/4))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-b*x**4+a)**(1/4),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(
1/4)*gamma(5/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{1}{4}}} - \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{1}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{8\,b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{b}-\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{8\,b^{\frac{1}{4}}}$$

input
```
integrate(1/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)
/b^(1/4))/b^(1/4) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(
-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(1/4) + 1/8*sqrt(2)*log(sqrt(b) + sqrt(2)*
(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(1/4) - 1/8*sqrt(2)
*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2
)/b^(1/4)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx=\frac{x\left(1-\frac{b\,x^4}{a}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{(a-b\,x^4)^{1/4}}$$

input
```
int(1/(a - b*x^4)^(1/4),x)
```

output
```
(x*(1 - (b*x^4)/a)^(1/4)*hypergeom([1/4, 1/4], 5/4, (b*x^4)/a))/(a - b*x^4
)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/(a - b*x**4)**(1/4),x)
```

3.683 $\int \frac{1}{x^4 \sqrt[4]{a-bx^4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}} dx = -\frac{(a-bx^4)^{3/4}}{3ax^3}$$

output
```
-1/3*(-b*x^4+a)^(3/4)/a/x^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}} dx = -\frac{(a-bx^4)^{3/4}}{3ax^3}$$

input
```
Integrate[1/(x^4*(a - b*x^4)^(1/4)),x]
```

output
```
-1/3*(a - b*x^4)^(3/4)/(a*x^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}} \, dx$$

$$\Big\downarrow \text{796}$$

$$-\frac{\left(a-bx^4\right)^{3/4}}{3ax^3}$$

input
```
Int[1/(x^4*(a - b*x^4)^(1/4)),x]
```

output
```
-1/3*(a - b*x^4)^(3/4)/(a*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(-b\,x^{4}+a)^{\frac{3}{4}}}{3a\,x^{3}}$	19
trager	$-\frac{(-b\,x^{4}+a)^{\frac{3}{4}}}{3a\,x^{3}}$	19
risch	$-\frac{(-b\,x^{4}+a)^{\frac{3}{4}}}{3a\,x^{3}}$	19
pseudoelliptic	$-\frac{(-b\,x^{4}+a)^{\frac{3}{4}}}{3a\,x^{3}}$	19
orering	$-\frac{(-b\,x^{4}+a)^{\frac{3}{4}}}{3a\,x^{3}}$	19

input
```
int(1/x^4/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-b*x^4+a)^(3/4)/a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{3}{4}}}{3\,ax^3}$$

input
```
integrate(1/x^4/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/3*(-b*x^4 + a)^(3/4)/(a*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.55

$$\int \frac{1}{x^4\sqrt[4]{a-bx^4}}\,dx = \begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^4}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{4a\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((b**(3/4)*(a/(b*x**4) - 1)**(3/4)*gamma(-3/4)/(4*a*gamma(1/4)),
Abs(a/(b*x**4)) > 1), (-b**(3/4)*(-a/(b*x**4) + 1)**(3/4)*exp(-I*pi/4)*gam
ma(-3/4)/(4*a*gamma(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt[4]{a-bx^4}}\,dx = -\frac{(-bx^4+a)^{\frac{3}{4}}}{3\,ax^3}$$

input
```
integrate(1/x^4/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/3*(-b*x^4 + a)^(3/4)/(a*x^3)
```

Giac [F]

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}\,dx = -\frac{(a-b\,x^4)^{3/4}}{3\,a\,x^3}$$

input
```
int(1/(x^4*(a - b*x^4)^(1/4)),x)
```

output
```
-(a - b*x^4)^(3/4)/(3*a*x^3)
```

Reduce [F]

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/x^4/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**4),x)
```

3.684 $\int \frac{1}{x^8 \sqrt[4]{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}} \, dx = -\frac{(a-bx^4)^{3/4}}{7ax^7} - \frac{4b(a-bx^4)^{3/4}}{21a^2x^3}$$

output
```
-1/7*(-b*x^4+a)^(3/4)/a/x^7-4/21*b*(-b*x^4+a)^(3/4)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}} \, dx = \frac{(-3a-4bx^4)\,(a-bx^4)^{3/4}}{21a^2x^7}$$

input
```
Integrate[1/(x^8*(a - b*x^4)^(1/4)),x]
```

output
```
((-3*a - 4*b*x^4)*(a - b*x^4)^(3/4))/(21*a^2*x^7)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}} \, dx$$
$$\Big\downarrow \text{803}$$
$$\frac{4b \int \frac{1}{x^4 \sqrt[4]{a-bx^4}}dx}{7a} - \frac{\left(a-bx^4\right)^{3/4}}{7ax^7}$$
$$\Big\downarrow \text{796}$$
$$-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3} - \frac{\left(a-bx^4\right)^{3/4}}{7ax^7}$$

input
```
Int[1/(x^8*(a - b*x^4)^(1/4)),x]
```

output
```
-1/7*(a - b*x^4)^(3/4)/(a*x^7) - (4*b*(a - b*x^4)^(3/4))/(21*a^2*x^3)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(4b\,x^{4}+3a\right)}{21a^{2}x^{7}}$	29
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(4b\,x^{4}+3a\right)}{21a^{2}x^{7}}$	29
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(4b\,x^{4}+3a\right)}{21a^{2}x^{7}}$	29
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(4b\,x^{4}+3a\right)}{21a^{2}x^{7}}$	29
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(4b\,x^{4}+3a\right)}{21a^{2}x^{7}}$	29

input
```
int(1/x^8/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(-b*x^4+a)^(3/4)*(4*b*x^4+3*a)/a^2/x^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^8\sqrt[4]{a-bx^4}}\,dx = -\frac{(4\,bx^4+3\,a)(-bx^4+a)^{\frac{3}{4}}}{21\,a^2x^7}$$

input
```
integrate(1/x^8/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/21*(4*b*x^4 + 3*a)*(-b*x^4 + a)^(3/4)/(a^2*x^7)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 303, normalized size of antiderivative = 6.59

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx = \begin{cases} -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^4}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{16ax^4\Gamma\left(\frac{1}{4}\right)} - \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^4}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{4a^2\Gamma\left(\frac{1}{4}\right)} & \text{for} \\ -\frac{3a^2b^{\frac{7}{4}}\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-16a^3bx^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+16a^2b^2x^8e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} - \frac{ab^{\frac{11}{4}}x^4\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-16a^3bx^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+16a^2b^2x^8e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} + \frac{4b^{\frac{15}{4}}x^8\left(-\frac{a}{bx^4}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-16a^3bx^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+16a^2b^2x^8e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} & \text{oth} \end{cases}$$

input
```
integrate(1/x**8/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-3*b**(3/4)*(a/(b*x**4) - 1)**(3/4)*gamma(-7/4)/(16*a*x**4*gamm
a(1/4)) - b**(7/4)*(a/(b*x**4) - 1)**(3/4)*gamma(-7/4)/(4*a**2*gamma(1/4))
, Abs(a/(b*x**4)) > 1), (-3*a**2*b**(7/4)*(-a/(b*x**4) + 1)**(3/4)*gamma(-
7/4)/(-16*a**3*b*x**4*exp(I*pi/4)*gamma(1/4) + 16*a**2*b**2*x**8*exp(I*pi/
4)*gamma(1/4)) - a*b**(11/4)*x**4*(-a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(-1
6*a**3*b*x**4*exp(I*pi/4)*gamma(1/4) + 16*a**2*b**2*x**8*exp(I*pi/4)*gamma
(1/4)) + 4*b**(15/4)*x**8*(-a/(b*x**4) + 1)**(3/4)*gamma(-7/4)/(-16*a**3*b
*x**4*exp(I*pi/4)*gamma(1/4) + 16*a**2*b**2*x**8*exp(I*pi/4)*gamma(1/4)),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{7\left(-bx^4+a\right)^{\frac{3}{4}}b}{x^3} + \frac{3\left(-bx^4+a\right)^{\frac{7}{4}}}{x^7}}{21\,a^2}$$

input
```
integrate(1/x^8/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/21*(7*(-b*x^4 + a)^(3/4)*b/x^3 + 3*(-b*x^4 + a)^(7/4)/x^7)/a^2
```

Giac [F]

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^8), x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx = -\frac{3\,a\,(a-b\,x^4)^{3/4}+4\,b\,x^4\,(a-b\,x^4)^{3/4}}{21\,a^2\,x^7}$$

input
```
int(1/(x^8*(a - b*x^4)^(1/4)),x)
```

output
```
-(3*a*(a - b*x^4)^(3/4) + 4*b*x^4*(a - b*x^4)^(3/4))/(21*a^2*x^7)
```

Reduce [F]

$$\int \frac{1}{x^8 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^8}dx$$

input
```
int(1/x^8/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**8),x)
```

3.685 $\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx=-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}-\frac{8b\left(a-bx^4\right)^{3/4}}{77a^2x^7}-\frac{32b^2\left(a-bx^4\right)^{3/4}}{231a^3x^3}$$

output
```
-1/11*(-b*x^4+a)^(3/4)/a/x^11-8/77*b*(-b*x^4+a)^(3/4)/a^2/x^7-32/231*b^2*(
-b*x^4+a)^(3/4)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx=\frac{\left(a-bx^4\right)^{3/4}\left(-21a^2-24abx^4-32b^2x^8\right)}{231a^3x^{11}}$$

input
```
Integrate[1/(x^12*(a - b*x^4)^(1/4)),x]
```

output
```
((a - b*x^4)^(3/4)*(-21*a^2 - 24*a*b*x^4 - 32*b^2*x^8))/(231*a^3*x^11)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{8b\int \frac{1}{x^8\sqrt[4]{a-bx^4}}dx}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{8b\left(\frac{4b\int \frac{1}{x^4\sqrt[4]{a-bx^4}}dx}{7a}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{8b\left(-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}$$

input
```
Int[1/(x^12*(a - b*x^4)^(1/4)),x]
```

output
```
-1/11*(a - b*x^4)^(3/4)/(a*x^11) + (8*b*(-1/7*(a - b*x^4)^(3/4)/(a*x^7) -
(4*b*(a - b*x^4)^(3/4))/(21*a^2*x^3)))/(11*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}+24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	40
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}+24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	40
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}+24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	40
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}+24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	40
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(32 b^{2} x^{8}+24 a b\,x^{4}+21 a^{2}\right)}{231 a^{3} x^{11}}$	40

input
```
int(1/x^12/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/231*(-b*x^4+a)^(3/4)*(32*b^2*x^8+24*a*b*x^4+21*a^2)/a^3/x^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx = -\frac{(32\,b^2x^8+24\,abx^4+21\,a^2)(-bx^4+a)^{\frac{3}{4}}}{231\,a^3x^{11}}$$

input
```
integrate(1/x^12/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/231*(32*b^2*x^8 + 24*a*b*x^4 + 21*a^2)*(-b*x^4 + a)^(3/4)/(a^3*x^11)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 1068, normalized size of antiderivative = 15.04

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**12/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-21*a**4*b**(19/4)*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma
(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*ex
p(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) + 18*a**
3*b**(23/4)*x**4*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(64*a
**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gam
ma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) - 5*a**2*b**(27/4)*x*
*8*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(64*a**5*b**4*x**8*
exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a
**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) + 40*a*b**(31/4)*x**12*(a/(b*x**4)
- 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gam
ma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*
exp(I*pi/4)*gamma(1/4)) - 32*b**(35/4)*x**16*(a/(b*x**4) - 1)**(3/4)*exp(-
3*I*pi/4)*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**
4*b**5*x**12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma
(1/4)), Abs(a/(b*x**4)) > 1), (-21*a**4*b**(19/4)*(-a/(b*x**4) + 1)**(3/4)
*gamma(-11/4)/(64*a**5*b**4*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x*
*12*exp(I*pi/4)*gamma(1/4) + 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) +
18*a**3*b**(23/4)*x**4*(-a/(b*x**4) + 1)**(3/4)*gamma(-11/4)/(64*a**5*b**4
*x**8*exp(I*pi/4)*gamma(1/4) - 128*a**4*b**5*x**12*exp(I*pi/4)*gamma(1/4)
+ 64*a**3*b**6*x**16*exp(I*pi/4)*gamma(1/4)) - 5*a**2*b**(27/4)*x**8*(-...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{77\,(-bx^4+a)^{\frac{3}{4}}b^2}{x^3}+\frac{66\,(-bx^4+a)^{\frac{7}{4}}b}{x^7}+\frac{21\,(-bx^4+a)^{\frac{11}{4}}}{x^{11}}}{231\,a^3}$$

input
```
integrate(1/x^12/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/231*(77*(-b*x^4 + a)^(3/4)*b^2/x^3 + 66*(-b*x^4 + a)^(7/4)*b/x^7 + 21*(
-b*x^4 + a)^(11/4)/x^11)/a^3
```

Giac [F]

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{12}}\,dx$$

input
```
integrate(1/x^12/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^12), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx = -\frac{21\,a^2\,{\left(a-b\,x^4\right)}^{3/4}+32\,b^2\,x^8\,{\left(a-b\,x^4\right)}^{3/4}+24\,a\,b\,x^4\,{\left(a-b\,x^4\right)}^{3/4}}{231\,a^3\,x^{11}}$$

input
```
int(1/(x^12*(a - b*x^4)^(1/4)),x)
```

output
```
-(21*a^2*(a - b*x^4)^(3/4) + 32*b^2*x^8*(a - b*x^4)^(3/4) + 24*a*b*x^4*(a
- b*x^4)^(3/4))/(231*a^3*x^11)
```

Reduce [F]

$$\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^{12}}dx$$

input
```
int(1/x^12/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**12),x)
```

3.686 $\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{15ax^{15}} - \frac{4b(a-bx^4)^{3/4}}{55a^2x^{11}} - \frac{32b^2(a-bx^4)^{3/4}}{385a^3x^7} - \frac{128b^3(a-bx^4)^{3/4}}{1155a^4x^3}$$

output
```
-1/15*(-b*x^4+a)^(3/4)/a/x^15-4/55*b*(-b*x^4+a)^(3/4)/a^2/x^11-32/385*b^2*
(-b*x^4+a)^(3/4)/a^3/x^7-128/1155*b^3*(-b*x^4+a)^(3/4)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = \frac{(a-bx^4)^{3/4}(-77a^3-84a^2bx^4-96ab^2x^8-128b^3x^{12})}{1155a^4x^{15}}$$

input
```
Integrate[1/(x^16*(a - b*x^4)^(1/4)),x]
```

output
```
((a - b*x^4)^(3/4)*(-77*a^3 - 84*a^2*b*x^4 - 96*a*b^2*x^8 - 128*b^3*x^12))
/(1155*a^4*x^15)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{4b\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}dx}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{4b\left(\frac{8b\int \frac{1}{x^8\sqrt[4]{a-bx^4}}dx}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{4b\left(\frac{8b\left(\frac{4b\int \frac{1}{x^4\sqrt[4]{a-bx^4}}dx}{7a}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{4b\left(\frac{8b\left(-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}$$

input
```
Int[1/(x^16*(a - b*x^4)^(1/4)),x]
```

output
```
-1/15*(a - b*x^4)^(3/4)/(a*x^15) + (4*b*(-1/11*(a - b*x^4)^(3/4)/(a*x^11)
+ (8*b*(-1/7*(a - b*x^4)^(3/4)/(a*x^7) - (4*b*(a - b*x^4)^(3/4))/(21*a^2*x
^3)))/(11*a)))/(5*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a \right)^{\frac{3}{4}} \left(128 b^{3} x^{12}+96 a\,b^{2} x^{8}+84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	51
trager	$-\frac{\left(-b\,x^{4}+a \right)^{\frac{3}{4}} \left(128 b^{3} x^{12}+96 a\,b^{2} x^{8}+84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	51
risch	$-\frac{\left(-b\,x^{4}+a \right)^{\frac{3}{4}} \left(128 b^{3} x^{12}+96 a\,b^{2} x^{8}+84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	51
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a \right)^{\frac{3}{4}} \left(128 b^{3} x^{12}+96 a\,b^{2} x^{8}+84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	51
orering	$-\frac{\left(-b\,x^{4}+a \right)^{\frac{3}{4}} \left(128 b^{3} x^{12}+96 a\,b^{2} x^{8}+84 a^{2} b\,x^{4}+77 a^{3}\right)}{1155 x^{15} a^{4}}$	51

input
```
int(1/x^16/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(-b*x^4+a)^(3/4)*(128*b^3*x^12+96*a*b^2*x^8+84*a^2*b*x^4+77*a^3)/x
^15/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = -\frac{(128\,b^3x^{12}+96\,ab^2x^8+84\,a^2bx^4+77\,a^3)(-bx^4+a)^{\frac{3}{4}}}{1155\,a^4x^{15}}$$

input
```
integrate(1/x^16/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/1155*(128*b^3*x^12 + 96*a*b^2*x^8 + 84*a^2*b*x^4 + 77*a^3)*(-b*x^4 + a)
^(3/4)/(a^4*x^15)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 1821, normalized size of antiderivative = 18.97

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**16/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((231*a**6*b**(39/4)*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-1
5/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*e
xp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*
a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) - 441*a**5*b**(43/4)*x**4*(a/(b*x
**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4
)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**1
1*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/
4)) + 225*a**4*b**(47/4)*x**8*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-1
5/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*e
xp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256*
a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) + 45*a**3*b**(51/4)*x**12*(a/(b*x
**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4
)*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b**1
1*x**20*exp(I*pi/4)*gamma(1/4) + 256*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/
4)) - 540*a**2*b**(55/4)*x**16*(a/(b*x**4) - 1)**(3/4)*exp(I*pi/4)*gamma(-
15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)*gamma(1/4) + 768*a**6*b**10*x**16*
exp(I*pi/4)*gamma(1/4) - 768*a**5*b**11*x**20*exp(I*pi/4)*gamma(1/4) + 256
*a**4*b**12*x**24*exp(I*pi/4)*gamma(1/4)) + 864*a*b**(59/4)*x**20*(a/(b*x*
*4) - 1)**(3/4)*exp(I*pi/4)*gamma(-15/4)/(-256*a**7*b**9*x**12*exp(I*pi/4)
*gamma(1/4) + 768*a**6*b**10*x**16*exp(I*pi/4)*gamma(1/4) - 768*a**5*b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{385\,(-bx^4+a)^{\frac{3}{4}}b^3}{x^3} + \frac{495\,(-bx^4+a)^{\frac{7}{4}}b^2}{x^7} + \frac{315\,(-bx^4+a)^{\frac{11}{4}}b}{x^{11}} + \frac{77\,(-bx^4+a)^{\frac{15}{4}}}{x^{15}}}{1155\,a^4}$$

input
```
integrate(1/x^16/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/1155*(385*(-b*x^4 + a)^(3/4)*b^3/x^3 + 495*(-b*x^4 + a)^(7/4)*b^2/x^7 +
 315*(-b*x^4 + a)^(11/4)*b/x^11 + 77*(-b*x^4 + a)^(15/4)/x^15)/a^4
```

Giac [F]

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{16}}\,dx$$

input
```
integrate(1/x^16/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^16), x)
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-b\,x^4)^{3/4}}{15\,a\,x^{15}} - \frac{4\,b\,(a-b\,x^4)^{3/4}}{55\,a^2\,x^{11}} - \frac{128\,b^3\,(a-b\,x^4)^{3/4}}{1155\,a^4\,x^3} - \frac{32\,b^2\,(a-b\,x^4)^{3/4}}{385\,a^3\,x^7}$$

input
```
int(1/(x^16*(a - b*x^4)^(1/4)),x)
```

output
```
- (a - b*x^4)^(3/4)/(15*a*x^15) - (4*b*(a - b*x^4)^(3/4))/(55*a^2*x^11) -
(128*b^3*(a - b*x^4)^(3/4))/(1155*a^4*x^3) - (32*b^2*(a - b*x^4)^(3/4))/(3
85*a^3*x^7)
```

Reduce [F]

$$\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^{16}}dx$$

input
```
int(1/x^16/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**16),x)
```

3.687 $\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{19ax^{19}} - \frac{16b(a-bx^4)^{3/4}}{285a^2x^{15}} - \frac{64b^2(a-bx^4)^{3/4}}{1045a^3x^{11}} - \frac{512b^3(a-bx^4)^{3/4}}{7315a^4x^7} - \frac{2048b^4(a-bx^4)^{3/4}}{21945a^5x^3}$$

output
```
-1/19*(-b*x^4+a)^(3/4)/a/x^19-16/285*b*(-b*x^4+a)^(3/4)/a^2/x^15-64/1045*b
^2*(-b*x^4+a)^(3/4)/a^3/x^11-512/7315*b^3*(-b*x^4+a)^(3/4)/a^4/x^7-2048/21
945*b^4*(-b*x^4+a)^(3/4)/a^5/x^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx = \frac{(a-bx^4)^{3/4}(-1155a^4-1232a^3bx^4-1344a^2b^2x^8-1536ab^3x^{12}-2048b^4x^{16})}{21945a^5x^{19}}$$

input
```
Integrate[1/(x^20*(a - b*x^4)^(1/4)),x]
```

output
```
((a - b*x^4)^(3/4)*(-1155*a^4 - 1232*a^3*b*x^4 - 1344*a^2*b^2*x^8 - 1536*a
*b^3*x^12 - 2048*b^4*x^16))/(21945*a^5*x^19)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {803, 803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{16b\int \frac{1}{x^{16}\sqrt[4]{a-bx^4}}dx}{19a}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{16b\left(\frac{4b\int \frac{1}{x^{12}\sqrt[4]{a-bx^4}}dx}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{16b\left(\frac{4b\left(\frac{8b\int \frac{1}{x^{8}\sqrt[4]{a-bx^4}}dx}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{16b\left(\frac{4b\left(\frac{8b\left(\frac{4b\int \frac{1}{x^4\sqrt[4]{a-bx^4}}dx}{7a}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

$$\Big\downarrow \text{796}$$

$$\frac{16b\left(\frac{4b\left(\frac{8b\left(-\frac{4b\left(a-bx^4\right)^{3/4}}{21a^2x^3}-\frac{\left(a-bx^4\right)^{3/4}}{7ax^7}\right)}{11a}-\frac{\left(a-bx^4\right)^{3/4}}{11ax^{11}}\right)}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{15ax^{15}}\right)}{19a}-\frac{\left(a-bx^4\right)^{3/4}}{19ax^{19}}$$

input
```
Int[1/(x^20*(a - b*x^4)^(1/4)),x]
```

output
```
-1/19*(a - b*x^4)^(3/4)/(a*x^19) + (16*b*(-1/15*(a - b*x^4)^(3/4)/(a*x^15)
 + (4*b*(-1/11*(a - b*x^4)^(3/4)/(a*x^11) + (8*b*(-1/7*(a - b*x^4)^(3/4)/(
a*x^7) - (4*b*(a - b*x^4)^(3/4))/(21*a^2*x^3)))/(11*a)))/(5*a)))/(19*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.51

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(2048 x^{16} b^{4}+1536 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1232 a^{3} b\,x^{4}+1155 a^{4}\right)}{21945 x^{19} a^{5}}$	62
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(2048 x^{16} b^{4}+1536 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1232 a^{3} b\,x^{4}+1155 a^{4}\right)}{21945 x^{19} a^{5}}$	62
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(2048 x^{16} b^{4}+1536 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1232 a^{3} b\,x^{4}+1155 a^{4}\right)}{21945 x^{19} a^{5}}$	62
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(2048 x^{16} b^{4}+1536 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1232 a^{3} b\,x^{4}+1155 a^{4}\right)}{21945 x^{19} a^{5}}$	62
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{3}{4}} \left(2048 x^{16} b^{4}+1536 a\,b^{3} x^{12}+1344 a^{2} b^{2} x^{8}+1232 a^{3} b\,x^{4}+1155 a^{4}\right)}{21945 x^{19} a^{5}}$	62

input
```
int(1/x^20/(-b*x^4+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/21945*(-b*x^4+a)^(3/4)*(2048*b^4*x^16+1536*a*b^3*x^12+1344*a^2*b^2*x^8+
1232*a^3*b*x^4+1155*a^4)/x^19/a^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx = -\frac{(2048\,b^4x^{16}+1536\,ab^3x^{12}+1344\,a^2b^2x^8+1232\,a^3bx^4+1155\,a^4)(-bx^4+a)^{\frac{3}{4}}}{21945\,a^5x^{19}}$$

input
```
integrate(1/x^20/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/21945*(2048*b^4*x^16 + 1536*a*b^3*x^12 + 1344*a^2*b^2*x^8 + 1232*a^3*b*
x^4 + 1155*a^4)*(-b*x^4 + a)^(3/4)/(a^5*x^19)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 2788, normalized size of antiderivative = 23.04

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**20/(-b*x**4+a)**(1/4),x)
```

output
```
Piecewise((-3465*a**8*b**(67/4)*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gam
ma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*
x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4
) - 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*e
xp(I*pi/4)*gamma(1/4)) + 10164*a**7*b**(71/4)*x**4*(a/(b*x**4) - 1)**(3/4)
*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4)
 - 4096*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*ex
p(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4)*gamma(1/4) + 1024
*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)) - 10038*a**6*b**(75/4)*x**8*(a/(
b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a**9*b**16*x**16*exp
(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)*gamma(1/4) + 6144*
a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**19*x**28*exp(I*pi/4
)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(1/4)) + 3204*a**5*b
**(79/4)*x**12*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-19/4)/(1024*a
**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17*x**20*exp(I*pi/4)
*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma(1/4) - 4096*a**6*b**
19*x**28*exp(I*pi/4)*gamma(1/4) + 1024*a**5*b**20*x**32*exp(I*pi/4)*gamma(
1/4)) - 585*a**4*b**(83/4)*x**16*(a/(b*x**4) - 1)**(3/4)*exp(-3*I*pi/4)*ga
mma(-19/4)/(1024*a**9*b**16*x**16*exp(I*pi/4)*gamma(1/4) - 4096*a**8*b**17
*x**20*exp(I*pi/4)*gamma(1/4) + 6144*a**7*b**18*x**24*exp(I*pi/4)*gamma...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx = -\frac{\frac{7315\,(-bx^4+a)^{\frac{3}{4}}b^4}{x^3}+\frac{12540\,(-bx^4+a)^{\frac{7}{4}}b^3}{x^7}+\frac{11970\,(-bx^4+a)^{\frac{11}{4}}b^2}{x^{11}}+\frac{5852\,(-bx^4+a)^{\frac{15}{4}}b}{x^{15}}+\frac{1155\,(-bx^4+a)^{\frac{19}{4}}}{x^{19}}}{21945\,a^5}$$

input
```
integrate(1/x^20/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
-1/21945*(7315*(-b*x^4 + a)^(3/4)*b^4/x^3 + 12540*(-b*x^4 + a)^(7/4)*b^3/x
^7 + 11970*(-b*x^4 + a)^(11/4)*b^2/x^11 + 5852*(-b*x^4 + a)^(15/4)*b/x^15
+ 1155*(-b*x^4 + a)^(19/4)/x^19)/a^5
```

Giac [F]

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{20}}\,dx$$

input
```
integrate(1/x^20/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^20), x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx=-\frac{(a-b\,x^4)^{3/4}}{19\,a\,x^{19}}-\frac{16\,b\,(a-b\,x^4)^{3/4}}{285\,a^2\,x^{15}}-\frac{2048\,b^4\,(a-b\,x^4)^{3/4}}{21945\,a^5\,x^3}-\frac{512\,b^3\,(a-b\,x^4)^{3/4}}{7315\,a^4\,x^7}-\frac{64\,b^2\,(a-b\,x^4)^{3/4}}{1045\,a^3\,x^{11}}$$

input
```
int(1/(x^20*(a - b*x^4)^(1/4)),x)
```

output
```
- (a - b*x^4)^(3/4)/(19*a*x^19) - (16*b*(a - b*x^4)^(3/4))/(285*a^2*x^15)
- (2048*b^4*(a - b*x^4)^(3/4))/(21945*a^5*x^3) - (512*b^3*(a - b*x^4)^(3/4
))/(7315*a^4*x^7) - (64*b^2*(a - b*x^4)^(3/4))/(1045*a^3*x^11)
```

Reduce [F]

$$\int \frac{1}{x^{20}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^{20}}dx$$

input
```
int(1/x^20/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**20),x)
```

3.688 $\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx=-\frac{7a^2(a-bx^4)^{3/4}}{40b^3x}-\frac{7ax^3(a-bx^4)^{3/4}}{60b^2}-\frac{x^7(a-bx^4)^{3/4}}{10b}+\frac{7a^{5/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{40b^{5/2}\sqrt[4]{a-bx^4}}$$

output
```
-7/40*a^2*(-b*x^4+a)^(3/4)/b^3/x-7/60*a*x^3*(-b*x^4+a)^(3/4)/b^2-1/10*x^7*
(-b*x^4+a)^(3/4)/b+7/40*a^(5/2)*(1-a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arcc
sc(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx=\frac{x^3\left(-7a^2+abx^4+6b^2x^8+7a^2\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)\right)}{60b^2\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^10/(a - b*x^4)^(1/4),x]
```

output
```
(x^3*(-7*a^2 + a*b*x^4 + 6*b^2*x^8 + 7*a^2*(1 - (b*x^4)/a)^(1/4)*Hypergeom
etric2F1[1/4, 3/4, 7/4, (b*x^4)/a]))/(60*b^2*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {843, 843, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{7a\int \frac{x^6}{\sqrt[4]{a-bx^4}}dx}{10b}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

$$\Big\downarrow \text{843}$$

$$\frac{7a\left(\frac{a\int \frac{x^2}{\sqrt[4]{a-bx^4}}dx}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}\right)}{10b}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

$$\Big\downarrow \text{840}$$

$$\frac{7a\left(\frac{a\left(-\frac{a\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{2b}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}\right)}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}\right)}{10b}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

$$\Big\downarrow \text{842}$$

$$\frac{7a\left(\frac{a\left(-\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{2b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}\right)}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}\right)}{10b}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

$\downarrow$ 858

$$\frac{7a\left(\frac{a\left(\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{2b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}\right)}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}\right)}{10b}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

$\downarrow$ 807

$$\frac{7a\left(\frac{a\left(\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{4b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}\right)}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}\right)}{10b}-\frac{x^7\left(a-bx^4\right)^{3/4}}{10b}$$

$$\downarrow 226$$

$$\frac{7a\left(\frac{a\left(\frac{\sqrt{a}x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{2bx}\right)}{2b}-\frac{x^3(a-bx^4)^{3/4}}{6b}\right)}{10b}-\frac{x^7(a-bx^4)^{3/4}}{10b}$$

input
```
Int[x^10/(a - b*x^4)^(1/4),x]
```

output
```
-1/10*(x^7*(a - b*x^4)^(3/4))/b + (7*a*(-1/6*(x^3*(a - b*x^4)^(3/4))/b + (
a*(-1/2*(a - b*x^4)^(3/4)/(b*x) + (Sqrt[a]*(1 - a/(b*x^4))^(1/4)*x*Ellipti
cE[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(1/4))))/(2
*b)))/(10*b)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 840
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{10}}{(-b\,x^{4}+a)^{\frac{1}{4}}}dx$$

input
```
int(x^10/(-b*x^4+a)^(1/4),x)
```

output
```
int(x^10/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{10}}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x^10/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx=\frac{x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(-b*x**4+a)**(1/4),x)
```

output
```
x**11*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/
(4*a**(1/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^{10}}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^10/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^{10}}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^10/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{10}}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

input
```
int(x^10/(a - b*x^4)^(1/4),x)
```

output
```
int(x^10/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^{10}}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^{10}}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^10/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**10/(a - b*x**4)**(1/4),x)
```

3.689 $\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx = -\frac{a(a-bx^4)^{3/4}}{4b^2x} - \frac{x^3(a-bx^4)^{3/4}}{6b} + \frac{a^{3/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4b^{3/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/4*a*(-b*x^4+a)^(3/4)/b^2/x-1/6*x^3*(-b*x^4+a)^(3/4)/b+1/4*a^(3/2)*(1-a/
b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccsc(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(
3/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^3\left(-a+bx^4+a\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)\right)}{6b\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^6/(a - b*x^4)^(1/4),x]
```

output
```
(x^3*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4
, (b*x^4)/a]))/(6*b*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {843, 840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{a\int \frac{x^2}{\sqrt[4]{a-bx^4}}dx}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}$$

$$\Big\downarrow \text{840}$$

$$\frac{a\left(-\frac{a\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{2b}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}\right)}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}$$

$$\Big\downarrow \text{842}$$

$$\frac{a\left(-\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{2b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}\right)}{2b}-\frac{x^3\left(a-bx^4\right)^{3/4}}{6b}$$

$$\Big\downarrow \text{858}$$

$$\frac{a\left(\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{2b\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{2bx}\right)}{2b}-\frac{x^3(a-bx^4)^{3/4}}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{a\left(\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{4b\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{2bx}\right)}{2b}-\frac{x^3(a-bx^4)^{3/4}}{6b}$$

$$\Big\downarrow \text{226}$$

$$\frac{a\left(\frac{\sqrt{a}x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{2bx}\right)}{2b}-\frac{x^3(a-bx^4)^{3/4}}{6b}$$

input
```
Int[x^6/(a - b*x^4)^(1/4),x]
```

output
```
-1/6*(x^3*(a - b*x^4)^(3/4))/b + (a*(-1/2*(a - b*x^4)^(3/4)/(b*x) + (Sqrt[
a]*(1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/
(2*Sqrt[b]*(a - b*x^4)^(1/4))))/(2*b)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 840
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^6}{(-b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(-b*x^4+a)^(1/4),x)
```

output
```
int(x^6/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^6}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x^6/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx=\frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\,\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(-b*x**4+a)**(1/4),x)
```

output
```
x**7*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
a**(1/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx=\int \frac{x^6}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^6}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^6}{\left(a-b\,x^4\right)^{1/4}}\,dx$$

input
```
int(x^6/(a - b*x^4)^(1/4),x)
```

output
```
int(x^6/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^6}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**6/(a - b*x**4)**(1/4),x)
```

3.690 $\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 86

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx=-\frac{(a-bx^4)^{3/4}}{2bx}+\frac{\sqrt{a}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx^4}}$$

output
```
-1/2*(-b*x^4+a)^(3/4)/b/x+1/2*a^(1/2)*(1-a/b/x^4)^(1/4)*x*EllipticE(sin(1/
2*arccsc(b^(1/2)*x^2/a^(1/2))),2^(1/2))/b^(1/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx=\frac{x^3\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{bx^4}{a}\right)}{3\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^2/(a - b*x^4)^(1/4),x]
```

output
```
(x^3*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (b*x^4)/a])/(3
*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {840, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{ 840}$$

$$-\frac{a\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{2b}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

$$\Big\downarrow \text{ 842}$$

$$-\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{2b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{2b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{ax\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{4b\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{\sqrt{a}x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{2\sqrt{b}\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{2bx}$$

input
```
Int[x^2/(a - b*x^4)^(1/4),x]
```

output
```
-1/2*(a - b*x^4)^(3/4)/(b*x) + (Sqrt[a]*(1 - a/(b*x^4))^(1/4)*x*EllipticE[
ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 840
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(1/4), x_Symbol] :> Simp[(a + b*x^4)^(3/4)
/(2*b*x), x] + Simp[a/(2*b)   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^2}{\left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(-b*x^4+a)^(1/4),x)
```

output
```
int(x^2/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^2}{\left(-bx^4+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)*x^2/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-b*x**4+a)**(1/4),x)
```

output
```
x**3*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(1/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^4 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^4 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^2}{(a-b\,x^4)^{1/4}}\,dx$$

input
```
int(x^2/(a - b*x^4)^(1/4),x)
```

output
```
int(x^2/(a - b*x^4)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^4}}\,dx = \int \frac{x^2}{(-b\,x^4+a)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(-b*x^4+a)^(1/4),x)
```

output
```
int(x**2/(a - b*x**4)**(1/4),x)
```

3.691 $\int \frac{1}{x^2 \sqrt[4]{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^4}} \, dx = -\frac{\sqrt{b}\sqrt[4]{1-\frac{a}{bx^4}}\, x E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^4}}$$

output
```
-b^(1/2)*(1-a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccsc(b^(1/2)*x^2/a^(1/2))
),2^(1/2))/a^(1/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^4}} \, dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}}\, \text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{bx^4}{a}\right)}{x\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^2*(a - b*x^4)^(1/4)),x]
```

output
```
-(((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x^4)/a])/(x*
(a - b*x^4)^(1/4)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^4}} \, dx$$

$$\Big\downarrow \text{ 842}$$

$$\frac{x \sqrt[4]{1-\frac{a}{bx^4}} \int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}} x^3} dx}{\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{x \sqrt[4]{1-\frac{a}{bx^4}} \int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}} x} d\frac{1}{x}}{\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{x \sqrt[4]{1-\frac{a}{bx^4}} \int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}} d\frac{1}{x^2}}{2 \sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{ 226}$$

$$-\frac{\sqrt{b} x \sqrt[4]{1-\frac{a}{bx^4}} E\left(\frac{1}{2} \arcsin\left(\frac{\sqrt{a}}{\sqrt{b} x^2}\right)\middle|2\right)}{\sqrt{a} \sqrt[4]{a-bx^4}}$$

input
```
Int[1/(x^2*(a - b*x^4)^(1/4)),x]
```

output
```
-((Sqrt[b]*(1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]
/2, 2])/(Sqrt[a]*(a - b*x^4)^(1/4)))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(-b \,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^2/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^2/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^6 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx=\frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{2\sqrt[4]{b}x^2}$$

input
```
integrate(1/x**2/(-b*x**4+a)**(1/4),x)
```

output
```
I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x**4))/(2*b**(1/4)*x**2)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx=-\frac{\left(1-\frac{a}{b\,x^4}\right)^{1/4}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{a}{b\,x^4}\right)}{2\,x\,\left(a-b\,x^4\right)^{1/4}}$$

input
```
int(1/(x^2*(a - b*x^4)^(1/4)),x)
```

output
```
-((1 - a/(b*x^4))^(1/4)*hypergeom([1/4, 1/2], 3/2, a/(b*x^4)))/(2*x*(a - b
*x^4)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^2}dx$$

input
```
int(1/x^2/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**2),x)
```

3.692 $\int \frac{1}{x^6 \sqrt[4]{a-bx^4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 86

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^4}} \, dx = -\frac{(a-bx^4)^{3/4}}{5ax^5} - \frac{2b^{3/2} \sqrt[4]{1-\frac{a}{bx^4}} x E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/5*(-b*x^4+a)^(3/4)/a/x^5-2/5*b^(3/2)*(1-a/b/x^4)^(1/4)*x*EllipticE(sin(
1/2*arccsc(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^4}} \, dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},\frac{bx^4}{a}\right)}{5x^5\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^6*(a - b*x^4)^(1/4)),x]
```

output
```
-1/5*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (b*x^4)/a])
/(x^5*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {847, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{2b\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{842}$$

$$\frac{2bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{807}$$

$$-\frac{bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2b^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}$$

input
```
Int[1/(x^6*(a - b*x^4)^(1/4)),x]
```

output
```
-1/5*(a - b*x^4)^(3/4)/(a*x^5) - (2*b^(3/2)*(1 - a/(b*x^4))^(1/4)*x*Ellipt
icE[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*a^(3/2)*(a - b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^6 \left(-b\,x^4+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^6/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^6/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{\left(-bx^4+a\right)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^10 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^4}}\,dx = -\frac{ie^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{6\sqrt[4]{b}x^6}$$

input
```
integrate(1/x**6/(-b*x**4+a)**(1/4),x)
```

output
```
-I*exp(-3*I*pi/4)*hyper((1/4, 3/2), (5/2,), a/(b*x**4))/(6*b**(1/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{x^6\,\left(a-b\,x^4\right)^{1/4}}\,dx$$

input
```
int(1/(x^6*(a - b*x^4)^(1/4)),x)
```

output
```
int(1/(x^6*(a - b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{\left(-b\,x^4+a\right)^{\frac{1}{4}} x^6}dx$$

input
```
int(1/x^6/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**6),x)
```

3.693 $\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{9ax^9} - \frac{2b(a-bx^4)^{3/4}}{15a^2x^5} - \frac{4b^{5/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/9*(-b*x^4+a)^(3/4)/a/x^9-2/15*b*(-b*x^4+a)^(3/4)/a^2/x^5-4/15*b^(5/2)*(
1-a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccsc(b^(1/2)*x^2/a^(1/2))),2^(1/2))
/a^(5/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},\frac{bx^4}{a}\right)}{9x^9\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^10*(a - b*x^4)^(1/4)),x]
```

output
```
-1/9*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, (b*x^4)/a])
/(x^9*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {847, 847, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx$$

$$\Big\downarrow \; 847$$

$$\frac{2b\int \frac{1}{x^6\sqrt[4]{a-bx^4}}dx}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \; 847$$

$$\frac{2b\left(\frac{2b\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \; 842$$

$$\frac{2b\left(\frac{2bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}$$

$$\Big\downarrow \; 858$$

$$\frac{2b\left(-\frac{2bx\sqrt[4]{1-\frac{a}{bx^4}}\int\frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{5a\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a-bx^4)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{807}$$

$$\frac{2b\left(-\frac{bx\sqrt[4]{1-\frac{a}{bx^4}}\int\frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{5a\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a-bx^4)^{3/4}}{9ax^9}$$

$$\Big\downarrow \text{226}$$

$$\frac{2b\left(-\frac{2b^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a-bx^4)^{3/4}}{9ax^9}$$

input
```
Int[1/(x^10*(a - b*x^4)^(1/4)),x]
```

output
```
-1/9*(a - b*x^4)^(3/4)/(a*x^9) + (2*b*(-1/5*(a - b*x^4)^(3/4)/(a*x^5) - (2
*b^(3/2)*(1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2
, 2])/(5*a^(3/2)*(a - b*x^4)^(1/4))))/(3*a)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{10} \left(-b\,x^{4}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^10/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^10/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^14 - a*x^10), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx = \frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{a}{bx^4}\right)}{10\sqrt[4]{b}x^{10}}$$

input
```
integrate(1/x**10/(-b*x**4+a)**(1/4),x)
```

output
```
I*exp(I*pi/4)*hyper((1/4, 5/2), (7/2,), a/(b*x**4))/(10*b**(1/4)*x**10)
```

Maxima [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^10), x)
```

Giac [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^10), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{x^{10}\,(a-b\,x^4)^{1/4}}\,dx$$

input
```
int(1/(x^10*(a - b*x^4)^(1/4)),x)
```

output
```
int(1/(x^10*(a - b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{(-b\,x^4+a)^{\frac{1}{4}}\,x^{10}}dx$$

input
```
int(1/x^10/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**10),x)
```

3.694 $\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx = -\frac{(a-bx^4)^{3/4}}{13ax^{13}} - \frac{10b(a-bx^4)^{3/4}}{117a^2x^9} - \frac{4b^2(a-bx^4)^{3/4}}{39a^3x^5} - \frac{8b^{7/2}\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{39a^{7/2}\sqrt[4]{a-bx^4}}$$

output
```
-1/13*(-b*x^4+a)^(3/4)/a/x^13-10/117*b*(-b*x^4+a)^(3/4)/a^2/x^9-4/39*b^2*(
-b*x^4+a)^(3/4)/a^3/x^5-8/39*b^(7/2)*(1-a/b/x^4)^(1/4)*x*EllipticE(sin(1/2
*arccsc(b^(1/2)*x^2/a^(1/2))),2^(1/2))/a^(7/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(-\frac{13}{4},\frac{1}{4},-\frac{9}{4},\frac{bx^4}{a}\right)}{13x^{13}\sqrt[4]{a-bx^4}}$$

input
```
Integrate[1/(x^14*(a - b*x^4)^(1/4)),x]
```

output
```
-1/13*((1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[-13/4, 1/4, -9/4, (b*x^4)/a
])/(x^13*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {847, 847, 847, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{10b\int \frac{1}{x^{10}\sqrt[4]{a-bx^4}}dx}{13a}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{847}$$

$$\frac{10b\left(\frac{2b\int \frac{1}{x^6\sqrt[4]{a-bx^4}}dx}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{847}$$

$$\frac{10b\left(\frac{2b\left(\frac{2b\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{5a}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{842}$$

$$\frac{10b\left(\frac{2b\left(\frac{2bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{10b\left(\frac{2b\left(-\frac{2bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{10b\left(\frac{2b\left(-\frac{bx\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{5a\sqrt[4]{a-bx^4}}-\frac{\left(a-bx^4\right)^{3/4}}{5ax^5}\right)}{3a}-\frac{\left(a-bx^4\right)^{3/4}}{9ax^9}\right)}{13a}-\frac{\left(a-bx^4\right)^{3/4}}{13ax^{13}}$$

$$\Big\downarrow \text{226}$$

$$\frac{10b\left(\frac{2b\left(-\frac{2b^{3/2}x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a-bx^4}}-\frac{(a-bx^4)^{3/4}}{5ax^5}\right)}{3a}-\frac{(a-bx^4)^{3/4}}{9ax^9}\right)}{13a}-\frac{(a-bx^4)^{3/4}}{13ax^{13}}$$

input
```
Int[1/(x^14*(a - b*x^4)^(1/4)),x]
```

output
```
-1/13*(a - b*x^4)^(3/4)/(a*x^13) + (10*b*(-1/9*(a - b*x^4)^(3/4)/(a*x^9) +
 (2*b*(-1/5*(a - b*x^4)^(3/4)/(a*x^5) - (2*b^(3/2)*(1 - a/(b*x^4))^(1/4)*x
*EllipticE[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(5*a^(3/2)*(a - b*x^4)^(1/
4))))/(3*a)))/(13*a)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{14} \left(-b\,x^{4}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^14/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/x^14/(-b*x^4+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{14} \sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{{\left(-bx^{4}+a\right)}^{\frac{1}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(-b*x^4+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(3/4)/(b*x^18 - a*x^14), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx = -\frac{ie^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{1}{4},\frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{a}{bx^4}\right)}{14\sqrt[4]{b}x^{14}}$$

input
```
integrate(1/x**14/(-b*x**4+a)**(1/4),x)
```

output
```
-I*exp(-3*I*pi/4)*hyper((1/4, 7/2), (9/2,), a/(b*x**4))/(14*b**(1/4)*x**14
)
```

Maxima [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(-b*x^4+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^14), x)
```

Giac [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{1}{4}}x^{14}}\,dx$$

input
```
integrate(1/x^14/(-b*x^4+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(1/4)*x^14), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{x^{14}\,{\left(a-b\,x^4\right)}^{1/4}} \,dx$$

input
```
int(1/(x^14*(a - b*x^4)^(1/4)),x)
```

output
```
int(1/(x^14*(a - b*x^4)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{14}\sqrt[4]{a-bx^4}}\,dx=\int \frac{1}{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} x^{14}}d x$$

input
```
int(1/x^14/(-b*x^4+a)^(1/4),x)
```

output
```
int(1/((a - b*x**4)**(1/4)*x**14),x)
```

3.695 $\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = -\frac{a^4\sqrt[4]{a-bx^4}}{b^5}+\frac{4a^3(a-bx^4)^{5/4}}{5b^5}-\frac{2a^2(a-bx^4)^{9/4}}{3b^5}+\frac{4a(a-bx^4)^{13/4}}{13b^5}-\frac{(a-bx^4)^{17/4}}{17b^5}$$

output
```
-a^4*(-b*x^4+a)^(1/4)/b^5+4/5*a^3*(-b*x^4+a)^(5/4)/b^5-2/3*a^2*(-b*x^4+a)^
(9/4)/b^5+4/13*a*(-b*x^4+a)^(13/4)/b^5-1/17*(-b*x^4+a)^(17/4)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.60

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a-bx^4}(-2048a^4-512a^3bx^4-320a^2b^2x^8-240ab^3x^{12}-195b^4x^{16})}{3315b^5}$$

input
```
Integrate[x^19/(a - b*x^4)^(3/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-2048*a^4 - 512*a^3*b*x^4 - 320*a^2*b^2*x^8 - 240*a*b^
3*x^12 - 195*b^4*x^16))/(3315*b^5)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{16}}{(a-bx^4)^{3/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^4}{b^4(a-bx^4)^{3/4}}-\frac{4\sqrt[4]{a-bx^4}a^3}{b^4}+\frac{6(a-bx^4)^{5/4}a^2}{b^4}-\frac{4(a-bx^4)^{9/4}a}{b^4}+\frac{(a-bx^4)^{13/4}}{b^4}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^4\sqrt[4]{a-bx^4}}{b^5}+\frac{16a^3(a-bx^4)^{5/4}}{5b^5}-\frac{8a^2(a-bx^4)^{9/4}}{3b^5}-\frac{4(a-bx^4)^{17/4}}{17b^5}+\frac{16a(a-bx^4)^{13/4}}{13b^5}\right)$$

input
```
Int[x^19/(a - b*x^4)^(3/4),x]
```

output
```
((-4*a^4*(a - b*x^4)^(1/4))/b^5 + (16*a^3*(a - b*x^4)^(5/4))/(5*b^5) - (8*
a^2*(a - b*x^4)^(9/4))/(3*b^5) + (16*a*(a - b*x^4)^(13/4))/(13*b^5) - (4*(
a - b*x^4)^(17/4))/(17*b^5))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(195 x^{16} b^{4}+240 a\,b^{3} x^{12}+320 a^{2} b^{2} x^{8}+512 a^{3} b\,x^{4}+2048 a^{4}\right)}{3315 b^{5}}$	59
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(195 x^{16} b^{4}+240 a\,b^{3} x^{12}+320 a^{2} b^{2} x^{8}+512 a^{3} b\,x^{4}+2048 a^{4}\right)}{3315 b^{5}}$	59
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(195 x^{16} b^{4}+240 a\,b^{3} x^{12}+320 a^{2} b^{2} x^{8}+512 a^{3} b\,x^{4}+2048 a^{4}\right)}{3315 b^{5}}$	59
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(195 x^{16} b^{4}+240 a\,b^{3} x^{12}+320 a^{2} b^{2} x^{8}+512 a^{3} b\,x^{4}+2048 a^{4}\right)}{3315 b^{5}}$	59
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(195 x^{16} b^{4}+240 a\,b^{3} x^{12}+320 a^{2} b^{2} x^{8}+512 a^{3} b\,x^{4}+2048 a^{4}\right)}{3315 b^{5} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	86

input
```
int(x^19/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/3315*(-b*x^4+a)^(1/4)*(195*b^4*x^16+240*a*b^3*x^12+320*a^2*b^2*x^8+512*
a^3*b*x^4+2048*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = -\frac{(195\,b^4x^{16}+240\,ab^3x^{12}+320\,a^2b^2x^8+512\,a^3bx^4+2048\,a^4)(-bx^4+a)^{\frac{1}{4}}}{3315\,b^5}$$

input
```
integrate(x^19/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/3315*(195*b^4*x^16 + 240*a*b^3*x^12 + 320*a^2*b^2*x^8 + 512*a^3*b*x^4 +
 2048*a^4)*(-b*x^4 + a)^(1/4)/b^5
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = \begin{cases} -\frac{2048a^4\sqrt[4]{a-bx^4}}{3315b^5} - \frac{512a^3x^4\sqrt[4]{a-bx^4}}{3315b^4} - \frac{64a^2x^8\sqrt[4]{a-bx^4}}{663b^3} - \frac{16ax^{12}\sqrt[4]{a-bx^4}}{221b^2} - \frac{x^{16}\sqrt[4]{a}}{17} \\ \frac{x^{20}}{20a^{\frac{3}{4}}} \end{cases}$$

input
```
integrate(x**19/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-2048*a**4*(a - b*x**4)**(1/4)/(3315*b**5) - 512*a**3*x**4*(a -
 b*x**4)**(1/4)/(3315*b**4) - 64*a**2*x**8*(a - b*x**4)**(1/4)/(663*b**3)
- 16*a*x**12*(a - b*x**4)**(1/4)/(221*b**2) - x**16*(a - b*x**4)**(1/4)/(1
7*b), Ne(b, 0)), (x**20/(20*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{17}{4}}}{17\,b^5} + \frac{4\,(-bx^4+a)^{\frac{13}{4}}a}{13\,b^5} - \frac{2\,(-bx^4+a)^{\frac{9}{4}}a^2}{3\,b^5} + \frac{4\,(-bx^4+a)^{\frac{5}{4}}a^3}{5\,b^5} - \frac{(-bx^4+a)^{\frac{1}{4}}a^4}{b^5}$$

input
```
integrate(x^19/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/17*(-b*x^4 + a)^(17/4)/b^5 + 4/13*(-b*x^4 + a)^(13/4)*a/b^5 - 2/3*(-b*x
^4 + a)^(9/4)*a^2/b^5 + 4/5*(-b*x^4 + a)^(5/4)*a^3/b^5 - (-b*x^4 + a)^(1/4
)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}a^4}{b^5} - \frac{195\,(bx^4-a)^4(-bx^4+a)^{\frac{1}{4}} + 1020\,(bx^4-a)^3(-bx^4+a)^{\frac{1}{4}}a + 2210\,(bx^4-a)^2(-bx^4+a)^{\frac{1}{4}}a^2 - 2652\,(-b}{3315\,b^5}$$

input
```
integrate(x^19/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-(-b*x^4 + a)^(1/4)*a^4/b^5 - 1/3315*(195*(b*x^4 - a)^4*(-b*x^4 + a)^(1/4)
 + 1020*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4)*a + 2210*(b*x^4 - a)^2*(-b*x^4 +
a)^(1/4)*a^2 - 2652*(-b*x^4 + a)^(5/4)*a^3)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.58

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = -\left(a-b\,x^4\right)^{1/4}\left(\frac{2048\,a^4}{3315\,b^5}+\frac{x^{16}}{17\,b}+\frac{16\,a\,x^{12}}{221\,b^2}+\frac{512\,a^3\,x^4}{3315\,b^4}+\frac{64\,a^2\,x^8}{663\,b^3}\right)$$

input
```
int(x^19/(a - b*x^4)^(3/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((2048*a^4)/(3315*b^5) + x^16/(17*b) + (16*a*x^12)/(221
*b^2) + (512*a^3*x^4)/(3315*b^4) + (64*a^2*x^8)/(663*b^3))
```

Reduce [F]

$$\int \frac{x^{19}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{19}}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^19/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**19/(a - b*x**4)**(3/4),x)
```

3.696 $\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 82

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = -\frac{a^3\sqrt[4]{a-bx^4}}{b^4}+\frac{3a^2(a-bx^4)^{5/4}}{5b^4}-\frac{a(a-bx^4)^{9/4}}{3b^4}+\frac{(a-bx^4)^{13/4}}{13b^4}$$

output
```
-a^3*(-b*x^4+a)^(1/4)/b^4+3/5*a^2*(-b*x^4+a)^(5/4)/b^4-1/3*a*(-b*x^4+a)^(9
/4)/b^4+1/13*(-b*x^4+a)^(13/4)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.62

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a-bx^4}(-128a^3-32a^2bx^4-20ab^2x^8-15b^3x^{12})}{195b^4}$$

input
```
Integrate[x^15/(a - b*x^4)^(3/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-128*a^3 - 32*a^2*b*x^4 - 20*a*b^2*x^8 - 15*b^3*x^12))
/(195*b^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^{12}}{(a-bx^4)^{3/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^3}{b^3(a-bx^4)^{3/4}}-\frac{3\sqrt[4]{a-bx^4}a^2}{b^3}+\frac{3(a-bx^4)^{5/4}a}{b^3}-\frac{(a-bx^4)^{9/4}}{b^3}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^3\sqrt[4]{a-bx^4}}{b^4}+\frac{12a^2(a-bx^4)^{5/4}}{5b^4}+\frac{4(a-bx^4)^{13/4}}{13b^4}-\frac{4a(a-bx^4)^{9/4}}{3b^4}\right)$$

input
```
Int[x^15/(a - b*x^4)^(3/4),x]
```

output
```
((-4*a^3*(a - b*x^4)^(1/4))/b^4 + (12*a^2*(a - b*x^4)^(5/4))/(5*b^4) - (4*
a*(a - b*x^4)^(9/4))/(3*b^4) + (4*(a - b*x^4)^(13/4))/(13*b^4))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(15 b^{3} x^{12}+20 a\,b^{2} x^{8}+32 a^{2} b\,x^{4}+128 a^{3}\right)}{195 b^{4}}$	48
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(15 b^{3} x^{12}+20 a\,b^{2} x^{8}+32 a^{2} b\,x^{4}+128 a^{3}\right)}{195 b^{4}}$	48
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(15 b^{3} x^{12}+20 a\,b^{2} x^{8}+32 a^{2} b\,x^{4}+128 a^{3}\right)}{195 b^{4}}$	48
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(15 b^{3} x^{12}+20 a\,b^{2} x^{8}+32 a^{2} b\,x^{4}+128 a^{3}\right)}{195 b^{4}}$	48
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}}\left(15 b^{3} x^{12}+20 a\,b^{2} x^{8}+32 a^{2} b\,x^{4}+128 a^{3}\right)}{195 b^{4}\left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	75

input
```
int(x^15/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/195*(-b*x^4+a)^(1/4)*(15*b^3*x^12+20*a*b^2*x^8+32*a^2*b*x^4+128*a^3)/b^
4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = -\frac{(15\,b^3x^{12}+20\,ab^2x^8+32\,a^2bx^4+128\,a^3)(-bx^4+a)^{\frac{1}{4}}}{195\,b^4}$$

input
```
integrate(x^15/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/195*(15*b^3*x^12 + 20*a*b^2*x^8 + 32*a^2*b*x^4 + 128*a^3)*(-b*x^4 + a)^
(1/4)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = \begin{cases} -\frac{128a^3\sqrt[4]{a-bx^4}}{195b^4}-\frac{32a^2x^4\sqrt[4]{a-bx^4}}{195b^3}-\frac{4ax^8\sqrt[4]{a-bx^4}}{39b^2}-\frac{x^{12}\sqrt[4]{a-bx^4}}{13b} & \text{for } b\neq 0 \\ \frac{x^{16}}{16a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**15/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-128*a**3*(a - b*x**4)**(1/4)/(195*b**4) - 32*a**2*x**4*(a - b*
x**4)**(1/4)/(195*b**3) - 4*a*x**8*(a - b*x**4)**(1/4)/(39*b**2) - x**12*(
a - b*x**4)**(1/4)/(13*b), Ne(b, 0)), (x**16/(16*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = \frac{(-bx^4+a)^{\frac{13}{4}}}{13\,b^4}-\frac{(-bx^4+a)^{\frac{9}{4}}a}{3\,b^4}+\frac{3\,(-bx^4+a)^{\frac{5}{4}}a^2}{5\,b^4}-\frac{(-bx^4+a)^{\frac{1}{4}}a^3}{b^4}$$

input
```
integrate(x^15/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/13*(-b*x^4 + a)^(13/4)/b^4 - 1/3*(-b*x^4 + a)^(9/4)*a/b^4 + 3/5*(-b*x^4
+ a)^(5/4)*a^2/b^4 - (-b*x^4 + a)^(1/4)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}a^3}{b^4} - \frac{15\,(bx^4-a)^3(-bx^4+a)^{\frac{1}{4}} + 65\,(bx^4-a)^2(-bx^4+a)^{\frac{1}{4}}a - 117\,(-bx^4+a)^{\frac{5}{4}}a^2}{195\,b^4}$$

input
```
integrate(x^15/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-(-b*x^4 + a)^(1/4)*a^3/b^4 - 1/195*(15*(b*x^4 - a)^3*(-b*x^4 + a)^(1/4) +
 65*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4)*a - 117*(-b*x^4 + a)^(5/4)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = -\left(a-b\,x^4\right)^{1/4}\left(\frac{128\,a^3}{195\,b^4}+\frac{x^{12}}{13\,b}+\frac{4\,a\,x^8}{39\,b^2}+\frac{32\,a^2\,x^4}{195\,b^3}\right)$$

input
```
int(x^15/(a - b*x^4)^(3/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((128*a^3)/(195*b^4) + x^12/(13*b) + (4*a*x^8)/(39*b^2)
 + (32*a^2*x^4)/(195*b^3))
```

Reduce [F]

$$\int \frac{x^{15}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{15}}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^15/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**15/(a - b*x**4)**(3/4),x)
```

3.697 $\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = -\frac{a^2\sqrt[4]{a-bx^4}}{b^3}+\frac{2a(a-bx^4)^{5/4}}{5b^3}-\frac{(a-bx^4)^{9/4}}{9b^3}$$

output
```
-a^2*(-b*x^4+a)^(1/4)/b^3+2/5*a*(-b*x^4+a)^(5/4)/b^3-1/9*(-b*x^4+a)^(9/4)/
b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = \frac{\sqrt[4]{a-bx^4}(-32a^2-8abx^4-5b^2x^8)}{45b^3}$$

input
```
Integrate[x^11/(a - b*x^4)^(3/4),x]
```

output
```
((a - b*x^4)^(1/4)*(-32*a^2 - 8*a*b*x^4 - 5*b^2*x^8))/(45*b^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{x^8}{(a-bx^4)^{3/4}}dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{a^2}{b^2 (a-bx^4)^{3/4}}-\frac{2\sqrt[4]{a-bx^4}a}{b^2}+\frac{(a-bx^4)^{5/4}}{b^2}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{4a^2\sqrt[4]{a-bx^4}}{b^3}-\frac{4(a-bx^4)^{9/4}}{9b^3}+\frac{8a(a-bx^4)^{5/4}}{5b^3}\right)$$

input
```
Int[x^11/(a - b*x^4)^(3/4),x]
```

output
```
((-4*a^2*(a - b*x^4)^(1/4))/b^3 + (8*a*(a - b*x^4)^(5/4))/(5*b^3) - (4*(a
- b*x^4)^(9/4))/(9*b^3))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(5 b^{2} x^{8}+8 a b\,x^{4}+32 a^{2}\right)}{45 b^{3}}$	37
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(5 b^{2} x^{8}+8 a b\,x^{4}+32 a^{2}\right)}{45 b^{3}}$	37
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(5 b^{2} x^{8}+8 a b\,x^{4}+32 a^{2}\right)}{45 b^{3}}$	37
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(5 b^{2} x^{8}+8 a b\,x^{4}+32 a^{2}\right)}{45 b^{3}}$	37
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(5 b^{2} x^{8}+8 a b\,x^{4}+32 a^{2}\right)}{45 b^{3} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	64

input
```
int(x^11/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(-b*x^4+a)^(1/4)*(5*b^2*x^8+8*a*b*x^4+32*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = -\frac{(5\,b^2x^8+8\,abx^4+32\,a^2)(-bx^4+a)^{\frac{1}{4}}}{45\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/45*(5*b^2*x^8 + 8*a*b*x^4 + 32*a^2)*(-b*x^4 + a)^(1/4)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = \begin{cases} -\frac{32a^2\sqrt[4]{a-bx^4}}{45b^3} - \frac{8ax^4\sqrt[4]{a-bx^4}}{45b^2} - \frac{x^8\sqrt[4]{a-bx^4}}{9b} & \text{for } b \neq 0 \\ \frac{x^{12}}{12a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**11/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-32*a**2*(a - b*x**4)**(1/4)/(45*b**3) - 8*a*x**4*(a - b*x**4)*
*(1/4)/(45*b**2) - x**8*(a - b*x**4)**(1/4)/(9*b), Ne(b, 0)), (x**12/(12*a
**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{9}{4}}}{9\,b^3} + \frac{2\,(-bx^4+a)^{\frac{5}{4}}a}{5\,b^3} - \frac{(-bx^4+a)^{\frac{1}{4}}a^2}{b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/9*(-b*x^4 + a)^(9/4)/b^3 + 2/5*(-b*x^4 + a)^(5/4)*a/b^3 - (-b*x^4 + a)^
(1/4)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}a^2}{b^3} - \frac{5\,(bx^4-a)^2(-bx^4+a)^{\frac{1}{4}} - 18\,(-bx^4+a)^{\frac{5}{4}}a}{45\,b^3}$$

input
```
integrate(x^11/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-(-b*x^4 + a)^(1/4)*a^2/b^3 - 1/45*(5*(b*x^4 - a)^2*(-b*x^4 + a)^(1/4) - 1
8*(-b*x^4 + a)^(5/4)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.63

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = -\left(a-b\,x^4\right)^{1/4}\left(\frac{32\,a^2}{45\,b^3}+\frac{x^8}{9\,b}+\frac{8\,a\,x^4}{45\,b^2}\right)$$

input
```
int(x^11/(a - b*x^4)^(3/4),x)
```

output
```
-(a - b*x^4)^(1/4)*((32*a^2)/(45*b^3) + x^8/(9*b) + (8*a*x^4)/(45*b^2))
```

Reduce [F]

$$\int \frac{x^{11}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{11}}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^11/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**11/(a - b*x**4)**(3/4),x)
```

3.698 $\int \frac{x^7}{(a-bx^4)^{3/4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 38

$$\int \frac{x^7}{(a-bx^4)^{3/4}} \, dx = -\frac{a\sqrt[4]{a-bx^4}}{b^2} + \frac{(a-bx^4)^{5/4}}{5b^2}$$

output
```
-a*(-b*x^4+a)^(1/4)/b^2+1/5*(-b*x^4+a)^(5/4)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(a-bx^4)^{3/4}} \, dx = \frac{(-4a-bx^4)\sqrt[4]{a-bx^4}}{5b^2}$$

input
```
Integrate[x^7/(a - b*x^4)^(3/4),x]
```

output
```
((-4*a - b*x^4)*(a - b*x^4)^(1/4))/(5*b^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{x^4}{(a-bx^4)^{3/4}}dx^4$$

$$\Big\downarrow \; 53$$

$$\frac{1}{4}\int \left(\frac{a}{b\,(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{b}\right)dx^4$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(\frac{4(a-bx^4)^{5/4}}{5b^2}-\frac{4a\sqrt[4]{a-bx^4}}{b^2}\right)$$

input
```
Int[x^7/(a - b*x^4)^(3/4),x]
```

output
```
((-4*a*(a - b*x^4)^(1/4))/b^2 + (4*(a - b*x^4)^(5/4))/(5*b^2))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(-b\,x^{4}+a)^{\frac{1}{4}}(b\,x^{4}+4a)}{5b^{2}}$	25
trager	$-\frac{(-b\,x^{4}+a)^{\frac{1}{4}}(b\,x^{4}+4a)}{5b^{2}}$	25
pseudoelliptic	$-\frac{(-b\,x^{4}+a)^{\frac{1}{4}}(b\,x^{4}+4a)}{5b^{2}}$	25
orering	$-\frac{(-b\,x^{4}+a)^{\frac{1}{4}}(b\,x^{4}+4a)}{5b^{2}}$	25
risch	$-\frac{(-b\,x^{4}+a)^{\frac{1}{4}}\left((-b\,x^{4}+a)^{3}\right)^{\frac{1}{4}}(b\,x^{4}+4a)}{5b^{2}\left(-(b\,x^{4}-a)^{3}\right)^{\frac{1}{4}}}$	52

input
```
int(x^7/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/4)*(b*x^4+4*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx = -\frac{(bx^4+4\,a)(-bx^4+a)^{\frac{1}{4}}}{5\,b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/5*(b*x^4 + 4*a)*(-b*x^4 + a)^(1/4)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx = \begin{cases} -\frac{4a\sqrt[4]{a-bx^4}}{5b^2} - \frac{x^4\sqrt[4]{a-bx^4}}{5b} & \text{for } b \neq 0 \\ \frac{x^8}{8a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-4*a*(a - b*x**4)**(1/4)/(5*b**2) - x**4*(a - b*x**4)**(1/4)/(5
*b), Ne(b, 0)), (x**8/(8*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx = \frac{(-bx^4+a)^{\frac{5}{4}}}{5\,b^2} - \frac{(-bx^4+a)^{\frac{1}{4}}a}{b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
1/5*(-b*x^4 + a)^(5/4)/b^2 - (-b*x^4 + a)^(1/4)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx = \frac{(-bx^4+a)^{\frac{5}{4}}}{5\,b^2} - \frac{(-bx^4+a)^{\frac{1}{4}}a}{b^2}$$

input
```
integrate(x^7/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
1/5*(-b*x^4 + a)^(5/4)/b^2 - (-b*x^4 + a)^(1/4)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx = -\frac{(a-b\,x^4)^{1/4}\,(b\,x^4+4\,a)}{5\,b^2}$$

input
```
int(x^7/(a - b*x^4)^(3/4),x)
```

output
```
-((a - b*x^4)^(1/4)*(4*a + b*x^4))/(5*b^2)
```

Reduce [F]

$$\int \frac{x^7}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^7}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^7/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**7/(a - b*x**4)**(3/4),x)
```

3.699 $\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 17

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{b}$$

output
```
-(-b*x^4+a)^(1/4)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{b}$$

input
```
Integrate[x^3/(a - b*x^4)^(3/4),x]
```

output
```
-((a - b*x^4)^(1/4)/b)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a-bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{793}$$

$$-\frac{\sqrt[4]{a-bx^4}}{b}$$

input
```
Int[x^3/(a - b*x^4)^(3/4),x]
```

output
```
-((a - b*x^4)^(1/4)/b)
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b}$	16
derivativedivides	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b}$	16
default	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b}$	16
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b}$	16
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b}$	16
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b}$	16
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}}}{b\left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	43

input
```
int(x^3/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-(-b*x^4+a)^(1/4)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}}{b}$$

input
```
integrate(x^3/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-(-b*x^4 + a)^(1/4)/b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = \begin{cases} -\frac{\sqrt[4]{a-bx^4}}{b} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{4}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-(a - b*x**4)**(1/4)/b, Ne(b, 0)), (x**4/(4*a**(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}}{b}$$

input
```
integrate(x^3/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-(-b*x^4 + a)^(1/4)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}}{b}$$

input
```
integrate(x^3/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-(-b*x^4 + a)^(1/4)/b
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{(a-b\,x^4)^{1/4}}{b}$$

input
```
int(x^3/(a - b*x^4)^(3/4),x)
```

output
```
-(a - b*x^4)^(1/4)/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a-bx^4)^{3/4}}\,dx = -\frac{(-b\,x^4+a)^{\frac{1}{4}}}{b}$$

input
```
int(x^3/(-b*x^4+a)^(3/4),x)
```

output
```
( - (a - b*x**4)**(1/4))/b
```

3.700 $\int \frac{1}{x(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

output
```
-1/2*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(3/4)-1/2*arctanh((-b*x^4+a)^(1/4)
/a^(1/4))/a^(3/4)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

$$\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)+\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}$$

input
```
Integrate[1/(x*(a - b*x^4)^(3/4)),x]
```

output
```
-1/2*(ArcTan[(a - b*x^4)^(1/4)/a^(1/4)] + ArcTanh[(a - b*x^4)^(1/4)/a^(1/4
)])/a^(3/4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {798, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx^4$$

$$\Big\downarrow \; 73$$

$$-\frac{\int \frac{1}{\frac{a}{b}-\frac{x^{16}}{b}}d\sqrt[4]{a-bx^4}}{b}$$

$$\Big\downarrow \; 756$$

$$-\frac{\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}}{b}$$

$$\Big\downarrow \; 216$$

$$-\frac{\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}}{b}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}}{b}$$

input
```
Int[1/(x*(a - b*x^4)^(3/4)),x]
```

output
```
-(((b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)) + (b*ArcTanh[(a - b*x
^4)^(1/4)/a^(1/4)])/(2*a^(3/4)))/b)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

method	result	size
pseudoelliptic	$-\frac{\ln\left(\frac{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{4a^{\frac{3}{4}}}$	62

input
```
int(1/x/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(3/4)*(ln((-(-b*x^4+a)^(1/4)-a^(1/4))/(-(-b*x^4+a)^(1/4)+a^(1/4)))+
2*arctan((-b*x^4+a)^(1/4)/a^(1/4)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

$$\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx = -\frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}}\log\left(a\frac{1}{a^3}^{\frac{1}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}\right)-\frac{1}{4}i\frac{1}{a^3}^{\frac{1}{4}}\log\left(i\,a\frac{1}{a^3}^{\frac{1}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}\right)+\frac{1}{4}i\frac{1}{a^3}^{\frac{1}{4}}\log\left(-i\,a\frac{1}{a^3}^{\frac{1}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}\right)+\frac{1}{4}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-a\frac{1}{a^3}^{\frac{1}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}\right)$$

input
```
integrate(1/x/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/4*(a^(-3))^(1/4)*log(a*(a^(-3))^(1/4) + (-b*x^4 + a)^(1/4)) - 1/4*I*(a^
(-3))^(1/4)*log(I*a*(a^(-3))^(1/4) + (-b*x^4 + a)^(1/4)) + 1/4*I*(a^(-3))^
(1/4)*log(-I*a*(a^(-3))^(1/4) + (-b*x^4 + a)^(1/4)) + 1/4*(a^(-3))^(1/4)*l
og(-a*(a^(-3))^(1/4) + (-b*x^4 + a)^(1/4))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x(a-bx^4)^{3/4}}\,dx = -\frac{e^{-\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\,\frac{a}{bx^4}\right)}{4b^{\frac{3}{4}}x^3\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(1/x/(-b*x**4+a)**(3/4),x)
```

output
```
-exp(-3*I*pi/4)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), a/(b*x**4))/(4*b**(3/
4)*x**3*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(a-bx^4)^{3/4}}\,dx = -\frac{\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{2\,a^{\frac{3}{4}}} + \frac{\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{4\,a^{\frac{3}{4}}}$$

input
```
integrate(1/x/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/2*arctan((-b*x^4 + a)^(1/4)/a^(1/4))/a^(3/4) + 1/4*log(((-b*x^4 + a)^(1
/4) - a^(1/4))/((-b*x^4 + a)^(1/4) + a^(1/4)))/a^(3/4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(41) = 82$.

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.37

$$\int \frac{1}{x(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}(-a)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{4\,a} - \frac{\sqrt{2}(-a)^{\frac{1}{4}}\log\left(\sqrt{2}(-bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4+a}+\sqrt{-a}\right)}{8\,a} + \frac{\sqrt{2}(-a)^{\frac{1}{4}}\log\left(-\sqrt{2}(-bx^4+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{-bx^4+a}+\sqrt{-a}\right)}{8\,a}$$

input
```
integrate(1/x/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-a)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(-b*x^4
 + a)^(1/4))/(-a)^(1/4))/a - 1/4*sqrt(2)*(-a)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a - 1/8*sqrt(2)*(-a)
^(1/4)*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt
(-a))/a + 1/8*sqrt(2)*(-a)^(1/4)*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4
) + sqrt(-b*x^4 + a) + sqrt(-a))/a
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{1}{x(a-bx^4)^{3/4}}\,dx = -\frac{\mathrm{atan}\left(\frac{(a-b\,x^4)^{1/4}}{a^{1/4}}\right)+\mathrm{atanh}\left(\frac{(a-b\,x^4)^{1/4}}{a^{1/4}}\right)}{2\,a^{3/4}}$$

input
```
int(1/(x*(a - b*x^4)^(3/4)),x)
```

output
```
-(atan((a - b*x^4)^(1/4)/a^(1/4)) + atanh((a - b*x^4)^(1/4)/a^(1/4)))/(2*a
^(3/4))
```

Reduce [F]

$$\int \frac{1}{x\left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x}dx$$

input
```
int(1/x/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x),x)
```

3.701 $\int \frac{1}{x^5(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{1}{x^5(a-bx^4)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{4ax^4}-\frac{3b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{3b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

output
```
-1/4*(-b*x^4+a)^(1/4)/a/x^4-3/8*b*arctan((-b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
-3/8*b*arctanh((-b*x^4+a)^(1/4)/a^(1/4))/a^(7/4)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5(a-bx^4)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{4ax^4}-\frac{3b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}-\frac{3b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{8a^{7/4}}$$

input
```
Integrate[1/(x^5*(a - b*x^4)^(3/4)),x]
```

output
```
-1/4*(a - b*x^4)^(1/4)/(a*x^4) - (3*b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(
8*a^(7/4)) - (3*b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(8*a^(7/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {798, 52, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a-bx^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{4} \int \frac{1}{x^8 \left(a-bx^4\right)^{3/4}} dx^4$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{4} \left(\frac{3b \int \frac{1}{x^4 (a-bx^4)^{3/4}} dx^4}{4a} - \frac{\sqrt[4]{a-bx^4}}{ax^4} \right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4} \left(-\frac{3 \int \frac{1}{\frac{a}{b}-\frac{x^{16}}{b}} d\sqrt[4]{a-bx^4}}{a} - \frac{\sqrt[4]{a-bx^4}}{ax^4} \right)$$

$$\Big\downarrow \text{ 756}$$

$$\frac{1}{4} \left(-\frac{3 \left(\frac{b \int \frac{1}{\sqrt{a}-x^8} d\sqrt[4]{a-bx^4}}{2\sqrt{a}} + \frac{b \int \frac{1}{x^8+\sqrt{a}} d\sqrt[4]{a-bx^4}}{2\sqrt{a}} \right)}{a} - \frac{\sqrt[4]{a-bx^4}}{ax^4} \right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}\left(-\frac{3\left(\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)$$

$\downarrow$ 219

$$\frac{1}{4}\left(-\frac{3\left(\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)$$

input
```
Int[1/(x^5*(a - b*x^4)^(3/4)),x]
```

output
```
(-((a - b*x^4)^(1/4)/(a*x^4)) - (3*((b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/
(2*a^(3/4)) + (b*ArcTanh[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4))))/a)/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

method	result	size
pseudoelliptic	$-\frac{3\left(\ln\left(\frac{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)b\,x^{4}+2\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b\,x^{4}+\frac{4\left(-b\,x^{4}+a\right)^{\frac{1}{4}}a^{\frac{3}{4}}}{3}\right)}{16a^{\frac{7}{4}}x^{4}}$	89

input
```
int(1/x^5/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-3/16*(ln((-(-b*x^4+a)^(1/4)-a^(1/4))/(-(-b*x^4+a)^(1/4)+a^(1/4)))*b*x^4+2
*arctan((-b*x^4+a)^(1/4)/a^(1/4))*b*x^4+4/3*(-b*x^4+a)^(1/4)*a^(3/4))/a^(7
/4)/x^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.43

$$\int \frac{1}{x^5 (a-bx^4)^{3/4}} \, dx =$$
$$-\frac{3\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(3\,a^2\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+3\,(-bx^4+a)^{\frac{1}{4}}b\right)+3i\,ax^4\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}\log\left(3i\,a^2\left(\frac{b^4}{a^7}\right)^{\frac{1}{4}}+3\,(-bx^4+a)^{\frac{1}{4}}b\right)-3i}{}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/16*(3*a*x^4*(b^4/a^7)^(1/4)*log(3*a^2*(b^4/a^7)^(1/4) + 3*(-b*x^4 + a)^
(1/4)*b) + 3*I*a*x^4*(b^4/a^7)^(1/4)*log(3*I*a^2*(b^4/a^7)^(1/4) + 3*(-b*x
^4 + a)^(1/4)*b) - 3*I*a*x^4*(b^4/a^7)^(1/4)*log(-3*I*a^2*(b^4/a^7)^(1/4)
+ 3*(-b*x^4 + a)^(1/4)*b) - 3*a*x^4*(b^4/a^7)^(1/4)*log(-3*a^2*(b^4/a^7)^(
1/4) + 3*(-b*x^4 + a)^(1/4)*b) + 4*(-b*x^4 + a)^(1/4))/(a*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^5 (a-bx^4)^{3/4}} \, dx = \frac{e^{\frac{i\pi}{4}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{a}{bx^4}\right)}{4b^{\frac{3}{4}}x^7\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(1/x**5/(-b*x**4+a)**(3/4),x)
```

output
```
exp(I*pi/4)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), a/(b*x**4))/(4*b**(3/4)*
x**7*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^5 (a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}b}{4\left((bx^4-a)a+a^2\right)} - \frac{3\left(\frac{2\,b\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}} - \frac{b\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{16\,a}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/4*(-b*x^4 + a)^(1/4)*b/((b*x^4 - a)*a + a^2) - 3/16*(2*b*arctan((-b*x^4
 + a)^(1/4)/a^(1/4))/a^(3/4) - b*log(((-b*x^4 + a)^(1/4) - a^(1/4))/((-b*x
^4 + a)^(1/4) + a^(1/4)))/a^(3/4))/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(61) = 122.

Time = 0.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^5 (a-bx^4)^{3/4}}\,dx = -\frac{\frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{6\sqrt{2}(-a)^{\frac{1}{4}}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^2} + \frac{3\sqrt{2}(-a)^{\frac{1}{4}}b^2\log\left(\sqrt{2}(\right.}{}}{32\,b}$$

input
```
integrate(1/x^5/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-1/32*(6*sqrt(2)*(-a)^(1/4)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2
*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 6*sqrt(2)*(-a)^(1/4)*b^2*arctan(-1/
2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^2 + 3*
sqrt(2)*(-a)^(1/4)*b^2*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqrt(-b
*x^4 + a) + sqrt(-a))/a^2 + 3*sqrt(2)*b^2*log(-sqrt(2)*(-b*x^4 + a)^(1/4)*
(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a) + 8*(-b*x^4 + a)^
(1/4)*b/(a*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5 (a-bx^4)^{3/4}}\,dx = -\frac{(a-b\,x^4)^{1/4}}{4\,a\,x^4} - \frac{3\,b\,\mathrm{atan}\left(\frac{(a-b\,x^4)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}} - \frac{3\,b\,\mathrm{atanh}\left(\frac{(a-b\,x^4)^{1/4}}{a^{1/4}}\right)}{8\,a^{7/4}}$$

input
```
int(1/(x^5*(a - b*x^4)^(3/4)),x)
```

output
```
- (a - b*x^4)^(1/4)/(4*a*x^4) - (3*b*atan((a - b*x^4)^(1/4)/a^(1/4)))/(8*a
^(7/4)) - (3*b*atanh((a - b*x^4)^(1/4)/a^(1/4)))/(8*a^(7/4))
```

Reduce [F]

$$\int \frac{1}{x^5 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}\,x^5}dx$$

input
```
int(1/x^5/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**5),x)
```

3.702 $\int \frac{1}{x^9\left(a-bx^4\right)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{1}{x^9\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{8ax^8}-\frac{7b\sqrt[4]{a-bx^4}}{32a^2x^4} -\frac{21b^2\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{21b^2\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}$$

output
```
-1/8*(-b*x^4+a)^(1/4)/a/x^8-7/32*b*(-b*x^4+a)^(1/4)/a^2/x^4-21/64*b^2*arct
an((-b*x^4+a)^(1/4)/a^(1/4))/a^(11/4)-21/64*b^2*arctanh((-b*x^4+a)^(1/4)/a
^(1/4))/a^(11/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^9\left(a-bx^4\right)^{3/4}}\,dx = \frac{\left(-4a-7bx^4\right)\sqrt[4]{a-bx^4}}{32a^2x^8} -\frac{21b^2\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}-\frac{21b^2\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{64a^{11/4}}$$

input
```
Integrate[1/(x^9*(a - b*x^4)^(3/4)),x]
```

output
```
((-4*a - 7*b*x^4)*(a - b*x^4)^(1/4))/(32*a^2*x^8) - (21*b^2*ArcTan[(a - b*
x^4)^(1/4)/a^(1/4)])/(64*a^(11/4)) - (21*b^2*ArcTanh[(a - b*x^4)^(1/4)/a^(
1/4)])/(64*a^(11/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {798, 52, 52, 73, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int \frac{1}{x^{12} (a-bx^4)^{3/4}}dx^4$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{7b\int \frac{1}{x^8 (a-bx^4)^{3/4}}dx^4}{8a}-\frac{\sqrt[4]{a-bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{4}\left(\frac{7b\left(\frac{3b\int \frac{1}{x^4 (a-bx^4)^{3/4}}dx^4}{4a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a-bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}\left(\frac{7b\left(-\frac{3\int \frac{1}{\frac{a}{b}-\frac{x^{16}}{b}}d\sqrt[4]{a-bx^4}}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a-bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \; 756$$

$$\frac{1}{4}\left(\frac{7b\left(-\frac{3\left(\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\int \frac{1}{x^8+\sqrt{a}}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a-bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{4}\left(\frac{7b\left(-\frac{3\left(\frac{b\int \frac{1}{\sqrt{a}-x^8}d\sqrt[4]{a-bx^4}}{2\sqrt{a}}+\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a-bx^4}}{2ax^8}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(\frac{7b\left(-\frac{3\left(\frac{b\arctan\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{b\text{arctanh}\left(\frac{\sqrt[4]{a-bx^4}}{\sqrt[4]{a}}\right)}{2a^{3/4}}\right)}{a}-\frac{\sqrt[4]{a-bx^4}}{ax^4}\right)}{8a}-\frac{\sqrt[4]{a-bx^4}}{2ax^8}\right)$$

input
```
Int[1/(x^9*(a - b*x^4)^(3/4)),x]
```

output
```
(-1/2*(a - b*x^4)^(1/4)/(a*x^8) + (7*b*(-((a - b*x^4)^(1/4)/(a*x^4)) - (3*
((b*ArcTan[(a - b*x^4)^(1/4)/a^(1/4)])/(2*a^(3/4)) + (b*ArcTanh[(a - b*x^4
)^(1/4)/a^(1/4)])/(2*a^(3/4))))/a))/(8*a))/4
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{-42\arctan\left(\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)b^{2}x^{8}-21\ln\left(\frac{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-a^{\frac{1}{4}}}{-\left(-b\,x^{4}+a\right)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)b^{2}x^{8}-28b\,x^{4}a^{\frac{3}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}-16a^{\frac{7}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{128a^{\frac{11}{4}}x^{8}}$	113

input
```
int(1/x^9/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-42*arctan((-b*x^4+a)^(1/4)/a^(1/4))*b^2*x^8-21*ln((-(-b*x^4+a)^(1/
4)-a^(1/4))/(-(-b*x^4+a)^(1/4)+a^(1/4)))*b^2*x^8-28*b*x^4*a^(3/4)*(-b*x^4+
a)^(1/4)-16*a^(7/4)*(-b*x^4+a)^(1/4))/a^(11/4)/x^8
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}} \, dx = -\frac{21\, a^2 x^8 \left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}} \log\left(21\, a^3 \left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}} + 21\, (-bx^4+a)^{\frac{1}{4}} b^2\right) + 21i\, a^2 x^8 \left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}} \log\left(21i\, a^3 \left(\frac{b^8}{a^{11}}\right)^{\frac{1}{4}} + 21\, (-bx^4 + \cdots\right.}{\cdots}$$

input
```
integrate(1/x^9/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/128*(21*a^2*x^8*(b^8/a^11)^(1/4)*log(21*a^3*(b^8/a^11)^(1/4) + 21*(-b*x
^4 + a)^(1/4)*b^2) + 21*I*a^2*x^8*(b^8/a^11)^(1/4)*log(21*I*a^3*(b^8/a^11)
^(1/4) + 21*(-b*x^4 + a)^(1/4)*b^2) - 21*I*a^2*x^8*(b^8/a^11)^(1/4)*log(-2
1*I*a^3*(b^8/a^11)^(1/4) + 21*(-b*x^4 + a)^(1/4)*b^2) - 21*a^2*x^8*(b^8/a^
11)^(1/4)*log(-21*a^3*(b^8/a^11)^(1/4) + 21*(-b*x^4 + a)^(1/4)*b^2) + 4*(7
*b*x^4 + 4*a)*(-b*x^4 + a)^(1/4))/(a^2*x^8)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}} \, dx = -\frac{e^{-\frac{3i\pi}{4}} \Gamma\left(\frac{11}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{3}{4}, \frac{11}{4} \\ \frac{15}{4} \end{matrix}\right| \frac{a}{bx^4}\right)}{4b^{\frac{3}{4}} x^{11} \Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(1/x**9/(-b*x**4+a)**(3/4),x)
```

output
```
-exp(-3*I*pi/4)*gamma(11/4)*hyper((3/4, 11/4), (15/4,), a/(b*x**4))/(4*b**
(3/4)*x**11*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}}\,dx = \frac{7\,(-bx^4+a)^{\frac{5}{4}}b^2 - 11\,(-bx^4+a)^{\frac{1}{4}}ab^2}{32\,\left((bx^4-a)^2a^2 + 2\,(bx^4-a)a^3 + a^4\right)} - \frac{21\left(\frac{2\,b^2\arctan\left(\frac{(-bx^4+a)^{\frac{1}{4}}}{a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}} - \frac{b^2\log\left(\frac{(-bx^4+a)^{\frac{1}{4}}-a^{\frac{1}{4}}}{(-bx^4+a)^{\frac{1}{4}}+a^{\frac{1}{4}}}\right)}{a^{\frac{3}{4}}}\right)}{128\,a^2}$$

input

```
integrate(1/x^9/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output

```
1/32*(7*(-b*x^4 + a)^(5/4)*b^2 - 11*(-b*x^4 + a)^(1/4)*a*b^2)/((b*x^4 - a)
^2*a^2 + 2*(b*x^4 - a)*a^3 + a^4) - 21/128*(2*b^2*arctan((-b*x^4 + a)^(1/4
)/a^(1/4))/a^(3/4) - b^2*log(((-b*x^4 + a)^(1/4) - a^(1/4))/((-b*x^4 + a)^
(1/4) + a^(1/4)))/a^(3/4))/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(84) = 168.

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}}\,dx = -\frac{\frac{42\,\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^3} + \frac{42\,\sqrt{2}(-a)^{\frac{1}{4}}b^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\,(-bx^4+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{a^3} + \frac{21\,\sqrt{2}(-a)^{\frac{1}{4}}b^3\log\left(\right.}{}}{}$$

input

```
integrate(1/x^9/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
-1/256*(42*sqrt(2)*(-a)^(1/4)*b^3*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) +
 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 + 42*sqrt(2)*(-a)^(1/4)*b^3*arctan(
-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(-b*x^4 + a)^(1/4))/(-a)^(1/4))/a^3 +
 21*sqrt(2)*(-a)^(1/4)*b^3*log(sqrt(2)*(-b*x^4 + a)^(1/4)*(-a)^(1/4) + sqr
t(-b*x^4 + a) + sqrt(-a))/a^3 + 21*sqrt(2)*b^3*log(-sqrt(2)*(-b*x^4 + a)^(
1/4)*(-a)^(1/4) + sqrt(-b*x^4 + a) + sqrt(-a))/((-a)^(3/4)*a^2) - 8*(7*(-b
*x^4 + a)^(5/4)*b^3 - 11*(-b*x^4 + a)^(1/4)*a*b^3)/(a^2*b^2*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}}\,dx = \frac{7\,(a-b\,x^4)^{5/4}}{32\,a^2\,x^8} - \frac{11\,(a-b\,x^4)^{1/4}}{32\,a\,x^8} - \frac{21\,b^2\,\mathrm{atan}\left(\frac{(a-b\,x^4)^{1/4}}{a^{1/4}}\right)}{64\,a^{11/4}} + \frac{b^2\,\mathrm{atan}\left(\frac{(a-b\,x^4)^{1/4}\,1\mathrm{i}}{a^{1/4}}\right)\,21\mathrm{i}}{64\,a^{11/4}}$$

input
```
int(1/(x^9*(a - b*x^4)^(3/4)),x)
```

output
```
(b^2*atan(((a - b*x^4)^(1/4)*1i)/a^(1/4))*21i)/(64*a^(11/4)) - (21*b^2*ata
n((a - b*x^4)^(1/4)/a^(1/4)))/(64*a^(11/4)) - (11*(a - b*x^4)^(1/4))/(32*a
*x^8) + (7*(a - b*x^4)^(5/4))/(32*a^2*x^8)
```

Reduce [F]

$$\int \frac{1}{x^9 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}\,x^9}dx$$

input
```
int(1/x^9/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**9),x)
```

3.703 $\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = -\frac{20a^2x^2\sqrt[4]{a-bx^4}}{77b^3} - \frac{10ax^6\sqrt[4]{a-bx^4}}{77b^2} - \frac{x^{10}\sqrt[4]{a-bx^4}}{11b} + \frac{40a^{7/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77b^{7/2}(a-bx^4)^{3/4}}$$

output

```
-20/77*a^2*x^2*(-b*x^4+a)^(1/4)/b^3-10/77*a*x^6*(-b*x^4+a)^(1/4)/b^2-1/11*
x^10*(-b*x^4+a)^(1/4)/b+40/77*a^(7/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(1/
2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(7/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \frac{x^2\left(-20a^3+10a^2bx^4+3ab^2x^8+7b^3x^{12}+20a^3\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\right.\right.}{77b^3(a-bx^4)^{3/4}}$$

input
```
Integrate[x^13/(a - b*x^4)^(3/4),x]
```

output
```
(x^2*(-20*a^3 + 10*a^2*b*x^4 + 3*a*b^2*x^8 + 7*b^3*x^12 + 20*a^3*(1 - (b*x
^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^4)/a]))/(77*b^3*(a - b*
x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {807, 262, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^{12}}{(a-bx^4)^{3/4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{10a\int \frac{x^8}{(a-bx^4)^{3/4}}dx^2}{11b}-\frac{2x^{10}\sqrt[4]{a-bx^4}}{11b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{6a\int \frac{x^4}{(a-bx^4)^{3/4}}dx^2}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)}{11b}-\frac{2x^{10}\sqrt[4]{a-bx^4}}{11b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{6a\left(\frac{2a\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx^2}{3b}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)}{11b}-\frac{2x^{10}\sqrt[4]{a-bx^4}}{11b}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{3b\left(a-bx^4\right)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)}{11b}-\frac{2x^{10}\sqrt[4]{a-bx^4}}{11b}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}\left(\frac{10a\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)}{11b}-\frac{2x^{10}\sqrt[4]{a-bx^4}}{11b}\right)$$

input
```
Int[x^13/(a - b*x^4)^(3/4),x]
```

output
```
((-2*x^10*(a - b*x^4)^(1/4))/(11*b) + (10*a*((-2*x^6*(a - b*x^4)^(1/4))/(7
*b) + (6*a*((-2*x^2*(a - b*x^4)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^4)/a)^
(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^4
)^(3/4))))/(7*b)))/(11*b))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{13}}{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(x^13/(-b*x^4+a)^(3/4),x)
```

output
```
int(x^13/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{\left(-bx^{4}+a\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^13/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x^13/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.22

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \frac{x^{14}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^{4}e^{2i\pi}}{a}\right)}{14a^{\frac{3}{4}}}$$

input
```
integrate(x**13/(-b*x**4+a)**(3/4),x)
```

output
```
x**14*hyper((3/4, 7/2), (9/2,), b*x**4*exp_polar(2*I*pi)/a)/(14*a**(3/4))
```

Maxima [F]

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^13/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^13/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^13/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^13/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^13/(a - b*x^4)^(3/4),x)
```

output
```
int(x^13/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{13}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{13}}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^13/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**13/(a - b*x**4)**(3/4),x)
```

3.704 $\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = -\frac{2ax^2\sqrt[4]{a-bx^4}}{7b^2} - \frac{x^6\sqrt[4]{a-bx^4}}{7b} + \frac{4a^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{7b^{5/2}(a-bx^4)^{3/4}}$$

output

```
-2/7*a*x^2*(-b*x^4+a)^(1/4)/b^2-1/7*x^6*(-b*x^4+a)^(1/4)/b+4/7*a^(5/2)*(1-
b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^
(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \frac{x^2\left(-2a^2+abx^4+b^2x^8+2a^2\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^4}{a}\right)\right)}{7b^2(a-bx^4)^{3/4}}$$

input
```
Integrate[x^9/(a - b*x^4)^(3/4),x]
```

output
```
(x^2*(-2*a^2 + a*b*x^4 + b^2*x^8 + 2*a^2*(1 - (b*x^4)/a)^(3/4)*Hypergeomet
ric2F1[1/2, 3/4, 3/2, (b*x^4)/a]))/(7*b^2*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}\int \frac{x^8}{(a-bx^4)^{3/4}}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{6a\int \frac{x^4}{(a-bx^4)^{3/4}}dx^2}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{6a\left(\frac{2a\int \frac{1}{(a-bx^4)^{3/4}}dx^2}{3b}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{3b(a-bx^4)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^4)^{3/4}}-\frac{2x^2\sqrt[4]{a-bx^4}}{3b}\right)}{7b}-\frac{2x^6\sqrt[4]{a-bx^4}}{7b}\right)$$

input
```
Int[x^9/(a - b*x^4)^(3/4),x]
```

output
```
((-2*x^6*(a - b*x^4)^(1/4))/(7*b) + (6*a*((-2*x^2*(a - b*x^4)^(1/4))/(3*b)
 + (4*a^(3/2)*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(3*b^(3/2)*(a - b*x^4)^(3/4))))/(7*b))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^{9}}{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(x^9/(-b*x^4+a)^(3/4),x)
```

output
```
int(x^9/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{9}}{(-bx^{4}+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^9/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x^9/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.27

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \frac{x^{10}{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{10a^{\frac{3}{4}}}$$

input
```
integrate(x**9/(-b*x**4+a)**(3/4),x)
```

output
```
x**10*hyper((3/4, 5/2), (7/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(3/4))
```

Maxima [F]

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^9}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^9/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^9/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^9}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^9/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^9/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^9}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^9/(a - b*x^4)^(3/4),x)
```

output
```
int(x^9/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^9}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^9}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^9/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**9/(a - b*x**4)**(3/4),x)
```

3.705 $\int \frac{x^5}{(a-bx^4)^{3/4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{x^5}{(a-bx^4)^{3/4}} dx = -\frac{x^2\sqrt[4]{a-bx^4}}{3b} + \frac{2a^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^4)^{3/4}}$$

output
```
-1/3*x^2*(-b*x^4+a)^(1/4)/b+2/3*a^(3/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(
1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a-bx^4)^{3/4}} dx = \frac{x^2\left(-a+bx^4+a\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^4}{a}\right)\right)}{3b(a-bx^4)^{3/4}}$$

input
```
Integrate[x^5/(a - b*x^4)^(3/4),x]
```

output
```
(x^2*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2
, (b*x^4)/a]))/(3*b*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {807, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a-bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{x^4}{(a-bx^4)^{3/4}} dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} \left(\frac{2a \int \frac{1}{(a-bx^4)^{3/4}} dx^2}{3b} - \frac{2x^2 \sqrt[4]{a-bx^4}}{3b} \right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2} \left(\frac{2a \left(1-\frac{bx^4}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}} dx^2}{3b (a-bx^4)^{3/4}} - \frac{2x^2 \sqrt[4]{a-bx^4}}{3b} \right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2} \left(\frac{4a^{3/2} \left(1-\frac{bx^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2} \arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right), 2\right)}{3b^{3/2} (a-bx^4)^{3/4}} - \frac{2x^2 \sqrt[4]{a-bx^4}}{3b} \right)$$

input
```
Int[x^5/(a - b*x^4)^(3/4),x]
```

output
```
((-2*x^2*(a - b*x^4)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^4)/a)^(3/4)*Ellip
ticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^4)^(3/4)))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x^5}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^5/(-b*x^4+a)^(3/4),x)
```

output
```
int(x^5/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^5}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^5}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x^5/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.34

$$\int \frac{x^5}{(a-bx^4)^{3/4}}\,dx = \frac{x^6 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{6a^{\frac{3}{4}}}$$

input
```
integrate(x**5/(-b*x**4+a)**(3/4),x)
```

output
```
x**6*hyper((3/4, 3/2), (5/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(3/4))
```

Maxima [F]

$$\int \frac{x^5}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^5}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^5/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^5}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^5}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^5/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^5/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^5}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^5/(a - b*x^4)^(3/4),x)
```

output
```
int(x^5/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^5}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^5}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^5/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**5/(a - b*x**4)**(3/4),x)
```

3.706 $\int \frac{x}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx = \frac{\sqrt{a}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a-bx^4)^{3/4}}$$

output
```
a^(1/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x^2/a^(1/2)),
2^(1/2))/b^(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx = \frac{x^2\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^4}{a}\right)}{2\,(a-bx^4)^{3/4}}$$

input
```
Integrate[x/(a - b*x^4)^(3/4),x]
```

output
```
(x^2*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^4)/a])/(2
*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {807, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-bx^4)^{3/4}}dx$$
$$\Big\downarrow \text{807}$$
$$\frac{1}{2}\int \frac{1}{(a-bx^4)^{3/4}}dx^2$$
$$\Big\downarrow \text{231}$$
$$\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{2(a-bx^4)^{3/4}}$$
$$\Big\downarrow \text{230}$$
$$\frac{\sqrt{a}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{b}(a-bx^4)^{3/4}}$$

input
```
Int[x/(a - b*x^4)^(3/4),x]
```

output
```
(Sqrt[a]*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a - b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{x}{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(x/(-b*x^4+a)^(3/4),x)
```

output
```
int(x/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx=\int \frac{x}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx = \frac{x^2 {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^4 e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}}$$

input
```
integrate(x/(-b*x**4+a)**(3/4),x)
```

output
```
x**2*hyper((1/2, 3/4), (3/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(3/4))
```

Maxima [F]

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx = \int \frac{x}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx = \int \frac{x}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx=\int \frac{x}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x/(a - b*x^4)^(3/4),x)
```

output
```
int(x/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x}{(a-bx^4)^{3/4}}\,dx=\int \frac{x}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x/(-b*x^4+a)^(3/4),x)
```

output
```
int(x/(a - b*x**4)**(3/4),x)
```

3.707 $\int \frac{1}{x^3(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{1}{x^3\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{2ax^2}+\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{a}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/2*(-b*x^4+a)^(1/4)/a/x^2+1/2*b^(1/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(
1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^3\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},\frac{bx^4}{a}\right)}{2x^2\left(a-bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^3*(a - b*x^4)^(3/4)),x]
```

output
```
-1/2*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (b*x^4)/a])/
(x^2*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {807, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a-b x^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(a-b x^4\right)^{3/4}}dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{b \int \frac{1}{\left(a-b x^4\right)^{3/4}}dx^2}{2 a}-\frac{\sqrt[4]{a-b x^4}}{a x^2}\right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2} \left(\frac{b \left(1-\frac{b x^4}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{b x^4}{a}\right)^{3/4}}dx^2}{2 a \left(a-b x^4\right)^{3/4}}-\frac{\sqrt[4]{a-b x^4}}{a x^2}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2} \left(\frac{\sqrt{b} \left(1-\frac{b x^4}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2} \arcsin \left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a} \left(a-b x^4\right)^{3/4}}-\frac{\sqrt[4]{a-b x^4}}{a x^2}\right)$$

input
```
Int[1/(x^3*(a - b*x^4)^(3/4)),x]
```

output
```
(-((a - b*x^4)^(1/4)/(a*x^2)) + (Sqrt[b]*(1 - (b*x^4)/a)^(3/4)*EllipticF[A
rcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^4)^(3/4)))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{3} \left(-b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^3/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/x^3/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^7 - a*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}}\,dx = -\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^4e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}x^2}$$

input
```
integrate(1/x**3/(-b*x**4+a)**(3/4),x)
```

output
```
-hyper((-1/2, 3/4), (1/2,), b*x**4*exp_polar(2*I*pi)/a)/(2*a**(3/4)*x**2)
```

Maxima [F]

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^3}\,dx$$

input
```
integrate(1/x^3/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{x^3 (a-b\,x^4)^{3/4}}\,dx$$

input
```
int(1/(x^3*(a - b*x^4)^(3/4)),x)
```

output
```
int(1/(x^3*(a - b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^3 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}\,x^3}dx$$

input
```
int(1/x^3/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**3),x)
```

3.708 $\int \frac{1}{x^7(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 108

$$\int \frac{1}{x^7\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{6ax^6}-\frac{5b\sqrt[4]{a-bx^4}}{12a^2x^2}+\frac{5b^{3/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/6*(-b*x^4+a)^(1/4)/a/x^6-5/12*b*(-b*x^4+a)^(1/4)/a^2/x^2+5/12*b^(3/2)*(
1-b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/
a^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^7\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},\frac{bx^4}{a}\right)}{6x^6\left(a-bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^7*(a - b*x^4)^(3/4)),x]
```

output
```
-1/6*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (b*x^4)/a])
/(x^6*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {807, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a-bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^8 (a-bx^4)^{3/4}} dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{5b \int \frac{1}{x^4 (a-bx^4)^{3/4}} dx^2}{6a} - \frac{\sqrt[4]{a-bx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{5b \left(\frac{b \int \frac{1}{(a-bx^4)^{3/4}} dx^2}{2a} - \frac{\sqrt[4]{a-bx^4}}{ax^2} \right)}{6a} - \frac{\sqrt[4]{a-bx^4}}{3ax^6} \right)$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}\left(\frac{5b\left(\frac{b\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{2a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{2}\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)$$

input
```
Int[1/(x^7*(a - b*x^4)^(3/4)),x]
```

output
```
(-1/3*(a - b*x^4)^(1/4)/(a*x^6) + (5*b*(-((a - b*x^4)^(1/4)/(a*x^2)) + (Sq
rt[b]*(1 - (b*x^4)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])
/(Sqrt[a]*(a - b*x^4)^(3/4))))/(6*a))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{7} \left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^7/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/x^7/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^7 \left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^7}\,dx$$

input
```
integrate(1/x^7/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^11 - a*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^7 \left(a-bx^4\right)^{3/4}}\,dx=-\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{6a^{\frac{3}{4}}x^6}$$

input
```
integrate(1/x**7/(-b*x**4+a)**(3/4),x)
```

output
```
-hyper((-3/2, 3/4), (-1/2,), b*x**4*exp_polar(2*I*pi)/a)/(6*a**(3/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^7 (a-bx^4)^{3/4}}\, dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}} x^7}\, dx$$

input
```
integrate(1/x^7/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^7), x)
```

Giac [F]

$$\int \frac{1}{x^7 (a-bx^4)^{3/4}}\, dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}} x^7}\, dx$$

input
```
integrate(1/x^7/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^7 (a - bx^4)^{3/4}} \, dx = \int \frac{1}{x^7 (a - b x^4)^{3/4}} \, dx$$

input
```
int(1/(x^7*(a - b*x^4)^(3/4)),x)
```

output
```
int(1/(x^7*(a - b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^7 (a - bx^4)^{3/4}} \, dx = \int \frac{1}{(-b x^4 + a)^{\frac{3}{4}} x^7} dx$$

input
```
int(1/x^7/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**7),x)
```

3.709 $\int \frac{1}{x^{11}(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{10ax^{10}}-\frac{3b\sqrt[4]{a-bx^4}}{20a^2x^6}-\frac{3b^2\sqrt[4]{a-bx^4}}{8a^3x^2}+\frac{3b^{5/2}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{8a^{5/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/10*(-b*x^4+a)^(1/4)/a/x^10-3/20*b*(-b*x^4+a)^(1/4)/a^2/x^6-3/8*b^2*(-b*
x^4+a)^(1/4)/a^3/x^2+3/8*b^(5/2)*(1-b*x^4/a)^(3/4)*InverseJacobiAM(1/2*arc
sin(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},\frac{bx^4}{a}\right)}{10x^{10}\left(a-bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^11*(a - b*x^4)^(3/4)),x]
```

output
```
-1/10*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (b*x^4)/a]
)/(x^10*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {807, 264, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{11} \left(a-b x^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2} \int \frac{1}{x^{12} \left(a-b x^4\right)^{3/4}}dx^2$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{9 b \int \frac{1}{x^8 \left(a-b x^4\right)^{3/4}}dx^2}{10 a}-\frac{\sqrt[4]{a-b x^4}}{5 a x^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2} \left(\frac{9 b \left(\frac{5 b \int \frac{1}{x^4 \left(a-b x^4\right)^{3/4}}dx^2}{6 a}-\frac{\sqrt[4]{a-b x^4}}{3 a x^6}\right)}{10 a}-\frac{\sqrt[4]{a-b x^4}}{5 a x^{10}}\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}\left(\frac{9b\left(\frac{5b\left(\frac{b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx^2}{2a}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)}{10a}-\frac{\sqrt[4]{a-bx^4}}{5ax^{10}}\right)$$

$$\Big\downarrow \; 231$$

$$\frac{1}{2}\left(\frac{9b\left(\frac{5b\left(\frac{b\left(1-\frac{bx^4}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^4}{a}\right)^{3/4}}dx^2}{2a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)}{10a}-\frac{\sqrt[4]{a-bx^4}}{5ax^{10}}\right)$$

$$\Big\downarrow \; 230$$

$$\frac{1}{2}\left(\frac{9b\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^4}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{ax^2}\right)}{6a}-\frac{\sqrt[4]{a-bx^4}}{3ax^6}\right)}{10a}-\frac{\sqrt[4]{a-bx^4}}{5ax^{10}}\right)$$

input
```
Int[1/(x^11*(a - b*x^4)^(3/4)),x]
```

output
```
(-1/5*(a - b*x^4)^(1/4)/(a*x^10) + (9*b*(-1/3*(a - b*x^4)^(1/4)/(a*x^6) +
(5*b*(-((a - b*x^4)^(1/4)/(a*x^2)) + (Sqrt[b]*(1 - (b*x^4)/a)^(3/4)*Ellipt
icF[ArcSin[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^4)^(3/4))))/(6*
a)))/(10*a))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{11} \left(-b\,x^{4}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^11/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/x^11/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{11} \left(a-b x^4\right)^{3/4}} \, dx=\int \frac{1}{{\left(-b x^{4} + a\right)}^{\frac{3}{4}} x^{11}} \,d x$$

input
```
integrate(1/x^11/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^15 - a*x^11), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^{11} \left(a-b x^4\right)^{3/4}} \, dx=-\frac{{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{4} e^{2 i \pi}}{a}} \right)}}{10 a^{\frac{3}{4}} x^{10}}$$

input
```
integrate(1/x**11/(-b*x**4+a)**(3/4),x)
```

output
```
-hyper((-5/2, 3/4), (-3/2,), b*x**4*exp_polar(2*I*pi)/a)/(10*a**(3/4)*x**1
0)
```

Maxima [F]

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^11), x)
```

Giac [F]

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{11}}\,dx$$

input
```
integrate(1/x^11/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^11), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{11}\left(a-bx^4\right)^{3/4}}\,dx = \int \frac{1}{x^{11}\left(a-b\,x^4\right)^{3/4}}\,dx$$

input
```
int(1/(x^11*(a - b*x^4)^(3/4)),x)
```

output
```
int(1/(x^11*(a - b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{11} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{\left(-b\,x^4 + a\right)^{\frac{3}{4}} x^{11}} dx$$

input
```
int(1/x^11/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**11),x)
```

3.710 $\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 202

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = -\frac{7ax^3\sqrt[4]{a-bx^4}}{32b^2} - \frac{x^7\sqrt[4]{a-bx^4}}{8b} - \frac{21a^2\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}b^{11/4}} + \frac{21a^2\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{64\sqrt{2}b^{11/4}} - \frac{21a^2\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{64\sqrt{2}b^{11/4}}$$

output
```
-7/32*a*x^3*(-b*x^4+a)^(1/4)/b^2-1/8*x^7*(-b*x^4+a)^(1/4)/b+21/128*a^2*arc
tan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(11/4)+21/128*a^2*arc
tan(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(11/4)-21/128*a^2*arct
anh(2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)*x^2/(-b*x^4+a)^(1/2)))*2
^(1/2)/b^(11/4)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = \frac{-4b^{3/4}x^3\sqrt[4]{a-bx^4}(7a+4bx^4)+21\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{-\sqrt{b}x^2+\sqrt{a-bx^4}}\right)-21\sqrt{2}a^2\text{arctan}}{128b^{11/4}}$$

input
```
Integrate[x^10/(a - b*x^4)^(3/4),x]
```

output
```
(-4*b^(3/4)*x^3*(a - b*x^4)^(1/4)*(7*a + 4*b*x^4) + 21*Sqrt[2]*a^2*ArcTan[
(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))/(-(Sqrt[b]*x^2) + Sqrt[a - b*x^4])]
- 21*Sqrt[2]*a^2*ArcTanh[(Sqrt[b]*x^2 + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*
x*(a - b*x^4)^(1/4))])/(128*b^(11/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {843, 843, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{7a\int \frac{x^6}{(a-bx^4)^{3/4}}dx}{8b}-\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$$\Big\downarrow \text{843}$$

$$\frac{7a\left(\frac{3a\int \frac{x^2}{(a-bx^4)^{3/4}}dx}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)}{8b}-\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$$\Big\downarrow \text{854}$$

$$\frac{7a\left(\frac{3a\int \frac{x^2}{\sqrt{a-bx^4}\left(\frac{bx^4}{a-bx^4}+1\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)}{8b}-\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$\downarrow$ 826

$$\frac{7a\left(\frac{3a\left(\frac{\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)}{8b}-\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$\downarrow$ 1476

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}\right.}{8b}$$

$$\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$\downarrow$ 1082

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)}{8b}-\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}\right)}{8b}-\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\cdots}{\sqrt[4]{b}\left(\cdots\right)}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{4b}\cdots\right)}{8b}\cdots$$

$$\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$\big\downarrow$ 25

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\cdots\right.}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}\cdots\right.}}{\cdots}}{2\sqrt{b}}\cdots\right.}{4b}\cdots\right.}{8b}$$

$$\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$$\Big\downarrow \; 27$$

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\cdots}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-\cdots}}\cdots}\cdots}{2\cdots}}{2\sqrt{b}}\cdots\right)}{4b}\cdots\right)}{8b}\cdots$$

$$\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

$\downarrow$ 1103

$$\frac{7a\left(\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{4b}-\frac{x}{}\cdots\right)}{8b}\cdots$$

$$\frac{x^7\sqrt[4]{a-bx^4}}{8b}$$

input
```
Int[x^10/(a - b*x^4)^(3/4),x]
```

output
```
-1/8*(x^7*(a - b*x^4)^(1/4))/b + (7*a*(-1/4*(x^3*(a - b*x^4)^(1/4))/b + (3
*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))
) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/(
2*Sqrt[b]) - (-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4
)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a -
 b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)))/(2*S
qrt[b])))/(4*b)))/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-32\left(-b\,x^{4}+a\right)^{\frac{1}{4}} x^{7} b^{\frac{7}{4}}-56\left(-b\,x^{4}+a\right)^{\frac{1}{4}} x^{3} a\,b^{\frac{3}{4}}-21\ln\left(\frac{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)\sqrt{2}\,a^{2}-42\arctan\left(\frac{\sqrt{2}\,(-\cdots}{\cdots}\right)\cdots}{256 b^{\frac{11}{4}}}$

input
```
int(x^10/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/256*(-32*(-b*x^4+a)^(1/4)*x^7*b^(7/4)-56*(-b*x^4+a)^(1/4)*x^3*a*b^(3/4)-
21*ln((b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2))/(-
b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+a)^(1/2)))*2^(1/2)*
a^2-42*arctan((2^(1/2)*(-b*x^4+a)^(1/4)+b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a^2-
42*arctan((2^(1/2)*(-b*x^4+a)^(1/4)-b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a^2)/b^(
11/4)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.19

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx =$$

$$-\frac{21\,b^2\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(\frac{21\left(b^3x\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}+(-bx^4+a)^{\frac{1}{4}}a^2\right)}{x}\right)-21\,b^2\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}\log\left(-\frac{21\left(b^3x\left(-\frac{a^8}{b^{11}}\right)^{\frac{1}{4}}-(-bx^4+a)^{\frac{1}{4}}a^2\right)}{x}\right)-}{}$$

input
```
integrate(x^10/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/128*(21*b^2*(-a^8/b^11)^(1/4)*log(21*(b^3*x*(-a^8/b^11)^(1/4) + (-b*x^4
 + a)^(1/4)*a^2)/x) - 21*b^2*(-a^8/b^11)^(1/4)*log(-21*(b^3*x*(-a^8/b^11)^
(1/4) - (-b*x^4 + a)^(1/4)*a^2)/x) - 21*I*b^2*(-a^8/b^11)^(1/4)*log(-21*(I
*b^3*x*(-a^8/b^11)^(1/4) - (-b*x^4 + a)^(1/4)*a^2)/x) + 21*I*b^2*(-a^8/b^1
1)^(1/4)*log(-21*(-I*b^3*x*(-a^8/b^11)^(1/4) - (-b*x^4 + a)^(1/4)*a^2)/x)
+ 4*(4*b*x^7 + 7*a*x^3)*(-b*x^4 + a)^(1/4))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.19

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = \frac{x^{11}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate(x**10/(-b*x**4+a)**(3/4),x)
```

output
```
x**11*gamma(11/4)*hyper((3/4, 11/4), (15/4,), b*x**4*exp_polar(2*I*pi)/a)/
(4*a**(3/4)*gamma(15/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.36

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = -\frac{\frac{11\,(-bx^4+a)^{\frac{1}{4}}a^2b}{x}+\frac{7\,(-bx^4+a)^{\frac{5}{4}}a^2}{x^5}}{32\left(b^4-\frac{2\,(bx^4-a)b^3}{x^4}+\frac{(bx^4-a)^2b^2}{x^8}\right)} - \frac{21\left(\frac{2\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\,(-bx^4+a)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}a^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\,(-bx^4+a)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}a^2\log\left(\sqrt{b}+\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}\cdots\right.}{b^{\frac{3}{4}}}\right.}{256\,b^2}$$

input
```
integrate(x^10/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/32*(11*(-b*x^4 + a)^(1/4)*a^2*b/x + 7*(-b*x^4 + a)^(5/4)*a^2/x^5)/(b^4
- 2*(b*x^4 - a)*b^3/x^4 + (b*x^4 - a)^2*b^2/x^8) - 21/256*(2*sqrt(2)*a^2*a
rctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3
/4) + 2*sqrt(2)*a^2*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(-b*x^4 + a)^
(1/4)/x)/b^(1/4))/b^(3/4) + sqrt(2)*a^2*log(sqrt(b) + sqrt(2)*(-b*x^4 + a)
^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) - sqrt(2)*a^2*log(sqrt(b)
 - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4))/b
^2
```

Giac [F]

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{10}}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^10/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^10/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{10}}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^10/(a - b*x^4)^(3/4),x)
```

output
```
int(x^10/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{10}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{10}}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^10/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**10/(a - b*x**4)**(3/4),x)
```

3.711 $\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 173

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = -\frac{x^3\sqrt[4]{a-bx^4}}{4b} - \frac{3a\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{7/4}} + \frac{3a\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{8\sqrt{2}b^{7/4}} - \frac{3a\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{8\sqrt{2}b^{7/4}}$$

output
```
-1/4*x^3*(-b*x^4+a)^(1/4)/b+3/16*a*arctan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^
(1/4))*2^(1/2)/b^(7/4)+3/16*a*arctan(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))
*2^(1/2)/b^(7/4)-3/16*a*arctanh(2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1
/2)*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = -\frac{4b^{3/4}x^3\sqrt[4]{a-bx^4}+3\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{\sqrt{b}x^2-\sqrt{a-bx^4}}\right)+3\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)}{16b^{7/4}}$$

input
```
Integrate[x^6/(a - b*x^4)^(3/4),x]
```

output
```
-1/16*(4*b^(3/4)*x^3*(a - b*x^4)^(1/4) + 3*Sqrt[2]*a*ArcTan[(Sqrt[2]*b^(1/
4)*x*(a - b*x^4)^(1/4))/(Sqrt[b]*x^2 - Sqrt[a - b*x^4])] + 3*Sqrt[2]*a*Arc
Tanh[(Sqrt[b]*x^2 + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))
])/b^(7/4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {843, 854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx$$

$$\Big\downarrow \text{843}$$

$$\frac{3a\int \frac{x^2}{(a-bx^4)^{3/4}}dx}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$$\Big\downarrow \text{854}$$

$$\frac{3a\int \frac{x^2}{\sqrt{a-bx^4}\left(\frac{bx^4}{a-bx^4}+1\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 826

$$\frac{3a\left(\frac{\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 1476

$$\frac{3a\left(\frac{\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b} \frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 1082

$$\frac{3a\left(\frac{\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 217

$$\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 1479

$$3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\cdots}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}\cdots\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$$\frac{\cdots}{4b}$$

$$\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 25

$$3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\cdots\right.}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\cdots\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$$\frac{\cdots}{4b}$$

$$\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$\Big\downarrow$ 27

$$3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\cdots}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-b\cdots}}\cdots}\cdots}{2\sqrt{b}\cdots}}{2\sqrt{b}}\cdots\right.$$

$$\frac{\cdots}{4b}$$

$$\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{4b}-\frac{x^3\sqrt[4]{a-bx^4}}{4b}$$

input
```
Int[x^6/(a - b*x^4)^(3/4),x]
```

output
```
-1/4*(x^3*(a - b*x^4)^(1/4))/b + (3*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(
a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a -
 b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[1 + (Sqrt[b]*x^2
)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4
)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^
4)^(1/4)]/(2*Sqrt[2]*b^(1/4)))/(2*Sqrt[b])))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{3\left(\frac{4\left(-b\,x^4+a\right)^{\frac{1}{4}}x^3 b^{\frac{3}{4}}}{3}+\arctan\left(\frac{\sqrt{2}\left(-b\,x^4+a\right)^{\frac{1}{4}}-b^{\frac{1}{4}}x}{b^{\frac{1}{4}}x}\right)\sqrt{2}\,a+\arctan\left(\frac{\sqrt{2}\left(-b\,x^4+a\right)^{\frac{1}{4}}+b^{\frac{1}{4}}x}{b^{\frac{1}{4}}x}\right)\sqrt{2}\,a+\frac{\ln\left(\frac{b^{\frac{1}{4}}\left(-b\,x^4+a\right)}{-b^{\frac{1}{4}}\left(-b\,x^4+a\right)}\right.}{}}{16 b^{\frac{7}{4}}}$

input
```
int(x^6/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-3/16/b^(7/4)*(4/3*(-b*x^4+a)^(1/4)*x^3*b^(3/4)+arctan((2^(1/2)*(-b*x^4+a)
^(1/4)-b^(1/4)*x)/b^(1/4)/x)*2^(1/2)*a+arctan((2^(1/2)*(-b*x^4+a)^(1/4)+b^
(1/4)*x)/b^(1/4)/x)*2^(1/2)*a+1/2*ln((b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b
^(1/2)*x^2+(-b*x^4+a)^(1/2))/(-b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*
x^2+(-b*x^4+a)^(1/2)))*2^(1/2)*a)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.24

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx =$$

$$-\frac{4\left(-bx^4+a\right)^{\frac{1}{4}}x^3+3\,b\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(\frac{3\left(b^2x\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}+\left(-bx^4+a\right)^{\frac{1}{4}}a\right)}{x}\right)-3\,b\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(b^2x\left(-\frac{a^4}{b^7}\right)^{\frac{1}{4}}-\left(-bx^4+\right.\right.}{x}\right.}{}$$

input
```
integrate(x^6/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/16*(4*(-b*x^4 + a)^(1/4)*x^3 + 3*b*(-a^4/b^7)^(1/4)*log(3*(b^2*x*(-a^4/
b^7)^(1/4) + (-b*x^4 + a)^(1/4)*a)/x) - 3*b*(-a^4/b^7)^(1/4)*log(-3*(b^2*x
*(-a^4/b^7)^(1/4) - (-b*x^4 + a)^(1/4)*a)/x) - 3*I*b*(-a^4/b^7)^(1/4)*log(
-3*(I*b^2*x*(-a^4/b^7)^(1/4) - (-b*x^4 + a)^(1/4)*a)/x) + 3*I*b*(-a^4/b^7)
^(1/4)*log(-3*(-I*b^2*x*(-a^4/b^7)^(1/4) - (-b*x^4 + a)^(1/4)*a)/x))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.23

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = \frac{x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**6/(-b*x**4+a)**(3/4),x)
```

output
```
x**7*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
a**(3/4)*gamma(11/4))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.28

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = -\frac{3\left(\frac{2\sqrt{2}a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2(-bx^4+a)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{2\sqrt{2}a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2(-bx^4+a)^{\frac{1}{4}}}{x}\right)}{2b^{\frac{1}{4}}}\right)}{b^{\frac{3}{4}}}+\frac{\sqrt{2}a\log\left(\sqrt{b}+\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\sqrt{-\ldots}\right.}{b^{\frac{3}{4}}}\right.}{32\,b} - \frac{(-bx^4+a)^{\frac{1}{4}}a}{4\left(b^2-\frac{(bx^4-a)b}{x^4}\right)x}$$

input
```
integrate(x^6/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-3/32*(2*sqrt(2)*a*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1
/4)/x)/b^(1/4))/b^(3/4) + 2*sqrt(2)*a*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4)
 - 2*(-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) + sqrt(2)*a*log(sqrt(b) + sqrt
(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) - sqrt(2)
*a*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x
^2)/b^(3/4))/b - 1/4*(-b*x^4 + a)^(1/4)*a/((b^2 - (b*x^4 - a)*b/x^4)*x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^6}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^6}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^6/(a - b*x^4)^(3/4),x)
```

output
```
int(x^6/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^6}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**6/(a - b*x**4)**(3/4),x)
```

3.712 $\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}b^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}b^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}\left(1+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}\right)}{2\sqrt{2}b^{3/4}}$$

output
```
1/4*arctan(-1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(3/4)+1/4*arct
an(1+2^(1/2)*b^(1/4)*x/(-b*x^4+a)^(1/4))*2^(1/2)/b^(3/4)-1/4*arctanh(2^(1/
2)*b^(1/4)*x/(-b*x^4+a)^(1/4)/(1+b^(1/2)*x^2/(-b*x^4+a)^(1/2)))*2^(1/2)/b^
(3/4)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}{-\sqrt{b}x^2+\sqrt{a-bx^4}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{b}x^2+\sqrt{a-bx^4}}{\sqrt{2}\sqrt[4]{b}x\sqrt[4]{a-bx^4}}\right)}{2\sqrt{2}b^{3/4}}$$

input
```
Integrate[x^2/(a - b*x^4)^(3/4),x]
```

output
```
(ArcTan[(Sqrt[2]*b^(1/4)*x*(a - b*x^4)^(1/4))/(-(Sqrt[b]*x^2) + Sqrt[a - b
*x^4])] - ArcTanh[(Sqrt[b]*x^2 + Sqrt[a - b*x^4])/(Sqrt[2]*b^(1/4)*x*(a -
b*x^4)^(1/4))])/(2*Sqrt[2]*b^(3/4))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {854, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{854}$$

$$\int \frac{x^2}{\sqrt{a-bx^4}\left(\frac{bx^4}{a-bx^4}+1\right)}d\frac{x}{\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{826}$$

$$\frac{\int \frac{\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}+\frac{\int \frac{1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\int \frac{1}{-\frac{x^2}{\sqrt{a-bx^4}}-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}}{\frac{bx^4}{a-bx^4}+1}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt[4]{b}\left(\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}\right)}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}-\frac{2\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}}{\frac{x^2}{\sqrt{a-bx^4}}-\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{2}\sqrt{b}}+\frac{\int\frac{\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1}{\frac{x^2}{\sqrt{a-bx^4}}+\frac{\sqrt{2}x}{\sqrt[4]{b}\sqrt[4]{a-bx^4}}+\frac{1}{\sqrt{b}}}d\frac{x}{\sqrt[4]{a-bx^4}}}{2\sqrt{b}}}{2\sqrt{b}}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+1\right)}{\sqrt{2}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}\right)}{\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a-bx^4}}+\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}+1\right)}{2\sqrt{2}\sqrt[4]{b}}}{2\sqrt{b}}$$

input

```
Int[x^2/(a - b*x^4)^(3/4),x]
```

output
```
(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4))) +
ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)))/(2*Sq
rt[b]) - (-1/2*Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x^4] - (Sqrt[2]*b^(1/4)*x)
/(a - b*x^4)^(1/4)]/(Sqrt[2]*b^(1/4)) + Log[1 + (Sqrt[b]*x^2)/Sqrt[a - b*x
^4] + (Sqrt[2]*b^(1/4)*x)/(a - b*x^4)^(1/4)]/(2*Sqrt[2]*b^(1/4)))/(2*Sqrt[
b])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{\sqrt{2}\left(\ln\left(\frac{b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}{-b^{\frac{1}{4}}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{b}\,x^{2}+\sqrt{-b\,x^{4}+a}}\right)-2\arctan\left(-\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)+2\arctan\left(\frac{\sqrt{2}\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{b^{\frac{1}{4}}x}+1\right)\right)}{8b^{\frac{3}{4}}}$

input
```
int(x^2/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^(3/4)*2^(1/2)*(ln((b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(
-b*x^4+a)^(1/2))/(-b^(1/4)*(-b*x^4+a)^(1/4)*2^(1/2)*x+b^(1/2)*x^2+(-b*x^4+
a)^(1/2)))-2*arctan(-2^(1/2)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1)+2*arctan(2^(1/2
)/b^(1/4)*(-b*x^4+a)^(1/4)/x+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx = -\frac{1}{4}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{bx\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+(-bx^4+a)^{\frac{1}{4}}}{x}\right) + \frac{1}{4}\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(-\frac{bx\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}-(-bx^4+a)^{\frac{1}{4}}}{x}\right) - \frac{1}{4}i\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{i\,bx\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+(-bx^4+a)^{\frac{1}{4}}}{x}\right) + \frac{1}{4}i\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}\log\left(\frac{-i\,bx\left(-\frac{1}{b^3}\right)^{\frac{1}{4}}+(-bx^4+a)^{\frac{1}{4}}}{x}\right)$$

input
```
integrate(x^2/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/4*(-1/b^3)^(1/4)*log((b*x*(-1/b^3)^(1/4) + (-b*x^4 + a)^(1/4))/x) + 1/4
*(-1/b^3)^(1/4)*log(-(b*x*(-1/b^3)^(1/4) - (-b*x^4 + a)^(1/4))/x) - 1/4*I*
(-1/b^3)^(1/4)*log((I*b*x*(-1/b^3)^(1/4) + (-b*x^4 + a)^(1/4))/x) + 1/4*I*
(-1/b^3)^(1/4)*log((-I*b*x*(-1/b^3)^(1/4) + (-b*x^4 + a)^(1/4))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-b*x**4+a)**(3/4),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(3/4)*gamma(7/4))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}+\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}} - \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}b^{\frac{1}{4}}-\frac{2\left(-bx^4+a\right)^{\frac{1}{4}}}{x}\right)}{2\,b^{\frac{1}{4}}}\right)}{4\,b^{\frac{3}{4}}} - \frac{\sqrt{2}\log\left(\sqrt{b}+\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{8\,b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\sqrt{b}-\frac{\sqrt{2}(-bx^4+a)^{\frac{1}{4}}b^{\frac{1}{4}}}{x}+\frac{\sqrt{-bx^4+a}}{x^2}\right)}{8\,b^{\frac{3}{4}}}$$

input
```
integrate(x^2/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*b^(1/4) + 2*(-b*x^4 + a)^(1/4)/x)
/b^(1/4))/b^(3/4) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*b^(1/4) - 2*(
-b*x^4 + a)^(1/4)/x)/b^(1/4))/b^(3/4) - 1/8*sqrt(2)*log(sqrt(b) + sqrt(2)*
(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2)/b^(3/4) + 1/8*sqrt(2)
*log(sqrt(b) - sqrt(2)*(-b*x^4 + a)^(1/4)*b^(1/4)/x + sqrt(-b*x^4 + a)/x^2
)/b^(3/4)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx=\int \frac{x^2}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx=\int \frac{x^2}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^2/(a - b*x^4)^(3/4),x)
```

output
```
int(x^2/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^4)^{3/4}}\,dx=\int \frac{x^2}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**2/(a - b*x**4)**(3/4),x)
```

3.713 $\int \frac{1}{x^2(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{1}{x^2(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{ax}$$

output
```
-(-b*x^4+a)^(1/4)/a/x
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{ax}$$

input
```
Integrate[1/(x^2*(a - b*x^4)^(3/4)),x]
```

output
```
-((a - b*x^4)^(1/4)/(a*x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a-b x^4\right)^{3/4}} \, dx$$
$$\Big\downarrow \ 796$$
$$-\frac{\sqrt[4]{a-b x^4}}{a x}$$

input
```
Int[1/(x^2*(a - b*x^4)^(3/4)),x]
```

output
```
-((a - b*x^4)^(1/4)/(a*x))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a x}$	19
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a x}$	19
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a x}$	19
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}}}{a x}$	19
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}}}{a x \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	46

input
```
int(1/x^2/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-(-b*x^4+a)^(1/4)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\left(-bx^4+a\right)^{\frac{1}{4}}}{ax}$$

input
```
integrate(1/x^2/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-(-b*x^4 + a)^(1/4)/(a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 4.00

$$\int \frac{1}{x^2 (a-bx^4)^{3/4}}\,dx = \begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)} & \text{for } \left|\frac{a}{bx^4}\right|>1 \\ -\frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^4}+1}e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{4a\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-1/4)/(4*a*gamma(3/4)),
Abs(a/(b*x**4)) > 1), (-b**(1/4)*(-a/(b*x**4) + 1)**(1/4)*exp(-3*I*pi/4)*g
amma(-1/4)/(4*a*gamma(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 (a-bx^4)^{3/4}}\,dx = -\frac{(-bx^4+a)^{\frac{1}{4}}}{ax}$$

input
```
integrate(1/x^2/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-(-b*x^4 + a)^(1/4)/(a*x)
```

Giac [F]

$$\int \frac{1}{x^2 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 (a-bx^4)^{3/4}}\,dx = -\frac{(a-b\,x^4)^{1/4}}{a\,x}$$

input
```
int(1/(x^2*(a - b*x^4)^(3/4)),x)
```

output
```
-(a - b*x^4)^(1/4)/(a*x)
```

Reduce [F]

$$\int \frac{1}{x^2 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(1/x^2/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**2),x)
```

3.714 $\int \frac{1}{x^6(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{5ax^5}-\frac{4b\sqrt[4]{a-bx^4}}{5a^2x}$$

output
```
-1/5*(-b*x^4+a)^(1/4)/a/x^5-4/5*b*(-b*x^4+a)^(1/4)/a^2/x
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx=\frac{\left(-a-4bx^4\right)\sqrt[4]{a-bx^4}}{5a^2x^5}$$

input
```
Integrate[1/(x^6*(a - b*x^4)^(3/4)),x]
```

output
```
((-a - 4*b*x^4)*(a - b*x^4)^(1/4))/(5*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a-b x^4\right)^{3/4}} \, dx$$
$$\Big\downarrow \text{803}$$
$$\frac{4 b \int \frac{1}{x^2 \left(a-b x^4\right)^{3/4}}dx}{5 a}-\frac{\sqrt[4]{a-b x^4}}{5 a x^5}$$
$$\Big\downarrow \text{796}$$
$$-\frac{4 b \sqrt[4]{a-b x^4}}{5 a^2 x}-\frac{\sqrt[4]{a-b x^4}}{5 a x^5}$$

input
```
Int[1/(x^6*(a - b*x^4)^(3/4)),x]
```

output
```
-1/5*(a - b*x^4)^(1/4)/(a*x^5) - (4*b*(a - b*x^4)^(1/4))/(5*a^2*x)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(4 b\,x^{4}+a\right)}{5 a^{2} x^{5}}$	27
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(4 b\,x^{4}+a\right)}{5 a^{2} x^{5}}$	27
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(4 b\,x^{4}+a\right)}{5 a^{2} x^{5}}$	27
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(4 b\,x^{4}+a\right)}{5 a^{2} x^{5}}$	27
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(4 b\,x^{4}+a\right)}{5 a^{2} x^{5} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	54

input
```
int(1/x^6/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b*x^4+a)^(1/4)*(4*b*x^4+a)/a^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6 \left(a-b x^4\right)^{3/4}}\,dx = -\frac{(4\,bx^4+a)(-bx^4+a)^{\frac{1}{4}}}{5\,a^2x^5}$$

input
```
integrate(1/x^6/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/5*(4*b*x^4 + a)*(-b*x^4 + a)^(1/4)/(a^2*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 311, normalized size of antiderivative = 6.76

$$\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx=\begin{cases} -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{5}{4}\right)}{16ax^4\Gamma\left(\frac{3}{4}\right)}-\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^4}-1}\Gamma\left(-\frac{5}{4}\right)}{4a^2\Gamma\left(\frac{3}{4}\right)} \\ -\frac{a^2b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{-16a^3bx^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+16a^2b^2x^8e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)}-\frac{3ab^{\frac{9}{4}}x^4\sqrt[4]{-\frac{a}{bx^4}+1}\Gamma\left(-\frac{5}{4}\right)}{-16a^3bx^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+16a^2b^2x^8e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)}+\frac{4b^{\frac{13}{4}}x^8 \cdots}{-16a^3bx^4e^{\frac{3i\pi}{4}} \cdots} \end{cases}$$

input
```
integrate(1/x**6/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-b**(1/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(16*a*x**4*gamma(
3/4)) - b**(5/4)*(a/(b*x**4) - 1)**(1/4)*gamma(-5/4)/(4*a**2*gamma(3/4)),
Abs(a/(b*x**4)) > 1), (-a**2*b**(5/4)*(-a/(b*x**4) + 1)**(1/4)*gamma(-5/4)
/(-16*a**3*b*x**4*exp(3*I*pi/4)*gamma(3/4) + 16*a**2*b**2*x**8*exp(3*I*pi/
4)*gamma(3/4)) - 3*a*b**(9/4)*x**4*(-a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(-
16*a**3*b*x**4*exp(3*I*pi/4)*gamma(3/4) + 16*a**2*b**2*x**8*exp(3*I*pi/4)*
gamma(3/4)) + 4*b**(13/4)*x**8*(-a/(b*x**4) + 1)**(1/4)*gamma(-5/4)/(-16*a
**3*b*x**4*exp(3*I*pi/4)*gamma(3/4) + 16*a**2*b**2*x**8*exp(3*I*pi/4)*gamm
a(3/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^6\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\frac{5\left(-bx^4+a\right)^{\frac{1}{4}}b}{x}+\frac{\left(-bx^4+a\right)^{\frac{5}{4}}}{x^5}}{5\,a^2}$$

input
```
integrate(1/x^6/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/5*(5*(-b*x^4 + a)^(1/4)*b/x + (-b*x^4 + a)^(5/4)/x^5)/a^2
```

Giac [F]

$$\int \frac{1}{x^6 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^6), x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^6 (a-bx^4)^{3/4}}\,dx = -\frac{(a-b\,x^4)^{1/4}\,(4\,b\,x^4+a)}{5\,a^2\,x^5}$$

input
```
int(1/(x^6*(a - b*x^4)^(3/4)),x)
```

output
```
-((a - b*x^4)^(1/4)*(a + 4*b*x^4))/(5*a^2*x^5)
```

Reduce [F]

$$\int \frac{1}{x^6 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}\,x^6}dx$$

input
```
int(1/x^6/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**6),x)
```

3.715 $\int \frac{1}{x^{10}(a-bx^4)^{3/4}} dx$

Optimal result

Integrand size = 16, antiderivative size = 71

$$\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{9ax^9}-\frac{8b\sqrt[4]{a-bx^4}}{45a^2x^5}-\frac{32b^2\sqrt[4]{a-bx^4}}{45a^3x}$$

output
```
-1/9*(-b*x^4+a)^(1/4)/a/x^9-8/45*b*(-b*x^4+a)^(1/4)/a^2/x^5-32/45*b^2*(-b*
x^4+a)^(1/4)/a^3/x
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx=\frac{\sqrt[4]{a-bx^4}(-5a^2-8abx^4-32b^2x^8)}{45a^3x^9}$$

input
```
Integrate[1/(x^10*(a - b*x^4)^(3/4)),x]
```

output
```
((a - b*x^4)^(1/4)*(-5*a^2 - 8*a*b*x^4 - 32*b^2*x^8))/(45*a^3*x^9)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{10} \left(a-b x^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{8 b \int \frac{1}{x^6 \left(a-b x^4\right)^{3/4}}dx}{9 a}-\frac{\sqrt[4]{a-b x^4}}{9 a x^9}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{8 b \left(\frac{4 b \int \frac{1}{x^2 \left(a-b x^4\right)^{3/4}}dx}{5 a}-\frac{\sqrt[4]{a-b x^4}}{5 a x^5}\right)}{9 a}-\frac{\sqrt[4]{a-b x^4}}{9 a x^9}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{8 b \left(-\frac{4 b \sqrt[4]{a-b x^4}}{5 a^2 x}-\frac{\sqrt[4]{a-b x^4}}{5 a x^5}\right)}{9 a}-\frac{\sqrt[4]{a-b x^4}}{9 a x^9}$$

input
```
Int[1/(x^10*(a - b*x^4)^(3/4)),x]
```

output
```
-1/9*(a - b*x^4)^(1/4)/(a*x^9) + (8*b*(-1/5*(a - b*x^4)^(1/4)/(a*x^5) - (4
*b*(a - b*x^4)^(1/4))/(5*a^2*x)))/(9*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(32 b^{2} x^{8}+8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	40
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(32 b^{2} x^{8}+8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	40
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(32 b^{2} x^{8}+8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	40
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(32 b^{2} x^{8}+8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9}}$	40
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(32 b^{2} x^{8}+8 a b\,x^{4}+5 a^{2}\right)}{45 a^{3} x^{9} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	67

input
```
int(1/x^10/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/45*(-b*x^4+a)^(1/4)*(32*b^2*x^8+8*a*b*x^4+5*a^2)/a^3/x^9
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{10} (a - bx^4)^{3/4}} \, dx = -\frac{(32 \, b^2 x^8 + 8 \, abx^4 + 5 \, a^2)(-bx^4 + a)^{\frac{1}{4}}}{45 \, a^3 x^9}$$

input
```
integrate(1/x^10/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/45*(32*b^2*x^8 + 8*a*b*x^4 + 5*a^2)*(-b*x^4 + a)^(1/4)/(a^3*x^9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 1110, normalized size of antiderivative = 15.63

$$\int \frac{1}{x^{10} (a - bx^4)^{3/4}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**10/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((-5*a**4*b**(17/4)*(a/(b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9
/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(
3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) + 2*a*
*3*b**(21/4)*x**4*(a/(b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**
5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*g
amma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/
4)*x**8*(a/(b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**5*b**4*x**
8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4)
 + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**12*(a/(
b*x**4) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi
/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b*
*6*x**16*exp(3*I*pi/4)*gamma(3/4)) - 32*b**(33/4)*x**16*(a/(b*x**4) - 1)**
(1/4)*exp(-I*pi/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4)
 - 128*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3
*I*pi/4)*gamma(3/4)), Abs(a/(b*x**4)) > 1), (-5*a**4*b**(17/4)*(-a/(b*x**4
) + 1)**(1/4)*gamma(-9/4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 12
8*a**4*b**5*x**12*exp(3*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi
/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x**4*(-a/(b*x**4) + 1)**(1/4)*gamma(-9/
4)/(64*a**5*b**4*x**8*exp(3*I*pi/4)*gamma(3/4) - 128*a**4*b**5*x**12*exp(3
*I*pi/4)*gamma(3/4) + 64*a**3*b**6*x**16*exp(3*I*pi/4)*gamma(3/4)) - 21...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^{10} \left(a-bx^4\right)^{3/4}} \, dx = -\frac{\frac{45 \left(-bx^4+a\right)^{\frac{1}{4}} b^2}{x} + \frac{18 \left(-bx^4+a\right)^{\frac{5}{4}} b}{x^5} + \frac{5 \left(-bx^4+a\right)^{\frac{9}{4}}}{x^9}}{45 \, a^3}$$

input
```
integrate(1/x^10/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/45*(45*(-b*x^4 + a)^(1/4)*b^2/x + 18*(-b*x^4 + a)^(5/4)*b/x^5 + 5*(-b*x
^4 + a)^(9/4)/x^9)/a^3
```

Giac [F]

$$\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{4}}x^{10}}\,dx$$

input
```
integrate(1/x^10/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^10), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\left(a-b\,x^4\right)^{1/4}\left(5\,a^2+8\,a\,b\,x^4+32\,b^2\,x^8\right)}{45\,a^3\,x^9}$$

input
```
int(1/(x^10*(a - b*x^4)^(3/4)),x)
```

output
```
-((a - b*x^4)^(1/4)*(5*a^2 + 32*b^2*x^8 + 8*a*b*x^4))/(45*a^3*x^9)
```

Reduce [F]

$$\int \frac{1}{x^{10}\left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(-b\,x^4+a\right)^{\frac{3}{4}}x^{10}}dx$$

input
```
int(1/x^10/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**10),x)
```

3.716 $\int \frac{1}{x^{14}(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{1}{x^{14}\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}-\frac{4b\sqrt[4]{a-bx^4}}{39a^2x^9}-\frac{32b^2\sqrt[4]{a-bx^4}}{195a^3x^5}-\frac{128b^3\sqrt[4]{a-bx^4}}{195a^4x}$$

output
```
-1/13*(-b*x^4+a)^(1/4)/a/x^13-4/39*b*(-b*x^4+a)^(1/4)/a^2/x^9-32/195*b^2*(
-b*x^4+a)^(1/4)/a^3/x^5-128/195*b^3*(-b*x^4+a)^(1/4)/a^4/x
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{14}\left(a-bx^4\right)^{3/4}}\,dx = \frac{\sqrt[4]{a-bx^4}\left(-15a^3-20a^2bx^4-32ab^2x^8-128b^3x^{12}\right)}{195a^4x^{13}}$$

input
```
Integrate[1/(x^14*(a - b*x^4)^(3/4)),x]
```

output
```
((a - b*x^4)^(1/4)*(-15*a^3 - 20*a^2*b*x^4 - 32*a*b^2*x^8 - 128*b^3*x^12))
/(195*a^4*x^13)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {803, 803, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{14} (a-bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 803}$$

$$\frac{12b \int \frac{1}{x^{10}(a-bx^4)^{3/4}}dx}{13a}-\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{12b \left(\frac{8b \int \frac{1}{x^6 \left(a-bx^4\right)^{3/4}}dx}{9a}-\frac{\sqrt[4]{a-bx^4}}{9ax^9}\right)}{13a}-\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 803}$$

$$\frac{12b \left(\frac{8b \left(\frac{4b \int \frac{1}{x^2 \left(a-bx^4\right)^{3/4}}dx}{5a}-\frac{\sqrt[4]{a-bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a-bx^4}}{9ax^9}\right)}{13a}-\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}$$

$$\Big\downarrow \text{ 796}$$

$$\frac{12b \left(\frac{8b \left(-\frac{4b \sqrt[4]{a-bx^4}}{5a^2 x}-\frac{\sqrt[4]{a-bx^4}}{5ax^5}\right)}{9a}-\frac{\sqrt[4]{a-bx^4}}{9ax^9}\right)}{13a}-\frac{\sqrt[4]{a-bx^4}}{13ax^{13}}$$

input
```
Int[1/(x^14*(a - b*x^4)^(3/4)),x]
```

output
```
-1/13*(a - b*x^4)^(1/4)/(a*x^13) + (12*b*(-1/9*(a - b*x^4)^(1/4)/(a*x^9) +
 (8*b*(-1/5*(a - b*x^4)^(1/4)/(a*x^5) - (4*b*(a - b*x^4)^(1/4))/(5*a^2*x))
)/(9*a)))/(13*a)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(128 b^{3} x^{12}+32 a\,b^{2} x^{8}+20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	51
trager	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(128 b^{3} x^{12}+32 a\,b^{2} x^{8}+20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	51
pseudoelliptic	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(128 b^{3} x^{12}+32 a\,b^{2} x^{8}+20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	51
orering	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(128 b^{3} x^{12}+32 a\,b^{2} x^{8}+20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 x^{13} a^{4}}$	51
risch	$-\frac{\left(-b\,x^{4}+a\right)^{\frac{1}{4}} \left(\left(-b\,x^{4}+a\right)^{3}\right)^{\frac{1}{4}} \left(128 b^{3} x^{12}+32 a\,b^{2} x^{8}+20 a^{2} b\,x^{4}+15 a^{3}\right)}{195 a^{4} x^{13} \left(-\left(b\,x^{4}-a\right)^{3}\right)^{\frac{1}{4}}}$	78

input
```
int(1/x^14/(-b*x^4+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/195*(-b*x^4+a)^(1/4)*(128*b^3*x^12+32*a*b^2*x^8+20*a^2*b*x^4+15*a^3)/x^
13/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{14} (a - bx^4)^{3/4}} \, dx = -\frac{(128 \, b^3 x^{12} + 32 \, ab^2 x^8 + 20 \, a^2 b x^4 + 15 \, a^3)(-bx^4 + a)^{\frac{1}{4}}}{195 \, a^4 x^{13}}$$

input
```
integrate(1/x^14/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
-1/195*(128*b^3*x^12 + 32*a*b^2*x^8 + 20*a^2*b*x^4 + 15*a^3)*(-b*x^4 + a)^
(1/4)/(a^4*x^13)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 1928, normalized size of antiderivative = 20.08

$$\int \frac{1}{x^{14} (a - bx^4)^{3/4}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**14/(-b*x**4+a)**(3/4),x)
```

output
```
Piecewise((45*a**6*b**(37/4)*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-
13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**1
6*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4)
 + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) - 75*a**5*b**(41/4)*x**4
*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*
exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) -
 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(
3*I*pi/4)*gamma(3/4)) + 51*a**4*b**(45/4)*x**8*(a/(b*x**4) - 1)**(1/4)*exp
(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 7
68*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*
I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) + 231*
a**3*b**(49/4)*x**12*(a/(b*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(-
256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*
I*pi/4)*gamma(3/4) - 768*a**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a
**4*b**12*x**24*exp(3*I*pi/4)*gamma(3/4)) - 924*a**2*b**(53/4)*x**16*(a/(b
*x**4) - 1)**(1/4)*exp(3*I*pi/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*
I*pi/4)*gamma(3/4) + 768*a**6*b**10*x**16*exp(3*I*pi/4)*gamma(3/4) - 768*a
**5*b**11*x**20*exp(3*I*pi/4)*gamma(3/4) + 256*a**4*b**12*x**24*exp(3*I*pi
/4)*gamma(3/4)) + 1056*a*b**(57/4)*x**20*(a/(b*x**4) - 1)**(1/4)*exp(3*I*p
i/4)*gamma(-13/4)/(-256*a**7*b**9*x**12*exp(3*I*pi/4)*gamma(3/4) + 768*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{14} (a-bx^4)^{3/4}} \, dx = -\frac{\frac{195 \, (-bx^4+a)^{\frac{1}{4}} b^3}{x} + \frac{117 \, (-bx^4+a)^{\frac{5}{4}} b^2}{x^5} + \frac{65 \, (-bx^4+a)^{\frac{9}{4}} b}{x^9} + \frac{15 \, (-bx^4+a)^{\frac{13}{4}}}{x^{13}}}{195 \, a^4}$$

input
```
integrate(1/x^14/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
-1/195*(195*(-b*x^4 + a)^(1/4)*b^3/x + 117*(-b*x^4 + a)^(5/4)*b^2/x^5 + 65
*(-b*x^4 + a)^(9/4)*b/x^9 + 15*(-b*x^4 + a)^(13/4)/x^13)/a^4
```

Giac [F]

$$\int \frac{1}{x^{14} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{4}} x^{14}} \, dx$$

input
```
integrate(1/x^14/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^14), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^{14} (a - bx^4)^{3/4}} \, dx = -\frac{(a - b\,x^4)^{1/4}}{13\,a\,x^{13}} - \frac{4\,b\,(a - b\,x^4)^{1/4}}{39\,a^2\,x^9} - \frac{128\,b^3\,(a - b\,x^4)^{1/4}}{195\,a^4\,x} - \frac{32\,b^2\,(a - b\,x^4)^{1/4}}{195\,a^3\,x^5}$$

input
```
int(1/(x^14*(a - b*x^4)^(3/4)),x)
```

output
```
- (a - b*x^4)^(1/4)/(13*a*x^13) - (4*b*(a - b*x^4)^(1/4))/(39*a^2*x^9) - (
128*b^3*(a - b*x^4)^(1/4))/(195*a^4*x) - (32*b^2*(a - b*x^4)^(1/4))/(195*a
^3*x^5)
```

Reduce [F]

$$\int \frac{1}{x^{14} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{(-b\,x^4 + a)^{\frac{3}{4}}\, x^{14}} dx$$

input
```
int(1/x^14/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**14),x)
```

3.717 $\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 134

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = -\frac{3a^2x\sqrt[4]{a-bx^4}}{8b^3}-\frac{3ax^5\sqrt[4]{a-bx^4}}{20b^2} -\frac{x^9\sqrt[4]{a-bx^4}}{10b}-\frac{3a^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{8b^{5/2}(a-bx^4)^{3/4}}$$

output
```
-3/8*a^2*x*(-b*x^4+a)^(1/4)/b^3-3/20*a*x^5*(-b*x^4+a)^(1/4)/b^2-1/10*x^9*(
-b*x^4+a)^(1/4)/b-3/8*a^(5/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*ar
ccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \frac{-15a^3x+9a^2bx^5+2ab^2x^9+4b^3x^{13}+15a^3x\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\right.}{40b^3(a-bx^4)^{3/4}}$$

input
```
Integrate[x^12/(a - b*x^4)^(3/4),x]
```

output
```
(-15*a^3*x + 9*a^2*b*x^5 + 2*a*b^2*x^9 + 4*b^3*x^13 + 15*a^3*x*(1 - (b*x^4
)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^4)/a])/(40*b^3*(a - b*x^4
)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {843, 843, 843, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{9a\int \frac{x^8}{(a-bx^4)^{3/4}}dx}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

$$\Big\downarrow \text{843}$$

$$\frac{9a\left(\frac{5a\int \frac{x^4}{\left(a-bx^4\right)^{3/4}}dx}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

$$\Big\downarrow \text{843}$$

$$\frac{9a\left(\frac{5a\left(\frac{a\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{2b}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

$$\Big\downarrow \text{768}$$

$$\frac{9a\left(\frac{5a\left(\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

$\downarrow$ 858

$$\frac{9a\left(\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

$\downarrow$ 807

$$\frac{9a\left(\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4b\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

$\downarrow$ 230

$$\frac{9a\left(\frac{5a\left(-\frac{\sqrt{a}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}\left(a-bx^4\right)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}\right)}{10b}-\frac{x^9\sqrt[4]{a-bx^4}}{10b}$$

input

```
Int[x^12/(a - b*x^4)^(3/4),x]
```

output
```
-1/10*(x^9*(a - b*x^4)^(1/4))/b + (9*a*(-1/6*(x^5*(a - b*x^4)^(1/4))/b + (
5*a*(-1/2*(x*(a - b*x^4)^(1/4))/b - (Sqrt[a]*(1 - a/(b*x^4))^(3/4)*x^3*Ell
ipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(3/4)))
)/(6*b)))/(10*b)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^{12}}{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(x^12/(-b*x^4+a)^(3/4),x)
```

output
```
int(x^12/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{\left(-bx^4+a\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^12/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x^12/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.29

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \frac{x^{13}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate(x**12/(-b*x**4+a)**(3/4),x)
```

output
```
x**13*gamma(13/4)*hyper((3/4, 13/4), (17/4,), b*x**4*exp_polar(2*I*pi)/a)/
(4*a**(3/4)*gamma(17/4))
```

Maxima [F]

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^12/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^12/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^12/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^12/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^12/(a - b*x^4)^(3/4),x)
```

output
```
int(x^12/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^{12}}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^{12}}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^12/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**12/(a - b*x**4)**(3/4),x)
```

3.718 $\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 109

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = -\frac{5ax\sqrt[4]{a-bx^4}}{12b^2} - \frac{x^5\sqrt[4]{a-bx^4}}{6b} - \frac{5a^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12b^{3/2}(a-bx^4)^{3/4}}$$

output
```
-5/12*a*x*(-b*x^4+a)^(1/4)/b^2-1/6*x^5*(-b*x^4+a)^(1/4)/b-5/12*a^(3/2)*(1-
a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2)
)/b^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \frac{-5a^2x+3abx^5+2b^2x^9+5a^2x\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{12b^2(a-bx^4)^{3/4}}$$

input
```
Integrate[x^8/(a - b*x^4)^(3/4),x]
```

output
```
(-5*a^2*x + 3*a*b*x^5 + 2*b^2*x^9 + 5*a^2*x*(1 - (b*x^4)/a)^(3/4)*Hypergeo
metric2F1[1/4, 3/4, 5/4, (b*x^4)/a])/(12*b^2*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {843, 843, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5a\int \frac{x^4}{(a-bx^4)^{3/4}}dx}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

$$\Big\downarrow \text{ 843}$$

$$\frac{5a\left(\frac{a\int \frac{1}{(a-bx^4)^{3/4}}dx}{2b}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{5a\left(\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

$$\Big\downarrow \text{807}$$

$$\frac{5a\left(-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

$$\Big\downarrow \text{230}$$

$$\frac{5a\left(-\frac{\sqrt{a}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}\right)}{6b}-\frac{x^5\sqrt[4]{a-bx^4}}{6b}$$

input

```
Int[x^8/(a - b*x^4)^(3/4),x]
```

output

```
-1/6*(x^5*(a - b*x^4)^(1/4))/b + (5*a*(-1/2*(x*(a - b*x^4)^(1/4))/b - (Sqr
t[a]*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2,
2])/(2*Sqrt[b]*(a - b*x^4)^(3/4))))/(6*b)
```

Defintions of rubi rules used

rule 230

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^8}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^8/(-b*x^4+a)^(3/4),x)
```

output
```
int(x^8/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^8}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x^8/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.36

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \frac{x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**8/(-b*x**4+a)**(3/4),x)
```

output
```
x**9*gamma(9/4)*hyper((3/4, 9/4), (13/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*
a**(3/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^8}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^8/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^8}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^8/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^8/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^8}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^8/(a - b*x^4)^(3/4),x)
```

output
```
int(x^8/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^8}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^8}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^8/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**8/(a - b*x**4)**(3/4),x)
```

3.719 $\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 86

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx=-\frac{x\sqrt[4]{a-bx^4}}{2b}-\frac{\sqrt{a}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2\sqrt{b}\,(a-bx^4)^{3/4}}$$

output
```
-1/2*x*(-b*x^4+a)^(1/4)/b-1/2*a^(1/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiA
M(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/b^(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx=\frac{x\left(-a+bx^4+a\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx^4}{a}\right)\right)}{2b\,(a-bx^4)^{3/4}}$$

input
```
Integrate[x^4/(a - b*x^4)^(3/4),x]
```

output
```
(x*(-a + b*x^4 + a*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4,
(b*x^4)/a]))/(2*b*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {843, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{a\int \frac{1}{(a-bx^4)^{3/4}}dx}{2b}-\frac{x\sqrt[4]{a-bx^4}}{2b}$$

$$\Big\downarrow \text{768}$$

$$\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{2b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}$$

$$\Big\downarrow \text{858}$$

$$-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{2b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}$$

$$\Big\downarrow \text{807}$$

$$-\frac{ax^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{4b(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}$$

$$\Big\downarrow \text{230}$$

$$-\frac{\sqrt{a}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{2\sqrt{b}(a-bx^4)^{3/4}}-\frac{x\sqrt[4]{a-bx^4}}{2b}$$

input
```
Int[x^4/(a - b*x^4)^(3/4),x]
```

output
```
-1/2*(x*(a - b*x^4)^(1/4))/b - (Sqrt[a]*(1 - a/(b*x^4))^(3/4)*x^3*Elliptic
F[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(2*Sqrt[b]*(a - b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^4}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(-b*x^4+a)^(3/4),x)
```

output
```
int(x^4/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^4}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)*x^4/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx = \frac{x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**4/(-b*x**4+a)**(3/4),x)
```

output
```
x**5*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^4}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-b*x^4 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^4}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^4 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^4}{(a-b\,x^4)^{3/4}}\,dx$$

input
```
int(x^4/(a - b*x^4)^(3/4),x)
```

output
```
int(x^4/(a - b*x^4)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^4)^{3/4}}\,dx = \int \frac{x^4}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(-b*x^4+a)^(3/4),x)
```

output
```
int(x**4/(a - b*x**4)**(3/4),x)
```

3.720 $\int \frac{1}{(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx = -\frac{\sqrt{b}\left(1-\frac{a}{bx^4}\right)^{3/4} x^3 \operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a-bx^4)^{3/4}}$$

output
```
-b^(1/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1
/2)),2^(1/2))/a^(1/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.75

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx = \frac{x\left(1-\frac{bx^4}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx^4}{a}\right)}{(a-bx^4)^{3/4}}$$

input
```
Integrate[(a - b*x^4)^(-3/4),x]
```

output
```
(x*(1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^4)/a])/(a -
 b*x^4)^(3/4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^4)^{3/4}}dx$$

$$\Big\downarrow \text{768}$$

$$\frac{x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{(a-bx^4)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{(a-bx^4)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{2(a-bx^4)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{\sqrt{b}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{\sqrt{a}(a-bx^4)^{3/4}}$$

input
```
Int[(a - b*x^4)^(-3/4),x]
```

output
```
-((Sqrt[b]*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2
)]/2, 2])/(Sqrt[a]*(a - b*x^4)^(3/4)))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^4 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx=\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{3}{4}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(-b*x**4+a)**(3/4),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/4), (5/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a**(
3/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx=\int \frac{1}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^4 + a)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^4 + a)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.60

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx = \frac{x\left(1-\frac{b\,x^4}{a}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ \frac{b\,x^4}{a}\right)}{(a-b\,x^4)^{3/4}}$$

input
```
int(1/(a - b*x^4)^(3/4),x)
```

output
```
(x*(1 - (b*x^4)/a)^(3/4)*hypergeom([1/4, 3/4], 5/4, (b*x^4)/a))/(a - b*x^4
)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/(a - b*x**4)**(3/4),x)
```

3.721 $\int \frac{1}{x^4(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{1}{x^4(a-bx^4)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{3ax^3}-\frac{2b^{3/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a-bx^4)^{3/4}}$$

output
```
-1/3*(-b*x^4+a)^(1/4)/a/x^3-2/3*b^(3/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacob
iAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(3/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^4(a-bx^4)^{3/4}}\,dx=-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},\frac{bx^4}{a}\right)}{3x^3(a-bx^4)^{3/4}}$$

input
```
Integrate[1/(x^4*(a - b*x^4)^(3/4)),x]
```

output
```
-1/3*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (b*x^4)/a])/
(x^3*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {847, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^4)^{3/4}} \, dx$$

$$\Big\downarrow \text{847}$$

$$\frac{2b\int \frac{1}{(a-bx^4)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{768}$$

$$\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{807}$$

$$-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}$$

input
```
Int[1/(x^4*(a - b*x^4)^(3/4)),x]
```

output
```
-1/3*(a - b*x^4)^(1/4)/(a*x^3) - (2*b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*Elli
pticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a - b*x^4)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(-b\,x^{4}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^4/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/x^4/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a-bx^4)^{3/4}}\, dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}} x^4}\, dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^8 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4 (a-bx^4)^{3/4}}\, dx = -\frac{ie^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{a}{bx^4}\right)}{6b^{\frac{3}{4}}x^6}$$

input
```
integrate(1/x**4/(-b*x**4+a)**(3/4),x)
```

output
```
-I*exp(3*I*pi/4)*hyper((3/4, 3/2), (5/2,), a/(b*x**4))/(6*b**(3/4)*x**6)
```

Maxima [F]

$$\int \frac{1}{x^4 (a - bx^4)^{3/4}} dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{4}} x^4} dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a - bx^4)^{3/4}} dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{4}} x^4} dx$$

input
```
integrate(1/x^4/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a - bx^4)^{3/4}} dx = \int \frac{1}{x^4 (a - b x^4)^{3/4}} dx$$

input
```
int(1/(x^4*(a - b*x^4)^(3/4)),x)
```

output
```
int(1/(x^4*(a - b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-b\,x^4+a)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/x^4/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**4),x)
```

3.722 $\int \frac{1}{x^8(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 111

$$\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\sqrt[4]{a-bx^4}}{7ax^7}-\frac{2b\sqrt[4]{a-bx^4}}{7a^2x^3}-\frac{4b^{5/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{7a^{5/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/7*(-b*x^4+a)^(1/4)/a/x^7-2/7*b*(-b*x^4+a)^(1/4)/a^2/x^3-4/7*b^(5/2)*(1-
a/b/x^4)^(3/4)*x^3*InverseJacobiAM(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2)
)/a^(5/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^8\left(a-bx^4\right)^{3/4}}\,dx=-\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},\frac{bx^4}{a}\right)}{7x^7\left(a-bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^8*(a - b*x^4)^(3/4)),x]
```

output
```
-1/7*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (b*x^4)/a])
/(x^7*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {847, 847, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 \left(a-bx^4\right)^{3/4}} \, dx$$

$$\Big\downarrow \; 847$$

$$\frac{6b\int \frac{1}{x^4(a-bx^4)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \; 847$$

$$\frac{6b\left(\frac{2b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \; 768$$

$$\frac{6b\left(\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \; 858$$

$$\frac{6b\left(-\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \text{807}$$

$$\frac{6b\left(-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{3a(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

$$\Big\downarrow \text{230}$$

$$\frac{6b\left(-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}(a-bx^4)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}$$

input
```
Int[1/(x^8*(a - b*x^4)^(3/4)),x]
```

output
```
-1/7*(a - b*x^4)^(1/4)/(a*x^7) + (6*b*(-1/3*(a - b*x^4)^(1/4)/(a*x^3) - (2
*b^(3/2)*(1 - a/(b*x^4))^(3/4)*x^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]
/2, 2])/(3*a^(3/2)*(a - b*x^4)^(3/4))))/(7*a)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{8} \left(-b\,x^{4}+a \right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^8/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/x^8/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^8 \left(a-bx^4\right)^{3/4}}\, dx = \int \frac{1}{{\left(-b x^{4} + a\right)}^{\frac{3}{4}} x^{8}}\, dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^12 - a*x^8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^8 (a-bx^4)^{3/4}}\,dx = \frac{ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{a}{bx^4}\right)}{10b^{\frac{3}{4}}x^{10}}$$

input
```
integrate(1/x**8/(-b*x**4+a)**(3/4),x)
```

output
```
I*exp(-I*pi/4)*hyper((3/4, 5/2), (7/2,), a/(b*x**4))/(10*b**(3/4)*x**10)
```

Maxima [F]

$$\int \frac{1}{x^8 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^8), x)
```

Giac [F]

$$\int \frac{1}{x^8 (a-bx^4)^{3/4}}\,dx = \int \frac{1}{(-bx^4+a)^{\frac{3}{4}}x^8}\,dx$$

input
```
integrate(1/x^8/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^8 \left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{x^8\,{\left(a-b\,x^4\right)}^{3/4}} \,d x$$

input
```
int(1/(x^8*(a - b*x^4)^(3/4)),x)
```

output
```
int(1/(x^8*(a - b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^8 \left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(-b\,x^{4}+a \right)^{\frac{3}{4}} x^{8}}d x$$

input
```
int(1/x^8/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**8),x)
```

3.723 $\int \frac{1}{x^{12}(a-bx^4)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}-\frac{10b\sqrt[4]{a-bx^4}}{77a^2x^7}-\frac{20b^2\sqrt[4]{a-bx^4}}{77a^3x^3} - \frac{40b^{7/2}\left(1-\frac{a}{bx^4}\right)^{3/4}x^3\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{77a^{7/2}\left(a-bx^4\right)^{3/4}}$$

output
```
-1/11*(-b*x^4+a)^(1/4)/a/x^11-10/77*b*(-b*x^4+a)^(1/4)/a^2/x^7-20/77*b^2*(
-b*x^4+a)^(1/4)/a^3/x^3-40/77*b^(7/2)*(1-a/b/x^4)^(3/4)*x^3*InverseJacobiA
M(1/2*arccsc(b^(1/2)*x^2/a^(1/2)),2^(1/2))/a^(7/2)/(-b*x^4+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}\,dx = -\frac{\left(1-\frac{bx^4}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},-\frac{7}{4},\frac{bx^4}{a}\right)}{11x^{11}\left(a-bx^4\right)^{3/4}}$$

input
```
Integrate[1/(x^12*(a - b*x^4)^(3/4)),x]
```

output
```
-1/11*((1 - (b*x^4)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, (b*x^4)/a
])/(x^11*(a - b*x^4)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {847, 847, 847, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{12}\left(a-bx^4\right)^{3/4}}dx$$

$$\Big\downarrow \text{847}$$

$$\frac{10b\int \frac{1}{x^8(a-bx^4)^{3/4}}dx}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{847}$$

$$\frac{10b\left(\frac{6b\int \frac{1}{x^4\left(a-bx^4\right)^{3/4}}dx}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{847}$$

$$\frac{10b\left(\frac{6b\left(\frac{2b\int \frac{1}{\left(a-bx^4\right)^{3/4}}dx}{3a}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

$$\Big\downarrow \text{768}$$

$$\frac{10b\left(\frac{6b\left(\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x^3}dx}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

$\Big\downarrow$ 858

$$\frac{10b\left(\frac{6b\left(-\frac{2bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^4}\right)^{3/4}x}d\frac{1}{x}}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

$\Big\downarrow$ 807

$$\frac{10b\left(\frac{6b\left(-\frac{bx^3\left(1-\frac{a}{bx^4}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}}d\frac{1}{x^2}}{3a\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

$\Big\downarrow$ 230

$$\frac{10b\left(\frac{6b\left(-\frac{2b^{3/2}x^3\left(1-\frac{a}{bx^4}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right),2\right)}{3a^{3/2}\left(a-bx^4\right)^{3/4}}-\frac{\sqrt[4]{a-bx^4}}{3ax^3}\right)}{7a}-\frac{\sqrt[4]{a-bx^4}}{7ax^7}\right)}{11a}-\frac{\sqrt[4]{a-bx^4}}{11ax^{11}}$$

input
```
Int[1/(x^12*(a - b*x^4)^(3/4)),x]
```

output
```
-1/11*(a - b*x^4)^(1/4)/(a*x^11) + (10*b*(-1/7*(a - b*x^4)^(1/4)/(a*x^7) +
 (6*b*(-1/3*(a - b*x^4)^(1/4)/(a*x^3) - (2*b^(3/2)*(1 - a/(b*x^4))^(3/4)*x
^3*EllipticF[ArcSin[Sqrt[a]/(Sqrt[b]*x^2)]/2, 2])/(3*a^(3/2)*(a - b*x^4)^(
3/4))))/(7*a)))/(11*a)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{x^{12} \left(-b\,x^{4}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^12/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/x^12/(-b*x^4+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^{12} \left(a-bx^4\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^{4}+a\right)^{\frac{3}{4}}x^{12}}\,dx$$

input
```
integrate(1/x^12/(-b*x^4+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^4 + a)^(1/4)/(b*x^16 - a*x^12), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^{12} \left(a-bx^4\right)^{3/4}}\,dx=-\frac{ie^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{a}{bx^{4}}\right)}{14b^{\frac{3}{4}}x^{14}}$$

input
```
integrate(1/x**12/(-b*x**4+a)**(3/4),x)
```

output
```
-I*exp(3*I*pi/4)*hyper((3/4, 7/2), (9/2,), a/(b*x**4))/(14*b**(3/4)*x**14)
```

Maxima [F]

$$\int \frac{1}{x^{12} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{4}} x^{12}} \, dx$$

input
```
integrate(1/x^12/(-b*x^4+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^12), x)
```

Giac [F]

$$\int \frac{1}{x^{12} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{(-bx^4 + a)^{\frac{3}{4}} x^{12}} \, dx$$

input
```
integrate(1/x^12/(-b*x^4+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^4 + a)^(3/4)*x^12), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^{12} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{x^{12} (a - b x^4)^{3/4}} \, dx$$

input
```
int(1/(x^12*(a - b*x^4)^(3/4)),x)
```

output
```
int(1/(x^12*(a - b*x^4)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^{12} (a - bx^4)^{3/4}} \, dx = \int \frac{1}{(-b\,x^4 + a)^{\frac{3}{4}} \, x^{12}} dx$$

input
```
int(1/x^12/(-b*x^4+a)^(3/4),x)
```

output
```
int(1/((a - b*x**4)**(3/4)*x**12),x)
```

3.724 $\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \frac{1}{bx\sqrt[4]{a-bx^4}} - \frac{\sqrt[4]{1-\frac{a}{bx^4}}\,xE\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^4}}$$

output
```
1/b/x/(-b*x^4+a)^(1/4)-(1-a/b/x^4)^(1/4)*x*EllipticE(sin(1/2*arccsc(b^(1/2
)*x^2/a^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/(-b*x^4+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.94 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \frac{x^3\sqrt[4]{1-\frac{bx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},\frac{bx^4}{a}\right)}{3a\sqrt[4]{a-bx^4}}$$

input
```
Integrate[x^2/(a - b*x^4)^(5/4),x]
```

output
```
(x^3*(1 - (b*x^4)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, (b*x^4)/a])/(3
*a*(a - b*x^4)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {814, 842, 858, 807, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^4)^{5/4}}dx$$

$$\Big\downarrow \text{814}$$

$$\frac{\int \frac{1}{x^2\sqrt[4]{a-bx^4}}dx}{b}+\frac{1}{bx\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{842}$$

$$\frac{x\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x^3}dx}{b\sqrt[4]{a-bx^4}}+\frac{1}{bx\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{bx\sqrt[4]{a-bx^4}}-\frac{x\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^4}}x}d\frac{1}{x}}{b\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{bx\sqrt[4]{a-bx^4}}-\frac{x\sqrt[4]{1-\frac{a}{bx^4}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x^2}}{2b\sqrt[4]{a-bx^4}}$$

$$\Big\downarrow \text{226}$$

$$\frac{1}{bx\sqrt[4]{a-bx^4}}-\frac{x\sqrt[4]{1-\frac{a}{bx^4}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x^2}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^4}}$$

input
```
Int[x^2/(a - b*x^4)^(5/4),x]
```

output
```
1/(b*x*(a - b*x^4)^(1/4)) - ((1 - a/(b*x^4))^(1/4)*x*EllipticE[ArcSin[Sqrt
[a]/(Sqrt[b]*x^2)]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^4)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 814
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4)^(5/4), x_Symbol] :> -Simp[(b*x*(a + b*x^4)
^(1/4))^(-1), x] - Simp[1/b   Int[1/(x^2*(a + b*x^4)^(1/4)), x], x] /; Free
Q[{a, b}, x] && NegQ[b/a]
```

rule 842
```
Int[1/((x_)^2*((a_) + (b_.)*(x_)^4)^(1/4)), x_Symbol] :> Simp[x*((1 + a/(b*
x^4))^(1/4)/(a + b*x^4)^(1/4))   Int[1/(x^3*(1 + a/(b*x^4))^(1/4)), x], x]
/; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{x^2}{(-b\,x^4+a)^{\frac{5}{4}}}dx$$

input
```
int(x^2/(-b*x^4+a)^(5/4),x)
```

output
```
int(x^2/(-b*x^4+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^4 + a)^(3/4)*x^2/(b^2*x^8 - 2*a*b*x^4 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \frac{x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^4e^{2i\pi}}{a}\right)}{4a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**2/(-b*x**4+a)**(5/4),x)
```

output
```
x**3*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**4*exp_polar(2*I*pi)/a)/(4*a
**(5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^4 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \int \frac{x^2}{(-bx^4+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^4+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^4 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \int \frac{x^2}{(a-b\,x^4)^{5/4}}\,dx$$

input
```
int(x^2/(a - b*x^4)^(5/4),x)
```

output
```
int(x^2/(a - b*x^4)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^4)^{5/4}}\,dx = \int \frac{x^2}{(-b\,x^4+a)^{\frac{1}{4}}\,a-(-b\,x^4+a)^{\frac{1}{4}}\,b\,x^4}dx$$

input
```
int(x^2/(-b*x^4+a)^(5/4),x)
```

output
```
int(x**2/((a - b*x**4)**(1/4)*a - (a - b*x**4)**(1/4)*b*x**4),x)
```

3.725 $\int x^7(a+bx^4)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x^7\left(a+bx^4\right)^p\,dx=-\frac{a\left(a+bx^4\right)^{1+p}}{4b^2(1+p)}+\frac{\left(a+bx^4\right)^{2+p}}{4b^2(2+p)}$$

output
```
-1/4*a*(b*x^4+a)^(p+1)/b^2/(p+1)+1/4*(b*x^4+a)^(2+p)/b^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int x^7\left(a+bx^4\right)^p\,dx=\frac{\left(a+bx^4\right)^{1+p}\left(-a+b(1+p)x^4\right)}{4b^2(1+p)(2+p)}$$

input
```
Integrate[x^7*(a + b*x^4)^p,x]
```

output
```
((a + b*x^4)^(1 + p)*(-a + b*(1 + p)*x^4))/(4*b^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^4\right)^p dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}\int x^4 \left(bx^4+a\right)^p dx^4$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}\int \left(\frac{\left(bx^4+a\right)^{p+1}}{b}-\frac{a\left(bx^4+a\right)^p}{b}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{\left(a+bx^4\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+bx^4\right)^{p+1}}{b^2(p+1)}\right)$$

input
```
Int[x^7*(a + b*x^4)^p,x]
```

output
```
(-((a*(a + b*x^4)^(1 + p))/(b^2*(1 + p))) + (a + b*x^4)^(2 + p)/(b^2*(2 +
p)))/4
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{(b\,x^4+a)^{p+1}(-x^4pb-b\,x^4+a)}{4b^2(p^2+3p+2)}$	42
orering	$-\frac{(b\,x^4+a)^{p}(-x^4pb-b\,x^4+a)(b\,x^4+a)}{4b^2(p^2+3p+2)}$	47
risch	$-\frac{(-b^2x^8p-b^2x^8-ap\,x^4b+a^2)(b\,x^4+a)^p}{4b^2(2+p)(p+1)}$	54
parallelrisch	$\frac{x^8(b\,x^4+a)^p b^2p+x^8(b\,x^4+a)^p b^2+x^4(b\,x^4+a)^p abp-(b\,x^4+a)^p a^2}{4b^2(p^2+3p+2)}$	80
norman	$\frac{x^8 e^{p\ln(b\,x^4+a)}}{4p+8}-\frac{a^2 e^{p\ln(b\,x^4+a)}}{4b^2(p^2+3p+2)}+\frac{pa\,x^4 e^{p\ln(b\,x^4+a)}}{4b(p^2+3p+2)}$	83

input
```
int(x^7*(b*x^4+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2*(b*x^4+a)^(p+1)/(p^2+3*p+2)*(-b*p*x^4-b*x^4+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x^7(a+bx^4)^p\,dx=\frac{((b^2p+b^2)x^8+abpx^4-a^2)(bx^4+a)^p}{4\,(b^2p^2+3\,b^2p+2\,b^2)}$$

input
```
integrate(x^7*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
1/4*((b^2*p + b^2)*x^8 + a*b*p*x^4 - a^2)*(b*x^4 + a)^p/(b^2*p^2 + 3*b^2*p
 + 2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(37) = 74$.

Time = 1.77 (sec) , antiderivative size = 418, normalized size of antiderivative = 8.71

$$\int x^7(a+bx^4)^p\,dx$$
$$= \begin{cases} \frac{a^px^8}{8} \\ \frac{a\log\left(x-\sqrt[4]{-\frac{a}{b}}\right)}{4ab^2+4b^3x^4}+\frac{a\log\left(x+\sqrt[4]{-\frac{a}{b}}\right)}{4ab^2+4b^3x^4}+\frac{a\log\left(x^2+\sqrt{-\frac{a}{b}}\right)}{4ab^2+4b^3x^4}+\frac{a}{4ab^2+4b^3x^4}+\frac{bx^4\log\left(x-\sqrt[4]{-\frac{a}{b}}\right)}{4ab^2+4b^3x^4}+\frac{bx^4\log\left(x+\sqrt[4]{-\frac{a}{b}}\right)}{4ab^2+4b^3x^4} - \\ -\frac{a\log\left(x-\sqrt[4]{-\frac{a}{b}}\right)}{4b^2}-\frac{a\log\left(x+\sqrt[4]{-\frac{a}{b}}\right)}{4b^2}-\frac{a\log\left(x^2+\sqrt{-\frac{a}{b}}\right)}{4b^2}+\frac{x^4}{4b} \\ -\frac{a^2(a+bx^4)^p}{4b^2p^2+12b^2p+8b^2}+\frac{abpx^4(a+bx^4)^p}{4b^2p^2+12b^2p+8b^2}+\frac{b^2px^8(a+bx^4)^p}{4b^2p^2+12b^2p+8b^2}+\frac{b^2x^8(a+bx^4)^p}{4b^2p^2+12b^2p+8b^2} \end{cases}$$

input
```
integrate(x**7*(b*x**4+a)**p,x)
```

output
```
Piecewise((a**p*x**8/8, Eq(b, 0)), (a*log(x - (-a/b)**(1/4))/(4*a*b**2 + 4
*b**3*x**4) + a*log(x + (-a/b)**(1/4))/(4*a*b**2 + 4*b**3*x**4) + a*log(x*
*2 + sqrt(-a/b))/(4*a*b**2 + 4*b**3*x**4) + a/(4*a*b**2 + 4*b**3*x**4) + b
*x**4*log(x - (-a/b)**(1/4))/(4*a*b**2 + 4*b**3*x**4) + b*x**4*log(x + (-a
/b)**(1/4))/(4*a*b**2 + 4*b**3*x**4) + b*x**4*log(x**2 + sqrt(-a/b))/(4*a*
b**2 + 4*b**3*x**4), Eq(p, -2)), (-a*log(x - (-a/b)**(1/4))/(4*b**2) - a*l
og(x + (-a/b)**(1/4))/(4*b**2) - a*log(x**2 + sqrt(-a/b))/(4*b**2) + x**4/
(4*b), Eq(p, -1)), (-a**2*(a + b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**
2) + a*b*p*x**4*(a + b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**2) + b**2*
p*x**8*(a + b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**2) + b**2*x**8*(a +
 b*x**4)**p/(4*b**2*p**2 + 12*b**2*p + 8*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x^7 (a + bx^4)^p \, dx = \frac{(b^2(p+1)x^8 + abpx^4 - a^2)(bx^4 + a)^p}{4(p^2 + 3p + 2)b^2}$$

input
```
integrate(x^7*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
1/4*(b^2*(p + 1)*x^8 + a*b*p*x^4 - a^2)*(b*x^4 + a)^p/((p^2 + 3*p + 2)*b^2
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^7 (a + bx^4)^p \, dx = \frac{(bx^4 + a)^2 (bx^4 + a)^p}{4b^2(p+2)} - \frac{(bx^4 + a)^{p+1} a}{4b^2(p+1)}$$

input
```
integrate(x^7*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
1/4*(b*x^4 + a)^2*(b*x^4 + a)^p/(b^2*(p + 2)) - 1/4*(b*x^4 + a)^(p + 1)*a/
(b^2*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int x^7 (a + bx^4)^p \, dx = (b\,x^4 + a)^p \left(\frac{x^8\,(p+1)}{4\,(p^2 + 3\,p + 2)} - \frac{a^2}{4\,b^2\,(p^2 + 3\,p + 2)} + \frac{a\,p\,x^4}{4\,b\,(p^2 + 3\,p + 2)} \right)$$

input
```
int(x^7*(a + b*x^4)^p,x)
```

output
```
(a + b*x^4)^p*((x^8*(p + 1))/(4*(3*p + p^2 + 2)) - a^2/(4*b^2*(3*p + p^2 +
 2)) + (a*p*x^4)/(4*b*(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int x^7 (a + bx^4)^p \, dx = \frac{(b\,x^4 + a)^p\,(b^2p\,x^8 + b^2x^8 + abp\,x^4 - a^2)}{4b^2\,(p^2 + 3p + 2)}$$

input
```
int(x^7*(b*x^4+a)^p,x)
```

output
```
((a + b*x**4)**p*( - a**2 + a*b*p*x**4 + b**2*p*x**8 + b**2*x**8))/(4*b**2
*(p**2 + 3*p + 2))
```

3.726 $\int x^3(a+bx^4)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int x^3\left(a+bx^4\right)^p\,dx=\frac{\left(a+bx^4\right)^{1+p}}{4b(1+p)}$$

output
```
1/4*(b*x^4+a)^(p+1)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^4\right)^p\,dx=\frac{\left(a+bx^4\right)^{1+p}}{4b(1+p)}$$

input
```
Integrate[x^3*(a + b*x^4)^p,x]
```

output
```
(a + b*x^4)^(1 + p)/(4*b*(1 + p))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^4\right)^p \, dx$$

$$\Big\downarrow \text{ 793}$$

$$\frac{\left(a+bx^4\right)^{p+1}}{4b(p+1)}$$

input
```
Int[x^3*(a + b*x^4)^p,x]
```

output
```
(a + b*x^4)^(1 + p)/(4*b*(1 + p))
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(b\,x^4+a)^{p+1}}{4b(p+1)}$	22
derivativedivides	$\frac{(b\,x^4+a)^{p+1}}{4b(p+1)}$	22
default	$\frac{(b\,x^4+a)^{p+1}}{4b(p+1)}$	22
risch	$\frac{(b\,x^4+a)\,(b\,x^4+a)^{p}}{4b(p+1)}$	27
orering	$\frac{(b\,x^4+a)\,(b\,x^4+a)^{p}}{4b(p+1)}$	27
parallelrisch	$\frac{x^4\,(b\,x^4+a)^{p}ab+(b\,x^4+a)^{p}a^2}{4ab(p+1)}$	43
norman	$\frac{x^4\mathrm{e}^{p\ln\left(b\,x^4+a\right)}}{4p+4}+\frac{a\,\mathrm{e}^{p\ln\left(b\,x^4+a\right)}}{4b(p+1)}$	45

input
```
int(x^3*(b*x^4+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b*x^4+a)^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^3\left(a+bx^4\right)^p\,dx=\frac{(bx^4+a)(bx^4+a)^p}{4\,(bp+b)}$$

input
```
integrate(x^3*(b*x^4+a)^p,x, algorithm="fricas")
```

output
```
1/4*(b*x^4 + a)*(b*x^4 + a)^p/(b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(15) = 30.

Time = 0.57 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.52

$$\int x^3(a+bx^4)^p\,dx = \begin{cases} \frac{x^4}{4a} & \text{for } b=0 \wedge p=-1 \\ \frac{a^p x^4}{4} & \text{for } b=0 \\ \frac{\log\left(x-\sqrt[4]{-\frac{a}{b}}\right)}{4b}+\frac{\log\left(x+\sqrt[4]{-\frac{a}{b}}\right)}{4b}+\frac{\log\left(x^2+\sqrt{-\frac{a}{b}}\right)}{4b} & \text{for } p=-1 \\ \frac{a(a+bx^4)^p}{4bp+4b}+\frac{bx^4(a+bx^4)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**4+a)**p,x)
```

output
```
Piecewise((x**4/(4*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**4/4, Eq(b, 0)), (lo
g(x - (-a/b)**(1/4))/(4*b) + log(x + (-a/b)**(1/4))/(4*b) + log(x**2 + sqr
t(-a/b))/(4*b), Eq(p, -1)), (a*(a + b*x**4)**p/(4*b*p + 4*b) + b*x**4*(a +
 b*x**4)**p/(4*b*p + 4*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^3(a+bx^4)^p\,dx = \frac{(bx^4+a)^{p+1}}{4\,b(p+1)}$$

input
```
integrate(x^3*(b*x^4+a)^p,x, algorithm="maxima")
```

output
```
1/4*(b*x^4 + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^3(a+bx^4)^p \, dx = \frac{(bx^4+a)^{p+1}}{4\,b(p+1)}$$

input
```
integrate(x^3*(b*x^4+a)^p,x, algorithm="giac")
```

output
```
1/4*(b*x^4 + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^3(a+bx^4)^p \, dx = \frac{(b\,x^4+a)^{p+1}}{4\,b\,(p+1)}$$

input
```
int(x^3*(a + b*x^4)^p,x)
```

output
```
(a + b*x^4)^(p + 1)/(4*b*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^3(a+bx^4)^p \, dx = \frac{(b\,x^4+a)^p\,(b\,x^4+a)}{4b\,(p+1)}$$

input
```
int(x^3*(b*x^4+a)^p,x)
```

output
```
((a + b*x**4)**p*(a + b*x**4))/(4*b*(p + 1))
```

3.727 $\int \frac{(a+bx^4)^p}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{(a+bx^4)^p}{x} \, dx = -\frac{(a+bx^4)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^4}{a}\right)}{4a(1+p)}$$

output
```
-1/4*(b*x^4+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^4/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^4)^p}{x} \, dx = -\frac{(a+bx^4)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^4}{a}\right)}{4a(1+p)}$$

input
```
Integrate[(a + b*x^4)^p/x,x]
```

output
```
-1/4*((a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^4)/a
])/(a*(1 + p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^4)^p}{x}\,dx$$

$$\Big\downarrow \; 798$$

$$\frac{1}{4}\int \frac{(bx^4+a)^p}{x^4}dx^4$$

$$\Big\downarrow \; 75$$

$$-\frac{(a+bx^4)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^4}{a}+1\right)}{4a(p+1)}$$

input
```
Int[(a + b*x^4)^p/x,x]
```

output
```
-1/4*((a + b*x^4)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^4)/a
])/(a*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [F]

$$\int \frac{(b\,x^4+a)^p}{x}dx$$

input
```
int((b*x^4+a)^p/x,x)
```

output
```
int((b*x^4+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^4)^p}{x}\,dx=\int \frac{(bx^4+a)^p}{x}\,dx$$

input
```
integrate((b*x^4+a)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^4 + a)^p/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^4)^p}{x}\,dx=-\frac{b^p x^{4p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\ 1-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^4}\right)}{4\Gamma\left(1-p\right)}$$

input
```
integrate((b*x**4+a)**p/x,x)
```

output
```
-b**p*x**(4*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x*
*4))/(4*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(a+bx^4)^p}{x}\,dx=\int \frac{(bx^4+a)^p}{x}\,dx$$

input
```
integrate((b*x^4+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^4 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^4)^p}{x}\,dx=\int \frac{(bx^4+a)^p}{x}\,dx$$

input
```
integrate((b*x^4+a)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^4 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^4)^p}{x}\,dx=\int \frac{(b\,x^4+a)^p}{x}\,dx$$

input
```
int((a + b*x^4)^p/x,x)
```

output
```
int((a + b*x^4)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^4)^p}{x}\,dx = \frac{(b\,x^4+a)^p + 4\left(\int \frac{(b\,x^4+a)^p}{b\,x^5+ax}dx\right)ap}{4p}$$

input
```
int((b*x^4+a)^p/x,x)
```

output
```
((a + b*x**4)**p + 4*int((a + b*x**4)**p/(a*x + b*x**5),x)*a*p)/(4*p)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file